ECE 147C: HW 2

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1 Exercise 3

Without knowing any prior parameters, the estimated transfer function is

$$H(s) = \frac{1.0074(s + 0.06421)}{(s + 0.4786)(s + 0.05168)}.$$

By knowing a zero at $s = -.5$ and a poles at $s = -.3$, the estimated transfer function is

$$H(s) = \frac{0.99295(s + 0.5)}{(s + 0.6995)(s + 0.3)}.$$

```
%% Exercise 3, part 1: Not knowing parameters

clear;
load('hw2_ex3_unknown_pole.mat');

Ts = t(2)-t(1);
data = iddata(y,u,Ts);
model = tfest(data,2,1);
sysc = tf(model);
zpk(sysc)

%% Exercise 3, part 2: Known parameters

clear;
load('hw2_ex3_unknown_pole.mat');

Ts = t(2)-t(1);
p1 = -0.3; % known pole
z1 = -0.5; % known zero
bary = (y(2:end)-y(1:end-1))/Ts - p1*y(1:end-1);
baru = (u(2:end)-u(1:end-1))/Ts - z1*u(1:end-1);
data = iddata(bary, baru, Ts);
model = tfest(data,1,0);
sysc = tf(model);
s = tf('s');
sysc = zpk(sysc*(s-z1)/(s-p1))
```

