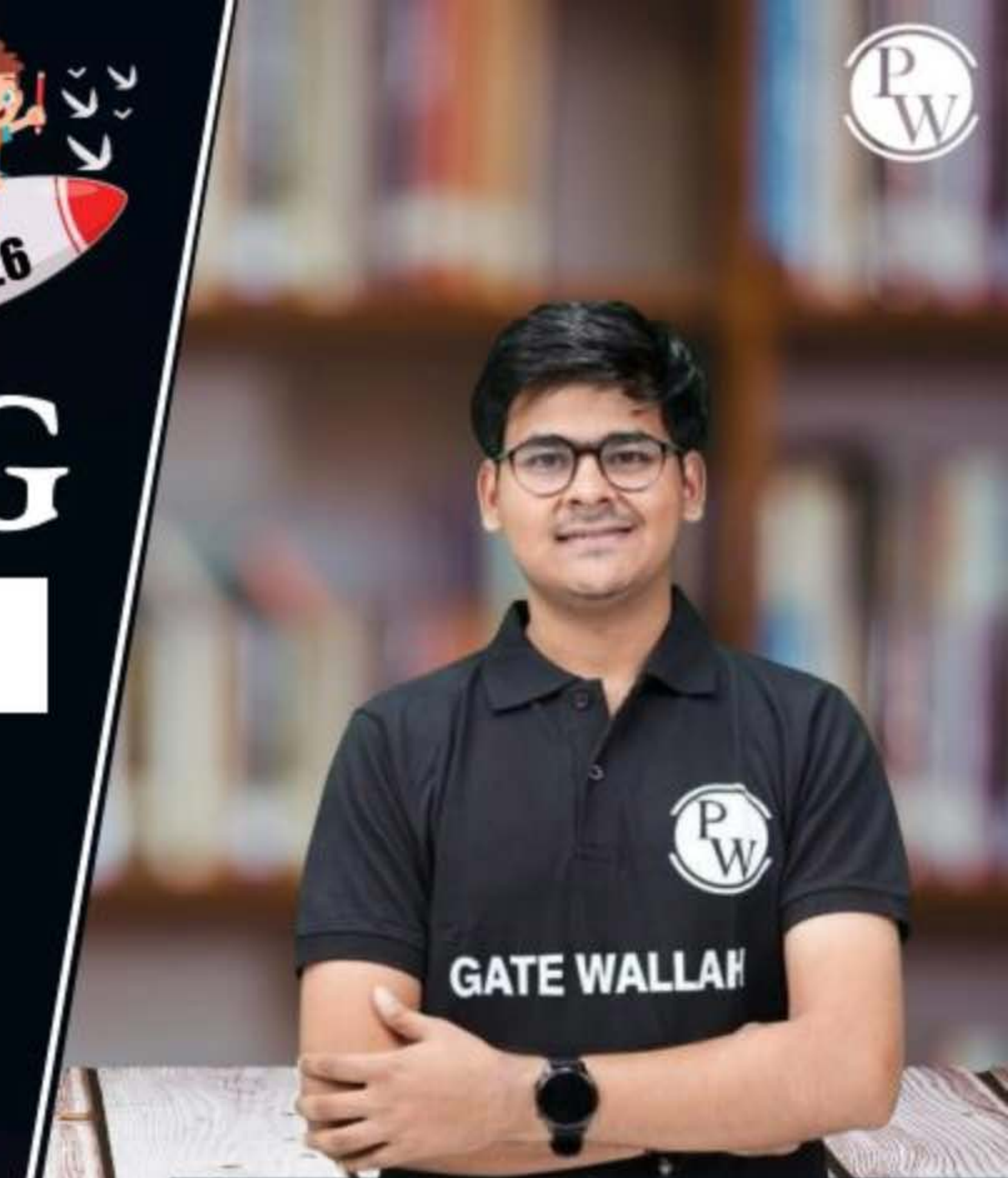
CS & IT ENGINEERING

Algorithms

Dynamic Programming (DP)

Lecture No.- 01



By- Aditya Jain sir

Recap of Previous Lecture

Topic

Topic Greedy Algo

Topics to be Covered

Topic

Topic

Topic Dynamic Prog Intro.

About Aditya Jain sir

1. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
2. Represented college as the first Google DSC Ambassador.
3. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored 12,000+ students & working professions in field of Data Science and Analytics
11. Have been mentoring & teaching GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

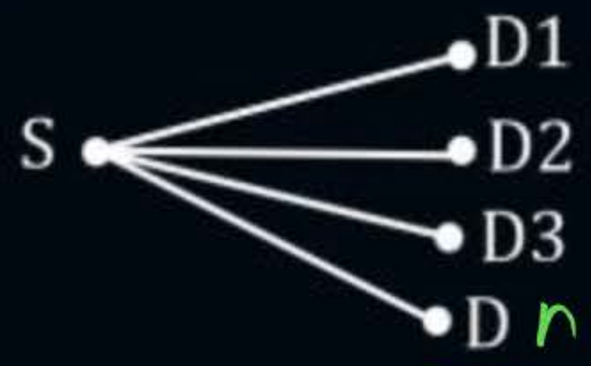
@ADITYASIR_PW

Telegram Link for Aditya Jain sir: https://t.me/AdityaSir_PW

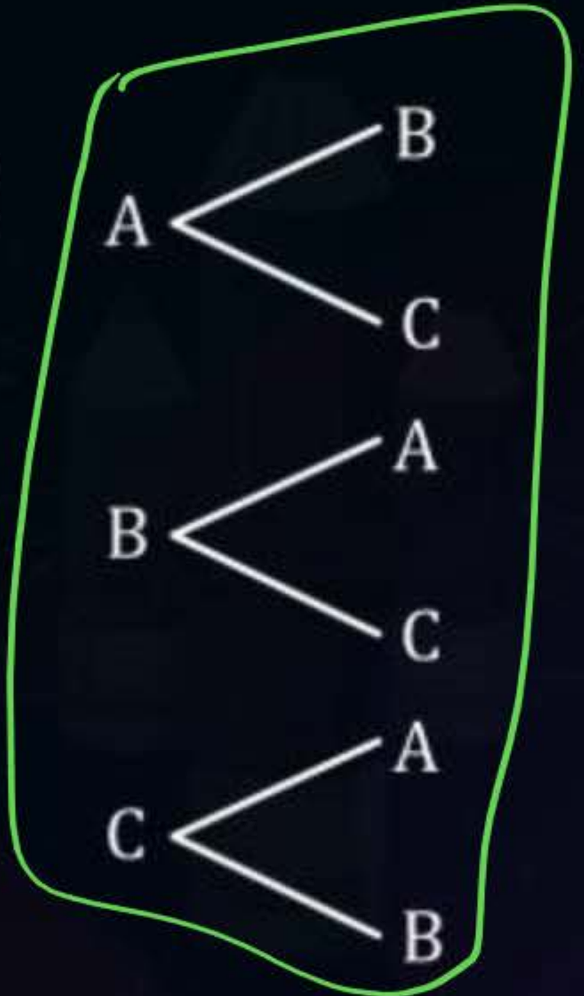
Topic : Greedy Techniques

Shortest Path Algorithms Summary

1. Single Pair shortest Path (SPSP):



2. Single source shortest Paths (SSSP)
   - (a) Dijkstra's SSSP Alog→ Greedy
   - (b) Bellman ford→ Dynamic Programming (DP)



