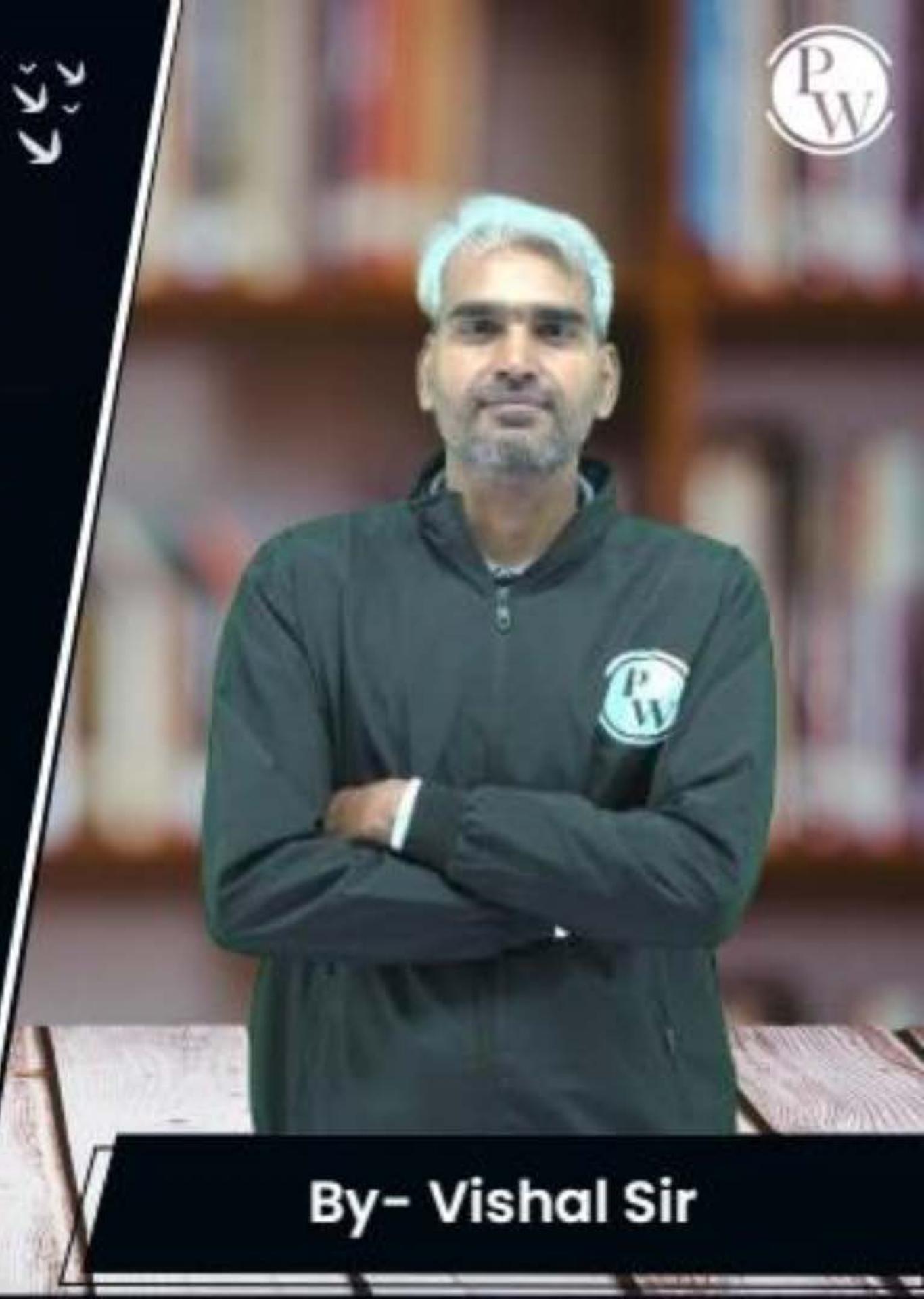


# Computer Science & IT

## Database Management System

Relational Model & Normal Forms

Lecture No. 05



By- Vishal Sir

# Recap of Previous Lecture



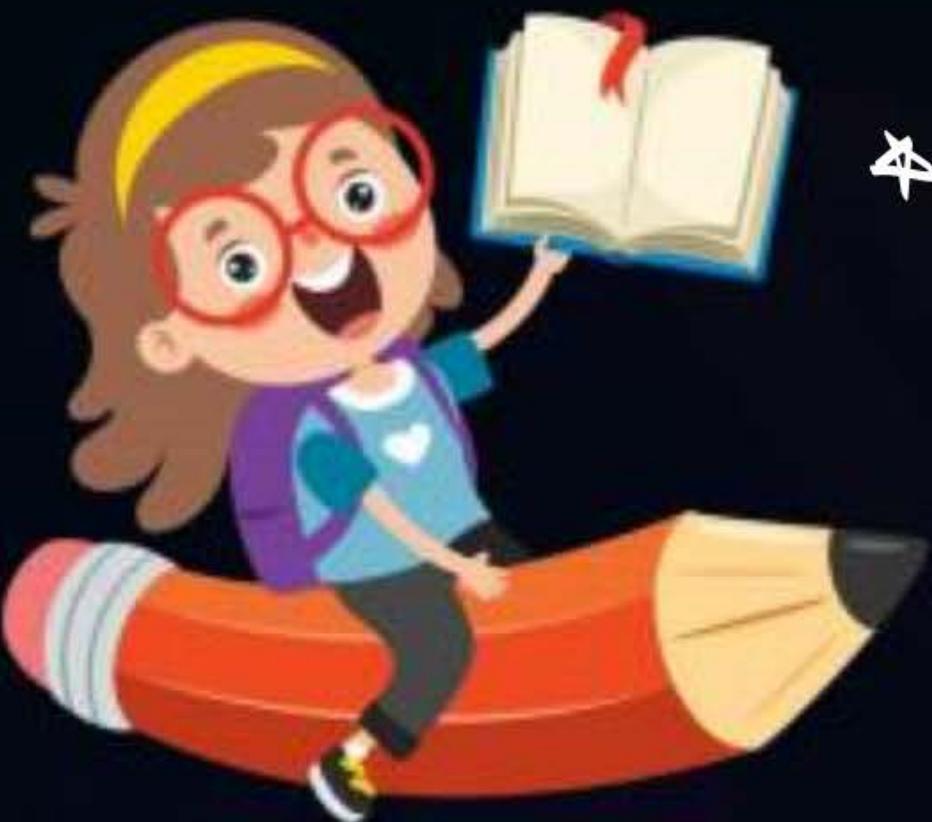
Topic

Closure of attribute Set

Topic

Candidate Key

# Topics to be Covered



Topic

Identification of candidate keys in a relation



Topic

Membership test

#Q. Assume a relation R (A, B, C, D, E) that has the following functional dependencies:

$AB \rightarrow C$ ,

$B \rightarrow E$ ,

$C \rightarrow D$

$E \rightarrow A$

Find the Candidate key of R.

Essential attribute = B

$(B)^+ = \{B, E, A, C, D\}$   
all : B is S.K

B is the only C.K OR well as C.K  
 $P.A = \{B\}$ ,  $N.P.A = \{A, C, D, E\}$

$(AB)^+ = \{A, B, C, D, E\}$   
all : AB is S.K  
check for C.K

Proper subset  
of  $\{A, B\}$

$\{A\}^+ = \{A\}$  : A is not a S.K  
 $\{B\}^+ = \{B, E, A, C, D\}$   
all : B is a S.K

$\{B\}$  is a S.K  
as well as C.K.  
B is the only C.K.  
 $\therefore P.A = \{B\}$

$N.P.A = \{A, C, D, E\}$

Proper subset is a S.K.  
 $\{A, B\}$  is not minimal  
 Hence  $\{A, B\}$  is just a S.K.  
but not a C.K

Topic : Note

If there exists any FD in the set of FDs  
of the type  $X \rightarrow$  Any of the prime attribute  
then we can replace that prime  
attribute in the corresponding  
Candidate key by 'X' {i.e. L.H.S of FD}  
and by doing so we may  
get a new Super Key of relation  
↳ then we will check  
for Candidate key

#Q. Assume a relation R (A, B, C, D) that has the following functional dependencies:

$$\begin{array}{l} AB \rightarrow C \\ AB \rightarrow CD \\ AB \rightarrow D \\ D \rightarrow A \end{array}$$

Find all the Candidate keys of R.

