CS & IT ENGINEERING

THEORY OF COMPUTATION

Pushdown Automata and CFL

Lecture – 03

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Recap of Previous Lecture

Topic ?????PDA Constructions

NPDA Construction

DPDA ≠ NPDA (CFL)

DCFL

PDA

Topics to be Covered

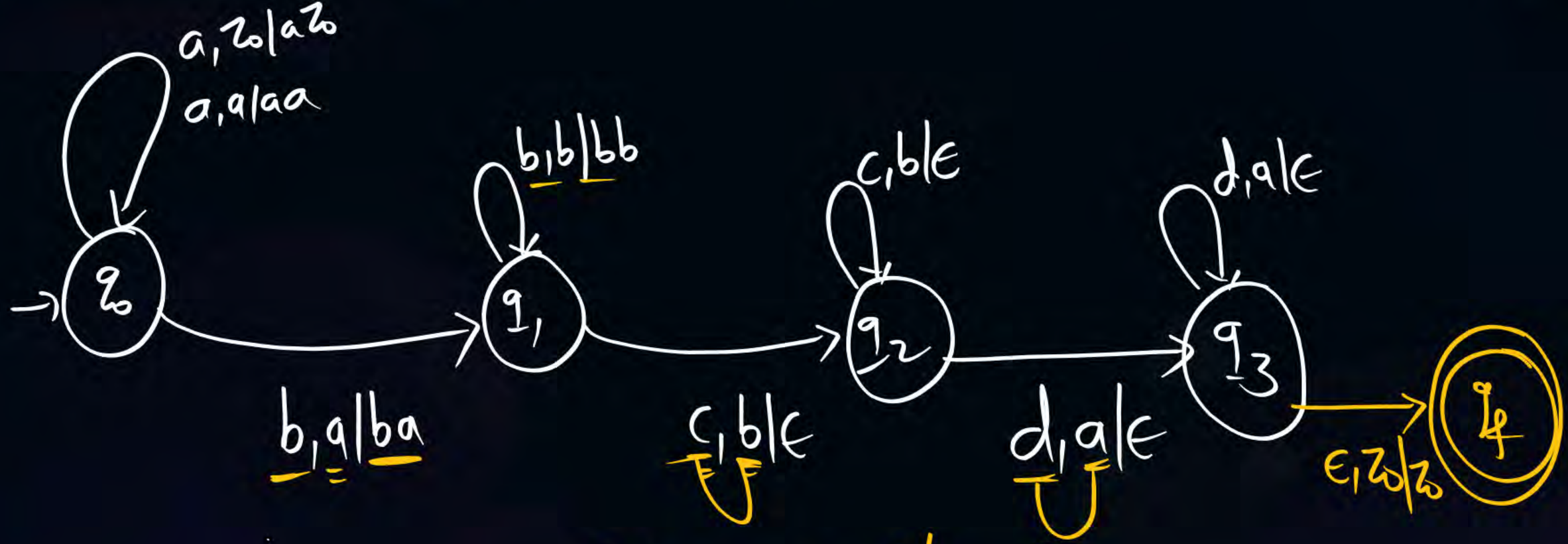
Topic Push down automata

Topic ?? CFL detection

Topic ?? closure properties of CFL and DCFL

Topic ??

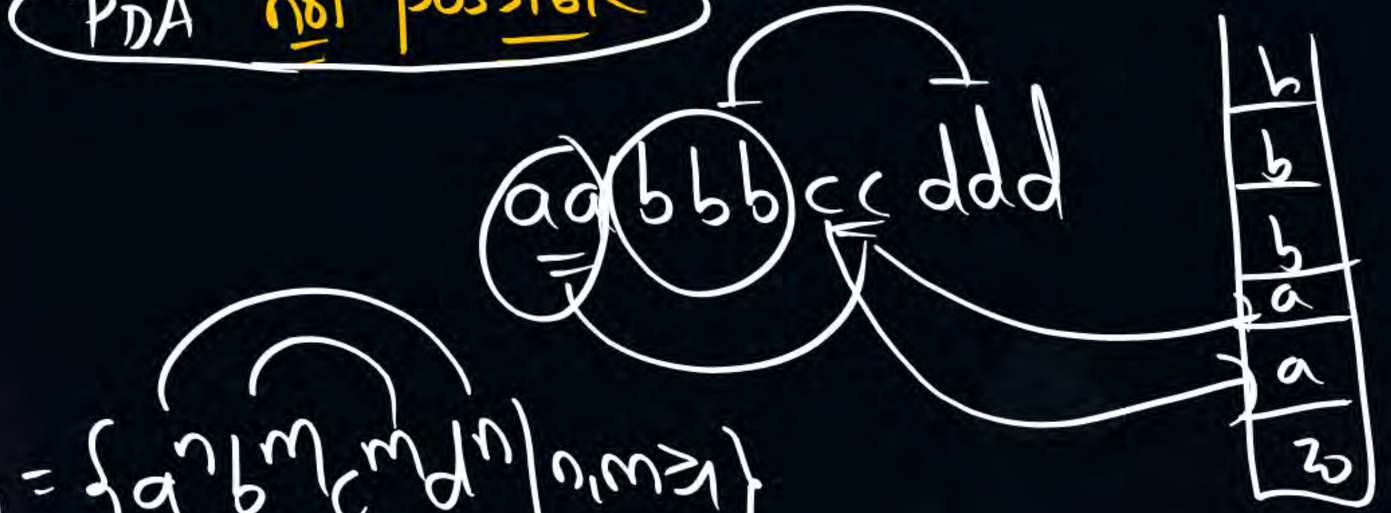
(Q) Identify language of following PDA?



