DATA SCIENCE
&
ARTIFICIAL INTELLIGENCE
& CS/IT

Calculus and Optimization

Lecture No. 11

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Recap of previous lecture

Topic: Maxima - Minima (Part 2)

Topics to be Covered

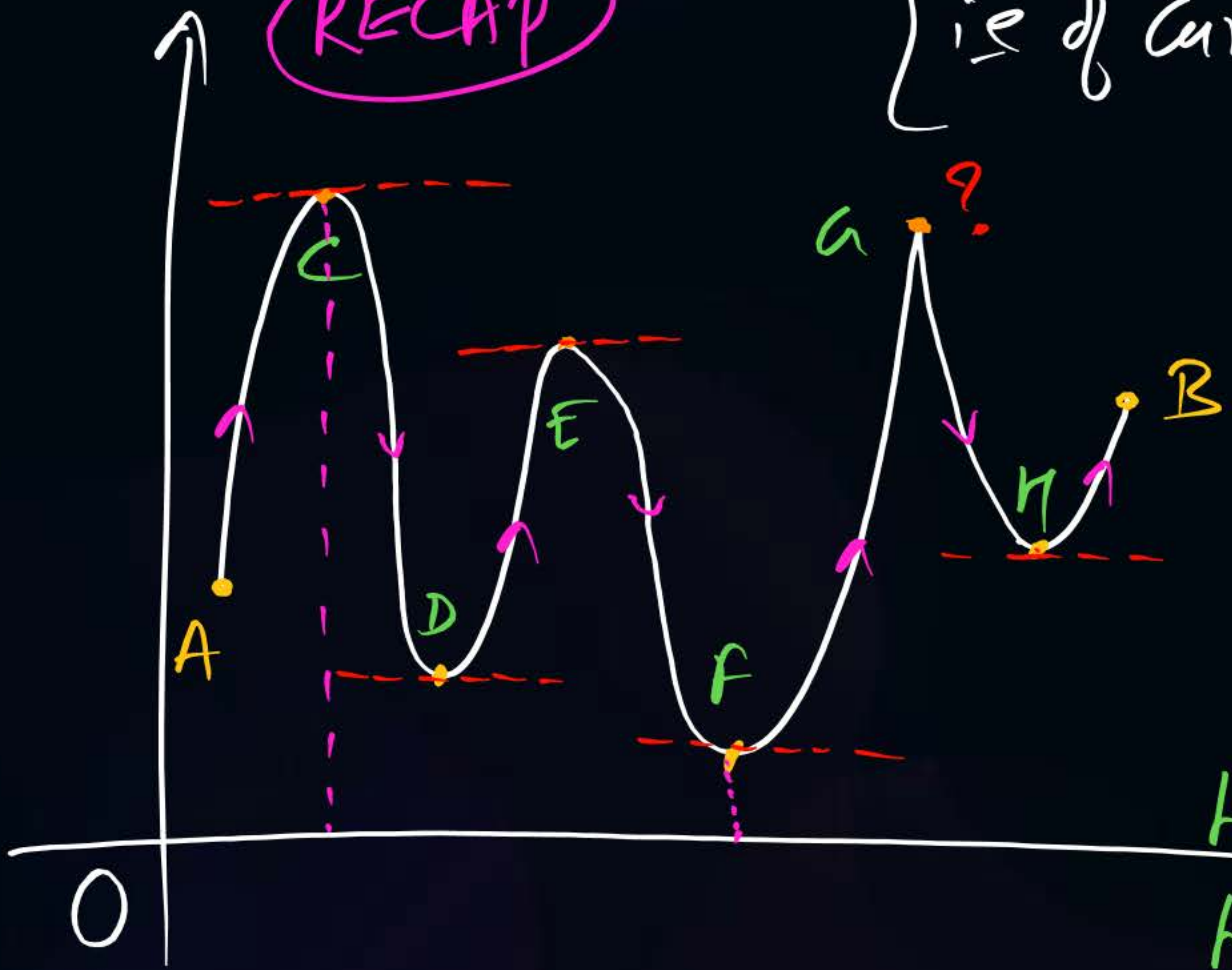
Topic

1. Maxima-Minima of Surface
2. Application Based Questions of Max-Minima
3. Integration (Part 1)

MAXIMA-MINIMA of func^n of Single Variable

[i.e. of curve $y = f(x)$.] → 12^{th} class.

