



DS & AI
CS & IT

Linear Algebra

Lecture No. 02



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Recap of previous lecture



Topic

BASICS of Determinants

Topics to be Covered



Topic

BASICS OF MATRICES

Remaining Portion of Det -

Analysis: if $A_{n \times n}$ then $\bar{A}^I = \frac{\text{adj } A}{|A|}$ where $(\text{adj } A)_{n \times n}$ & $|A|$ is any const.

(PODCAST)

$$\text{Now, } A\bar{A}^I = \frac{A\text{adj } A}{|A|} \Rightarrow I = \frac{A\text{adj } A}{|A|} \Rightarrow A\text{adj } A = |A| \cdot I_n$$

$$\text{Now } A(\text{adj } A) = |A| \cdot I_n = |A| \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{n \times n} = \begin{bmatrix} |A| & 0 & 0 & \cdots & 0 \\ 0 & |A| & 0 & \cdots & 0 \\ 0 & 0 & |A| & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & |A| \end{bmatrix}_{n \times n} = \text{scalar Mat.}$$

$$\text{So } |A\text{adj } A| = |A| \cdot |A| \cdot |A| \cdot \cdots \cdot |A| = |A|^n \text{ term}$$

$$\& \text{Tr}(A\text{adj } A) = |A| + |A| + |A| + \cdots + |A| = n|A| \text{ term}$$

Q) If $A_{n \times n}$, then Evaluate $|\text{adj} A|, |\text{cof} A|, |\bar{A}'| = ?$

Proof: w.k.t that $|A \cdot (\text{adj } A)| = |A|^n$

$$\boxed{|A| \cdot |\text{adj } A| = |A|^n}$$

$$\boxed{|\text{adj } A| = |A|^{n-1}}$$

Now again w.k.t that, $|B| = |B^T|$

So $|\text{cof } A| = |(\text{cof } A)^T| = |\text{adj } A| = |A|^{n-1}$

$$\text{i.e. } \boxed{|\text{cof } A| = |A|^{n-1}}$$

Now, $\bar{A}' = \frac{\text{adj } A}{|A|}$

$$\Rightarrow |\bar{A}'| = \left| \frac{(\text{adj } A)}{|A|} \right| = \frac{1}{|A|} \cdot |\text{adj } A|$$

$$|\bar{A}'| = \frac{|A|^{n-1}}{|A|^n} = \frac{1}{|A|}$$

Hence $\det \bar{A}' = \frac{1}{\det A}$

Imp Point: if $A_{n \times n}$ then $|\text{adj} \text{adj} \text{adj} \dots \text{adj} A| = |A|^{(n-1)^r}$ Learn.

e.g. if $|A|_{4 \times 4} = 5$ then evaluate $|\text{adj} \text{adj} \text{adj} A| = ?$

- (a) 5^9 $n=4, r=3$
- (b) ~~5^{27}~~ So $|\text{adj} \text{adj} \text{adj} A| = |A|^{(n-1)^r}$
 $= (5)^{(4-1)^3} = (5)^{3^3} = 5^{27}$
- (c) 5^4 Note: $(a^b)^c = a^{bc}$, $(a^b)^c = (a)^{\underbrace{b \times b \times b \times \dots \times b}_{c \text{ times}}}$
- (d) 5^3

MATRIX

→ "It is a rectangular arrangement of m-n numbers"

$$A = [a_{ij}]_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

$1 \leq i \leq m$

$1 \leq j \leq n$

H-lines = Rows

V-lines = Columns.

→ Sq. Mat = Defⁿ ① No. of Rows = No. of Columns.

Defⁿ ② If in a Mat, Diag exist, then it Must be Square

Defⁿ ③ if \forall element, \exists Corresponding Element then Mat is Square Mat.

