VIJAY

DATA SCIENCE & ARTIFICIAL INTELLIGENCE

& CS

Calculus and Optimization

Lecture No. 04

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Recap of previous lecture

Topic

Limit / Continuity / Differentiability

(Part - 1)

Topics to be Covered

Topic

Limit, Continuity & Differentiability

(PART-2)

Method of Solving Questions →

(M-I) By Direct Substitution (Best Method).

(M-II) By factorisation

(M-III) By Rationalisation

(M-IV) By using IND form Concept

(M-V) By using Standard Results

(M-VI) using common sense.

Sp. formulaes :→

① $\sum_{N=1}^{N} N = 1+2+3+\dots+N = \frac{N(N+1)}{2}$

② $\sum_{N=1}^{N} N^2 = 1^2+2^2+3^2+\dots+N^2 = \frac{N(N+1)(2N+1)}{6}$

③ $\sum_{N=1}^{N} N^3 = 1^3+2^3+3^3+\dots+N^3 = \left(\frac{N(N+1)}{2}\right)^2$

④ $\sum_{N=1}^{N} (a) = \underbrace{a+a+a+\dots+a}_{N\text{ times}} = N\cdot a$

Type 3 (Rationalisation) →

Q $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)=?$

Ans $=\frac{1}{2}$

$$=\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)\times\left(\frac{\sqrt{x^2+x+1}+\sqrt{x^2+1}}{\sqrt{x^2+x+1}+\sqrt{x^2+1}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{(x^2+x+1)-(x^2+1)}{\sqrt{x^2+x+1}+\sqrt{x^2+1}}\right)$$

$$=\lim_{x\to\infty}\left[\frac{\cancel{x}}{\cancel{x}\left[\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}\right]}\right]$$

$$=\frac{1}{\sqrt{1+0+0}+\sqrt{1+0}}=\frac{1}{1+1}=\frac{1}{2}$$

Type IV (IND form Concept) $\longrightarrow$.

$\underbrace{\frac{0}{0}, \frac{\infty}{\infty}}$, $\underbrace{0\times\infty, \infty-\infty}$, $\underbrace{0^0, \infty^0, 1^\infty}$

Directly use L-Hospital Rule

use L-Hosp Rule after converting them into one of the 1st two forms.

use log Concept

Note: $\frac{d}{dx}(x^a) = ax^{a-1}$, $\boxed{\frac{d}{dx}(x^x) = x^x(1+\log x)}$

$\frac{d}{dx}(a^x) = a^x \log_e a$, $\frac{d}{dx}(a^a) = 0$

Prove that $\boxed{\frac{d}{dx}(x^x) = x^x(1+\log x)}$

Proof: Let $y = x^x \Rightarrow \frac{dy}{dx} = ?$

$$\log y = \log(x^x)$$

$$\log y = x \log x$$

$$\frac{d}{dx}(\log y) = \frac{d}{dx}(x \log x)$$

$$\frac{d}{dy}(\log y)\cdot\frac{dy}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x(1)$$

$$\frac{dy}{dx} = y(1+\log x) = x^x(1+\log x)$$

Note:

$$\frac{d}{dx}(x^3) = 3x^2$$

$$\frac{d}{dy}(y^3) = 3y^2$$

$$\frac{d}{dx}(y^3) = \frac{d}{dy}(y^3)\cdot\frac{dy}{dx}$$

$$= (3y^2)\frac{dy}{dx}$$

Q1. $\lim_{x\to a}\left(\frac{x^x-a^a}{x^a-a^x}\right)=?\leftarrow\frac{0}{0}$ form $=\lim_{x\to a}\left(\frac{x^x(1+\log x)-0}{a x^{a-1}-a^x\log_e a}\right)=\frac{a^a(1+\log a)}{a a^{a-1}-a^a\log a}$

