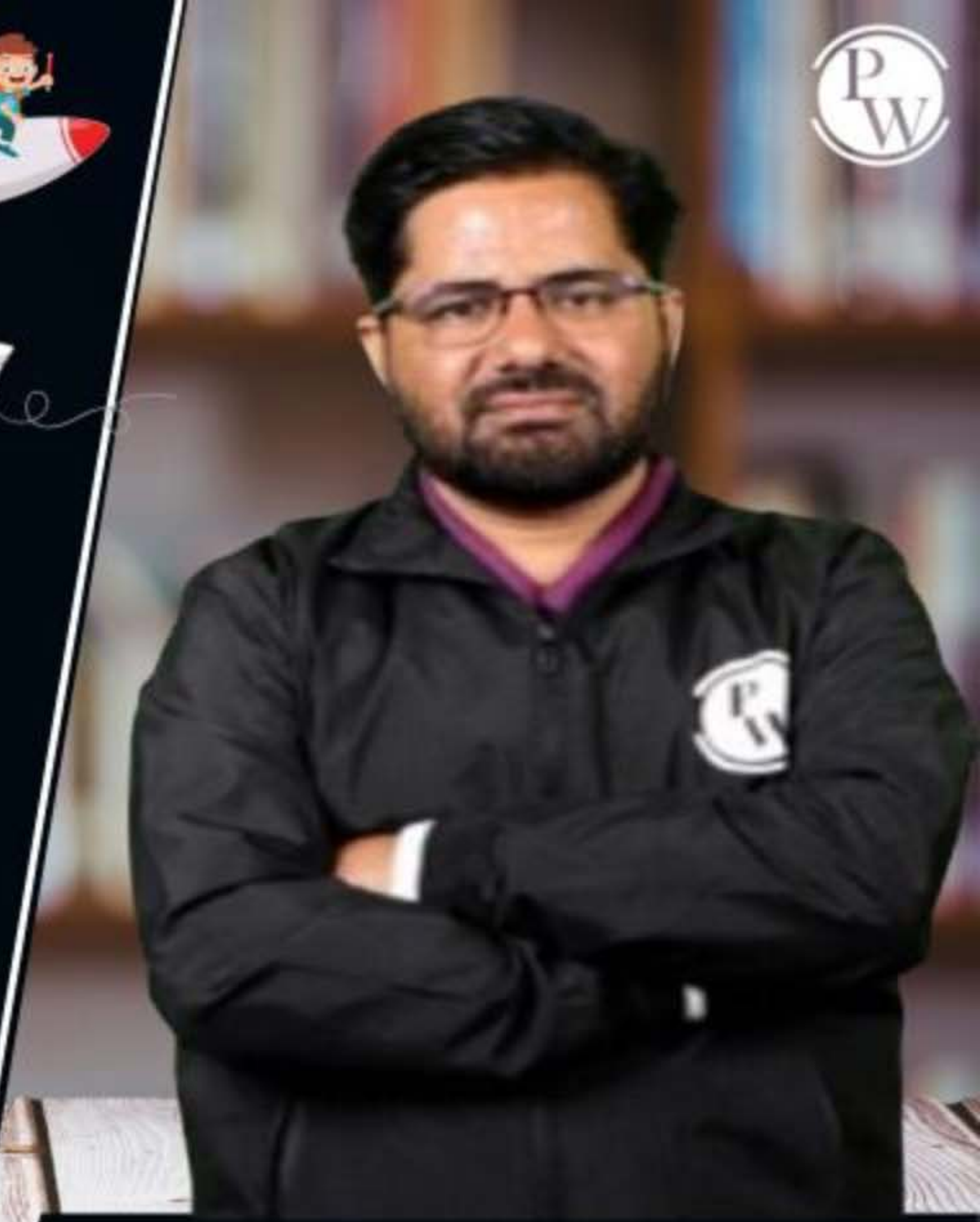
DATA SCIENCE & ARTIFICIAL INTELLIGENCE

& CS/IT

Calculus and Optimization

Lecture No. 12



By- Dr. Puneet Sharma Sir

Recap of previous lecture

Topic

INTEGRATION (Part-1)

(Indefinite Integration)

Topics to be Covered

Topic

INTEGRATION (Part 2)

- → Definite Integration
- → Leibnitz Rule of Diff under Integration
- → Application of Definite Integration (Finding Length of Curve & Volume of Revolution)

Standard Results :

1. Power formula: $\boxed{\int x^a dx = \frac{x^{a+1}}{a+1}, a \neq -1}$

2. $\int \frac{1}{x} dx = \log x + C$

3. $\int a^x dx = \frac{a^x}{\log_e a} + C$

4. $\int e^x dx = e^x + C$

5. $\int \sin x \, dx = -\cos x + C$

6. $\int \cos x \, dx = \sin x + C$

7. $\int \tan x \, dx = \log \sec x + C$

8. $\int \cot x \, dx = \log \sin x + C$

9. $\int \sec x \, dx = \log(\sec x + \tan x) + C$

10. $\int (\operatorname{cosec} x) dx = \log(\operatorname{cosec} x - \cot x) + C$

11. $\int \sec^2 x \, dx = \tan x + C$

12. $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

13. $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$

14. $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx)$

(15) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

(16) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + C$

(17) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right) + C$

(18) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left(x+\sqrt{x^2+a^2}\right) + C$

(19) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left(x+\sqrt{x^2-a^2}\right) + C$

(20) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

(21) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right) + C$

(22) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right) + C$

(23) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

(24) $\int \sec x \tan x\,dx = \sec x + C$, (25) $\int (\operatorname{cosec} x \cot x)\,dx = -\operatorname{cosec} x + C$

DEFINITE INTEGRAL

Fundamental Theorem of Integral Calculus →

if $\frac{d}{dx}(f(x)+c) = \phi(x)$

then $\int \phi(x)dx = f(x) + c$

F.Th: $\int_a^b \phi(x)\,dx = \Big[f(x)+c\Big]_{x=a}^{x=b}$

$= (f(b)+c) - (f(a)+c)$

$= \boxed{f(b) - f(a)}$

Q. $I = \int_0^{\pi/4} \tan^2(x)\,dx = ?$

$= \int_0^{\pi/4} (\sec^2 x - 1)\,dx$

$= (\tan x - x)_0^{\pi/4}$

$= (1-0) - \left(\frac{\pi}{4} - 0\right) = 1 - \frac{\pi}{4}$

Q. $I=\int_0^{\pi/2}\sqrt{\cos\theta}\,\sin^3\theta\,d\theta=?$ (a) 21 (b) 8/3 (c) $\frac{8}{21}$ (d) 21/8

Put $\cos\theta=t$ → At $\theta=0,\ t=1$; At $\theta=\frac{\pi}{2},\ t=0$

$\sin\theta\,d\theta=-dt$

$$I=\int_0^{\pi/2}\sqrt{\cos\theta}\cdot\sin^2\theta\cdot\sin\theta\,d\theta$$

$$=\int_1^0\sqrt{t}\cdot(1-t^2)\,(-dt)$$

$$=\int_0^1\left(t^{1/2}-t^{5/2}\right)dt=\left(\frac{t^{3/2}}{3/2}-\frac{t^{7/2}}{7/2}\right)_0^1=\frac{8}{21}$$ (c)

Properties of Definite Integral →

(1) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(2) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(3) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

where $a < c < b$

(4) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(5) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

Property (6) $\rightarrow$

$$\int_0^{2a} f(x)\,dx = \boxed{\int_0^{a} \{f(x)+f(2a-x)\}\,dx} = \begin{cases} 2\int_0^{a} f(x)\,dx, & \text{if } f(2a-x)=f(x) \\ 0, & \text{if } f(2a-x)=-f(x) \end{cases}$$

Property (7) $\rightarrow$

$$\int_{-a}^{a} f(x)\,dx = \boxed{\int_0^{a} \{f(x)+f(-x)\}\,dx} = \begin{cases} 2\int_0^{a} f(x)\,dx, & \text{if } f(-x)=f(x) \text{ i.e. } f(x) \text{ is Even} \\ 0, & \text{if } f(-x)=-f(x) \text{ i.e. } f(x) \text{ is odd} \end{cases}$$

Prop (1) $I=\int_1^3 x\,dx = ? = \left(\frac{x^2}{2}\right)_1^3 = \frac{9}{2}-\frac{1}{2} = 4$

$I=\int_1^3 t\,dt = ? = \left(\frac{t^2}{2}\right)_1^3 = \frac{9}{2}-\frac{1}{2} = 4$

Prop (2) $I=\int_3^1 x\,dx = ? = \left(\frac{x^2}{2}\right)_3^1$

$= \frac{1}{2}-\frac{9}{2} = -4$

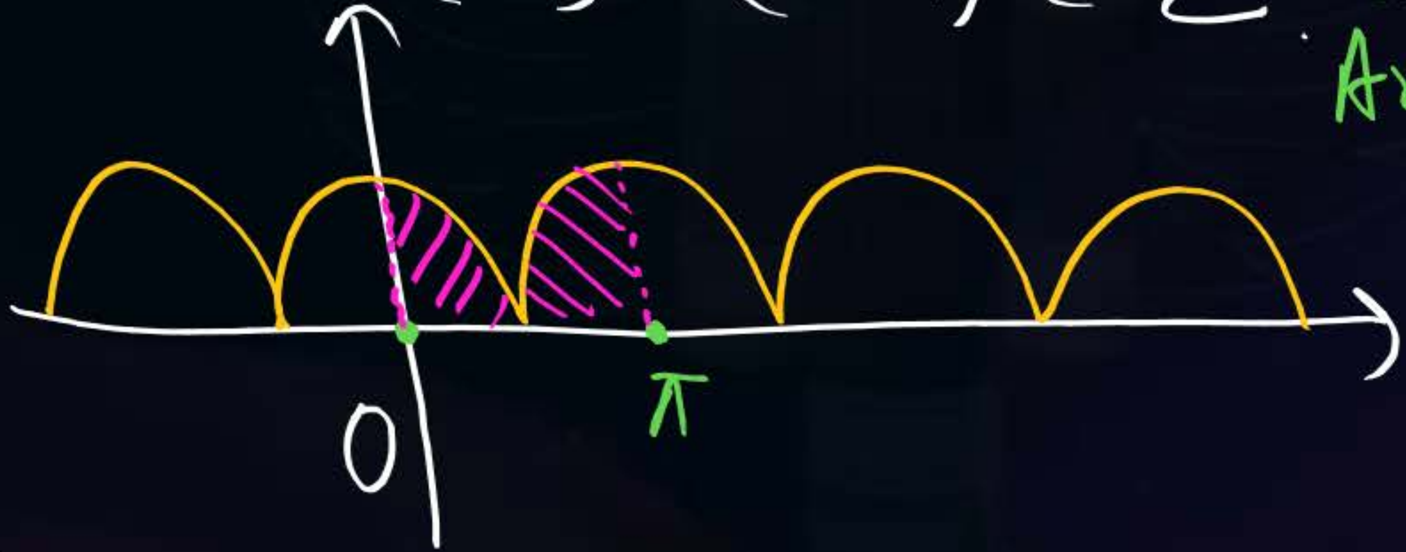
Prop (3): $I=\int_0^{\pi} |\cos x|\,dx = ?$ (Ans = 2)

