

CS & IT ENGINEERING

THEORY OF COMPUTATION

Regular Language

Lecture No.- 02



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Recap of Previous Lecture



Topic

?????

Regular Language Detection

Topics to be Covered



Topic

✓ Regular Language Detection

Topic

Pumping Lemma

{ closure properties of Regular Languages }

$x, y \in (a+b)^*$

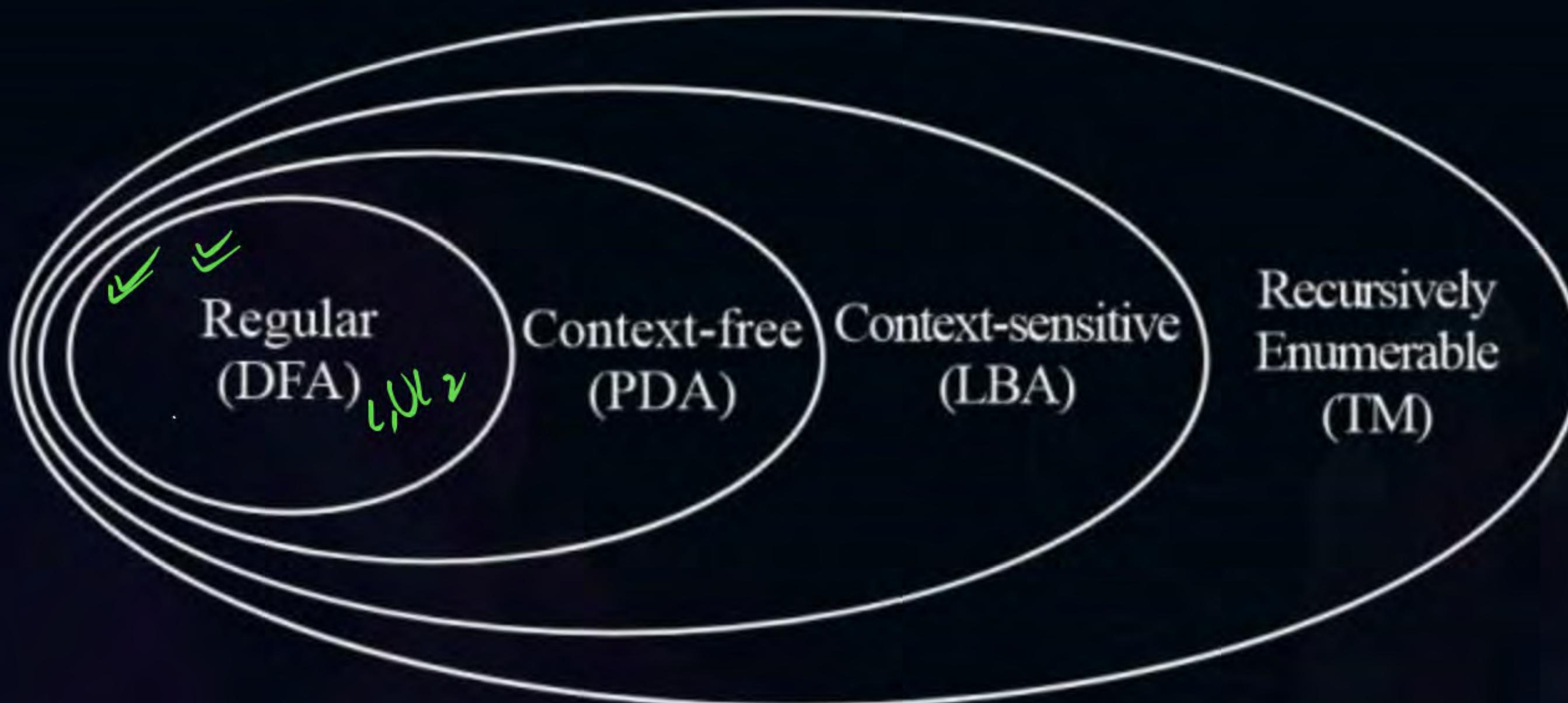
$$L_1 = \{x \$ y \mid n_a(x) = n_b(y)\}$$
 Non regular
$$L_2 = \{xy \mid n_a(x) = n_b(y)\} \Rightarrow (a+b)^* \Rightarrow \underline{\text{regular}}$$
$$x \in (a+b)^*$$