3. All pairs shortest Path: (APSP)
   - (a) Floyd warshall algo
   - (b) Dynamic Programming (DP)

Topic : Greedy Techniques

#Q. Dijkstra's Single source shortest Path Algorithm (SSSP)

Approaches:-

1. Based on Matrix → given only SSSP cost but not paths
2. Based spanning Tree→ Gives both costs as well as paths.

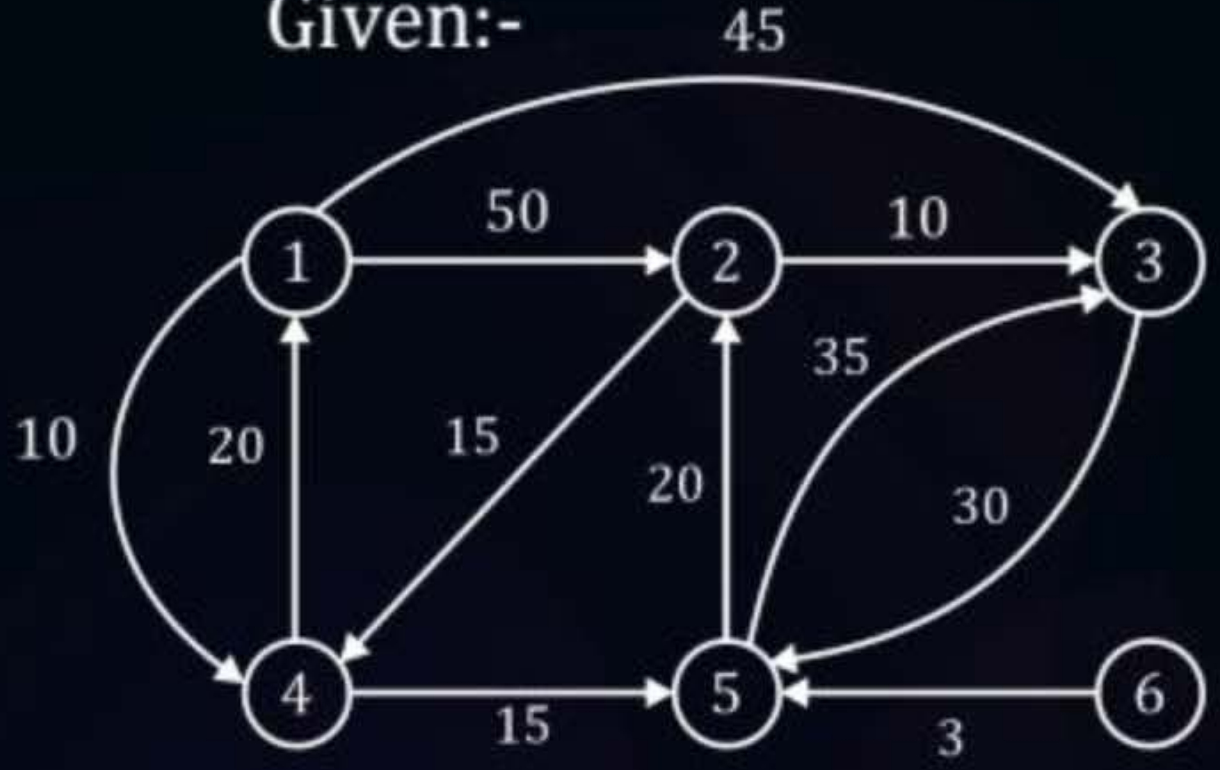
Topic : Greedy Techniques

1. Matrix Based

Relaxation

1

Given:-



vertex Set= {}	1	2	3	4	5	6
S1{1}	-	50	45	10	∞	∞
S2{1,4}	-	50	45	10	25	∞
S3:{1,4,5}	-	45	45	10	25	∞
S4{1,4,5,2}	-	45	45	10	25	∞
S5{1,4,5,2,3}	-	45	45	10	25	∞
S6{1,4,5,2,3,6}	-0	45	45	10	25	∞

1

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$d(x)$ = distance from source (V_0) to a vertex 'x' known so far.