$AB$  &  $DB$  are the only Candidate keys of relation  
 $\therefore$  Prime Attributes:  $\{A, B, D\}$   
Non-prime attribute:  $\{C\}$

$$(AB)^+ = \{ \underbrace{A, B, C, D} \}$$

all attributes  $\therefore AB$  is a S.K.  
as well as a C.K.

$AB$  is a C.K.

Prime attribute:  $\{A, B\}$

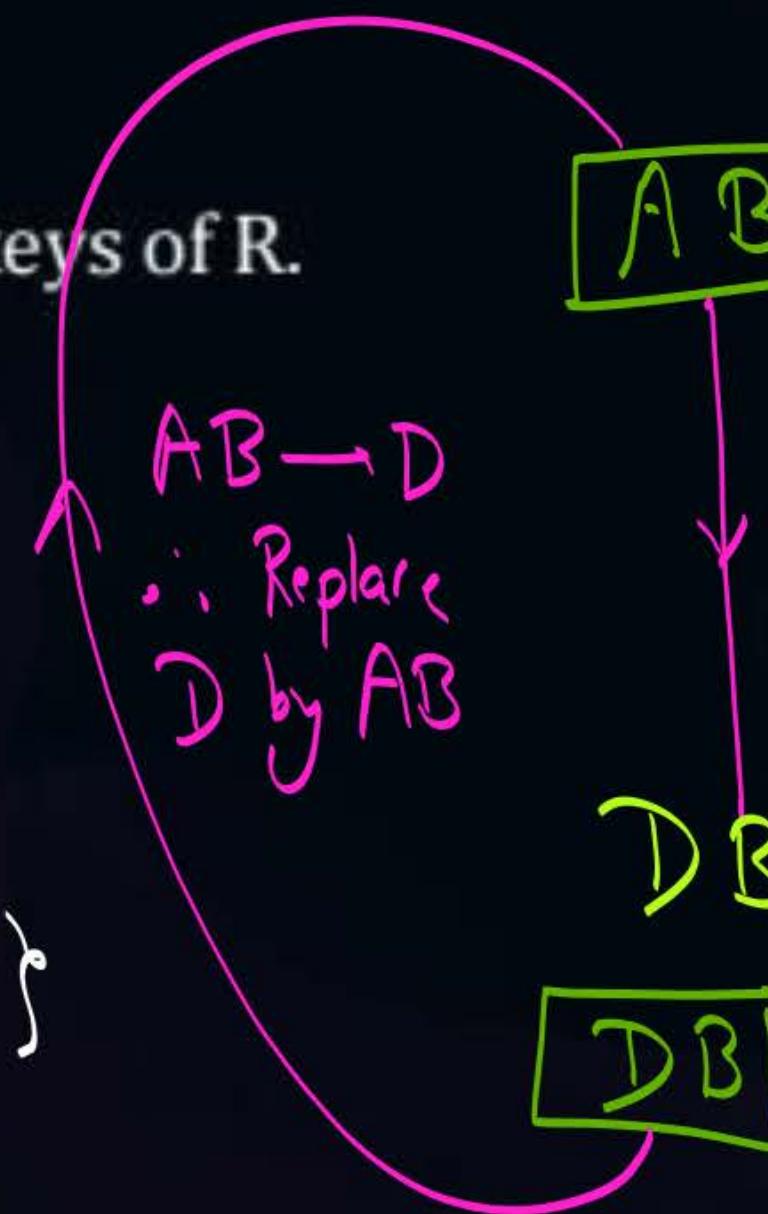
$D \rightarrow A$  is present s.t.

$\therefore$  Replace A by D RHS Contains  
Prime attribute 'A'

$DB$  is a S.K.  $\langle D \rangle^+ = \{D, A\}$

as well as a C.K.  $(B)^+ = \{B\}$

$DB$  is a C.K.



#Q. Assume a relation R (M, N, O, P, Q) that has the following functional dependencies:

$$\begin{array}{l} MNO \rightarrow P \\ MNO \rightarrow PQ \text{ and } MNO \rightarrow Q \\ P \rightarrow MN \end{array}$$

Find the Candidate keys of R.