RECAP



Point ≃ x

Value ≃ y

Local Max points → C, E, G, B

Local Max Values → $f(C), f(E), f(G), f(B)$

Absolute Maxima / Global → $f(C)$

Local min Points → A, D, F, H

Local Min Values → $f(A), f(D), f(F), f(H)$

Absolute Minima / Global: $f(F)$

Q: The Maximum slope of $y = x^3 - 9x^2 + 24x + 5$ will be?

HW

(a) 2

(b) -3

(c) 4

(d) DNE

Sol: $f(x) = x^3 - 9x^2 + 24x + 5 \longrightarrow (1)$

$f'(x) = 3x^2 - 18x + 24 \longrightarrow (2)$

so we will try to find the Max value of $f'(x)$

Let $f'(x) = g(x) = 3x^2 - 18x + 24$

$g'(x) = 6x - 18 = \boxed{6(x-3)}$

T. Point of $g(x)$ is $x = 3$

i.e. At $x = 3$, we will get minima

& Min Value $= g(3) = (3x^2 - 18x + 24)_{x=3}$

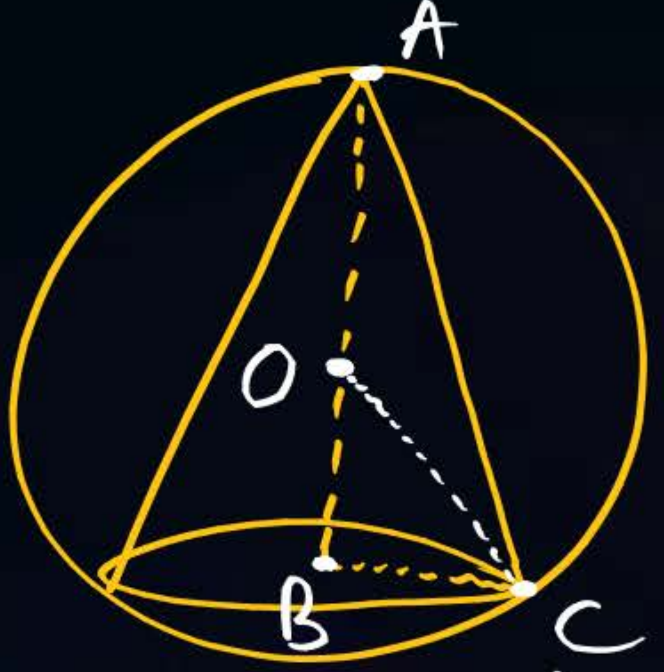
i.e. Min slope of $f(x) = -3$ Ans

& Max Value of $g(x) =$ DNE

i.e. Max slope of $f(x) =$ DNE Ans

Qs (Type II) A Right Circular Cone of Maximum Volume is to be inscribed in a Sphere of Radius 1 mtr then find the Height of such Cone.

(Application Based Questions)



given, $OC = OA = 1$ mtr

let $AB = h$

$BC = r$

In ΔOBC,

$$OC^2 = OB^2 + BC^2$$

$$1^2 = (h-1)^2 + r^2$$

$V = \frac{1}{3}\pi r^2 h$ {i.e. $V = f(r, h)$ Surface}

$$V = \frac{\pi}{3}\left[1-(h-1)^2\right]\cdot h$$

$$= \frac{\pi}{3}\left[1-(h^2+1-2h)\right]h$$

$$V = \frac{\pi}{3}\left[2h^2-h^3\right]$$ {Now $V = f(h)$ curve}

for Max. Volume, $\frac{dV}{dh} = 0$ .... $\Rightarrow h = 0, \frac{4}{3}$

Neglect $h=0$ so Ans is $h = 4/3$.