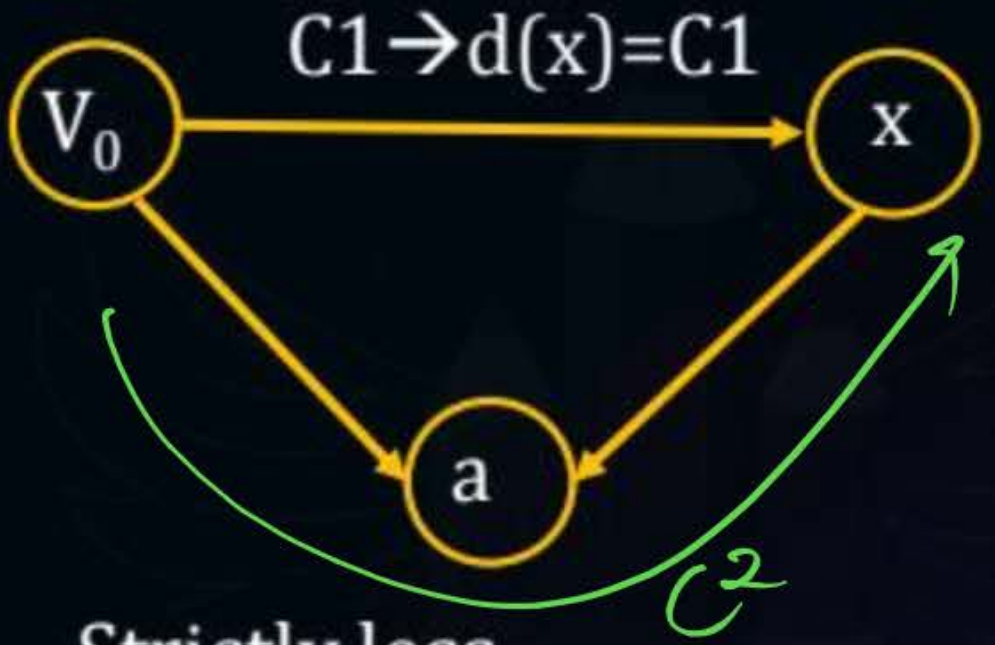
Relaxation

Final answer

$1 \to 2 : 45$ $\quad$ $1 \to 4 : 10$

$1 \to 3 : 45$ $\quad$ $1 \to 5 : 25$

$1 \to 6 : \infty$



Strictly less
If (C2 < C1)
Update d(x) = C2
}

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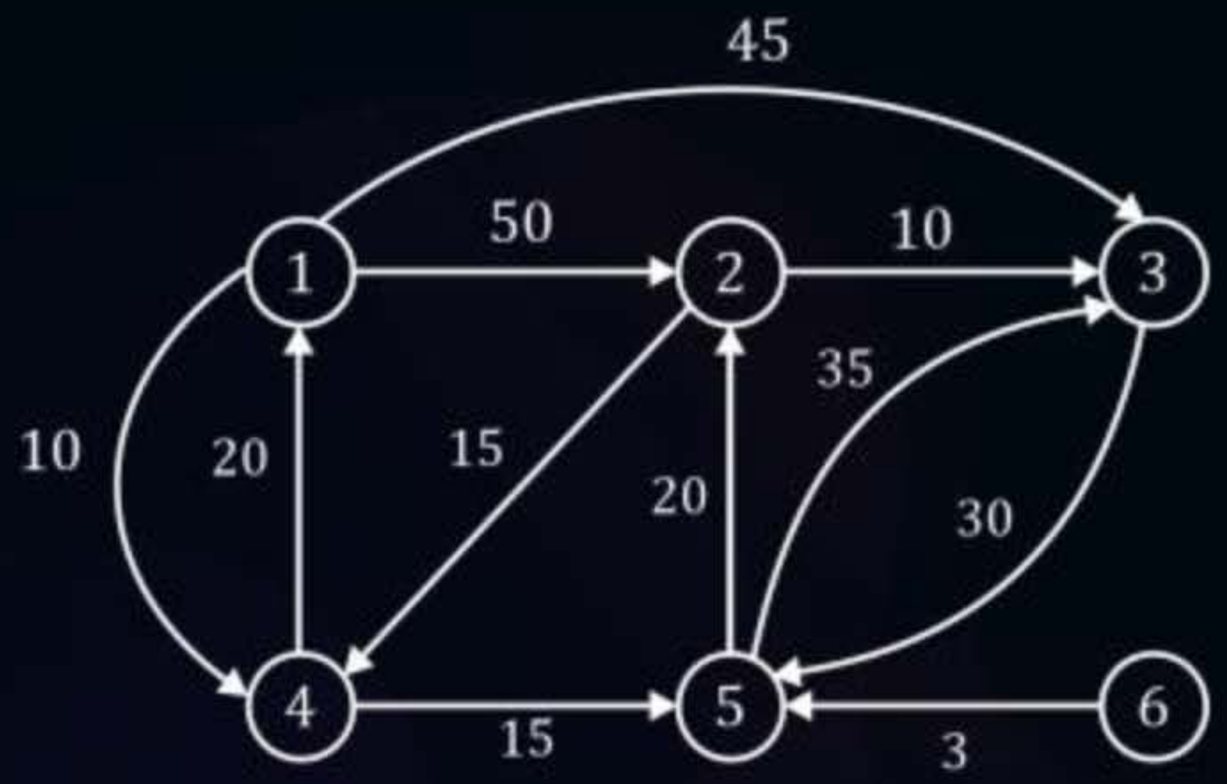
Practice:- Source $V_0 = 6$

Given:-



vertex Set= {}	1	2	3	4	5	6
S1{6}	∞	∞	∞	∞	3	-
{6,5}	∞	23	38	∞	3	-
{6,5,2}	∞	23	33	38	3	-
{6,5,2,3}	∞	23	33	38	3	-
{6,5,2,3,4}	58	23	33	38	3	-
{6,5,2,3,4,1}	58	23	33	38	3	-

Note :- once a ~~matrix~~ vertex is selected it does not gets furthers relaxed.