$\because |\cos x| = \begin{cases} +\cos x, & 0<x<\frac{\pi}{2} \\ -\cos x, & \frac{\pi}{2}<x<\pi \end{cases}$

$I=\int_0^{\pi} |\cos x|\,dx = \int_0^{\pi/2} (+\cos x)\,dx + \int_{\pi/2}^{\pi} (-\cos x)\,dx$

$= (\sin x)_0^{\pi/2} - (\sin x)_{\pi/2}^{\pi}$

$= (1-0)-(0-1) = 2$ = Shaded Area



Q. $I = \int_{-1}^{2} (1+|x|)\,dx = ?$ (a) 5.5 (b) 4.5 (c) $\frac{2}{11}$ (d) 3

$$I = \int_{-1}^{0} (1+|x|)\,dx + \int_{0}^{2} (1+|x|)\,dx$$

$$= \int_{-1}^{0} (1-x)\,dx + \int_{0}^{2} (1+x)\,dx$$

$$= \left(x - \frac{x^2}{2}\right)_{-1}^{0} + \left(x + \frac{x^2}{2}\right)_{0}^{2}$$

$$= \left[(0-0) - \left(-1-\frac{1}{2}\right)\right] + \left[(2+2) - (0+0)\right] = 5.5$$

Property 4 →

$$\int_a^b f(x)\,dx = \int_a^b f(\underbrace{a+b-x})\,dx$$

Property 5 →

$$\int_0^a f(x)\,dx = \int_0^a f(\underbrace{a-x})\,dx$$

Q. $I = \int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}\,dx = ?$ —— (1)

$a=1, b=2$, so $a+b-x = (3-x)$

i.e. By Prop(4); $x \longrightarrow (3-x)$

$I = \int_1^2 \frac{\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}}\,dx$ —— (2)

$2I = \int_1^2 \left(\frac{\sqrt{x}+\sqrt{3-x}}{\sqrt{x}+\sqrt{3-x}}\right)dx$

$2I = (x)_1^2 = 2-1 = 1$

$I = \frac{1}{2}$

Q. $I = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx = ?$ —— (1)

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$, (c) π (d) 0

$a=0, b=\frac{\pi}{2}$ so $a+b-x = \frac{\pi}{2}-x$

so by Prop(5); $x \longrightarrow \frac{\pi}{2}-x$

$I = \int_0^{\pi/2} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}\,dx$

$= \int_0^{\pi/2} \left(\frac{\cos x}{\cos x+\sin x}\right)dx$ —— (2)

$2I = \int_0^{\pi/2} (1)\,dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

Q. $I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx = ?$ (a) $\frac{\pi}{4}$ (b) $\frac{\pi^2}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi^2}{2}$

By Prop (5); $x \longrightarrow (\pi-x)$

$$I=\int_0^{\pi}\frac{(\pi-x)\cdot\sin(\pi-x)}{1+\cos^2(\pi-x)}\cdot dx$$

$$=\int_0^{\pi}\frac{(\pi-x)\cdot\sin x}{1+(-\cos x)^2}dx$$

$$I=\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx-\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$

$$I= \quad\downarrow\quad -I$$

$$2I=\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

Put $\cos x = t$ → At, $x=0$, $t=1$; At, $x=\pi$, $t=-1$

$-\sin x\,dx = dt$

$$=\pi\int_{+1}^{-1}\frac{(-dt)}{1+t^2}=+\pi\int_{-1}^{1}\left(\frac{1}{1+t^2}\right)dt$$

$$=+2\pi\int_0^1\left(\frac{1}{1+t^2}\right)dt=+2\pi\left(\tan^{-1}t\right)_0^1$$

$$2I=+2\pi\left(\frac{\pi}{4}-0\right)\Rightarrow I=+\pi^2/4$$

Property (6) →

$$\int_0^{2a} f(x)\,dx = \boxed{\int_0^{a}\{f(x)+f(2a-x)\}\,dx} = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(2a-x)=f(x) \\ 0, & \text{if } f(2a-x)=-f(x) \end{cases}$$

Property (7) →

$$\int_{-a}^{a} f(x)\,dx = \boxed{\int_0^{a}\{f(x)+f(-x)\}\,dx} = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(-x)=f(x) \text{ i.e. } f(x) \text{ is Even} \\ 0, & \text{if } f(-x)=-f(x) \text{ i.e. } f(x) \text{ is odd} \end{cases}$$

$$\int_0^{\alpha} f(x)\,dx = \begin{cases} 2\int_0^{\alpha/2} f(x)\,dx, & \text{if } f(\alpha-x)=f(x) \\ 0, & \text{if } f(\alpha-x)=-f(x) \end{cases}$$

Q. $I=\int_{-\pi/2}^{\pi/2}\underbrace{|\sin x|}_{\text{even func}^n}dx=?\;=2\int_0^{\pi/2}|\sin x|\,dx$

Prop 7

(a) 0

(b) 1

(c) 2

(d) 4

$=2\int_0^{\pi/2}(+\sin x)\,dx$

$=2(-\cos x)_0^{\pi/2}$

$=-2[0-1]=2$

M-II $I=\int_{-\pi/2}^{0}(-\sin x)\,dx+\int_0^{\pi/2}(+\sin x)\,dx$

$=(+\cos x)_{-\pi/2}^{0}-(\cos x)_0^{\pi/2}$

$=(1-0)-(0-1)=2$

Q $\int_{-\pi/3}^{\pi/3}\underbrace{x^5.\sin^4 x}_{\text{odd func}^n}dx=\;?=0$

(By Prop 7)

$f(x)=x^5\sin^4 x$

$f(-x)=(-x)^5\sin^4(-x)$

$=-(x^5).(-\sin x)^4$

$=-x^5.\sin^4 x$

$=-f(x)$ so $f(x)$ is odd func^n

Q. $I = \int_{-\pi/2}^{\pi/2} (\underbrace{\sin|x| + \cos|x|}_{\text{even func}^n}) \cdot dx = ?$

(a) 2 (b) 0 (c) 4 (d) 1

By Prop (7);

$I = 2\int_0^{\pi/2} (\sin|x| + \cos|x|)\, dx$

$= 2\int_0^{\pi/2} (\sin x + \cos x)\, dx$

$= 2\left[-\cos x + \sin x\right]_0^{\pi/2}$

$= 2\left[\{-0+1\} - \{-1+0\}\right] = 4$

Q. $I = \int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin x}{2+\sin x}\right) dx = ?$

$f(x) = \log\left(\frac{2-\sin x}{2+\sin x}\right)$

$f(-x) = \log\left[\frac{2+\sin x}{2-\sin x}\right] = \log\left(\frac{2-\sin x}{2+\sin x}\right)^{-1}$

$= -\log\left(\frac{2-\sin x}{2+\sin x}\right) = -f(x)$

i.e. $f(x)$ is an odd func^n so $I = 0$

By Prop (7)

Q: $I = \int_0^{2\pi} \cos^5 x\,dx = ?$ $= 2\int_0^{\pi} \cos^5 x\,dx = 2[\,0\,] = 0$

Imp 6

(a) 0

(b) 1

(c) 2

(d) 8

$f(x) = \cos^5 x$

$f(2\pi - x) = \cos^5(2\pi - x)$

$= (\cos x)^5$

$= \cos^5 x$

$= f(x)$

Let $f(x) = \cos^5 x$

$f(\pi - x) = [\cos(\pi - x)]^5$

$= (-\cos x)^5 = -\cos^5 x$

$= -f(x)$

Q. $I = \int_0^{2\pi} \sin x \, dx = ?$ (a) 0 (b) 1 (c) 2 (d) 4

$= \int_0^{\pi} (\sin x) dx + \int_{\pi}^{2\pi} (\sin x) dx = 2 + (-2) = 0$

Q. if $f(x) = \sin x$ then find area bounded by $f(x)$ b/w $x=0$ & $x = 2\pi$

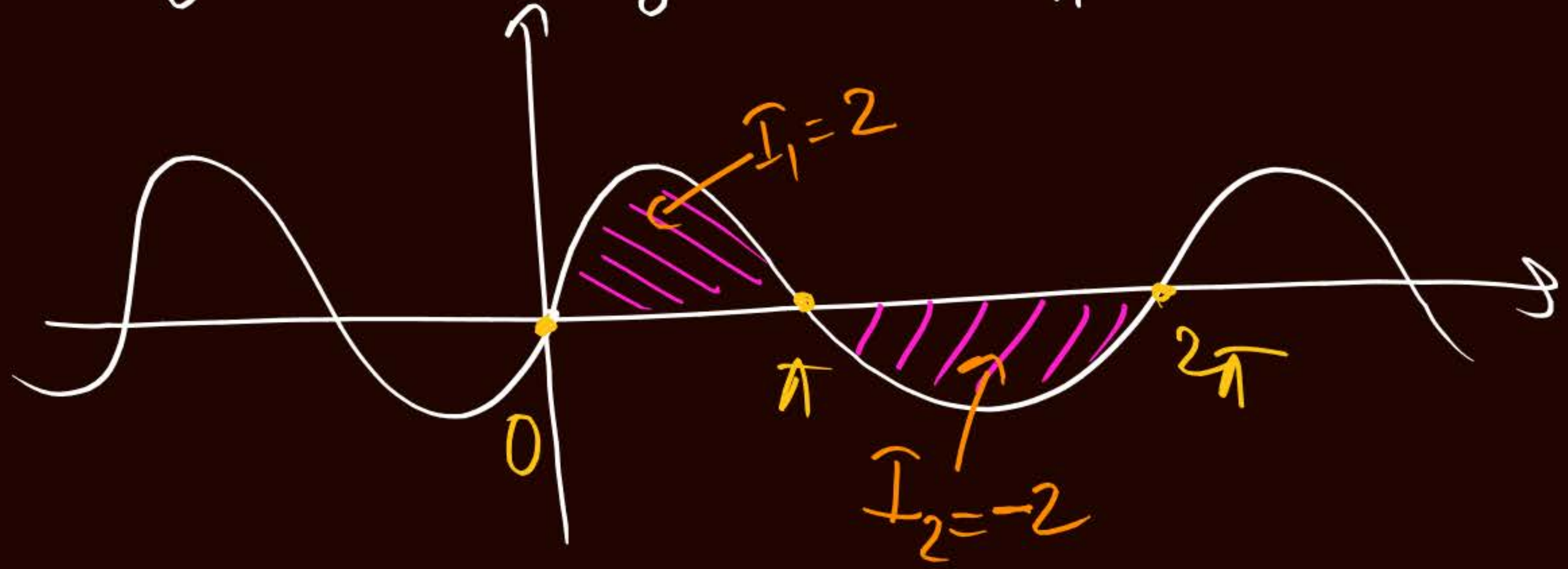
(a) 0

(b) 1

(c) 2

(d) 4

$\text{Area} = \int_0^{2\pi} |\sin x| dx = \int_0^{\pi} (+\sin x) dx + \int_{\pi}^{2\pi} (-\sin x) dx = 2 - (-2) = 4$