Topic : Theory of Computation



$L_1 \quad L_2$



① Subset op~~X~~

$$\overbrace{\{a^n b^n \mid n \geq 1\}}^{\text{a}^* b^*} \subset \overbrace{(a+b)^*}^{\text{closed?}}$$

Subset of regular lang by many (a) may not regular.
Hence regular language not closed under subset.

finite
② Union op

$F_1 \uparrow$
 L_1
 $|$
 $F_2 \uparrow$
 L_2

$L_1 \cup L_2 =$ always regular } closed
 $F_1 \times F_2$

(OR)

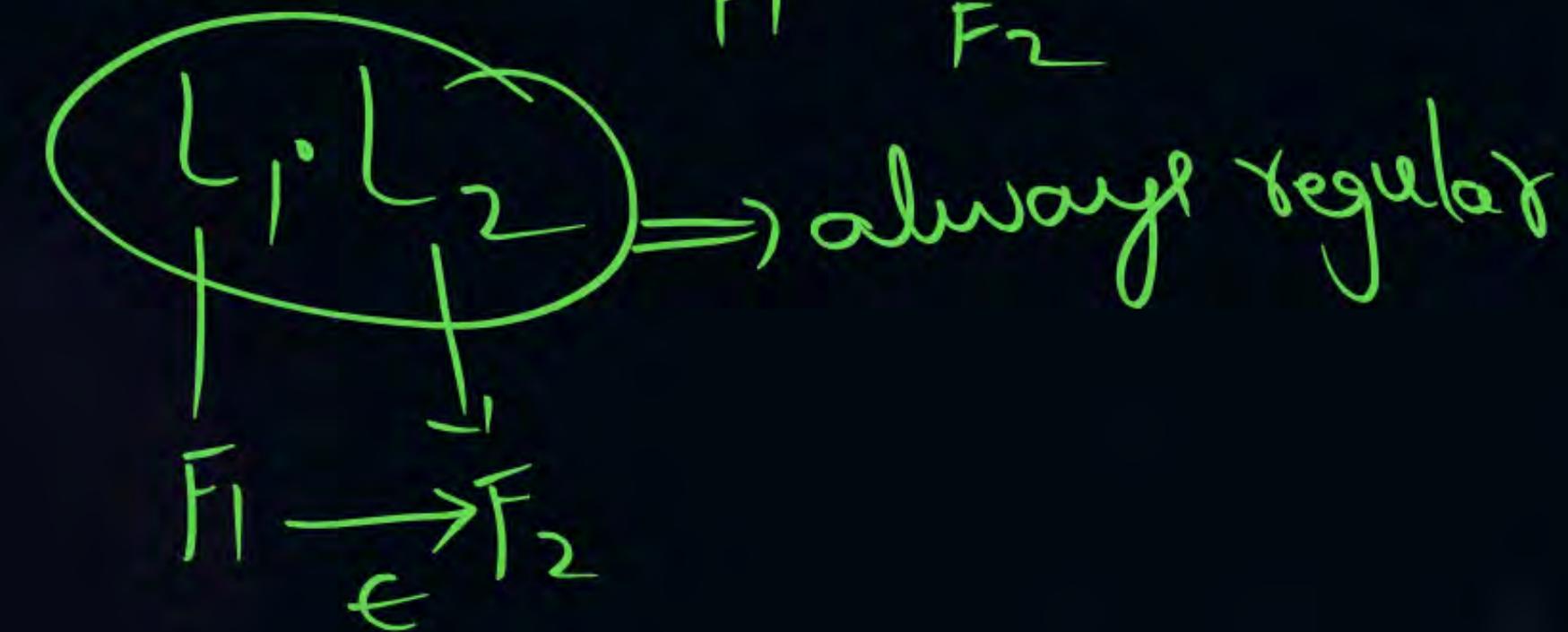
closed

{ a^n } { b^m }

③ Concatenation op

close)

$$\begin{matrix} L_1 \\ \vdash \\ F_1 \end{matrix} \quad = \quad \begin{matrix} L_2 \\ \vdash \\ F_2 \end{matrix}$$

