DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE
& CS / IT

Calculus and Optimization

Lecture No. 13

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Recap of previous lecture

Topic

INTEGRATION

Topics to be Covered

Topic

→ <u>Beta & Gamma functions</u>

→ Nature of Roots

→ PRACTICE QUESTIONS

BETA & GAMMA func^n

Properties of Gamma func^n →

① $\boxed{\Gamma(n+1) = \begin{cases} n! & , n \in +\text{Integer} \\ n\Gamma n & , n \in +\text{ve Rational.} \end{cases}}$

eg $\Gamma 5 = \Gamma(4+1) = 4!$, $\Gamma 4 = 3!$, $\Gamma 3 = 2!$

$\Gamma 2 = 1!$, $\Gamma 1 = 0!$, $\boxed{\Gamma 0 = \text{N.D}}$

Note: $\boxed{\Gamma\tfrac{1}{2} = \sqrt{\pi}}$, $\boxed{\Gamma(-\text{ve int}) = \text{N.D}}$

$\Gamma(-\text{ve Rational})$ = Defined by formula ②

eg $\Gamma\frac{3}{2} = \Gamma\left(\frac{1}{2}+1\right) = \frac{1}{2}\Gamma\frac{1}{2} = \frac{\sqrt{\pi}}{2}$

eg $\Gamma\frac{5}{2} = \Gamma\left(\frac{3}{2}+1\right) = \frac{3}{2}\cdot\Gamma\frac{3}{2} = \frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma\frac{1}{2} = \frac{3\sqrt{\pi}}{4}$

eg $\Gamma\frac{7}{2} = ? = \frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma\frac{1}{2}$

eg $\Gamma\frac{9}{2} = ? = \frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma\frac{1}{2}$

eg $\Gamma\frac{17}{4} = ?$

$= \cancel{\frac{15}{4}\cdot\frac{13}{4}\cdot\frac{11}{4}\cdot\frac{9}{4}\cdot\frac{7}{4}\cdot\frac{5}{4}\cdot\frac{3}{4}\cdot\frac{1}{4}\Gamma\frac{1}{4}}$

$= \frac{13}{4}\cdot\frac{9}{4}\cdot\frac{5}{4}\cdot\frac{1}{4}\cdot\Gamma\frac{1}{4}$

② $\boxed{\Gamma n \cdot \Gamma(1-n) = \dfrac{\pi}{\sin n\pi}}, \; n \notin I$ — ①

eg Evaluate $\Gamma\frac{1}{2}, \Gamma\left(-\frac{1}{2}\right), \Gamma\left(-\frac{3}{2}\right), \Gamma\frac{1}{4}\cdot\Gamma\frac{3}{4} = ?$

Sol ① Put $n = \frac{1}{2}$ in ①

$$\Gamma\frac{1}{2}\cdot\Gamma\left(1-\frac{1}{2}\right) = \frac{\pi}{\sin\left(\frac{\pi}{2}\right)}$$

$$\Gamma\frac{1}{2}\cdot\Gamma\frac{1}{2} = \frac{\pi}{(1)}$$

$$\left(\Gamma\frac{1}{2}\right)^2 = \pi \Rightarrow \Gamma\frac{1}{2} = \sqrt{\pi} \quad \text{Ans}$$

Neglecting -ve sign

Sol ② Put $n = \frac{3}{2}$ in ①.

$$\Gamma\frac{3}{2}\cdot\Gamma\left(1-\frac{3}{2}\right) = \frac{\pi}{\sin\left(\frac{3\pi}{2}\right)}$$

$$\frac{1}{2}\Gamma\frac{1}{2}\cdot\Gamma\left(-\frac{1}{2}\right) = \frac{\pi}{(-1)}$$

$$\frac{\sqrt{\pi}}{2}\cdot\Gamma\left(-\frac{1}{2}\right) = -\pi \Rightarrow \Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}$$

Sol 3: Put $n = \frac{5}{2}$ in ①.

$$\Gamma\frac{5}{2}\cdot\Gamma\left(1-\frac{5}{2}\right) = \frac{\pi}{\sin\left(\frac{5\pi}{2}\right)}$$

$$\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma\frac{1}{2}\cdot\Gamma\left(-3/2\right) = \frac{\pi}{(1)}$$

$$\frac{3}{4}\sqrt{\pi}\,\Gamma\left(-3/2\right) = \pi \Rightarrow \Gamma\left(-\frac{3}{2}\right) = \frac{4}{3}\sqrt{\pi}$$

