CS & IT ENGINEERING

THEORY OF COMPUTATION

Pushdown Automata

Lecture - 02

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Recap of Previous Lecture

Topic ?????

PDA Construction

Topics to be Covered

Topic Push down automat

Topic ??

Topic ??

Topic ??

PDA $(Q, \Sigma, \delta, q_0, F, Z_0, \Gamma)$

Q :- Finite number of states

Σ :- Input alphabet

q_0 :- initial state

F :-set of final states

Z_0 :-initial stack symbol

Γ :-stack alphabet

δ :- transition function

$$Q \times \Sigma \cup \{\varepsilon\} \times \Gamma \rightarrow Q \times \Gamma)$$

Topic : Note:

Note:- The following operation possible with PDA stack.

Push operation:- Moving i/p symbol from i/p buffer stack.

POP operation:- removing element from stack.

By pass operation:- don't push & don't pop (just reading symbol only)

SKIP

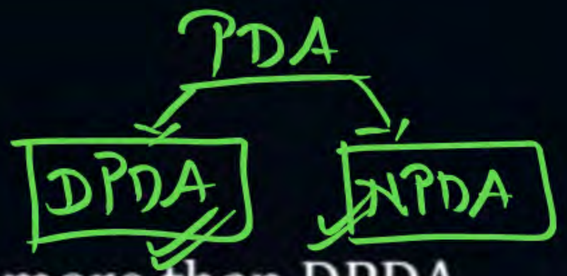
Topic : Empty Stack

By reading the string from let to Right by end of the string, if stack of the PDA is empty, then given string is accepted and inrelicent of No of final state.

Topic : Final State

By reading the string from left to right, end the string PDA enters into final state then given string is accepted and irradiant about stack is empty or not.

Topic PDA



PDA

1) The expressive power of NPDA is more than DPDA.
2) By Default PDA means NPDA.
3) PDA practically used in compilers as parser.
4) There are two types of acceptance method in PDA they are acceptance by empty stack and acceptance by final stack.

Notations:-

- Transition diagram
- Transitions

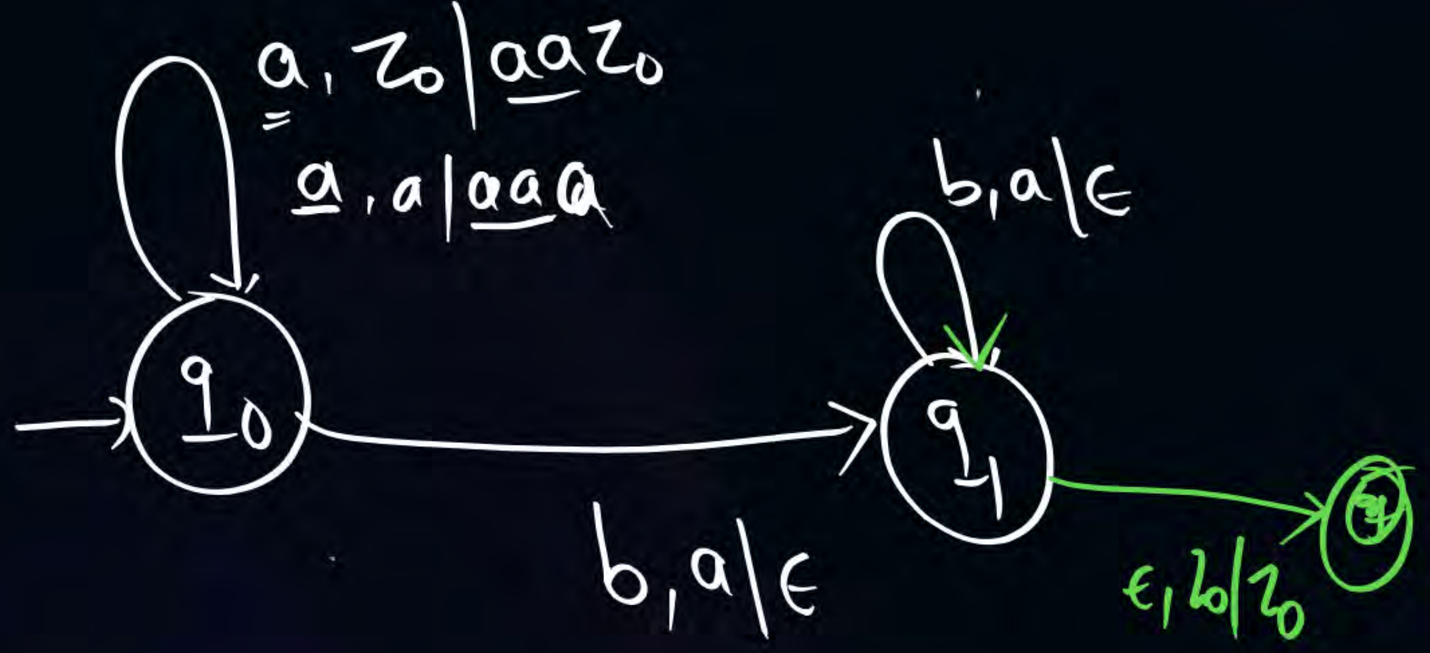
PDA (Acceptor)