Sq Mat

eg $A = [a_{ij}]_{n \times n} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{pmatrix}_{n \times n}$

$1 \leq i \leq n$
 $1 \leq j \leq n$

- ① for Diagonal elements, $i=j \neq i \neq j$
- ② for upper Diag elements, $i < j \neq i \neq j$
- ③ for Lower " ", $i > j " "$
- ④ for off Diag elements, $i \neq j " "$
- ⑤ Corresponding elements are a_{ij} & a_{ji}

$\text{Trace(Sq Mat)} = \sum a_{ii}$

i.e. $T_8(A) = \text{sum of diag elements}$

eg $A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

$T_8(A) = 3$, & $|A| = -16$

Some Special types of Sq. Matrices -

eg $\begin{bmatrix} 2 & 4 & -1 & 3 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -4 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 2 & -1 & 3 & -3 \end{bmatrix}$,

U.T.M L.T.M

$\text{Tr} = -2, \text{Det} = 8$ $\text{Tr} = 0, \text{Det} = 0$

$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Diag. Mat. Scalar Mat Identity Mat

$\text{Tr} = 4, \text{Det} = 0$ $\text{Tr} = 20$
 $\text{Det} = 5^4$ $\text{Tr} = 9$
 $\text{Det} = 1$

(*) Diag Mat: $A = [a_{ij}]_{n \times n}$ s.t $a_{ij} = \begin{cases} 0 & i \neq j \\ \text{at least one Diag element should be Non zero, } i=j \end{cases}$

(*) To find Determinant of U.T.M, L.T.M, Diag Mat, Scalar Mat, Identity Mat,
 just Multiply the Diag elements ie $\boxed{\text{Det} = \text{Product of Diag elements}}$.

$$\text{Ques } |A| = \begin{vmatrix} 0 & 1 & 2 & 0 \\ -1 & 0 & 4 & 0 \\ -2 & -4 & 0 & 6 \\ 0 & 0 & -6 & 0 \end{vmatrix} = ?$$

4x4

(a) 0

(b) 16

~~(c) 36~~

(d) 21

Given Mat is neither U.T.M, Nor L.T.M, not a Diag Mat

So we can not use Shortcut Method as discussed in Previous slide.

Hence we will follow Conventional approach.

$$|A| = \dots \textcircled{HW} \dots = 36$$

A matrix $A = [a_{ij}]_{n \times n}$ is said to be lower triangular

if

- (a) $a_{ij} = 0$ for $i > j$
- (b) $a_{ij} = 0$ for $i < j$
- (c) $a_{ij} = 0$ for $i \geq j$
- (d) $a_{ij} = 0$ for $i \leq j$

$$g A = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3 \times 3} = L.T.M.$$

$$a_{12} = a_{13} = a_{23} = 0.$$

ie $a_{ij} = 0 \text{ } \forall i < j$ ie L.T.M

~~Q~~ If $A = [a_{ij}]_{n \times n}$ s.t $a_{ij} = 0 \text{ } \forall i > j$ then A is U.T.M

Singular Mat: If $|A|=0$ then A is called Singular Mat

Non Singular Mat: If $|A| \neq 0$ then A is called Non Singular Mat

Invertible Mat - If \bar{A}^{-1} exist then A is called Invertible Mat

& it is defined as $\boxed{\bar{A}^{-1} = \frac{\text{adj } A}{|A|}}$ or $\frac{(\text{Cof } A)^T}{|A|}$

N. Condition for a Matrix to be invertible is $|A| \neq 0$ i.e. A must be Non Sing.

Sp. Conclusion: w.k. that, $A\bar{A}^{-1} = \bar{A}^{-1}A = I$

so if we have,

$$\boxed{AB = BA = I} \Rightarrow$$

Inverse of $A = B$ i.e. $\bar{A}^{-1} = B$

Inverse of $B = A$ i.e. $\bar{B}^{-1} = A$

i.e. Both are the Inverses of each other.

Q: If $A_{n \times n}$ s.t $A^2 = I$ then $A^{-1} = ?$

Ⓐ I Ⓑ A
Ⓒ O Ⓒ Not possible

Sol: Given, $A \cdot A = I \Rightarrow A^{-1} = A$ i.e A is self Invertible
on Comparison with,
 $(\text{Mat}) (\text{Mat})^{-1} = I$ or A is inverse of itself.

