CS & IT ENGINEERING

Algorithms

Graph Algorithms

Lecture No.- 03

By- Aditya Jain sir

Recap of Previous Lecture

Topic

Topic

DFS on Directed.

→ DAG

→ Topological ordering

Topics to be Covered

- Topic
- Topic — Applications of DFS
- Topic

About Aditya Jain sir

1. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
2. Represented college as the first Google DSC Ambassador.
3. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored 12,000+ students & working professions in field of Data Science and Analytics
11. Have been mentoring & teaching GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

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Topic : Graph Algorithms

Time complexity:-

1. Time complexity of both DFS & BFS dependents on how the graph is represented.

   (a) Adjacency matrix: $O(n^2)$

   (b) Adjacency list: $O(n + e)$

   n = number of vertices

   e = number of edges

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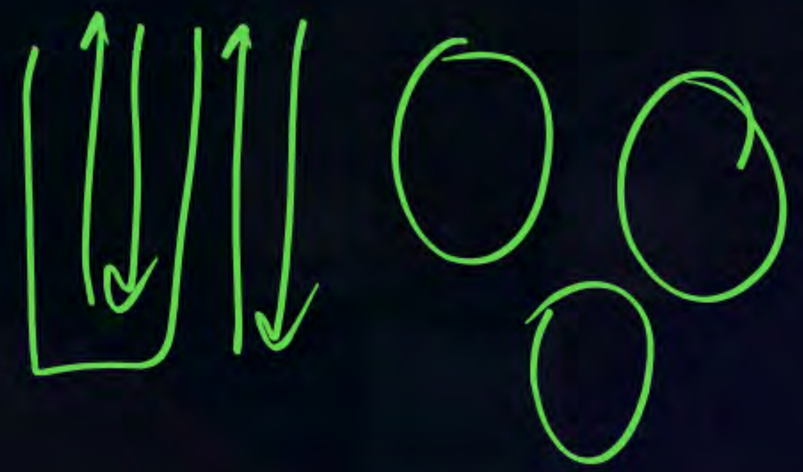
Application of DFS and BFS:

1. Both DFS and BFS can be used to determine whether the graph is connected on disconnected.
2. Both DFS and BFS can be need to check/ detect whether the two vertices u and v connected or not.
3. Cycle detection: Both DFS and BFS can be used to detect the presence of a cycle in the given graph.



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4. DFS can also be used to determine:

   (a) Connected components in a graph→ [undirected graph] (CC)
   (b) Strongly connected components → [directed graphs] (SCC)
   (c) Bi- connected components (BCC)
   (d) Articulation points/cut vertex. (AP)

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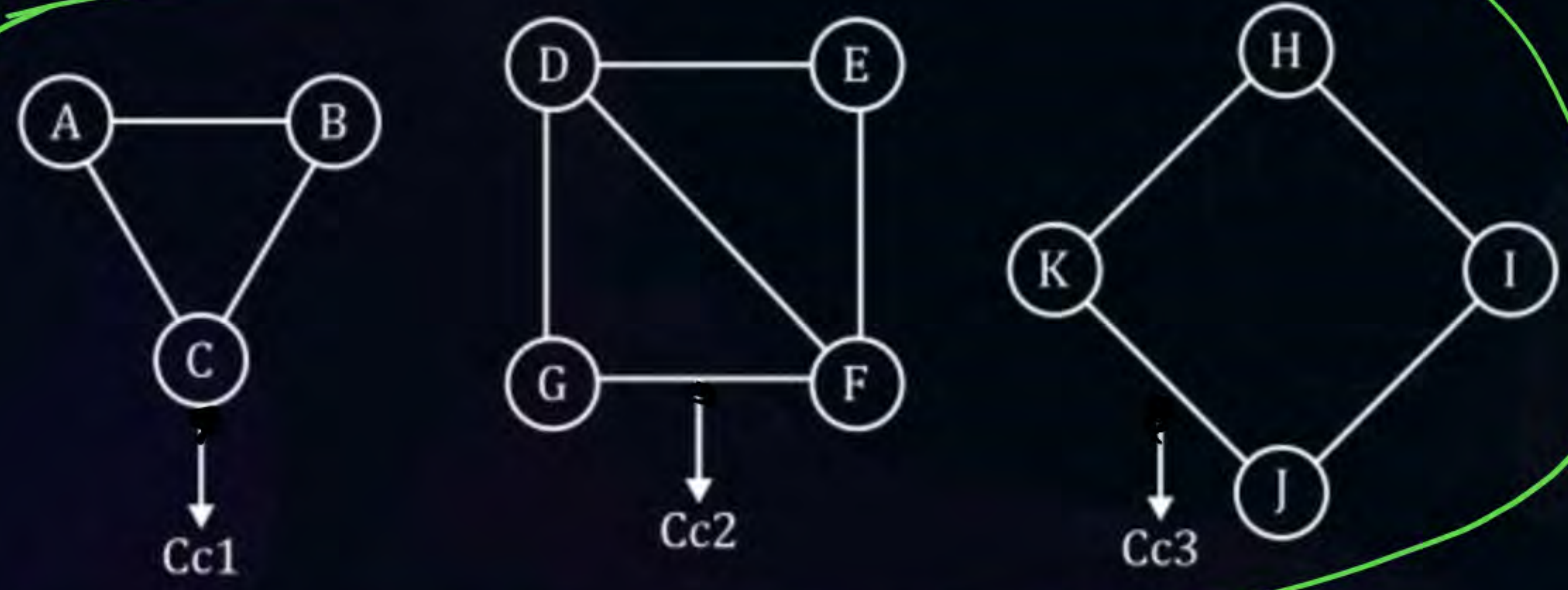
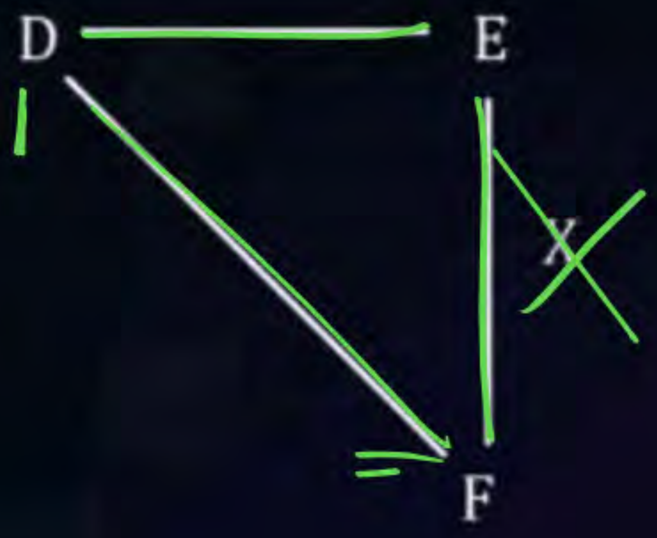
1. Connected components (CC) → [undirected graphs]

Maximal subgraph that is connected.

Given: G(V,E)

G(V,E)

