VIJAY

DATA SCIENCE & ARTIFICIAL INTELLIGENCE

& CS

Calculus and Optimization

Lecture No. 03

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Recap of previous lecture

Topic

FUNCTIONS & GRAPHS

Topics to be Covered

Topic

Limit_ Continuity & Differentiability

(PART-1)

RECAP

- Types of functions
  - ALGEBRAIC function
    1. Polynomial funcⁿ
    2. Rational funcⁿ
    3. Irrational funcⁿ
    4. Piecewise funcⁿ
       - Mod funcⁿ
       - Signum funcⁿ
       - G.I.F.
       - L.I.f
       - F.P.f
  - TRANSCEDENTAL function
    1. Exponential funcⁿ
    2. log function.
    3. Trigonometric funcⁿ
    4. Inverse Trig. functions

G.I.f = Greatest Integer funcⁿ (Floor funcⁿ)

LIf = Least Integer funcⁿ (Ceiling funcⁿ)

FPF = Fractional Part funcⁿ.

Function → If $\forall x \in A$ $\exists$ unique $y \in B$ s.t $f(x) = y$ then f is called funcⁿ from A to B & it is denoted as $f : A \to B$

$$f : \underset{\text{Domain}}{A} \longrightarrow \underset{\text{Codomain}}{B}$$

⊛ In $y = f(x)$

y → Dependent Variable (OUTPUT)

x → Independent Variable (INPUT)

Domain of $y=f(x) \Rightarrow$ Set of permissible values of x is called Domain

Range of $y=f(x) \Rightarrow$ Set of permissible values of y is called Range

i.e. Restrictions imposed on Inputs (x) is called Domain

& " " " outputs (y) " " Range.

Note: Vertical line Test → If any Random line $\parallel^r$ to y axis, cuts the graph only at one point, then it is a func^n.

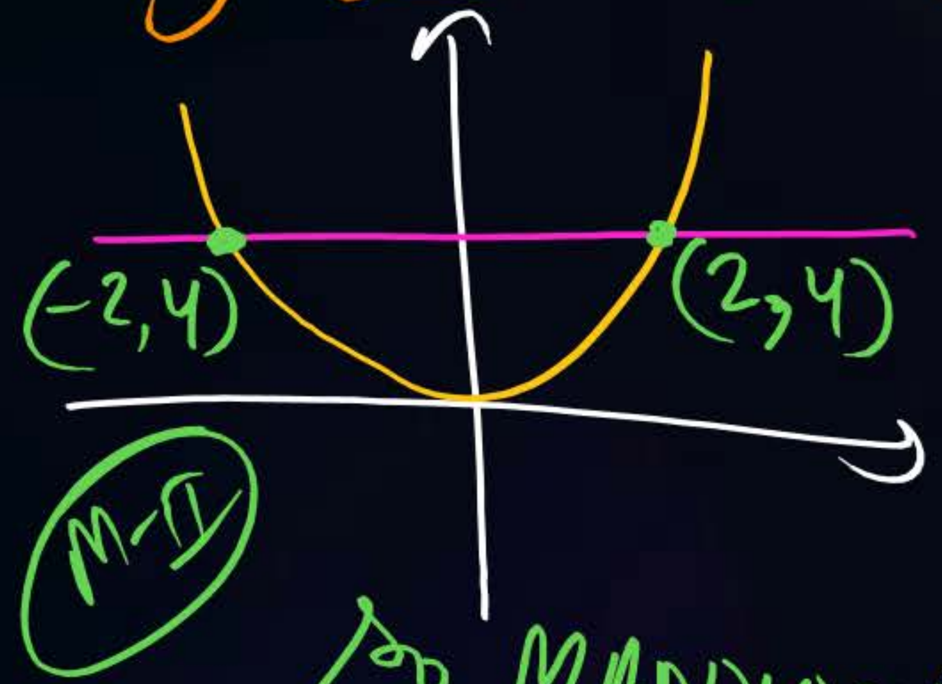
& if this line cuts the graph at more than one point, then it is not a func^n.

Horizontal Line Test (Shortcut of checking ONE-ONE OR MANY ONE).

If Any Random line $\parallel^r$ to x axis cuts the graph only at one point then it is one-one otherwise MANY-ONE

eg $y = x^2$ (M-I) : At $x = 2, y = 4$

& at $x = -2, y = 4$



$\Rightarrow$ MANYONE.

MANYONE

eg $y = x^3$ = ONE-ONE

Conclusions: ① Vertical line test → To check validity of func^n.

② Horizontal line test → To check validity of one-one

③ If $\boxed{\text{Range} = \text{Codomain}}$ → Then func^n is ONTO.

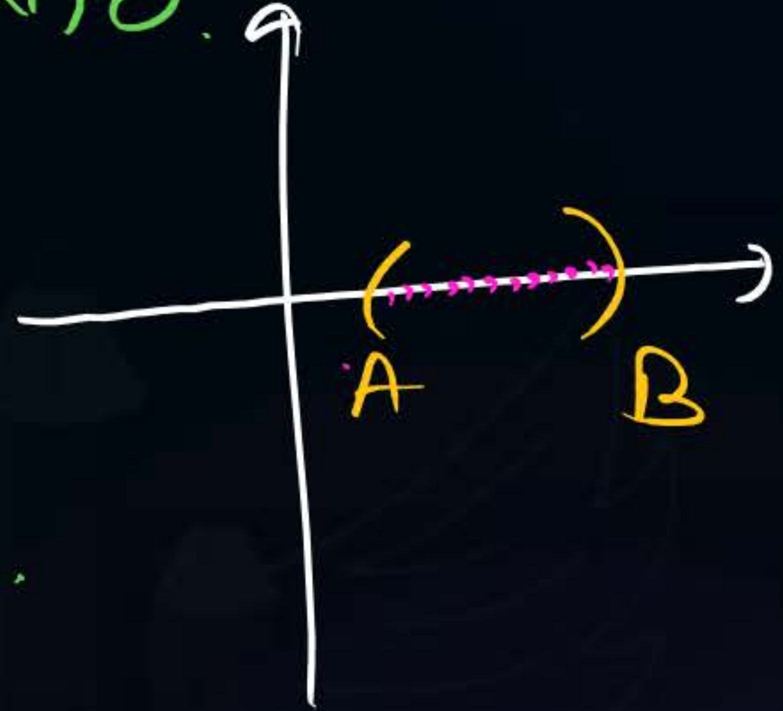
④ ONE-ONE func^n ⟹ INJECTIVE MAPPING

⑤ ONE-ONE/ONTO ⟹ BIJECTIVE MAPPING / one-one correspondence.

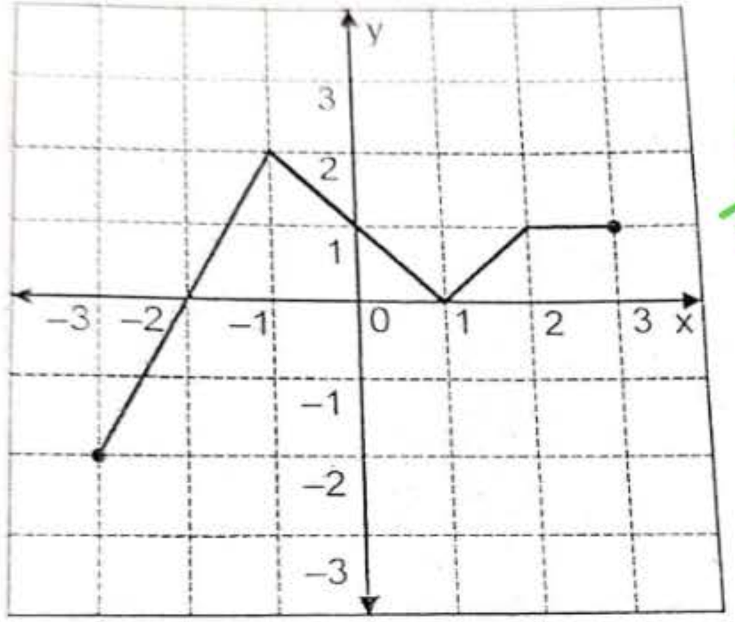
⑥ one one-one/ONTO func^n have Inverse.

i.e. if $f(x)$ is one one/ONTO then only $f^{-1}(x)$ exist

⟺



Which of the following function(s) is an accurate description of the graph for the range(s) indicated?



