DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE
& CS / IT

Calculus and Optimization

Lecture No. 05

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Recap of previous lecture

Topic Limit & Continuity

Topics to be Covered

Topic

1. Differentiability
2. Taylor & Maclaurin series

Q. $\lim_{x\to 0}\left\{\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}\right\} = ?$ (a) 0 (b) 1 (c) −1 (d) DNE ✓

Sol: $\simeq \dfrac{e^{\infty}-1}{e^{\infty}+1} = \dfrac{\infty-1}{\infty+1} = \dfrac{\infty}{\infty}$ form

But we can't apply L-Hospital's Rule because $f(x)$ is not diff at $x=0$

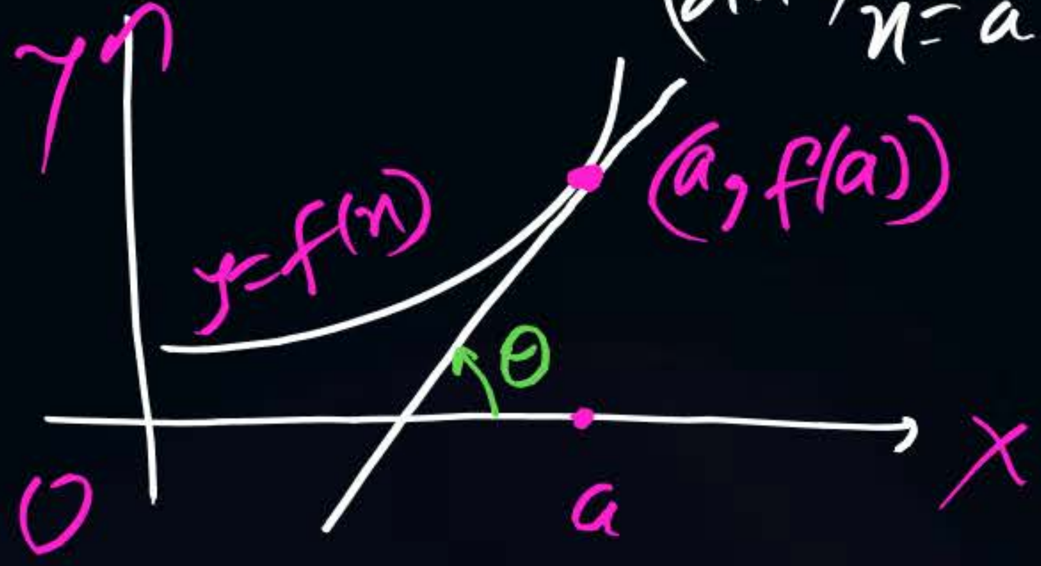
Now $LHL = f(0^-) = f(0-h) = \lim_{h\to 0}\left(\dfrac{e^{\frac{1}{0-h}}-1}{e^{\frac{1}{0-h}}+1}\right) = \dfrac{e^{-\infty}-1}{e^{-\infty}+1} = \dfrac{0-1}{0+1} = -1$

$RHL = f(0^+) = f(0+h) = \lim_{h\to 0}\left(\dfrac{e^{\frac{1}{0+h}}-1}{e^{\frac{1}{0+h}}+1}\right) = \lim_{h\to 0}\left(\dfrac{1-e^{-\frac{1}{h}}}{1+e^{-\frac{1}{h}}}\right) = \dfrac{1-e^{-\infty}}{1+e^{-\infty}} = \dfrac{1-0}{1+0} = 1$

$\because$ LHL $\neq$ RHL so limit DNE.

G. Meaning of Derivative

for $y=f(x)$, $\left(\frac{dy}{dx}\right)_{x=a} = \boxed{f'(a)=\tan\theta}$ = slope of tangent at $(x=a)$



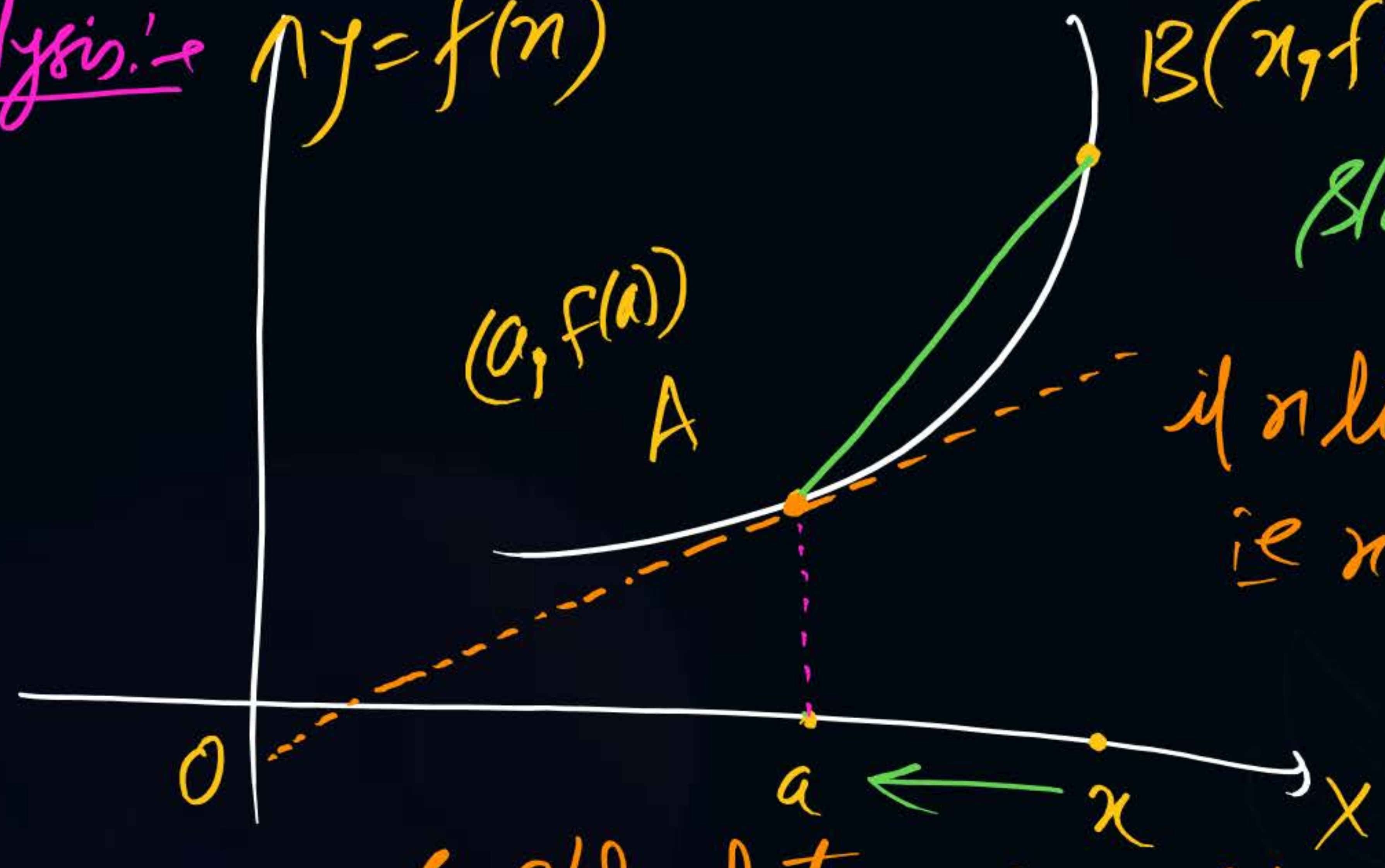
$\because$ $x=a$ is any Random point in the Domain of $f(x)$

