DS & AI
CS & IT

Probability & Statistics

Lecture No. 07

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Recap of previous lecture

Topic BASICS of PROBABILITY (Part-2)

Topics to be Covered

Topic

PROBABILITY (Part-3)

- Conditional Prob
- Law of Total Prob
- Baye's Theorem

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

Short RECAP

Operation	P&C	Prob	Formula	ME	Ind.
Either or	Add	union	Addition Th	$P(A\cup B)=P(A)+P(B)$	—
AND	Multiply	Intersection	Multi Th	$P(A\cap B)=0$	$P(A\cap B)=P(A)\cdot P(B)$

Addition Th: $\boxed{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$

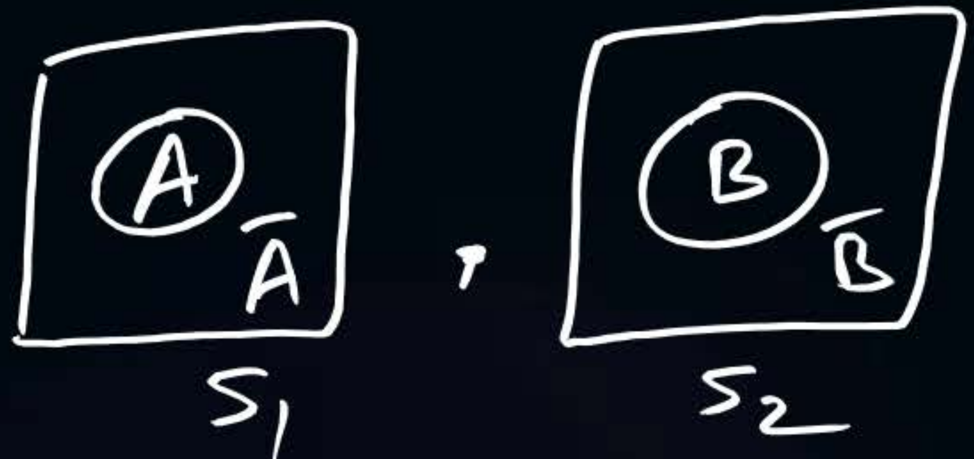
for Independency: $P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$

for ME: $P(A\cup B)=P(A)+P(B)-0$

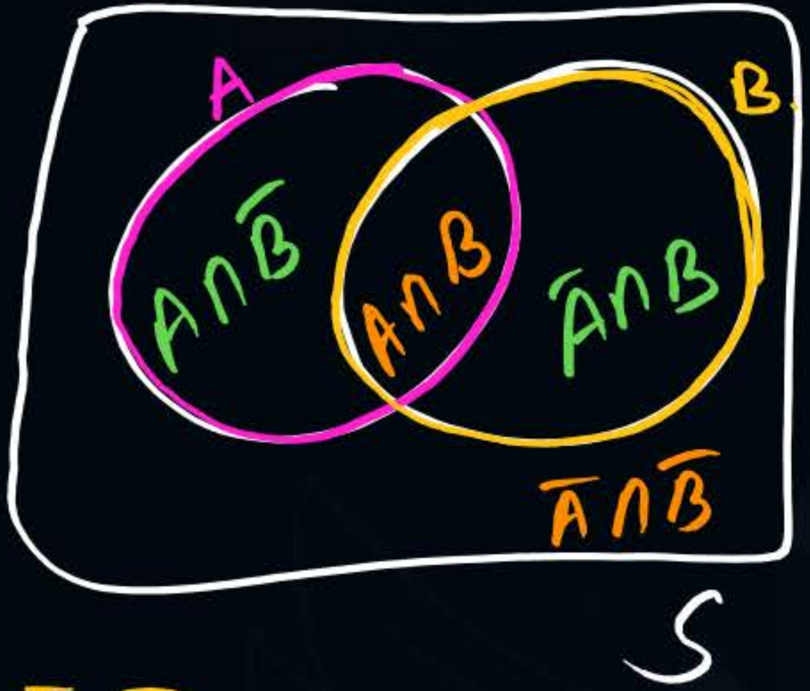
Concept of ME and Independency in a Single Question →

eg Two Persons A & B fire at the target once then write it's s. space ?

Sol:



$\Longrightarrow$



A & B are Independent (T)

A & $\bar{B}$ " " (T)

$\bar{A}$ & B " " (T)

$\bar{A}$ & $\bar{B}$ " " (T)

$S = \{\bar{A}\cap\bar{B},\ A\cap\bar{B},\ \bar{A}\cap B,\ A\cap B\}$

$= \{E_1, E_2, E_3, E_4\}$

So E_1, E_2, E_3, E_4 are ME Events.

Analysis: Various possibilities are ; $S=\{\underbrace{\bar{A}\cap\bar{B}}_{=E_1}, \underbrace{A\cap\bar{B}}_{=E_2}, \underbrace{\bar{A}\cap B}_{=E_3}, \underbrace{A\cap B}_{=E_4}\}$

(None will hit) or (A hit & B missed) or (A missed & B hit) or (Both hit) = Total possibilities

$$(\bar{A}\cap\bar{B}) \cup \underbrace{(A\cap\bar{B}) \cup (\bar{A}\cap B)}_{\text{only one will hit / Exactly one will hit.}} \cup (A\cap B) = S$$

At least one will hit.

$$E_1 \cup E_2 \cup E_3 \cup E_4 = S \Rightarrow P(E_1\cup E_2\cup E_3\cup E_4) = P(S)$$

$\because E_1, E_2, E_3, E_4$ are ME so $P(E_1)+P(E_2)+P(E_3)+P(E_4) = 1$

i.e $P(\text{something occurs}) = 1$

F. Q3 →

There are two Gangsters Munna Mobile (A) & Pappu Pazer (B), They both fire at the target once with probability of their hitting is $\frac{4}{5}$ & $\frac{3}{4}$ resp.

$P(A)=\frac{4}{5}$, $P(\bar{A})=\frac{1}{5}$, $P(B)=\frac{3}{4}$, $P(\bar{B})=\frac{1}{4}$, A & B are Independent.

