DS & AI
CS & IT

Linear Algebra

DPP-03 Discussion Notes

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[MCQ]

#Q. The eigen values of the matrix $\begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix}$ are : $\Rightarrow$ $|A| = 0 \Rightarrow \lambda = 0$

$\lambda_1 + \lambda_2 = a+1$

$\lambda_2 = a+1$

A (a + 1), 0

B a, 0

C (a – 1), 0

D 0, 0

[MCQ]

$\Rightarrow \rho(M) = \text{one} \; \text{so No. of Non Zero Evalues} \le \rho(A)$

#Q. Consider $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. The eigenvalues of M are

$= 3 \quad 3 \quad 3$

$\Rightarrow$ so Non Zero Evalues = one

which is $\lambda = 3$

$\Rightarrow \lambda_2 = \lambda_3 = 0$

A. 0, 1, 2

B. 0, 0, 3

C. 1, 1, 1

D. –1, 1, 3

[MCQ]

#Q. Consider the matrix $M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$ $\Rightarrow |M| = 0 \Rightarrow \lambda = 0$ .The eigenvalues of M are

A ✗ −5, −2, 7

B ✓ −7, 0, 7

C ✗ −4i, 2i, 2i

D ✗ 2, 3, 6

[MCQ]

#Q. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are

$Tr = 2$

$|A| = 1 - (-i^2) = 1 - 1 = 0 \Rightarrow \lambda = 0$

A +1 and +1

B Zero and +1

C Zero and +2

D −1 and +1

[MCQ]

#Q. The eigenvalues of $\boxed{(A^4 + 3A - 2I)}$, where $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$, are $= U.T.M \Rightarrow \lambda = 1, 2, 3$

A. 2, 20, 88

B. 1, 2, 3

C. 2, 20, 3

D. 1, 20, 88

$B = A^4 + 3A - 2I$

$= 1^4 + 3(1) - 2(1) = 2$

$= 16 + 3(2) - 2(1) = 20$

$= 81 + 3(3) - 2(1) = 88$

[MCQ]

#Q. Eigen values of matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are $\Rightarrow Tr(A) = 0 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$

A ✓ $-2, -1, 1, 2$

B ✗ $-1, 1, 0, 2$

C $1, 0, 2, 3$ ✗

D ✗ $-1, 1, 0, 3$

[MCQ]

#Q. The eigenvalues of a matrix are i, –2i and 3i. The matrix is

A. Unitary

B. Anti-Unitary

C. Hermitian

D. Anti-Hermitian

[MCQ]

#Q. The eigenvalue of the matrix $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are $\Rightarrow \lambda^2 - (Tr)\lambda + (|A|) = 0$

$\lambda^2 - 0 + 1 = 0 \Rightarrow \lambda^2 = -1$

$\lambda = \pm i$

A. Real and Distinct

B. Complex and Distinct ✓

C. Complex and Coinciding

D. Real and Coinciding

[MCQ]

#Q. The eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are $Tr(5)$ & $|A| = -5$

A. 5, 2, –2

B. –5, –1, 1 ✗

C. 5, 1, –1 ✓

D. –5, 1, 1 ✗

[MCQ]

PW

#Q. A 3×3 matrix has element such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is:

A 18

B 12

C 9

D 6

$\lambda_1 + \lambda_2 + \lambda_3 = 11$

$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 36$

$\boxed{6 \times 3 \times 2 = 36}$

[MCQ]

#Q. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

$Tr = 0$

A ✗ 0, 1, 1

B ✓ $0, -\sqrt{2}, \sqrt{2}$

C ✗ $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$

D ✗ $\sqrt{2}, \sqrt{2}, 0$

[MCQ]

#Q. The trace of a 2 × 2 matrix is 4 and its determinant is 8. If one of the eigenvalue is $2(1 + i)$, the other eigenvalue is

$\lambda_1 = 2 + 2i$

$\lambda_2 = 2 - 2i$

$\lambda_1 + \lambda_2 = Tr = 4$

A $2(1 - i)$

B $2(1 + i)$

C $(1 + 2i)$

D $(1 - 2i)$

[MCQ]

