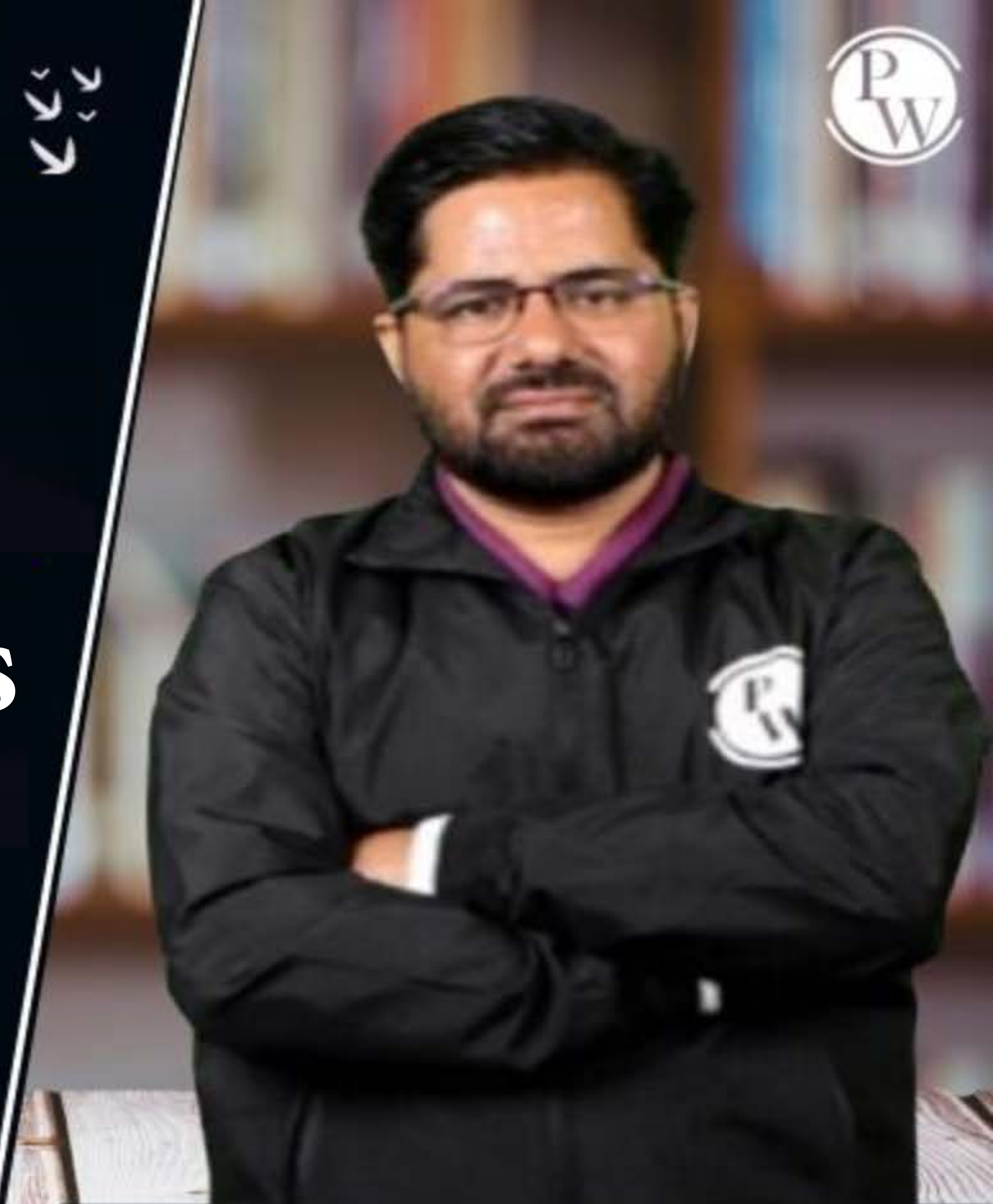
CS & DA

Probability and Statistics



DPP 02 Discussion Notes

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[NAT]

#Q. A jar contains three marbles numbered 1, 2, and 3. If two marbles are drawn without replacement, what is the probability that the sum of the numbers is 5?

