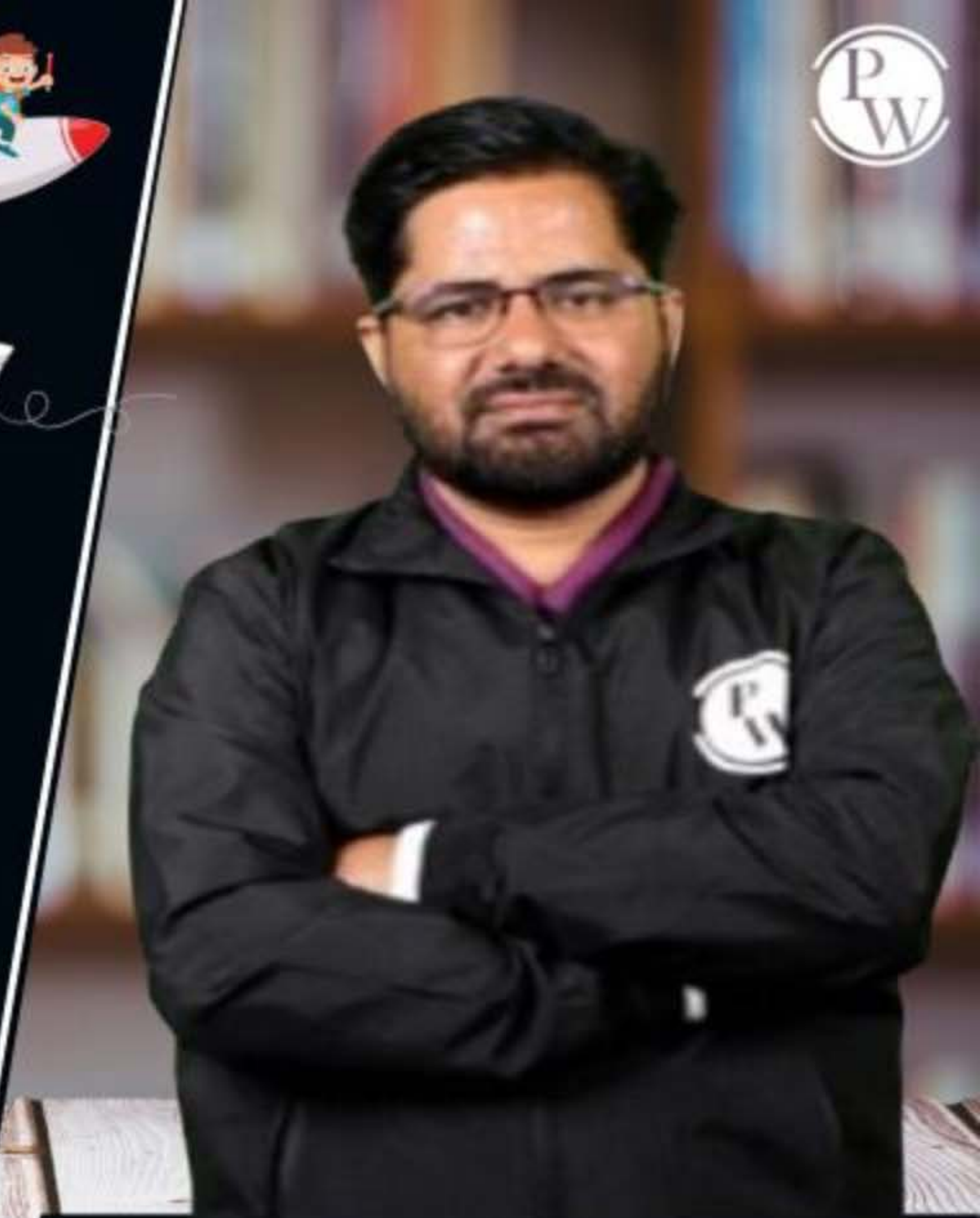
DS & AI
CS & IT

Probability & Statistics

Lecture No. 05



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Recap of previous lecture

Topic PERMUTATION - COMBINATION

Topics to be Covered

Topic PROBABILITY (Part-1)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

(1) The No. of factors of $5^2 6^3 7^4$ are ?