HW8.

(i) $y = 2x + 4$ for $-3 \le x \le -1$

(ii) $y = |x - 1|$ for $-1 \le x \le 2$

(iii) $y = ||x| - 1|$ for $-1 \le x \le 2$

(iv) $y = 1$ for $2 \le x \le 3$

(a) (i), (ii) and (iii) only

(b) (i), (ii) and (iv) only

(c) (i) and (iv) only

(d) (ii) and (iv) only

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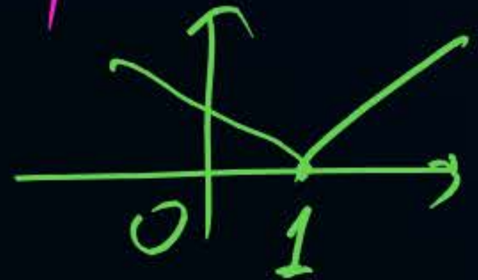
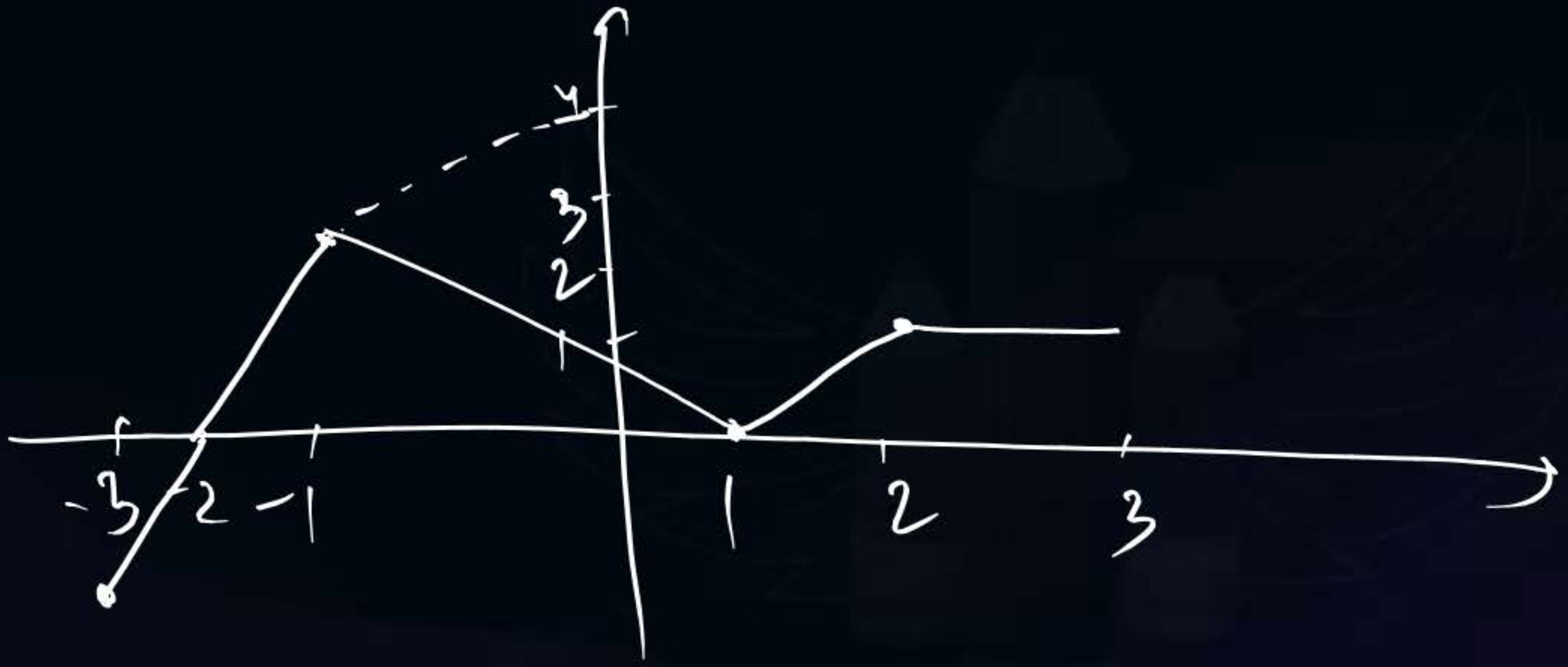
Case I: $y = 2x+4 \Rightarrow -2x + y = 4 \Rightarrow \frac{x}{-2} + \frac{y}{4} = 1$

$\boxed{-3 \le x \le -1}$, $y(-3) = -3.5$, $y(0) = 4$

Case II: $\boxed{-1 \le x \le 2}$, $y = |x-1|$



Case IV: $\boxed{2 \le y \le 3}$, $y = 1$



MS8

Let max {a, b} denote the maximum of two real numbers a and b. Which of the following statement(s) is/are TRUE about the function $\boxed{f(x) = \max\{3 - x,\ x - 1\}}$?

(a) It is continuous on its domain. ✓

(b) It has a local minimum at $x = 2$. ✓

(c) It has a local maximum at $x = 2$. ✗

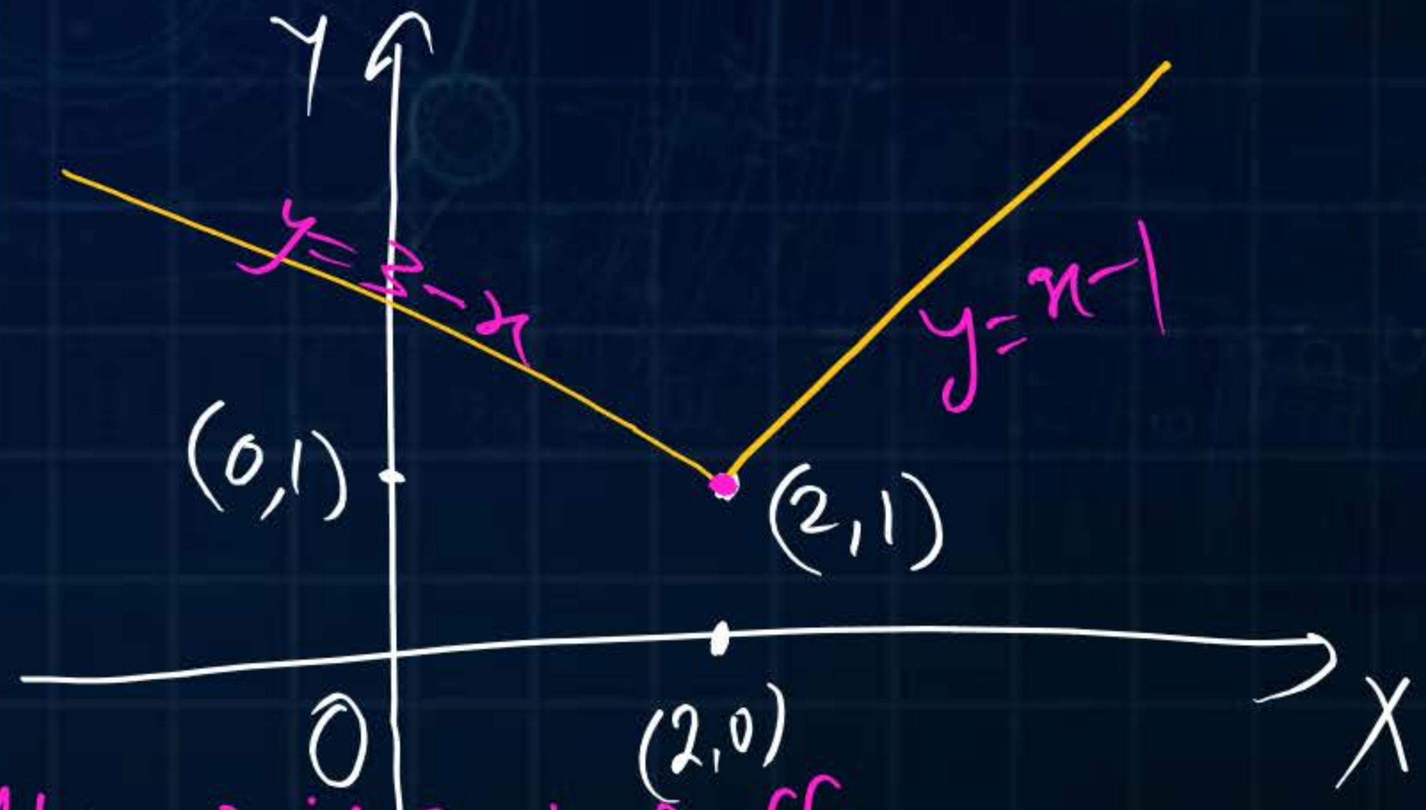
(d) It is differentiable on its domain. ✗

