CS & IT ENGINEERING

Algorithms

Greedy Method

Lecture No.- 06

By- Aditya Jain sir

Recap of Previous Lecture

Topic MCST

Topic HE

Topics to be Covered

- Topic
- Topic
- Topic

MCST Problem Solving

About Aditya Jain sir

1. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
2. Represented college as the first Google DSC Ambassador.
3. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored 12,000+ students & working professions in field of Data Science and Analytics
11. Have been mentoring & teaching GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

Topic : Greedy Algorithm

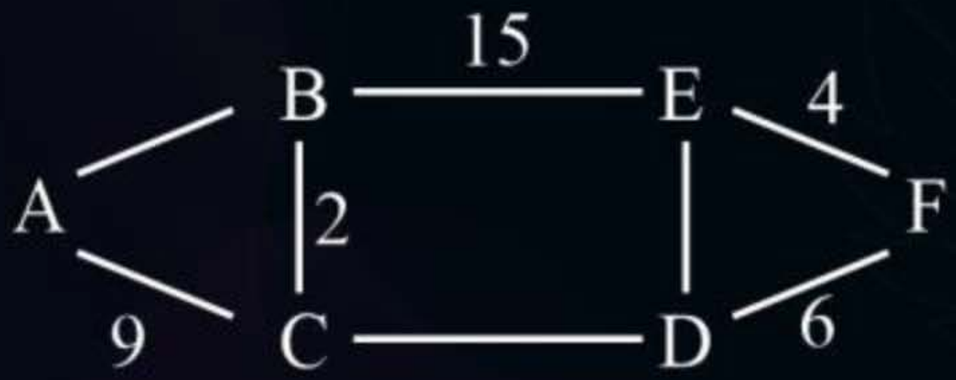
[NAT]

#Q. Consider the following graph whose minimum cost spanning tree marked with edges values have a weight of 36.

Minimum possible cost of all the edges of the graph G is _____?

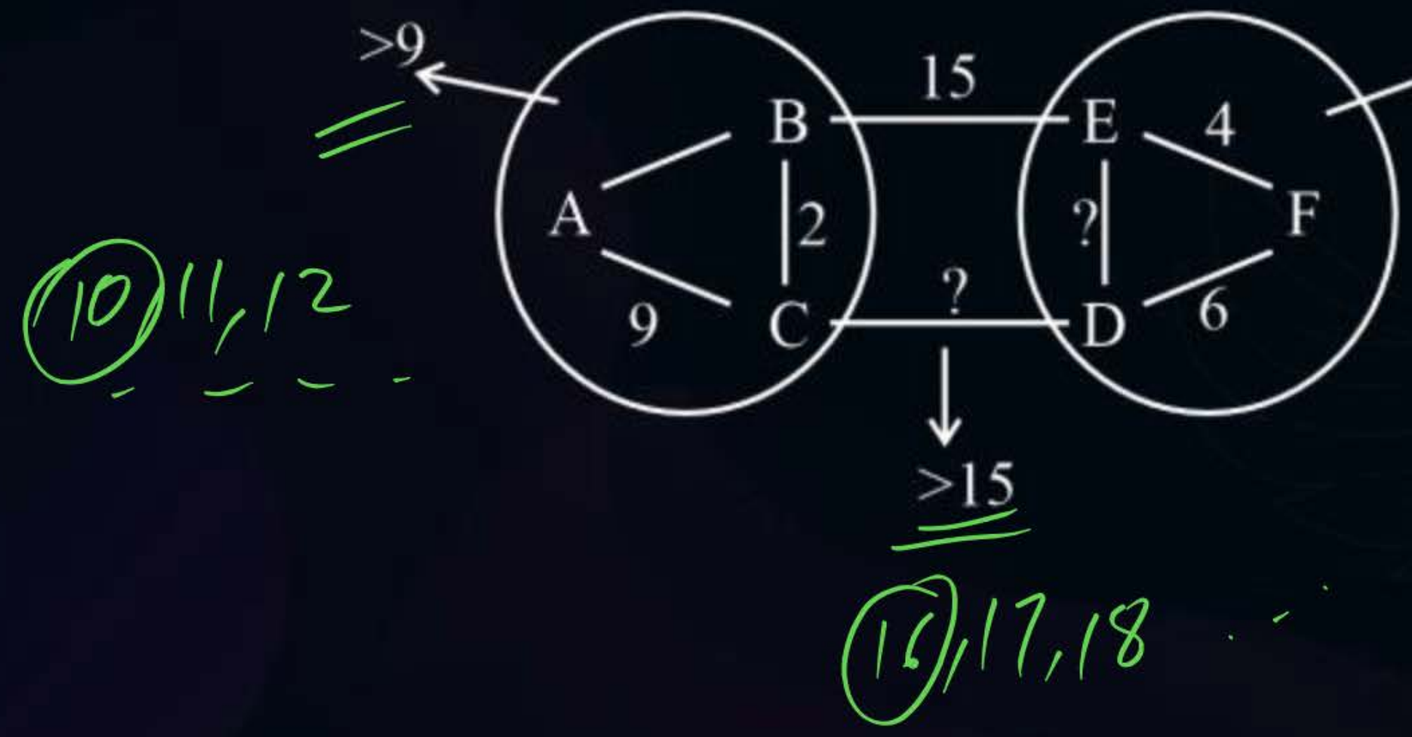
Assume that all the edges have distinct cost.

27



Topic : Greedy Algorithm

As per Dijkstra MCST Approach:



10, 11, 12 . . .

7, 8, 9 . . .

16, 17, 18 . . .

$\Rightarrow 7+10+16+36$

$= 36+33$

$= 69$

Topic : Greedy Algorithm

[NAT]

#Q. Let G be a complete undirected graph with 4 vertices and edge weights are {1, 2, 3, 4, 5, 6} The maximum possible weight that a minimum weight spanning tree can have is _____.

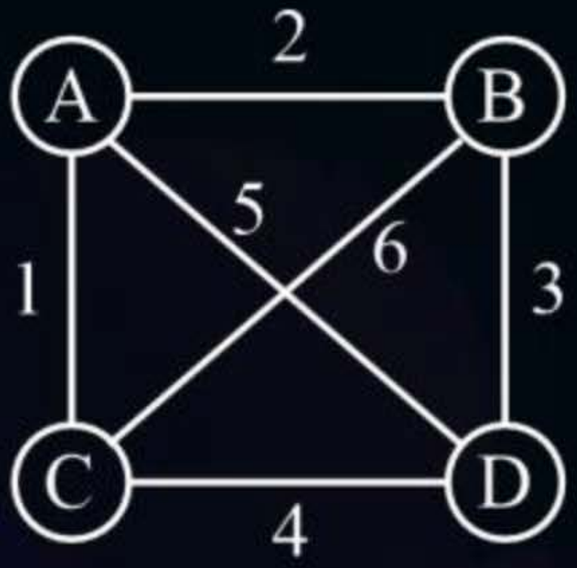
36%

Ans - 7

Topic : Greedy Algorithm

Solution:

G1:

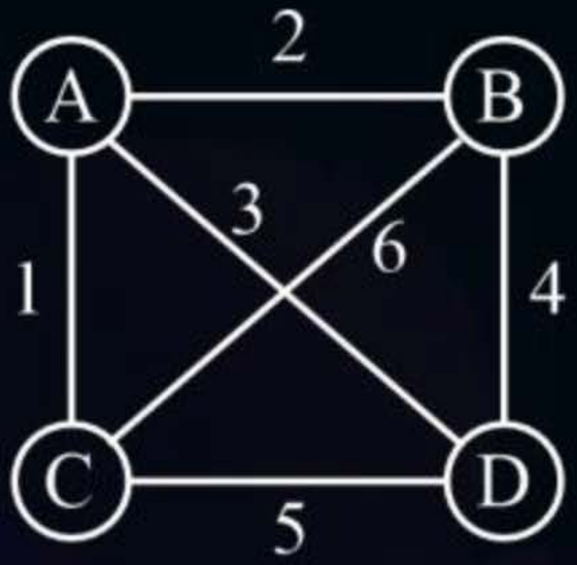


MCST →

Cost = 1 + 2 + 3 = 6

Topic : Greedy Algorithm

G2:

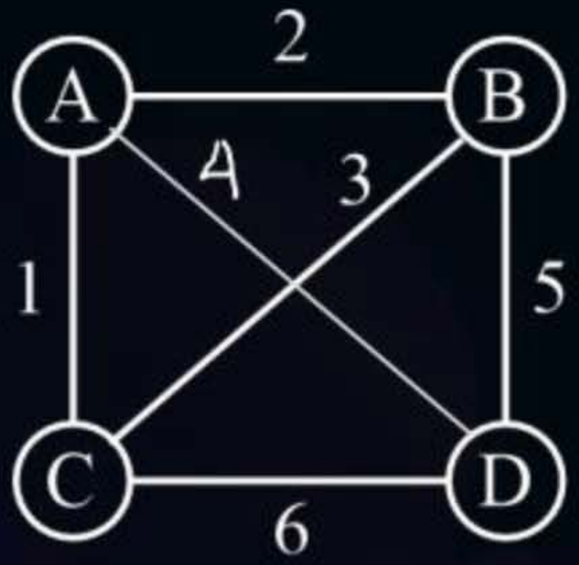


MCST →

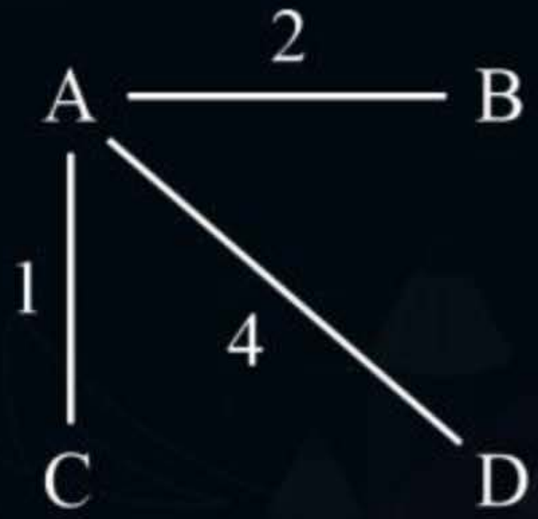
Cost = 1 + 2 + 4 = 7

Topic : Greedy Algorithm

G3:



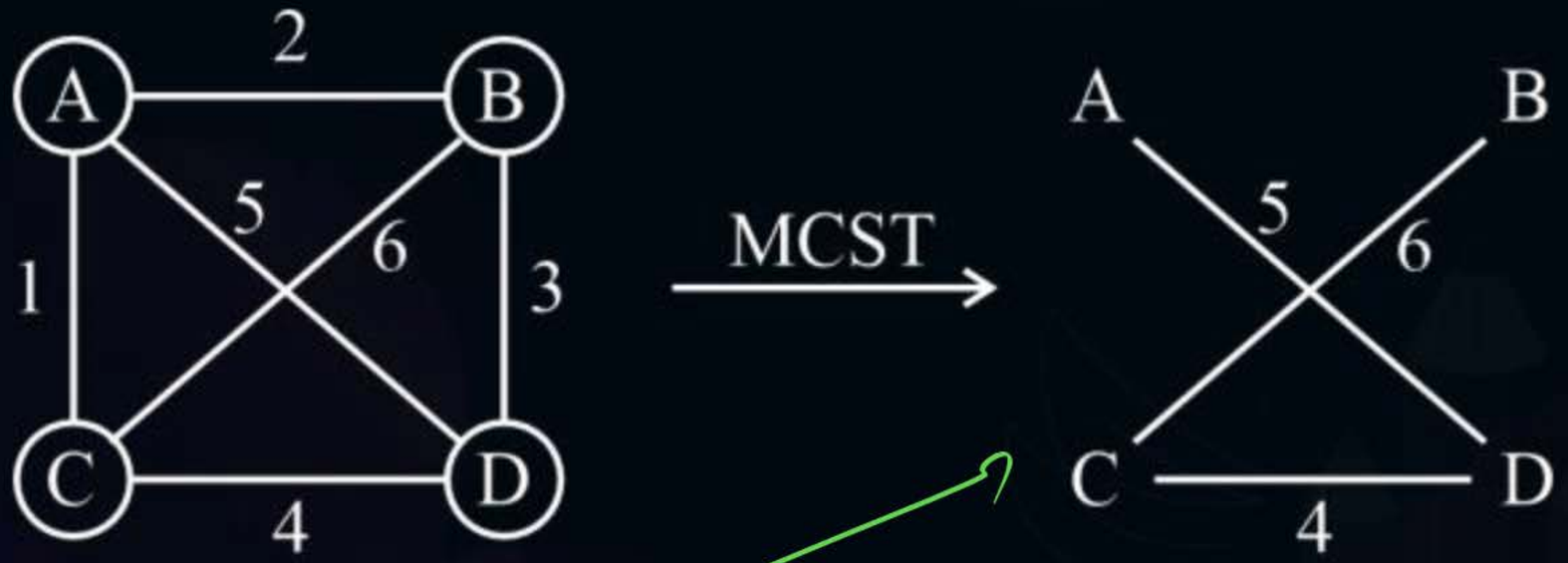
MCST



Cost = 1 + 2 + 4 = 7

Topic : Greedy Algorithm

G4:



Cost = 6 + 5 + 4 = 15

- Not a valid MCST

Topic : Greedy Algorithm

[MCQ]

#Q. Consider a complete weighted graph with n vertices numbered V_1 to V_n. Two vertices V_i and V_j having edge between them has a cost value of $2\,|i - j|$. The weight of minimum cost spanning tree of such a graph is _____.

