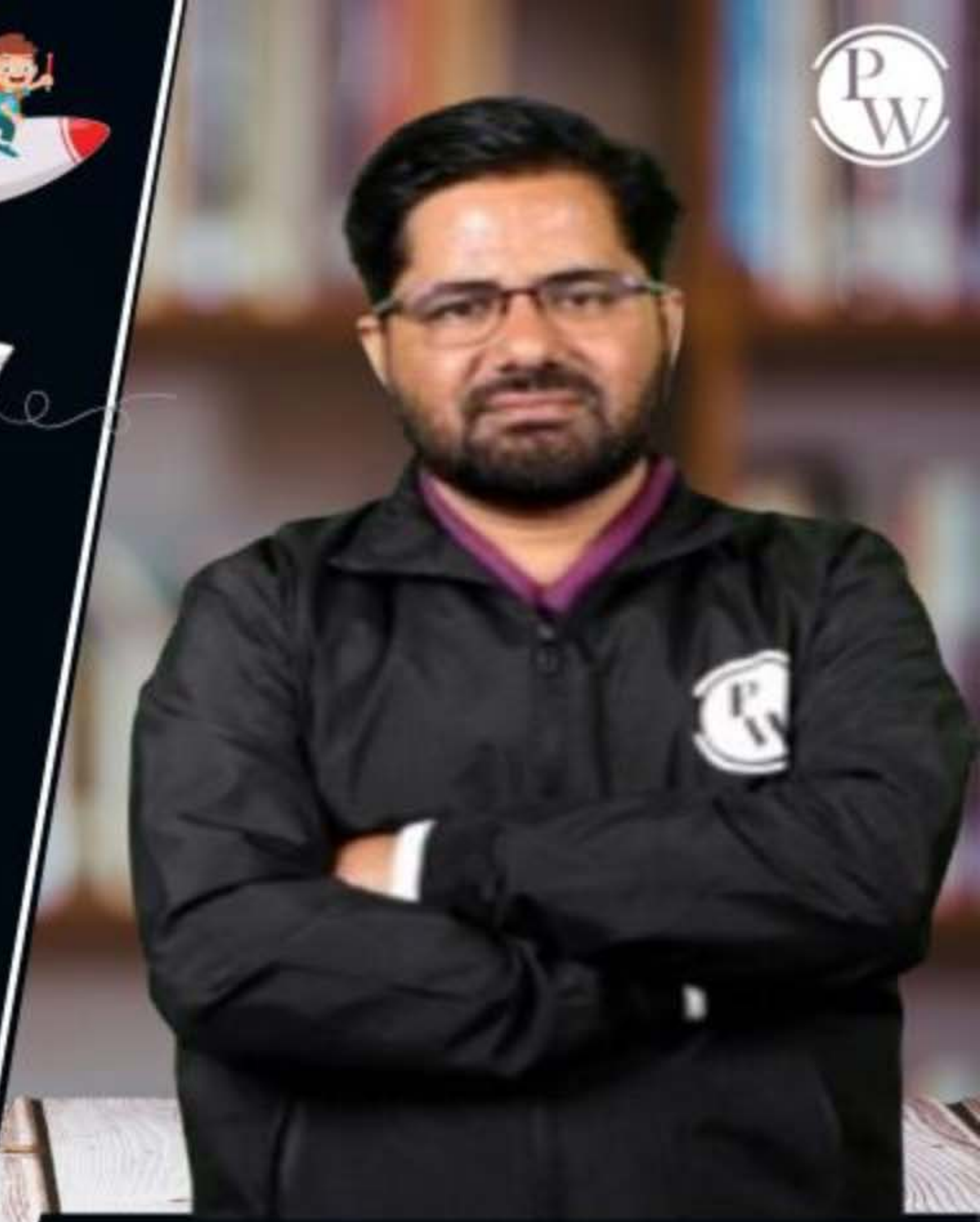
DS & AI
CS & IT

Probability & Statistics

Lecture No. 04



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Recap of previous lecture

Topic

PERMUTATION - COMBINATION

(Part- 3)

Topics to be Covered

Topic

"Remaining Part of Permutation-Combination"

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

" If, what if, AGAR, YADI, TOH, ... "

OR

Don't Try to develop Question (by your little mind until you have a complete understanding of the chapter) & try to solve the Quest.

