# Project 1 Report - ME155C/ECE147C

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**Abstract**

In this project, we design a controller for a two mass - one spring, cart system. The goal is to optimize the performance of the second cart's response to a step input to the reference through control of the first cart's behavior, with respect to several key control metrics. We look at optimizing for a very short settling time along with minimizing the amount of overshoot. The 'undershoot' is treated as an acceptable behavior since the cart will pass through this region, so we give ourselves the difficulty of treating the overshoot as a highly undesirable set. We achieve our goals with a settling time under three seconds along with less than one percent overshoot.

# 1 Introduction

We begin our control journey with the search of an optimal controller. The first idea here is LQR, but it is clear that we have noise in our system and uncertainty in our data, so we consider going to LQG. We try to decrease our uncertainty by doing a sinusoidal sweep of 20 frequencies.

# 2 System Identification

## 2.1 Process to be controlled

The process we are controlling is a two cart system connected by a spring. The system is driven by a motor that's given a voltage $V$ [Volt], which produces a force $F$ [N] applied to the first cart with mass $m_1$ [kg], and the second cart with mass $m_2$ [kg] is connected to the first cart via a spring with spring constant $k$ [N/m]. In this system, $x_1$ [m] is the position of the first cart, and $x_2$ [m] is the position of the second cart. The control input is the voltage $u := V$ [Volt] applied to the motor, and the measured output is the position $y := x_2$ [m] of the second cart.

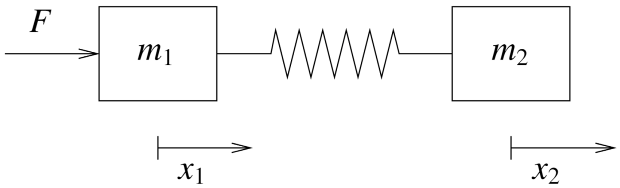


Figure 1: Two cart system

## 2.2 Non-Parametric Identification

The non-parametric identification method used was sine-wave testing in conjunction with the correleation method. This strategy consists of applying sinusoidal inputs at distinct frequencies to calculate the magnitude and phase of the frequency response at that frequency from the output of the system.

