DS & AI
CS & IT

Probability & Statistics

Lecture No. 13

By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic BASICS of C.R.V

(Continuous Random Variable)

Topics to be Covered

Topic

1. PRACTICE QUESTIONS of p.d.f
2. " " " C.D.F
3. Exponential Distribution.

Q. The variance of R.V x for which p.d.f is $f(x) = \frac{1}{2}|x|e^{-|x|}$ will be ?

HW

$= \text{Even func}^n.$

(a) 0

(b) 2

(c) 6

(d) $\sqrt{6}$

$$Var(x) = E(x^2) - E^2(x)$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left(\int_{-\infty}^{\infty} \underbrace{x\cdot f(x)}_{\text{odd.}}\,dx\right)^2$$

$$= 2\int_0^{\infty} x^2\left(\frac{1}{2}|x|e^{-|x|}\right)dx - (\;0\;)^2$$

$$= \int_0^{\infty} x^3 e^{-x}\,dx = ?$$

M-I using Int By Part = 6

M-II " Gamma func^n = 6

Note: $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots\ldots$

eg $\int \underset{u}{x^2}.\underset{v}{\sin x}\,dx = x^2(-\cos x) - 2x(-\sin x) + 2(+\cos x) - 0 + 0 - 0 \ldots$

M-I $\int_0^\infty x^3 e^{-x}\,dx = x^3(-e^{-x}) - 3x^2(+e^{-x}) + 6x(-e^{-x}) - 6(+e^{-x}) + 0$

$= -\left[(x^3+3x^2+6x+6)\,e^{-x}\right]_0^\infty = -\left[0 - (0+0+0+6)e^{-0}\right]$

$= 6$ Ans

M-II $\int_0^\infty e^{-x}.x^{n-1}\,dx = \Gamma n$

$\int_0^\infty e^{-x}x^3\,dx = \int_0^\infty e^{-x}x^{4-1}\,dx = \Gamma 4 = 3! = 6$

$\Gamma(n+1) = \begin{cases} n!, & n \in I^+ \\ n\Gamma n, & n \in Q^+ \end{cases}$

eg $\Gamma 5 = 4!,\ \Gamma 6 = 5!,\ \Gamma 2 = 1! \ldots$

HW

Q→ if $f(x)=\begin{cases} kx+1, & 0\le x\le 4 \\ 0, & \text{otherwise} \end{cases}$ is p.d.f for x then $k=?$

(a) $-3/8$ (b) $8/3$ (c) $\frac{16}{3}$ (d) f(x) can't be a p.d.f

Sol: ∵ f(x) is p.d.f of x so $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0}(0)\,dx+\int_{0}^{4}(kx+1)\,dx+\int_{4}^{\infty}(0)\,dx=1$$

$$0+\left[k\left(\frac{x^2}{2}\right)+x\right]_{x=0}^{4}+0=1$$

------ $k=-3/8$

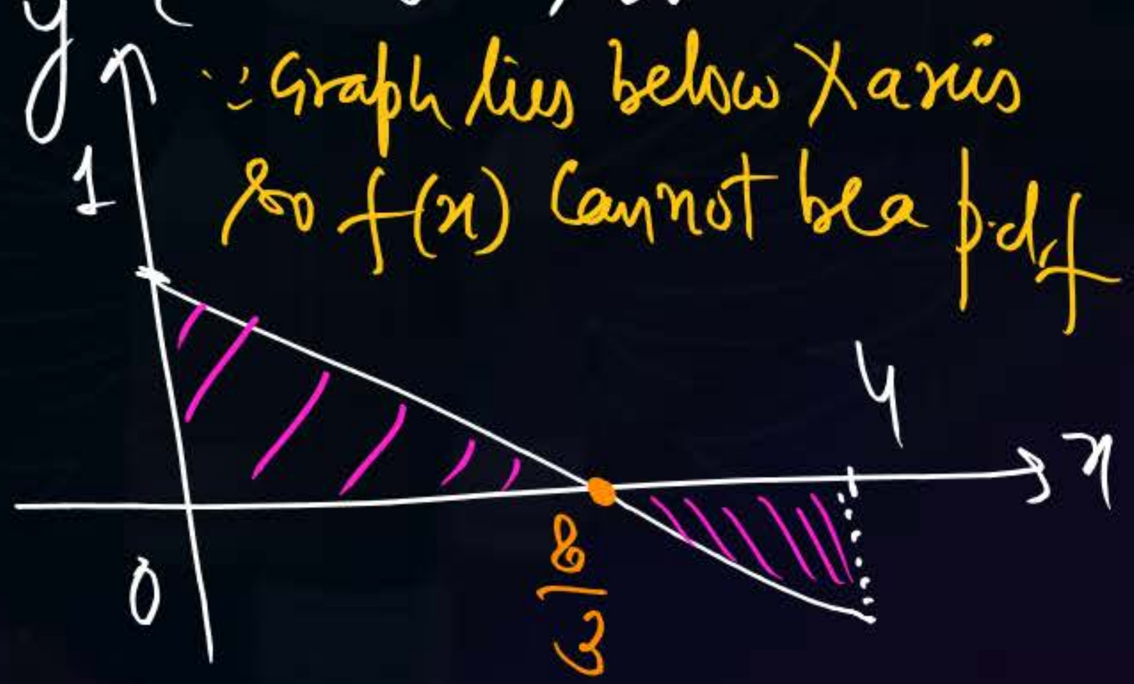
So pdf is $y=f(x)=\begin{cases} -\frac{3}{8}x+1, & 0\le x\le 4 \\ 0, & \text{otw.} \end{cases}$