- DPDA
- NPDA

Topic : Pushdown Automata

Empty stack

(@) Construct PDA for $L = \{a^n b^{2n} \mid n \geq 1\}$



Logic

① for 1 a → push 2 a's

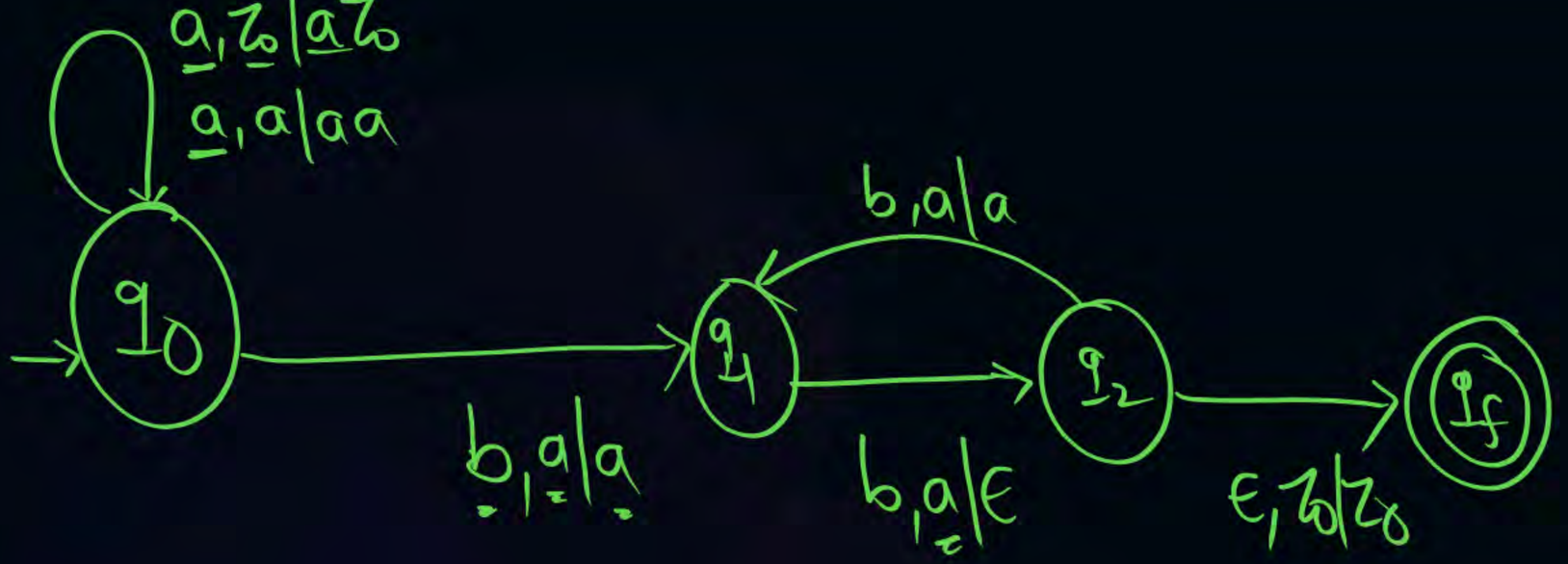
② for 1 b → pop 1 a

③ accept

Topic : Pushdown Automata

{final state}

(@) Construct PDA for $L = \{a^n b^{2n} \mid n \geq 1\}$



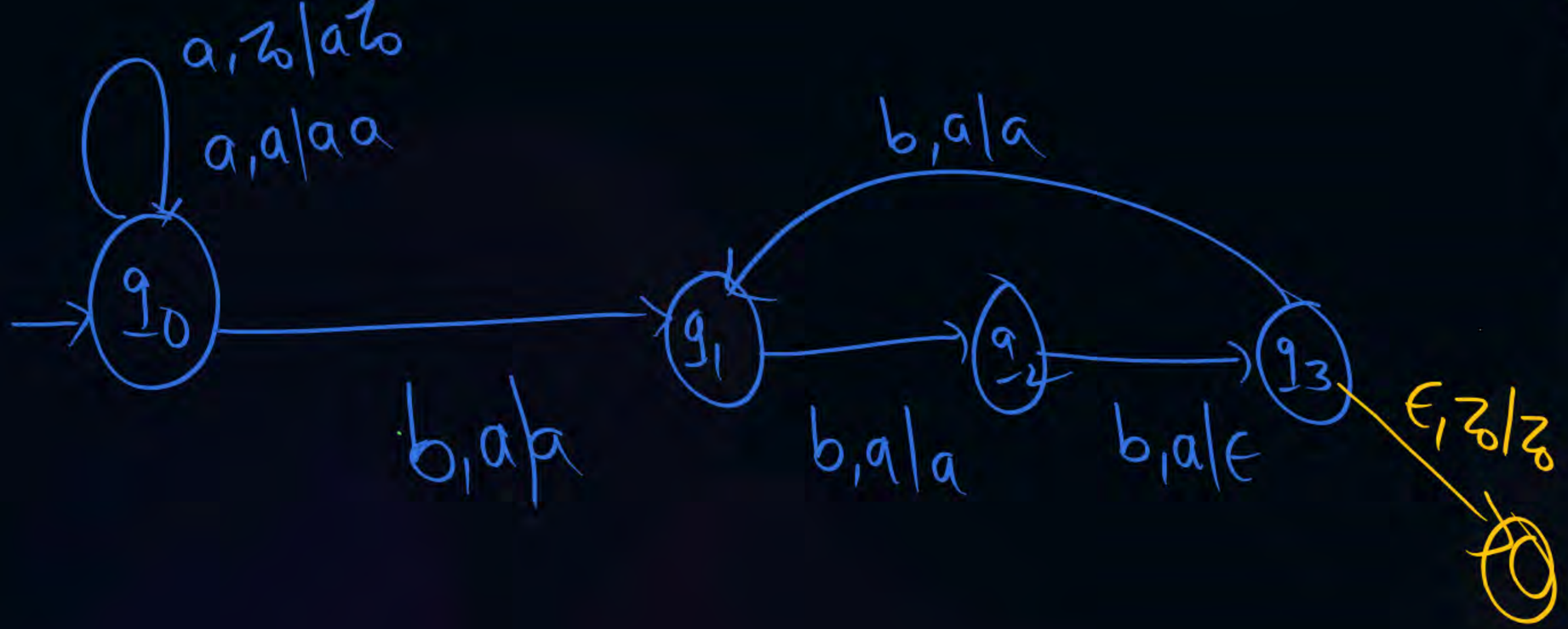
Logic

① for 1a → push 1a

② b^{2n} → First b → skip
Second b → pop

Topic : Pushdown Automata

(Q) Construct PDA for $L=\{a^n b^{3n} \mid n \geq 1\}$



Logic

① 1 a → push 1 a

② 1 b → SKIP
2 b → SKIP
3 b → pop

Topic : Pushdown Automata

(Q) Construct PDA for $L = \{a^n b^{2^n} \mid n \geq 1\} = \{a^1 b^2, a^2 b^4, a^3 b^8 \ldots\}$

$\{b^2, b^4, b^8, b^{16} \ldots\}$ Logic

PDA not possible

$\{$non CFL$\}$

(Q) which of the following is CFL ?