Topic : Greedy Techniques

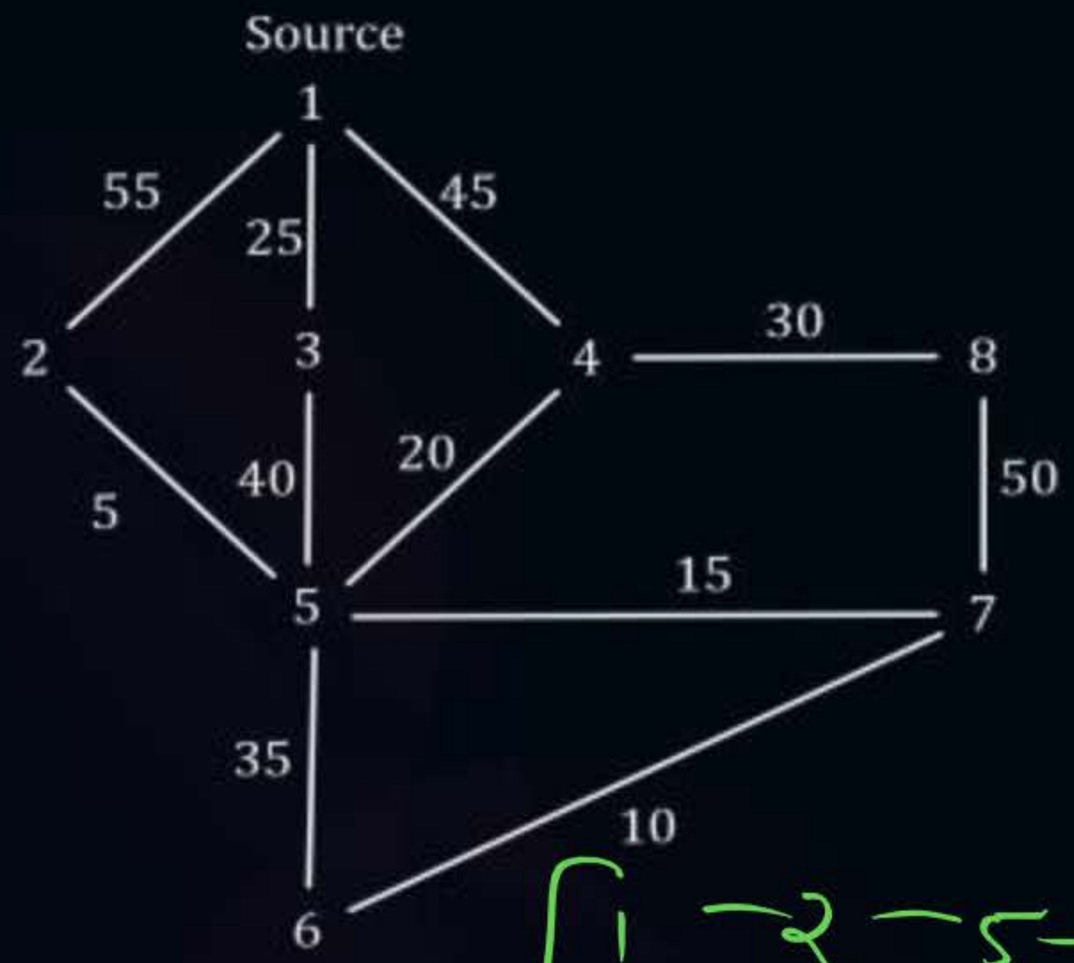
SSSP ST

source

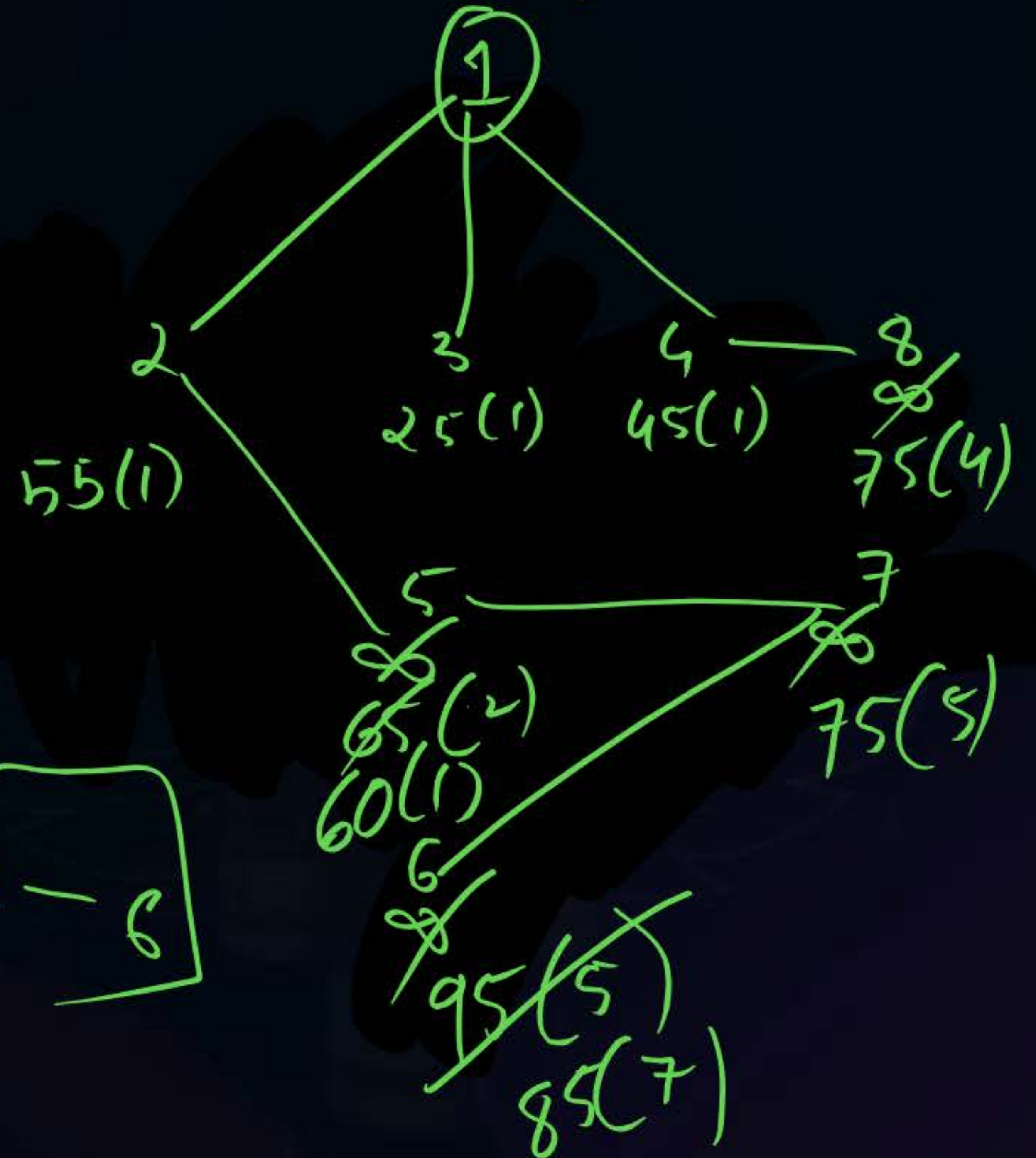
2. Spanning Tree based approach: Source $V_0 = 1$

Given:

$1 \to 6$



[1 — 2 — 5 — 7 — 6]



$1 \to 6$ cost: 85

Path given By DJ SSSP: $[1 \to 2 \to 5 \to 7 \to 6]$

#Q. Is the spanning tree by above approach of DJ SSSP same as MCST?

No

Topic : Greedy Techniques

Time Complexity analysis:-

1. Non-heap based implementation

   TC→ $O(n^2)$

2. Heap based implementation

   TC→ $(n+e) \log n$

PYQ

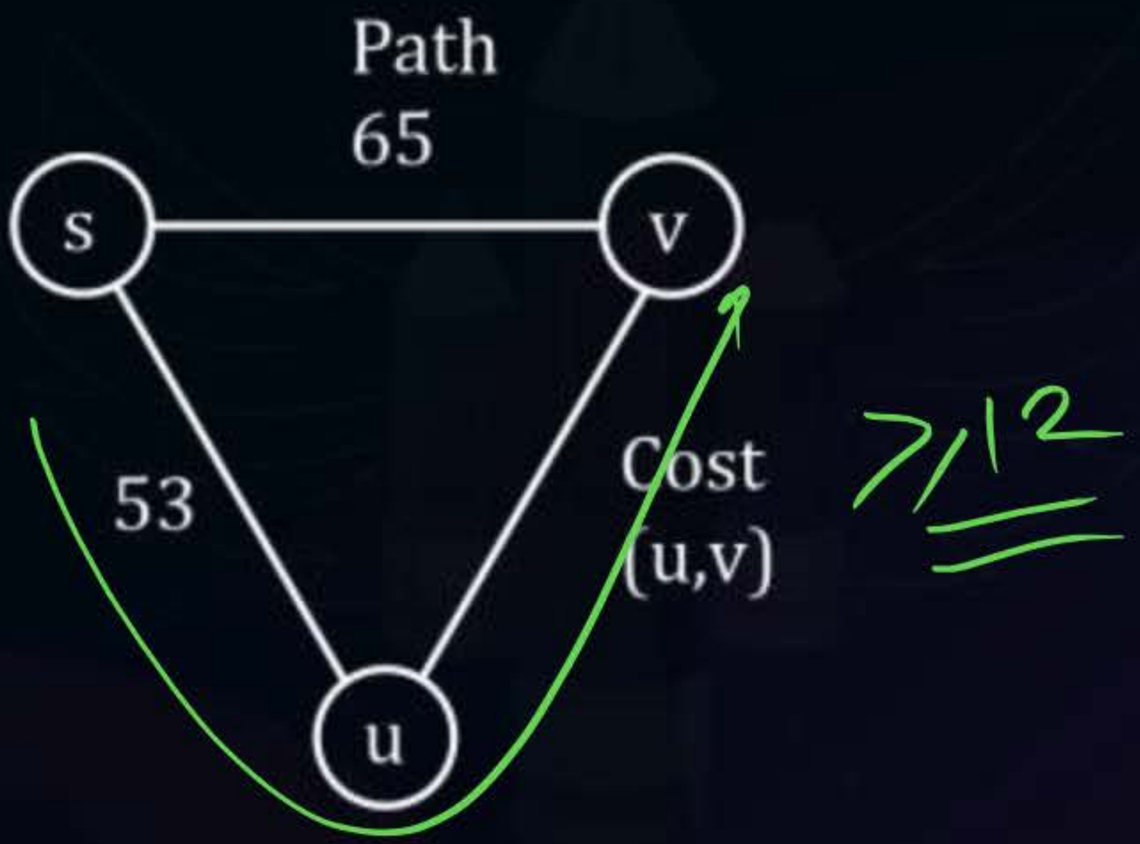
#Q. Consider a weighted undirected graph with positive edge weights and let uv be an edge in the edge in the graph. It is known that the shortest path from the sources vertex s to u has weight 53 and the shortest path from s to v has weight 65. Which one of the following statements is always true?