2 Exercise 4

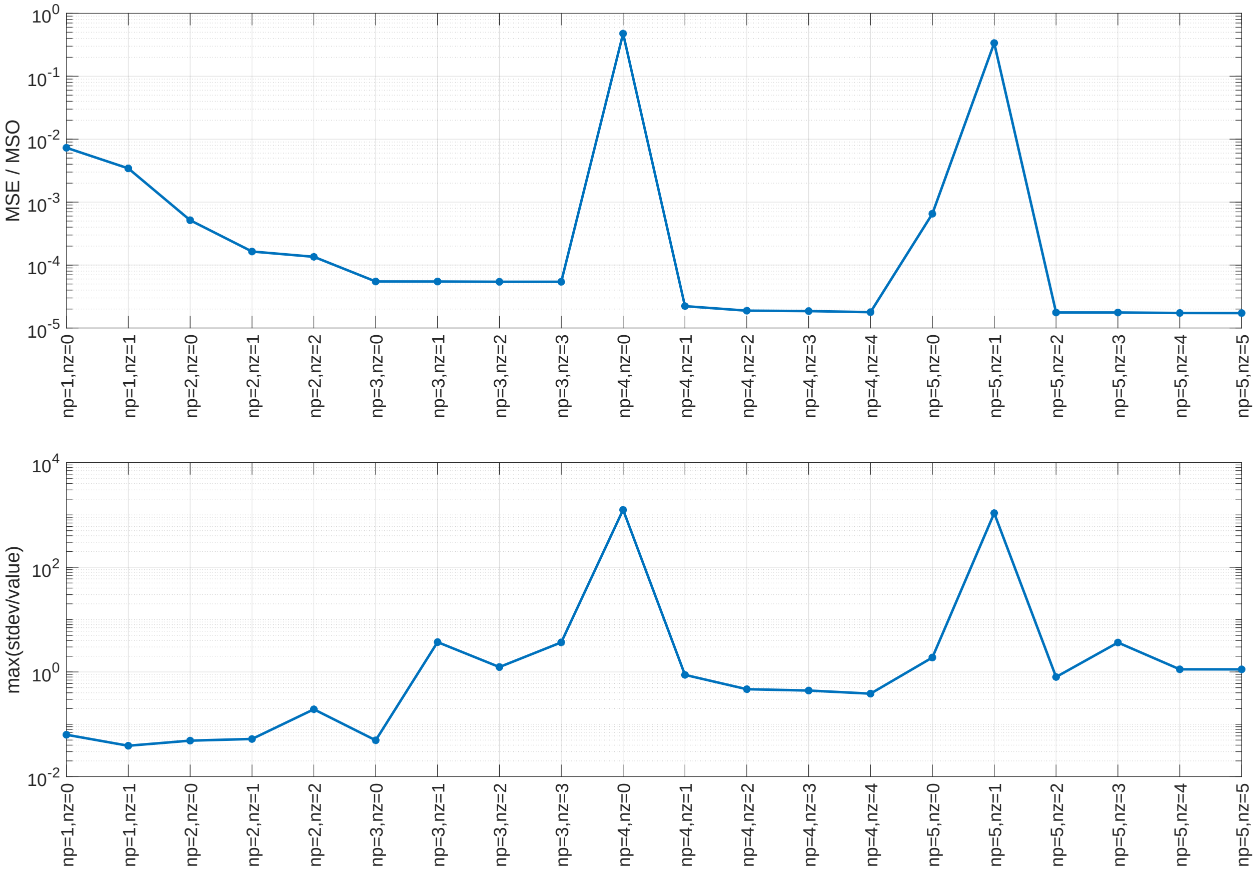

Figure 1: Normalized MSE and largest normalized parameter standard deviations for different choices of model order.

