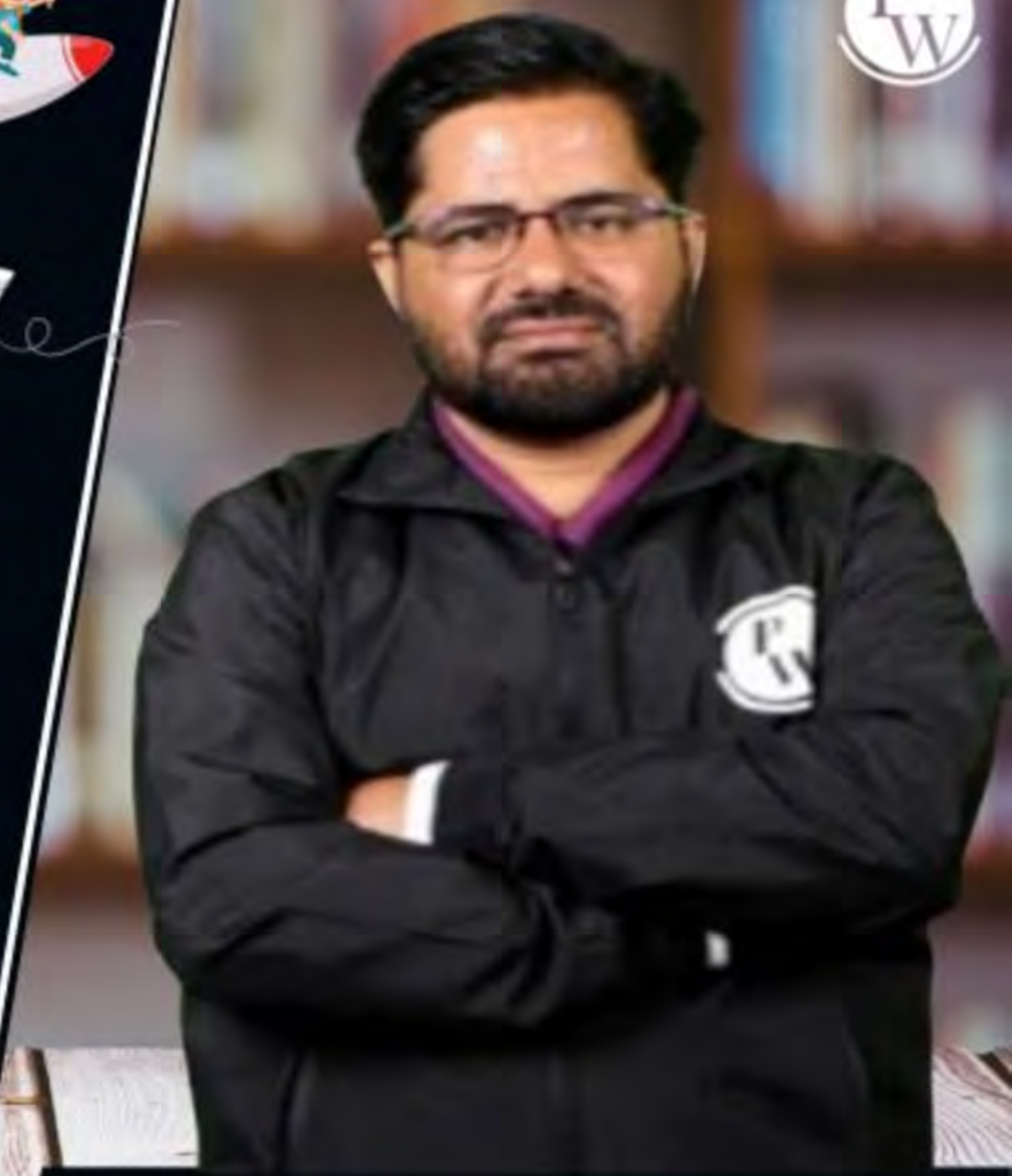
DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE
& CS & IT

Linear Algebra

Lecture No. 01



By- Dr. Puneet Sharma Sir

Topics to be Covered

Topic

→ BASICS of Determinants

Start: 10^{th} March, END - 31^{st} MAY, 7:00 - 9:30 AM (SUNDAY off)

Syllabus	ESE	GATE (EC EE IN CE ME CH PI XE)
(1) Linear Algebra	✓	✓
(2) Calculus	✓	✓
(3) V. Calculus	✓	✓
(4) Complex	✓	Not for CE
(5) D-Equn	✓	✓
(6) N Tech	✓	✓
(7) LT & FS	✓	Not for EC/EE
(8) Prob & Stats	✓	✓

Don't Judge yourself before solving P.Y.Q.