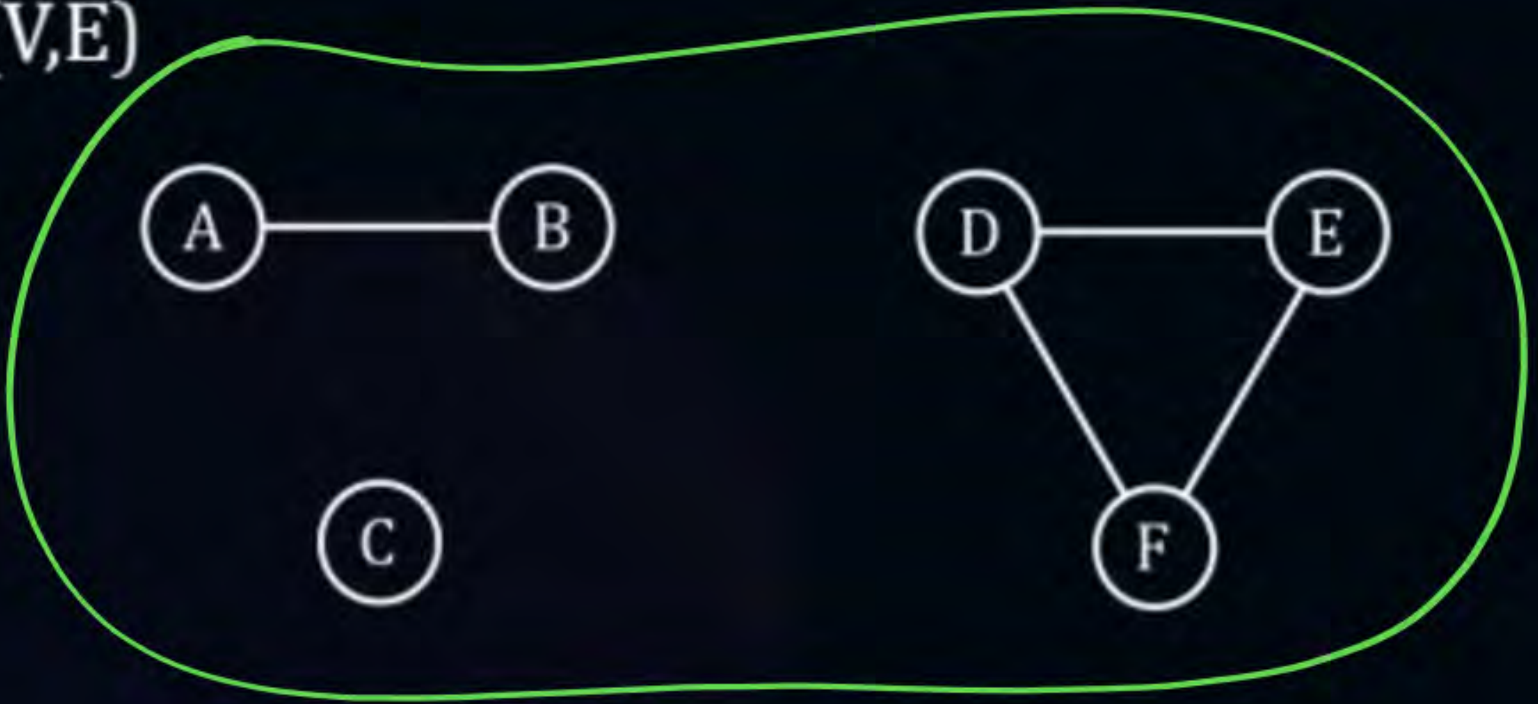


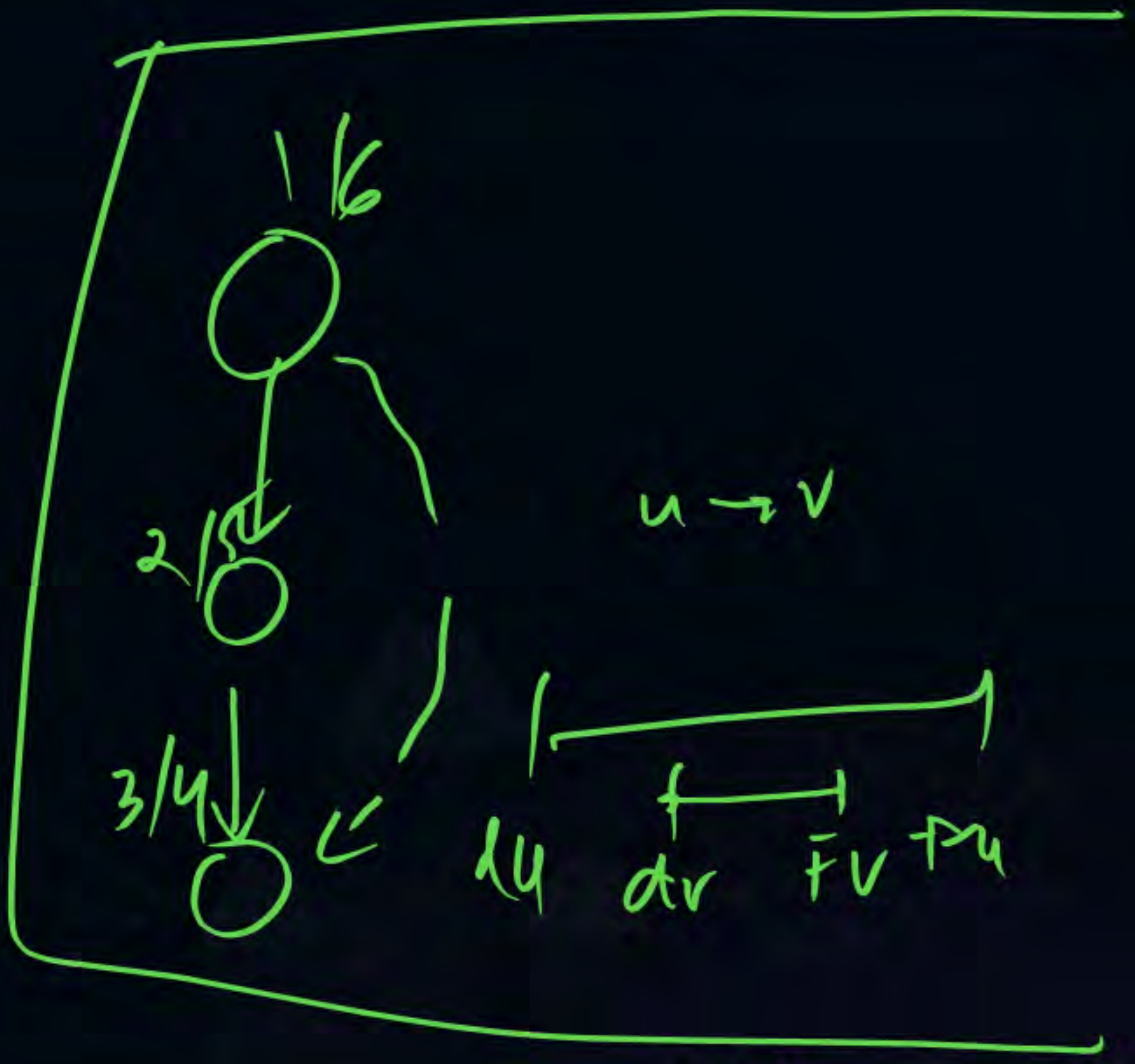


3 connected components

E.g. 2.

G(V,E)





3 connected components

Single vertex can also be connected components

doubt

2. Strongly connected components (SCC)

Directed graph

V= [set of all vertices in G]

Definition:-

1. Two nodes 'x' and 'y' of directed graph are connected, if there is a path from 'x' to 'y' and there is also a path 'y' to 'x'.

$x \rightleftarrows y$

2. The relation, partitions the vertex set V into disjoint set which are known as strongly connected components (SCC).

$S_1 \cap S_2 = \phi$

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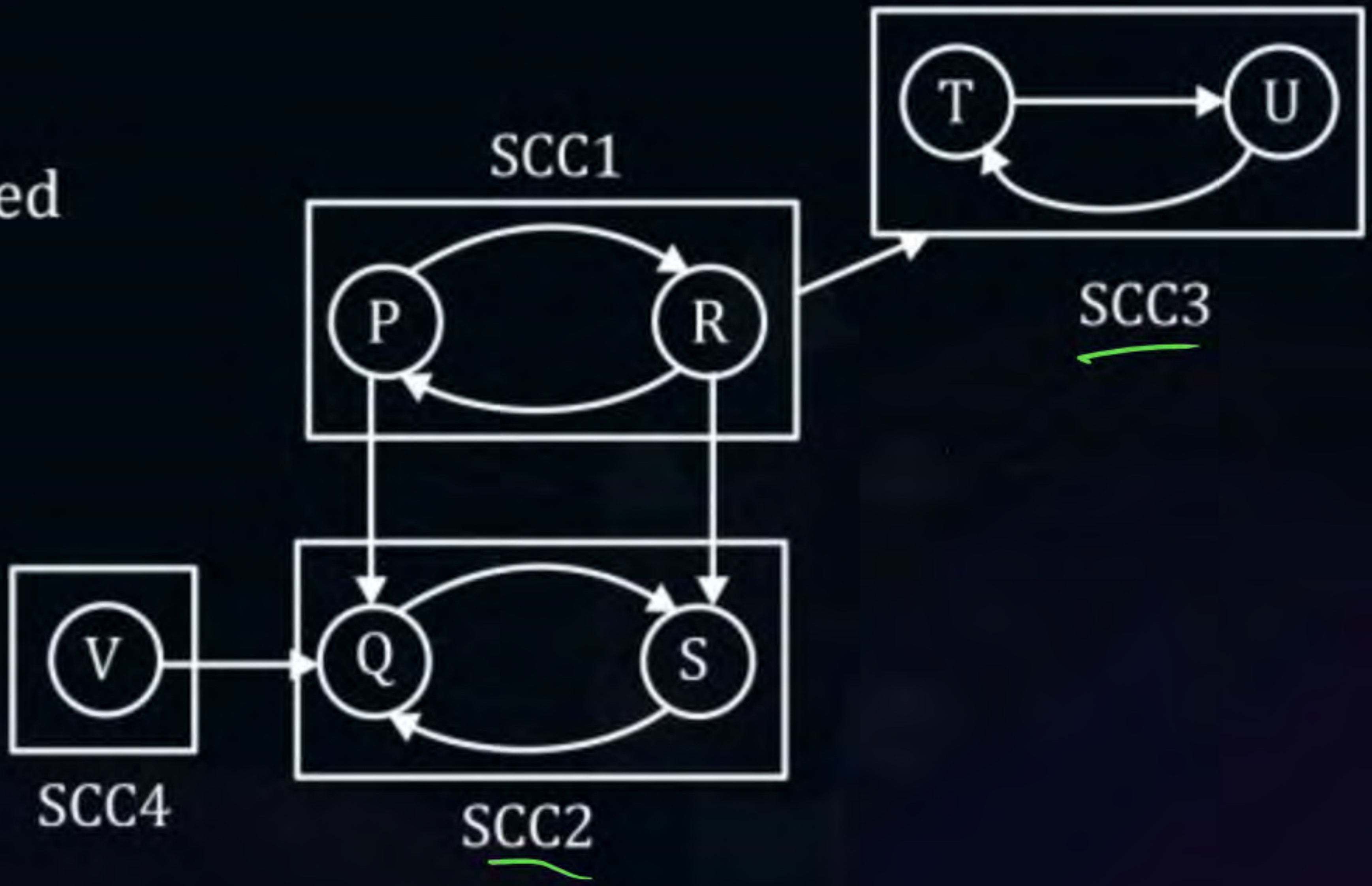
E.g.3. Given: (V,E)

Strongly connected components

↓

CC

maximal subgraph that is connected



In above eg. There are 4 strongly connected components.

SCC1	SCC2	SCC3	SCC4
{P,R}	{Q,S}	{T,U}	{V}

Combine then you get set of all vertices
{P,Q,R,S,T,U,V}

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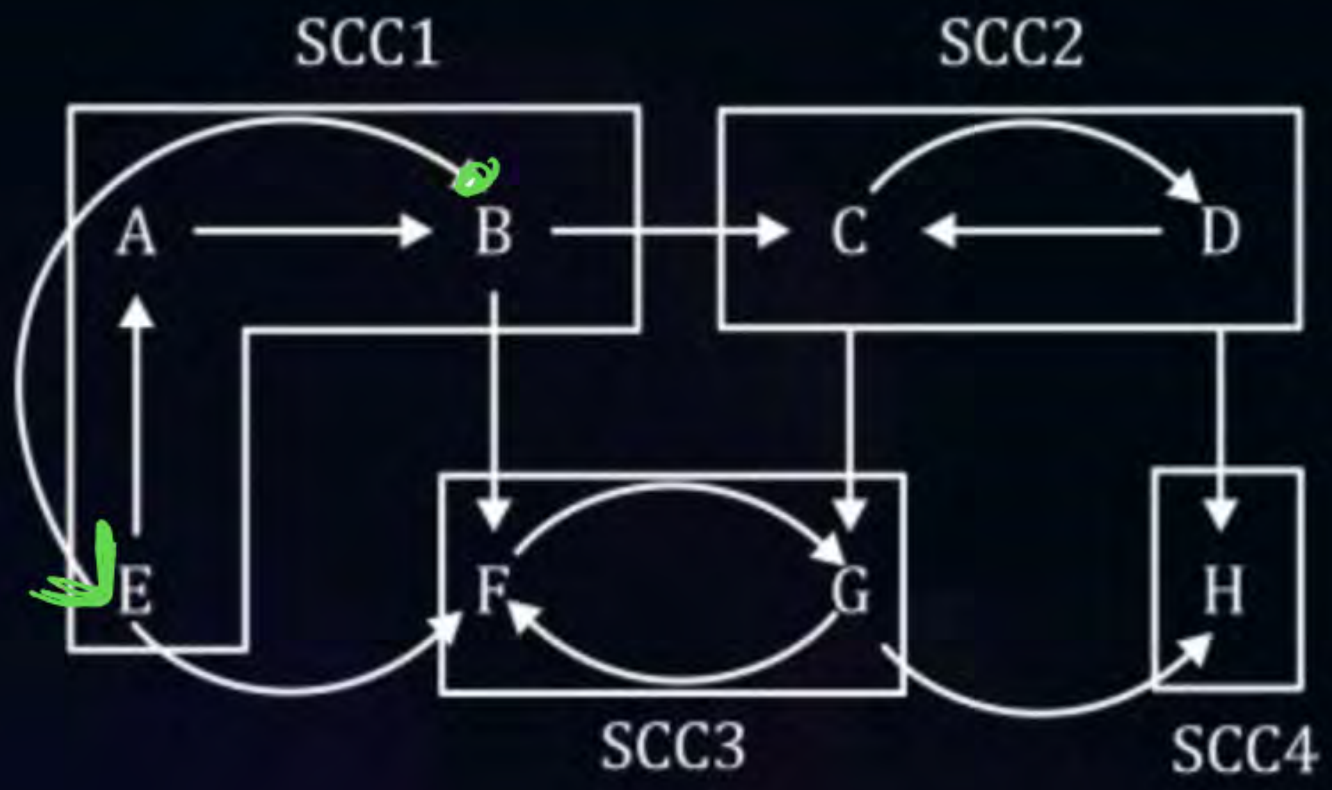
Validate property:-

#Q. How many strongly connected components are there in the given graph G?