$y=-\frac{3}{8}x+1$

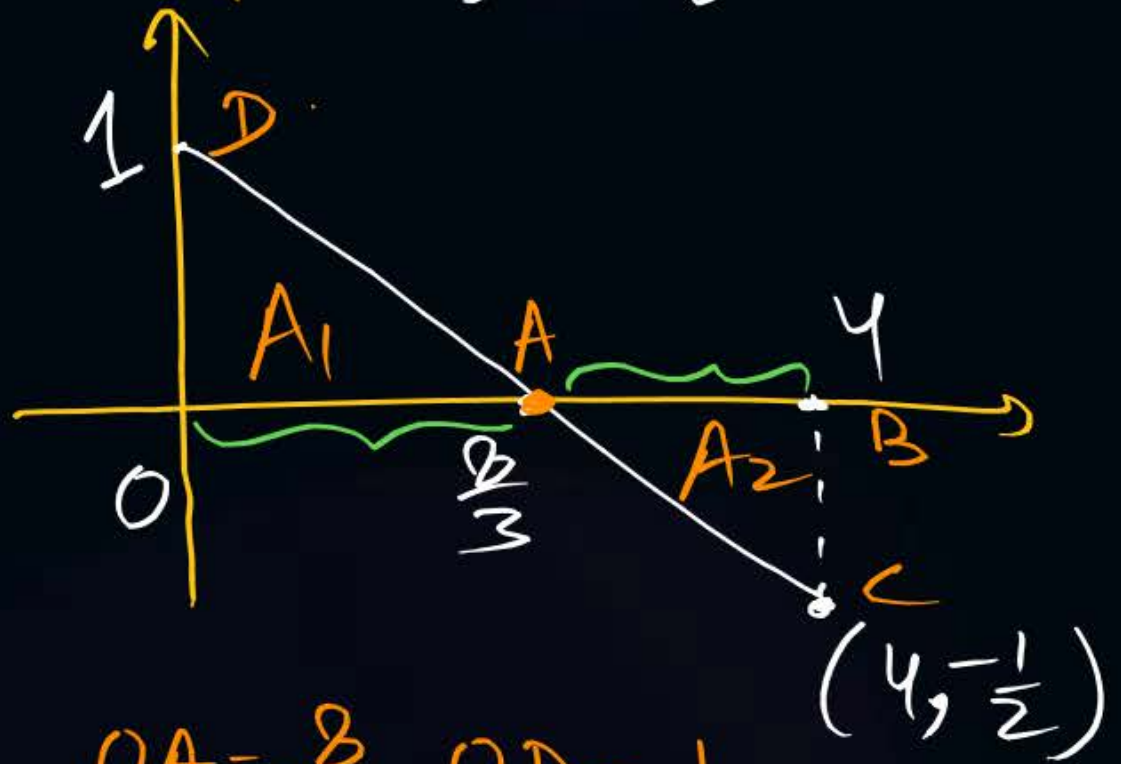
OR

$\frac{x}{8/3}+\frac{y}{1}=1$

∵ Graph lies below X axis so f(x) Cannot be a p.d.f



CROSS Check: $y = -\frac{3}{8}x + 1$



$OA = \frac{8}{3}, \; OD = 1$

$AB = 4 - \frac{8}{3} = \frac{4}{3}$ & $BC = -\frac{1}{2}$

Area = Area of A_1 + Area of A_2

$= \frac{1}{2}\left(\frac{8}{3}\right)(1) + \frac{1}{2}\left(\frac{4}{3}\right)\left(+\frac{1}{2}\right) = \frac{4}{3} + \frac{1}{3} = \frac{5}{3} > 1$, Not possible

(M-II) $\int_{-\infty}^{\infty} f(x)\,dx = \int_0^4 f(x)\,dx = \int_0^{8/3} f(x)\,dx + \int_{8/3}^{4} |f(x)|\,dx$

$= \int_0^{8/3} \left(-\frac{3}{8}x + 1\right) dx + \int_{8/3}^{4} \left|\left(-\frac{3}{8}x + 1\right)\right| dx$

$= \frac{4}{3} + \left|-\frac{1}{3}\right| = \frac{4}{3} + \frac{1}{3} = \frac{5}{3} > 1$

Not possible.

Find the value of λ such that the function f(x) is a valid probability density function. ________

$$f(x) = \begin{cases} \lambda(x-1)(2-x), & \text{for } 1 \le x \le 2 \\ = 0, & \text{otherwise} \end{cases}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_1^2 \lambda(x-1)(2-x)\,dx = 1$$

$$\lambda \int_1^2 -(x^2-3x+2)\,dx = 1$$

$$-\lambda\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2 = 1$$

$$-\lambda(\,?-?\,) = 1$$

$$\cdots\cdots \lambda = 6$$

Let X be a random variable with probability density function

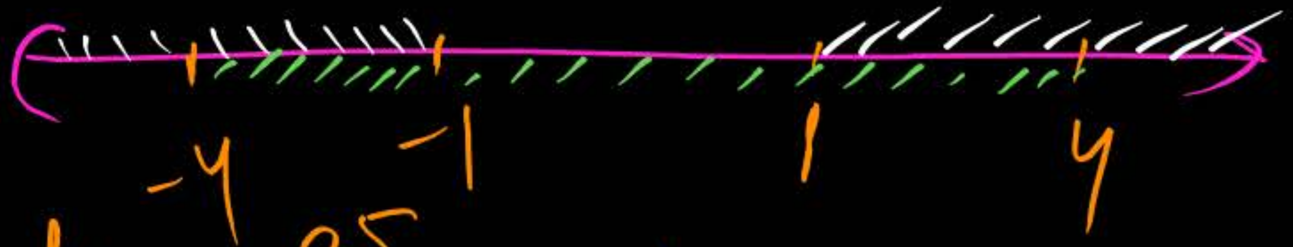
$$f(x) = \begin{cases} 0.2, & \text{for } |x| \le 1 \Rightarrow \boxed{-1 \le x \le 1} \\ 0.1, & \text{for } 1 < |x| \le 4 \\ 0, & \text{otherwise} \end{cases}$$

$|x| > 1 \Rightarrow x < -1$ or $x > 1$

$|x| \le 4 \Rightarrow -4 \le x \le 4$

$\Rightarrow \boxed{-4 \le x < -1 \cup 1 < x \le 4}$

The probability $\boxed{P(0.5 < X < 5)}$ is ______.



(a) 0.1

(b) 0.3

(c) 0.4

(d) 0.5

$$P(0.5 < x < 5) = \int_{0.5}^{1} 0.2\, dx + \int_{1}^{4} 0.1\, dx + \int_{4}^{5} 0 \cdot dx$$

$$= 0.2(1 - 0.5) + 0.1(4 - 1)$$

$$= 0.2 \times 0.5 + 0.3 = 0.10 + 0.30 = 0.40$$

Cummulative Density func^n (C.d.f) / Distribution func^n.

Let x is C.R.V and $f(x)$ is it's p.d.f then it's C.d.f is denoted by $F(x)$ and is defined as

$$\boxed{F(x) = \int_{-\infty}^{x} f(x)\,dx}$$

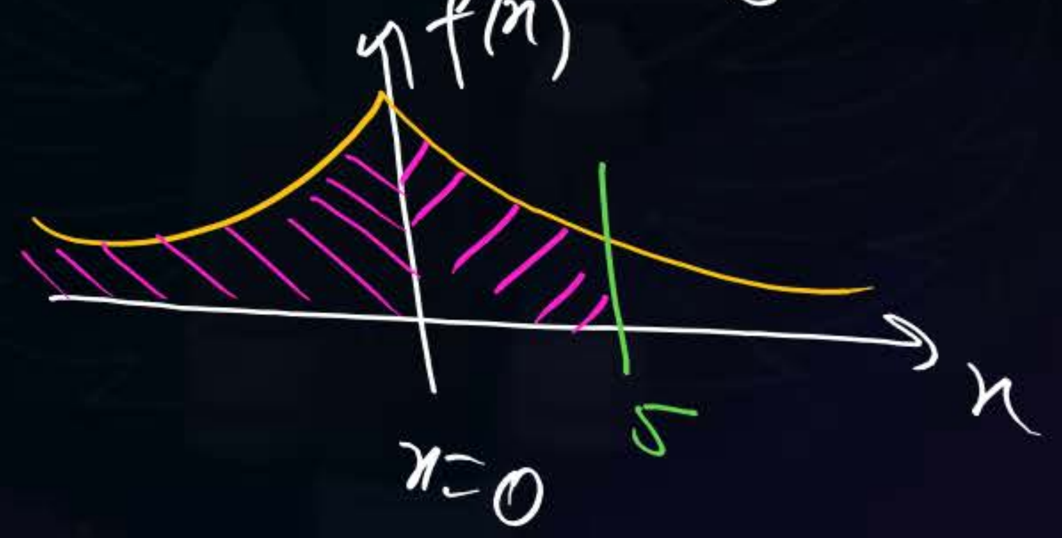
$$F(-\infty) = \int_{-\infty}^{-\infty} f(x)\,dx = 0$$

$$F(\infty) = \int_{-\infty}^{\infty} f(x)\,dx = 1$$

for eg, $F(5) = \int_{-\infty}^{5} f(x)\,dx$ = Sum of all the probabilities from starting point upto $x = 5$

$F(5)$ ↓ (C.d.f at 5)

= Shaded area.



