GATE

CS & DA

DPP: 2

Probability and Statistics

Q1 A jar contains three marbles numbered 1, 2, and 3. If two marbles are drawn without replacement, what is the probability that the sum of the numbers is 5?

Q2 A jar contains three marbles numbered 1, 2, and 3. If two marbles are drawn with replacement, what is the probability that the sum of the numbers is at least 4 ?

Q3 If two dice are rolled, find the probability that the sum of the faces of the dice is 7.

Q4 Two dice are thrown at a time , find the probability of the following :
(i) The number shown are equal
(ii) The difference of number shown is 1

Q5 Three coins are tossed together, then find the probabilities of the following :
(i) Getting exactly two head
(ii) Getting at least two tails.

Q6 A box contains cards numbered 3,5,7,9,....35, 37. A card is drawn at random form the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Q7 The probability that a person will get an electrification contract is 3/5 and the probability that he will not get plumbing contract is 5/8. The probability of getting atleast one contract is 5/7. What is the probability that he will get both ?

Q8 Neha has 4 yellow t-shirts, 6 black t-shirts, and 2 blue t-shirts to choose for her outfit today. She chooses a t-shirt randomly with each t-shirt equally likely to be chosen. Find the probability that a black or blue t-shirt is chosen for the outfit.
(A) $\frac{8}{13}$ (B) $\frac{5}{6}$
(C) $\frac{2}{3}$ (D) $\frac{7}{12}$

Q9 The probabilty of an event A occurring is 0.5 and of B occurring is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B occuring is :
(A) 0.6 (B) 0.5
(C) 0.7 (D) 0.2

Q10 A and B are two events and $\overline{A}$ denotes the complements of A.
Consider the following statements :
(i) $P(A \cup B) \leq P(B) + P(A)$ (ii) $P(A) + P(\overline{A} \cup B) \leq 1 + P(B)$
Which of the above statements is/are correct ?

(A) Only I
(B) Only II
(C) Both I and II
(D) Neither I nor II

Q11 Three intergers are chosen at random from the first 20 integers. The Probability that their product is even ?
(A) 2/19 (B) 3/29
(C) 17/29 (D) 4/29

Q12 If the letters of word 'REGULATIONS' be arranged at random, the probability that there will be exactly 4 letters between R and E is :
(A) 6/55 (B) 3/55
(C) 49/55 (D) None of these

Q13 A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match both of them win a prize. The probability that they will not win a prize in a single trial is :

(A) 1/25 (B) 24/25

(C) 2/25 (D) None of these

Q14 The probability that out of 10 persons, all born in April, at least two have the same birthday is :

(A) $\frac{^{30}C_{10}}{(30)_{10}}$

(B) $1-\frac{^{30}C_{10}}{(30)!}$

(C) $\frac{(30)^{10}-{}^{30}C_{10}}{(30)^{10}}$

(D) None of these

Q15 A bag contains 5 white and 8 red balls. Two successive drawings of 3 balls are made such that the balls are replaced before the second drawing. Find the probability that the first drawing will give 3 white and the second 3 red balls in each case.

Q16 A bag contains 5 white and 3 red balls and four balls are sucessively drawn and are not placed. What is the chance that (i) white and red balls appear alternatively .

Q17 Suppose a box contains 10 white balls and 8 black balls. What is the probability of drawing 2 white and 1 black balls for the following three cases ?

a)All the three balls picked in a single draw

b)All the three balls drawn one after another with (replacement case)

c)All the three balls picked one after another (without replacement case)

Q18 A can hit a target 4 times in 5 shots . B 3 times in 4 shots and C twice in 3 shots. They fire a volley. What is the probability that :

(a) Two shots hit the target.

(b) At least two shots hit the target.

Q19 A problem is given to students A, B, C whose chances of solving it is $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}$. What is the probability that the problem will be solved ?

Q20 There are two packs of card. One card is drawn at random from each pack. What is the probability that :

(i) Both of them are black (ii) They are of different in colour

Q21 In a batch Of 100 resistors Of 1kiloohm resistance, 80 numbers are within the required tolérance values and II numbers are below the required tolerance values, the remaining are above the required tolerance values. If two resistors are drawn one after the other without replacement, the probability of the first one drawn is below and the second one drawn is above the required tolerance value is :

(A) 0.01 (B) 0.09

(C) 0.11 (D) 0.89

Q22 A bag contains 5 white and 4 black balls. A ball is drawn from this bag and is replaced and then second draw of a ball is made. What is the probability that two balls are of different colors.