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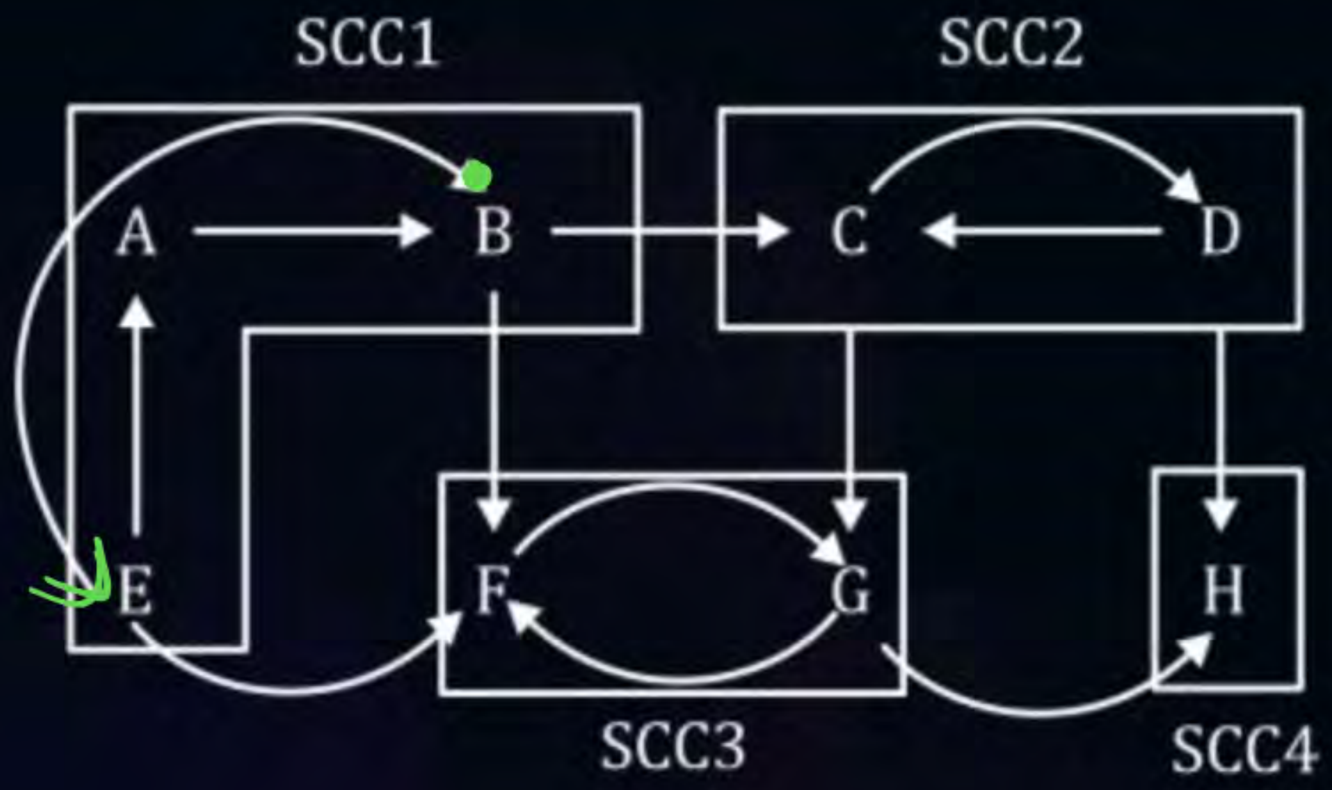

SSC 1 → {A, B, E}
SSC 2 → {C, D}
SSC 3 → {F, G}
SSC 4 → {H}
'V' = {A,B,C,D,E,F,G,H}

Disjoint

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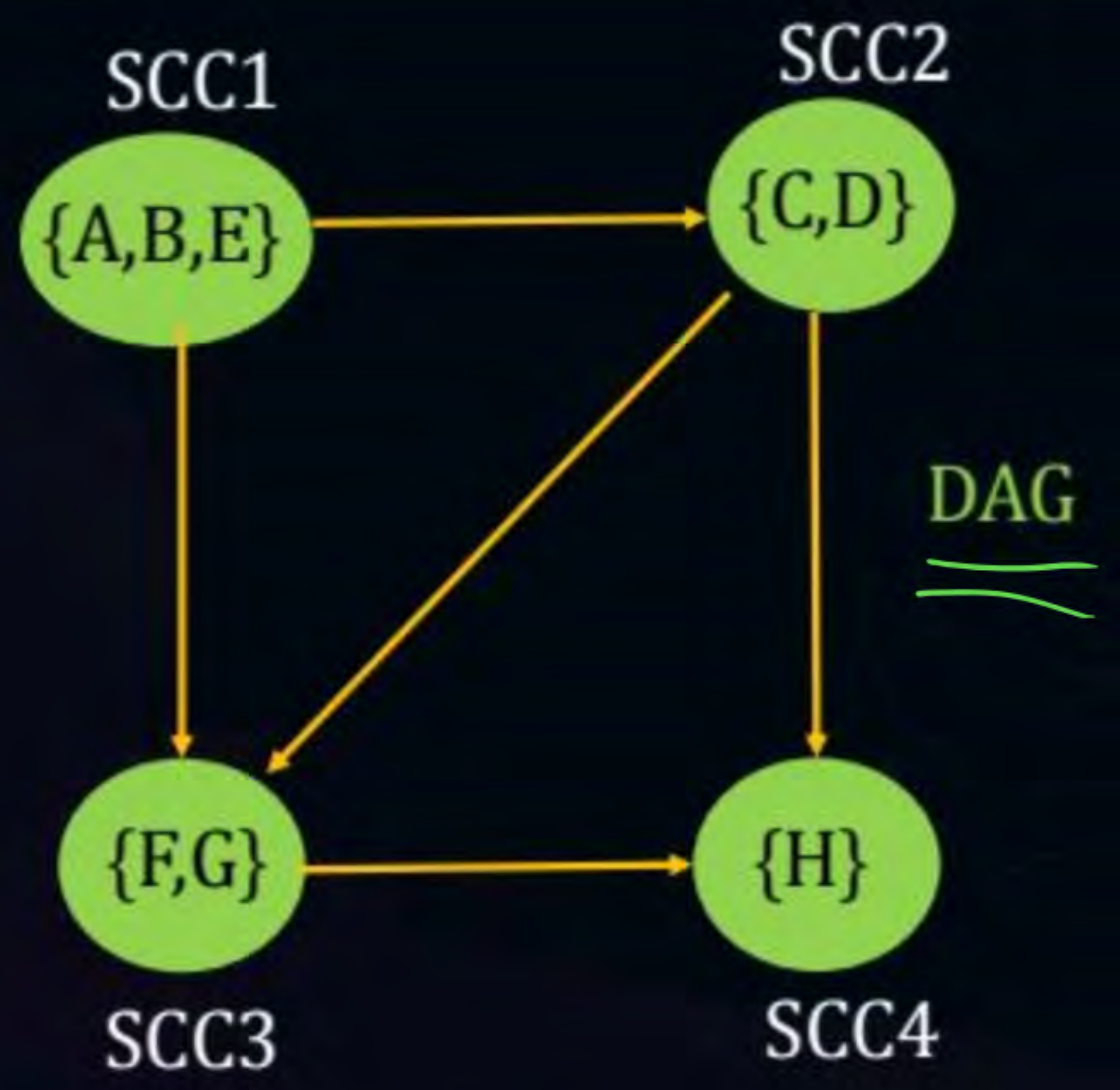
Property:-1



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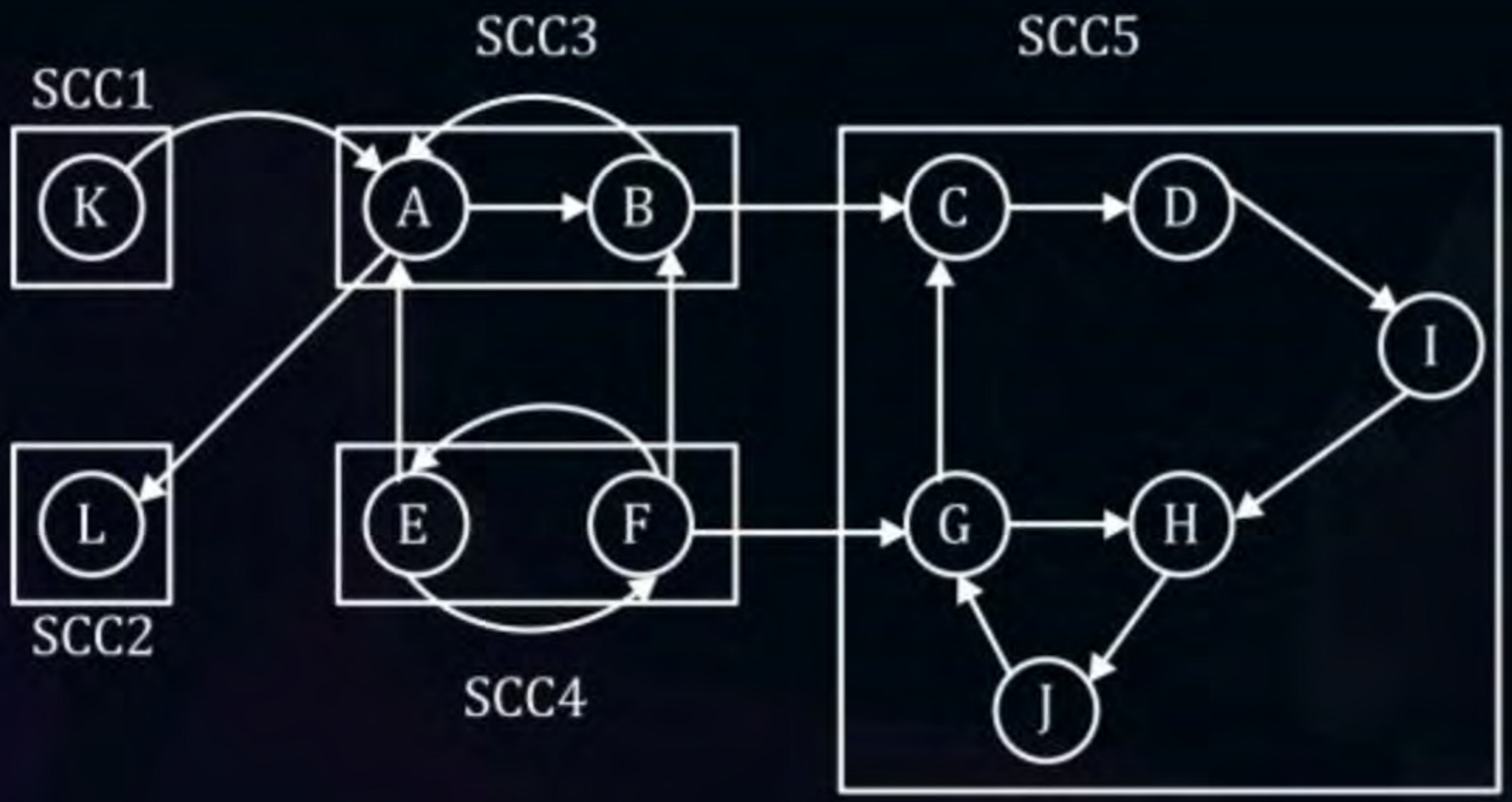
#Q. How many strongly connected components are present in given graph and what are those?