Sol: (4) Put $x = \frac{1}{4}$ in (1) $\Rightarrow \Gamma\frac{1}{4} \cdot \Gamma\left(1-\frac{1}{4}\right) = \frac{\pi}{\sin\left(\frac{\pi}{4}\right)} \Rightarrow \Gamma\frac{1}{4} \cdot \Gamma\frac{3}{4} = \pi\sqrt{2}$ Ans

Property (3) $\displaystyle\int_0^{\pi/2} \sin^m\theta \cos^n\theta \, d\theta = \frac{\Gamma\frac{m+1}{2} \cdot \Gamma\frac{n+1}{2}}{2\,\Gamma\frac{m+n+2}{2}}$

Qe $\displaystyle I = \int_0^{\pi/2} \sin^3\theta \cos^4\theta \, d\theta = ?$

Here, $m = 3, n = 4$

$$= \frac{\Gamma\frac{3+1}{2} \cdot \Gamma\frac{4+1}{2}}{2\,\Gamma\frac{3+4+2}{2}} = \frac{\Gamma 2 \cdot \Gamma\frac{5}{2}}{2\,\Gamma\frac{9}{2}} = \frac{\lfloor 1 \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \sqrt{\pi}}{2 \cdot \frac{7}{2} \cdot \frac{5}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} \cdot \sqrt{\pi}}$$

$$= \frac{2}{35}$$

Q. $I=\int_0^{\pi/2}\sqrt{\cot\theta}\,d\theta = ? = \int_0^{\pi/2}\sqrt{\frac{\cos\theta}{\sin\theta}}\,d\theta = \int_{\theta=0}^{\pi/2}\sin^{-\frac{1}{2}}\theta\,\cos^{\frac{1}{2}}\theta\,d\theta$

Here $m=-\frac{1}{2},\ n=\frac{1}{2}$

(a) $\pi/\sqrt{2}$

(b) $\pi\sqrt{2}$

(c) 2π

(d) $\pi/2$

$$= \frac{\Gamma\left(\frac{-\frac{1}{2}+1}{2}\right)\cdot\Gamma\left(\frac{\frac{1}{2}+1}{2}\right)}{2\,\Gamma\left(\frac{-\frac{1}{2}+\frac{1}{2}+2}{2}\right)} = \frac{\Gamma\frac{1}{4}\cdot\Gamma\frac{3}{4}}{2\,\Gamma 1} = \frac{\pi\sqrt{2}}{2(1)} = \frac{\pi}{\sqrt{2}}$$

PYQ
2M

$$I=\int_0^{\pi/6}\cos^4(3\theta)\cdot\sin^3(6\theta)\,d\theta=?=\int_0^{\pi/2}\cos^4(t)\cdot\sin^3(2t)\cdot\frac{dt}{3}$$

(a) 3

(b) 1/90

(c) 1/15

(d) 0

Put $3\theta=t$

$d\theta=\frac{dt}{3}$

At $\theta=0,\ t=0$

At $\theta=\frac{\pi}{6},\ t=\frac{\pi}{2}$

$$=\frac{1}{3}\int_0^{\pi/2}\cos^4(t)\cdot[2\sin t\cos t]^3\cdot dt$$

$$=\frac{8}{3}\int_0^{\pi/2}\sin^3 t\cos^7 t\,dt=\frac{8}{3}\cdot\frac{\Gamma\frac{3+1}{2}\cdot\Gamma\frac{7+1}{2}}{2\Gamma\frac{3+7+2}{2}}$$

$$=\frac{4}{3}\cdot\frac{\Gamma 2\cdot\Gamma 4}{\Gamma 6}=\frac{4}{3}\cdot\frac{\lfloor 1\lfloor 3}{\lfloor 5}=\frac{1}{15}\ \text{(c)}$$

$$I=\int_0^{\pi} x\cos^2 x\,dx = ?$$

use property 5 of Definite Integration

ie $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I=\int_0^{\pi} x\cdot\cos^2 x\,dx = \int_0^{\pi}(\pi-x)\cos^2 x\,dx$$

$$I = \pi\int_0^{\pi}\cos^2 x\,dx - \int_0^{\pi} x\cos^2 x\,dx$$

The value of the integral $\int_0^{\pi} x\cos^2 x\,dx$ is

(a) $\frac{\pi^2}{8}$

(b) $\frac{\pi^2}{4}$

(c) $\frac{\pi^2}{2}$

(d) π^2

$$I=\pi\int_0^{\pi}\cos^2 x\,dx - I$$

$$2I = 2\pi\int_0^{\pi/2}\cos^2 x\,dx$$

$\because \int_0^{\alpha} f(x)\,dx = 2\int_0^{\alpha/2} f(x)\,dx$ if $f(\alpha-x)=f(x)$

$$I=\pi\int_0^{\pi/2}\sin^0 x\cdot\cos^2 x\,dx = \pi\cdot\frac{\Gamma\frac{0+1}{2}\,\Gamma\frac{2+1}{2}}{2\,\Gamma\frac{0+2+2}{2}}$$