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Strategy:
1. Live Class.
2. Revision
3. Short Notes (in Later Phase)
4. D.P.P
5. Chapterwise test (Sund)
6. P.Y.Q. → Judge

Book: → No Book is Needed only PYQ Book is required.

eg: L-Algebra: Class (150-200 Q) | DPP (70-90) | WT (13 Q) | OTS 100 Q | PYQ (200 Q) | 700-800 Q

Doubts:
→ Conceptual Doubts → you can ask anytime.
→ Generic Doubt → will be discussed after class.

Parachute Landing → Conceptual Doubts are also not allowed.

PRERUISITE of Engg Maths: → ✓ (25 Lectures)
(Foundation Series of Engg Maths)

⊛ Engg Maths — CS: $\frac{7-8}{100}$ — DSAI: $\frac{40+}{100}$

⊛ Maths: is the Language of Engg.

⊛ Maths
- → Language → understanding of Symbols.
- → Concept
  - → Information Based Concept (No Brain should be used)
  - → Analysis Based Concept (Pure Maths) (useful to develop Neurons)
  - → Application Based Concept. (Applied Maths) in LIVE Class.

⊛ M.M. Imp Point → Try to have Patience as much as possible

Determinants

$\begin{vmatrix} + & - \\ - & + \end{vmatrix}$, $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$, $\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{vmatrix}$

$\boxed{\text{sign of } a_{ij} = (-1)^{i+j}}$ $\forall\, i \,\&\, j$

eg sign of $a_{23} = (-1)^{2+3} = -ve$

eg sign of $a_{42} = (-1)^{4+2} = +ve$.

(*) $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

eg $|A| = \begin{vmatrix} 2 & -4 \\ 5 & 3 \end{vmatrix} = (2)(3) - (5)(-4) = 6 + 20 = 26$

eg $|A| = \begin{vmatrix} 2+3i & -i \\ i & 2-3i \end{vmatrix} = ?$

where $i = \sqrt{-1}$

$= (2+3i)(2-3i) - (i)(-i)$

$= 2^2 - (3i)^2 + i^2$

$= 4 - 9i^2 + i^2 = 4 - 9(-1) + (-1) = 4 + 9 - 1 = 12$

eg $|A| = \begin{vmatrix} 2 & -3 & 4 \\ -1 & 2 & 0 \\ 3 & 4 & 1 \end{vmatrix} = ? = -(-1)\begin{vmatrix} -3 & 4 \\ 4 & 1 \end{vmatrix} + (2)\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} - (0)\begin{vmatrix} 2 & -3 \\ 3 & 4 \end{vmatrix}$

$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

$= 1[-3-16] + 2[2-12] - 0 = -39$ Ans

Shortcut (SARRUS Method) $\rightarrow$

$$\begin{matrix} 2 & -3 & 4 & 2 & -3 \\ -1 & 2 & 0 & -1 & 2 \\ 3 & 4 & 1 & 3 & 4 \end{matrix}$$

$|A| = \left(4 + 0 + (-16)\right) - \left(24 + 0 + 3\right) = -39$

eg $|A| = \begin{vmatrix} 1 & 4 & 16 \\ 1 & 3 & 9 \\ 1 & 2 & 4 \end{vmatrix} = ? \Rightarrow \begin{matrix} 1 & 4 & 16 & 1 & 4 \\ 1 & 3 & 9 & 1 & 3 \\ 1 & 2 & 4 & 1 & 2 \end{matrix} \Rightarrow |A| = [12 + 36 + 32] - \{48 + 18 + 16\}$

$= 80 - 82 = -2$

M-II $|A| = \begin{vmatrix} 1 & 4 & 4^2 \\ 1 & 3 & 3^2 \\ 1 & 2 & 2^2 \end{vmatrix} = ? = (4-3)(3-2)(2-4) = (1)(+1)(-2) = -2$