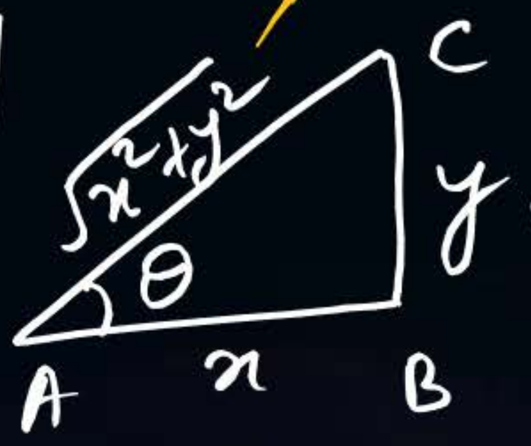
Q. In a Right angle Δ, if sum of the Hypotenuse & one side is
2015 kept constant, in order to find Maximum area then find the angle b/w them?

(a) 30°

(b) 60°

(c) 45°

(d) 90°



$\theta = ?$

ATQ: $x+\sqrt{x^2+y^2}=K$

$x^2+y^2=(K-x)^2$

$x^2+y^2=K^2+x^2-2Kx$

$y^2=K^2-2Kx$ —— (1)

$A=\frac{1}{2}xy$ $\{A=f(x,y)$ surface$\}$

$A^2=\frac{1}{4}x^2y^2=\frac{1}{4}x^2\left[K^2-2Kx\right]$

Let $A^2=u=\frac{K^2}{4}x^2-\frac{K}{2}x^3$ $\{$Now $u=f(x)$ curve$\}$

where $A=\sqrt{u} \Rightarrow A_{max}=\sqrt{u_{max}}=?$

Now for Max u, $\frac{du}{dx}=0$

$$U = \frac{K^2}{4}x^2 - \frac{K}{2}x^3 \Rightarrow \frac{du}{dx} = \frac{K^2}{2}x - \frac{3K}{2}x^2$$

for Max U, $\frac{du}{dx} = 0 \Rightarrow \frac{K^2}{2}x - \frac{3K}{2}x^2 = 0$

$$\frac{Kx}{2}(K - 3x) = 0 \Rightarrow x = \frac{K}{3}$$

Now $y^2 = K^2 - 2Kx = K^2 - 2K\left(\frac{K}{3}\right) = \frac{3K^2 - 2K^2}{3} = \frac{K^2}{3} \Rightarrow y = \frac{K}{\sqrt{3}}$

In $\triangle ABC$, $\tan\theta = \frac{y}{x} = \frac{K/\sqrt{3}}{K/3} = \sqrt{3} \Rightarrow \theta = 60^\circ$

Note: $(\sin\theta, \cos\theta, \tan\theta) = \frac{P.\ B.\ P}{H\ H\ B}$

A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation y $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

$y_{max} = ?$

(a) 8 meters
(b) 10 meters
(c) 12 meters
(d) 14 meters

$y = f(x) = 2x - 0.1x^2$ — (1)

$f'(x) = 2 - 0.2x = 0.2[10 - x]$

T. Point is $x = 10$

So Max Value $= f(10)$

$= \left[2x - 0.1x^2\right]_{x=10}$

$= 20 - 10 = 10$ mtrs

Max-Min of func^n of two Variables (i.e of $z = f(x,y)$)

Point $\simeq (x, y)$

Value $\simeq z$

N. Cond^n for Max-Min: $\rightarrow \dfrac{\partial z}{\partial x} = 0 \rightarrow (1), \quad \dfrac{\partial z}{\partial y} = 0 \rightarrow (2)$

Stationary Point / Critical Point $\rightarrow$ Points obtained by Solving (1) & (2).

for eg after solving we get $x = a$ & $y = b$ then $P(a,b)$ is critical Point.

Sufficient Cond^n for Max-Min $\rightarrow$ function must satisfy Lagrange's cond^n.