Q. $I = \int_0^{\pi/2} \log(\sin x)\,dx = ?$ —— (1)

By Prop (5), $x \longrightarrow \frac{\pi}{2} - x$

$$I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx = \int_0^{\pi/2} \log \cos x \, dx \quad \text{—— (2)}$$

$$2I = \int_0^{\pi/2} (\log \sin x + \log \cos x)\, dx = \int_0^{\pi/2} \log(\sin x \cos x)\, dx$$

$$= \int_0^{\pi/2} \log\left(\frac{2\sin x \cos x}{2}\right) dx = \int_0^{\pi/2} (\log \sin 2x - \log 2)\, dx$$

$$= \int_0^{\pi/2} \log \sin 2x \, dx - \log 2 \int_0^{\pi/2} (1)\, dx$$

$$2I = I_1 - \log 2\left(\frac{\pi}{2} - 0\right) \quad \text{—— (3)}$$

Now $I_1 = \int_0^{\pi/2} \log \sin(2x)\, dx$

Put $2x = t$
$dx = \frac{dt}{2}$

At $x = 0,\ t = 0$

At $x = \frac{\pi}{2},\ t = \pi$

$$I_1 = \int_0^{\pi} \log \sin t \cdot \frac{dt}{2} = \int_0^{\pi/2} \log \sin t \, dt = I$$

So By (3), $2I = I_1 - \frac{\pi}{2}\log 2$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = -\frac{\pi}{2}\log 2$$

(*) $\boxed{\int_0^{\pi/2} \log \sin x\, dx = \int_0^{\pi/2} \log \cos x\, dx = -\frac{\pi}{2}\log 2}$ Learn

Q. $I = \int_0^{\pi/2} \log(\tan x + \cot x)\, dx$

(a) $\frac{\pi}{2}\ln 2$

(b) $\pi \ln 2$ ✓

(c) $-\frac{\pi}{2}\ln 2$

(d) 0

$$= \int_0^{\pi/2} \log\left\{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right\} dx$$

$$= \int_0^{\pi/2} \log\left\{\frac{1}{\sin x \cos x}\right\} dx$$

$$= \int_0^{\pi/2} \left[\log 1 - \underbrace{\log(\sin x \cos x)}\right] dx$$

$$I = \int_0^{\pi/2} \left\{0 - \underbrace{(\log \sin x + \log \cos x)}\right\} dx$$

$$= -\left[\int_0^{\pi/2} \log \sin x\, dx + \int_0^{\pi/2} \log \cos x\, dx\right]$$

$$= -\left[-\frac{\pi}{2}\log 2 - \frac{\pi}{2}\log 2\right]$$

$$= \pi \log 2$$

Leibnitz Rule of Diff under the Sign of Integration $\longrightarrow$

$$\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(t)\,dt = \boxed{\frac{d}{dx}(\psi)\cdot f(\psi) - \frac{d}{dx}(\phi)\, f(\phi)}$$

eg $I = \dfrac{d}{dx}\displaystyle\int_{x^2}^{x^3}\left(\frac{1}{\log t}\right)dt = ?$

$$= \frac{d}{dx}(x^3)\frac{1}{\log(x^3)} - \frac{d}{dx}(x^2)\frac{1}{\log(x^2)}$$

$$= 3x^2\left(\frac{1}{3\log x}\right) - 2x\left(\frac{1}{2\log x}\right)$$

$$= \frac{x^2 - x}{\log x}$$

Q. Consider the func^n $\boxed{f(x)=\int_0^x e^{-\left(\frac{x^2}{2}\right)}dx}$ & let it's T.S. Exp about $x=0$ is given as $f(x)=a_0+a_1x+\underline{a_2}x^2+a_3x^3+\ldots\ldots$ then $a_2=\underline{?}$

(a) 0

(b) 0.5

(c) 1

(d) 2

In Maclaurin series Exp, Coeff of $x^2=a_2=\frac{f''(0)}{2!}$

$$\frac{d}{dx}f(x)=\frac{d}{dx}\int_0^x e^{-\left(\frac{x^2}{2}\right)}dx=\boxed{\frac{d}{dx}(x)\cdot e^{-\frac{x^2}{2}}-\frac{d}{dx}(0)e^{\frac{0^2}{2}}}=e^{-\frac{x^2}{2}}-0$$

$$f''(x)=\frac{d}{dx}(f'(x))=\frac{d}{dx}\left(e^{-\frac{x^2}{2}}\right)=e^{-\frac{x^2}{2}}\left(-\frac{2x}{2}\right)=-xe^{-x^2/2}$$

$$a_2=\frac{f''(0)}{2!}=0$$

Let $f(x) = \int_0^x e^t (t-1)(t-2)dt$. Then $f(x)$ decreases in the interval.

(a) $x \in (1, 2)$ (b) $x \in (2, 3)$

(c) $x \in (0, 1)$ (d) $x \in (0.5, 1)$

$$f(x) = \int_0^x e^t (t-1)(t-2)dt$$

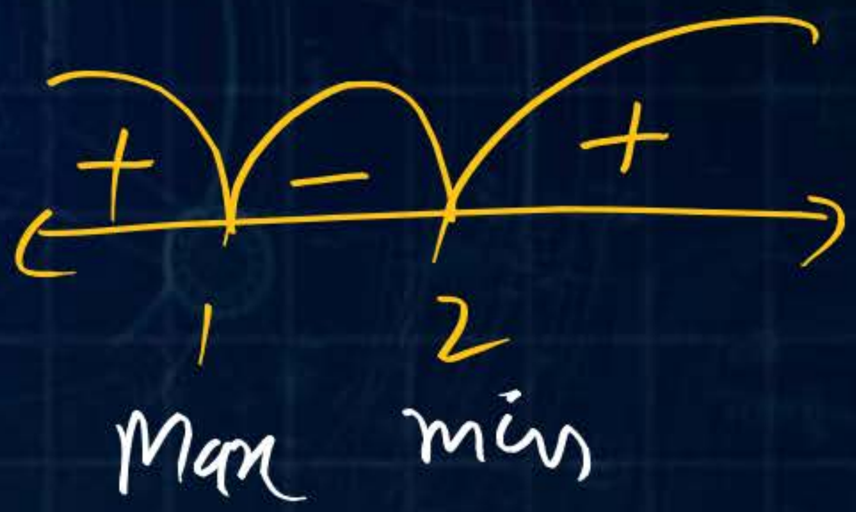
$$f'(x) = \frac{d}{dx}\int_0^x e^t (t-1)(t-2)dt$$

$$= \frac{d}{dx}(x)\left\{e^x(x-1)(x-2)\right\} - \frac{d}{dx}(0)\left\{e^0(0-1)(0-2)\right\}$$

$$= e^x(x-1)(x-2) - 0$$

$$\boxed{f'(x) = e^x(x-1)(x-2)}$$

T. Points are $x = 1$ & 2

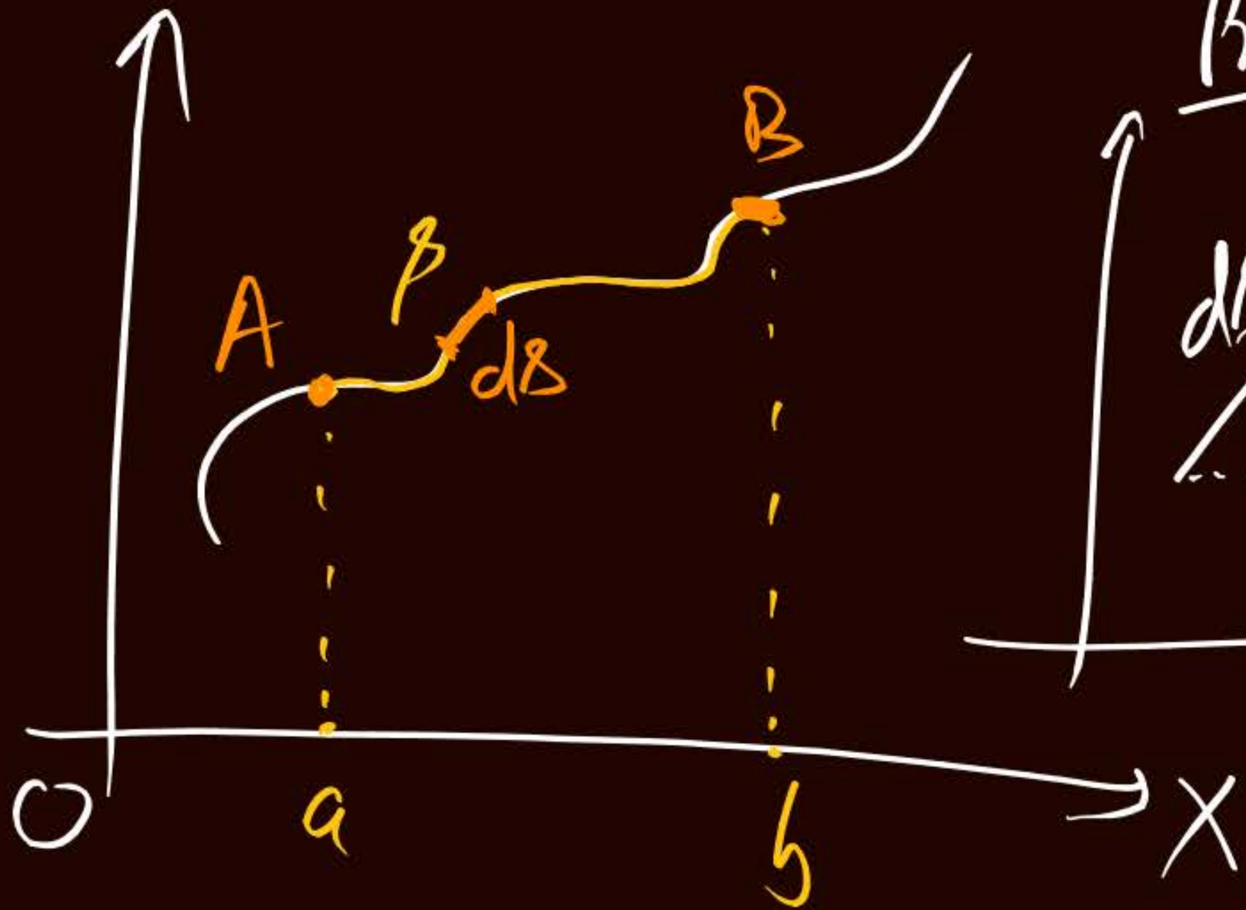


$f(x)$ Dec in $(1, 2)$

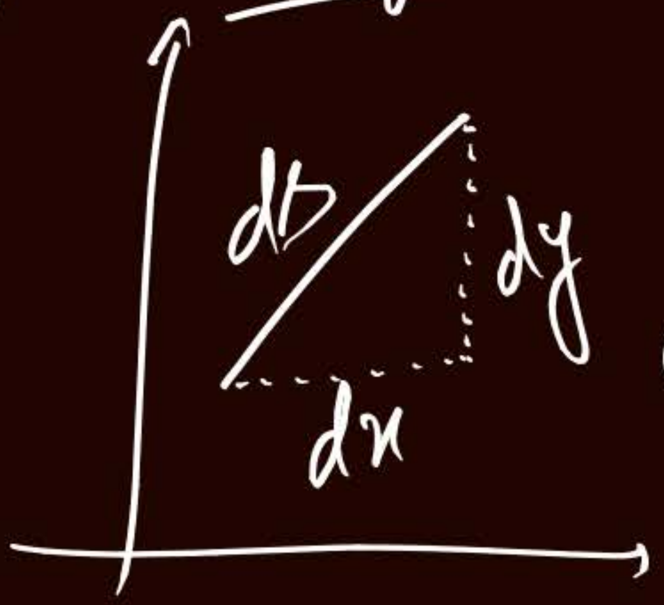
Application of Definite Integration → To find Length of Curve
→ To find Volume of solid formed by Revolution.

