CS & IT ENGINEERING

Theory of Computation

DFA

Lecture No.- 03

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Recap of Previous Lecture

Topic — DFA Construction

Topic — DFA ⇒ Language [Condition]

Topics to be Covered

- Topic
- Topic ??
- Topic ??

DFA

① minimization of DFA

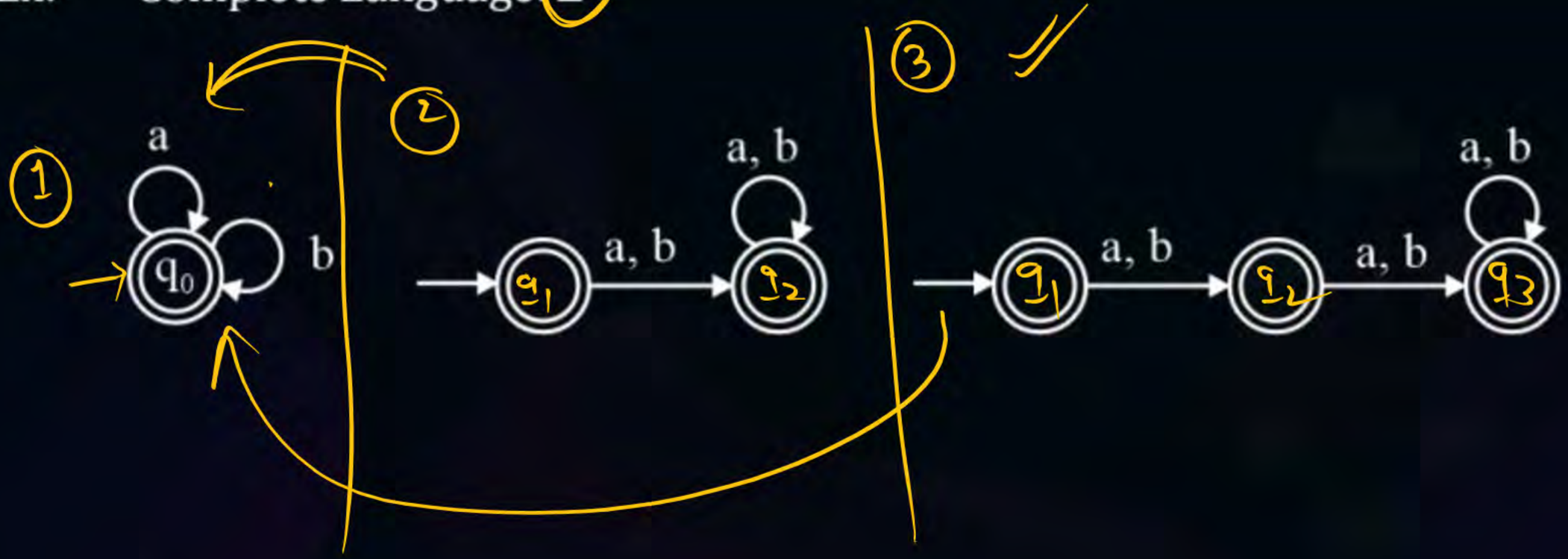
② Complement of DFA

③ Product DFA Construction

Topic : Minimization of DFA

→ For a given regular language even though many DFA exist but minimal state DFA is unique.

Ex: Complete Language Σ^*



① In any DFA if all states are final then it accepts Complete Language.

② In any DFA if all states are non final then it accepts. empty Language

Topic : Minimization Algorithm

1. State equivalence algorithm
2. Table filling algorithm

Equivalent States:

Two states q_1, q_2 are said to be equivalent both $\delta(q_1, x)$ and $\delta(q_2, x)$, $\forall\ x \in \Sigma^*$ should result either final state or non final state.