A. Weight (u, v) < 12

B. Weight (u, v) ≤ 12

C. Weight (u, v) > 12

D. Weight (u, v) ≥ 12



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PYQ

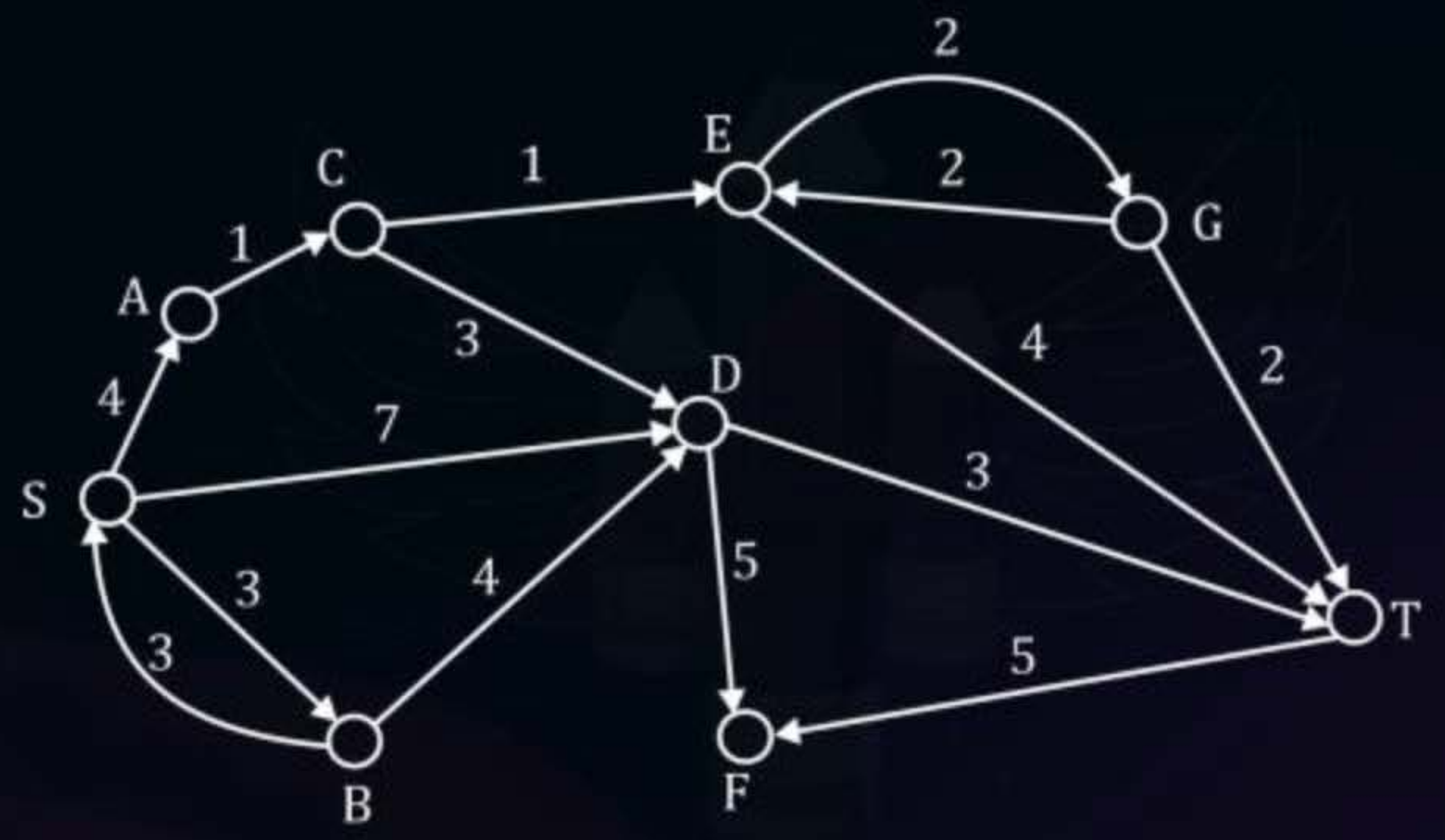
#Q. Applying Dijkstra's Algoirthm over the given Graph, which path is reported for 'S' to 'T';

