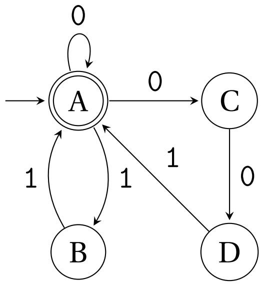
DISCRETE MATHEMATICS (MAT 2138) ASSIGNMENT 2

Answer at least two questions.

1. Let M be the non-deterministic finite automaton shown below.



   (a) Give an example of a string that is **not** accepted by M.
   (b) Convert M to a deterministic finite automaton.
   (c) Convert M to a regular expression.

2. Let G be a graph of order $n > 1$. Prove the following:
   (a) If $\delta(G) \geq \frac{n-1}{2}$, then G is connected and $\operatorname{diam} G \leq 2$.
   (b) If G and $\overline{G}$ are both k-regular, for some positive integer k, then $n = 4t + 1$, for some integer $t \geq 0$.
   (c) If G and $\overline{G}$ are both k-regular, for some positive integer k, then $\operatorname{diam} G = 2$.

3. Let G be a graph with vertex set $\{v_1, v_2, \ldots, v_n\}$, and $A = A(G)$. Let $c_4(G)$ denote the total number of cycles of length 4 in G, and $c_4(v_i)$ the number of cycles of length 4 containing the vertex v_i. Prove that the i^{th} diagonal entry of A^4 is

$$(A^4)_{ii} = 2c_4(v_i) + \sum_{j:\ v_j \sim v_i} \deg v_j + 2\binom{\deg v_i}{2}$$

   and hence prove that

$$c_4(G) = \frac{1}{8} \operatorname{tr} A^4 - \frac{1}{4} \sum_{i=1}^{n} (\deg v_i)^2 + \frac{|E(G)|}{4}$$

4. Let G and H be two graphs on the same vertex set $\{v_1, \ldots, v_n\}$, and let A and B be their adjacency matrices, respectively. Show that AB is the adjacency matrix of some graph if and only if the following conditions are satisfied:
   (a) G and H have no common edges.
   (b) For any two vertices v_i and v_j, there exists at most one vertex v_k such that $v_i \sim_G v_k \sim_H v_j$.
   (c) If $v_i \sim_G v_k \sim_H v_j$, then $v_i \sim_H v_l \sim_G v_j$ for some l.