$\forall x \in \Sigma^*$

Topic : Procedure of minimization

1. Elimination of inaccessible states.

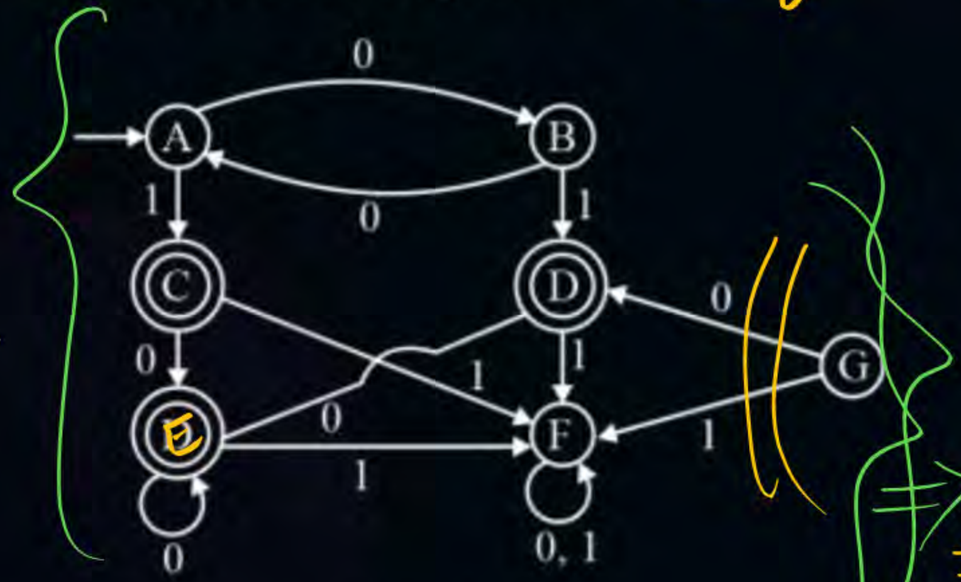
inaccessible state:

Any State which is not reachable from initial state is inaccessible state.

2. Apply algorithm steps {equivalent state}
3. Merge single group into one state
4. Construct new minimized DFA

Topic : Procedure of minimization

1. Reduce states of following DFA



Step-1: Elimination inaccessible state.

Note: Dead state is different from inaccessible state.

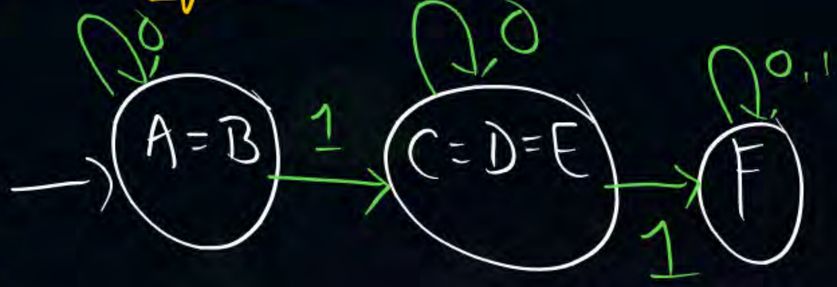
Don't remove Dead state

Language ?

① Inaccessible state

{G}

② Equivalent state



③

min DFA

min DFA

3 State

Topic : Procedure of minimization

Step:2

transition table

State	0	1
A	B	C
B	A	D
F	F	F
C	E	F
D	E	F
E	E	F

Equivalent state

① {A, B, F} {C, D, E}

② {A, B} {F} {C, D, E}

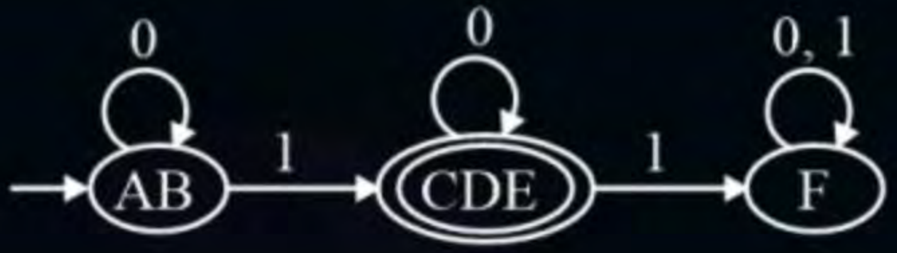
③ A≠B | F | C≠D≠E

Algorithm:

1. {A, B, F} {C, D, E}
2.
3.