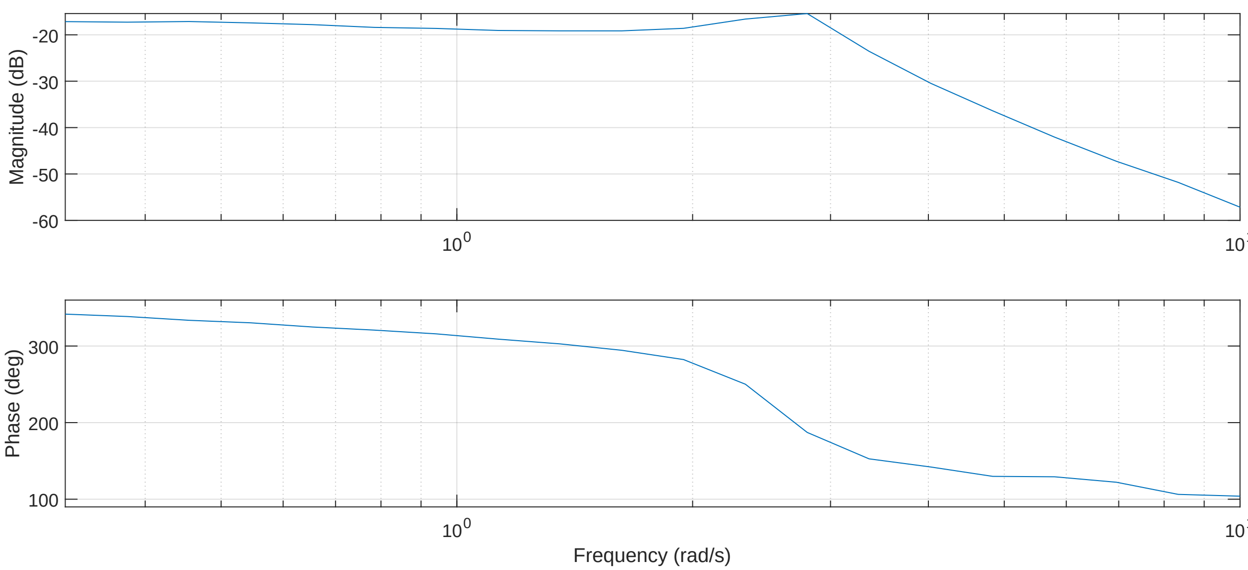


Figure 2: Bode plot of the identified system using non-parametric identification

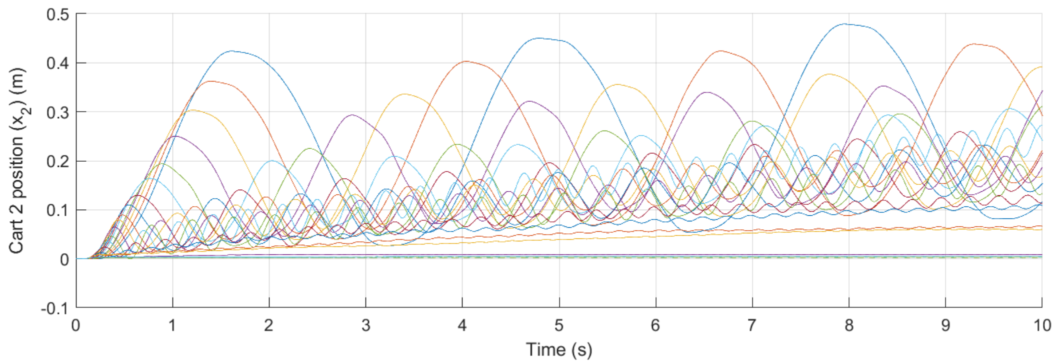


Figure 3: Output signals of all experiments for parametric identification

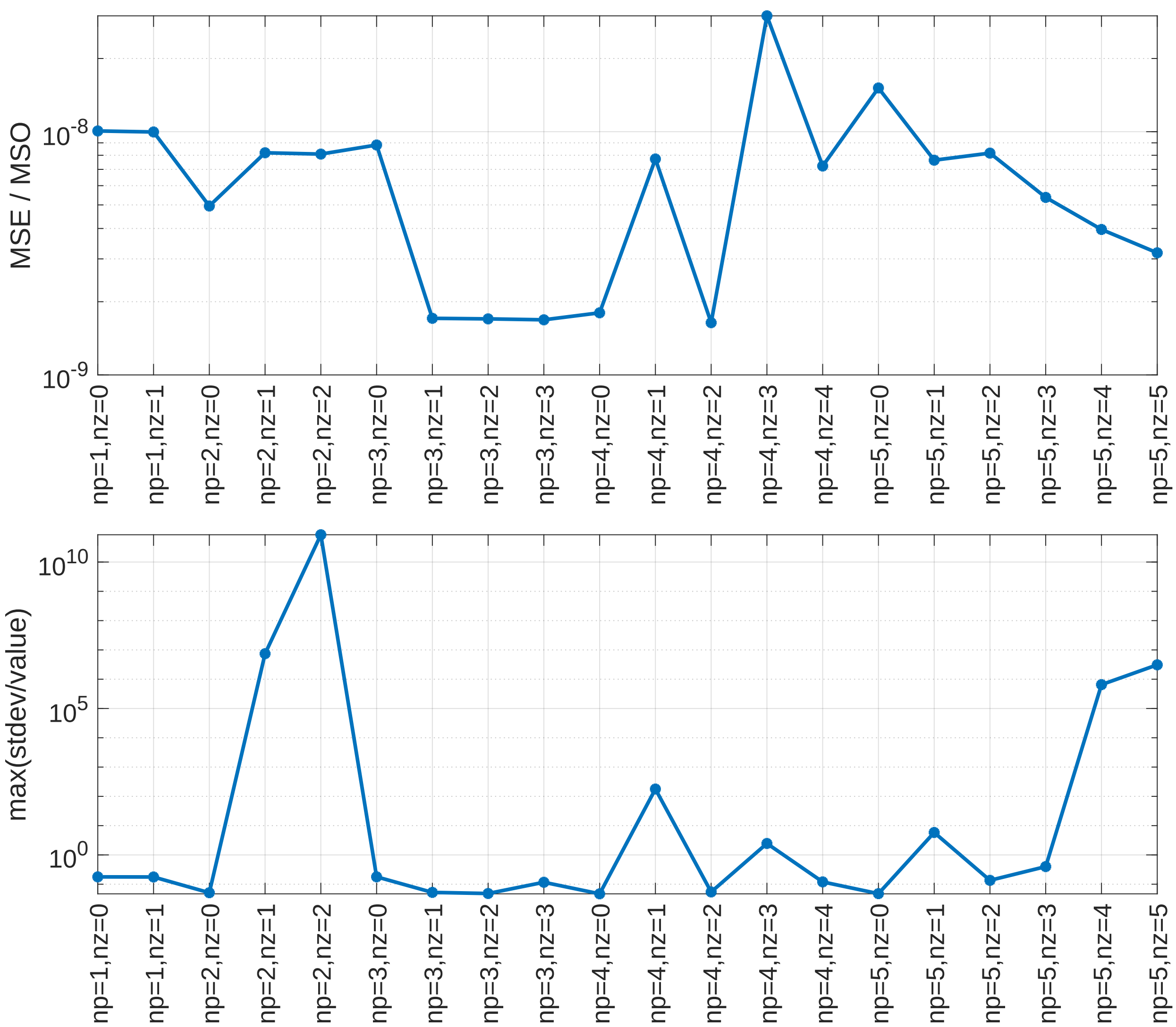


Figure 4: Normalized MSE and worst parameter standard deviation for different model orders

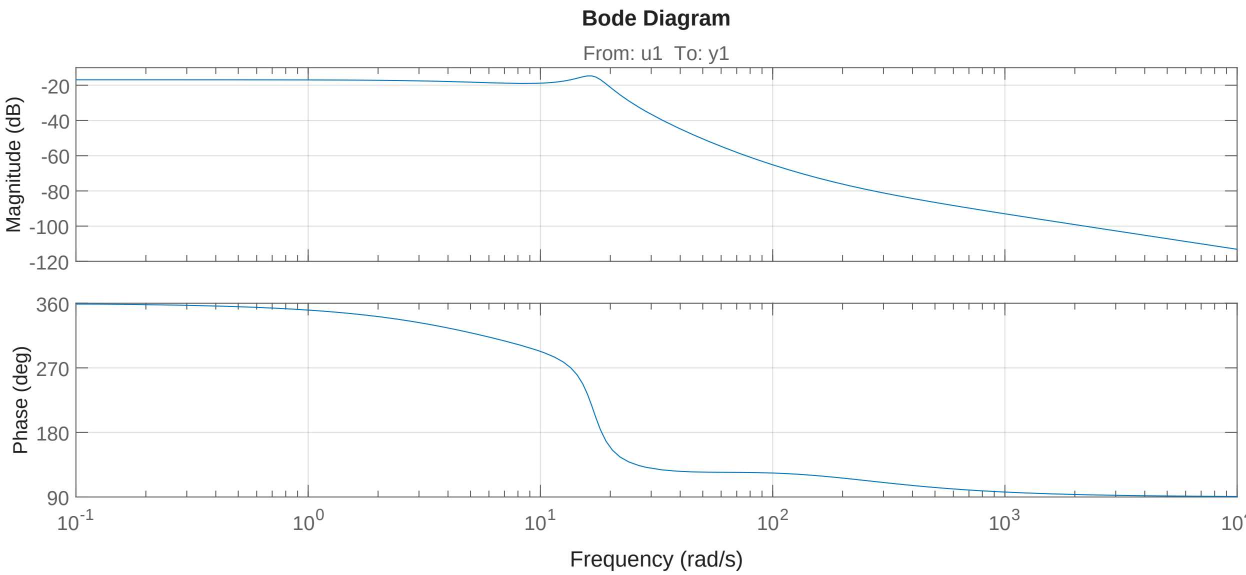


Figure 5: Bode plot of the identified system using parametric identification

### 2.3 Parametric Identification

## 3 Controller Design

### 3.1 Design Methodology

### 3.2 Simulation Results

## 4 Closed-loop Testing

| **Parameter** | **Value** |
|---|---|
| Overshoot | ¡Insert Value¿ |
| Rise Time | ¡Insert Value¿ |
| Settling Time | ¡Insert Value¿ |
| Max Control Input | ¡Insert Value¿ |