'MNO' & 'PO' are the only C.K.s of the relation

$$\begin{array}{l} \therefore \text{Prime Attributes} = \{M, N, O, P\} \\ \text{Non-Prime attributes} = \{Q\} \end{array}$$

'O' is essential,  $(O)^+ = \{O\}$

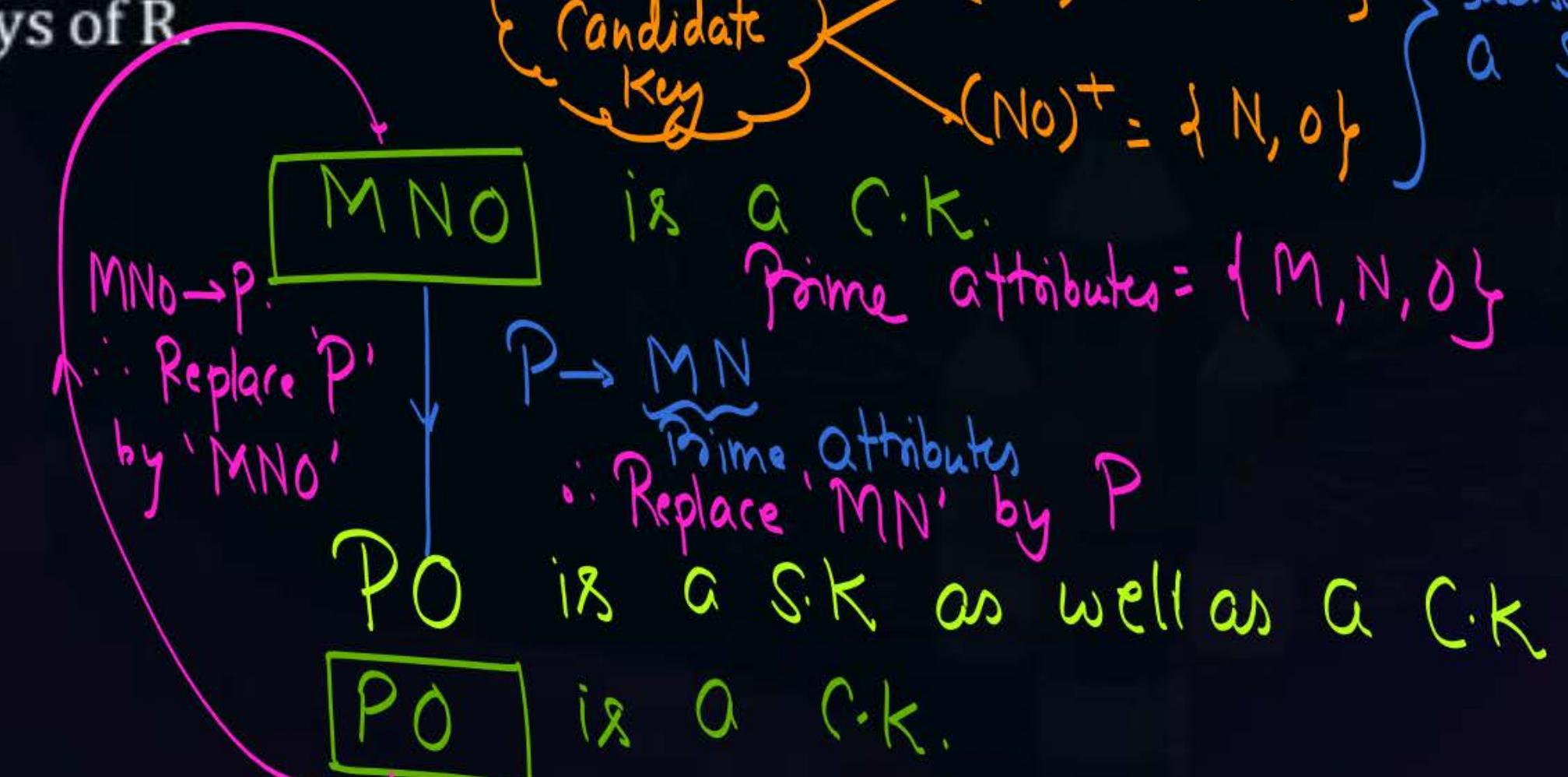
$$(MNO)^+ = \{M, N, O, P, Q\}$$

all attributes  $\therefore$  S.K.

$$\begin{array}{l} \text{Check for candidate key} \\ \therefore (MO)^+ = \{M, O\} \\ \quad \quad \quad (NO)^+ = \{N, O\} \end{array} \left. \begin{array}{l} \text{No Proper} \\ \text{Subset is} \\ \text{a S.K.} \end{array} \right\}$$

is a C.K.