Q: If $M^4 = I$ then Various Conclusions are?

$$(i) M \cdot M^3 = I \Rightarrow M^{-1} = M^3$$

$$(ii) M^2 \cdot M^2 = I \Rightarrow (M^2)^{-1} = M^2$$

$$(iii) M^3 \cdot M = I \Rightarrow (M^3)^{-1} = M$$

$$\begin{array}{l} M^{-1} = ? \\ (M^2)^{-1} = ? \\ (M^3)^{-1} = ? \end{array}$$

Shortcut Method of finding Inverse of 2×2 Mat

$$\text{if } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } \bar{A}^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Ex: $A = \begin{bmatrix} 2 & 3 \\ -4 & 1 \end{bmatrix}$, then $\bar{A}^{-1} = ? = \frac{1}{(14)} \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$

Ex: $A = \begin{bmatrix} 3+4i & -i \\ i & 3-4i \end{bmatrix}$ then $\bar{A}^{-1} = ? = \frac{1}{24} \begin{bmatrix} 3-4i & +i \\ -i & 3+4i \end{bmatrix}$

$\therefore |A| = (3+4i)(3-4i) - (i)(-i)$

=

$= 24 \because |A| \neq 0 \Rightarrow \bar{A}^{-1} \text{ exist}$

Shortcut Method of finding Inverse of 3×3 Mat

eg $A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 0 & 3 \\ -3 & 1 & 4 \end{bmatrix}$ then $\bar{A}^{-1} = ?$ $= \frac{\text{adj } A}{|A|} = \frac{(\text{adj } A)^T}{|A|} = \frac{1}{35} \begin{bmatrix} -3 & 10 & -6 \\ -17 & 10 & 1 \\ 2 & 5 & 4 \end{bmatrix}$

sol: $|A| = \dots = 35$

$\therefore |A| \neq 0 \Rightarrow \bar{A}^{-1}$ exists.

1	-2	2	1	-2
2	0	3	2	0
-3	1	4	-3	1
1	-2	2	1	-2

$$\text{Cof } A = ? = (\text{adj } A)^T = \begin{bmatrix} -3 & -17 & 2 \\ 10 & 10 & 5 \\ -6 & 1 & 4 \end{bmatrix}$$

Top Row of $\bar{A}^{-1} = ? = [-3 \ 10 \ -6] \times$

$$= \left[\frac{-3}{35} \ \frac{10}{35} \ \frac{-6}{35} \right] \checkmark$$

If $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$. Then top row of R^{-1} is

- (a) $[5 \ 6 \ 4]$
- (b) $[5 \ -3 \ 1]$
- (c) $[2 \ 0 \ -1]$
- (d) $\left[2 \ -1 \ \frac{1}{2} \right]$

$$|R| = 1[2+3] + (-1)[6-2] \\ = 5 - 4 = 1.$$

$$R^{-1} = \frac{\text{adj } R}{|R|} = \frac{1}{1} \begin{bmatrix} 5 & -3 & 1 \\ - & - & - \\ - & - & - \end{bmatrix}$$

$$\begin{array}{c|cccc} & 1 & 0 & -1 & 1 & 0 \\ \hline 2 & & 1 & -1 & 2 & 1 \\ 2 & & 3 & 2 & 2 & 3 \\ 1 & & 0 & -1 & 1 & 0 \\ 2 & & 1 & -1 & 2 & 1 \end{array}$$

(ii) If $\alpha_{21} = ? = -3$ Ans

$\therefore \text{adj } A = \begin{bmatrix} 5 & - & - \\ -3 & - & - \\ 1 & - & - \end{bmatrix}$

~~P8~~ $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & -6 \\ -10 & -14 & 6 \end{bmatrix}$ Then $\bar{A}^1 = ?$, $\text{adj} A = ?$

~~sol~~ $\because |A| = \dots = 0$ 😞

ie A is singular so $\bar{A}^1 = \text{DNE}$

(ii)

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 1 & 2 \\ \hline 4 & 5 & -6 & 4 & 5 \\ -10 & -14 & 6 & -10 & -14 \\ \hline 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & -6 & 4 & 5 \end{array}$$

$\text{adj} A = (\text{cof} A)^T = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

HW

Methods of finding Inverse of 4x4, 5x5, 6x6.....Matrices

Not in a syllabus by Conventional Approach.