So In general,

$\boxed{f'(x)=\tan\theta}$ at any Random Point x

Mathematical Defn of Differentiability → $f(x)$ is said to be differentiable at $x=a$ if $\lim\limits_{x\to a}\left(\frac{f(x)-f(a)}{x-a}\right)$ exist & Value in this limit in called Derivative of $f(x)$ at $x=a$ & it is denoted as $f'(a)$ i.e. we have $\boxed{f'(a)=\lim\limits_{x\to a}\left(\frac{f(x)-f(a)}{x-a}\right)}$

Analysis :-



Slope of line AB $= \dfrac{f(x) - f(a)}{x - a}$

if x lies in the Nbd of a

i.e x lies very-very close to a

i.e $x \to a$ then

Chord will convert into tangent

& Slope of tangent = Slope of chord (AB)

$$f'(a) = \lim_{x \to a} \left(\frac{f(x) - f(a)}{x - a} \right)$$

Mathematical Defn → $f(x)$ is said to be differentiable at $x=a$, if it is possible to find the value of $\lim_{x \to a}\left(\frac{f(x)-f(a)}{x-a}\right)$

& Value of this limit is called Derivative of $f(x)$ at $x=a$ & it is denoted by $f'(a)$ i.e

$$\boxed{f'(a) = \lim_{x \to a}\left(\frac{f(x)-f(a)}{x-a}\right)}$$

$$LHD = \lim_{h \to 0}\left(\frac{f(a-h)-f(a)}{-h}\right)$$

$$RHD = \lim_{h \to 0}\left(\frac{f(a+h)-f(a)}{h}\right)$$

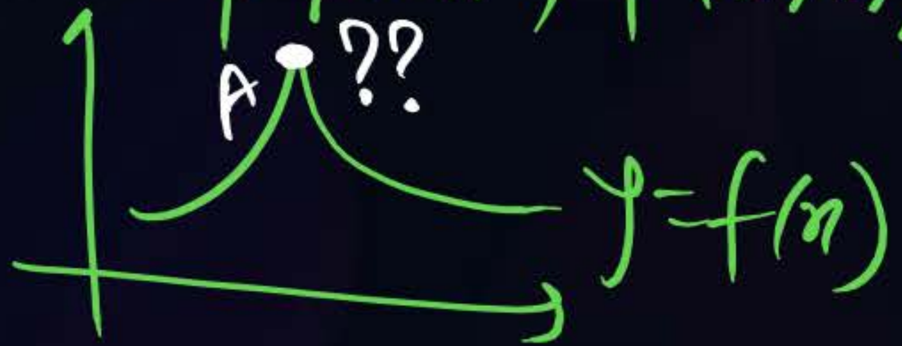
when $LHD = RHD$ then $f(x)$ is said to be Differentiable at $x=a$

Note ① Continuity is the Necessary Condition for Differentiability.
But it is not sufficient condition.

② N Condⁿ is ⟶ Continuity
Sufficient Condⁿ is ⟶ $\lim_{x \to a}\left(\frac{f(x)-f(a)}{x-a}\right)$ must exist.

Q: $\underbrace{\text{Differentiability} \Longrightarrow \text{Continuity}}_{\text{True}}$ or $\underbrace{\text{Continuity} \Longrightarrow \text{Differentiability}}_{\text{False}}$.