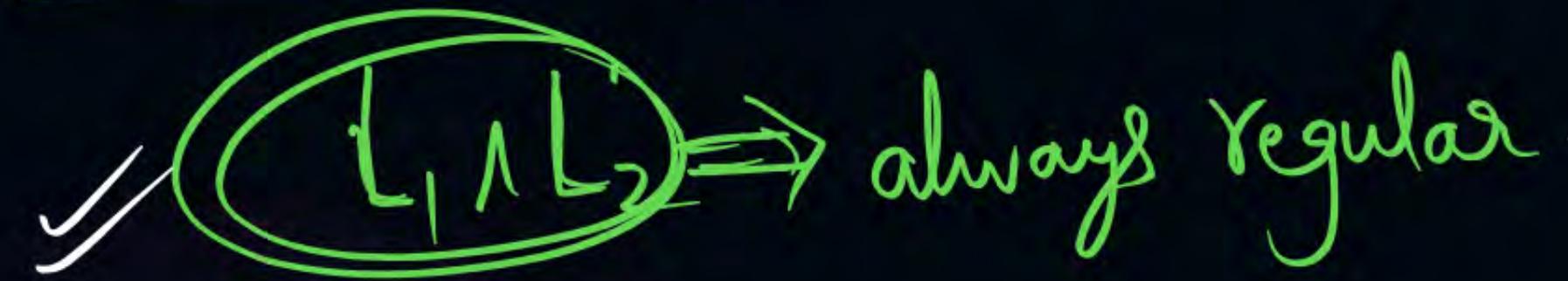


always regular

④

Intersection of $F_1 \uparrow$
 L_1 $F_2 \uparrow$
 L_2

(and)

 $L_1 \cap L_2 \Rightarrow$ always Regular $F_1 \times F_2$

{closed}

⑥ Infinite Intersection of (not closed)

Yes

No

$$\overline{L_1 \cap L_2 \cap L_3 \cap L_4 \cap \dots} = \overline{\bigcup_{i=1}^{\infty} L_i}$$