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SSC → CC: Maximal subgraph that is connected

SSC 1 → {K}

SSC 2 → {L}

SSC 3 → {A,B}

SSC 4 → {E, F}

SSC 5 → {C, D, G, H, I, J }

Disjoint

Combine the get 'v'

Maximal

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Property:

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Imp

Property-1: Every directed graph is a DAG of its strongly connected components.

Property-2 : Let C and C^1 be distinct strongly connected components in directed graph $G = (V, E)$, let $u, v, \in C$ and $u^1, v^1 \in C^1$, suppose that there is a path $u \sim u^1$ in G, then there cannot also be a path $v^1 \sim v$ in G.

Property-3: If 'C' and 'C^1' are strongly connected components of G and there is an edge from a node in C to a node in C^1, then the highest post number in C is bigger than the highest post number in C^1.

→ Finishing time

Finishing time

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Property 2 explanation:

In $u \to u^1$ is a path then there cannot be a path from $v^1 \to v$ (as it will no longer be a DAG)

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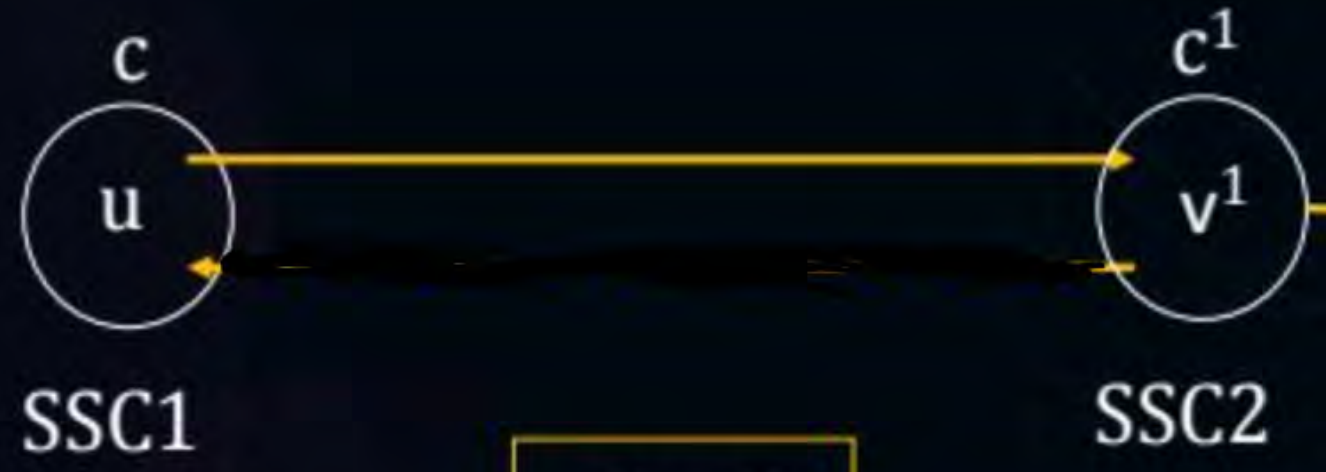
Property-3 explanation:



The highest finishing time of c^1 be F^1

The highest finishing time of c is F

$\boxed{F > F^1}$

Max among all finishing time in it

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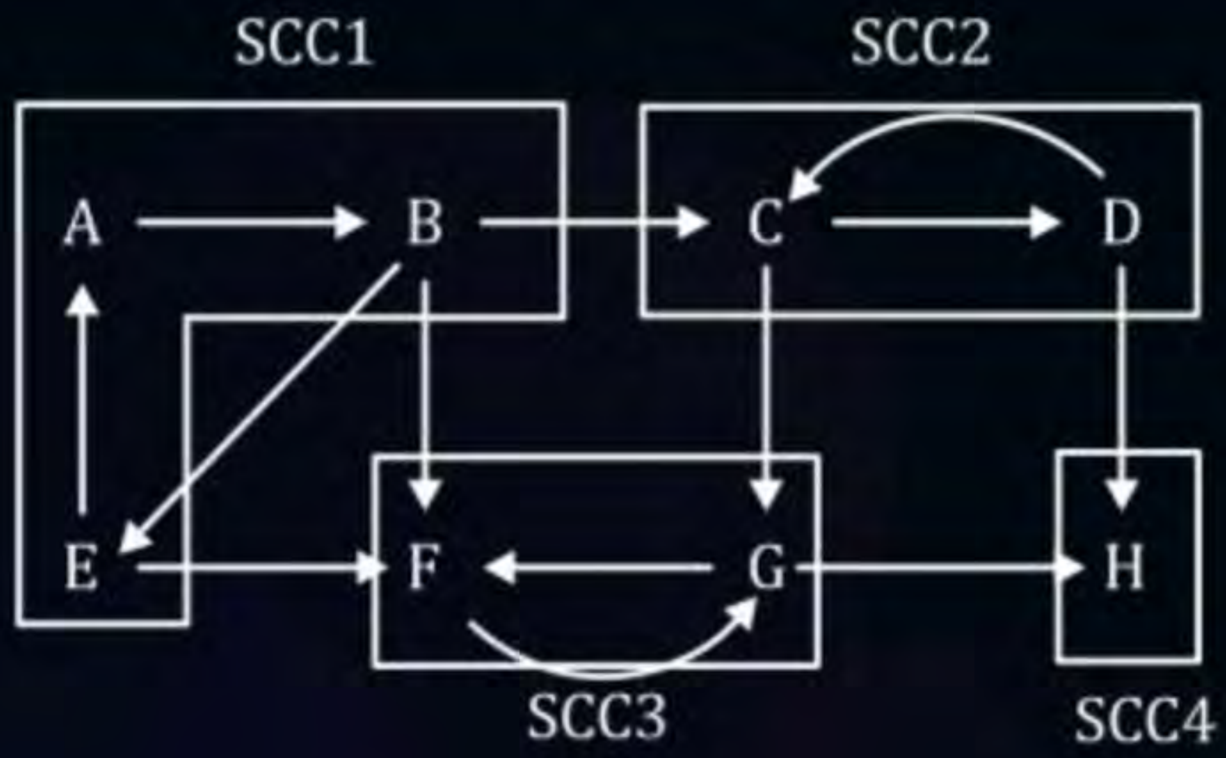
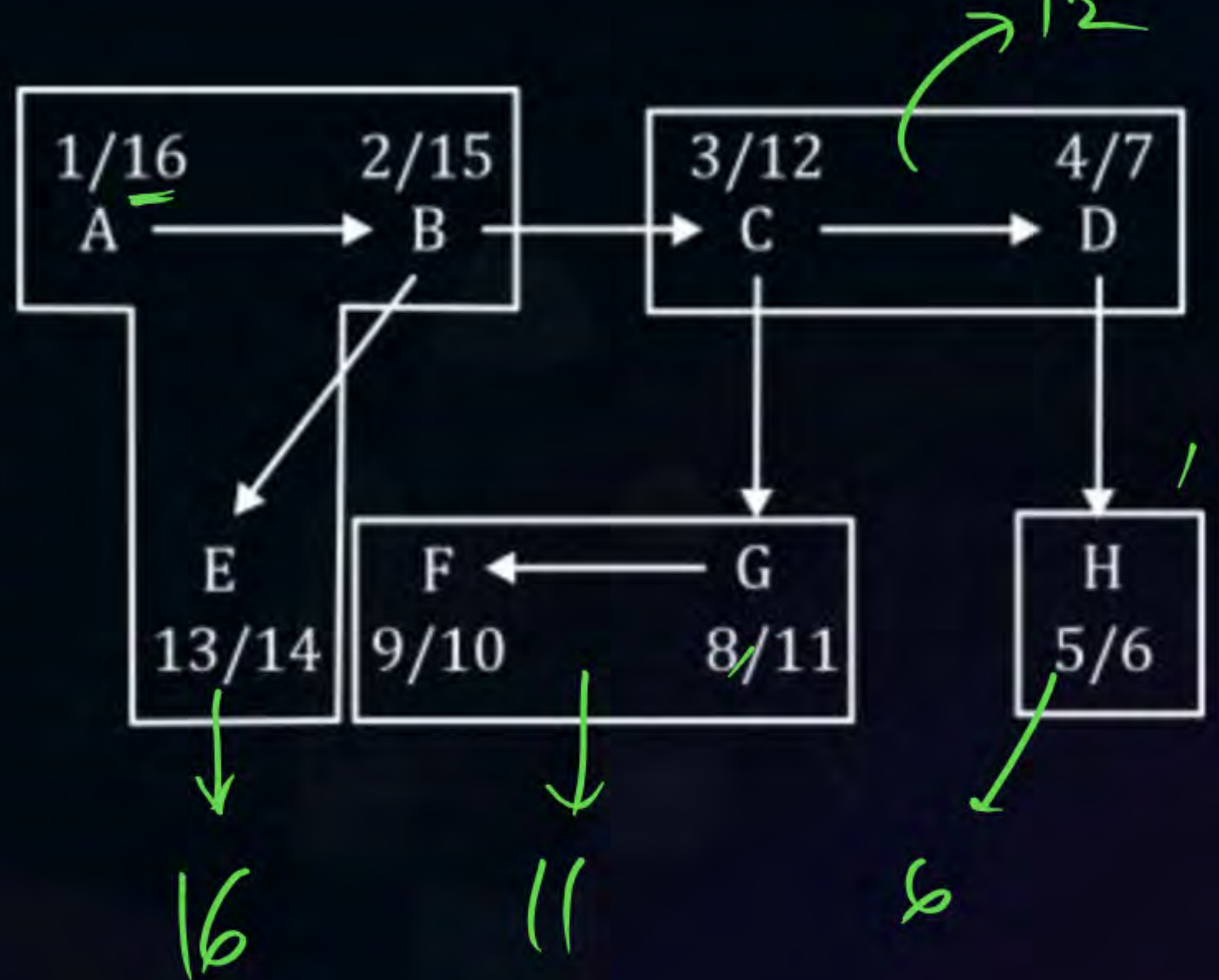
Example to understand this property -3:



8×2
=16

DFS Starting at vertex A.