(a) $L = \{a^P b^P \mid P \text{ is prime number}\} \Rightarrow$ Non CFL

(b) $L = \{a^{n!} b^{n!} \mid n \geq 1\} \Rightarrow$ Non CFL

(c) $L = \{a^n b^{n^2} \mid n \geq 1\} \Rightarrow$ Non CFL

(d) none

Topic : Pushdown Automata

(Q) Construct PDA for $L = \{a^n b^{n^2} \mid n \geq 1\}$

CFL

PDA not possible

$\{b^1, b^4, b^9, b^{16} \ldots\}$

3 5 7

F.A + stack

non CFL

Topic : Pushdown Automata

non CFL

(Q) Construct PDA for $L=\{a^n b^n c^n \mid n \geq 1\}$

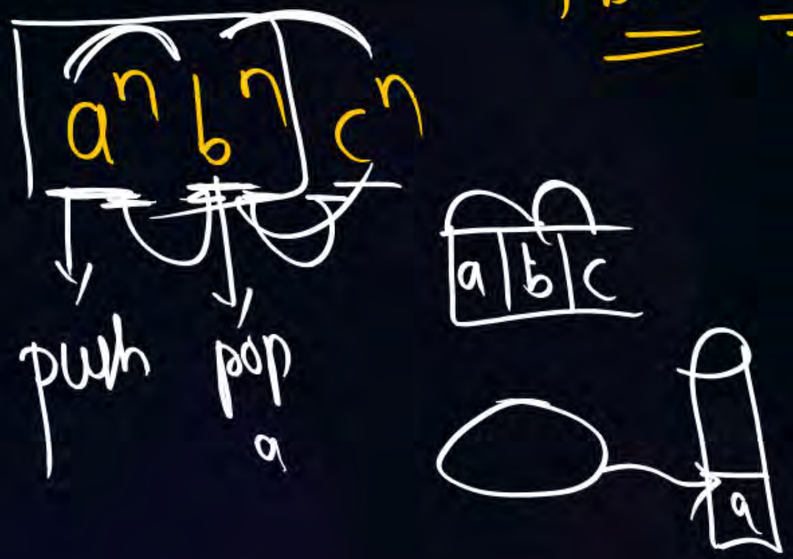
{Logic}

aa bbb c ✗

PDA not possible

Yes

NO

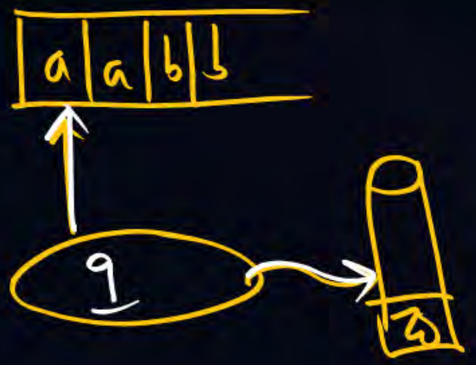


Non CFL

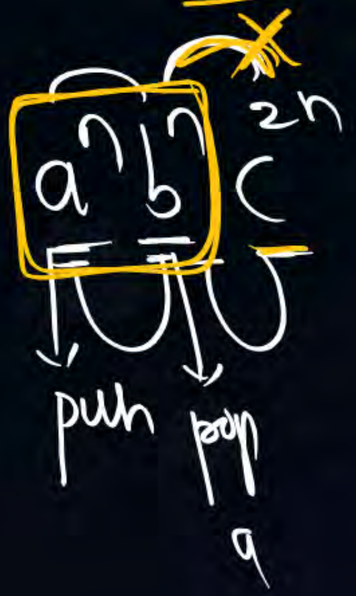
(Q) Construct PDA for $L = \{a^n b^n c^{2n} \mid n \geq 1\}$