Length of Curve: $y=f(x)$ b/n $x=a$ & $x=b$ is

$$\boxed{S=\int_{x=a}^{b}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx}$$



Proof:



$$(ds)^2=(dx)^2+(dy)^2$$

$$(ds)^2=\left[1+\left(\frac{dy}{dx}\right)^2\right](dx)^2$$

$$\int_0^S ds=\int_{x=a}^{b}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\cdot dx$$

$S=$ Hence Prooved.

Case I: length of curve $y = f(x)$ b/n $x = a$ to b is $s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case II: ,, ,, $x = f(y)$ b/n $y = c$ to d is $s = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \cdot dy$

w/o proof

Case III: if $x = x(t)$, $y = y(t)$, $z = z(t)$ then

w/o proof

length of curve b/n t_1 & t_2 is $s = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \cdot dt$

Case IV: if $r = f(\theta)$ then length of curve b/n θ_1 & θ_2 is $s = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \cdot d\theta$

w/o proof

(Not in a syllabus of CS & DSAI)

Q. The length of curve $y = \frac{2}{3} x^{3/2}$ b/w $x = 0$ & 1 is ?____

(a) 1.732

(b) 1.414

(c) 1.22

(d) 3.14

$$\frac{dy}{dx} = \frac{2}{3} \cdot \left[\frac{3}{2} \cdot x^{\frac{3}{2}-1}\right] = x^{1/2} = \sqrt{x}$$

$$S = \int_{x=0}^{1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \int_{x=0}^{1} \sqrt{1+x}\, dx = \left[\frac{(1+x)^{3/2}}{3/2}\right]_{x=0}^{1}$$

$$= \frac{2}{3}\left[2^{3/2} - 1^{3/2}\right] = \frac{2}{3}\left[\sqrt{2^3} - 1\right] = \frac{2}{3}\left[2\sqrt{2} - 1\right]$$

$$= \frac{2}{3}\left[2 \times 1.414 - 1\right] = 0.66\left[2.828 - 1\right] = 0.66 \times 1.828$$

$$= 1.22$$

Consider a spatial curve in three-dimensional space given in parametric form by

$$x(t) = \cos t,\ y(t) = \sin t,\ z(t) = \frac{2}{\pi}t,\ 0 \le t \le \frac{\pi}{2}$$

The length of the curve is ________.

Use Case III

(a) 1

(b) 2

(c) 1.86

(d) 3.14

A parabolic cable is held between two supports at the same level. The horizontal span between the supports is L. The sag at the mid-span is h. The equation of the parabola is $y = 4h\frac{x^2}{L^2}$, where x is the horizontal coordinate and y is the vertical coordinate with the origin at the centre of the cable. The expression for the total length of the cable is

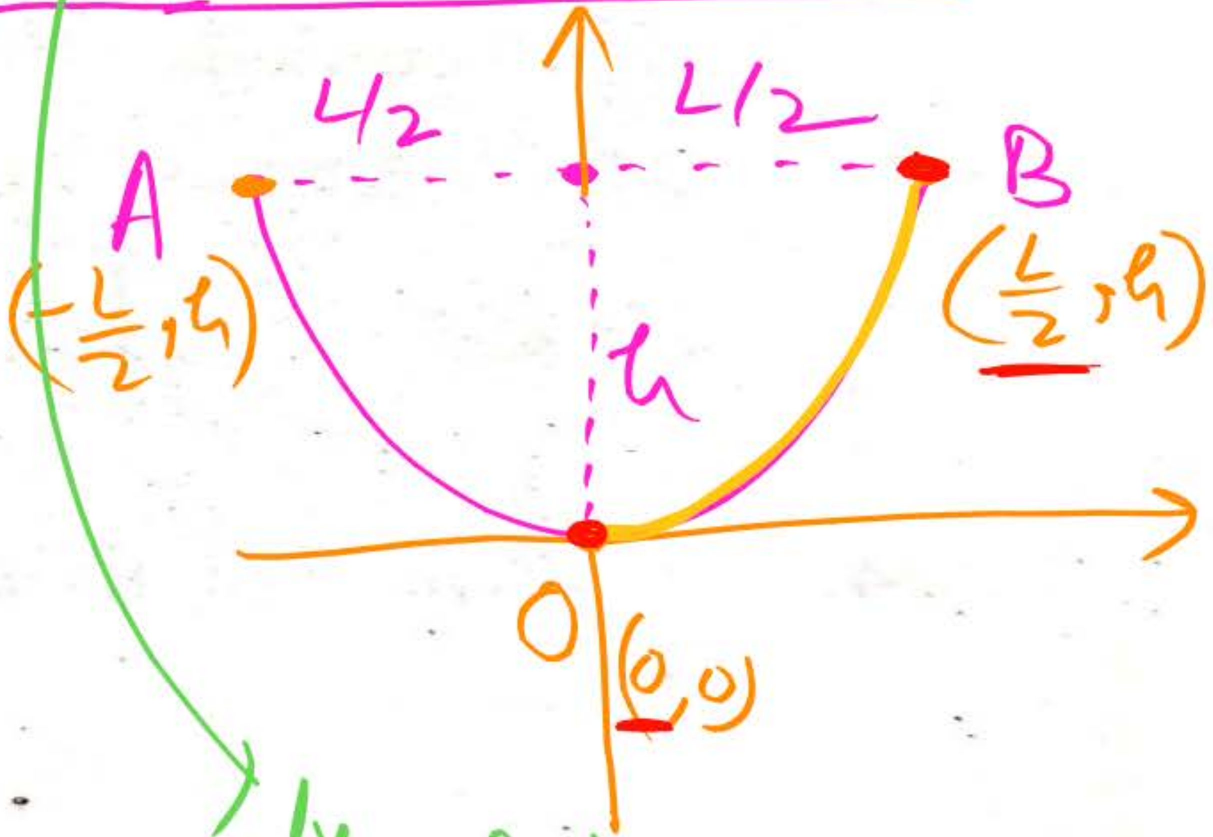
depth

(a) $\int_0^{L} \sqrt{1 + 64\frac{h^2x^2}{L^4}}\,dx$

(b) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^3x^2}{L^4}}\,dx$

(c) $\int_0^{L/2} \sqrt{1 + 64\frac{h^2x^2}{L^4}}\,dx$

(d) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^2x^2}{L^4}}\,dx$



$\frac{dy}{dx} = \frac{8xh}{L^2}$

Total Length AOB

$= 2$ length of OB

$= 2\int_{x=0}^{L/2} \sqrt{1+\left(\frac{dy}{dx}\right)^2} \cdot dx$

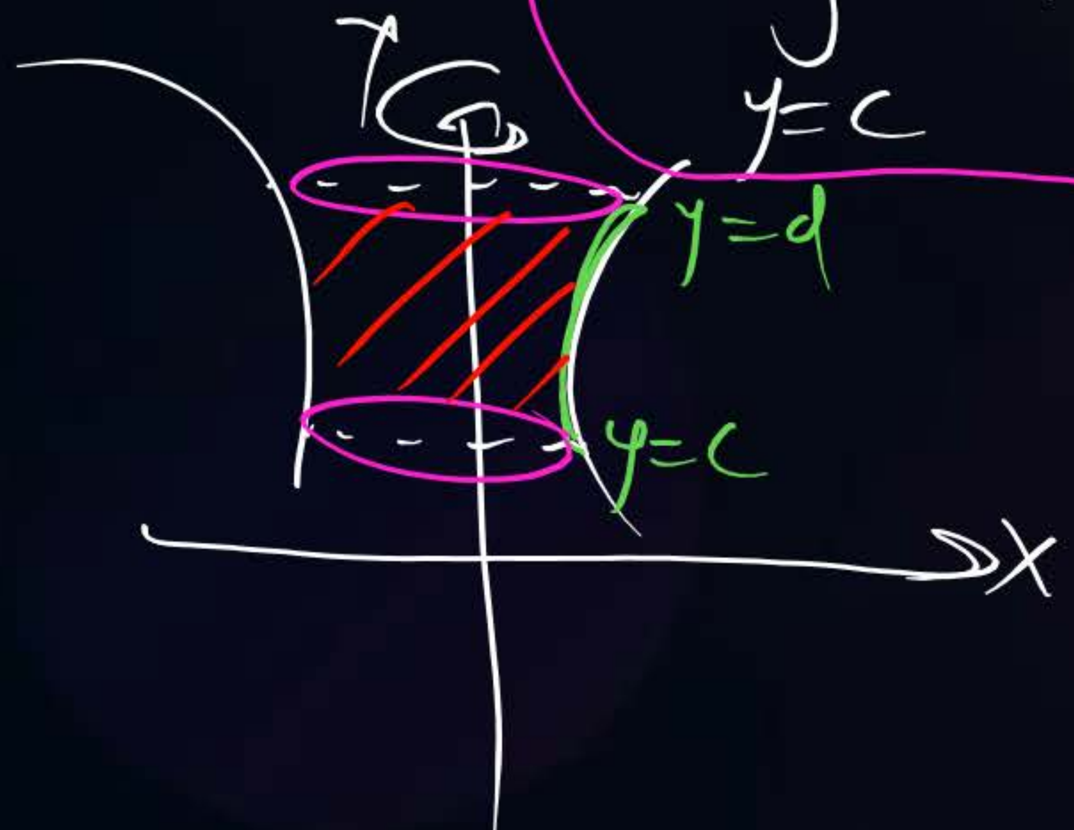
$= 2\int_0^{L/2} \sqrt{1+\frac{64x^2h^2}{L^4}} \cdot dx$

