as n goes to infinity, where $\{\mu_n\}$ and $\{\sigma_n\}$ are sequences of real numbers and $\sigma_n > 0$. The Fisher–Tippett–Gnedenko theorem states that if there exist such sequences, then:

$$\mathbb{P}[l_n^* \le z] \xrightarrow[n\to\infty]{} G(z) \propto \exp\left[-(1+\xi z)^{-1/\xi}\right] \tag{61}$$

where the term

$$z = \frac{l-\mu}{\sigma} \tag{62}$$

is normalized by the location parameter $\mu \in \mathbb{R}$ and scale parameter $\sigma > 0$. The parameter ξ governs the tail shape of the limiting distribution; the generalized extreme value (GEV) distribution has a cumulative density function:

$$G(z) = \exp\left[-(1+\xi z)^{-1/\xi}\right] \tag{63}$$

for $1+\xi z > 0$. Depending on the value of ξ, the $G(z)$ belongs to one of the following distribution families:

1. if $\xi < 0$: Weibull family
$$G(z) = \begin{cases} \exp\left[-(1+\xi z)^{-1/\xi}\right] & \text{if } z < -1/\xi \\ 1 & \text{otherwise} \end{cases} \tag{64}$$
2. if $\xi = 0$: Gumbel family (the function $G(z)$ takes the limit as $\xi \to 0$)
$$G(z) = \exp[-\exp(-z)], \qquad z \in \mathbb{R} \tag{65}$$
3. if $\xi > 0$: Frechet family
$$G(z) = \begin{cases} \exp\left[-(1+\xi z)^{-1/\xi}\right] & \text{if } z > -1/\xi \\ 0 & \text{otherwise} \end{cases} \tag{66}$$

Figure 11 shows the probability density functions of the three types of the extreme value distributions:

$$g(z) = \begin{cases} \exp\left[-(1+\xi z)^{-1/\xi}\right](1+\xi z)^{-1/\xi-1} & \text{if } \xi \ne 0 \\ \exp[-\exp(-z)]\exp(-z) & \text{if } \xi = 0 \end{cases} \tag{67}$$

which are obtained by differentiating the $G(z)$ with respect to z.

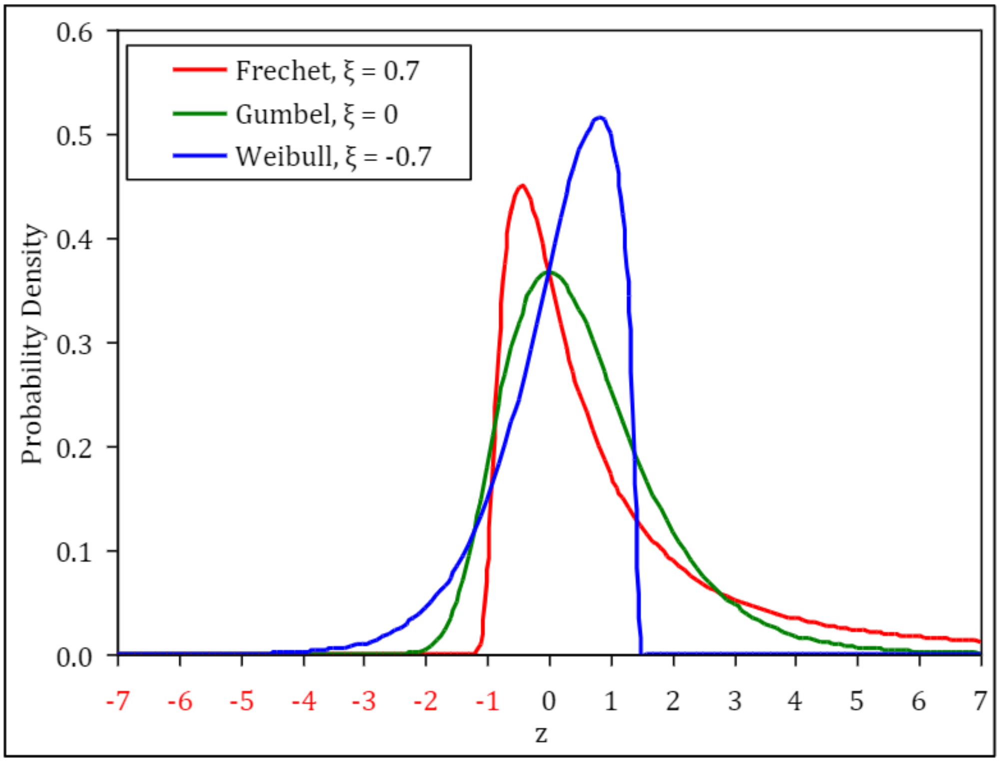


Figure 11. Probability density functions of extreme value distributions

The extreme value theorem suggests that the maximum of a large but finite number of independent and identically distributed random variables asymptotically (if there is convergence) follows a generalized extreme value distribution regardless of the original distribution of the random variables. This offers good analogy with the better-known central limit theorem, which states that the mean of a large number of random variables asymptotically follows a normal distribution regardless of the original distribution of the random variables.

Block Maxima Approach

Classical EVT proves the existence of a limiting distribution for the maxima of i.i.d. random samples. Approaches have been developed for extreme value analysis based on the limiting distribution. One is called the Block Maxima (BM) approach, which estimates the distribution for rare extremes by parametric modeling of maxima taken from large blocks of independent data. The idea is to divide the full dataset into equal sized blocks of data, and then determine the maximum for each block. The GEV distribution (63) can then be fitted to the sampled maxima by, say, maximum likelihood estimation. The statistical analysis and inference can then be derived from the fitted GEV distribution.

It is obvious that for block maxima to become close to i.i.d., the blocks must be sufficiently long. Since the statistical analysis is performed on only maxima, BM requires a large set of data, which is often unavailable for VaR estimation (e.g., if a block is defined over 50 days to give a single maximum, a common VaR window, say 2 years of data, provides only 10 usable points, far less than enough for a reliable calibration of BM analysis). For practical reasons, one has to consider other approaches that exploit data more cleverly.

Peak over Threshold Approach

Peak over Threshold (POT) is another popular method used during extreme value analysis. It focuses on parametric modeling of independent exceedances above a large threshold in a dataset. It is convenient to illustrate this concept with an example. Suppose we have a set of data $\{x_i\}$ for $i = 1, \cdots, N$ that are i.i.d., taken from a cumulative distribution function $F(x)$. This can be the daily hypothetical P&L of a portfolio from historical simulation over the past 500-day scenarios ($N = 500$). To simplify notation for the P&L data, positive numbers denote losses. Define a threshold u such that it is the 21st largest loss in the past 500 days. In other words, analysis focuses on losses beyond the 4% tail. We define $y = x - u$ as the exceedance beyond the threshold. The CDF of the exceedances can then be derived from $F(x)$":

$$\begin{aligned} E_u(y) &= \mathbb{P}[X < u + y | X > u] \\ &= \frac{\mathbb{P}[u < X < u + y]}{\mathbb{P}[X > u]} \\ &= \frac{F(u + y) - F(u)}{1 - F(u)} \end{aligned} \tag{68}$$

The asymptotic theorem for exceedance suggests that if the block maxima of $F(x)$ asymptotically converges to GEV distribution, the distribution of exceedance $E_u(y)$ asymptotically converges to a generalized Pareto distribution (GPD) as u goes to infinity:

$$E_u(y) \xrightarrow[u \to \infty]{} P(y) = \begin{cases} 1 - \left(1 + \xi \dfrac{y}{\beta}\right)^{-1/\xi} & \text{if } \xi \neq 0 \\ 1 - e^{-y/\beta} & \text{if } \xi = 0 \end{cases} \tag{69}$$

where the shape parameters ξ in GPD is identical to that in GEV, and scale parameter β relates to the scale parameter σ in GEV. Differentiating (69) with respect to y, we get the probability density function of GPD:

$$p(y) = \begin{cases} \dfrac{1}{\beta}\left(1 + \xi \dfrac{y}{\beta}\right)^{-1/\xi - 1} & \text{if } \xi \neq 0 \\ \dfrac{1}{\beta} e^{-y/\beta} & \text{if } \xi = 0 \end{cases} \tag{70}$$

Figure 12 shows the probability density functions of GPD with various shape parameters.

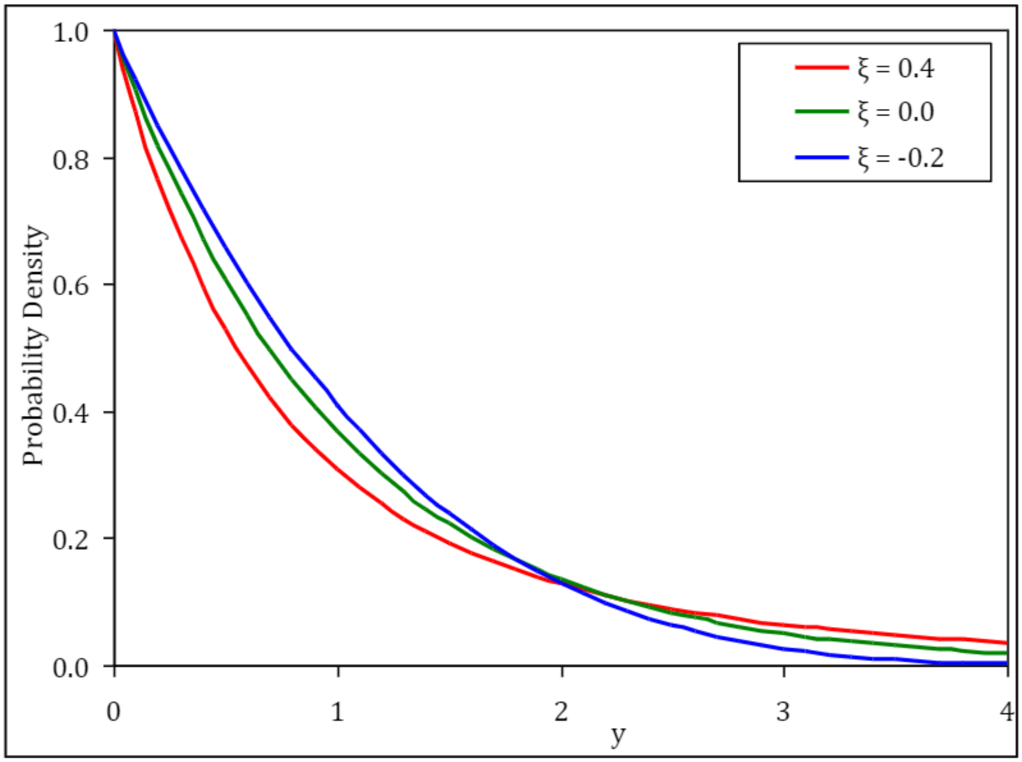


Figure 12. Probability density functions of generalized Pareto distribution ($\beta = 1$) with different shape parameters

To calibrate the GPD to our 500-day P&L data, we extract the exceedances from the dataset (e.g., there are 20 loss exceedances [$n = 20$] of 500 P&L). The GPD distribution function $P(y)$ can be calibrated to the exceedances using MLE by maximizing the log-likelihood function:

$$\ln \mathcal{L} = \begin{cases} -n \ln \beta - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^{n} \ln \left(1 + \beta \frac{y_i}{\xi}\right) & \text{if } \xi \neq 0 \\ -n \ln \beta - \frac{1}{\beta} \sum_{i=1}^{n} \ln y_i & \text{if } \xi = 0 \end{cases} \tag{71}$$

In the spreadsheet [VaR_EVT.xls], we perform the calibration using Microsoft Excel Solver, which yields estimates for parameters $\hat{\xi} = 0.031$ and $\hat{\beta} = 2{,}062$. Once the density function is calibrated, we can derive the conditional distribution:

$$\begin{aligned} \mathbb{P}[X > u + y | X > u] &= 1 - \mathbb{P}[X < u + y | X > u] \\ &= 1 - P(y) \end{aligned} \tag{72}$$

Since $x = u + y$, the unconditional distribution is:

$$\begin{aligned}\mathbb{P}[X > x] &= \mathbb{P}[X > x | X > u] \cdot \mathbb{P}[X > u] \\ &= [1 - P(x - u)] \cdot \mathbb{P}[X > u]\end{aligned} \tag{73}$$

where $\mathbb{P}[\mathrm{X} > u]$ can be estimated as n/N from the empirical distribution. The VaR is then calculated from the unconditional distribution, $\mathbb{P}[X > x]$. For example, if the confidence level is α, then by definition:

$$\mathbb{P}[X > \mathrm{VaR}_\alpha] = [1 - P(\mathrm{VaR}_\alpha - u)]\frac{n}{N} = (1 - \alpha) \tag{74}$$

where VaR_α in this case is a positive number denoting the size of a loss. After rearrangement:

$$\begin{aligned}P(\mathrm{VaR}_\alpha - u) &= 1 - \left(1 + \xi\frac{\mathrm{VaR}_\alpha - u}{\beta}\right)^{-\frac{1}{\xi}} \\ &= 1 - (1 - \alpha)\frac{N}{n}\end{aligned} \tag{75}$$

The last step is to solve for the VaR_α from (75), which gives:

$$\mathrm{VaR}_\alpha = u + \frac{\beta}{\xi}\left(\left[(1 - \alpha)\frac{N}{n}\right]^{-\xi} - 1\right) \tag{76}$$

Referring to the spreadsheet, we use the formula in (76) to calculate 1-day VaR at confidence levels from 95.0% to 99.9%. Results appear in Figure 13. As a comparison, the parametric VaR is also included in the plot, where pVaR is estimated by assuming a normal distribution for P&L. Clearly, the VaR estimated from EVT becomes increasingly fatter than normal as the confidence level increases.

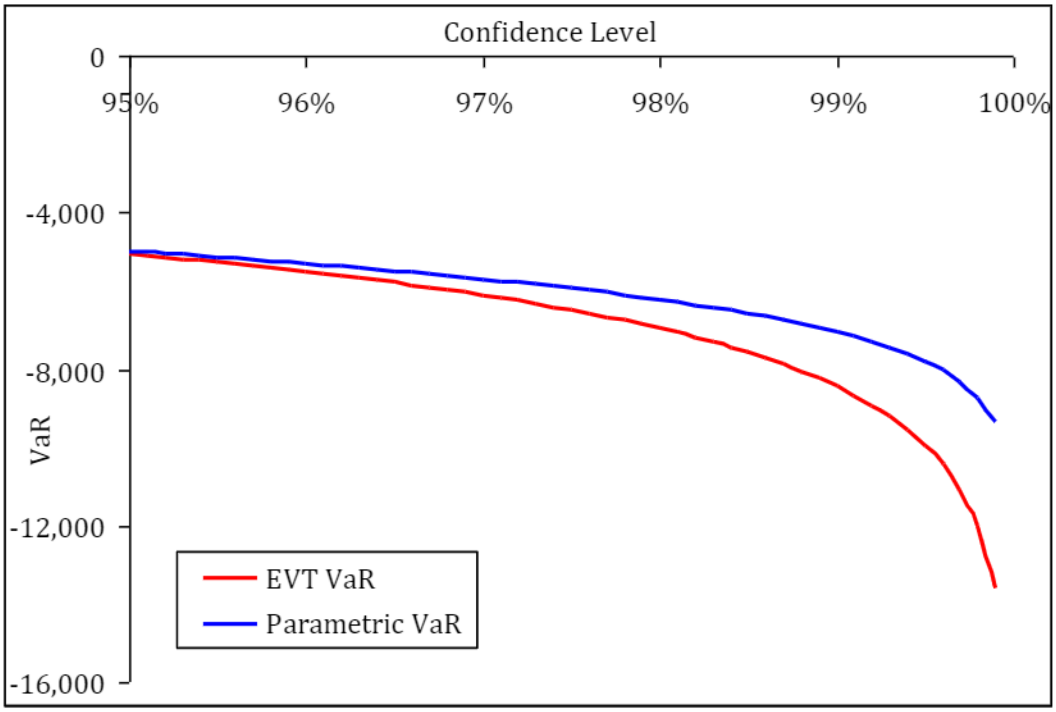


Figure 13. EVT VaR versus Parametric VaR at different confidence levels

A natural question is: how do we choose the value for the threshold, u? We want u to be sufficiently large so it meets the asymptotic criteria for POT. We also want it to be small so the MLE has enough exceedances to produce reliable model estimation. This is not an issue for large datasets. However, in VaR applications, the size of the dataset is almost always limited; we typically have only about 500 P&L data points to decide an optimum value for u. A practical rule is to make the u equal to the 95% percentile of the empirical distribution as an initial guess. When we calibrate the GPD distribution, we need to ensure the estimated parameters $\hat{\xi}$ and $\hat{\beta}$ are positive. If the estimated $\hat{\xi}$ is negative, the reason could be a) the u is improperly chosen, or b) the empirical distribution does not have heavier tails than that of the normal distribution. This effect can be seen from our example. If we choose $u = 95\%$ percentile (i.e., the 26th loss in the P&L data), the calibrated $\hat{\xi}$ turns out to be negative, and as a result, we might raise the u to the 96% percentile to perform EVT analysis.

Expected Shortfall

The VaR metric is where a single number is used to represent an entire distribution. Clearly, a scalar can never represent the detailed features of the tail of a multi-variant distribution fully, but it does provide a great deal of convenience and intuition for risk management. In 1999, Artzner et al. introduced coherence, a list of desirable properties for any such risk measure. A risk measure *F* (here *F* is assumed to be positive; the larger the *F*, the higher the risk) is coherent if it has the following four properties:

1. Monotonicity: if $X_1 \le X_2$ then $F(X_1) \ge F(X_2)$
   If a portfolio has values (*X*) lower than another under all possible scenarios, its risk as measured by *F* must be larger.

2. Homogeneity: $F(aX) = aF(X)$
   Increasing the size of the portfolio by factor *a* linearly scales its risk measure *F* by the same factor.
3. Translation invariance: $F(X + k) = F(X) - k$
   Adding riskless assets or cash *k* to the portfolio lowers the risk measure *F* by *k*. This property justifies imposing regulatory capital buffers and reserves at banks.
4. Sub-additivity: $F(X_1 + X_2) \leq F(X_1) + F(X_2)$
   This property reflects the intuition of risk diversification such that the risk of a portfolio of securities should always be less than (or equal to) the sum of risks of its components (i.e., adding sub-portfolios does not increase risks, and splitting portfolios does not decrease risks). If the latter were true, a bank could simply book deals into separate portfolios and total risks would become lower, a clear fallacy.

Coherence is often discussed in academia because it is found that VaR is not sub-additive. This is rarely an issue in practice. First, sub-additivity is rarely violated in bank portfolios, and even when it is violated, in a large portfolio its effect might not be obvious (or material) enough for a risk manager to notice. Violation of sub-additivity typically occurs when the portfolio contains concentrated long-short positions that are hedged, and the market goes into a stressful period. In that situation, a risk manager might get a nonsensical VaR decomposition.

In 1999, Artzner et al. (1999) [9] proposed an alternative measure called expected shortfall, which satisfies all criteria of coherence. This is sometimes called conditional VaR (cVaR), expected tail loss (ETL), or Tail Average (TA), defined as the expectation of the loss at the tail of the distribution beyond the VaR quantile:

$$ES = \mathbb{E}[-X | X \leq -VaR] \tag{77}$$

Practically, this is computed as the average of all the loss points in the sample distribution that is larger than VaR, and can thus be incorporated easily into existing VaR architectures, especially if a bank uses historical simulated VaR. For example, the ES at 97.5% confidence using a 250-day observation period is given by the average of the 6 (rounded from 6.25) largest losses in the left tail. The ES, in some sense, considers the shape of the loss tail. In contrast, VaR is oblivious to the tail beyond the quantile level. Thus, the latest BCBS consultative paper "Fundamental Review of the Trading Book" (2013) calls for adoption of expected shortfall as a replacement for VaR. In particular, the 99% VaR should be replaced by 97.5% ES at a 10-day risk horizon.

The expected shortfall is a special case of a spectral risk measure (SRM), introduced by Acerbi in 2002 [10]. The return sample is first ranked. Then each observation (each quantile $L(u)$) is multiplied by a weighting function $w(u)$:

$$SRM = \int_0^1 w(u)L(u)du \tag{78}$$

To qualify as a SRM, $w(u)$ must be a function that strictly increases over $[0, 1]$ and integrates (i.e., sums) to one. The expected shortfall (of confidence level $(1-\alpha)$) is a special case of a SRM, where the weights are equal beyond a certain quantile α, and beneath α, the weights are zero (i.e., it is the average loss that occurs for all losses that are above VaR). Every reasonable weighting function can be approximated by a step function, and a spectral risk measure based on a step function is simply a weighted average of the cVaRs at the jump points, so spectral risk measures do not add much beyond what can be achieved with intuitively easier to grasp cVaR measures.

Advanced VaR Models – Multivariate

Risk factors are usually codependent among themselves, and the dependence structure can influence the total risk at the portfolio level strongly due to diversification effects, or lack of them. In previous sections, we assumed a Gaussian model. This assumption plays a role in financial risk modeling, mostly because of its simplicity and computational efficiency, and since practitioners think it is good enough, especially given the paucity of underlying data that often do not allow fitting more complex models. In a Gaussian model, the codependence structure can be described by the covariance matrix (or correlation matrix plus variances, which is the same). This makes risk aggregation of a portfolio simple.

However, the financial crisis of 2008 was yet another example of the shortcomings of a Gaussian model in a crisis environment. First, it fails to fit to the empirical distributions of risk factors, notably their fat tails and skewness. Second, a single covariance matrix is insufficient to describe the fine codependence structure among risk factors; it cannot capture non-linear dependencies or tail correlations, for example.

To overcome such issues, a more advanced approach to portfolio risk analysis is required. A well-known method is called the copula function, which is a useful extension and generalization of approaches for modeling joint distribution of risk factors. A word of warning: determining the correct covariance matrix is impossible for high-dimensional data (the amount of data available scales linearly with the number of dimensions, while the number of coefficients in a covariance matrix scales quadratically). We introduce additional degrees of freedom here so this problem becomes worse, and choice of a good (implicit) prior is essential to obtaining reasonable results.

Joint Distribution of Risk Factors

We need to introduce basic concepts related to joint distribution of random variables. Consider a d-dimensional vector of variables $(f_1, f_2, \cdots, f_d)$ (e.g., returns of risk factors). Their randomness (hence risks) is characterized by a joint distribution. For example, the joint distribution for a pair of continuous random variables, (X, Y), can be expressed by a 2-D cumulative density function:

$$F_{X,Y}(x,y) = \mathbb{P}[X \leq x, Y \leq y] \qquad \text{and} \qquad F_{X,Y}(x,y) \in [0,1] \tag{79}$$

We define a probability density function, $f_{X,Y}(x,y)$, for the distribution, which by definition, must satisfy (i.e., probabilities sum to one):

$$\begin{gathered} \int_x \int_y f_{X,Y}(x,y) dy\, dx = 1 \\ \text{where} \qquad f_{X,Y}(x,y) \geq 0 \end{gathered} \tag{80}$$

In (79) and (80), we denote the random variables by capital letters and their values by lower case letters. By definition, the joint CDF can be obtained from the joint PDF via integration:

$$F_{X,Y}(a,b) = \int_{-\infty}^{a} \int_{-\infty}^{b} f_{X,Y}(x,y) dy\, dx \tag{81}$$

Marginal and Conditional Densities

If we consider the probability density function $f_X(x)$ of random variable X alone, it is called a marginal density function, and it gives the probability of variable X without referencing (or independent of) values of the other variable Y. The joint density relates to the marginal density through the equation:

$$\begin{aligned} f_{X,Y}(x,y) &= f_{Y|X}(y|x) f_X(x) \\ &= f_{X|Y}(x|y) f_Y(y) \end{aligned} \tag{82}$$

where $f_{Y|X}(y|x)$ and $f_{X|Y}(x|y)$ are the conditional probability density functions that define the distribution of a random variable given the occurrence of the other random variable. In a special case that X is independent of Y, the $f_{Y|X}(y|x)$ and $f_{X|Y}(x|y)$ both reduce to their respective margins, and give:

$$f_{X,Y}(x,y) = f_Y(y) f_X(x) \tag{83}$$

From (82), the joint distribution among random variables can be described by their conditional PDF, along with the marginal PDF.

Sklar's Theorem

Sklar’s theorem provides a theoretical foundation for application of copulas when describing the dependence structure of random variables. It states that every continuous multivariate distribution $F(x_1, \cdots, x_n)$ of a random vector $(X_1, \cdots, X_n)$ can be decomposed into marginal distributions $F_i(x) = \mathbb{P}[X_i \leq x]$, linked by a unique function C called copula such that:

$$F(x_1, \cdots, x_n) = C\big(F_1(x_1), \cdots, F_n(x_n)\big) \tag{84}$$

or equivalently:

$$F\left(F_1^{-1}(u_1), \cdots, F_n^{-1}(u_n)\right) = C(u_1, \cdots, u_n) \tag{85}$$

where $x_i \in \mathbb{R}$, $u_i = F_i^{-1}(x_i) \in [0, 1]$ and F_i^{-1} is the inverse function of the marginal CDF F_i. In other words, an m-copula is an m-dimensional distribution function in which all m univariate margins are distributed uniformly on $[0, 1]$. In practice, data for marginal distributions of random variables is far easier to obtain than their joint distribution. Rather than considering only explicit multivariate distributions, it is wise to model the joint distribution by the margins along with a suitable copula function measuring their dependence structure.

A prominent feature of the copula-based approach is that it models the dependence structure independent of the margins. This is especially useful when the variables are not Gaussian (as is often the case in finance). We might choose appropriate univariate distributions (e.g., fat-tailed Student-*t* distributions) for the margins and link them by a copula function most suitable for the implied dependence structure.