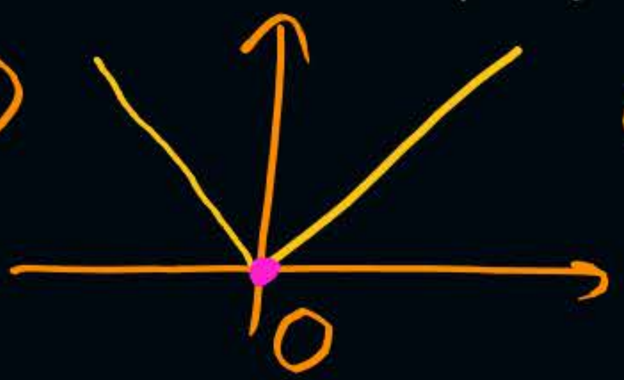
③ At sharp point, $f(x)$ is said to be Non Differentiable.

eg 

at A, $f(x)$ is Non Diff, ∵ it is not possible to find unique tangent at A.

eg At $x=0$, $f(x)=|x|$ is Continuous but Not Diff

M-I



M-II $f(x)=|x|=\begin{cases}-x, & x<0\\ x, & x\geq 0\end{cases}$, $f'(x)=\begin{cases}-1, & x<0\\ 1, & x>0\end{cases}$

LHL = 0, RHL = 0, $f(0)=0$ | LHD = -1, RHD = 1, Not Diff

i.e. Cont at $x=0$

(v) Shortcut Method of finding LHD and RHD of $f(x)$ →

W.K. that $LHL=\lim_{x\to a^-} f(x)$ & $RHL=\lim_{x\to a^+} f(x)$

Similarly, LHD of $f(x)$ = LHL of $f'(x)$ & RHD of $f(x)$ = RHL of $f'(x)$

$=\lim_{x\to a^-} f'(x)$ $\qquad$ $=\lim_{x\to a^+} f'(x)$

Q: find the Derivative of $y = |x|$?

Sol: $\frac{dy}{dx} = \frac{d}{dx}|x| = \frac{d}{dx}\left(\sqrt{x^2}\right) = \frac{1}{2\sqrt{x^2}}\frac{d}{dx}(x^2) = \frac{2x}{2\sqrt{x^2}} = \frac{x}{|x|}$

$\begin{cases} \frac{-x}{x}, & x<0 \\ \frac{+x}{x}, & x>0 \end{cases}$

ie $\frac{d}{dx}|x| = \frac{x}{|x|} = \frac{x|x|}{|x|^2} = \frac{x|x|}{x^2} = \frac{|x|}{x}$, $x \neq 0 = \begin{cases} -1, & x<0 \\ \text{DNE}, & x=0 \\ +1, & x>0 \end{cases}$

$\underbrace{\left(\frac{d}{dx}|x|\right)_{x=0} = \text{DNE}}_{\substack{LHD=-1 \\ RHD=+1}}$, $\underbrace{\left(\frac{d}{dx}|x|\right)_{x=5} = +1}_{\substack{LHD=1 \\ RHD=1}}$, $\underbrace{\left(\frac{d}{dx}|x|\right)_{x=-3} = -1}_{\substack{LHD=-1 \\ RHD=-1}}$, $\underbrace{\left(\frac{d}{dx}|x|\right)_{x=-5} = -1}_{\substack{LHD=-1 \\ RHD=-1}}$

ie $f(x) = |x|$ is diff everywhere except at $x=0$

Q. if $y = \sin|x|$ then evaluate $\frac{dy}{dx}$ at $x = 0$ & $x = -\frac{\pi}{4}$

Sol:

$$y = f(x) = \sin|x| = \begin{cases} \sin(-x), & x < 0 \\ \sin(+x), & x > 0 \end{cases}$$

$$= \begin{cases} -\sin x, & x < 0 \\ \sin x, & x > 0 \\ 0, & x = 0 \end{cases}$$

At $x = 0$:
- LHL $= -0$
- RHL $= +0$ ie Continuous
- $f(0) = 0$

$$f'(x) = \begin{cases} -\cos x, & x < 0 \\ +\cos x, & x > 0 \end{cases}$$

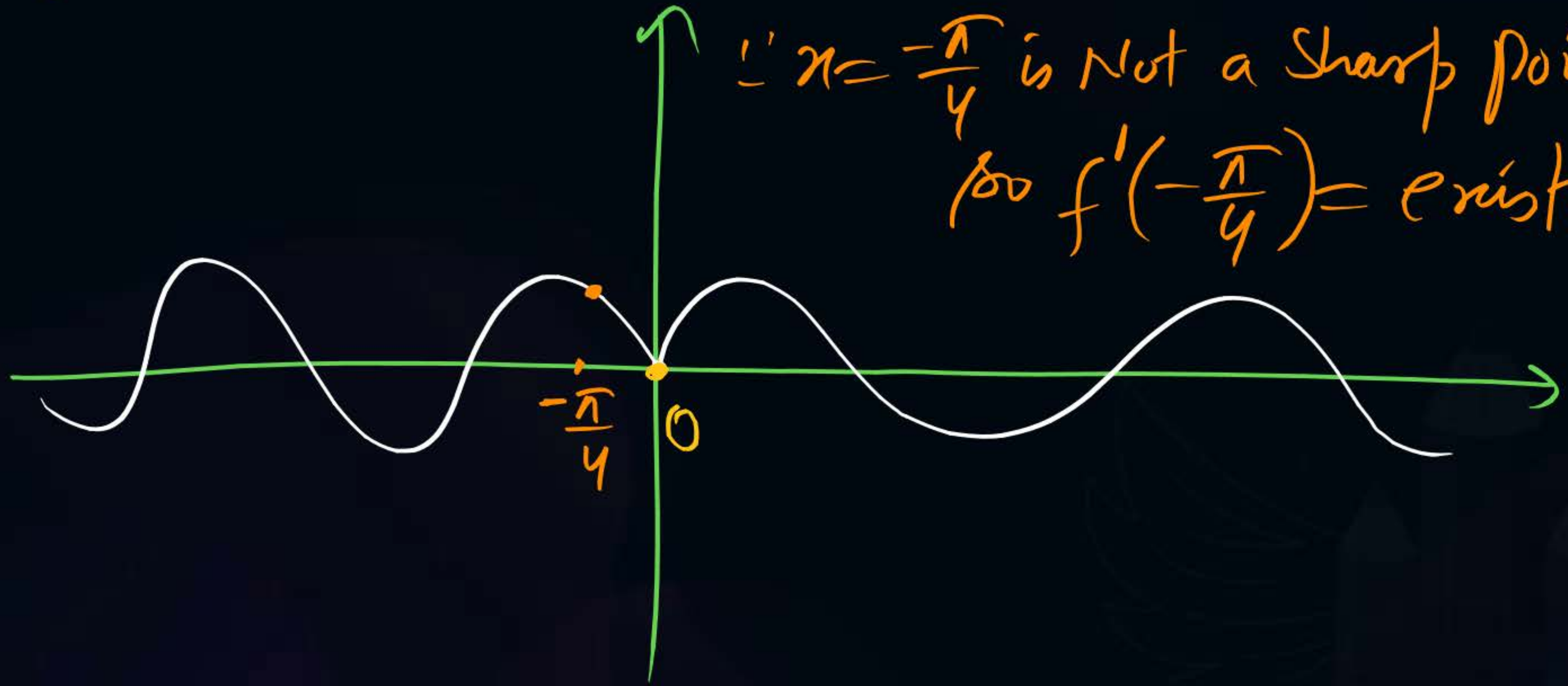
At $x = 0$:
- LHD $= -\cos 0 = -1$
- RHD $= +\cos 0 = +1$