Procedure: write $A = I \cdot A$

↓ ↓ |
I = BA Using E-operation

$$I = BA \Rightarrow A^{-1} = B = \underline{Ans}$$

② Addition & Subtraction in Matrix →

This is **defined** only when Matrices are of **Same** order

for eg $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & 2 \end{bmatrix}_{2 \times 3}$, $B = \begin{bmatrix} -1 & 3 & 0 \\ 1 & 2 & 5 \end{bmatrix}_{2 \times 3}$, $C = \begin{bmatrix} 2 & -1 & 2 \\ 1 & 3 & 1 \\ 0 & 0 & 4 \end{bmatrix}_{3 \times 3}$

$$A+B = B+A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 2 & 7 \end{bmatrix}_{2 \times 3}$$

$$A-B = \begin{bmatrix} 3 & -4 & 3 \\ 3 & -2 & -3 \end{bmatrix} \text{ i.e } A-B \neq B-A$$

$$B-A = \begin{bmatrix} -3 & 4 & -3 \\ -3 & 2 & 3 \end{bmatrix}$$

$A+C = N.D$ & $B+C = N.D.$

" Matrix Addition is Commutative
But Matrix Subtraction is
not Commutative ?? "

Ex) Matrix Multiplication → is defined when

"Number of Columns in 1st Mat = Number of Rows in 2nd Mat"

for eg, consider $A_{m \times n}$ & $B_{n \times p}$ Then $AB = []_{m \times p}$.

$BA = B_{n \times p} A_{m \times n} = N.D.$

e.g.: $A = \begin{bmatrix} 1 & 2 & -4 \end{bmatrix}_{1 \times 3}$ & $B = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}_{3 \times 1}$ then $AB = ?$ & $BA = ?$

$$\text{SOL: } AB = \begin{bmatrix} 1 & 2 & -4 \end{bmatrix}_{1 \times 3} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} (2+2+(-12)) \end{bmatrix} = \begin{bmatrix} -8 \end{bmatrix}_{1 \times 1} \quad \begin{array}{l} \text{Multi = 3 times} \\ \text{Add = 2 times} \end{array}$$

$$BA = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}_{3 \times 1} \begin{bmatrix} 1 & 2 & -4 \end{bmatrix}_{1 \times 3} = \begin{bmatrix} 2 & 4 & -8 \\ 1 & 2 & -4 \\ 3 & 6 & -12 \end{bmatrix}_{3 \times 3} \quad \begin{array}{l} \text{No. of Multi used = 9 times} \\ \text{No. of Addition used = 0 times} \end{array}$$

The value of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \times [3 \ 2 \ 4] \times \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix}$ equals

BY8

A B C

(a) $\begin{bmatrix} 52 \\ -104 \\ 156 \end{bmatrix}$

(b) $[52 \ -104 \ 15]$

(c) $\begin{bmatrix} 52 \\ 104 \\ 156 \end{bmatrix}$

(d) None of these

M-D $BC = []_{1 \times 1}$

$$A(BC) = \begin{bmatrix} - \\ - \\ - \end{bmatrix} \begin{bmatrix} - \end{bmatrix}_{1 \times 1} = \begin{bmatrix} 52 \\ 104 \\ 156 \end{bmatrix}_{3 \times 1}$$

$$AB = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3 \times 1} \begin{bmatrix} 3 & 2 & 4 \end{bmatrix}_{1 \times 3} = \begin{bmatrix} 3 & 2 & 4 \\ 6 & 4 & 8 \\ 9 & 6 & 12 \end{bmatrix}_{3 \times 3}$$

$$(AB)C = \begin{bmatrix} 3 & 2 & 4 \\ 6 & 4 & 8 \\ 9 & 6 & 12 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 4 \\ 6 \\ 7 \end{bmatrix}_{3 \times 1} \longrightarrow$$

$$= \begin{bmatrix} (12+12+28) \\ (24+24+56) \\ (36+36+84) \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 52 \\ 104 \\ 156 \end{bmatrix}_{3 \times 1}$$

Q: $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 1 & 2 \end{bmatrix}_{2 \times 3}$ & $B = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 0 & 4 \\ 1 & 2 & -2 \end{bmatrix}_{3 \times 3}$ Then $AB = ?$