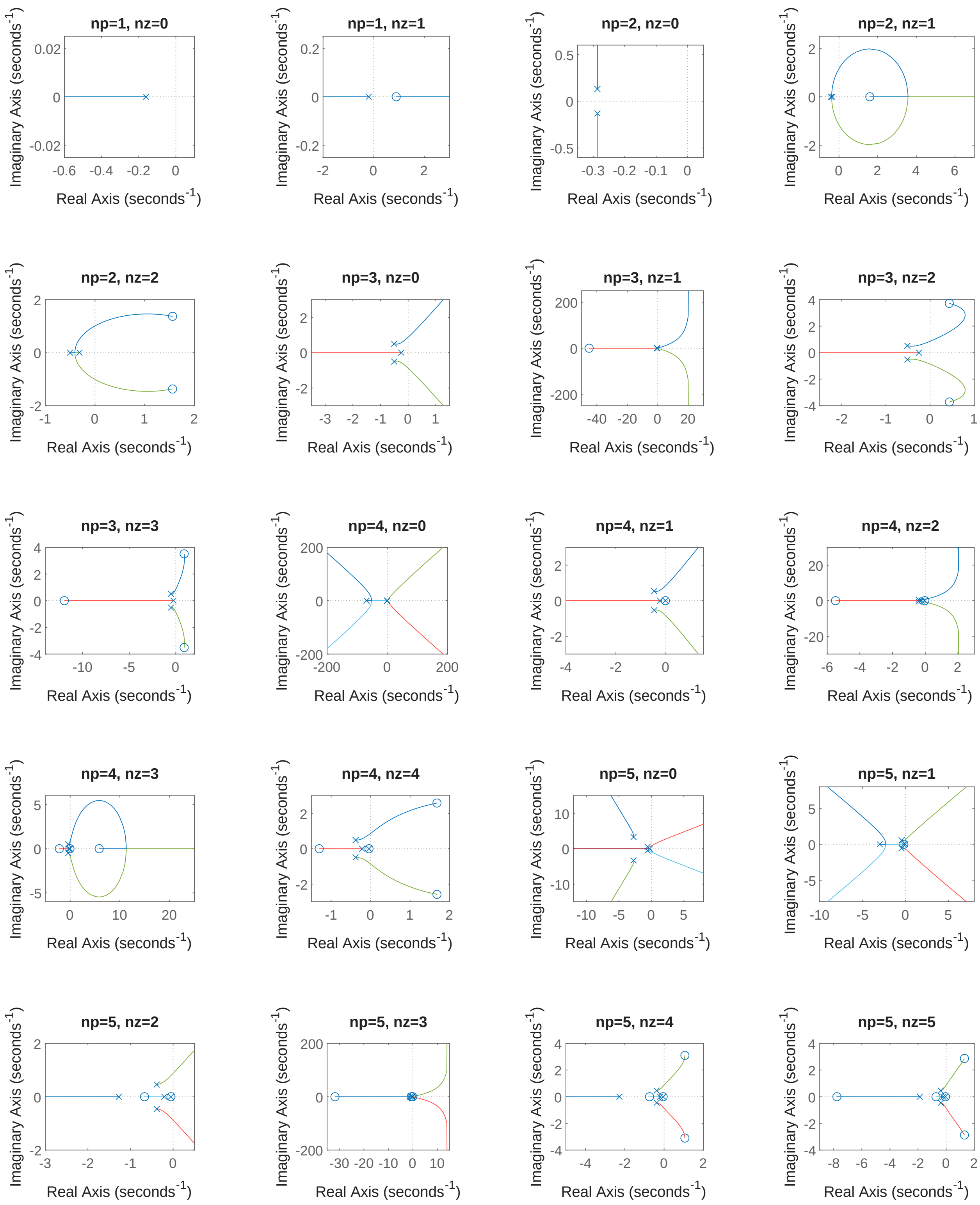

Figure 2: Root locus of estimated transfer functions for different choices of model order.

```
clear;
load("hw2_ex4_id_order.mat");

npnz = [1 0; 1 1;
        2 0; 2 1; 2 2;
        3 0; 3 1; 3 2; 3 3;
        4 0; 4 1; 4 2; 4 3; 4 4;
        5 0; 5 1; 5 2; 5 3; 5 4; 5 5];
MSEs = zeros(length(npnz),1);
StdDevs = zeros(length(npnz),1);
figure(2); clf;

Ts = t(2)-t(1);
data = iddata(y,u,Ts);
for i = 1:size(npnz,1)
    np = npnz(i,1);
    nz = npnz(i,2);
    model = tfest(data,np,nz);
    normalizedMSE = model.Report.Fit.MSE/((y'*y)/length(y));
    MSEs(i) = normalizedMSE;
    num = model.Numerator;
    den = model.Denominator(2:end); % excludes leading coeff (always 1)
    std_num = sqrt(diag(model.Report.Parameters.FreeParCovariance(1:nz+1,1:nz+1)))';
    std_den = sqrt(diag(model.Report.Parameters.FreeParCovariance(nz+2:end,nz+2:end)))';
    maxStdDev = max([std_num,std_den]./[num,den]);
    StdDevs(i) = maxStdDev;
    subplot(5,4,i); hold on; rlocus(model); title(sprintf('np=%d, nz=%d',np,nz));
end
hold off;
set(gcf,'Position',[0,0,1200,1400]);
% exportgraphics(gcf,'./figure2.pdf');

figure(1); clf;
subplot(2,1,1);
semilogy(MSEs,'-*','LineWidth',2);
xticklabels(arrayfun(@(i) sprintf('np=%d,nz=%d', npnz(i,1), npnz(i,2)), 1:length(npnz),
    'UniformOutput', false));
xticks(1:length(npnz)); xtickangle(90); xlim([1, length(npnz)]);
ylabel("MSE / MSO");
grid on;
set(gca,'FontSize',14);
subplot(2,1,2);
semilogy(StdDevs,'-*','LineWidth',2);
xticklabels(arrayfun(@(i) sprintf('np=%d,nz=%d', npnz(i,1), npnz(i,2)), 1:length(npnz),
    'UniformOutput', false));
xticks(1:length(npnz)); xtickangle(90); xlim([1, length(npnz)]);
ylabel("max(stdev/value)");
grid on;
set(gca,'FontSize',14);
set(gcf,'Position',[50,50,1600,1000]);
% exportgraphics(gcf,'./figure1.pdf');
```

Observing the upper plot of Figure 1, 3–5 poles mostly lead to an acceptably low normalized MSE, and 1–2 poles never lead to a good MSE. Looking at the root locus plots in Figure 2, 4–5 poles lead to a transfer function with an overlapping pole and zero, so we should focus on 3 poles. Observing the lower plot of Figure 1, 3 poles and no zeros lead to a parameter standard deviation of less than 10 percent. So, the best choice is 3 poles and no zeros. This produces the following transfer function:

$$H(s) = \frac{0.2561}{s^3 + 1.26s^2 + 0.7626s + 0.1277}.$$

3 Exercise 5

3.1 Part 1

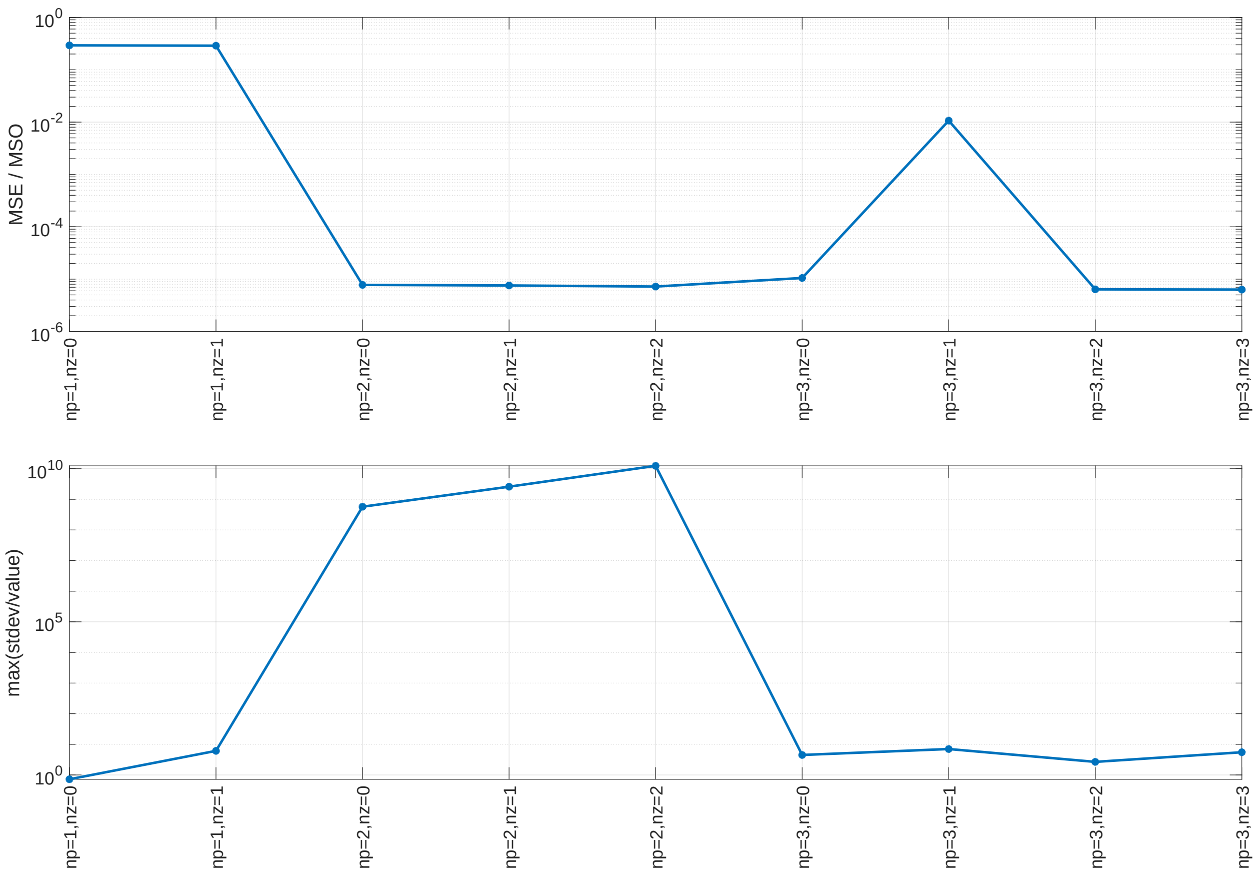

Figure 3: Normalized MSE and largest normalized parameter standard deviations for different choices of model order (0.1 step amplitude, no noise).