ie $\left[\frac{d}{dx}(\sin|x|)\right]_{x=0} = \text{DNE}$

$\because x = -\frac{\pi}{4}$ is not a problem creating Point

so $f'\left(-\frac{\pi}{4}\right) = \left(-\cos x\right)_{x=-\frac{\pi}{4}} = -\cos\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Analysis $y = \sin|x|$, $\because x = 0$ is Sharp point so $f'(0) = DNE$



$\because x = -\frac{\pi}{4}$ is Not a Sharp point

so $f'\left(-\frac{\pi}{4}\right) =$ exist $= \ldots\ldots = -\frac{1}{\sqrt{2}}$

Q: At $x=1$, $f(x)=|\log_e x|$ is

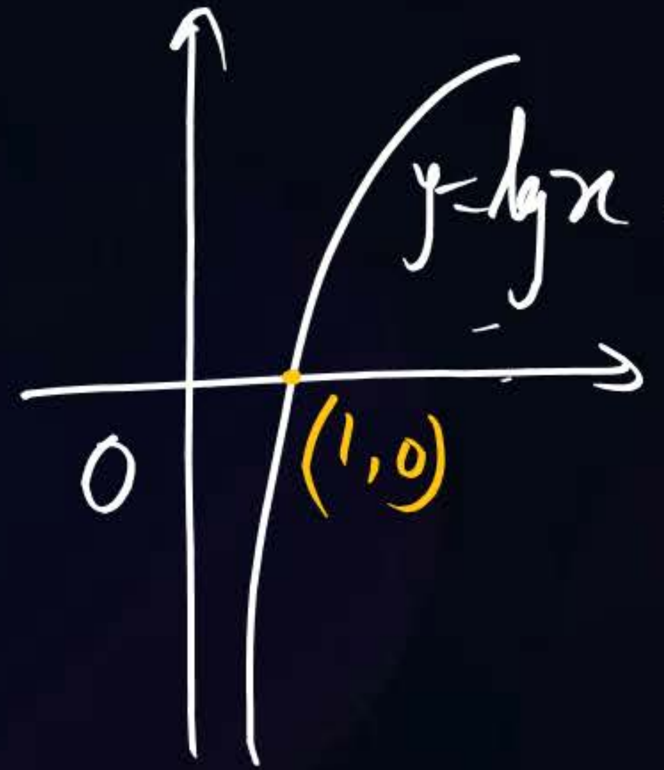
(a) Cont but Not Diff

(b) Diff but Not Cont (PAAP)

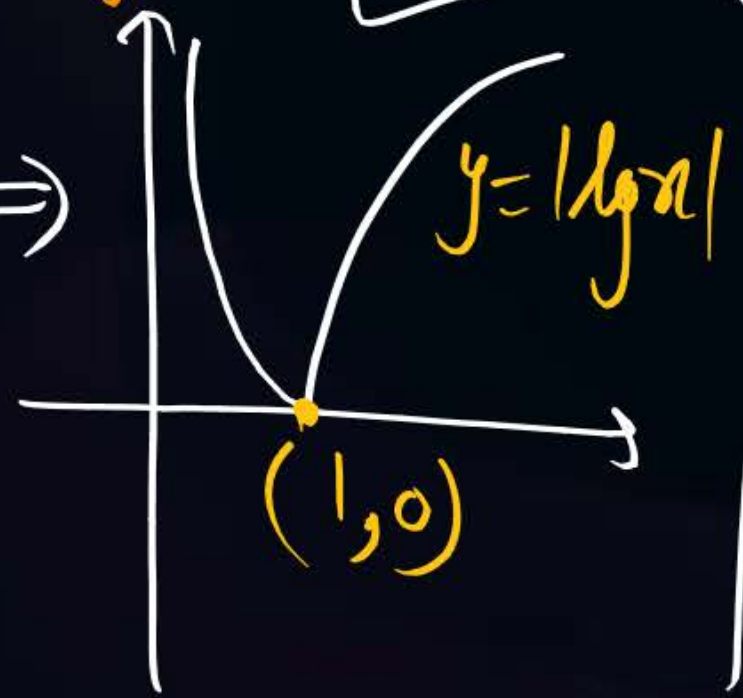
(c) Neither Cont Nor Diff

(d) Both Cont as well as Diff

M-I



$\Longrightarrow$



Dom $=(0,\infty)$

M-II $f(x)=|\log x| = \begin{cases} -\log x, & 0<x<1 \\ +\log x, & x>1 \end{cases}$

$f'(x)=\begin{cases} -1/x, & 0<x<1 \\ 1/x, & x>1 \end{cases}$

At $x=1$ $\begin{cases} LHD = f'(1^-) = -1 \\ RHD = f'(1^+) = +1 \end{cases}$

i.e. $LHD \neq RHD$

So $f(x)$ is Not Diff at $x=1$

Q. find points of Discontinuity and Non Diff points of $f(x) = \frac{x - |x-1|}{x}$

HW Q

[Ans: Discont at $x = 0$ only & Non Diff at $x = 0$ & 1]

Which of the following function is differentiable at $x = 0$?

