Project 1 Report - ME155C/ECE147C

Cole Giusto, Raaghav Thirumaligai, Tien Nguyen

May 11, 2025

Abstract

In this project, we design a controller for a two mass - one spring, cart system. The goal is to optimize the performance of the second cart's response to a step input to the reference through control of the first cart's behavior, with respect to several key control metrics. We look at optimizing for a very short settling time along with minimizing the amount of overshoot. The 'undershoot' is treated as an acceptable behavior since the cart will pass through this region, so we give ourselves the difficulty of treating the overshoot as a highly undesirable set. We achieve our goals with a settling time under three seconds along with less than one percent overshoot.

1 Introduction

We begin our control journey with the search of an optimal controller. The first idea here is LQR, but it is clear that we have noise in our system and uncertainty in our data, so we consider going to LQG. We try to decrease our uncertainty by doing a sinusoidal sweep of 20 frequencies.

2 System Identification

2.1 Process to be controlled

The process we are controlling is a two cart system connected by a spring. The system is driven by a motor that applies a force F to the first cart with mass m_1 [kg], and the second cart with mass m_2 [kg] is connected to the first cart via a spring with spring constant k [N/m]. In this system, x_1 [m] is the position of the first cart, and x_2 [m] is the position of the second cart. The control input is the voltage $u := V$ [Volt] applied to the motor, and the measured output is the position $y := x_2$ [m] of the second cart.

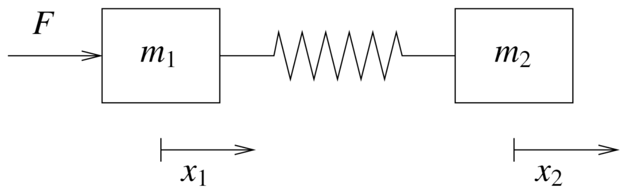

Figure 1: Two cart system

2.2 Non-Parametric Identification

The non-parametric identification method used was sine-wave testing in conjunction with the correleation method. This strategy consists of applying sinusoidal inputs at distinct frequencies to calculate the magnitude and phase of the frequency response at that frequency from the output of the system.