$y = 3 - x$ — (1) $\quad y = x - 1$ — (2)

Solving, $3 - x = x - 1$

$2x = 4 \Rightarrow x = 2,\ y = 1$

i.e. Intersecting Point $= (2, 1)$



At $x = 2$, it is Non Diff.

$f(x) = \max\{3 - x,\ x - 1\}$, $D_f = (-\infty, \infty)$

for eg at $x = 1$, $\max(2, 0) = 2$

at $x = 3$, $\max(0, 2) = 2$

$$y = f(x) = \begin{cases} 3 - x, & x < 2 \\ x - 1, & x > 2 \end{cases}$$

BASICS of limits & Continuity

Infinity → I don't know, Not unique,

$\boxed{\underset{\text{I}}{\infty} + \underset{\text{you}}{\infty} = \underset{\text{we}}{\infty}}$, $\frac{\text{Something}}{0} = \text{N.D}$, $\boxed{\frac{\text{Something}}{\infty} = 0}$

$\left.\begin{array}{l} \infty + \infty = \infty \\ \infty \times \infty = \infty \\ \infty^{\infty} = \infty \end{array}\right\}$ N.D, $\underbrace{\frac{0}{0}, \frac{\infty}{\infty}, 0\times\infty, \infty-\infty, 0^0, \infty^0, 1^{\infty}}_{\text{IND Forms}}$

eg $\frac{10}{2} = 5$ Similarly, $\frac{0}{0} = \underbrace{1, 2, -\frac{1}{3}, -4, \sqrt{5}, 7, \text{------} \ldots}_{\text{Multiple Ans exist.}}$

$$\frac{\infty}{\infty} = \frac{1/0}{1/0} = \frac{0}{0} = \text{IND Form.}$$

$$0 \times \infty = 0 \times \left(\frac{1}{0}\right) = \frac{0}{0} = \text{IND Form}$$

$\infty - \infty = ?$ (Have patience)

(it will be explained after 2 hrs)

WRONG App. :e

$$\cancel{\infty - \infty = \frac{1}{0} - \frac{1}{0} = \frac{0-0}{0} = \frac{0}{0} = \text{IND}}$$

if this app. is correct then why not

$$\cancel{\infty + \infty = \frac{1}{0} + \frac{1}{0} = \frac{0+0}{0} = \frac{0}{0}} \quad ???$$

Let $\boxed{0^0 = K}$

or $K = 0^0$

$$\log_e K = \log(0^0)$$
$$= 0 \times \log_e 0$$
$$= 0 \times (-\infty)$$
$$= -0 \times \left(\frac{1}{0}\right)$$
$$\log_e K = \frac{0}{0}$$
$$K = e^{\frac{0}{0}} = \text{IND Form}$$

(x) Let $\boxed{\infty^0 = K}$

or $K = \infty^0$

$\log_e K = \log(\infty^0)$

$= 0 \times \log \infty$

$= 0 \times (\infty)$

$= 0 \times \left(\frac{1}{0}\right)$

$\log_e K = \frac{0}{0}$

$K = e^{\frac{0}{0}} =$ IND Form

(x) $1^{20} = 1 \times 1 \times 1 \times \dots \times 1 = 1$

$1^{50} = 1 \times 1 \times 1 \times \dots \times 1 = 1$

$1^{\infty} \neq 1$

Let $1^{\infty} = K$

or $K = 1^{\infty}$

$\log_e K = \log(1^{\infty})$

$= \infty \times \log 1$

$= \infty \times 0$

$= \frac{1}{0} \times 0$

$\log_e K = \frac{0}{0}$

$K = e^{\frac{0}{0}} =$ IND Form

Check the Nature of 0^{∞}

Let $0^{\infty} = K$

or $K = 0^{\infty}$

$\log_e K = \log(0^{\infty})$

$= \infty \times \log(0)$

$= \infty \times (-\infty)$

$= -(\infty \times \infty)$

$\log_e K = -\infty$

$K = e^{-\infty} = 0$

ie $\boxed{0^{\infty} = 0}$ Hence Determinate

→ If we are getting ∞ or $-\infty$ or both, our conclusion is "it is N.D form"

→ If we are getting Multiple answers, " " is "it is IND Form"

eg $\left(\frac{0}{0}, \frac{\infty}{\infty}, 0\times\infty, \infty-\infty, 0^0, \infty^0, 1^\infty\right)$.

(*) Check the nature of ∞^∞?

Sol: let $K = \infty^\infty$

$\log_e K = \log(\infty^\infty) = \infty \times (\log \infty)$

$= \infty \times \infty$

$\log_e K = \infty$

$K = e^\infty = \infty$ i.e $\boxed{\infty^\infty = \infty}$

N.D.

(*) $\frac{\text{Something}}{\infty} = 0$, $\boxed{\frac{0}{\infty} = 0}$

$\because \frac{0}{\infty} = 0 \times \frac{1}{\infty} = 0 \times 0 = 0$

$\boxed{\infty \times \infty = \infty}$

let $K = \infty \times \infty$

$\log K = \log(\infty \times \infty) = \log(\infty)^2 = 2\log\infty$

$= 2 \times \infty = \infty$

$K = e^\infty = \infty$ i.e $\boxed{\infty \times \infty = \infty}$

Useful Concepts

Infinity :- Infinity is Not a very large Number. it is the presentation of that Concept which is beyond Imagination.

→ & Infinity is Not unique, it depends upon the Imagination capacity of an Individual.

→ $\boxed{\dfrac{\text{Something}}{\infty} \cong 0}$, $\boxed{\dfrac{\text{Something}}{0} = \text{N.D.}}$ (FACT)

Assumption

GOD GOD GOD

$\rightarrow \boxed{\infty + \infty = \infty} \approx$ N.D

I don't know | you don't know | we don't know

$\rightarrow \boxed{\infty \times \infty = \infty} =$ N.D

$\rightarrow \boxed{\infty^{\infty} = \infty} \approx$ N.D

Note: if final ans is ∞ or $-\infty$ then we say that it is Not Defined

But if final ans is $\frac{0}{0}$ then we say that it is IND form

$\rightarrow \infty - \infty = ? =$ IND form.