A SBDT → 10

B SDT → 10

C SACEGT → 10

D SACET → 10



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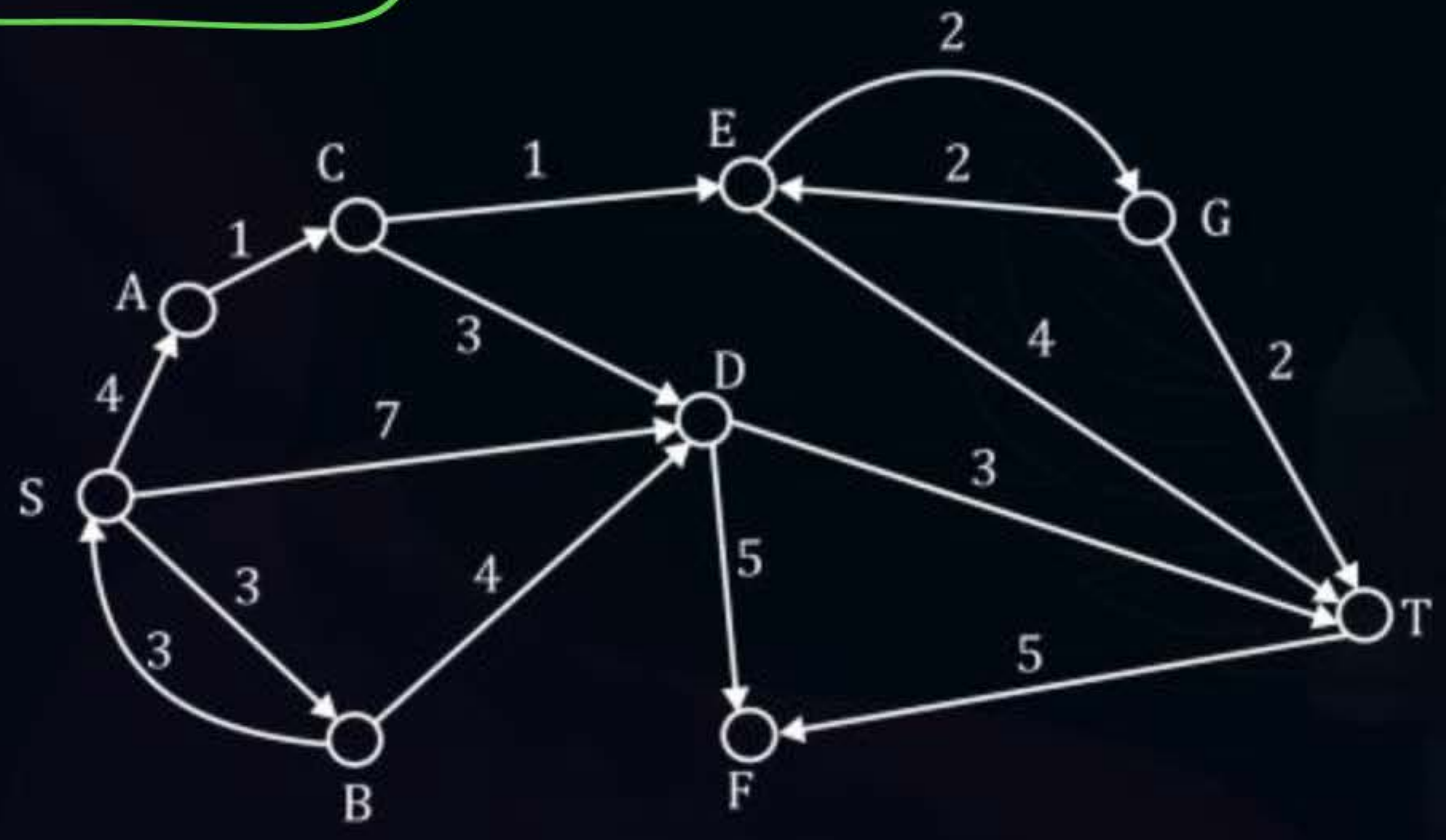
Applying Dijkstra's SSSP Sapnning Tree approach

Spanning: Path reported by DJ SSSP is

Only S→A → C → E → T



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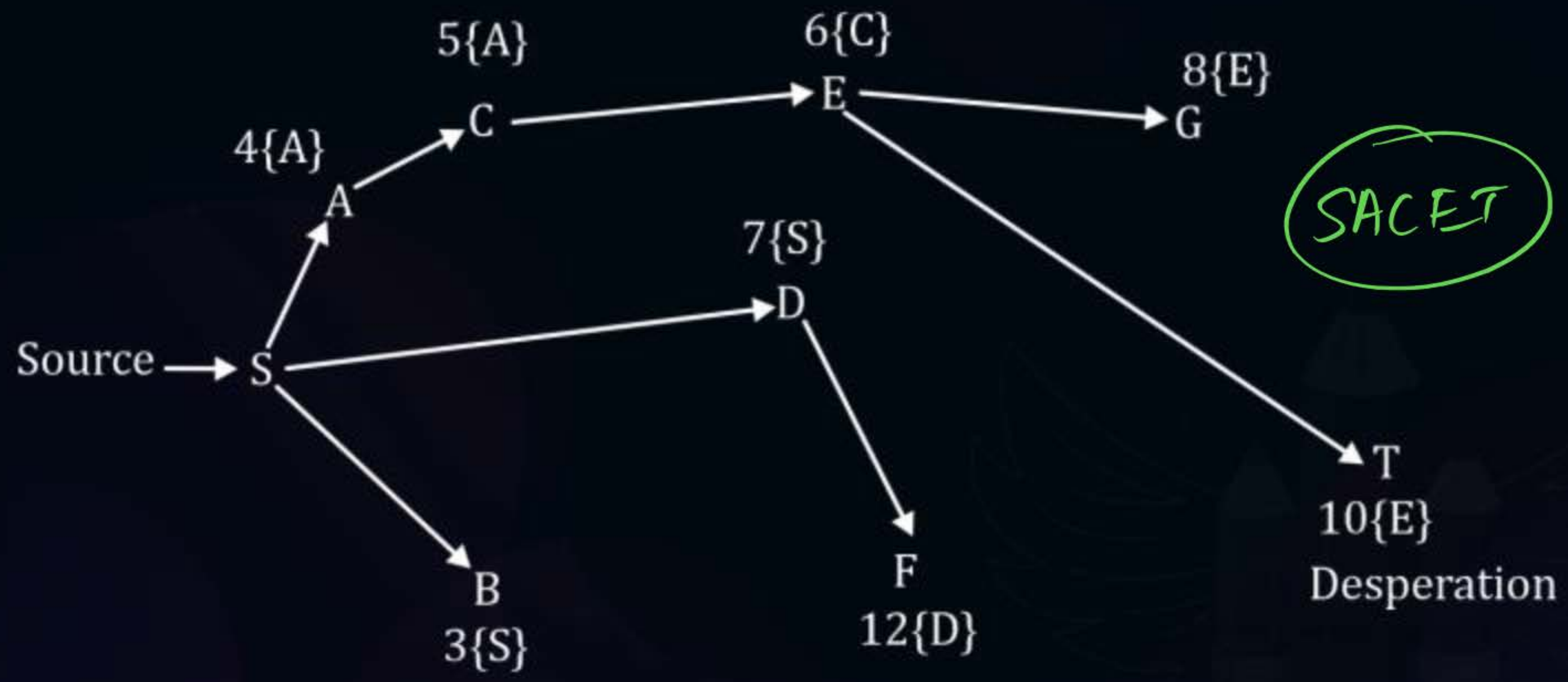

Final Spanning Tree By DJ SSSP.

Relaxation happen only when a strictly lesser path cost is found.

Topic : Greedy Techniques

PYQ

#Q. Let G be weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are true?

