1.A *Problem Statement*

A. Construct a free-body diagram of this situation.

Given:

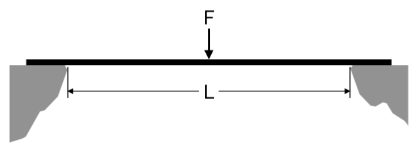


Figure 1: Beam spanning two cliffs

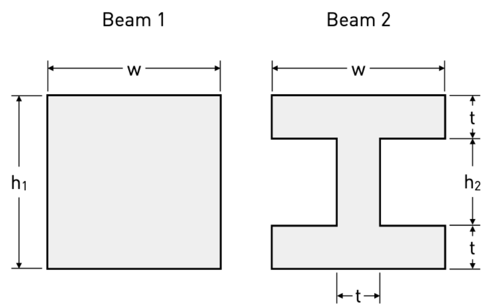
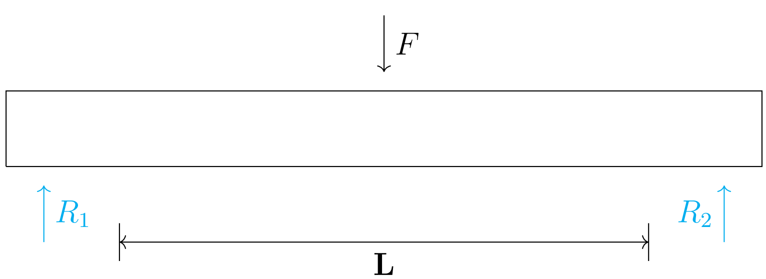


Figure 2: Beam geometries

We can consider this beam to be simply supported.

Solution:

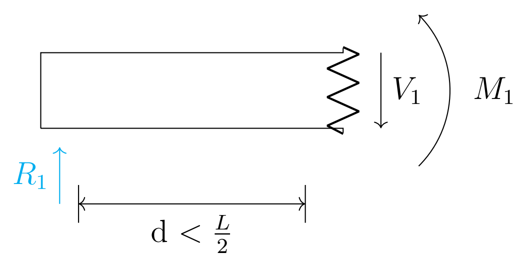


$$\begin{aligned}\Sigma F_y = 0 \quad &\therefore \quad F - R_1 - R_2 \\ F \quad &= \quad R_1 + R_2 \\ \Sigma M_1 = 0 \quad &\therefore \quad F * \frac{L}{2} = R_2 * L \\ R_1 \quad &= \quad R_2 = \frac{F}{2}\end{aligned}$$

1.B *Problem Statement*

Construct shear and moment diagrams of the beam and label with known quantities.

Solution:



$$\begin{aligned}\Sigma F_y = 0 \quad &\therefore \quad V_1 = R_1 \\ V_1 \quad &= \quad \frac{F}{2} \\ \Sigma M_1 \quad &= \quad 0 \therefore F * \frac{L}{2} = R_2 * L \\ R_1 \quad &= \quad R_2 = \frac{F}{2}\end{aligned}$$

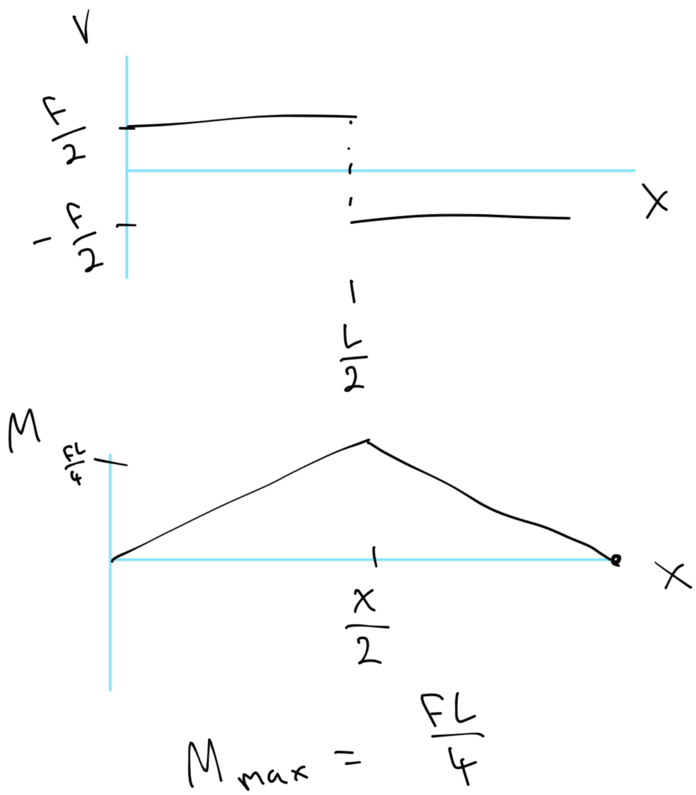


Figure 3: Shear and Moment Diagrams

1.C *Problem Statement*

Calculate the maximum bending stress as a function of the area moment of inertia, I, and the height of the beam, h.

Solution:

We can use the formula for shear and plug in the maximum bending moment.

$$\begin{aligned} \sigma &= \frac{-My}{I} \\ M_{max} &= \frac{FL}{4} \\ y_{max} &= \frac{h}{2} \\ \sigma &= \frac{-FLh}{8I} \end{aligned}$$

1.D *Problem Statement*

Calculate the area moment of inertia, I_1, for Beam 1.