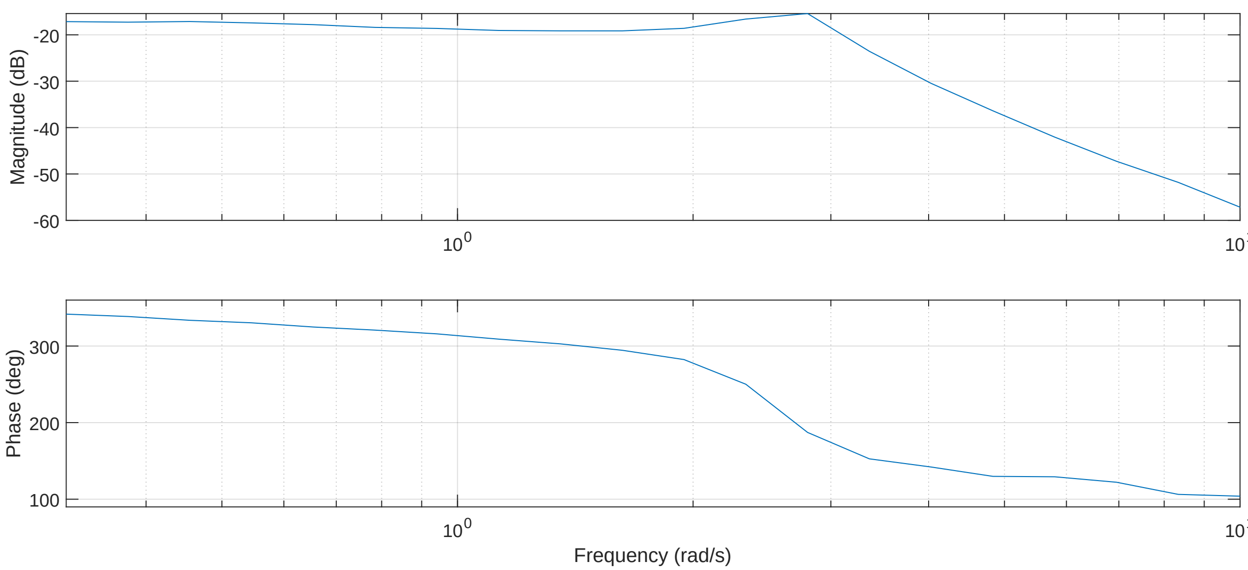

Figure 2: Bode plot of the identified system using non-parametric identification

2.3 Parametric Identification

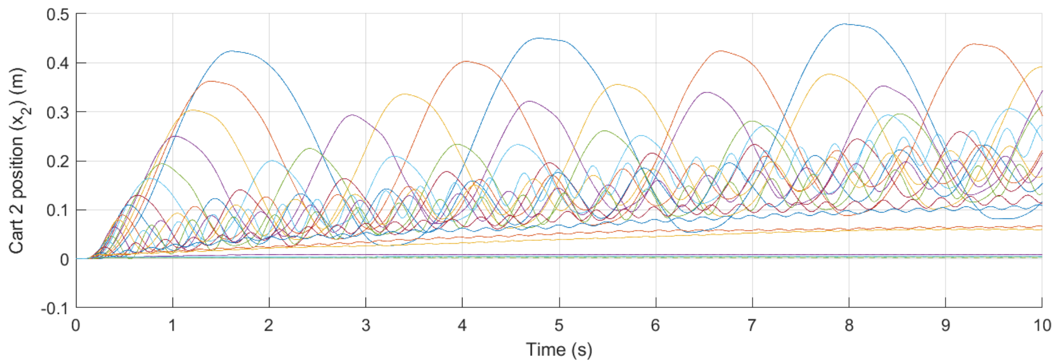

Figure 3: Output signals of all experiments for parametric identification

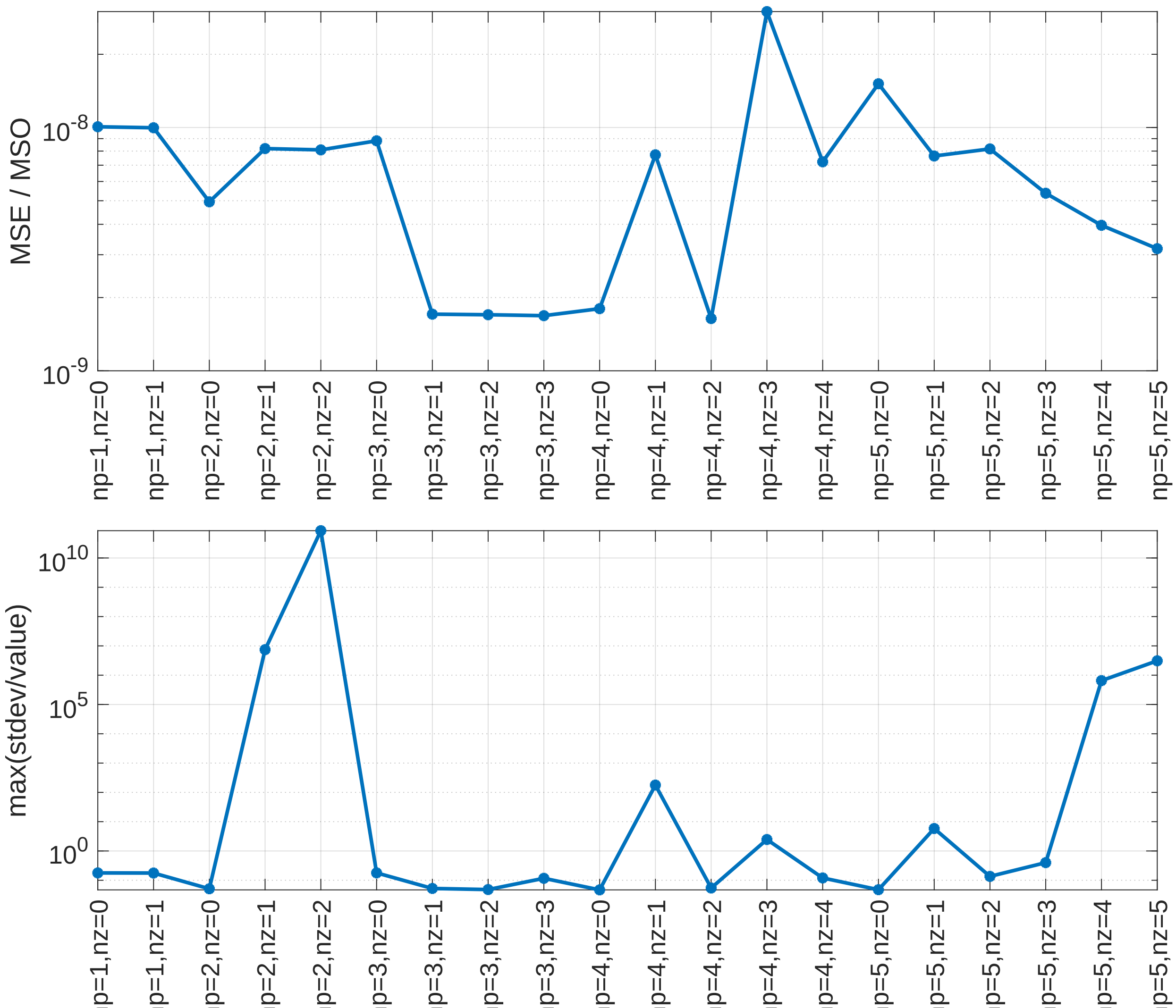