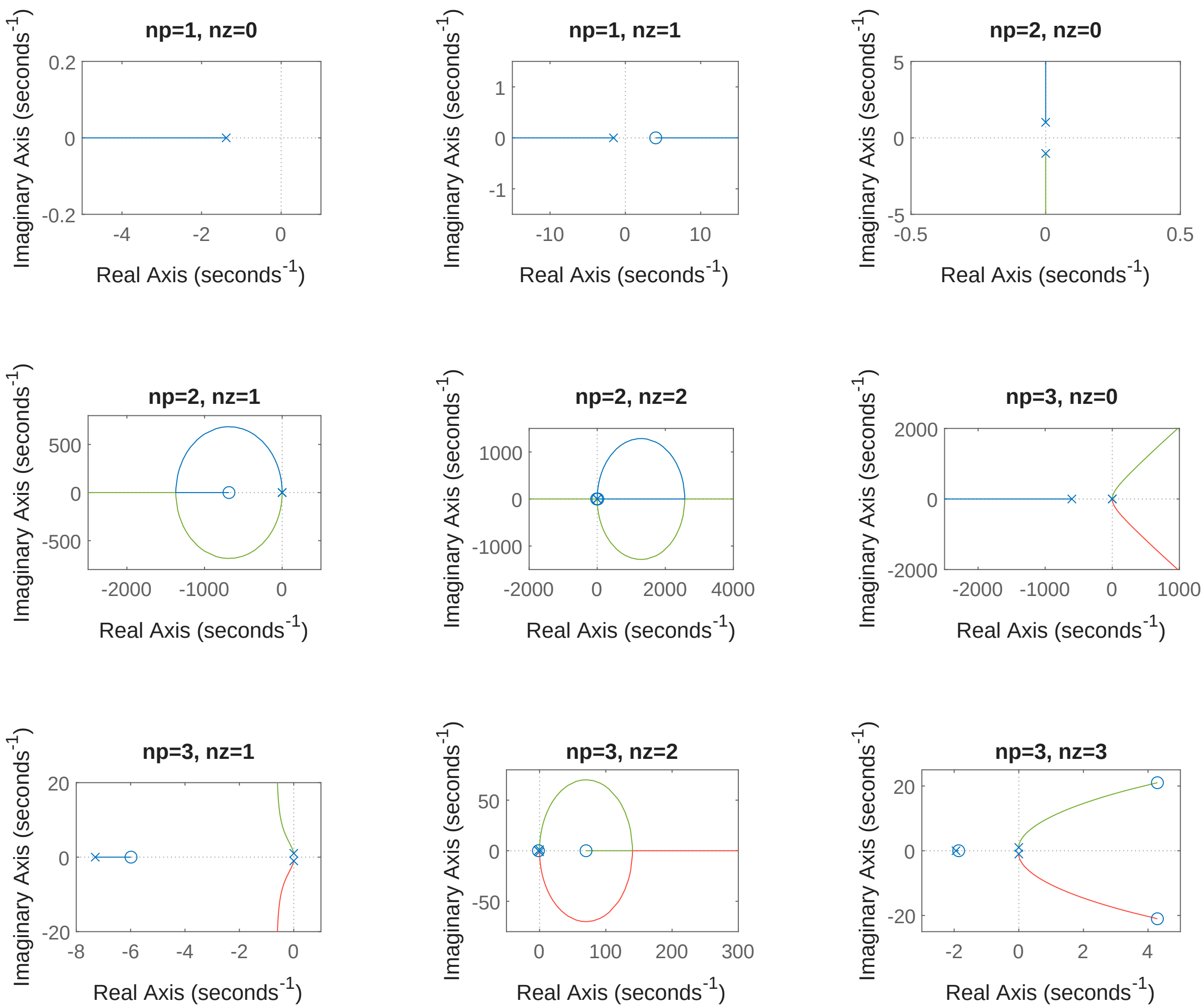

Figure 4: Root locus of estimated transfer functions for different choices of model order (0.1 step amplitude, no noise).