PDA not possible

nonCFL



Logic



Topic : Pushdown Automata

(Q) Construct PDA for $L = \{ W C W^R \mid W \in (a+b)^* \}$

{WCW^R}

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, b, Z_0) = (q_0, bZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

$\delta(q_0, b, b) = (q_0, bb)$

$\delta(q_0, a, b) = (q_0, ab)$

$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, c, Z_0) = (q_1, Z_0)$

$\delta(q_0, c, a) = (q_1, a)$

$\delta(q_0, c, b) = (q_1, b)$

$\delta(q_1, a, a) = (q_1, \epsilon)$

$\delta(q_1, b, b) = (q_1, \epsilon)$

$\delta(q_1, \epsilon, Z_0) = (q_f, Z_0)$

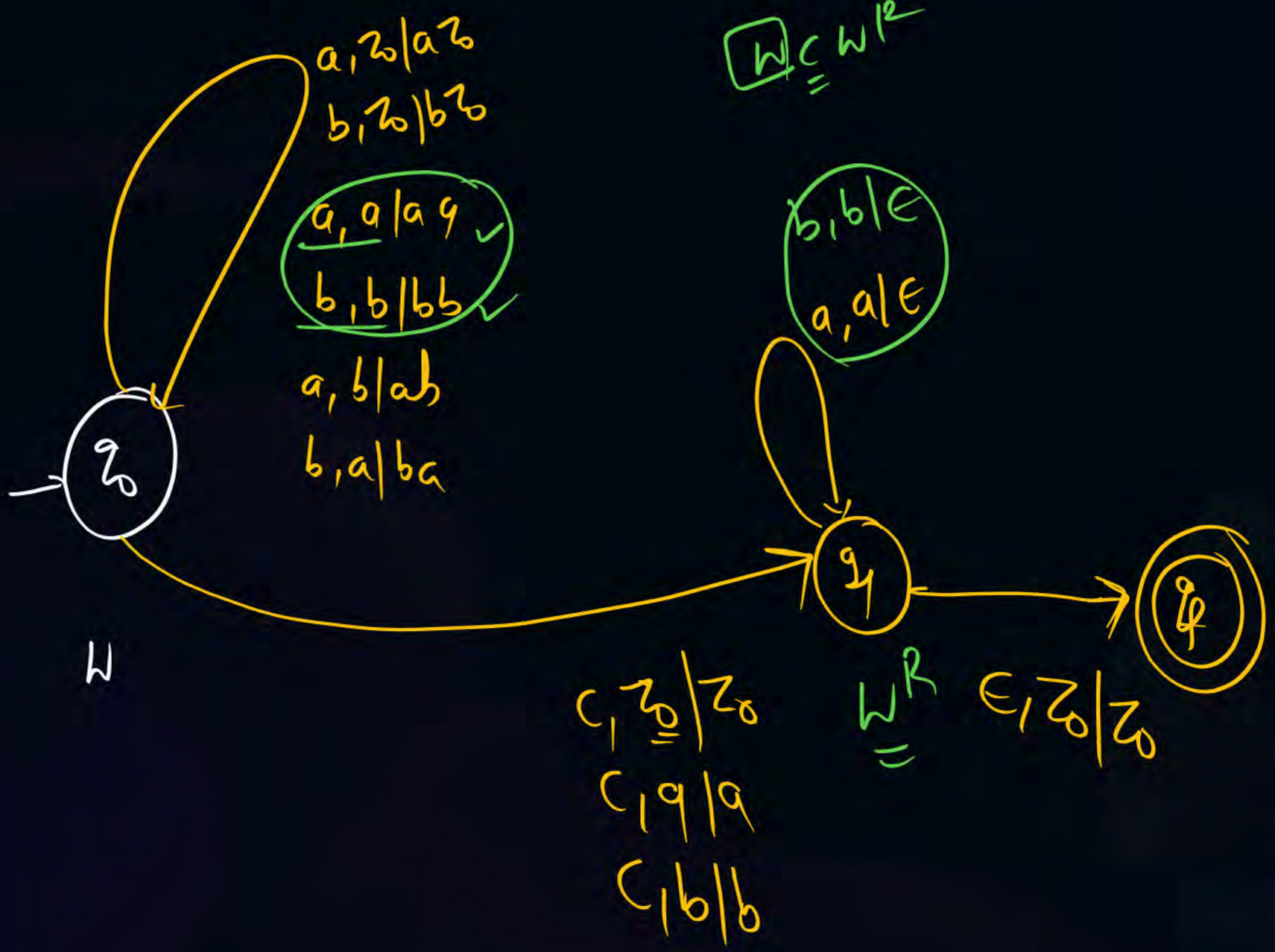
Logic

ab c b a

① push W into stack

② SKIP C

③ {a, a → pop ; b, b → pop}



$W \subseteq W^R$

(Q) Construct PDA for $L = \{ W W^R \mid W \in (a+b)^* \}$

