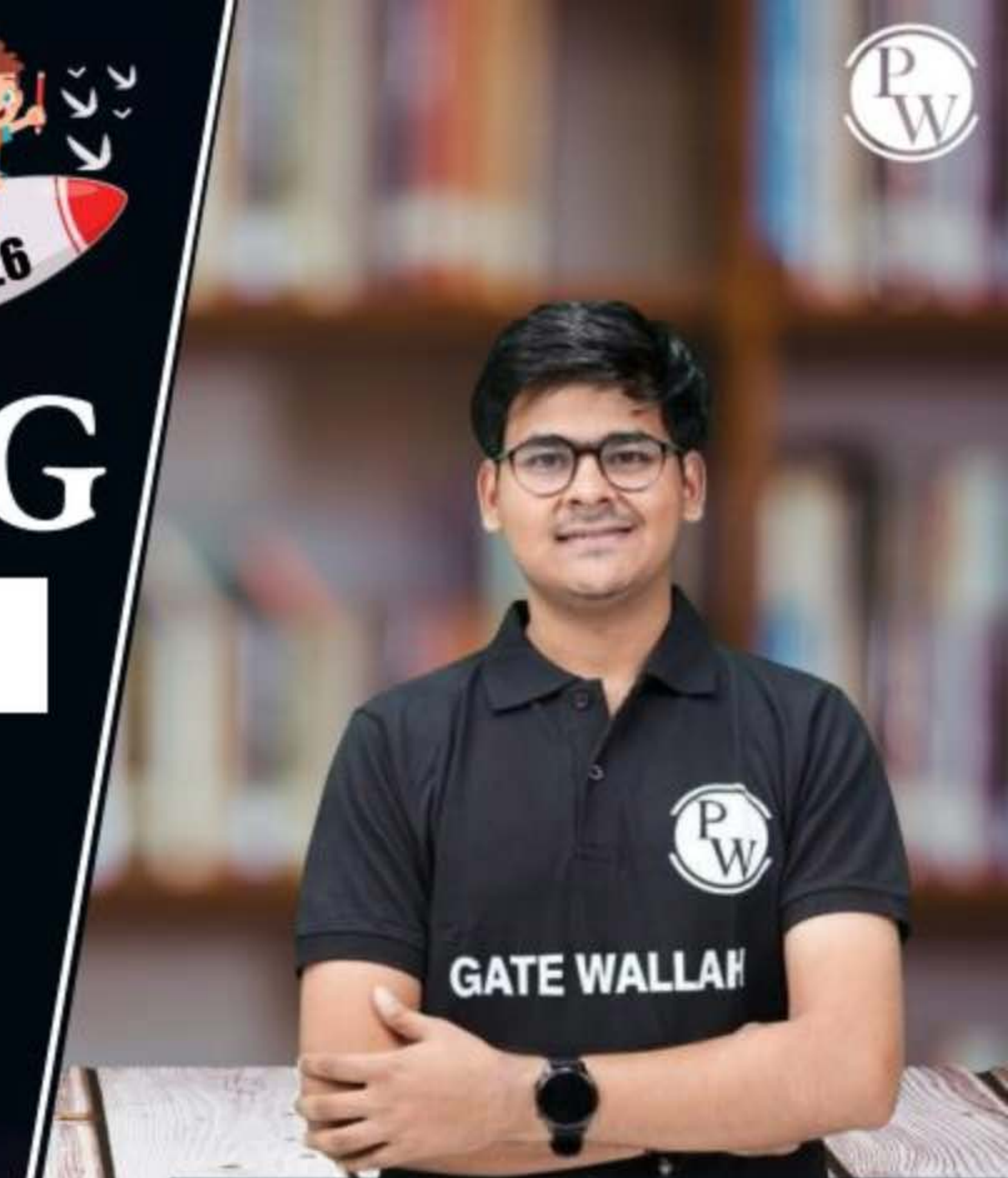
CS & IT ENGINEERING

Algorithms

Greedy Method

Lecture No.- 05



By- Aditya Jain sir

Topics to be Covered

Topic

Topic HE

Topic MCST

About Aditya Jain sir

1. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
2. Represented college as the first Google DSC Ambassador.
3. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored 12,000+ students & working professions in field of Data Science and Analytics
11. Have been mentoring & teaching GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

@ADITYASIR_PW

Telegram Link for Aditya Jain sir: https://t.me/AdityaSir_PW

Tomorrow: Algo at 8 AM.

Algo ends on 17th.

Topic : Greedy Algorithms

#Q. The characters 'A' to 'H' have the set of frequencies based on the first 8 Fibonacci number as follows:

A =1, B =1, C=2, D=3, E= 5, F= 8, G = 13, H=21

A Huffman code is used to represent the characters. What is the sequence of characters corresponding to the following code: 110111100111010

A FDHEG

B ECGDF

C DCHFG

D FEHDG

Topic : Greedy Algorithms

$A \rightarrow 1111100$

$B \rightarrow 1111101$

$C \rightarrow 111111$

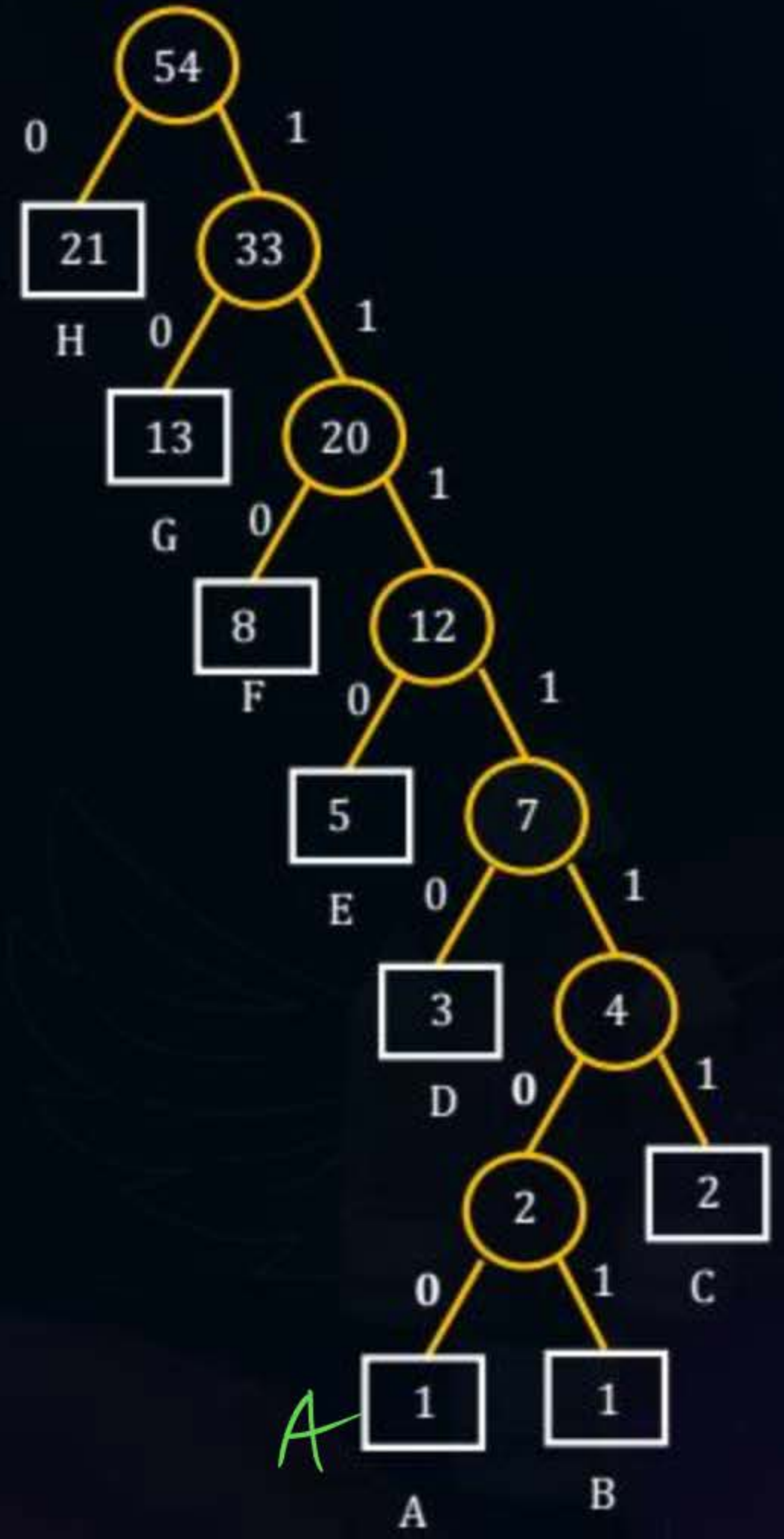
$D \rightarrow 11110$

$E \rightarrow 1110$

$F \rightarrow 110$

$G \rightarrow 10$

$H \rightarrow 0$



Topic : Greedy Algorithms

110	11 11 0	0	111	010
F	D	H	E	G

Decode :

Text : "FDHEG"

PYQ

#Q. A message is made up entirely of characters from the set X = {P ,Q , R, S, T}. The table probabilities for each of the characters is shown below:

Characters	Probability
P	0.22
Q	0.34
R	0.17
S	0.19
T	0.08
Total	1.00

If a message of 100 characters over X is encoded using Huffman coding, then the expected length of the encoded message in bits is______.