#Q. The eigenvalues of the matrix representing the following pair of linear equations $x + iy = 0$ and $ix + y = 0$ are

$$\begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{where } A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$$

$$AX = 0$$

A. $1 + i, 1 + i$ ✗

B. $1 - i, 1 - i$ ✗

C. $1, i$ ✗

D. $1 + i, 1 - i$ ✓

[MCQ]

u.T.M $\Rightarrow \lambda = -1, 3, -2$

#Q. If the matrix $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then the eigen values of $A^2 + 5A + 8I$, are :

A. $-1, 27, -8$

B. $1, 3, -2$

C. $2, 32, 4$

D. $2, 50, 10$

$B = A^2 + 5A + 8I$

$= 1 + 5(-1) + 8(1) = 4$

$= 9 + 5(3) + 8(1) = 32$

$= 4 + 5(-2) + 8(1) = 2$

[MCQ]

#Q. Two of the eigen values of a 3×3 matrix, whose determinant equals 4, are -1 and +2 the third eigen value of the matrix is equal to :

A. -2

B. -1

C. 1

D. 2

$\lambda_1 \lambda_2 \lambda_3 = 4$

$(-1)(2)\lambda_3 = 4$

$\lambda_3 = -2$

[MCQ]

#Q. If A is a singular Hermitian matrix, then the least eigen value of A^2 is :

$\Downarrow$ $\lambda = 0$, $\Downarrow$ $\lambda \in R$

A 0

B 1

C 2

D None of these

$A \to \lambda_1\ \lambda_2\ \lambda_3 \cdots \lambda_n$

$A^2 \to \lambda_1^2, \lambda_2^2 \cdots \lambda_n^2$

$= 0, +ve$

[MCQ]

#Q. If λ is an eigen value of matrix 'M' then for the matrix $(M - \lambda I)$, Which of the following statement (s) is/ are correct ?

A. Skew symmetric

B. Nonsingular

C. Singular

D. None of these

$$MX = \lambda X$$
$$MX - \lambda I X = 0$$
$$(M - \lambda I)X = 0$$

$$(M - \lambda I)X = 0 \Rightarrow \boxed{|M - \lambda I| = 0}$$

X → Non Zero E Vector

$$\frac{\text{Non Zero E Vector}}{\text{Non Zero Null / } \infty \text{ Soln.}} \Rightarrow Det = 0$$

[MCQ]

#Q. Let A be a matrix whose characteristic roots are 3, 2, −1. If $B = A^2 - A$ then $|B|$ = ______.

A 24

B −2

C 12

D −12

$B = A^2 - A$

$= 9 - 3 = 6$

$= 4 - 2 = 2$

$= 1 + 1 = 2$

So $|B| = 6 \times 2 \times 2$

$= 24$

[MCQ]

#Q. The Eigen vector corresponding to the largest Eigen value of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is____

A $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

B $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

C $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

D None.

(a) $AX = \lambda X$

$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \lambda\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda \\ \lambda \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda \\ \lambda \\ 0 \end{bmatrix}$

$\Rightarrow \lambda = 3, \lambda = 3, 0 = 0$

(c) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda \\ 0 \\ \lambda \end{bmatrix}$

[MCQ]

#Q. The Eigen vector of the matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ are :

(a) $Ax = \lambda x$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} +1 \\ -1 \end{bmatrix} = \begin{bmatrix} \lambda \\ 0 \end{bmatrix}$

$\lambda = 1, \; -1 = 0$ ✗

A ✗ (1, 0)

B ✗ (0, 1)

C ✓ (1, 1)

D ✗ (1, –1)

(c) $Ax = \lambda x$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda \\ \lambda \end{bmatrix} \Rightarrow \lambda = 0, \lambda = 0$

[MCQ]

#Q. The vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an eigen vector of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then corresponding eigen values of A is :

A 1

B 2

C 5

D -1

$AX = \lambda X$

$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$\begin{bmatrix} 5 \\ 10 \\ -5 \end{bmatrix} = \begin{bmatrix} \lambda \\ 2\lambda \\ -\lambda \end{bmatrix}$