Note : Let $P(a,b)$ is critical point then we can use following symbols.

$$\left(\frac{\partial^2 z}{\partial x^2}\right)_{P(a,b)} = r \quad , \left(\frac{\partial^2 z}{\partial y^2}\right)_{P(a,b)} = t \quad , \left(\frac{\partial^2 z}{\partial x \partial y}\right)_{P(a,b)} = s$$

Lagrange's Conditions :-

(1) if $rt - s^2 > 0$ & $r > 0$ then $P(a,b)$ is point of minima

(2) if $rt - s^2 > 0$ & $r < 0$ " " " " " Maxima

(3) if $rt - s^2 < 0$ then " " " " Inflexion

(4) if $rt - s^2 = 0$ then case fails i.e L-Conditions are unable to give an idea about Maxima or minima & we need further investigation.

Q. $\boxed{z = f(x,y)}$ s.t $f_x(a,b) = 0$, $f_y(a,b) = 0$ N. Cond^n

and also we have

$f_{xy}^2(a,b) - f_{xx}(a,b) f_{yy}(a,b) < 0$ & $f_{xx}(a,b) < 0$

S. Cond^n

then $P(a,b)$ is Point of ______

(a) Minima

(b) Maxima

(c) Inflexion

(d) Data Insufficient

$s^2 - rt < 0$ & $r < 0$

or $rt - s^2 > 0$ & $r < 0$

So $P(a,b)$ is Point of Maxima

$Z = 4x^2 + 6y^2 - 8x - 4y + 8$

$Z_x = 8x - 8$, $Z_{xx} = 8$

$Z_y = 12y - 4$, $Z_{yy} = 12$

$Z_{xy} = \frac{\partial}{\partial x}\left(\frac{\partial Z}{\partial y}\right) = \frac{\partial}{\partial x}(12y - 4) = 0$

Given a function $f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$. The optimum value of $f(x, y)$ is

(a) a minimum equal to 10/3

(b) a maximum equal to 10/3

(c) a minimum equal to 8/3

(d) a maximum equal to 8/3

Critical Points: Put $Z_x = 0$ & $Z_y = 0$

$8x - 8 = 0$ & $12y - 4 = 0$

$x = 1$ & $y = 1/3$

So $P(1, \frac{1}{3})$ is C.Point.

Here $r = (Z_{xx})_P = 8$, $t = (Z_{yy})_P = 12$,

$s = (Z_{xy})_P = 0$

$\therefore rt - s^2 = (8)(12) - (0)^2 = 96 > 0$ & $r > 0$

So $P(1, \frac{1}{3})$ is Point of Minima.

Min. Value $= (f(x,y))_P = (4x^2 - 6y^2 - 8x - 4y + 8)_{P(1, \frac{1}{3})}$

$= 10/3$

Find the absolute maxima and minima of the function respectively

$f(x, y) = x^2 - xy - y^2 - 6x + 2$ on the rectangular plate $0 \le x \le 5$, $-3 \le y \le 0$

(a) 7, −3
(b) 3, −7
(c) −3, 7
(d) DNE, DNE

$z = x^2 - xy - y^2 - 6x + 2$ — (1)

$z_x = 2x - y - 6$, $z_{xx} = 2$

$z_y = -x - 2y$, $z_{yy} = -2$

$z_{xy} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(-x-2y) = -1$

for critical points, $z_x = 0$ & $z_y = 0$

$2x - y = 6$ & $x + 2y = 0$

After solving we get $x = \frac{+12}{5}$ & $y = \frac{-6}{5}$

So stationary point is $P\left(\frac{+12}{5}, \frac{-6}{5}\right)$.

$r = 2$, $t = -2$, $s = -1$

$\because rt - s^2 = (2)(-2) - (-1)^2 = -5 < 0$

So $P\left(\frac{12}{5}, \frac{-6}{5}\right)$ is P.O.I ie At P, NMNM occurs

How we will check Max & Minima at corner points.

$0 \le x \le 5$ & $-3 \le y \le 0$ so $P(0,-3)$, $Q(0,0)$, $R(5,-3)$, $S(5,0)$ are corner points.