Sol: $5^2 6^3 7^4 = 5^2 \times 2^3 \times 3^3 \times 7^4$

$= 5^{0,1,2} \times 2^{0,1,2,3} \times 3^{0,1,2,3} \times 7^{0,1,2,3,4}$

Total factors $= 3 \times 4 \times 4 \times 5 = 240$ factors.

(2) In how many ways one or more selections can be made from

5, 5, 6, 6, 6, 7, 7, 7, 7

Total No. of selections or Rejections $= (2+1)(3+1)(4+1) = 3 \times 4 \times 5 = 60$ ways.

Hence No. of ways in which we can make one or more selections $= 60 - 1 = 59$ ways.

(3) How many 9 digit Numbers can be made using the digits

5, 5, 6, 6, 6, 7, 7, 7, 7

Total 9 digit Nos $= \dfrac{9!}{2!\,3!\,4!}$

(4) How many <u>4 digit</u> Numbers (> 6000) can be made using 5, 5, 6, 6, 6, 7, 7, 7, 7

Sol: Total 4 digit Nos (> 6000) $= \underset{P_1}{\underline{2 \text{ way}}} \times \underset{P_2}{\underline{3 \text{ way}}} \times \underset{P_3}{\underline{3 \text{ way}}} \times \underset{P_4}{\underline{3 \text{ way}}} = 54$ Nos.

(6 or 7) (5 or 6 or 7) (5 or 6 or 7) (5 or 6 or 7) (WILD GUESS)

But these 54 Nos include some wrong Nos which are as follows;

6 5 5 5 ✗, 6 6 6 6 ✗, 7 5 5 5 ✗

So Req Ans $= 54 - 3 = 51$ Nos

Q. How many 4 digit Nos (> 3000) can be made using the digits
2, 2, 3, 3, 3, 4, 4, 4, 4

(a) $\frac{9!}{2!\,3!\,4!}$

(b) 54

(c) 51

(d) 60

Total digit No (> 3000) = ?

$= \underset{\substack{P_1 \\ (3 \text{ or } 4)}}{\underline{2 \text{ way}}} \times \underset{\substack{P_2 \\ (2 \text{ or } 3 \text{ or } 4)}}{\underline{3 \text{ way}}} \times \underset{\substack{P_3 \\ (2 \text{ or } 3 \text{ or } 4)}}{\underline{3 \text{ way}}} \times \underset{\substack{P_4 \\ (2 \text{ or } 3 \text{ or } 4)}}{\underline{3 \text{ way}}} = 54$ Nos

4 it includes following wrong NOS

3 2 2 2, 3 3 3 3, 4 2 2 2

✗ ✗ ✗

So Req Ans = 54 − 3 = 51

PROBABILITY (CHANCE)

Prob → Base is of 1 unit
% → " " " 100 units.
Proportion → " " " 1 unit

In M & W sol^n, $m : w = 3 : 4$ then

$$\text{Prop of } M = \frac{3}{3+4} = \frac{3}{7} = \frac{3/7}{(1)} = p_1$$

$$\text{Prop of } w = \frac{4}{3+4} = \frac{4}{7} = \frac{4/7}{1} = p_2$$

then $\boxed{p_1 + p_2 = 1}$

Q In 70 ltrs of M & W sol^n, $\boxed{M : W = 3 : 4}$ then find exact Quantity of M & W.

Sol: Prop of $M = \frac{3}{7}$ so Q of $M = \frac{3}{7} \times 70 = 30$ ltrs

Prop of $W = \frac{4}{7}$ Q of $W = \frac{4}{7} \times 70 = 40$ ltrs.

eg: A Baby is going to take Birth in a family then $P(\text{Boy}) = \frac{5000\,Cr}{10000\,Cr} = \frac{1}{2}$

eg: In a family there are $\underbrace{3B \;\&\; 2G}$ & we are selecting a kid then $P(\text{Boy}) = \frac{3}{5} \simeq$ Proportion.

(2) Random Experiment → whenever we are not sure about the outcome of an Experiment then such types of Experiments are called R. Exp. for eg; Tossing a coin, throwing a die, selection of card from pack of cards etc.

(3) Sample space → If we write total possible outcomes of any Random Exp in set form then this set is called sample space.