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, b, Z_0) = (q_0, bZ_0)$

$\delta(q_0, a, b) = (q_0, ab)$

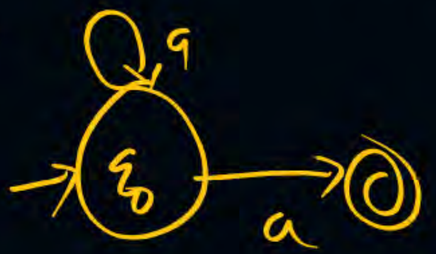
$\delta(q_0, b, a) = (q_0, ba)$

$\delta(q_0, a, a) = (\underline{\text{push}})(a)(\underline{\text{pop}})$

$\delta(q_0, b, b) = (\text{push})(a)(\text{pop})$

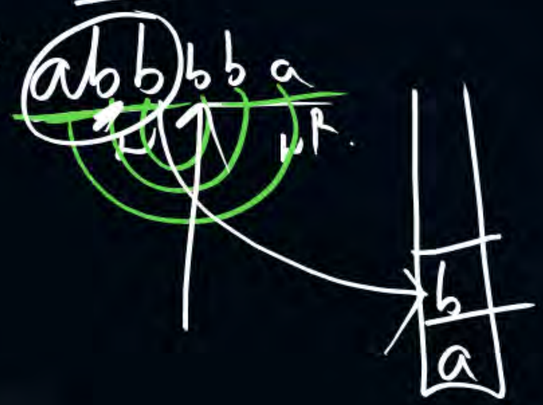
~~DPDA~~ NPDA

PDA



Yes / No

Logic



CFL but not DCFL

NPDA

(Q) Construct NPDA for $L=\{WW^R \mid W\in(a+b)^+\}$

$\delta(q_0,a,Z_0)=(q_0,aZ_0)$

$\delta(q_0,b,Z_0)=(q_0,bZ_0)$

$\delta(q_0,a,b)=(q_0,ab)$

$\delta(q_0,b,a)=(q_0,ba)$

W $\quad$ W^R

$\delta(q_0,a,a)=(q_0,aa)(w)(q_1,\epsilon)$

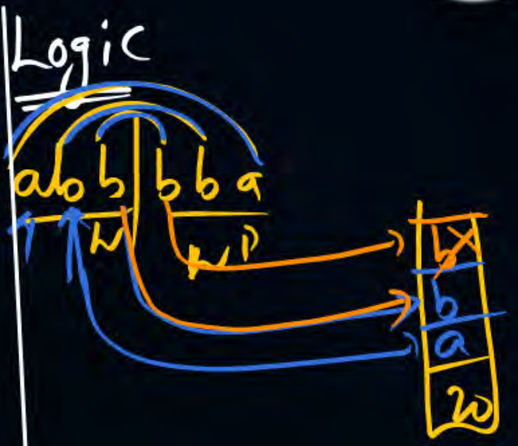
$\delta(q_0,b,b)=(q_0,bb)(a)(q_1,\epsilon)$