$S = \{(1\,2), (2\,1), (1\,3), (3\,1), (2\,3), (3\,2)\} \Rightarrow n(S) = 6$

$fav = \{(2\,3), (3\,2)\} \Rightarrow n(f) = 2 \Rightarrow$ Req Prob $= \frac{f}{T} = \frac{2}{6} = \frac{1}{3}$

$= 0.33$

[NAT]

#Q. A jar contains three marbles numbered 1, 2, and 3. If two marbles are drawn with replacement, what is the probability that the sum of the numbers is at least 4 ?

$$S = \left\{ \begin{matrix} (11), & (12), & (13) \\ (21), & (22), & (23) \\ (31), & (32), & (33) \end{matrix} \right\} = 9, \quad fav = \left\{ \begin{matrix} (13), & (31), & (22) \\ (23), & (32), & (33) \end{matrix} \right\} = 6$$

$$\therefore \text{Req Prob} = \frac{f}{T} = \frac{6}{9} = \frac{2}{3} = 0.666$$

[NAT]

#Q. If two dice are rolled, find the probability that the sum of the faces of the dice is 7.

Shortcut: Req Prob $= P(\text{Sum} = 7) = \frac{\text{fav}}{\text{Total}} = \frac{(s-1)}{6^2} = \frac{7-1}{36} = \frac{6}{36} = \frac{1}{6}$

M-II $S = \begin{Bmatrix} (11)(12)(13) - - (16) \\ (21)(22) - - - - (66) \end{Bmatrix} \Rightarrow n(S) = 36$

$\text{fav} = \begin{Bmatrix} (16), (61), (25), (52), \\ (34), (43) \end{Bmatrix} = 6 \Longrightarrow$ Req Prob $= \frac{f}{T} = \frac{6}{36} = \frac{1}{6}$

[NAT]

#Q. Two dice are thrown at a time, find the probability of the following :

(i) The number shown are equal

(ii) The difference of number shown is 1

(i) $n(S) = 6 \times 6 = 36$ pair,

$fav = \{(11)\ (22)\ (33)\ (44)\ (55)\ (66)\} = 6$

Req. Prob $= \frac{6}{36} = \frac{1}{6}$

(ii) $fav = \{(12), (21)\ (23)\ (32), (34), (43), (45)\ (54), (56), (65)\} = 10$

Req. Prob $= \frac{10}{36} = \frac{5}{18}$

[NAT]

#Q. Three coins are tossed together, then find the probabilities of the following : (i) Getting exactly two head (ii) Getting at least two tails.

$S = \{(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)\} \Rightarrow n(S) = 8$

(i) Req Prob $= \frac{F}{T} = \frac{3}{8}$

(ii) Req Prob $= \frac{4}{8} = \frac{1}{2}$

[NAT]

#Q. A box contains cards numbered 3,5,7,9,....35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

$\underbrace{3, 5, 7, 9, \ldots 35, 37}_{A.P}$

$a = 3,\ d = 2,\ l = 37$

$l = a + (n-1)d$

$37 = 3 + (n-1)2$

$2n = 36$

$n = 18$ terms

$A = \{\text{drawn card is Multiple of } 7\} = \{7, 14, 21, 28, 35\} = 5$

$B = \{\text{'' '' is Prime No}\} = \{\cancel{2}, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\} = 11$

$A \cap B = \{7\} = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{18} + \frac{11}{18} - \frac{1}{18} = \frac{15}{18} = \frac{5}{6}$

[NAT]

#Q. The probability that a person will get an electrification contract is 3/5 and the probability that he will not get plumbing contract is 5/8. The probability of getting at least one contract is 5/7. What is the probability that he will get both ?

$P(A) = \frac{3}{5}$

$P(\bar{B}) = \frac{5}{8}$

$P(B) = 1 - \frac{5}{8} = \frac{3}{8}$

$P(A \cup B) = \frac{5}{7}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = \frac{3}{5} + \frac{3}{8} - \frac{5}{7} = \frac{73}{280}$

[MCQ]

#Q. Neha has 4 yellow t-shirts, 6 black t-shirts, and 2 blue t-shirts to choose for her outfit today. She chooses a t-shirt randomly with each t-shirt equally likely to be chosen. Find the probability that a black or blue t-shirt is chosen for the outfit.

