GATE

CS & DA

Probability and Statistics

DPP: 1

Permutations and Combinations

Q1 Homer, Gomer, Plato, Euclid, Socrates, Aristotle, Homerina and Gomerina form the board of directors of the Lawyer and Poodle Admirers Club. They will choose from amongst themselves a Chairperson, Secretary, and Treasurer. No person will hold more than one position. How many different outcomes are possible?

(A) 336 (B) 24

(C) 512 (D) 21

Q2 Erasmus is trying to guess the combination to his combination lock. The "combination" is a sequence of three numbers, where the numbers range from 1 to 12, with no numbers repeated. How many different "combinations" are possible if he knows that the last number in the combination is either 1 or 11?

(A) 264 (B) 1320

(C) 220 (D) 288

Q3 The Egotists' Club has 6 members: A, B, C, D, E, and F. They are going to line up, from left to right, for a group photo. After lining up in alphabetical order (ABCDEF), Mr. F complains that he is always last whenever they do things alphabetically, so they agree to line up in reverse order (FEDCBA) and take another picture. Then Ms. D complains that she's always stuck next to Mr. C, and that she never gets to be first in line. Finally, in order to avoid bruised egos, they all agree to take pictures for every possible left-to-right line-up of the six people. How many different photos must be taken?

Q4 There are 6 multiple choice questions in an examination. How many sequences of answers are possible, if the first three questions have 4 choices each and the next three have 2 each ?

Q5 How many five-digit number license plates can be made if

(i) first digit cannot be zero and the repetition of digits is not allowed.

Q6 The number of 4 digit even numbers that can be formed using 0, 1, 2, 3, 4, 5, 6 without repetition is-

(A) 120

(B) 300

(C) 420

(D) More than one of the above

Q7 If ${}^{18}C_{15} + 2\left({}^{18}C_{16}\right) + {}^{17}C_{16} + 1 = {}^{n}C_3$, then n =

(A) 19

(B) 20

(C) 18

(D) More than one of the above

Q8 Three Men have 6 coats, 5 belts and 4 caps. The number of ways the can wear them is

(A) 172800

(B) 172500

(C) 174800

(D) None of the above

Q9 If there are 6 girls and 5 boys who sit in a row the the possible number of ways in which no two boys sit together-

(A) $\frac{6!6!}{2!11!}$ (B) $\frac{7!5!}{2!11!}$

(C) $\frac{6!7!}{2!}$ (D) None of these

Q10 Find the number of different permutation of the letters of the word MISSISSIPPI.

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Q11 If a coin is tossed six times, how many different outcomes consisting of 4 heads and 2 tails are there?

Q12 A shopping mall has a straight row of 5 flagpoles at its main entrance plaza. It has 3 identical green flags and 2 identical yellow flags. How many distinct arrangements of flags on the flagpoles are possible?

Q13 How many words can be formed by using the letters from the word "DRIVER" such that all the vowels are never together?

Q14 In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls and in 190 games both the players were boy. The number of games in which one player was a boy and other was girl is

(A) 200 (B) 216

(C) 235 (D) 256

Q15 18 guests have to be seated, half on each side of a long table. Four particular guests desired to sit on one particular and three others on the other side, then how many seating arrangements can be made?

(A) ${}^{18}C_4.{}^{14}C_3.9!.9!$

(B) ${}^{2}\mathrm{C}_1.{}^{9}\mathrm{P}_4.{}^{9}\mathrm{P}_3.11!$

(C) ${}^{9}\mathrm{P}_4.{}^{9}\mathrm{P}_3.{}^{9}\mathrm{P}_3.11!$

(D) ${}^{2}C_1.\frac{9!}{4!}.\frac{9!}{3!}$

Q16 In how many ways can 8 Directors, Vice-Chairman & Chairman of a firm be seated at a round table, if the Chairman has to sit between Vice-Chairman & Director

(A) 2 × 9! (B) 2 × 8!

(C) 2 × 7! (D) 3! × 9!