Copula Functions

Given the definition above, we introduce a few copula functions. Any multivariate distribution CDF can serve as a copula function. However, copula functions in high dimension are generally non-trivial and difficult to visualize. For illustrative purposes, we consider a bivariate copula function, defined as $C(u, v) \in [0,1]\ \forall\, u, v \in [0,1]$. The copula function, by definition, must satisfy the following three properties:

1. The copula must be zero if one of the arguments is zero.

$$C(u, 0) = C(0, v) = 0 \tag{86}$$

2. The copula is equal to u if one argument is u and the other is 1.

$$C(1, v) = v \qquad \text{and} \qquad C(u, 1) = u \tag{87}$$

3. The copula is m-increasing (i.e., when $u_1 < u_2$ and $v_1 < v_2$

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0) \tag{88}$$

The simplest example of copula function is the product copula. For the bivariate case, it is:

$$C_{Prod}(u, v) = uv \tag{89}$$

It is easy to check that the product copula satisfies the three properties mentioned above. It describes the case in which there is no codependence among the random variables, and therefore is the generalization of the zero-correlation situation into non-Gaussian marginal variables.

Another example is the Gaussian copula, which has been used broadly in financial applications, defined as:

$$C_{Gauss}(u_1, \cdots, u_n; \rho) = \Phi(\phi^{-1}(u_1), \cdots, \phi^{-1}(u_n); \rho) \tag{90}$$

where $\phi^{-1}(u_i)$ is the inverse of the univariate standard Gaussian CDF and $\Phi(x_1, \cdots, x_n; \rho)$ is the joint standard Gaussian CDF with an $n \times n$ correlation matrix ρ. The term *standard* means that for a univariate normal distribution, it has a mean of zero and variance of one, and for a joint normal distribution, it has a mean of zero and covariance of ρ (i.e., all marginal normals have a unit variance of 1). The standard joint Gaussian CDF is:

$$\begin{aligned} &\Phi(x_1, \cdots, x_n; \rho) \\ &= \int_{-\infty}^{x_1} \cdots \int_{-\infty}^{x_n} \frac{1}{\sqrt{(2\pi)^n |\rho|}} \exp\left(-\frac{1}{2} z^T \rho^{-1} z\right) dz_n \cdots dz_1 \end{aligned} \tag{91}$$

in which we write variable $z = (z_1, \cdots, z_n)^T$ as a column vector and $|\rho|$ is the determinant of the correlation matrix. Since the copula function behaves like a CDF, we can differentiate (90) to derive an equivalent PDF for the Gaussian copula:

$$c_{Gauss}(u_1, \cdots, u_n; \rho) = \frac{1}{\sqrt{|\rho|}} \exp\left[-\frac{1}{2} x^T (\rho - I)^{-1} x\right] \tag{92}$$

where I is an $n \times n$ identity matrix and x is a column vector given by $x = \left(\phi^{-1}(u_1), \cdots, \phi^{-1}(u_n)\right)^T$. In the bivariate case, the Gaussian copula reduces to:

$$C_{Gauss}(u, v; \rho) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \varphi_{X,Y}(x, y) dy\, dx \tag{93}$$

where $\varphi_{X,Y}(x, y)$ denotes the PDF of a bivariate standard Gaussian. If X and Y are independent, we can split the double integral, and the Gaussian copula in (93) further reduces to a product copula such that:

$$C_{Gauss}(u, v; I) = \int_{-\infty}^{\Phi^{-1}(u)} \varphi(x) dx \int_{-\infty}^{\Phi^{-1}(v)} \varphi(y) dy = uv \tag{94}$$

where I is a 2×2 identity matrix (implying zero correlation between X and Y) and $\varphi(x)$ is the univariate standard Gaussian PDF.

In financial markets, concurrent bad events are common. A good copula function must model such extreme co-movements regardless of the shape of margins. Embrechts et al. [**Error! Bookmark not defined.**] introduce the coefficient of tail dependence to measure the probability of joint extreme events among random variables. They show that Gaussian copula has zero tail dependence regardless of which high correlation we choose. This suggests that Gaussian copula is unsuitable for modeling joint distributions with strong tail dependence. A better choice is to use the Student t copula. It differs from Gaussian copula by assuming t distributions for the joint and marginal densities, which has the form:

$$C_t(u_1, \cdots, u_n; \nu, \rho) = t(t_\nu^{-1}(u_1), \cdots, t_\nu^{-1}(u_n); \nu, \rho) \tag{95}$$

where $t_\nu^{-1}(u)$ is the inverse of a univariate standard Student t CDF with degrees of freedom ν, and $t(x_1, \cdots, x_n; \nu, \rho)$ is a multivariate standard Student t CDF with mean of zero, correlation matrix of ρ, and degrees of freedom ν (although we still call it a correlation matrix, ρ in a joint Student t distribution has a different meaning from that in a joint normal distribution):

$$\begin{aligned} &t(x_1, \cdots, x_n; \nu, \rho) \\ &= \int_{-\infty}^{x_1} \cdots \int_{-\infty}^{x_n} \frac{1}{\sqrt{(v\pi)^n |\rho|}} \frac{\Gamma\left(\frac{v+n}{2}\right)}{\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{z^T \rho^{-1} z}{v}\right)^{-\frac{v+n}{2}} dz_n \cdots dz_1 \end{aligned} \tag{96}$$

where $\Gamma(\cdot)$ denotes the well-known Gamma function. Similarly, differentiating (95), we derive a PDF for the Student t copula:

$$c_t(u_1, \cdots, u_n; \nu, \rho) = \frac{1}{\sqrt{|\rho|}} \frac{\Gamma\left(\frac{v+n}{2}\right) \Gamma\left(\frac{v}{2}\right)^{n-1}}{\Gamma\left(\frac{v+1}{2}\right)^n} \frac{\left(1 + \frac{z^T \rho^{-1} z}{v}\right)^{-\frac{v+n}{2}}}{\prod_{i=1}^n \left(1 + \frac{z_i^2}{v}\right)^{-\frac{v+1}{2}}} \tag{97}$$

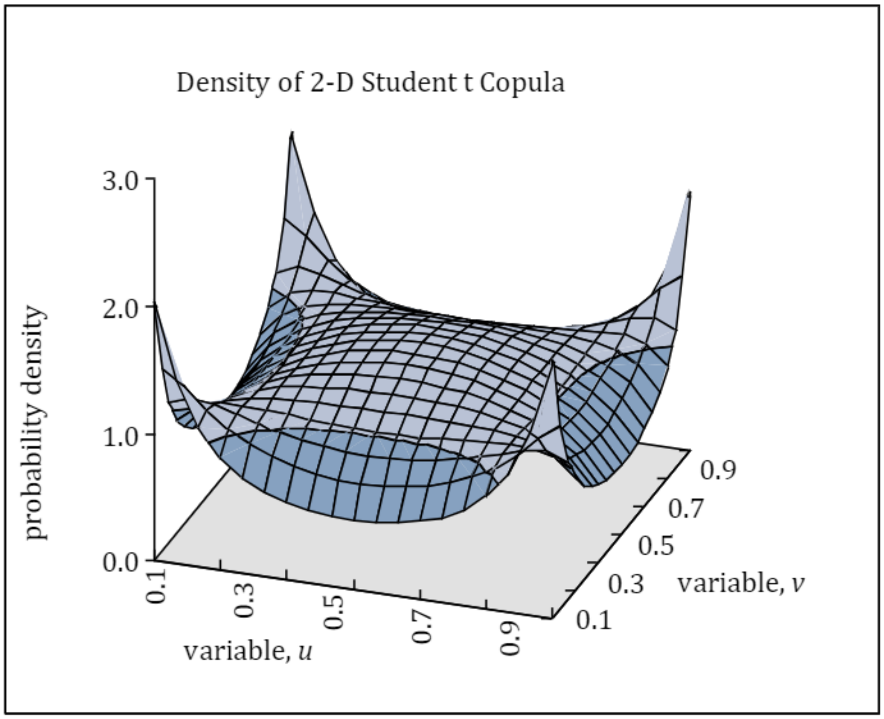


Figure 14. Density of 2-D Student *t* Copula with correlation zero and 2 degrees of freedom

To compare the two copula functions, we can look at their density functions with correlation set to zero. The density of Gaussian copula (92) in this case becomes uniform. The tail dependence is now determined only by the tail density of margins, which can vanish quickly. In contrast, the density of the Student *t* Copula (97) (Figure 14) has increasingly large density in the tails, which elevates the overall tail dependence.

Two points are noteworthy. First, the Student *t* copula converges to a Gaussian copula as the degrees of freedom, ν, goes to infinity. When the degrees of freedom are chosen properly (usually calibrated from historical data), the Student *t* copula gives a better fit to empirical joint distributions than the Gaussian copula. Second, in our derivation, we assume all associated margins have the same degrees of freedom. Although this simplification might limit the flexibility of the Student *t* copula when modeling tail dependence, it reduces the complexity of calibration greatly and simplifies its application during VaR estimation.

VaR Estimation using Copula

The dependence structure of risk factors plays a role in determining the VaR of portfolios. In practice, the joint distribution of risk factors can be far from a multivariate normal, and a single correlation matrix estimated from historical data might be insufficient to capture the full dependence structure in risk factors. Sklar's theorem suggests we can use a unique copula function, along with a set of marginal density functions, to describe the joint distribution. In this section, we use an example, referring to spreadsheet [VaR_Copula.xls], to demonstrate application of copula functions during VaR estimation.

Suppose a Gaussian copula is used to describe the dependence structure. Based on (84) and (90), the true joint distribution F can be approximated by:

$$\hat{F}(x_1, \cdots, x_n) = \Phi(\phi^{-1}(F_1(x_1)), \cdots, \phi^{-1}(F_n(x_n)); \rho) \tag{98}$$

where we use a hat on F to denote estimation. The first step is to calibrate the copula model (98). Ideally, we assume parametric marginals for $F_i(x)$ and use an estimation method such as MLE to calibrate the correlation matrix, ρ, and the parameters in $F_i(x)$ simultaneously to historical returns of risk factors. However, a large number of parameters might overwhelm calibration. Instead, we use empirical cumulative density function $\hat{F}_i(x) \in [0,1)$, defined by:

$$\hat{F}_i(x) = \frac{\text{number of elements in the sample} \leq x}{\text{total number of elements in the sample} + 1} \tag{99}$$

as estimations of the marginals $F_i(x)$, and calibrate only the correlation matrix, ρ. The MLE of ρ for the Gaussian copula is just the correlation matrix estimated from historical returns of risk factors after mapping to standard normal random space. In the previous example, we have daily returns $r_{i,t}$ for risk factor $i = 1, \cdots, 4$ and historical scenario $t = 1, \cdots, 500$. For the *i*-th risk factor, we have a vector of 500 returns that defines empirical distribution $\hat{F}_i(\cdot)$. The empirical distribution is used to transform the 500 returns to a vector of decimal numbers $u_{i,t} = \hat{F}_i(r_{i,t})$, valued between zero and 1.

The transformation is similar to the Rosenblatt transformation mentioned earlier. However, the difference is that in this case, the value transformed belongs to the sample that forms the empirical distribution (i.e., we are performing in-the-sample transformation). After the transformation, the series of $u_{i,t}$ is then further mapped to standard Gaussian random space by the inverse of the standard Gaussian CDF $x_{i,t} = \phi^{-1}(u_{i,t})$. Repeating the transformation for all risk factors, we then use the resulting data (e.g., four vectors of $x_{i,t}$ for $i = 1, \cdots, 4$) to estimate the correlation matrix, ρ, for the Gaussian copula.

With the Gaussian copula calibrated, we estimate the VaR using Monte Carlo simulation. The idea is to draw samples from the calibrated copula function, which is then inverted by the empirical CDF of the corresponding marginal to generate simulated risk factor returns. The detailed procedure is:

1. Sample from a multivariate standard Gaussian distribution $\Phi(0, \rho)$ to obtain $z = (z_1, \cdots, z_4)^T$. Again, the *Cholesky* decomposition of the correlation matrix, ρ, is used here to correlate the independent standard normal samples.
2. Apply standard Gaussian CDF ϕ to each component of z to obtain $v = (v_1, \cdots, v_4)^T = \left(\phi(z_1), \cdots, \phi(z_n)\right)^T$ such that each entry of v is valued between zero and 1. The sample, v, follows a distribution given by the Gaussian copula $C_{Gauss}(v_1, \cdots, v_n; \rho)$.

3. Apply the respective inverse function of the marginal empirical cumulative density to each component of v to obtain $\hat{r} = \left(\hat{F}_1^{-1}(v_1), \cdots, \hat{F}_1^{-1}(v_n)\right)^T$. The $\hat{r}$ is a vector of simulated risk factor returns.
4. Calculate portfolio P&L using the simulated $\hat{r}$ by full revaluation, the P&L resulting from one simulated scenario.

We repeat the simulation many times to derive a P&L vector that characterizes the P&L distribution, from which we estimate the portfolio VaR.

The Student *t* copula can be applied during VaR estimation similarly. An example of a Student *t* copula during VaR estimation is included in the spreadsheet [VaR_Copula.xls]. Due to additional parameters in the model, it requires more sophisticated calibration and sampling procedures, which lie outside the scope of this book. Interested readers can refer to Trivedi and Zimmer (2007) [11] for more information.

Chapter 4 – Market Risk in the Trading Books: Business Specific Context

Douglas Bongartz-Renaud

Contextual Introduction to Bank Trading Activities & Historical Development of Financial Product Markets

Since the late 1970s, financial markets have developed dramatically and off-balance-sheet, trading book activities have grown rapidly. To understand present market conditions and modern trading book, market risk technology and challenges, we begin by summarizing key developments in financial markets over the past four decades.

1970s: Globalization of FX & Debt Markets, International Banking Expansion & Growing Financial & Commodity Market Volatility:

In 1971, the United States exited the Bretton Wood Agreement guaranteeing the convertibility of the U.S. dollar into gold, which had been the basis for a global, fixed foreign exchange rate regime since 1944. Global trade and investment, gradual development of debt and equity markets in major economies, and international expansion of western banks flourished during the 1950s and 1960s. The combined cost of the war in Vietnam and expanding social programs incurred growing trade and current account deficits in the United States, leading to the floating of foreign exchange rates, the end of stable interest rates, the formation of OPEC, and increased volatility in commodity prices. The cold war also led to the formation of offshore markets, notably the London-based Eurodollar financial market. By the end of the 1970s, several international trends were developing that led to financial innovation and rapid growth in financial derivatives:

- Growing volatility in financial markets in foreign exchange and interest rates, equity markets, and oil and other commodity prices;
- Growth in international trade, investment, multinational corporations, and international banking;
- Growth in debt capital markets in major western economies, Japan, and Australia;
- Growing academic interest in financial economics, laying the foundations of modern portfolio management theory and derivatives valuation, and simultaneously in the commercial availability of electronic calculators/desktop computers and software, including spreadsheet programs;
- These trends created the initial drivers of modern financial markets trading and product expansion in the early 1980s, notably:
    - Arbitrage between debt capital markets by banks directly and banks on behalf of clients as counterparties;

- Hedging - FX, interest rate, equity, and commodity price exposures.

By the end of the 1970s, a number of financial exchanges already existed in North America and Europe, providing a limited range of standardized futures and options contracts in FX, equity, fixed income, equity, and commodities. OTC (over-the-counter) private, bilateral, off-exchange transactions were limited to banks trading foreign exchange forwards.

1980s: Growth of Swaps, Options and Focus on Counterparty Credit Risk & Basel I:

In the early 1980s, banks began arbitraging for themselves and clients in the debt capital market through OTC interest rate and cross-currency swap transactions. Initially, swaps were arranged as corporate finance cash flow exchange transactions on a back-to-back basis, with complex documentation. In 1985 however, banks formed ISDA (International Swaps & Derivatives Association), a trade association, which developed a standardized master netting agreement incorporating transaction definitions for many forms of swaps and other derivatives under the concept of multi-confirmations subject to umbrella agreement close-outs, netting on counterparty default or other methods agreed on in early termination. At the same time, a number of banks moved derivatives structuring groups to their trading floors and began warehousing swaps in trading books using government bonds (both long and short positions) to hedge the open swap exposures. With standardized documentation and the ability to transact immediately, the swaps market immediately took off, and by late 1986, ISDA reported USD 865 bn notional amounts in swaps on the books of its members. By 1990, the amount outstanding had grown steadily to USD 3.5 tn.

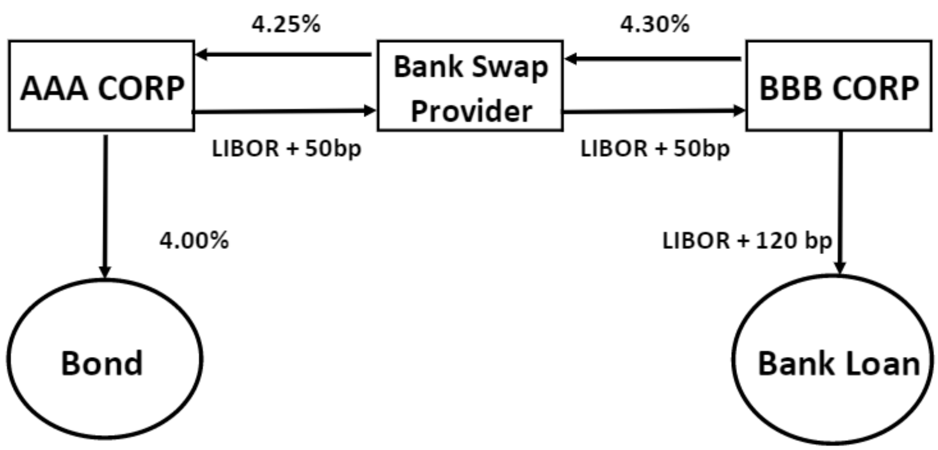
Typical of the type of debt arbitrage transactions being arranged by banks for clients in the mid-1980s was an Australian dollar bond issue underwritten for a Dutch insurance company by a large bank. This was swapped by the bank on behalf of the insurance company issuer into Dutch guilders. The all-in cost was about 40 basis points below the issuer’s direct cost of borrowing through a Dutch guilder bond issue. The issue was then sold to Australian investors who were willing to pay a yield trade-off premium for the opportunity of diversifying their investment portfolio outside of Australian companies without assuming foreign exchange risk.

During this early period of explosive swaps and FX derivatives growth, OECD banking regulators developed concerns about banks’ growing off-balance-sheet counterparty credit risk exposures. They jointly convened a working group - the Basel Committee on Banking Supervision - under the framework of the Bank for International Settlements to formulate bank capital standards that included risk coverage of both on- and off-balance-sheet counterparty credit exposures. The result was agreement and implementation of Basel I, which became effective in 1988.

Example 1: Debt Capital Market Arbitrage Using Swaps

Debt capital markets arbitrage has been one of the primary drivers of the explosive growth of swaps during the past three decades. The following example illustrates the concept of the use of swaps to lower cost of borrowing by sharing competitive advantage of different counterparties in different markets:

Borrowing Counterparty	Fixed-income Market Borrowing Cost – 5yrs	Bank Loan Floating Rate Borrowing Cost – 5yrs
AAA-rated Corporation	4.0%	6 month LIBOR + 0.5%
BBB-rated Corporation	5.2%	6 month LIBOR + 1.2%



Counterparty	Total Borrowing Cost for 5 years	Swap-based Gain Per Annum (5yrs)
AAA Corporation	LIBOR + 25bp p.a.	25 bp p.a.
BBB Corporation	5.00% p.a.	20 bp p.a.
Swap Provider	5 bp p.a.	5 bp p.a.

All three parties gain from the transaction—the two borrowers and swap provider. The gain results from the difference in credit spreads required by fixed-income bond investors versus the spread difference required by banks from floating-rate lending. In this example, the AAA Corporation has a 120 basis point borrowing advantage over the BBB Corporation in the 5-year bond market, versus a 70 basis point advantage in the 5-year floating-rate bank lending market. Assuming the AAA Corporation wants to borrow on a floating-rate basis and the BBB Corporation on a fixed-rate basis, they are able to share the difference in the two markets, and both benefit from swapping their debt costs through an intermediating, swap-providing bank. The swap-providing bank is also able to make an annual margin in the swap flows between the two corporate counterparties.

In the early days of the swaps market (1980s and 1990s), there were substantial arbitrage opportunities in both the interest rate swap market (as in the above example, for same-currency borrowings) and through arbitraging across debt capital markets through cross-currency interest rate swaps. Over the past two decades, debt market arbitrage opportunities diminished as borrowing costs converged across markets through swaps.

Growth of derivatives trading at banks led to the expansion of the product range, risk classes (adding more FX, equity, commodity, and interest rate based products), and the addition of option-based products (i.e., contingent claim derivatives) to the original forwards-based contracts. In a number of currencies, including the U.S. dollar, the interbank swaps markets gradually became more liquid than the underlying government bond markets, and banks gradually moved to LIBOR curves, zero-coupon discounting, and the ability to price and risk manage complex contractual and contingent cash flows in trading book. By the end of the 1980s, many banks were using variations of the Black-Sholes-Merton (BSM) model for pricing, trading, and risk management of a range of options products such as interest rate caps/floors/swaptions and bond options, and equity, FX, and commodity options in a range of currencies.

The banks increasingly used derivatives for not only arbitraging bond markets and designing liability structures for clients, but also structuring financial risk exposure hedges for corporate and institutional clients. Volatility remained an ongoing concern and was evidenced by many examples. Included in this period were items such as, the 1987 stock market crash and the FX and interest rate instability during the breakup of the Soviet Union & re-unification of Germany. In addition, application of derivatives as risk exposure management solutions for clients continued to grow as the arbitrage gain opportunities in the debt capital markets. These were exploited aggressively but then gradually slowed in availability and execution.

Late in the 1980s, the growth of debt capital markets began influencing large bank balance sheets and net interest margins since many corporate clients in the U.S. and parts of Europe were increasingly able to bypass bank balance sheets and issue debt to institutional investors. Development of high-yield bond markets, started initially by Mike Milken at Drexel Burnham Lambert, increased the disintermediation pressure on large western banks, incentivizing them to increase their derivatives and financial markets trading as an alternative source of profiting from client relationships.

Additional important events during this decade were:

- The lifting of U.S. withholding tax on foreign-owned U.S. treasury securities, which was a further catalyst for international capital markets expansion; and
- The U.S. Saving & Loan Crisis, which gave birth to the options-embedded, U.S. dollar, mortgage-backed securities market, which was an additional driver of derivatives expansion.

1990s: Trading Books, Exotic Derivatives, Basel II and Market Risk

Early in the 1990s, with banks hiring more quants to join their trading desks and development teams, the number and complexity of products began to grow rapidly. Banks developed a range of exotic derivatives in the underlying risk classes, starting with barrier, binary, and average rate payout options, and progressing to extremely complex and often path-dependent pay-outs. Concurrently, banks were forced to go beyond the simplicity of the approaches for derivatives in different underlying risk asset classes. A division between flow and non-flow (i.e., more complex, less liquid) products developed, and model risk joined market risk in trading. The organization of trading activities and alignment of trading desks, trading books, and mapping of market risk exposure measurement, reporting, and limits (sometimes called trading book structural risk hierarchy) rapidly became more complex at major banks.

The banks began packaging complex products into structured-notes, deposits, and other investment vehicle wrappers to add higher or alternative-yield assets as a growth driver of the derivatives market. These products gained in popularity toward the end of the decade when the introduction of the Euro reduced yields for investors in Italy, France, Spain, and other markets.

Earlier in the decade, speculation became a driver of derivatives trading growth, and events such as the unexpected 1994 interest rate increases and the Osaka earthquake triggered a number of large derivatives losses from derivatives speculation such as the collapse of Baring Brothers, major losses by Orange County in the U.S., and others. Market developments and derivatives growth prompted international regulators to further develop the Basel bank risk capital framework, introducing Basel II, which formally divided bank trading book activities from traditional balance sheet or banking book activities and created market risk management standards and capital requirements.

According to ISDA membership data, the derivatives notional contract amounts at large banks had grown to USD 17 trillion by 1995, and further tripled to USD 65 trillion by 2000. With rising volumes, counterparty credit exposures, and increasing regulatory capital requirements, banks began exchanging collateral as an additional credit risk mitigation tool with the help of ISDA, which developed standardized Credit Support Annex (CSA) documentation for bilateral exchanges of collateral on close-out exposures under the derivatives master agreements. During the decade, large banks increasingly operated trading book activities on a hub and spoke basis, operating trading hubs in financial centers in North America (usually New York), Europe (usually London), and Asia (often Singapore, Hong Kong, and/or Tokyo). Distribution to clients is then generally managed through the spokes, comprised of decentralized sales teams based in or near clients.

The decade culminated in a series of market crisis events—the 1997 Asian Credit Crisis, the 1998 Russian debt crisis, and the collapse of Long Term Capital Markets (LTCM). On a positive note, the decade also witnessed the development of exchange traded derivatives markets, with the most active globally traded contract being the CME LIBOR Futures contacts, which major banks use during hedging of swap and fixed-income trading books.

2000 - Present: Advent of Securitization, Credit Derivatives and Default Correlation Products, Financial Crisis, Global Regulatory Reform

Following the market crises in the late 1990s, low-interest rates and surplus liquidity existed in major markets during the first half of the 2000 to 2010 decade, enhancing market stability and low volatilities but at the cost of creating a difficult financial environment for insurance companies, pension funds and other institutional investors. Higher-yield and alternative-investment products came into increasing demand, and many large banks became increasingly active in originating, warehousing, repackaging, and distributing securitization and re-securitization with higher yields based largely on multi-obligor, credit default correlation risk. Structures such as credit default swaps (CDS), collateralized debt obligations (CDOs), asset-backed securities (ABS), and then CDOs on ABS, CDOs on CDOs, etc., with waterfall structures and individually rated tranches became increasingly commonplace in financial markets. During the decade, retail and commercial real estate loans became increasingly used for these complex debt-investment vehicles.

In 2007, the financial crisis started, and a number of large banks failed. Many were rescued with government capital injections. As a result, on a global level, regulatory risk management and capital requirements are being revised and strengthened (Basel 2.5, III, and Fundamental Review of Trading Book). Additional requirements being implemented include the Dodd-Frank Act in the United States and the European Markets & Infrastructure Regulations (EMIR) in the European Union.