A $\frac{8}{13}$

B $\frac{5}{6}$

C $\frac{2}{3}$

D $\frac{7}{12}$

$P(A \cup B) = P(A) + P(B) - 0$

$= \frac{^6C_1}{^{12}C_1} + \frac{^2C_1}{^{12}C_1}$

$= \frac{6}{12} + \frac{2}{12} = \frac{8}{12}$

$= \frac{2}{3}$

[MCQ]

#Q. The probability of an event A occurring is 0.5 and of B occurring is 0.3. If A and B are mutually exclusive events, then the probability of neither A nor B occurring's is :

$$P(A \cup B) = 0.5 + 0.3 - 0 = 0.8$$

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - 0.8 = 0.2$$

A) 0.6

B) 0.5

C) 0.7

D) 0.2

[MCQ]

#Q. A and B are two events and $\bar{A}$ denotes the complements of A. Consider the following statements :

(i) $P(A \cup B) \leq P(B) + P(A)$

(ii) $P(A) + P(\bar{A} \cup B) \leq 1 + P(B)$

Which of the above statements is/are correct ?

A Only I

B Only II

C Both I and II

D Neither I nor II

(i) $LHS = P(A \cup B) = \underbrace{P(A) + P(B)} - P(A \cap B)$

$\leq P(A) + P(B)$ (T)

(ii) $LHS = P(A) + P(\bar{A} \cup B)$

$= \underbrace{P(A) + P(\bar{A})} + P(B) - P(\bar{A} \cap B)$

$= \underbrace{1 + P(B)} - P(\bar{A} \cap B) \leq 1 + P(B)$ (T)

[MCQ]

#Q. Three integers are chosen at random from the first 20 integers. The Probability that their product is even ?

P(Product is Even) = P(at least one number is even)

= 1 − P(None)

= 1 − P(No Number is even)

= 1 − P(all three numbers are odd)

$$= 1 - \frac{{}^{10}C_3}{{}^{20}C_3} = \frac{17}{19}$$

A) 2/19

B) 3/29

C) ~~17/29~~ $\frac{17}{19}$

D) 4/29

[MCQ]

#Q. If the letters of word 'REGULATIONS' be arranged at random, the probability that there will be exactly 4 letters between R and E is :

$\Rightarrow n = 11$ different

fav = $\underbrace{(R \_\,\_\,\_\,\_\, E)}_{1}\ \underset{2}{\_}\ \underset{3}{\_}\ \underset{4}{\_}\ \underset{5}{\_}\ \underset{6}{\_}$ $= 6! \times 2! \times {}^9C_4 \times 4!$

Total = $11!$

So Req Prob $= \dfrac{f}{T} = \dfrac{6! \times 2! \times {}^9C_4 \times 4!}{11!} = \dfrac{6}{55}$

A. 6/55

B. 3/55

C. 49/55

D. None of these

[MCQ]

#Q. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match both of them win a prize. The probability that they will not win a prize in a single trial is :

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & \cdots & (1,25) \\ (2,1) & (2,2) & (2,3) & \cdots & (2,25) \\ \cdots & & & & (25,25) \end{Bmatrix} \Rightarrow \text{Total} = \underset{A}{25} \times \underset{B}{25} = 625 \text{ pair}$$

A. 1/25

B. 24/25

C. 2/25

D. None of these

$$\text{un fav} = \begin{Bmatrix} (1,1) & (2,2) & (3,3) & \cdots \\ \cdots & & & (25,25) \end{Bmatrix} \Rightarrow \text{fav} = 25$$

$$P(\text{un fav.}) = \frac{25}{625} = \frac{1}{25} \Rightarrow P(\text{fav}) = 1 - \frac{1}{25} = \frac{24}{25}$$

[MCQ]

PW

#Q. The probability that out of 10 persons, all born in April, at least two have the same birthday is :

Total ways of falling B'day $= \underset{P_1}{30} \times \underset{P_2}{30} \times \underset{P_3}{30} \times \cdots \times \underset{P_{10}}{30} = 30^{10}$

A) $\dfrac{{}^{30}C_{10}}{(30)_{10}}$

B) $1 - \dfrac{{}^{30}C_{10}}{(30)!}$

C) $1 \cancel{- \dfrac{C_{10}}{(30)_{10}}}$ $1 - \dfrac{{}^{30}C_{10}}{(30)^{10}}$

D) None of these

No of ways in which all will take Birth on different dates $= {}^{30}C_{10}$

$P(\text{all will take Birth on diff dates}) = \dfrac{f}{T} = \dfrac{{}^{30}C_{10}}{30^{10}}$

$P(\text{at least two persons will take Birth on same day}) = 1 - \dfrac{{}^{30}C_{10}}{30^{10}}$ //

[NAT]

#Q. A bag contains 5 white and 8 red balls. Two successive drawings of 3 balls are made such that

(a) the balls are replaced before the second drawing

Find the probability that the first drawing will give 3 white and the second will give 3 red balls ~~in each case~~.