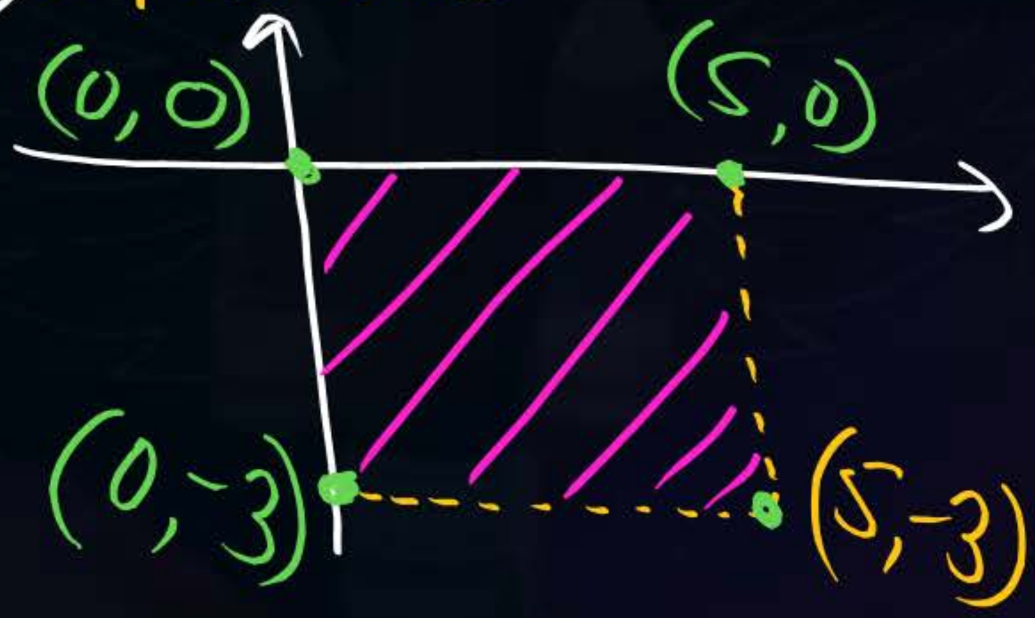
Given, $\boxed{f(x,y) = x^2 - xy - y^2 - 6x + 2}$

At P, $f(0,-3) = 0 - 0 - (-3)^2 - 0 + 2 = -7$ = min. Value

At Q, $f(0,0) = 2$

At R, $f(5,-3) = 25 - (-15) - 9 - 30 + 2 = 3$ = Max Value

At S, $f(5,0) = 25 - 0 - 0 - 30 + 2 = -3$



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EE

$\boxed{f(x,y) = (x+y-1)^2 + (x+y)^2}$ & $S = \{(x,y) : x \in R, y \in R\}$

where S contains set of points which minimizes $f(x,y)$ then

(a) S contains exactly one point

(b) S is empty

(c) ✓ S contains Infinitely many points

(d) S Contains finite Number of Points.

At $P(\frac{1}{2}, 0)$; $z = (\frac{1}{2})^2 + (\frac{1}{2})^2 = 1/2$

At $P(\frac{1}{4}, \frac{1}{4})$; $z = (\frac{1}{2})^2 + (\frac{1}{2})^2 = 1/2$ & so on.....

$z_x = 2(x+y-1) + 2(x+y)$

$z_{xx} = 2+2 = 4 = r$

$z_y = 2(x+y-1) + 2(x+y)$

$z_{yy} = 2+2 = 4 = t$

$z_{xy} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = 2+2 = 4 = s$

$\because rt - s^2 = (4)(4) - (4)^2 = 0$ Case fails.

For Stationary Points: $\frac{\partial z}{\partial x} = 0 \Rightarrow 2(x+y-1) + 2(x+y) = 0 \Rightarrow 4x+4y = 2$

$\frac{\partial z}{\partial y} = 0 \Rightarrow 2(x+y-1) + 2(x+y) = 0 \Rightarrow 4x+4y = 2$

i.e. By both the eqn, we have only one conclusion, $\boxed{x+y = \frac{1}{2}}$ — (1)

So $(x, y) = \left(\frac{1}{4}, \frac{1}{4}\right), \left(\frac{3}{2}, -1\right), \left(\frac{1}{8}, \frac{3}{8}\right), \left(\frac{1}{2}, 0\right) \ldots\ldots$