(2) Volume of Solid formed by Revolution →

Case I: (Revolution about Y axis) →

if the curve $x=f(y)$ is to be Revolved about Y axis b/n $y=c$ to d then Volume of Solid formed is

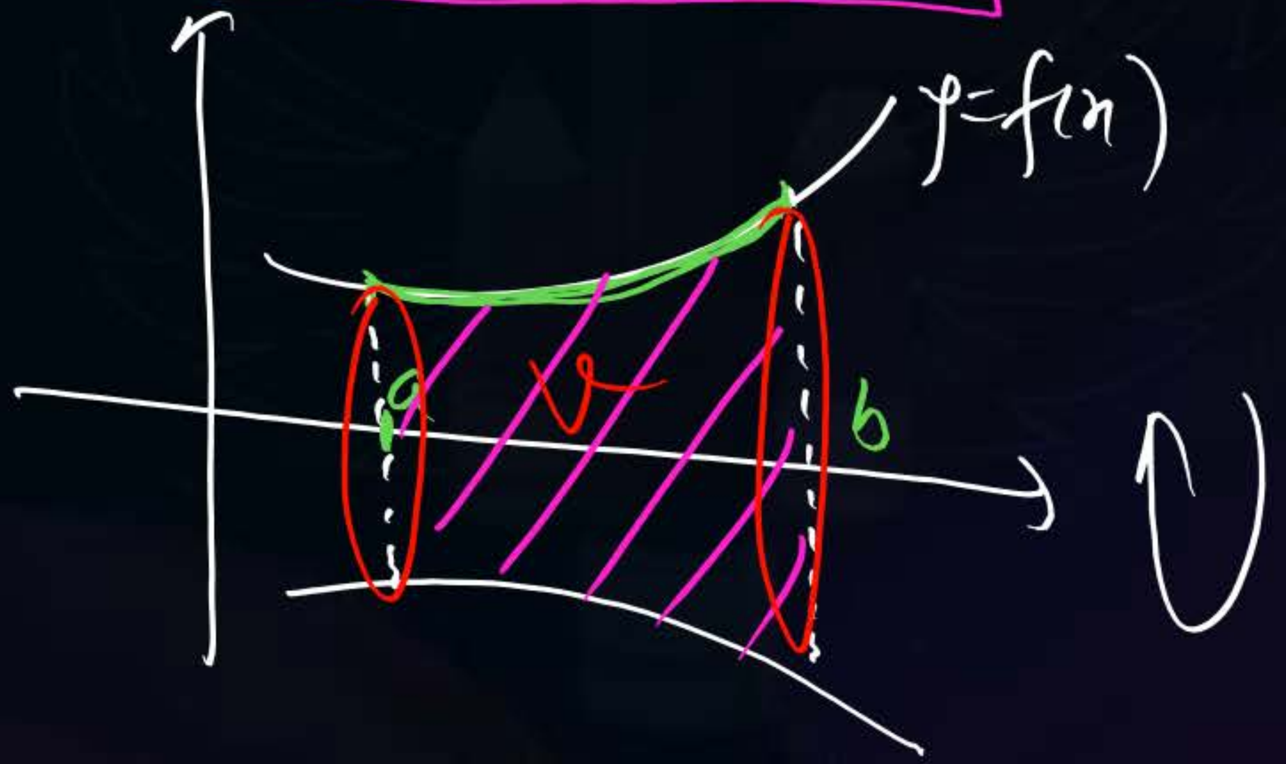
$$\boxed{V=\int_{y=c}^{d}\pi x^2\,dy}$$



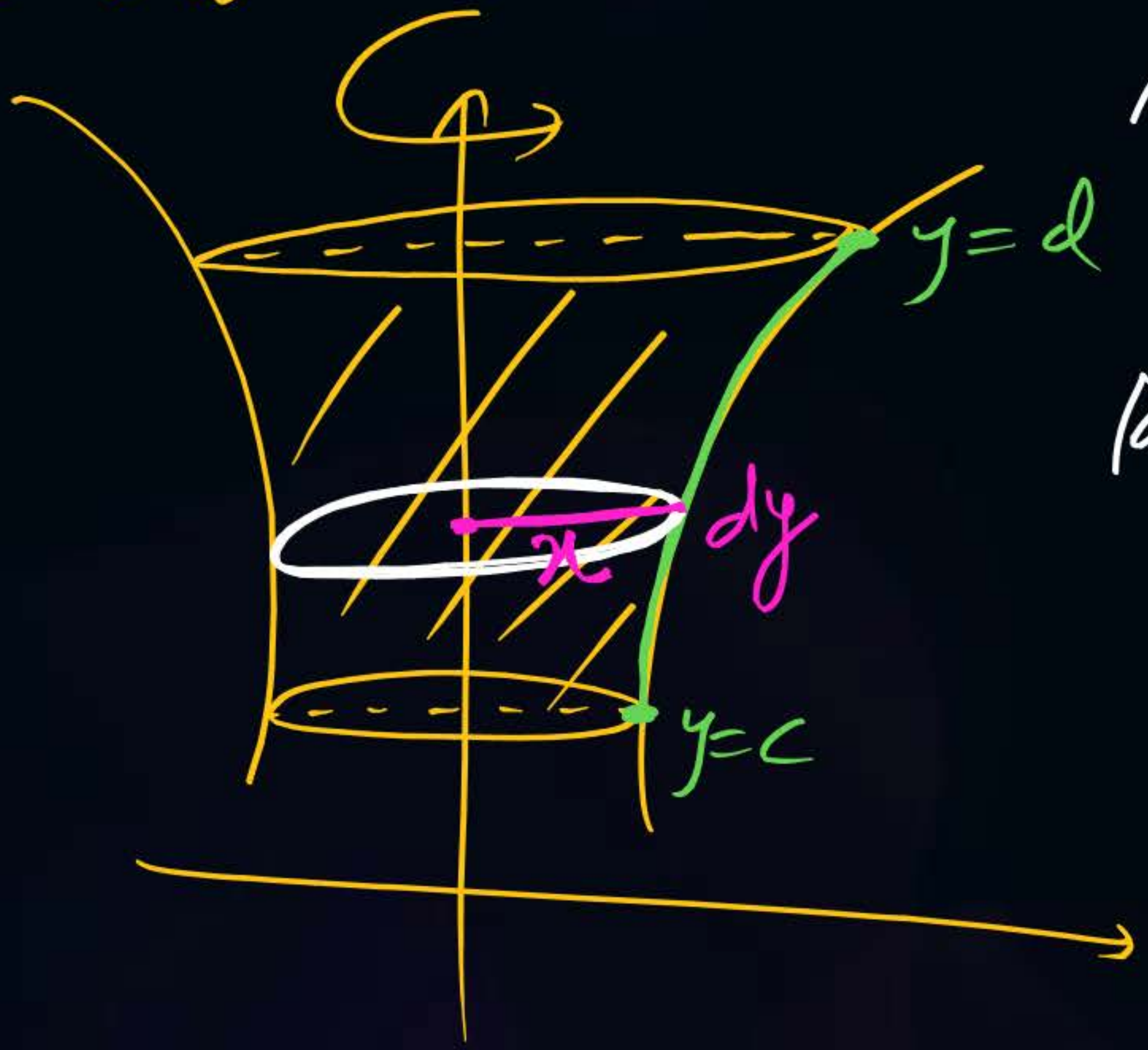
Case II: (Revolution about X axis) →

if the curve $y=f(x)$ is to be Revolved about X axis b/n $x=a$ to b then Volume of solid formed is

$$\boxed{V=\int_{x=a}^{b}\pi y^2\,dx}$$



Proof of Case I:



Consider the small disc as shown by white marker.

so we can assume this disc as a cylinder of Radius x & height dy

so Volume of this disc is

$$dV = \pi x^2 dy$$

$$\int_0^V (1)\,dV = \int_{y=c}^{d} \pi x^2 dy$$

$$V = \int_c^d \pi x^2 dy$$ Hence Proved

Q. Find the Volume of Solid formed by Revolution of the Arc $y=\sqrt{x}$ b/w $x=1$ & 2 about x axis ?

(a) $3\pi/2$

(b) $\pi/2$

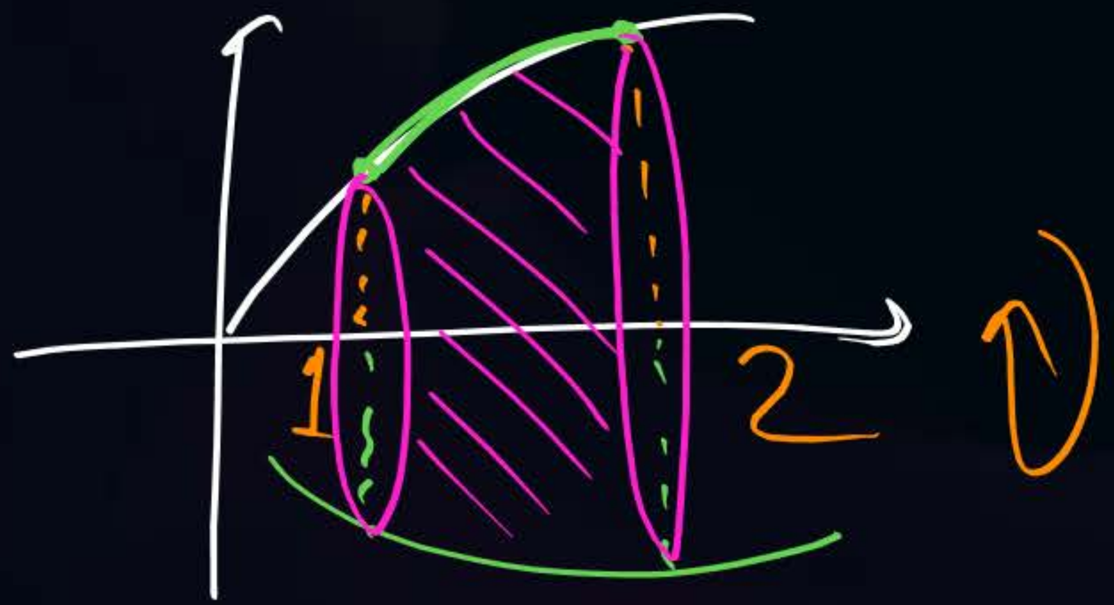
(c) $3/2$

(d) 3π

$$\text{Req Volume} = \int_{x=a}^{b} \pi y^2 \, dx = \int_{x=1}^{2} \pi (\sqrt{x})^2 \, dx = \pi \int_{1}^{2} x \, dx$$