Dearrangements → when No one goes at Right place assigned for him then such types of arrangements are called Dearrangements.

If there are n persons & n directed places. then

Total No. of arrangements $= n!$ this Result is applicable when RNA

All Correct " $= 1$

All wrong " $= \boxed{n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\cdots\cdots+(-1)^n\frac{1}{n!}\right]}$

For $n=2$, $n=3$, $n=4$, $n=5$, $n=6$ - - -

$D=1$, $D=2$, $D=9$, $D=44$, $D=265$ - - - -

$f:\{1,2,3,4,5\}\to\{1,2,3,4,5\}$ that are ONTO and $f(i)\neq i\ \forall i$ the no. of such functions will be?

?

(a) 9 (b) 44 (c) 119 (d) 120

Sol.

HW

for $n=5$, De-Arrangement $= 44$

ONTO $\Longrightarrow$ RNA

NWQ There are 5 Balls & 5 distinct cells. Then find the number of arrangements in which $\underbrace{\text{Ball } B_i \text{ is not placed in cell } C_i \ \forall i}_{\text{Dearrangements for } n=5}$ & $\underbrace{\text{No cell Remains empty}}_{\text{RNA}}$.

Gate

(a) 44

(b) 1

(c) 119

(d) 120

Dearrangements for $n=5$.

RNA

So $A_n = 44$

These 44 DeArrangements do not include following types of De-Arrang...

Such types of Questions are asked only in CAT type of Exams.

Distribution of Different items → No formula exist

& only use Multiplication Rule

Q: In how many ways 10 different pencils can be distributed among 4 kids if

(1) there is No Restriction = ? (RA)

$= \underline{4\text{ways}} \times \underline{4\text{ways}} \times \underline{4\text{ways}} \times \underline{4} \times \underline{4} \times \underline{4} \times \underline{4} \times \underline{4} \times \underline{4} \times \underline{4\text{ways}} = 4^{10}\text{ ways}$

WRONG: ~~Req Ans $= \frac{10}{K_1} \times \frac{10}{K_2} \times \frac{10}{K_3} \times \frac{10}{K_4} = 10^4$ ways.~~

Here ~~kids are picking pencils, you are not distributing.~~

Q If we have to distribute 7 pencils among 3 kids w/o any Restriction then Ans = ?

Total ways $= \underset{P_1}{\underline{3\text{ways}}} \times \underset{P_2}{\underline{3\text{ways}}} \times \underset{P_3}{\underline{3}} \times \underset{P_4}{\underline{3}} \times \underset{P_5}{\underline{3}} \times \underset{P_6}{\underline{3}} \times \underset{P_7}{\underline{3\text{ways}}} = 3^7\text{ ways}$

Q. In how many ways 10 $\boxed{\text{different}}$ pencils can be distributed among 4 kids if -

(1) there is no Restriction = ? $= 4^{10}$ ways (Max Ans)

(2) one child gets 3 particular pencils = ? $= \left({}^4C_1 \times {}^3C_3 \times 1\right) \times$ (Dist of 7 P among 3 kids) $= 3^7$ ways

$= {}^4C_1 \times 3^7$ ways

(3) 1st child gets exactly 3 pencils = ?

$= \left({}^1C_1 \times {}^{10}C_3 \times 1\right) \times 3^7$ ways

(4) 1st child gets 3 Particular Pencils = ? $= \left({}^1C_1 \times {}^3C_3 \times 1\right) \times 3^7$ ways

(5) Exactly one child gets exactly 3 Pencils = ? $= \cancel{\left({}^4C_1 \times {}^{10}C_3 \times 1\right)} \times 3^7$ ways. WRONG QUEST

Q. In how many ways ③ prizes can be distributed among ④ Boys s.that

① No Boy $\underbrace{\text{gets more than one prize}}_{RNA}$ = ? $\underset{P_1}{\underline{4}} \times \underset{P_2}{\underline{3}} \times \underset{P_3}{\underline{2}} = {}^4P_3 = 24$ ways