In case of D.R.V if $x = \{$No. of Heads in 3 tosses$\} = \{0, 1, 2, 3\}$

x:	0	1	2	3
$p(x)$:	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

p.m.f.

then C.D.f at $(X=2) = ? = \underbrace{p_0 + p_1 + p_2} = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

Similarly CDF at $(X=1) = ? = \underbrace{p_0 + p_1} = \frac{1}{8} + \frac{3}{8} = \frac{1}{4}$

Similarly in C.R.V if $f(x)$ is p.d.f then C.D.f at $x = a$ is

$$F(a) = \int_{-\infty}^{a} f(x)\,dx = \text{Area under } f(x) \text{ b/w Starting Point \& } a.$$

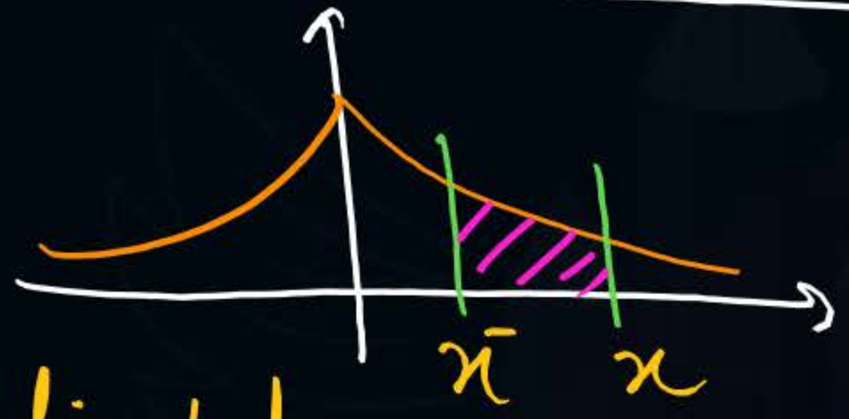
ASLI Conclusion ;-

(1) $f(x) =$ p.d.f of $x \neq$ Probability. & $\boxed{0 \le f(x) < \infty}$

But $\int_{-\infty}^{\infty} f(x)\,dx =$ Total area $= 1$ unit.

(2) $F(x) =$ C.D.F at $x =$ Probability. & $\boxed{0 \le F(x) \le 1}$

$$\boxed{f(x) = F(x) - F(\bar{x})}$$

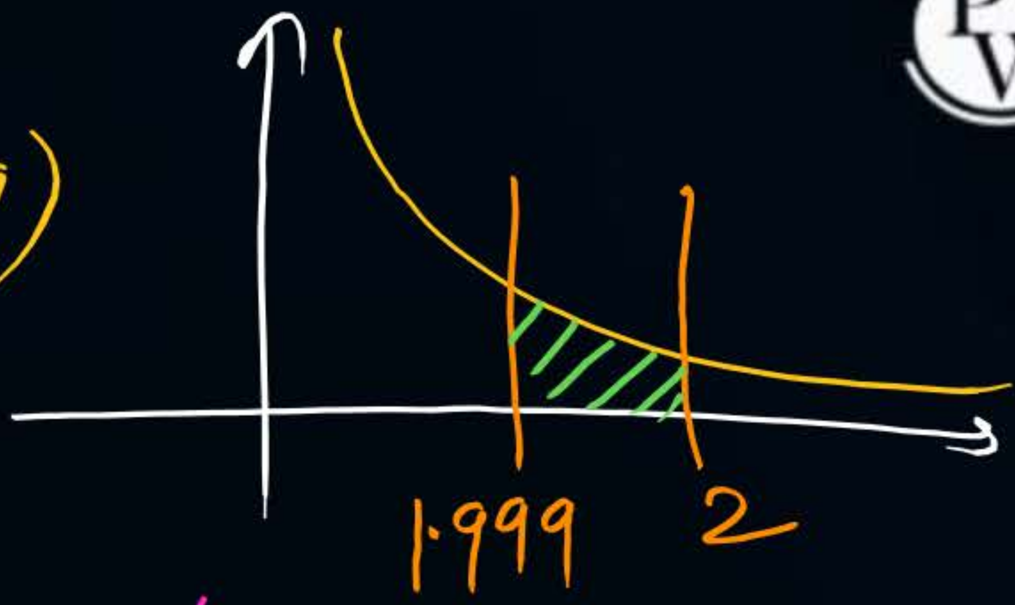


Note (1) Graph of p.d.f as well as c.d.f can not lies below X axis.

(2) Graph of p.d.f can go from $y = 0$ to $y = \infty$

(3) Graph of C.d.f can go only in b/w $y = 0$ & $y = 1$

$\circledast$ $f(2) = \int_{1.999}^{2} f(x)\,dx$ or $F(2) - F(1.999)$



$\circledast$ $P(a < x < b) = ?$

M-I $= \int_a^b f(x)\,dx$

M-II $= \boxed{F(b) - F(a)} = \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx$

$\circledast$ $\underset{(C.D.F)}{F(x)} = \int_{-\infty}^{x} f(x)\,dx$ is By Integrating p.d.f we can find c.d.f

& $\underset{(p.d.f)}{f(x)} = \frac{d}{dx} F(x)$ is By Differentiating c.d.f we can find p.d.f.

