



DS & AI
CS & IT

Linear Algebra

Lecture No. 07



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Recap of previous lecture



Topic

Non Homogeneous System of Equⁿ

Topics to be Covered



Topic

- Homogeneous System of Equⁿ
- Basics of Eigen Values.

Homogeneous system of linear Equ'

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eg $\begin{cases} 2x - y + 4z = 0 \\ x + 2y - 2z = 0 \\ -x - y + 3z = 0 \\ x + 2y - 4z = 0 \end{cases} \Rightarrow \begin{bmatrix} 2 & -1 & 4 \\ 1 & 2 & -2 \\ -1 & -1 & 3 \\ 1 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow A_{4 \times 3} X_{3 \times 1} = 0_{4 \times 1}$

By observation, we can see that $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is a solution of above system.
because these values of x, y, z satisfying all 4 equ' at a time.

& this type of solution always exist. & this sol. is known as TRIVIAL sol.

(*) Homog system is always consistent OR Homog system Never inconsistent.
because Reason 1: Trivial sol always exist.

Reason 2: $P(A) = P(A: 0)$ always. Hence there is No Need to write Aug Mat further.

why zero vector is treated as LD vector →

vectors $x_1, x_2, x_3, \dots, x_r$ are called LD if \exists a relationship of the type $k_1x_1 + k_2x_2 + \dots + k_rx_r = 0$ ————— ①

where $k_1, k_2, k_3, \dots, k_r$ should not all zero simultaneously.

Eg(i) Let us consider $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ then we have

$$2x_1 + x_2 + x_3 = 0 \text{ i.e } x_1, x_2, x_3 \text{ are LD}$$

Eg(ii) Let us consider $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ then we have

$$0x_1 + 0x_2 + kx_3 = 0 \text{ i.e relationship exist where not}$$

all constants are zero Hence x_1, x_2, x_3 are LD

Eg(iii) Let us consider $x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ then we can write

$$kx = 0 \text{ where } k \text{ is non zero} \text{ Hence relationship again exist so } x \text{ is LD}$$

Note: ① underdetermined, Homog system always consist ∞ solutions

$$\text{unique sol} \Leftrightarrow \text{DNE} \quad \text{No sol} \Leftrightarrow \text{DNE}$$

② unique sol \Leftrightarrow Trivial sol \equiv zero solution (always exist)

③ ∞ sol \Leftrightarrow Non Trivial sols also exist \equiv Non zero but also exist.

④ ZERO sol \neq No sol.

$$\therefore X = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

Methods of solving Homog. system

$$(A_{m \times n} X_{n \times 1} = O_{m \times 1})$$

RANK Method

($m > n, m = n, m < n$)

- | | |
|--|---|
| ① If $\beta(A) = \text{No. of Variables} \Rightarrow$ unique soln.
② If $\beta(A) < \Rightarrow \infty$ soln. | ① If $ A \neq 0 \Rightarrow$ unique soln exist
② If $ A = 0 \Rightarrow \infty$ soln exist |
|--|---|

Note - ① Unique soln = Trivial soln. = zero soln. always exist.
 ② ∞ soln = Non Trivial soln = Non zero soln always exist.

Q Find k for which

$$\text{MSQ} \quad (3k-8)x + 3y + 3z = 0$$

$$3x + (3k-8)y + 3z = 0$$

$$3x + 3y + (3k-8)z = 0$$

has Non Trivial soln.

- (a) $\frac{2}{3}$ (b) $\frac{11}{3}$ (c) $\frac{4}{3}$ (d) $\frac{8}{3}$ (e) $3k-2$

for Non Trivial soln $\Leftrightarrow |A| = 0$

$$\begin{vmatrix} (3k-8) & 3 & 3 \\ 3 & (3k-8) & 3 \\ 3 & 3 & (3k-8) \end{vmatrix} = 0 \rightarrow \begin{vmatrix} (3k-2) & 3 & 3 \\ (3k-2) & (3k-8) & 3 \\ (3k-2) & 3 & (3k-8) \end{vmatrix} = 0$$

$$(3k-2) \begin{vmatrix} 1 & 3 & 3 \\ 1 & (3k-8) & 3 \\ 1 & 3 & (3k-8) \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 3 & 3 \\ 0 & (3k-11) & 0 \\ 0 & 0 & (3k-11) \end{vmatrix} = 0 \Rightarrow (3k-2)(3k-11)^2 = 0$$

$$k = \frac{2}{3}, \frac{11}{3}, \frac{11}{3}$$

The value of α for which the system of equation

$$x + y + z = 0$$

$$y + 2z = 0$$

$$\alpha x + z = 0$$

PYB

has more than one solution is

(a) -1

(c) $\frac{1}{2}$

(b) 0

(d) 1

$$|A| = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ \alpha & 0 & 1 \end{vmatrix} = 0$$

