CS & IT ENGINEERING

Algorithms

Greedy Method

Lecture No.- 01

By- Aditya Jain sir

Topics to be Covered

Topic

Topic — Intro to Greedy

Topic — Knapsack

About Aditya Jain sir

1. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
2. Represented college as the first Google DSC Ambassador.
3. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored 12,000+ students & working professions in field of Data Science and Analytics
11. Have been mentoring & teaching GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

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Topic : Divide and Conquer

[MCQ]

#Q. For Constant $a \geq 1$ and $b > 1$, consider the following recurrence defined on the non-negative integers:

$$T(n) = a.T\left(\frac{n}{b}\right) + f(n)$$

Which one of the following options is correct about the recurrence T(n)?

A. If f(n) is $\theta\left(n^{\log_b(a)}\right)$ then T(n) is $\theta\left(n^{\log_b(a)}\right)$

B. If f(n) is $\theta\left(n^{\log_b(a)-e}\right)$ for some $e > 0$, then T(n) is $\theta\left(n^{\log_b(a)}\right)$

C. If f(n) is $\frac{n}{\log_2(n)}$ then T(n) is $\theta(n\log_2(n))$

D. If f(n) is $n\log_2(n)$, then T(n) is $\theta(n\log_2(n))$

Topic : Divide and Conquer

[MCQ]

#Q. For parameters a and b, both of which are $\omega(1)$, $T(n) = T(n^{1/a}) + 1$, and $T(b) = 1$ ~~= 1~~. Then T(n) is :

Back-sub

A. $O(\log_2 \log_2 n)$

B. $O(\log_b \log_a n)$

C. $O(\log_a \log_b n)$

D. $O(\log_{ab} n)$

$$T(n) = T(n^{1/a}) + 1$$

$$T(n^{1/a}) = T(n^{1/a^2}) + 1$$

$$T(n) = T(n^{1/a^2}) + 2$$

$$T(n) = T(n^{1/a^3}) + 3$$

$$\vdots$$

General Term

$$T(n) = T(n^{1/a^k}) + k \quad \text{—} \ 1$$

$$n^{1/a^k} = b$$

$$\frac{1}{a^k} \log n = \log b$$

$$\frac{\log n}{a^k} = \log b$$

$$a^k = \frac{\log n}{\log b} = \log_b n$$

$$k \log_a a = \log_a(\log_b n)$$

$$\boxed{k = \log_a(\log_b n)}$$

$$T(n) = T(1) + k$$

$$\boxed{T(n) = c + \log_a(\log_b n)}$$

Topic : Divide and Conquer

QS

QS: Middle element as pivot

Input:

Worst case TC = ?

A) $O(n)$

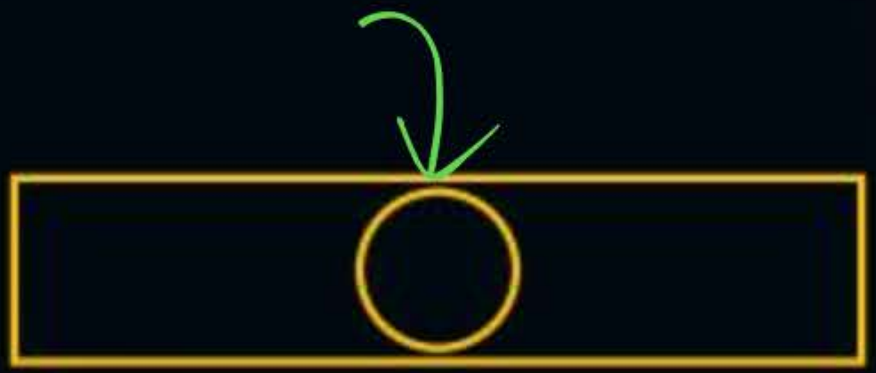
B) $O(\log n)$

C) $O(n \log n)$

D) $O(n^2)$

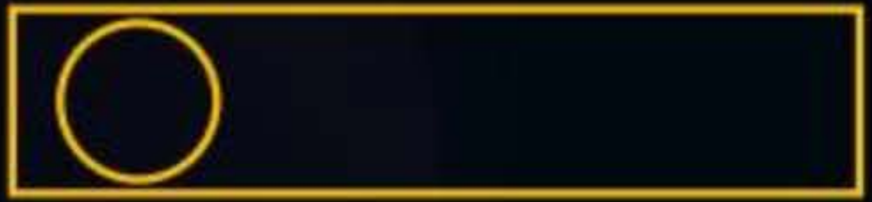
Topic : Divide and Conquer

Before partition:



3 4 1 9 2

After partition:



or

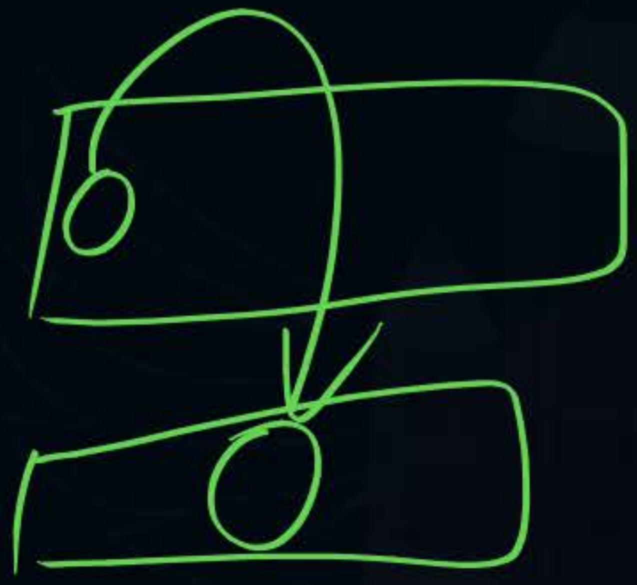
- "Divide happens as per where the pivot get placed after partition."

  Hence, worst case : $O(n^2)$

Topic : Greedy Algorithm

In QS, if median is always selected as pivot then the worst case TC is?

Median → middle element in sorted order



Topic : Greedy Algorithm

Example:

10, 2, 9, 15, 13   pivot $\rightarrow$ 10

Median:

2, 9, 10, 13, 15   median $\rightarrow$ 10

Time Complexity:

$$T(n) = T(n/2) + T(n/2) + O(n) + O(n)$$

$$= O(n \log n)$$

Topic : Divide and Conquer

[MCQ]

#Q. When $n = 2^{2k}$ for some $k \geq 0$, then recurrence relation

$$T(n) = \sqrt{2}T(n/2) + \sqrt{n}, \qquad T(1) = 1$$

Evaluates to :