$\rightarrow \frac{\infty}{\infty} = ? =$ IND form.

$\rightarrow \infty^0 = ? =$ IND form.

$\rightarrow 1^{\infty} = ? =$ IND form.

$\rightarrow 0 \times \infty = ? =$ IND form.

$\rightarrow 0^0 = ? =$ IND form.

$\rightarrow \frac{0}{0} = ? =$ IND form.

PROPERTIES of LOG → let $a>1$

① $\boxed{\log_a x = b \Rightarrow x = a^b}$

eg $\log_4 64 = ? = 3 \Rightarrow 64 = 4^3$

eg $\log_5 625 = ? = 4 \Rightarrow 625 = 5^4$

eg $\log_a 1 = 0$, eg $\log_a 0 = -\infty$ where $a>1$

$\left(\because a^{-\infty} = \frac{1}{a^{\infty}} = \frac{1}{\infty} = 0\right)$

eg $\log_a \infty = \infty$ eg $\boxed{\log_a a = 1}$, $a \neq 1$ & $\log_1 1 = \text{IND}$

② $\boxed{\log_a x > \log_a y \Rightarrow x > y}$

Provided $a>1$ i.e when Base is greater than 1, $\log_a x$ is an Increasing func^n.

Domain $= (0, \infty)$

Range $= (-\infty, \infty)$

③ Product formula;

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

④ Quotient formula →

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

⑤ Power formula –

$$\log_a (x^y) = y \log_a x$$

⑥ Base change formula →

$$\log_y x = \frac{\log_a x}{\log_a y},$$

eg $\log_\infty x = \frac{\log_e x}{\log_e \infty} = \frac{\log_e x}{\infty} = 0$

eg $\log_1 1 = \frac{\log_e 1}{\log_e 1} = \frac{0}{0} = ? = \text{IND form}$

Note – $\log_1 x = \frac{\log_e x}{\log_e 1} = \frac{\log_e x}{0} = \text{N.D.}$ (i.e $\log_1 x$ is not defined)

(7) Reciprocal formula →

$$\log_y x = \frac{1}{\log_x y}$$

(8) $$\log_{a^b}(x) = \frac{1}{b}\log_a x$$

(9) $$a = e^{\log_e a}$$

(10) $$a^n = e^{\log_e(a^n)} = e^{n(\log_e a)}$$

(11) $$a^{\log_e b} = b^{\log_e a}$$

(8) Some Useful Results

(i) $a^3+b^3=(a+b)(a^2+b^2-ab)$

(ii) $a^3-b^3=(a-b)(a^2+b^2+ab)$

(iii) $(a+b)^3=a^3+b^3+3ab(a+b)$

(iv) $(a-b)^3=a^3-b^3-3ab(a-b)$

(V) $a^2-b^2=(a-b)(a+b)$

Some Usefull Results :→

① $\boxed{|x-a|<l \Rightarrow a-l < x < a+l}$

eg $|x|<l \Rightarrow -l < x < l$

② $|x-a|>l \Rightarrow x < a-l$ or $x > a+l$

eg $|x|>l \Rightarrow x < -l$ or $x > l$

③ if $a<b$ s.t $(x-a)(x-b)<0$

$\Rightarrow a < x < b$

④ if $a<b$ s.t $(x-a)(x-b)>0$

$\Rightarrow x < a$ or $x > b$

Proof ① $|x-a|<l$

$\pm(x-a)<l$

$-(x-a)<l$ & $+(x-a)<l$

$(x-a)>-l$ & $(x-a)<l$

$$\underbrace{x>a-l \quad \& \quad x<a+l}_{\boxed{a-l<x<a+l}}$$

INDETERMINATE form → There are exactly 7 Indeterminate forms for eg $\frac{0}{0}, \frac{\infty}{\infty}, 0\times\infty, \infty-\infty, 0^0, \infty^0, 1^\infty$

"If any Mathematical Expression has Multiple Answers then that expression is said to be in INDETERMINATE form"

L-Hospital Rule → this Rule provides unique answer out of ∞ answers & this Rule is applicable only for $\frac{0}{0}$ and $\frac{\infty}{\infty}$ form.

Rule: Separately Differentiate N^r & D^r, equal number of times, until we are free from $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form.

i.e. $\lim_{x\to a}\left\{\frac{f(x)}{g(x)}\right\} \simeq \frac{0}{0} = \lim_{x\to a}\frac{f'(x)}{g'(x)} \simeq \frac{0}{0} = \lim_{x\to a}\frac{f''(x)}{g''(x)} = \cdots\cdots = \text{unique ans}$

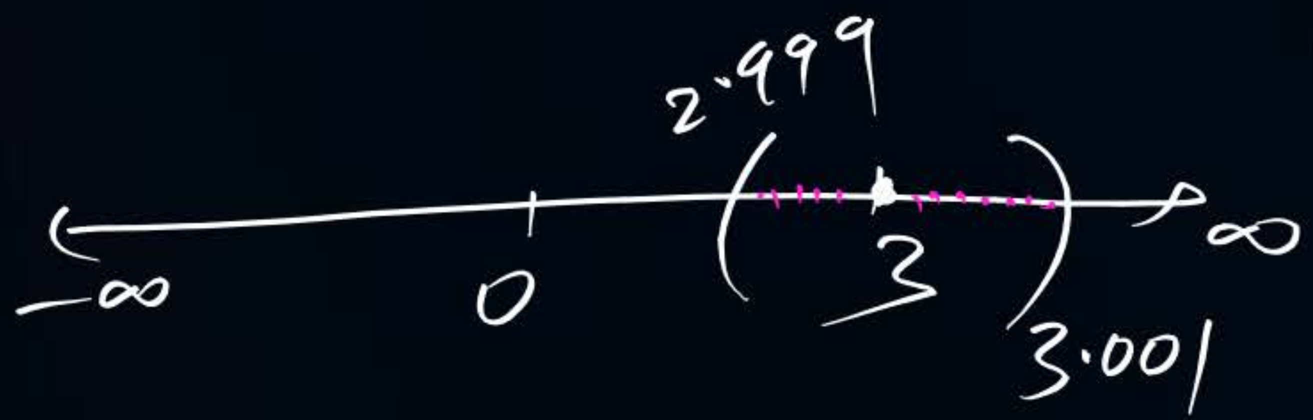
eg $\lim_{x\to\infty}\left(\frac{x^3}{e^x}\right) = ? = \frac{\infty}{\infty} = \lim_{x\to\infty}\left(\frac{3x^2}{e^x}\right) = \frac{\infty}{\infty} = \lim_{x\to\infty}\left(\frac{6x}{e^x}\right) = \frac{\infty}{\infty} = \lim_{x\to\infty}\left(\frac{6}{e^x}\right) = \frac{6}{e^\infty} = \frac{6}{\infty} = 0$

i.e. unique & constant. Hence exist

Note: limit exist means "There exist unique, constant value that must be free from x"

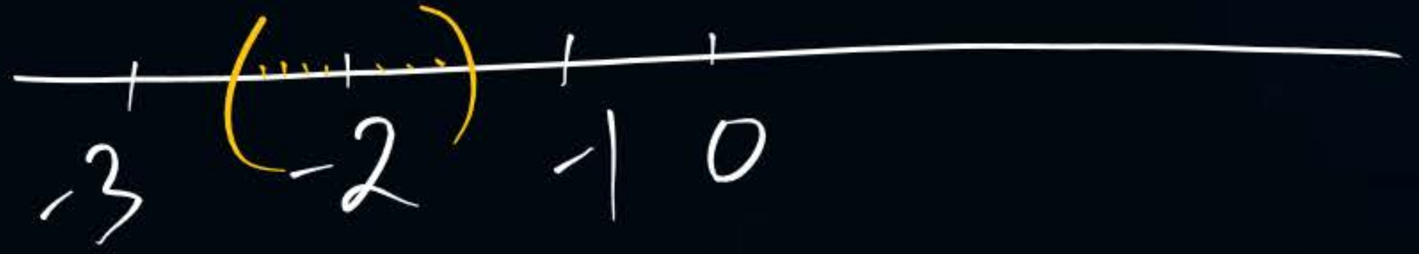
Neighbourhood of Real No. 'a' →

Nbd of $3 = (2.999, 3.001)$



Nbd of $0 = (-0.001, 0.001)$

Nbd of $-2 = (-2.001, -1.999)$



Neighbourhood of Real Number 'a' $\longrightarrow$ means we are considering an open interval of length $2h$ with centered at point 'a'

i.e Nbd of $a = (a-h, a+h)$ where $h > 0$ & h depends on our choice. & h is very small +ve Number.