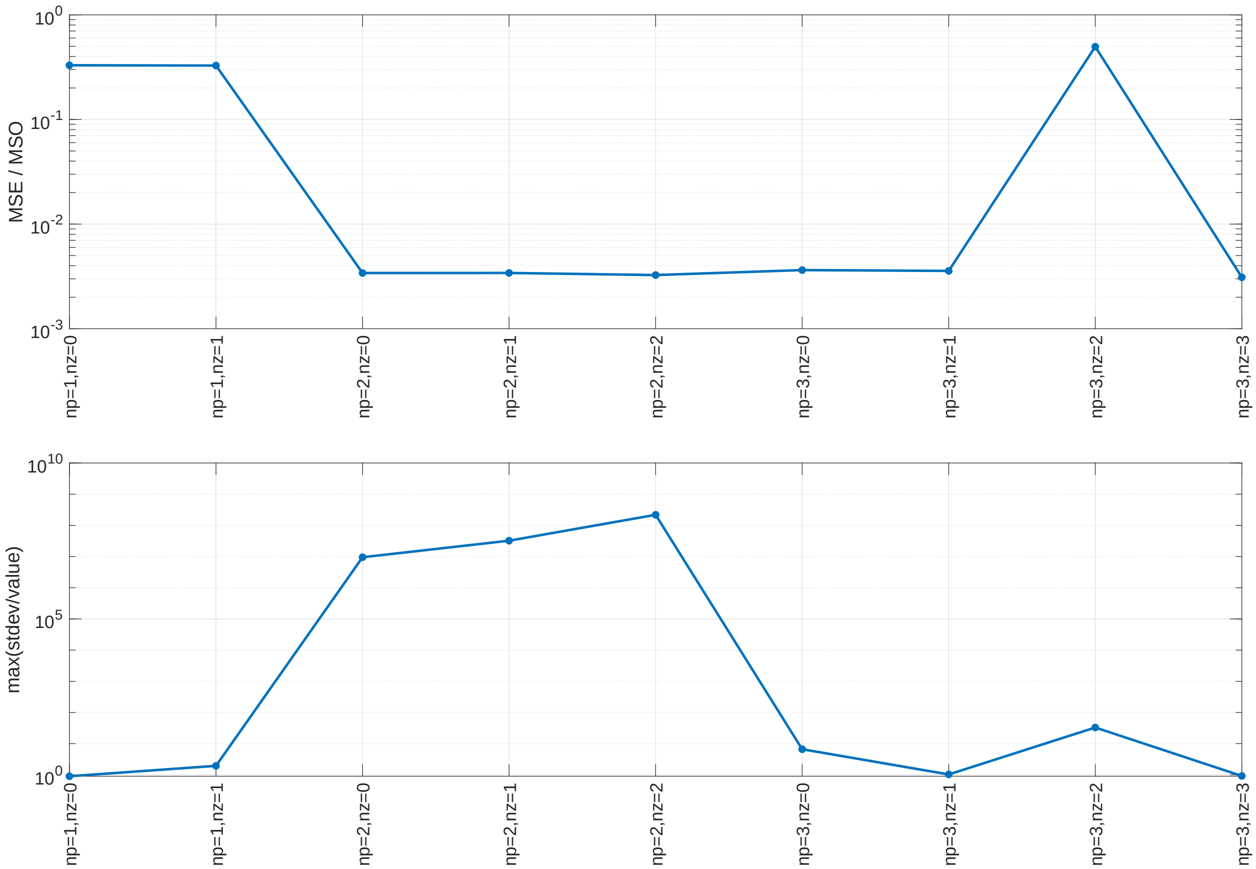

Figure 5: Normalized MSE and largest normalized parameter standard deviations for different choices of model order (2 step amplitude, no noise).