$(m=0, n=2)$

$$=\frac{\pi}{2}\cdot\frac{\sqrt{\pi}\cdot\frac{1}{2}\sqrt{\pi}}{\Gamma 2}=\frac{\pi^2}{4}$$

(4) Def^n of Gamma func^n (2nd Eulerian Integral) →

$$\boxed{\int_0^\infty e^{-x}\cdot x^{n-1}\,dx = \Gamma n}$$

eg $I = \int_0^\infty \frac{x^3}{e^x}dx = ? = \int_0^\infty e^{-x}\cdot x^3 dx = \int_0^\infty e^{-x}\cdot x^{4-1}dx = \Gamma 4 = 3! = 6$

eg $I = \int_{x=0}^\infty e^{-\frac{x^2}{8}}dx = ? = \int_0^\infty e^{-y}\cdot \frac{\sqrt{2}\,dy}{\sqrt{y}} = \sqrt{2}\int_0^\infty e^{-y}\cdot y^{-\frac{1}{2}}dy$

Put $\frac{x^2}{8} = y \Rightarrow x = \sqrt{8}\cdot\sqrt{y}$

$dx = \sqrt{8}\cdot\frac{1}{2\sqrt{y}}dy = \sqrt{2}\cdot\frac{dy}{\sqrt{y}}$

$= \sqrt{2}\int_0^\infty e^{-\infty}\cdot y^{\frac{1}{2}-1}dy$

$= \sqrt{2}\cdot\Gamma\frac{1}{2} = \sqrt{2}\sqrt{\pi} = \sqrt{2\pi}$

Q. $\int_{-\infty}^{\infty} e^{-(x^2/2)}\,dx = ?$ Even func^n $= 2\int_0^{\infty} e^{-\left(\frac{x^2}{2}\right)}dx = 2\int_0^{\infty} e^{-y}\cdot\frac{1}{\sqrt{2}\cdot\sqrt{y}}dy = \sqrt{2}\int_0^{\infty} e^{-y}\cdot y^{-\frac{1}{2}}dy$

(a) $\frac{1}{2}$ (b) $\sqrt{2\pi}$

(c) 1 (d) ∞

Put $\frac{x^2}{2} = y \Rightarrow x = \sqrt{2}\cdot\sqrt{y}$

$dx = \sqrt{2}\cdot\frac{1}{2\sqrt{y}}dy$

$= \sqrt{2}\cdot\int_0^{\infty} e^{-y}\cdot y^{\frac{1}{2}-1}dy = \sqrt{2}\cdot\Gamma\frac{1}{2} = \sqrt{2\pi}$

Question of Country :-

$I = \int_0^{\infty} e^{-x^2}dx = ? = \int_0^{\infty} e^{-y}\cdot\frac{dy}{2\sqrt{y}} = \frac{1}{2}\int_0^{\infty} e^{-y}\cdot y^{-\frac{1}{2}}dy = \frac{1}{2}\int_0^{\infty} e^{-y}\cdot y^{\frac{1}{2}-1}dy$

Put $x^2 = y \Rightarrow x = \sqrt{y}$

$dx = \frac{dy}{2\sqrt{y}}$

$= \frac{1}{2}\Gamma\frac{1}{2} = \frac{\sqrt{\pi}}{2}$ i.e. $\boxed{\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}}$

(5) Def^n of Beta func^n

(1st Eulerian Integral)

$$\boxed{\int_0^1 x^{m-1}\cdot(1-x)^{n-1}\,dx = B(m,n)} = \frac{\Gamma m\cdot\Gamma n}{\Gamma m+n}$$

Note (1) Relationship b/w Beta & Gamma func^n is $B(m,n) = \frac{\Gamma m\cdot\Gamma n}{\Gamma m+n}$

(2) Beta func^n is symmetrical about m & n i.e $B(m,n) = B(n,m)$

(3) $$\boxed{\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = B(m,n)} = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}$$