P
W

$$AB = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 1 & 2 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 2 & -1 & 3 \\ -1 & 0 & 4 \\ 1 & 2 & -2 \end{bmatrix}_{3 \times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2 \times 3} = \begin{bmatrix} 7 & 5 & -11 \\ 9 & 0 & 12 \end{bmatrix}$$

$$BA = ? = \text{N.D.}$$

$$a_{11} = (1)(2) + (-2)(-1) + (3)(1) = 7$$

$$a_{12} = (1)(-1) + (-2)(0) + (3)(2) = 5$$

$$a_{13} = (1)(3) + (-2)(4) + (3)(-2) = -11$$

$$a_{21} = (4)(2) + (1)(-1) + (2)(1) = 9$$

$$a_{22} = (4)(-1) + (1)(0) + (2)(2) = 0$$

$$a_{23} = (4)(3) + (1)(4) + (2)(-2) = 12$$

Number of terms, symbol of Multiplication

is used to find $AB = 18$ times

Number of terms, symbol of addition is used

to find $AB = 12$ times

Shortcut: Consider $A_{m \times n}$ & $B_{n \times p}$ then to find $(AB)_{m \times p}^{m(n-1)p}$

Number of Multiplications Required = $m \cdot n \cdot p$ & Number of Additions Req = n

e.g if $A_{2 \times 3}$ & $B_{3 \times 3}$ then to find AB

\rightarrow No. of Multi Req = $2 \times 3 \times 3 = 18$ times

$m=2, n=3, p=3$

\rightarrow No. of Additions Req = $2(3-1)3 = 12$ times

e.g if $A_{3 \times 4}$ & $B_{4 \times 3}$ then to find (AB)

\rightarrow No. of Multi Req = $3 \times 4 \times 3 = 36$ times

$m=3, n=4, p=3$

\rightarrow No. of Additions Req = $3(4-1)3 = 27$ times

& to find (BA)

$B_{4 \times 3} A_{3 \times 4} = \text{Defined. } m=4, n=3, p=4$

\rightarrow No. of Multi Req = $4 \times 3 \times 4 = 48$

\rightarrow No. of Additions Req = $4(3-1)4 = 32$

MAJEDAAR QUESTION → Consider $A_{2 \times 3}$, $B_{3 \times 4}$ & $C_{4 \times 2}$ then find the
 minimum number of Multiplications & Additions that will be Required

To find Matrix Product (ABC) ?

sol: we can find (ABC) either by using

Case I: $(AB)C$

Multi = 40

Addition = 28

Case II: $A(BC)$

Multi = 36

Addition = 26

i.e Min. Multi Required = 36

Min Addition Required = 26 (Obtained by Case II)

Commutative Law

$$A+B = B+A$$

But $AB \neq BA$

Associative Law

$$A+(B+C) = (A+B)+C$$

$$A(BC) = (AB)C$$

Matrix Addition is Commutative as well as Associative.

while Matrix Multiplication is associative but not commutative in general.

Note Although $AB \neq BA$ in general But $\text{Tr}(AB) = \text{Tr}(BA)$ always.

$$\text{e.g } A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 0 & 0 \end{bmatrix} \quad AB = \begin{bmatrix} 5 & 17 \\ 2 & -7 \end{bmatrix} \quad \text{Tr}(AB) = -2$$

$$B = \begin{bmatrix} 2 & 1 \\ 0 & -2 \\ 1 & 4 \end{bmatrix} \quad \text{then} \quad BA = \begin{bmatrix} 3 & 0 & 6 \\ -2 & -8 & 0 \\ 5 & 14 & 3 \end{bmatrix} \quad \text{Tr}(BA) = -2$$

Hence verified

If A & B are two matrices of same order then
 which of the following is true.

- (a) ~~$(A + B)^2 = A^2 + 2AB + B^2$~~
- (b) ~~$(A - B)^2 = A^2 - 2AB + B^2$~~
- (c) $(A + B)^2 + (A - B)^2 = 2A^2 + 2B^2$
- (d) $(A + B)(A - B) = A^2 - B^2$

\downarrow
 $AB \neq BA$ both defined

④ $(A+B)(A-B) = A^2 - \underbrace{AB}_{BA} + BA + B^2$

④ $(A+B)^2 = (A+B)(A+B)$
 $= A^2 + AB + BA + B^2$

⑤ $(A-B)^2 = A^2 - \underbrace{AB - BA}_{(AB \neq BA)} + B^2$
 ?

④ $(A+B)^2 + (A-B)^2$
 $= (A^2 + B^2 + AB + BA) + (A^2 + B^2 - AB - BA)$
 $= 2A^2 + 2B^2$



thank
you

Keep Hustling!

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