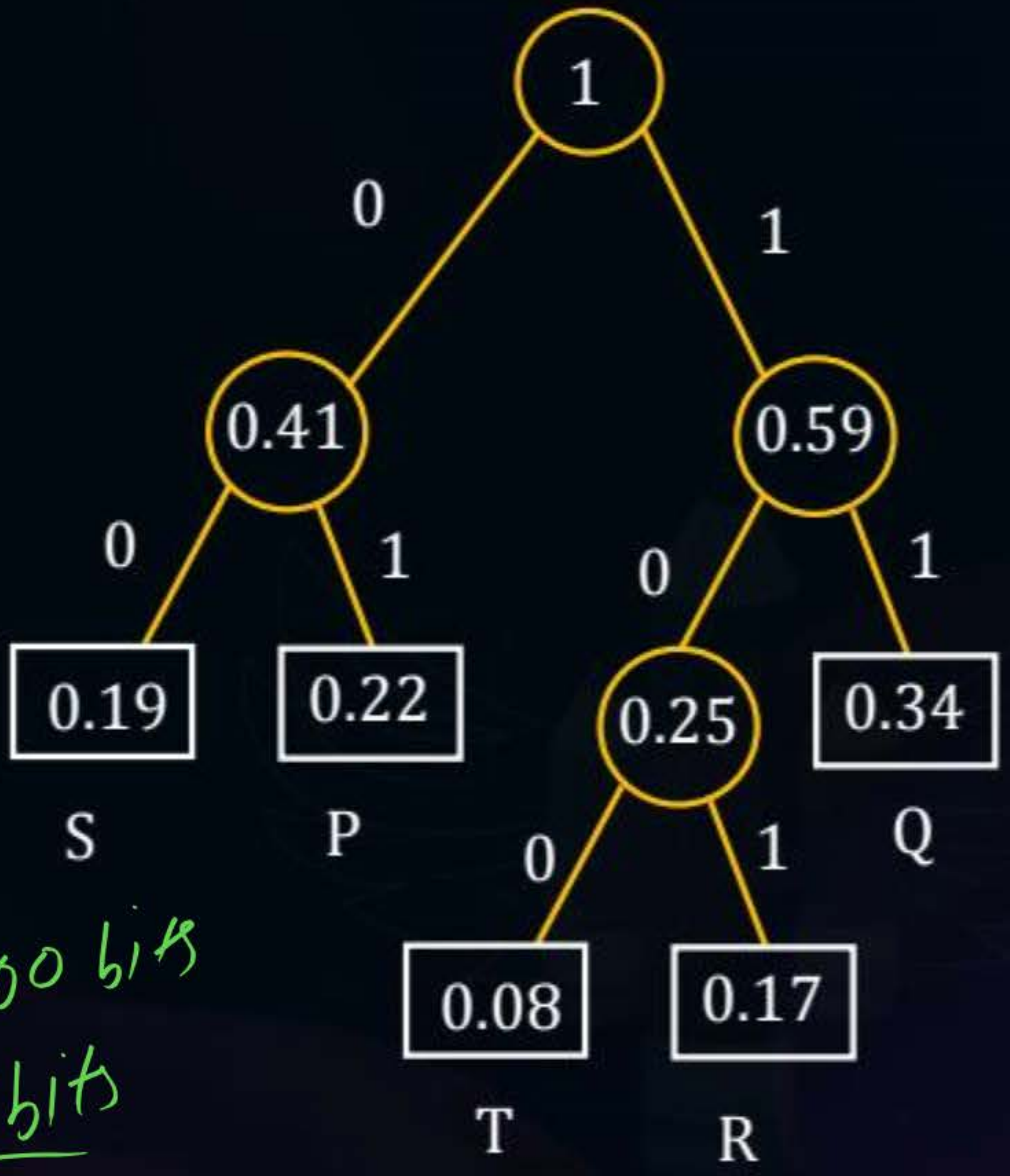
$P \rightarrow 01$

$Q \rightarrow 11$

$S \rightarrow 00$

$R \rightarrow 101$

$T \rightarrow 100$



Avg bits/char:

$= \sum d_i q_i = \underline{\underline{2.25}}$

For 100 char $= 2.25 \times 100$ bits

$= \underline{225 \text{ bits}}$

Topic : Greedy Algorithm

[MCQ]

#Q. Consider the string abbccddeee. Each letter in the string must be assigned a binary code satisfying the following properties:

1. For any two letters, the code assigned to one letter must not be a prefix of the code assigned to the other letter.

HE

2. For any two letters of the same frequency, the letter which occurs earlier in the dictionary order is assigned a code whose length is at most the length of the code assigned to the other letter.

Among the set of all binary code assignment which satisfy the above two properties, what is the minimum length of the encoded string?

$b \leq c \leq d$

A. 25

B. 23

C. 21

D. 30

Topic : Greedy Algorithm

Freq

a - 1 — 1

b - 2 — 4

c - 2 — 3

d - 2 — 2

e - 3 — 5

Encoding:

a → 100 (3)

b → 00 (2)

c → 01 (2)

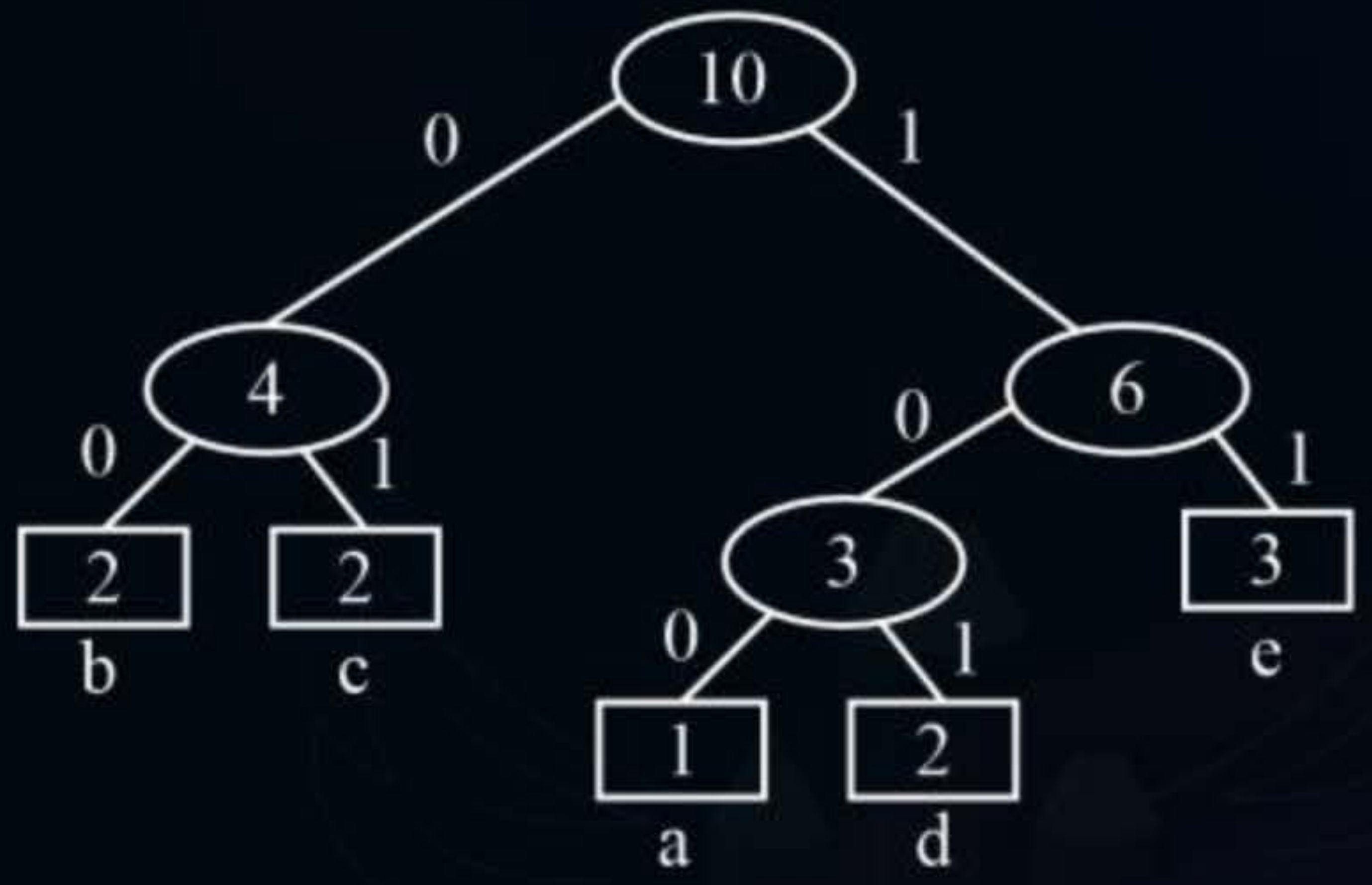
d → 101 (3)

e → 11 (2)

Text "abbccddeee"

Total length = 3 × 1 + 2 × 2 + 2 × 2 + 3 × 2 + 2 × 3

= 23



MCST

Topic : Greedy Algorithm

Minimum Cost Spanning Tree (MCST):

- Graph Based Problem
- Graph : $G \equiv (V, E)$

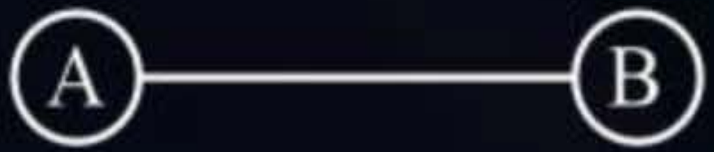
where, V = Set of vertices

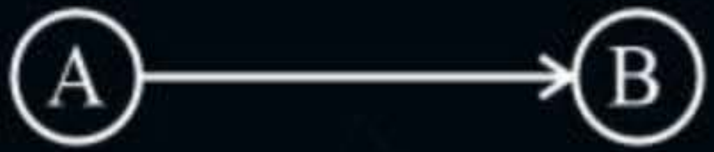
E = Set of edges

Topic : Greedy Algorithm

Types:

(1) Undirected vs Directed:



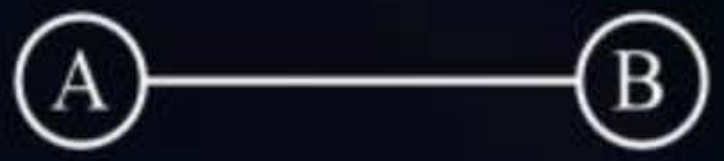


$2 \times 2 = 4$

Topic : Greedy Algorithm

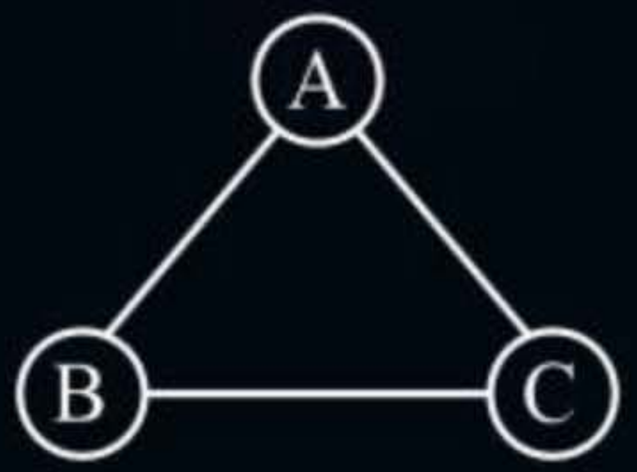
Types:

(2) Weighted vs Un-weighted:

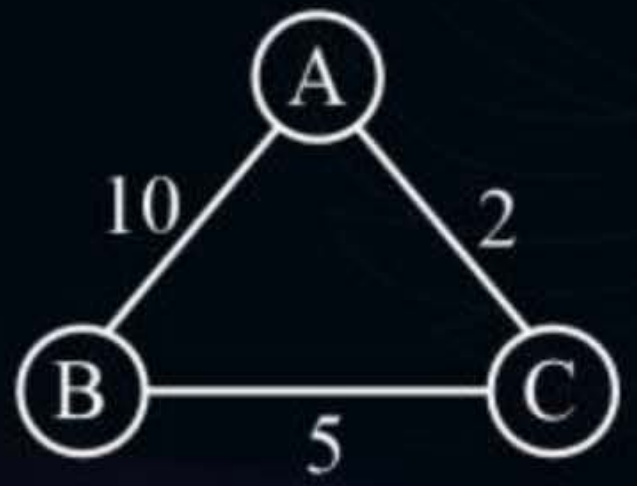


Topic : Greedy Algorithm

(1) Undirected Unweight Graph:

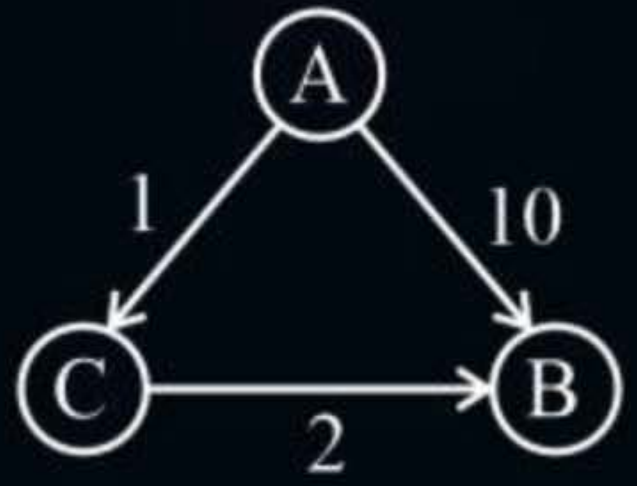


(2) Undirected Weight Graph:

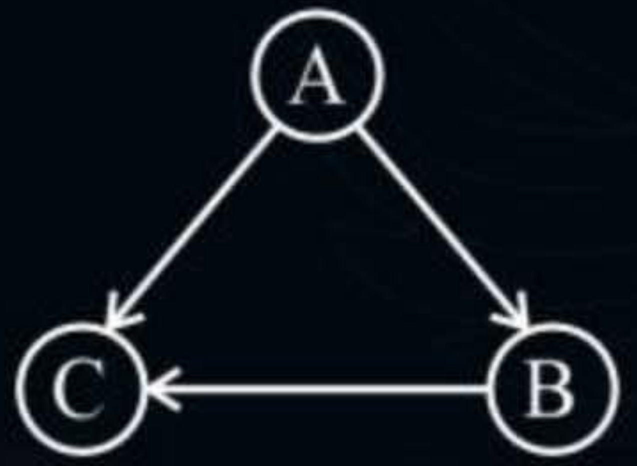


Topic : Greedy Algorithm

(3) Directed weight Graph:



(4) Directed Unweight Graph:



Topic : Greedy Algorithm

Representation of Graph:

(1) Adjacency matrix → (Array)

(2) Adjacency List → (List based)

Topic : Greedy Algorithm

(1) Adjacency matrix:

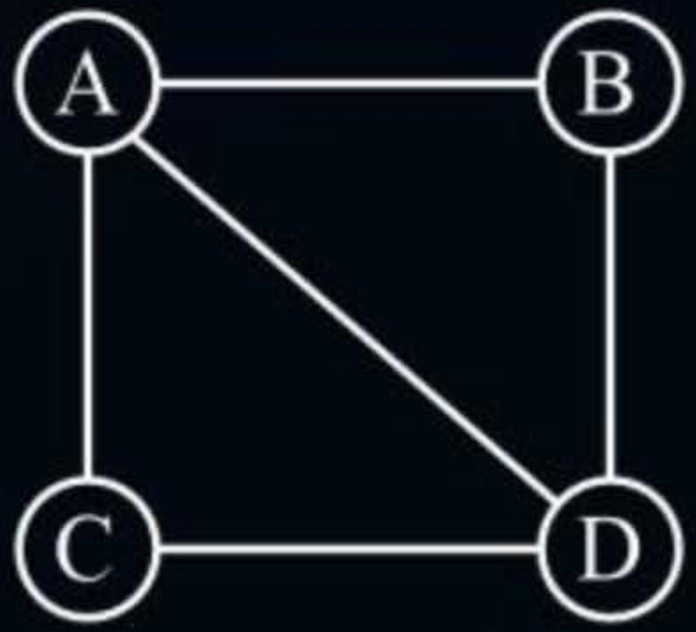
n = no. of vertices = 4

e = no. of edges = 5

Size = $O(n^2)$

$Adj[i, j] = 1 \qquad (i, j) \in E$

$Adj[i, j] = 0 \qquad (i, j) \notin E$

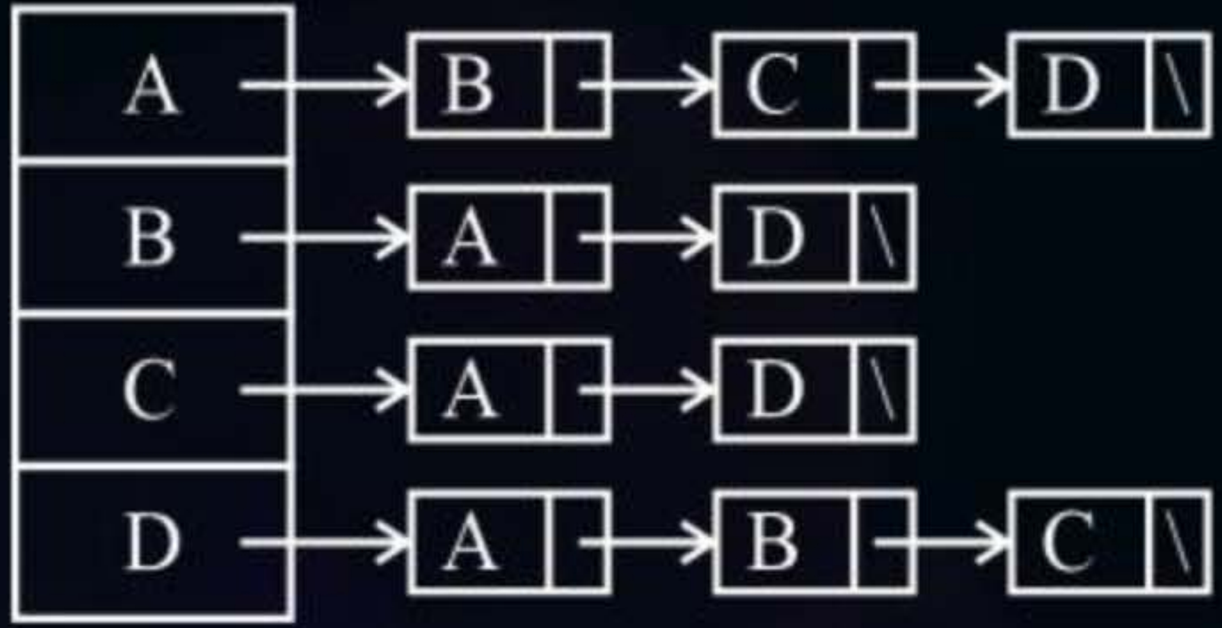


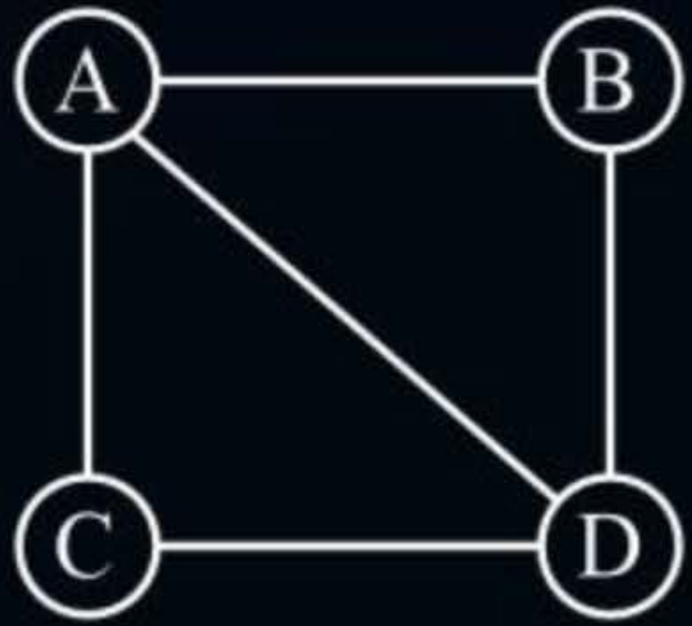
Adj	A	B	C	D
A	0	1	1	1
B	1	0	0	1
C	1	0	0	1
D	1	1	1	0

Topic : Greedy Algorithm

(2) Adjacency List:

Array of linked list





(i) Undirected graph: Size = (n + 2e)

(ii) Directed graph: Size = (n + e)

$$\left.\begin{array}{l}(n + 2e)\\(n + e)\end{array}\right\} = O(n + e)$$

Topic : Greedy Algorithm

(1) Directed Complete Graph:

$$\text{Total edges}(e) = n * (n - 1)$$

$$= O(n^2)$$

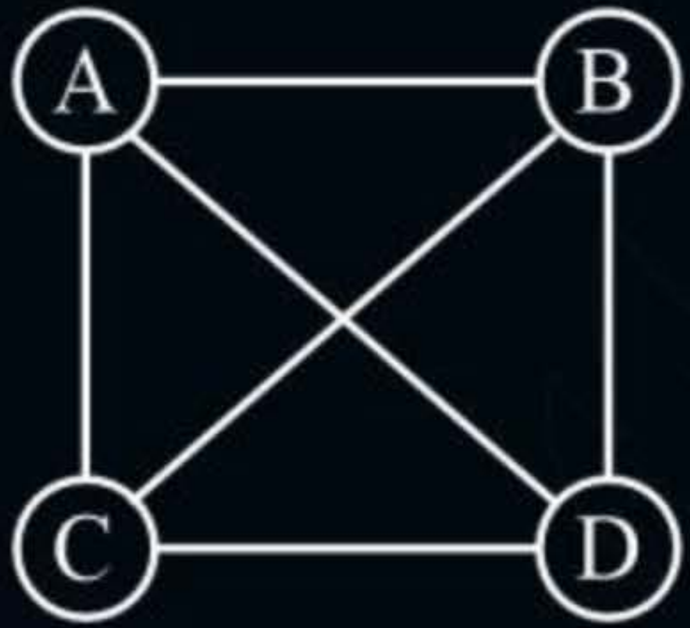
WC

$e = O(n^2)$

(2) Un-Directed Complete Graph:

$$e = n(n-1)/2$$

$$= O(n^2)$$



[NAT]

#Q. Which representation to use when?

Adj Matrix $\rightarrow O(n^2)$

Adj List $\rightarrow O(n + e)$

Topic : Greedy Algorithm

Dense Graph:

Complete Graph:

- $e \approx O(n^2)$
- Adjacency Matrix is preferred.

Sparse Graph:

Non-Complete Graph:

$e \neq O(n^2)$

- Adjacency list is preferred.
- $O(n + e)$

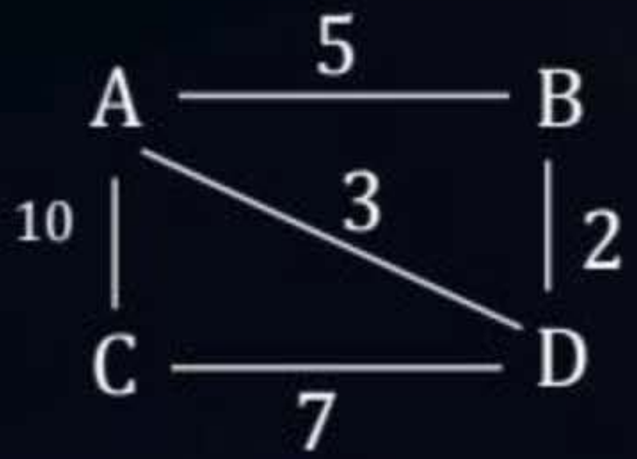
Spanning Tree:

- A subgraph $T(V, E')$ of a given graph (V, E) where $E' \subset E$ is a spanning Tree. if ' T ' is a Tree.

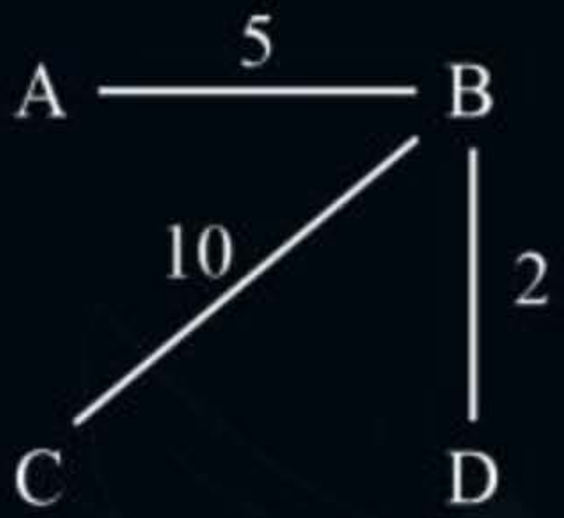
Topic : Greedy Algorithm

Example:

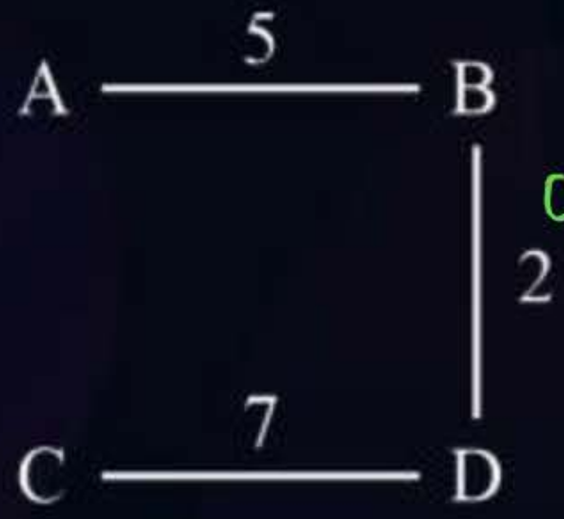
Q. Which of the following are valid Spanning Tree of G?



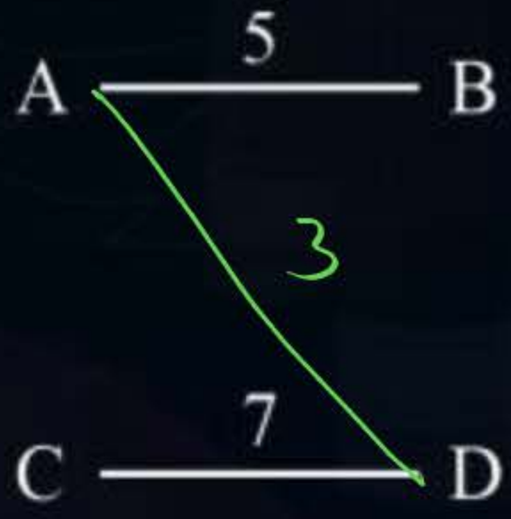
B



A



C



Topic : Greedy Algorithm

Important Observations:

1) For a given graph Multiple Spanning Trees are possible. (cost can be different)