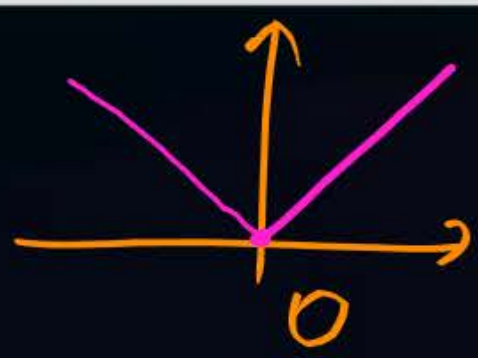
(a) $f(x) = |x|$ ✗

(b) $f(x) = |x| + |x-1|$

(c) $f(x) = x|x|$ ✓

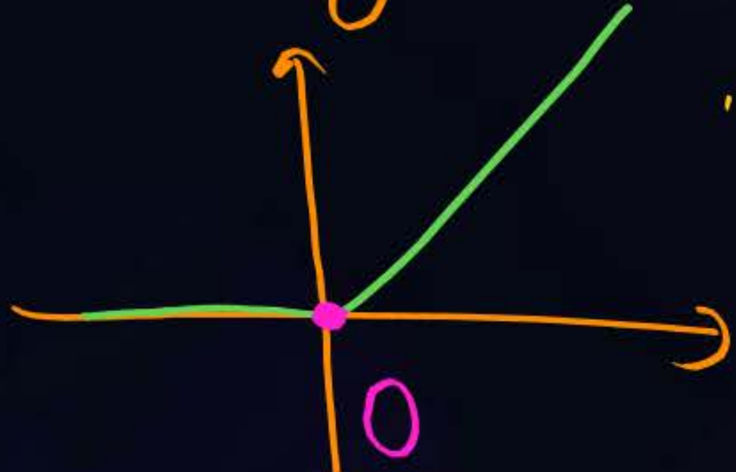
(d) $f(x) = \begin{cases} 0 \text{ if, } & x \le 0 \\ x \text{ if, } & x > 0 \end{cases}$ ✗

(a)



Not Diff at $x=0$

(d)



" " " "

(c) $y = f(x) = x|x| = \begin{cases} -x^2 & , x < 0 \\ x^2 & , x > 0 \end{cases}$

(M-I)

$\because x = 0$ is not a sharp point so $f(x)$ is Diff at $x = 0$

(M-II) $f'(x) = \begin{cases} -2x & , x < 0 \\ 2x & , x > 0 \end{cases}$

LHD of $f(x) = f'(0^-) = 0$

RHD of $f(x) = f'(0^+) = 0$

$\because$ LHD = RHD so $f(x)$ is diff at $x = 0$

Taking (b): $\boxed{f(x) = |x| + |x-1|}$

Analysis

$$= \begin{cases} -x + (1-x), & x < 0 \\ +x + (1-x), & 0 \le x < 1 \\ +x + (x-1), & x \ge 1 \end{cases}$$

$$f(x) = \begin{cases} 1 - 2x, & x < 0 \\ 1, & 0 \le x < 1 \\ 2x - 1, & x \ge 1 \end{cases}$$

At $x=0$, $LHL = 1$, $RHL = 1$, $f(0) = 1$ Hence cont at $x=0$

At $x=1$, $LHL = 1$, $RHL = 1$, $f(1) = 1$ " " " $x=1$

$$f'(x) = \begin{cases} -2, & x < 0 \\ 0, & 0 < x < 1 \\ 2, & x > 1 \end{cases}$$

At $x=0$: $LHD = -2$, $RHD = 0$ is Not Diff at $x=0$

At $x=1$: $LHD = 0$, $RHD = 2$ is Not Diff at $x=1$

A real function

PYQ.

$$f(x)=\begin{cases}\alpha x^2+\beta x, & \text{for } x<0\\ \alpha x^3+\beta x^2+5\sin x, & x\ge 0\end{cases}$$

If $f(x)$ is twice differentiable then

(a) $\alpha = 1, \beta = 0$ (b) $\alpha = 1, \beta = 5$

(c) $\alpha = 5, \beta = -10$ (d) $\alpha = 5, \beta = 5$

$$f'(x)=\begin{cases}2\alpha x+\beta & , x<0\\ 3\alpha x^2+2\beta x+5\cos x, & x>0\end{cases}$$

At $x=0$, LHD = RHD

$\beta=5$

$$f''(x)=\begin{cases}2\alpha & , x<0\\ 6\alpha x+2\beta-5\sin x & , x>0\end{cases}$$

LHD = RHD

$2\alpha=2\beta \Rightarrow \alpha=\beta=5$

Q $\lim_{x \to 0}\left(\frac{\sin x}{x}\right) = ? = \lim_{x \to 0}\left(\frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\cdots}{x}\right) = \lim_{x \to 0}\left[1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots\right]$

$= 1 - 0 + 0 - 0 + \cdots = 1$

$\because \sin x = x - \frac{x^3}{3!} - \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\cdots$

Note: $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\cdots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots$

TAYLOR & MACLAURIN SERIES

(POWER SERIES)