$=\frac{1+\log a}{1-\log a}$ Ans

Q. $\lim_{x\to\infty}\left(\frac{x^n}{e^x}\right)=?\leftarrow\frac{\infty}{\infty}$ form $=\lim_{x\to\infty}\frac{D^n(x^n)}{D^n(e^x)}=\lim_{x\to\infty}\left(\frac{n!}{e^x}\right)=\frac{n!}{e^\infty}=\frac{n!}{\infty}=0$ Ans

$n\in$ +ve integer

w.k.that $\boxed{D^n(x^n)=n!\ \&\ D^{n+1}(x^n)=0}$

eg $D(x^2)=2x$, $D(x^3)=3x^2$

$\boxed{D^2(x^2)=2=2!}$, $D^2(x^3)=6x$

$\boxed{D^3(x^3)=6=3!}$

$D^4(x^3)=0$

$D(x^4)=4x^3$

$D^2(x^4)=12x^2$

$D^3(x^4)=24x$

$\boxed{D^4(x^4)=24=4!}$

$D^5(x^4)=0$

& so on.....

Detailed Exp:-

$$\lim_{x\to\infty}\left(\frac{x^n}{e^x}\right)=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{n(x)^{n-1}}{e^x}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{n(n-1)x^{n-2}}{e^x}=\frac{\infty}{\infty}$$

$$=\lim_{x\to\infty}\frac{n(n-1)(n-2)x^{n-3}}{e^x}=\frac{\infty}{\infty}=\cdots\cdots=\lim_{x\to\infty}\left(\frac{n(n-1)\ldots3.2.x^1}{e^x}\right)$$

$$=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{n!}{e^x}\neq\frac{\infty}{\infty}=\left(\frac{n!}{e^\infty}\right)=\frac{n!}{\infty}=0$$

Note:

$$D(x) = 1 = 1!$$

$$D^2(x^2) = D(2x) = 2\times 1 = 2!$$

$$D^3(x^3) = D^2(3x^2) = D(3\times 2x) = 3\times 2\times 1 = 3!$$

$$D^4(x^4) = D^3(4x^3) = D^2(4\times 3x^2) = D(4\times 3\times 2x) = 4\times 3\times 2\times 1 = 4!$$

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In general $\boxed{D^n(x^n) = n!}$ & $D^{n+1}(x^n) = D\left(D^n(x^n)\right)$

$$= D(n!) = 0$$

ie $\boxed{D^{n+1}(x^n) = 0}$

Type V Standard Results of limits →

(1) $$\boxed{\lim_{x\to 0}(1+ax)^{\frac{1}{x}} = e^{a} = \lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{x}}$$

eg $$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e = \lim_{x\to 0}\left(1+\frac{1}{x}\right)^{x}$$

eg $$\lim_{x\to 0}(1-x^2)^{\frac{1}{x}} = ? = \lim_{x\to 0}\left[(1-x)(1+x)\right]^{\frac{1}{x}} = \lim_{x\to 0}(1-x)^{\frac{1}{x}} \cdot \lim_{x\to 0}(1+x)^{\frac{1}{x}}$$

$$= e^{-1}\times e^{1} = e^{0} = 1$$

eg $$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n} = ? = \lim_{n\to\infty}\left[\left(1-\frac{1}{n}\right)^{n}\right]^{2} = \left(e^{-1}\right)^{2} = e^{-2}$$ Ans

Some More Standard Results: $\rightarrow$

(1) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1$

(2) $\lim_{x\to\infty}\left(\frac{\sin x}{x}\right) = \frac{\sin\infty}{\infty} = \frac{\text{Any No. b/n } -1 \,\&\, 1}{\infty} = 0$

(3) $\lim_{x\to 0}\left(\frac{\cos x}{x}\right) = \frac{\cos 0}{0} = \frac{1}{0} = \text{DNE}$, (4) $\lim_{x\to\infty}\left(\frac{\cos x}{x}\right) = \frac{\cos\infty}{\infty} = \frac{\text{Any No b/n } -1 \,\&\, 1}{\infty} = 0$