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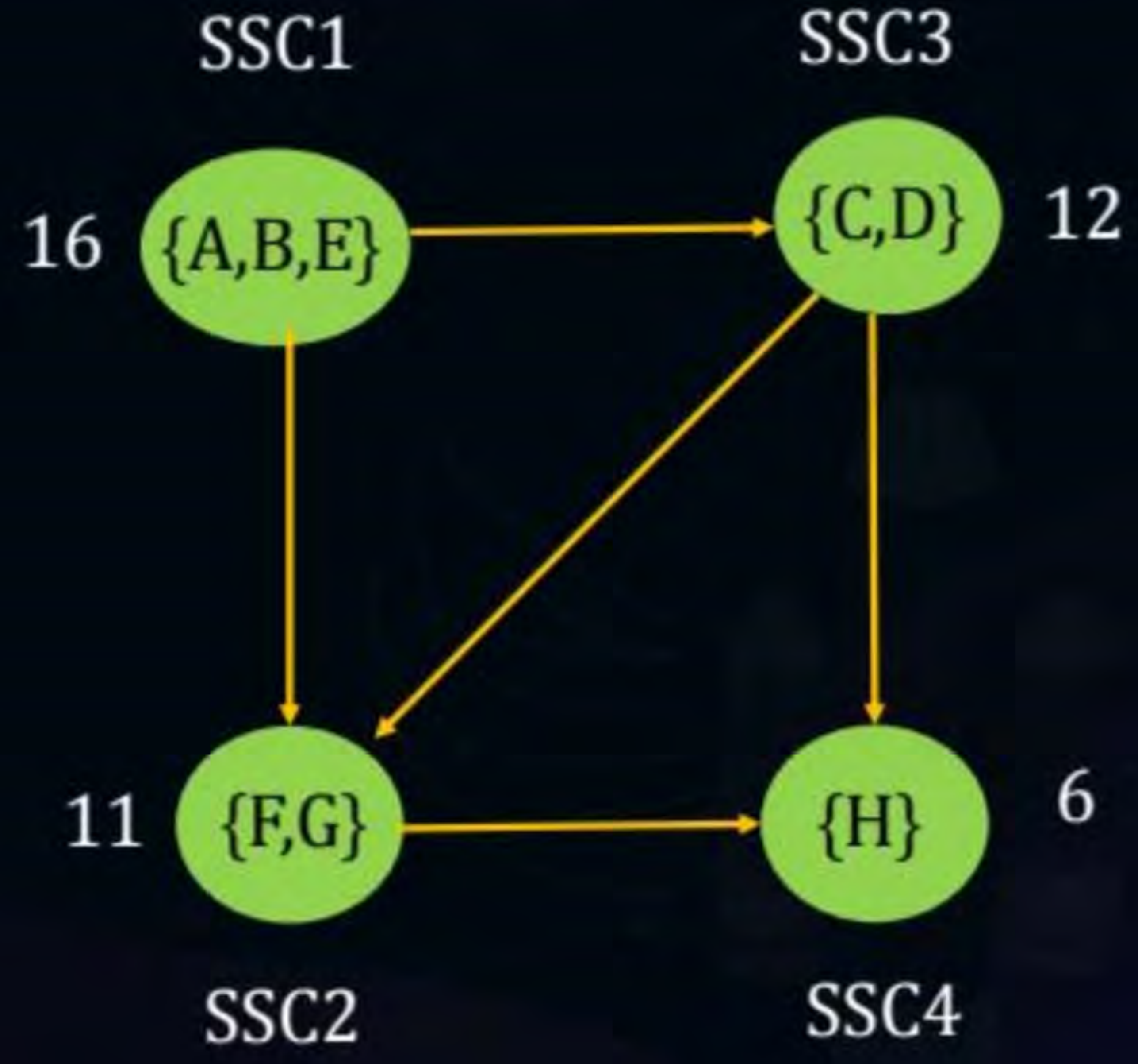
$SSC\ 1 \rightarrow SSC\ 3: 16 > 12$

$SSC\ 1 \rightarrow SSC\ 2: 16 > 11$

$SSC\ 3 \rightarrow SSC\ 2: 12 > 11$

$SSC\ 3 \rightarrow SSC\ 4: 12 > 6$

$SSC\ 2 \rightarrow SSC\ 4: 11 > 6$



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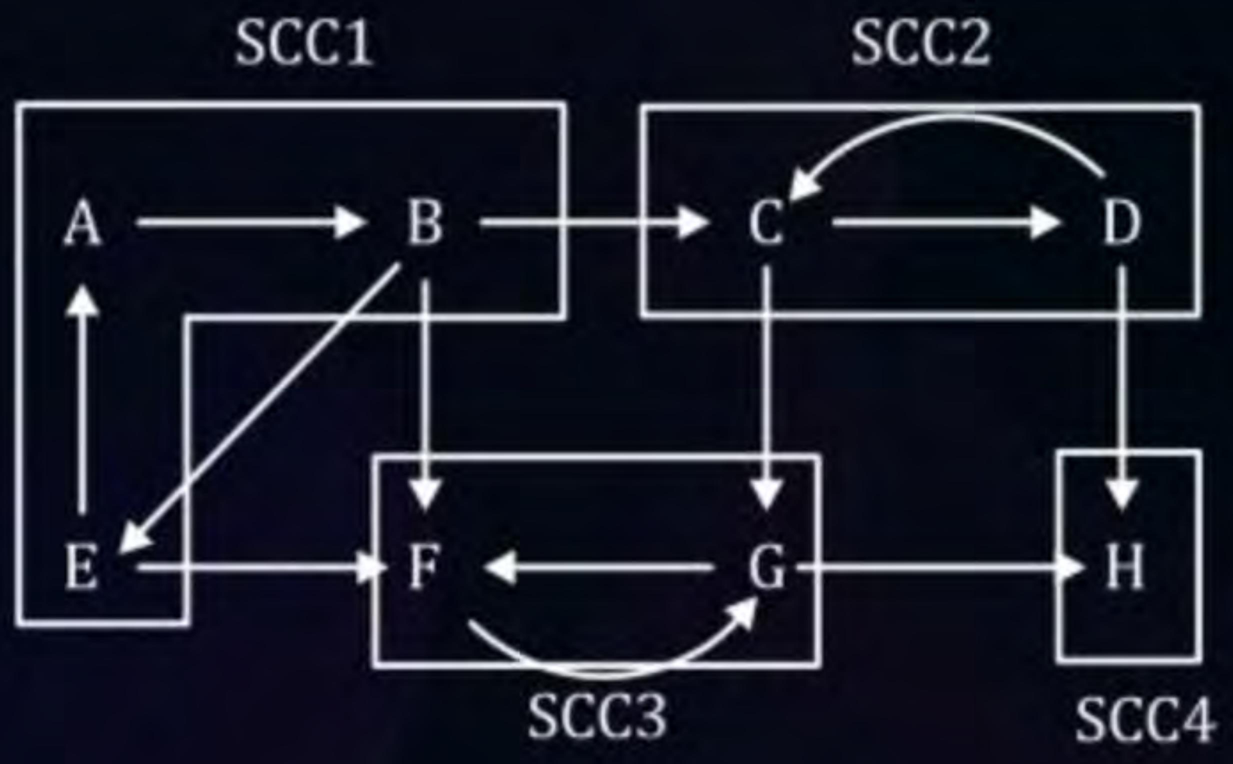
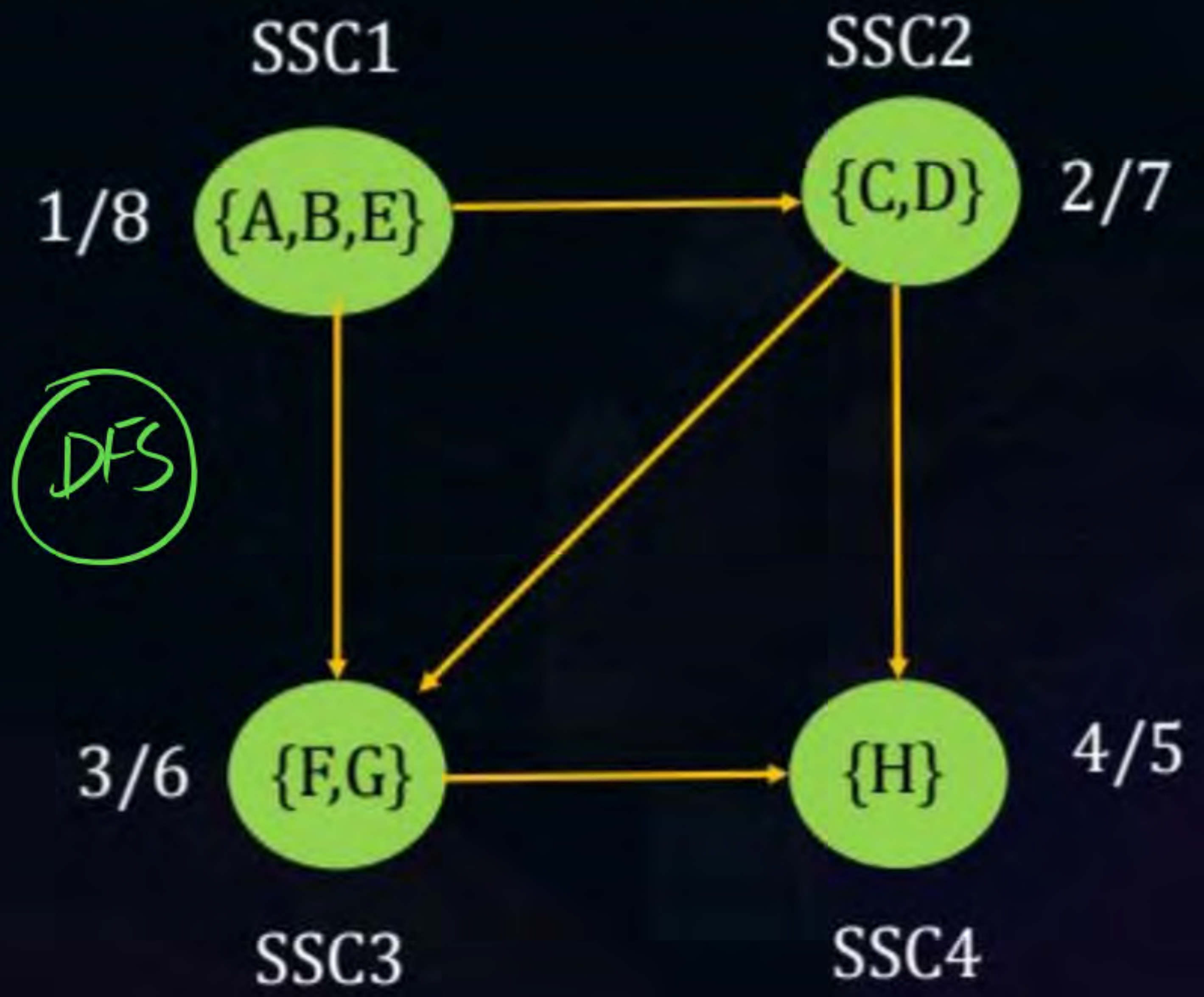
Bonus concept for property-3

If we apply DFS on the SSC DAG, then also property 3 can be validated.