Prime attributes = {M, N, O}



#Q. Assume a relation R (A, B, C, D ) that has the following functional dependencies:

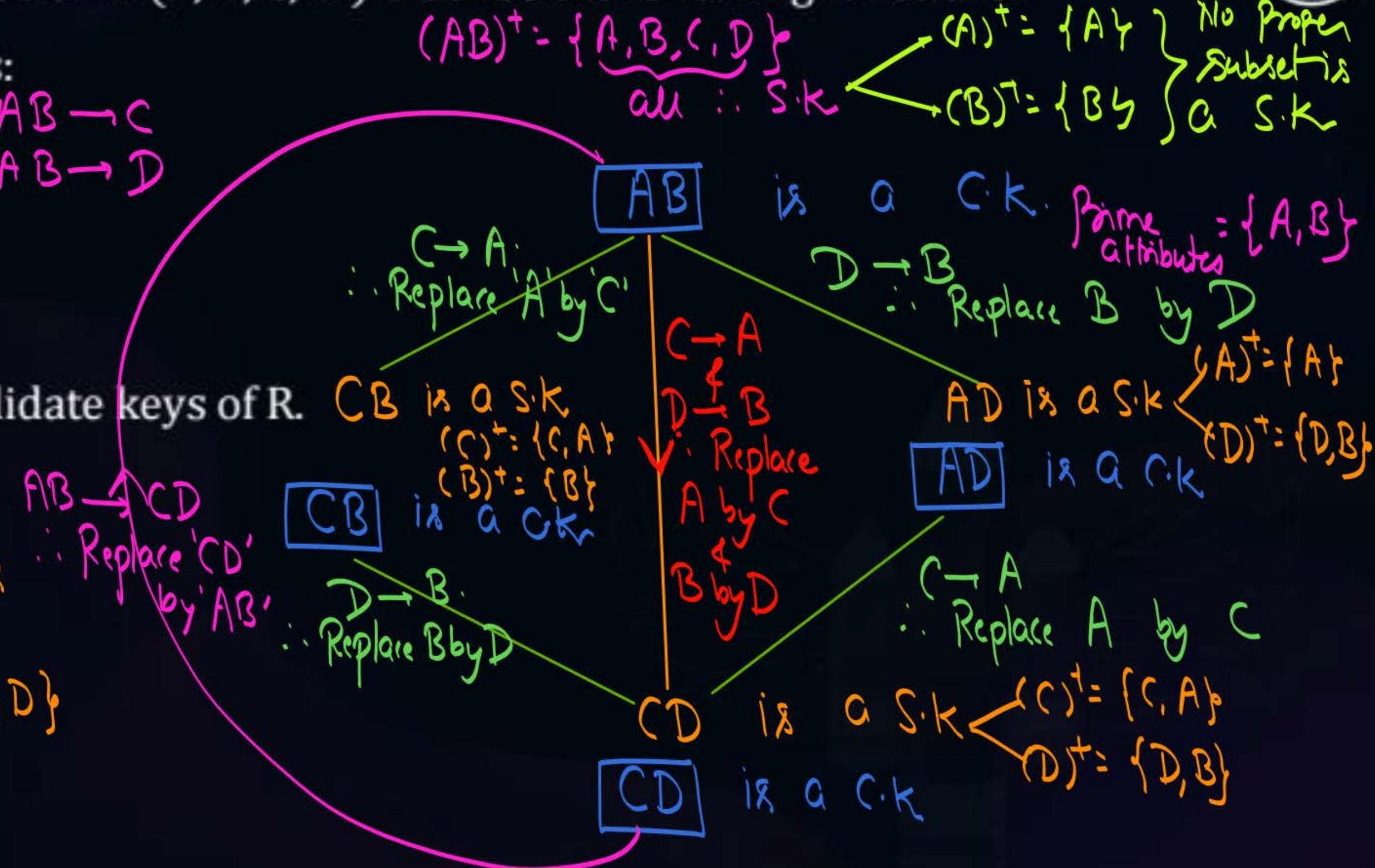
$$\begin{array}{c} AB \rightarrow CD \Rightarrow AB \rightarrow C \\ AB \rightarrow D \\ C \rightarrow A \\ D \rightarrow B \end{array}$$

Find the Candidate keys of R

AB, CB, AD, CD  
are the candidate keys  
of the relation

Prime Attribute = {A, B, C, D}

Non-prime = { }



#Q. Assume a relation R (A, B, C, D, E, H) that has the following functional dependencies:

$$A \rightarrow B$$

$$BC \rightarrow D$$

$$E \rightarrow C$$

$$D \rightarrow A$$

Find the Candidate keys of R.