$\boxed{\begin{matrix} 5W \\ 8R \end{matrix}}$

$$\text{Req Prob} = \frac{{}^5C_3}{{}^{13}C_3} \times \frac{{}^8C_3}{{}^{13}C_3}$$

[NAT]

#Q. A bag contains 5 white and 3 red balls and four balls are successively drawn and are not placed. What is the chance that ~~(i)~~ white and red balls appear alternatively? (in that order)

5W
3R

w/o Replacement

W R W R

$$\text{Req Prob} = \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5}$$

[MCQ]

$\boxed{\begin{matrix}10W\\8B\end{matrix}}$ $\boxed{\begin{matrix}2W\\1B\end{matrix}}$

#Q. Suppose a box contains 10 white balls and 8 black balls. What is the probability of drawing 2 white and 1 black balls for the following three cases? (in that order for 2nd & 3rd)

A. All the three balls picked in a single draw $= \frac{f}{T} = \frac{{}^{10}C_2 \times {}^{8}C_1}{{}^{18}C_3}$

B. All the three balls drawn one after another with (replacement case) $= \frac{10}{18} \times \frac{10}{18} \times \frac{8}{18}$

C. All the three balls picked one after another (without replacement case)

$$\frac{10}{18} \times \frac{9}{17} \times \frac{8}{16}$$

[NAT]

#Q. A can hit a target 4 times in 5 shots ,B 3 times in 4 shots and C twice in 3 shots. They fire a volley. What is the probability that :

(a) Two shots hit the target.

(b) At least two shots hit the target.

$A = \frac{4}{5},\ \bar{A} = \frac{1}{5}$

$B = \frac{3}{4},\ \bar{B} = \frac{1}{4}$

$C = \frac{2}{3},\ \bar{C} = \frac{1}{3}$

(1) Req Prob $= (A \cap B \cap \bar{C})$ or $(A \cap \bar{B} \cap C)$ or $(\bar{A} \cap B \cap C)$

Exactly two $= \left(\frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}\right) + \left(\frac{4}{5} \times \frac{1}{4} \times \frac{2}{3}\right) + \left(\frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}\right) = \frac{26}{60}$

(2) At least two $= \downarrow + \downarrow + \downarrow + (A \cap B \cap C)$

$= \frac{12}{60} + \frac{8}{60} + \frac{6}{60} + \frac{24}{60} = \frac{50}{60} = \frac{5}{6}$

[NAT]

#Q. A problem is given to students A, B, C whose chances of solving it is $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}$. What is the probability that the problem will be solved?

P(Problem will be solved) = P(at least one student will solve it)

$= 1 - P(\text{None})$

$= 1 - P(\text{No student will solve it})$

$= 1 - \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right) = 1 - \frac{3}{32} = \frac{29}{32}$

[NAT]

#Q. There are two packs of card. One card is drawn at random from each pack. What is the probability that :

(i) Both of them are black

(ii) They are of different in colour

(1) Req Prob $= B \cap B$

$= \frac{26}{52} \times \frac{26}{52}$

$= \frac{1}{4}$

(2) Req Prob $= (B \cap R)$ or $(R \cap B)$

$= \frac{26}{52} \times \frac{26}{52} + \frac{26}{52} \times \frac{26}{52}$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

[MCQ]

#Q. In a batch Of 100 resistors Of 1kilo ohm resistance, 80 numbers are within the required tolerance values and 11 numbers are below the required tolerance values, the remaining are above the required tolerance values. If two resistors are drawn one after the other without replacement, the probability of the first one drawn is below and the second one drawn is above the required tolerance value is :

A 0.01

B 0.09

C 0.11

D 0.89

100 → 80 within

100 → 11 Below

100 → 9 above

one by one w/o

Req Prob $= \frac{11}{100} \times \frac{9}{99} = \frac{1}{100} = 0.01$

[NAT]

#Q. A bag contains 5 white and 4 black balls. A ball is drawn from this bag and is replaced and then second draw of a ball is made. What is the probability that two balls are of different colors.

