

# CS & IT ENGINEERING



**THEORY OF COMPUTATION**

✓ Grammar

Lecture - 03



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# Recap of Previous Lecture



Topic

?????

Grammar Construction

Grammar  $\Rightarrow$  Language

Types of Grammar

$F.A \Leftrightarrow \text{Regular grammar} \Leftrightarrow \text{Regular Expression}$

Ambiguous Grammar





# Topics to be Covered



SPDA?



Topic

Ambiguous Grammar

Topic

?? Simplification of grammar

Topic

?? Normal form of grammar

Topic

?? Decision Properties of Grammar



# Ambiguous Grammar

Ex:-

$S \rightarrow AaAb \mid BbBa$

$A \rightarrow \epsilon$

$B \rightarrow \epsilon$

$= \{ab, ba\}$

{Unambiguous Grammar}

ab



ba





Ambiguous Grammar

②

$S \rightarrow AB|BC$

$A \rightarrow BA|a$

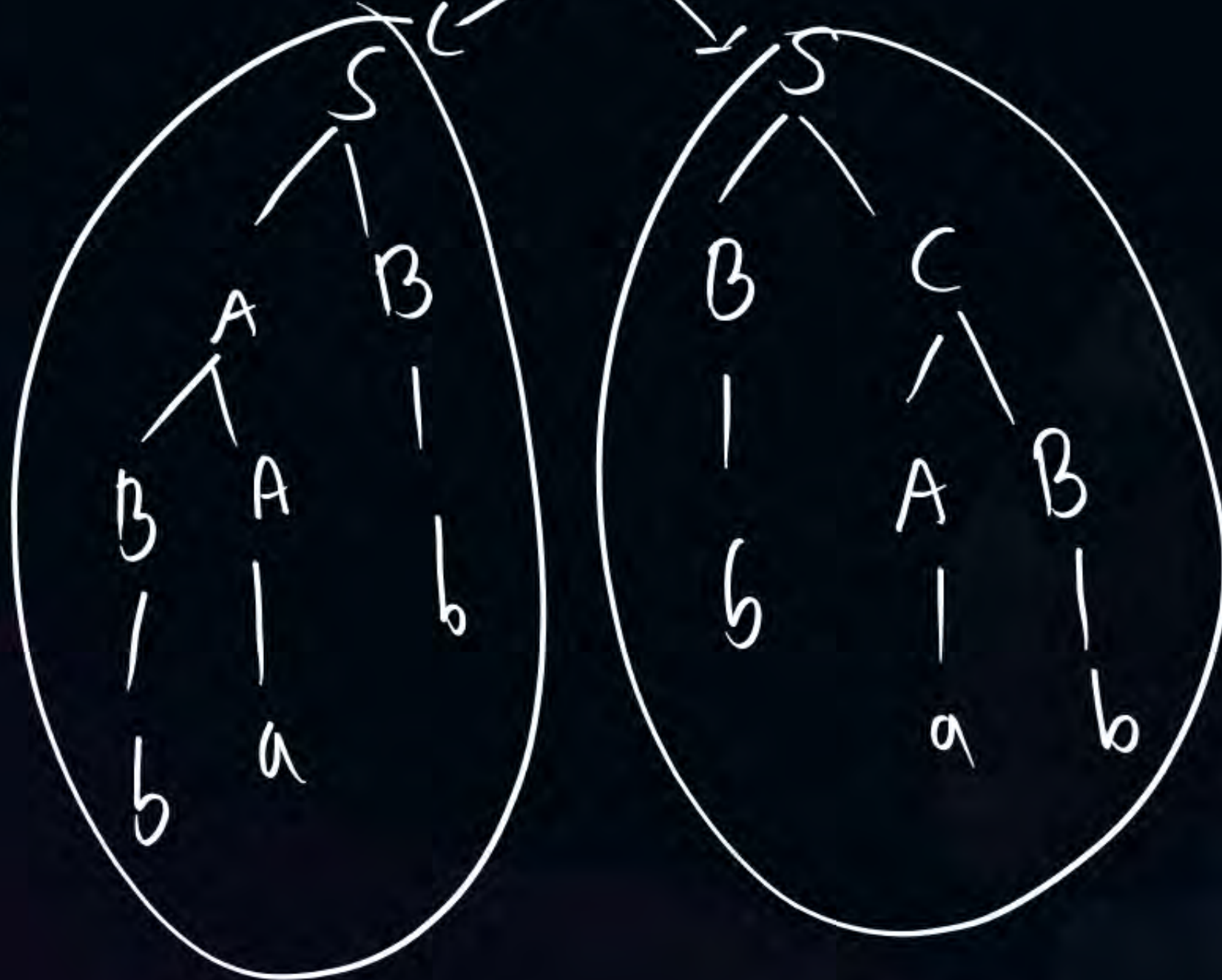
$B \rightarrow CC|b$

$C \rightarrow a|AB$

Ambiguous or not?