S_1. Minimum spanning Tree of the graph does not change. ✓

S_2. Shortest path between any pair of vertices does not change. ✗

34%

A. Only S_1 true

B. Only S_2 true

C. Both S_1 and S_2 true

D. Both S_1 and S_2 are False

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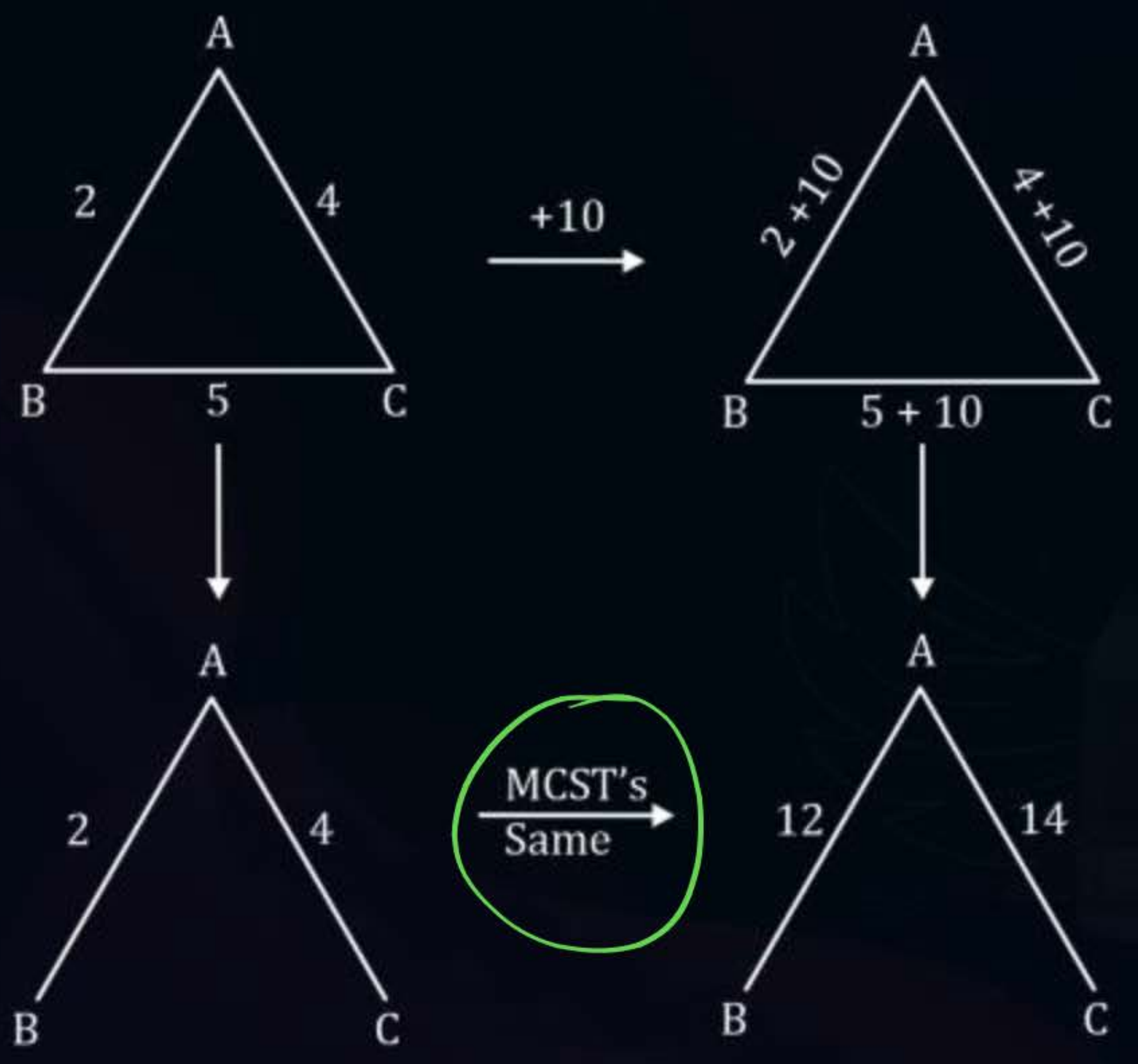
S2:

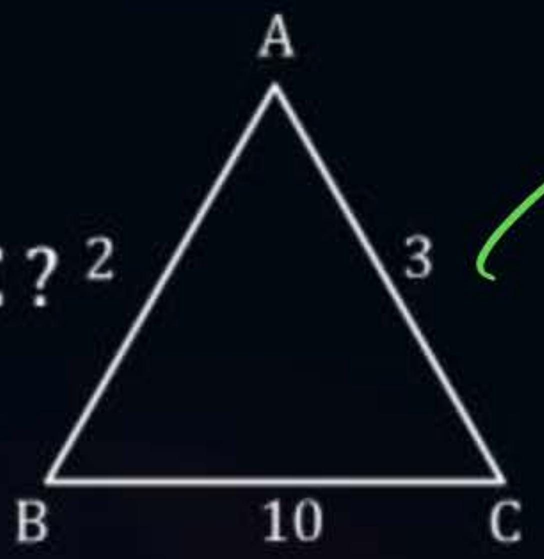


Topic : Greedy Techniques

S2:

Shortest Path

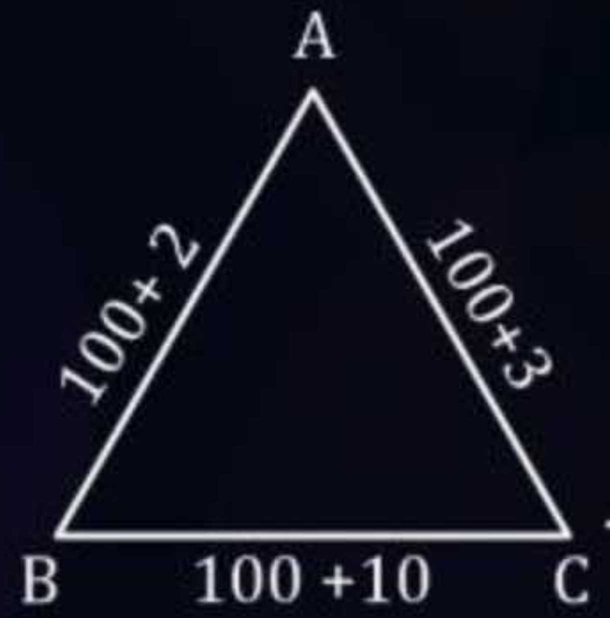
from B→ C ?



$B \xrightarrow[2]{} A \xrightarrow[3]{} C = 2 + 3 = 5$ (BAC)

$\downarrow + 100$

B to C



$\longrightarrow$ $B \xrightarrow[110]{} C$

BAC → 205
BC → 110

P48

#Q. Let G = (V, E) be any connected undirected edge- weighted graph. The weights of the edges in E are positive and distinct. Consider the following statements:

✓ I. Minimum Spanning Tree of G is always unique

✗ II. Shortest path between any two vertices of G is always unique.