Q23 In a clase 40% student read mathematics, 25% Biology and 15% both Mathematics and Biology. One student was selected at random. Find

(i) The probability that he reads Mathematics if it is known that he reads Biology.

(ii) The probability that he reads Biology if it is known that he reads Mathematics.

Q24 A dice is rolled twice and the sum of the numbers appearing on them is observed to be 7. What is the conditional probability that the number 2 has appeared at least once ?

Q25 A family has 2 children. Given that one of the children is a boy , what is the probability that the other child is also a boy ?

Q26 A coin is tossed once. If it shows head,it is tossed again and if ir showns tail, then a dice is thrown. Let E_1 be the event the first throw of coin shows tail and E_2 be the dice shows a number greater than 4. Find P $(E_2|E_1)$.

Q27 It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%.
Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email ?

Q28 Three persons A, B and C have applied for a job in a private company. The chance of their selections is in the ratio 1:2:4. The probabilities that A, B and C can introduce changes to improve the profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

Q29 A bag contains 4 balls. Two balls are drawn at random and are found to be blue. What is the probability that all balls in the bag are blue ?

Q30 In a bolt factory, machines A, B, C manufactures respectively 25%, 35%, 40% of the total bolts of there output and 5%, 4%, 2% are respectively defective bolts. A bolt is drawn at random from the product. What is the probability that the bolt drawn is defective ?

Q31 Suppose the test for HIV is 99% accurate in both directions and 0.3% of the population is HIV positive. If someone tests positive, what is the probability they actually are HIV positive ?

Q32 In a neighbourhood, 90% children were falling sick due flu and 10% due to measles and no other disease. The probability of observing rashes for measles is 0.95 and for flu is 0.08. If a child develops rashes, find the child's probability of having flu.

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Answer Key

Q1 1 / 3

Q2 2 / 3.

Q3 1/6.

Q4 $\left(\frac{5}{18}\right)$

Q5 $\frac{1}{2}$

Q6 $\mathrm{P}(\mathrm{A} \cup \mathrm{B})=5/6$

Q7 P(A n B) = 73 / 280

Q8 (C)

Q9 (B)

Q10 (C)

Q11 (D)

Q12 (A)

Q13 (B)

Q14 (C)

Q15 $=\frac{140}{20{,}449}$

Q16 $\frac{1}{14}$

Q17 $\frac{15}{34}, \frac{100}{729}, \frac{5}{34}$

Q18 $\frac{5}{6}$

Q19 $\frac{29}{32}$

Q20 $\frac{1}{2}$

Q21 (A)

Q22 $\frac{40}{81}$

Q23 $\frac{3}{8}$

Q24 $\frac{1}{3}$

Q25 1/3.

Q26 $\frac{1}{3}$

Q27 $\frac{5}{104}$

Q28 0.7

Q29 3/5.

Q30 0.00345

Q31 23%

Q32 0.43

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Hints & Solutions

Q1 Text Solution:

Since two marbles are drawn without replacement, the sample space consists of the following six possibilities.

S = {(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)}

Note that (1,1), (2,2) and (3,3) are not listed in the sample space. These outcomes are not possible when drawing without replacement, marble is drawn but not replaced into the jar, that marble is not available in the jar to be selected again on the second draw.

Let the event E represent that the sum of the numbers is five. Then

E = {(2, 3), (3, 2)}

Therefore, the probabilty of E is :

P(E) = 2/6 or 1/3.

Q2 Text Solution:

The sample space when drawing with replacement consists of the following nine possibilities.

S = (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)

Let the event F represent that the sum of the number is at least four. Then

F = (1, 3), (3, 1), (2, 3), (3, 2), (2, 2), (3, 3)

Therefore, the probability of F is :

P(F) = 6 / 9 or 2/3.

Note that in Example 8.1. 10 when we selected marbles with replacement , the probability is the same as in Example 8.1.8 where we selected marbles without replacement.

Q3 Text Solution:

Let E represent the event that the sum of the faces of two dice is 7.