$$= \frac{3\pi}{2}$$

M-II $y=\sqrt{x}$



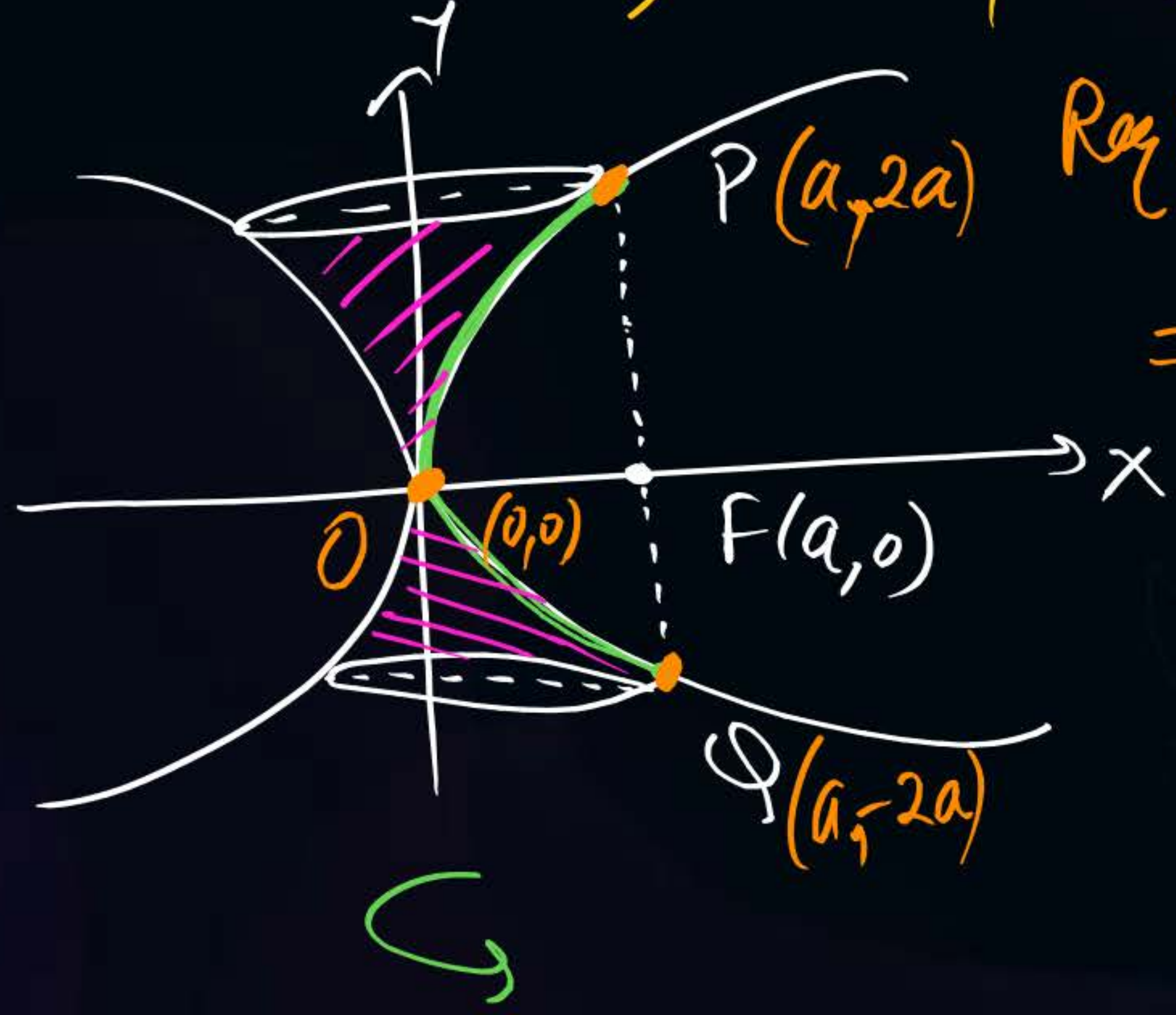
Q: Find the Volume of Reel Shaped Solid formed by Revolution of arc of the $y^2=4ax$ (cut off by it's latus Rectum) about Y axis?

(a) $32\pi a^3$

(b) $\frac{16\pi a^3}{5}$

(c) $\frac{32}{5}\pi a^3$

(d) $\frac{4}{5}\pi a^3$



Req Volume = 2 Volume of above portion.

$$= 2\int_{y=0}^{2a} \pi x^2\, dy = 2\pi \int_{y=0}^{2a} \left(\frac{y^2}{4a}\right)^2 dy$$

$$= \frac{2\pi}{16a^2}\left(\frac{y^5}{5}\right)_0^{2a} = \frac{\pi}{8\times 5a^2}\left[2^5 a^5 - 0\right]$$

$$= \frac{4\pi a^3}{5}$$

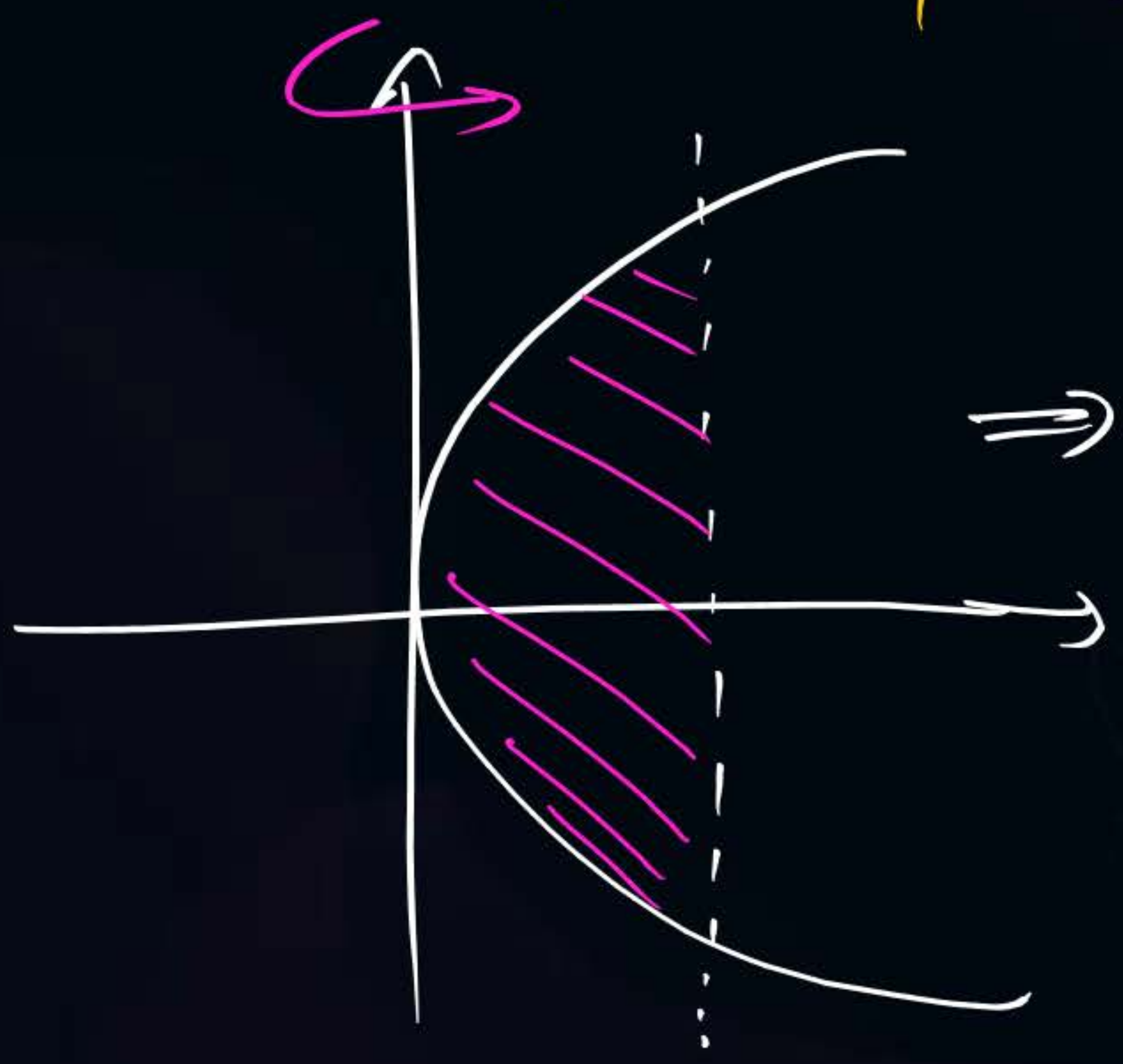
Q: Find the volume of Solid formed by Revolution of area of the $y^2 = 4ax$ (cut off by it's latus Rectum) about Y axis?