The impact of the Basel framework changes alone increased trading-book capital requirement by several hundred percent at large banks active in financial markets. The crisis also, at least initially, decreased market risk and product complexity risk appetite, also having an adverse impact on bank financial market transaction volumes. Market events such as the failure of Lehman Brothers underlined counterparty credit risk exposure as a significant risk and an incremental pricing factor. Other elements of the Basel III requirements, notably additional capital requirements, leverage, and liquidity ratio rules are prompting many large, western banks to deleverage and alter their business models, With the result that global, trading-book activities are becoming more expensive now than before the crisis. These trends are beginning to affect product pricing, volumes, availability, and liquidity.

Micro - Organization of Trading Books and Risk Factor Management Responsibilities

Bank trading book activities are normally organized on a product-specific basis. Products are based on one or more market risk factors, on which their market value is based, and are either developed within the banking institution or provided to the bank in its externally provided trading system software. In either case, best market practice (and increasingly regulators) requires that the product valuation method/model be assessed and validated independently, usually by qualified staff members in a bank's risk management department. Valuation of the product is key to daily mark-to-market trading book valuation, and to market and counterparty credit risk measurement and reporting. A validated product can then be approved for use in specific trading books, on a primary, secondary (hedging), or back-to-back basis. Normally, only one trading desk is assigned primary trading responsibility for a validated product, and can then trade the product in its trading books, subject to additional approval conditions such as currency, tenor, or other restrictions. Trading approval in a validated product might be requested by other trading desks on a secondary (or hedging use) basis. Trading desks/books with secondary product trading approval must then trade the product internally with the authorized primary trading desk, and not with the external market. Back-to-back trading approvals might require stringent validation since the approved trading desk/book is required to match the position in its trading position with an offsetting position acquired externally, but subject to counterparty credit risk limits and the ability to calculate counterparty credit risk exposure for the product.

Products are primarily assigned based on risk factor management of the trading desk, and trading books within a trading desk group are normally organized in terms of a structural risk hierarchy, separating the complex from simpler products, and using simpler products as secondary (hedging) products in the more complex product trading books. Market risk factor limits are then mapped, measured, reported, and managed against the trading book structural risk hierarchy, starting at the lowest level with individual trading books and aggregating up to trading desks, trading units, and ultimately to total bank trading book exposure.

One of the goals of designing an effective and efficient structural risk hierarchy within a bank's trading book is to impose aggregation and separation of market risk exposure systemically, so foreign exchange, interest rate equity, and commodity risks are reported and managed by under limits provided to the desks primarily responsible for those risks. Separation of risks is imposed by risk factor limits, which require trading desks to trade internally with other trading desks to remain within their assigned risk limits. For example, a foreign exchange trading desk has primary FX risk management responsibility and is allocated most of the bank's approved FX risk limits. When a cross-currency interest rate swap trader on the long-term rates trading desk transacts a swap, he/she must hedge FX exposure in the trading position with the FX desk through execution of an approved product trade with that desk. The FX risk is thereby moved from the swaps trading book to the FX trading books. Internal allocation of risk factor limits designed within the structural risk hierarchy mapping enforces this separation/aggregation of risks across trading books.

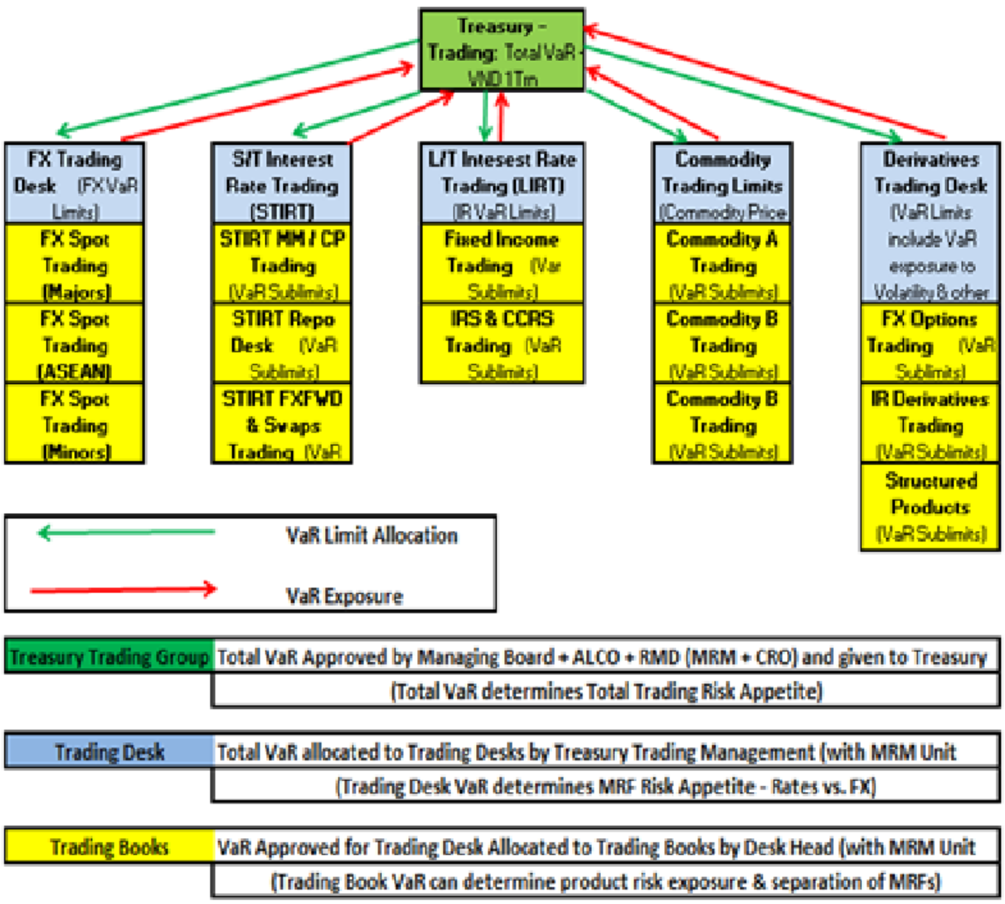
In the analysis below, products are discussed in terms of frequently used policies of assigning primary trading responsibilities. Individual banks might vary in terms of their structural risk hierarchy assignments of primary product trading groups and product responsibilities.

Macro - Organization of Trading Books and Bank Trading Activities

Larger banks with a wholesale client focus and significant investment banking activities usually have extensive financial product and trading. These banks often split products and trading activities into groups—FICC (Fixed Income, Currencies, and Commodities, which was labeled formerly as Debt Capital Markets) and Equity Capital Markets. The focus of large investment banks was traditionally on securities trading, on primary (underwriting) and secondary (market making) in bonds and equities. These were among the first banks to get involved in swaps and other financial products in the early 1980s because of the initial close link between swap (both interest rate and cross-currency) and debt capital markets. Many of these larger investment and universal banks initially developed a separate derivatives trading group and divided the group into separate underlying risk class trading desks, which were integrated into the larger risk class trading desks that managed the simpler trading products. Gradually, the derivatives products became more dominant in many of these banks.

Example 2: Trading Book Structural Risk Hierarchy (SRH)

Bank trading books must be organized coherently to enable orderly transaction flows, daily mark-to-market, and market risk management. An example of a trading floor organization by risk class for allocation of trading book, and trading book product responsibilities is shown below:



Bank trading books are organized by risk class trading desks to establish defined front-, middle-, back-office, and risk measurement and management structures. The trading desk structure, defined by dominant risk class responsibilities, varies depending on a bank's activities and trading book product rate, but as in the diagram above, can be divided generally into:

- Foreign Exchange Trading, responsible for most of the bank's FX risks exposures;
- Short-term Interest Rate Trading (STIRT), responsible for all LIBOR-based risk (1- to 2-year horizon, depending on currency);

- Fixed Income, responsible for products based on yield curves with maturities (longer than STIRT desk limits), including all bond trading, interest rate, and cross-currency swap, and credit-related products such as CDSs, CDOs, etc.;
- Equity Trading, responsible for all equity-related products and risk exposures;
- Commodity Trading, responsible for all commodity-related products and risk exposures;
- Options/Derivatives – in small banks with limited product ranges and derivatives activities, a separate derivatives trading group might have responsibility for all or most derivatives. In most banks active in derivatives trading, however, derivative products are assigned to the risk-class-based trading desks above.

The organization of trading books, called structural risk hierarchy (SRH), begins with an approval process for products and assignment of primary trading and risk management responsibilities per product by trading book, with secondary authorization for internal use of some of the primary products by other trading books as market risk-hedging instruments. Market risk measurement and risk limits are mapped using SRH.

Since many trading book products have multiple market risk factors, SRH must be designed carefully. A well-conceived SRH facilitates future additions of new products.
Consider several common FX-linked products such as FX spot, FX swaps, FX forwards, and FX vanilla European options. These products are commonly assigned to trading desks and mapped to a bank's trading book SRH as follows:

Products	**Market Risk Factors**	**Trading Desk Assignment**	**Market Risk Factor Limits Allocation**
FX Spot	FX spot rate (per currency pair)	FX Trading Desk	FX desk; allocated most of the bank's FX limits forcing other desks to transfer FX risk to FX Trading
FX Swaps/Forwards	FX spot rate (per currency pair); Interest rates in both underlying currencies	Short-term Interest Rate Trading (STIRT)	
FX Options	FX spot rate (per currency pair); Interest rates in both underlying currencies; FX rate volatility (per currency pair)	FX Trading (or Derivatives Trading Desk)	Options trading books have FX volatility (i.e., vega) limits and small FX (i.e., delta) and interest rate (i.e., rho) limits, forcing options trading books to transfer (i.e., hedge) most FX and interest rate exposure to FX Trading and STIRT

Banks with conventional retail and commercial banking origins and activities might organize trading differently, and their derivatives activities might be less dominant since the products are less dominant in their client business models.

In the market-risk-specific context of trading book products and market analyses that follow, products are arbitrarily covered under four headings: Fixed Income, FX and Rates, Equities, and Commodities. Under each heading, the structural risk hierarchy organization applied by different types of banks in organizing the trading and risk management of the products covered is outlined briefly.

Fixed Income

Cash Bonds & Bond Derivatives

On-balance-sheet, fixed-income banking books for credit risk/investment (i.e., loan substitute) activity and liquidity/cash management (i.e., liquidity buffer) are core activities of all banks. Fixed-income trading and trading book activities are core activities of investment banks, and are less important to conventional, commercial banks. Investment banks developed debt capital market underwriting and liquidity provisions as core client activities, retaining a service relationship for which banks have less balance sheet/credit intermediation value for clients. Investment banks use their trading books to originate debt market securities largely for government and clients, and provide investments and market liquidity for institutional investor (i.e., buy side) clients.

Introduction - Fixed Income Trading in the Basel Framework

Bonds must be categorized on acquisition by banks as either Banking book (i.e., Available for Sale or Hold to Maturity) or Trading book (i.e., Held for Trading). All banks use bonds of cash investment and liquidity management (i.e., banking book). Banks active in investment banking activities such as debt structuring, securitization, underwriting, and distribution have larger, fixed-income trading books. Bonds in trading books are subject to holding period limits under the Basel framework to enforce a business intent-based and credit-risk-treatment boundary between banking and trading book capital treatment, and are subject to risk charges as part of adding book market risk capital requirements. Under recent Basel revisions (including 2.5 and Basel Fundamental Review of Trading Book), the division of trading and banking books is changing, and market and counterparty risk requirements for bonds and bond derivatives such as credit default swaps (CDS) are subject to much more stringent capital treatment, including:

- Comprehensive Risk Measure (CRM), which requires banking-book, risk-capital treatment for credit correlation risk products such as securitizations and re-securitizations of debt instruments regardless of holding in trading books or banking book based on business intent;
- Incremental Risk Charge (IRC), which requires advanced (Internal Models Approach) banks to model and calculate required capital for credit default and migration risk in fixed-income trading books over a one-year time horizon (more in line with bond risk capital treatment in the banking book). Liquidity and correlation risks must also be factored into the IRC model;

Fixed Income - Products & Valuation Models

Bonds are cash instruments traded in both OTC and regulated (i.e., organized and exchange-traded) markets globally. There are numerous international and market specific debt markets. Some fixed-income derivatives such as U.S. Treasury Bond futures and options are available on exchanges such as CBOT and CME in Chicago.

Shorter-term bonds (original maturity longer than one year) are commonly called notes, and the term bond is generally used for longer-term debt instruments. Shorter-term debt issues commonly include instruments such as commercial paper and treasury bills. Special-purpose forms of debt such as trade bills exist, and other instruments often backed by collateral. Bonds differ from bank loans in their documentation, registration, and the rights of the debt holder. Loans are documented as private agreements between borrowers and lenders, whereas bonds are documented with indentures that define the rights and seniority of claim of the bondholders, and are typically exchange registered to enable resale in a secondary market.

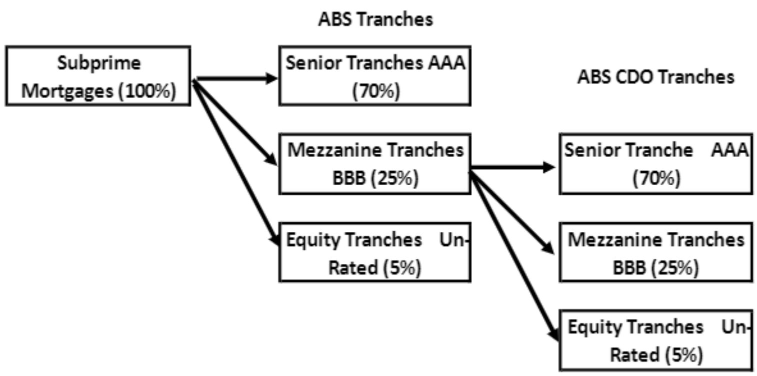
Products are normally classified by borrower type (i.e., government versus corporate issuer/obligor) and credit rating (i.e., public ratings such as provided by S&P, Moody's', Fitch, or individual internal bank rating models). Bonds vary in issue size and liquidity.

The most common form of bond is fixed interest rate with bullet redemption, but many bond cash flow structures are issued and traded, including amortizing bonds, floating rate bonds (with coupon resets based on reference indices such as LIBOR), and securitized, or pass-through, asset-backed structures. A common form of asset-backed, pass-through structure is found in mortgage-backed securities (MBS) in which the bond is collateralized by pools of individual mortgage loans whose interest repayments are passed through to the MBS holder. MBS structures often have prepayment risk, or maturity and cash flow uncertainty, requiring interest rate/behavioral valuation models.

During the low-interest rate environment of the past decade, banks engaged in packaging and distributing securities with pass-through bond structures for sale to institutional clients as higher or alternative yield investments. These products ranged from simple asset-backed, pass-through securities backed by pools of debt instrument assets such as car loans or credit card receivables to more complex, multi-tranche collateralized debt obligations (CDOs) in which investor return is based on credit default correlation risk in the underlying pool of loan assets. CDOs might sometimes be thought of as purchase of small loan portfolios in security form. This peaked in 2006 and 2007, and since the financial crisis, both regulation and market risk appetite for these structures have subsided.

Example 3: Collateralized Debt Obligation (CDO) Structures

Consider securities from which an investor earns a yield in exchange for taking levels of credit default risk. Generally, several classes of securities or tranches are constructed from a portfolio of underlying credit assets. A structure is then defined for allocation of default risk to the tranches (i.e., waterfall). Investors are effectively taking credit default correlation risk. Banks package several types of debt obligations into CDOs, including assets such as corporate bonds and mortgage-backed securities. A very simple example of a CDO structure is outlined below:



ABS Tranches:			
Losses to Subprime Porttfolio	Losses to Senior AAA of ABS	Losses to Mezzanine Tranche of ABS	Losses to Equity Tranche of ABS
2.5%	0%	0%	50%
5%	0%	0%	100%
10%	0%	20%	100%
15%	0%	40%	100%
20%	0%	60%	100%
25%	0%	80%	100%
30%	0%	100%	100%
35%	7%	100%	100%
40%	14%	100%	100%

ABS CDO Tranches:				
Losses to Subprime Porttfolio	Losses to Mezzanine of ABS	Losses to Equity Tranche of ABS CDO	Losses to Mezzanine Tranche of ABS CDO	Losses to Senior Tranche of ABS CDO
2.5%	0%	0%	0%	0%
5%	0%	0%	0%	0%
10%	20%	100%	60%	0%
15%	40%	100%	100%	14%
20%	60%	100%	100%	43%
25%	80%	100%	100%	72%
30%	100%	100%	100%	100%
35%	100%	100%	100%	100%
40%	100%	100%	100%	100%

A typical CDO structure begins with the collection of a pool of assets such as a portfolio of subprime mortgages in the example above. The portfolio is placed into a Special Purpose Vehicle (SPV), which issues tranches of securities based on a defined allocation of cash flows from the underlying portfolio. The tranche structure is generally multi-level and in the example above, we show two levels: a first ABS tranche level and a second ABS CDO tranche level created from the Mezzanine, BBB-rated tranche of the ABS level. Payments are made to tranches at each level of the structure from payments emanating from the underlying portfolio (in this example, subprime mortgages). Generally, the levels are derived by dividing mezzanine tranches at each level into further tranches at an additional level, and ultimately, the tranches at levels farther removed from the original portfolio in the waterfall structure bear increasing default correlation risk.

Banking book-based capital treatment for CDOs, initiated as part of the 2009 Basel reforms and diminished investor risk appetite after the 2007/2008 financial crisis, led to a general decrease in securitization, specifically in CDOs based on mortgage portfolios.

Bonds are largely wholesale, financial-institution (so-called investment bank buy-side client) products, but bonds can be packaged into funds, and retain other client-oriented investment products.

Callable and puttable bonds represent fixed-income products with embedded optionality, and many variants of such embedded option bonds exist. More complex forms of bonds exist: for example, convertibles (i.e., an equity hybrid, fixed income hybrid) and dual-currency bonds (i.e., an FX, fixed income hybrid construction with coupon and principal payments in different currencies).

Existing bonds are also often restructured through interest rate or currency swaps into alternative cash flow packages such as asset swaps (i.e., bond plus swap structures converting a fixed income bond into a floating rate note, inverse floating rate or other cash flow structure).

A new form of derivative appeared in the markets in the late 1990s, called credit default swaps (CDS), which can be thought of as put options on bonds. Under a CDS contract, the protection buyer pays the protection seller a periodic fee for default (or credit event) protection on a reference name party. The CDS initially referenced specific outstanding bonds issued by the reference name party, enabling the protection buyer to deliver the bonds to the protection seller on the occurrence of the credit event. CDSs are referenced to as specific bonds, even though they are more commonly based on cash settlement (i.e., based on auction pricing of defaulted reference party bonds) and not restricted to physical delivery. In the event of reference party default, the CDS protection seller buys the bonds from the CDS buyer at par and is exposed to the loss given default (LGD) on the bonds. Both a bond buyer and seller of a CDS referenced on the bonds receive a credit-risk-based spread based on PD*LGD. The CDS as a bond put option enables the protection buyer to transfer credit risk on the reference name party to the CDS (protection) seller.

Fixed Income - Trading Activities & Hedging Strategies

Investment banks trade and market-make in government and non-government bonds as primary or secondary dealers in a number of markets and currencies. Proprietary trading was previously a driver of bond trading, but banks are largely stopping speculative trading in the aftermath of the 2007/2008 financial crisis, either voluntarily or due to regulatory prohibition (such as under the U.S. Dodd-Frank Act Volker Rule). Client debt underwriting and debt market making remains a primary bond trading-based client service and revenue activity for banks in the investment-banking sector.

Banks divide bond trading by currency into bond-issuer, credit-risk-rating categories, which each credit risk-rating group being valued on a credit-rating-specific yield curve. These rating-based valuation yield curves are determined as market-based credit spreads over the market government (i.e., riskless) yield curve. The spread levels, which increase inversely with rating levels, are in part determined and vary according to investor risk appetite and other bond market supply versus demand factors. However, the base level of credit spreads is determined (for both bonds and CDSs) by market probability (PD) of default times loss given default (LGD) expectations (i.e., PD * LGD, or expected loss). Rating agencies provide historical data for calculation of real-world probabilities of default measures (used for risk management models such as value-at-risk, or VaR), whereas market makers put their own price on PD * LGD (i.e., risk-neutral measures used for trading book position and derivatives valuations in trading books).

Normal fixed-income bonds have traditionally been valued using simple price-yield to maturity calculations and calculators. As the bond products get more complex with irregular cash flow structures and/or embedded optionality, more modern, swap-based, zero-coupon curves based valuation techniques are applied to trading position valuation and market risk measurement (described in the swaps and interest rate derivatives sections below). Yield given price, Price given Yield (i.e., clean and dirty price), duration, and convexity are common traditional valuation and risk measurement and management tools used during fixed-income trading. Increasingly, banks using modern trading systems use zero-coupon curve and PV01-based, yield curve, time-bucketing, interest-rate exposure technology (also linked to their VaR measurement models) when risk managing fixed-income trading portfolios.

Fixed Income - Markets & Product Liquidity

Banks interact with bond issuers and bond investors in both a product supply and liquidity-providing role, and are underwriting and trading intermediaries in bond markets. Bond underwriting and sales are important revenue-generating activities for investment banks (a core element of bank FICC market strategy). Both asset (i.e., market) and funding (i.e., liability) risks factor into fixed-income trading. The recent financial crisis examples of Bear Stearns, Lehman Brothers, Merrill Lynch, and other active investment banking firms underline these risks. A number of the Basel III regulatory revisions recognize the importance of managing liquidity exposure to fixed-income trading book activities and require stronger risk measurement and trading book capital to cover these risks.

Bond products have obligor/issuer-specific and general market risks. Liquidity risk varies by market, issuer and currency, and general market liquidity and risk appetite levels, which vary during normal economic/business expansion and contraction cycles, and that can create liquidity exposure to liquidity crisis events. Normal procedures for measuring and managing concentration and liquidity risk in fixed-income trading books include:

- Bond specific holding period limits (i.e., forced replacement of illiquid positions);
- Counterparty/obligor limits (i.e., issuer default/migration/event exposure diversification);
- Size of issue percentage holding limits (i.e., concentration risk);
- Valuation adjustments (i.e., size of market bid/offer spread, and revaluation of illiquid positions).

Additionally, fixed-income trading books are being brought into many banks more directly in asset & liability management (ALM) and funds transfer pricing (FTP) policies and procedures of the bank. Prior to the recent financial crisis, trading books, including fixed-income, were commonly treated as overnight funding positions on a roll-over cost basis. Trading books and fixed-income, ongoing, long-position funding requirements are now being treated like many banking book asset pools, with determination of core and volatile funding requirements and application of higher funding and liquidity cost profitability adjustments. In some cases, product middle office/product control departments apply liquidity-based valuation haircuts to fixed-income positions in trading books. In many banks, fixed-income trading is linking more closely to collateral management and securities lending.

CDSs have become important credit risk management instruments since their appearance over a decade ago. Demand for CDS protection has been strong, especially on the part of large banks, which are allowed under the Basel framework to reduce counterparty credit risk capital with CDS protection. Unfortunately, CDS supply in the market declined substantially since the financial crisis. Many of the CDS credit protection sellers were insurance companies, and the failure and bailout of AIG in 2008, which was one of the most active protection sellers, had a continuing negative impact on CDS availability. Some of the CDS indices used by banks for referencing credit spreads and hedging risks have encountered difficulty in finding sufficient liquidity in the underlying CDS market to maintain liquidity in the indices.

Fixed Income - Market Trends & Developments

The changing regulatory capital framework adversely influences the future profitability of bank fixed-income trading, since requirements for more default and migration risk capital and liquidity premia are applied to trading books for specific risk instruments such as bonds and bond derivatives under the Incremental Risk Charge. Under Basel 2.5 requirements, the capital requirement for securitization warehousing in trading books has increased dramatically. Both will impact bank trading book activities directly, and debt markets indirectly.

Fixed Income - Swaps & Swap Products

Swaps, both single currency interest rate swaps (IRS) and cross-currency interest rate swaps (CCIRS) as debt derivatives, take cash flows arranged by banks in the popular back-to-back and parallel loan structures for multi-national clients into off-balance-sheet transaction structures. In the early 1980s, IRS and CCIRS transactions were essentially bond derivatives used to transform existing bond obligations and exchanges of debt cash-flow contracts. The swaps were widely executed on the back of new Eurobond and domestic bond issues.

In the early days, IRS transactions were often economically described as equivalent to floating rate/fixed rate bond exchange contracts, in which the principle exchange would offset, leaving only fixed and floating coupon cash flows on an agreed-on swap notional principal amount. CCIRS transactions were economically described as equivalent to long-dated, foreign forwards on a par-forward cash flow structure.

Bankers quickly applied swaps to engineering cash flows in a multitude of client financial applications beyond capital market arbitrage transactions. One widely used strategy was structuring long-dated, customized, irregular, aircraft lease financing cash flows to suit the needs of both lessors and lessees.

In the 1990s, banks began developing exotic IRS products, frequently used examples of which are LIBOR-in-Arrears (LIA) swaps, Constant Maturity Treasury (CMT), and Constant Maturity Swaps (CMS). These swaps require convexity adjustments during pricing and risk-exposure hedging. CMS and CMT are products often offered by banks to institutional investors. A CMT swap can be structured, for example, as a yield curve steepening trade for a fund manager by offering to exchange floating, semi-annual payments in, say, a 3-year CMT contract, in which the bank resets the coupons paid semi-annually to the fund manager at the then current 5-year treasury bond yield, and the semi-annual rate paid by the fund to the bank is reset at the current 2-year treasury bond market yield. The fund manager then receives a payment every six months, calculated as the positive difference between the 5- and 2-year treasury yields, and benefits if the yield curve steepens, increasing the yield difference.