Note if $\boxed{x \text{ lies in the Nbd of } a} \implies x \in (a-h, a+h)$

or

$a-h < x < a+h$

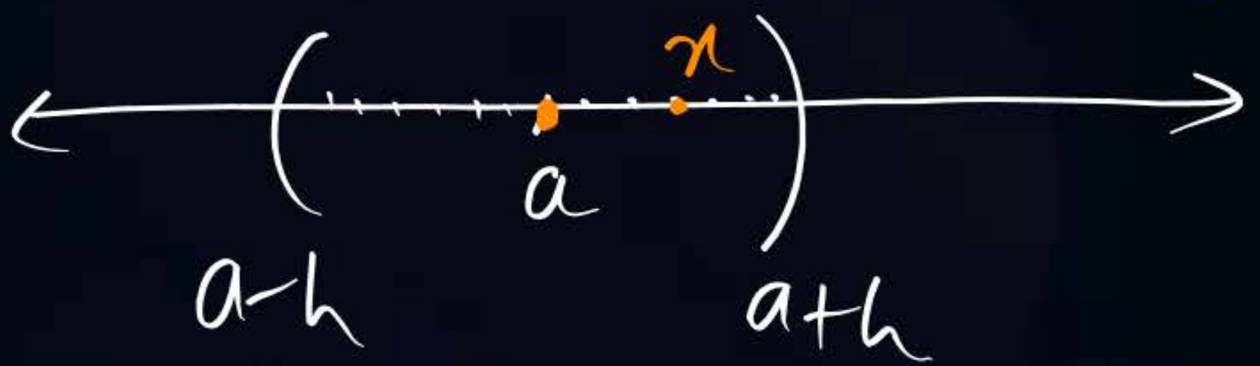
or

$|x-a| < h$

or

$x \to a$

or x is about a



Understanding of limit →

eg $f(x) = \dfrac{x^2-9}{x-3}$

$f(3) = \text{DNE}$ (i.e. exact Value at 3 DNE)

$\text{LHL} = 6$ (i.e. App. Value in the left Nbd of 3)

$\text{RHL} = 6$ (i.e. App. " " Right Nbd of 3)

$\lim\limits_{x\to 3} f(x) = 6$ (i.e. $\underbrace{\text{Approx Value in the Nbd of 3}}$)

$$f(x) = \frac{(x-3)(x+3)}{(x-3)}$$

$$f(2.999) = \frac{-0.\cancel{00}1 \times 5.999}{-0.\cancel{00}1} = 5.999 \approx 6$$

$$f(3.001) = \frac{0.\cancel{00}1 \times 6.001}{0.\cancel{00}1} = 6.001 \approx 6$$

(M-II) $\lim\limits_{x\to 3}\left(\dfrac{x^2-9}{x-3}\right) = \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\because x \to 3$

$\Rightarrow (x-3) \to 0$

i.e. $(x-3) \neq 0$

$= \lim\limits_{x\to 3} (x+3)$

$\approx 3+3 = 6$

eg $\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = ?$

$LHL = f(-0.001) = \frac{\sin(-0.001)}{-0.001} = ?$

$RHL = f(0.001) = \frac{\sin(0.001)}{0.001} = ?$

M-II $\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = \frac{0}{0}$ form $= \lim_{x\to 0}\left(\frac{\cos x}{1}\right) = \frac{\cos(0)}{1} = \frac{1}{1} = 1$ ✓

eg: $\lim_{x\to \infty}\left(\frac{\sin x}{x}\right) = ?$

$LHL = f(\infty^-) = ?$

$RHL = f(\infty^+) = ?$

M-II $\lim_{x\to \infty}\left(\frac{\sin x}{x}\right) = \frac{\sin \infty}{\infty} = \frac{\text{Any No. b/n } -1 \ \& \ 1}{\infty} = 0$ ie limit exist.

Continuity → if $\lim\limits_{x\to a} f(x)$ exist & is equal to $f(a)$ then $f(x)$ is cont at a

i.e if $\boxed{\lim\limits_{x\to a} f(x) = f(a)}$ $\Longleftrightarrow$ then $f(x)$ is cont at $x=a$

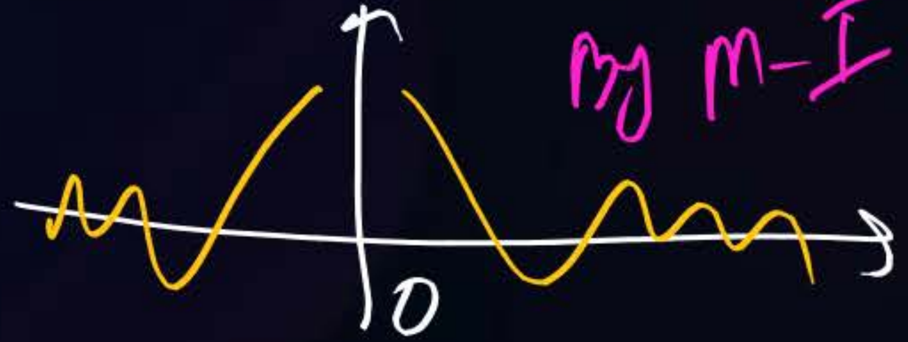
i.e for cont func^n, App Value = Exact Value

eg Check the continuity of the following func^n's; at $x=0$

① $f(x) = \dfrac{\sin x}{x}$, $D_f = R - \{0\}$

Dis cont at $x=0$ $\because$ Exact Value = DNE

By M-I



② $f(x) = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ $D_f = R$

Cont. at $x=0$

Def^n of limit $\rightarrow$ $\boxed{\lim_{x \to a} f(x) = l}$ $\Longrightarrow$ whenever x lies in the Nbd of 'a'

$f(x)$ lies in the Nbd of l

eg $\lim_{x \to 0} \left(\frac{\sin x}{x}\right) = 1$ $\Longrightarrow$ whenever x lies in the Nbd of '0'

$\left(\frac{\sin x}{x}\right)$ lies in the Nbd of 1

Continuity $f(x)$ is said to be continuous at $x = a$ if

$\boxed{\lim_{x \to a} f(x) = f(a)}$ $\Longrightarrow$ whenever x lies in the Nbd of a

$f(x)$ lies " " of $f(a)$

App Value = Exact Value

i.e $\boxed{(LHL = RHL) = f(a)}$

Sp. Note: To check Discontinuity of $f(x)$ at $x=a$ we have 3 methods;

(M-I) If Exact Value = DNE then $f(x)$ is Discontinuous

(M-II) if Approx Value = DNE then " " "

(M-III) if Both exist But not equal then $f(x)$ is Discont.

ie $\boxed{\lim_{x \to a} f(x) \neq f(a)} \Rightarrow f(x)$ is Discont at $x=a$

Method of Solving Questions →

(M-I) By Direct Substitution (Best Method).