$S=\{\bar{A}\cap\bar{B}, A\cap\bar{B}, \bar{A}\cap B, A\cap B\}$

(1) Find the prob that Both will hit = ? $= P(A\cap B) = \frac{4}{5}\times\frac{3}{4} = \frac{3}{5}$

(2) " " " " None will hit = ? $= P(\bar{A}\cap\bar{B}) = \frac{1}{5}\times\frac{1}{4} = \frac{1}{20}$

(3) " " " " Atleast one will hit = ? $= 1 - P(\text{None will hit}) = 1 - \frac{1}{20} = \frac{19}{20}$

(4) " " " " target will be hit = ? $= P(\text{at least one will Hit}) = \frac{19}{20}$.

(5) Find the prob that either of them will hit = ?

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{4}{5} + \frac{3}{4} - \frac{3}{5} = \frac{19}{20}.$$

(6) find the prob that A hit & B missed = ? $= P(A\cap\bar{B}) = P(A)\cdot P(\bar{B})$

$$= \frac{4}{5}\times\frac{1}{4} = \frac{1}{5}$$

(7) Find the prob that only one will hit = ?

$$= P(\underbrace{A\cap\bar{B}}\cup\underbrace{\bar{A}\cap B})$$

$$= P(E_2\cup E_3)$$

$$= P(E_2) + P(E_3) \quad (\because E_2 \,\&\, E_3 \text{ are ME})$$

$$= P(A\cap\bar{B}) + P(\bar{A}\cap B)$$

$$= P(A)\cdot P(\bar{B}) + P(\bar{A})\cdot P(B)$$

$$= \underbrace{\frac{4}{5}\times\frac{1}{4}}_{\text{Ind}} + \underbrace{\frac{1}{5}\times\frac{3}{4}}_{\text{Ind.}} = \frac{7}{20}$$

(8) If $\underbrace{\text{exactly one person hit}}_{\text{only one person hit} = 7^{th}}$ then Find the prob that A hit & B missed = ?

6th Part

Sol:

original Prob = $P(S) = 1$

Reduced Prob = P(Condition) = P(only one person hit) = $\frac{7}{20}$ (By Part 7)

fav Prob = $P(A \cap \bar{B}) = \frac{1}{5} = \frac{1/5}{1}$ (By Part 6)

Now Conditional Prob = $\frac{\text{fav Prob}}{\text{R. Prob}} = \frac{1/5}{7/20} = \frac{4}{7}$

Conditional Probability

(just Reduce the S. Space according to Condition)

eg: A Couple has 3 kids & 1st child is known to be a Boy then Find the prob of having exactly 2 Boys?

Sol: original S. Sp = {(BBB), (BBG), (BGB), (BGG), (GBB), (GBG), (GGB), (GGG)} = 8 Trip

App I

Reduce S.Sp = { 1st child is a Boy } = {(BBB), (BBG), (BGB), (BGG)} = 4 Triplets.

fav Cases = { Exactly 2 Boys } = { (BBG), (BGB) } ≡ 2 Triplets.

Conditional Prob = $P\left[\dfrac{\text{exactly 2B}}{\text{1st child is Boy}}\right] = \dfrac{\text{fav Cases}}{\text{R.S.Sp.}} = \dfrac{2}{4} = \dfrac{1}{2} \neq \dfrac{4}{8}$

App III Req Prob = P[exactly 2B] = P[(B B G) or (B G B)] ← ME →

$$= \underbrace{1\times\frac{1}{2}\times\frac{1}{2}}_{\text{Ind.}} + \underbrace{1\times\frac{1}{2}\times\frac{1}{2}}_{\text{Ind.}} \quad \{\because E_2 \,\&\, E_3 \text{ are ME}\}$$

$$= \frac{2}{4} = \frac{1}{2} \neq \frac{4}{8}$$ (Avoid)

Note: Had the Condition was not there then answer would have been = ?

① A couple has 3 kids then Find (OR) the Prob of having exactly 2 Boys = ? $\frac{\text{fav}}{\text{Total}} = \frac{3}{8}$

② while writing Ans of Conditional Prob we should not generalise our Final Ans.

eg: A Die is thrown twice. If $\underbrace{\text{Sum of the outcomes is 6}}_{\text{condition}}$, then Find the prob that number 4 has appeared at least once?

Sol: original S.Sp = {(11), (12), (13) .. (66), (21) (22) ....(66)} ⇒ n(S) = 36 pair

R.S.Sp = {Sum is 6} = {(15), (51), (24), (42), (33)} = 5 pair

fav Cases = {digit 4 should appear at least once} = {(24), (42)} = 2 pair

$$\text{Conditional Prob} = P\left(\frac{\text{4 appeared at least once}}{\text{Sum is 6}}\right) = \frac{\text{fav}}{\text{R.S.Sp}} = \frac{2}{5}$$

Note: A Die is thrown twice then find the prob that 4 has appeared at least once = ? $= \frac{f}{T} = \frac{11}{36}$

fav Cases = { (41) (42) (43) (44) (45) (46)
(14) (24) (34) $\cancel{(44)}$ (54) (64) } = 11 pair

Q: A coin is tossed thrice then Find the prob of getting exactly 1 H if 1st outcome is known to be Head?

Sol: (M-I) Aamir KHAN method:

original S.Sp = {(HHH)(HHT) ... (TTT)} = 8

R.SSp = {1st outcome is Head}

= {(HHH)(HHT)(HTH)(HTT)} = 4

fav cases = {Exactly 1 H} = {(HTT)} = 1

$$\text{Cond}^n \text{ Prob} = \frac{\text{fav}}{\text{R.SSp}} = \frac{1}{4}$$

(M-II) (SALMAAN KHAN method) →

$$\text{Req Prob} = P[\text{H T T}]$$

$$= 1 \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Eg: A Coin is tossed 6 times & 1^{st} three outcomes are H H H then Find the prob of occurring Tail in Remaining tosses ?

condition

Sol: App III: Req Prob $= P[\underbrace{HHH}_{\text{given}} TTT] = 1^3 \times \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

App I original S.Sp $= \{(HHHHHH), (HHHHHT), \ldots\ldots, (TTTTTT)\} = 2^6 = 64$ Tuples

R.S.Sp $= \{1^{st}$ three outcomes are Head$\} = \{(HHH___) \ldots\ldots\} = ??$ (Not easy)

App II R.S.Sp $= \{$ eg $(\underset{1}{H} \times \underset{1}{H} \times \underset{1}{H} \times \underset{2}{H/T} \times \underset{2}{H/T} \times \underset{2}{H/T}) \ldots \} = 1 \times 1 \times 1 \times 2 \times 2 \times 2 = 8.$

Fav Cases $= \{$ eg $(\underline{H}\,\underline{H}\,\underline{H}\,\underline{T}\,\underline{T}\,\underline{T})\} = 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$ Tuple.