Q. $|A| = \begin{vmatrix} 1 & -2 & 3 & 0 \\ 0 & 1 & 2 & 4 \\ -1 & 2 & 0 & 3 \\ 0 & 2 & -2 & 1 \end{vmatrix} = ?$

$\begin{vmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{vmatrix}$

$= (+1)\begin{vmatrix} 1 & 2 & 4 \\ 2 & 0 & 3 \\ 2 & -2 & 1 \end{vmatrix} - (0)\begin{vmatrix} & ? & \end{vmatrix} + (-1)\begin{vmatrix} -2 & 3 & 0 \\ 1 & 2 & 4 \\ 2 & -2 & 1 \end{vmatrix} - (0)\begin{vmatrix} & ? & \end{vmatrix}$

$= 1\left[-2\{2+8\} - 3\{-2-4\}\right] - 0 - (1)\left[(-2)\{2+8\} - 3\{1-8\}\right] - 0$

$= -20 + 18 - [-20 + 21] = -2 - 1 = -3$

Q. $|A| = \begin{vmatrix} -3 & 1 & 1 & 2 \\ 1 & -3 & 1 & 2 \\ 1 & 1 & -3 & 2 \\ 2 & 1 & 1 & -3 \end{vmatrix} = ?$

M-I Directly open it using conventional Approach.

(TIME CONSUMING APP)

M-II $C_1 \rightarrow C_1 + C_2 + C_3 + C_4$

$\begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & -3 & 1 & 2 \\ 1 & 1 & -3 & 2 \\ 1 & 1 & 1 & -3 \end{vmatrix} \xrightarrow[R_4 \rightarrow R_4 - R_1]{R_2 \rightarrow R_2 - R_1,\ R_3 \rightarrow R_3 - R_1} \begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & -5 \end{vmatrix} = 1[(-4)(-4)(-5)]$

$= -80$

The determinant value of the matrix

$$\begin{vmatrix} 13 & 2 & 1 & 3 \\ 31 & 4 & 5 & 6 \\ 26 & 3 & 7 & 4 \\ 10 & 1 & 3 & 2 \end{vmatrix}$$ is ______.

(a) 55 (b) 101

(c) 126 (d) −10

$$\xrightarrow{C_2 \to C_2 - C_4} \begin{vmatrix} 13 & -1 & 1 & 3 \\ 31 & -2 & 5 & 6 \\ 26 & -1 & 7 & 4 \\ 10 & -1 & 3 & 2 \end{vmatrix} \xrightarrow[\substack{R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1}]{R_2 \to R_2 - 2R_1} \begin{vmatrix} 13 & -1 & 1 & 3 \\ 5 & 0 & 3 & 0 \\ 13 & 0 & 6 & 1 \\ -3 & 0 & 2 & -1 \end{vmatrix}$$

$$= (-1) \begin{vmatrix} 5 & 3 & 0 \\ 13 & 6 & 1 \\ -3 & 2 & -1 \end{vmatrix}$$

$$= (+1)\left[+5\{-6-2\} - 3\{-13+3\}\right]$$

$$= +\left[-40+30\right] = -10$$

Q $|A| = \begin{vmatrix} 3 & -3 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 \\ 0 & 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 3 & -3 \\ -3 & 0 & 0 & 0 & 3 \end{vmatrix}_{5\times 5} = \underline{? = 0}$

$\xrightarrow{C_1 \to C_1 + (C_2 + C_3 + C_4 + C_5)} \begin{vmatrix} 0 & - & - & - & - \\ 0 & - & - & - & - \\ 0 & - & - & - & - \\ 0 & - & - & - & - \\ 0 & - & - & - & - \end{vmatrix}_{5\times 5}$ = Expanding along C_1

$= 0$

Q $|A| = \begin{vmatrix} 2 & -1 & 2 & 3 \\ -3 & 2 & -3 & 2 \\ 4 & 1 & 4 & 1 \\ 0 & 4 & 0 & 4 \end{vmatrix}_{4\times 4} = ? \xrightarrow{C_1 \to C_1 - C_3} \begin{vmatrix} 0 & - & - & - \\ 0 & - & - & - \\ 0 & - & - & - \\ 0 & - & - & - \end{vmatrix} = 0$