$$\overline{L_1 \cap L_2} = \overline{L_1} \cup \overline{L_2}$$

$$F_1 \times F_2 \times F_3 \dots \dots$$

not closed

⑦ Complement of

$\Sigma^* - L$

is also regular

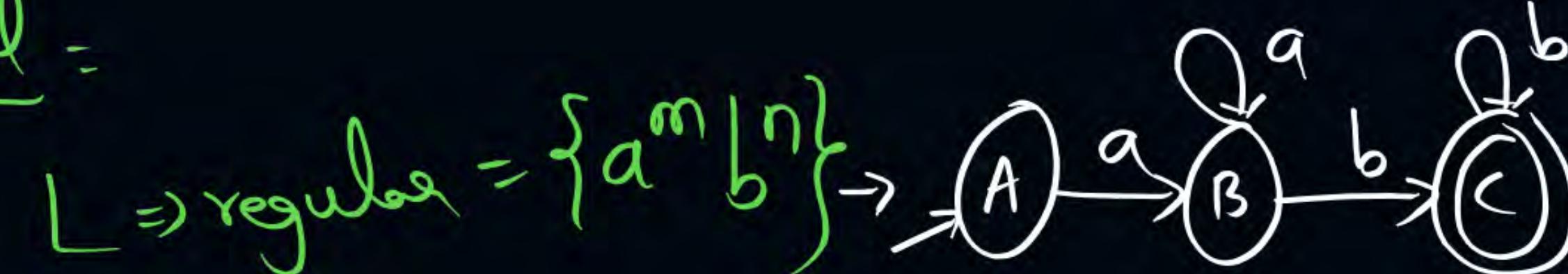
$$L = \text{regular} = F$$

$$L' = (\underline{\Sigma^* - L}) = F'$$

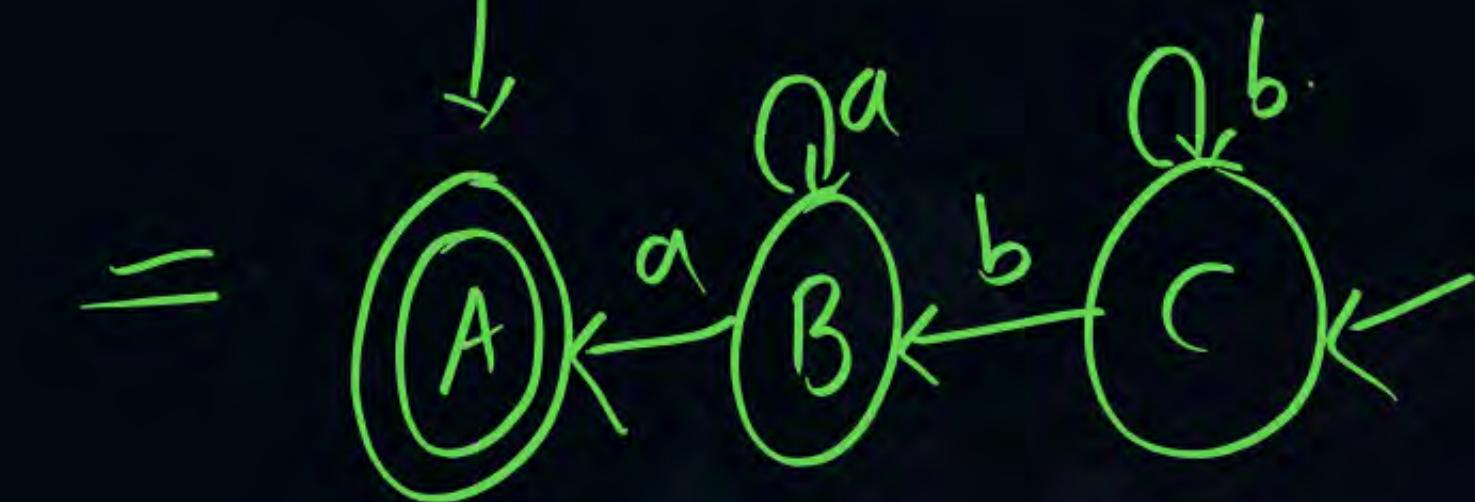
Interchange final and Non final states

⑧ Reversal - Closed

$$L \Rightarrow \text{regular} = \{a^m b^n\}$$



$$\underline{\text{regular}}^R = \underline{\underline{\{b^n a^m\}}}$$



Interchange Initial state and Final state
Reverse transition Direction

(9)

Kleene closureClosed $L \rightarrow \text{regular} = \emptyset$ $\vdash^* \Rightarrow \gamma^* \Rightarrow \text{always regular}$

(10)