(a) $\frac{4\pi}{5}a^3$

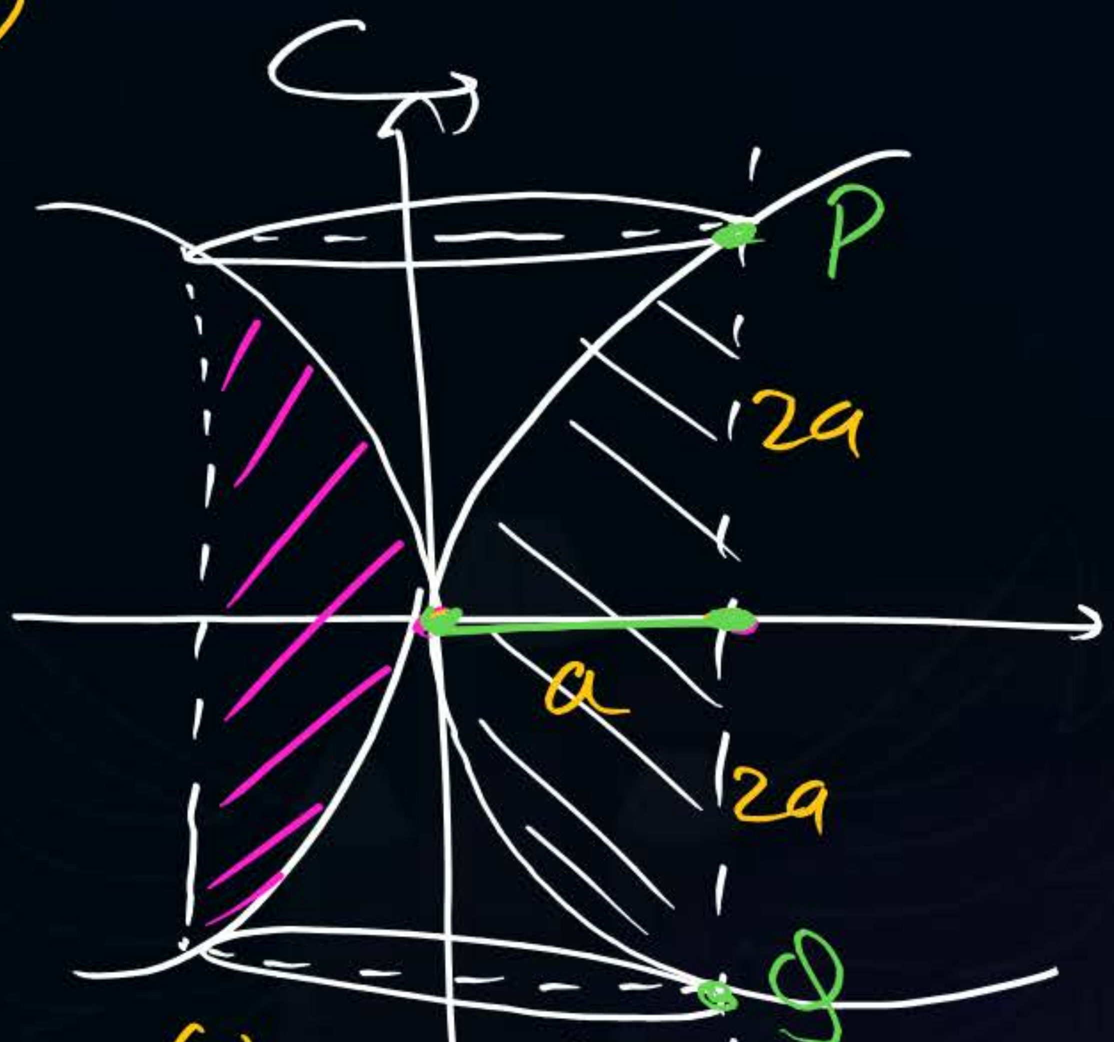
(b) $\frac{2\pi}{5}a^3$

(c) $\frac{128\pi}{5}a^3$

(d) $\frac{16\pi a^3}{5}$



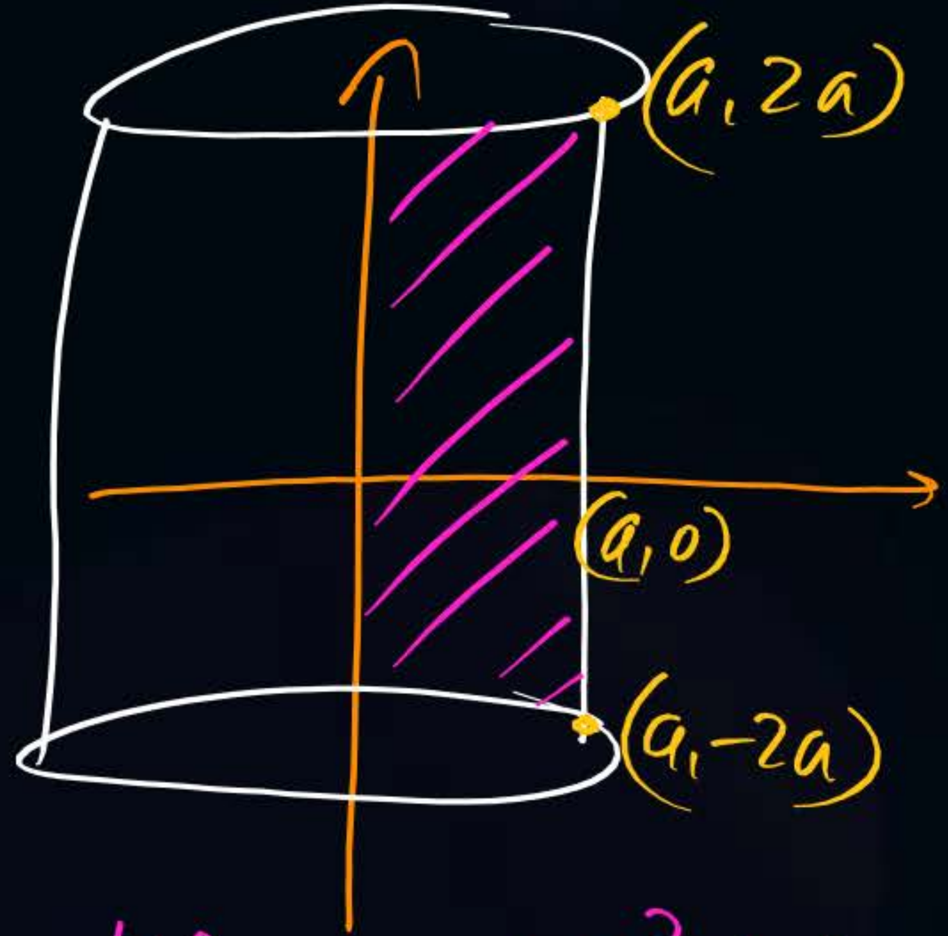
$\Longrightarrow$

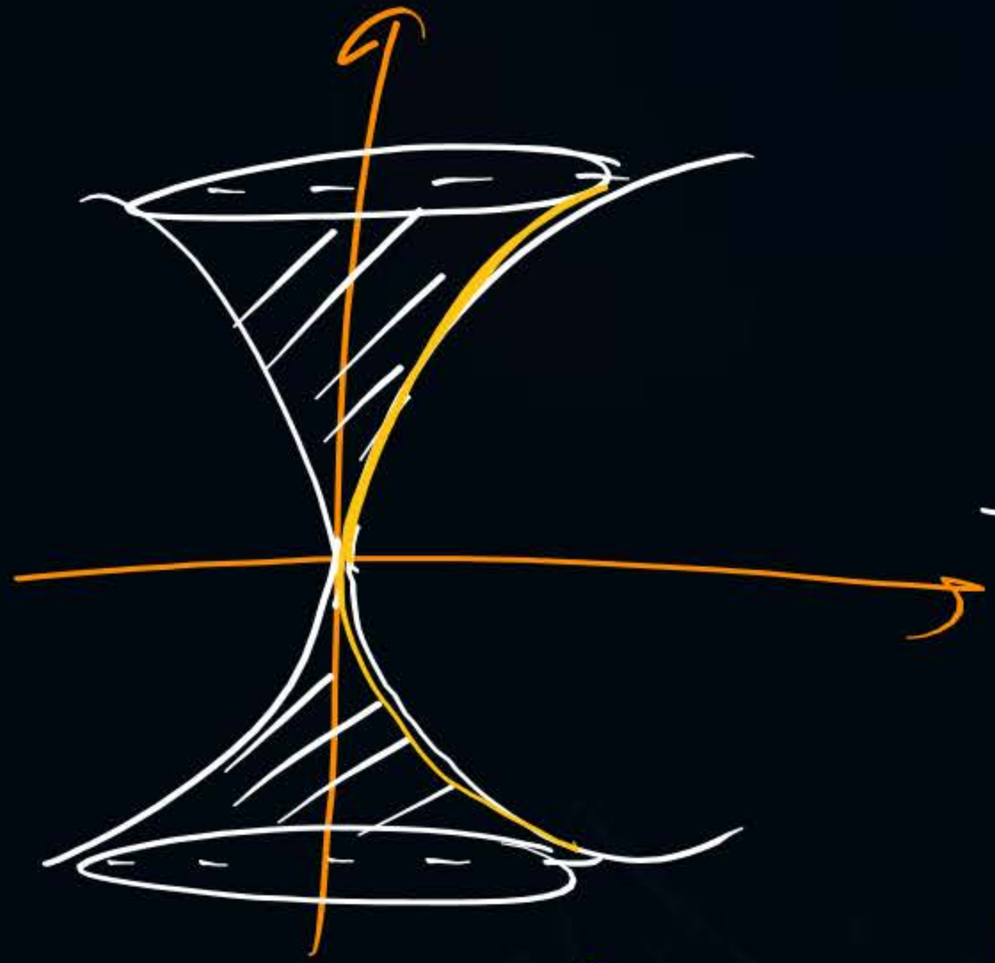


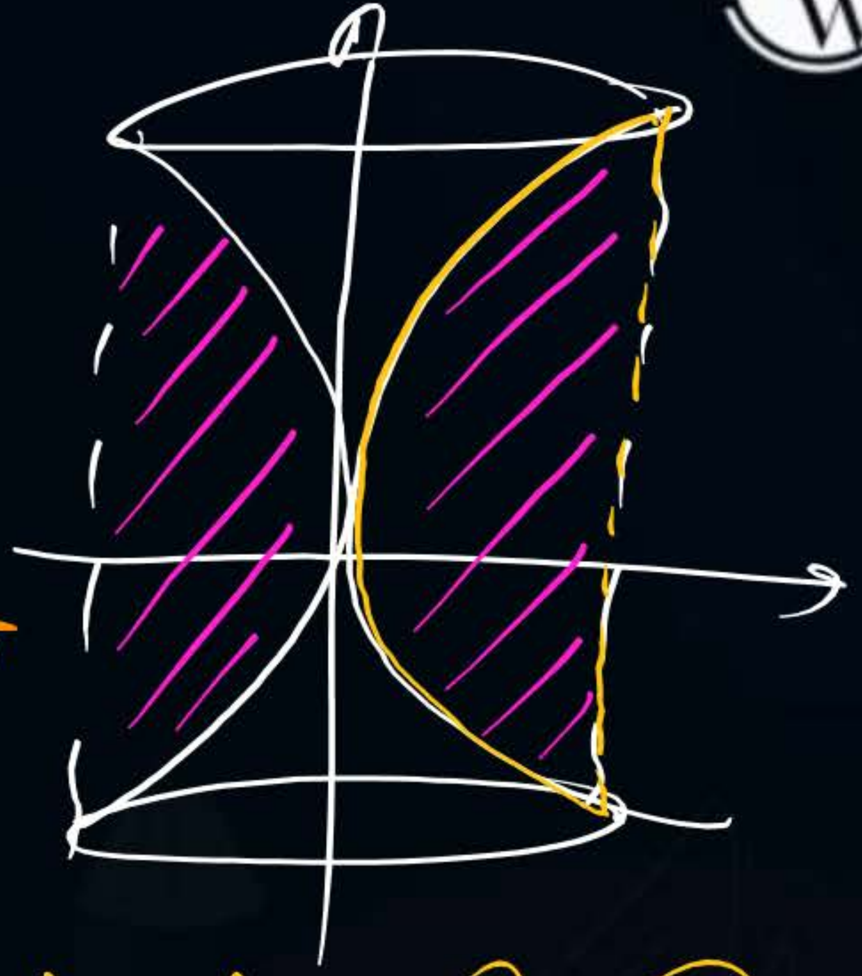
Cylinder with Cavity

$r = a$ & $h = 2a$

Explanation :-