60%

A. $2n$

B. $3n - 2$

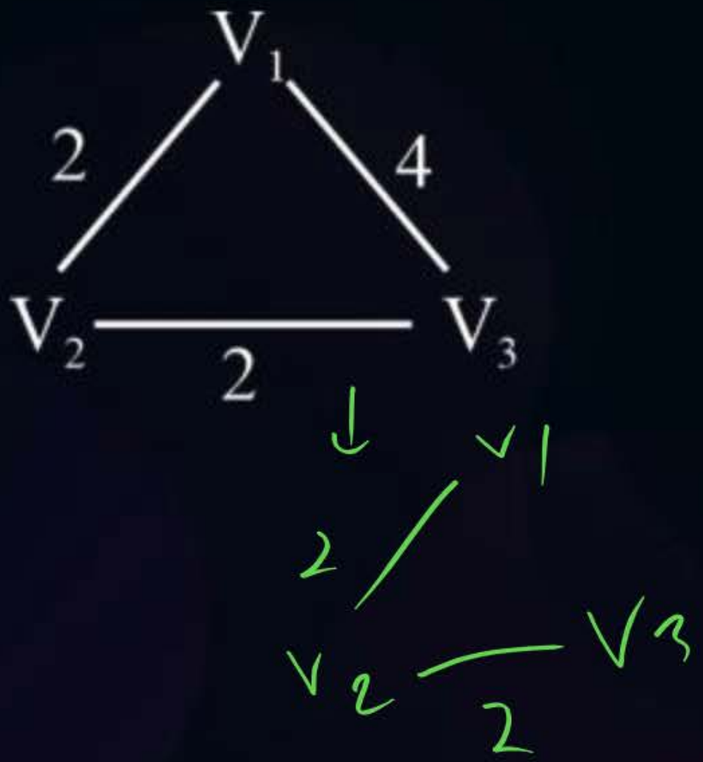
C. $2n - 2$

D. $2n - 1$

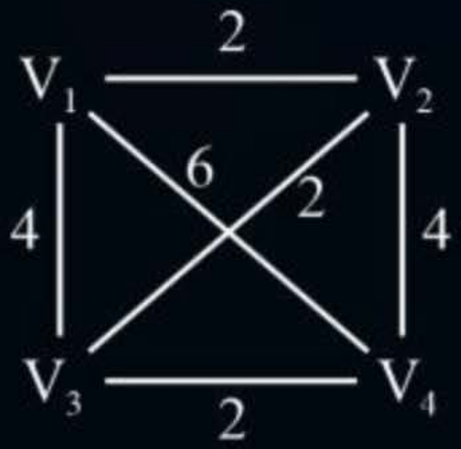
Topic : Greedy Algorithm

Solution:

1)



2)



Logic : $v_1 \overset{2}{—} v_2 \overset{2}{—} v_3 \overset{2}{—} v_4 \overset{2}{—} \cdots v_n$

cost of MCST = $2*(n-1)$ = $2n-2$

Topic : Greedy Algorithm

[MCQ]

#Q. Consider a graph with 'n' vertices $n > 2$. The vertices are numbered V_1 to V_n. Two vertices V_i and V_j are adjacent iff $0 < |i - j| \leq 2$. The weight of such an edge is $i + j$. The weight of minimum cost spanning tree of such a graph for a value of n is _____.

A $n^2 - n$ $\Rightarrow 9-3=6$ ✗

B $n^2 + n + 1$ $\Rightarrow 9+3+1=13$ ✗

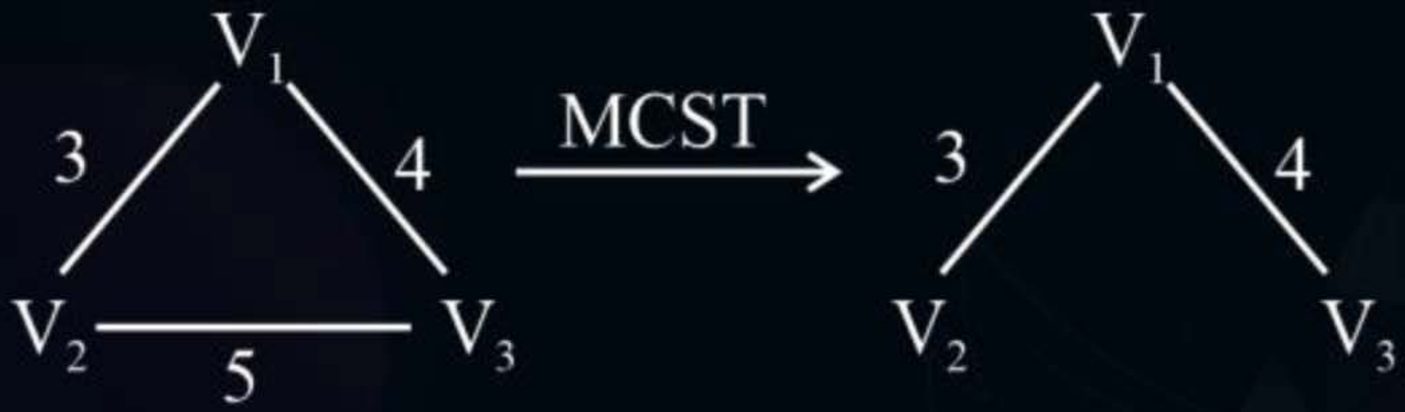
C $n^2 - n - 1$ $\Rightarrow 9-3-1=5$ ✗

D $n^2 - n + 1$ $\Rightarrow 9-3+1=7$ ✓

Topic : Greedy Algorithm

Solution:

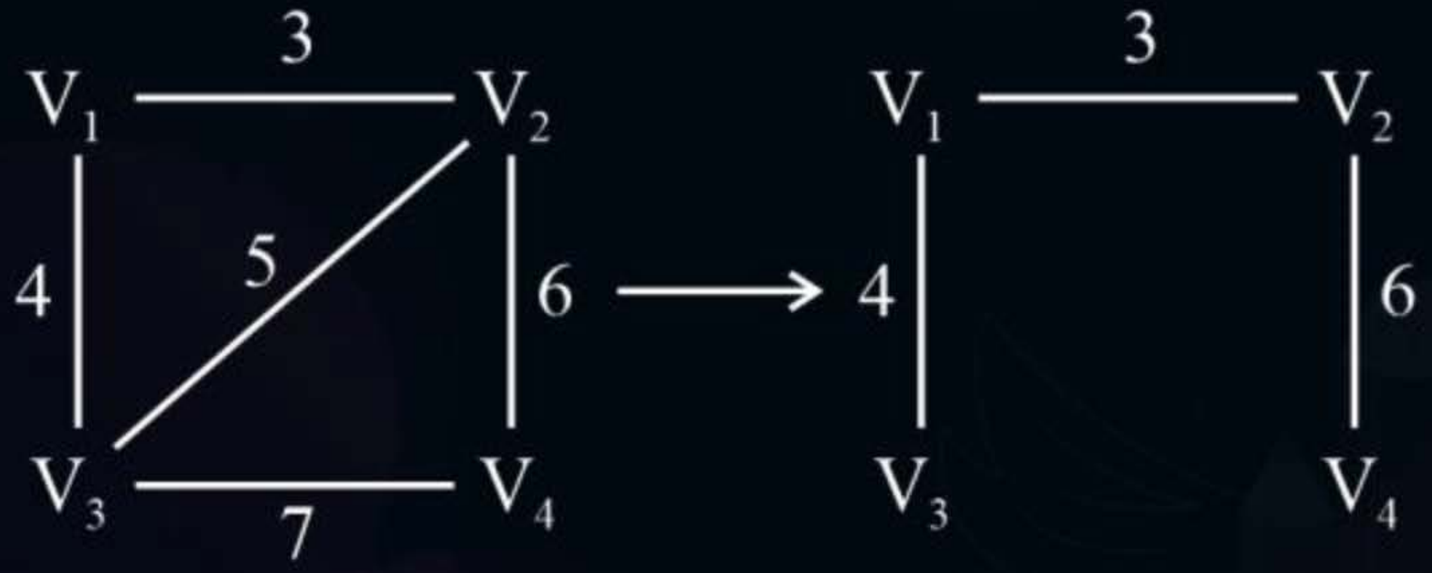
1) n = 3



cost = 7

Topic : Greedy Algorithm

2) n = 4



Topic : Greedy Algorithm

[MCQ]

#Q. Consider a graph whose vertices are points in a plane with integer co-ordinates (x, y) where $1 \le x \le n$, $n > 2$ is an integer. 2 vertices $\langle x_1, y_1 \rangle$ & $\langle x_2, y_2 \rangle$ are adjacent iff $|x_1 - x_2| \le 1$ and $|y_1 - y_2| \le 1$. The cost of such an edge is given by the distance between them. Compute the weight of min cost spanning tree of such graph for a given value of n.