(4) Event → Any subset of sample space is called an event.

if No. of elements in S. Space = N then

Total No of Events associated with S = Total No. of subsets = 2^N

eg $S_{Die} = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

let $E_1 = \{1, 3, 5\} = \{$odd No occurs$\}$,

$E_2 = \{2, 4, 6\} = \{$Even No " $\}$

$E_3 = \{1, 2, 3, 4\} = \{$No ≤ 4 occurs$\}$

$E_4 = \{3, 6\} = \{$No divisible by 3$\}$

⋮

& so on - - - - .

these are called Events associated with S.

& Total No. of events $= ? = 2^6 = 64$ Events

Note: $A = \{a, b, c\} \Rightarrow n(A) = 3$

Various subsets are ;

$\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}$

$\underbrace{\{a, b, c\}}_{\downarrow \text{ Sure Event}}, \underbrace{\phi}_{\downarrow \text{ Impossible Event.}}$

Total subsets of $A = ? = 2^3 = 8$

Hence Total Events $= ? = 8$

Impossible Event → $\because \phi \subset S$ so ϕ is also an event & it is called Impossible Event

& $\boxed{P(\phi) = 0}$

Sure Event / Certain Event → $\because S \subseteq S$ so S is also an Event

& it is called Sure Event i.e. $\boxed{P(S) = 1}$

Note: (1) $0 \le P(E) \le 1$, (2) $P(\text{Nothing occurs}) = 0$

(3) $P(\text{something occurs}) = 1$ (4) $P(\text{given statement}) = 1$

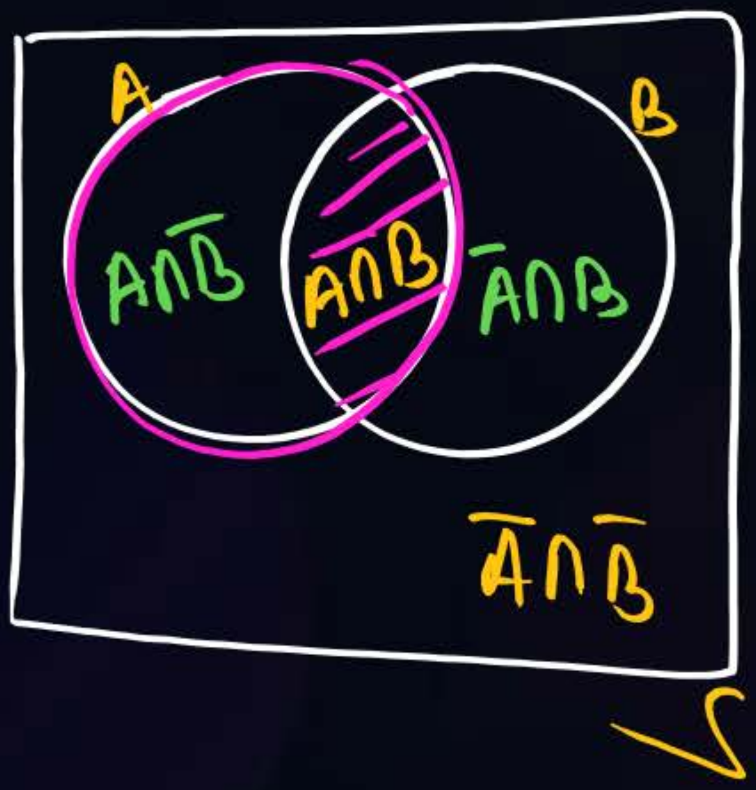
(5) $P(\text{Death}) = 1$, (6) $P(\text{GOD}) = 1$

Some Special Discussion →

(1) P(either A or B or Both) = P(at least one of A or B) = $P(A\cup B)$

(2) P(Both A & B occurs) = P(simultaneous occurrence of A & B) = $P(A\cap B)$

(3) P(Neither A Nor B) = P(None of A & B) = $P(\bar{A}\cap\bar{B})$



(i) $\boxed{A\cup B = (A\cap\bar{B})\cup(A\cap B)\cup(\bar{A}\cap B)}$ Tough job.