Q. $I = \int_0^2 x(8-x^3)^{1/3}\,dx = ?$ $= \int_0^1 2\cdot y^{\frac{1}{3}}\cdot(8-8y)^{\frac{1}{3}}\cdot\frac{2}{3}y^{-\frac{2}{3}}\,dy$

(a) 16

(b) 16π

(c) $\frac{16\pi}{3\sqrt{3}}$

(d) $\frac{16\pi}{9\sqrt{3}}$ ✓

Put $x^3 = 8y$

$x = (8y)^{\frac{1}{3}} = 2y^{\frac{1}{3}}$

$dx = 2\cdot\frac{1}{3}y^{-2/3}\,dy$

At $x=0,\ y=0$

At $x=2,\ y=1$

$= \frac{8}{3}\int_0^1 y^{-\frac{1}{3}}(1-y)^{\frac{1}{3}}\,dy = \frac{8}{3}\int_0^1 y^{\frac{2}{3}-1}\cdot(1-y)^{\frac{4}{3}-1}\cdot dy$

$= \frac{8}{3}\cdot B\left(\frac{2}{3},\frac{4}{3}\right) = \frac{8}{3}\cdot\frac{\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{2}{3}+\frac{4}{3}\right)}$

$= \frac{8}{3}\cdot\frac{\Gamma\left(\frac{2}{3}\right)\cdot\frac{1}{3}\Gamma\left(\frac{1}{3}\right)}{\Gamma(2)} = \frac{8}{9}\Gamma\left(\frac{1}{3}\right)\cdot\Gamma\left(1-\frac{1}{3}\right) = \frac{8}{9}\cdot\frac{\pi}{\sin\left(\frac{\pi}{3}\right)}$

$= \frac{8}{9}\frac{\pi}{(\sqrt{3}/2)} = \frac{16\pi}{9\sqrt{3}}$

Q. $I=\int_0^\infty \frac{dx}{1+x^4}=?=\int_0^\infty \frac{\frac{1}{4}y^{-\frac{3}{4}}dy}{(1+y)^1}=\frac{1}{4}\int_0^\infty \frac{y^{\frac{1}{4}-1}}{(1+y)^{\frac{1}{4}+\frac{3}{4}}}\cdot dy$

(a) $\pi/2$

(b) 0

(c) $\pi/2\sqrt{2}$

(d) $\frac{\pi}{4}$

Put $x^4=y \Rightarrow x=y^{\frac{1}{4}}$

$dx=\frac{1}{4}y^{-3/4}dy$

$=\frac{1}{4}B\left(\frac{1}{4},\frac{3}{4}\right)=\frac{1}{4}\frac{\Gamma\frac{1}{4}\cdot\Gamma\frac{3}{4}}{\Gamma\left(\frac{1}{4}+\frac{3}{4}\right)}$

$=\frac{1}{4}\left[\frac{\frac{\pi}{\sin\left(\frac{\pi}{4}\right)}}{\Gamma 1}\right]=\frac{1}{4}(\pi\sqrt{2})=\frac{\pi}{2\sqrt{2}}$

W.k.that $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx=B(m,n)$

Question of Country:

Learn by ♡

① $\int_0^{\pi/2} \log(\sin x)\,dx = -\frac{\pi}{2}\log 2$, ② $\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$

③ $\int_0^{\infty}\left(\frac{\sin ax}{x}\right)dx = \frac{\pi}{2}$ $(a>0)$, ④ $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy = \frac{\pi}{4}$

(using Laplace Transform) (using Double Integral)

eg $\boxed{\int_0^{\infty}\frac{\sin x}{x}dx = \frac{\pi}{2},}$ $\int_0^{\infty}\left(\frac{\sin 2x}{x}\right)dx = \frac{\pi}{2}$, $\int_0^{\infty}\left(\frac{\sin 3x}{x}\right)dx = \frac{\pi}{2}$ - - - - -

Proof of ④: $I = \int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy = \int_0^{\infty} e^{-x^2}dx \times \int_0^{\infty} e^{-y^2}dy = \frac{\sqrt{\pi}}{2}\times\frac{\sqrt{\pi}}{2} = \frac{\pi}{4}$

Q8) $I = \int_0^{\infty} \frac{12 \cos \pi t \sin 2\pi t}{\pi t} dt = ?$ $= \frac{6}{\pi}\int_0^{\infty} \frac{2 \cdot \sin(2\pi t)\cos(\pi t)}{t} dt$