A n^2

B $(n^2 - 1) * 2$

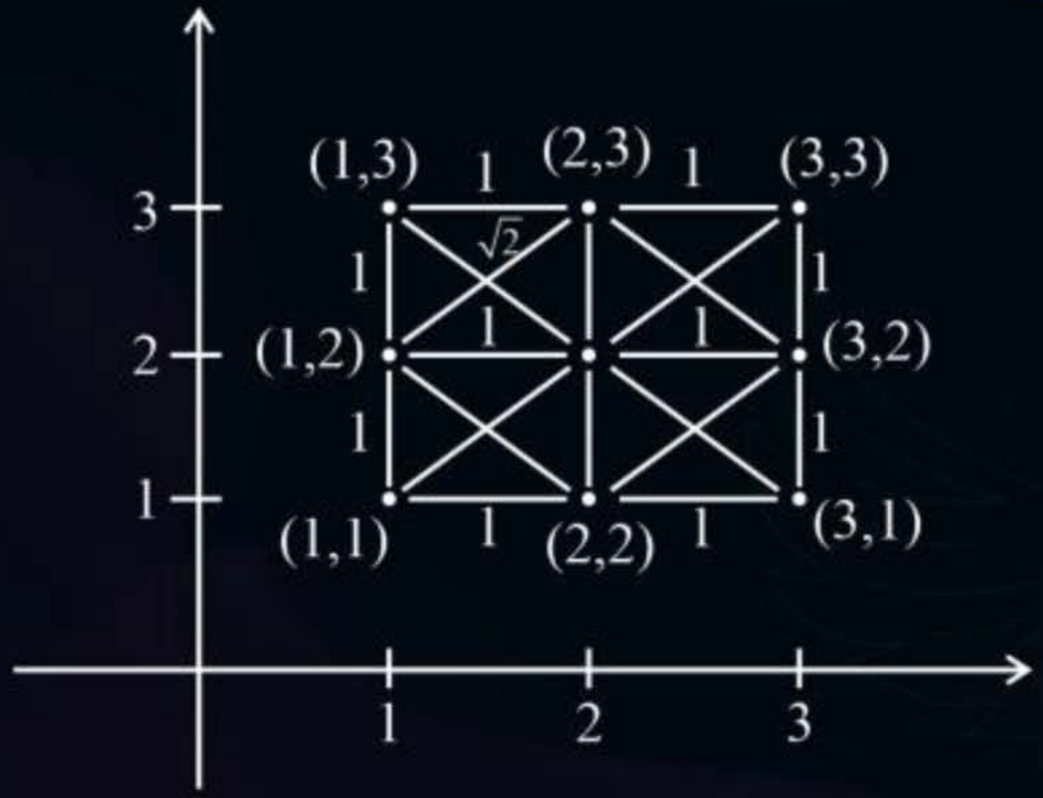
C $n^2 - 1$

D $n(n - 1)$

Topic : Greedy Algorithm

Solution:

n = 3



n → n^2 vertices

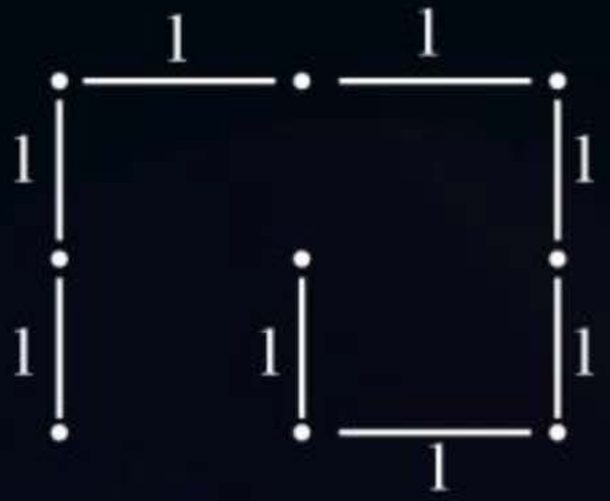
$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

(2,2)

(1,1)

Topic : Greedy Algorithm

Kruskal



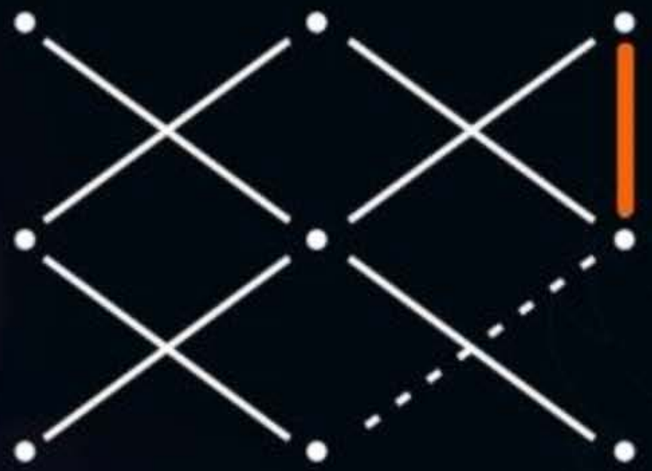
$n^2 - 1$

$\rightarrow$ Cost $= (n^2 - 1) * 1 = n^2 - 1$

Topic : Greedy Algorithm

Variant 2:

#Q. What is the max cost possible for Spanning Tree?



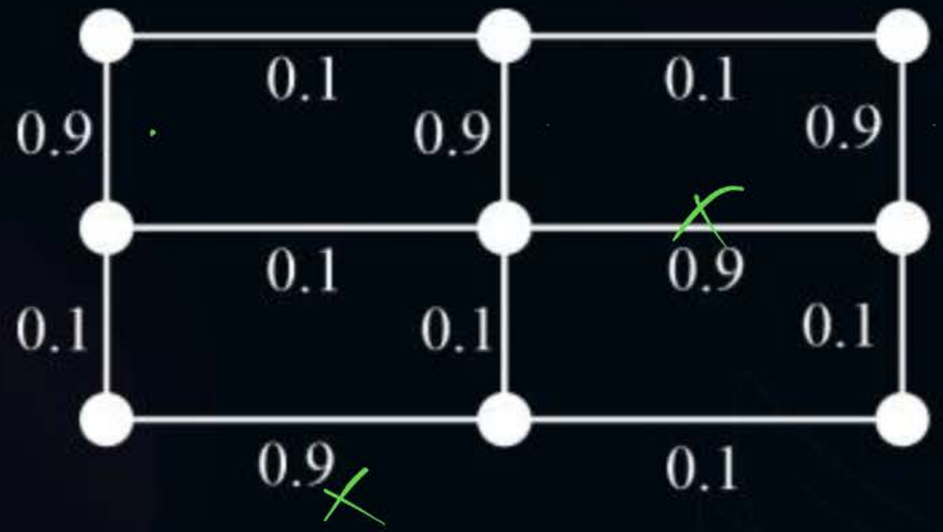
Max

$(n^2-2)\sqrt{2}+1$

Topic : Greedy Algorithm

[NAT]

#Q. Consider the following undirected graph with edge weights as shown



25%

count of MCSTs = 3

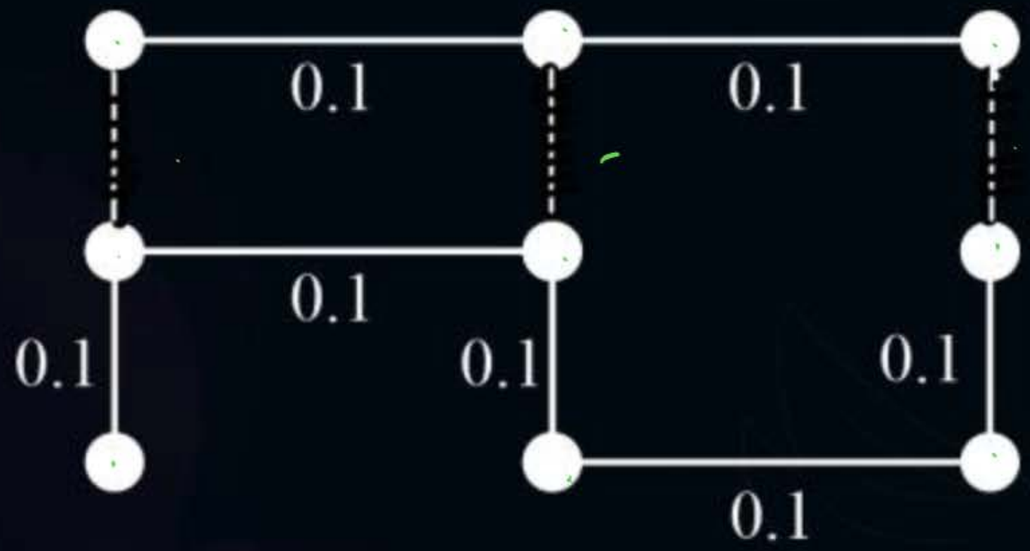
The number of minimum-weight spanning trees of the graph is _____.