② A Boy may get $\underbrace{\text{any number of prizes}}_{RA}$ = ? $= \underset{P_1}{\underline{4}} \times \underset{P_2}{\underline{4}} \times \underset{P_3}{\underline{4}} = 64$ ways

③ No Boy gets all the prizes = ? $= 64 - ({}^4C_1 \times {}^3C_3 \times 1) = 64 - 4 = 60$ ways

Various Case in which single Boy gets all the prizes are as follows,

$\begin{pmatrix} P_1 & P_2 & P_3 \\ B_1 & B_1 & B_1 \end{pmatrix}$ or $\begin{pmatrix} P_1 & P_2 & P_3 \\ B_2 & B_2 & B_2 \end{pmatrix}$ or $\begin{pmatrix} P_1 & P_2 & P_3 \\ B_3 & B_3 & B_3 \end{pmatrix}$ or $\begin{pmatrix} P_1 & P_2 & P_3 \\ B_4 & B_4 & B_4 \end{pmatrix} \longrightarrow$ unfav. Cases.

If eight different biscuits are distributed among 6 beggars, find the number of ways in which particular beggar will get 3 biscuits

(a) $^8P_6$ (b) $^8C_6$ (c) $^8C_3 \times 5!$ (d) $^8C_3 \times 5^5$

Req ways of distributing $= \left(\underbrace{{}^1C_1}_{\text{Beggar}} \times \underbrace{{}^8C_3}_{\text{Biscuits}} \times 1\right) \times$ (Remaining Biscuits = 5, Beggars = 5 & No Rept.)

$$= {}^8C_3 \times \underset{B_1}{5} \times \underset{B_2}{5} \times \underset{B_3}{5} \times \underset{B_4}{5} \times \underset{B_5}{5} = {}^8C_3 \times 5^5 \text{ ways}$$

In how many ways 12 ~~balls~~ diff Balls can be distributed in 3 boxes, such that exactly one box contains 3 balls

(a) $^{12}C_3.2^9$ (b) 3^{12} (c) $^{12}C_3 \times ^3C_1 \times 2^9$ (d) $^3C_1 \times 12^3$ ~~(c)~~ 0

WRONG QUESTION

~~Total ways of distributing w/o any Restriction = 3^{12} ways (Max Ans)~~

~~fav cases = $\left(\underset{(\text{Box})}{^3C_1} \times \underset{(\text{Balls})}{^{12}C_3} \times 1\right) \times$ (Remaining Balls = 9, Remaining Boxes = 2, No Rest.)~~

~~$= \left(^3C_1 \times ^{12}C_3 \times 1\right) \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = ^3C_1 \times ^{12}C_3 \times 2^9$~~

Distribution of identical items →

If n identical items are to be distributed among r persons then possible number of Distributions will be, if

(1) Blank groups are not allowed (BNA) $= {}^{n-1}C_{r-1}$ (formula starts from 1)

(2) Blank groups are allowed (BA) $= {}^{n+r-1}C_{r-1}$ (Formula starts from Zero)

eg: In how many ways 4 identical pencils can be distributed among 2 kids

(1) if Blank groups are not allowed?

(M-I) Total Cases = 3 ways.

(3,1), (1,3), (2,2)

(M-II) $n=4, r=2$

$$BNA = {}^{n-1}C_{r-1} = {}^{4-1}C_{2-1} = {}^{3}C_{1} = 3$$

(2) If Blank groups are allowed?

(M-I) Total Cases = 5 ways.

(3,1), (1,3), (2,2), (4,0), (0,4)

(M-II) $BA = {}^{n+r-1}C_{r-1} = {}^{4+2-1}C_{2-1} = {}^{5}C_{1} = 5$

eg:- In how many ways 4 identical pencils can be distributed among 3 kids

① if Blank groups are not allowed?

M-I Various Cases:

(2 1 1), (1 2 1), (1 1 2)

So Ans = 3 ways.

M-II $n = 4, r = 3$

$$BNA = {}^{n-1}C_{r-1} = {}^{4-1}C_{3-1} = {}^{3}C_{2} = 3$$

② if Blank groups are allowed?