Elementary Operations →

E-Row Operations

(i) $R_i \leftrightarrow R_j$

(ii) $R_i \to kR_i$

(iii) $R_i \to R_i + kR_j$

E-Column Operations

(i) $C_i \leftrightarrow C_j$

(ii) $C_i \to kC_i$

(iii) $C_i \to C_i + kC_j$

$R_3 \to R_3 + 5R_2$ (E-operation of 3rd type)

$R_3 \to 5R_3 + R_2$ (it is not an E-operation, it is Mixed operation

$R_3 \to 5R_3$ & then $R_3 \to R_3 + R_2$)

Note ① we are free to apply only $\underline{3^{rd}}$ E-operations while solving determinant

② If we interchange any two Rows (or any two Columns) in a Matrix then value of it's determinant changes by $\underline{-ve}$ sign GATE

③ If we multiply with constant K in any Row (or in any Column) in a Mat then value of it's determinant changes by $\underline{K}$ times

i.e we are not free to Multiply by any constant in Determinant

$\boxed{\text{But}}$ we can take out Common Rowwise or Columnwise

PROPERTIES of Determinants →

① If in a Mat all the elements in a Row (or in a Column) are zero then it's $\text{Det} = 0$

② If " " any two Rows (or any two Columns) are identical then it's $\text{Det} = 0$

③ $|ABC| = |A| \cdot |B| \cdot |C|$

④ $|A^m| = |A|^m$, $m \in N$.

⑤ $|A^T| = |A|$

⑥ $|(A+B+C)| \neq |A| + |B| + |C|$

No formula exist, Calculate Manually

⑦ $|A|^{-1} = \dfrac{1}{|A|}$ for eg, if $|A| = 5$ then $|A|^{-1} = ? = \dfrac{1}{5}$

⑧ $|A^{-1}| = \dfrac{1}{|A|}$ or $\boxed{\det(A^{-1}) = \dfrac{1}{\det(A)}}$

where $A^{-1} = ? = \dfrac{1}{A}$ PAAP, $A^{-1} = \dfrac{\text{adj} A}{|A|} = \dfrac{(\text{Cof} A)^T}{|A|}$

(9) Area of $\triangle ABC$ formed by points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$:→

$$\text{Area} = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$$

eg Area of $\triangle$ formed $(1,0), (2,2), (4,3)$?

$$A = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{vmatrix} = \ldots\ldots = -3/2$$

So Req Area $= \frac{3}{2}$ Ans

(10) Differentiation of Determinant :

$$\frac{d}{dx}\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = ? = \begin{vmatrix} a' & b' & c' \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ d' & e' & f' \\ g & h & i \end{vmatrix} + \begin{vmatrix} a & b & c \\ d & e & f \\ g' & h' & i' \end{vmatrix}$$

(11) $\begin{vmatrix} a+l & b & c \\ d+m & e & f \\ g+n & h & i \end{vmatrix} = ? = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} l & b & c \\ m & e & f \\ n & h & i \end{vmatrix}$

& $\begin{vmatrix} (a+l) & (b+m) & (c+n) \\ d & e & f \\ g & h & i \end{vmatrix} = ? = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} l & m & n \\ d & e & f \\ g & h & i \end{vmatrix}$

eg $\begin{vmatrix} 1+1 & -3 & 0 \\ 2+3 & 1 & 2 \\ 2+2 & 0 & 3 \end{vmatrix} = ? = \begin{vmatrix} 1 & -3 & 0 \\ 2 & 1 & 2 \\ 2 & 0 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -3 & 0 \\ 3 & 1 & 2 \\ 2 & 0 & 3 \end{vmatrix}$

$= 27$ $\qquad = 9 \qquad = 18$

(12) $A=\begin{bmatrix}1 & -2 & 3\\ 2 & 1 & 4\\ 0 & -1 & 2\end{bmatrix}$ & $2A=\begin{bmatrix}2 & -4 & 6\\ 4 & 2 & 8\\ 0 & -2 & 4\end{bmatrix} = 2\cdot\begin{bmatrix}1 & -2 & 3\\ 2 & 1 & 4\\ 0 & -1 & 2\end{bmatrix}$

$\Rightarrow |A| = 8$ $\qquad |2A| = 64 = 8.8 = 2^3.|A|$

ie $|2A| = \begin{vmatrix}2 & -4 & 6\\ 4 & 2 & 8\\ 0 & -2 & 4\end{vmatrix} = 2\times2\times2\begin{vmatrix}1 & -2 & 6\\ 2 & 1 & 4\\ 0 & -1 & 2\end{vmatrix} = 2^3.|A|$

M. Imp

(*) if $A_{n\times n}$ & K is any scalar then

$[KA] = K[A]$

$\boxed{|KA| = K^n.|A|}$

(13) Max Number of terms in the General Expansion of $|A|_{n\times n} = ? = n!$ term.