With Replace

5W
4B

Req Prob = $(B \cap W)$ or $(W \cap B)$

$$= \left(\frac{4}{9} \times \frac{5}{9}\right) + \left(\frac{5}{9} \times \frac{4}{9}\right) = \frac{40}{81}$$

[NAT]

#Q. In a class 40% student read mathematics, 25% Biology and 15% both Mathematics and Biology. One student was selected at random. Find

(i) The probability that he reads Mathematics if it is known that he reads Biology.

(ii) The probability that he reads Biology if it is known that he reads Mathematics.

$P(M) = 0.4$

$P(B) = 0.25$

$P(M \cap B) = 0.15$

① $P\left(M/B\right) = \dfrac{P(M \cap B)}{P(B)} = \dfrac{0.15}{0.25} = \dfrac{3}{5}$

② $P\left(B/M\right) = \dfrac{P(B \cap M)}{P(M)} = \dfrac{0.15}{0.40} = \dfrac{3}{8}$

[NAT]

#Q. A dice is rolled twice and the sum of the numbers appearing on them is observed to be 7. What is the conditional probability that the number 2 has appeared at least once ?

R. Cases = {Sum is 7} = {(1 6), (6 1), (2 5), (5 2), (3 4), (4 3)} = 6

fav Cases = {(5 2), (2 5)} = 2

Condn Prob = $\frac{f}{R} = \frac{2}{6} = \frac{1}{3}$

[NAT]

#Q. A family has 2 children. Given that one of the children is a boy, what is the probability that the other child is also a boy?

Total $S = \{(BB), (BG), (GB), \cancel{(GG)}\} = 4$ Cases

R. S Sp $= \{(BB), (BG), (GB)\} = 3$ Cases

fav Cases $= \{(BB)\} = 1$

Condⁿ Prob $= \frac{f}{R} = \frac{1}{3}$

[NAT]

#Q. A coin is tossed once. If it shows head, it is tossed again and if it shown tail, then a dice is thrown. Let E_1 be the event, the first throw of coin shows tail and E_2 be the dice shows a number greater than 4. Find P $(E_2|E_1)$.

$S = \{\cancel{(HHH.....)}, \underbrace{T1, T2, T3, T4, T5, T6}\}$

E_1 = Reduced Cases = 6

E_2 = fav. Cases = {T5, T6} = 2

$$P(E_2/E_1) = \frac{fav}{R} = \frac{2}{6} = \frac{1}{3}$$

[NAT]

#Q. It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email ?

A = { email detected as spam by software }

E-Mail

1/2 SPAM → 0.99

1/2 Not SPAM → 0.05

A :

$$P(A) = \left(\frac{1}{2} \times 0.99\right) + \left(\frac{1}{2} \times 0.05\right)$$

$$P\left(\frac{E_2}{A}\right) = \frac{\frac{1}{2} \times 0.05}{P(A)} = \frac{5}{104}$$

[NAT]

#Q. Three persons A, B and C have applied for a job in a private company. The chance of their selections is in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve the profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

A

A = {Profit is not occurring due to changes}

$P(A) = \frac{k}{7}, P(B) = \frac{2k}{7}, P(C) = \frac{4k}{7}$

Persons

	Person A	Person B	Person C
	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{4}{7}$
	E_1	E_2	E_3
A :	0.2	0.5	0.7

$P(A) = \frac{1}{7} \times 0.2 + \frac{2}{7} \times 0.5 + \frac{4}{7} \times 0.7$ — (1)

$P(E_3/A) = \frac{\frac{4}{7} \times 0.7}{P(A)} = 0.7$

NAT:

\# Q. A Bag contains 4 balls. Two balls are drawn at random and are found to be blue. What is the probability that all balls in the bag are Blue?