Yes

no





① { Ambiguity checking is Undecidable Problem }

② Eliminating ambiguity is also Undecidable Problem





# C.F.G ( $A \rightarrow \alpha$ ) $\alpha \in (V+T)^*$

① G, not regular CFG

$S \rightarrow AB$   
 $A \rightarrow a$   
 $B \rightarrow b$

~~regular~~  
 $L = \{ab\}$

②  $S \rightarrow aSb/ab$

$L = \{a^n b^n \mid n \geq 1\}$

non regular

③ {CFG}

$S \rightarrow AB/BC$   
 $A \rightarrow BA/a$   
 $B \rightarrow CC/b$   
 $C \rightarrow AB/a$

$= \text{Language} = \text{Regular or not?}$

Regularity Problem

Undecidable Problem



# Simplification of CFG

Removal of

① useless variable (N.T)

② Unit Productions ( $A \rightarrow B$ )

③ Null Productions ( $A \rightarrow \epsilon$ )



# Unit Production (A $\rightarrow$ B)

- ①  $S \rightarrow A$
- ②  $A \rightarrow \textcircled{B}$
- ③  $B \rightarrow \textcircled{C}$
- ④  $C \rightarrow \textcircled{\underline{D}}$
- $D \rightarrow a$

$S \rightarrow a$   
 $\gamma A \rightarrow a$   
 $\gamma B \rightarrow a$   
 $\gamma C \rightarrow a$   
 $\gamma D \rightarrow a$

$S \rightarrow a$

4



# Null Productions ( $A \rightarrow \epsilon$ )

① Null



② Unit



③ When Variable

$S \rightarrow AaB$

$A \rightarrow a | \epsilon$

$B \rightarrow b | \epsilon$

$S \rightarrow \underline{A}a\underline{B} | Aa | aB | a$

$A \rightarrow a$

$B \rightarrow b$



## Elimination of useless Variable

Remove

① Any Variable not generating string

(useless  
variable)