The possible cases for the sum to be equal to 7 are: (1, 6), (2,5), (3, 4), (4, 3), (5, 2), and (6, 1)

So event E is :

E= {(1, 6), (2,5), (3, 4), (4, 3), (5, 2), (6, 1)}

The probability of the event E is

P(E) = 6/36 or 1/6.

Q4 Text Solution:

The sample space in a throw of two dice is :

$$S = \left\{\begin{matrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6.5) & (6,6) \end{matrix}\right\}$$

So, n(S) = $6 \times 6 = 36$

(i) Here E_1 = the event of showing equal number of both dice.

= [(1, 1),(2, 2),(3, 3),(4, 4),(5, 5),(6, 6)]

$\therefore$ n(E_1) = 6

$\therefore$ P(E_1) = $\frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) Here E_2 = the event of showing number whose difference is 1.

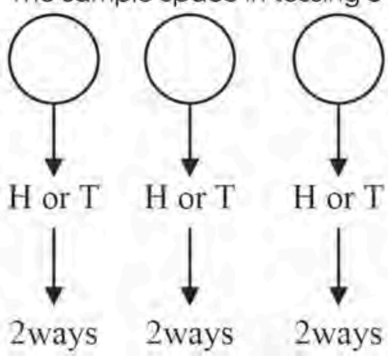
= {(1, 2),(2, 1),(2, 3),(3, 2),(3, 4),(4, 3),(4, 5),(5, 4),(5, 6),(6, 5)}

$\therefore$ n(E_2) = 10

$\therefore$ P(E_1) = $\frac{n(E_2)}{n(S)} = \frac{10}{36} = \left(\frac{5}{18}\right)$

Q5 Text Solution:

The sample space in tossing 3 coins.



So n(S) = 8

(i) Let E_1 = event corresponding to getting exactly two heads.

means E_1 = {HHT, THH, HTH}

$\therefore n(E_1) = 3$

$\Rightarrow \mathrm{P}(\mathrm{E}_1) = \frac{\mathrm{n}(\mathrm{E}_1)}{\mathrm{n}(\mathrm{S})} = \frac{3}{8}$

(ii) E_2 = {HTT, THT, TTH, TTT}

$\therefore n(E_2) = 4$

$\therefore \mathrm{P}(\mathrm{E}_2) = \frac{4}{8} = \frac{1}{2}$

Q6 Text Solution:

Cards in the box = {3, 5, 7, 9, ...37}

number of cards = n = [(1 - a) /d] + 1

n = [(37 - 3) / 2] + 1

n = (34 / 2) + 1

n = (34 / 2) + 1

n = 18.

n (S) = 18

Let " A " be the event of selecting a number which is multiple of 7.

A = {7, 14, 21, 28, 35}

n(A) = 5

P(A) = n(A) / n(S)

P(A) = 5/18

Let "B" be the even of selecting a prime number

B = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}

n(B) = 11

P(B) = n(B)/n(S)

P(B) = 11/18

A n B = {7}

n(A n B) = 1

P(A n B) = n (A n B) /n (S)

P(A n B) = 1/18

$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A\,n\,B})$

$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = (5/18) + (11/18) - (1/18)$

$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = 15/18$

$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = 5/6.$

Q7 Text Solution:

Let " A " and " B " be the event of getting electrification contract, plumbing contract respectively.

P(A) = 3/5

P(B bar) = 5/8

P(B) = 1 — P (B bar)

= 1 — (5 / 8)

P(B) = 3/8

P (A u B) = 5/7, we need to find P (A n B) = ?

P (A n B) = P (A) + P (B) — $\mathrm{P}(\mathrm{A} \cup \mathrm{B})$

= $(3/5) + (3/8) - (5/7)$

= $(168 + 105 - 200)/280$

P (A n B) = 73 / 280

Q8 Text Solution:

Explanation: Define the events A and B as follows: A - Neha chooses a black t-shirt. B - Neha chooses a blue skirt. Neha cannot choose both a black t-shirt and a blue t - shirt, so the addition theorem of probability applies:

$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) = \left(\frac{6}{12}\right) + \left(\frac{2}{12}\right).$

$= \frac{2}{3}$

Q9 Text Solution:

For 2 events to be mutually exclusive

$\Rightarrow \mathrm{P}(\mathrm{A} \cap \mathrm{B}) = 0\ldots \quad \text{(i)}$

$\Rightarrow$ and we know that,

$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B}) \ldots \text{(ii)}$

From eq. (i) & (ii)