(M-II) By factorisation

(M-III) By Rationatisation

(M-IV) By using I/D form Concept

(M-V) By using Standard Results

(M-VI) using Common sense.

Sp. formulaes :-

(1) $\sum_{N=1}^{N} N = 1+2+3+....+N$

$= \frac{N(N+1)}{2}$

(2) $\sum_{N=1}^{N} N^2 = 1^2+2^2+3^2+.....+N^2$

$= \frac{N(N+1)(2N+1)}{6}$

(3) $\sum_{N=1}^{N} N^3 = 1^3+2^3+3^3+....+N^3$

$= \left(\frac{N(N+1)}{2}\right)^2$

(4) $\sum_{N=1}^{N}(a) = \underbrace{a+a+a+....+a}_{N \text{ times}} = N.a$

Q1: $\lim_{x\to 2}\left(\frac{x^2-4}{x+3}\right) = ? = \frac{2^2-4}{2+3} = \frac{4-4}{5} = \frac{0}{5} = 0$ i.e unique so limit exist.

Q2: $\lim_{n\to\infty}\left(\frac{1+2+3+4+\cdots\cdots+n}{n^2}\right) = ? = \lim_{n\to\infty}\left[\frac{\frac{n(n+1)}{2}}{n^2}\right] = \lim_{n\to\infty}\frac{1}{2}\left(1+\frac{1}{n}\right)$

PYQ

$= \frac{1}{2}(1+0) = \frac{1}{2}$

Q3: $\lim_{n\to\infty}\left[\frac{1^2+2^2+3^2+\cdots\cdots+n^2}{n^3}\right]$

$= \lim_{n\to\infty}\left[\frac{\frac{n(n+1)(2n+1)}{6}}{n^3}\right] = \lim_{n\to\infty}\frac{1}{6}\left[\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = \frac{1}{6}(1+0)(2+0)$

$= \frac{2}{6} = \frac{1}{3}$

Q. $\lim_{n\to\infty}\left[\dfrac{1-2+3-4+5-6+\ldots\ldots+(2n-1)-2n}{\sqrt{n^2+1}+\sqrt{n^2-1}}\right]=?$

(a) $1/2$

(b) $-1/2$

(c) 1

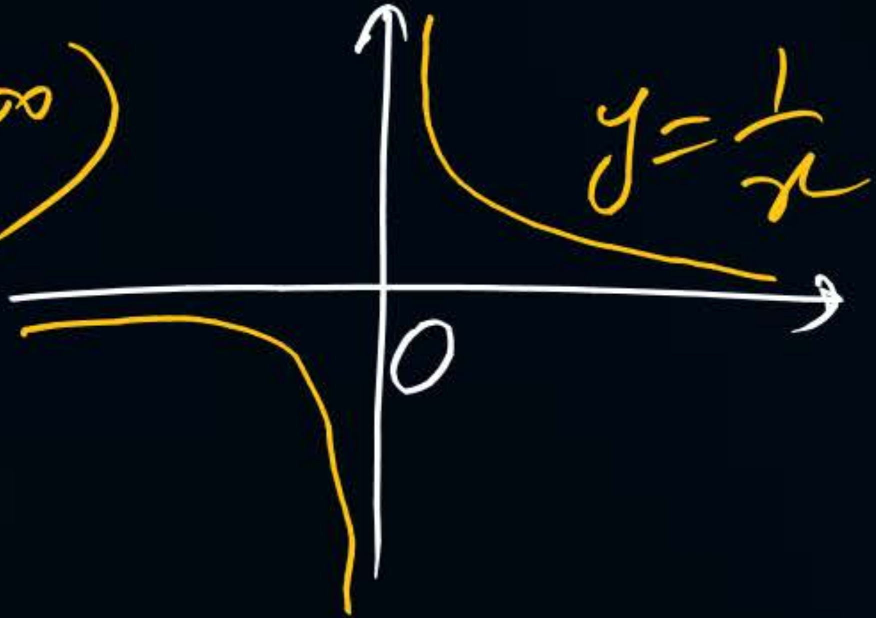
(d) D.N.E

$$=\lim_{n\to\infty}\left[\frac{(1-2)+(3-4)+(5-6)+\ldots\ldots+((2n-1)-2n)}{\sqrt{n^2+1}+\sqrt{n^2-1}}\right]$$

$$=\lim_{n\to\infty}\left[\frac{-1+(-1)+(-1)+\ldots\ldots+(-1)\ \text{n times}}{\sqrt{n^2+1}+\sqrt{n^2-1}}\right]$$

$$=\lim_{n\to\infty}\left[\frac{-n}{n\left[\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{1}{n^2}}\right]}\right]=\frac{-1}{\sqrt{1+0}+\sqrt{1-0}}=-\frac{1}{2}$$

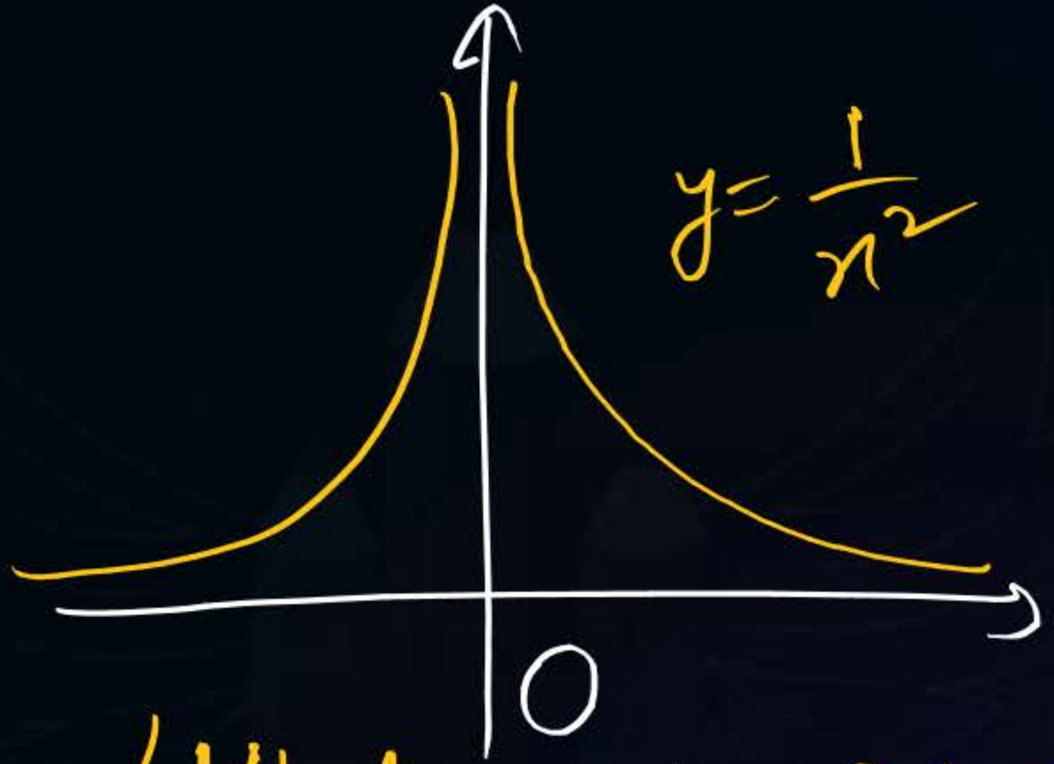
Q1 $\lim_{x\to 0}\left(\frac{1}{x}\right)=\frac{1}{0}=$ N.D ie limit DNE $\left(\because \begin{array}{l} LHL=-\infty \\ RHL=+\infty \end{array}\right)$