Given:





Finishing time

SCC 1 $\rightarrow$ 8

SCC 2 $\rightarrow$ 7

SCC 3 $\rightarrow$ 6

SCC 4 $\rightarrow$ 5

$1 \rightarrow 2$: $8 > 7$ ✓

$2 \rightarrow 4$: $7 > 5$ ✓

$2 \rightarrow 3$: $7 > 6$ ✓

$1 \rightarrow 3$: $8 > 6$ ✓

$3 \rightarrow 4$: $6 > 5$ ✓

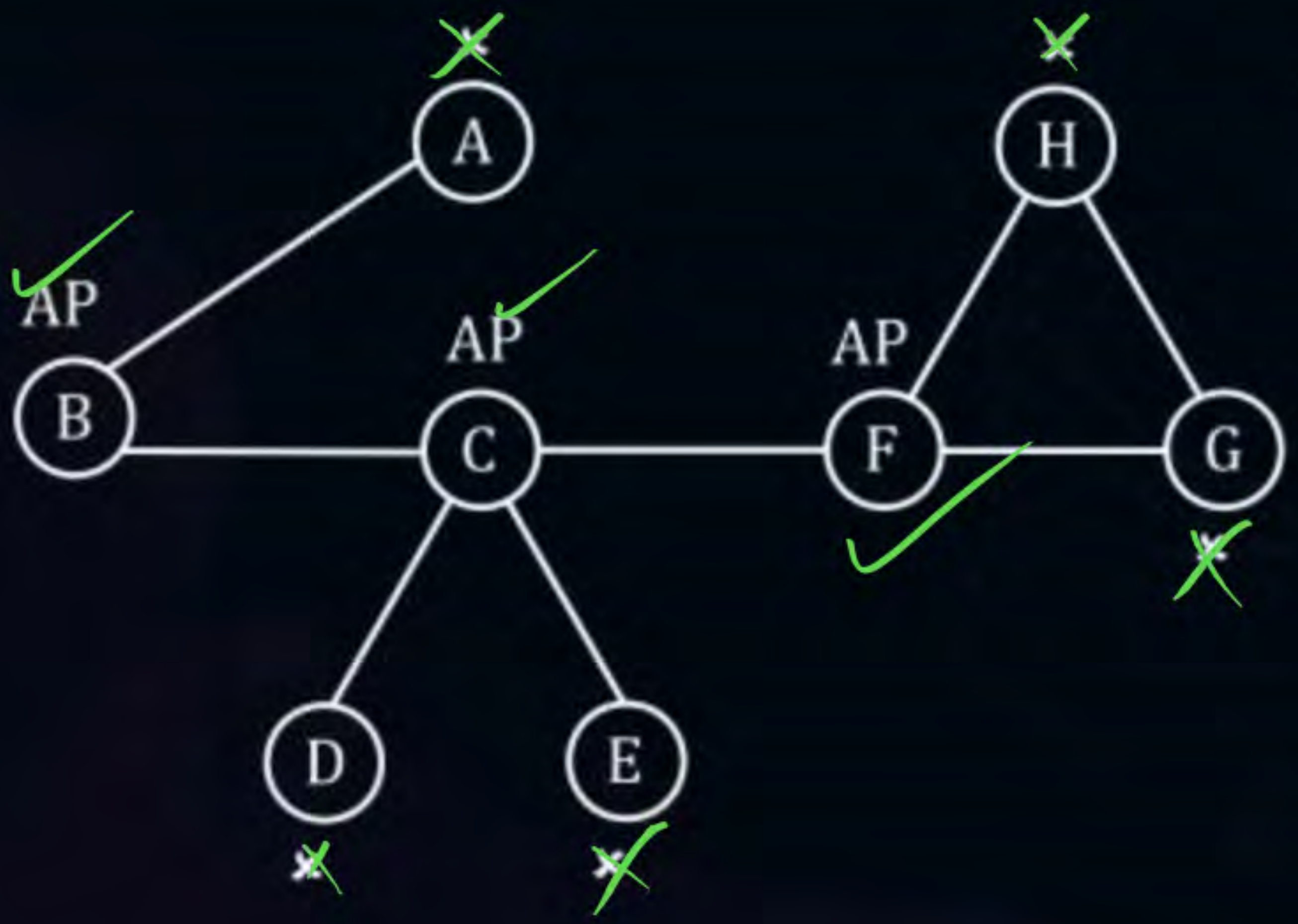
Articulation point / cut vertex

When do use call a vertex as a cut vertex / articulation point?

Articulation point in a graph in such a vertex, which when removed along with all of its edges from the given graph , then it partitions the graph into 2 or more non-empty components.

E.g.4. How many articulation point are there in the below graph? What are those?

Given: (G(V,E))



→ 3 articulation points

B, C and F

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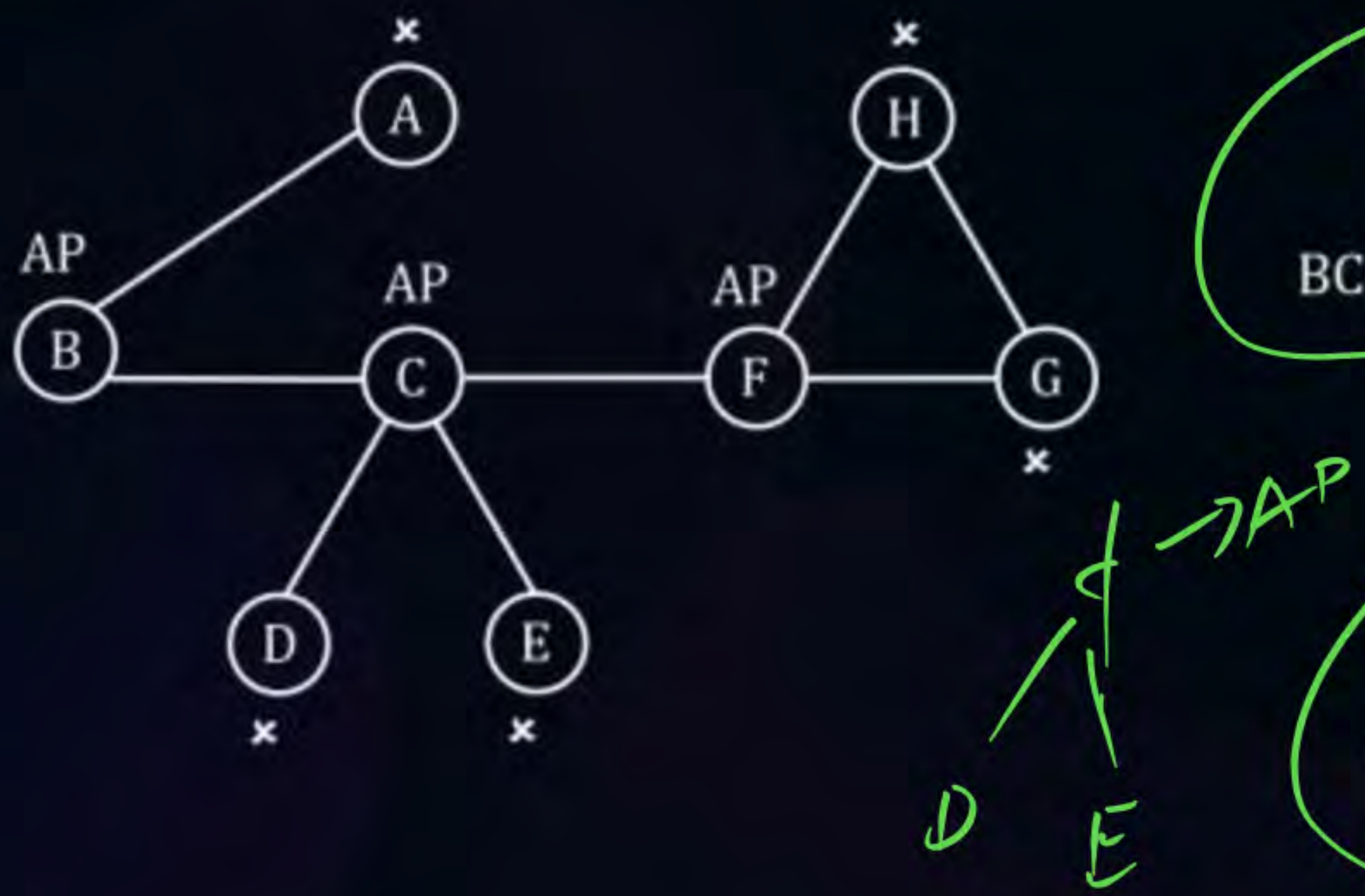
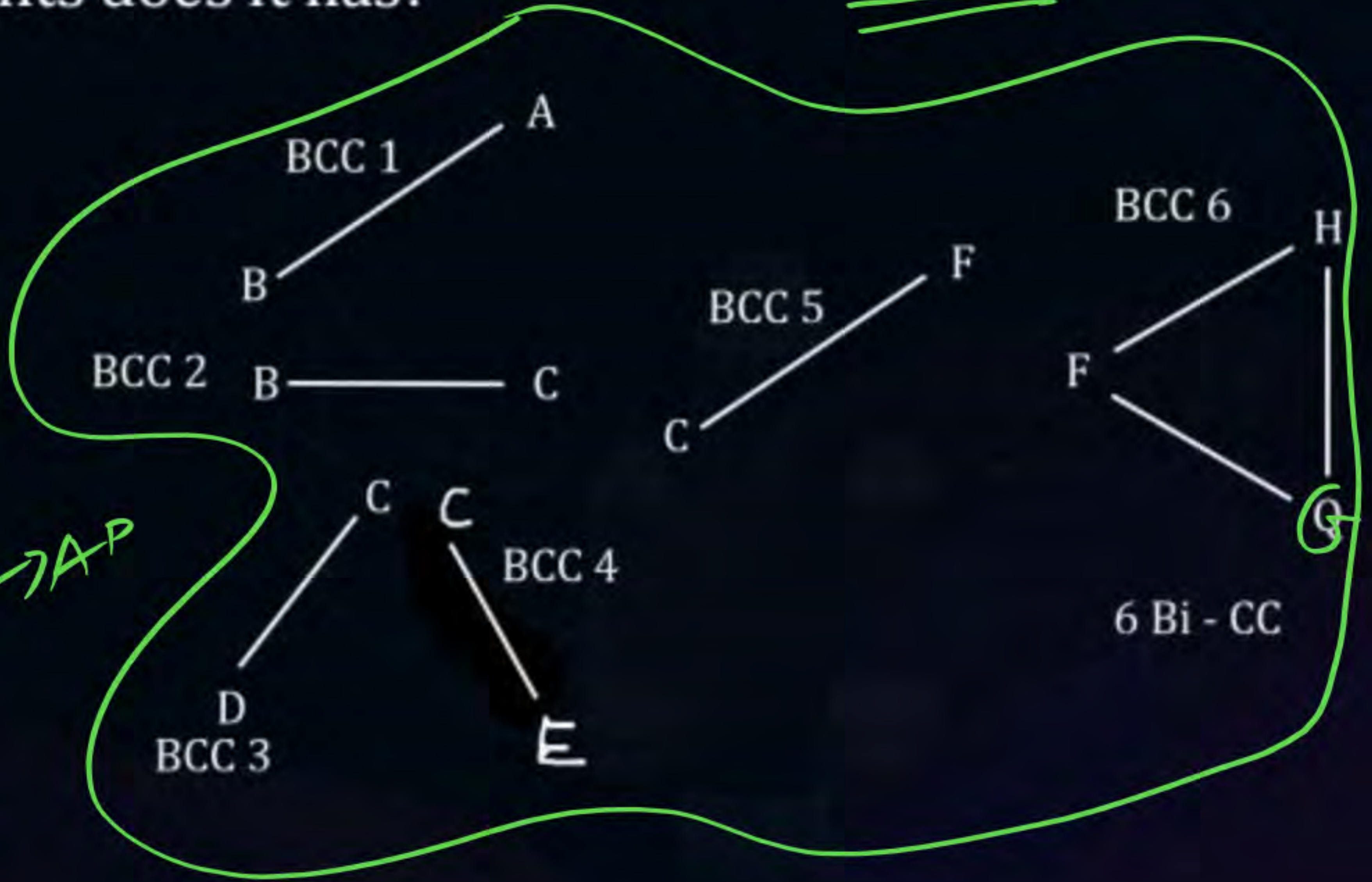
- AS 3 APs not a bi-connected graph:
- How many bi-connected components does it has?