   ST – A → Cost = 5 + 2 + 7 = 14

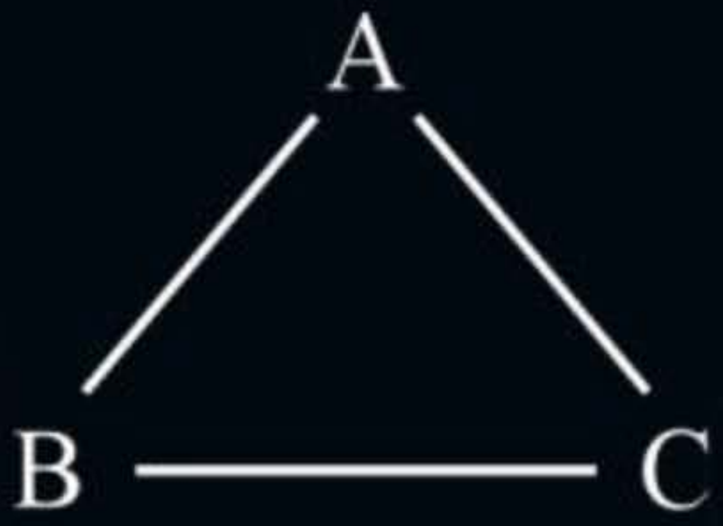
   ST – B → Cost = 5 + 3 + 7 = 15

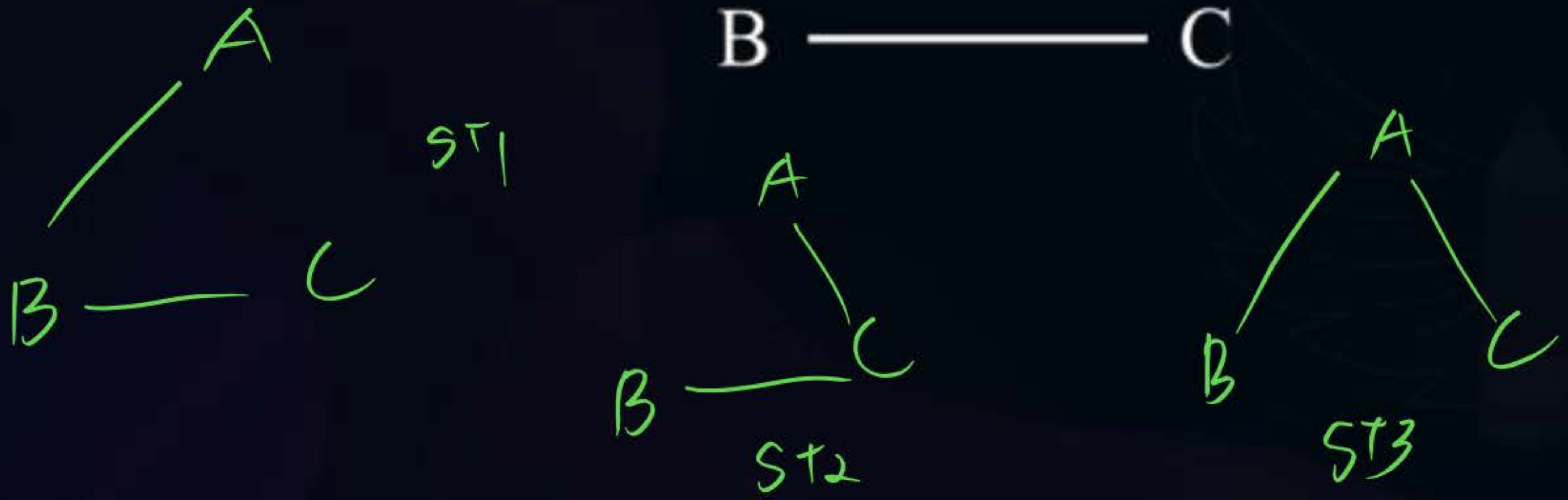
2) A Spanning Tree with n-vertices will always have (n – 1) edges.

Topic : Greedy Algorithm

[NAT]

#Q. For a given graph with n-vertices and e-edges the maximum number of Spanning Tree =?





#Q. Graph: $n \rightarrow$ vertices,

$e \rightarrow$ edges

The no. of edges that must be removed from this graph to get its Spanning Tree?

$V = n$ $\quad G : e$

$T \rightarrow n-1$

$e = (n-1) + \text{Removed} \Rightarrow \boxed{R = \underline{e-(n-1)}}$

Topic : Greedy Algorithm

- Spanning Tree of 'n' vertices → (n – 1) edges required.

Total edges = (required + removal)

$$\begin{array}{ccccc} \downarrow & & \downarrow & & \downarrow \\ e & = & (n-1) & + & x \end{array}$$

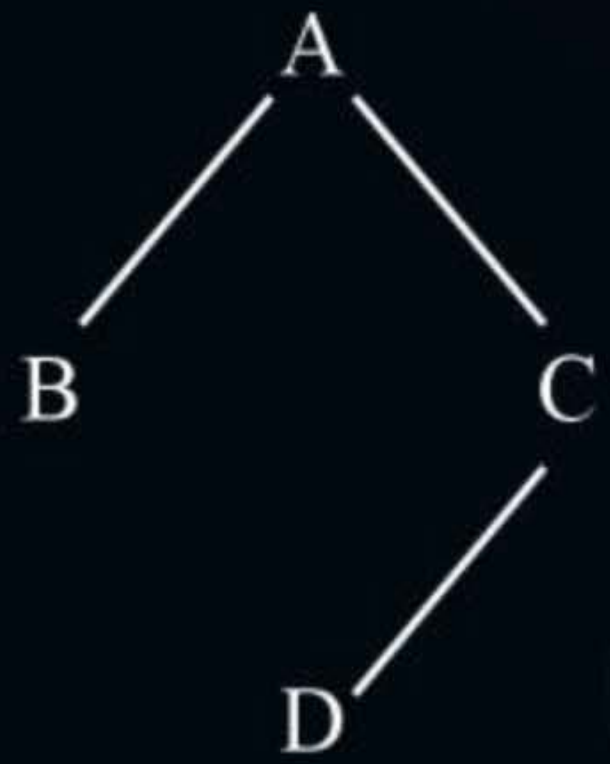
$$x = e - (n - 1)$$

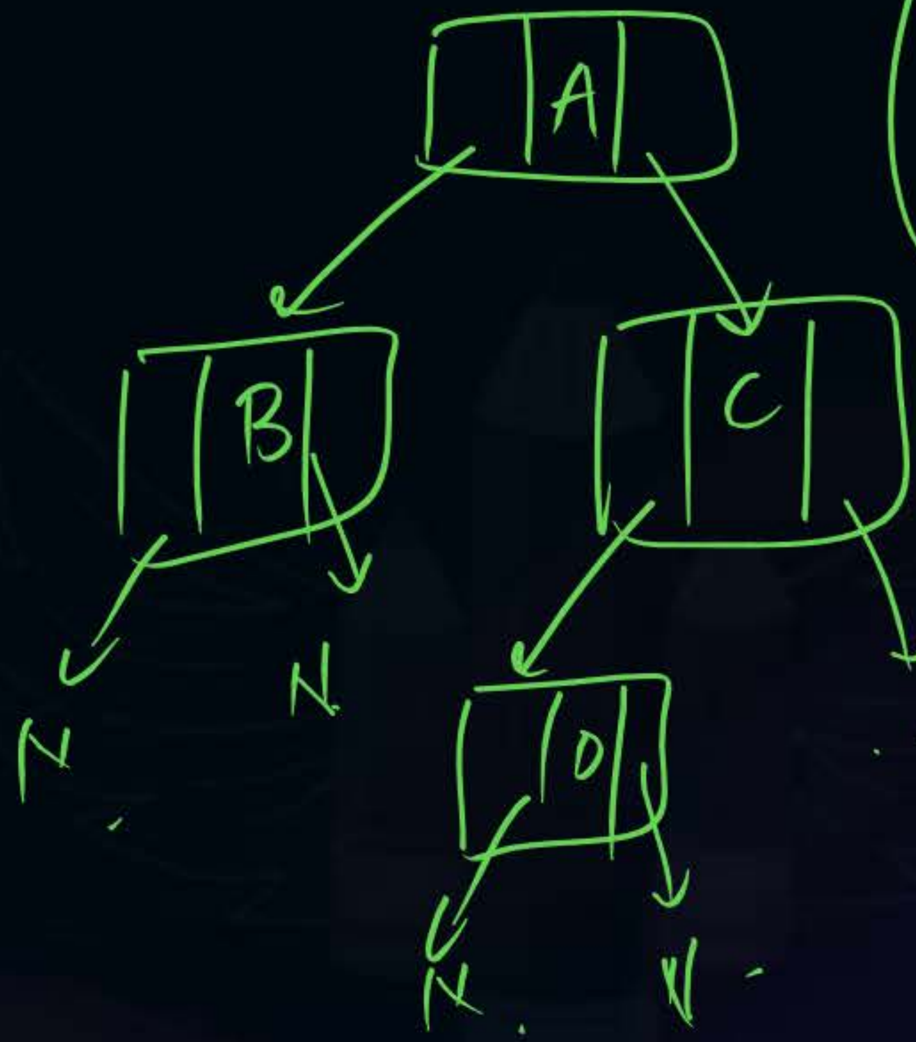
$$x = e - n + 1$$

Topic : Greedy Algorithm

[NAT]

#Q. A Tree with 'n' nodes/vertices will have ______ no. of null link?





$n = 4$

5

Topic : Greedy Algorithm

Logic: Total links in a tree with n vertices = 2 * n

edges (used) = (n – 1) → non-null links

unused/null-links = Total – edges

$$= 2n - (n - 1)$$

$$= 2n - n + 1$$

$$= n + 1$$

Topic : Applications (real-life) of spanning Trees

Networking:

1) Broadcasting
2) Multicasting
3) Electronics: Circuits (Embeded System)

MCST → Optimization Problem → Unique min cost

Topic : Applications (real-life) of spanning Trees

Algorithms for Construction of Minimum Cost Spanning Trees:

1) Prim's Algorithm
2) Kruskal Algorithm
3) Dijkstra's Algorithm for MCST

V.imp

Different from Dijkstra's Single Source Shortest Path Algo.