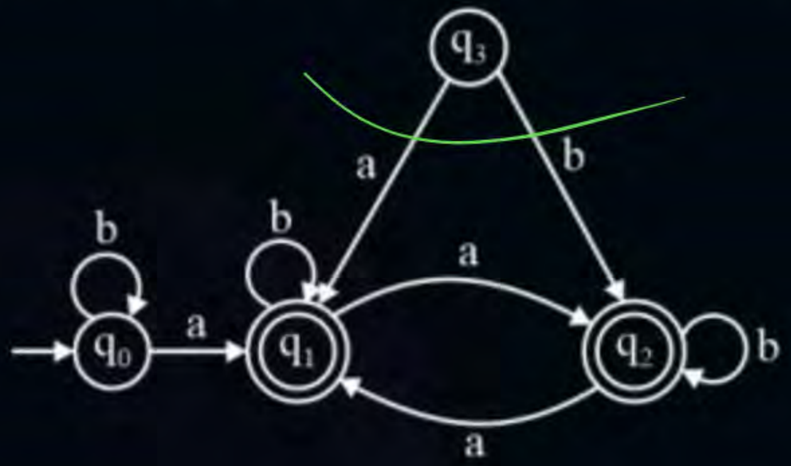
Topic : Procedure of minimization

Minimized DFA



Topic : Procedure of minimization

Consider the following Finite State Automation



① In accessible state

q_3

② Equivalent states

min DFA ?

②

Topic : Procedure of minimization

Step 1: Eliminate q_3

Step 2:

	a	b
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_1	q_2

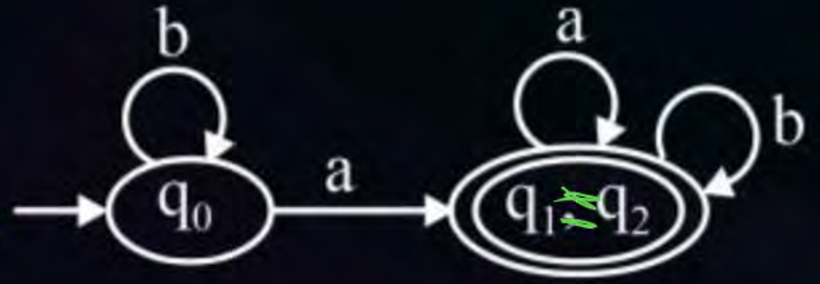
① $\{q_0\}$ $\{q_1, q_2\}$

② $\{q_0\}$ $\{q_1, q_2\}$

Algorithm step

1. $\{q_0\}$ $\{q_1, q_2\}$
2. $\{q_0\}$ $\{q_1, q_2\}$

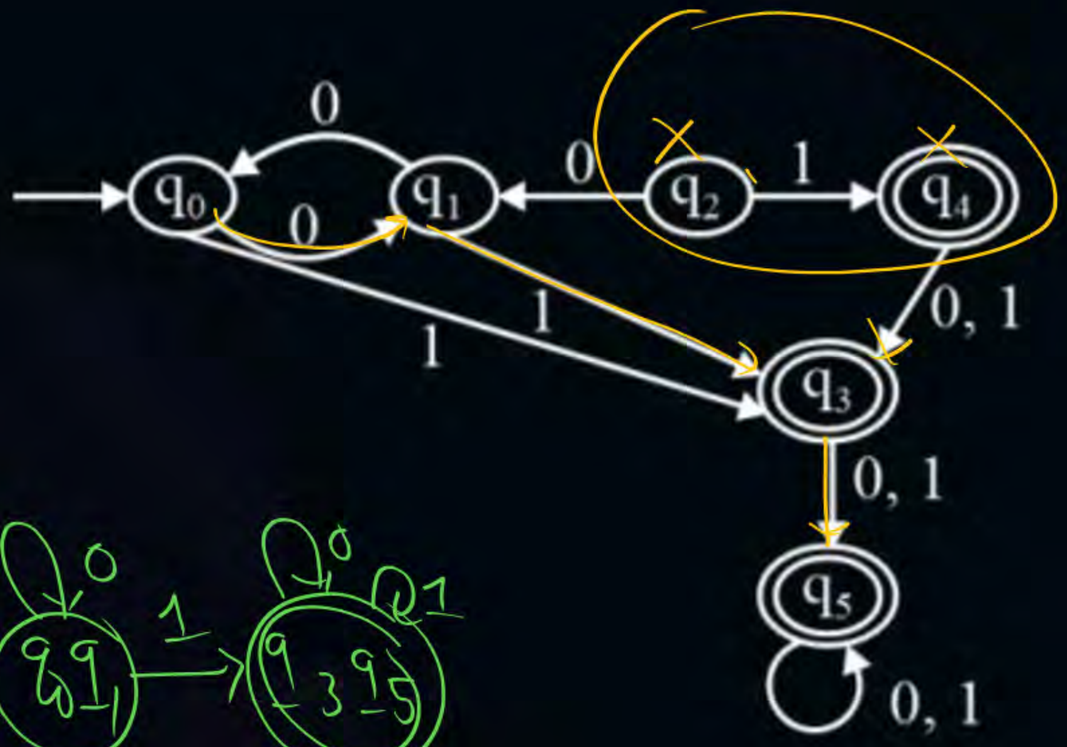
Minimum DFA



Topic : Procedure of minimization

① Inaccessible state

$\{q_2, q_4\}$



② Equivalent

$q_0 = q_1$

$q_3 = q_5$

②

min DFA?

②

Minimize given DFA

Topic : Procedure

Step 1: Eliminate

Step 2:

	a	b
→ q_0	q_1	q_3
q_1	q_0	q_3
(q_3)	q_5	q_5
(q_5)	q_5	q_5

Algorithm

1. $\{q_0, q_1\}\ \{q_3, q_5\}$

2.

Minimum DFA

① $\{q_0, q_1\}\ \{q_3, q_5\}$

② $\{q_0, q_1\}\ \{q_3, q_5\}$

Topic : Procedure of minimization

min DFA?

(1) state

How many inaccessible states present in given DFA

(1) $\{q_4, q_5, q_6, q_7\}$ ✓

(2)

	0	1
→ q_0	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
(q_3)	q_3	q_0

(a) $\{q_0, q_1, q_2\}\{q_3\}$

(b) $\{q_0, q_1\}\{q_2\}\{q_3$

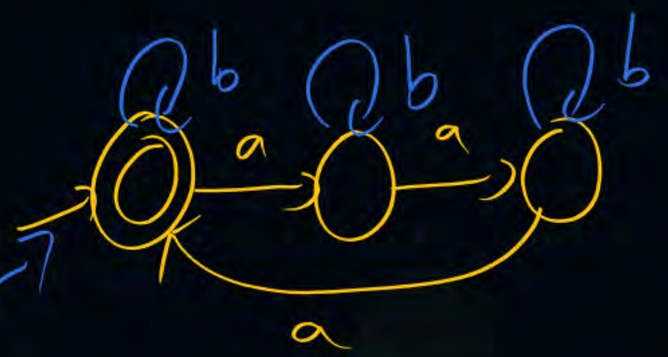
(c) $\{q_0\}\{q_1\}\{q_2\}\{q_3\}$

Topic : DFA

$\Sigma = \{a, b\}$

Construct the minimal DFA that accept all string a's and b's where

1. Length of string exactly 4.
2. Number of a's length of string atleast 4.
3. Length of string atmost 4.
4. Length of string divisible by 4.
5. Number of a's exactly 5. ⟹ (7)
6. Number of b's exactly 2.
7. Number of a's divisible by 3. ⟹ (3)
8. Number of b's not divisible by 4. ⟹ (4)
9. Length of the string even.