Positive closure $L^+ : \gamma^+ \Rightarrow \text{always regular}$

⑪ Differentieel op

Closed

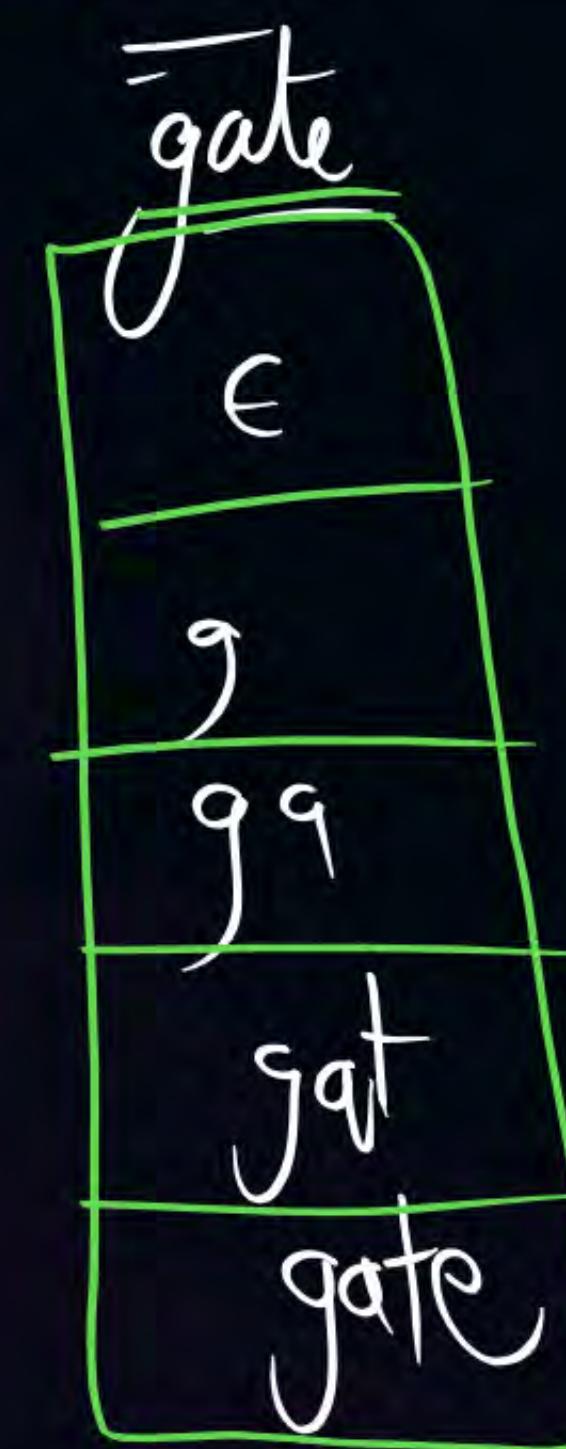
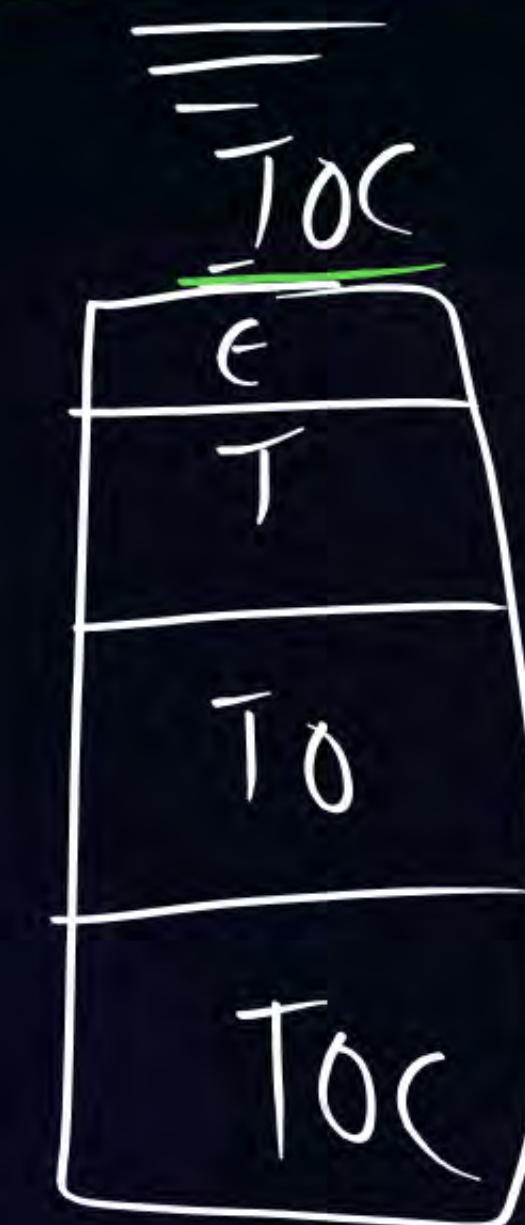
$L_1, L_2 \}$ regular

$$L - L_2 = L_1 \Delta L_2^c$$
$$= L_1 \Delta L_2$$

reg ∩ reg

⑫ Prefix op (closed)

Prefix of a string} Sequence of leading symbols over given string



- - - abc - - n

How many prefixes?
 $(n+1)$ prefixes

⑬ Suffixes (closed)

Suffix of a string:

Sequence of trailing symbols over given string

T
O
C

ε
c
o
T
O
C

gate

e

e

te

de

gate

abc...n

⑭ Suffixes

⑯ Substitution op

closed

$$\sum \xrightarrow{=} \Delta$$

$$\{a\} \xrightarrow{=} \{0, 1\}$$

$$L = \underline{aa}$$

$$S(L) = \underline{S(a)} \underline{S(a)}$$

$$S(L) = \underline{(0+1)} \underline{(0+1)} (0+1) \underline{(0+1)}$$

$$S(a) = \{00, 01, 10, 11\}$$

$$S(a) = \underline{(0+1)} \underline{(0+1)}$$

$$L = \underline{a}^*$$

$$S(L) = (S(a))^*$$

$$= \underline{(0+1)} \underline{(0+1)}^*$$

⑯ Homomorphism op} It is a substitution in which symbol is replaced by single string

↓
loop

$$L = a^* b^*$$

$$\begin{aligned} h(L) &= h(a)^* h(b)^* \\ &= (01)^* (00)^* \end{aligned}$$

$$\left. \begin{array}{l} h(a) = 01 \\ h(b) = 00 \end{array} \right\}$$

⑯ Inverse Homomorphism \Leftrightarrow { Applying Homomorphism in reverse)
String replaced by symbol.