AEH, DEH & BEH are the Candidate keys.

$\therefore$  Prime Attribute = {A, E, H, D, B}

Non-prime attribute = {C}

$A \rightarrow B$   
 $\therefore$  Replace B by A

$$(AEH)^T = \{A, E, H, B, C, D\}$$

Essential attributes = E & H

$$(EH)^T = \{E, H, C\}$$

AEH

DEH

DEH

BCEH

BEH

all attributes  $\therefore$  S.K  
 as well as a C.K

is a C.K

$D \rightarrow A$   
 $\therefore$  Replace A by D

DEH is a S.K as well as C.K

is a C.K

$BC \rightarrow D$

$\therefore$  Replace D by BC

BCEH is a S.K, but not a C.K

is a C.K

Prime Attributes = {A, E, H, D, B}

Prime Attributes = {A, E, H}

Prime attributes = {A, E, H, D}

if E & H  
 are present  
 then C  
 can be  
 removed



## Topic : Membership test

- \* ➤ Membership test is used to check whether a given FD is a member of given FD set or not.

➤ To check whether  $X \rightarrow Y$  is a member of FD set F or not  
(i.e.,  $F \models X \rightarrow Y$  or not)

*infers/yields*

We first obtain  $X^+$  (closure of X) w.r.t. FD set F.

If  $Y \in X^+$ , then  $X \rightarrow Y$  is a member of FD set F  
otherwise not a member of FD set F

- \* If  $x \rightarrow y$  is a member of FD set F,  
then we can say that  
 $x \rightarrow y$  can be inferred from F

#Q. Let FD set  $F = \{ A \rightarrow B, B \rightarrow C \}$

Check whether  $A \rightarrow C$  is a member of  $F$  or not?

$$\begin{aligned} & (A)^+ = \{ A, B, C \} \\ & C \in (A)^+ \therefore "A \rightarrow C" \text{ is a member of } F \end{aligned}$$

#Q. Let FD set  $F = \{ A \rightarrow B, B \rightarrow C \}$

Check whether  $B \rightarrow A$  is a member of  $F$  or not?

$$\begin{array}{c} \downarrow \\ (B)^+ \text{ w.r.t. } F = \{ B, C \} \\ A \notin (B)^+ \text{ w.r.t. } F \\ \therefore B \rightarrow A \text{ is not a member of } F \end{array}$$

#Q. Let FD set  $F = \{ AB \rightarrow C, BC \rightarrow D \}$

Check whether  $AB \rightarrow D$  is a member of  $F$  or not?



$(AB)^F$  w.r.t.  $F = \{ A, B, C, D \}$

$D \in (AB)^F$  w.r.t.  $F$ .

∴  $AB \rightarrow D$  is a member of  $F$

#Q. Let FD set  $F = \{ AB \rightarrow C, C \rightarrow A\}$

Check whether  $C \rightarrow B$  is a member of  $F$  or not?

$$(C)^T \text{ wrt } F = \{ C, A \}$$

B \notin (C)^T \text{ wrt } F

∴  $C \rightarrow B$  is not a member of  $F$

Note:-

- \* Let  $F$  is a non-empty set of FDs  
The set of all functional dependencies  
that can be inferred from  $F$  can  
be denoted by  $F^+$   
 $\hookrightarrow$  closure of FD set  $F$

#Q. Let FD set  $F = \{ A \rightarrow B, B \rightarrow C \}$  over relation  $R(A, B, C)$