Topic : DFA

6 states

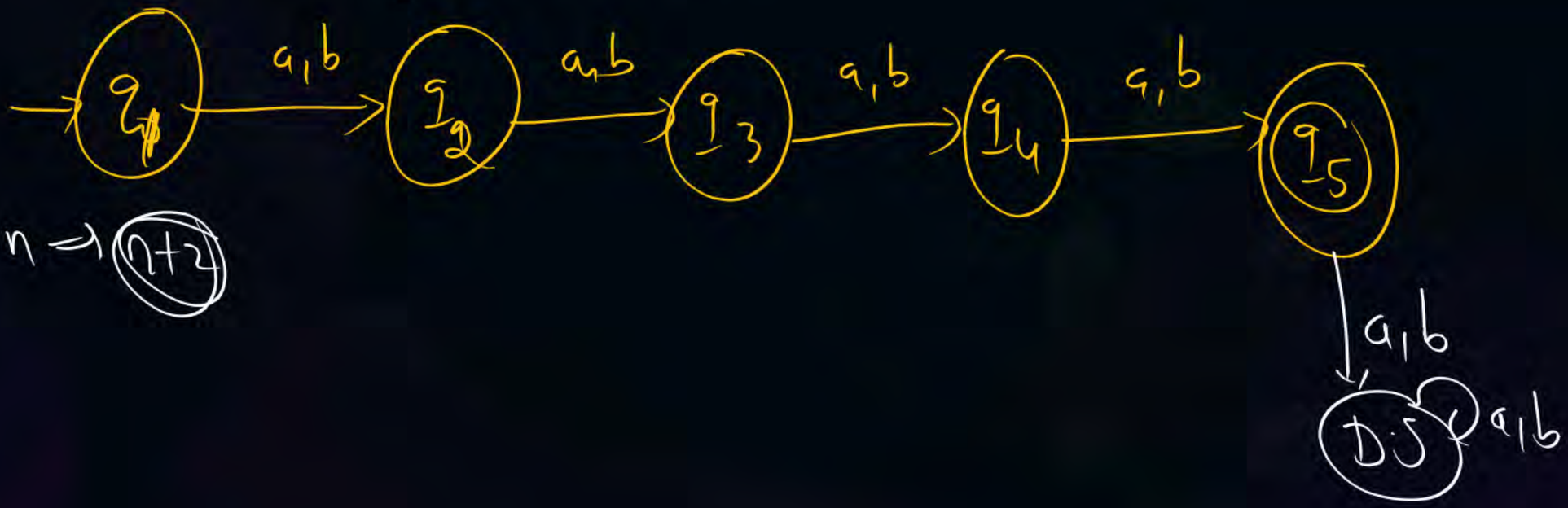
#Q. Length of string exactly 4. → 6 states