$$L = \{a^n b^m c^m d^n \mid m, n \geq 1\}$$

(Q) Construct PDA for $L = \{a^n b^m c^n d^m / n, m \geq 1\}$

PDA not possible

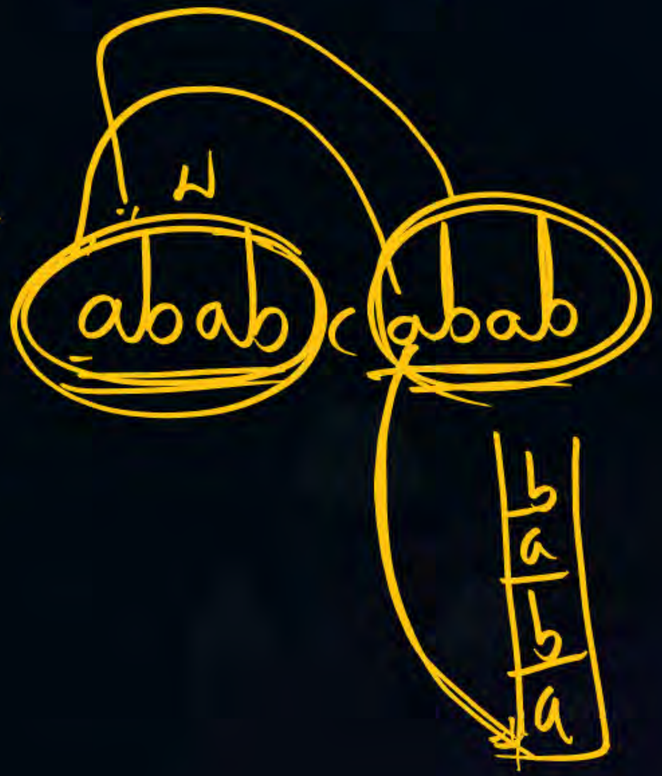


$L_1 = \{a^n b^m c^m d^n / n, m \geq 1\}$

$L_2 = \{a^n b^m c^n d^m / n, m \geq 1\}$

(Q) Construct PDA for $L = \{WCW / W \in (a+b)^*\}$

PDA not possible



$aa\,C\,bb$

(W = aa, W = bb)

not possible

NOTE

$$L_1 = \Sigma^* - \{ww \mid w \in (a+b)^*\}$$

$$L_2 = \Sigma^* - \{wcw \mid w \in (a+b)^*\}$$

CFL

(Q) Construct PDA for $L = \{WW \mid W \in (a+b)^*\}$

e

F.A. + Stack = PDA

Topic : PDA

- Finite Automata having additional power form of stack known as Push down automata.
- Size of stack in Push Down automata is infinite
- There exist only one type of push down automata i.e. “language recognisor”
- Push down automata can accept language in deterministic way or non-deterministic way

Topic : Note

Note:- By reading the input string by the end of the string stack is non empty or starting is not ended is-

Whenever m/c is halted then that i/p is rejected.

- the input is valid only string is ended end 2 not be in there stack.
- In final state mechanism i/p is valid only when automata enters into final state whenever m/c is halted.