Table 1: Step Response Data

For the frequency response identification of the closed loop system, we ran a sinusoidal sweep across frequencies: $\omega \in 2\pi[10^{-0.5}, 10]$ The theoretical and experimental bode diagrams seem to disagree in phase and magnitude. This can be due to an inaccurate estimation of the order of the transfer function but ultimately we end up with a real closed loop that performs well so it is not too important.

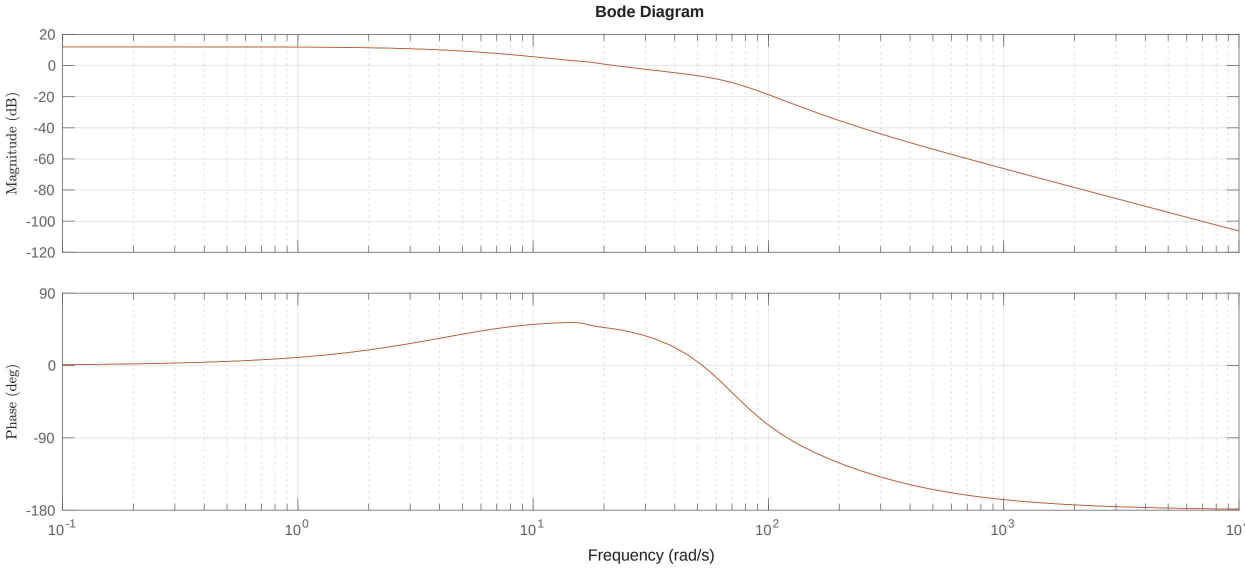


Figure 6: Bode plot of complementary sensitivity function of simulated controller