Hom

$$L = \{ \underline{\underline{00}}, \underline{\underline{10}} \}$$

$$h^{-1}(L) = \{ \underline{\underline{a}}, \underline{\underline{b}} \}$$

$$\begin{array}{l} h(a) = \overset{\curvearrowleft}{00} \\ h(b) = \underline{\underline{10}} \end{array}$$

⑮ Homomorphism of

⑯ Inverse Homomorphism oP

⑤ Infinite Union of
L₁, L₂, L₃, ... = regular

$$L_1 \cup L_2 \cup L_3 \cup L_4 \cup \dots$$
$$\{ \underline{ab} \} \cup \{ \underline{a^2b^2} \} \cup \{ \underline{a^3b^3} \} \cup \{ \underline{a^4b^4} \} \cup \dots = \overline{\{ a^n b^n \}}$$

Normal

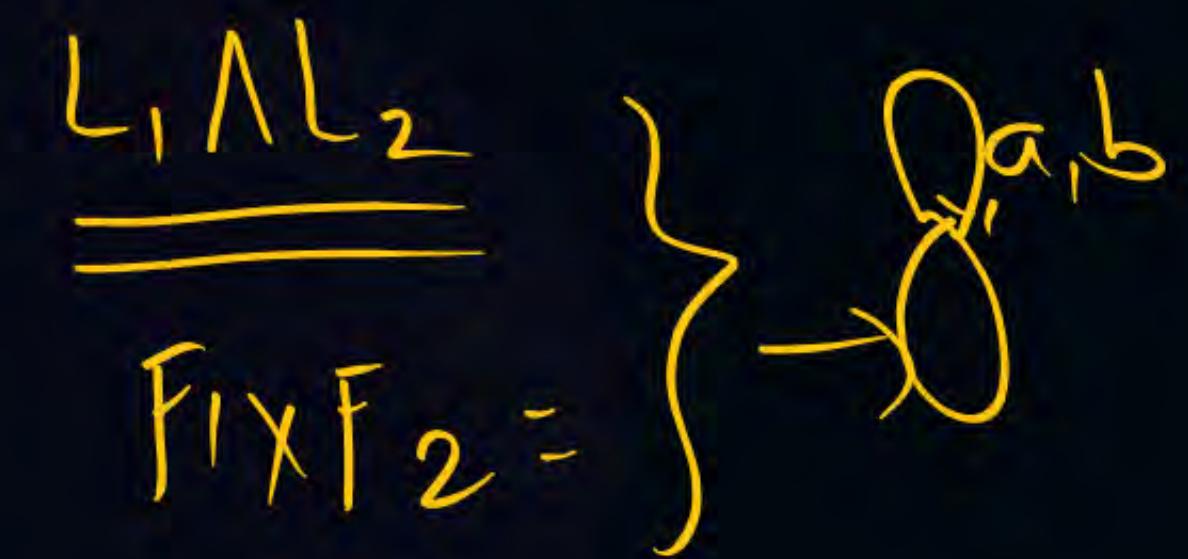
May (a) may not regular

not closed

(Q) How many states in $L_1 \cap L_2$ min DFA where

$$\emptyset \leftarrow L_1 = \frac{(a+b)^* a}{\quad} \rightarrow F_1$$

$$L_2 = \frac{(a+b)^* b}{\quad} \rightarrow F_2$$



(Q) Which of the following is true?

- a) Subset of $\{a^n b^n\} \subseteq \{a^n b^n c^m\}$ is Regular
- b) Subset of any Non regular language is Regular
- c) ~~Subset of any finite language is regular~~
- d) none



Topic : Pumping Lemma



$a^n b^n | n \geq 1$



#Q. To Prove a Language L is Non-Regular

- ① Assume L is Regular ✓
2. There exist F.A for L and n is no. of states in that F.A
3. Select some string W from L such that $|W| > n$.
4. Divide W into XYZ such that $|xy| \leq n$ and $|y| > 0$.
5. Find a suitable integer i such that UV^iW is not belongs to L.