Q② $\lim_{x\to\infty}\left(\frac{1}{x}\right)=\frac{1}{\infty}=0$ ie unique so limit exist

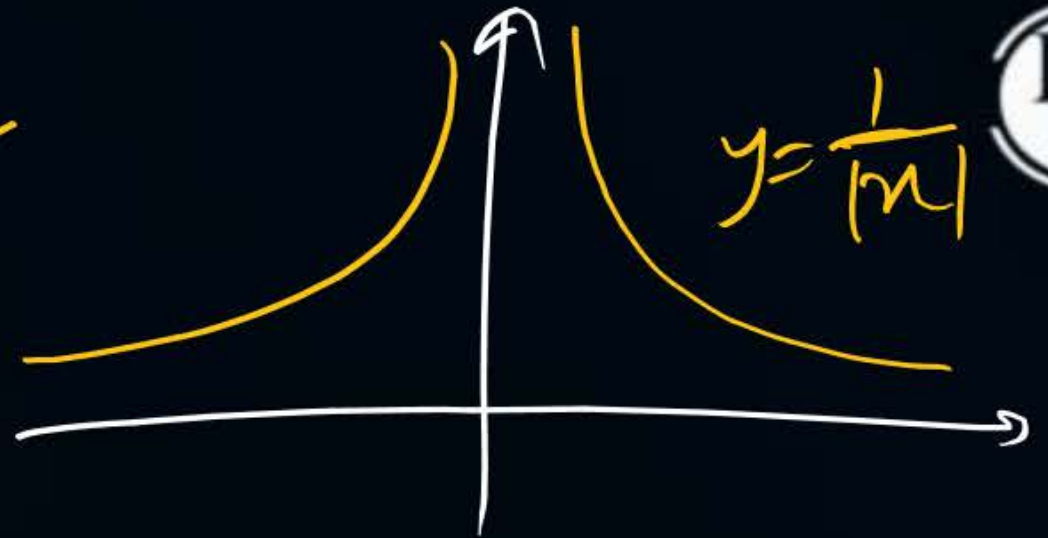
Q③ $\lim_{x\to 0}\left(\frac{1}{x^2}\right)=\frac{1}{0^2}=\frac{1}{0^+}=+\infty$ so limit DNE

Q④ $\lim_{x\to\infty}\left(\frac{1}{x^2}\right)=\frac{1}{\infty^2}=\frac{1}{\infty}=0$ ie unique so limit exist.



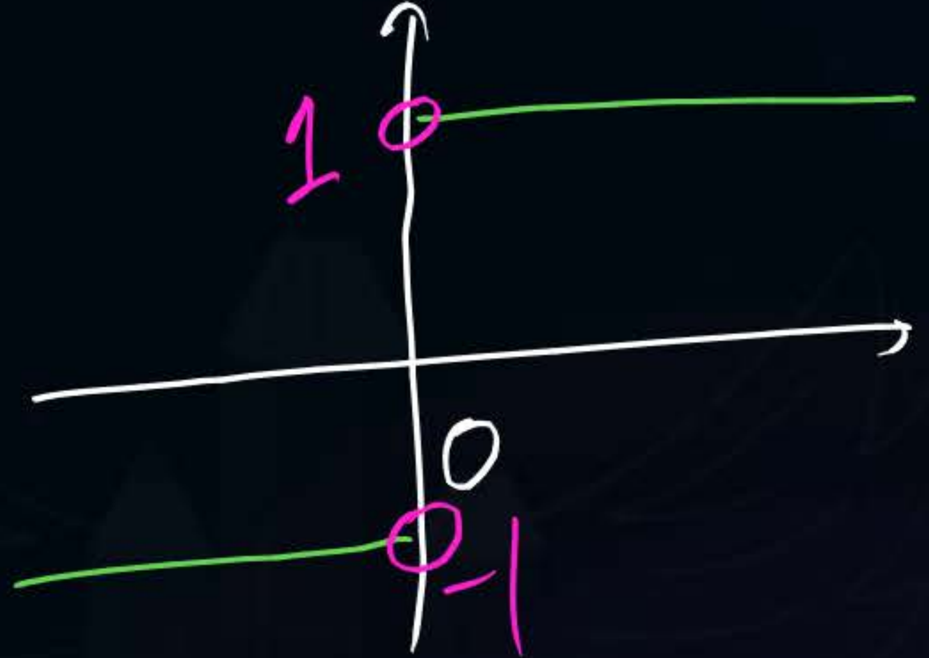
$\left.\begin{array}{l} \text{LHL at } 0=+\infty \\ \text{RHL at } =0=+\infty \end{array}\right\}$ limit DNE.

Q. $\lim_{x\to 0}\left(\frac{1}{|x|}\right) = \frac{1}{|0|} = \frac{1}{0^+} = +\infty$ i.e limit DNE



Q. $\lim_{x\to \infty}\left(\frac{1}{|x|}\right) = \frac{1}{|\infty|} = \frac{1}{\infty} = 0$ i.e limit exist

Q. $\lim_{x\to 0}\left(\frac{|x|}{x}\right) =$ LHL $= -1$, RHL $= 1$ ∵ LHL $\neq$ RHL so limit DNE.



Q. $\lim_{x\to \infty}\left(\frac{|x|}{x}\right) = +1$ is unique so limit exist.

Draw the Graph of following func^n:

① $f(x) = \frac{|x|}{x}$, $D_f = R - \{0\}$

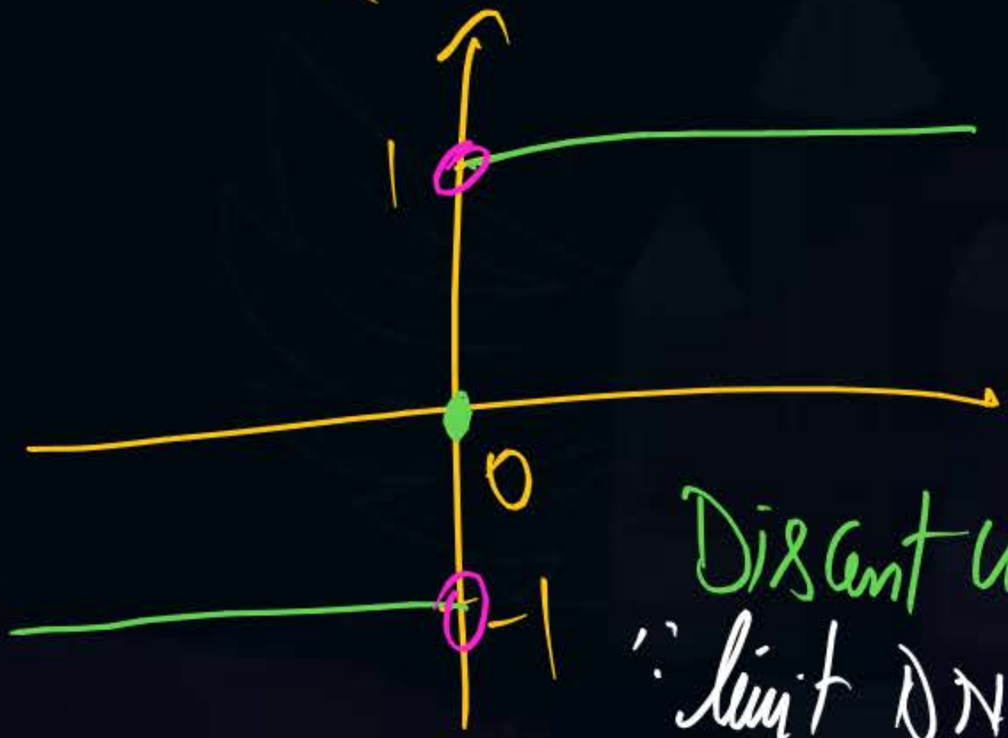
$$= \begin{cases} -1 & , x < 0 \\ 1 & , x > 0 \end{cases}$$

Discont using M-I

∵ exact value DNE

② $Sgn(x) = \begin{cases} \frac{|x|}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$

$$= \begin{cases} -1 & ; x < 0 \\ +1 & ; x > 0 \\ 0 & ; x = 0 \end{cases}$$



Discont using M-II

∵ limit DNE.