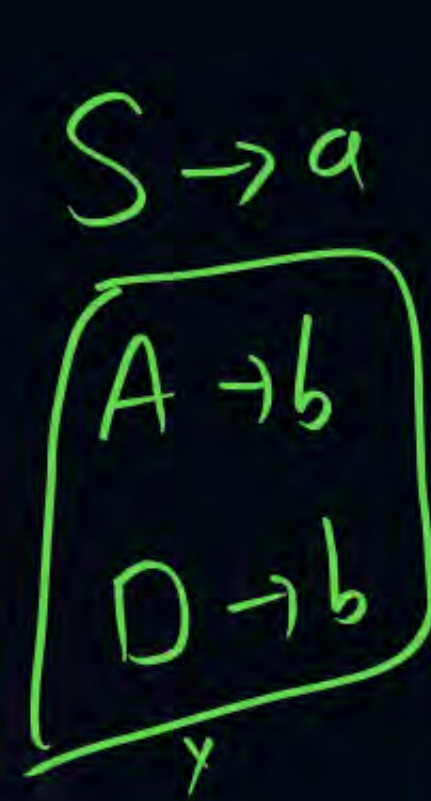
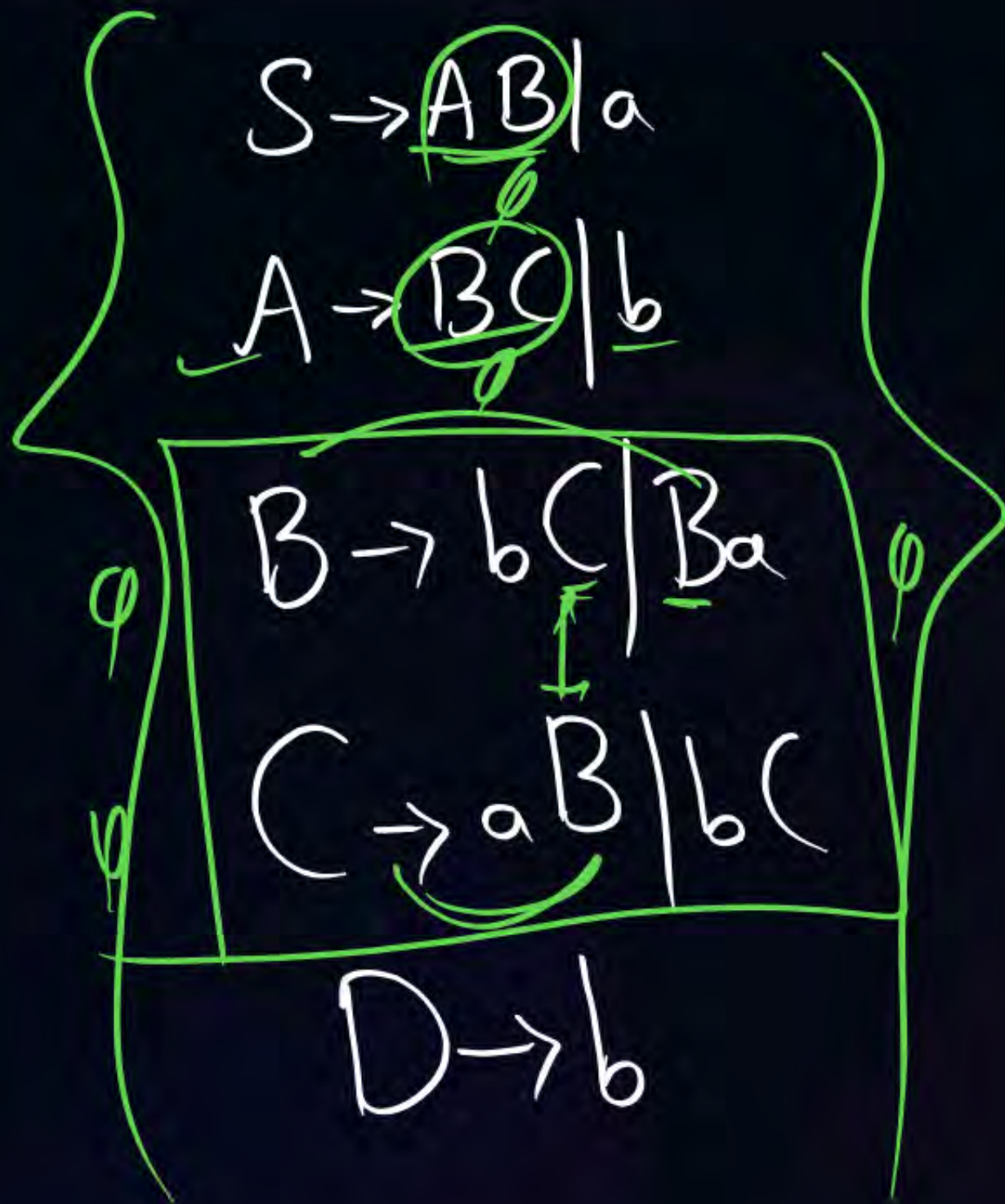
② Any variable not required for derivation