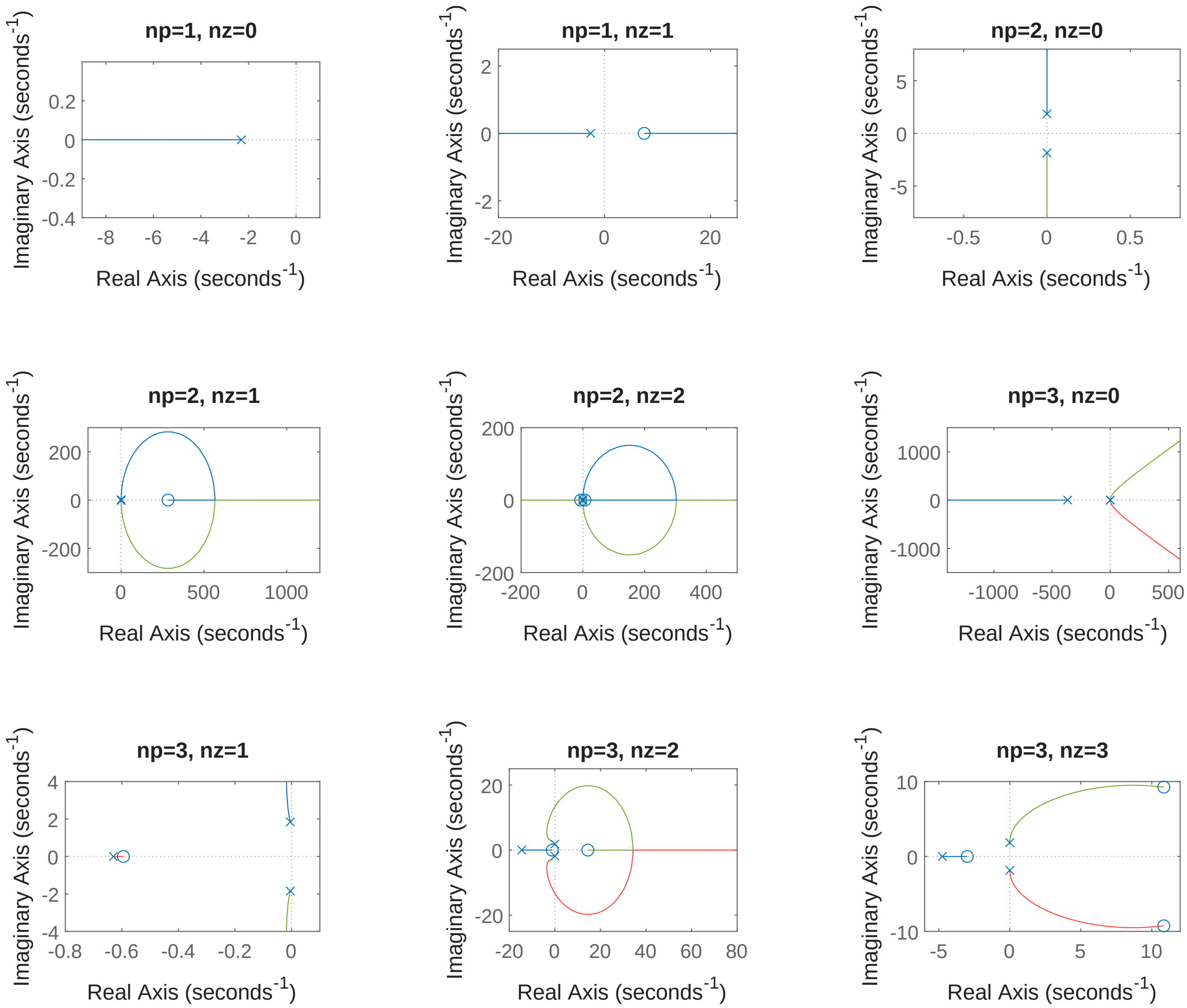

Figure 6: Root locus of estimated transfer functions for different choices of model order (2 step amplitude, no noise).

Observing Figures 3 and 5, the only combination of poles and zeros for which both the MSE and the parameter standard deviation is low across both step magnitudes is (np $=3$, nz $=0$) and (np $=3$, nz $=3$). But observing Figures 4 and 6, 3 poles and no zeros lead to an extremely large pole, so the best choice is 3 poles and 3 zeros. Do note that for 3 poles and 3 zeros, a 0.1 step magnitude leads to an overlapping pole and zero, while a 2 step magnitude does not have this. The corresponding transfer function for 3 poles, 3 zeros, and a step magnitude of 2 is:

$$H(s) = \frac{0.451s^3 + 2.141s^2 + 2.76s + 5.351}{s^3 + 4.146s^2 + 3.446s + 14.22}.$$

3.2 Part 2

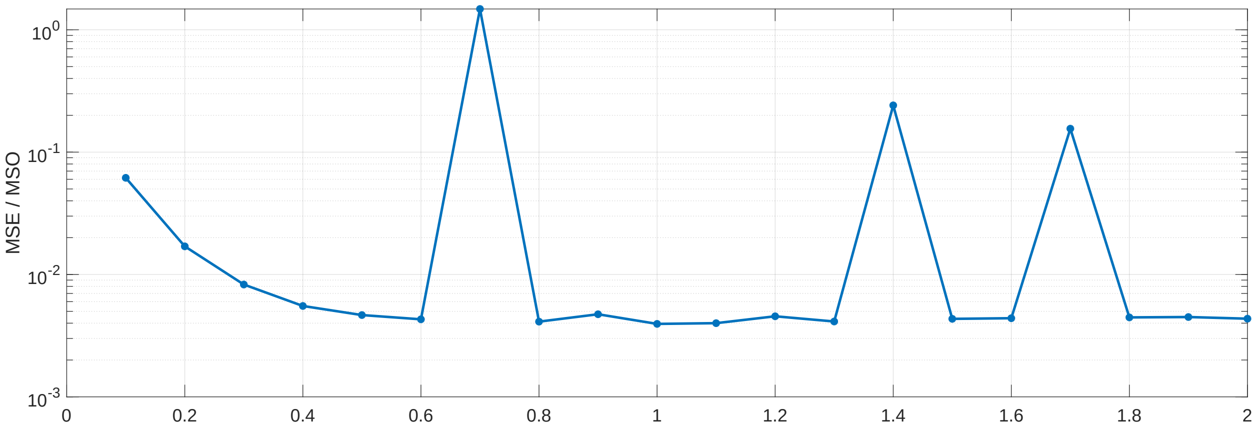

Figure 7: Normalized MSE for different choices of step amplitude (3 poles, 3 zeros).

Observing Figure 7, a step amplitude of 1 leads to the best model with the lowest MSE, assuming the model has 3 poles and 3 zeros.