$$\text{Volume} = \pi(a)^2(2a) \quad \text{--- (1)}$$

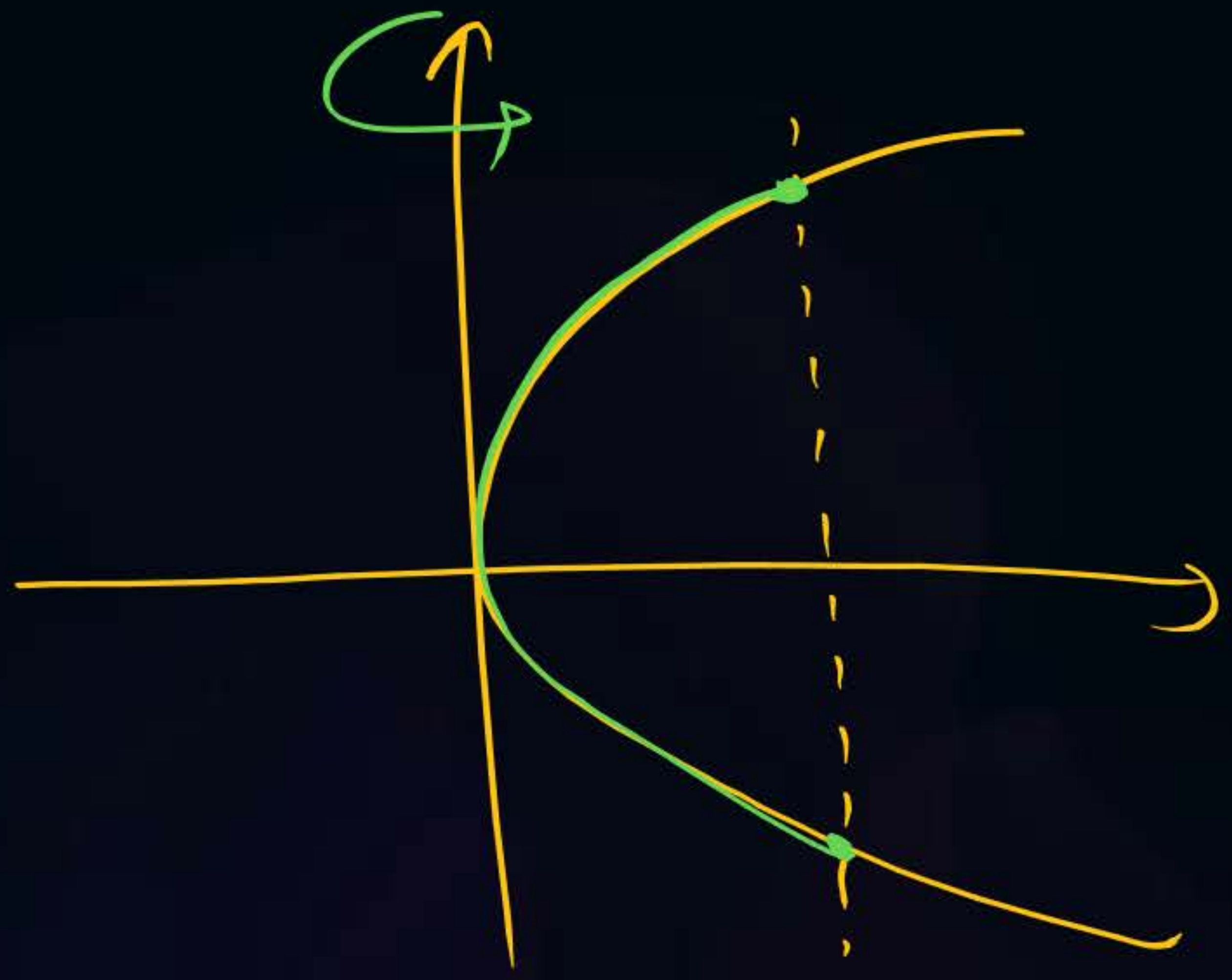
$$\text{Volume} = \frac{4\pi a^3}{5} \quad \text{--- (2)}$$

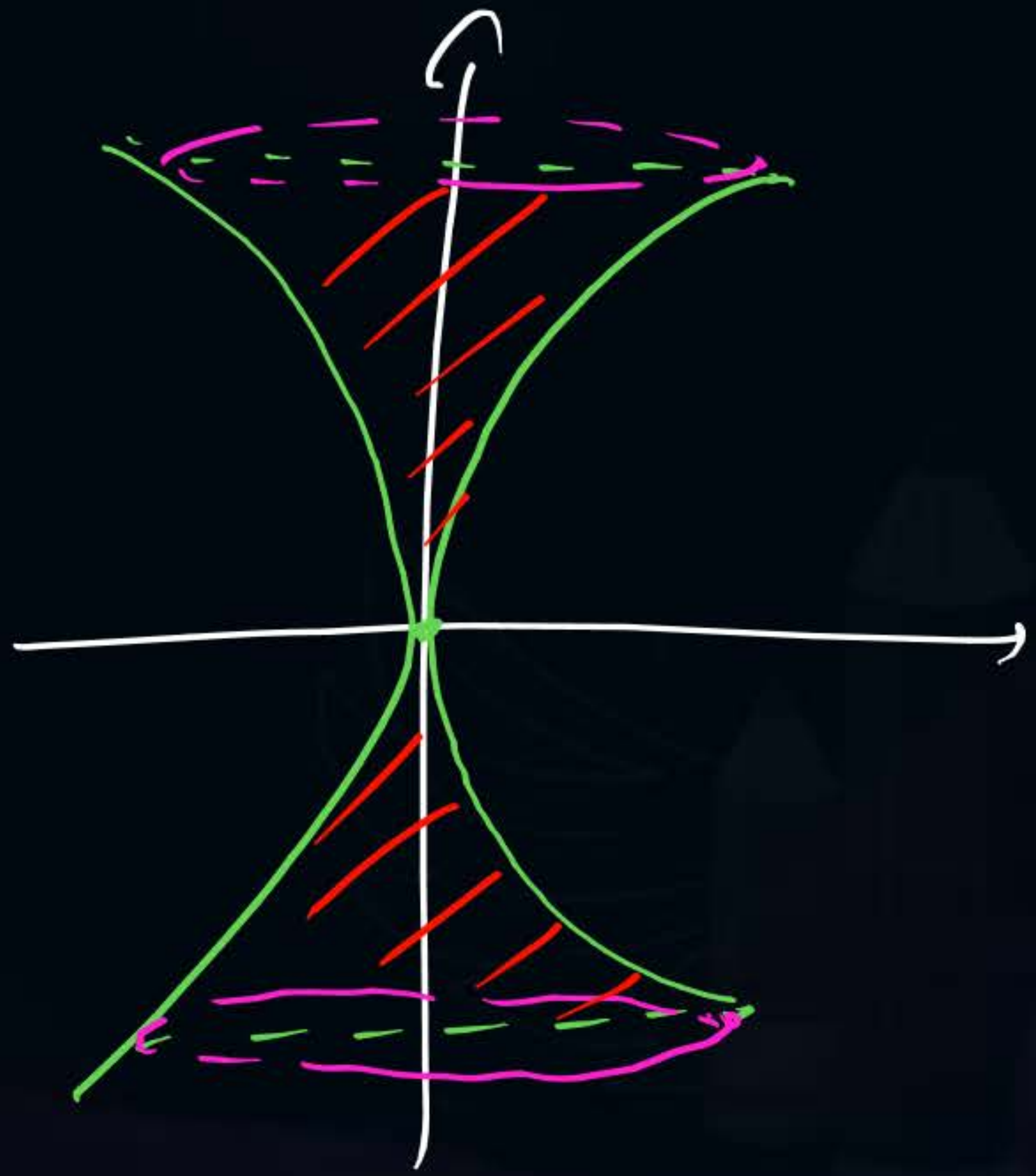
$$\text{Req Volume} = (1) - (2)$$

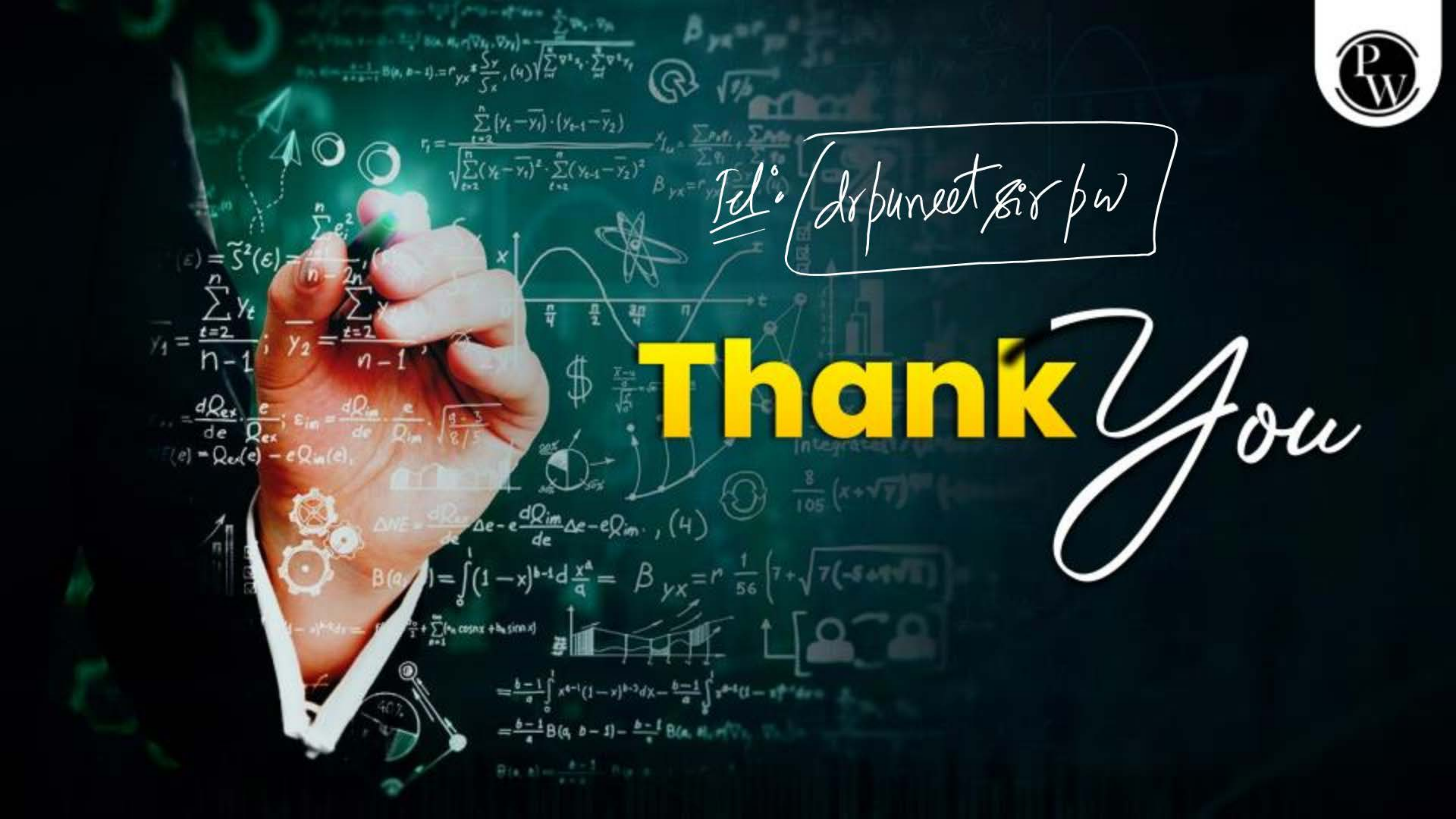
$$= \frac{16}{5}\pi a^3$$

$y^2 = 4ax$, LR $x=a$

length of LR $= 4a$



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Thank You