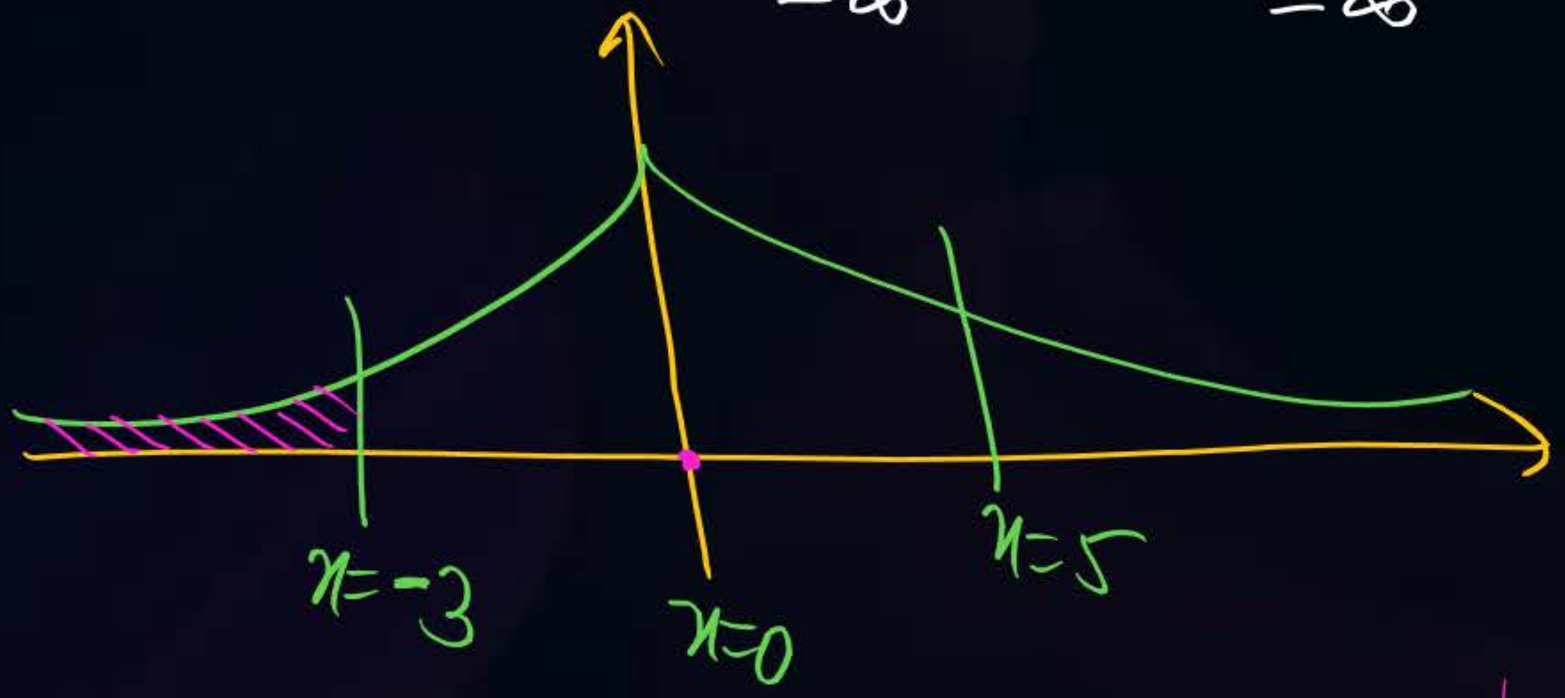
Q: if $f(x) = e^{-2|x|}$ is p.d.f for x then evaluate (1) $F(-3)$, (2) $F(0)$, (3) $F(5) = ?$

Sol: (1) $F(-3) = \int_{-\infty}^{-3} f(x)\,dx = \int_{-\infty}^{-3} e^{-2|x|}\,dx = \int_{-\infty}^{-3} e^{-2(-x)}\,dx = \left(\frac{e^{2x}}{2}\right)_{-\infty}^{-3} = \frac{1}{2}\left(e^{-6} - e^{-\infty}\right) = \frac{1}{2e^{6}} = 0.00125$

(2) $F(0) = \int_{-\infty}^{0} f(x)\,dx = \int_{-\infty}^{0} e^{-2|x|}\,dx = \int_{-\infty}^{0} e^{-2(-x)}\,dx = \left(\frac{e^{2x}}{2}\right)_{-\infty}^{0} = \frac{1}{2}\left(e^{0} - e^{-\infty}\right) = \frac{1}{2} = 0.5$

(3) $F(5) = \int_{-\infty}^{5} f(x)\,dx = \int_{-\infty}^{5} e^{-2|x|}\,dx = \int_{-\infty}^{0} e^{-2|x|}\,dx + \int_{0}^{5} e^{-2|x|}\,dx = \frac{1}{2} + \int_{0}^{5} e^{-2x}\,dx$

$= 0.5 + \left(\frac{e^{-2x}}{-2}\right)_{0}^{5} = 0.5 - \frac{1}{2}\left(e^{-10} - 1\right) = 0.9999$

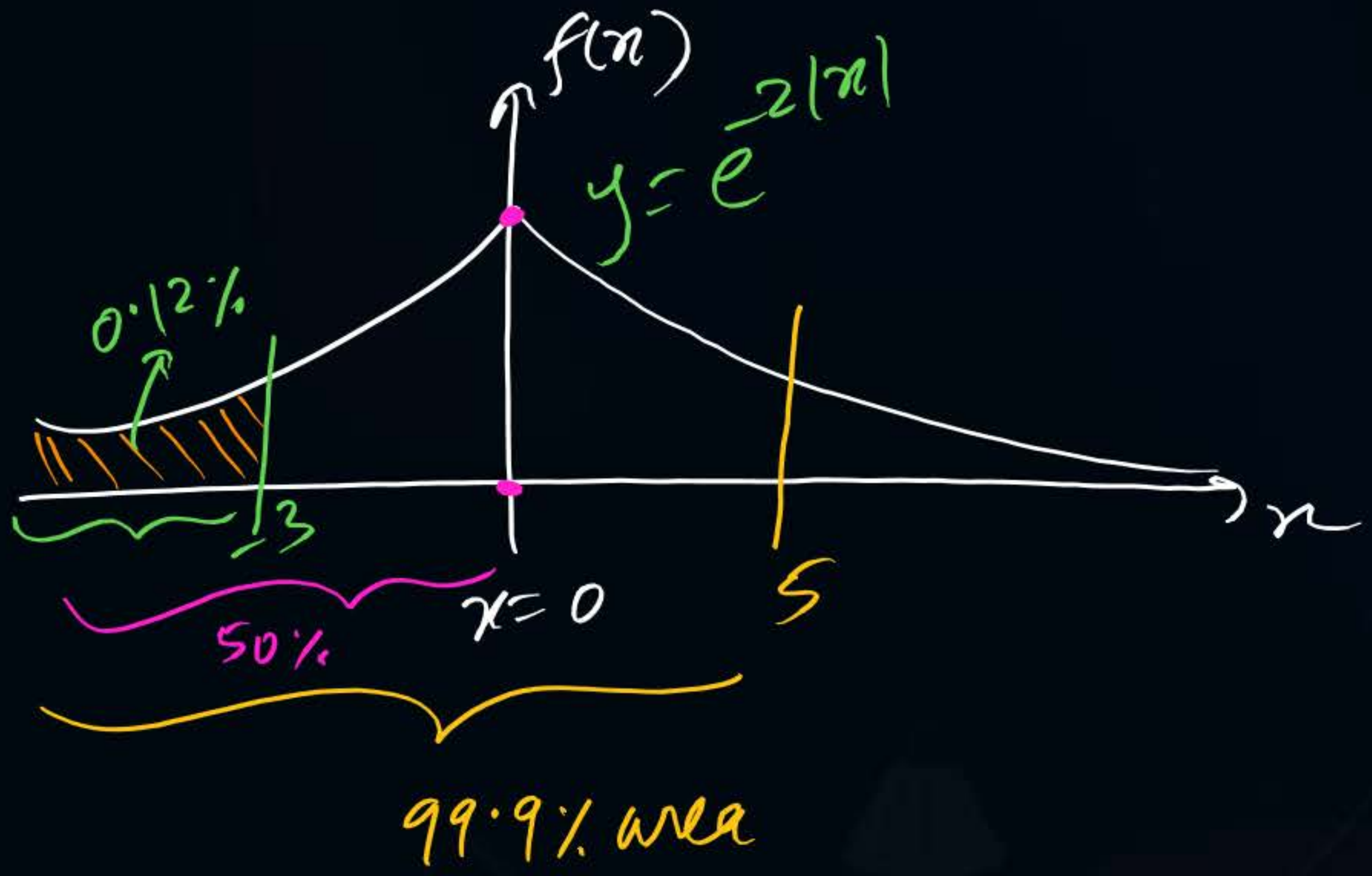


Note: ① $F(-3) = 0.0012 \simeq 0.12\%$

② $F(0) = 0.5 = 50\%$

③ $F(5) = 0.999 = 99.9\%$

ANALYSIS



Q: Find the General formula for calculating c.d.f for which p.d.f is given as $f(x)=e^{-2|x|}$