Interest rate and cross-currency interest rate swaps are products, which are primarily applicable to wholesale financial institutions and corporate clients for interest rate and long-dated currency exposure hedging, and investment banking clients in connection with debt capital markets. Swaps require the execution of ISDA master agreements between counterparties, and banks often require counterparties to enter ISDA CSA (collateral exchange) agreements to minimize credit risk capital requirements further.

Swaps are frequently embedded in other financial transactions, which are provided to smaller clients such as SMEs. An example is an amortizing loan product offered to an SME or retail client by a bank, for which the client can request the bank to convert interest payments during the remaining life of the loan from a fixed to a floating or a floating to fixed rate, at which the bank is willing to do so. The client effectively enters a transaction swap with the bank but views the change in rates as part of the loan agreement and not as a separate transaction. The bank books the rate switch as an IRS on its trading books. If the client seeks to prepay the loan, the embedded swap must be revalued, closed out on the trading book, and added to the client's loan close-out pricing cost.

Trading Activities & Hedging Strategies

Swaps were booked in increasing volumes and from a trading perspective, were not viewed as matched positions in client-engineered transactions but as cash flow positions in trading books. The banks developed zero-coupon yield curves initially as extensions of government bond yield curves with bank credit-based swap spreads. By the late 1980s various items, such as the interbank LIBOR-based deposit/IRS curves, had become more liquid. In addition, swaps trading had become very liquid with narrow interbank swap bid showing offers with spreads of 4 to 5 basis points in many currencies out to 10-year maturities. The interbank swaps market, as an extension of LIBOR curves in major currencies, had become a dominant market benchmark for other rates. The banks developed them in-house or purchased trading systems that were using LIBOR IRS-based zero-coupon curves and discount factors, to value all trading book cash. The economic concept of IRS fixed rates had become the annual annuity (average) cash flow price that a bank would pay (receive) in exchange for LIBOR rates to be set and paid in the future.

Swap markets became economically interpreted as an interest rate forwards market, the basis for pricing and exchanging LIBOR FRAs. At this point, it became possible for banks in liquid IRS currencies to price, trade, and risk manage cash flows independently from the underlying money market and bond cash markets. Swaps have become and remain both interest rate risk hedging instruments (hedging forward interest rate exposures) and powerful cash flow transformation and engineering tools. They are used widely worldwide, frequently by banks themselves as ALM hedges for balance-sheet interest, repricing mismatch exposures.

Swaps trading is an example of the close synergy between OTC and exchange-traded (ET) markets. ET short-term futures are used by swap traders to manage interest rate exposures in their trading books. A prime example is the USD IRS market, whose active use of the Chicago Mercantile Exchange (CME) LIBOR Futures strips has made this the highest globally exchange traded contract. On the CME, it is possible to trade these futures for maturities longer than 20 years. Hedging swaps with futures requires convexity adjustment in designing effective hedging strategies.

Swaps are forwards rather than options-based transactions, and are slightly non-linear with limited convexity (or interest rate gamma), as is the case with fixed-income securities. Irregular transactions such as CMT/CMS swap structures have more non-linearity, requiring pricing and hedging convexity adjustments. This can be explained by the difference in behavior between the swap itself and its hedging portfolio. In the example noted above of a 3-year CMT swap entered into by a bank with a fund management client, the semi-annual payments are based on the yield difference between the then current maturity 5- and 2-year treasury bonds at each reset and payment date. The hedging of this swap requires the bank to buy and sell treasury bonds to replicate cash flows in the swap. The swap, however, has linear payments in yield differences based on yield changes times swap notional principle, whereas the bank's bond hedge portfolio changes in value based on price differences given yield changes (duration and convexity), and are non-linear. The bank must correct its hedge and pricing for convexity in its hedging strategy.

Swaps - Markets & Product Liquidity

Increasing liquid market conditions in IRS transactions prevailed from the onset of the 2007 till the 2009 financial crisis. The failure of Lehman Brothers and other financial firms ended the perception of interbank markets and LIBOR as a risk remote benchmark. Counterparty-specific credit risk spreads were added to financial transactions, and interbank liquidity in swaps and other financial derivatives decreased in many markets.

The liquidity of CCIRS transactions decreased in the early 1990s. Under Basel II's counterparty capital requirements, the re-exchange of principle amounts in currency swaps at the maturity of the transactions created significant exposure at default (EAD) in assessing the future risk of these contracts; and so, required large amounts of capital, which was not the case with IRS transactions during which there is no exchange of (notional) principal amounts.

IRS and CDS have been mandated in the United States by the Commodity Futures Exchange (CFTC) and the Securities Exchange Commission (SEC) under the Dodd-Frank Act, and in the European Union by the ESMA (European Securities and Markets Authority) under EMIR, as subject to central clearing. Central clearing requires initial margin posting and, therefore, adds additional expenses to banks and client counterparties. Other G-20 jurisdictions, which endorsed OTC central clearing, are likely to follow suit, and it is expected that the imposition of initial margin requirements on OTC products such as swaps will slow growth and might negatively affect the popularity and use of these products.

The failure of Lehman Brothers and other financial crisis events changed the market perception of counterparty credit risk in swaps and other bilateral trading book exchange contracts by demonstrating that financial authorities would no longer let it be assumed that they would bail out or rescue financial institutions. The result has been recognition and pricing of counterparty credit risk (probability of default times loss given default or expected loss) into swaps and other derivatives, as both an IAS required fair value accounting adjustment (known as credit value adjustment, or CVA) and for Basel capital calculations. During a bilateral exchange transaction such as an IRS, both counterparties have credit exposure to each other and, therefore, both parties seek to add CVA to the pricing of the transaction. In such a case, parties confront large bids—offer spread differences. Although professional counterparties often agree to net the credit spread difference, so-called bilateral CVA or BCVA, recognition of CVA decreases market liquidity for swaps and other trading book derivatives. CVA is frequently calculated by each party using the CDS spread (or estimate). Due to counterparty credit risk and CVA, LIBOR ceased to be a liquid benchmark for interbank transaction pricing and cash flow discounting as an interbank based risk-free rate. Overnight Swap Index (OIS) rates, based on exchange of collateral, such as EONIA in European markets and the U.S. Fed Funds rate for the U.S. dollar markets, are being used by many banks in many respects in place of U.S. dollar LIBOR and Euro EURIBOR. The OIS curves are not very liquid to maturities beyond one year. Overall, interbank liquidity and the reference rates use by banks for financial market transaction pricing have decreased since before the financial crisis.

Example 4: Applications of Swap Transactions

Swap Application	**Application Example**
Hedging Debt Issuance (cross-currency interest rate swap)	A corporation issues a bond in U.S. dollars and swaps the U.S. dollar liability into Swiss francs at a lower all-in cost than issuing a bond in Swiss francs by entering into a cross-currency interest rate swap, receiving U.S. dollars and paying Swiss francs (principal + interest).
Loan Hedging (fixed/floating interest rate swap)	A corporation borrows from a bank at a floating rate for five years, with a view that rates will decline. One year later, rates have fallen and the corporation converts the floating rate loan obligation into fixed rate for the remaining four years through an interest rate swap.
Creating Asset Swaps (fixed/floating interest rate swap)	A bank finds a fixed-rate bond in the market with an attractive yield to maturity, but the bank lacks long-term, fixed-rate funding. The bank buys the bond and enters an interest rate swap, paying fixed/receiving floating and funding the asset on short-term LIBOR deposits, converting the bond into a synthetic FRN with spread over LIBOR.
Bank ALM Management (fixed floating interest rate swap)	A bank has an interest rate repricing gap on its balance sheet with more rate-sensitive liabilities than rate-sensitive assets, and is concerned rates will rise. The bank enters an interest rate swap, paying fixed and receiving to lessen its balance sheet repricing gap exposure.
Investment Portfolio Management (equity-based total return swap)	A bank has some unlisted equity share investments on its balance sheet and wants to lower its equity portfolio exposure. The bank finds an institutional investor who is willing to take the equity risk, but does not want to take illiquid equity assets on its balance sheet. The bank enters a long-term total return swap with the institutional investor, receiving LIBOR plus a spread in exchange for the equity cash flows paid to the investor and matching the bank's funding of shares remaining on its balance sheet.

Fixed Income - Interest Rate Option Products

Interest Rate Option Products & Valuation Models

In the mid-1980s, during the explosive growth in swaps transactions, banks were initiating option products in other risk classes such as equity and foreign exchange, using applications of the Black-Scholes-Merton (BSM) model and the related Cox-Ross-Rubenstein (CRR) binomial trees valuation approach. Swaps were then being used as floating interest rate protection instruments (i.e., interest rate forwards on LIBOR and other floating interest rate fixing indices).

The banks quickly developed two interest rate option products based on IRS: interest rate caps/floors and interest rate swaptions.

We examine the basics of forward transactions and their relationship to options, showing the relationships fundamental to the Black-Scholes-Merton model. We then examine the valuation of simple interest rate and fixed income derivatives within the BSM framework.

Example 5: European-style Options – Put Call Parity

Options and forward contracts are closely related. A long call option, for example, can be viewed as a conditional long forward contract for the buyer of a call option (or seller of a put option) and short forward contract for the seller of a call option (or buyer of a put option) at a pre-agreed forward striking price, commonly given the formula abbreviation of K (or sometimes X). In the forward contract, one party always pays the other at expiration of the forward contract in bilateral settlement, if the value of the underlying is greater or less than K at expiration of the forward contract. In the case of the option, the option buyer has the right to exercise the option and receive positive value, but no obligation to exercise the option if there is no positive value (i.e., if the contract expires out-of-the-money).

A long (short) forward contract at striking price K and time to expiration t (in years) can be decomposed into a long call/short put (short call/long put) pair of options with the same K and t. We can describe this relationship for the long forward (using the common financial formula notations we used previously and will continue using below) as:

$$(F - K) = \max(F - K, 0) + \max(K - F, 0), \text{ before expiration, and}$$

$$(S - K) = \max(S - K, 0) + \max(K - S, 0), \text{ at expiration}$$

where max (S – K, 0) is the payout on a call option with strike price K expiring in t years, or $C_{kK,t}$, and max(K – S, 0) is the put, or $P_{K,t}$.

Recalling the spot-forward relationship we examined previously:

$$\boldsymbol{F = S * \frac{(1 + r_d t)}{(1 + r_f t)}}$$

for t <= 1 year[20], where r_d and r_f are discrete rates of return (i.e., LIBOR-based rates for time t in FX forward calculations), which are commonly translated in continuous compounding rate

[20] If t > 1 year, the discrete rate version of the forwards calculation equation becomes $F = S * \frac{(1+r_d)^t}{(1+r_f)^t}$.

equivalents, here below r and δ, respectively.[21] We can now shift to continuous rate calculation mode going forward, and the forward calculation formula becomes:

$$\boldsymbol{F = Se^{(rc-\delta c)t}}$$

where the spot (S) and forward (F) price of an asset (such as an FX rate, or a share or commodity prices) differ by the relative returns (r for the base currency and δ on the underlying asset) during holding period t in years (or time to expiration of the forward contract). We can then couple these relationships into one of the most important foundations of derivatives, the Put-Call Parity relationships, expressed as:

$$\boldsymbol{C_{K,t} - P_{K,t} = Se^{-\delta t} - Ke^{-rt}}$$

This formally states the relationship between European-style call options, put options, and forward transactions, where $C_{K,t}$ and $P_{K,t}$ represent the current (premium) value of the call and put option. From this relationship, we can intuitively conclude:

1. European-style options and forwards can be structured/decomposed into a range of combined structures, the simplest being decomposition of a forward contract into a zero-cost put/call combination;
2. There is a strong relationship between options and forwards, with the formula representation of put-call parity relationship above being the foundation of the Black-Scholes-Merton (BSM) model and option valuation, and by extension, the basis of delta hedging options for portfolio/trading book profitability and risk management (described further below).

Other parity relationships exist for non-vanilla options and are frequently used by option traders and structuring professionals. For example, barrier options can be decomposed. A long down-and-out barrier call option is equivalent to a long vanilla call option and short down-and-in barrier call option when the time to expiration, strike prices, and barrier levels are the same for all.

Example 6: Black-Scholes-Merton (BSM) Model Calculations

The BSM model is very similar to the continuous rate version of the financial forwards calculation derived in Example 5 above. The additional terms in the formula, notably the N(.)

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[21] Continuous interest rates (or other rates of return such as dividend yields for equity shares) are commonly used in financial calculations, notably within the BSM model, because the continuous rate equivalents are simple linear functions of time, whereas discrete rates are subject to different compounding and day count convention calculation complexities. Continuous rate r_c

equivalents of discrete rates r_d can be derived using the formula $r_c = m * Ln(\frac{e^{r_d}}{m} - 1)$

where m is the annual compounding frequency of r_d and r_d is converted to an actual/actual day rate equivalent. Ln() and e, also commonly noted as exp(), are logarithmic mathematical functions.

related terms, are necessary because of the conditional nature of the forward contract in the option. Since the option buyer has the right but not the obligation to exchange assets at expiration, the option contract, unlike the forward, is an asymmetrical, bilateral exchange agreement. If the option expires in-the-money, the buyer exercises the option to the disadvantage of the option seller, but if the option expires out-of the-money for the buyer, the option is not exercised, and unlike the case of the forward, the option selling counterparty does not benefit from the move in the underlying asset. The N(.) terms calculate the probability of the option expiring in-the-money, based on the assumption of a normal distribution in the return on a stochastic evolution of the underlying asset price (S) over the life of the option:

$$BSM_{Premium} = \varphi S e^{-\delta t} N(\varphi d_1) - \varphi K e^{-rt} N(\varphi d_2)$$
$$d_1 = \frac{Ln\left(\frac{S}{K}\right) + (r - \delta + \sigma^2/2)t}{\sigma\sqrt{t}}$$
$$d_2 = d_1 - \sigma\sqrt{t}$$

where:
$BSM_{Premium}$ = the option value in base currency per unit of underlying asset in the exchange contract;
S = the underlying asset in base units of currency;
K = the striking price of option at expiration (in base currency per unit of underlying asset);
ϕ = +1 for call options, and -1 for put options;
r = the interest rate of base currency (compounded continuously);
δ = the continuously compounded rate of return on S (counter-currency interest rate for FX option, dividend for equity option, convenience yield and storage costs for commodity options);
t = the time to option expiration in years;
σ = the annualized volatility of S (standard deviation);
N(x) = the cumulative normal distribution of x (which can be calculated in Microsoft Excel using the function =NORMSDIST(.).

The value of the option at expiration is max(ϕS − ϕK, 0). The value of the option before expiration, the option premium value on a market-to-market basis, is the probability of a positive payoff to the buyer discounted to a present value. The BSM model is essentially performing a numerical integration calculation on the expected payout of the option based on the simplifying cumulative normal distribution assumption in expected payoff value at expiration. The normal distribution assumption is flawed regarding financial asset returns, and we examine skew-adjustment calculations for the model in Example 10. In addition to being a flaw, the normal distribution assumption is also the fundamental strength of the BSM model since the normal distribution is defined by two parameters – mean and standard deviation. If a more complex distribution assumption is used (requiring higher-order parameters such as kurtosis and skew), we lose the computational simplicity of the semi-closed form BSM model and require more complex and computationally challenging numerical method solutions.

We use the BSM model to calculate the value of a 1-year European-style ATM put option on Australian dollar/call on U.S. dollar (AUD/USD) foreign exchange rate, assuming:

- Spot rate (S) = USD 0.9246/AUD1;
- Time to expiration (t) = 1.0 years;
- Base currency rate of return (r in USD) = 0.75%;
- Underlying asset rate of return (δ in AUD) = 0.0313%;
- ATM option volatility (σ) = 10.60% AUD / USD rate p.a.

The at-the-money (ATM) strike rate (K) will be set not to equal the ATM forward rate (F) using the formula derived in Example 5:

- K = F = USD 0.9036 / AUD 1;
- Call / Put (ϕ) = (-1).

Now we can value the option in the BSM model to calculate the AUD premium required on a 1-year call option on USD 1 at strike of AUD 0.9036/USD 1 expiring in 1-year. First, calculating $\mathbf{d_1}$:

$$d_1 = \frac{Ln\left(\frac{0.9246}{0.9036}\right)+[(0.0075-0.0313+0.1060^2/2)*1.0]}{(0.1060*\sqrt{1.0})} = 0.4521$$

Second, calculating $\mathbf{d_2}$:

$$d_2 = 0.5473 - 0.1060 * \sqrt{1.0} = -0.06079$$

Next, calculating **N(d_1)** and **N(d_2)** using Microsoft Excel function **=NORMSDIST(d_x)**:

N(d_1) = +1* Excel Application.NORMSDIST(0.45212) = + 0.51803
N(d_2) = +1* Excel Application.NORMSDIST(-0.06079) = + 0.47576

Finally, plugging these results and the other parameters into the $\mathbf{BSM_{premium}}$ formula above to calculate the premium value of the ATM USD/AUD call option expiring in 1-year:

$BSM_{PutlPremium}$ = [- 1*0.9246*exp(- 0.0313*1)*(-1*0.45212)] – [- 1*0.9036 * exp(- 0.0075*1)*(-1*0.47576)] = USD xx / AUD 1

If we decide to buy the AUD call/USD put at the premium price above on an option to sell buy AUD 10,000,000/sell 11,066,844 (at K) in 1-year, against payment of USD 0.0370/AUD 1 = 0.03753*10,000,000 = USD 375,300 up-front. We use this AUD call/USD put option while structuring a dual-currency note (DCN) in Example 8.

Interest rate caps are a European-style strip of individual call options on a strip floating rate, usually 3- or 6-month LIBOR, in a given currency at an agreed on strike price, and floors are put options on the rates at a given strike price. A client exposed, for example, to 6-month LIBOR re-settings in a 3-year floating rate note liability buys a 3-year cap contract with a 4% strike price on the 6-month LIBOR reference setting rate. If the LIBOR rate goes above 4%, the cap buyer's exposure in the floating rate note is protected by the cap contract payoffs.

Interest rate swaptions differ from caps and floor options since they offer protection not on individual floating rate exposures, but on the basket of floating rates. A payer's right swaption gives the buyer a European style right to pay fixed and receive floating in an interest rate swap at an agreed on fixed rate (i.e., strike price), and a receiver's right swaption gives the right to enter a swap from which he/she will receive the fixed rate and pay the floating.

In a swaption contract, parties agree on the option period (i.e., expiration date), tenor of the underlying swap that will be entered into (i.e., physical settlement basis) or used to calculate a cash settlement amount and fixed rate on the underlying swap that can be entered. Using the floating rate note/cap purchase example above as an alternative strategy to purchasing a cap contract, a client might consider buying a six-month payer's swaption on a 2.5-year underlying swap with a fixed rate strike price on the swap of 4%. If the market swap rate goes above 4%, the swaption buyer enters the swap and converts the floating rate note exposure to a 4% fixed interest rate. The cap offers a different risk/reward hedging profile since the cap buyer hedges each floating rate exposure separately and can gain more than the swaption buyer from future decreases in LIBOR.

Caps/floors/swaptions remain used and are flow products, bought and sold as corporate client interest rate protection products, and as hedges by banks for more complex interest rate derivatives.

In the early 1990s, trading desks in more advanced banks started providing clients with more complex derivatives such as limit and flex caps, and Bermudan swaptions. Flex and limit caps allow a cap protection buyer to select a number of caplets (i.e., floorlets) to exercise in a cap contract. For example, a flex cap buyer might agree to choose six floating rates to hedge 10 semi-annual rate fixings in a 5-year cap contract. In a similar limit cap contract, the buyer buys the same contract, with the exercise of the six possible caplets not selectively determined by the limit cap buyer, but subject to automatic exercise when in-the-money (as is the case in a normal cap contract). Flex and limit caps are path-dependent, contingent claim (option) contracts representing a higher degree of complexity than can be handled in the BSM framework.

In the case of Bermudan swaptions, payer's right buyer is not constrained to a single, at-option expiration (i.e., European-style) exercise decision, but is given the right at specific dates during the life of the contract up to and including the option expiration date. Bermudan swaption exercise dates are normally set on underlying swap payment dates. The Bermudan swaption was initially developed in connection with step-up coupon callable notes, which were very popular with wealthy retail investors in the U.S. and other markets. Investment banks arranged the note issues for issuers such as the then AAA rate Fannie Mae, buying a Bermudan Swaption mirroring the step-up callable note terms from Fannie Mae in connection with the debt structure. The notes were sold by banks to retail investors as initially higher-than-normal coupon fixed income investments. By selling the Bermudan swaption to the bank, the note issuer receives an option premium, converting the cost of borrowing through the notes to a deeply attractive sub-LIBOR level in exchange for the bank having the right to exercise the swaption, forcing the issuer to call the note before maturity. The profit to the bank is derived from the pricing of the note sold to the retail investor, who was attracted to the high initial coupons and unable to value the call option embedded in the note and independently monetized by the bank in the Bermudan swaption transaction.

The Bermudan swaption is a fundamental and enduring structure that has been widely applied by banks in a variety of structured product formats over the past twenty years. It is a complex product to price and risk manage and requires ongoing efforts to build increasingly complex interest rate derivatives and yield curve-based valuation models.

An example of a popular retail client targeted structured note embedded application of the Bermudan swaption is a callable LIBOR range accrual note. In this type of structure, the investor buys a note from the bank that pays periodic coupons based on an agreed on LIBOR reference, staying inside of a given floor and ceiling range during the coupon calculation periods over the life of the note. The more days the LIBOR rate is within the range, the higher the coupon. The attraction to the investor is that if LIBOR remains within the range for a substantial portion of the note's life, the investor has potential for a return from the note considerably higher than what the investor would receive from a normal, interest-bearing note for the same maturity. However, the investor sold the right to call the note before maturity to the bank. The bank monetizes the note call option by extracting the call option in a Bermudan swaption, whose value to the bank on its trading books (or sold to a third party) is higher than options embedded in the note and bought by the investor, which can offer the investor an attractive coupon payout. In this case, the investor bought in the note a complex strip of daily European-style, double-barrier, binary, digital options with payout or no payout based on the level of the LIBOR reference on each business day during the life of the note. This type of callable note structure based on Bermudan swaptions is a form of zero cost structure. The investor buys and sells options in the structure to the bank. The bank can choose a range of option types to sell to the client in the structure to make the structure more attractive to investors with different market views or investment preferences. For example, the digital option range payouts can be based on a commodity price (such as gold, oil, or an FX rate such as USD/JPY or EUR/USD) in or out of a given range, or above or below a single level. The structure is path-dependent since digital options disappear if the Bermudan option is exercised calling the note. In this case, the note is principal-protected since the investor receives the initial investment back in the case of either early call by the bank of the note or normal repayment at note maturity. By embedding the options in notes or deposits (i.e., wrapper-based structures), the bank avoids having to take counterparty credit exposure to the client since the client provides the bank with collateral through purchase of the note, or placement of deposit.

Interest Rate Options - Trading Activities & Hedging Strategies

Caps, floors, and swaptions are the standard interest rate derivatives flow products, which are managed in the trading book using the BSM modeling framework, with volatility skew adjustments. IRSs are used as a standard hedging product in these books. As in the case of currency, equity, commodity, and other risk class derivatives, trading book management is based on delta, gamma, and vega exposure measurement, hedging, and market risk limits. Some banks replaced or enhanced the BSM modeling framework for these flow products with more sophisticated models such as the SABR stochastic volatility model (developed by Hagan et al., 2002), an extension of the BSM framework, relaxing the BSM constant volatility assumption constraint. Caps, floors, and swaptions are still quoted by interbank brokers using BSM volatility quotes as the market trade pricing standard. Volatility skew adjustments are then applied to these at-the-money (ATM) reference quotes.

Example 7: Pricing Interest Rate Options with BSM Model

The Black-Scholes-Merton (BSM) is commonly used to price and risk manage simple interest rate options such as call and floor contracts, European-style swaptions, and European-style bond options. We use the Black-76 version of BSM, which uses the forward rather than spot rate or price as the model input. By using the forward (F) in place of the spot (S) in the model, the interest rates, $r_{domestic}$ (r) and $r_{foreign}$ (δ), do not have to be used in the model for calculating the forward price, as:

$$\boldsymbol{F = Se^{(r-\delta)t}}$$

The r remains in the Black-76 version of BSM model for discounting the probability-based expected option pay-off to a net present value premium price. The Black-76 model variation of BSM is:

$$\boldsymbol{Black76 = [\varphi FN(\varphi d_1) - \varphi KN(\varphi d_2)]}$$
$$\boldsymbol{d_1 = \frac{Ln\left(F/_K\right) + (\sigma^2/2)t}{\sigma\sqrt{t}}}$$
$$\boldsymbol{d_2 = d_1 - \sigma\sqrt{t}}$$

The model can then be applied to valuing simple European-style, interest-rate-based option contracts using market-traded implied (or historical) volatilities for each type of instrument:

- Interest rate caps/floors (respectively, call and puts on forward LIBOR rates) where caps and floor are a portfolio of options on the underlying forward rate agreements (FRAs) and each FRA in the contract is priced separately using the individual FRA rate as the F in the formula. Each FRA is priced as:

$$FRA_{1,2} = \frac{r_2 t_2 - r_1 t_1}{t_2 - t_1}$$

where t_1 and t_2 are the beginning and ending future times of the underlying LIBOR FRA (e.g., for a 3x6 month FRA, which is a 3-month LIBOR rate set in three months, t_1 = 025 (years) and t_2 = 0.50 (years), and r_1 and r_2 are the current yield curve rates for t_1 and t_2 converted to continuous rate basis. For caps ϕ = (+) and for floors ϕ = (-).

- Payer's right/Receiver's right swaptions (respectively, calls and puts on a forward interest rate swap), where the forward value of the underling swap to be delivered at the expiration of the option is calculated separately and input as F in the Black-76 model. For payer's right swaptions, ϕ = (+), and for receiver's right swaptions, ϕ = (-).
- Bond option calls and puts can be valued using the Black-76 model, calculating the forward bond price as F. For bond options, there are two approaches to applying the model: a) model the yield volatility of the bond and convert the F and K to bond prices from yields, and b) model the price volatility of the bond and the bond (i.e., clean price) directly. The yield volatility approach is preferable since the price volatility of a bond changes as it approaches maturity (i.e., pull-to-par effect).