48%

Which of the above statements is/are necessarily true?

A I only

B II only

C Both I and II

D Neither I and II

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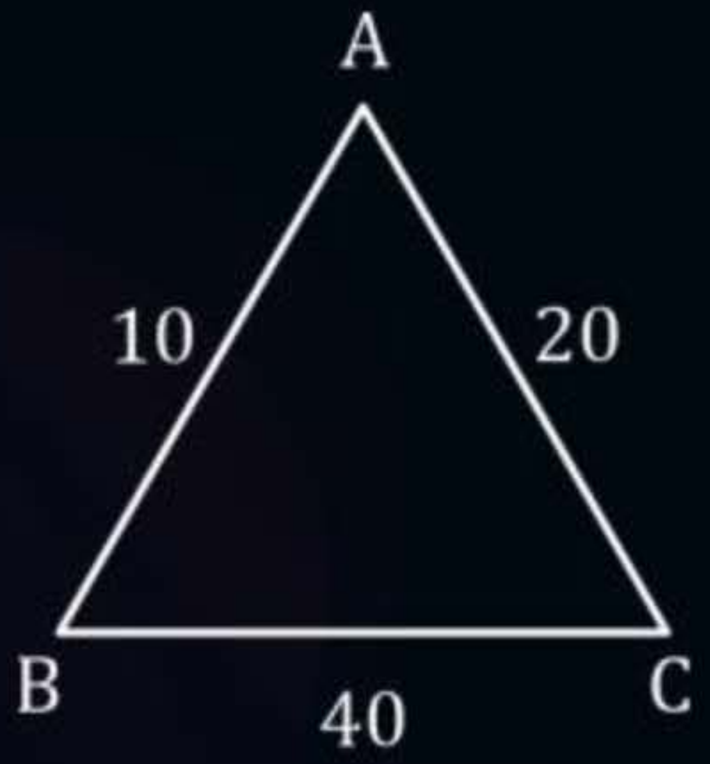
Eg.1.

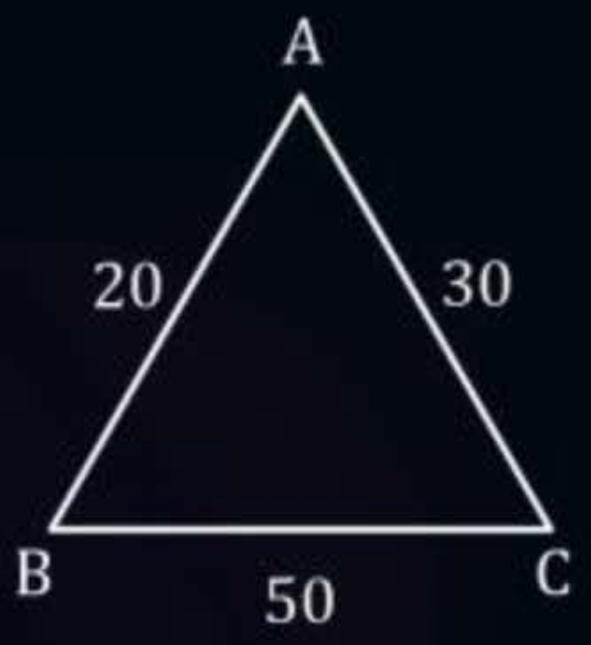


$B \longrightarrow C$: 40

$B \xrightarrow[10]{} A \xrightarrow[20]{} C$

$10 + 20 = 30$

Eg.2.



Shortest Path

$B \longrightarrow C$

(1) $B \xrightarrow[20]{} A \xrightarrow[30]{} C$

$20 + 30 = 50$

(2) $B \longrightarrow C = 50$

Shortest Path Is NOT unique in this case.

Topic : Dynamic Programming

Introduction to Dynamic Programming:

Sorting/ Tabulating the values or Results of the sub- Problems.

Problems

- Decision
- Optimization

Topic : Dynamic Programming

Dynamic Programming (DP) is an algorithm design method used for solving problems, whose solutions are viewed as a result of making a set / sequence of decisions.

- One way of making these decisions is to make them one at a time in a step-wise (sequential) step-by-step manner and never make an erroneous decision. This is true of all problems solvable by Greedy method.)
- For many other problems it is not possible to make step-wise decisions based on local information available at every step, In such a manner that the sequence of decision made is optimal.

Topic : Dynamic Programming

Eg. 0/1 Knapsack

→ Main diff. between greedy & DP.

Sometimes greedy appr. Does not give optimal solution of some problems.

Consider the weights and values of times listed below: Note that there is only one unit of each item.

M = 11 (Capacity)

Item number	Weight (in kgs)	Profit (in Rupees)
1	10	60
2	7	28
3	4	20
4	2	24

$V_{DP} = 60$ ✓

Binary Knapsack

Topic : Dynamic Programming

	W	P
01	10	60
02	7	28
03	4	20
04	2	24

$\rightarrow 60/10 = 6 \rightarrow 2\times x_1 = 0$

$\rightarrow 27/7 = 4 \rightarrow 4 \times x_2 = 0$

$\rightarrow 40/4 = 5 \rightarrow 3 \times x_3 = 1$

$\rightarrow 24/2 = 12 \rightarrow 4 \times x_1 = 1$

$M = 11$

$11 - 2 = 9$

$10 > 9 \rightarrow$ Skip

$9 - 4 = 5$

Maximum Profit By Queued approach to 0/1 Knapsack:

$= \sum_{i=1}^{4} P_i \times X_i$

$= 60 * 0 + 28 * 0 + 20 * 1 + 24 * 1$

$= 20 + 24$

$= 44$

Topic : Dynamic Programming

But optimal profit

when taken only object O_1

$= 60$

Hence, greedy approach fails to give optimal solution in 0/1 knapsack (may or not give)

Eg.1. Coin change Problem (DP vs Greedy)

Problem:- Given a set of coin values, construct a sum of money using as few coins as possible.

You can use each coin value any number of times.

Coin Values: $\{C_1, C_2 \ldots C_K\}$

Target Money (Sum) : N

Topic : Dynamic Programming

Eg.1. Coins = {1,2,5}, Sum = N =12

Greedy Method: 5 + 5 + 2 = 12

Number of coins required = 3

in this case, Greedy gives optimal answer.

Topic : Dynamic Programming

Eg.1. Coins = {1, 3, 4 } , N = 6 (sum)

Greedy approach: 4 + 1+ 1 = 6 (Number of coins = 3)

in general, optimal sol. 3 + 3 = 6 (number of coins = 2)

Topic : Dynamic Programming

In above example , greedy approach failed to give optimal solution.

Hence, greedy approach doesn't guarantee (may or may not give) optimal solution to coins change problem.

THANK - YOU