Req Prob $= \frac{\text{Fav}}{\text{R.Cases}} = \frac{1}{8} \neq \frac{8}{64}$

eg: Two Integers are to be Selected from integers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

If $\underbrace{\text{their sum is Even}}_{\text{Condition}}$ then Find the prob that both the selected integers are odd?

Sol: $\underbrace{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}_{^{11}C_2 = 55 \text{ pair}}$

$\to 2, 4, 6, 8, 10 \longrightarrow {}^5C_2 = 10$ pair

$\to 1, 3, 5, 7, 9, 11 \longrightarrow {}^6C_2 = 15$ pair

App II Total ways of selecting 2 integers $= {}^{11}C_2 = 55$ pair

R. ways $= \{$their sum should be Even$\}$

$= \{$either $\underbrace{\text{Both are Even}}$ or $\underbrace{\text{Both are odd}}\} = {}^5C_2 + {}^6C_2 = 10 + 15 = 25$ pair

fav ways $= \{$Both are odd$\} = {}^6C_2 = 15$ pair

Hence Conditional Prob $= \dfrac{\text{fav}}{\text{R. Cases}} = \dfrac{15}{25}$.

M-II

$S = \{ (1\,2)\,(1\,3)\,(1\,4) \ldots (1,11) \to 10$

$(2\,3)\,(2\,4) \ldots (2,11) \to 9$

$(3\,4)\,(3\,5) \ldots (3\,11) \to 8$

$(4\,5) \ldots \to 7$

$(5\,6) \ldots \quad 6$

$\vdots \quad 5 \quad 4 \quad 3$

$(9\,10), (9\,11) \to 2$

$(10,11) \} \to 1$

So $n(S) = 1+2+3+\ldots+9+10$

$= \frac{10 \times 11}{2} = 55$ Cases

$= {}^{11}C_2$

$R.SSp = \{$ Both are even or Both are odd $\}$

$= \{ (2\,4), (2\,6), (2\,8), (2\,10), (4\,6), (4\,8), (4\,10)$

$(6\,8)\,(6\,10)\,(8\,10)$

$(1\,3), (1\,5), (1\,7), (1\,9), (1\,11), (3\,5), (3\,7), (3\,9), (3,11)$

$(5\,7), (5\,9), (5\,11), (7\,9), (7\,11), (9\,11) \}$

$= 10 + 15 = 25$ Cases

$= {}^{5}C_2 + {}^{6}C_2$

$$\text{fav Cases} = \left\{ \text{Both are odd} \right\} = \left\{ \begin{matrix} (1,3)\ (1,5) \ldots (1,11) \\ (3,5)\ (3,7) \ldots (3,11) \\ \ldots\ldots\ (9,11) \end{matrix} \right\} = 15 \text{ Cases.}$$

$$\text{So Cond}^n \text{ Prob} = \frac{\text{fav cases}}{\text{R. Cases}} = \frac{15}{25} \quad \underline{\text{Ans}}$$

<u>Note</u> Two int are to selected from int 1, 2, 3, 4 .... 10, 11.
Find the prob that both the selected Int are odd?

$$\text{Req Prob} = \frac{\text{fav Cases}}{\text{Total Cases}} = \frac{{}^6C_2}{{}^{11}C_2} = \frac{15}{55}$$

Formulatic Approach of Conditional Prob →

① $\boxed{P(A/B) = \dfrac{P(A \cap B)}{P(B)}}$ it is the prob of A when B has already occurred.

② $P(B/A) = \dfrac{P(B \cap A)}{P(A)}$ it is the prob of B when A has already occurred

③ $P(A \cap B/C) = \dfrac{P(A \cap B \cap C)}{P(C)}$ it is the prob of simultaneous occurrence of A & B when C has already occurred.

Sp. Note: If A & B are Ind then

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A) \cdot P(B)}{P(B)} = P(A)$

$P(B/A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{P(B) \cdot P(A)}{P(A)} = P(B)$

i.e In case of Independency, condition has NO significance.

V. V. Special Note :- To check the Independency of Events, we have following three methods;

(M-I) By def^n. (Best Method)

(M-II) if $\left[P(A\cap B)=P(A)\cdot P(B)\right] \Longleftrightarrow$ then [A & B are Called Ind]

(M-III) if $\left[P(A/B)=P(A)\right] \Longleftrightarrow$ then [A & B are Ind.]

(*) the Relation of Dependency or Independency is a Vice-Versa Relation.

eg $C = \{H, T\}$, $D = \{1, 2, 3, 4, 5, 6\}$, Coin & Die are Ind.

$A = \{H\}$, $B = \{\text{No} \le 4\}$

$P(A) = \frac{1}{2}$, $P(B) = \frac{4}{6}$

$P(A/B) = ? = P(A) = \frac{1}{2}$ ($\because$ A & B are Ind.)

& $P(B/A) = ? = P(B) = \frac{4}{6}$ (" " ")

Qs If $P(A)=1$, $P(B)=\frac{1}{2}$ then find $P(A/B)=?$, $P(B/A)=?$ respectively.

PYQ

(a) 1, 0

(b) 0, 1

(c) $1, \frac{1}{2}$

(d) Data Inadequate.

A & B are Ind By M-I

$P(A/B)=P(A)$
$=1$

$P(B/A)=P(B)$
$=\frac{1}{2}$

Qs If $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$, $P(A/B)=\frac{1}{6}$ then find the prob of their simultaneous occurrence ?

(a) $\frac{1}{24}$

(b) $\frac{1}{12}$

(c) 0

(d) 1

$P(A\cap B)=?$

M-I $\cancel{P(A\cap B)=?=P(A)\cdot P(B)=\frac{1}{3}\times\frac{1}{4}}=\frac{1}{12}$

M-II $\because P(A/B)\neq P(A) \implies$ A & B are not Ind

i.e we can not use Multiplication Rule.

i.e we will use Multi Theorem as follows.

$$P(A\cap B)=P(A/B)\cdot P(B)$$

$$=\frac{1}{6}\times\frac{1}{4}=\frac{1}{24}$$

If P(A) = 1/3, P(B) = 1/4, P(A/B) = 1/6, then what is P(B/A) equal to?