(MACLAURIN SERIES) → Let $f(x)$ is continuous func^n defined in a Domain D s.t it's all derivative exist then $f(x)$ can be expanded in the Nbd of 0 $(x \to 0)$ as follows;

$$\boxed{f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots}$$

Note ① this series provides Approx value of $f(x)$ in the Nbd of 0

② Coeff of x^n in the M.S.Exp $= \dfrac{f^n(0)}{n!}$

③ In this series we are getting Increasing powers of x

Some Standard Results of Maclaurin Series

→ All these Results are Valid only when $x \to 0$

① $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$

$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} \cdots$

② $a^x = 1 + x(\ln a) + \frac{x^2}{2!}(\ln a)^2 + \frac{x^3}{3!}(\ln a)^3 + \cdots$

Gate ✓ $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

③ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$

Gate

$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots = \frac{e^x - e^{-x}}{2}$

(4) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots\cdots \infty$

$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots\cdots = \frac{e^x + e^{-x}}{2}$

(5) $\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots\cdots$

<u>Note</u>: $\sinh x = \frac{e^x - e^{-x}}{2}$,

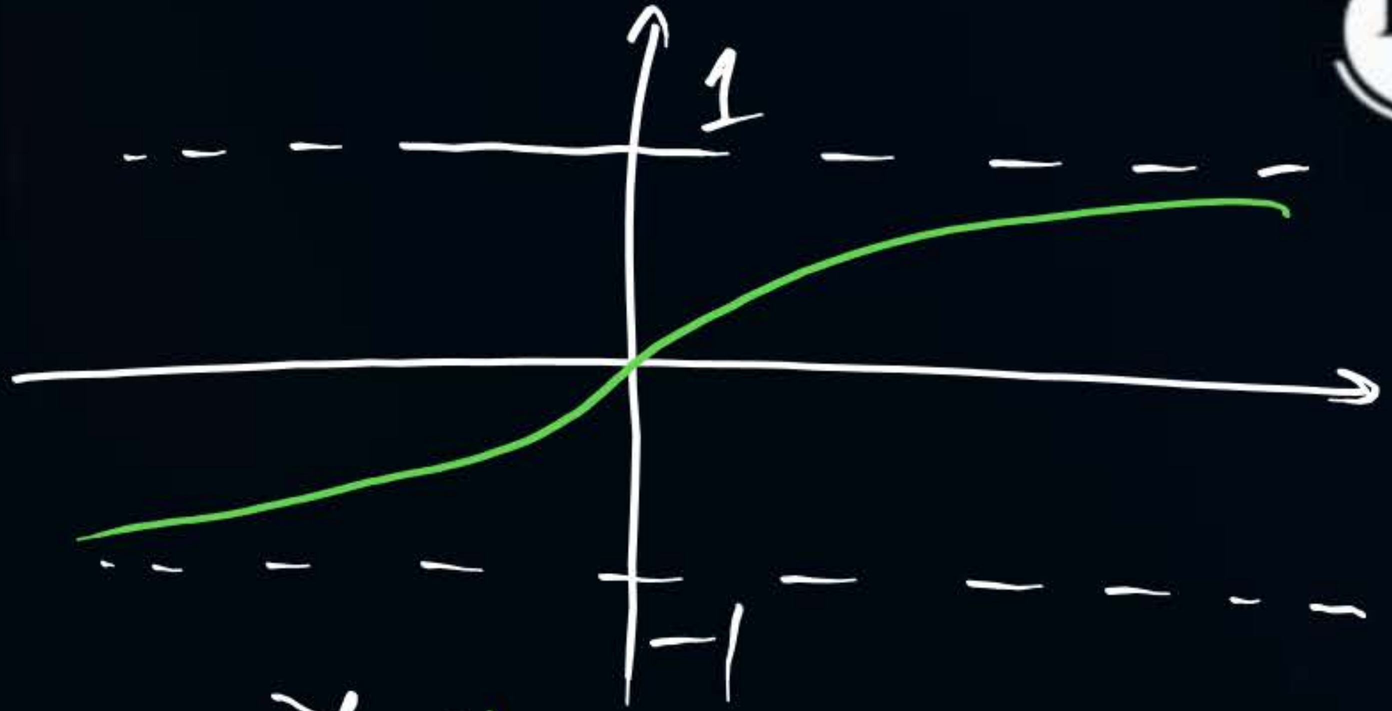
$\frac{d}{dx}(\sinh x) = \int \sinh x\,dx = \cosh x$