Topic : Greedy Algorithm

Solution:

$^3C_1$

$= 3$



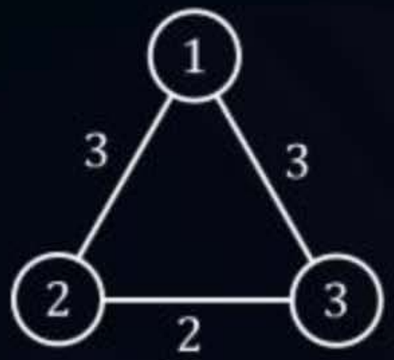
$n = 9$

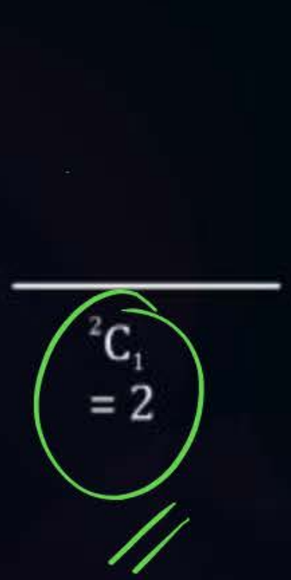
MCST edges $= 8$

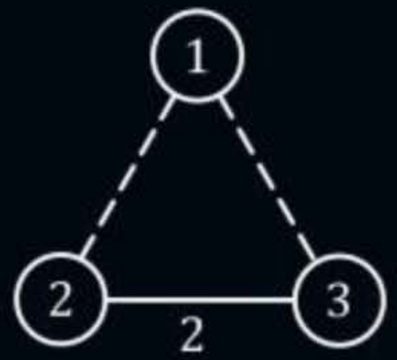
Topic : Greedy Techniques

Concept:-

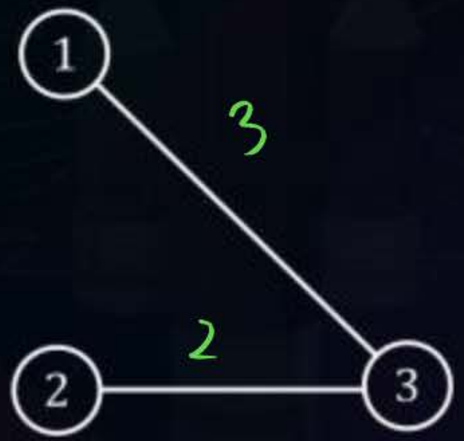
How many MCST?





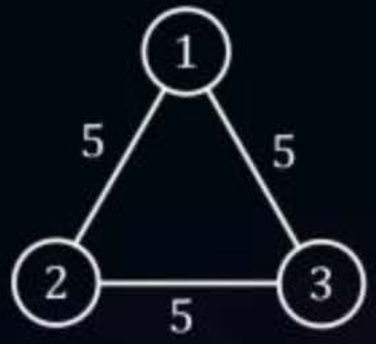


OR



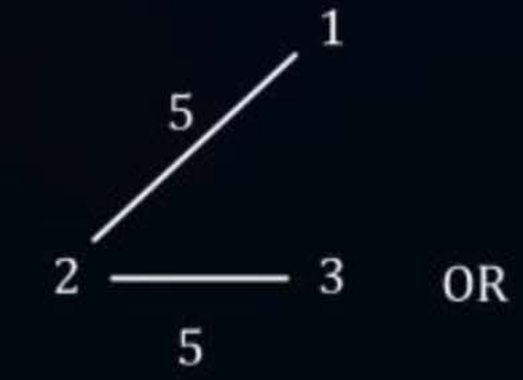
Topic : Greedy Techniques

Eg.1.