Gate.

(a) n

(b) n^2

(c) $2n$

(d) $n!$ ✓

eg $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}_{2\times 2} = ad - bc$ i.e Max term $= 2$ terms $= 2!$

eg $|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}_{3\times 3} = a(ei - hf) - b(di - gf) + c(dh - eg)$

i.e Max term $= 6$ terms $= 3!$ terms.

eg $|A| = \begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix}_{4\times 4} = a\begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix} - b\begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix} + c\begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix} - d\begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix}$

$= (6 \text{ term}) + (6 \text{ term}) + (6 \text{ term}) + (6 \text{ terms})$

i.e Max term $= 24$ terms $= 4!$ & so on....

(*) Total Number of terms in Mat $A_{n\times n} = ? = n^2$

HWQ ① $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = ? = (a-b)(b-c)(c-a)$ Learn

HWQ ② $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = ? = (a-b)(b-c)(c-a)(a+b+c)$

CW ③ $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = ? = -(a+b+c)[a^2+b^2+c^2-ab-bc-ca]$

OR

$= -(a^3+b^3+c^3-3abc)$

OR

$= -\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$

Proof ③ $|A| = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \xrightarrow{C_1 \to C_1 + C_2 + C_3} \begin{vmatrix} (a+b+c) & b & c \\ (b+c+a) & c & a \\ (c+a+b) & a & b \end{vmatrix} = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$

$\xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} (a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = (a+b+c)\left[(c-b)(b-c) - (a-b)(a-c)\right]$

$|A| = -\left[\underset{\text{Ans}}{a^3+b^3+c^3-3abc}\right] = -\left[\underset{\text{Ans}}{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}\right]$

$= -\frac{1}{2}\left[(a+b+c)(2a^2+2b^2+2c^2-2ab-2bc-2ca)\right]$

$= -\frac{1}{2}(a+b+c)\left[(a^2+b^2-2ab)+(b^2+c^2-2bc)+(c^2+a^2-2ca)\right]$

$= -\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$ Ans

If $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ then which of the following is a factor of Δ.

(a) $a + b$

(b) $a - b$

(c) abc

(d) $a + b + c$

$R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & (a-c)b \end{vmatrix} = (a-b)(a-c)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & -1 & b \end{vmatrix} \xrightarrow{R_3 - R_2} (a-b)(a-c)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & 0 & b-c \end{vmatrix}$$

$$= (a-b)(b-c)(c-a)$$

Qs if $A=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$ then $|A^3|=?$

Sol: $|A|=(3)(3)-(2)(2)=5$

so $|A^3|=|A|^3=(5)^3=125$

(using Prop 4)

Qs If $A=\begin{bmatrix}\alpha & 2\\ 2 & \alpha\end{bmatrix}$ s.t $|A^3|=125$

then $\alpha=?$

(a) 3 (b) -3 (c) ± 3 (d) 0

Sol: $|A^3|=125$

$|A|^3=(5)^3$

$|A|=5$

$\because |A|=5$

$\alpha^2-4=5$

$\alpha^2=9 \Rightarrow \alpha=\pm 3$

Qs if $A=\begin{bmatrix}1 & -\tan x & -\sec x\\ \tan x & 0 & 3\\ \sec x & 3 & \sin x\end{bmatrix}_{3\times 3}$

then $|A^T \cdot A^{-1}|=?$

$=|A^T|\cdot|A^{-1}|$ (using Prop 3)

$=|A|\cdot\dfrac{1}{|A|}$ (using Prop 5 & 8)