(a) 0

(b) $6/\pi$

(c) 6

(d) $\pi/2$

$2 \sin A \cos B$

$= \sin(A+B) + \sin(A-B)$

$= \frac{6}{\pi}\int_0^{\infty}\left[\frac{\sin(3\pi t) + \sin(\pi t)}{t}\right] dt$

$= \frac{6}{\pi}\left[\int_0^{\infty}\left(\frac{\sin 3\pi t}{t}\right) dt + \int_0^{\infty}\left(\frac{\sin \pi t}{t}\right) dt\right]$

$= \frac{6}{\pi}\left[\frac{\pi}{2} + \frac{\pi}{2}\right] = 6$

$$\int_0^1 x^6\sqrt{1-x^2}\,dx =$$

(a) $\frac{5\pi}{256}$ (b) $\frac{5\pi}{128}$

(c) $\frac{5\pi}{512}$ (d) $\frac{3\pi}{512}$

$$I=\int_0^1 x^6\sqrt{1-x^2}\,dx$$

$$=\int_0^1 x^6(1-\underline{x^2})^{\frac{1}{2}}\,dx$$

Put $x^2=y \Rightarrow x=\sqrt{y}$, $dx=\frac{1}{2\sqrt{y}}dy$

$$I=\int_0^1 (\sqrt{y})^6(1-y)^{\frac{1}{2}}\cdot\frac{1}{2\sqrt{y}}dy$$

$$=\frac{1}{2}\int_0^1 y^3\cdot y^{-\frac{1}{2}}\cdot(1-y)^{\frac{1}{2}}\cdot dy$$

$$I=\frac{1}{2}\int_0^1 y^{\frac{5}{2}}\cdot(1-y)^{\frac{1}{2}}\,dy$$

$$=\frac{1}{2}\int_0^1 y^{\frac{7}{2}-1}\cdot(1-y)^{\frac{3}{2}-1}\,dy$$

$$=\frac{1}{2}B\left(\frac{7}{2},\frac{3}{2}\right)=\frac{1}{2}\cdot\frac{\Gamma\frac{7}{2}\cdot\Gamma\frac{3}{2}}{\Gamma\left(\frac{7}{2}+\frac{3}{2}\right)}$$

Ans.

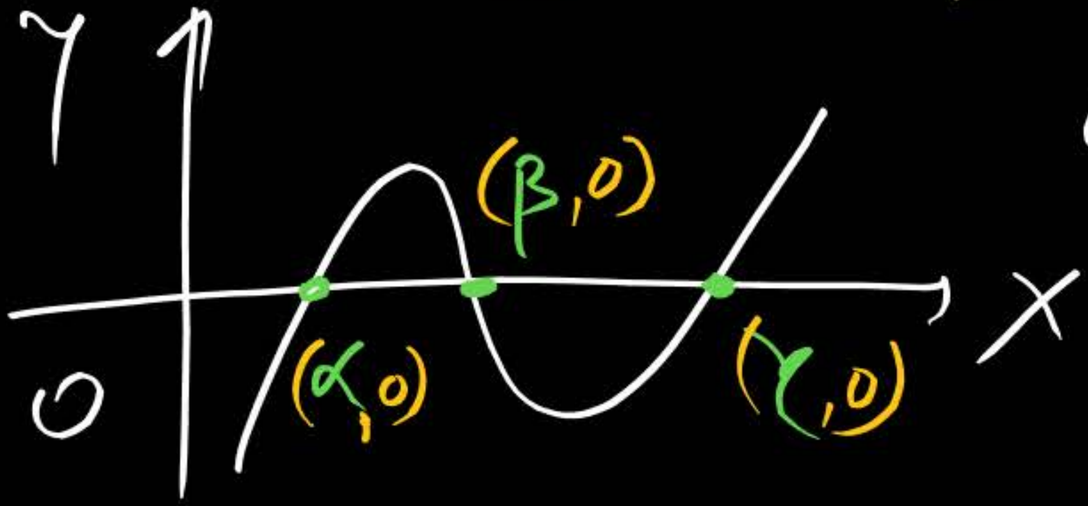
$$=\frac{1}{2}\cdot\frac{\left(\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\right)\cdot\left(\frac{1}{2}\cdot\sqrt{\pi}\right)}{\Gamma 5}$$

$$=\frac{5\pi}{256}$$ Ans.