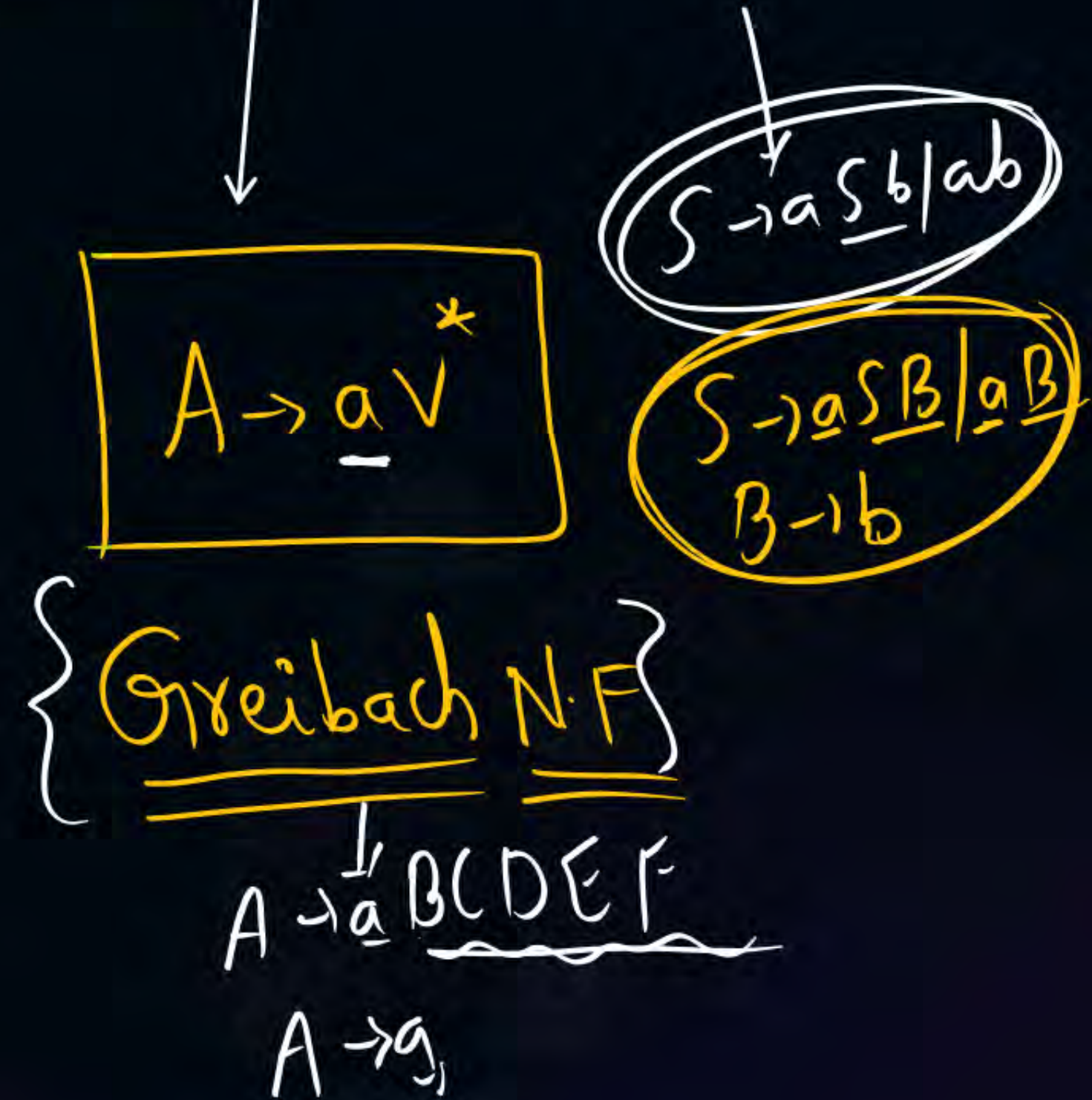
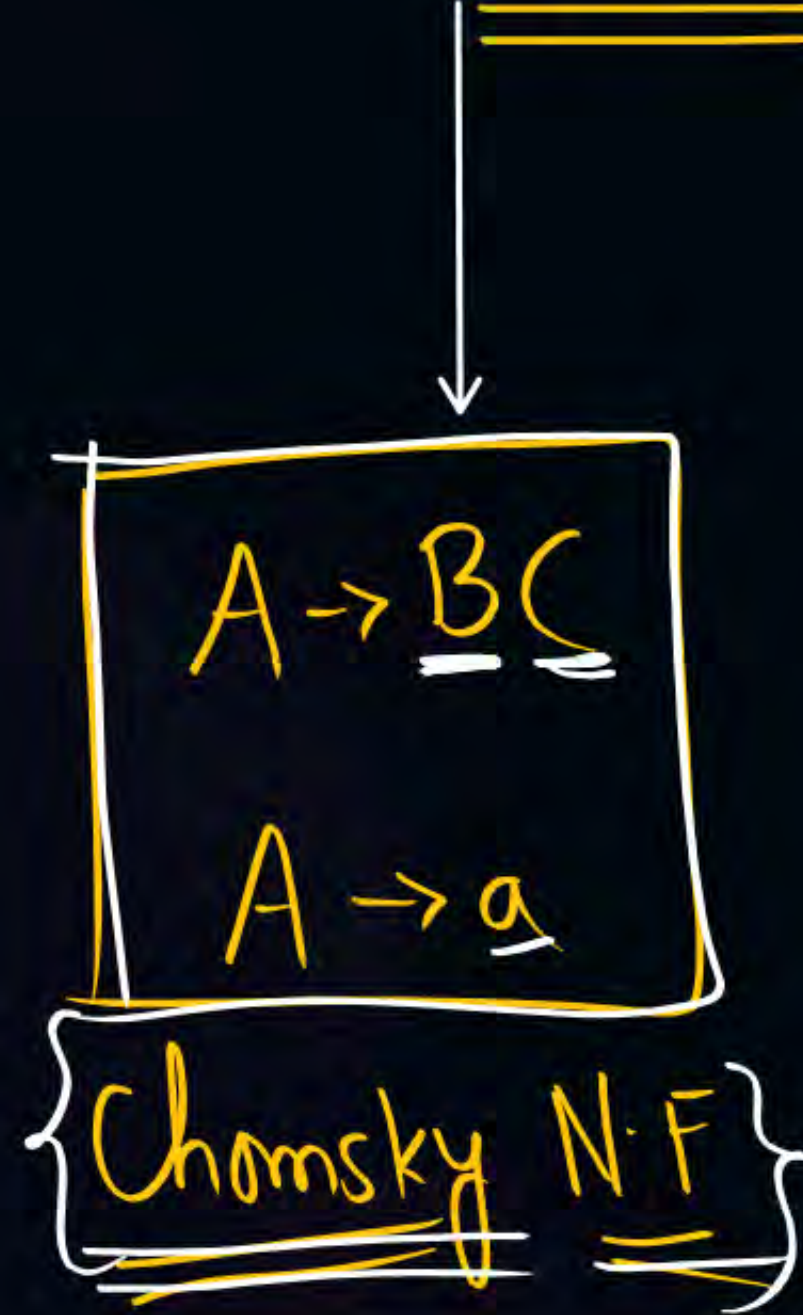
④ How many useless variables present in following grammar