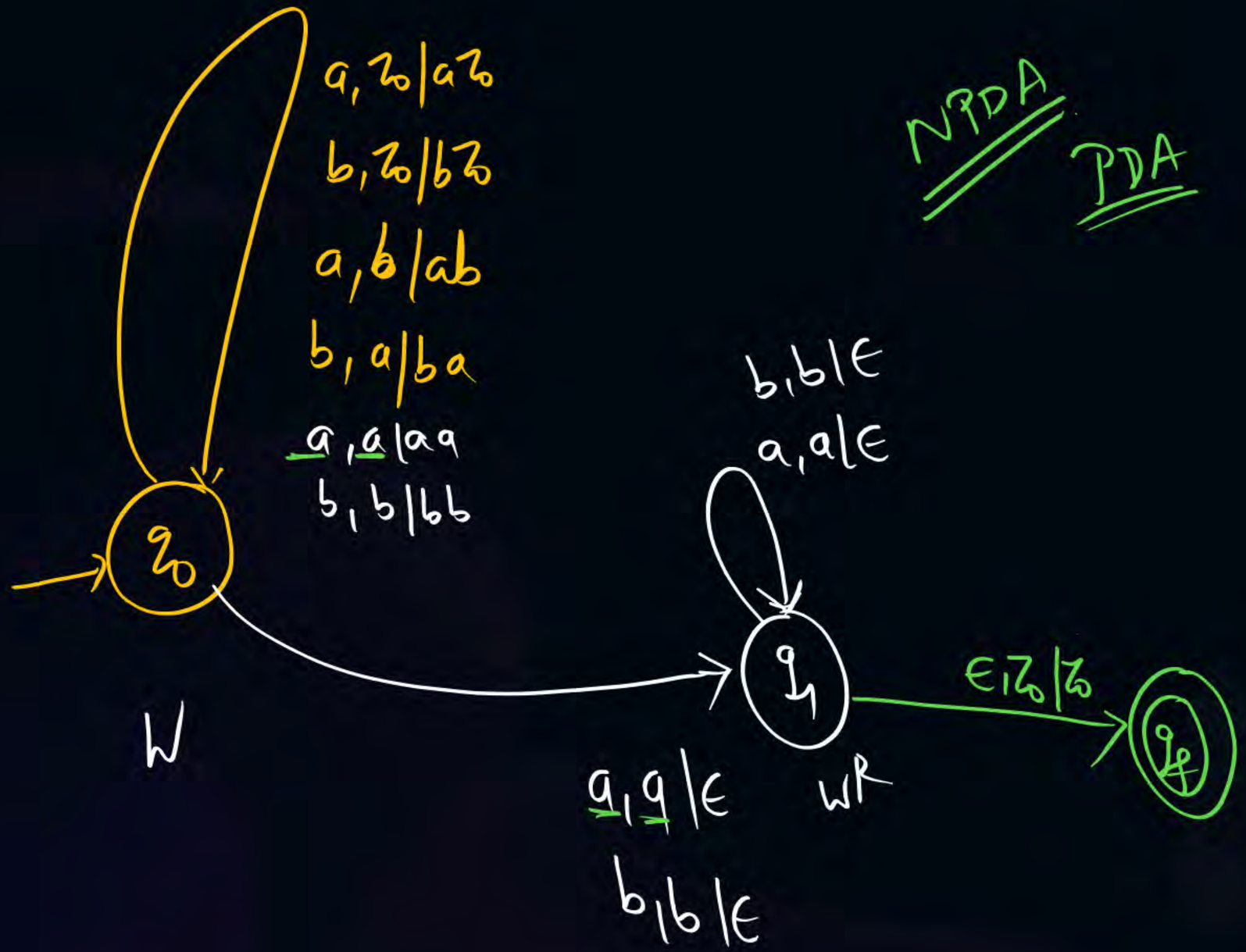
$\delta(q_1,a,a)=(q_1,\epsilon)$

$\delta(q_1,b,b)=(q_1,\epsilon)$

$\delta(q_1,\epsilon,Z_0)=(q_f,Z_0)$

Logic




a, Z0 | aZ0
b, Z0 | bZ0
a, b | ab
b, a | ba
a, a | aa
b, b | bb
q0
W
b, b | ∈
a, a | ∈
q1
q1, q | ∈
b, b | ∈
WR
∈, Z0 | Z0
qf