Figure 4: Normalized MSE and worst parameter standard deviation for different model orders

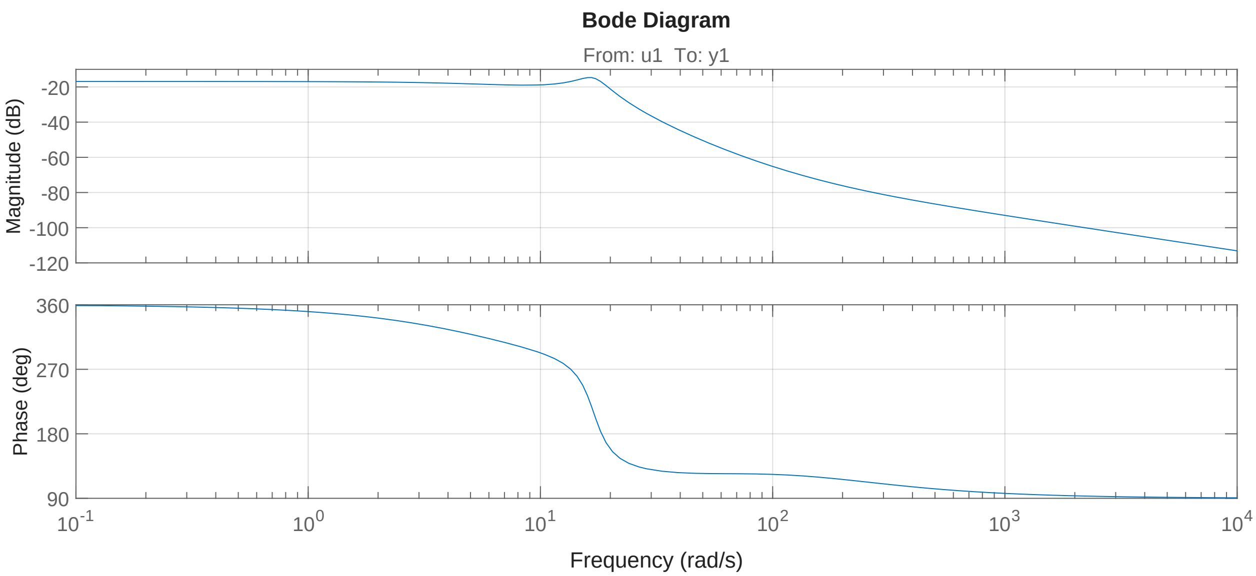

Figure 5: Bode plot of the identified system using parametric identification

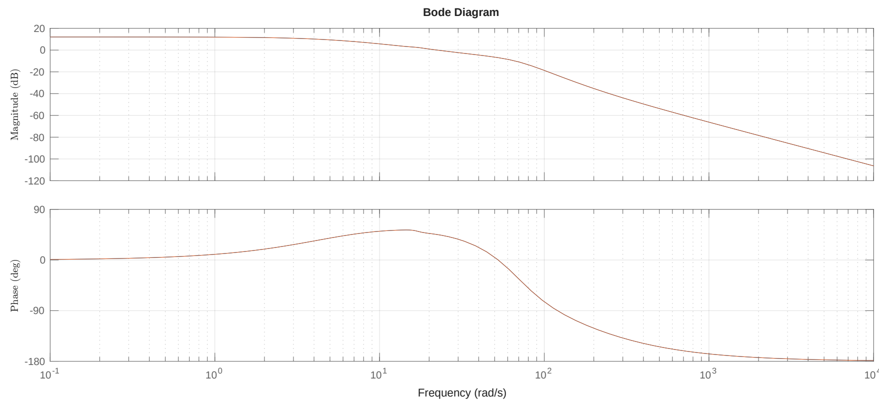

Figure 6: Closed-loop frequency response (simulated)