$$1[1-0] - 0 + \alpha[2-1] = 0$$

$$1 + \alpha = 0$$

$\alpha = -1$

M-II

Expanding along R_3

$$\alpha[2-1] - 0 + 1[1-0] = 0$$

$$\alpha + 1 = 0 \Rightarrow \alpha = -1$$

MW8

for the system to have Infinite sol, which of the following is/are true

MS8

$$px + qy + rz = 0$$

$$qx + ry + pz = 0$$

$$rx + py + qz = 0$$

~~a~~ $p = q = r$ (By observation)

~~b~~ $p + q + r = 0$

~~c~~ $p^3 + q^3 + r^3 - 3pqr = 0$

~~d~~ None

$$|A| = 0 \Rightarrow \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$$

$$\therefore p^3 + q^3 + r^3 - 3pqr = 0 \quad \text{Ans}$$

$$(p+q+r) \cdot [(p-q)^2 + (q-r)^2 + (r-p)^2] = 0$$

ie $p+q+r=0$ or $p=q=r$ Ans

~~Q8~~ Consider $(A_{m \times n} X_{n \times 1} = B_{m \times 1})$ then which one is false ?

- (a) if $m > n$, $B \neq 0$, $\rho(A) < \rho(A; B)$ then system has NO sol. (T)
- (b) if $m = n$, $B = 0$, $|A| \neq 0$ then system has only Trivial sol. (T)
- (c) if $m = 3$, $n = 5$, $B = 0$ then system has also Non Zero sol. (T)
- (d) if $m = 5$, $n = 3$, $B = 0$ & $\rho(A) = 3$ then system has only zero sol (T)
- (e) if $m = n$, $B = 0$, then system has sol. (T) (obviously True)
- (f) if $m > n$, $B = 0$ then system has Multiple sol. (False)
- (g) if $m > n$, $B = 0$, $\rho(A) < n$ then system has Multiple sol. (True)
- (h) Sun Rises from east (T) (obviously True)

Analysis (PODCAST) → Consider $A_{1 \times 3} X_{3 \times 1} = 0_{1 \times 1}$ →

$$A_{1 \times 3} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}_{1 \times 1}$$

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i.e $A_{1 \times 3} X_{3 \times 1} = 0_{1 \times 1}$ then if '8 solns' are;

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} 7/2 \\ 0 \\ 7/2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}, \dots \dots \dots \infty \text{ solns.}$$

Trivial sol.

Non Trivial solns

Null space

X

$X_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 1/3 \\ 0 \end{bmatrix}, \begin{bmatrix} -0.2 \\ +0.2 \\ 0 \end{bmatrix} \dots \dots \text{But all are L.D. on } X_1$

$X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} -\sqrt{5} \\ 0 \\ \sqrt{5} \end{bmatrix}, \dots \dots \text{Not all are L.D. on } X_2$

$X_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = X_1 + X_2, X_4 = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} = 2X_1 + X_2 \dots \text{i.e again all are L.D.}$

i.e. out of ∞ solns only x_1 & x_2 are L.I & rest are L.D on them.

Nullity (Dimension of Nullspace) \rightarrow

"the Counting of L.I solns in any Homog system $A_{m \times n} X_{n \times 1} = O_{m \times 1}$ " is called Nullity & it is given as $N(A)$.

Rank-Nullity Th.: Nullity = Number of columns - Rank(A)

$$\text{i.e. } N(A) = N(\text{columns}) - R(A)$$

e.g. find Nullity of $\begin{bmatrix} x+y+z=0 \\ x+y=0 \end{bmatrix}$? Here $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{2 \times 3}$ & $R(A) = 1$.

$$\text{Hence Nullity}(A) = 3 - 1 = 2$$

Is above system has two L.I solns & rest are L.D on them.

PODCAST:

Consider $(A_{m \times n} X_{n \times 1} = O_{m \times 1})$

Row Space: Set of all linear combinations of Row vectors of A is called Row space

& Dimension of Row Space = Max No. of L.I Row vectors = $\delta(A)$

Column Space (Range Space) Set of all linear combinations of Column vectors of A is called Column space

& Dimension of Column space = Max No. of L.I Column vectors = $\delta(A)$

Null Space: Set of all solutions of $AX=0$ is called Null Space

& Dimension of Null Space = Nullity - No. of L.I equations in terms of X.