Condition = A

$A = \{$ Both the Balls (drawn) are Blue $\}$

Balls

- $\frac{1}{3}$ → 2 Blue Balls (E_1) → $\frac{{}^2C_2}{{}^4C_2}$
- $\frac{1}{3}$ → 3 Blue Balls (E_2) → $\frac{{}^3C_2}{{}^4C_2}$
- $\frac{1}{3}$ → 4 Blue Balls (E_3) → $\frac{{}^4C_2}{{}^4C_2}$

A:

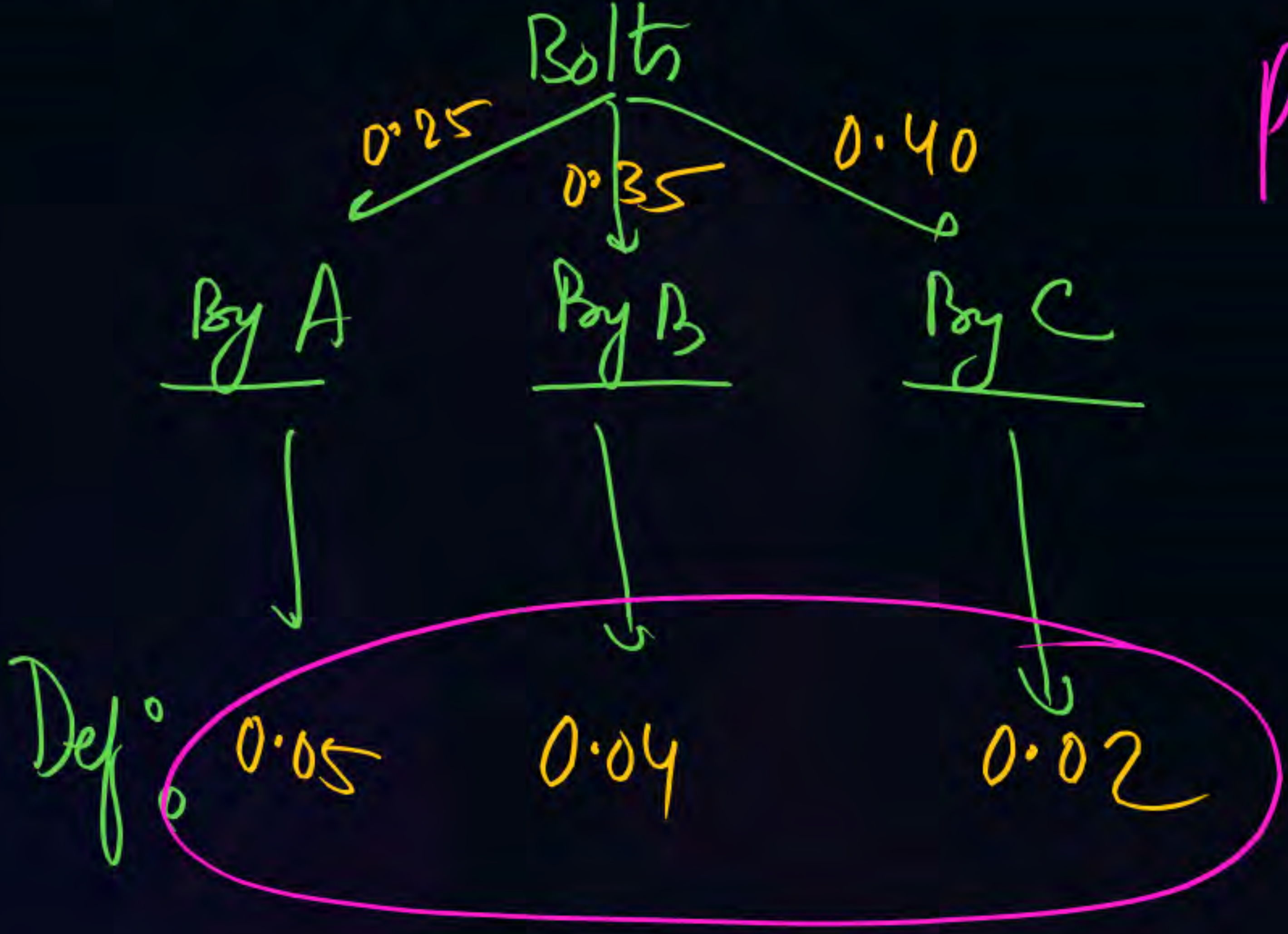
$$P(A) = \left(\frac{1}{3} \times \frac{1}{{}^4C_2}\right) + \left(\frac{1}{3} \times \frac{3}{{}^4C_2}\right) + \left(\frac{1}{3} \times 1\right)$$