M-I Various Cases:

(2 1 1), (1 2 1), (1 1 2), (4 0 0), (0 4 0), (0 0 4),

(2 2 0), (2 0 2), (0 2 2), (3 1 0), (3 0 1),

(1 3 0), (1 0 3), (0 1 3), (0 3 1)

So Ans = 15 ways.

M-II $n = 4, r = 3$

$$BA = {}^{n+r-1}C_{r-1} = {}^{4+3-1}C_{3-1} = {}^{6}C_{2} = 15$$

Q. In how many ways 10 identical pencils can be distributed among 4 kids?

$n=10,\ r=4$

(1) There is No Restriction = ? $\;{}^{n+r-1}C_{r-1} = {}^{10+4-1}C_{4-1} = {}^{13}C_3$

(BA)

(2) A child can get $\underbrace{\text{any number of pencils}}_{BA}$ = ? $= {}^{13}C_3$

(3) A child may get $\underbrace{\text{nothing}}_{BA}$ = ? $= {}^{13}C_3$

(4) Each child gets $\underbrace{\text{at least one pencil}}_{BNA}$ = ? $= {}^{n-1}C_{r-1} = {}^{10-1}C_{4-1} = {}^{9}C_3$

(5) Each child gets $\underbrace{\text{something}}_{BNA}$ = ? $= {}^{9}C_3$

Q. In how many ways (16) Mangoes can be distributed among (3) Beggars s.that each beggar gets at least 3 Mangoes ?

(a) ${}^{16}C_3$

(b) ${}^{15}C_2$

(c) ${}^{18}C_2$

(d) ${}^{9}C_2$

BNA

(M-I) First we will distribute 3-3 Mangoes to each Beggar & this can be done by one way

Now Remaining Mangoes $(n) = 16-9 = 7$

& Beggars are still $(r) = 3$

Now we have No Restriction to distribute these 7 Mangoes Hence

$$BA = {}^{n+r-1}C_{r-1} = {}^{7+3-1}C_{3-1} = {}^{9}C_2$$

M-II $\boxed{x+y+z=16}$, $(x,y,z) \geq 3$. —— (1)

$(x-3)+(y-3)+(z-3)=(16-9)$

$\boxed{u+v+w=7}$; $u,v,w \geq 0$ —— (2)

i.e. $n=7, r=3$, &

$BA = ? = {}^{7+3-1}C_{3-1} = {}^{9}C_{2}$

M-III $\boxed{x+y+z=16}$, $(x,y,z) \geq 3$

$(x-2)+(y-2)+(z-2)=(16-6)$

$\boxed{u+v+w=10}$; $(u,v,w) \geq 1$.

$(x-2) \geq 1$, $(y-2) \geq 1$, $(z-2) \geq 1$

$u \geq 1$, $v \geq 1$, $w \geq 1$

$n=10$, $r=3$, &

$BNA = {}^{n-1}C_{r-1} = {}^{10-1}C_{3-1} = {}^{9}C_{2}$

Here it is not mentioned that Mangoes are identical, still we have assumed them as identical Mangoes ∵ we are performing NOBLE job.

Q: How many $\underbrace{\text{non-ve integral sol}^n\text{s}}_{(x,y,z,w)\geq 0}$ are there of $\boxed{x+y+z+w=100}$

let $n=100$ Mangoes, $r=4$ Beggars,

$$BA = {}^{n+r-1}C_{r-1} = {}^{100+4-1}C_{4-1} = {}^{103}C_3$$

ANALYSIS: Various sol^n are as follows. $\boxed{x+y+z+w=100}$

$(x,y,z,w) = \left.\begin{matrix}(50, 40, 5, 5) \\ (40, 29, 1, 30) \\ (4, 6, 88, 2) \\ \vdots \\ (46, 54, 0, 0) \\ (100, 0, 0, 0) \\ (0, 0, 0, 100)\end{matrix}\right\}$ 103 $\to$ 3

BA

$(0\ 0\ 0\ 0) \longrightarrow$??

Blanks are allowed But all Blanks (at a time) are not allowed just because of ?

SECURITY REASONS

In how many ways sum of upper faces of four dices can be six?