⑥ $N(A) \perp R(A)$ i.e Row Space is \perp to Null Space (Null Space)

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"Every vector in Null Space is orthogonal to every vector in Row space"

$$\because AX = 0 \Rightarrow \begin{bmatrix} a_1 & a_2 & a_3 & \dots & a_n \\ b_1 & b_2 & b_3 & \dots & b_n \\ c_1 & c_2 & c_3 & \dots & c_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d_1 & d_2 & d_3 & \dots & d_n \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{mx1}$$

$$a_1n_1 + a_2n_2 + \dots + a_nn_n = 0 \text{ ie } R_{1 \times n}^{mxn} \cdot \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_n \end{bmatrix} = 0$$

again $b_1n_1 + b_2n_2 + \dots + b_nn_n = 0 \text{ ie } R_{2 \times n}^{mxn} \cdot \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_n \end{bmatrix} = 0$ ie $R_2 \nparallel X$ are orthogonal

& so on.... i.e hence proved.

Ques If every soln of system $A_{4 \times 5} X = 0$ ① is a scalar multiplication of $\begin{bmatrix} 2 \\ -3 \\ 1 \\ 2 \end{bmatrix}$ then $\beta(A) = ?$

ATQ, No. of LI solns of ① = one = $N(A)$

By Rank-Nullity Th. ② $N(A) = N(C) - \beta(A)$

$$1 = 5 - \beta(A) \Rightarrow \beta(A) = 4 \quad A_n$$

Solution set $X = \begin{bmatrix} 2 \\ -3 \\ 1 \\ 2 \end{bmatrix} + \text{span} \left\{ \begin{bmatrix} 4 \\ -6 \\ 8 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -4 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1.5 \\ 2 \\ 0.5 \end{bmatrix}, \dots \right\}$.

The nullity of system of equations:

$$\left. \begin{array}{l} x_1 + x_2 - x_3 + x_4 = 0 \\ 2x_1 + 3x_2 + x_3 + 4x_4 = 0 \\ 3x_1 + 2x_2 - 6x_3 + x_4 = 0 \end{array} \right\}$$

- (a) 1
 (b) 2
 (c) 3
 (d) 4

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 1 & 4 \\ 3 & 2 & -6 & 1 \end{bmatrix} \xrightarrow{\substack{R_2 - 2R_1 \\ R_3 - 3R_1}} \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & -1 & -3 & -2 \end{bmatrix}$$

$$S(A) = 2 \quad N(\text{Columns}) = 4$$

$$\therefore N(A) = N(C) - S(A) = 4 - 2 = 2$$

$$A_{3 \times 4} X_{4 \times 1} = [0]_{3 \times 1} \Rightarrow X = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} = \begin{array}{l} \text{only two Non zero} \\ \text{Rest are LD on them.} \end{array}$$

The number of linearly independent solutions of
the system of equations

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{0} \text{ is equal to}$$

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{pmatrix}$$

$$\because |A|=0 \Rightarrow f(A) \neq \emptyset$$

$$\text{ie } g(A) = 2$$

$$N(A) = N(C) - \rho(A) \\ = 3 - 2 = 1.$$

Ques Solve $\begin{cases} 2x+y+z=0 \\ x+y=0 \\ x+z=0 \end{cases}$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{\text{Row Operations}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{Row Operations}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$R(A) = 2 < \text{No of Variables}$

$\Rightarrow \infty \text{ sol exist.}$

$\& N(A) = 3 - R(A) = 3 - 2 = 1$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (x+y) \\ (-y+z) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x+y=0 \Rightarrow x=-y$$

$$-y+z=0 \Rightarrow y=z \quad \text{Let } z=k, \text{ then } y=k$$

$$\text{So } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k \\ k \\ k \end{bmatrix} = k \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \quad k = \text{arbitrary const.}$$

i.e. ∞ sol exist which are $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ (1)

Eigen Values / Eigen Vectors (Now they will ask Q.)