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

6

M-I $\cancel{P(B/A) = \frac{1}{P(A/B)}} = \frac{1}{1/6} = 6$??

M-II $P(A/B) = \frac{1}{6}$

$\frac{P(A\cap B)}{P(B)} = \frac{1}{6}$

$P(A\cap B) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

So $P(B/A) = \frac{P(B\cap A)}{P(A)} = \frac{1/24}{1/3}$

$= \frac{1}{8}$

If A and B are events such that

$$\boxed{P(A \cup B) = 0.5,}\ P(\bar{B}) = 0.8 \text{ and } P(A/B) = 0.4,$$

just to disturb us.

What is $P(A \cap B)$ equal to?

(a) 0.08 (b) 0.02 (c) 0.8 (d) 0.2

W.K. that $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A \cap B) = P(A/B) \cdot P(B)$

$= 0.4(0.2)$

$= 0.08$

$\because P(\bar{B}) = 0.8$

$1 - P(B) = 0.8$

$P(B) = 0.2$

An examination consists of two papers, Paper 1 and Paper 2. The probability of failing in Paper 1 is 0.3 and that in Paper 2 is 0.2. Given that a student has failed in Paper 2, the probability of failing in Paper 1 is 0.6. The probability of a student failing in both the papers is

(a) 0.5 (b) 0.18

(c) 0.12 (d) 0.06

$P(I) = 0.3$

$P(II) = 0.2$

$P(I/II) = 0.6$

$P(I \cap II) = ?$

$= P(I/II) \cdot P(II)$

$= 0.6 \times 0.2$

$= 0.12$

Qs (2009/EE) Consider an unbalanced die numbered 1 to 6.
The prob of an odd face is 90% the prob of an Even face. &
The prob of any even numbered face is same
The prob of an Even face given that face Value exceeds 3 is 0.75
then find the prob that face Value exceeds 3 ?

(a) 10/19
(b) 10/57
(c) 8/3 > 1
(d) 80/171

$P(\text{odd } f) = 90\% \text{ of } P(\text{Even } f)$

$P(2) = P(4) = P(6) = x$ (let)

$P(\text{Even}f / f>3) = 0.75$

$P(f>3) = ?$

By Commonsense, $P(\text{odd}f) + P(\text{Even}f) = 1$

$\frac{90}{100} P(\text{Even}f) + P(\text{Even}f) = 1$

$P(\text{Even}f) = \frac{10}{19}$

$P(2) + P(4) + P(6) = \frac{10}{19}$

$x + x + x = 10/19 \Longrightarrow x = \frac{10}{19 \times 3}$

$$P(\text{Even}f / f>3) = 0.75$$

$$\frac{P\{\text{Even}f \cap f>3\}}{P(f>3)} = \frac{3}{4}$$

$$\frac{P(4 \text{ or } 6)}{P(f>3)} = \frac{3}{4}$$

$$\frac{P(4)+P(6)}{P(f>3)} = \frac{3}{4}$$

$$\frac{x+x}{P(f>3)} = \frac{3}{4}$$

$$\frac{2x}{P(f>3)} = \frac{3}{4} \Rightarrow 8x = 3 \times P(f>3)$$

$$\text{or } P[\text{face} > 3] = \frac{8x}{3} = \frac{8}{3} \times \frac{10}{57}$$

$$= \frac{80}{171}$$

A box contains 4 while balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is

(a) 1/3 (b) 3/7

(c) 1/2 (d) 4/7

App I

$S.Sp = \{(WW), (WR), (RW), (RR)\}$

$R.SSp = \{(WW), (WR)\} = 2$

$fav\ cases = \{(WR)\} = 1$

so Cond^n Prob $= \frac{fav}{R.SSp} = \frac{1}{2}$

App III

Req Prob $= P[W \cap R]$

$= 1 \times \left(\frac{3}{6}\right) = \frac{1}{2}$

(x)

$$\begin{array}{|c|}\hline 4W \\ 3R \\ \hline\end{array} \longrightarrow \begin{array}{|c|}\hline 3W \\ 3R \\ \hline\end{array}$$

R.Exp 1 $\Downarrow$ $P(W) = 1$

R Exp 2 $\Downarrow$ $P(R) = \frac{3}{6} = \frac{1}{2}$

$\because$ Both the R Exp are $\boxed{\text{Ind}}$ so Req Prob $= P(W \cap R)$

$= 1 \times \frac{1}{2} = \frac{1}{2}$

Exhaustive Events →

if $\boxed{E_1 \cup E_2 \cup E_3 = S}$ $\Longleftrightarrow$ then [E_1, E_2, E_3 are called Exhaustive events]

M.E Events : if $\boxed{E_i \cap E_j = \phi}$ $\forall\, i \neq j$ then $\Longleftrightarrow$ [E_1, E_2, E_3 are called ME Events.]

ME & Exhaustive Events →

if $\boxed{\underbrace{E_i \cap E_j = \phi}_{\text{ME Nature}} \;\&\; \underbrace{E_1 \cup E_2 \cup E_3 = S}_{\text{Exhaustive Nature}}}$ $\Longleftrightarrow$ then [Events are called ME as well as Exhaustive]

Conclusion :→ $P(E_1 \cup E_2 \cup E_3) = P(S) \Longrightarrow \boxed{P(E_1) + P(E_2) + P(E_3) = 1}$

it is the Necessary & Sufficient Condition ME & Exhaustive events.

eg: $S_{Die} = \{1, 2, 3, 4, 5, 6\}$ ⟶ $E_1 = \{1, 3, 5\}$, $P(E_1) = \frac{1}{2}$

$E_2 = \{2, 4, 6\}$, $P(E_2) = \frac{1}{2}$

$\because E_1 \cap E_2 = \phi \Rightarrow$ ME & $E_1 \cup E_2 = S$ is Exhaustive

$\because \boxed{P(E_1) + P(E_2) = 1}$ Hence verified.

eg $S_{Die} = \{1, 2, 3, 4, 5, 6\}$ ⟶ $E_1 = \{1, 2\}$, $P(E_1) = \frac{2}{6} = \frac{1}{3}$