(5) $\lim_{x\to 0}\left[x.\sin\left(\frac{1}{x}\right)\right] =$

$\rightarrow = 0\times\sin\infty = 0\times(\text{Any No. b/n } -1 \,\&\, 1) = 0$

$\rightarrow = \lim_{x\to 0}\frac{\sin\left(\frac{1}{x}\right)}{\left(1/x\right)} = \lim_{y\to\infty}\left(\frac{\sin y}{y}\right) = 0$

Put $\left(\frac{1}{x} = y\right)$; when $x\to 0,\ y\to\infty$

(6) $\lim_{x\to 0}\sin\left(\frac{1}{x}\right) = ? = \sin(\infty) = \text{Any No b/n } -1 \,\&\, 1 = \text{Not unique} = \text{DNE}$

Explanation of (6) $\to$

$$\lim_{x\to 0}\left(\sin\frac{1}{x}\right) = ? = \lim_{x\to 0}\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}\times\left(\frac{1}{x}\right)$$

$$= \lim_{y\to\infty}\left(\frac{\sin y}{y}\right)\times(y), \quad \text{Put } \frac{1}{x} = y$$

$$= 0\times\infty$$

Still it is IND form.

So this method is not good enough.

(7) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = \lim_{x\to 0}\left(\frac{x}{\sin x}\right) = \lim_{x\to 0}\left(\frac{\sin^{-1} x}{x}\right) = \lim_{x\to 0}\left(\frac{\tan x}{x}\right) = \lim_{(x-a)\to 0}\frac{\sin(x-a)}{(x-a)} = 1$

(8) $\lim_{x\to 0}\left(\frac{a^x-1}{x}\right) = \log_e a$, eg $\lim_{x\to 0}\left(\frac{e^x-1}{x}\right) = 1$

Gate (17 8):

$\lim_{a\to 0}\left(\frac{x^a-1}{a}\right) = ? = \log_e x$

using formula (8)

(9) $\lim_{x\to 0}\frac{\log(1+x)}{x} = 1$, (10) $\lim_{x\to 0}\frac{\log(1-x)}{x} = -1$

(11) $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right) = \frac{1}{2}$

Note: All the above Results can be calculated using L-Hospital's Rule.

Q. $\lim_{x \to \infty} \left(\dfrac{3x - \sin x}{2x + 5\cos x} \right) = ?$ $= \lim_{x \to \infty} \dfrac{\cancel{x}}{\cancel{x}} \left[\dfrac{3 - \left(\frac{\sin x}{x}\right)}{2 + 5\left(\frac{\cos x}{x}\right)} \right]$

(a) 3

(b) 1.5 ✓

(c) 1

(d) 0

$$= \frac{3 - 0}{2 + 0} = 1.5$$

L-Hospital's Rule →

Q. $\lim_{x\to a}\left(\frac{x\sqrt{x}-a\sqrt{a}}{x-a}\right)=?$

(a) $\frac{3}{2}\sqrt{a}$ (b) $\frac{3}{2}$ (c) $\sqrt{a}$ (d) $\frac{3}{4}\sqrt{a}$

$$=\lim_{x\to a}\left(\frac{x^{3/2}-a^{3/2}}{x-a}\right)=\frac{0}{0}$$

$$=\lim_{x\to a}\left(\frac{\frac{3}{2}x^{1/2}-0}{1-0}\right)$$

$$=\frac{3}{2}a^{1/2}=\frac{3}{2}\sqrt{a}$$

Q. $\lim_{x\to 0}\left[\frac{(1-x)^n-1}{x}\right]=?$ = $\frac{0}{0}$ form.

(a) 0 (b) n (c) $-n$ (d) $2n$

Put $1-x=t$

when $x\to 0$, $t\to 1$

$$=\lim_{t\to 1}\left(\frac{t^n-1}{1-t}\right)=-\lim_{t\to 1}\left(\frac{t^n-1}{t-1}\right)=\frac{0}{0}\text{ form}$$