PODCAST

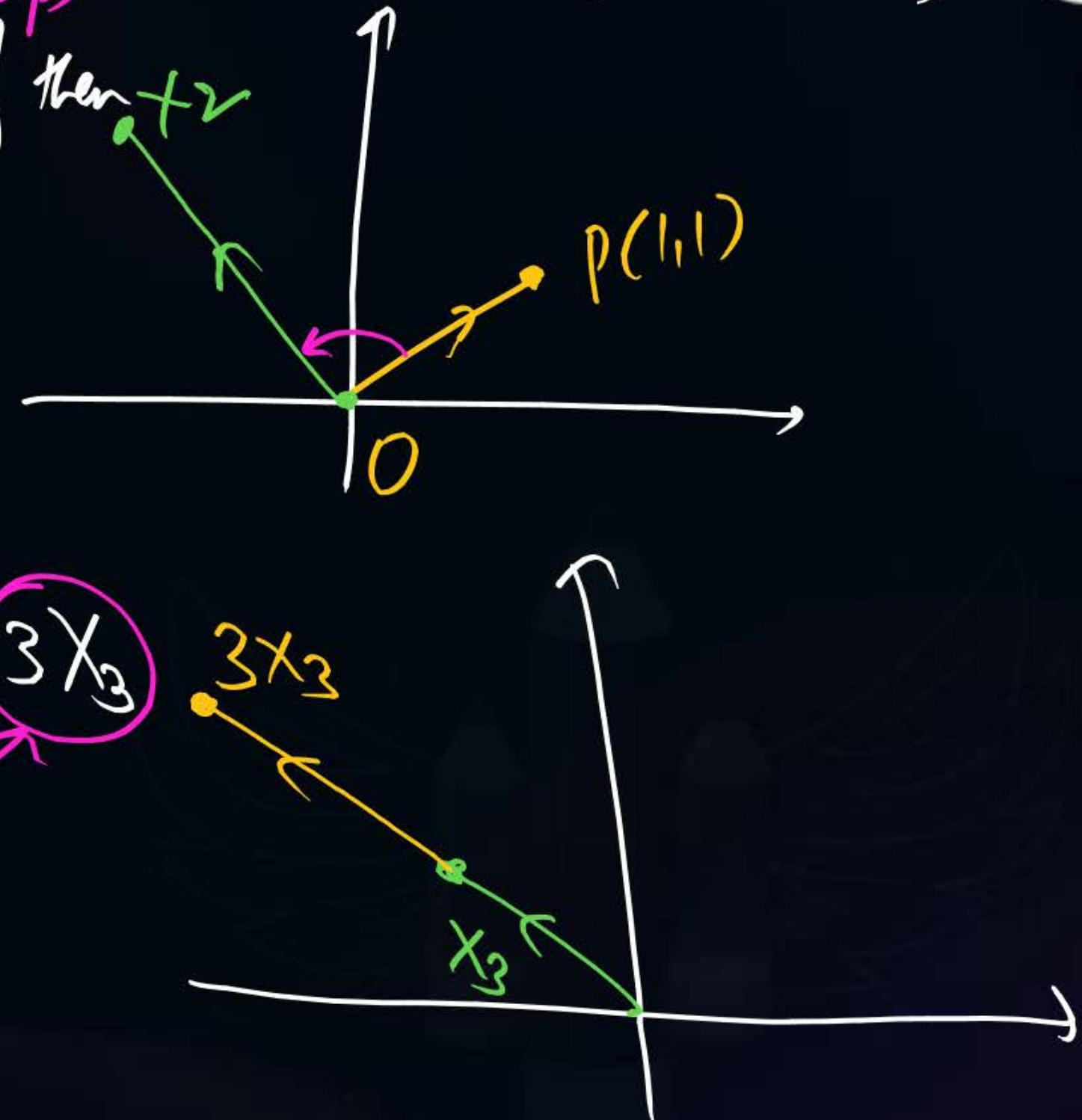
Consider $A = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$ & $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ then

$$AX_1 = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix} = x_2$$

again Consider $x_3 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

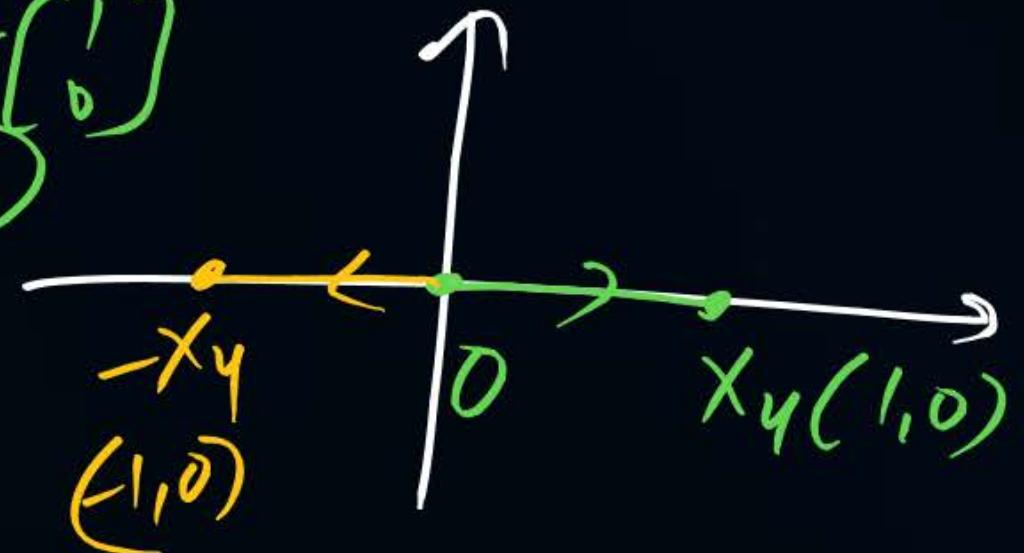
$$AX_3 = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix} = 3 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = 3x_3$$