Sol: $f(x)=e^{-2|x|}\ ;\ -\infty<x<\infty$ = Even func^n

Case I: if $x<0$:

$$F(x)=\int_{-\infty}^{x} f(x)\,dx=\int_{-\infty}^{x} e^{-2|x|}\,dx$$

$$=\int_{-\infty}^{x} e^{-2(-x)}\,dx=\int_{-\infty}^{x} e^{2x}\,dx$$

$$=\left(\frac{e^{2x}}{2}\right)_{-\infty}^{x}=\frac{1}{2}\left(e^{2x}-e^{-\infty}\right)=\frac{e^{2x}}{2}$$

Case II if $x=0$

$$F(0)=\int_{-\infty}^{0} f(x)\,dx=\int_{-\infty}^{0} e^{-2|x|}\,dx=\int_{-\infty}^{0} e^{2x}\,dx=\frac{1}{2}$$

Case III if $x>0$;

$$F(x)=\int_{-\infty}^{x} f(x)\,dx=\int_{-\infty}^{0} f(x)\,dx+\int_{0}^{x} f(x)\,dx$$

$$=\frac{1}{2}+\int_{0}^{x} e^{-2|x|}\,dx=\frac{1}{2}+\int_{0}^{x} e^{-2(+x)}\,dx$$

$$=\frac{1}{2}+\left(\frac{e^{-2x}}{-2}\right)_{0}^{x}=\frac{1}{2}-\frac{1}{2}\left[e^{-2x}-e^{0}\right]=1-\frac{1}{2e^{2x}}$$

Hence C.D.F is $F(x)=\begin{cases}\dfrac{e^{2x}}{2}, & x<0\\ 1/2, & x=0\\ 1-\dfrac{1}{2e^{2x}}, & x>0\end{cases}$

ANALYSIS.

So c.d.f is $F(x) = \begin{cases} \dfrac{e^{2x}}{2}, & x<0 \\ \dfrac{1}{2}, & x=0 \\ 1-\dfrac{1}{2e^{2x}}, & x>0 \end{cases}$

Note: $\dfrac{d}{dx}F(x) = \begin{cases} e^{2x}, & x<0 \\ e^{-2x}, & x>0 \end{cases}$

$\boxed{f(x) = e^{-2|x|}}$; $-\infty<x<\infty$

Note: Evaluate $F(-3) = \dfrac{e^{-6}}{2} = 0.0012$

$F(0) = \dfrac{1}{2} = 0.5$

$F(5) = 1-\dfrac{1}{2e^{10}} = 0.9999$

$F(-2) = \dfrac{e^{-4}}{2} = \dfrac{1}{2e^{4}}$

$F(4) = 1-\dfrac{1}{2e^{8}}$ //

& so on....

Q. Qs $f(x)=\begin{cases}\frac{1}{8}, & 0<x\le 2\\ 3/8, & 2<x\le 4\\ 0, & \text{otw}\end{cases}$ is p.d.f then find c.d.f $F(x)=?$

Sol: **Case I** if $(x<0)\rightarrow$

$$F(x)=\int_{-\infty}^{x} f(x)\,dx=\int_{-\infty}^{x}(0)\,dx=0$$

Case II if $(0<x\le 2)\rightarrow$

$$F(x)=\int_{-\infty}^{x} f(x)\,dx=\int_{-\infty}^{0}(f(x))\,dx+\int_{0}^{x} f(x)\,dx$$

$$=0+\int_{0}^{x}\left(\frac{1}{8}\right)dx=\frac{x}{8}$$

Case III if $(2<x\le 4)$

$$F(x)=\int_{-\infty}^{x} f(x)=\int_{-\infty}^{0} f(x)\,dx+\int_{0}^{2} f(x)\,dx+\int_{2}^{x} f(x)\,dx$$

$$=0+\int_{0}^{2}\left(\frac{1}{8}\right)dx+\int_{2}^{x}\left(\frac{3}{8}\right)dx$$

$$=0+\frac{1}{8}(2-0)+\frac{3}{8}(x-2)=\frac{3x}{8}-\frac{1}{2}$$

Case IV if $x>4$

See Next Slide.

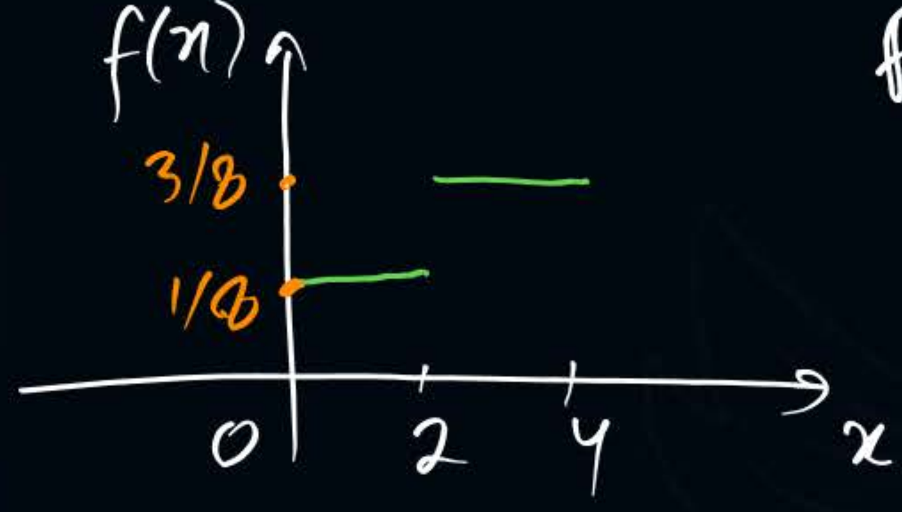
Case IV if $(x > 4) \to$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{0} (0)\,dx + \int_{0}^{2} \left(\frac{1}{8}\right) dx + \int_{2}^{4} \left(\frac{3}{8}\right) dx + \int_{4}^{\infty} (0)\,dx$$

$$= 0 + \frac{1}{8}(2-0) + \frac{3}{8}(4-2) + 0 = \dots = 1$$

i.e $f(x) = \begin{cases} 1/8, & 0 < x \le 2 \\ 3/8, & 2 < x \le 4 \\ 0, & \text{otw} \end{cases}$