HW

A ~~$O(\log_2 \log_2 n)$~~

B ~~$O(\log_b \log_a n)$~~

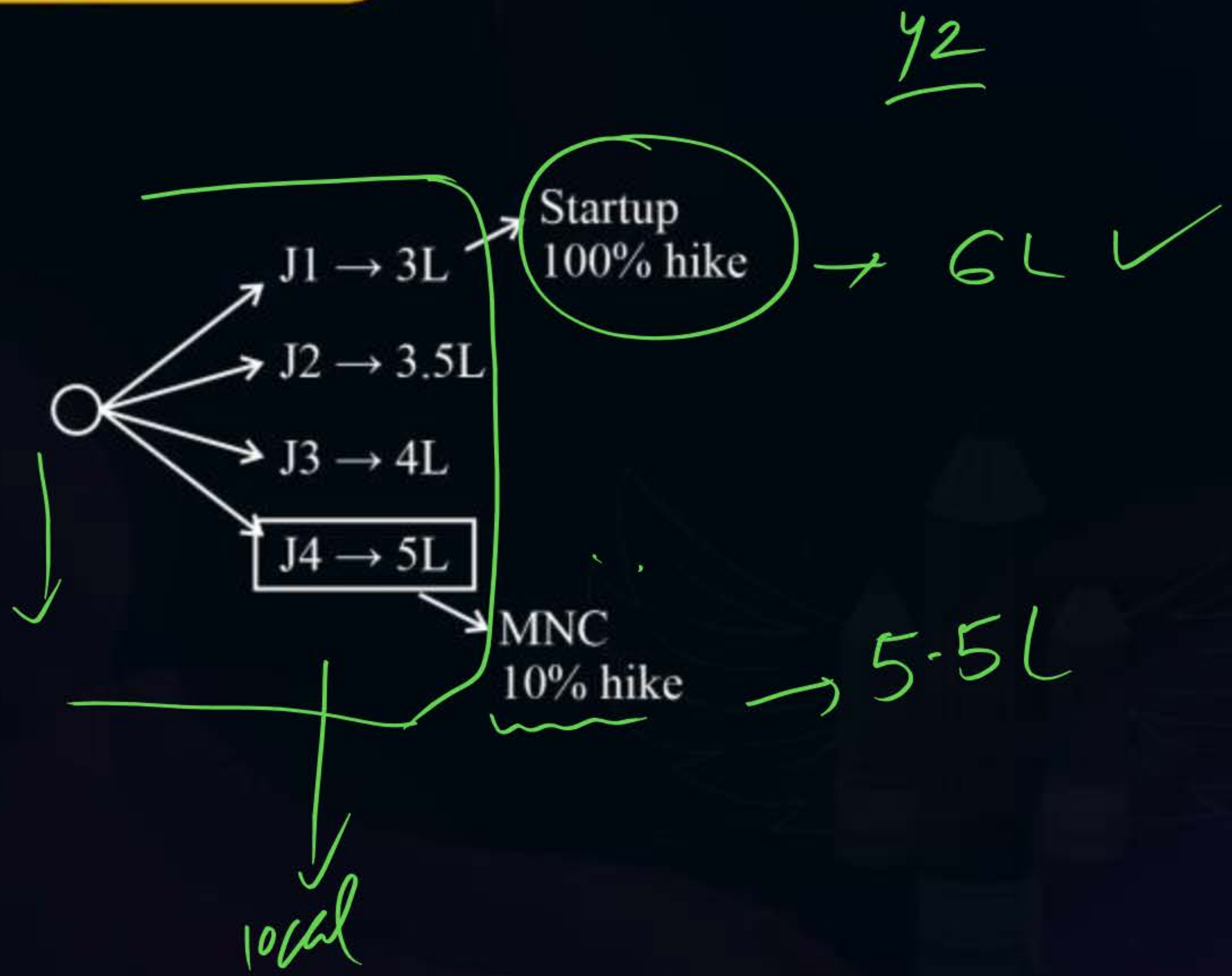
C ~~$O(\log_a \log_b n)$~~

D ~~$O(\log_{ab} n)$~~

Greedy Algo

Topic : Greedy Algorithm

Idea:



Topic : Greedy Algorithm

Steps:

(1) Used for problems whose solutions are viewed as a set/sequence of decisions.

(2) These decisions are made in a step-wise manner.

(3) At each step, out of all the available options, gradually select these option, which satisfies the criteria of the problem.

1 Knapsack

Topic : Knapsack Problem

Description:

(1) Given a Knapsack bag of capacity 'm'.

(2) Given n objects:

Object (i) → Weight (W_i)

Object (i) → Profit (P_i)

Object (i) → Decision (X_i)

Topic : Knapsack Problem

- You have to maximize the profit subject to the criteria that the total weight put into the knapsack does not exceed its capacity (M).

Topic : Knapsack Problem

Knapsack Problem

Fractional Knapsack (Pickle/Aachar)

- Partial objects are allowed
- Greedy
- $0 \leq x_i \leq 1$

$x_i = 0.5$

Binary Knapsack (Laptop)

$x_i = \{0, 1\}$

- No partial objects
- Dynamic Programming

1/0

Topic : Knapsack Problem

$O_1 \rightarrow P_1 \quad W_1 \quad X_1$

$O_2 \rightarrow P_2 \quad W_2 \quad X_2$

$O_3 \rightarrow P_3 \quad W_3 \quad X_3$

$\cancel{\Sigma P_i \rightarrow \text{maximize}}$

$\Sigma x_i * P_i \rightarrow \text{Maximize}$

Subject to the criteria:

$\Sigma x_i * W_i \leq M$

ϕ

Topic : Knapsack Problem

Decision x_i on object O_i

Profit: $x_i * P_i$

Weight: $x_i * W_i$

Topic : Knapsack Problem

Example:

Fractional Knapsack:

No. of objects, $n = 3$

Knapsack Capacity, $M = 20$

$$[w_1\ w_2\ w_3] = [18, 15, 10]$$

$$[p_1\ p_2\ p_3] = [25, 24, 15]$$

1) Max profit

2) min wt.