$\Rightarrow$For mutually Exclusive Events,

$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B})$

Given P(A) = 0.5, P(B) = 0.3

$\Rightarrow$Probability of Either A or B, where A & B are mutually exclusive Events = P (A) + P(B)

= 0.5 + 0.3

= 0.8

$\Rightarrow$ Probability of Neither A nor B = $P\left(\overline{A \cup B}\right)$

= $1 - \mathrm{P}(\mathrm{A} \cup \mathrm{B})$

= $1 - 0.8$

= 0.2

Q10 Text Solution:

We know that,

$\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$

as $0 \leq \mathrm{P}(\mathrm{A} \cap \mathrm{B}) \leq 1,$ so,

$\mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) \geq \mathrm{P}(\mathrm{A} \cup \mathrm{B})$

So, Statement (I) is correct.

As $\mathrm{P}\left(\overline{\mathrm{A}} \cup \mathrm{B}\right) = \mathrm{P}\left(\overline{\mathrm{A}}\right) + \mathrm{P}(\mathrm{B}) + \mathrm{P}\left(\overline{\mathrm{A}} \cap \mathrm{B}\right)$

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So, $\mathrm{P}(\mathrm{A})+\mathrm{P}(\overline{\mathrm{A}} \cup \mathrm{B})=\mathrm{P}(\mathrm{A})+\mathrm{P}(\overline{\mathrm{A}})$
$+\mathrm{P}(\mathrm{B})+\mathrm{P}(\overline{\mathrm{A}} \cup \mathrm{B})$
$=1+\mathrm{P}(\mathrm{B})+\mathrm{P}(\overline{\mathrm{A}} \cup \mathrm{B})=$
$\mathrm{P}(\because \mathrm{P}(\mathrm{A})+\mathrm{P}(\overline{\mathrm{A}})=1) \quad \leq 1+\mathrm{P}(\mathrm{B})$
$(\because 0 \leq \mathrm{P}(\overline{\mathrm{A}} \cup \mathrm{B}) \leq 1)$
Hence, both the statements are correct.

Q11 Text Solution:

The total number of ways in which 3 integers can be chosen from first 20 integers is ${}^{20}_{3}C$
The product of three integers will be even if at least one of the integers is even.
Therefore, the required probability =1- Probability of three integers is even
$1-\frac{{}^{10}_{3}C}{{}^{20}_{3}C}$=17/19

Q12 Text Solution:

There are 11 letters in the word.
n(S)=$11!$
R and E can occur at (1,6),(2,7)...........(6,11) positions.
There are 6 possibilities-
And as they can interchange their positions and remaining 9 letters can be arranged in 9!.
n(E)=2*6*9!
P(E)=$\frac{6}{55}$

Q13 Text Solution:

The correct option is **B**
$\frac{24}{25}$

Explanation for the correct options:
Probability:
$\text{Totalnumberofcases} = 25 \times 25$
$\text{Favourablecases} = 25$
The probability of winning a prize
$=\frac{25}{25 \times 25}$
$=\frac{1}{25}$
Hence, the probability of not winning prize
$=1-\frac{1}{25}$
$=\frac{24}{25}$
Hence, option(B) is correct .

Q14 Text Solution:

Total no. of ways of 10 persons birthday in April(30days)= 30^{10}
Total ways of single birthday, single day=${}^{30}_{10}C$
Total ways of atleast two have same birthday = Total ways - single days
$=30^{10}-{}^{30}_{10}C$
Probability of atleast two have the sme birthday is
$=\frac{\textit{Atleast two birthday}}{\textit{Total ways}}$
$=\frac{30^{10}-{}^{30}_{10}C}{30^{10}}$

Q15 Text Solution:

When balls are replaced.
Total balls in the bag = 8 + 5 = 13
3 balls can be drawn out of total of 13 balls in ${}^{13}C_3$ ways.
3 white balls can be drawn out of 5 white balls in ${}^{5}C_3$ ways.
Probabilty of 3 white balls = P(3W) = $\frac{{}^{5}C_{3}}{{}^{13}C_{3}}=\frac{10}{286}$
Since the events are replaced after the first draw so again there are 13 balls in the bag 3 red balls can be drawn out of 8 red balls in ${}_{8}C_{3}$ ways.
Probability of 3 red balls = P(3R) = $\frac{{}^{8}C_{3}}{{}^{13}C_{3}}=\frac{56}{286}$
Since the events are independent , the requirec probability is :
P(3W and 3R) =
$\mathrm{P}(3\mathrm{W}) \times \mathrm{P}(3\mathrm{R})=\frac{{}^{5}C_{3}}{{}^{13}C_{3}} \times \frac{{}^{8}C_{3}}{{}^{13}C_{3}}=\frac{10}{286}$
$\times \frac{56}{286}=\frac{140}{20,449}$

Q16 Text Solution:

(i) The probability of drawing a white ball = 5 / 8
The probabilty of drawing a white ball = 4/6 and the probabilty of drawing a red ball = 2/5.
Since the events are dependent , therefore the required probability is :
P(W and R and W and R) =
$\mathrm{P}(\mathrm{W} \cap \mathrm{R} \cap \mathrm{W} \cap \mathrm{R})$
= P(W) xP(R|W) xP (W|WR) xP (R|WRW) =
$=\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5}=\frac{1}{14}$

Q17 Text Solution:

Lets analyze the problem first. First case is simple probability problem. Second case tells that ball is drawn drafter another (by replacing the ball after presbility problem, Second calis available in the present draw). Third case tells that ball is drawn one after another (without replacing the ball after first draw, that mean in the present draw one ball is less).

(a) Probability (All the ball in a single draw) = $\frac{{}^{10}C_2{}^{8}C_1}{{}^{18}C_3} = \frac{10\times9\times8\times6}{2\times18\times17\times16} = \frac{15}{34}$

(b) Probability [One after another (replacement)] = $\frac{{}^{10}C_1{}^{10}C_1{}^{8}C_1}{{}^{18}C_1{}^{18}C_1{}^{18}C_1} = \frac{10\times10\times8}{18\times18\times18} = \frac{100}{729}$

(c) Probability [One after another (without replacement)] = $\frac{{}^{10}C_1{}^{9}C_1{}^{8}C_1}{{}^{18}C_1{}^{17}C_1{}^{16}C_1} = \frac{10\times9\times8}{18\times17\times16} = \frac{5}{34}$

Q18 Text Solution:

(A) There are three possibilities that two shots hit the target

(i) A and B hit the target and C could not hit i. e. $A\cap B\cap\overline{C}$

(i) B and C hit the target and A could not hit i. e. $\overline{A}\cap B\cap C$

(i) A and C hit the target and B could not hit i. e. $A\cap\overline{B}\cap C$

Since the events are inpendent, so using multiplication theorem

(i)

$P\left(A\cap B\cap\overline{C}\right) = \frac{4}{5}\times\frac{3}{4}\times\left(1-\frac{2}{3}\right) = \frac{4}{5}\times\frac{3}{4}\times\frac{1}{3} = \frac{12}{60}$

(ii)

$P\left(\overline{A}\cap B\cap C\right) = \left(1-\frac{4}{5}\right)\times\frac{3}{4}\times\frac{2}{3} = \frac{1}{5}\times\frac{3}{4}\times\frac{2}{3} = \frac{6}{60}$

(iii)

$P\left(A\cap\overline{B}\cap C\right) = \frac{4}{5}\times\left(1-\frac{3}{4}\right)\times\frac{2}{3} = \frac{4}{5}\times\frac{1}{4}\times\frac{2}{3} = \frac{8}{60}$

Since the above three possibilities are mutually exclusive, so by using addition theorem, we have

P(Two shots hit the target) = $P\left(A\cap B\cap\overline{C}\right)+P\left(\overline{A}\cap B\cap C\right)+P\left(A\cap\overline{B}\cap C\right)$

$= \frac{12}{60}+\frac{6}{60}+\frac{8}{60} = \frac{26}{60} = \frac{13}{30}$

There are two possibilities that at least two shots hit the target

(i) Two could hit the target

(ii) Three could hit the target i.e. A and B and C could hit i.e. $A\cap B\cap C$

Since the events are independent, so by using multiplication theorem

(i) P(Two could hit the target) = $\frac{13}{30}$

(ii) $P(A\cap B\cap C) = \frac{4}{5}\times\frac{3}{4}\times\frac{2}{3} = \frac{24}{60}$