6 BCC





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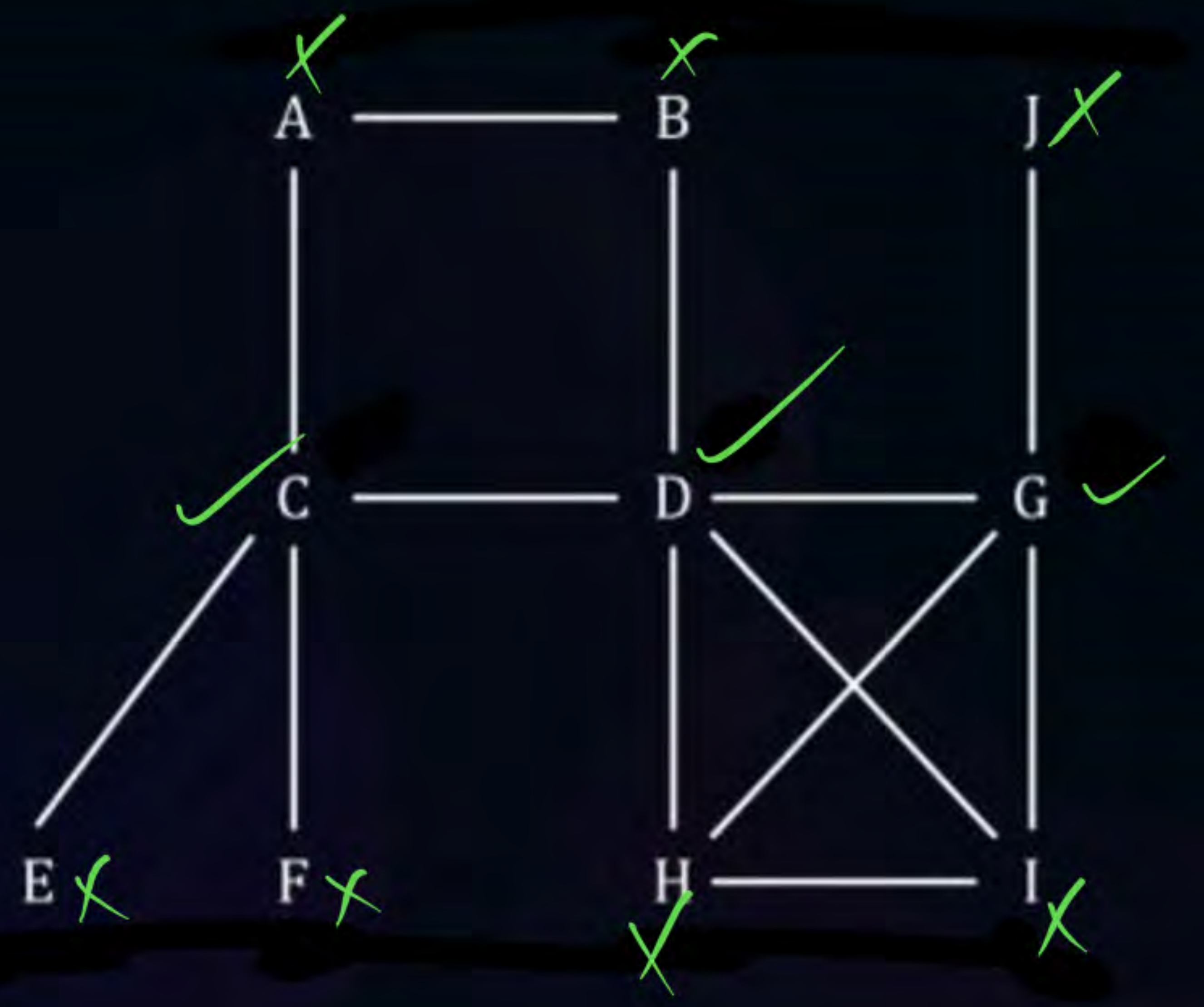
#Q. How many articulation point and what are these?

89%



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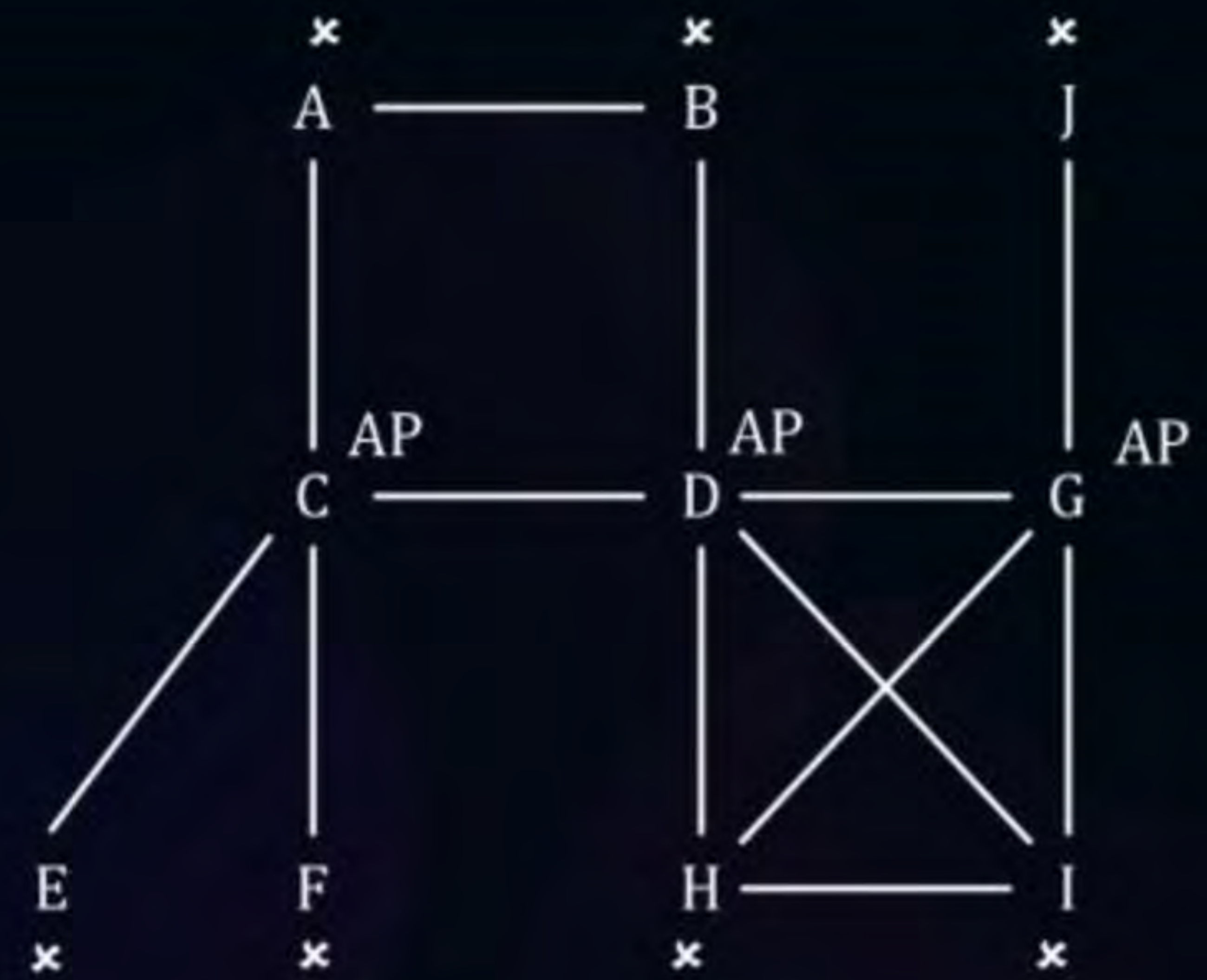
#Q. How many articulation point and what are these?