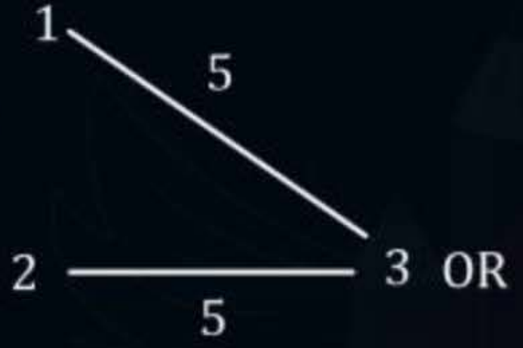


$^3C_2$

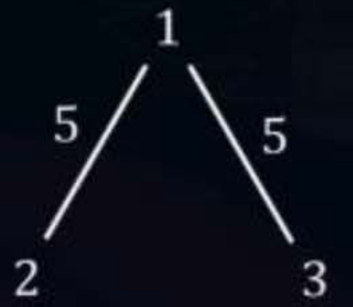
3. MCSTs



OR

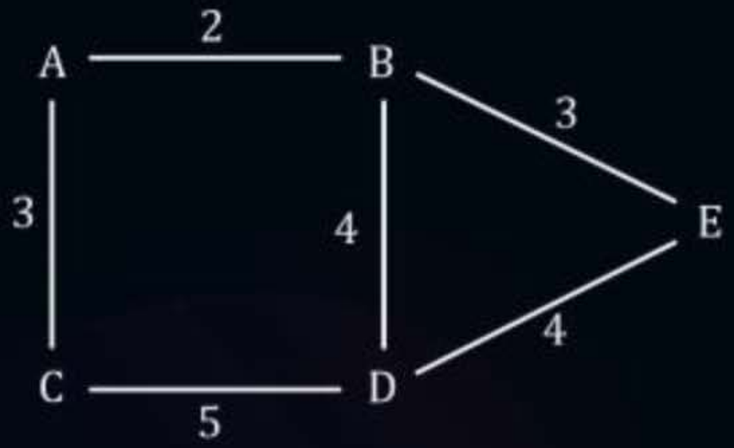


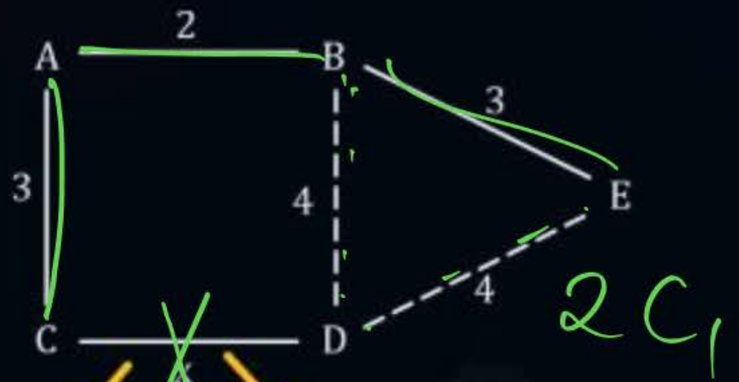
OR

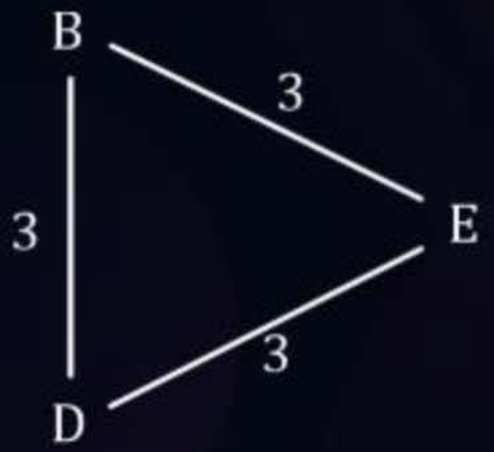


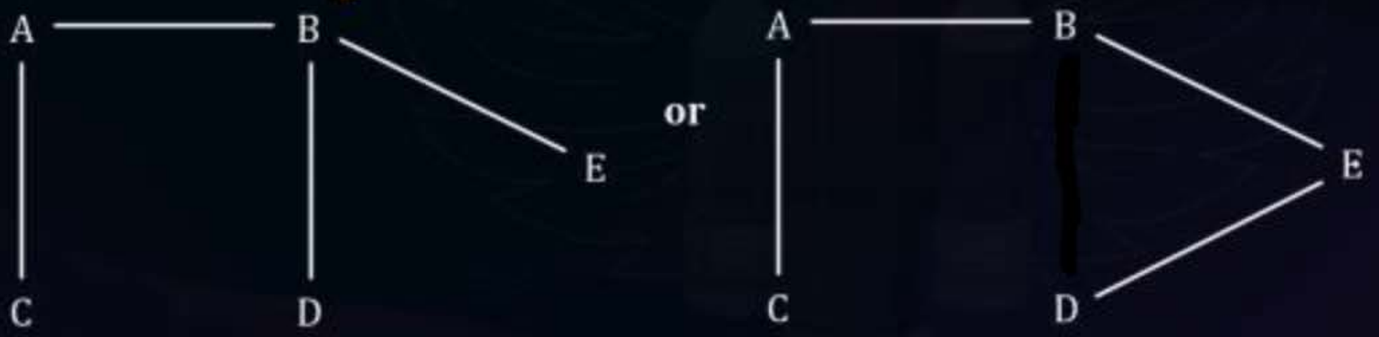
Topic : Greedy Techniques

Eg.2.



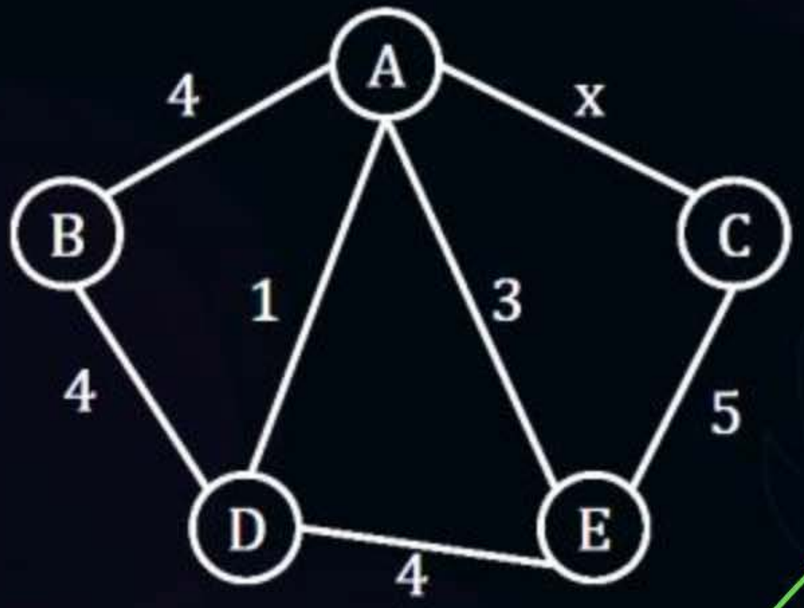






Topic : Greedy Techniques

#Q. Consider the following undirected graph G:



Choose a value for x that maximize the number of minimum weight spanning tree (MWSTs) of G. The number of MWST of G this value of x is_____.

Topic : Greedy Techniques

Possible Case:

1. $x > 5$: Then AC will never be a part go MCST and CE will be always a part.

   → Left → 2G Right →1 ⇒ Total = $^2C_1 * 1 = 2$MCSTs

2. $x < 5$: Then AC will always be a part of MCSTs and CE will never be a part.

   Left→ $^2C_1$ Right → 1 ⇒ Total $^2C_1 * 1 = 2$ MCSTs

3 $x = 5$

⇒Right $^2C_1$ Left $^2C_1$ ⇒Total $= {}^2C_1 \times {}^2C_1$

$= 2 \times 2$

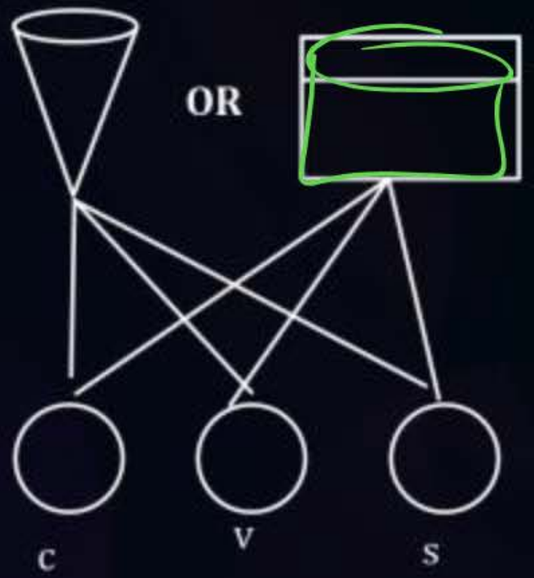
$= 4$ MCSTs

Topic : Greedy Techniques

When to mul and add

Ice Cream

How many ways Ice cream can be chosen?



$2 \times 3 = 6$

Topic : Greedy Techniques

#Q. Let 'w' be the minimum weight among all edge weights in an undirected connected graph. Let 'e' be a specific edge of weight 'w'. Which of the following is false?

Ans: C

i. There is a minimum spanning tree containing 'e' always (T)

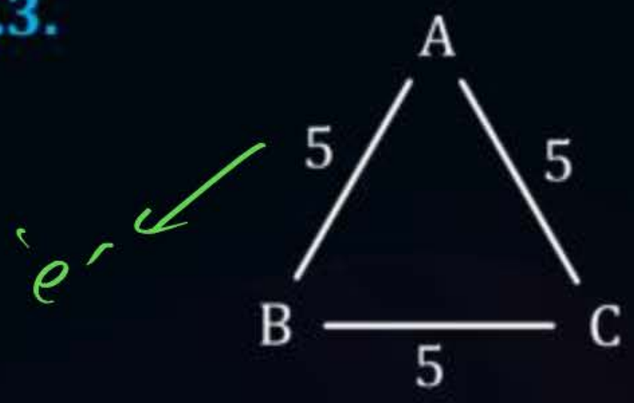
ii. Every minimum spanning tree has an edges of weight 'w'. (T)

iii. 'e' is present in every minimum spanning tree. (F)

iv. If 'e' is not present in a minimum spanning tree named 'T' then there will be a cycle formed by adding 'e' to T. (T)

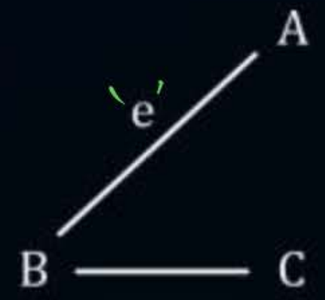
Topic : Greedy Techniques

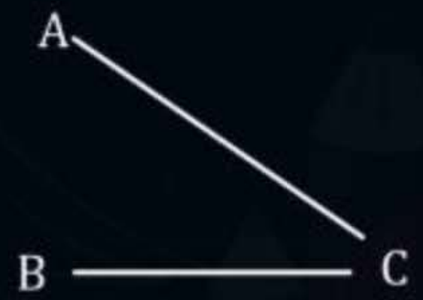
Eg.3.