Find  $F^+$

$$\{A\}^+ = \{A, B, C\} = A \rightarrow BC$$

$$\{A, B\}^+ = \{A, B, C\}$$

$$\{A, B, C\}^+ = \{A, B, C\}$$

$$\{B\}^+ = \{B, C\}$$

$$\{A, C\}^+ = \{A, B, C\}$$

$$\{C\}^+ = \{C\}$$

$$\{B, C\}^+ = \{B, C\}$$

$$F^+ = \left\{ \begin{array}{l} A \rightarrow BC, \quad B \rightarrow C, \\ AB \rightarrow C, \quad AC \rightarrow B \end{array} \right\}$$

Let  $A$  &  $B$  are two sets then,

①.  $A = B$  iff  $A \subseteq B$  &  $B \subseteq A$

②. If  $A \subseteq B$  but  $B \not\subseteq A$ , then  $A \subset B$  &  $A \neq B$

③. If  $A \not\subseteq B$  but  $B \subseteq A$ , then  $B \subset A$  &  $A \neq B$

④. Neither  $A \subseteq B$  nor  $B \subseteq A$  {  $A$  &  $B$  are not comparable }  $\Leftrightarrow A \neq B$

## Relationship between two sets of FDs

Let  $F$  &  $G$  are two sets of functional dependencies:

- ① If all FDs of set  $F$  are member of FD set  $G$   
then  $F \subseteq G$ , { i.e., all FDs of  $F$  can be inferred from FD set  $G$  }  
(Or) we can say  $G$  covers  $F$
- ② If all FDs of set  $G$  are member of FD set  $F$   
then  $G \subseteq F$ , { i.e., all FDs of  $G$  can be inferred from FD set  $F$  }  
(Or) we can say  $F$  covers  $G$

Q: Consider two FD sets

$$F_1 = \{A \rightarrow B, B \rightarrow C, AB \rightarrow D\}$$

$$\text{&} F_2 = \{A \rightarrow B, B \rightarrow C, A \rightarrow C, A \rightarrow D\}$$

Check if  $F_1$  covers  $F_2$  or not  
i.e., check if  $F_2 \subseteq F_1$  or not

FDs of $F_2$	Closure w.r.t. $F_1$	Member or not	Relationship
$A \rightarrow B$	$(A)^+ \text{ w.r.t. } F_1 = \{A, B, C, D\}$	$B \in A^+ \checkmark$	All FDs of $F_1$ are members of $F_2$
$B \rightarrow C$	$(B)^+ \text{ w.r.t. } F_1 = \{B, C\}$	$C \in B^+ \checkmark$	
$A \rightarrow C$	$(A)^+ \text{ w.r.t. } F_1 = \{A, B, C, D\}$	$C \in A^+ \checkmark$	
$A \rightarrow D$	$(A)^+ \text{ w.r.t. } F_1 = \{A, B, C, D\}$	$D \in A^+ \checkmark$	

$\therefore F_2 \subseteq F_1$

Find the relationship b/w  $F_1 \& F_2$

Check if  $F_2$  covers  $F_1$  or not  
i.e., Check if  $F_1 \subseteq F_2$  or not

FDs of $F_1$	Closure w.r.t. $F_2$	Member or not	Relationship
$A \rightarrow B$	$(A)^+ \text{ w.r.t. } F_2 = \{A, B, C, D\}$	$B \in A^+ \checkmark$	All FDs of $F_1$ are members of $F_2$
$B \rightarrow C$	$(B)^+ \text{ w.r.t. } F_2 = \{B, C\}$	$C \in B^+ \checkmark$	
$AB \rightarrow D$	$(AB)^+ \text{ w.r.t. } F_2 = \{A, B, C, D\}$	$D \in (AB)^+ \checkmark$	

$F_1 \subseteq F_2 \& F_2 \subseteq F_1 \therefore F_1 = F_2$

Q: Consider two FD sets

$$F_1 = \{ A \rightarrow B, B \rightarrow C, AB \rightarrow D \}$$

$$\& F_2 = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C, A \rightarrow D \}$$