(ii) only $A = A\cap\bar{B} = A - A\cap B$

(iii) only $B = \bar{A}\cap B = B - A\cap B$

(iv) $\bar{A}\cap\bar{B}$ = Neither A Nor B

(1) Addition Theorem of Prob. :→ $\boxed{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

(2) Multiplication Theorem of Prob → $\boxed{P(A\cap B)=P(A/B)\cdot P(B)}$

(3) P(Neither A Nor B) = 1 − P(Either A or B or Both)

$\boxed{P(\bar{A}\cap\bar{B})=1-P(A\cup B)}$

(4) P(either A or B or Both) = 1 − P(Neither A Nor B)

$\boxed{P(\text{at least one of } A \text{ or } B)=1-P(\text{None})}$

Mutually Exclusive Events →

If two events can't occur simultaneously then these are called M.E. Events

OR

If occurrence of one event prevents the occurrence of other event & vice versa than events are called ME Events . ie

If A & B are ME then only one can occur at a time

Mathematically: if E_1 & E_2 are ME events then $\boxed{E_1 \cap E_2 = \phi}$

& $P(E_1 \cap E_2) = 0$, $P(E_1 \cup E_2) = P(E_1) + P(E_2) - 0$

eg $S_D = \{1, 2, 3, 4, 5, 6\}$ & let us consider following events

$E_1 = \{1, 3, 5\}$ ∵ $E_1 \cap E_2 = \phi \Rightarrow E_1$ & E_2 are M.E & $P(E_1 \cap E_2) = 0$

$E_2 = \{2, 4, 6\}$ ∵ $E_1 \cap E_3 \neq \phi \Rightarrow E_1$ & E_2 are Not M.E

$E_3 = \{1, 2, 3, 4\}$ ∵ $E_2 \cap E_3 \neq \phi \Rightarrow E_2$ & E_3 are Not ME

$E_4 = \{2, 4\}$, ∵ $E_1 \cap E_4 = \phi \Rightarrow E_1$ & E_4 are also ME But $E_1 \cup E_4 \neq S$

i.e it is not Necessary that, in Case of ME Events, you will get their union as S. Space

$E_4 = \{x : 1 < x < 5$ & x is divisible by $2\}$

Independent Events → If occurrence or non occurrence of one event does not alter the occurrence or non occurrence of other Event then Events are called Independent events

$\Longleftrightarrow$

Mathematically: If A & B are Ind Events then $\Longleftrightarrow$ $\boxed{P(A\cap B) = P(A).P(B)}$

eg: $S_{\text{Coin}} = \{H, T\}$, $S_{\text{Die}} = \{1, 2, 3, 4, 5, 6\}$

$A = \{H\}$, $P(A) = \frac{1}{2}$, $B = \{1, 2, 3, 4\}$, $P(B) = \frac{4}{6}$

By observation, A & B are Independent & $P(A\cap B) = \frac{1}{2} \times \frac{4}{6} = \frac{1}{3}$

Note –
1. If we are taking two Events from different S.Sp then Quest of their M.E. Nature doesn't arise.
2. " " " " " " same " " " " Independency " "

PODCAST:-

① ME events are associated with SAME sample space
while Ind " " " " Different sample spaces

② In case of Ind events, we can observe that

$\boxed{A \cap B = \phi}$ then why not these are ME ??

∵ we are forcefully drawing a wrong conclusion.
as the concept of Intersection is also applicable with in the same S.Sp.
→ that's why it is WRONG conclusion.

③ Same Sample Space
- A & B may be ME
- " " may not be ME

(4) $\boxed{\text{Events formed by individual elements of S. Space are ME}}$ (T)

eg $S_D = \{1, 2, 3, 4, 5, 6\}$, Total Events $= 2^6 = 64$

$E_1 = \{1\}$, $E_2 = \{2\}$, $E_3 = \{3\}$, $E_4 = \{4\}$, $E_5 = \{5\}$, $E_6 = \{6\}$

$\because E_i \cap E_j = \phi \; \forall \; i \,\&\, j \Rightarrow E_i \,\&\, E_j$ are ME.

eg $S_{coin} = \{H, T\}$, $E_1 = \{H\}$, $E_2 = \{T\}$

$\because E_1 \cap E_2 = \phi \Rightarrow E_1 \,\&\, E_2$ are ME

(5) If two Events E_1 & E_2 are associated with different S Space then Question of their ME Nature doesn't arise.