$\cosh x = \frac{e^x + e^{-x}}{2}$

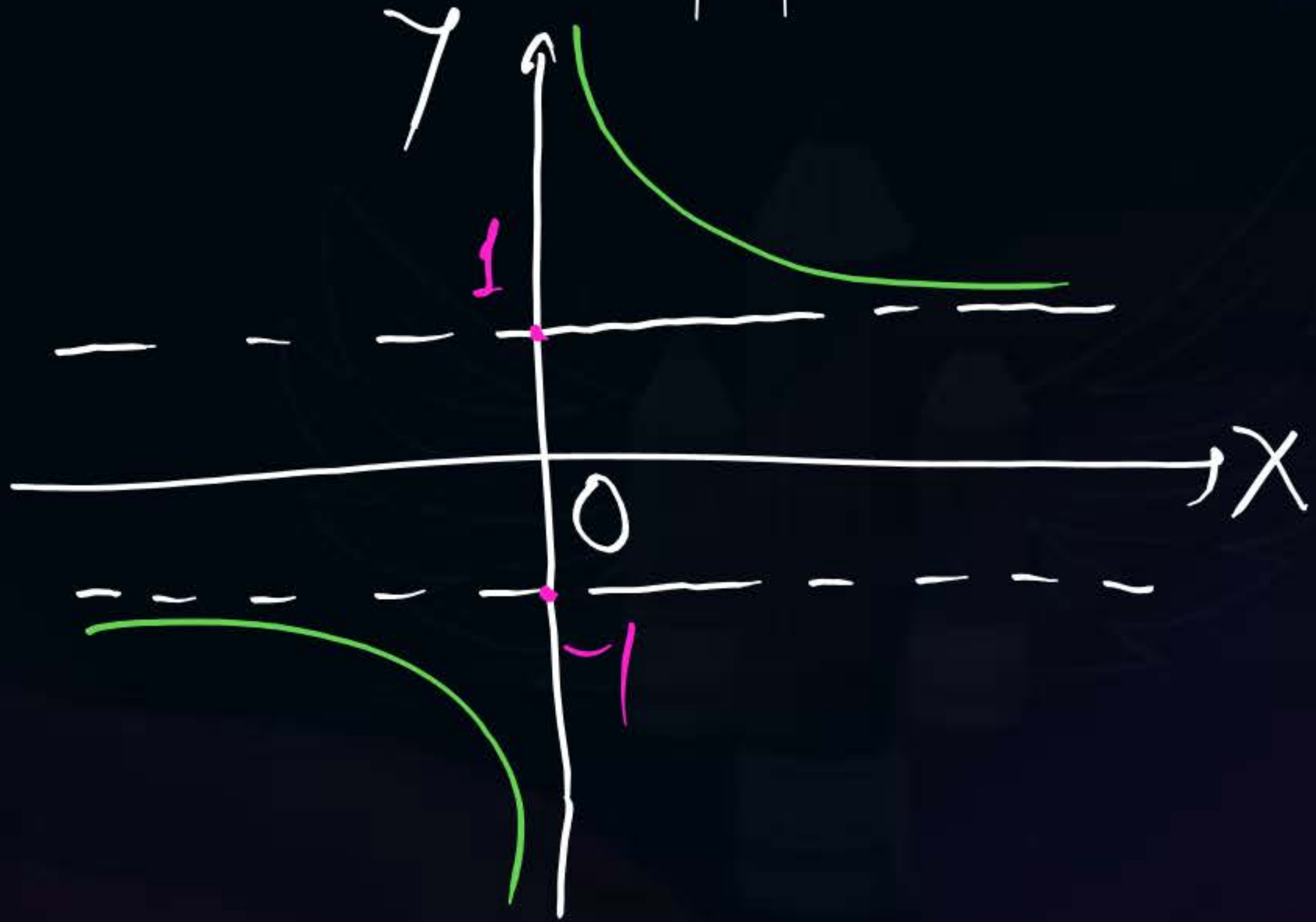
$\frac{d}{dx}(\cosh x) = \int \cosh x\,dx = \sinh x$

Note:

① $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$



② $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$



Q. Expand $\tan x$ in the nbd of $x=0$

Sol: $f(x)=\tan x$, $f'(x)=\sec^2 x$, $f''(x)=2\sec x(\sec x\tan x)=2\sec^2 x\tan x$

$f(0)=0$, $f'(0)=1$, $f''(0)=0$

$f'''(x)=2\left[2\sec^2 x\tan x\right]\tan x+2\sec^2 x(\sec^2 x)$

$=4\sec^2 x\tan^2 x+2\sec^4 x \Longrightarrow f'''(0)=0+2=2$

$f'''(0)=0+2=2$

Mac. series is given as, $f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\ldots\ldots$

$\tan x=(0)+x(1)+\frac{x^2}{2!}(0)+\frac{x^3}{3!}(2)+\ldots$

$\tan x=x+\frac{x^3}{3}+\frac{2}{15}x^5+\ldots\ldots$

Qs for $x <<< 1$, $\coth x$ can be approximated as ?

(a) x

(b) $1/x$

(c) $1/x^2$

(d) e^x

$\Downarrow$

x is very very very less than 1

So we can take x is very small $\Rightarrow$ x lies in the Nbd of 0

or $x \to 0$

& Mac. Series expansion of $\coth x$ is given as,

$$f(x) = \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots}{x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots}$$

$$= \frac{1 + \text{Neglect}}{x + \text{Neglect}} \simeq \frac{1}{x}$$

Q. The Taylor Series Expansion of $(3\sin x + 2\cos x)$ is ?

PYQ

(a) $2+3x-x^2-\frac{x^3}{2}+\cdots\cdots$ ✓

(b) $2-3x+x^2-\frac{x^3}{2}+\cdots\cdots$ ✗

(c) $2+3x+x^2+\frac{x^3}{2}+\cdots\cdots$ ✗

(d) $2-3x-x^2+\frac{x^3}{2}+\cdots\cdots$ ✗

we will expansion $f(x)$ in the nbd of $x=0$

i.e. $f(x) = 3\sin x + 2\cos x$

$$= 3\left\{x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots\cdots\right]$$

$$+2\left[1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots\cdots\right]$$

$$f(x) = 2+3x-x^2-\frac{x^3}{2}+\cdots\cdots \quad (a)$$

Q. Consider the Differential Equation $\boxed{\frac{dy}{dx} = x^2y - 1}$, $y(0)=1$ then value of y at $x = 0.1$ is

(a) 1.0000

(b) 0.90000

(c) 0.90033

(d) 0.8133

$\because$ 0.1 lies in the nbd of '0' so we will follow Mac. Series expansion for $\boxed{y = f(x)}$ in the nbd of $x = 0$

$$y(0) = f(0) = 1, \quad f'(0) = \left(x^2y - 1\right)_{x=0} = 0\times 1 - 1 = -1$$

$$f''(x) = \frac{d^2y}{dx^2} = x^2\frac{dy}{dx} + y\cdot(2x) = x^2\left(x^2y - 1\right) + 2xy$$

$$\boxed{f''(x) = x^4y - x^2 + 2xy} \Longrightarrow f''(0) = 0$$

$$f'''(x) = x^4\frac{dy}{dx} + y(4x^3) - 2x + 2x\frac{dy}{dx} + y(1)$$

$$f'''(x) = x^4(x^2y - 1) + 4x^3y - 2x + 2x(x^2y - 1) + y \Longrightarrow f'''(0) = 1$$

Now the Maclaurin Series Exp of $f(x)$ in the Nbd of $x=0$ is as follows.