$$=-\left(\lim_{t\to 1}\left(\frac{nt^{n-1}}{1}\right)\right)=-n$$

Q. $\lim_{x\to 5}\left[\frac{\log_e x-\log_e 5}{x-5}\right]=\ ?\ =\frac{0}{0}$

$=\lim_{x\to 5}\left[\frac{1/x-0}{1-0}\right]$

$=\frac{1}{5}$ ✓

Note: $(x\to 0)\Rightarrow x$ is very small

i.e. $\sin x$ is also very small

i.e. $(\sin x\to 0)$

Q. $\lim_{x\to 0}\left[\frac{e^{\sin x}-1}{x}\right]=\ ?\ =\frac{0}{0}$ form

(M-I) $=\lim_{x\to 0}\left(\frac{e^{\sin x}(\cos x)}{1}\right)=e^{0}\cdot\cos 0=1\times 1=1$

(M-II)

$=\lim_{x\to 0}\left(\frac{e^{\sin x}-1}{\sin x}\right)\times\left(\frac{\sin x}{x}\right)$

$=\lim_{\sin x\to 0}\left(\frac{e^{\sin x}-1}{\sin x}\right)\times\lim_{x\to 0}\left(\frac{\sin x}{x}\right)$

$=\quad 1\times 1=1$

Q. $\lim_{x\to 0}(\tan x \cdot \log x) = ?$ $= 0\times(-\infty)$ form

(a) 0 (b) 1 (c) -1 (d) DNE

$= \lim_{x\to 0}\left(\frac{\log x}{\cot x}\right) = \frac{-\infty}{\infty}$ form

$= \lim_{x\to 0}\left[\frac{\left(\frac{1}{x}\right)}{-\operatorname{cosec}^2 x}\right] = -\lim_{x\to 0}\left(\frac{\sin^2 x}{x}\right)$

$= -\lim_{x\to 0}\left(\frac{\sin x}{x}\right)\times\lim_{x\to 0}(\sin x)$

$= -1\times 0 = 0$

Q. $\lim_{x\to 0}\left[x\cdot\sin\frac{1}{x}\right] = ?$

(a) 1 (b) 0 (c) -1 (d) ∞

$\lim_{x\to 0}\left[x\sin\frac{1}{x}\right] = 0\times\sin\infty$

$= 0\times(\text{Any } \underline{\text{No}} \text{ b/w } -1 \,\&\, 1) = 0$

M-II $\lim_{x\to 0}\left[\frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}\right] = \lim_{y\to\infty}\left(\frac{\sin y}{y}\right)$

Put $y = \frac{1}{x}$

when $x\to 0$, $y\to\infty$

$= \frac{\sin\infty}{\infty}$

$= \frac{\text{No b/w } -1 \,\&\, 1}{\infty} = 0$

Q. $\lim_{x\to 0}(\sin x)^{\tan x} = ? = 0^0$ form.

Let $K = \lim_{x\to 0}(\sin x)^{\tan x}$

$$\log_e K = \lim_{x\to 0}\log(\sin x)^{\tan x}$$

$$= \lim_{x\to 0}\left[\tan x(\log \sin x)\right] = 0\times(-\infty)$$

$$= \lim_{x\to 0}\left[\frac{\log \sin x}{\cot x}\right] = \frac{-\infty}{\infty} \text{ form.}$$

$$= \lim_{x\to 0}\left(\frac{\frac{1}{\sin x}\cdot\cos x}{-\operatorname{cosec}^2 x}\right) = \lim_{x\to 0} -(\cos x\sin x) = -1\times 0 = 0 \Rightarrow K = e^0 = 1 \text{ Ans}$$