3 Controller Design

3.1 Design Methodology

Next, we were tasked to design a controller for the system. We shot for an overshoot of less than 15% and a settling time of less than 2 seconds. To achieve this, we chose to use an LQR/LQG design. Using our identified model, we used Matlab to calculate the optimal LQR gains and Kalman filter.

However, even with the optimal linear controller, the unmodelled dead zone of the motor still caused there to be a large steady state error. To fix this, we added an integrator to the controller, which successfully eliminated the steady state error. The final controller is an LQR/LQG controller in parallel with an integrator, which led to a closed loop system with an adequate performance.

In order to combat integral windup and saturation of the motor, we added a clamping function to the integrator and controller output. This saturation allowed us to achieve the limited steady state error of the integrator without suffering from the overshoot and instabilities that come from integral windup.

3.2 Simulation Results

We simulated the closed-loop system in Simulink using the identified model. The frequency response can be seen in Figure 6. The simulated step response is unstable, as seen in Figure 7. However, as we will show, the controller still performs well in the actual system, as we tuned the parameters using the real setup in the lab.

4 Closed-loop Testing

For the frequency response identification of the closed loop system, we ran a sinusoidal sweep across frequencies: $\omega \in 2\pi[10^{-0.5}, 10]$ The theoretical and experimental bode diagrams seem to disagree in phase and magnitude. This can be due to an inaccurate estimation of the order of the transfer function but ultimately we end up with a real closed loop that performs well so it is not too important.

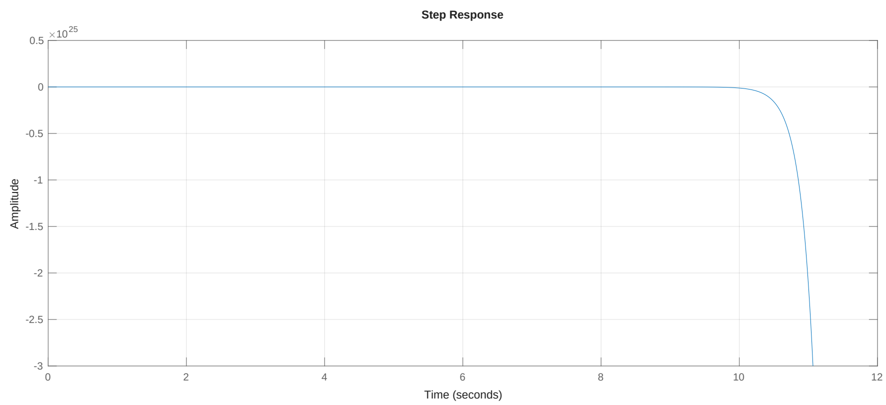

Figure 7: Closed-loop step response (simulated)

Parameter	**Value**
Overshoot	0.3%
Rise Time	1.81 sec
Settling Time	2.23 sec
Max Control Input	5 V

Table 1: Step Response Data

4.1 Step Response Experiments

4.2 Closed-loop Frequency Response

The closed-loop frequency response was identified using a sinusoidal sweep of the system. The results are shown in Figure 9 and Figure 10. The first figure shows the transfer function from reference to output, while the second figure shows the transfer function from reference to error.

5 Conclusions and Future Work

We are happy with the performance of our controller and would love to improve it in the following ways. First, we ran a small sweep of frequencies but if we had more time, it may make sense to test many more frequencies with a finer mesh. Additionally, we could test a wider range to get better data about the noise and for higher frequency data. The sinusoidal sweep tests are simple, but it would also be interesting to try a test using a white noise input. Next, for the controller performance, we see that around resonance there is a large tracking error. This is a place which could use more work, but part of the issues could come from optimizing for that ideal step response behavior.