a/b a/b a/b a/b
1 2 3 4

exactly 5 ⇒ 7

⇓

exactly n ⇒ (n+2) states

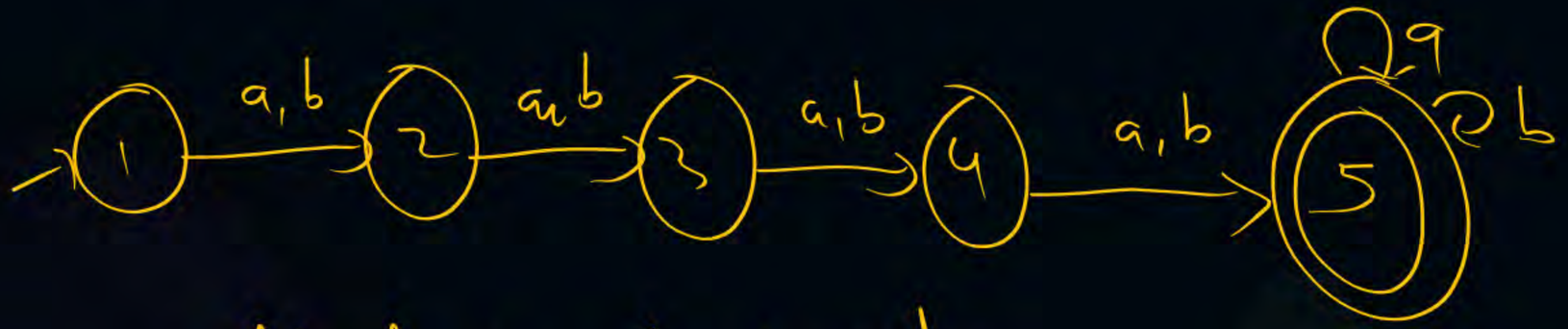


exactly n ⇒ (n+2)

Topic : DFA

min DFA

#Q. Length of string atleast 4. $\Rightarrow$ 5 states



atleast $n \Rightarrow (n+1)$ states

Topic : DFA

min DFA {0, 1, 2, 3, 4}

#Q. Length of string atmost 4.

⇒ 6 states

5 ⇒ 7 states

atmost n ⇒ (n+2) States

min DFA

exactly $n \Rightarrow (n+2)$

atleast $n \Rightarrow (n+1)$

atmost $n \Rightarrow (n+2)$

Divisible by $n \Rightarrow n$ state

Topic : DFA

$\{0, 4, 8, 12, 16 \cdots\}$

$= \cup \cup \cup \cup \cup$

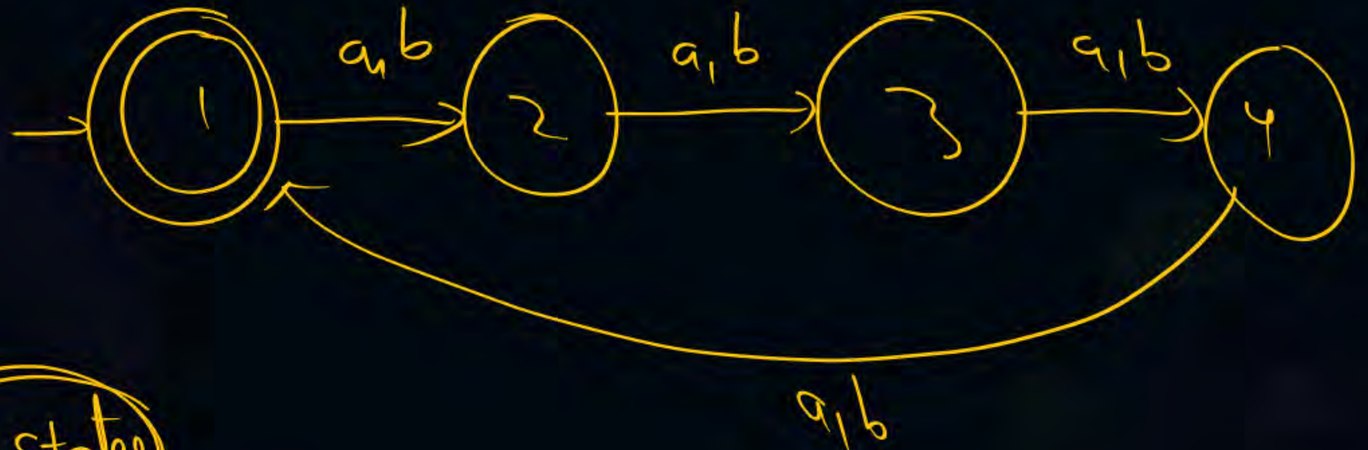
$4 \quad 4 \quad 4 \quad 4$

#Q. Length of string divisible by 4.

Div by 4 => (4)

Div by 6 => 6

Div by n => (n states)



Topic : DFA

NOTE:

→ Minimal DFA that accept exactly N length string requires (N + 2) states includes dead state.

→ Minimal DFA that accept atleast N length string requires (N + 1) states.

→ Minimal DFA that accept atmost N length string requires (N + 2) states includes dead states.