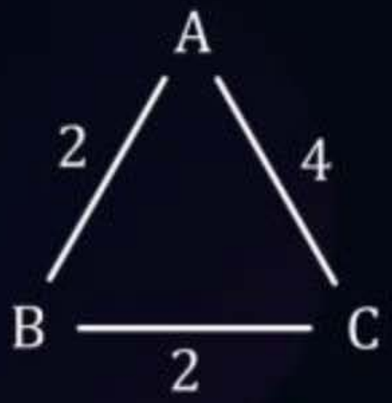
W = 5

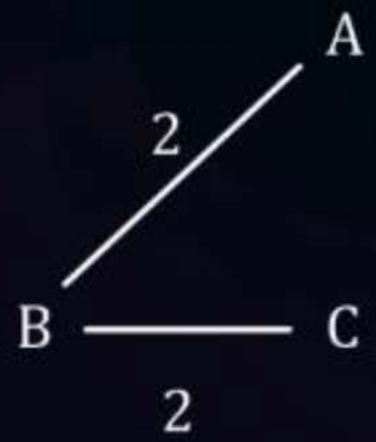
e = AB

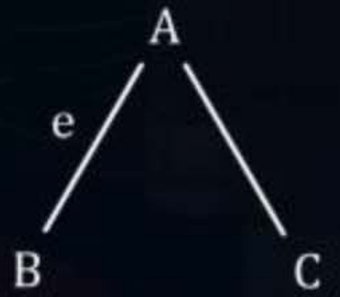




Eg.4.

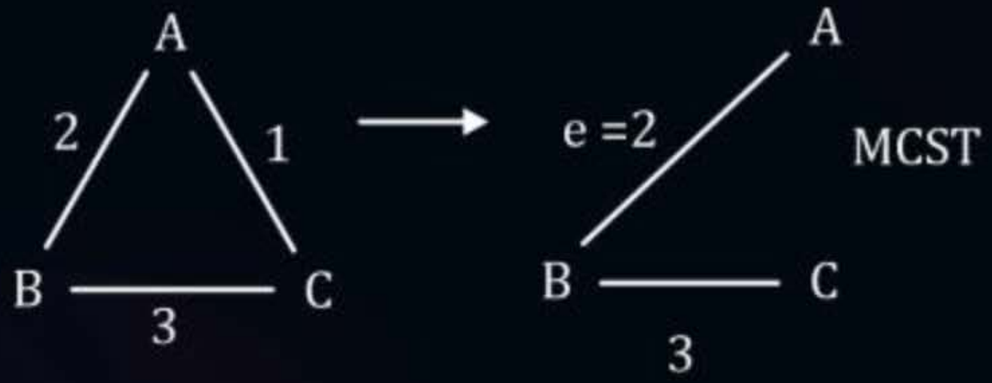




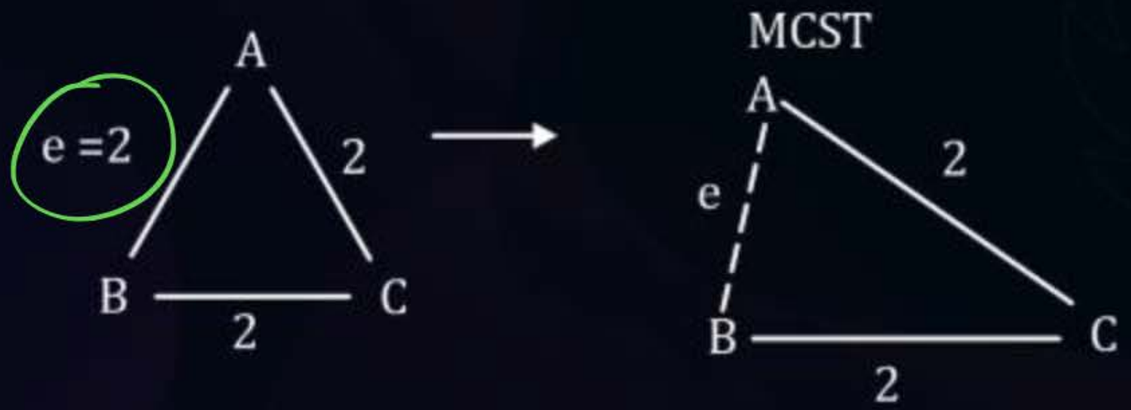


Topic : Greedy Techniques

Eg.5.



Eg.6.



Topic : Greedy Techniques

#Q. G= (V.E) is an undirected simple graph in each edge has a distinct weight, and e is particular edge of G. Which of the following statement about the minimum spanning tree (MSTs) of G is/are True?