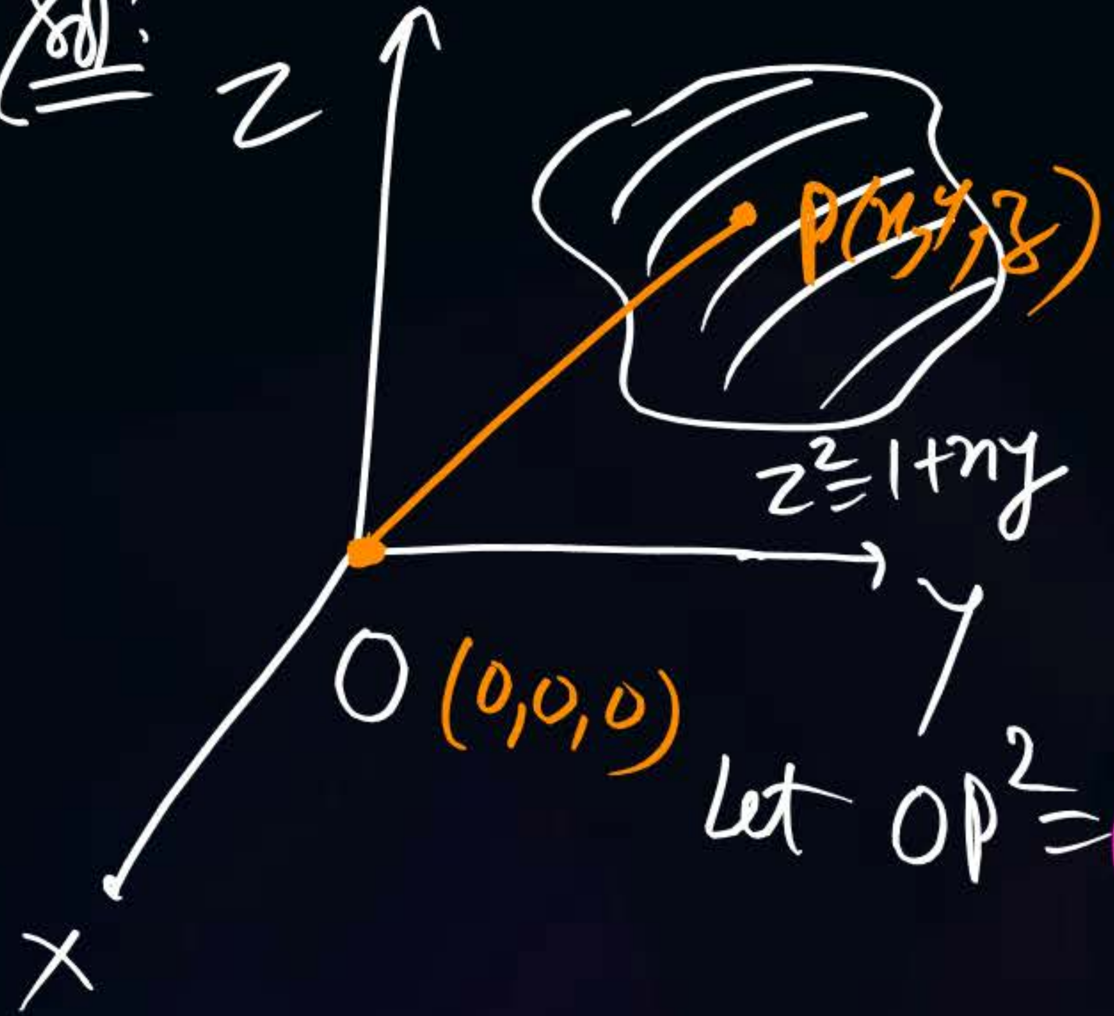
are the possible solutions of (1)

$\Rightarrow f(x, y)$ has $\underline{\infty}$ Number of stationary Points.