$=1$

Qs if $|A|_{3\times3} = 6$ then ① $|(2A)^{-1}| = ?$ ② $|2\cdot A^{-1}| = ?$

$\because A_{3\times3} \Rightarrow (A^{-1})_{3\times3}$ Sol ① $|(2A)^{-1}| = \dfrac{1}{|2A|} = \dfrac{1}{2^3\cdot|A|} = \dfrac{1}{8\times6} = \dfrac{1}{48}$

Sol ② $|2\cdot A^{-1}| = 2^3\cdot|A^{-1}| = 2^3\cdot\dfrac{1}{|A|} = \dfrac{8}{6} = \dfrac{4}{3}$

Qs if $|A|_{3\times3} = -1$, $|B|_{3\times3} = 4$ then $|5AB| = ?$ (a) 320 (b) -20 (c) -500 (d) -320

Sol: $\because A_{3\times3}$ & $B_{3\times3} \Rightarrow (AB)_{3\times3}$

$|5AB| = 5^3\cdot|A\cdot B|$

$= 125\ |A|\cdot|B|$

$= 125(-1)(4) = -500$

If A and B are two matrices of order 3×5 and 5×3 respectively then determinant of the matrix $4BA$ equals

(a) $4|B|/|A|$ (b) $4^3|B||A|$

(c) $4^5|A||B|$ (d) None $= 4^5 \cdot |BA|$

The concept of Determinant is valid only for Sq Mat

& A & B are Rectangular

so $|A| = DNE$

$|B| = DNE$

$\because B_{5\times 3} A_{3\times 5} = (BA)_{5\times 5}$

so $|4BA| = 4^5|BA| = 4^5 \cdot |B| \cdot |A|$

(d)

Let a 3×3 matrix A have determinant value 5. If $B = 4A^2$ then the determinant valueo f B is equal to ______.

(a) 20 (b) 100

(c) 320 (d) 1600

$|A| = 5$

$|A^2| = |A|^2 = 5^2 = 25$

$\& \ (A^2)_{3\times 3}$

$B = 4A^2$ then

$|B| = |4A^2| = 4^3 |A^2|$

$= 64 \cdot |A|^2 = 64(5)^2 = 6400.$

Q. Find the area of the $\triangle$ formed by $(1,0)$, $(2,2)$, $(4,3)$?

Sol: Area $= \frac{1}{2}\left|\begin{matrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{matrix}\right| = \cdots\cdots = -3/2$ So Req Area $= +\frac{3}{2}$.

Q. Show that $(a, b+c)$, $(b, c+a)$, $(c, a+b)$ are collinear.

Sol: $(a, b+c) = A$, $(b, c+a) = B$, $(c, a+b) = C$

Points A, B, C are collinear if Area of the $\triangle ABC = 0$

So Area $= \frac{1}{2}\left|\begin{matrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{matrix}\right| = \frac{1}{2}\left|\begin{matrix} (a+b+c) & b+c & 1 \\ (a+b+c) & c+a & 1 \\ (a+b+c) & a+b & 1 \end{matrix}\right| = \frac{(a+b+c)}{2}\left|\begin{matrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{matrix}\right|$

$= 0$ Hence proved

Q. If $\Delta = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2 & 3x \\ 0 & 2 & 5x \end{vmatrix}$ then $\dfrac{d\Delta}{dx} = ?$

Sol: $\dfrac{d\Delta}{dx} = \begin{vmatrix} 1 & 2x & 3x^2 \\ 1 & 2 & 3x \\ 0 & 2 & 5x \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ 0 & 0 & 3 \\ 0 & 2 & 5x \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2 & 3x \\ 0 & 0 & 5 \end{vmatrix}$

$= 1[10x - 6x] - 1[10x^2 - 6x^2] + (-3)[2x] + 5[2x - x^2]$

$= 4x - 4x^2 - 6x + 10x - 5x^2$

$-9x^2 + 8x$

Q. (MSQ) If $f(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2 & 3x \\ 0 & 2 & 5x \end{vmatrix}$ s.t. $f'(x) = -1$ then $x = ?$

(a) 1 (b) −1

(c) $-1/9$ (d) 0

$-9x^2 + 8x = -1$

$9x^2 - 8x - 1 = 0$

$(9x+1)(x-1) = 0$

$x = 1$

$x = -1/9$

Which one of the following does NOT equal

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} ? = |A| = (x-y)(y-z)(z-x)$$