NATURE of ROOTS

(*) Roots / Solutions / ZEROCROSSING / ZEROS $\longrightarrow$ (Points touching X axis)

(Real Roots) (Complex Roots)



Consider $y = f(x)$ s.t α, β, γ are the Roots

then $y = f(\alpha) = f(\beta) = f(\gamma) = 0$

(*) Complex Roots occurs in pair only when coefficients are Real.

eg Consider $x^2 - (i+1)x + i = 0$

$$x^2 - ix - x + i = 0$$

$$x(x-i) - 1(x-i) = 0 \Rightarrow (x-i)(x-1) = 0 \Rightarrow x = 1 \ \& \ i$$

(*) n^{th} degree poly has exactly <u>n</u> roots whether Real or Complex.

(*) n^{th} degree poly (with Real Coeff) has exactly <u>n</u> roots in which at least <u>one</u> will be Real & at Most <u>n−1</u> will complex. (where n = odd)

bcoz in that Situation Complex Roots will be in pair.

(*) <u>Descartes Rule of Sign</u> →

(i) No. of +ve Real Roots of $f(x) \leq$ No. of times Sign changes in $f(x)$

(ii) No. of −ve Real Roots of $f(x) \leq$ No. of times Sign changes in $f(-x)$

Q: choose the possible correct options for $f(x) = x^9 + 5x^3 - x^2 + 7x + 2$

MSQ

(a) $f(x)$ has at most 2 +ve roots ($\because$ No. of times sign changes in $f(x) = 2$)

(b) $f(x)$ has at most 3 -ve roots

$f(-x) = (-x)^9 + 5(-x)^3 - (-x)^2 + 7(-x) + 2$

$= -x^9 - 5x^3 - x^2 - 7x + 2$

(c) $f(x)$ has at least 4 complex roots

so No. of times sign changes in $f(-x)$ = one

(d) $f(x)$ has at least one real root ($\because$ $f(x)$ is an odd degree poly with real coeff)

The polynomial $p(x) = x^5 + x + 2$ has

(a) all real roots

(b) 3 real and 2 complex roots

(c) 1 real and 4 complex roots

(d) all complex roots

$P(x) = x^5 + x + 2$

No sign change in $P(x)$

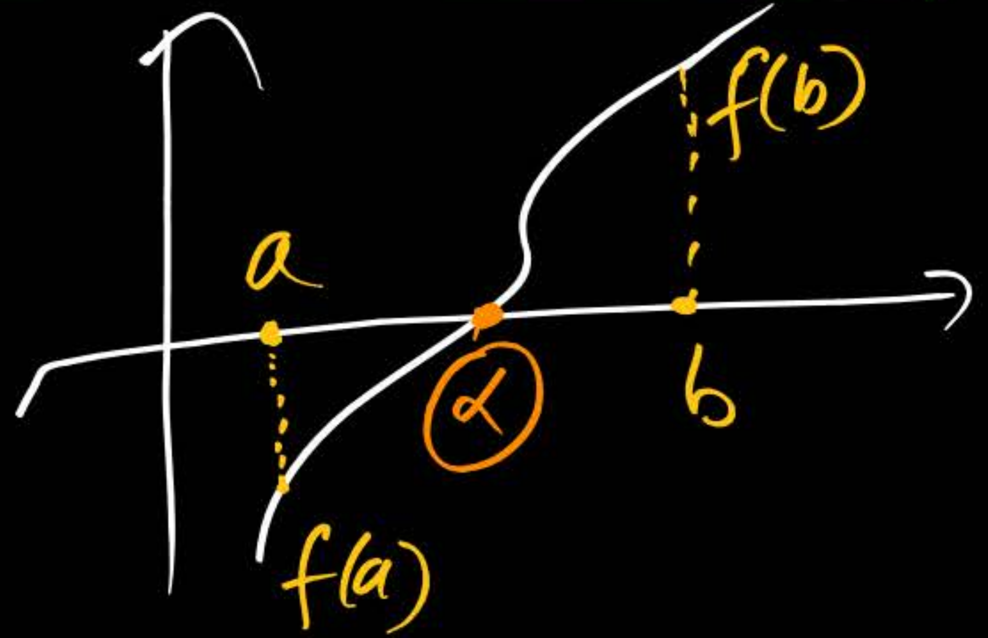
$\Rightarrow$ No +ve Root.