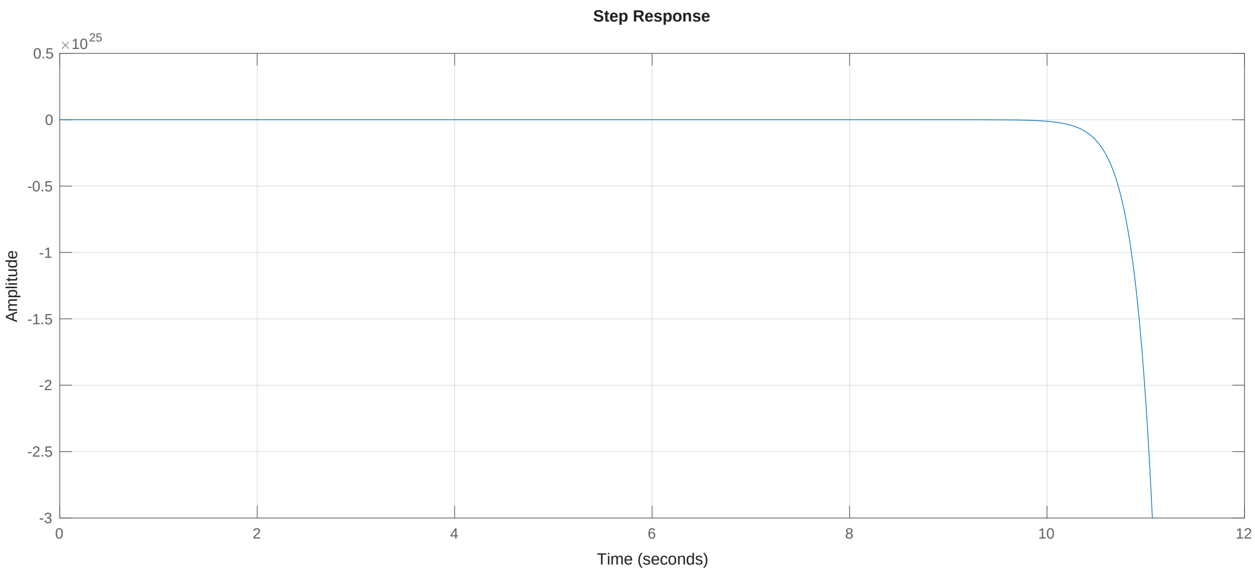


Figure 7: Simulated closed-loop step response

### 4.1 Step Response Experiments

### 4.2 Closed-loop Frequency Response

## 5 Conclusions and Future Work

## References

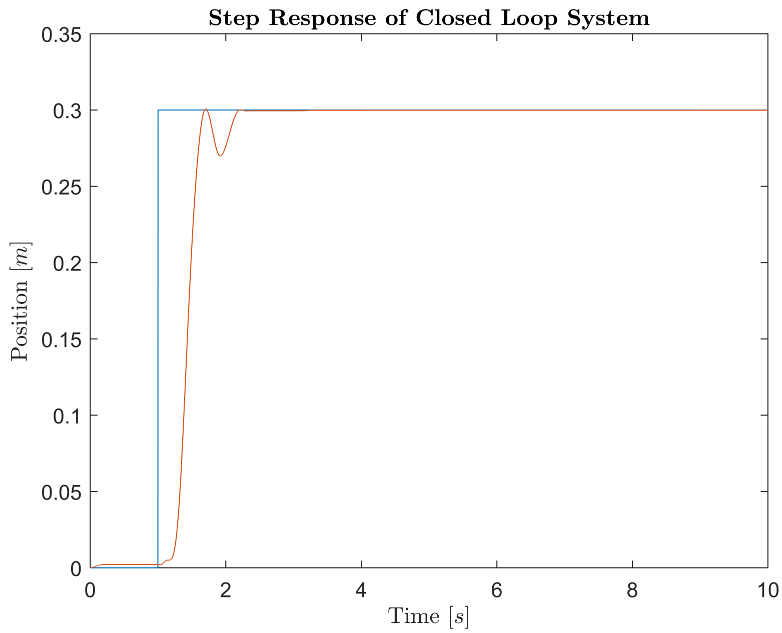


Figure 8: Closed-loop step response