SSSP

Topic : Applications (real-life) of spanning Trees

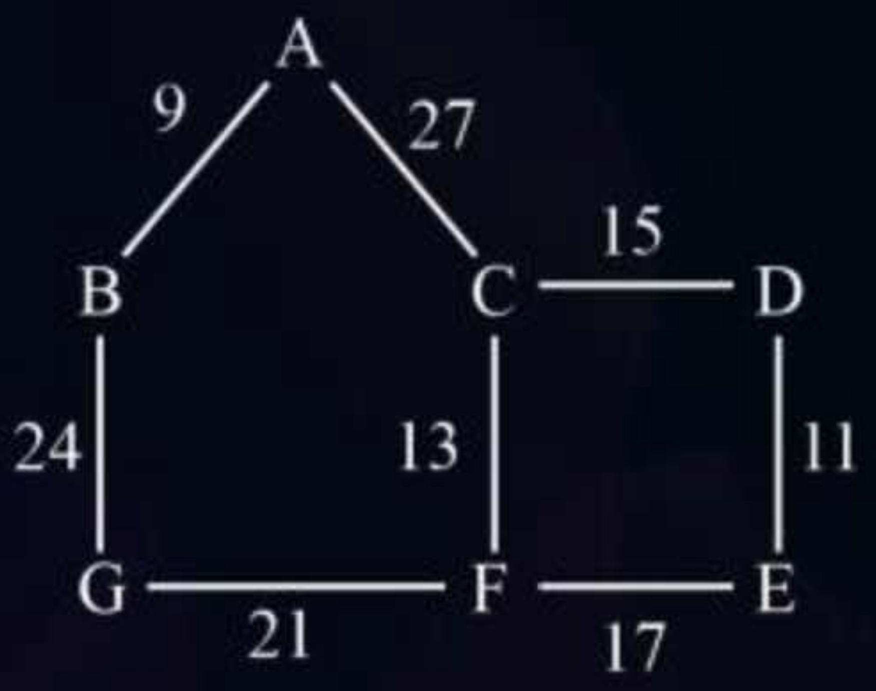
Prim's Algorithm: Construction of MCST.

Given Graph, $G = (v, e)$

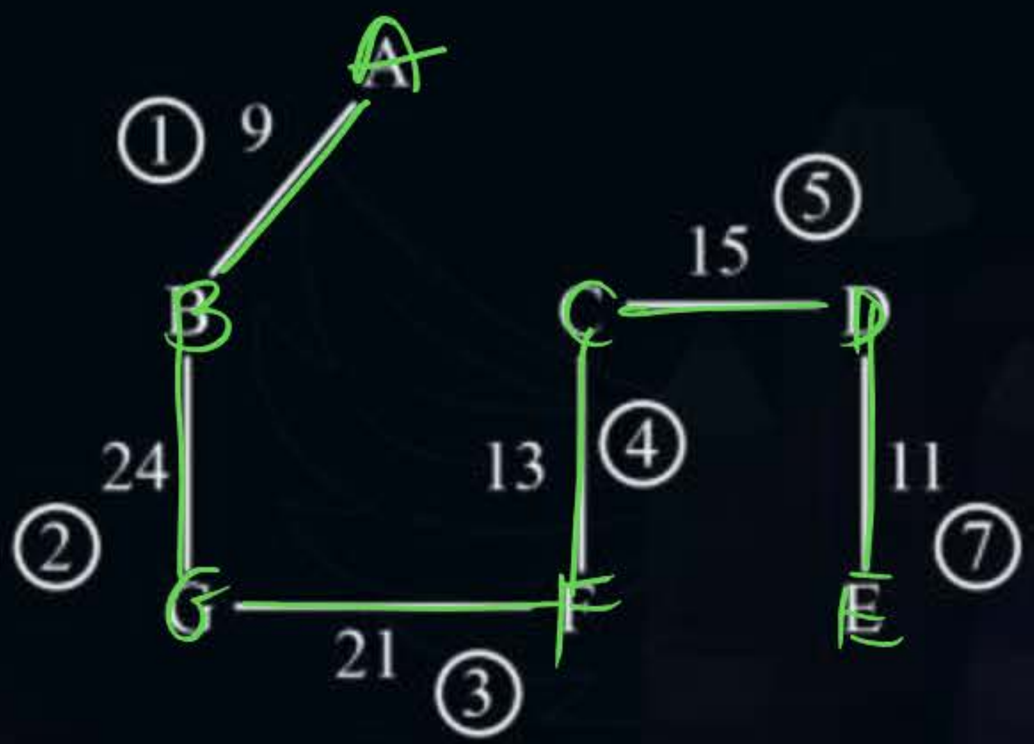
$v \to n, e \to (n-1)$

MCST

HW



$\Rightarrow$

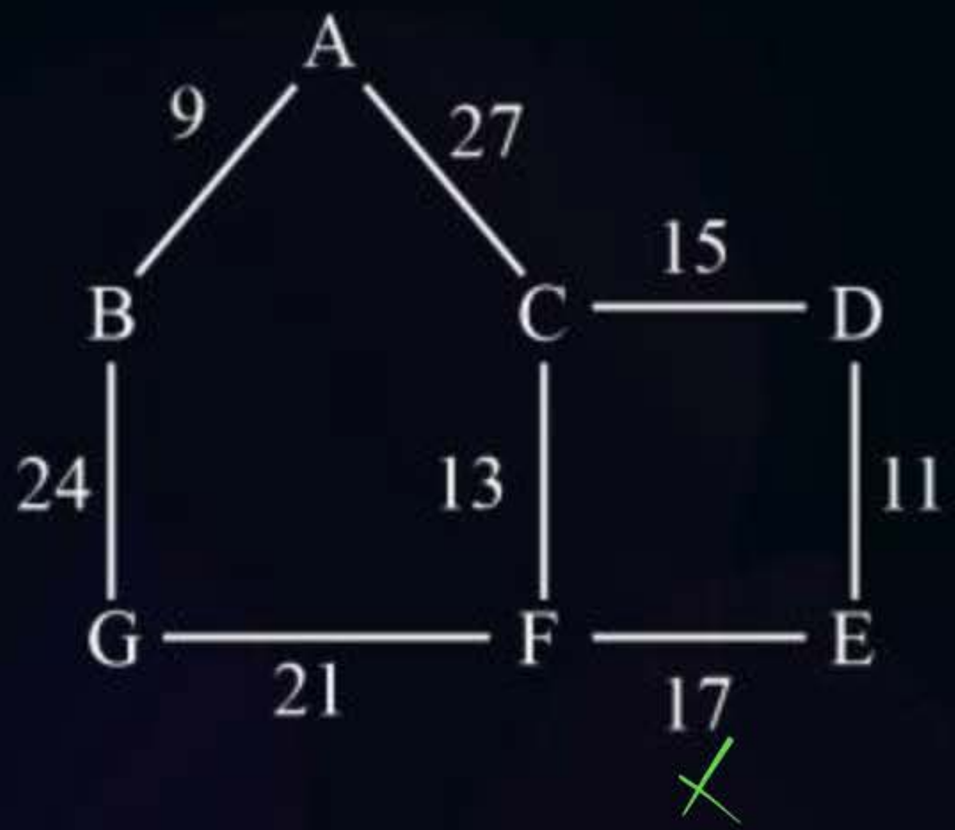


Cost = 9 + 24 + 21 + 13 + 15 + 11 = 93

Topic : Applications (real-life) of spanning Trees

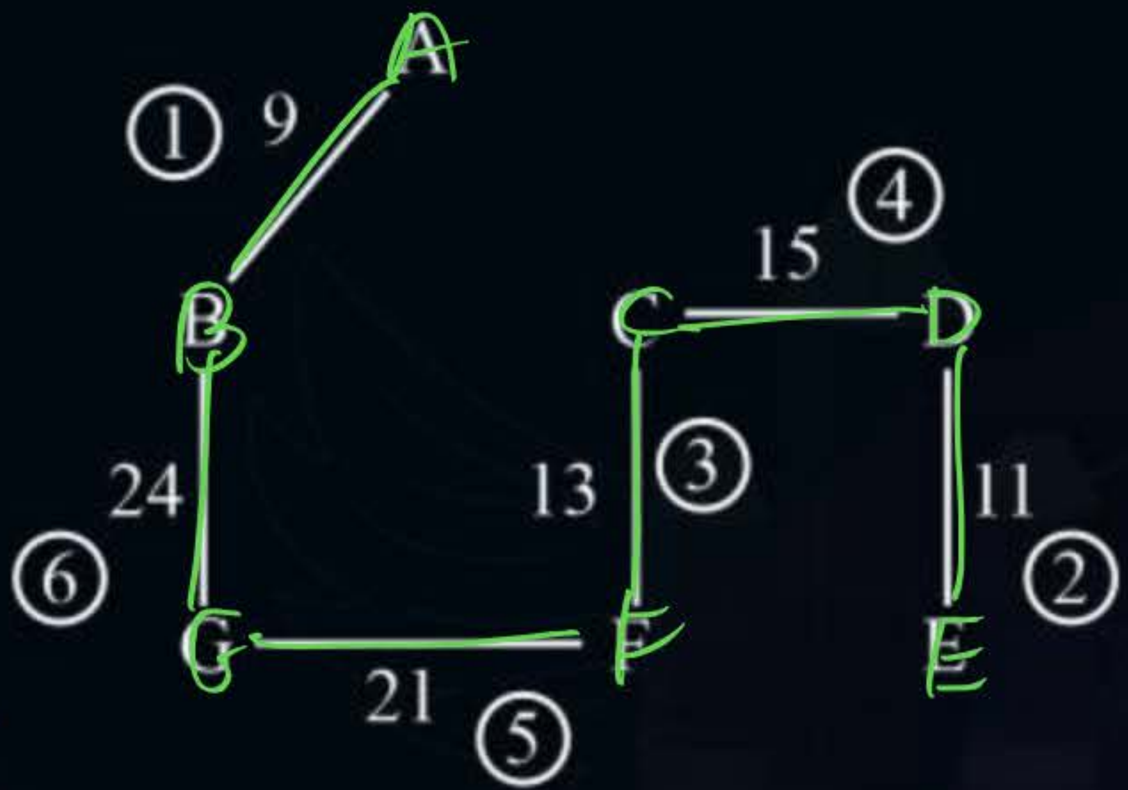
Kruskal Algorithm: Min Heap + Set

[9, 11, 13, 15, 17, 21, 24, 27]



⇒

MCST



Cost = 9 + 24 + 21 + 13 + 15 + 11 = 93

Topic : Greedy Algorithm

For a given Graph : $G(V, E)$, $|V| = n$, $|E| = e$

1) Prims Algo: TC = $O(n^2)$ → Non-Heap Mechanism

2) Kruskal Algo: TC = $O(e \log e)$

3) Prims Algo always maintains Tree Structure property at each step, whereas Kruskal may or may not.