Q. $\lim_{x\to 0}\left(\frac{1}{x}\right)^x = ?$ Ans $= 1$

HWQ $= (\infty^0)$ form

Q $\lim_{x\to 0}(\cos 2x)^{\frac{1}{x^2}} = ? \simeq 1^{\infty}$

(a) e (b) e^{2} (c) $\sqrt{e}$ (d) e^{-2}

Sol: Let $K = \lim_{x\to 0}(\cos 2x)^{\frac{1}{x^2}}$

$\log K = \lim_{x\to 0}\log(\cos 2x)^{\frac{1}{x^2}}$

$= \lim_{x\to 0}\left[\frac{\log(\cos 2x)}{x^2}\right] \simeq \frac{0}{0}$ form.

$= \lim_{x\to 0}\left[\frac{\frac{1}{\cos 2x}(-\sin 2x(2))}{2x}\right] = -2\lim_{2x\to 0}\left(\frac{\sin 2x}{2x}\right)\times\lim_{x\to 0}\left(\frac{1}{\cos 2x}\right) = -2(1)\times\frac{1}{1} = -2$

$\log_e K = -2 \Rightarrow K = e^{-2}$ Ans

HW Q

Q $\lim_{x\to 1}[\log_3(3x)]^{\log_x 3} = ? \simeq 1^{\infty}$ form.

(a) 0 (b) 1 (c) e (d) e^{2}

Q. $\lim_{x\to 0}\left(\frac{\log(1+x^3)}{\sin^3 x}\right) = ? \quad \frac{0}{0}$

(a) 0 (b) 1 (c) e (d) ∞

M-I Using L-Hosp Rule :→ Irritating

M-II Using standard Results :

$$\lim_{x\to 0}\frac{\log(1+x^3)}{x^3} \times \lim_{x\to 0}\left(\frac{x^3}{\sin^3 x}\right)$$

$$\lim_{x^3\to 0}\left[\frac{\log(1+x^3)}{x^3}\right] \times \lim_{x\to 0}\left(\frac{x}{\sin x}\right)^3$$

$$= 1 \times 1^3 = 1$$

Q. $\lim_{x\to 0}\frac{\sin[\pi\cos^2 x]}{x^2} = ?$

(a) 1 (b) e (c) π (d) 0

$$= \lim_{x\to 0}\frac{\sin[\pi(1-\sin^2 x)]}{x^2} = \lim_{x\to 0}\frac{\sin(\pi - \pi\sin^2 x)}{x^2}$$

$$= \lim_{x\to 0}\left[\frac{\sin(\pi\sin^2 x)}{x^2}\right] \times \frac{\pi\sin^2 x}{\pi\sin^2 x}$$

$$= \lim_{x\to 0}\left[\frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\right] \times \lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 \times \pi$$

$$= 1 \times 1^2 \times \pi = \pi$$

Q. $\lim_{x\to 0}\left(\dfrac{\tan x-\sin x}{x^3}\right)=?$

(a) 1 (b) 0.5 (c) −0.5 (d) −1

(M-I) → Lengthy

(M-II) $\lim_{x\to 0}\left(\dfrac{\dfrac{\sin x}{\cos x}-\sin x}{x^3}\right)=\lim_{x\to 0}\dfrac{\sin x(1-\cos x)}{\cos x\cdot x^3}$

$=\lim_{x\to 0}\left(\dfrac{\sin x}{x}\right)\times\lim_{x\to 0}\left(\dfrac{1-\cos x}{x^2}\right)\times\lim_{x\to 0}\left(\dfrac{1}{\cos x}\right)$

$=1\times\dfrac{1}{2}\times 1=\dfrac{1}{2}$

Q. $\lim_{x\to 0}\left[\dfrac{e^x+e^{-x}-2}{x^2}\right]=?$

(a) 1 (b) 0 (c) 1/2 (d) e

(M-I) → Imitating

(M-II) $\lim_{x\to 0}\left(\dfrac{e^x+\dfrac{1}{e^x}-2}{x^2}\right)$

$=\lim_{x\to 0}\left(\dfrac{e^{2x}+1-2e^x}{e^x\cdot x^2}\right)=\lim_{x\to 0}\dfrac{(e^x-1)^2}{e^x\cdot x^2}$