M. Imp

Nature of Elements in S. Space → If our R-Exp is Repeated n times then elements of S. Space are in the form of ordered n-tuple.

eg If Die is thrown once, then $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

eg if " " twice, then $S = \begin{Bmatrix} (11), (12), (13) \ldots (16) \\ (21), (22) \ldots (26) \\ (31), \ldots (66) \end{Bmatrix} \Rightarrow n(S) = \underset{D_1}{6} \times \underset{D_2}{6}$

$= 36$ pair.

eg if a coin is tossed 5 times then $S = ?$

$\{(HHHHH), (HHHHT), (HHHTT), (HHTTT), (HTTTT), (TTTTT)\} \Rightarrow n(S) = \underset{C_1}{2} \times \underset{C_2}{2} \times \underset{C_3}{2} \times \underset{C_4}{2} \times \underset{C_5}{2} = 2^5$

$= 32$ ordered 5 tuples

eg A couple has 3 kids, then $S = \{(BBB), (BBG), (BGB), (BGG), (GBB), (GBG), (GGB), (GGG)\}$

$n(S) = 2 \times 2 \times 2 = 2^3 = 8$ Triplets.

Note

(1) A coin is tossed thrice

&

3 coins are tossed simultaneously

$\Bigg\}$ In both cases S.Sp would be **SAME**

$S = \{(HHH), (HHT), \ldots, (TTT)\}$

$n(S) = \underset{C_1}{\underline{2}} \times \underset{C_2}{\underline{2}} \times \underset{C_3}{\underline{2}} = 8$

(2) **Favourable Event:-** Assume that event as fav. Event, "which is Required"

③ Methods of Solving Questions →

App I By writing all the elements of S.Space & Favourable event (E) in Set form, we can find $\boxed{P(E) = \frac{n(E)}{n(S)}}$

App II If it is not easy to write S.Space then directly find fav. number of cases and Total Number of cases by using the concept of P & C & then $\boxed{\text{Req Prob} = \frac{\text{fav Cases}}{\text{Total Cases}}}$

App III By using some standard Results & Standard Definitions.

whenever in a Question, given information is in the form of Probability then use App III.

Short RECAP

Operation	P&C	Prob	Formula	ME	Ind.
Either or	Add	union	Addition Th	$P(A\cup B)=P(A)+P(B)$	—
AND	Multiply	Intersection	Multi Th	$P(A\cap B)=0$	$P(A\cap B)=P(A)\cdot P(B)$

Addition Th: $\boxed{P(A\cup B)=P(A)+P(B)-P(A\cap B)}$

for Independency: $P(A\cup B)=P(A)+P(B)-P(A)\cdot P(B)$

for ME: $P(A\cup B)=P(A)+P(B)-0$

F.Q-1 A Die is thrown twice then write it's S.Space.

$$S = \begin{Bmatrix} (11)(12)(13)(14)(15)(16),(21)(22)\ldots(26) \\ (31)(32)\ldots\ldots(36),(41)\ldots\ldots(66) \end{Bmatrix} \Rightarrow n(S) = 6 \times 6 = 36 \text{ pair}$$

① Find the prob that sum of outcomes is 8?

App I

$$A = \{\text{Sum is } 8\} = \{(62),(26),(53),(35),(44)\} \Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

② Find the prob that sum of outcomes is 9?

App I

$$B = \{\text{Sum is } 9\} = \{(63)(36),(54),(45)\} \Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{4}{36}$$

③ Find the prob that $\underbrace{\text{sum is Both 8 \& 9}}_{A \cap B}$? $\Rightarrow P(A \cap B) = P(\phi) = 0$

App III

$\because A \cap B = \phi$ ie A & B are M.E.

(4) Find the Prob that Sum is either 8 or 9 ?

App III

$\because A \cap B = \phi \Rightarrow A$ & B are ME,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{5}{36} + \frac{4}{36} - 0 = \frac{1}{4}$$

(5) Find the Prob that Sum is Neither 8 Nor 9 ? $= 1 - P(\text{either } 8 \text{ or } 9)$

App III Gate

$$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - \frac{1}{4} = \frac{3}{4}$$

(6) Find the Prob that both the outcomes are identical ?

App I Gate

$$C = \{(11), (22), (33), (44), (55), (66)\} \Rightarrow P(C) = \frac{6}{36} = \frac{1}{6}$$

(7) Gate Find the Prob that Product of the outcomes will be a perfect square ?