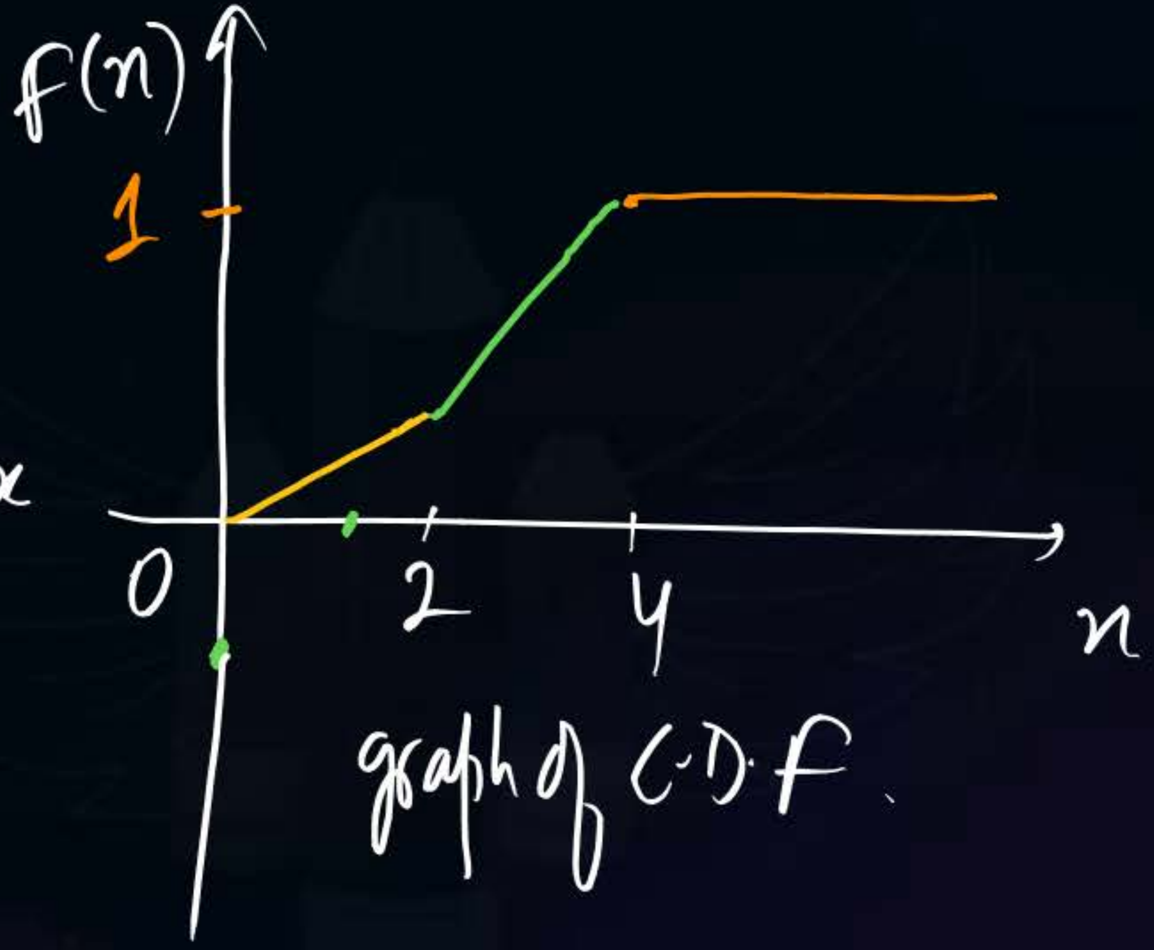
p.d.f



Graph of p.d.f.

$F(x) = \begin{cases} 0, & x < 0 \\ x/8, & 0 < x \le 2 \\ \frac{3x}{8} - \frac{1}{2}, & 2 < x \le 4 \\ 1, & x > 4 \end{cases}$

c.d.f

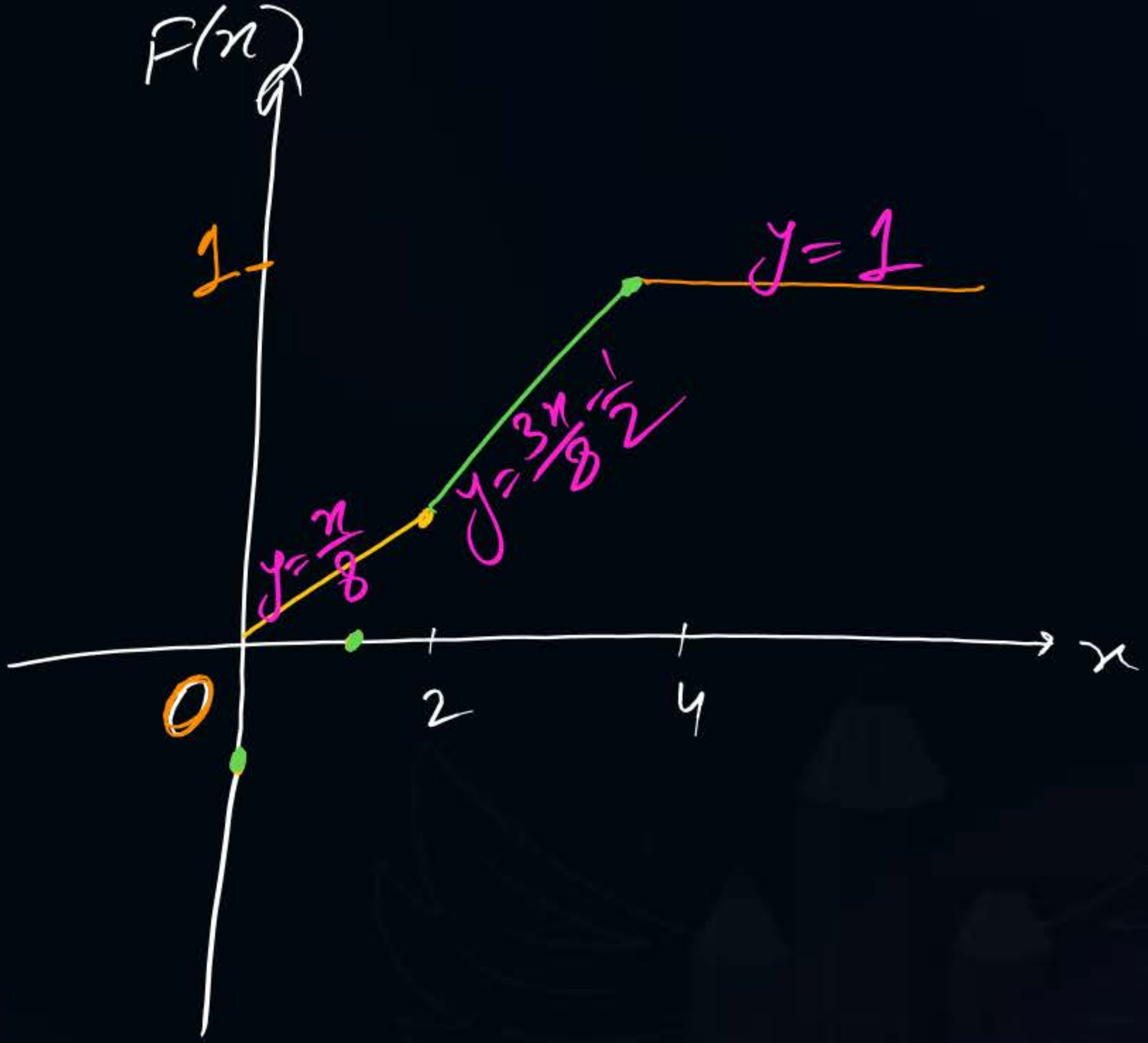


graph of C.D.F.