The following formulas convert bond prices to yield volatilities, and vice versa:

$$\boldsymbol{\sigma_{yield}} = \frac{\boldsymbol{\sigma_{price}}}{\boldsymbol{yield} * \left(\frac{\boldsymbol{bond\ duration}}{(\mathbf{1} + \boldsymbol{yield})}\right)}$$

$$\boldsymbol{\sigma_{price}} = \boldsymbol{\sigma_{yield}}\left[\frac{\boldsymbol{\sigma_{yield}}}{\boldsymbol{yield} * \frac{\boldsymbol{bond\ duration}}{(\mathbf{1} + \boldsymbol{yield})}}\right]$$

For example, we can calculate the yield volatility of a bond with an implied price volatility of 8%, a duration of 5 years, and a yield to maturity of 6%:

$$\boldsymbol{\sigma_{yield}} = \frac{\mathbf{0.08}}{\mathbf{0.06} * \frac{\mathbf{5}}{(\mathbf{1} + \mathbf{0.06})}} = \mathbf{28.27}\%$$

The Black-76 model can then be applied to valuation of these instruments using the same calculation approach as described in Example 6.

In the early stages of development of more complex derivatives, academicians and derivatives practitioners began developing more complex and realistic modeling techniques. Immediately, a different approach was taken for interest rate based derivatives than for derivatives on underlyings in other risk classes. Whereas FX rates and equity and commodity prices can be treated as an underlying asset price distributed stochastically over time, interest rate products such as bonds and swaps are valued based on interest rate terms structures of yield curves, which are a collection of tenor-based stochastic rates. Under Basel standards, banks are required to calculate interest rate risk exposures using a minimum of 10 tenor-based rates.

The challenge faced when valuing more complex rate derivatives (and modeling the simple rate derivatives more realistically) is modeling the future shape of the yield curve. The evolution of rate derivatives modeling began in the early 1990s with simpler short-rate models (with familiar historical names such as Ho-Lee, Hull-White, Black-Derman-Toy, Black-Karasinsky, etc.). As the underlying derivatives became more complex (notably with the appearance of Bermudan swaptions in the mid-1990s), the models became more complex, and we now have a set of yield curve models with similarities and differences with longer names, but known by their acronyms such as LMM, BGM, HJM, etc. These models are base extensions of the BSM portfolio replication basis for valuing and trading derivatives on a portfolio basis. The models are based on mathematical assumptions on the stochastic movement of the interest rates comprising the yield curve, deriving various outcomes of the yield curve in the future and then valuing all cash-flow-based interest rate derivatives in a given trading portfolio based on the evolution of yield curves within the model.

Since the models are based on portfolio replication, it is important that a model comes up with correct market prices for standard flow products used for hedging optionality risk in the portfolio (trading book)—swaps, caps, floors, and swaptions. An ideal model has a realistic and simple stochastic process (i.e., formula) assumption on the yield curve reference interest rates, an efficient calibration algorithm/process (to fit the model to derive correct market prices on the flow hedging products just mentioned), and a lattice outcome representation, if possible, of recombining nodes structure.

A limited number of large global investment banks have the infrastructure and technology (including systems, validated models, and risk management capabilities) to produce the range of structures available in the market. They are frequently provided to other banks with these structures on a back-to-back basis, enabling a wider population of banks to package and distribute complex structured products, primarily to retail clients in structured investment note form. The underlying option structures have limited market liquidity and are based on complex yield curve models. The models developed and used by large banks fall into several classes, and their implementation, especially calibration of the models to a reference set of hedging instruments, is not standardized. The large derivatives trading firms have each developed a number of interest rate models, with disparate implementations. Having developed a basic model, the bank has several market challenges, leading to developing adaptation, variants, or additional, alternative models. These challenges are:

- The complexity of the underlying derivative(s) in the trading book, in terms of the number of risk factors that need to be modeled for hedging the payouts. For example, if we structure a callable FX range accrual note, we must incorporate FX rates, along with interest rates in the base Bermudan swaption model (and FX/rates correlation as a modeling risk factor), and the hedging portfolio for this derivative includes most probably FX options, and interest rate caps and swaptions.
- The modeling application required to value types of derivatives varies. For example, path-dependent derivatives structures (e.g., average rate based) and early exercisable derivatives (i.e., Bermudan swaptions) are solved using different approaches. Path-

dependent derivatives lend themselves to efficient valuation through Monte Carlo simulation (MCS), and with MCS, we can use more models whose representation is based on either recombining or non-recombining nodes. Early exercisable derivatives require a lattice-based solution with which we test for the optimum exercise time during the life of the derivative. MCS does not solve that for us. Then, during the lattice-based calculation, the process becomes computationally unfeasible if the model's lattice representation structure is non-recombining.

So, in a large bank, which is active in a range of interest rate derivatives trading and structuring, a range of models might be required, and the bank's risk managers and traders will want to have as much consistency as possible in the modeling framework across products and across the range of the bank's trading books.

Interest Rate Options - Markets & Product Liquidity

In a continuing low-interest-rate environment, structured products built on the Bermudan swaption base of callability of the underlying investment vehicle (i.e., the note or deposit wrapper) have continuing retail and institutional demand. The callable structured notes, mentioned above, can be packaged with option-based payouts (i.e., coupons) based on a variety of risk classes: interest rate (e.g., LIBOR range accruals), foreign exchange, commodities, equity, or even more exotic references such as weather indices (e.g., rain fall or temperature degree days). The products then become hybrid derivative structures, which the banks normally base in the Bermudan swaptions trading books of the interest rate derivatives trading desk as the base derivative in the structure. The Bermudan swaption trading books in turn buys or sells FX, equity, or other risk class components of the structure internally with other trading desks/books in their financial products trading unit.

With increasing complexity of its activities and products, a bank will become more reliant on marking trading book to model derived values than direct market prices, and will require more valuation adjustments and reserves.

Interest Rate Options - Market Trends & Developments

In the aftermath of the financial crisis, risk appetite for and availability of complex rate based and hybrid derivatives structures subsided, and a number of the large global investment banks began scaling back activities in these areas. Local and regional banks in regions such as Asia and the Middle East are showing a growing interest in filling the space being opened by the deleveraging western banks in terms of producing structured products. A number of banks active in developing and distributing structured notes see expansion on the trading side as a natural and potentially profitable extension of their current back-to-back derivatives activities. Unfortunately, as discussed in subsequent sections covering FX, equity, and commodity products, the interest rate derivatives market modeling requirements are substantially more complex than those for structures in the other risk classes. Bermudan swaptions particularly represent a technically difficult modeling challenge. No available standard technology can meet the challenge of reliably pricing and risk managing a range of structures, and giving easy and quick access to smaller banks. The large, active banks in the complex interest rate derivatives invested heavily in internally developed yield curve models, systems infrastructure, and front-to-back trading book processes, including market risk management of complex trading books. Banks with limited experience with derivatives product development and trading should consider starting with structures involving FX and other risk classes, and with simple but popular products such as dual currency notes/deposits (DCD). Tackling more complex, yield curve, model-based challenges such as Bermudan swaption might be analogous to a novice swimmer jumping into the deeper end of the swimming pool.

FX & Rates Trading

FX & Rates Trading - Overview

Whereas trading in interest rated derivatives, equity, and commodity products have largely developed at large banks and followed the growth of international capital markets, FX and short-term interest rate product trading are traditional bank treasury activities. The complexity of bank activities and products, especially in the areas of FX derivatives, has substantially expanded in the past three decades, beyond the needs of traditional bank balance sheet management requirements.

Banks are key players in both global and domestic money markets, with interbank deposit trading being a core component of most markets since in many markets, banks manage a large portion of the liquidity and savings of households, corporations, and institutional clients.

FX trading is the largest global OTC trading market with no global regulation. Activity in FX is linked to international payments and settlements systems in which banks play the dominant role, with the result of nearly all electronic FX transactions going through the global banking systems through bank Nostro and Vostro, or internal bank account transfers. Foreign exchange activity is simply cash payment exchanges in various currencies, pre-agreed on market rates. Some countries impose varying forms of FX controls, ranging from rules regarding on or offshore bank account ownership and activities to requirements for FX transaction approvals or complete prohibition on foreign exchange transactions outside the central bank. When foreign exchange controls are in place, non-deliverable forwards (NDFs, a form of contract on differences) often arise, enabling offshore parties to take or exchange risk exposure positions in controlled currencies.

Financial products/derivatives often contain several underlying risk factors—foreign exchange swaps, futures, and forwards are examples. In the following sections, FX forward products are discussed both as a short-term rate, trading book, risk management product and a foreign exchange product in terms of hedging of foreign exchange options trading books and of client application as part of the bankers' FX client solutions hedging and investment structuring tool kit. FX options, in turn, are FX forward-based transactions with conditional or asymmetrical payoff profiles, and the simplest European-style FX options can be seen as FX forwards with an additional risk factor in addition to the FX and underlying interest rates, that being the expected or market-implied volatility of the FX rate over the life of the option contract.

Money Markets & Short-term Interest Rate Trading (STIRT)

STIRT - Products & Markets

A common practice in large banks with active financial market trading books is to separate short- and longer-term interest rate trading activities into separate trading desks. Short-term rates (i.e., maturities out one to sometimes two years) link to money markets and traditional bank treasury deposits and interbank funding of the balance sheet. Longer-term rates link to debt capital markets.

The STIRT desk is assigned responsibility for the short-end of yield curves, which can be defined as out to two years for liquid global currencies (e.g., USD, EUR, GBP, and JPY) and a one-year time horizon for less liquid currencies. In terms of products, the STIRT desk normally has primary product responsibility for the following trading book product-types across a number of currency yield curves:

- Cash money market instruments; primarily LIBOR (interbank) deposits (but not including the bank's funding position, which is a banking book responsibility);
- Rate Forwards; LIBOR FRAs and Bond Repos;
- Rate Hedging; exchange-traded futures (e.g., CME LIBOR Futures);
- FX swap & FX forwards;
- Other LIBOR-based products (such as asset swaps).

STIRT - Trading Activities & Hedging Strategies

FX swaps and FX forwards are hedged with the FX Trading Desk through spot FX transactions, thereby decomposing the instruments into long and short LIBOR deposit positions in the STIRT books. The FX exposure is hedged with the FX Trading Desk, which books exposure under its FX trading.

The STIRT desk manages bank-trading risk in the short end of interest rate yield curves. It aggregates the PV01 risks of all its approved products under its rate time bucket limits for each currency-based LIBOR curve. The desk might manage the wrappers used by the bank in structured products. For example, when the bank structures a dual currency deposit (or note), the STIRT Desk manages the underlying deposit/note in its trading books.

The STIRT desk operates largely as an internal risk management and liquidity management function within a bank's trading book financial markets department. Most of its products have internal hedging applications rather than retail or wholesale client applications. The exception is FX swaps and FX forwards, which have direct wholesale client application and counterparty credit risk positions on the trading books. Structured product wrappers also represent client positions on the trading books.

STIRT Markets & Product Liquidity

Product liquidity is determined largely by currency. Yield curves and money markets/interbank placement markets are very liquid in some currencies (e.g., U.S. dollar, Euro, British pound, and Japanese yen), and in a number of important regional currencies.

Other currency yield curves might be defined as less liquid or illiquid, and trading limits are established according to liquidity. In many smaller and less-developed economies, the domestic money market can often be illiquid, with a limited range or traded instruments available. In these markets, it is often difficult to define a relevant reference yield curve.

STIRT - Market Trends & Developments

Interbank trading and LIBOR curve-based products are less liquid and have been subject to counterparty risk exposure spreads since 2008 (i.e., the Lehman Brothers failure). Banks are increasingly using OIS curves. The OIS curves are used in collateral-backed transactions such as derivatives covered under ISDA CSA agreements, and in repo market transactions.

Until the onset of the financial crisis in 2007, banks were becoming more active in trading a wider range of local currency products since availability and liquidity of local-currency, money-market-denominated instruments is a key factor in pricing and hedging derivative products in those currencies (e.g., FX swaps and forwards). In many smaller, local, currency markets, FX swaps and forwards might be more liquid than in money markets, and implied local interest rates in traded FX products are often used then to define the short end of the yield curve. Regional currency trading centers have developed in Singapore, Hong Kong, Tokyo, New York (for Latin American currencies), Dubai, Johannesburg, London (for all of Eastern and Western Europe), and elsewhere.

FX Spot & Forward Trading Products

FX spot trading is an OTC market. In the late 1970s and early 1980s, FX futures and options were available as exchange-traded products. Most FX activity is now in the bank-based OTC market. Banks offer complete flexibility to wholesale and investment banking clients not provided in standardized exchange contracts, and a range of FX products and services to retail clients.

The global FX markets are generally divided into several categories: majors, crosses, and exotics. The majors are those currencies most actively traded against the U.S. dollar, including EUR/USD, GBP/USD, and USD/JPY. Crosses can be very large currencies traded in pairs, not including the U.S. dollar such as EUR/JPY or MYR/IDR, and exotics that are less active or developing market currencies traded against the USD such as USD/CNY and USD/BRL. Crosses in less liquid currency pairs can be created by buying and selling currencies against the U.S. dollar. Banks normally trade currencies and currency derivatives (i.e., forwards and options) in separate currency pair trading books. Trading currencies in groups is difficult since correlations between the currencies are volatile and difficult to manage. Banks often divide the separate currency pair trading books into groups for risk management purposes, and their internal limits structures might allow for taking risk between books within the groups. Ten currencies can theoretically be divided into 45 currency pairs. Large banks often trade in 50 or more currencies, with positions against both the U.S. dollar and some crosses. The number of currencies a bank's FX trading desk trades can be large, but a small number of currency pairs are traded very actively, and the majority is traded much less actively. FX spot/forwards/swaps trading is increasingly conducted electronically through systems in major banks.

Many banks trade gold as a currency (XAU) on the foreign exchange desk, not as a commodity though storage costs are used to determine forward prices and option values.

Discussed earlier, in terms of trading and market risk management, OTC traded FX forwards and swaps are a hybrid FX interest rates product that can easily be replicated or synthetically created by a bank on its balance sheet as a long position in a deposit of one currency in the pair, and a corresponding short deposit position to maturity of the forward to the second currency. Hence, a common practice is to treat FX forward products in the STIRT trading areas as interest rate risk positions and create the long and short synthetic deposit positions in the STIRT trading desk by doing an internal spot FX trade with the FX Spot trading desk. FX options products tend to be managed at the FX trading desk in larger banks, but as volatility (i.e., vega) limit-based trading book products, or in a separate trading group for currency derivatives or all derivatives, depending on the bank's commercial bank/treasury versus investment banking/financial markets organizational and business model.

In terms of marketing and distribution, banks sell FX spot, forward, and options products as FX services, or solutions, to clients. Liquid flow products are services, and more complex FX derivatives along with flow products are structured by the bank sales force into client FX exposure hedging forwards (or synthetic forwards) and into client structured products (FX-linked investment products). FX (or FX-linked) products are applicable (and sold) to nearly all client groups on both the asset and liability side of the bank's balance sheet. The ALM/ALCO of the bank itself is a client of the FX trading books in hedging and managing the bank balance sheet (FX swaps, for example, are a primary bank balance sheet funding related product). Again, the client driver in FX transactions, as with other bank trading book products, can be hedging, investment, arbitrage, or speculation. In the past two decades, suitability, compliance, and KYC (know your client) have become bank selling responsibilities since the *caveat emptor* approach, which was prevalent until the early 1990s is no longer accepted by markets practitioners and regulators.

FX Options, Synthetic Forwards & Structured Products

FX Options - Products & Valuation Models

In the mid 1980s, when derivatives markets were beginning to emerge, banks started trading OTC currency options. Larger western investment and universal banks were quick to recognize that the BSM model framework (adapted for FX options in the Garman-Kohhagen model) could be easily applied to foreign exchange to price and risk manage European-style calls and puts on actively traded currency pairs. The banks were already very active in foreign exchange and had large numbers of clients with FX exposures. Currencies markets had grown increasingly volatile since the collapse of the Bretton Woods Agreement a decade earlier, so currency options developed quickly after the banks started down the path of derivatives trading with swaps in 1980.

Vanilla currency options (i.e., European-style calls and puts) in the major currencies were widely traded and made available from banks by 1990, and banks were using the BSM model to price mark-to-market market risk exposures of FX options trading books. A software vendor provided an FX options calculator product, FENICS, which became the market standard for pricing the initial delta hedge on options, and a number of interbank brokerage firms began collecting and posting FX volatility curves from major bank trading desks. Large banks started delta hedging trading books from the outset since the Cox-Ross-Rubenstein binomial tree (CRR) numerical model of the BSM process concurrently explained the hedging basis of the BSM process and provided a pricing solution for American-style and other early exercise derivatives. Mark Rubenstein and other academicians began publishing BSM-based solutions for the first generation exotic options (i.e., average rate, compound, lookback, and digital and barrier options). Trading desk practitioners began applying volatility skew adjustments.

By the early 1990s, FX derivatives were well entrenched in global markets and very liquid in major currency pairs. A broad set of first-generation exotic options were becoming available and used to structure client FX exposure hedges. The barrier and digital options have been used extensively and have become flow products, largely in the case of barriers because they can be used to lower the cost of option structures. A European-style call or put with a knock-out barrier condition will always be worth less than the call or put without a barrier. Banks quickly started applying exotic options to asset-based applications to provide investment products, and structured products quickly grew and were spread to equity and commodity applications, providing clients with higher yield, alternative-risk investments.

During the early 1990s, currency options trading desks also started reaching limitations in using the BSM model framework for the more exotic options that were being developed and becoming popular in the markets for structures marketed to clients. New exotics were becoming more complex in terms of risk factors, path-dependency, and (sometimes early) exercise conditions. Currency options traders followed the trend of their interest rate derivatives colleagues in seeking more robust modeling solutions for pricing, hedging, and risk managing trading books with more complex, exotic options.

Whereas the problem with using the BSM model for all but the simplest interest rate options is the inability to model the yield curve itself, the problem encountered by the currency option traders (and by commodity and equity options trading desks) lay in the BSM model normal distribution and constant volatility assumptions. The development path of interest rate derivatives and currency/equity/commodity derivatives at this point went into radically different types of modeling solutions. In the case of rates derivatives, yield curve models were developed, and in the case of options on the other underlying risk classes, development went in two directions or approaches: local and stochastic volatility models. In 1993, several local and stochastic model papers were published, and during the remainder of the decade, large derivatives trading banks independently developed a number of proprietary implementations of these types of models.

As with the case of rate options, the BSM framework with volatility skew adjustment is still widely used for simpler flow products—both for pricing and trading book hedging and market risk measurement. The BSM framework provides a standardized valuation framework for active interbank trading of flow products since the interbank trades are based on volatility prices, which align with the BSM mode. More complex derivatives and trading books are managed using the more advanced but not standardized modeling approaches. Traders have been forced to compromise convenience and simplicity for more robust but complex pricing, hedging, and risk management.

In the past decade, more complex products such as Target Redemption Forwards (TARFs) and their variants stimulated further model development in the area of stochastic local volatility models, which attempt to mix the calculation processes and advantages of local and stochastic volatility models into a single framework. There is no consensus on standardizing such models, and the situation is similar to that described earlier regarding rate derivatives and yield curve models: a small number of banks with individually developed models and model calibration methods for complex derivatives, and a large group of banks distributing complex products using back-to-back hedging with the larger banks. Several of the larger trading system software vendors have been building more complex models into their systems, but for banks lacking their own complex model development and validation abilities, these represent potentially black-box solutions if used for hedging and risk managing trading books.

FX Options - Markets & Product Liquidity

High volatilities in FX (currency pair specific, but generally in the 7% to 15% range for actively traded floating rate currencies) promote both corporate client hedging protection and institutional and retail FX-linked investment interest. Banks structure both assets and liabilities based transactions for clients.

Corporate and SME clients' active, cross-border activities such as importing and exporting must decide whether to hedge their foreign exchange exposures. The simplest hedge is transacting a foreign exchange forward with a bank. Options, however, enable the bank to offer a client a range of hedging possibilities through synthetic forward transactions in which the client can buy exposure protection at a cheaper price in exchange for selling some of the potential upside gain in the exposed position. A simple example is the collar or risk reversal, a structure in which a Japanese importing client with Japanese yen sales revenues and U.S. dollar expenses seeks to hedge U.S. dollar exposure. The client might alternatively buy U.S. dollar forwards from the bank against the Japanese yen, buy USD/JPY call options, or enter a risk reversal structure with the bank in which he/she buys out-of-the-money USD/JPY calls and sells out-of-the-money USD/JPY puts.

The strike prices on the long and short options positions determine whether and how much option premium exchanges there are between the bank and client. Often, these structured hedge transactions are arranged on a zero-cost basis (i.e., the value of purchased and sold options are equal and offsetting). A risk reversal is among the simplest synthetic forward structures. Using a wider range of FX options during structuring such as barrier options (i.e., single, double, normal, or reverse barrier,, window-style barriers, etc.) and/or binary/digital options (i.e., digital calls or puts, one touch, no touch, double one touch, double no touch, etc.), the bank can offer an unlimited range of risk-reward profiles to a client and assist the client in selecting the most appropriate structure for the exposure, risk-reward appetite, and hedging cost budget. Banks can offer a range of alternatives to protection-buying clients. As publicized, companies such as the McDonald's Corporation use products such as average rate and currency basket options frequently during FX exposure management. The most active hedging (i.e., protection-buying) clients are corporate treasurers, and they are net buyers of options from banks.

The same techniques are used by banks when structuring FX-linked, asset–based structured product investment solutions for retail, high net worth private banking, and institutional investor clients. In asset-based, investment structures, the bank clients can be net buyers or net sellers of options to the bank, depending on whether the structure (most often packaged, and sold to the client in a wrapper such as a deposit or note) is principal protected or not principal protected.

In a principal protected note structure, the investor is a net buyer of options. The interest coupon that would be paid on a note (i.e., market-based time value of money) is used to buy options, and the options offer the investor a return (i.e., coupon) if they expire in-the-money, and no return on the note if they expire out-of-the-money. In either case, the investor receives back the amount of money at note maturity that he/she invested. The risk on the structure is limited to possible loss of return on the investment, but the return can be higher than interest offered in the market.

In a non-principal-protected structure, an investor is usually selling options to a note arranger. The investor's return on the note is attractive (in comparison to market interest rates) if the embedded options in the note expire while out-of-the-money. If the options expire in-the-money, the investor has a riskier position than the buyer of a principal-protected note. A very popular FX-linked wealth client structured product is the dual-currency deposit (DCD) or note (DCN). DCD is an example of a non-principal-protected structure. A common structure in the Asian markets is an AUD/USD DCD structure in which the bank offers a higher-than-market interest rate to the investor on a deposit for fixed maturity in exchange for the bank be able to repay the investor the currency market equivalent value of the U.S. dollar deposit in Australian dollars at maturity of the deposit. The investor (i.e., depositor) sells to the bank a call on the U.S. dollar/put on the Australian dollar embedded in the structure of the deposit contract. AUD/USD DCDs are a common product in Asia in part because of regional familiarity with and use of the underlying currencies, but mostly because of the high value of the embedded AUD/USD FX option. Volatilities of AUD/USD options have been well over 10%, on average, whereas, in most other active currency pairs, volatilities have been averaging well below 10%.

Example 8: FX Structured Note – Dual-currency Note

This example illustrates using derivatives to structure an alternative investment for a private wealth client of a bank who wishes to invest USD 10,000,000 for 1 year. We use the market rates and calculations from Example 6 to structure a dual-currency note alternative investment proposal for the client as an investment opportunity. DCNs have been one of the most actively chosen structured investment products used by clients for over two decades in many markets. Since this note structure is not a principal-protected investment (discussed in Example 11), the bank must determine whether the product is suitable for the client during its compliance process.

From the market rates in Example 6, the bank, at best, is willing to offer the client the USD interbank interest rate 0.75% per annum on a USD 10,000,000 1-year deposit. The client is disappointed at the low rate and asks the bank if there are alternative 1-year USD investment instruments available. The bank responds by suggesting a 1-year USD/AUD dual-currency note structure in which the bank can pay a higher USD interest rate on the USD 10,000,000 investment for 1 year, in exchange for the client giving the bank the option on expiration of the note to repay the investor in AUD at the current 1-year forward rate of USD 0.9036/AUD 1, or principal repayment of AUD 11,066.844 instead of USD 10,000,000 at maturity.

The bank structures the note with an embedded USD call option on the investor principal of USD 10, 000,000/AUD 11,066,844 put option, sold by the investor to the note-issuing bank. Based on the USD 0.03753/AUD 1 premium value we calculated on this option in Example 6, the bank is willing to pay, at most, a USD 415,325 frontend premium for the option, or 4.15% of the USD 10,000,000 note principal, which together with the USD 0.75% interest rate on the USD principal enables the bank to provide a return to the investor of up to 4.90% in USD interest, over six times the interest payable on the USD deposit without the embedded option.

In exchange for the substantially enhanced USD interest rate alternative return, the investing client takes risk on the USD 10 million note principal redemption value. If the AUD/USD rate is above AUD 0.9036 per USD 1, the bank exercises the embedded USD call option, keeping the

USD note principal amount invested by the client and redeeming the note by payment of AUD 11,066,473 principal plus the amount of USD interest agreed on in the note structure. The client then has an AUD principal amount worth less in the market than his/her original USD 10 million principal amount. The note therefore offers no principal protection, but the opportunity for the client is to earn a substantial return for the USD investment if he/she is confident that the value of the USD in one year will be at USD 0.9036 or less against the Australian dollar.

