EE3025-Assignment 1

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Download all python codes from

```
https://github.com/raagini99/EE3025/tree/main/
    Assignment%201/codes
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and latex-tikz codes from

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https://github.com/raagini99/EE3025/tree/main/
    Assignment%201
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1 Problem

5.3 The system h(n) is said to be stable if

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{1.0.1}$$

Is the system defined by (3.2) stable for impulse response in (5.1)?

2 Solution

BIBO (Bounded-Input, Bounded-Output) Stability: A system is said to be BIBO stable, if the output of the system is bounded for every input to the system that is bounded, i.e.,

$$|x(n)| \le B_x < \infty \implies |y(n)| \le B_y < \infty \tag{2.0.1}$$

Condition for BIBO Stability in Time Domain: Let the input $x(n)$ be bounded, i.e.,

$$|x(n)| \le B_x < \infty \tag{2.0.2}$$

W.k.t.,

$$y(n) = \sum_{-\infty}^{\infty} h(k)x(n-k) \tag{2.0.3}$$

Applying modulus on both sides, we get

$$|y(n)| = |\sum_{-\infty}^{\infty} h(k)x(n-k)| \tag{2.0.4}$$

$$\implies |y(n)| \le |\sum_{-\infty}^{\infty} h(k)B_x| \tag{2.0.5}$$

$$\implies |y(n)| \le B_x|\sum_{-\infty}^{\infty} h(k)| \tag{2.0.6}$$

For the system to be BIBO stable, $|y(n)| < \infty$. This holds only if

$$|\sum_{-\infty}^{\infty} h(k)| < \infty \tag{2.0.7}$$

i.e., for the system to be BIBO stable, it's impulse response in time domain must be absolutely summable.

In given system,

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{2.0.8}$$

$$y(n) = 0 \text{ for } n < 0 \tag{2.0.9}$$

Applying Z-transform on both sides,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{2.0.10}$$

$$\implies Y(z) = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}}X(z) \tag{2.0.11}$$

$$\implies \frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{2.0.12}$$

But,

$$H(z) = \frac{Y(z)}{X(z)} \tag{2.0.13}$$

Plugging 2.0.13 into 2.0.12

$$\implies H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{2.0.14}$$

$$\implies H(z) = \frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{2.0.15}$$

Applying inverse Z-transform on both sides, we get,

$$\implies h(n) = \left[\frac{-1}{2}\right]^{n} u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \tag{2.0.16}$$

Now checking if the system is BIBO stable using 2.0.7,

$$\sum_{n=-\infty}^{\infty} |\left[\frac{-1}{2}\right]^{n} u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2)| < \infty \tag{2.0.17}$$

$$\implies \sum_{n=-\infty}^{\infty} |\left[\frac{1}{2}\right]^{n} u(n) + \left[\frac{1}{2}\right]^{n-2} u(n-2)| < \infty \tag{2.0.18}$$

$$\implies 2\sum_{n=-\infty}^{\infty} |\left[\frac{1}{2}\right]^{n} u(n)| < \infty \tag{2.0.19}$$

$$\implies 2\left[\frac{1}{1-\frac{1}{2}}\right] < \infty \tag{2.0.20}$$

$$\implies 4 < \infty \tag{2.0.21}$$

which is true. Therefore, the system is BIBO stable.

Additional:
When h(n) is absolutely summable, i.e.,

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \tag{2.0.22}$$

$$\sum_{n=-\infty}^{\infty} |h(n)z^{-n}|_{|z|=1} < \infty \tag{2.0.23}$$

$$\tag{2.0.24}$$

From triangle inequality,

$$|\sum_{n=-\infty}^{\infty} h(n)z^{-n}|_{|z|=1} < \sum_{n=-\infty}^{\infty} |h(n)z^{-n}|_{|z|=1} \tag{2.0.25}$$

Plugging 2.0.25 into 2.0.24, we get,

$$\implies |H(z)|_{|z|=1} < \infty \tag{2.0.26}$$

Therefore, the Z transform of the system's ROC includes the unit circle.
Solving, we get,

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{2.0.27}$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{2.0.28}$$

$$Poles = 0, -\frac{1}{2} \tag{2.0.29}$$

$$Zeros = +1j, -1j \tag{2.0.30}$$

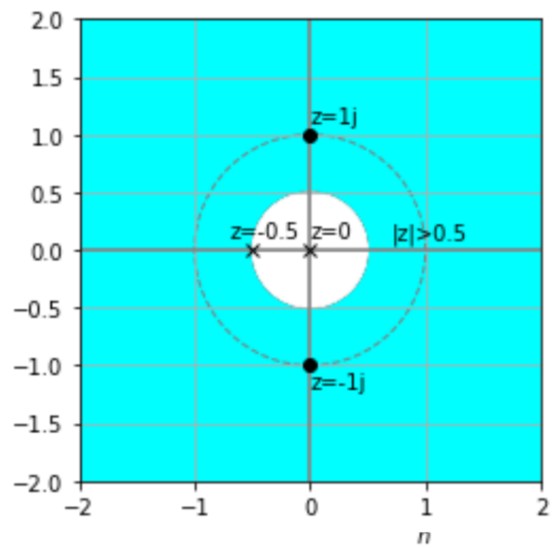

Fig. 0: Pole-Zero Plot

Therefore, from the solved equations or Z plane plot, the same is verified.

Verification of BIBO stability of given system for an example:
Given system:

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{2.0.31}$$

$$y(n) = 0 \text{ for } n < 0 \tag{2.0.32}$$

Let input be given as follows:

$$x(n) = \{1, 2, 3, 4, 2, 1\} \tag{2.0.33}$$

$B_x = 4$ for the input signal.

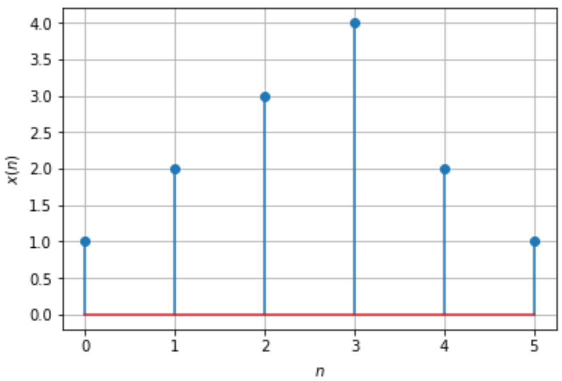

Fig. 0: Input signal, x(n)

Passing the signal through the system, we get the following signal:
$B_y = 4.375$ for the output signal.

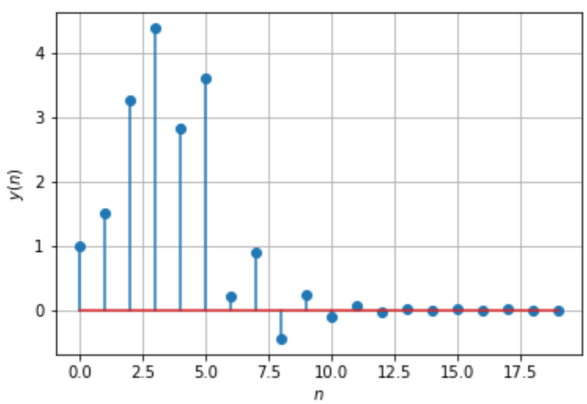

Fig. 0: Output signal, y(n)

Therefore, since the system is BIBO stable,
$B_x < \infty \implies B_y < \infty$