Topic : Context Free Language

Which of the following language are

1. CLF and Regular
2. CFL but not regular
3. Non CFL

Topic : Context Free Language

1. $L = \{a^n b^n c^n / n \leq 10\}$ → finite → regular → CFL

2. $L = \{a^n b^n c^m / n \neq m\}$ → Non CFL

3. $L = \{a^n b^m c^n / n > m \mid n, m \leq 1000\}$ → finite → regular → CFL

4. $\{L = a^n b^m / n - m = 4\} = \{a^{m+4} b^m\}$ = CFL
   $n = m+4$

5. $L = \{a^n b^m / n/m = 4\}$ $n = 4m$ $= \{a^{4m} b^m\} =$ CFL

6. $L = \{a^n b^m / n = 2m + 1\}$ $= \{a^{2m+1} b^m\} \Rightarrow$ CFL

7. $L = \{a^n b^m / n \neq m\}$ $=$ CFL

8. $L = \{a^n b^m / n \neq 2m\}$ $=$ CFL

9. $L = \{a^n b^m / n = m^2\}$ $= \{a^{m^2} b^m\} =$ Non CFL

Topic : Context Free Language

10. $\{a^{n!}\, b^{n!} / n \geq 13\}$ → नहीं CFL

11. $L = \{a^n b^m / n \leq m\}$ = CFL

12. $\{a^n b^m c^{n+m} / n, m \geq 1\}$ → CFL

13. $L = \{L = a^n b^{n+m} / n, m = 1\}$ → CFL

14. $L = \{a^{m^2} b^{n^3} c^{k^5} / m, n, k > 1\}$ → non CFL

15. $L = \{a^{3^n} b^{5^k} c^{2^\ell} / n, k, \ell \geq 1\}$ ⇒ non CFL

16. $L = \{a^i b^j c^k / j = i + k\} = \{a^i b^{i+k} c^k\} = \{a^i b^i b^k c^k\}$ = CFL

17. $L = \{a^i b^j c^k / i > j \text{ (or) } j < k\}$ =

Topic : Context Free Language

17. $L = \{a^i b^j c^k \mid i \geq j \text{ or } j \geq k\} \Rightarrow$ CFL

18. $L = \{a^i b^j c^k / i > j > k\} =$ non CFL

19. $L = \{a^i b^j c^k / j = \max(i, k)\} \Rightarrow$ non CFL

20. $L = \{a^i b^j c^k / j = i^2 + k^2\} \Rightarrow$ Non CFL

21. $L = \{a^i b^j c^k\ d^\ell / i = \ell \text{ and } j = k\} \Rightarrow$ CFL

22. $L = \{a^i b^j c^k\ d^\ell / i = k \text{ and } i = \ell\} =$ non CFL

$\{a^n b^m c^m d^n\}$

23. $L = \{a^i b^j c^k / d^\ell / \ i = k \text{ or } j = \ell\}$

24. $L = \{a^i b^j c^k / d^\ell / \ i = 2k \text{ or } j \neq 5\ \ell\}$

25. $L = \{a^i b^j c^k / d^\ell / \ i + j = k + \ell\}$

26. $L = \{a^i b^j c^k / d^\ell / \ i = 4\ \ell \text{ and } j = 3\ell\}$

Topic : Context Free Language

27. ~~$L = \{a^i b^j / (i + j) \bmod 5 = 0\}$~~

28. $\{a^{2^n} / n \geq 1\}$ → Non CFL

29. $\{a^{n^2} / n \geq 1\}$ → Non CFL

30. $L = \{1^{2n+1} / n \geq 1\}$ → CFL

31. $L = \{a^P / P \text{ is prime number}\}$ → non CFL

32. $L = \{a^k / k \text{ is odd number}\}$ → CFL

Topic : Context Free Language

33. $L = \{wxw / w \in \{a, b\}^*\}$ → Non CFL

34. $L = \{wxw / w, x \in \{a, b\}^*\} = (a+b)^* = \text{Reg} = \text{CFL}$

35. $\{ww^Rx / \{w,x, \in \{a, b\}^+\} = \{WW^Rx\} = \text{CFL}$

36. $L = \{\Sigma^* - \{ww / w \in \{a, b\}^+\} \rightarrow \text{CFL}$

37. $L = \{ww^Rw / w \in \{a, b\}^+\} \rightarrow$ Non CFL

38. $L = \{ww^Rww^R / w \in \{a, b\}^+\} \rightarrow$ non CFL.