1

$$P(E_3/A) = \frac{\frac{1}{3} \times 1}{P(A)} = ? = \frac{3}{5}$$

[NAT]

#Q. In a bolt factory, machines A, B, C manufactures respectively 25%, 35%, 40% of the total bolts of there output and 5%, 4%, 2% are respectively defective bolts. A bolt is drawn at random from the product. What is the probability that the bolt drawn is defective ?



$$P(A) = (0.25 \times 0.05) + (0.35 \times 0.04) + (0.4 \times 0.02)$$

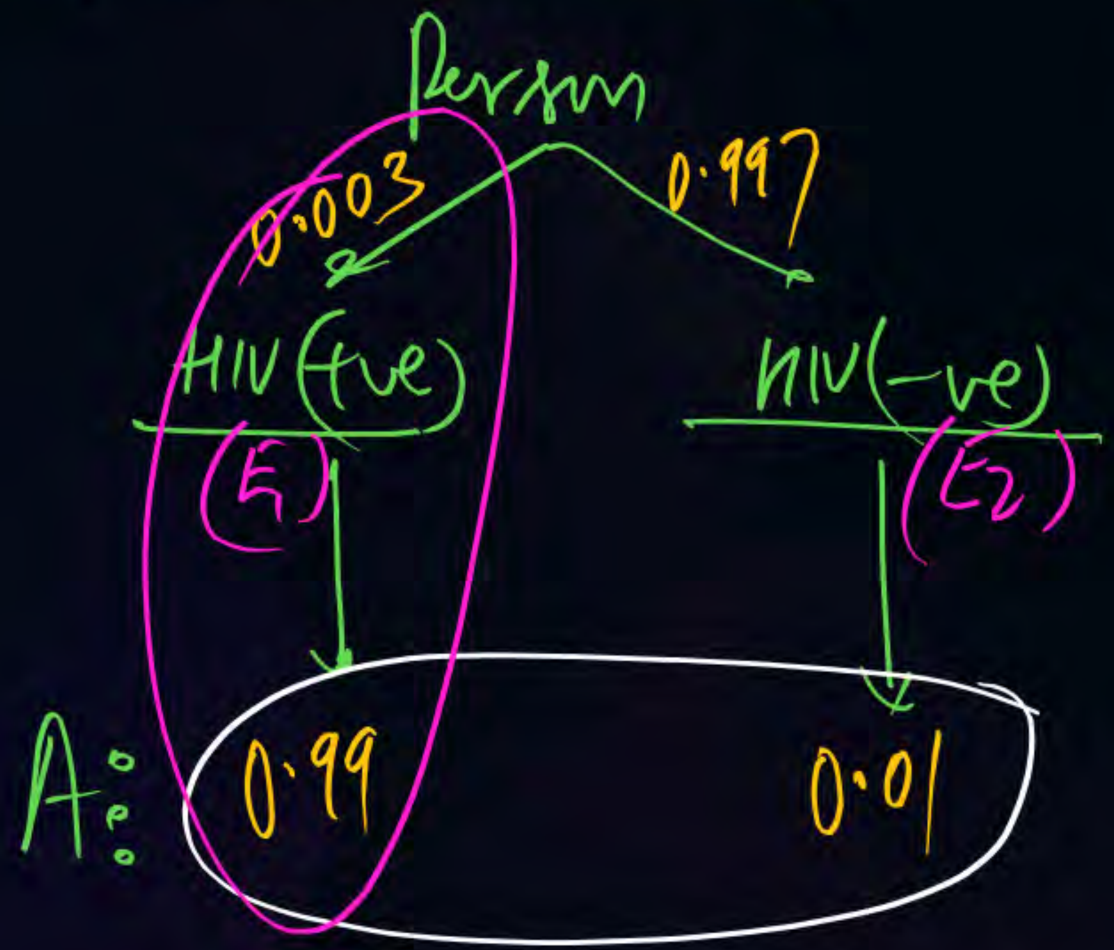
$$= 0.00345$$

[NAT]

#Q. Suppose the test for HIV is 99% accurate in both directions and 0.3% of the population is HIV positive. If someone tests positive, what is the probability they actually are HIV positive ?