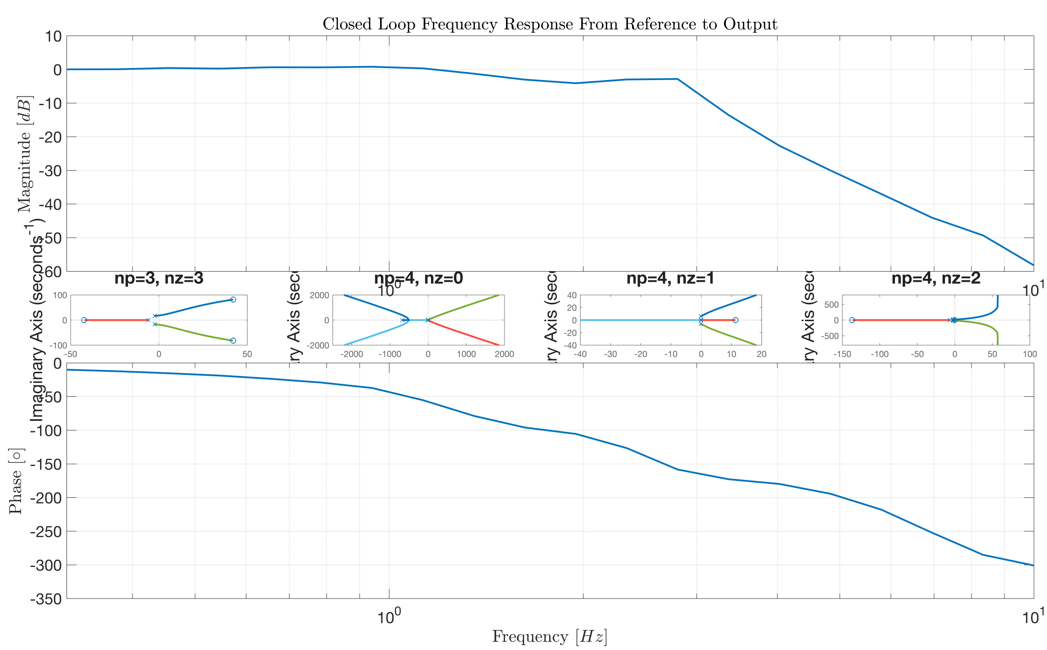


Figure 9: Closed-loop frequency response of the transfer function from reference to output

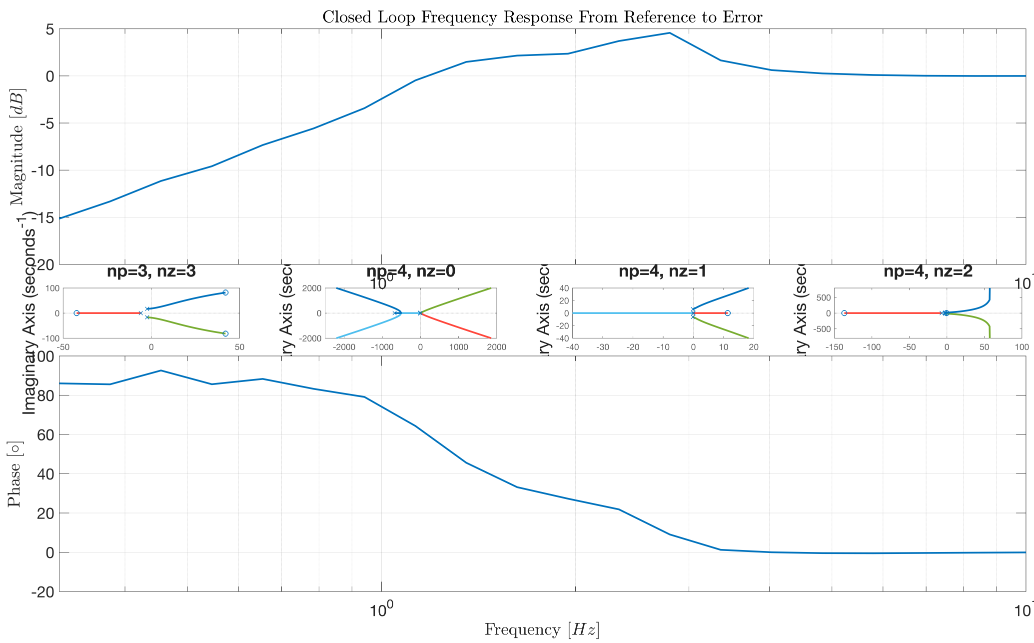


Figure 10: Closed-loop frequency response of the transfer function from reference to error