The bank can of course increase the strike price in the AUD redemption option in the note structure to the current spot rate, but any increase, while giving the investor more protection, decreases the enhanced USD interest rate yield in the note. The bank tries to structure notes such as this DCN investment to buy options in high-volatility assets such as the AUD/USD, with a 1-year implied volatility of 10.60% as the premium value of the option. The yield enhancement of passing on the premium to the investing client makes the return look more attractive. In Example 10, we examine FX option structures using U.S. dollars/Singapore dollars as the underlying currency pair, and we see that the USD/SGD with an implied 1-year market volatility (of 6.60% in the example, or about half the AUD/USD rate under normal market conditions) offers less yield enhancement structuring potential. This accounts for DCNs often being structured using AUD/USD options, almost never using USD/SGD options.

As noted in the FX options product section above, with development of more complex derivatives and models over the past two decades, came the popularity of a number of more complicated and often path-dependent structured products. In the FX risk class, products such as TARFs (target redemptions forwards), KODAs (knock-out discount accumulators), Fade In and Fade Out options, and time range accrual structures are used in structures. These require more complex models such as the stochastic local volatility models mentioned above. Since the number of banks with advanced models is limited, many of the structures are arranged by banks on a back-to-back basis with larger global banks.

FX Options - Trading Activities & Hedging Strategies

FX options with maturities of up to two years are generally traded in FX options books and hedged with sport and forward FX delta hedges. The interest rate in derivatives, including those FX-based, grows with maturity, and the banking industry trend has been to place trading of most long-dated (i.e., maturity beyond 2-years) instruments into interest rate derivatives trading using yield curve models. Currency options trading books are portfolios with FX, interest rate, and equity exposure. Exposures in each trading book portfolio to each of these risk factors are measured in terms of the Greeks' sensitivities of the portfolio to changes in the market risk factors.

The Greeks are measured and managed on a portfolio basis. The portfolio risk is the sum of the risks in each of the transactions in the portfolio. Greeks used for portfolio risk management include:

- FX rate: **delta and gamma**: the sensitivities of position/portfolio value for changes in the FX rate. The positions and portfolio are sensitive to FX rate changes, and therefore delta hedging is an important component of trading book risk management. Normally, new options positions booked into the portfolio, long or short, are delta-hedged at the time of transaction. The delta is not a linear exposure function for options, and therefore the gamma or second-order change in delta, given a change in FX rates, is measured.
- Option volatility; options trading books are marked-to-market at the close of each trading day, and the market trading level of implied volatility is used for valuation. Option traders are making markets in volatility, and there are bid-offer levels for options in the market in terms of both volatility and value (premium). From the traded price, we can determine the implied volatility, and inversely, from the traded (implied) volatility, we can calculate the option/portfolio value. The sensitivity to changes in market-traded, implied volatility is known as **vega**. In the market, traders use different volatilities for different maturities, or option expiration horizons; and therefore volatility—like interest rates—has a term structure. The vega exposure in an options trading portfolio (such as interest rate exposure) should be measured in terms of time buckets. An option portfolio with two options positions—a long (bought) 6-month option with a positive vega and a short (sold) 3-month option with equal and offsetting negative vega—is not hedged against a change in volatilities since the vega exposure is on different time buckets on the market implied volatility curve.

Example 9: Pricing Interest Rate Options with BSM Model

The Black-Scholes-Merton (BSM) is commonly used to price and risk manage simple interest rate options such as call and floor contracts, European-style swaptions, and European-style bond options. We use the Black-76 version of BSM, which uses the forward rather than spot rate or price as the model input. By using the forward (F) in place of the spot (S), the interest rates, $r_{domestic}$ (r) and $r_{foreign}$ (δ), do not have to be used in the model for calculating the forward price:

$$F = Se^{(r-\delta)t}$$

The r remains in the Black-76 version of BSM model for discounting the probability-based expected option pay-off to a net present value premium price. The Black-76 model variation of BSM is:

$$Black76 = [\varphi FN(\varphi d_1) - \varphi KN(\varphi d_2)]$$

$$d_1 = \frac{Ln\left({}^{F}/_{K}\right) + \left(\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

The model can then be applied to valuing simple European-style interest rate based option contracts using market traded, implied (or historical) volatilities for each type of instrument:

- Interest rate caps/floors, respectively—calls and puts on forward LIBOR rates—where the caps and floors are a portfolio of options on the underlying forward rate agreements (FRAs), and where each FRA in the contract is priced separately using the individual FRA rate as the F in the formula. Each FRA is priced as:

$$FRA_{1,2} = \frac{r_2 t_2 - r_1 t_1}{t_2 - t_1}$$

  where t_1 and t_2 are the beginning and ending future times of the underlying LIBOR FRA (e.g., for a 3x6 month FRA, which is a three month LIBOR rate set in three months, $t_1 =$ 025 (years) and $t_2 = 0.50$ (years), and r_1 and r_2 are the current yield curve rates for t_1 and t_2, when converted to continuous rate basis. For caps, $\phi = (+)$, and for floors, $\phi = (-)$;
- Payer's right/Receiver's right swaptions, respectively—calls and puts on a forward interest rate swap—where the forward value of the underling swap to be delivered at the expiration of the option is calculated separately and is plugged in as the F in the Black-76 model. For payer's right swaptions, $\phi = (+)$, and for receiver's right swaptions, $\phi = (-)$;
- Bond option calls and puts can be valued using the Black-76 model, by calculating the forward bond price as F. For bond options, there are two approaches for applying the model: a) model the yield volatility of the bond and convert the F and K to bond prices from yields, or b) model the price volatility of the bond and the bond (i.e., clean price) directly. The yield volatility approach is preferable, since the price volatility of a bond changes as it approaches maturity (i.e., the pull-to-par effect).

The following formulas convert bond prices to yield volatilities, and vice versa:

$$\boldsymbol{\sigma_{yield}} = \frac{\boldsymbol{\sigma_{price}}}{\boldsymbol{yield} * (\frac{\boldsymbol{bond\ duration}}{\boldsymbol{(1 + yield)}})}$$

$$\boldsymbol{\sigma_{price}} = \boldsymbol{\sigma_{yield}}[\frac{\boldsymbol{\sigma_{yield}}}{\boldsymbol{yield} * \frac{\boldsymbol{bond\ duration}}{\boldsymbol{(1 + yield)}}}]$$

For example, we can calculate the yield volatility of a bond with an implied price volatility of 8%, for a duration of 5 years, and a yield to maturity of 6% as:

$$\boldsymbol{\sigma_{yield}} = \frac{\boldsymbol{0.08}}{\boldsymbol{0.06} * \frac{\boldsymbol{5}}{\boldsymbol{(1 + 0.06)}}} = \boldsymbol{28.27\%}$$

The Black-76 model can then be applied to valuation of these instruments using the same calculation approach as described in Example 6 on BSM model calculations.

Vega risk can be hedged only with options—spot and forward positions in the underlying do not have volatility exposure (i.e., no vega). Option volatility (measured with gamma and vega) is the risk factor, in which the option desk is trading and market making. Option convexity is distributed between gamma and vega. The longer the option tenor, the more that the convexity of the option will lie in its time value (i.e., vega). When the option's remaining life becomes shorter, its vega sensitivity decreases and gamma might become more important. An option that is at-the-money and near expiration has very large gamma exposure, and is very sensitive to changes in the underlying spot rate. Market volatilities for options on FX and other risk classes are unstable, and vary stochastically, which practitioners recognize by expanding their models beyond the BSM framework.

Example 10: Volatility Skew Adjustments in the BSM Model

As discussed earlier, in the late 1980s, practitioners recognized that the BSM model is flawed in its simplifying assumption that financial assets (e.g., FX rates, share and commodity prices, etc.) have constant volatility and can be modeled reliably assuming a normal distribution of returns during stochastic pricing. Financial markets are not so well behaved, and the BSM model was arbitraged early on by practitioners buying out-of-the-money options (primarily in FX) and selling at-the-money options, then delta hedging the options and exploiting the real-world financial conditions of leptokurtic (fat-tail) distributions of asset returns.

Practitioners have been very reluctant to give up the BSM option valuation framework in favor of more complex, non-closed solution models. They retain the BSM approach, with its embedded normal distribution assumption, for simpler flow products such as European-style vanilla, barrier, digital, and other options because of its simplicity as a standard model and ease in calculation. They choose instead to adjust the volatility inputs to the BSM model, with higher volatilities for out-of-the-money options.

For example, in the FX options market, bank dealers and interbank brokers quote options in terms of at-the-money (ATM), based on implied volatilities for input into the BSM model by time to maturity (i.e., as a term structure based on expiration maturity of the option). They also give the implied volatilities being quoted by active bank dealers in three iconic, out-of-the-money option combination put and call structures for the same maturities. The three quoted structures are:

- 50-delta (i.e., at-the money) volatility for options (with ATM puts and calls trading at the same BSM based implied volatility);
- 25-delta (i.e., out-of the money) strangles (or butterflies) as an implied volatility adjustment (i.e., increase) in cost of buying the combination of a 25-delta put and a 25-delta call over buying the combination of two 50-delta (ATM) puts or calls;
- 25-delta risk reversal as the implied volatility difference between buying a 25-delta call and selling a 25-delta put.

In FX options, for example, all three structures are quoted by market makers, and shown on broker screens, for a range of maturities, as in the example of quotations below, quoted in U.S. dollar/Singapore dollar FX option BSM volatilities (i.e., annualized normal distribution, standard deviations):

Option Maturity	**ATM Volatility**	**25-delta Strangles**	**25-delta Risk Reversals (USD call – USD put)**
1 day	6.75%	+0.100%	+0.375%
1 week	6.75%	+1.000%	+0.375%
2 weeks	6.05%	+0.115%	+0.475%
1 month	5.60%	+0.155%	+0.725%
2 months	5.80%	+0.190%	+0.950%
3 months	6.15%	+0.245%	+1.325%
6 months	6.25%	+0.365%	+1.940%
9 months	6.30%	+0.415%	+2.200%
1 year	6.60%	+0.465%	+0.245%
2 years	7.40%	+0.470%	+3.115%
4 years	8.95%	+0.470%	+3.115%
5 years	9.20%	+0.470%	+3.115%

The standardized market prices in ATM and OTM options above enable FX options traders to calculate a volatility term structure skew surface by individual currency pairs. Taking the chart above and looking at the 3-month maturity quotes, for example, the interpretation is:

As a bank trader, I can arrange to buy an ATM put or call at 6.15% volatility, a 25-delta strangle at 6.395% volatility (i.e., buying the put and call at an implied average volatility for each of 0.245% higher than buying two ATM options), and I can buy the risk reversal (defined in the chart as the USD call minus the USD put) at an implied volatility of buying the call and selling the put, and paying +1.325% difference in volatility (or 1.3 times the option vega). The strangle implied volatility pricing adjustment effectively indicates the kurtosis (fat-tailedness) adjustment to the normal distribution being charged in the market for OTM options by maturity, and the risk reversal volatility price adjustment indicates the skew adjustment, if any. The risk reversals in the example above indicate that at this time, there was more market demand for USD/SGD calls than puts, or market sentiment that the USD would appreciate against the SGD, and dealers were pricing that bias into the volatility skew pricing term structure.

We can use these market quotations to calculate volatility curve reference points for each maturity as three defined volatility prices:

- 25-delta Put volatility price, as $\sigma_{25\delta P}$;
- 50-delta Call/Put volatility price, as σ_{ATM}.

We calculate the implied volatility reference prices for each maturity by using the chart quotes to set up and solve the following set of two simultaneous equations in two unknowns:

$$\boldsymbol{\sigma_{25\delta C} = \sigma_{ATM} + \sigma_{STR} + \frac{1}{2}\sigma_{RR}}$$

$$\boldsymbol{\sigma_{25\delta P} = \sigma_{ATM} + \sigma_{STR} + \frac{1}{2}\sigma_{RR}}$$

Solving these equations for the 3-month market quotes above gives a 3-month volatility skew curve of:

- $\sigma_{25\delta P}$ = 5.7325%;
- σ_{ATM} = 6.1500%;
- $\sigma_{25\delta C}$ = 7.0575%.

Solving the three-point, reference-adjusted volatilities for each maturity provides the basis for calculating a volatility surface for options on the currency pair, and an interpolation/extrapolation methodology is required for calculating the surface for options with different deltas and maturities. This method provides a simple basis for estimating the skew surface though other approaches such as the Vanna-Volga method, described in advanced options papers, are also used by professionals to deal with the interpolation/extrapolation issue and for calculating skews on more complex flow products such as barrier and binary options. For more complex derivatives, market-making professions move out of BSM to more complex local, stochastic, local stochastic, and yield curve models that abandon the BSM normal distribution assumption.

Markets are aware that FX rates and other financial risk factors/prices are not well-behaved and normally distributed, but prone to larger moves than normal distributions predict. Practitioners use traded prices of out-of-the-money option structures (i.e., delta risk reversals and strangles) to make strike-based volatility skew adjustments for vanilla options in the BSM framework, and adjust similarly for exotic options in BSM. For exotic options, a standard approach is to make Vanna-Volga-based adjustments to the pricing of the options. Vanna and Volga are second-order risk sensitivity measures; respectively, the sensitivity of the option vega to change in delta of the option and sensitivity of the vega to change in the volatility level. Local volatility and stochastic volatility models are designed to value options without the simplifying assumptions of the BSM model. The more complex models model these factors implicitly in their structure, without the need for external adjustments:

- Interest rate exposure: valuation of FX options is based on the FX forward rate, which in turn is based on the interest rate differential between the two currencies. Option trading books are therefore sensitive to changes in the term structure of interest rates. Interest rate sensitivity to the individual interest rates is termed rho and phi for domestic and foreign rates, respectively. For longer-dated options and forwards, rho and phi exposures become significant factors, explaining why many banks trade longer-dated FX products (including CCIRS, mentioned previously) in their interest rate derivatives (IRD) trading group using yield curve models to manage rate risk. The IRD desk hedges the option vega risk with the FX options desk;

- Two additional factors are very important for both trading and structuring options in all risk classes, including FX:
  - Put-Call Parity: options are components of forwards. A long (short) forward position for a given maturity can be decomposed into a long (short) call and short (long) put with the same maturity and string price. Synthetic forwards, structured products, and other derivatives-based constructions are based on this underlying relationship. If the put-call relationship is violated in the pricing of a structure, an arbitrage opportunity will emerge;
  - Gamma-Theta relationship: An option buyer (seller) pays (receives) premiums for the convexity of the option. The premium value of the option declines over time as the option approaches expiration (and the time decay of the option premium is called theta). The buyer (seller) of an option has positive (negative) gamma and negative (positive) theta. Positive (negative) gamma puts the trader in the position of gaining (losing) money by delta hedging. Delta hedging with positive gamma puts the trader in the position of buying the underlying at declining prices and selling at higher prices; negative gamma does the opposite. Traders make bid-offers with implied volatility below and above the level at which they break even in the gamma-theta relationship. Positive gamma/negative theta (i.e., net long options) is a less risky position over time than short options with negative gamma/positive theta. The theta is quantifiable, but potential loss from a large market move and negative gamma is undefined, and the loss depends on the magnitude of the market move in the underlying.

Barrier and digital options have become flow products in FX trading. These options have discontinuities in their payout and hedging profiles and are sometimes difficult to manage under volatile market conditions. Traders tend to price these flow exotics conservatively, and market risk management generally establishes parity limits for option trading books. Parity limits measure the concentration risk created in trading books from barrier and digital options in terms of barrier levels and expiration times. The goal of parity option limits is to require traders to keep the portfolios diversified and less vulnerable to exposure from jump risk in value during volatile markets.

FX derivatives are well established and simple, and there is deep liquidity in most products under normal market conditions for derivatives in the major currency pairs and crosses on the majors. Under stressed market conditions (such as the period following the collapse of Lehman Brothers in 2008), markets can immediately become illiquid in all but the flow products. Complex structures such as TARFs have expanded rapidly into less liquid regional and local currency markets during the past two decades. These products are readily available in normal markets, but can quickly become illiquid in changing market conditions. TARFs denominated in USD/CNY purchased by Asian companies to sell USD against the CNY are an interesting example. From 2011 to 2013, the CNY appreciated at a steady pace against the USD, and many bank clients entered TARF structures, enabling them to sell USD at attractive rates. Anticipating continued appreciation of the Chinese Yuan, many TARFs were structured on zero cost, attractive but with capped upside potential and unlimited downside risk in the unlikely event of CNY depreciation against the USD. Unfortunately, the unlikely event became a possibility in February 2014, when the Chinese authorities intervened to reverse the steady upward trend of the Yuan, and the market immediately became less liquid with the estimated amounts of USD/CNY TARFs being very large.

FX Options - Market Trends & Developments

FX derivatives markets dominated by banks will likely be impacted by the higher Basel trading book capital requirements being put in place in the aftermath of the financial crisis. Increased recognition of counterparty credit risk exposure and risk valuation adjustments (notably credit value adjustment, or CVA, and Funding Value Adjustment, or FVA) are influencing the pricing of all derivative products, including FX. The G-20 intention to move standardized derivatives into OTC central clearing and be subject to initial margin costs will also have an impact at some future point on FX trading products.

Equity Market Trading

Equity Trading Markets - Overview

Equity product trading has its roots in investment banking and evolved separately in many banks as part of an equity capital market group. Equity products are also traded both OTC and on exchanges by smaller regional and local securities companies, and by private banks or private banking (e.g., wealth management) departments or subsidiaries in large banks. Bank are often clearing members of equity exchanges, providing clients with exchange-traded products. Equity product trading desks at banks interact closely with exchanges—either directly or through other avenues. Exchange-traded products provide hedging and liquidity support to OTC equity trading books. The underlying markets are exchange traded for listed shares and share index instruments. In equity markets, there is a much closer overall relationship between exchanges-traded and OTC activities and the exchanges because shares are underlyings in equity derivatives and are largely exchange traded (i.e., listed shares and exchange based equity indices).

Only a small percentage of all banks are active in equities since the activity is investment and private banking centered. Under the Basel capital framework, equity product-based counterparty credit exposures are taxed heavily in terms of risk capital, with equity positions attracting a 200% risk weighting. Banks active in equity trading products—cash equities exchanges and/or OTC market, equity OTC forward structures, and Delta One trading, and OTC and exchange-traded options—are a small subset of the total banking system. These banks are usually the largest global investment banks and smaller local or regional market firms with securities licenses.

Demand for equity products is also concentrated in some segments of bank clients, predominantly large wholesale institutional investors and money managers (e.g., insurance companies, pension funds, hedge and investment funds, and equity funds managers) and in private-banking (i.e., high net worth), clients looking for retail or structured investment products.

Equity Trading Products – Overview

In addition to the problem of forward and option value modeling (i.e., dividends and path-dependency), equity products involve not only derivatives on individual shares, but also options involving two of more share construction (i.e., best of, baskets, larger share indices, etc.) and currency protection (i.e., quanto and composite structures). Therefore, correlation and dispersion trading (explained below) are often involved. Therefore, equity derivatives have developed on a very different track.

Retail Clients & Wholesale (Delta One Desk Clients)

Bank clients using equity-trading products include:

- Wholesale financial institutions;
- Retail clients, structured notes standardized into large offerings sold down in smaller denominations to retail clients, often in the form of principal-protected savings deposits or structured (note) investment products (constructed with embedded equity derivatives, marketed as equity-linked);
- Private banking institutions as wholesale back-to-back clients for structured products and equity derivatives, which the private banking client uses to create, or white-label, structured products for distribution to his/her own wealth/retail clients;
- Private banking (wealth) clients investing or position-taking in structured products with their own retail/wealth clients.

Equity Trading Products & Valuation Models

Equities, as underlying assets for derivatives, have much more specific risk and are less liquid than other financial derivatives underlyings such as currencies, interest rates, and commodities. Equities—individual shares and to a lesser degree share indices—are difficult to value and model because of dividends. Dividends are uncertain until declared by the issuing company annually shortly before payment, and the lump sum payments at specific times makes derivatives on the share and share indices process particularly challenging to model. First is the forecasting uncertainty, and then the value jump modeling ex-dividend.

Equity trading, in line with bank trading of interest rate/fixed income, foreign exchange, and commodities, can be divided into cash, forwards (i.e., futures) and option products. Ranges of equity exchange-traded products are globally available from stock and derivatives exchanges, predominantly in cash and futures instruments, but also in options (e.g., American and European-style puts and calls on individual listed shares and share indices). A limited number of exotic exchange-traded products are available. Some of the more active OTC trading book products offered by large banks and securities companies include:

OTC equity cash products:

- Non-listed shares, Delta One cash products (share baskets and tracker products)
- Delta One products: ETFs (exchanged-traded funds), trackers, and customized share basket structures;
- Share lending and repo transactions.

OTC equity forwards products:

- Delta One products: non-optional equity derivatives (i.e., products with low or no delta) such as equity swaps and equity forwards-type products, incorporating a number of underlying equity securities and designed to provide institutional clients with an easy way to take exposure to a basket of securities in a single product;
- Dividend swaps;
- Total return swaps.

OTC equity options products (the following list is only representative; the range and delivery of equity options and structured product is extensive):

- Building blocks vanilla and flow exotic options: barriers (on shares or indices), barrier, digital, Asian, forward starting options, etc.;
- Building-block-based structures: ladder options and cliquet (comprised of forward start options), structured in notes and designed to lock in gains of the underlying share or index price over the life of the investment;
- Fixed income/share hybrids; convertible/reverse convertible bonds; callable/puttable convertibles (i.e., bonds with share conversion optionality);
- Auto-callable structures: a note-based structure that—in its simplest form—pays a coupon to the investor only if the underlying share (or share index) is above or below an agreed on level, and if the underlying price breaches a second defined hurdle, the note is automatically redeemed (i.e., in-the-money knock-out with principle repayment), representative of a range of equity; knock-in/knock-out (KIKO structures, including the Accumulators and TARFs with equity-linked payouts);
- FX-protected equity/index options: quanto & composite options, designed for institutional investors; a quanto fixes a positive payout in shares in and options on a share in a foreign currency at a pre-agreed exchange rate in the selected currency, and a composite gives the product at the expiration FX rate and the positive payout on a gain in the option on the shares in the forcing currency (The quanto protects the payout and the composite enables the investor to take FX and share/index price exposures);

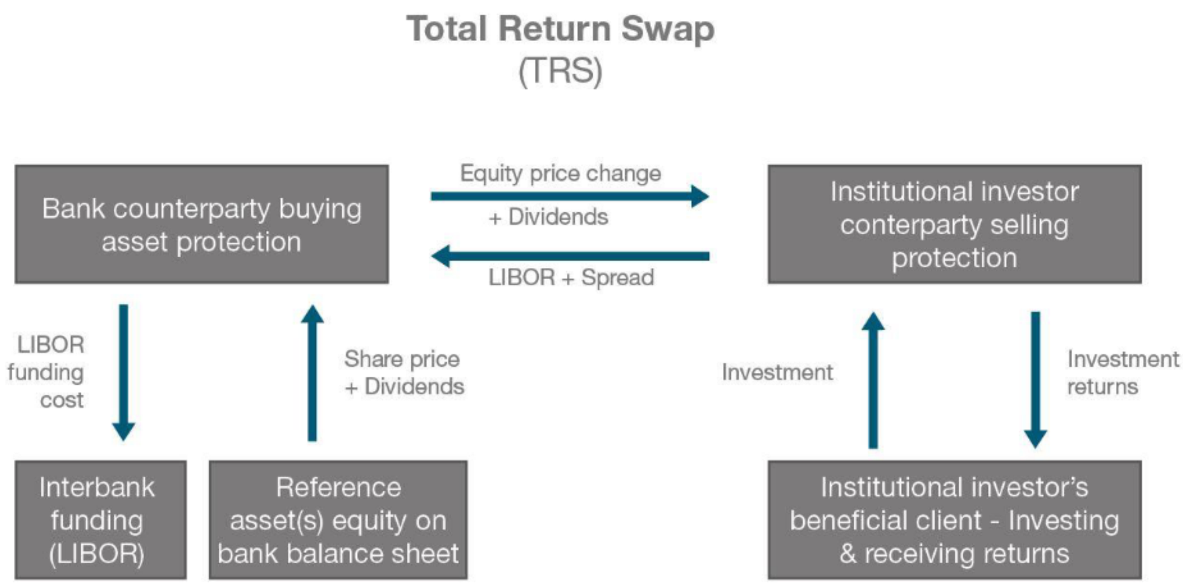
Options on several shares/equity indices (correlation trading products: out performance options, best of, or worst of option:

- Mountain range options: highly complex, path-dependent structures on the performance of defined collections of share prices;
- Basket options (hedges for clients and equity options trading desks a dispersion trading strategy with correlation and vega risk/rewards, trading a basket of options against the basket itself).

As demonstrated by this limited but representative product list, OTC equity derivatives trading has followed a different development path, with roots unsurprisingly in equity capital markets.

Example 11: Total Return Swap (TRS) Structure

A total return swap (TRS) is a financial transaction during which the risk and return on an asset on one counterparty's balance sheet is exchanged with another counterparty for bonds and equities, the usual assets involved, which are exchanged for agreed on cash flows, usually at LIBOR in the currency of the asset plus a spread. The return on the asset in the TRS includes income such as coupons or dividends and the change in the value of the asset.



When used with equity assets, TRS transactions have been historically used for either transfer of asset risk and return, as in the transaction structure illustrated above, or in some jurisdictions, because of preferential tax treatments of dividends received through a TRS in comparison to those received physically from assets held directly on a firm's balance sheet.

In the example above, a bank might have been holding some equity assets that are illiquid and wish to transfer the risk and return on the equities to another party. The bank finds an institutional investor that is interested in enhancing its investment portfolio returns by taking the price and dividend yield risk of the bank's equity holdings in exchange for paying the bank a cash flow to cover its funding costs plus some return for the indirect use of the bank's balance sheet. The institutional investor does not have to take the illiquid assets on its own balance sheet. Revaluation risk/return on the asset and dividends are passed on to the institutional investor above in the TRS cash flow structure. TRS transactions can be structured as longer-term repurchase (i.e., repo) agreements.