Q17 f:{1, 2, 3, 4, 5} → {1, 2, 3, 4, 5} that are onto and f(i) $\neq \mathrm{i}\forall\mathrm{i}$, the number of such functions will be?

(A) 9 (B) 44

(C) 119 (D) 120

Q18 One red flag, three white flags and two blue flags are arranged in a row such that,

(i) No two adjacent flags are of same color.

(ii) The flags at the two ends of the line are of different color.

In how many different ways can the flags be arranged?

(A) 6 (B) 4

(C) 10 (D) 2

Q19 The number of words of four letters containing equal number of vowels and consonants (Repetition allowed)

(A) 60 × 210

(B) 210 × 243

(C) 210 × 315

(D) 630

Q20 The total numbers of four letter words that can be formed out of the letters of the word "COMMITTEE"

(A) 360 (B) 738

(C) 414 (D) None

GATE

Answer Key

Q1	(A)
Q2	(C)
Q3	720
Q4	512
Q5	27216
Q6	(C)
Q7	(B)
Q8	(A)
Q9	(C)
Q10	34,650
Q11	15.
Q12	10
Q13	240
Q14	(A)
Q15	(C)
Q16	(B)
Q17	(B)
Q18	(A)
Q19	(C)
Q20	(B)

GATE

Hints & Solutions

Q1 Text Solution:

Choosing a Chairperson, Secretary and Treasurer from among these 8 people requires us to make three decisions. For instance, the choice we make when we select the chairperson affects which options are available when we go to choose the Secretary, since the person selected to be Chairperson cannot also be selected to be Secretary.

i. Choose Chairperson: 8 options;

ii. Choose Secretary: 7 options (one person has already been chosen to be Chairperson);

iii. Choose Treasurer: 6 options (two people have already been chosen to be Chairperson and Secretary, respectively).

According to the Fundamental Counting Principle the number of outcomes is: (8)(7)(6) = 336.

Q2 Text Solution:

Choosing a three-number "combination" having no repeated numbers requires that we make three dependent decisions. One of these decisions, however, has a special condition attached to it (the third number must be either 1 or 11).

Three dependent decisions:

1. Choose third number (two options);
2. Choose first number (11 options);
3. Choose second number (10 options).

According to the Fundamental Counting Principle the number of possible outcomes is (2)(11)(10) = 220.

The correct choice is C.

Q3 Text Solution:

To arrange the six people in a line we need to make six dependent decisions:

1. Choose first person (6 options)
2. Choose second person (5 options)
3. Choose third person (4 options)
4. Choose fourth person (3 options)
5. Choose fifth person (2 options)
6. Choose last person (1 option)

According to the Fundamental Counting Principle the number of outcomes is (6)(5)(4)(3)(2)(1) = 720.

Q4 Text Solution:

To answer the first question, we have 4 ways.

To answer the second question, we have 4 ways.

To answer the third question, we have 4 ways.

To answer the fourth question, we have 2 ways.

To answer the fifth question, we have 2 ways.

To answer the sixth question, we have 2 ways.

$= 4 \times 4 \times 4 \times 2 \times 2 \times 2 = 512$

Q5 Text Solution:

Numbers can be filled in the places are 0,1,2,3,4,........9

Number of options we have for first place = 9 (except 0)

Since repetition is not allowed the second dash is having 9 options (including 0 except the number filled in the first dash).

Like wise the third, fourth and fifth dashes are having 8, 7 and 6 options respectively.

Total number of ways $= 9 \times 9 \times 8 \times 7 \times 6 = 27216$

Q6 Text Solution:

4 digit even number have to be formed using 0,1,2,3,4,5,6 without repetition.

Here, the last digit could be 0,2,4,6 and the first digit cannot be 0.

Case-I when the last digit is 0.

The first digit has the remaining 6 choices (except 0), the second has 5 and the third has 4.

$\Rightarrow$ The number of way $= 6 \times 5 \times 4 = 120$

Case-II when the last digit is 2.