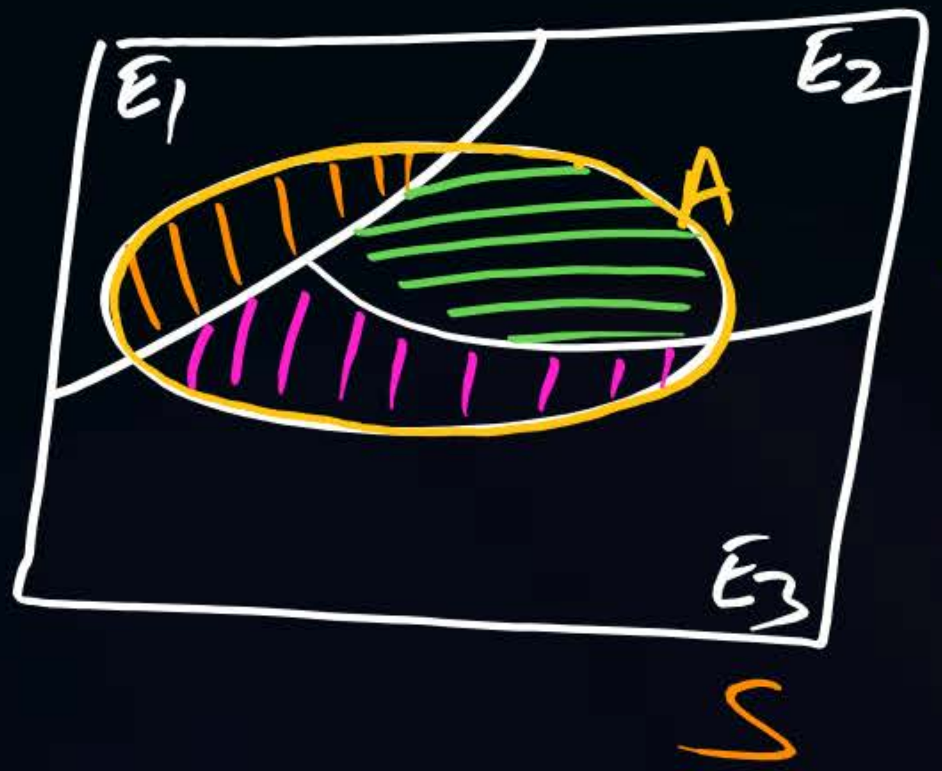
$E_2 = \{3\}$, $P(E_2) = \frac{1}{6}$

$E_3 = \{4, 5, 6\}$, $P(E_3) = \frac{3}{6} = \frac{1}{2}$

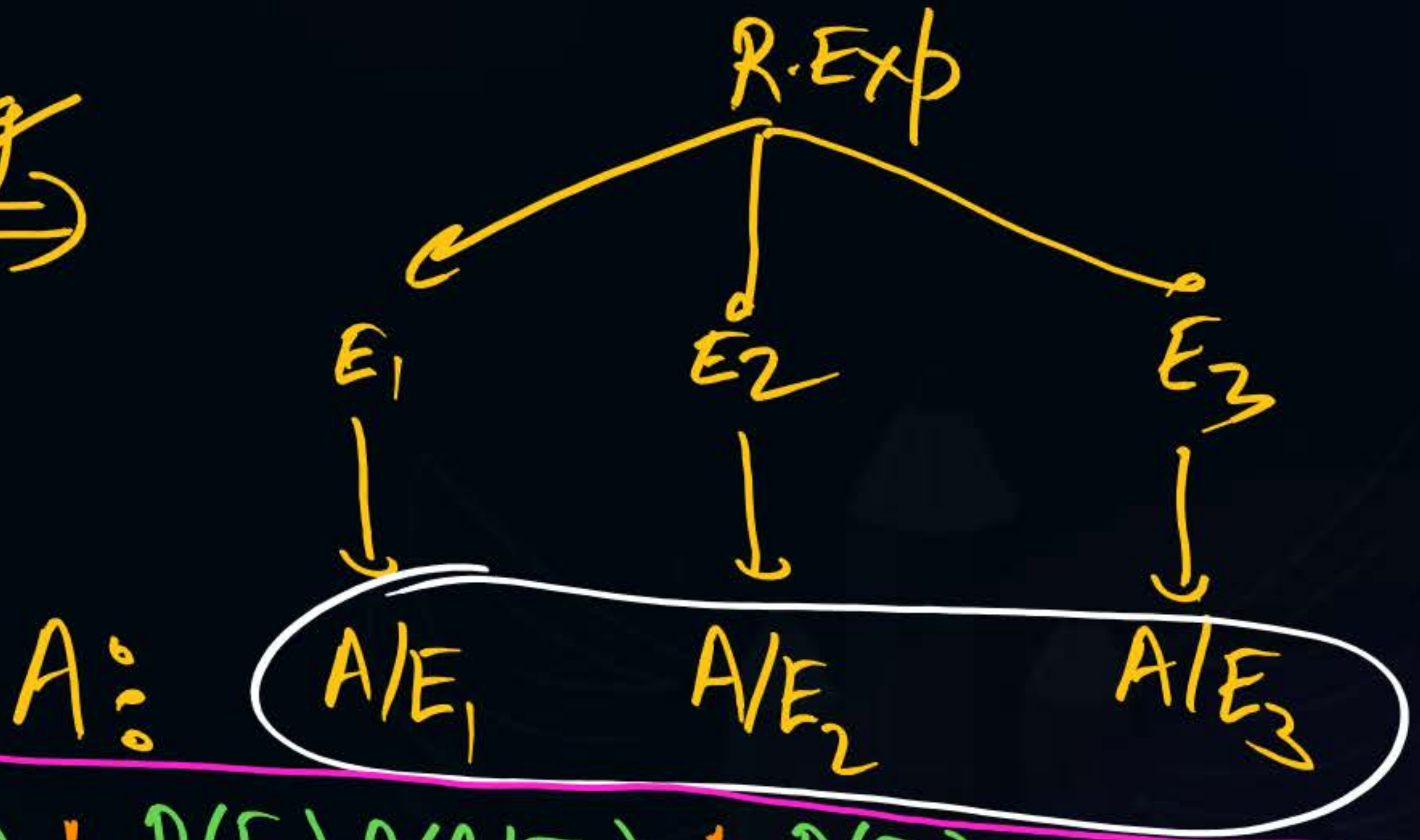
$\because E_i \cap E_j = \phi$ is ME, & $E_1 \cup E_2 \cup E_3 = S$ is Exhaustive

$\because P(E_1) + P(E_2) + P(E_3) = \frac{1}{3} + \frac{1}{6} + \frac{1}{2} = 1$ Hence Verified