3 Articulation Point

C, D, and G

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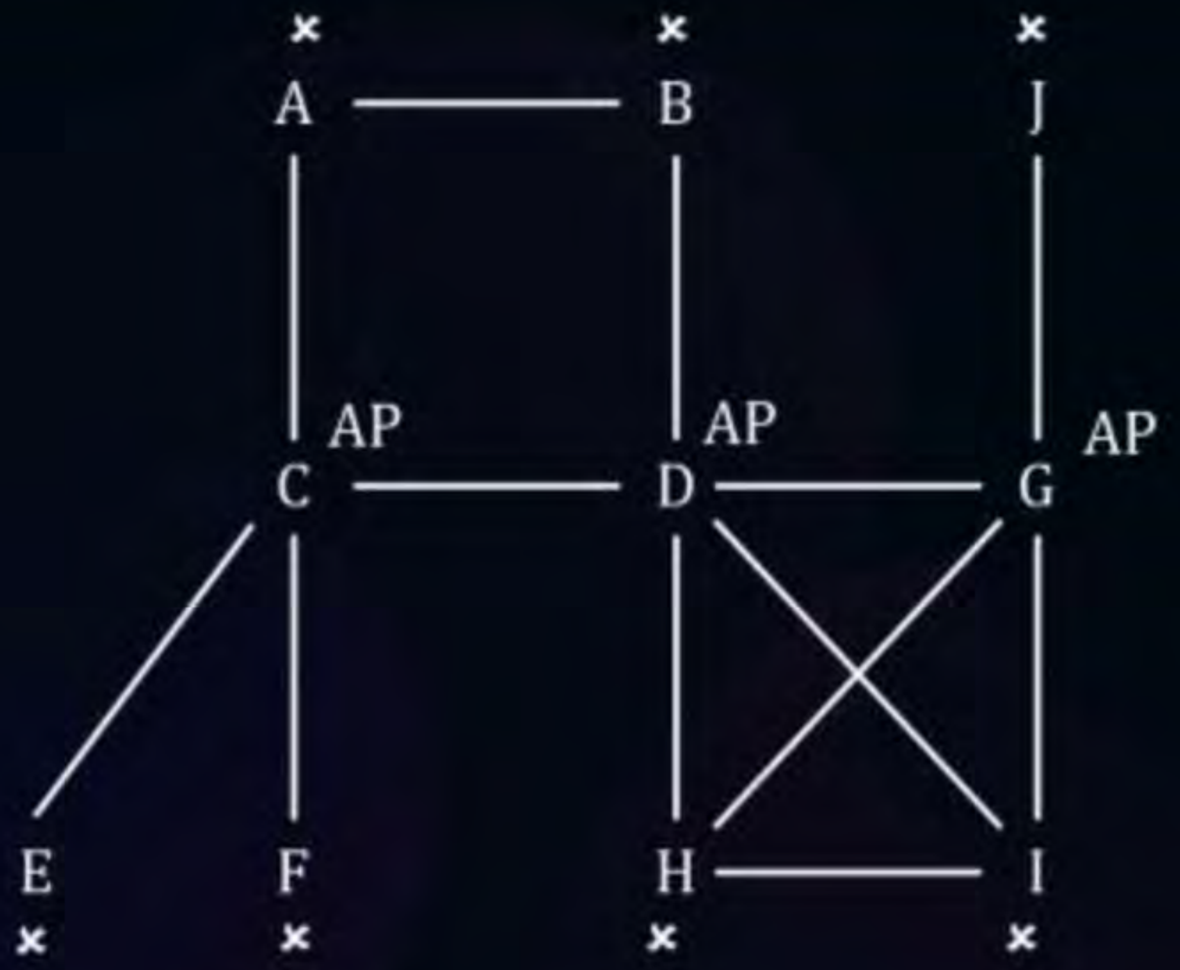
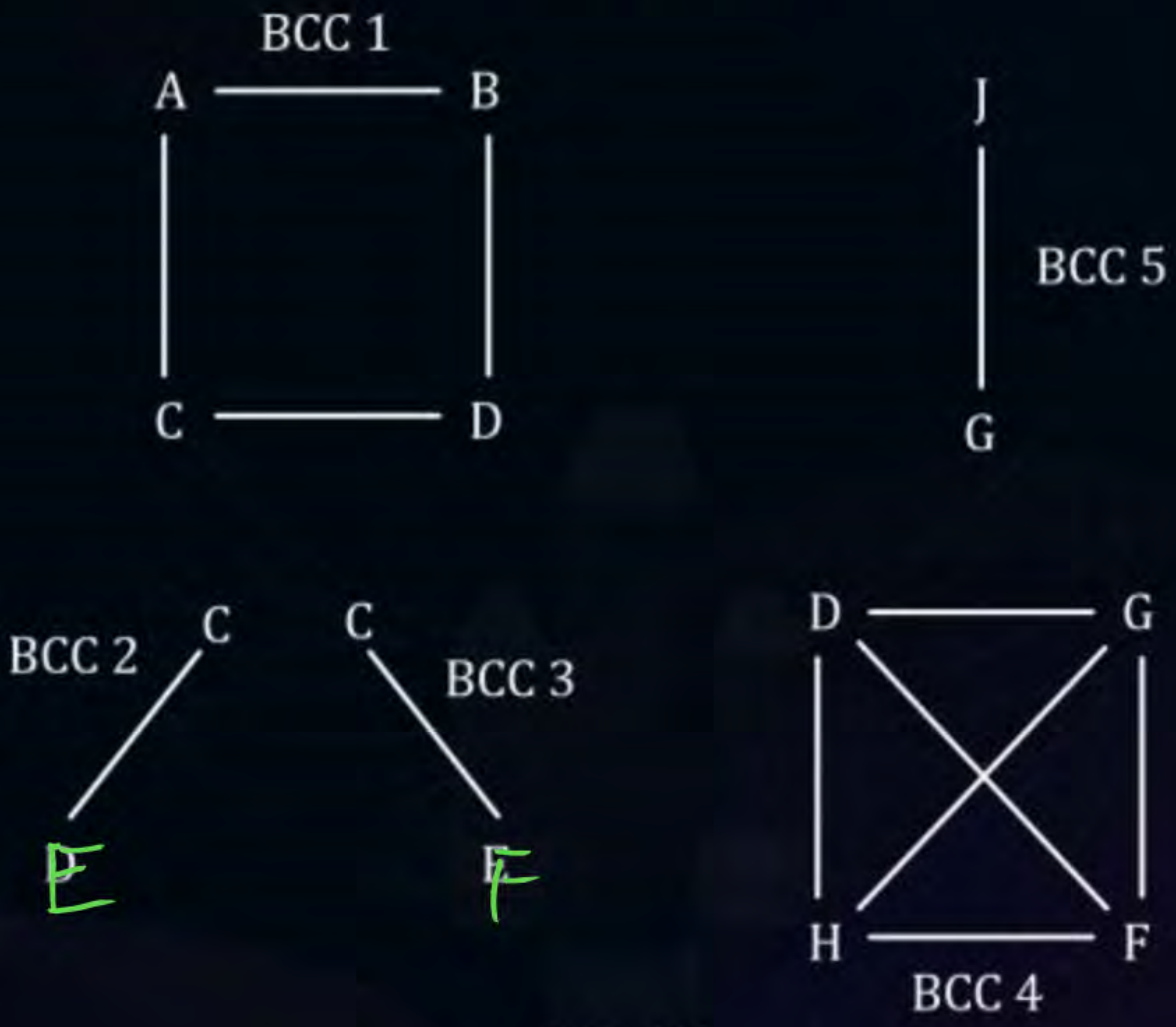
12.5%

#Q. How many bi-connected components?

Bi connected graph?→ No (3AP)





5 Bi- CC

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#Q. How many Aps? And how many Bi connected components are there ?

50%

BCC

1

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#Q. How many Aps? And how many Bi connected components are there ?

- As there are 0 Aps in given graph, if is a Bi- Connected graph.
- Only 1 Bi-cc (graph itself)

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Bi- connected components:

A graph is considered to be bi-connected if it does not have any articulation point in it.

Property:

If a graph is not bi-connected then its maximal sub-graph which is bi-connected is a bi-connected component.

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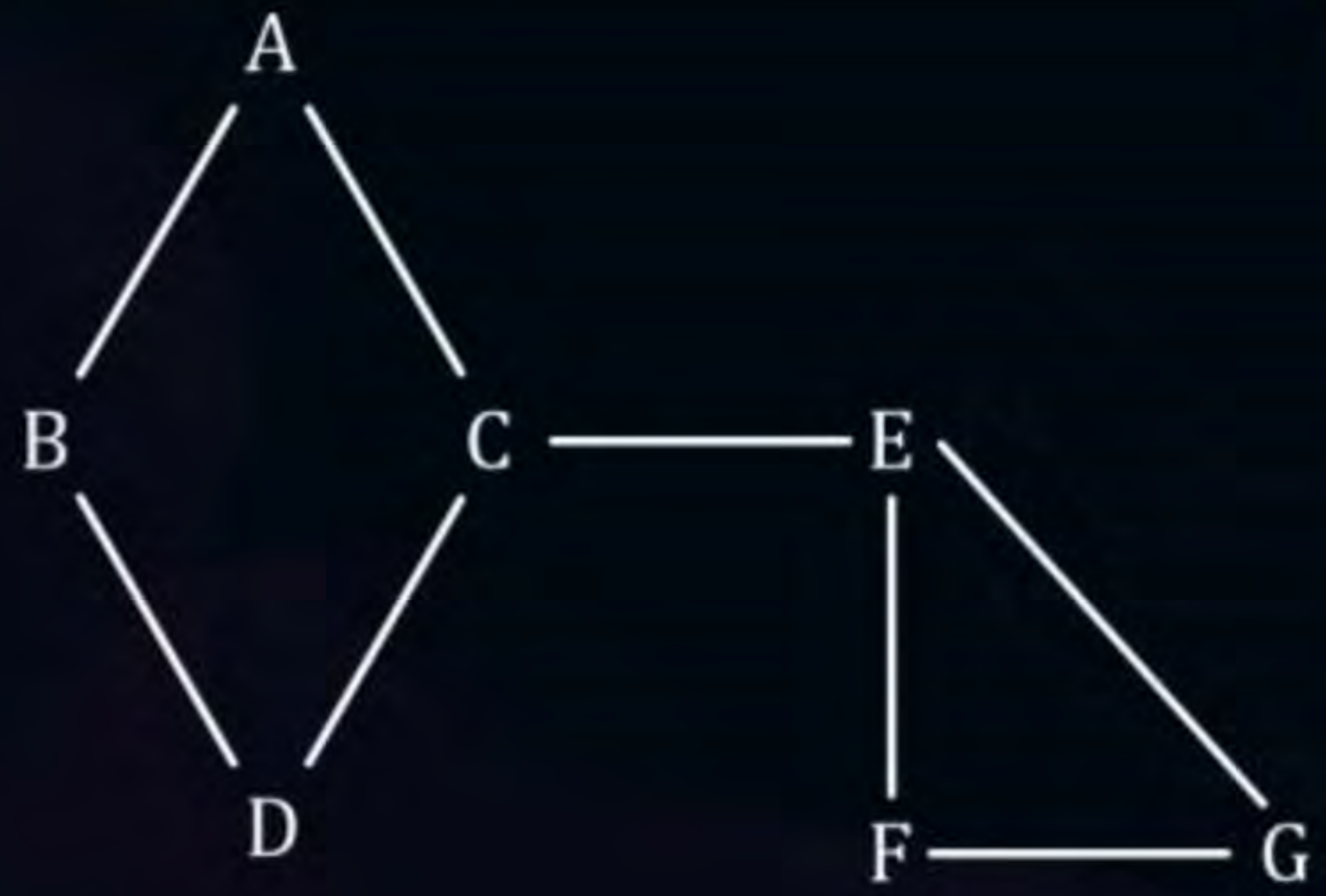
#Q. Given a graph G and it has A articulation points and B bi-connected components then what is the value of $(A)^B$?

Given (V,E)

HW



2 mins Summary

Topic

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Topic

Graph Traversal

THANK - YOU