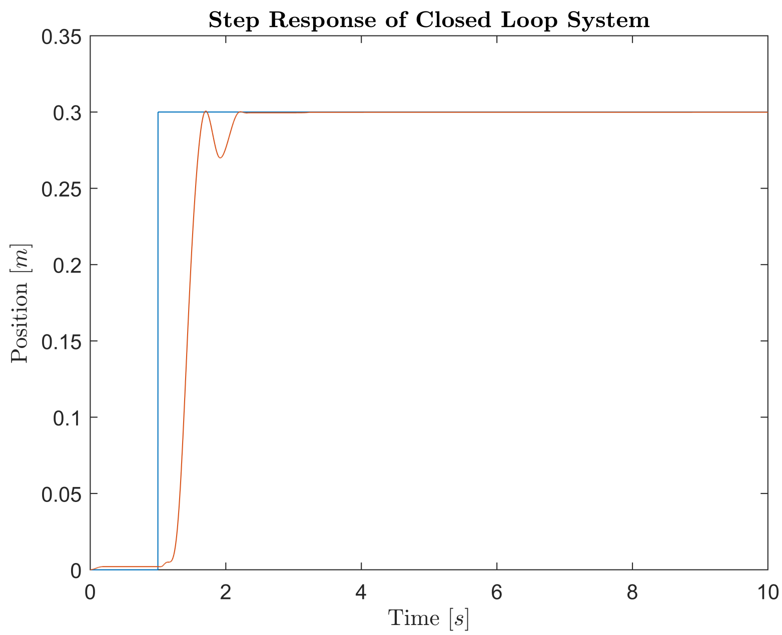

Figure 8: Closed-loop step response

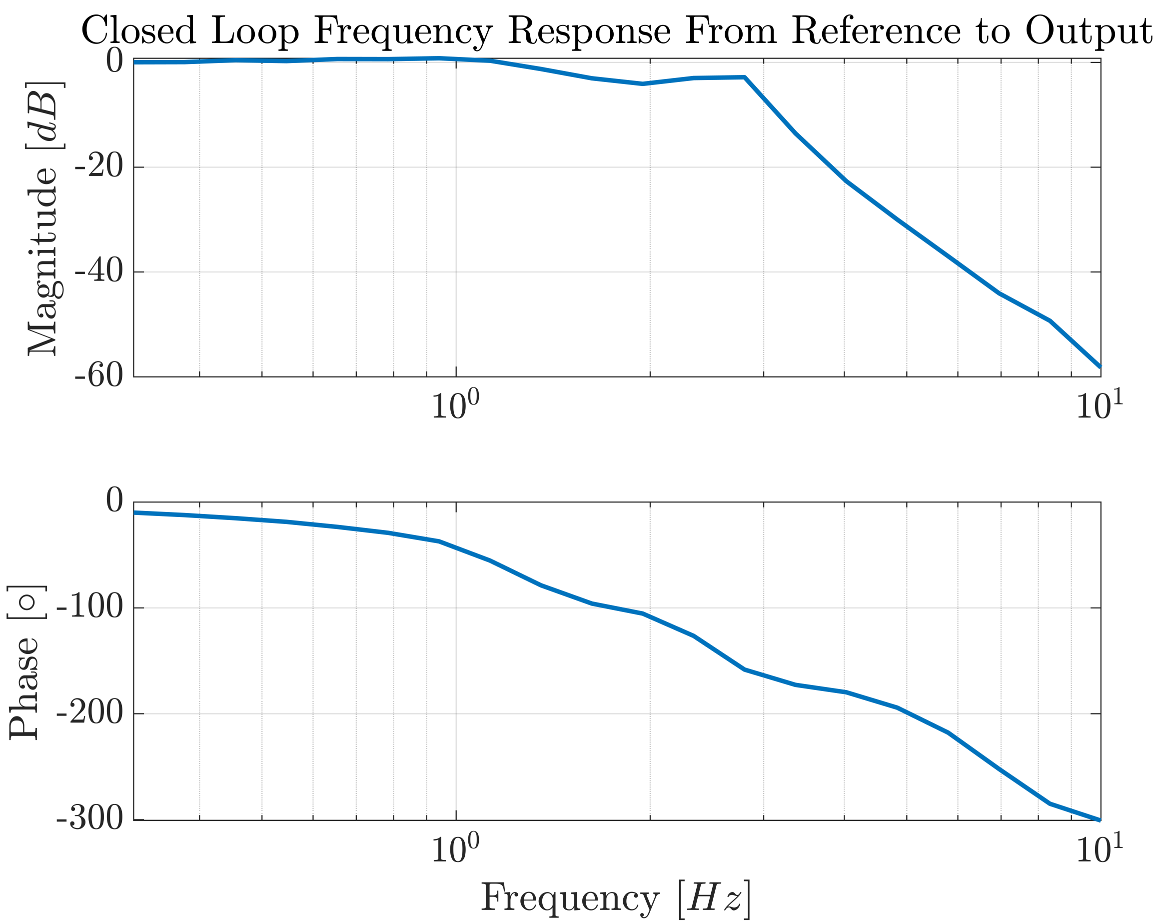

Figure 9: Closed-loop frequency response of the