(a) $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix} = -|A|$

(b) $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix} = |A|$

(c) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix} = |A|$

(d) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix} = |A|$

Taking (a):

$$\begin{vmatrix} 1 & x^2+x & x+1 \\ 1 & y^2+y & y+1 \\ 1 & z^2+z & z+1 \end{vmatrix} = \begin{vmatrix} 1 & x^2 & x+1 \\ 1 & y^2 & y+1 \\ 1 & z^2 & z+1 \end{vmatrix} + \begin{vmatrix} 1 & x & x+1 \\ 1 & y & y+1 \\ 1 & z & z+1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} + \begin{vmatrix} 1 & x^2 & 1 \\ 1 & y^2 & 1 \\ 1 & z^2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & x & x \\ 1 & y & y \\ 1 & z & z \end{vmatrix} + \begin{vmatrix} 1 & x & 1 \\ 1 & y & 1 \\ 1 & z & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x^2 & x \\ 1 & y^2 & y \\ 1 & z^2 & z \end{vmatrix} + 0 + 0 + 0$$

$$= -\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = -|A| \quad \text{ie (a)}$$

Taking (d): $|A| = \begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z+0 & z^2+0 \end{vmatrix} = \begin{vmatrix} 2 & x & x^2+y^2 \\ 2 & y & y^2+z^2 \\ 1 & z & z^2+0 \end{vmatrix} + \begin{vmatrix} 2 & y & x^2+y^2 \\ 2 & z & y^2+z^2 \\ 1 & 0 & z^2+0 \end{vmatrix}$

$$= \begin{vmatrix} 2 & x & x^2 \\ 2 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 2 & x & y^2 \\ 2 & y & z^2 \\ 1 & z & 0 \end{vmatrix} + \begin{vmatrix} 2 & y & x^2 \\ 2 & z & y^2 \\ 1 & 0 & z^2 \end{vmatrix} + \begin{vmatrix} 2 & y & y^2 \\ 2 & z & z^2 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & x-2z & x^2-2z^2 \\ 0 & y-2z & y^2-2z^2 \\ 1 & z & z^2 \end{vmatrix} + \begin{vmatrix} 0 & x-2z & y^2 \\ 0 & y-2z & z^2 \\ 1 & z & 0 \end{vmatrix} + \begin{vmatrix} 0 & y & x^2-2z^2 \\ 0 & z & y^2-2z^2 \\ 1 & 0 & z^2 \end{vmatrix} + \begin{vmatrix} 0 & y & y^2 \\ 0 & z & z^2 \\ 1 & 0 & 0 \end{vmatrix}$$

$= \text{- - - - - - - - - -} = |A|$ so (d) ✗

(M-II) $|A| = \begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix} \xrightarrow{\text{use E-operations}} \text{- - -} = |A|$ so (d) ✗

The value of the determinant $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$ will be ______.

(a) $2a^2b^2c^2$ (b) $-2a^2b^2c^2$

(c) $4a^2b^2c^2$ (d) $-4a^2b^2c^2$

$= (abc)\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$ $R_2 \to R_2 + R_1$, $R_3 \to R_3 + R_1$

$= (abc)\begin{vmatrix} -a & b & c \\ 0 & 0 & 2c \\ 0 & 2b & 0 \end{vmatrix}$

$= (abc)\left[(-a)\{0 - 4bc\}\right]$

$= (abc)(4abc) = 4a^2b^2c^2$

M-II $|A| = abc\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$

$= (abc)(abc)\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$= (a^2b^2c^2)(4)$

If A and B are matrices of determinant 1 then

(a) Determinant of A + B is 2

(b) Determinant of A + B is 1

(c) Determinant of A + B is 0

(d) Nothing can be concluded about the determinant of A + B

ATQ, $|A| = 1$ & $|B| = 1$ (given)

so $|A+B| = ? \neq |A| + |B|$

while $|A+B| \leq |A| + |B|$

$|A+B| \leq 1 + 1$

or $|A+B| \leq 2$

The determinant of a matrix has 720 terms (in the unsimplified form). The order of the matrix is ____.

(a) 5 (b) 6 (c) 7 (d) 8

Let A is of order $n \times n$

then A.T.Q,

$$n! = 720$$

$$n! = 6!$$

$$n = 6$$

So $A_{6\times 6}$

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Thank you

Keep Hustling!