Type (2): FACTORISATION

Q. $\lim_{x\to 2}\left(\frac{x^2-5x+6}{x^2-4}\right) = ? = \frac{0}{0}$

(M-I) using L-Hospital's Rule ....

(M-II) $\lim_{x\to 2}\frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x+2)}$

$= \lim_{x\to 2}\left(\frac{x-3}{x+2}\right) = \frac{2-3}{2+2} = \frac{-1}{4}$

$x\to 2 \Rightarrow (x-2)\to 0$ i.e. $(x-2)\neq 0$

that's why we can cancel out $(x-2)$

Q. $\lim_{y\to 1}\left(\frac{y^3-1}{y-1}\right) = ? = \lim_{y\to 1}\frac{(y-1)(y^2+1+y)}{(y-1)}$

$= \lim_{y\to 1}(y^2+1+y) = 3$ Ans

Q. $\lim_{y\to 1}\left(\frac{y^{\frac{1}{4}}-1}{y^{\frac{1}{3}}-1}\right) = ? = \lim_{x\to 1}\left(\frac{x^3-1}{x^4-1}\right)$

Put $y = x^{12}$

when $y\to 1$, $x\to 1$

$= \lim_{x\to 1}\frac{(x-1)(x^2+1+x)}{(x^2-1)(x^2+1)}$

$= \lim_{x\to 1}\frac{x^2+1+x}{(x+1)(x^2+1)} = \frac{3}{4}$

Type 3 (Rationalisation) →

Q $\lim_{x \to \infty} \left(\sqrt{x^2+x+1} - \sqrt{x^2+1}\right) = ?$

Ans $= \frac{1}{2}$

Q. $\lim_{x \to \infty} \left[\sqrt{x^2+x} - x\right] = ? = (\infty - \infty)$ form.

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$$= \lim_{x \to \infty} \left(\sqrt{x^2+x} - x\right) \frac{\left(\sqrt{x^2+x} + x\right)}{\left(\sqrt{x^2+x} + x\right)}$$

$$= \lim_{x \to \infty} \left[\frac{(x^2+x) - x^2}{\sqrt{x^2+x} + x}\right]$$

$$= \lim_{x \to \infty} \left[\frac{x}{\sqrt{x^2+x} + x}\right] = \frac{\infty}{\infty} \text{ form}$$

$$= \lim_{x \to \infty} \left[\frac{1}{\sqrt{1+\frac{1}{x}} + 1}\right]$$

$$= \frac{1}{\sqrt{1+0} + 1} = \frac{1}{2}$$

Q. $\lim_{x \to 4} \left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right] = ? = \frac{0}{0}$ form

M-I using L'Hosp Rule

M-II using Rationalisation

(a) $1/2$

(b) $-1/2$

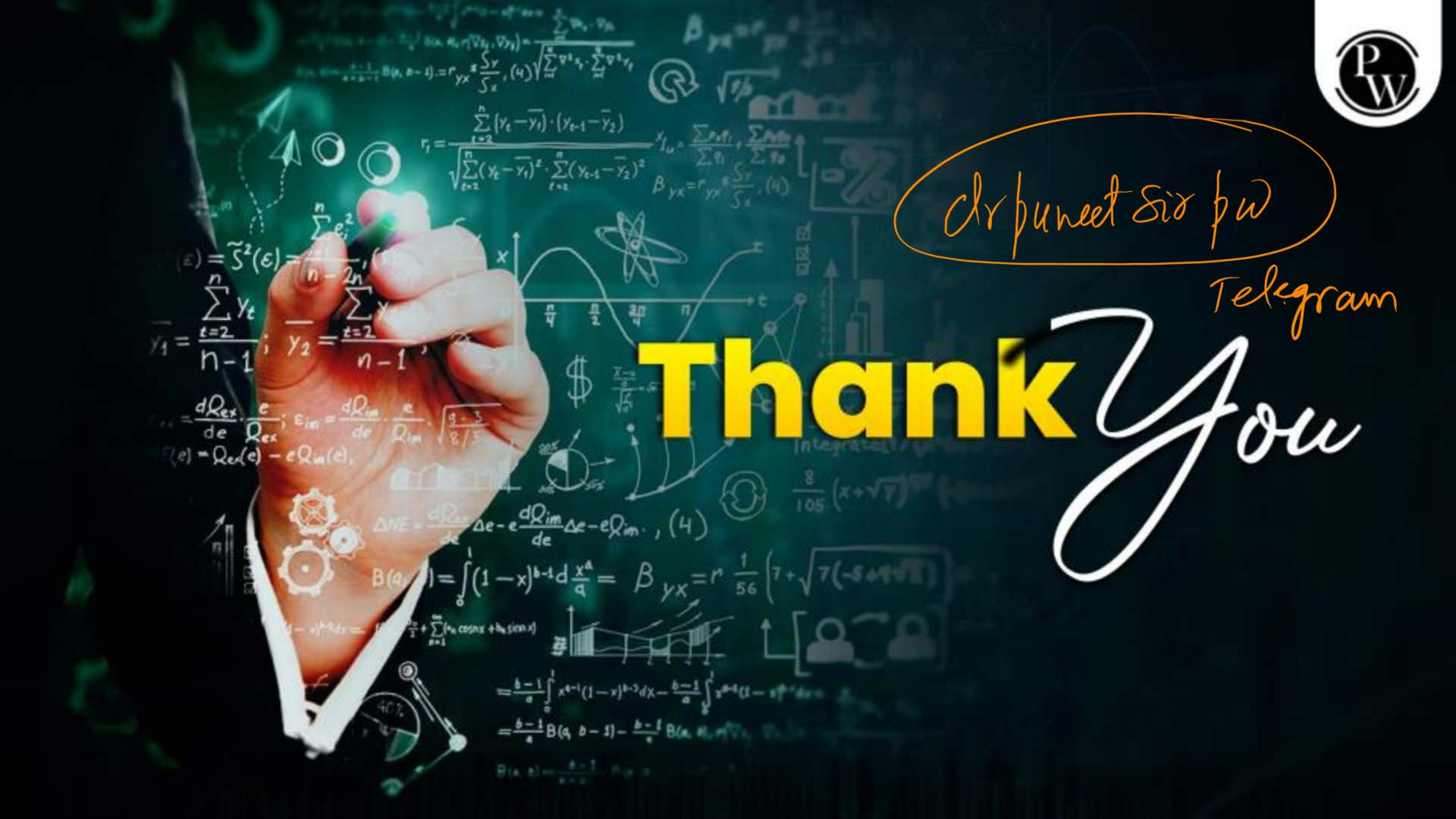
(c) $-1/3$

(d) 3

M-II $\lim_{x \to 4} \left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right) \times \left(\frac{1+\sqrt{5-x}}{1+\sqrt{5-x}}\right) \times \left(\frac{3+\sqrt{5+x}}{3+\sqrt{5+x}}\right)$

$= \lim_{x \to 4} \left[\frac{9-(5+x)}{1-(5-x)}\right] \times \left(\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right)$

$= \lim_{x \to 4} \left(\frac{4-x}{-4+x}\right) \times \left(\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right) = -1 \times \frac{2}{6} = -\frac{1}{3}$


drpuneet sir pw
Telegram
Thank You