The first digit has the remaining 5 choices (except 0 and 2), the second has 5, and the third has 4.

$\Rightarrow$ The number of ways $= 5 \times 5 \times 4 = 100$

Case-III: when the last digit is 4.

The first digit has the remaining 5 choices (except 0 and 4), the second has 5, and the third has 4.

$\Rightarrow$The number of ways = $5 \times 5 \times 4 = 100$

Case IV: when the last digit is 6

The first digit has the remaning 5 choices (except 0 and 6), the second has 5, and the third has 4.

$\Rightarrow$The number of ways = $5 \times 5 \times 4 = 100$

So, The total number of ways in which 4 digits even number from given digit is formed = 120 + 100 + 100 + 100 = 420

$\therefore$ The correct answer is option (3).

Q7 Text Solution:

If $\quad {}^{18}C_{15} + 2\left({}^{18}C_{16}\right) +^{17} C_{16} + 1 =^{n} C_3$

$\Rightarrow \ {}^{18}C_{15} +^{18} C_{16} +^{18} C_{16} +^{17} C_{16} + 1$

$= \ {}^{n}C_3$

$\Rightarrow \ {}^{19}C_{16} +^{18} C_{16} +^{17} C_{16} + 1 \ = \ {}^{n}C_3$

$\Rightarrow \ {}^{19}C_{16} +^{18} C_{16} +^{17} C_{16} +^{17}C_{17}$

$= \ {}^{n}C_3$

$\Rightarrow \ {}^{19}C_{16} +^{18} C_{16} +^{18} C_{17} \ = \ {}^{n}C_3$

$\Rightarrow \ {}^{19}C_{16} +^{19} C_{17} \ = \ {}^{n}C_3$

$\Rightarrow \ {}^{20}C_{17} = \ {}^{n}C_3$

$\Rightarrow \ {}^{20}C_{3} = \ {}^{n}C_3$

$\Rightarrow \ n = \ 20$

$\therefore$The correct answer is option is (2).

Q8 Text Solution:

Given: 6 coats 5 belts and 4 caps

First man can wear any of the 6 coats.

Second man can wear any of the remaining 5 coats.

Third man can wear any of the remaining 4 coats.

Therefore the number of ways in which 3 men can wear 6 coats = $6 \times 5 \times 4 = 120$

Similarly,

Number of ways in which 3 men can wear 5 belts = $5 \times 4 \times 3 = 60$

Similarly,

Number of ways in which 3 men can wear 4 caps = $4 \times 3 \times 2 = 24$

Therefore, required number of ways = $120 \times 60 \times 24 = 172800$

Q9 Text Solution:

Six girls and five boys can sit in a row in 11 ! ways.

$\therefore$ exhaustive number of cases = 11 !

Six girls can sit in a row in 6 ! ways and in each such arrangement there are 7 places between them in which 5 boys can be seatd in

${}_{5}^{7}P$ ways.

Therefore, the total number of ways in which no two boys sit together = 6 ! ${}_{5}^{7}P$

Q10 Text Solution:

The word MISSISSIPPI has 11 letters. If the letters were all different there would have been 11! different permutations. But MISSISSIPPI has 4 S's, 4 I's, and 2 P's that are alike.

so the answer is $\frac{11!}{4!4!2!} = 34,650$

Q11 Text Solution:

Again, we have permutations with similar elements.

We are looking for permutations for the letters HHHHTT.

The answer is $\frac{6!}{4!2!} = 15.$

Q12 Text Solution:

The problem can be thought of as distinct permutations of the letters GGGYY; that is arrangements of 5 letters, where 3 letters are similar, and the remaining 2 letters are similar:

$\frac{5!}{3!2!} = 10$

Just to provide a little more insight into the solution, we list all 10 distinct permutations:

GGGYY, GGYGY, GGYYG, GYGGY, GYGYG, GYYGG, YGGGY, YGGYG, YGYGY, YYGGG

Q13 Text Solution:

we assume all the vowels to be a single character, i.e., "IE" is a single character. So, now we have 5 characters in the word, namely, D, R, V, R, and IE. But, R occurs 2 times. => Number of possible arrangements = 5!/2! = 60 Now, the two vowels can be arranged in 2! = 2 ways. => Total number of possible words such that the vowels are always together = 60 x 2 = 120, total number of possible words = 6!/2! = 720/2 = 360 .Therefore, the total number of possible words such that the vowels are never together is 240

Q14 Text Solution:

Let there are m boys and n girls.