$P(-x) = -x^5 - x + 2$

No of −ve Real Roots ≤ 1

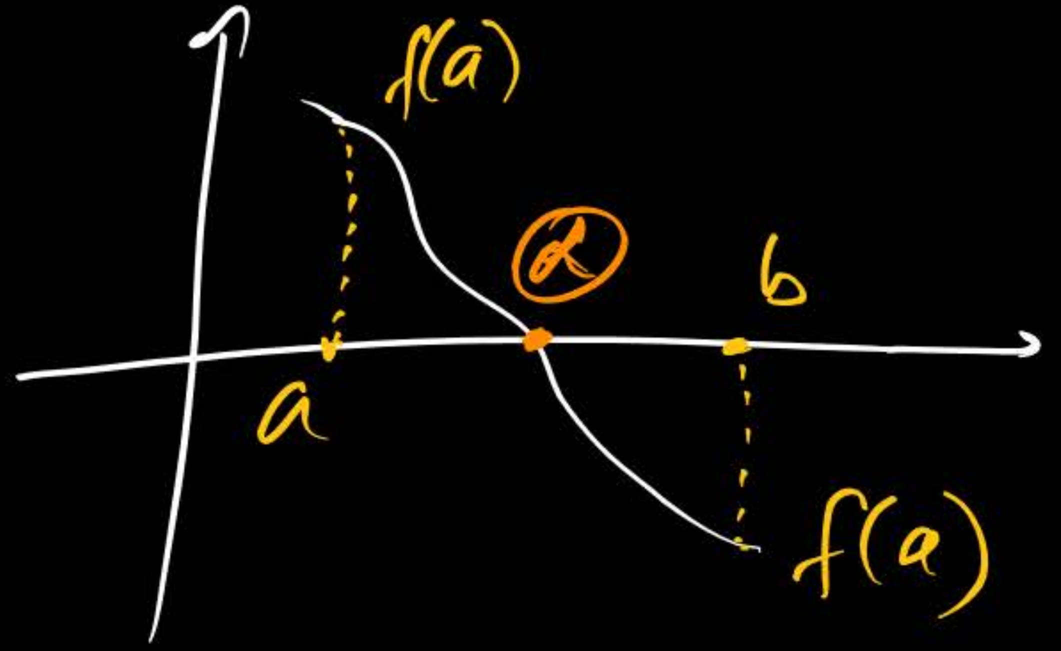
So No of Complex Roots ≤ 4

BOLZANO THEOREM →



$\Rightarrow \quad f(a).f(b) < 0$

$\Rightarrow \exists\ \alpha \in (a,b)$



$f(a).f(b) < 0$

$\exists\ \alpha \in (a,b)$

"if at $x=a$ & at $x=b$, $f(a)$ & $f(b)$ are of opposite sign then $\exists$ at least one Root α of $f(x)$ b/w a & b"

or "if $f(a).f(b)<0$ then $\exists$ at least one $\alpha \in (a,b)$ s.t $f(\alpha)=0$."

Q: Consider $\boxed{f(x) = x^4 - x^3 - x^2 - 4}$, $[1, 9]$ then $\alpha = ?$ where $\alpha \in (1, 9)$

Sol: $\because f(1) = -ve$

$f(9) = +ve$ So By B.Th $\alpha \in (1, 9)$

Again $f(5) = +ve$ So By B.Th $\alpha \in (1, 5)$

Again $f(3) = +ve$ " " " $\alpha \in (1, 3)$

Again $\boxed{f(2) = 0}$ So $\alpha = 2$ Ans

Q. if $xe^x = \cos x$ then one of the sol of this eq^n lies in b/w

(a) $(2, 3)$

(b) $(-1, 0)$

(c) $(0.56, 0.60)$

(d) $(1, 2)$

Let $\boxed{f(x) = xe^x - \cos x}$

(a) $f(2) = +ve$, $f(3) = +ve \Rightarrow \alpha \notin (2, 3)$

(b) $f(-1) = -1.e^{-1} - \cos(-1) \simeq \frac{-1}{e} - \cos\left(\frac{\pi}{3}\right) = -ve$

$f(0) = 0 - \cos(0) = -1 = -ve \Rightarrow \alpha \notin (-1, 0)$

(d) $f(1) = e - \cos(1) = 2.71 - (\text{value b/w } -1 \,\&\, 1) = +ve$

$f(2) = 2e^2 - \cos(2) = +ve \Rightarrow \alpha \notin (1, 2)$

(c) $f(0) = -\cos 0 = -1$, $f(1) = e - \cos(1) = +ve \Rightarrow \alpha \in (0, 1)$ ✓

A non-zero polynomial f(x) of degree 3 has roots at $x = 1$, $x = 2$ and $x = 3$. Which one of the following must be TRUE?