App I

(a) 0 (b) $\frac{1}{6}$ (c) $\frac{2}{9}$ (d) 1

$$D = \{(11), (22), (33), (44), (55), (66), (14), (41)\} \Rightarrow P(D) = \frac{8}{36} = \frac{2}{9}$$

(8) P (Sum of outcomes is divisible by 4) = ? = P (Sum = 4 or 8 or 12)

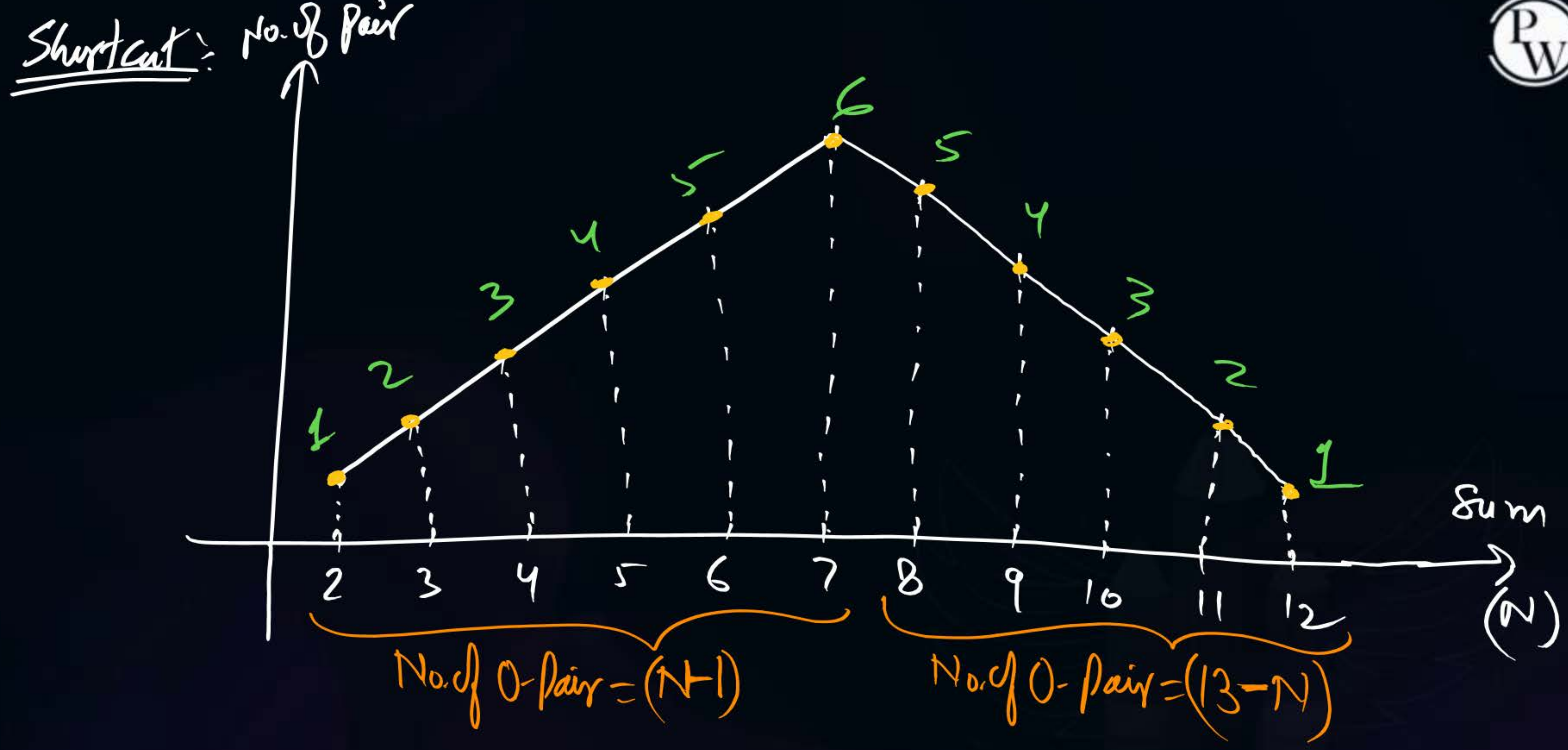
$$= \frac{3+(13-8)+(13-12)}{36} = \frac{3+5+1}{36} = \frac{9}{36} = \frac{1}{4}$$

(9) P (Sum of outcomes is prime number) = ?