4) Cost of Spanning Tree (MCST) by both the approaches will always be same.

5) The tree structure of the MCST obtained by both may or may not be same.

V. imp

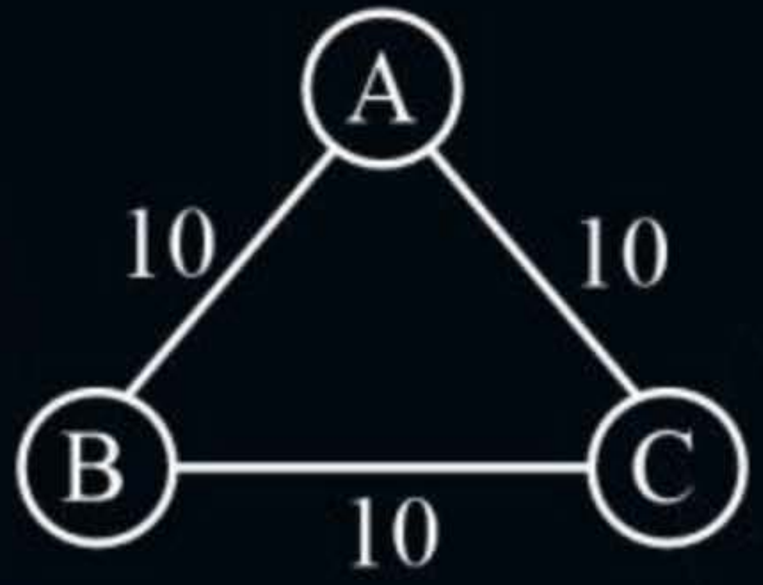
5) The tree structure of the MCST obtained by both may or may not be same.

a) If all the edges have unique/distinct cost, then the structure of MCST by both algo is also same.

b) If there are duplicate weighted edges, then the structure of MCST may or may not be the same.

Topic : Greedy Algorithm

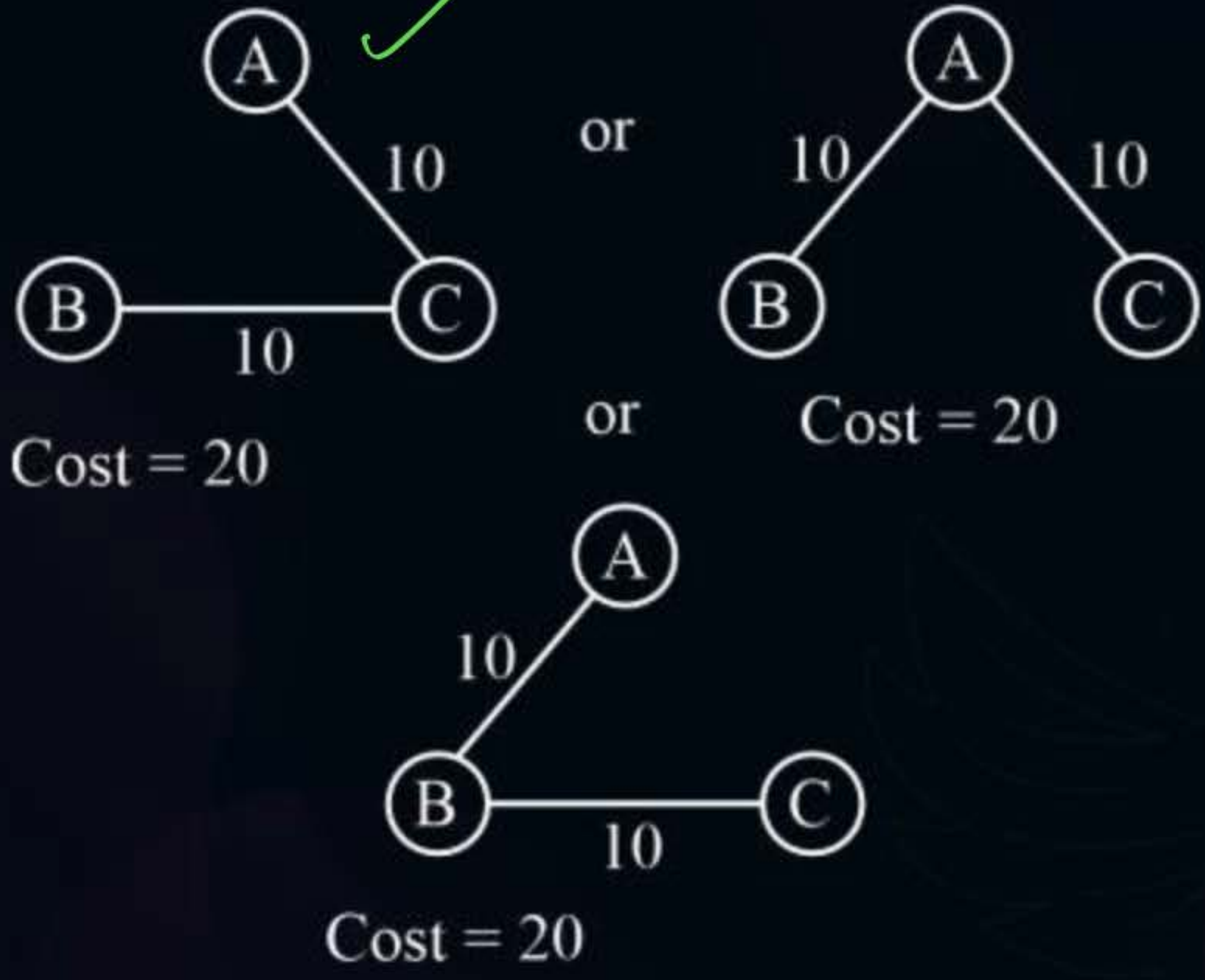
Example: n = 3, e = 3



Cost (MCST) = 20

Topic : Greedy Algorithm

MCST:



- Hence, cost of MCST is always unique. But, there can be multiple MCSTs (as there are duplicate weighted edges)

Important:

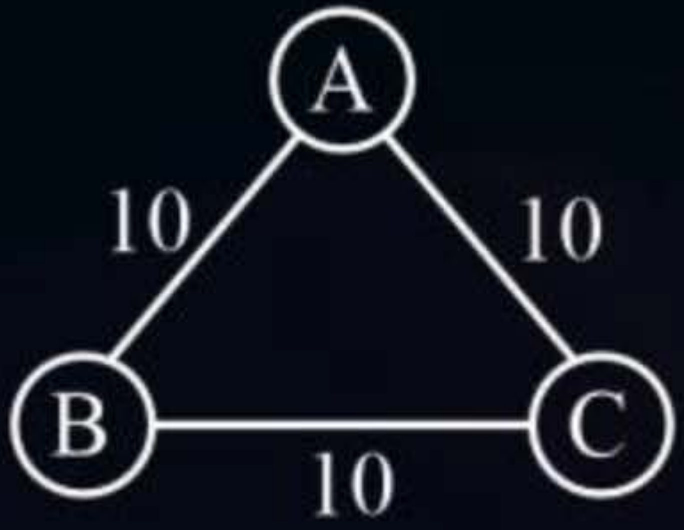
#Q. If all the edges in the G(V, E) are equal (equal-weighted graph), then the cost of MCST = ?