transfer function from reference to output

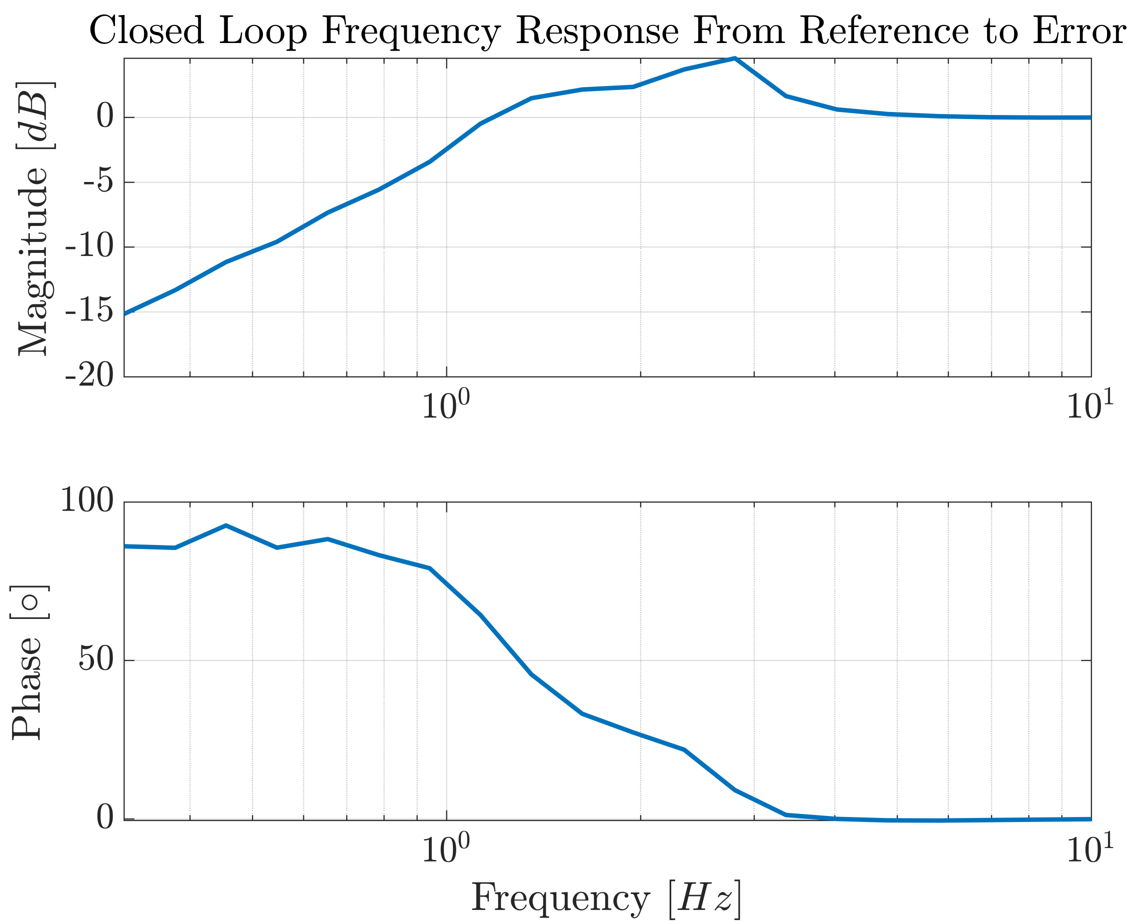

Figure 10: Closed-loop frequency response of the transfer function from reference to error