(a) 4 (b) 6 (c) 1 (d) 10

M-I Various possibilities are

(1113), (1131), (1311), (3111), (2211), (2121), (2112), (1122), (1212), (1221) i.e Ans = 10

M-II $\boxed{x+y+z+w=6}$; $(x, y, z, w) \geq 1$

$n=6, r=4$, BNA $= {}^{n-1}C_{r-1} = {}^{6-1}C_{4-1} = {}^{5}C_{3} = 10$ Ans

The number of possible positions of P will be ? if its position vector is given as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $x, y, z \in N$ and $\hat{r}.\vec{a} = 10$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

(a) 36 (b) 72 (c) 66 (d) None

Sol.

$$\hat{r} \cdot \vec{a} = 10$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 10$$

$$\boxed{x + y + z = 10}, \quad (x, y, z) \geq 1$$

$$n = 10,\ r = 3,\ BNA = {}^{n-1}C_{r-1} = {}^{10-1}C_{3-1} = {}^{9}C_{2} = \frac{9 \times 8}{2} = 36$$

Summation of Numbers: $\rightarrow$ Sum of n digit Nos that can be formed using RNA

$$\boxed{\text{Required Sum} = (\text{Sum of digits}) \times (\underbrace{1111\ldots\ldots1}_{n \text{ times}}) \times (n-1)!}$$

1. Take Care $\rightarrow$ this formula is applicable only for RNA
2. this formula is also valid when one of the digit is 0 but with an open mind (common sense)

eg: Find the sum of all three digit Nos that can be formed using 1, 2, 3 w/o Repetition?

Sol: Total 3 digit Nos (RNA) $= \underset{P_1}{\underline{3}} \times \underset{P_2}{\underline{2}} \times \underset{P_3}{\underline{1}} = 3! = 6$ Nos

Various Nos are

$$\begin{array}{r} 123 \\ 132 \\ 213 \\ 231 \\ 312 \\ +\ 321 \\ \hline 1332 \end{array}$$

Ans

M-II Req. sum

$= (1+2+3) \times (111) \times (3-1)!$

$= 6 \times 111 \times 2$

$= 1332$

eg: Find the sum of all five digit Nos that can be formed using 1, 3, 5, 7, 9 w/o Repetition ?

2014 hate

Apti

Total 5 digit Nos = $5! = 120$ Nos.
(RNA)

Req sum of these 120 Nos = ? $= (1+3+5+7+9) \times (11111) \times (5-1)!$

$= 25 \times 11111 \times 24$

$= 6666600$

The sum of all the numbers greater than 10000 formed by using digits 0, 2, 4, 6, 8 (no digit repeated in any number) is equal to?

RNA

(a) 5199960 (b) 5209960 (c) 5199980 (d) 5299960

Sol.

Total 5 digit Nos (>10000) that can be formed using given digits are $= \underset{P_1}{\underline{4 \text{ way}}} \times \underset{P_2}{\underline{4 \text{ way}}} \times \underset{P_3}{\underline{3 \text{ way}}} \times \underset{P_4}{\underline{2 \text{ way}}} \times \underset{P_5}{\underline{1 \text{ way}}} = 96$ Nos

(RNA)

(excluding = 0)

(1) sum of 120 Nos (including zero as complete digit) $= (0+2+4+6+8) \times (11111) \times \lfloor 5-1$

$= 20 \times 11111 \times 24 = 5333280$

(2) sum of 24 Nos (i.e. total 4 digit Nos using 2, 4, 6, 8) $= (2+4+6+8) \times 1111 \times \lfloor 4-1$

$= 20 \times 1111 \times 6 = 133320$

Hence sum of 96 No $=$ (1) $-$ (2) $= 5199960$

only Selection/Rezection Based Questions →

if we have p alike items of 1^{st} kind, q alike items of 2^{nd} kind, r alike items of 3^{rd} kind & n different items then

Total Number of selections & Rezection $= \boxed{(p+1)(q+1)(r+1)2^n}$

& No. of ways in which we can select at least one item $= \boxed{(p+1)(q+1)(r+1)2^n - 1}$

where $p+q+r+$ Rest(n) = Total (?)