Solution:

For this we can simply solve the integral.

I for Beam1

$$I_1 = \int y^2 \, dA$$

$$I_1 = \int_{-\frac{h}{2}}^{\frac{h}{2}} x y^2 \, dy$$

$$I_1 = w \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 \, dy$$

$$I_1 = w \left(\frac{y^3}{3} \Big|_{-\frac{h}{2}}^{\frac{h}{2}} \right)$$

$$I_1 = \frac{w}{3} \left(\frac{h^3}{8} + \frac{h^3}{8} \right)$$

$$I_1 = \frac{w}{3} \left(\frac{\overset{1}{\cancel{2}} h^3}{4\cancel{8}} \right)$$

$$I_1 = \frac{w h_1^3}{12}$$

☆ about y axis:
flip x & y

$$\downarrow$$

$$I_1 = \frac{w^3 h_1}{12}$$

Figure 4: Derivation for I_1

Final Answer:

$$I_1 = \frac{wh_1^3}{12}$$

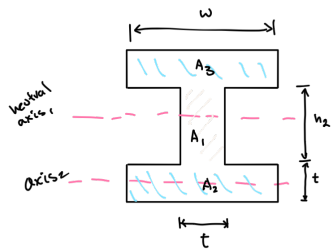
1.E *Problem Statement*

Calculate the area moment of inertia, I_2, for Beam 2.

Solution:

For this, I broke it down into the center piece and the two bars above and below, then added them together.

*at the bottom is work I did to check from a formula online.



Using x neutral axis

$$I_{A_2} = I_{A_3}$$

$$\frac{w-t}{2}$$

I_{A2} about axis 2

$$I_{A2} = \frac{wt^3}{12}$$

$$A = wt \qquad d = \frac{t+h_2}{2}$$

-> Parallel axis theorem

$$I_{A2} = \frac{wt^3}{12} + Ad^2$$

$$I_{A2} = \frac{wt^3}{12} + wt(t+h_2)^2\frac{1}{4}$$

$$I_{A2} = \frac{1}{4}wt\left(\frac{t^2}{3} + t^2 + 2th_2 + h_2^2\right)$$

$$I_{A2} = \frac{wt}{4}\left(\frac{4}{3}t^2 + 2th_2 + h_2^2\right)$$

$$I_{A2} = \frac{wt^3}{3} + \frac{wt^2h_2}{2} + \frac{wth_2^2}{4}$$

$$I_{A_1} = \int_{-\frac{h_2}{2}}^{\frac{h_2}{2}} y^2 \, x \, dy$$

$$I_{A1} = \frac{th_2^3}{12}$$

$$I_{Beam} = I_{A1} + I_{A2} + I_{A3}$$

$$= I_{A1} + 2I_{A2}$$

$$I_2 = \frac{th_2^3}{12} + \frac{2wt^3}{3} + wt^2h_2 + \frac{wth_2^2}{2}$$

from formula online:

$$\left[w\cdot(h_2+2t)^3 - (w-t)\cdot(h_2)^3\right]/12$$

$$w(h_2+2t)(h_2^2+4h_2t+4t^2) - wh_2^3 + th_2^3$$

$$w(\cancel{h_2^3} + 4h_2^2t + 4h_2t^2 + 2h_2^2t + 8h_2t^2 + 8t^3) - \cancel{wh_2^3} + th_2^3$$

$$\frac{6wh_2^2t}{12} + \frac{12wh_2t^2}{12} + \frac{8wt^3}{12} + \frac{th_2^3}{12}$$

$$\frac{th_2^3}{12} + \frac{wh_2^2t}{2} + \frac{2wt^3}{3} + wh_2t^2$$

Figure 5: Derivation for I_2

Final Answer:

$$I_1 = \frac{t(h_2)^3}{12} + \frac{2wt^3}{3} + wt^2h_2 + \frac{wt(h_2)^2}{2}$$

1.F *Problem Statement*

If you set $I_1 = I_2$, what is h_1 as a function of the other variables?

Solution:

$$I_1 = I_2$$

$$\frac{w h_1^3}{12} = \frac{t h_2^3}{12} + \frac{2wt^3}{3} + wt^2h_2 + \frac{wth_2^2}{2}$$

$$h_1 = \left(\frac{12 h_2^3}{w} + 8t^3 + t^2h_2 + 6th_2^2\right)^{1/3}$$

Figure 6: Derivation for h_1 from $I_1 = I_2$

Final Answer:

$$h_1 = \sqrt[3]{\frac{12(h_2)^3}{w} + 8t^3 + t^2h_2 + 6t(h_2)^2}$$

1.G *Problem Statement*

Calculate h_2.

Given:

10 = h1 = w = 10t

Solution:

Plugging in 10 for h_1 and w, and 1 for t.

$$10 = \sqrt[3]{\frac{12x^3}{10} + 8 + x + 6x^2}$$

Final Answer:

An online calculator gives x = 7.9558

1.H *Problem Statement*

Although Beam 1 and Beam 2 can carry the same load, their design differences mean they use a different amount of material. Calculate the volume per length for Beam 1 and Beam 2.

Solution:

We can easily find this from the volume formula.

Volume is just Length times Cross-sectional area. So, Volume per Length is just the Cross-sectional area.

$$\begin{aligned}\frac{V}{L} &= A \\ \frac{V_1}{L} &= w * h_1 \\ \frac{V_2}{L} &= (2w + h_2)t\end{aligned}$$