Law of Total Prob: Let E_1, E_2, E_3 are ME & Exhaustive events & A is an event which can occur with all E_1, E_2, E_3 then



Tree Diag $\Longrightarrow$



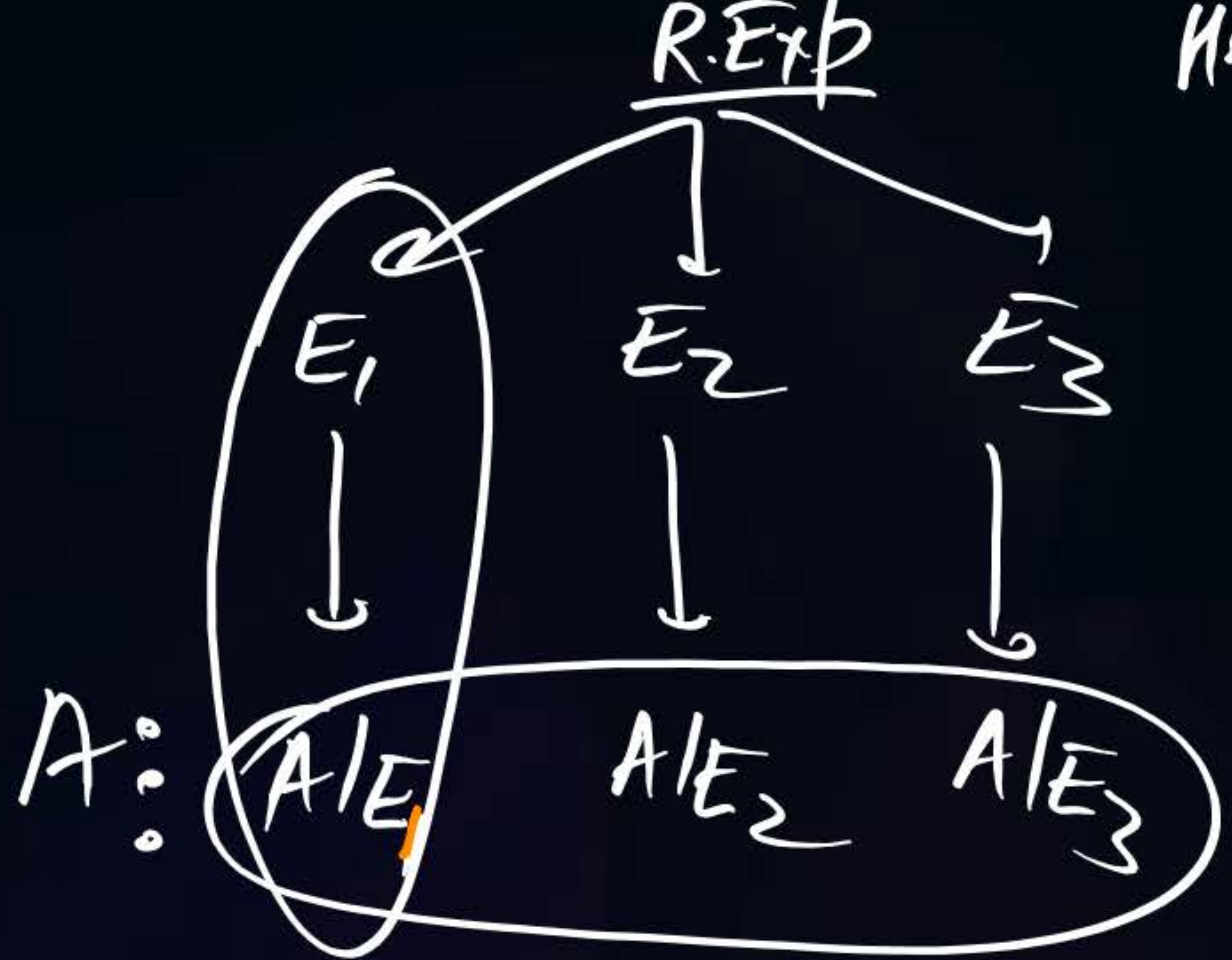
So $$\boxed{P(A) = P(E_1)\cdot P(A|E_1) + P(E_2)\cdot P(A|E_2) + P(E_3)\,P(A|E_3)}$$

this formula of finding A is called Law of Total Probability.

Baye's Theorem (Inverse probability theorem) →

(this Theorem is useful to solve complex Questions of Conditional Probability.)

Theory same as above



Here $\boxed{P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$ — (1)

$$\left.\begin{aligned} P(E_1/A) &= \frac{P(E_1)\cdot P(A/E_1)}{P(A)} \\ P(E_2/A) &= \frac{P(E_2)P(A/E_2)}{P(A)} \\ P(E_3/A) &= \frac{P(E_3)P(A/E_3)}{P(A)} \end{aligned}\right\} \text{Baye's Th.}$$

$P(A/E_1)$ = Prob of A when E_1 has already occurred

$P(E_1/A)$ = Prob of E_1 when A ,, ,, ,,

Important Points :-

1. Necessary condition for the existence of Law of Total Prob & Baye's Th is
   Associated events must be ME & Exhaustive.

2. In Law of Total Prob ; $A = \{$ Assume that event as A which is Required $\}$

3. In Baye's Th : $A = \{$ Assume that event as A which is given as condition $\}$

4. If in a Question, there is a feeling of $\boxed{\text{CROSS Check}}$ the given condition we can use Baye's Th.
   & if we have No Condition in a Question (or No Feeling of CROSS Check) then use Law of Total Prob.