$=\lim_{x\to 0}\left(\dfrac{e^x-1}{x}\right)^2\times\lim_{x\to 0}\left(\dfrac{1}{e^x}\right)=1\times 1=1$

Q. If $2-\frac{x^2}{3} < \frac{x\sin x}{1-\cos x} < 2$ then Evaluate $\lim_{x\to 0}\left(\frac{x\sin x}{1-\cos x}\right) = \underline{\ ?\ }$

M-I $\lim_{x\to 0}\left(\frac{x\sin x}{1-\cos x}\right) = \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\times\left(\frac{x^2}{1-\cos x}\right) = 1\times 2 = 2$

SANDWITCH THEOREM → if $f(x)\le g(x)\le h(x)$

then $\boxed{\lim_{x\to a} f(x) \le \lim_{x\to a} g(x) \le \lim_{x\to a} h(x)}$

M-II By S.Th, $\lim_{x\to 0}\left(\frac{x\sin x}{1-\cos x}\right) = 2$

Continuity → f(x) is said to be cont if,

$$\boxed{\lim_{x\to a} f(x) = f(a)}$$

App. Value = Exact Value

$$\boxed{(LHL = RHL) = f(a)}$$

M-I Either exact Value DNE

M-II or App. Value " "

M-III Both exist But Not equal.

Methods to Check Discontinuity

Q. Check the continuity of the following functions

① $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ (cont)

② $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ (cont)

Sol: At $x=0$, Exact Value $= f(0) = 0$ (given)

App Value $= \lim_{x \to 0} f(x) = \lim_{x \to 0} \left(x \sin\frac{1}{x}\right)$

$= 0 \times \sin\infty = 0 \times (\text{No b/w } -1 \,\&\, 1)$

$= 0$

$\because$ Approx Value = Exact Value

So $f(x)$ is Cont at $x=0$

Already Solved YESTERDAY

Q. $f(x)=\begin{cases} 1+x^2, & 0\le x\le 1 \\ 2-x, & x>1 \end{cases}$ (Discont)

At $x=1$, Exact Value $= f(1) = \left(1+x^2\right)_{x=1} = 2$

LHL at $(x=1) = f(1^-) = \lim\limits_{x\to 1^-} f(x)$

$= \lim\limits_{x\to 1} (1+x^2) = 1+1^2 = 2$

RHL at $(x=1) = f(1^+) = \lim\limits_{x\to 1^+} f(x)$

$= \lim\limits_{x\to 1} (2-x) = 2-1 = 1$

$\because$ LHL $\neq$ RHL So limit DNE $\Rightarrow f(x)$ is Discont at $x=1$ by M-II

Q. $f(x)=\begin{cases} \dfrac{|x|}{x}, & x\neq 0 \\ 1, & x=0 \end{cases}$ (Discont)

$=\begin{cases} -1, & x<0 \\ +1, & x>0 \\ 1, & x=0 \end{cases}$

At $x=0$:

- Exact Value $= f(0) = 1$
- LHL $= f(1^-) = -1$
- RHL $= f(1^+) = +1$

$\because$ LHL $\neq$ RHL So Discont at $x=0$

Note → Here $f(x)$ is Not a Signum func^n

Q. HW Q. If $f(x)=\begin{cases}\dfrac{ae^{x}-b\cos x+ce^{-x}}{x\sin x}, & x\neq 0\\ 2, & x=0\end{cases}$

is continuous everywhere then correct option is?