A = Condition

A = { Test declares +ve }



$$P(A) = (0.003 \times 0.99) + (0.997 \times 0.01) \quad \text{①}$$

$$P(\text{actually +ve}) = P(E_1/A) = \frac{0.003 \times 0.99}{P(A)}$$

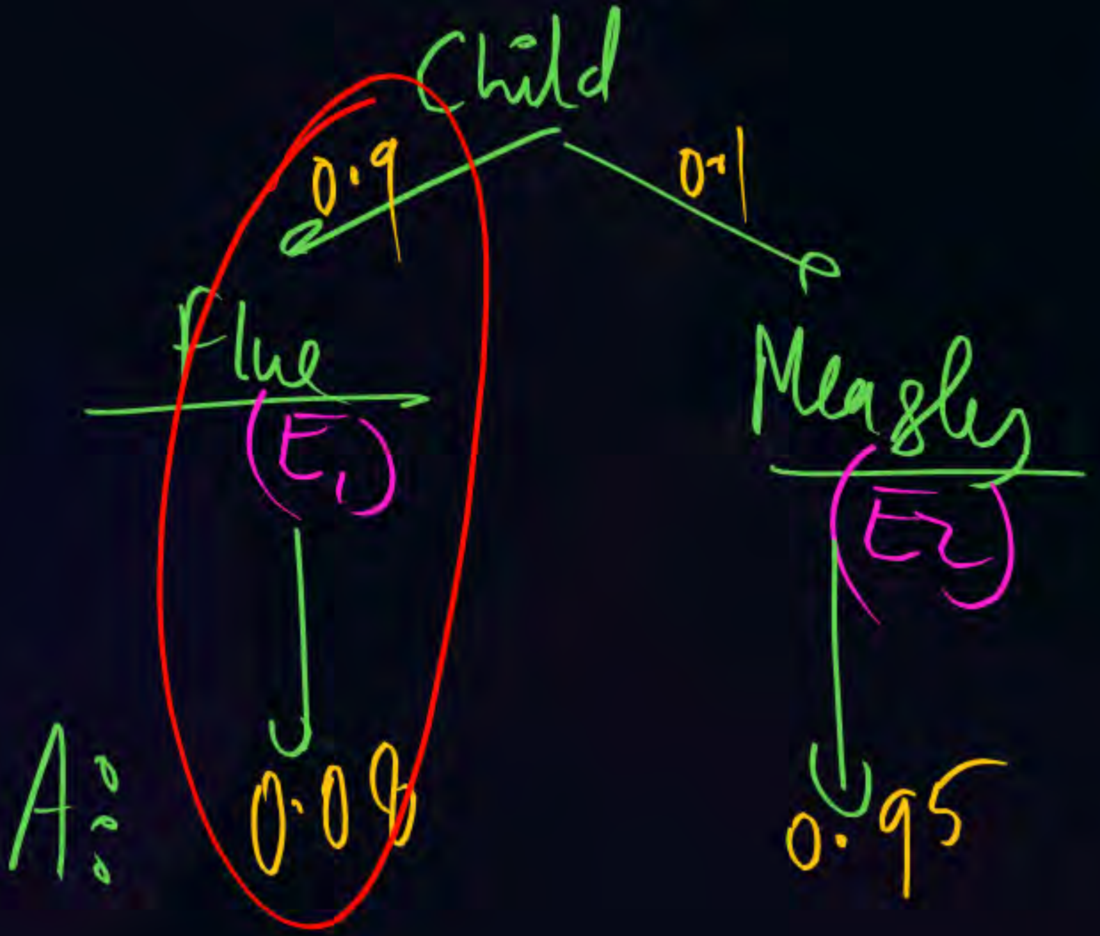
$$= 0.23 \simeq 23\%$$

[NAT]

#Q. In a neighbourhood, 90% children were falling sick due to flu and 10% due to measles and no other disease. The probability of observing rashes for measles is 0.95 and for flu is 0.08. If a child develops rashes, find the child's probability of having flu.

A

$A = \{\text{Child develops Rashes}\}$



$$P(E_1/A) = \frac{0.9 \times 0.08}{(0.9 \times 0.08) + (0.1 \times 0.95)}$$

$$= 0.43$$

Ans

THANK - YOU