Since the above two possibilities are mutually exclusive, so by addition theorem

P (At least two shots hit the target) = $\frac{26}{60}+\frac{24}{60} = \frac{50}{60} = \frac{5}{6}$

Q19 Text Solution:

Approach : 1

Probability that A solves the problem

$P(A) = \frac{1}{2}, P(A') = \frac{1}{2}$

$P(B) = \frac{3}{4}, P(B') = \frac{1}{4}$

$P(C) = \frac{1}{4}$

$P(C') = \frac{3}{4}$

Probability the A, B, C can't solve the problem = $\left(1-\frac{1}{2}\right)\left(1-\frac{3}{4}\right)\left(1-\frac{1}{4}\right) = \frac{1}{2}\times\frac{3}{4}\times\frac{1}{4} = \frac{3}{32}$

Probability that the problem will be solved = $1-\frac{3}{32} = \frac{29}{32}$.

Q20 Text Solution:

(i) Let A = event of drawing a black card from the 1st pack.

B = event of drawing a black card from the 2nd pack.

So, P(A) = $\frac{26}{52}$ and P(B) = $\frac{26}{52}$

Since both are independent event.

$P(A\cap B) = P(A)\times P(B) = \frac{1}{4}$

(ii) Probability of drawing a red card form the 1^{st} pack and a black card from the 2^{nd} pack = $\frac{1}{4}$

Probability of drawing a black card form the 1^{st} pack and a red card from the 2^{nd} pack = $\frac{1}{4}$

These two are mutually exclusive events. So Required probability = $\frac{1}{4}+\frac{1}{4} = \frac{1}{2}$

Q21 Text Solution:

The total number of resistors is 100. Among them, 80 are within the required tolerance values, 11 are below, and the remaining (100-80-119) are above the required tolerance values.

The probability of drawing a resistor below the tolerance value first is the number of such resistors divided by the total number of resistors, which is 11/100.

After drawing one resistor, we have 99 resistors left. The probability of drawing a resistor above the tolerance value next is the number of such resistors divided by the remaining number of resistors, which is 9/99.

The probability of both events happening (drawing a resistor below the tolerance value first and drawing a resistor above the tolerance value next) is the product of the probabilities of the individual events, which is

(11/100) * (9/99) = 0.01.

Q22 Text Solution:

There are two possibilities

i) First ball is white and the second ball drawn is black.

ii) First ball is black and the second ball drawn is white.

Since the events are independent, so by using multiplication theorem we have

(i) Probability of drawing First ball white and the second ball black =$\frac{5}{9} \times \frac{4}{9}$

(ii) Probability of drawing First ball black and the second ball white =$\frac{4}{9} \times \frac{5}{9}$

Since these probabilities are mutually exclusive, by using addition theorem

Probability that two balls are of different colors=$\frac{20}{81} + \frac{20}{81} = \frac{40}{81}$

Q23 Text Solution:

Here, n(S) = 100

M = set of student who read mathematics

then, n(M) = 40

P(M) = 0.4

B = set of student who read Biology

P(B) = 0.25

$\mathrm{P}(\mathrm{M} \cap \mathrm{B}) = \frac{15}{100} = 0.15$

(i) $\mathrm{P}(\mathrm{M}/\mathrm{B}) = \frac{\mathrm{P}(\mathrm{M} \cap \mathrm{B})}{\mathrm{P}(\mathrm{B})} = \frac{0.15}{0.25} = \frac{3}{5}$

(ii) $\mathrm{P}(\mathrm{B}/\mathrm{M}) = \frac{\mathrm{P}(\mathrm{M} \cap \mathrm{B})}{\mathrm{P}(\mathrm{M})} = \frac{0.15}{0.4} = \frac{3}{8}$

Q24 Text Solution:

Let A = (No. 2 has appeared at least once)

= (1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2), (6, 2)) i.e., n(A) = 11

B {Sum of the numbers is 7}

B = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)} i.e. n (B)1.6, n(B) = 6