So ${}^{m}C_2 = 190$ and ${}^{n}C_2 = 45 \Rightarrow$m = 20 and n = 10

So number of games when one player is boy & one player is girl = ${}^{20}C_1 \times {}^{10}C_1 = 200$

Q15 Text Solution:

No need to select 4 & 3 guest as they are given particular. similarly sides are also particular, so no need to select side also. Now 4 particular persons can be arranged in 9 places by ${}^{9}\mathrm{P}_4$ ways (on one side) and particular persons can be arranged in 9 places by ${}^{9}\mathrm{P}_3$ ways. (on other side). Now remaining 11 persons can be arranged in 11 places by 11! ways (as there is no restriction for them).

So Required Number of arrangements = ${}^{9}\mathrm{P}_4 \times {}^{9}\mathrm{P}_3 \times 11!$

Q16 Text Solution:

Total persons n = 10

According to question, Chairman & Vice-Chairman has to sit adjacently so overall we have 9 persons.

So Required Arrangements = (9 – 1)! × 2! = 2 × 8! (option b). [because we can arrange chairman & Vice-Chairman also]

Q17 Text Solution:

$\therefore$ f is onto & finite → it is one-one also. (By property of function)

So f is one-one/onto such that f(i) $\neq \mathrm{i} \forall \mathrm{i}$

So question is based on number of De-arrangement when n = 5

so required Answer = $5!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right]=44$

Q18 Text Solution:

For desired arrangements, we have following two cases:-

Either Case-I: $\mathrm{W} \times \mathrm{W} \times \mathrm{W} \times \rightarrow \frac{3!}{2!} = 3$ ways ($\because$out of remaining 3 flags two blue flags are alike)

Case-2: $\mathrm{W} \times \mathrm{W} \times \mathrm{W} \times \rightarrow \frac{3!}{2!} = 3$ ways (out of remaining 3 flags two blue flags are alike)

Now by F.P. of Addition:

Required number of arrangements = 3 + 3 = 6

[Since two blue and one red flags can be arranged by 3!/2! ways]

Q19 Text Solution:

Case-I : When both vowels and consonants are different = ${}^{5}C_2 \times {}^{21}C_2 \times 4!$

Case-II : When both vowels are alike & both consonants are alike = ${}^{5}C_1 \times {}^{21}C_1 \times \frac{4!}{2!2!}$

Case-III: When vowels alike but consonants are different = ${}^{5}C_1 \times {}^{21}C_2 \times \frac{4!}{2!} = 5 \times 210 \times 12$

Case-IV : When vowels are different but consonants alike = ${}^{5}C_2 \times {}^{21}C_1 \times \frac{4!}{2!} = 10 \times 21 \times 12$

So Total number of four letter words

${}^{5}C_2 \times {}^{21}C_2 \times 4! + {}^{5}C_1 \times {}^{21}C_1 \times \frac{4!}{2!2!} + 5 + \times 210 \times 12$

10 × 21 × 12 = 210 × 315.

Q20 Text Solution:

Given letters are MM, TT, EE, C, O, I

For four letter words, we have following cases:

Case-I : (2 alike & 2 alike) In this case, number of words = ${}^{3}C_2 \times \frac{4!}{2!2!} = 18$

Case-II : (2 alike & two different) In this case, number of words = ${}^{3}C_1 \times {}^{5}C_2 \times \frac{4!}{2!} = 360$

Case-III : (all different) In this case, number of words = ${}^{6}C_4 \times 4! = 360$

So total 4 letter words = 18 + 360 + 360 = 738.