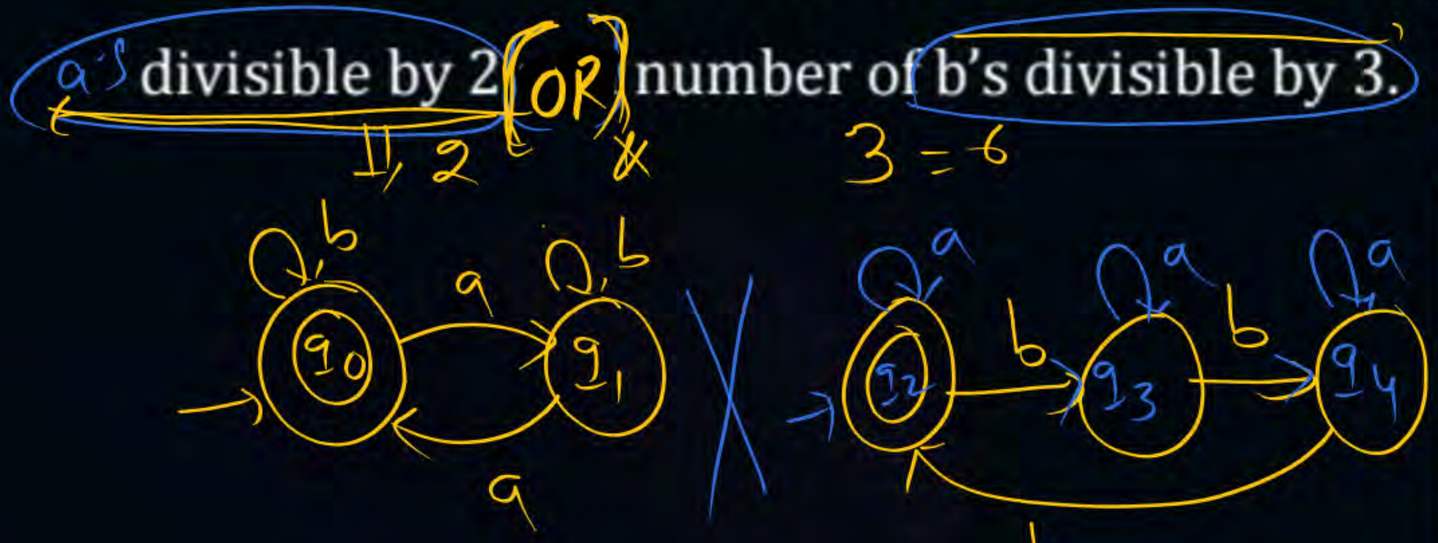
→ The minimal DFA that accept length of the string divisible by N then requires N states.

Topic : DFA

Product DFA Construction

$\Sigma = \{a, b\}$

Construct a minimal DFA that accept all string a's and b's. where number of a's divisible by 2 (OR) number of b's divisible by 3.