$$= P(\text{Sum} = 2 \text{ or } 3 \text{ or } 5 \text{ or } 7 \text{ or } 11) = \frac{1+2+4+6+(13-11)}{36} = \frac{15}{36}$$

(10) P (Sum exceeds 9) = ? = P (Sum = 10 or 11 or 12)

$$= \frac{(3)+(2)+1}{36} = \frac{6}{36}$$



Shortcut: No. of Pair
1
2
3
4
5
6
5
4
3
2
1
Sum
2
3
4
5
6
7
8
9
10
11
12
(N)
No. of O-Pair = (N-1)
No. of O-Pair = (13-N)

Q. Four Dice are thrown simultaneously then find the Prob that sum of the outcomes is 22 ?

Sol: AppII $S = \{(1111), (1112), \ldots (1116)$
$(2111), (2112), \ldots (2116)$
$\ldots \ldots (6666)\}$ $\Rightarrow n(S) = \underset{D_1}{\underline{6}} \times \underset{D_2}{\underline{6}} \times \underset{D_3}{\underline{6}} \times \underset{D_4}{\underline{6}} = 6^4$

$= 1296$ Quadruples.

AppI fav outcomes $= \{\text{sum is } 22\} = \{(6664), (6646), (6466), (4666), (6655), (6565), (6556), (5566), (5656), (5665)\} = 10.$

AppII: fav cases $= \{\text{sum is } 22\} = \{(6664) \ldots, (6655) \ldots\} = \frac{4!}{3!} = 4$, $\frac{4!}{2!2!} = 6$

So $P = \frac{10}{1296}$

Q: 7 Surgical Strikes occurred in a week from INDIA on PAKISTAN then find the prob that all will occur on a same day ?

Sol: App I :→ Not easy to write S.Space

App II : Total ways of occurring S.Strikes (R.A) $= \underset{S_1}{\underline{7}} \times \underset{S_2}{\underline{7}} \times \underset{S_3}{\underline{7}} \times \underset{S_4}{\underline{7}} \times \underset{S_5}{\underline{7}} \times \underset{S_6}{\underline{7}} \times \underset{S_7}{\underline{7}}$

7 Days & 7 SS.

$= 7^7$ ways.

fav ways of occurring S Strikes = All will occur Either on M or T or W or Th or F or Sat or Sunday.

$= 7$ ways Hence Req Prob $= \frac{f}{T} = \frac{7}{7^7} = \frac{1}{7^6}$.

Note: Here it is obvious that S-Strikes are different.

Analysis:

$$\begin{array}{lccccccc} & \underline{S_1} & \underline{S_2} & \underline{S_3} & \underline{S_4} & \underline{S_5} & \underline{S_6} & \underline{S_7} \end{array}$$

(1)

$$\left.\begin{array}{lccccccc} & (M & M & M & M & M & M & M) \\ \text{or} & (T & T & T & T & T & T & T) \\ \text{or} & (W & W & W & W & W & W & W) \\ \text{or} & (Th & Th & Th & Th & Th & Th & Th) \\ \text{or} & (F & F & F & F & F & F & F) \\ \text{or} & (S & S & S & S & S & S & S) \\ \text{or} & (Su & Su & Su & Su & Su & Su & Su) \end{array}\right\} \text{7 choices} = \text{fav. choices.}$$

(2) Here all the 5 strikes are different.

A fair dice is tossed two times. The probability that the second toss results in a value that is higher than the first toss is . .

(a) 2/36 (b) 2/6

(c) 5/12 (d) 1/2

for upper diag elements $i<j$

$$A = \begin{bmatrix} (11) & (12) & (13) & (14) & (15) & (16) \\ (21) & (22) & (23) & \cdots & & (26) \\ (31) & (32) & (33) & \cdots & & (36) \\ \vdots & \vdots & & & & \vdots \\ (61) & (62) & \cdots & & & (66) \end{bmatrix}_{6\times 6}$$

Appr I $S = \{(11), (12) \ldots (66)\}$

$n(S) = 36$ pair

fav $(E) = \{(12)(13) \ldots (16), (23)(24) \ldots (26), \ldots\ldots ??\}$ = upper diag elements $= \frac{(36-6)}{2} = 15$ pair

Tough

So Req Prob $= \frac{15}{36} = \frac{5}{12}$

Thank you

Keep Hustling!