$\mathrm{A} \cap \mathrm{B}$ = {(2,5), (5, 2)} i.e. n $(\mathrm{A} \cap \mathrm{B})$ = 2

and S = {(1,1), (1,2),......(6 ,6)} i.e. n(S) = 36

$\mathrm{P}(\mathrm{A}) = \frac{\mathrm{n(A)}}{\mathrm{n(S)}} = \frac{11}{36}, \mathrm{P}(\mathrm{B}) = \frac{\mathrm{n(B)}}{\mathrm{n(S)}} = \frac{6}{36} = \frac{1}{6}$

$\mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{n(S)}} = \frac{2}{36} = \frac{1}{18}$

So, $\mathrm{P}(\mathrm{A}/\mathrm{B}) = \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P(B)}} = \frac{1/18}{1/6} = \frac{1}{3}$

Q25 Text Solution:

Let B be the event that one child is a boy, and A the event that both children are boys. The possibilities are bb, bg. gb, gg, each with probability 1/4. P$(\mathrm{A} \cap \mathrm{B})$ = P(bb) = 1/4 and P(B) = P (bb, bg, gb) = 3/4. So the answer is $\frac{1/4}{3/4} = 1/3.$

Q26 Text Solution:

In this problem the random experiment was carried out in two stages.

(a) A coin is tossed (b) If the first stage shows a head, coin is tossed again and if it shows a tail, a dice is thrown. The sample space is {HH, HT, T1, T2, T3, T4, T5, T6}.

P(HH) = P(HT) = 1/2 x 1/2 = 1/4

and P(T1) P(T2) = P(T3) = P(T4) = P(T5) = P(T6) = 1/6 X 1/2 = 1/12

E_1 : the event the first throw of coin shows tail = {T1, T2, T3, T4, T5, T6}

E_2 : the event the dice shows a number greater than 4 = {T5, T6}

P(E_1) = 6 x 1/12 = 1/2 and P (E_2) = 2 x 1/12 = 1/6

Also $\mathrm{P}(\mathrm{E}_1 \cap \mathrm{E}_2) = 2 \times \frac{1}{12} = \frac{1}{6}$

$\mathrm{P}(\mathrm{E}_2|\mathrm{E}_1) = \frac{\mathrm{P}(\mathrm{E}_1 \cap \mathrm{E}_2)}{\mathrm{P}(\mathrm{E}_1)} = \frac{1/6}{1/2} = \frac{1}{3}$

Q27 Text Solution:

Define events.

A = event that an email is detected as spam,

B = event that an email is spam,

B^c = event that an email is not spam,

We know P(B) = P(B^c) = 5, P(A | B) = 0.99, P(A | B^c) = 0.05.

Hence by the Bayes's formula we have.

$P(B^c|A) = \frac{P(A|B^c)P(B^c)}{P(A|B)P(B)+P(A|B^c)P(B^c)}$

$= \frac{0.05\times0.5}{0.05\times0.5+0.99\times0.5}$

$= \frac{5}{104}$

Q28 Text Solution:

Let E_1): person A get selected

E_2: person B get selected

E_3 :person C get selected

A: Changes introduced but profit not happened

Now, P(E_1) = 1 / (1 + 2 + 4) = 1/7

P(E_2) = 2/7 and P(E_3) = 4/7

P(A|E_1) = 1 (Profit not happened by the changes introduces by A) = 1 — P(Profit happened by the changes

introduces by A) 1 — 0.8 = 0.2

P(A|E_2) = P(Profit not happened by the changes introduces by B) = 1 — P(Profit happened by the changes introduces

by B) 1 — 0.5 — 0.5

P(A|E_3) = 1 (Profit not happened by the changes introduces by C) = 1 — P(Profit happened by the changes

introduces by C) = 1 — 0.3 = 0.7

We have to find the probability of not happening profit due to selection of C.

$P(E_3|A)$

$= \frac{P(A|E_3)P(E_3)}{P(A|E_1)P(E_1)+P(A|E_2)P(E_2)+P(A|E_3)P(E_3)}$

$\mathrm{P}(\mathrm{E}_3|\mathrm{A}) = \frac{0.7\times\frac{4}{7}}{0.2\times\frac{1}{7}+0.5\times\frac{2}{7}+0.7\times\frac{4}{7}}$

= 7/10.

∴ The required probability is 0.7.