$\Rightarrow \lambda = 5, \lambda = 5, \lambda = 5$

[MCQ]

#Q. The column vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is a simultaneous eigenvector of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

$AX = BX$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$AX = \lambda X$

& $BX = \lambda X$

(A) b = 0 or a = 0

(B) b = 0 or a = –c ✓

(C) b = 2a or b = –a

(D) b = a/2 or b = –a/2

$$\begin{bmatrix} c \\ b \\ a \end{bmatrix} = \begin{bmatrix} b+c \\ a+c \\ a+b \end{bmatrix} \Rightarrow$$

$b + c = c \Rightarrow b = 0$

$c + a = b \Rightarrow a = -c$

$a + b = a \Rightarrow b = 0$

[MCQ]

$T_{2\times 3}\, X_{3\times 1} = B_{2\times 1}$

#Q. A linear transformation T, defined as $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_{3\times 1} = \begin{pmatrix} x_1 & + & x_2 \\ x_2 & - & x_3 \end{pmatrix}_{2\times 1}$, transform a vector $\vec{x}$ three-dimensional space to a two-dimensional real space. The transformation matrix T is

A $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ ✓

B $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

C $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$

D $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(a) $TX = B$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$

$\begin{bmatrix} (x_1 + x_2) \\ (x_2 - x_3) \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_3 \end{bmatrix}$

[MCQ]

#Q. The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

$Tr = 7 = \lambda_1 + \lambda_2$

$|A| = 6 = \lambda_1 \cdot \lambda_2$

A. 6,1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

B. 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

C. 6, 2 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

D. 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

[MCQ]

#Q. Match the following-

(i) The eigen vectors X of a matrix A, is not	(a) $X'_1 X_2 = 0$
(ii) Two eigen vectors X_1 and X_2 are called orthogonal if	(b) $\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$
(iii) Normalized form of vectors $\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$ is obtained on dividing each element by.	(c) unique
(iv) Every square matrix satisfies its own	(d) Characteristic equation

A (i)-c (ii)-a (iii)-b (iv)-d

B (i)-a (ii)-c (iii)-b (iv)-d

C (i)-c (ii)-a (iii)-d (iv)-b

D (i)-c (ii)-d (iii)-b (iv)-a

[NAT]

$A^4 = I_{4\times4}$ Ans.

#Q. Calculate the matrix A^4, by the use of Cayley – Hamilton theorem (or) otherwise

where ~~det~~ $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & 1 \\ 0 & 0 & 0 & -i \end{bmatrix}$ $\Rightarrow$ $\because A$ is U.T.M so $\lambda = 1, -1, i, -i$

so it's C.Eqn

$$\underbrace{(\lambda-1)(\lambda+1)}\underbrace{(\lambda-i)(\lambda+i)} = 0$$

$$(\lambda^2-1)(\lambda^2+1) = 0$$

$$\lambda^4 - 1 = 0 \quad \text{—— (1)}$$

By C.H.T: $\lambda \to A$ $\quad A^4 - I = 0 \Rightarrow A^4 = I = I_{4\times4}$

[MCQ]

#Q. The matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ is given and the eigen values of $4A^{-1} + 3A + 2I$ are.

L.T.M $\Rightarrow$ $\lambda = 1, 4$

$4\left(\frac{1}{1}\right) + 3(1) + 2(1)$
$= 9$

$4\left(\frac{1}{4}\right) + 3(4) + 2(1)$
$= 1 + 12 + 2$
$= 15$

A) 6, 15

B) 9, 12

C) 9, 15

D) 7, 15

[MCQ]

#Q. In matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ if $\overset{Tr}{a + d} = \overset{|A|}{ad - bc} = 1$ then $A^3 =$ ______.