Note: while in case of Arrangements $= \dfrac{n!}{p!\,q!\,r!}$ where $p+q+r+$ Rest (?) = Total(n)

The total number of selections of fruits which can be made from 3 bananas, 4 apples, and 2 oranges is

(a) 39 (b) 315 (c) 512 (d) None

Total selections or Rejections $= (\underbrace{\text{Bananas}}_{=3}) \times (\underbrace{\text{Apples}}_{=4}) \times (\underbrace{\text{oranges}}_{=2})$

$= [0 \text{ or } 1 \text{ or } 2 \text{ or } 3] \times [0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \text{ or } 4] \times [0 \text{ or } 1 \text{ or } 2]$

$\equiv$ 4 ways $\times$ 5 ways $\times$ 3 ways $=$ 60 ways.

these 60 ways includes, that case also in which we have Rejected all the fruits.

So Total No. of selections $= 60 - 1 = 59$ Ans

M-II $p = 3, q = 4, r = 2, n = 0$ so Req Ans $= (p+1)(q+1)(r+1)2^n - 1 = (3+1)(4+1)(2+1)2^0 - 1$

$= 59$

Q. The number of factors of 7875 will be?

(a) 10

(b) 12

(c) 22

(d) 24

(e) 23

5	7875
5	1575
5	315
3	63
3	21
7	7
	1

$\boxed{7875 = 3^2 \times 5^3 \times 7^1}$ Prime factorisation form.

$\cong 3^{0,1,2} \times 5^{0,1,2,3} \times 7^{0,1}$

i.e. Total factors = 3 ways × 4 ways × 2 ways

= 24 ways = 24 factors Ans

& No. of Proper factors = 24 − 2 = 22

M-II $p = 2, q = 3, r = 1$, Total factors $= (p+1)(q+1)2^r = (2+1)(3+1)2^1 = 24$.

Note

eg 36 → 1, 36 (Improper factors)

36 → 2, 3, 4, 6, 9, 12, 18 (Proper factors)

ie Any no has exactly Two Improper Factors 1 & no itself.

$$36 = 4 \times 9 = 2^2 \times 3^2 \equiv 2^{0,1,2} \times 3^{0,1,2}$$

ie Total factors = 3 ways × 3 ways = 9 ☺

2014 The numbers of factors of 2014 are ____?

(a) 2

(b) 6

(c) 8

(d) 12

(e) 1007

2	2014
19	1007
53	53
	1

$2014 = 2^1 \times 19^1 \times 53^1$

$= 2^{0 \text{ or } 1} \times 19^{0 \text{ or } 1} \text{ or } 53^{0 \times 1}$

So Total factors = 2 way × 2 ways × 2 ways = 8

M-II $n = 3$, So Total factors $= 2^n = 2^3 = 8$

$p = 0, q = 0, r = 0$, Rest $(n) = 3$ So T.f $= (0+1)(0+1)(0+1)2^3 = 8$

2014 = 1, 2, 19, 53, 38, 106, 1007, 2014

Qs Find Total factors of $5^2\times 6^3\times 7^4$?

(a) 60

(b) 24

(c) 160

(d) 240

(M-I) $\cancel{\text{T. factors} = 5^{0,1,2}\times 6^{0,1,2,3}\times 7^{0,1,2,3,4}}$

$\cancel{= 3\times 4\times 5} = 60$

(M-II) Given No $= 5^2\times 6^3\times 7^4$ (Not in prime form)

$\boxed{= 5^2\times 2^3\times 3^3\times 7^4}$ (No in prime form)

$= 5^{0,1,2}\times 2^{0,1,2,3}\times 3^{0,1,2,3}\times 7^{0,1,2,3,4}$

So Ans $= 3\times 4\times 4\times 5 = 240$

Q. Find the Total $\boxed{\text{9 digit Nos}}$ that can be formed using 5, 5, 6, 6, 6, 7, 7, 7, 7

Toyp = RNA.

Arrangement.

$= 5^2\ 6^3\ 7^4$

So Total 9 digit Nos $= \dfrac{9!}{2!\,3!\,4!}$

Q. Find the total $\boxed{\text{4 digit Nos}}$ (>6000) that can be formed using 5, 5, 6, 6, 6, 7, 7, 7, 7 ?

Toyp (RNA)

(Now it can be solved only by making Cases)

HW

Ans = 51

Thank

you

Keep Hustling!