$|V| = n$

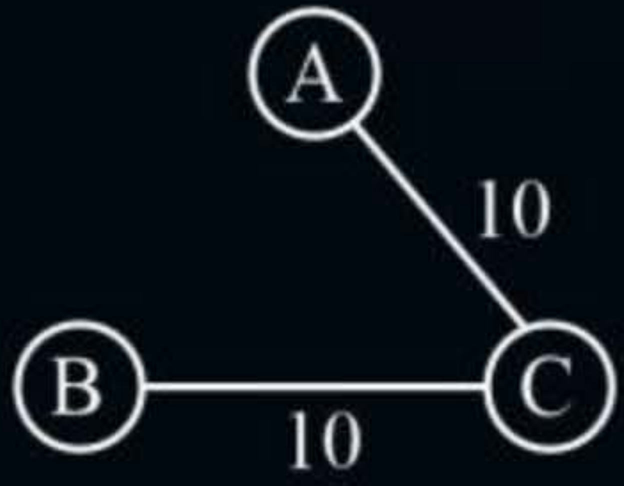
$(n-1) * c$

Topic : Greedy Algorithm

10, 10, 10

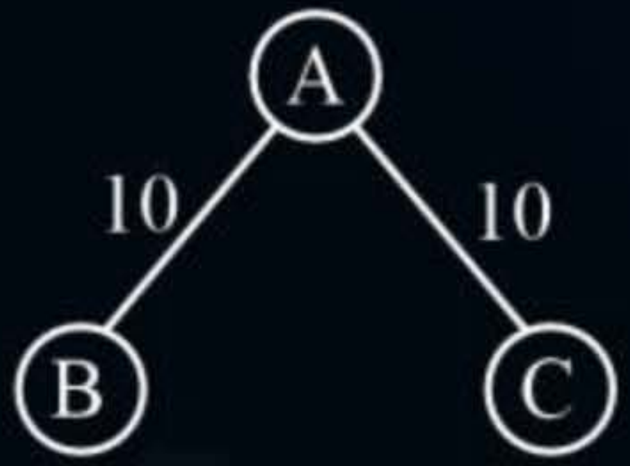


$(3-1)\times 10$
$= \underline{20}$



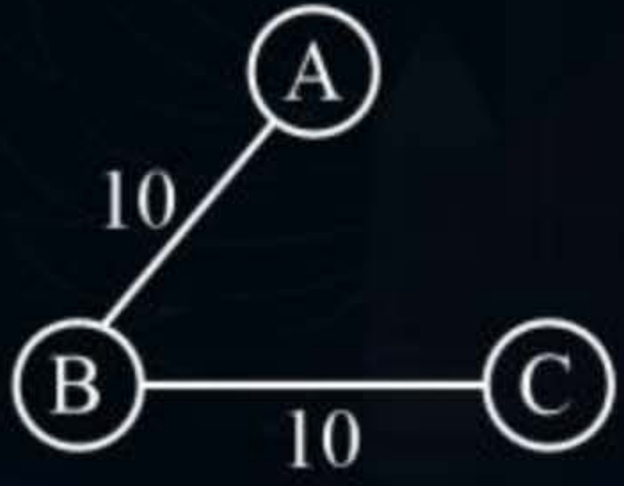
Cost = 20

or



Cost = 20

or



Cost = 20

Topic : Greedy Algorithm

Dijkstra's MCST Construction Algorithm:

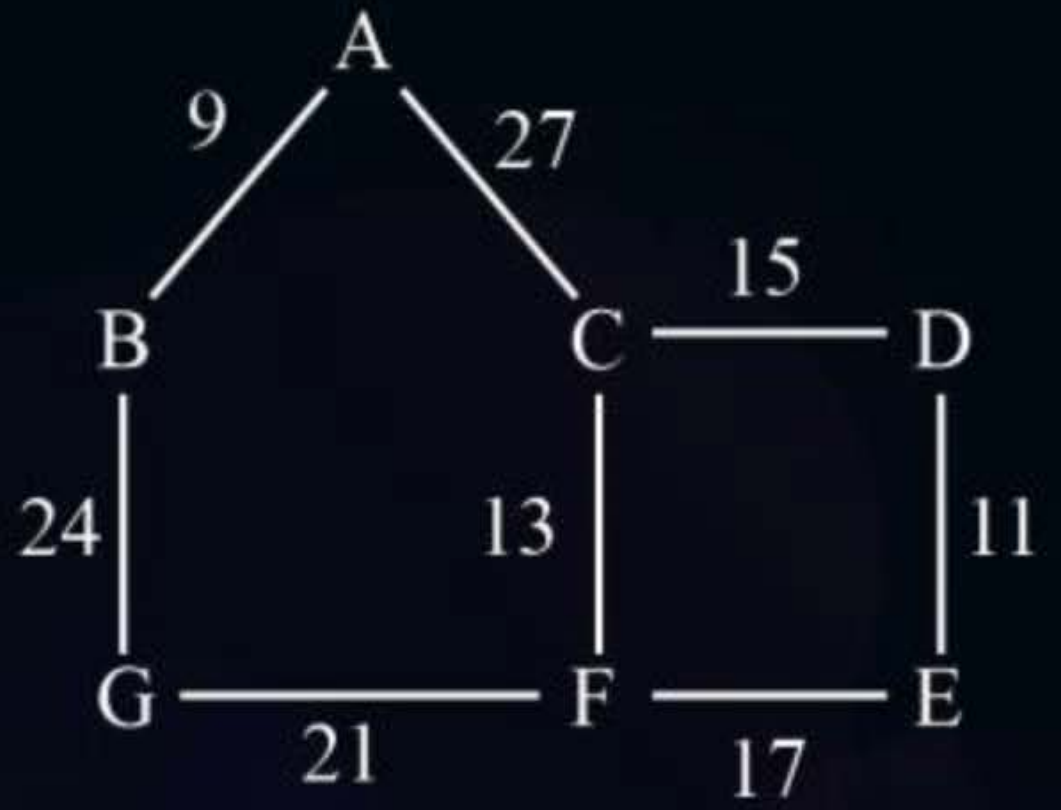
Steps:

1) Randomly Pick edges (consider all of them)
2) Only when cycle gets formed, remove the max cost edge from that cycle and continue until all edges are considered.

$n-1$ ×

Topic : Greedy Algorithm

Dijkstra's MCST Construction Algorithm:



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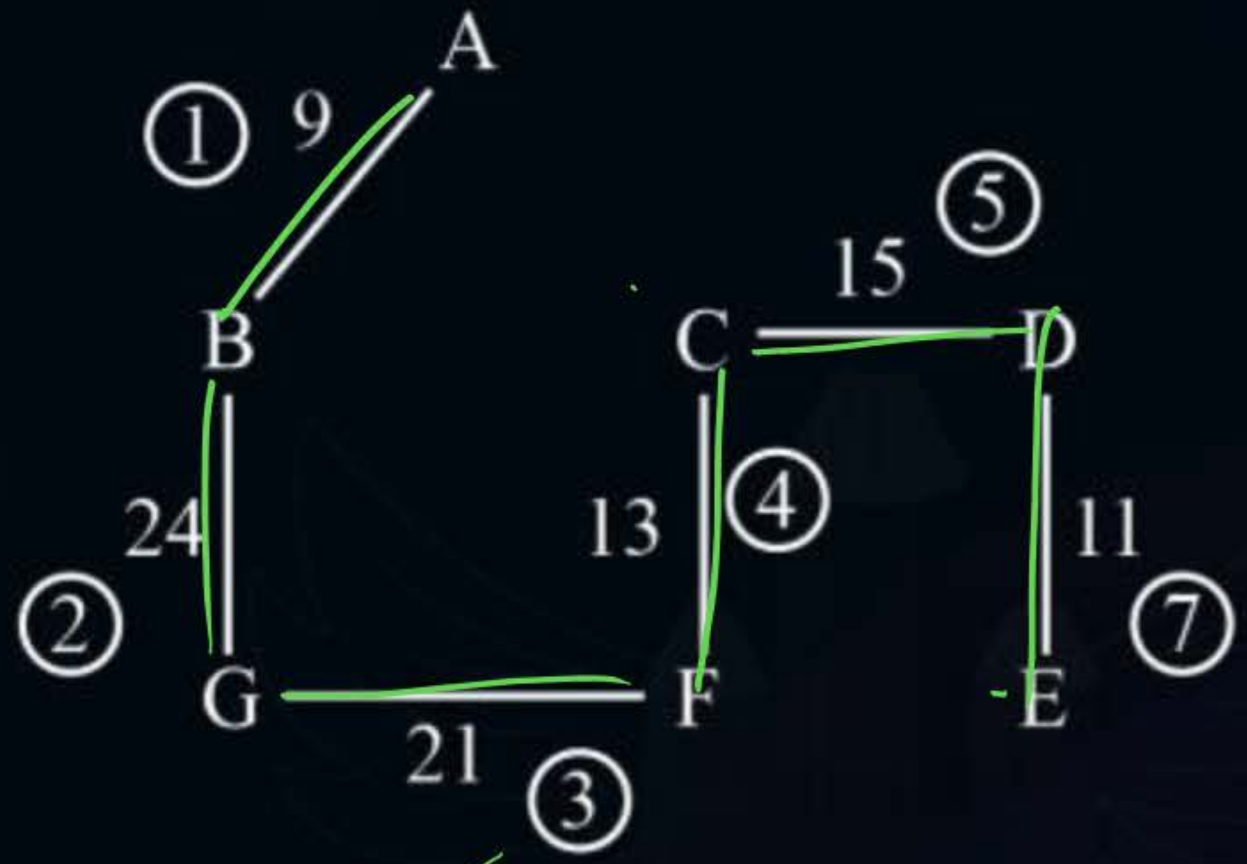


MCST Cost = 9 + 24 + 21 + 13 + 15 + 11 = 93

Possible mistake as stopped after (n – 1) edges

Topic : Greedy Algorithm

PYQ

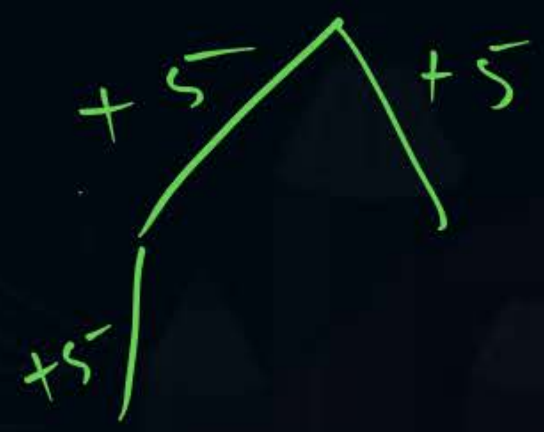
[NAT]

#Q. Let G a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes ________.

$|V| = 100$

$|E| = 300$

$ST = 500$



$ST' = (n-1) \times 5 + ST$

$= (100-1) \times 5 + 500 = 995$

Tomorrow - 8 AM

THANK - YOU