39. $L = \{wwww / w \in \{a, b\}^*\}$ = non CFL

40. $\{L = \{x / x \in \{a, b, c\}^* / n_a(x) = n_b(x) = n_c(x)\}\}$ = non CFL

41. $L = \{x / x \in \{a, b, c\}^* / n_a^2(x) = n_b^2(x) + n_c^2(x)\}$ → non CFL

42. $L = \{x / x \in \{a, b, c\}^* / n_a(x) = n_b^2(x) + n_c^2(x)\}$ = non CFL

Topic : Context Free Language

43. $L = \{x/x \in \{a, b\}^* \; n_a(x) \bmod 5 = 0 \text{ and } n_b(x) \bmod 4 = 0\}$

44. $L = \{a^n b^{2n} c^{3n} / \; n \geq 1\}$ ⇒ non CFL

45. $L = \{a^n b^n c a^m b^m / n, m \geq 0\}$ ⇒ CFL

push pop ≡ push pop

46. $L = \{a^n b^m c^k / n \neq m \text{ or } m \neq k\}$ ⇒ CFL

Topic : Context Free Language

47. Set of all odd length palindrome string of Hindi language → CFL

48. Set of all even length palindrome string of ~~are~~ language (English) ⇒ CFL

49. Set of all balanced parenthesis ⇝ CFL ((())) (())

50. ~~Set of all lexical error produced by compiler~~

Q

Consider the following languages:

$L_1 = \{a^n b^m c^{n+m} : m, n \geq 1\}$

$L_2 = \{a^n b^n c^{2n} : n \geq 1\}$

Which one of the following is TRUE?

[2016(Set-2): 2 Marks]

A. Both L_1 and L_2 are context-free.

B. L_1 is context-free while L_2 is not context-free

C. L_2 is context-free while L_1 is not context-free

D. Neither L_1 nor L_2 is context-free

Q

Consider the following language over the alphabet $\Sigma = \{a, b, c\}$.

Let $L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$ and

$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$.

Which of the following are context-free languages?

I. $L_1 \cup L_2$

II. $L_1 \cap L_2$

[2017(Set-1): 2 Marks]

A. I only

B. II only

C. I and II

D. Neither I nor II

Q

Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?

I. $L_1 \cup L_2$ is context-free – CFL

II. L_1 is context-free → ✗

III. $L_1 - R$ is context-free → ✓ CFL

IV. $L_1 \cap L_2$ is context-free ✗

[2017(Set-2): 1 Marks]

A. I, II and IV only

B. I and III only

C. II and IV only

D. I only

Q

Consider the following languages:

$L_1 = \{a^p \mid p \text{ is a prime number}\}$ → non CFL

$L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$ — CFL

$L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$ → non CFL

$L_4 = \{a^n b^n \mid n \geq 1\}$ → CFL

Which of the following are CORRECT?

I. L_1 is context-free but not regular. X

II. L_2 is not context-free. X

III. L_3 is not context-free but ~~recursive~~.

IV. L_4 is deterministic context-free. ✓

[2017(Set-2): 2 Marks]

A. I, II and IV only

B. II and III only

C. I and IV only

D. III and IV only ✓

Q

Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?

[2021(Set-1): 2 Marks]

CFL . CFL
reg . CFL

A $L_1 \cdot L_2$ = CFL

CFL

B $L_1 \cup L_2$

C $L_1 - L_2$

reg − CFL

reg ∧ CFL^C =

$L_1 - L_2 = L_1 \wedge L_2^C$

D $L_1 \cap L_2$

reg ∧ CFL = CFL

Closure Properties of CFL and DCFL.

$\Rightarrow$ always CFL (may (or) may not regular)

$\{a^n b^n\} \wedge (a+b)^* = \{a^n b^n\}$

$\Rightarrow$ always DCFL

Operation	CFL	DCFL
① Union	✓	✗
② Concatenation	✓	✗
③ Intersection	✗	✗
④ Complement	✗	✓
⑤ Kleene closure	✓	✗
⑥ Positive closure	✓	✗

Operation	CFL	DCFL
⑦ Intersection with regular	✓	✓
⑧ Difference $(L_1 - L_2 = L_1 \cap L_2^c)$	X	X
⑨ L − R $(L \cap R^c) = L \cap \text{reg}$	✓	✓
⑩ R − L $(R \cap L^c)$	X	✓
⑪ Subset	X	X
⑫ Reversal	✓	X

Operation	CFL	DCFL
(13) Union with regular (LUR)	✓	✓
(14) Substitution	✓	✗
(15) Homomorphism	✓	✗
(16) Inverse Homomorphism	✓	✓

Q

Which of the following languages are context-free?

$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$

$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$

$L_3 = \{a^m b^n \mid m = 2n + 1\}$

Home work

[2015(Set-3): 1 Marks]

A L_1 and L_2 only

B L_1 and L_3 only

C L_2 and L_3 only

D L_3 only

2 mins Summary

- Topic One
- Topic Two
- Topic Three
- Topic Four
- Topic Five

Grammar
Types of grammar
Ambigue grammar
Simplification, Normal form
PDA Construction
CFL detection
Closure properties

THANK - YOU