$x_3 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ is a vector & its value = 3



Now consider $A = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$, $x_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ Sp. vector = $x_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ & Sp. value = -1

$$AX_4 = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -x_4$$



Consider $A = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$, $x_5 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow AX_5 = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = x_5$

But x_5 is not an eigen vector \therefore it is a zero vector.
i.e. E vector can not be zero vector.

Consider $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $x_6 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ then $AX_6 = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 0 \cdot X_6$

So x_6 is also Special vector where Sp. Value = 0

i.e. E value can be zero also.

~~Def^{n!}~~ Consider Sq. Mat Anxn then Non Zero Vector X is called

Eigen Vector, Corresponding to Eigen Value λ (Real/Complex/zero)
 if we are able to find a relationship of the type,

$$\boxed{AX = \lambda \cdot X} \quad \begin{cases} \lambda = \text{Eigen value} \\ X = \text{Eigen Vector} \end{cases}$$

LHS is the Multi of Two Matrices = RHS is the Scalar Multi in a Mat
 (Tough) (Easy)

⊗ Here we are considering Homogeneous system as follows

$$AX = \lambda X \Rightarrow AX - \lambda X = 0 \Rightarrow \boxed{(A - \lambda I)X = 0}$$

So it will satisfy all the prop of Homog system.

* Consider $AX = \lambda X$

$$(A - \lambda I)X = 0 \quad \text{①}$$

$$MX = 0$$

Non zero Eigen Vector

Non zero solution

$\Rightarrow \infty \text{ R.S.}$

$$\rho(M) < n \text{ or } |M| = 0$$

$$\Rightarrow \rho(A - \lambda I) < n \text{ or } |A - \lambda I| = 0$$

So Necessary Condition for the existence
of Non zero Eigen Vector is

$$\rho(A - \lambda I) < n \text{ or } |A - \lambda I| = 0 \quad \text{②}$$

Characteristic Eqn of A →

eqn ② is called C.Eq of A & Roots of
this eqn ie values of λ are called
E. Values / E. Roots / Char Values
/ Char Roots / Latent Roots / Sp. Values

Ex: Find the E-values of $\textcircled{1} A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\textcircled{2} A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\textcircled{3} A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

$$\textcircled{1} A - \lambda I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (2-\lambda) & 1 & 1 \\ 1 & (2-\lambda) & 1 \\ 0 & 0 & (\lambda) \end{bmatrix}$$

$\neq C\text{-Equ}^n$ But Char Mat

Now C-Equ's $|A - \lambda I| = 0$

$$\begin{vmatrix} (2-\lambda) & 1 & 1 \\ 1 & (2-\lambda) & 1 \\ 0 & 0 & (\lambda) \end{vmatrix} = 0$$

$$(1-\lambda)[(2-\lambda)^2 - 1] = 0$$

$$(1-\lambda)[(2-\lambda+1)(2-\lambda-1)] = 0$$

$$\boxed{(1-\lambda)(\lambda-1)^2 = 0}$$

$(\lambda-3)(\lambda-1)^2 = 0$ \rightarrow (Eqn of A)

& $\lambda = 3, 1, 1 \rightarrow$ Char Values.

$$\textcircled{2} \quad A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ then}$$

C.Equⁿ is $|A - \lambda I| = 0$

$$\begin{vmatrix} (0-\lambda) & -1 \\ 1 & (0-\lambda) \end{vmatrix} = 0$$

$$\lambda + 1 = 0 \rightarrow \text{C.Equ}^n$$

$$\lambda = 1, -1 \quad E \text{ Values}$$

$$\textcircled{3} \quad A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

C.Equⁿ is $|A - \lambda I| = 0$

$$\begin{vmatrix} (8-\lambda) & 6 & 2 \\ -6 & (7-\lambda) & -4 \\ 2 & -4 & (3-\lambda) \end{vmatrix} = 0$$

HW

$$\lambda(\lambda-3)(\lambda-15) = 0 \rightarrow \text{C.Equ}^n$$

$$\lambda = 0, 3, 15 \rightarrow E \text{ Values}$$



THANK - YOU

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