NPDA

PDA

(Q) Construct PDA

$L = \{a^n b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$

DPDA ↑ DPDA ↑ NPDA

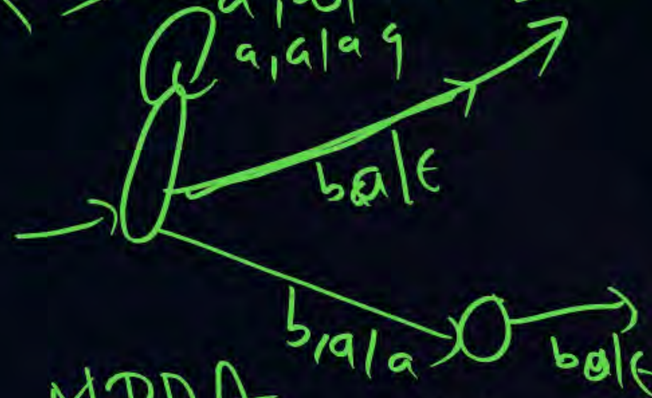
Logic

Homework

push 1 a

push 2 a's

X DPDA



$a^n b^n$

NPDA

$L = \{aaa---b-bb--\}$

$\delta(q_0, a, z_0) = (q_0, az_0)\ (a)\ (q_0, aaz_0)$

$(a^n b^{2n})$

CFL but not DCFL

(Q)

$L = \{a^n b^n c^m\} \cup \{a^n b^m c^m\}$

push DPDA — DPDA — SKIP

✗ DPDA

{ aa ... bb.. cc ... }

NPDA

CFL but not DCFL

PDA ⇔ CFL

DCFL

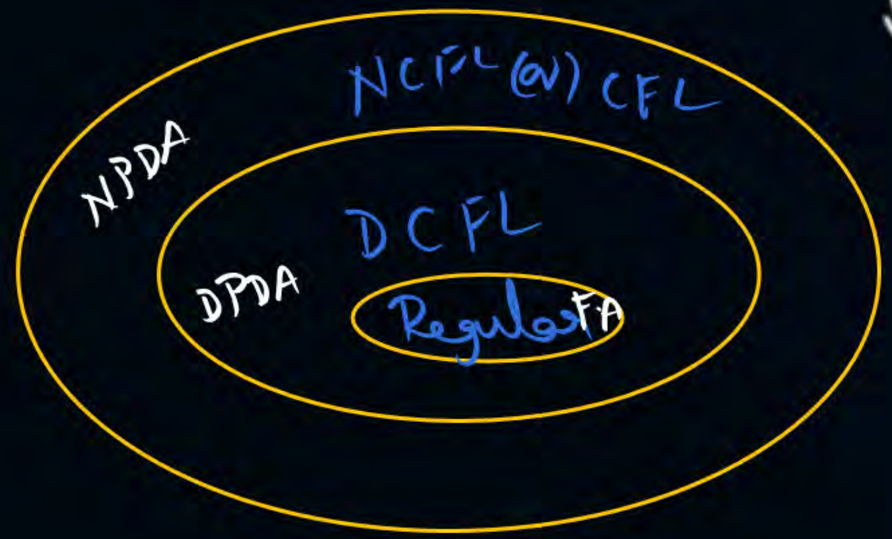
DPDA

$(Q, \Sigma, \Gamma) \to 1$

NCFL

NPDA (CFL)

$(Q, \Sigma, \Gamma) \to 1, 2, 3 \cdots n$



① Every DPDA is NPDA but every NPDA need not be DPDA

② Expressive power of NPDA is more than DPDA

Every DCFL is CFL

But every CFL need not be DCFL

THANK - YOU