As mentioned, underlying shares and share indices are often much less liquid than FX and interest rate underlyings and are more difficult to model. Products are often more path-dependent and/or correlation risk related. Equity option derivatives activities at banks also started initially in the BSM modeling framework, but from the start, the equity market required modeling dividends and availability of American-style options. Due to these initial difficulties or special equity requirements; from early days, banks active in equity derivatives concentrated on replacing the BSM model with numerical modeling implementations, particularly the Monte Carlo simulation. Whereas FX, interest rate, and commodities practitioners attempted to hang on to the BSM framework for as long as possible (and still use it for trading books with simpler flow products), equity derivatives practitioners, when confronted with the difficulties of modeling dividends, started moving away from the BSM closed form approximation framework to numerical method solution modeling. Many equity products lend themselves to an MCS approach because of the path-dependency of options. In equity derivatives markets, hedging and yield enhancements are client demand factors or market drivers, but liquidity provision and tailored client solutions are also requirements.

Markets & Product Liquidity

Equity market underlyings—shares and indices—have higher general volatility than FX and interest rate markets, but lower than some of the more volatile traded commodities. Under normal uncertainty (i.e., business as usual) market conditions, equity volatility might range from 20% to 30%. In stressed or crisis markets, equity markets can become quickly illiquid (e.g., panic sell off), and the implied volatilities can go to 40% or higher. The volatility skews of equity market options (unlike those in other markets) are nearly always skewed dramatically on the downside, demonstrating risk aversion on the part of market makers to sell out-of-the-money puts (i.e., market selloff protection). Liquidity in individual share derivatives can vary considerably, and depends on the underlying share trading volumes and availability of actively traded exchange products such as share options. Liquidity in OTC share index-based derivatives tends to be much better, as expected.

Institutional clients often come to banks for large transactions, for products on non-listed shares, and to provide derivatives on specific baskets of shares reflecting the client's exposure (for hedging), or to take an unfunded exposure position (i.e., investment). Institutional clients look to banks to provide customization and liquidity unavailable on exchanges, which places risks, of course, on bank trading books.

Example 12: Equity Derivatives Structure Applications

A number of products have been developed by investment banks that are designed for retail and/or institutional equity investment clients. Some of these products are described below.

Equity Derivatives Structure	*Brief Description*
Ladder Options	Ladder options are designed for investors who want to get exposure to upside share price movement, while at the same time lock in the upward performance of the share at agreed on levels during the life of the option. This structure is popular with retail investors.
Lookback Options	The objective of lookback options is to give the investor the maximum move in a share price over a predefined period looking back at expiration of the option. There are a number of variants, including lookback on the strike or trading price of the share.
Cliquets	Cliquet options have resetting of the option strike price at pre-specified times during the life of the option. A popular cliquet structure is the ratchet option in which the strike price resets at pre-determined points while at the same time locking in the share performance during the previous setting period.
Autocallables	Autocallable is a note structure that pays a coupon only if an agreed on underlying share index is above (i.e., call) or below (i.e., put) a certain level and the note redeems automatically before the expiration of the share index breaches a defined hurdle.
Composite Options & Composite Options	A composite option is designed for investors wanting a put or call option on a foreign currency share, but wanting the strike price fixed and payout on the option made in a base domestic currency. Investors often use composite options to protect the value of their own currency in a foreign investment.
Quanto Options	In contrast to a composite option, a quanto option holder gets a percentage return from a put or call in a base currency if the share expires in-the-money in its foreign currency. A quanto option holder takes share price risk but no currency risk in the share's performance.
Best of / Worst of Options	In a best of (worst of) option, an investor receives the gain (loss) amount of the best (worst) performing share in a pre-determined set (two or more) of shares.
Outperformance Options	An outperformance option measures the upside or downside (as agreed) performance of one share price (or share index) against another and pays the investor the difference in performance.
Basket Options	A basket option is a put or call on the value of an agreed on basket of shares, tailored by the option provider to the portfolio-hedging requirements or risk preferences of the investor.
Variance Swaps	A variance swap is a contract that pays a holder the difference between the annualized variance of a share or share index and the annualized variance strike level agreed on with a seller at the inception of the trade, as agreed on from one counterparty during the swap to the other.

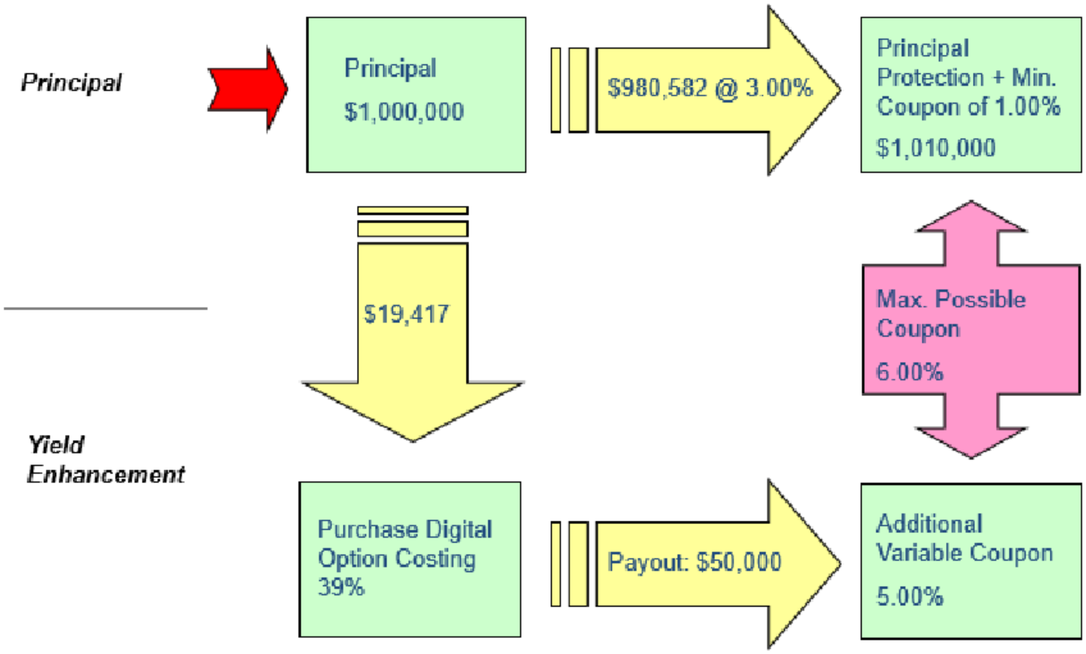
Banks create other structured products for equity investment clients using combinations of barrier options, digital options, average rate, and other option structures, with share price or share indexes as underlyings similar to structures provided in FX and commodity underlyings for other clients.

Banks like to enter dividend swaps with institutional clients as a dividend hedge on their trading books (facilitating pricing and trading book risk management). Clients often like to enter total return (TRS) or equity swaps with banks as an off-balance-sheet exposure position in equities (TRS positions were often treated more favorably in terms of taxation—dividends and capital gains—than investment in shares).

Example 13: Principal Protected Note Structuring

In Example 8, a dual-currency note (DCN) structure was illustrated. The DCN is an example of a non-principal note structure in which the investor sells the note issuer one or more options embedded in the note, and the note issuer offers the investor an enhanced yield. The enhanced yield is a premium paid to the investor for his/her short option position. The more risk the investor is willing to take in the structure, the higher the enhanced coupon on the note, but the higher the potential loss in the overall return on the structure borne by the investor. The note issuer has effectively collateralized his/her counterparty risk exposure by embedding the option bought from the investor in a note issuance structure in which the investor buys the note, paying principal up-front. The investor can lose some or the entire principal invested in the note.

In principal protected structures, illustrated below, the investor effectively buys some form of optionality embedded in the note structure. The amount of optionality that can be provided to the investor is limited by the market interest rate value of the principal amount paid by the investor when purchasing the note. The note issuer uses the present value of the interest payable on the principal amount to purchase the embedded option and invests the remainder of the principal in a zero-coupon instrument to repay the principal amount to the investor at maturity of the note. The investor hopes to earn a return on the embedded options, but in the worst case, risks losing only the interest that would have been paid on the principal. The embedded options can be structured on different risk classes (e.g., FX, equity, commodities, credit default, etc.), depending on the market view and risk preferences of the investor.



In the example above, the investor bought an embedded digital option on some risk class in the risk class (e.g., FX) that has a possible fixed payout of either USD 50,000 or USD 0 at maturity of the structured note. The note is designed to repay at minimum the USD 1 million principal amount plus a small return of 1%, and might pay out a maximum of USD 1 million principal plus 5% if the embedded digital option expires in-the-money on maturity of the structured note.

Chapter 5 – Commodities Market Risk Management

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Introduction

Commodity markets are the oldest hedging markets worldwide. Farmers used futures contracts to lock in prices long before equity markets developed, and contemporary commodity markets have strong links with the real economy. The presence of financial actors such as banks, hedge funds, institutional investors, and dedicated trading companies is at times controversial, and subject to considerable scrutiny by banking regulators. Commodity markets divide into four categories:

- Energy (i.e., oil, natural gas, and electricity/power);
- Metals (i.e., base metals and alloys such as iron, steel, aluminum, and copper, and precious metals like gold, silver, and palladium);
- Bulks (i.e., iron ore and coal);
- Agricultural (i.e., food products such as orange juice, sugar, and cacao, and softs like cotton and lumber/wood).

The taxonomy is imperfect since coal is as much a fuel for deriving energy as a raw material for making carbon, and products such as palm oil and ethanol straddle food and energy. Broadly, it is a useful way to understand the reasons products in these categories relate and why their prices correlate positively—these markets reflect the supply and demand of physical products produced and consumed in industrial, as opposed to financial, markets. Financial markets fulfill their traditional role as intermediaries, described in more detail below, but producers, processors, and consumers of a commodity, and the products and services they enable set the fundamentals.

End markets, especially demand markets, drive much of the price of commodities. In turn, these markets have disparate characteristics that are the provenance of economics more than of trading; demand for energy correlates strongly with GDP per capita, but the energy intensity (i.e., energy consumption per unit of GDP) of advanced economies is lower than that of early-stage economies. Food consumption also shifts with stages of economic development, from grains and legumes to meats, for example. Demand for base metals is driven by industrial production and construction, while the gold market is dominated by jewelry. However, the greatest driver of the platinum demand is the market for catalytic converters in gasoline-engine cars.

Traders and risk managers in commodities need to understand the role those fundamental supply and demand drivers play in short-term price dynamics. Seasonality is a huge factor in commodity markets; gas derives from demand for heat, largely in households, but grains derive from growing and harvest cycles. Weather is important for agricultural and softs markets, whereas storms wreak havoc on waterborne deliveries of crude oil and influence prices of refined products such as gasoline and diesel. Intraday prices for electricity are the result of a complex dance between demand (driven by factors like time of day and temperature) and the marginal cost of supply (driven by available capacity by fuel type such as nuclear or coal, gas, and renewables). Each market has its own drivers of both long- and short-term prices and price changes, and understanding the state of underlying factors is important.

A second differentiating factor between commodity and financial markets is the physical delivery of transactions. Although markets such as equities and bonds feature delivery of an underlying security, delivery is largely electronic and generally close to a true delivery-versus-payment mechanism. It is sufficiently close that for measuring market risk, settlement and delivery can be ignored with minimal impact on the accuracy or relevance of the outcome. This is not the case in commodity markets. An over-the-counter (OTC) contract for delivery of crude oil specifies the delivery location and timing window (with ten-day windows being common). And the pricing of cargo (e.g., a quantity on a ship) is often the average of daily spot prices recorded over a five-day window around the time the oil is loaded onto the ship.

Location matters. Most commodities are consumed a considerable distance from the production location. Transport or shipping (more generally, logistics that include storage and documentation) is a necessary part of trading physical commodities, and there are markets for shipping, or freight, that link to commodities. With shipping comes paperwork. Traded commodities are often exported, and this requires licenses and export and freight documentation, including bills of lading, import licenses and documentation, storage agreements, inspections for quality and safety, etc. Physical operations are a source of differentiation; access to ships is not as easy as access to a clearinghouse or broker, and the availability of transport and storage is a critical factor when trading commodities.

Market Participants

Players in commodity markets categorize as producers, processors, consumers, and traders. Producers include mining companies, upstream (i.e., exploration and production) oil and gas companies, farmers and agricultural cooperatives, etc. They generally have high fixed costs from investment in drilling platforms, mines, property and equipment, and lower marginal costs, giving most producers a strong incentive to keep producing irrespective of prices, up to, or rather down to, the point at which net prices are consistently below marginal costs. In the short-term, producers are classic price takers, and to the extent that owners and investors are comfortable with that, there is little reason for a producer to attempt to smooth out price fluctuations by hedging. An exception is highly leveraged producers – a growing category in many commodity markets due to the inflow of capital from private-equity funds and other institutional investors. Since these producers need a minimum cash flow to service debt, they are active hedgers, especially at the time of spinoffs, buyouts, and takeovers, and the first few years following those events. Producers who hedge, tend to do so for medium-term cash flow predictability (i.e., highly levered ones) or short-term cash flow certainty for a large contract. This leads to common maturities of 12 to 18 months at the short end and five to seven years at the long end, though all maturities show at least some demand. Producers are located wherever the commodity is present, for obvious reasons.

Processors include oil refiners, gold refiners, agricultural producers that crush seeds for oil, and smelters that turn bauxite into aluminum. Like producers, these companies have high capital costs, but generally, their marginal costs are higher than producers' (per unit of throughput) and are often dominated by energy costs. Processors seek a business model that insulates against the cost of raw materials; many prefer a tolling model in which they earn a fee per unit of throughput that is independent of raw material costs. In practice, they achieve a blend of a tolling model and dependency on the value of production. Processors are not natural hedgers in derivative markets, preferring commercial contracts with suppliers and customers that provide price protection, but if they do hedge, then the instruments most likely to be used are short-dated hedges on the margin, or a spread, between the price of the product and raw materials. Demand for margin hedges is short- to medium-term, generally under three years. Whereas energy costs are a primary driver of profitability, processors seek locations with structurally lower energy costs, hence the prevalence of aluminum smelters in Iceland.

Consumers active in hedging markets include airlines, steel companies, and industrial producers like car companies. Their business models allow them to pass on permanent increases in input prices but expose them to shorter-term fluctuations because their product markets do not tolerate frequent price changes or companies in the industry have more alternatives due to production, as often is the case with chemicals. Those companies for which commodity price is a large part of the costs (e.g., airlines) hedge, though, in many industries, there is a large variation in practices. For that reason (there is a penalty for getting it wrong relative to competitors), consumers hedge for shorter periods, 6 to 18 months being common.

Commodity traders are traditional intermediaries between producers and processors or consumers. Their business model is predicated on sourcing the commodity, arranging for shipping, simple blending or processing like crushing, and financing the flow of commodities between customers on both sides. They are also key in smoothing fluctuations in quality (by sourcing from multiple producers and blending), supply (through storage facilities), aggregating small quantities of supply into parcels large enough to ship economically, breaking up those larger parcels into quantities required by processors or the consumer, and managing considerable documentation flows. Physical commodity markets are far less transparent than financial markets, resembling commercial markets in their ongoing potential to profit from better information and access to contracts with a large number of companies. For an analog with financial markets, think retail brokerage, rather than interdealer trading.

Physical commodity transactions require large sums of money or working capital; in a typical deal, a trader buys from a producer and sells to a processor or consumer, and the full nominal value of the transaction needs to be settled on both ends. Banks traditionally finance the trade with a combination of letters of credit (that transfer the credit risk of the buyer from the seller to the bank) and lending, usually secured by the cargo itself. Some participants prefer repo financing, during which the bank buys cargo from a producer and executes a forward sale to a trader (a reverse repo from the bank's perspective). During such deals, the financing bank also carries short-term price risk on the cargo, and typically hedges this using a futures contract, or short-dated derivative. Generally, the price of a cargo of oil, metal, or other commodity relates, but is not identical, to the price that determines the value of the futures contract, leaving the bank with basis risk. Basis risk is an integral part of a commodity trading book's market risk. Like processors, banks are intermediaries and prefer not to take directional (i.e., outright or flat price) risk in their physical activities. This transactional finance and risk management is a traditional role for banks.

A second role of banks is financing inventory, and providing price risk management for the inventory. Companies hold inventory to reduce the risk of supply disruptions (which may cause them to stop production), and easily tradable commodities are a good collateral for banks to lend against. Price risk management on inventories is naturally longer-dated than purely transactional hedging, and so, is more suited to OTC swaps and options than using futures. The basis risk between the precise quality of the commodity and the underlying for the futures contract (explained below) is also greater, and can lead clients to prefer a custom derivative contract.

A third role banks play is providing long-term price risk management solutions to clients, often linked to the financing of a company rather than transactions or inventory. These are more strategic deal, and are often more highly structured, with embedded options and other features to tailor the transaction to a client's requirements.

Key products and instruments

As commodity markets developed, products and instruments of increasing sophistication evolved to support the risk management requirements of market participants. In many of these markets, development closely followed how instruments evolved in FX and equity markets. Many instruments developed to reflect the unique characteristics of their underlying commodities. Broadly, commodity instruments can be classified as financial or physical instruments, and as the names suggest, financial instruments usually lead to cash settlement between counterparties while physical settlement usually involves delivering a physical commodity.

Financial instruments can be traded through a commodity exchange or bilaterally between counterparties as OTC products. Commodity exchanges are markets in which standardized contracts on various commodities are executed through an exchange. The most commonly traded instrument on an exchange is a futures contract, which entitles a holder to take delivery of a predefined quantity and quality of commodity. The specification of a futures contract lays out quantity, grade of commodity, settlement dates, settlement mechanisms, etc. On execution of a futures contract with an exchange, the market participant directly faces the exchange, with a counterparty on the other side of the transaction staying anonymous to the market participant. As prices of the underlying commodity increase and decline, the value of the contract increases or decreases for the market participant, with the exchange managing the value transfer between the long and short market participant through a mechanism of initial and variation margin.

The standardized nature of the exchange contract is both an advantage and a disadvantage. Standardization allows for larger numbers of market participants to be involved in the trading of a contract, improving the depth and liquidity of the market. However, a standardized product might not meet the exact hedging requirements of a hedger. The market participant could be hedging an underlying position that differs in size, quality, or settlement mechanism from that specified in the underlying exchange contract. This risk of divergence between the underlying position and the hedge (called basis risk) might lead to imperfect hedges that might, at worst, lead to losses on both sides of a trade.

OTC product markets provide the hedger an alternative market to standardized offerings from exchange-traded markets. OTC products are bilateral transactions executed between counterparties and are often executed through a live or online broker market. The legal underpinning of such transactions is usually standardized, and might be governed by standard contracts such as ISDA (2012). The bilateral nature of these contracts means that as the value of these contracts changes (with changes in prices throughout the life of the contract), the credit risks of these contracts are managed through bilateral negotiations between counterparties. OTC products also permit high customization since they are negotiated bilaterally. Market participants can enter customized contracts that meet a hedger's requirements much more closely, in comparison to exchange-traded products.

In addition to market-based classifications, products can be classified based on the relationship between the value of the instrument to the underlying prices of the commodity. The instruments can thus be classified as linear or non-Linear products. As the name suggests, the value of a linear product varies by a linear relationship with underlying prices. The most common examples of linear products include instruments such as commodity futures, commodity swaps, and commodity forwards.

Commodity Futures

Futures are traded on exchanges and are derivative contracts that counterparties enter into with exchanges. A counterparty that wants to enter a contract that makes it long on the underlying commodity cannot do so until another counterparty executes an equivalent short contract with an exchange. The two counterparties remain mutually anonymous, with an exchange acting as the intermediary during the transaction. Despite being an intermediary, the exchange does not bear risk during the transaction. As prices of the underlying commodities increase or decrease, one of the counterparties loses money in the position (i.e., contract) while the other counterparty gains. This transfer of value due to change in price is managed by the exchange through a mechanism of margins. When a contract is entered, each counterparty is required to post an initial margin for the contract. As prices change and the value of one contract rises while the other declines, an amount from the initial margin equivalent to the change in value is transferred from the account of the counterparty whose contract value declined to the counterparty whose contract value increased. The exchange also levies a variation margin as and when the initial margin dips below a threshold. If either counterparty does not post margin within a prescribed time, the exchange can dissolve the contract at market price and thereby limit exposure to a counterparty. Commodity futures have expiration dates specified for each contract. Prior to expiration, a holder of a futures contract can close out the contract by entering into an offsetting transaction with the exchange. Alternately, the holder can also take the contract to physical delivery; wherein the exchange matches the longs and shorts, which can then exchange the underlying commodity of a quality and at a location as outlined in the exchange contract.

For example, the London Metals Exchange allows for physical delivery of a specified quantity and quality of material, delivered to LME-approved storage facilities and conforming to specifications on quality, shape, and weight. The specification for copper lists 20 other elements, with maximum percentages allowable for conforming metals. Specifications for other commodities including those for oil and refined products, other metals, and agricultural/soft commodities, are equally detailed. Delivery under LME contracts occurs in one of more than 500 approved storage facilities worldwide. All material held in LME warehouses links to warrants and proof of ownership of an amount of metal in the LME system. Whereas holding a futures contract gives a holder exposure to the price of the generic metal, a warrant links to a batch in a warehouse. Both the future and warrant are tradable, and in effect, the warrant is the mechanism for affecting the transfer of ownership during physical delivery.

Commodity Swaps

Swaps are traded in bilateral OTC markets, and are yet another popular, linear instrument like commodity futures. Commodity swaps and futures account for the majority of financial exposures in traded commodity markets. The specifications and legal basis of an OTC swap are governed by industry-wide specifications such as the ISDA (2012) agreement. Swaps contracts specify an underlying commodity price (i.e., index) on which a swap is written. The contracts specify a long counterparty (i.e., fixed-price payer or index-price receiver) and a short counterparty (i.e., index-price payer or fixed-price receiver). Similar to a futures contract, as prices rise, the value of the long contract increases in value, with an equivalent decline in the short contract. The contract specifies net cash settlement between counterparties at regular intervals. Counterparty exposures associated with forward exposures that have not yet settled occur under an agreed-on margining framework between counterparties, which considers exposure associated with the universe of all other contracts executed between them.

Commodity Forwards

Commodity forwards are forward contracts between counterparties that obligate a buyer to purchase a fixed volume of commodity of a specified quality at a fixed price and at a specified location. Unlike OTC swaps, which are cash-settled, commodity forwards settlement usually leads to a physical commodity supplied from a seller to a buyer. The logistics of arranging the supply of a physical commodity is called scheduling. From a valuation and risk perspective, a commodity forward behaves much similarly to a commodity swap, but unlike a commodity swap, which is settled through a net exchange of cash, commodity forwards must undergo scheduling to enable physical settlement. Depending on the type of commodities involved, scheduling can occur through either bilateral settlement or pooled settlement through a system operator.

Bilateral Settlement

This type of settlement is popular in oil and natural gas markets, involving a buyer and seller submitting a buy and sell schedule to an intermediary such as a pipeline operator. The scheduler has information such as the point of delivery into the pipeline by the seller, and point of receipt by the buyer. The seller of the physical product schedules with the pipeline operator to deliver oil at a previously agreed on receipt point. The buyer has the responsibility of contracting for transport capacity and/or storage capacity of the pipeline before delivery begins. Once the receipt point, delivery point, and logistics related to transport and storage are determined, the pipeline operator has a fixed number of days to effect the transfer and transport specified by the buyer. Once the transfer is complete, the settlement is complete. It is possible that a buyer and seller must schedule through multiple pipelines to deliver crude oil at its destination. This type of settlement involves much interaction between the buyer's and seller's schedulers and the pipeline operator.

Pooled Settlement through a System Operator

In real-time markets such as power, in which operators must ensure constant system reliability while also ensuring accurate match-up of buyers and sellers, it might be impossible to conduct a bilateral settlement. Pooled settlement through a system operator is the solution. Every seller and buyer submit an hourly schedule to a pool operator, which the pool operator uses to determine the amount of generation resources required at any given hour. If any variation occurs in this schedule because of a buyer/seller using/delivering more power than what was originally scheduled, the pool operator makes up the difference by starting up generation resources the operator has at its disposal. This ensures not only that the integrity of a bilateral contract is maintained, but also that by ramping up or down the resources available to the operator, the operational reliability of the system is maintained. The variation volumes are charged at a punitive imbalance rate, and paid by the erring counterparty that under/overscheduled respective volumes. The pool operator, therefore, plays a role both as a financial/settlement intermediary and system operator that ensures reliable operations of networks. Several examples of pooled settlement systems exist in U.S. power markets, including the PJM (Pennsylvania, Jersey, and Maryland) system in the northeast, ERCOT (Electric Reliability Council of Texas) in the south, and NEPOOL (New England Power Pool) in New England.

Non-Linear Instruments

Nonlinear instruments include products such as vanilla and exotics options, whose values change nonlinearly in relation to an underlying commodity price.

Vanilla Options

An options contract provides a holder the right but not the obligation to buy/sell a commodity contract, physical or financial, at a fixed price in a previously specified period or on a particular date. If a holder of an option can exercise it in a previously specified period, it is called an American option. If an option must be exercised on a specific date, it is a European option. Call options confer a holder the right to purchase a commodity while a put option gives a holder the right to sell a commodity at a fixed price. Both American and European options are called vanilla options, and these classes comprise the majority traded in the market.

Exotic Options

Vanilla options do not lend themselves to easy customization to fit a client's hedging requirements. To overcome this limitation, options with much more customized payoff structures appeared in the late 1960s and early 1970s. These types of tailored options with customized payoff structures were called boutique or exotic options. Whereas the market for commodity exotic options is not as liquid as that for rates, FX, or equity exotics, it benefitted greatly from expertise developed in these markets and crossing to the structuring desks in commodity trading firms. Over the years, exotic product offerings for commodities developed steadily, and the liquidity of many of these products improved. Broadly, exotic options classify into the following categories:

Basket Options

A basket option provides a holder with the option to choose among various commodities in a basket of commodities, based on previously defined payoff criteria. Criteria might include payoff based on best of, worst of, or averaging of commodities from the basket. Basket options are the most popular of the exotic structures in the market. In addition to the Greeks normally associated with vanilla options such as delta and vega, it is important to monitor many second-order Greeks. Correlation sensitivity is one of the most important of the exotic Greeks a risk manager must monitor during an exotic trade.