Q29 Text Solution:

Let E_1 = Bag contains two blue balls

E_2 = Bag contains three blue balls

E_3 = Bag contains four blue balls

A = event of getting two blue balls

P(E_1) = P(E_2) = P(E_3) = 1/3

P(A|E_1) = ${}^2C_2/{}^4C_2$ = 1/ 6

P(A|E_2) = ${}^3C_2/{}^4C_2$ = 1/ 2

P(A|E_3) = ${}^4C_2/{}^4C_2$ = 1

$P(E_3|A)$

$= \frac{P(A|E_3)P(E_3)}{P(A|E_1)P(E_1)+P(A|E_2)P(E_2)+P(A|E_3)P(E_3)}$

$= [1/3 \times 1]$

$/ [1/3 \times 1/6 + 1/3 \times 1/2 + 1/3 \times 1]$

= 3/5.

Q30 Text Solution:

Let M = {Both drawn in defective}

and Let E_1 = {Bolt is manufactured by machine A } $\Rightarrow \mathrm{P}(\mathrm{E}_1) = \frac{25}{100} = \frac{1}{4}$

Let E_2 = {Bolt is manufactured by machine A } $\Rightarrow \mathrm{P}(\mathrm{E}_2) = \frac{35}{100} = \frac{7}{20}$

Let E_1 = {Bolt is manufactured by machine A } $\Rightarrow \mathrm{P}(\mathrm{E}_3) = \frac{40}{100} = \frac{2}{5}$

Now. P(M/E_1) = P (Defective bolt when it is manufactured by A) = $\frac{5}{100}$

P(M/E_2) = P (Defective bolt when it is manufactured by B) = $\frac{4}{100}$

P(M/E_3) = P (Defective bolt when it is manufactured by C) = $\frac{2}{100}$

$\therefore \mathrm{E_i} \cap \mathrm{E_j} = \phi\forall\mathrm{i}$ and j soE_1 , E_2, E_3 are mutually exclusive . Again $\mathrm{E}_1 \cup \mathrm{E}_2 \cup \mathrm{E}_3 = 100\%$ Bolts i.e

$\mathrm{E}_1 \cup \mathrm{E}_2 \cup \mathrm{E}_3$ = sample space so they are exhaustive also. So we can apply law of total prob i.e.

$\mathrm{P}(\mathrm{M}) = \mathrm{P}(\mathrm{E}_1).\mathrm{P}\left(\frac{\mathrm{M}}{\mathrm{E}_1}\right) + \mathrm{P}(\mathrm{E}_2)\left(\frac{\mathrm{M}}{\mathrm{E}_2}\right)$

$+ \mathrm{P}(\mathrm{E}_3)\mathrm{P}\left(\frac{\mathrm{M}}{\mathrm{E}_3}\right)$

$= \frac{1}{4} \times \frac{5}{100} + \frac{7}{20} \times \frac{4}{100} + \frac{2}{5} \times \frac{2}{100} = 0.00345$

Q31 Text Solution:

Let D is the event that a person HIV positive, and T is the event that the person tests positive.

$\mathrm{P}(\mathrm{D}/\mathrm{T}) = \frac{\mathrm{P}(\mathrm{D}\cap\mathrm{T})}{\mathrm{P}(\mathrm{T})} = \frac{(0.99)(0.003)}{(0.99)(0.003)+(0.01)(0.997)}$.

$\approx 23\%$

As D and T are independent thus $P(D \cap T) = P(D) * P(T)$ where P(D)=0.99,P(T)=0.003

Q32 Text Solution:

Let,

F: children with flu

M: children with measles

R: children showing the symptom of rash

P(F) = 90% = 0.9

P(M) = 10% = 0.1

P(R|F) = 0.08

P(R|M) = 0.95

$$\mathrm{P}\left(\mathrm{F}/\mathrm{R}\right) = \frac{\mathrm{P}(\mathrm{R}/\mathrm{F})\mathrm{P}(\mathrm{F})}{\mathrm{P}(\mathrm{R}|\mathrm{M})\mathrm{P}(\mathrm{M})+\mathrm{P}(\mathrm{R}|\mathrm{F})\mathrm{P}(\mathrm{F})}$$

$$\mathrm{P}\left(\mathrm{F}/\mathrm{R}\right) = \frac{0.08\times0.9}{0.95\times0.1+0.08\times0.9}$$

= 0.072 / (0.095 + 0.072) = 0.072/0.167 $\approx$ 0.43

$\Rightarrow$P(F|R) = 0.43.