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+\cdots$$

$$f(x)=(1)+x(-1)+\frac{x^2}{2!}(0)+\frac{x^3}{3!}(1)+\cdots$$

$$f(x)=1-x+\frac{x^3}{3!}+\cdots$$

So $f(0.1)=\underbrace{1-(0.1)+\frac{(0.1)^3}{3!}}_{\approx 0.90033}+\text{Neglect}$

≈ 0.90033 (C)

Note: x lies in the Nbd of 'a' $\Rightarrow x \in (a-h, a+h)$

OR

$a-h < x < a+h$ where $h \to 0$

OR

$|x-a| < h$

OR

$x \to a$

OR

x is about a

Note: $|x-a| < h$

$\pm(x-a) < h$

$-(x-a) < h$ & $+(x-a) < h$

$x-a > -h$ & $x < h+a$

$x > -h+a$ & $x < a+h$

$x > a-h$ & $x < a+h$

$\Rightarrow \underbrace{}_{a-h < x < a+h}$

x is at 'a'

$\Rightarrow$ we are considering Exact value

Taylor Series: If we want to Expand our function $f(x)$ in the Nbd of 'a' then expansion is called T.S. Expansion & it is defined as follows;

$$f(x) = \underbrace{f(a) + (x-a)f'(a)}_{\text{Linear Approximation}} + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \cdots$$

Note ① this series will provide Approx value of $f(x)$ in the Nbd of a

② Coeff of $(x-a)^n = \frac{f^n(a)}{n!}$

③ Here we are getting Increasing powers of $(x-a)$

④ To find Approx Value of $f(x)$ in LINEAR Form, Neglect 2^{nd} & higher degree term

⑤ " " " " " " " Quadratic form, Neglect 3^{rd} " " " "

Linear Interpolation → it is nothing but the L. Approximation of T-series

$$f(x)=\underbrace{f(a)+(x-a)f'(a)}+\frac{(x-a)^2}{2!}f''(a)+\cdots$$

for L. App$_n$: $\boxed{f(x)=f(a)+(x-a)f'(a)}$ + Neglect

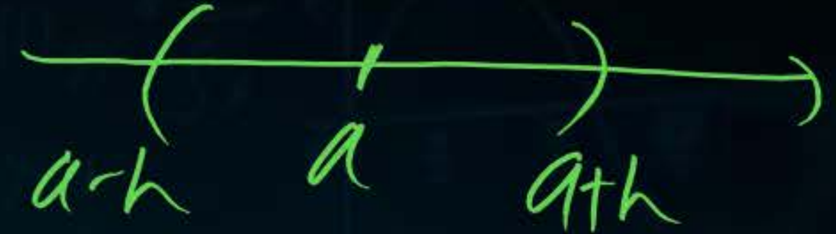
$$y=y_1+(x-x_1)\left(\frac{dy}{dx}\right)_{(x_1,y_1)}$$

$$\boxed{y=y_1+(x-x_1)\left(\frac{y_2-y_1}{x_2-x_1}\right)}$$

eq^n of straight line passing through (x_1,y_1) & (x_2,y_2)

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

Various forms of Taylor Series →



$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots\cdots$$

Here $x = a+h$ so we have

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \frac{h^3}{3!}f'''(a) + \cdots\cdots \quad \text{(Another form)}$$

$\because$ a is any Random point in the Domain of $f(x)$ so we can write

$$a \approx x$$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots\cdots$$

(Another form)

Q: Expand $\log_e x$ in the Nbd of 1 & Hence Evaluate $\log_e(1.1) = ?$

Sol: $f(x) = \log_e x$, $f'(x) = \frac{1}{x}$, $f''(x) = \frac{-1}{x^2}$, $f'''(x) = \frac{2}{x^3}$ ....

$f(1) = 0$, $f'(1) = 1$, $f''(1) = -1$, $f'''(1) = 2$ ......

So T.S. Exp of $f(x)$ in the Nbd of $x=1$ is given as,

$$f(x) = f(1) + (x-1) f'(1) + \frac{(x-1)^2}{2!} f''(1) + \frac{(x-1)^3}{3!} f'''(1) + \cdots$$

$$\log x = (0) + (x-1)(1) + \frac{(x-1)^2}{2!}(-1) + \frac{(x-1)^3}{3!}(2) + \cdots$$

$$\log_e x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \cdots \quad \text{Ans}$$

(ii) $\log_e x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \cdots\cdots$

Put $x = 1.1$ in above expression

$\log_e(1.1) = (1.1-1) - \frac{(1.1-1)^2}{2} + \frac{(1.1-1)^3}{3} - \cdots\cdots$

$= (0.1) - \frac{(0.1)^2}{2} + \frac{(0.1)^3}{3} +$ Neglect

$\cong 0.0953$

Shortcut: $\log x = \log(1 + (x-1)) = \log(1+X) = X - \frac{X^2}{2} + \frac{X^3}{3} - \frac{X^4}{4} + \cdots\cdots$

$x \to 1$

$(x-1) \to 0$

$X \to 0$

$\log_e x = (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} \cdots\cdots$

Q. In the power series Expansion of $f(x)=\frac{x-1}{x+1}$ about $x=1$, 3^{rd} term will be?

(a) $(x-1)^2/2$

(b) $(x-1)^2/4$

(c) $(x-1)^3/8$

(d) $(x-1)^3/4$

Q. if $\boxed{f(x) = x^3 + 8x^2 + 15x - 24}$ then $f\left(\frac{11}{10}\right) = ?$ using T.S. Exp Method.

(a) 0

(b) 3.5111

(c) 5.312

(d) 2.179

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