Profit : ?

int( )

Topic : Knapsack Problem

Aim: Maximize Profit:

Prefer objects with max Profit:

	1	2	3
P_i :	25	24	15
W_i :	18	15	10
X_i :	1	$\frac{2}{15}$	0

$m = 20 - 18$

$= 2 - 2 = 0$

$15 \longrightarrow 24$

$1 \longrightarrow \frac{24}{15}$

$2 \longrightarrow 2 \times \frac{24}{15}$

Topic : Knapsack Problem

$M = 20$

$18 \leq 20$

$M = 20 - 18 = 2$

$M = 2 - 2 = 0$

Hence Total profit by this approach

$$\boxed{= \sum_{i=1}^{3} P_i * x_i}$$

$$= P_1 \times x_1 + P_2 \times x_2 + P_3 \times x_3$$

$$= 25 * 1 + 24 * \frac{2}{15} + 15 * 0$$ $= 25 + 3.2 = 28.2$

$$= 28.2$$

Topic : Knapsack Problem

Approach 2:

Greedy about weight: Prefer less weight objects first

P_i :	25	24	15
	3	2	1
W_i :	18	15	10
X_i :	0	$\frac{10}{15}$	1

$M = 20$

$= 20 - 10$

$= 10 - 10 = 0$

$O \to w$, $O \to P$

Topic : Knapsack Problem

$M = 20$

$M = 20 - 10 = 10$

$M = 10 - 10 = 0$

Hence Total profit by this approach

$$= \sum_{i=1}^{3} P_i * x_i$$

$$= P_1 \times x_1 + P_2 \times x_2 + P_3 \times x_3$$

$$= 0 * 25 + 24 * \frac{10}{15} + 1 * 15$$

$$= 31$$

$0 \rightarrow 1$

$0 \rightarrow 1$

$0 \rightarrow 1$

Topic : Knapsack Problem

Approach 3:

Combines both logic (optimal): Profit per unit weight

	Wt	Profit
O1 →	1	P_1
O2 →	1	P_2
O3 →	1	P_3

Topic : Knapsack Problem

Weight adjusted Profits:

Profit per unit weight: P_i/W_i

$01 \rightarrow W_1 \rightarrow P_1$	$1 \rightarrow P_1/W_1 \rightarrow$ 25/18 = 1.38	3
$01 \rightarrow W_2 \rightarrow P_2$	$1 \rightarrow P_2/W_2 \rightarrow$ 24/15 = 1.6	1
$01 \rightarrow W_3 \rightarrow P_3$	$1 \rightarrow P_3/W_3 \rightarrow$ 15/10 = 1.5	2

$20 - 15 = 5$

$x_2 = 1$

$x_3 = 0.5$

$x_1 = 0$

Topic : Knapsack Problem

$M = 20$

$M = 20 - 15 = 5$

$M = 5 - 5 = 0$

Hence Total profit by this approach

$$= \sum_{i=1}^{3} P_i * x_i$$

$$= P_1 \times x_1 + P_2 \times x_2 + P_3 \times x_3$$

$$= 25 * 0 + 24 * 1 + 15 * \frac{1}{2}$$

$$= 31.5$$

[NAT]

#Q. Find max/optimal solution to the fractional knapsack problem:

Given: n = 7, M = 15

Object	Weight	Profit
01	4	18
02	1	3
03	3	5
04	7	7
05	1	6
06	5	15
07	2	10

P/W $\qquad x_i$

$18/4 = 4.5$ — 3 — 1

$3/1 = 3$ — 5 — 1

$5/3 = 1.67$ — 6 — $2/3$

$7/7 \to 1$ — 7 — 0

$6/1 \to 6$ — 1 — 1

$15/5 \to 3$ — 4 — 1

$10/2 \to 5$ — 2 — 1

int(ans)

$m = 15 - 1$
$= 14 - 2$
$= 12 - 4$
$= 8 - 5$
$= 3 - 1$
$= 2 - 2 = 0$

$\underline{18+3} + 5 \times 2/3 + 0 + \underbrace{6+15+10}_{\to 31} = 52 + 3.33 = 55.33$

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HW

1. Greedy abt max Profit
2. Greedy about min Weight.

THANK - YOU