$\{A, B, C, D\}$



# Normal form of CFG ( $A \rightarrow \alpha$ )





(Q) Convert following Grammar into CNF Grammar?

$$S \rightarrow \underline{a} \underline{S} \underline{b} / \underline{a} \underline{b}$$

①

$$S \rightarrow \boxed{ASB} / \underline{AB}$$

$$A \rightarrow a \checkmark$$

$$B \rightarrow b \checkmark$$

②

$$S \rightarrow ASB$$


---


$$\boxed{S \rightarrow AX / AB}$$

$$X \rightarrow SB$$

$$A \rightarrow a$$

$$B \rightarrow b$$

① Convert total R.H.S into only N.T

by assuming terminals as new N.T

$$S \rightarrow AX / AB$$

$$X \rightarrow SB$$

$$A \rightarrow a \checkmark$$

$$B \rightarrow b$$



$\boxed{aabb}$



(Q) Convert following Grammar into CNF Grammar?

$$\begin{aligned} S &\rightarrow \underline{b}A | a\underline{B} \\ A &\rightarrow b\underline{A}A | a\underline{S} | a \\ B &\rightarrow aBB | b\underline{S} | b \end{aligned}$$

①

$$\begin{aligned} S &\rightarrow \underline{\gamma}A | \underline{x}B \\ A &\rightarrow \underline{\gamma}A\underline{A} | \underline{x}\underline{S} | a \\ B &\rightarrow \underline{x}B\underline{B} | \underline{\gamma}\underline{S} | b \end{aligned}$$

$$\begin{aligned} X &\rightarrow a \checkmark \\ \gamma &\rightarrow b \checkmark \end{aligned}$$

②

$$\begin{aligned} S &\rightarrow \gamma A | xB \\ A &\rightarrow \gamma P | xS | a \\ P &\rightarrow AA \\ B &\rightarrow xQ | \gamma S | b \\ Q &\rightarrow BB \end{aligned}$$

③

$$\begin{aligned} X &\rightarrow a \\ \gamma &\rightarrow b \end{aligned}$$



## NOTE

$$\begin{matrix} (2 \times 4 - 1) \\ (8 - 1) = 7 \end{matrix} \quad \textcircled{aabb}$$

- ① For generating  $n$  length string from CNF grammar total number of productions required is  $\boxed{2n-1}$   
(Derivation steps)
- ② For generating  $n$  length string from GNF grammar total number of productions required is  $\boxed{n \text{ only.}}$



(Q) Convert following Grammar into CNF Grammar?





## Decision Properties of CFG

Decidable

① Emptyness Problem [if  $L(CFG) = \emptyset$ ?

② Finiteness Problem [if  $L(CFG) = \text{finite}$ ?

③ Membership Problem [if  $w \in CFG$ ?



$$\textcircled{1} \left\{ \begin{array}{l} S \rightarrow \underline{AB} - \phi \\ A \rightarrow a \\ B \rightarrow \underline{bB} - \phi \end{array} \right\} \begin{array}{l} \text{empty (a) not?} \\ \underline{\text{empty}} \end{array}$$



Thank You!