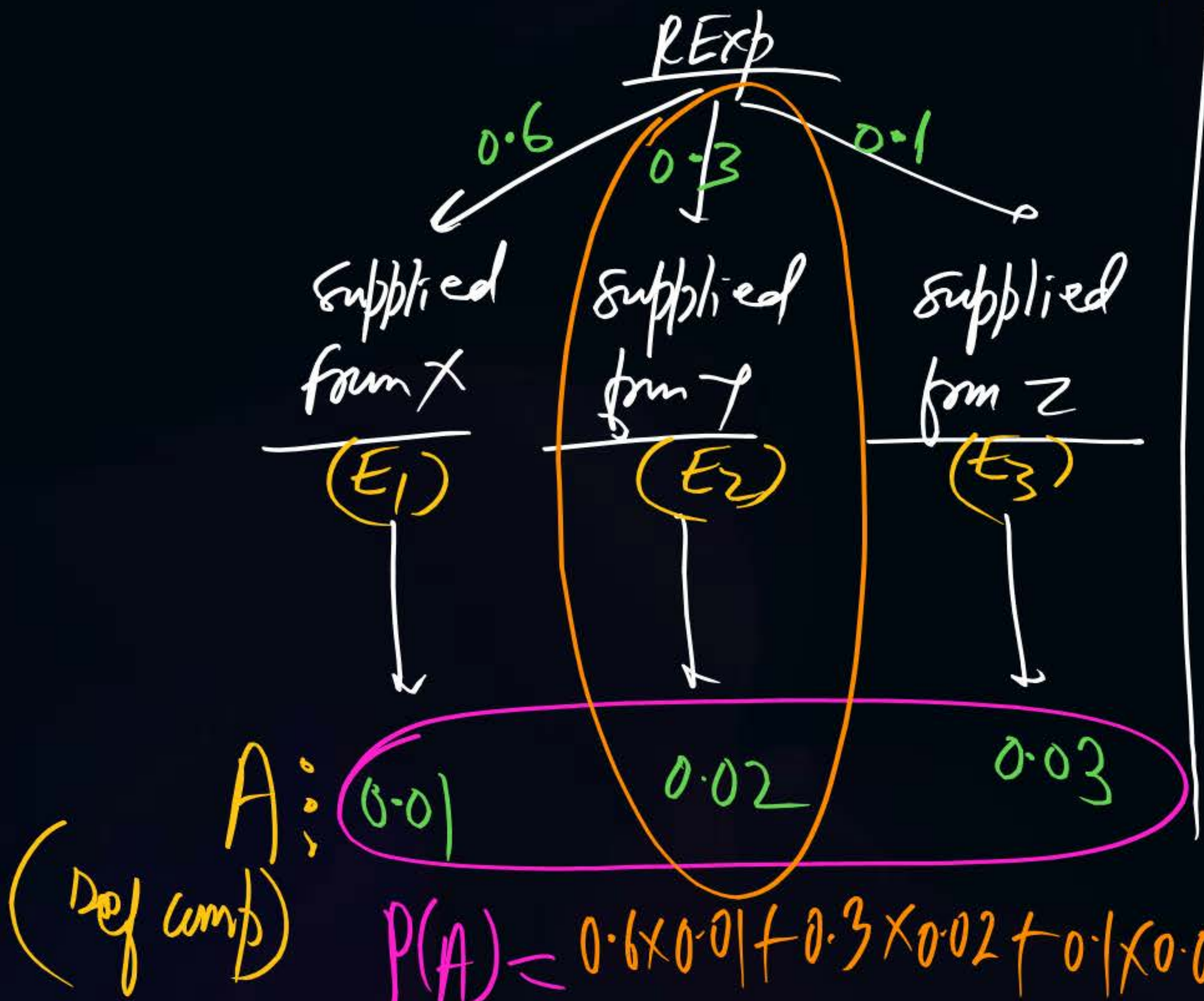
Q. Computers are supplied to an organisation according to chart; R. Exp.

Company	% of Computers supplied	Prob of being Defective
X	60%	0.01
Y	30%	0.02
Z	10%	0.03

1. Find the Prob that supplied Computer is defective.
2. If supplied Comp is Defective then Find the Prob that it was supplied from Company Y.

A

Sol: $A = \{\text{Supplied Comp is Defective}\}$ $\because P(E_1)+P(E_2)+P(E_3) = 0.6+0.3+0.1 = 1$



(ii) original Prob $= P(S) = 1$.

Reduced Prob = Total Prob = P(Cond^n)

$= P(A) = 0.015$

$$P(E_2/A) = \frac{\text{fav Path}}{\text{R. Path}} = \frac{\text{fav Prob}}{\text{R. Prob}}$$

$$= \frac{0.3 \times 0.02}{0.015} = \frac{6}{15}$$

$$P(A) = 0.6 \times 0.01 + 0.3 \times 0.02 + 0.1 \times 0.03 = 0.015 = \frac{15}{1000}$$

Note:

(1) Also Evaluate $P(E_1/A) = ? = \frac{0.6 \times 0.01}{0.015} = \frac{6}{15}$

& $P(E_3/A) = ? = \frac{0.1 \times 0.03}{0.015} = \frac{3}{15}$

(2) out of 1000 supplied computers only 15 are Defective & out of 15 Defective computers, 6 are supplied from X, 6 from Y & 3 from Z.

Note: If we are using Given Method to ans an objective type question having 4 choices in which only one is correct then $P(\text{Choosing Correct Ans}) = \frac{{}^1C_1}{{}^4C_1} = \frac{1}{4}$

(HW) Parcels are sending from Sender S to Receiver R sequentially through two post offices. The probability of loosing an incoming parcel by each P.O is $\frac{1}{5}$ independently of all other parcels.

Given that parcel is lost, then Find the prob that it was lost by 2^{nd} P.O?