(a) $f(0)\,f(4) < 0$ (b) $f(0)\,f(4) > 0$

(c) $f(0) + f(4) > 0$ (d) $f(0) + f(4) < 0$

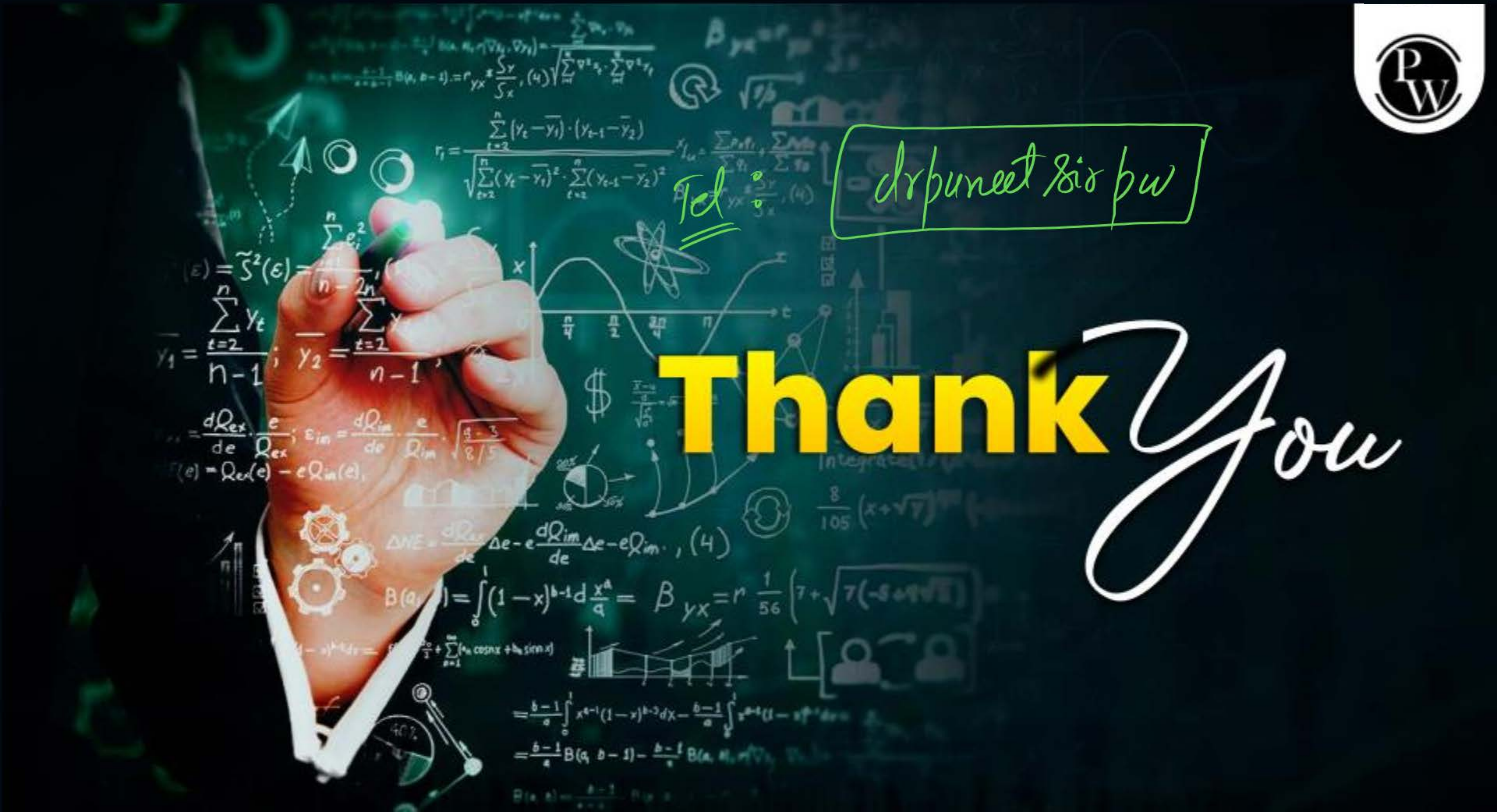
$f(x)$: $\alpha = 1$, $\beta = 2$, $\gamma = 3$

if $f(0) \cdot f(4) < 0$

i.e. $f(0)$ & $f(4)$ are of opposite sign

So By B. Th, $\exists$ at least one root α of $f(x)$ b/w 0 & 4

& Here $\alpha = 1, 2, 3 \in (0, 4)$ So (a) ✓



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