ANALYSIS :

Hence a formula for C.D.F is

$$y = F(x) = \begin{cases} 0 & , x < 0 \\ \frac{x}{8} & , 0 < x \le 2 \\ \left(\frac{3x}{8} - \frac{1}{2}\right) & , 2 < x \le 4 \\ 1 & , x > 4 \end{cases}$$



when, $2 < x \le 4 \Rightarrow y = \frac{3x}{8} - \frac{1}{2}$

$$-\frac{3x}{8} + y = -\frac{1}{2}$$

$$-3x + 8y = -4 \Rightarrow \frac{x}{4/3} + \frac{y}{-1/2} = 1$$

Q. if $f(x)=\begin{cases} \frac{1}{8}, & 0<x\le 2 \\ \frac{3}{8}, & 2<x\le 4 \\ 0, & \text{otherwise} \end{cases}$ is density function (p.d.f) for Random Variable x then it's distribution function (C.D.F.) will be?

(a) $F(x)=\begin{cases} 0, & x\le 0 \\ \frac{x}{8}, & 0<x\le 2 \\ \frac{3x}{8}+\frac{1}{4}, & 2<x\le 4 \\ 1, & x>4 \end{cases}$

(b) $F(x)=\begin{cases} 0, & x\le 0 \\ \frac{x}{8}, & 0<x\le 2 \\ \frac{3x}{8}+\frac{1}{2}, & 2<x\le 4 \\ 1, & x>4 \end{cases}$

(c) $F(x)=1 \ \forall x$

(d) $F(x)=\begin{cases} 0, & x\le 0 \\ x/8, & 0<x\le 2 \\ \frac{3x}{8}-\frac{1}{2}, & 2<x\le 4 \\ 1, & x>4 \end{cases}$

M-I By differentiating each option we can find p.d.f

(But Not always $\because$ options are not good enough).

C.D.F when $2<x\le 4$

$$F(x)=\int_{-\infty}^{x} f(x)\,dx=\int_{-\infty}^{0}(0)\,dx+\int_{0}^{2}\left(\frac{1}{8}\right)dx+\int_{2}^{x}\left(\frac{3}{8}\right)dx$$

$$=\cdots\cdots=\frac{3x}{8}-\frac{1}{2}$$

So (d) ✓

(HW) if $f(x)=ae^{-b|x|}$ is p.d.f for R.V x then find c.d.f for $x>0$

Sol: $F(x)=\int_{-\infty}^{x} f(x)\,dx=\int_{-\infty}^{0} f(x)\,dx+\int_{0}^{x} f(x)\,dx$

$=\frac{1}{2}+\ ?\ =------=$ Ans.

(HW) if $f(x)=ae^{-b|x|}$ is p.d.f then find C.D.F for $x<0$

Sol: $F(x)=\int_{-\infty}^{x} ae^{-b|x|}\,dx=------?=$ Ans

Exponential Distribution.

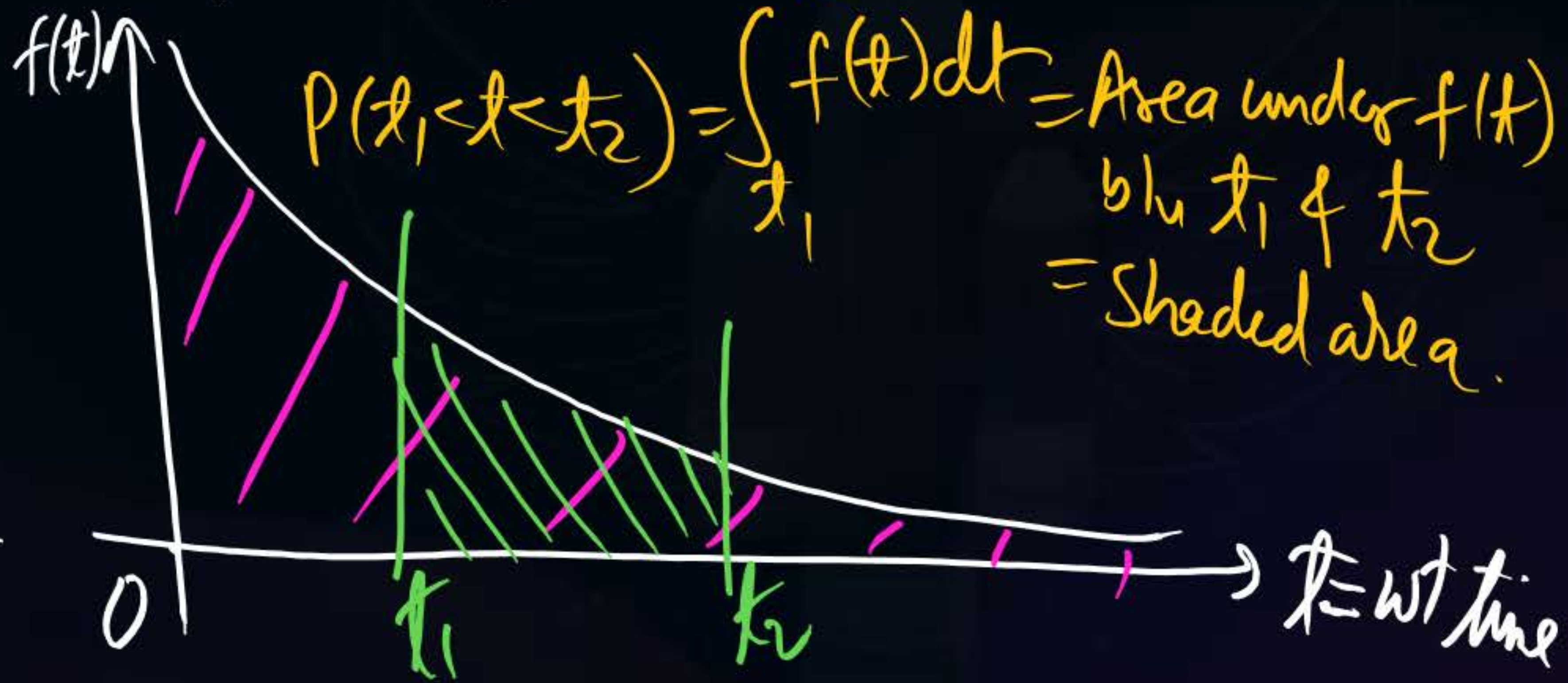
"Whenever in a Question there is a feeling of waiting time, we can follow E.D."

for eg, Arrivals of persons in a Queue is governed by POISSON. Dist.
& their waiting time in a Queue is governed by Exponential Distribution

Def^n: Let t is C.R.V s.t it's p.d.f is given as,

$$\boxed{f(t) = \begin{cases} \mu e^{-\mu t}, & t \geq 0 \\ 0, & t < 0 \end{cases}}$$

then t is called E.R.V with parameter μ
i.e $t \sim E\{\mu\}$



$$P(t_1 < t < t_2) = \int_{t_1}^{t_2} f(t)\,dt$$ = Area under $f(t)$ b/w t_1 & t_2
= Shaded area.