(a) × $a=2,\ b=1,\ c=1$

(b) × $a=1,\ b=-2,\ c=1$

(c) × $a=1,\ b=1,\ c=2$

(d) ✓ $a=1,\ b=2,\ c=1$

(e) None

$\because f(x)$ is cont at $x=0$ so

App Value = Exact Value

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0}\left(\frac{ae^{x}-b\cos x+ce^{-x}}{x\sin x}\right)=2$$

$$\frac{a-b+c}{0}=2 \Rightarrow \boxed{a-b+c=0}$$

Now, if we take $a-b+c=0$ then LHS will be in $\frac{0}{0}$ form.

$$\lim_{x\to 0}\left(\frac{ae^{x}+b\sin x-ce^{-x}}{x\cos x+\sin x}\right)=2$$

$$\frac{a-c}{0}=2 \Rightarrow \boxed{a=c}$$

Now when we take $a = c$, LHS will become again $\frac{0}{0}$ form.
So again applying L-Hosp Rule in LHS.

$$\lim_{x \to 0} \left[\frac{ae^{x} + b\cos x + ce^{-x}}{\underbrace{x(-\sin x) + \cos x} + \cos x} \right] = 2$$

$$\frac{a+b+c}{0+1+1} = 2 \Rightarrow \boxed{a+b+c = 4}$$

Solving these three eqn's, $a = 1, b = 2, c = 1$

Q. $\lim_{x \to 0} \left(\frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} \right) = ?$ = DNE

HWQ

Q. find K for which $f(x)$ is continuous at $x=0$

(1) $f(x)=\begin{cases}\dfrac{\log(1+3x)-\log(1-2x)}{x}, & x\neq 0\\ K, & x=0\end{cases}$

(a) 1 (b) 0 (c) -1 (d) 5 ✓

(2) $f(x)=\begin{cases}\sin\frac{1}{x}, & x\neq 0\\ K, & x=0\end{cases}$

(a) 0 (b) 1 (c) $(-\infty,\infty)$

(d) No such value of K

Sol: At $x=0$, Exact Value $=f(0)=K$ (given)

Now, App. Value $=\lim_{x\to 0} f(x)=$

$$=\lim_{x\to 0}\left[\frac{\log(1+3x)-\log(1-2x)}{x}\right]=\lim_{x\to 0}\left[\frac{\left(\frac{3}{1+3x}\right)-\left(\frac{-2}{1-2x}\right)}{1}\right]=5$$

for Continuity at $x=0$, Exact Value $=$ App Value $\Rightarrow K=5$

(M-II) $\lim_{x\to 0} f(x) = \lim_{x\to 0}\left[\frac{\log(1+3x)-\log(1-2x)}{x}\right]$

$$= \lim_{3x\to 0}\left(\frac{\log(1+3x)}{3x}\right)\times 3 - \lim_{2x\to 0}\left(\frac{\log(1-2x)}{2x}\right)\times 2$$

$$= 1\times 3 - (-1)2 = 5$$

$x=1$ & $x=3$ are problem creating points.

PYQ

The values of a and b for which the function

$$f(x) = \begin{cases} 2x+1, \text{ if } x \le 1 \\ ax^2+b \text{ if } 1 < x < 3 \\ 5x+2a \text{ if } x \ge 3 \end{cases}$$ is continuous every where

(a) $a = 2,\ b = 1$ (b) $a = 1,\ b = 2$

(c) $a = 3,\ b = 2$ (d) $a = 2,\ b = 3$

At $x=1$, LHL = RHL = $f(1)$

$(2x+1)_{x=1} = (ax^2+b)_{x=1} = (2x+1)_{x=1}$

$3 = \boxed{a+b=3}$ — (1)

At $x=3$, LHL = RHL = $f(3)$

$(ax^2+b)_{x=3} = (5x+2a)_{x=3} = 15+2a$

$9a+b = 15+2a \Rightarrow \boxed{7a+b=15}$ — (2)

$a=2,\ b=1$

Thank

THANK

Keep Hustling!