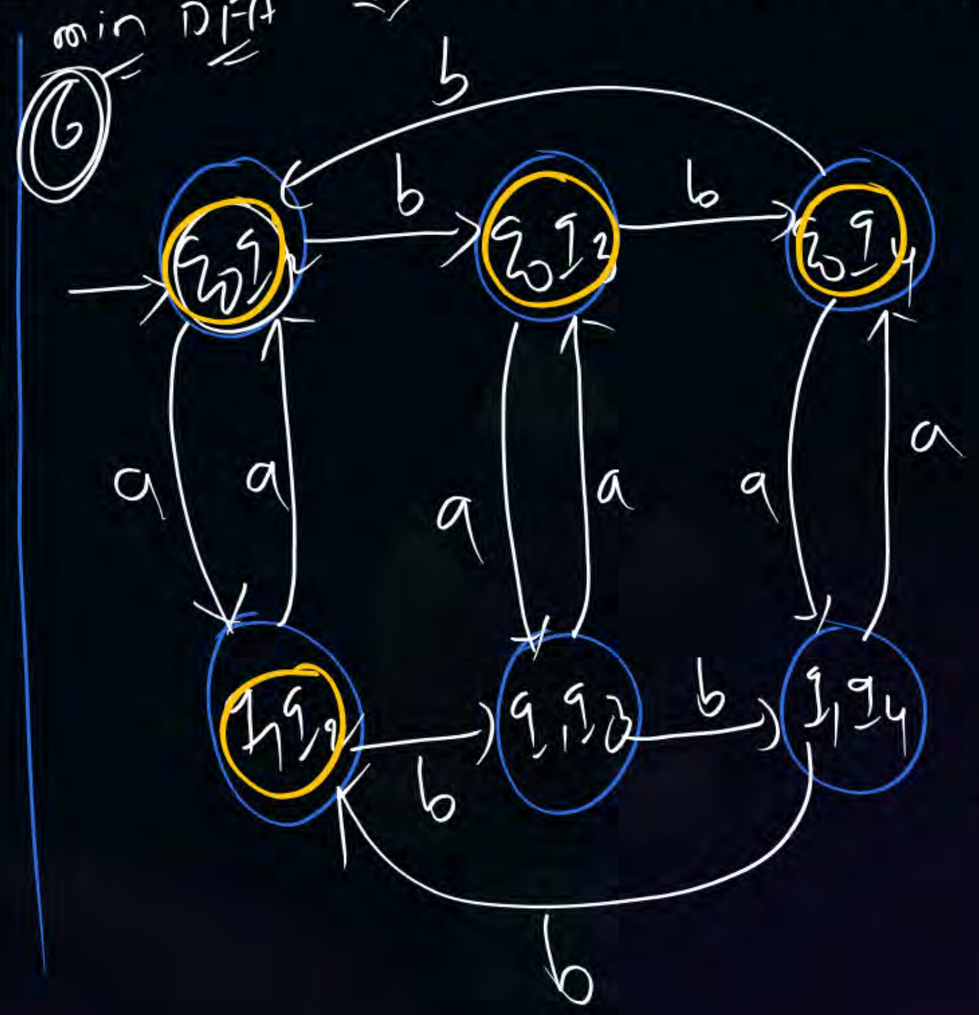
q_0 q_2	q_0 q_2
a ↓ ↓ a	b ↓ ↓ b
q_1 q_3	q_0 q_3

$Q = 6$

$\Sigma = \{a, b\}$

$q_0 = (q_0 q_2)$

$F = (q_0 q_1)$



Topic : DFA

Construct a minimal DFA that accept all string a's and b's. where number of a's divisible by 2 and number of b's divisible by 3.

Topic : DFA

$\Sigma = \{a, b\}$

How many number of state are there with minimum DFA for the following state.

(a) Number of a's divisible by 2 (and) number of b's not divisible by 3. $2 \times 3 = 6$ = 6 states

(b) Number of a's divisible by 2 (and) number of b's atleast 3. $2 * 4$ = 8 states

(c) Number of a's atleast 2 (and) number of b's atleast 3. $3 * 4$ = 12 states

(d) Number of a's exactly 2 (and) number b's atleast 2. $3 * 3 = (9+1) = 10$ states

(e) Number of b's atmost 3 (and) number b's exactly 3. $4 * 4 = (16+1) = 17$ states

(f) Number of a's not divisible by 2 (or) number of b's exactly 3.

$2 + 4 \Rightarrow (8+1) = 9$ states

0's exactly 4 and # 1's atleast 2

$5 * 3 = 15+1$

16 states

Topic : DFA

$\Sigma = \{a, b\}$ $\{0, 4, 8, 12, \ldots\}$

1. Length of the string divisible by 2 and divisible by 4 $\Rightarrow$ (4)
2. Length of string divisible by 2 OR divisible by 4. $\Rightarrow$ (2) $= \{0, 2, 4, 6, 8 \ldots\}$
3. Length of string divisible by 3 divisible by 4
4. Length of string divisible by 3 OR divisibly by 4
5. Length of string divisible by 6 OR divisibly by 8
6. Number of a's divisible by 6 and number of divisible by 8.

Home Work

THANK - YOU