Then L is not Regular.

Q

P
W

If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

- I. $L_1 \cdot L_2$ is a regular language ✓
II. $L_1 \cdot L_2 = \{a^n b^n \mid n \geq 0\} \rightarrow$ false

Which one of the following is CORRECT?

$a^* b^*$

Only I ✓

[2014-Set2: 1 Mark]

Only II

Both I and II

Neither I nor II

Q

P
W

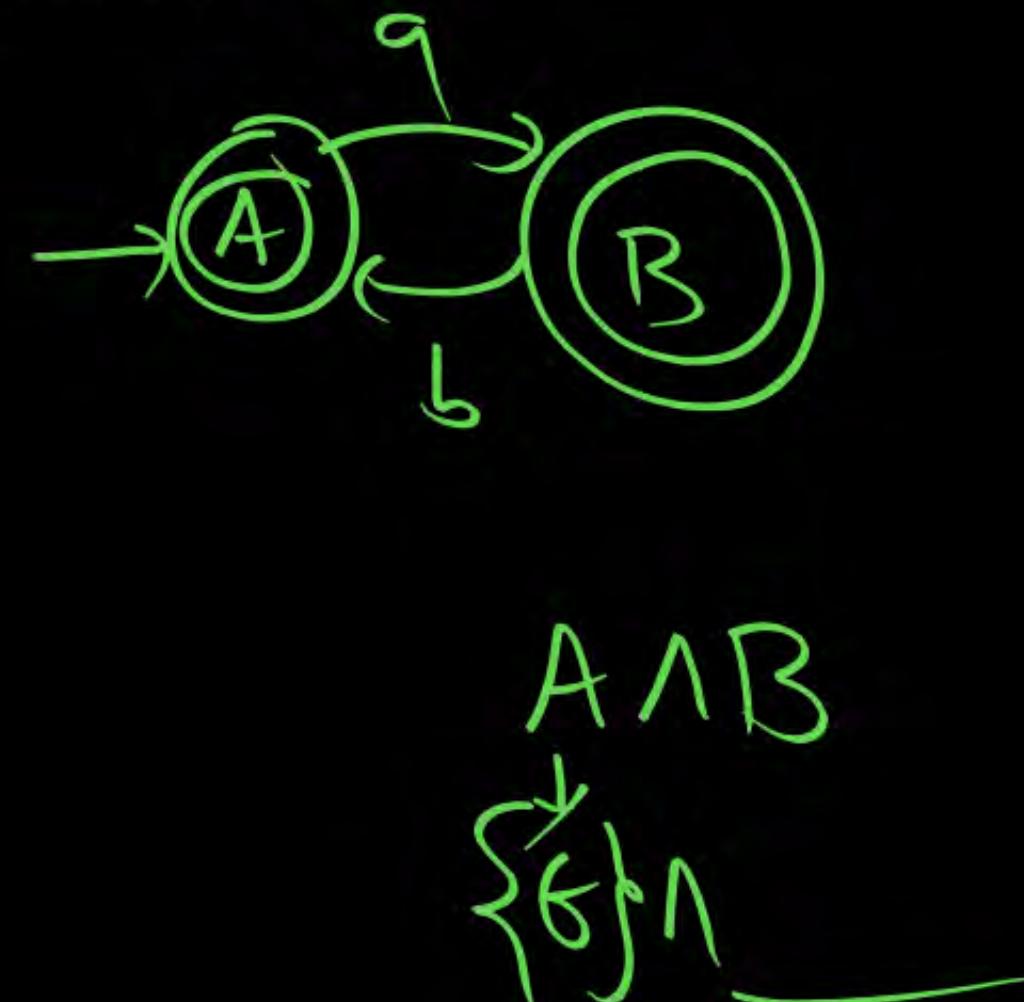
Consider the following two statements:

- I. If all states of an NFA are accepting states then the language accepted by the NFA is Σ^* → false
- II. There exists a regular language A such that for all language B, $A \cap B$ is regular. → true

[2016-Set2: 2 Marks]

Which one of the following is CORRECT

- A Only I is true
- B Only II is true
- C Both I and II are true
- D Both I and II are false





2 mins Summary



Topic One

Topic Two

Topic Three

Topic Four

Topic Five



THANK - YOU