① Verification ! Total area $= \int_{-\infty}^{\infty} f(t)\,dt = \int_0^{\infty} \mu e^{-\mu t}\,dt = \mu\left[\frac{e^{-\mu t}}{-\mu}\right]_{t=0}^{\infty} = -\left[e^{-\infty} - e^{0}\right] = 1$

② Mean$(t) = E(t) = \int_{-\infty}^{\infty} t \cdot f(t)\,dt = \int_0^{\infty} t\,(\mu e^{-\mu t})\,dt = \text{.......} = \frac{1}{\mu}$ Learn

$\because$ t = waiting time So $E(t)$ = Average waiting time $= \frac{1}{\mu}$

③ $\text{Var}(t) = E(t^2) - E^2(t) = \int_0^{\infty} t^2 \cdot f(t)\,dt - \left(\frac{1}{\mu}\right)^2 = \text{.......} = \frac{1}{\mu^2}$

④ $SD(\sigma) = \frac{1}{\mu}$ i.e In Poisson Dist, Mean = Variance.

& In Exponential Dist, Mean = SD

(5) In poisson Dist, Arrival Rate = λ per unit Data

& In Exp Dist, $\boxed{\text{Service Rate} = \mu}$ " " "

If Doctor is taking 5 mins on an average to check one patient then Average waiting time = 5 mins per patient. $= \frac{1}{\mu}$

& service Rate of Doctor $= \mu = \frac{1}{5}$ patients per minute.

Conclusion :- Inter arrival time of two successive Arrivals follow Exp Dist

(6) $P(t_1 < t < t_2) = \int_{t_1}^{t_2} f(t)\,dt$ = Area under $f(t)$ b/w t_1 & t_2

Analysis (TRAILOR)

eg $\lambda = \frac{1}{10}$ per min $\Rightarrow$ In every 10 mins, one patient is coming

So Average waiting time for service provider

$$= 10 \text{ mins} = \frac{1}{\mu}$$

$$\Rightarrow \mu = \frac{1}{10} \text{ per min.}$$

7 C.D.F of Exp Dist — w.k. that pdf of E.D is $f(t) = \begin{cases} \mu e^{-\mu t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

So it's C.D.F will be

$$F(t) = \int_{-\infty}^{t} f(t)\,dt = \int_{0}^{t} f(t)\,dt = \int_{0}^{t} \mu e^{-\mu t}\,dt = \mu \left[\frac{e^{-\mu t}}{-\mu}\right]_{t=0}^{t}$$

$$= -\left[e^{-\mu t} - e^{0}\right] = 1 - e^{-\mu t}, \quad t \geq 0$$

Hence proved.

p.d.f $\boxed{f(t) = \begin{cases} \mu e^{-\mu t}, & t \geq 0 \\ 0, & \text{o/w} \end{cases}}$ & C.D.F $\boxed{F(t) = 1 - e^{-\mu t}}, \; t \geq 0$

Qs If the Average length of phone call at public Telephone Bhooth is of 10 mins & when you are about to start your call, someone arrives immediately ahead of you and starts calling (with your permission) then find the prob that you will have to wait b/w 10 & 20 mins?

(a) 0.124

(b) 0.413

(c) 0.232

(d) 0.5

$t = \{\text{length of the phone call}\} = \{\text{waiting time of that person}\}$

& Av waiting time $= \frac{1}{\mu} = 10$ mins $\Rightarrow \mu = \frac{1}{10}$ per min.

$$f(t) = \begin{cases} \mu e^{-\mu t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

So $P(10 < t < 20) = ?$

M-I - - - -

M-II - - - - -

M-I $P(10<t<20) = \int_{10}^{20} f(t)\,dt = \int_{10}^{20} \mu e^{-\mu t}\,dt = \mu\left[\frac{e^{-\mu t}}{-\mu}\right]_{10}^{20}$

$= -\left[e^{-20\mu} - e^{-10\mu}\right] = \frac{1}{e^{10\mu}} - \frac{1}{e^{20\mu}} = \frac{1}{e} - \frac{1}{e^2} = 0.232$

M-II $P(10<t<20) = F(20) - F(10) = \left(1 - e^{-20\mu}\right) - \left(1 - e^{-10\mu}\right)$

C.D.F of Exp Dist is

$F(t) = \begin{cases} 1 - e^{-\mu t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$= e^{-10\mu} - e^{-20\mu} = \frac{1}{e} - \frac{1}{e^2} = 0.232$

$= \frac{0.232}{1} = \frac{232}{1000}$

ie out of 1000 persons facing same problem,
232 " will have to wait b/w 10 & 20 mins.

Q. If x is E.R.V with parameter 1 and it's C.D.F is given as

$$F(x)=\begin{cases}\boxed{1-e^{-x}}, & x\geq 0\\ 0, & x<0\end{cases}$$ then Evaluate $P\left(\frac{x>2}{x>1}\right)=?$

(a) e

(b) $1/e$

(c) e^2

(d) $1/e^2$

$$P\left\{\frac{x>2}{x>1}\right\}=\frac{P\{x>2\cap x>1\}}{P\{x>1\}}=\frac{P(x>2)}{P(x>1)}=\frac{1-P(x\leq 2)}{1-P(x\leq 1)}$$

$$\left[\because P(A/B)=\frac{P(A\cap B)}{P(B)}\right]$$

$$=\frac{1-F(2)}{1-F(1)}=\frac{1-(1-e^{-2})}{1-(1-e^{-1})}=\frac{1}{e}$$

Note: $f(x)=\frac{d}{dx}F(x)=\begin{cases}e^{-x}, & x\geq 0\\ 0, & x<0\end{cases}$

Here $\mu=1$

HWQ Traffic is moving at the Rate of 360 veh/hr on a specific highway location. If the Arrivals of the vehicle at the location follows Poisson Distribution then find the probability that GAP b/n two successive vehicles lies in b/n 6 & 10 seconds?

(a) 0.2

(b) 0.1

(c) 0.18

(d) 0.5

Arrival Rate $(\lambda) = 360$ veh/hr $= 6$ veh/min $= \frac{1}{10}$ veh/sec

i.e. $\underbrace{\text{Av. time GAP}}$ b/n two successive vehicles $= 10$ sec

i.e. Av. waiting time for observer $= \frac{1}{\mu} = 10 \Rightarrow \mu = \frac{1}{10}$ per sec.

$$f(t) = \begin{cases} \mu e^{-\mu t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$t \to$ {time Gap b/n two successive Vehicles}

So $P(6 < t < 10) = \int_6^{10} f(t)\,dt = \ldots\ldots = 0.18$

Assume that the duration in minutes of a telephone conversation follows the exponential distribution $f(x) = \frac{1}{5}e^{-x/5}, x \geq 0$. The probability that the conversation will exceed five minutes is

(a) $\frac{1}{e}$ (HW)

(b) $1-\frac{1}{e}$

(c) $\frac{1}{e^2}$

(d) $1-\frac{1}{e^2}$

$\mu = \frac{1}{5}, \; P(x>5) = ? = \int_5^\infty f(x)\,dx = \dots = \frac{1}{e}$

Vehicles arriving at an intersection from one of the approach roads follow the Poisson distribution. The mean rate of arrival is 900 vehicles per hour. If a gap is defined as the time difference between two successive vehicle arrivals (with vehicles assumed to be points), the probability (up to four decimal places) that the gap is greater than 8 seconds is ______.

HW

Ans = 0.135

Thank

YOU

$\boxed{\text{6:30–10:00 AM}}$

Sunday