(a) $1/5$

(b) $\frac{1}{25}$

(c) $\frac{4}{25}$

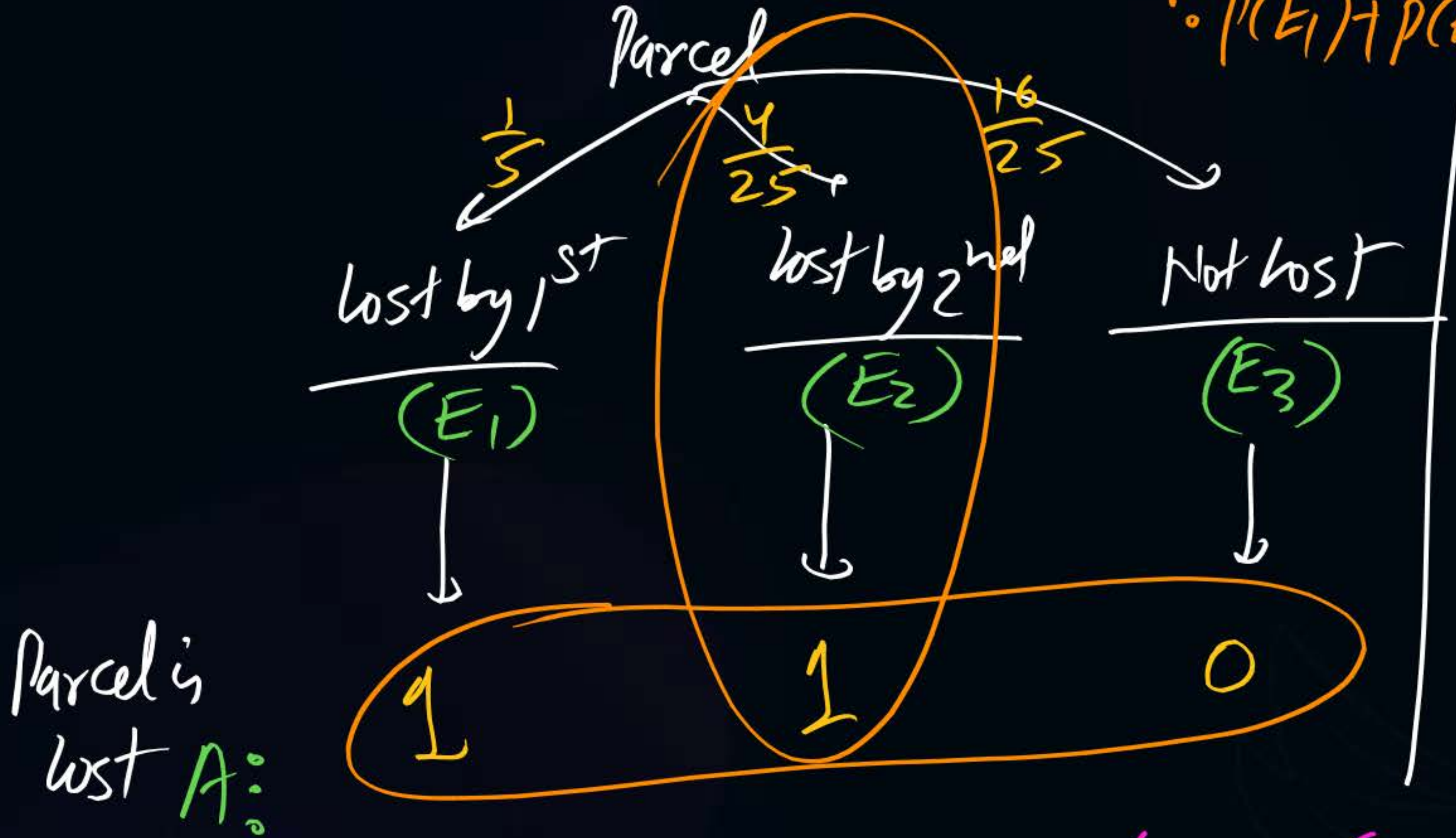
(d) $\frac{4}{9}$

$\underbrace{\text{parcel is lost}}_{A = \text{Condition}}$

$E_1 = \{\text{lost by } 1^{st}\}, \; P(E_1) = \frac{1}{5}$

$E_2 = \{\text{lost by } 2^{nd}\}, \; P(E_2) = P(NL \text{ by } 1^{st} \;\&\; L \text{ by } 2^{nd}) = \frac{4}{5} \times \frac{1}{5} = \frac{4}{25}$

$E_3 = \{NL \text{ by any P.O}\}, \; P(E_3) = P(NL \text{ by } 1^{st} \;\&\; NL \text{ by } 2^{nd}) = \frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$



$\because P(E_1) + P(E_2) + P(E_3) = 1$

$$P(E/A) = \frac{\frac{4}{25} \times 1}{9/25} = \frac{4}{9}$$

$$= \frac{\text{fav Prob}}{\text{R}\cdot\text{Prob}} = \frac{\text{fav Prob}}{P(\text{cond}^n)}$$

$$P(A) = P(\text{parcel is lost}) = \left(\frac{1}{5} \times 1\right) + \left(\frac{4}{25} \times 1\right) + \left(\frac{16}{25} \times 0\right)$$

Reduced Prob $= 9/25$

Analysis:

$$\text{Sender} \longrightarrow \text{PO-1} \xrightarrow[20\,NL]{5\,L} \text{PO-2} \xrightarrow[16\,NL]{4\,L} \text{Receiver}$$

eg (25P) (25P) (20P) (16P)

$$\text{eg} \quad (50P) \longrightarrow (50P) \xrightarrow[40\,NL]{10\,L} (40P) \xrightarrow[32\,NL]{8\,L} (32P)$$

S (PO-1) (PO-2) R

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter driver, car driver and a truck driver is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

Condition

(a) 0.019

(b) 0.086

(c) 0.19

(d) 0.86

Total drivers = 2000 + 4000 + 6000 = 12000

$E_1 = \{\text{Scooter driver}\}$, $P(E_1) = \frac{2000}{12000} = \frac{1}{6}$, $P(A/E_1) = 0.01$

$E_2 = \{\text{Car Driver}\}$, $P(E_2) = \frac{4000}{12000} = \frac{1}{3}$, $P(A/E_2) = 0.03$

$E_3 = \{\text{Truck Driver}\}$, $P(E_3) = \frac{6000}{12000} = \frac{1}{2}$, $P(A/E_3) = 0.15$

A : {one of the person meets with an Accident}

R.Exp.

$\frac{1}{6}$ Scooter (E_1) — $\frac{1}{3}$ Car (E_2) — $\frac{1}{2}$ Truck (E_3)

$\because P(E_1)+P(E_2)+P(E_3)=\frac{1}{6}+\frac{1}{3}+\frac{1}{2}=1$

$\therefore$ Events are ME & Exhaustive.

Person meets with an Accident A: $A/E_1 = 0.01$ $\quad A/E_2 = 0.03$ $\quad A/E_3 = 0.15$

$$P(A)=\left(\frac{1}{6}\times 0.01\right)+\left(\frac{1}{3}\times 0.03\right)+\left(\frac{1}{2}\times 0.15\right)$$

$$=\frac{0.01+0.06+0.45}{6}=\frac{52}{600}$$

$$P(E_1/A)=\frac{P(\text{V. Path})}{P(\text{H. Path})}$$

$$=\frac{P(\text{fav Path})}{P(\text{Reduced Path})}$$

$$=\frac{P(\text{fav Path})}{P(\text{Cond}^n)}=\frac{\frac{1}{6}\times 0.01}{52/600}$$

$$=\frac{1}{52}$$

Thank you

Keep Hustling!