Barrier Options

Barriers typically trigger payoff when a price is breached. The payoff could be a fixed payment as in the case of a digital option, or involve a path-dependent payoff for the option. Path dependency is an important method of classification for barriers since it determines whether a payoff occurs if a price target is touched or achieved through a path-dependent process such as averaging over a period.

Lookback/Accumulator Options

This style of exotics typically allows the holder to exercise an option based on some form of averaging of settlement prices. The simplest form of this type of option is an Asian option, in which exercise occurs based on a simple average of settled prices over a period. This option can take other forms such as in the case of an accumulator; if cumulative returns/prices exceed a threshold, a predefined payoff occurs.

Compound Options/Swaptions

A compound option is an option to enter a call or put option at a previously defined strike and expiration date. The holder of a compound option, when exercised, becomes the holder of a put or call option that when exercised again, makes the holder long (for calls) or short (for puts) the underlying commodity. A Swaption is an option on a swap in which a holder, on expiration of the swaption, is long (for calls) or short (for puts) a commodity swap. On expiration of both instruments, the holder continues to be subject to market risk since he/she continues to be holder of live swaps or options.

Risk Implications of Physical Nature of Commodities

In this section, we consider what the physical nature of commodities means for traders, risk managers, and intermediaries.

Arbitrage Pricing Theory Review

Many aspects of financial markets are influenced strongly by arbitrage, or rather, non-arbitrage conditions. Forward FX rates, for example, are determined by spot rates and interest rates in both currencies. This works because it is easy and cheap to profit from a misalignment of forward rates: executing a spot transaction, borrowing one currency, and lending another are all standard transactions with low cost, and crucially, no restrictions on the volumes that can be transacted. If a misalignment occurs and one trade for, say, $100 million does not close the arbitrage opportunity, another trade might, or a third, but at some stage, enough trades are executed to close the arbitrage opportunity.

In some markets, there are limitations on the ability of traders to execute arbitrage-closing trades, usually due to limited liquidity. Small cap equities and corporate bonds are good examples in which trading to close an arbitrage opportunity might not work. Generally, a misalignment does not have to be too big for the trade to work: the price incentive a trader can offer to a holder of a financial instrument to close an opportunity is usually sufficient to ensure the availability of the instrument(s). The power of this condition is difficult to understate. Arbitrage trading ensures that theoretical price relationships hold, with small error rates. This matters to traders and risk managers: if the price of an instrument or security is determined through arbitrage pricing arguments (i.e., derived from a replicating portfolio), then generally, so is the market risk.

This is fundamentally different in commodity markets. The principles behind arbitrage pricing apply, but the ability to close arbitrage opportunities is far more limited than in most financial markets. The equivalent of the forward FX relationship (i.e., the relationship between spot and forward prices) is determined, in theory, by arbitrage. Using this argument, a forward price can be derived by taking the spot price (i.e., buy the commodity) and adding the cost of storage (i.e., interest costs to finance the inventory, working capital, and costs of renting warehouse space). This relationship is not generally true. In fact, forward prices are often lower than spot prices despite positive storage costs and interest rates, a condition known as backwardation. The opposite, in which forward prices are above spot prices, is called contango.

The term convenience yield is often used to deal with this situation. It is a theoretical implied yield, taking the observed spot and forward prices, and storage costs, and computing the interest rate necessary to complete an arbitrage condition. The most useful interpretation of a convenience yield is to view it as the yield that can be earned from the ability to execute an arbitrage closing deal. If that yield exceeds that which is available in money markets and related investments, it is unsurprising that traders attempt to execute arbitrage-closing trades. Usually, this occurs when forward prices are too high (i.e., above the level determined by spot prices, storage, and interest costs). In such a steep contango market, all storage is used, and the rent available to warehouse owners, the demand for working capital financing and the rates charged, and spot prices all increase. This leads to situations like floating storage for crude oil, where tankers are used to store oil instead of for transport.

Price relationships in commodity markets are more complicated than in financial markets, but they are not irrational or random; they are the consequences of the physical nature of commodities and consequent limitations on the ability of many market participants to execute trades that close arbitrage opportunities. The remainder of this chapter explores consequences of the physical nature of commodities.

Basis Risk

Commodities are much less fungible than money and securities. This creates significant basis risk. Spot and forward price relationships are a consequence of the need to source and store a physical commodity to close a theoretical arbitrage, but storage is only one physical aspect that influences price relationships. Despite the term *commodity*, many commodities are not easily fungible. Crude oil comes in hundreds of specifications, depending on the source of the oil (i.e., the field from which it was extracted). Although the viscosity (*heavy* and *light* denote opposite ends of the spectrum) and sulfur content (*sweet*, low in sulfur and *sour*, high in sulfur) are the most important characteristics for determining the price of crude oil, dozens of other variables matter.

Why do crude oil prices differ across specifications? Crude oil is nearly useless in its raw form; it is a base material to make useful products like gasoline, diesel, jet fuel, bitumen (asphalt), and carbon. Those products have narrow ranges of specifications: your car must run well on gasoline from any brand available in your country or region, and airplanes rely on jet fuel worldwide. Making a standard-specification diesel, with very low sulfur content, is much harder and more expensive when starting with a sour crude oil. For some refineries, it is impossible. Refineries are expensive installations, with costs running into many billions of dollars for a refinery with a throughput of 200,000 barrels a day. Upgrading a refinery to enable processing of different crude oils can cost tens or hundreds of millions of dollars, and take months or years to implement.

Extracting oil is also a very expensive business, and the cost of finding and producing oil varies greatly. The lowest cost producers with very large established fields, mostly in the Middle East, might have costs of around $10 per barrel, whereas costs for deep-water reservoirs, or tar sands and other unconventional sources, might approach $100 per barrel. By comparison, transport costs rarely exceed $3 per barrel.

Matching the world's refining capacity to various specifications of the world's supply of crude oils is a long-term, strategic exercise undertaken by private and state-run companies – both producers and refiners. Include changing demands for products (e.g., tax incentives can shift the market share of diesel and gasoline dramatically, and environmental rules have steadily reduced sulfur content) and the oil market is very complex. Other commodity markets have similar drivers of value, though oil is the most complicated. Ore varies in its content of valuable metal, whether iron, copper, gold, or any of the other base or precious metal. Natural gas varies in its calorific content (i.e., the energy per volume unit of gas at atmospheric pressure), and agricultural commodities vary by tastes and other attributes.

Implications for Arbitrage Pricing

Typical relationships between prices are fraught with difficulties:

Deriving Prices for Illiquid Commodities from More Liquid Types

The proliferation of specifications that drive prices, whether in energy or other types of commodities, complicates both pricing and risk management. The standard approach used in rates and FX when constructing a single, arbitrage-free, forward curve and deriving that curve from a limited number of factors (whether prices for varying maturities or principal components) does not work well for many commodities due to the limited power of the no-arbitrage condition. Treating each maturity for each specification as a different pricing point leads to many thousands of prices that need to be collected, exhibiting such high correlations that mathematical techniques to derive variance-covariance matrices or other inputs for simulations becomes difficult.

A better approach is to see prices as deriving from a hierarchy, or tree, starting with benchmark prices such as Brent or WTI for crude oil, API 2 for coal, LME for copper, etc. Prices of related commodities can then be expressed in terms of spreads to benchmark prices. Since, in most cases, commodity price risk must be integrated with market risk measurement for other asset classes, it is appealing to take the most liquid futures contracts as a definition of benchmark prices. Price data for futures is easy to obtain, and of high quality and consistency, and exchanges and clearinghouses set out detailed specifications and have an interest in ensuring their markets operate well and produce relevant prices.

Deriving Forward Prices from a Price

Futures prices for commodities can be treated like other futures. They are fixed maturity values so care must be taken when deriving tenors such as 3 months or 2 years. The majority is cash-settled, and so, the maturity date can be taken as the date on which final cash flows occur, but there are important differences for some contracts, and in general, forward curves derived from futures are a proxy for forward curves for the underlying physical commodity. There are also important characteristics to consider for market risk measurement. Futures can, on expiry, be settled physically, that is, through delivery of the underlying commodity. In such a case, the futures contract turns, on expiry, into a contract for delivery of a specified quantity and quality of physical product at a specified location and over a specified period, or point in time, against a cash payment. For example, in the case of the U.K. natural gas contract, or NBP (national balancing point), physical delivery means that a specified quantity of gas is delivered into the national grid over the calendar month following the expiry of the contract. Payment for the delivered gas is 20 days after the delivery month, so the final cash flow occurs nearly 60 days after the expiry of the futures contract. The consequences are that market risk does not stop on expiry of the contract when physical delivery occurs; the value of the gas being delivered continues to fluctuate and needs to be marked against balance-of-month OTC market prices. Credit risk is also fundamentally different. The two parties in the delivery phase are exposed to non-delivery of or non-payment for the physical gas, which is approximately equal to the full notional amount of the futures contract.

Contracts have different delivery specifications, and each contract must be modeled according to that specification, or the market risk manager must accept an error rate in market risk measurement that can be considerably higher than typical. The choice of whether to model in detail depends in practice on whether the organization participates in the physical delivery market, and if so, to what extent:

- Parcel size matters (e.g., barges versus cargos);
- Location matters;
- Transaction costs matter.

Price Risk Management

All of the above entails that most market participants have thousands, if not millions, of trades in their portfolios, executed as part of their normal market making or risk management. The challenge to the market participant is to ensure that the characteristic of the portfolio is maintained as such that under various possible scenarios in the market, losses that accrue from the portfolio stay within a previously determined stress loss value, hence, robust price risk management is needed.

Prior to delving into price risk management, we consider various forms of residual risks a market maker has in its portfolio. We exclude participants such as hedge funds and private equity that typically carry large outright risks in a commodity, with the intent of benefitting from a directional price move. Most market makers enter a transaction with a producer or consumer to hedge natural exposure to a commodity. If the hedged commodity is a liquid product, the market maker hedges exposure immediately, thereby leaving little residual risk in the portfolio. However, hedges are typically in either an illiquid commodity/product or an illiquid tenor, which cannot be hedged immediately. For example, a producer wants to hedge 10 million barrels of production for the next 20 years of a crude grade, of which pricing links closely to a West African physical grade of crude such as Bonny Light. The Bonny Light grade of physical crude typically trades a couple of months out in the forward, and the volumes traded tend to fall off quickly in a few months forward. The hedging, market-making trader must, therefore, hedge with a grade of crude that meets the following criteria:

- The hedging grade trades in sufficient liquidity in the forward markets;
- The hedging grade correlates sufficiently with the hedged grade (in this case, Bonny Light), and the relationship is maintained in various stressed market conditions.

Assume that the trader's analysis reveals that the Brent grade of crude meets these criteria, and, therefore, the trader carries out the following trading actions:

1) Hedge as much of the Bonny Light exposure with like grade in the front month as the liquidity in the market allows;
2) Hedge as much of the remaining exposure with the Brent grade of crude through the tenors as liquidity in the Brent market allows;
3) For the remaining unhedged Bonny Light exposure through the tenor, the trader establishes a stack and roll hedge wherein the trader assumes a correlation between unhedged, outer tenor of Bonny Light with Brent grade in a liquid tenor;
4) The trader typically maintains and manages these hedges as market conditions change through the life of the transactions until such time that liquidity becomes available to establish like hedges in Bonny Light.

Hedging strategies of this type, which are typical for market makers, introduce residual risks to portfolios such as basis risk (i.e., spread risks between two grades or locations, as explained in 2) and calendar risks (i.e., spread risks between two tenors introduced in the portfolio from trading strategies outlined in 3). When thousands of such trades are added to the portfolio, the importance of having a good framework to manage basis, calendar, and other residual risks becomes clearer.

The first step toward setting up an appropriate risk framework begins with senior business and risk management outlining the risk appetite of a business unit. Risk appetite is defined as the maximum risk that managers are willing to tolerate within a well-defined confidence interval (such as a 99.97%, or defined in terms of 1 in 10 years frequency) and a defined holding period (e.g., weekly, monthly, or annual). From risk appetite, appropriate risk limits are calculated and established for various types of risks in a portfolio.

Example

During the annual risk review, the risk management committee of Market Maker XYZ communicates that the board's risk appetite for the market-making operation of any desk is $20MM, and the risk appetite for the entire market-making operation is $100MM. The risk committee states that the frequency of loss is 1 in 10 years, and the holding period is assumed to be 20 days. Market Maker XYZ plans to setup a NYMEX natural gas trading desk and you, as the market risk manager, have been tasked with coming up with a net delta limit for the NYMEX natural gas desk. Assume that the desk will only take outright (i.e., flat price) risk, and the percentage stress move (1 in 10 years) for the 6-month contract for a holding period of 20 days is 30%. 6-month natural gas trades at $5 per MMBtu. Assume that the 6-month contract is a proxy for the tenor, where the desk is expected to take flat price risk.

> Risk Appetite for the desk: $20MM
> 1 in 10 year percentage stress for a 20 day holding period: 30%
> Notional value of the NYMEX gas delta position that produces the stress loss above:
> $20MM/30% = $66MM
> To convert the notional $ delta to MMBTu equivalent, divide by cost of gas:
> $66MM/$5 = 13MM MMBtus
> The delta limit in NYMEX contract equivalent:
> 13MM/10,000 = 1333 contracts (10,000 MMBtu is equivalent to 1 NYMEX contract)

Typical risk factors monitored in a market maker's portfolio are listed below.

Sensitivities

Delta risk. This Greek is monitored to limit the amount of outright (i.e., flat price) risk in a portfolio. Delta risk is a first-order Greek, defined as the change in PV for a 1% change in underlying prices. For a market maker, delta risk tends to be small since much of the flat price risk is hedged with like commodity hedges (liquidity permitting), or cross-hedged with closely correlated commodities or locations. Nevertheless, it important to monitor this in a portfolio to assess concentration risks that might arise for an individual commodity or location.

Vega risk. Vega risk is the sensitivity of a portfolio to change in underlying volatility, typically quantified as the change in PV of a portfolio to a change in underlying volatility by 1% (1 vol pt). In portfolios dominated by non-linear instruments such as options, this is an important measure to monitor. Market makers might carry small net Vega exposures due to cross-hedging with other, more liquid vega positions. However, as in the case of delta monitoring, Vega is important to track for large concentrations that might occur in portfolios.

Location spread risks. Location spread risk typically arises when a market maker hedges an illiquid position with a more liquid and well-correlated commodity exposure. When establishing this position, a trader assumes that historical relationships (i.e., correlations) between two locations typically hold through such time that the spread risk is carried in the portfolio. If a correlation breaks out of correlation-range assumptions, a desk could lose money from this location spread position. Consequently, it is important to monitor the magnitude of the location spread risk carried in the book.

Spread Risks

Cross-commodity spread risks. Often, a desk cannot find liquidity to hedge in the same commodity, and requires hedges to be established in other correlated commodities. A good example occurs in U.S. power markets, in which if liquidity does not exist to establish like hedges, cross-hedges are established in natural gas power markets. Such hedges typically work in markets in which there is strong correlation between power and gas. The risk for the market maker is similar to that for location spread risk in that the historical relationship between the two commodities disconnects, resulting in losses for the desk.

Calendar spread risks. Calendar spread risk arises when a market maker establishes hedges in a more-liquid tenor, resulting in a time spread position between the underlying and hedged exposures. These hedges are established for the same commodities but in different tenors. As a result, the net delta exposure for the commodity might be zero or near zero, but the risk to the position is that the two tenors move unexpectedly, resulting in a book incurring large losses. This risk is much more pronounced for seasonal commodities such as natural gas or agricultural products where unexpected changes in inventories result in spread prices breaking out unexpectedly and resulting in large losses.

Value at Risk

Value at Risk (VaR) has steadily gained popularity among risk managers since its introduction in the late 1980s, following its use as the RiskMetrics variance model by JP Morgan. The VaR process creates a distribution of possible P&L outcomes for a current portfolio of exposures. The distribution is generated using various price processes, derived statistically using historical prices of the various commodities represented in the portfolio. The VaR model also requires a holding period (1-day, 5-day, etc.) assumption. Once a distribution of P&L outcome is generated, the VaR for the portfolio is the worst P&L outcome at a confidence interval (typically 95th or 99th percentile) for the defined holding period. The greatest attraction of the VaR methodology from a risk manager's perspective is its simplicity. VaR reduces the risk of a portfolio to a single number that provides risk and business managers with a standardized view of the downside risk of a portfolio. However, the greatest downside of VaR as a risk management tool is its exclusive reliance on statistical analysis of historical prices to generate the distribution of P&L, which generates VaR. This limitation means that VaR is a good predictive tool for portfolio downside as long as future price processes behave similarly to how they did in the past. The history of financial crises is littered with examples in which this assumption broke down, as during the 2008 financial crises, the Amaranth hedge fund blowout, etc. This is particularly true for commodities vulnerable to infrequent events driven by physical disruptions that produce short periods of extreme volatility during which a desk could take large losses while the desk VaR continues to indicate low risk. This lack of predictability opened VaR to substantial criticisms, especially since the financial crisis of 2008, with many risk management experts blaming the tool for creating a sense of complacency for desks (by understating risks) as markets changed dramatically, resulting in substantial losses.

Example

The NYMEX desk at Market Maker XYZ is now active and operating under the risk framework designed by you – its risk manager. The desk is considering hedging a large position from a client, which will be unable to hedge for at least a week. The risk management committee is concerned about the risk in the book during the period the desk holds the exposure. The committee asks you to compute the VaR for this position. The position to be executed is a 10,000 NYMEX natural gas December 2015 contract. Your analysis of the contract indicates that the 5-day, 99th-percentile move is about 3%. XYZ computes a 99th-percentile VaR for a 1-day holding period. The December 2015 NYMEX gas price is $5 per MMBtu.

Position in the book from transaction (in contracts): 10,000 contracts
Notional $ position in the book from transaction (in $MM): 10,000 x 10,000 x 5 = $500MM
5-day, 99th-percentile move for NYMEX December 2015 contract: 3%
1-day, 99th-percentile move for NYMEX December 2015 contract: 3%/Sqrt(5) = 1.35%
1-day, 99th-percentile standalone VaR for December 2015 exposure: $500 x 1.35% = $6.*75MM*

Stress Testing

In addition to Greek limits and VaR, the third part of the risk-management framework, and one of the most important tools in the risk-management toolkit, is the stress-testing framework (see the stress-testing chapter of this handbook for details).

Although Greek limits help control concentrations in the portfolio and through VaR by providing a snapshot of the day changes in the systemic risk of the portfolio, neither has a good predictive capability. A robust, stress-testing framework that is properly designed for a portfolio helps to fill the predictive gap, and can be an effective tool in a risk manager's kit.
A good stress-testing framework provides information on two fronts; it should:

- Highlight stress P&L outcome for the portfolio during historical stress events;
- Highlight stress P&L outcome for predictive scenarios – outlined by risk managers – that considers market fundamentals and concentrations in the portfolios.

Historical stress testing. The mechanics of historical stress testing is straightforward: it involves taking current portfolio exposures and computing P&L for the exposure (for a holding period defined by a risk manager) using historical prices through periods for which reliable historical data is available. Price data should be granular since greater granularity highlights basis risk in a portfolio. The P&L outcome from this historical analysis is ranked from the worst loss outcome through various percentile outcomes, along with various dates when these loss events occurred. Stress analysis of this nature offers many advantages. It highlights how a portfolio would have behaved during past stress events. Risk managers and traders could have lived through many of these stress events and would be able to relate directly to the historical stress data. This enables them to develop meaningful strategies on what changes are necessary during similar stress events. This also helps desks to develop actions during market-changing events such as hurricanes, which can disrupt the energy commodity production in the Gulf of Mexico, massive price spikes related to extreme winters, large volatility changes as investors abandon commodity markets following a massive sell off in financial markets, etc.

Predictive stress testing. The obvious downside of the historical stress framework is that it does nothing to highlight a potential stress event that has not occurred in the past. Historical stress frameworks do not highlight stress losses associated with concentrations in a portfolio that are affected by large, systemic stress moves in a market. The predictive stress-testing framework provides risk and trading managers with a tool to predict and quantify what could go wrong with existing positions in a portfolio, and the analysis is forward- rather than backward-looking.

The first step involved in a predictive stress-test analysis is to identify concentrated positions in a portfolio that could cause significant P&L losses in case of an adverse move. As part of this process, concentrations in basis positions such as locations, cross-commodities, and calendar spreads need to be identified. The next step is more complicated, and should involve trading managers, analysts, and risk managers. The group should keep the concentrated positions in the book in context and jointly develop potential adverse scenarios for the commodity positions in the book. The group should also develop probabilities of the scenarios occurring simultaneously. For example, a book could have a large basis position in natural gas and a cross-commodity position in crude oil. Although an adverse stress scenario can be developed for the natural gas basis and crude commodity positions, a joint probability of the two adverse scenarios occurring is necessary to determine a combined stress loss for a book. Scenarios could also be developed for near-, medium-, and long-term since fundamentals of the adverse scenarios can change based on the period of analysis.

Stress P&L for each concentrated position is determined by using these data. Then, using joint probabilities for each event, a consolidated stress number is created at the book, desk, and business levels. The subjective nature of the predictive stress analysis means that a mechanism should exist whereby participants meet regularly and update assumptions, considering changes to exposure concentrations, market liquidity, market fundamentals, etc.

Reverse stress testing is a subset of predictive stress testing. It begins with a defined stress loss threshold for a business, typically a loss appetite provided by a risk management committee, or more often, a capital loss threshold provided by a treasury function that renders a trading business insolvent. The process continues with this loss threshold, identifying a combination of stress loss events for concentrated positions discussed above that exceeds the threshold. The largest advantage is that the test provides risk and business managers with a set/combination of scenarios that lead to losses that would have severe repercussions for the business. This allows the organization to focus its risk-management resources and expertise on managing losses, should one of the scenarios occur:

- Respect arbitrages;
- History as a source of scenarios;
- Developing forward-looking scenarios: reverse stress testing as a less-subjective alternative.

Chapter 6 – Market Risk Stress Testing - Beyond the VaR Threshold

David M. Rowe, PhD

Introduction

Before VaR

The financial crisis that began in September 2008 prompted considerable soul searching among risk managers. Much criticism appeared on Value-at-Risk (VaR) as a tool for risk estimation and reporting. VaR was an important but limited advance in risk-management methodology. Before the early 1990s, traders were constrained by a complex array of micro-limits, including:

- Total gross open positions;
- Gross open positions at a variety of fixed maturities;
- Total absolute value of open positions summed across all maturity buckets;
- Limits across multiple option types on:
    - Delta;
    - (negative) gamma;
    - Vega (possibly asymmetrical limits on long and short vega).

Market-risk policy committees faced a stream of recurring requests for increases in these limits. However, committees had no meaningful method of weighing the risk implicit in the existing structure of these limits, let alone what further risk was implied by requested increases. VaR was the first effective means of communicating risk implications between traders and general managers.

Language Matters

Unfortunately, general managers found VaR to be such a compelling advance that they viewed it as an all-encompassing measure of risk. For their part, financial risk managers did far too little to counter the misconception; many risk managers fell into the sloppy practice of calling VaR the worst-case loss. They seemed to believe that everyone would realize this was not to be taken literally; what they really meant was a loss that will be exceeded only two or three times a year. That belief was often unfounded. The problem with calling VaR a worst-case loss was that many people take you at your word, believing that VaR is the most that can be lost on any given day.

A far better alternative, shorthand description is to call VaR the minimum twice-a-year loss. This terminology conveys two things. It indicates the approximate rarity of the stated loss threshold being breached, and begs the question, "How big could the loss be on those two days a year?" The answer is that VaR conveys nothing about what lurks beyond the 1% threshold.

Dangerous Unknowns

Risk Versus Uncertainty

A central weakness of financial risk management has been to neglect the distinction between risk and uncertainty, which Frank Knight enunciated in his 1921 book *Risk, Uncertainty and Profit*.[22] Knight defines risk as randomness that can be analyzed using a distributional framework, and uncertainty as randomness that cannot be so analyzed.

Situations in the risk domain are characterized by repeated realizations of random events generated by a process that exhibits stochastic stability, or at least a high degree of stochastic inertia. In non-technical language, this means that the nature of the randomness is constant or changes only slowly over time.

Events in the domain of uncertainty are rare but not necessarily historically unprecedented. They are not frequent enough to allow creation of a distribution, and typically, they result from a system building up structural stresses that eventually cannot be sustained. One part of the system gives way, causing increased stress elsewhere in the system and leading to a self-reinforcing feedback loop until the source of the stress is relieved.

An extremely important point to realize is that low levels of risk in the narrow sense do not imply the absence of dangerously unsustainable systemic stresses. In many cases, the opposite is true. Periods of sustained, low, short-term volatility can induce behaviors that increase structural stresses, making the system prone to a major crisis.[23]

The Basis for Normality

The conceptual basis for the occurrence of black swans (i.e., rare and large events far removed from the normal range of fluctuations) is the core of classical statistical theory. Many types of recurring, random events exhibit a pattern, which approximate the bell curve, or normal distribution. The proposition in classical statistics that gives rise to the normal distribution is the central limit theorem. Too often, we remember this vaguely from our statistics education but forget the details.

The Central Limit Theorem

The mean of a sufficiently large number of independent random draws from a stable distribution, with finite mean and variance, will be approximately normally distributed.

Two crucially important words are underlined that risk managers must always remember: stable and independent. All social scientific systems, including finance, are characterized by

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[22] *Knight, Frank; Risk*, *Uncertainty, and Profit*, Hart, Schaffner and Marx; Houghton Mifflin, 1921.

[23] The thinking of the late economist Hyman Minsky gained renewed appreciation in the aftermath of the global financial crisis. His famous dictum was "stability is destabilizing."