C Equⁿ is $\lambda^2 - (Tr)\lambda + (|A|) = 0$

$\lambda^2 - \lambda + 1 = 0$

By C.H.T $\Rightarrow$ $A^2 - A + I = O$

$\boxed{A^2 = A - I}$

$A^3 = A^2 \cdot A$

$= (A - I)A$

$= A^2 - IA$

$= A - I - A$

$= -I$

A. $A - I$

B. $A + I$

C. $-I$ ✓

D. 0

[MCQ]

#Q. Let A be an $n \times n$ complex matrix whose characteristic polynomial is: $f(t) = t^n + c_{n-1}t^{n-1} + \ldots + c_1 t + c_0$ then ______.

Constant term $(c_0) = (-1)^n |A|$

$(-1)^n c_0 = (-1)^{2n} |A| \implies |A| = (-1)^n c_0$

A. $\det A = C_{n-1}$

B. $\det A = C_0$

C. $\det A = (-1)^{n-1} C_{n-1}$

D. $\det A = (-1)^n C_0$

[NAT]

#Q. The constant term of the characteristic polynomial of the matrix.

$$A = \begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 3 & 2 & -5 \\ 2 & 4 & 12 & 10 \\ 3 & -2 & 1 & -4 \end{bmatrix}_{4\times 4} \text{ is\_\_\_\_\_.}$$

Constant term $= (-1)^n |A|$

$= (-1)^4 |A| = |A| = 0$

$R_3 \to R_3 - 2R_1$

$$A \sim \begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 3 & 2 & -5 \\ 0 & 0 & 0 & 0 \\ 3 & -2 & 1 & -4 \end{bmatrix} \Rightarrow |A| = 0$$

[MCQ]

#Q. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is NOT TRUE?

A. Det A = Det B (T)

B. Trace of A = Trace of B (T)

C. A and B have the same eigenvectors (F)

D. A and B have the same eigenvalues (T)

[NAT]

#Q. If A and P be square matrices of the same type and if P is invertible, then the matrices A and $P^{-1}AP$ have Same characteristic roots

$P^{-1}AP = B$ (let)

$$\text{C Eq of } B = |B - \lambda I|$$
$$= |P^{-1}AP - \lambda I|$$
$$= |P^{-1}AP - \lambda I P^{-1}P|$$
$$= |P^{-1}AP - P^{-1}(\lambda I)P|$$
$$= |P^{-1}(A - \lambda I)P|$$

$$|B - \lambda I| = |P^{-1}| \cdot |A - \lambda I| \cdot |P|$$
$$= \frac{1}{|P|} \cdot |A - \lambda I| \cdot |P|$$
$$= |A - \lambda I|$$

is A & B has same C Eqn
or A & $P^{-1}AP$ " " "

[MCQ]

#Q. If $A_{2\times2}$ s. t $A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$ & $A\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}4\\2\end{bmatrix}$ then find A = ?

$A\begin{bmatrix}1\\0\end{bmatrix}=1.\begin{bmatrix}1\\0\end{bmatrix}$ & $A\begin{bmatrix}2\\1\end{bmatrix}=2\begin{bmatrix}2\\1\end{bmatrix}$

$\Rightarrow \lambda_1=1, x_1=\begin{bmatrix}1\\0\end{bmatrix}$ & $\lambda_2=2, x_2=\begin{bmatrix}2\\1\end{bmatrix}$

A $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$

B $\begin{bmatrix}1 & 0\\2 & 2\end{bmatrix}$

C $\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}$ ✓

D $\begin{bmatrix}1 & 2\\2 & 2\end{bmatrix}$

$D=\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$

$P=[x_1\ x_2]=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$

$P^{-1}=\frac{1}{(1)}\begin{bmatrix}1 & -2\\0 & 1\end{bmatrix}$

$P^{-1}AP=D$

$\Rightarrow A=PDP^{-1}$

$A=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}1 & -2\\0 & 1\end{bmatrix}$

$=\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}$

[MCQ]

#Q. If $A_{3\times3}$ then number of L. I Eigen vectors of A to have diagonalization possible will be ?

N. Condⁿ for diagonalisation = ?

$\boxed{\text{No. of L I E vectors = order of A}}$

A) 1

B) 2

C) 3

D) Less than 3.

THANK - YOU