I. If e is the lightest edge of some cycle in G, then every MST of G includes. it

II. If e is the heaviest edge of some cycle in G, then every MST of G excluded e.

Ans :- B

A. I only

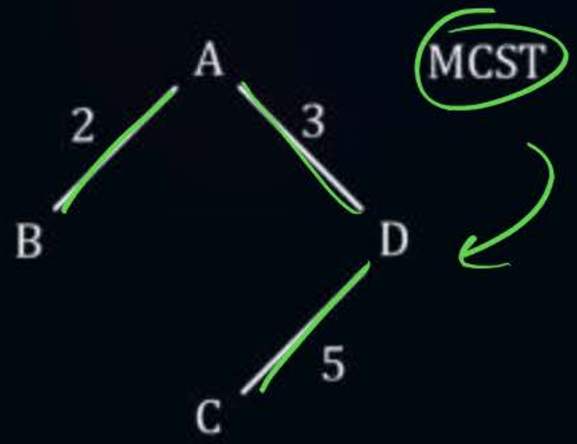
B. II only

C. Both I and II

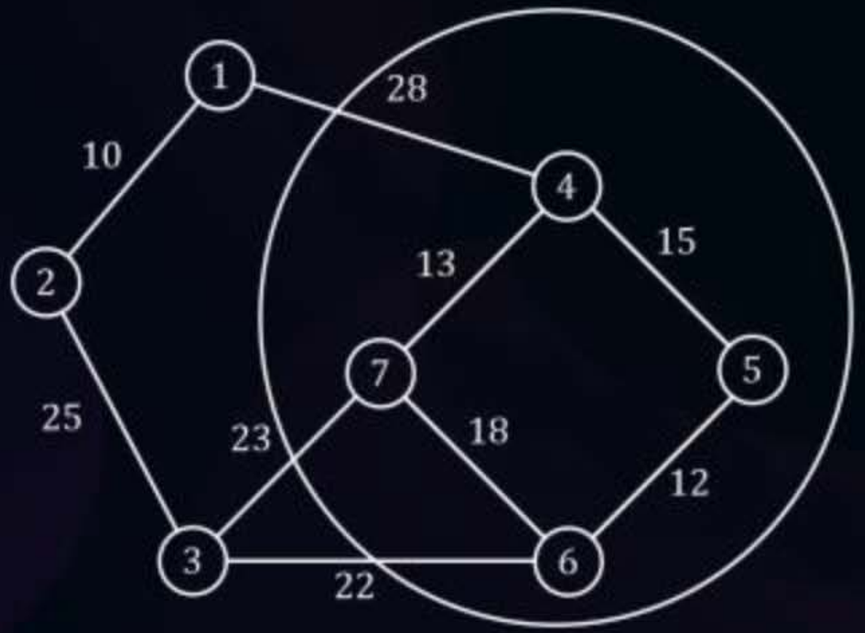
D. Neither I nor II

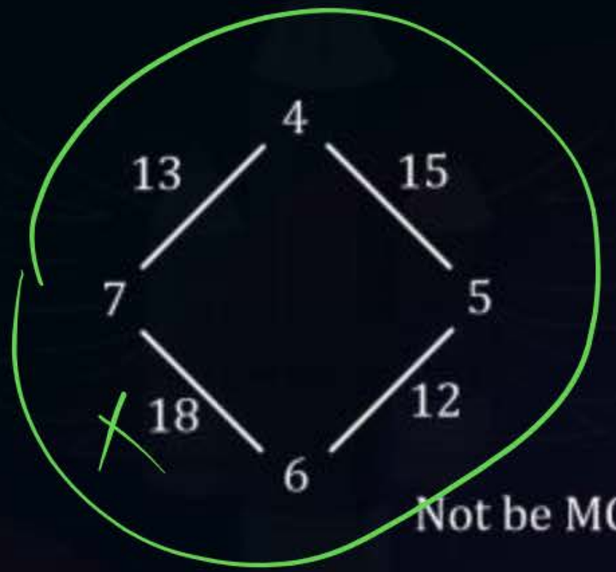
Topic : Greedy Techniques

Eg.7.



Eg.8.





Not be MCST

Topic : Greedy Techniques

HW

#Q. Let G be a connected undirect weight graph. Consider the following two statements.

S_1: There exits a minimum weight edge in G which is present in every minimum spanning tree of G.

S_2: If every edge in G has distinct weight, then G has a unique minimum spanning tree.

A Both S_1 and S_2 are true

B S_1 is true and S_2 false

C S_1 is false and S_2 is true

D Both S_1 and S_2 are false

Topic : Greedy Techniques

HW

```
1.  Algorithm Prime (E, cost, n, t)
2.  {
3.  Let (k,l) be an edge of minimum cost in E;
3.    mincost: cost[k,l]
4.    t[1,1]: =k; t[1,2]=l;
5.    for i:=1 to n do // Initialize near.
6.    If (cost [i, l]< cos [i,k]) then near [i] :=l;
7.    else near [i]: =k;
8.    near[k] := near [l] :=0;
9.    For i:=2 to n–1 do
```

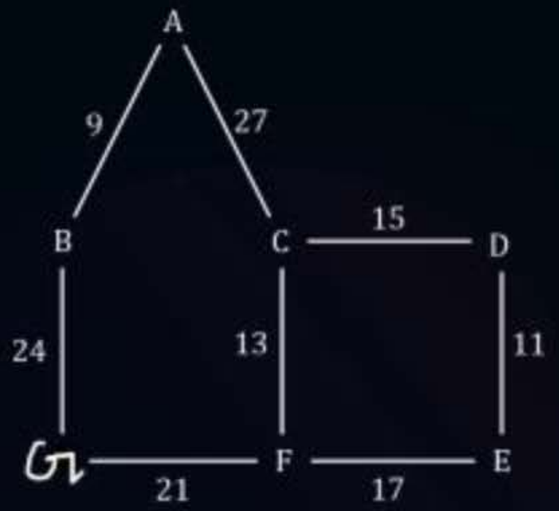
Topic : Greedy Techniques

HW

```
10.  { // Find n-2 additional edges for t.
11.  Let J be an index such that near [j] ≠0 and
12.  cost[j, near [j]] is minimum;
13.  t[i,l]:=j; t[i,2]= near [j];
14.  mincost:=mincost + cost[j , near [j]];
15.  near[j]:=0;
16.    for k:=1 to n do// Update near [].
17.    If ((near[k]≠0) and (cost [k, near[k]]>cos[k , j ]))
18.    then near [k]:=j;
19.    }
20. Return mincost;
```

Topic : Greedy Techniques

Prims algo dry run & Explanation



HW

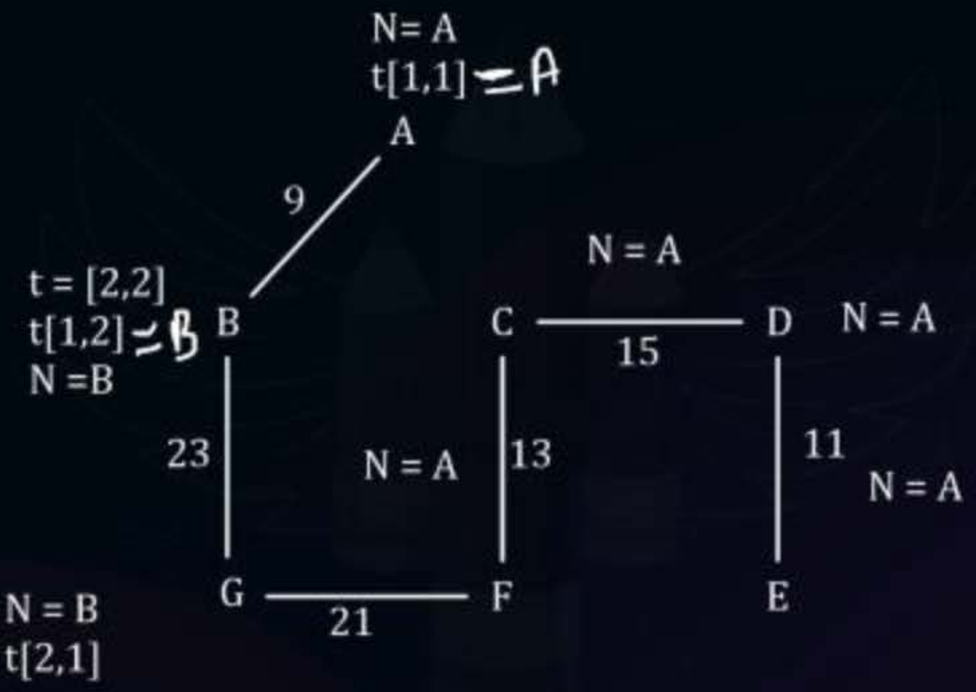
$A \to C : 27$

$A \to D : \infty$

$A \to E : \infty$

$A \to F : \infty$

$A \to G : 24$



t[i, 1]

t[i, 2]

Topic : Greedy Techniques

$\min(k, l) \rightarrow (A, B) = 9$

t [1,1] = A

t[1,2] = B

After adding F: to MCST

C →F : 13

D → A : ∞ → D- C : 15

E → F: 17 → E-D : 11

Topic : Greedy Techniques

Minicost = 9 + 24 + 21 + 13 + 15+11 = 93

Topic : Greedy Techniques

Time complexity Analysis of Prime Algorithm:

1. Prev. Implementation (non-Heap based)

   $= e + n + c + n\ (n + c + n)$

   $= O(n^2)$

2. If heap is used in Prime algo implementation

   $TC = (n + e) \log n$

   If $e = O(n^2)$ then $n^2 \log n$

Telegram Link for Aditya Jain sir: https://t.me/AdityaSir_PW

2 mins Summary

- Topic
- Topic
- Topic
- Topic
- Topic

Greedy Algo

THANK - YOU