} Find the relationship b/w  
 $F_1 \& F_2$

Q: Consider two FD sets

$$F_1 = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C \}$$

$$\& F_2 = \{ A \rightarrow B, B \rightarrow C, A \rightarrow D \}$$

} Find the relationship b/w  
 $F_1 \& F_2$

$$F_1 = \{ \cancel{A \rightarrow B}, \cancel{B \rightarrow C}, \cancel{A \rightarrow C} \}$$

$$(A)^+ \text{ wrt } F_2 = \{ \cancel{A}, \cancel{B}, \cancel{C}, D \}$$

$$\therefore F_1 \subseteq F_2$$

$$F_2 = \{ \cancel{A \rightarrow B}, \cancel{B \rightarrow C}, \cancel{A \rightarrow D} \}$$

$$(A)^+ \text{ wrt } F_1 = \{ A, B, C \}$$

D.F.A<sup>+</sup>  
∴ A → D is not inferred by  $F_1$

$$\therefore F_2 \not\subseteq F_1$$

$F_1 \subseteq F_2$  but  $F_2 \not\subseteq F_1$ , ∴  $F_1 \neq F_2$  but  $F_1 \subset F_2$

Q: Consider two FD sets

$$F_1 = \{ A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E \}$$

$$\& F_2 = \{ A \rightarrow BC, D \rightarrow AE \}$$

Find the relationship b/w  
 $F_1 \& F_2$

$$F_1 = \left\{ \begin{array}{l} A \rightarrow B \\ AB \rightarrow C \\ D \rightarrow AC \\ D \rightarrow E \end{array} \right.$$

$$\therefore F_1 \subseteq F_2$$

$$F_2 = \left\{ \begin{array}{l} A \rightarrow BC \\ D \rightarrow AE \end{array} \right.$$

$$\therefore F_2 \subseteq F_1$$

Hence  $F_1 = F_2$

## FD set of a Sub-relation

- Let  $R$  be the relation with FD set  $F$ , and  $R_1$  is any sub-relation of  $R$ .  
Concept of membership test can be used to identify the FDs of sub-relation
- Let  $R(A, B, C, D, E)$  is a relation, then  $R_1(A, B, E)$  can be called a subrelation of relation  $R$

Q: Let  $R(A, B, C, D, E)$  is a relation with FD set F.

$$F = \{ A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E \}$$

And let  $R_L(A, B, E)$  is a sub-relation of relation  $R(A, B, C, D, E)$

Find the Candidate keys of sub-relation  $R_L(A, B, E)$

Soln. To find the candidate keys of any relation  
we need the set of functional dependencies  
w.r.t. that relation

- a. First we need to identify the FDs  
that exists in sub-relation  $R_L(A, B, E)$

Q: Let  $R(A,B,C,D,E)$  is a relation with FD set  $F$ .

$$F = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$$

and let  $R_1(A, B, E)$  is a sub-relation of relation  $R(A, B, C, D, E)$

We need to find FD set  $F_1$  w.r.t. sub-relation  $R_1$   
i.e., we need identify the relationship b/w  $A, B \nmid E$

$$(A)^+ \text{ wrt } F = \{A, B, C\}$$

$$\text{i.e. } A \rightarrow \cancel{A} \quad B \cancel{\rightarrow} \quad C$$

trivial

$$\therefore \boxed{A \rightarrow B}$$

Not in relation  $R_1$

$$(B)^+ \text{ wrt } F = \{\cancel{B}\} \quad \therefore \text{No useful FD}$$

$$(E)^+ \text{ wrt } F = \{\cancel{E}\} \quad \therefore \text{No useful FD}$$

$$(AB)^+ \text{ wrt } F = \{\cancel{A}, \cancel{B}, C\} \quad \therefore \text{No useful FD}$$

$$(AE)^+ \text{ wrt } F = \{\cancel{A}, \cancel{E}, B, C\} \quad \therefore \boxed{AE \rightarrow B}$$

$$(BE)^+ \text{ wrt } F = \{\cancel{B}, \cancel{E}\} \quad \therefore \text{No useful FD}$$

Hence FD set  $F_1$  w.r.t  
sub relation  $R_1(A, B, E)$  is

$$F_1 = \{A \rightarrow B\}$$

so Ck of  $R_1$  is  $(AE)$

H.W.

Q: Let  $R(A, B, C, D, E, F)$  is a relation with FD set  $F$ .  
 $F = \{AB \rightarrow C, B \rightarrow D, BC \rightarrow A, D \rightarrow EF\}$

And let  $R_1(A, B, C, D)$  is a sub-relation of relation  $R(A, B, C, D, E, F)$   
Find the Candidate keys of sub-relation  $R_1(A, B, C, D)$



## 2 mins Summary



- Topic** Identification of candidate keys in a relation
- Topic** Membership test

# THANK - YOU