S: Contains ∞ Points

Q. The minimum distance of any point on the surface $\boxed{z^2 = 1 + xy}$ from origin will be ? ______

Sol:



Let $P(x, y, z)$ be any random point on this surface

$OP = \sqrt{x^2 + y^2 + z^2}$ $\{\because OP = f(x, y, z)\}$

$OP = \sqrt{x^2 + y^2 + 1 + xy}$ $\{$Now $OP = f(x, y)$? $\}$

Let $OP^2 = \boxed{u = x^2 + y^2 + 1 + xy}$ — (1) where $OP = \sqrt{u}$

& $OP_{min} = \sqrt{u_{min}} = ?$

$u = x^2 + y^2 + 1 + xy$ —— (1)

For S. Points: $\dfrac{\partial u}{\partial x} = 0$, $\dfrac{\partial u}{\partial y} = 0$ $\Big\} \Rightarrow x = 0,\ y = 0$.

i.e. $P(0,0)$ is Stationary Point.

Now $r = 2,\ t = 2,\ s = 1$

$\because rt - s^2 > 0$ & $r > 0$

So $P(0,0)$ is Point of Minima

So $u_{min} = \left(x^2 + y^2 + 1 + xy\right)_{P(0,0)}$

$= (0 + 0 + 1 + 0) = 1$

So $OP_{min} = \sqrt{u_{min}} = \sqrt{1} = 1$

INTEGRATION

* $y = \sin(e^{x^2})$

$\rightarrow \frac{dy}{dx} = \cos(e^{x^2}) \cdot e^{x^2} \cdot (2x)$

$\rightarrow \int \sin(e^{x^2})\,dx = \ ? =$ Not possible.

i.e Diff is easy just because of the presence of chain Rule.

& Int is tough bcoz chain Rule DNE.

* $\boxed{\text{Area} = \iint (1)\,dx\,dy} \simeq \int y\,dx = \int_{x=a}^{b} f(x)\,dx =$ Area under $f(x)$ b/n $x=a$ & $x=b$

Volume $= \iiint (1)\,dx\,dy\,dz \simeq \iint z\,dx\,dy = \iint f(x,y)\,dx\,dy$

Mass $= \iiint \rho \cdot dx\,dy\,dz \quad \because D = \frac{M}{V}$

(3) Linear Algebra, Prob & Stats, Calculus $\begin{cases} \to \text{Diff} \\ \to \text{Integration} \end{cases}$

→ Single Int
Multiple Int,

$\int_0^\infty \frac{\sin x}{x}\,dx = ?$ (Laplace Transf), $\int_0^\infty e^{-x^2}\,dx = ?$ (β-γ func^n), $\int$(Complex func^n) $dz = ?$ (C-I Th / C-R Th)

$\int$(vector) $d\vec{r}$ (G-Div / G-Th / S Th), C.R.V., $P(a<x<b) = \int_a^b f(x)\,dx$ (Prob & Stats), $\frac{d^2y}{dx^2} + y = 0$ (D Eq) → (twice Integrate)

$\int_1^5 (?)\,dx = ?$
N.Tech (T Rule / Simpson's)

For CS & DA :-

(1) L.Algebra
(2) Calculus
(3) Prob & Stats

INDEFINITE INT → Collection of all the Antiderivatives (Primitives) is called Indefinite Int.

if $\frac{d}{dx}\left(f(x)+c\right) = \varphi(x) \implies \boxed{\int \varphi(x)\,dx = f(x)+c}$

$f(x)+c$ → Antiderivative

$\varphi(x)$ → Derivative of $f(x)$

eg

$$\left.\begin{aligned} &\frac{d}{dx}(x^2) = 2x \\ &\frac{d}{dx}(x^2-5) = 2x \\ &\frac{d}{dx}(x^2+7) = 2x \\ &\frac{d}{dx}(x^2+11) = 2x \\ &\ldots\ldots \end{aligned}\right\} \implies \int 2x\,dx = x^2 + c$$

Methods of Solving Indefinite Integration →

Methods

1. Using Standard Result.
2. Using substitution (M. Imp).
3. Using Integration by part
4. Using partial Fraction

Standard Results :

(1) Power formula: $\boxed{\int x^a dx = \frac{x^{a+1}}{a+1}, a \neq -1}$

(2) $\int \frac{1}{x} dx = \log x + C$

(3) $\int a^x dx = \frac{a^x}{\log_e a} + C$

(4) $\int e^x dx = e^x + C$

(5) $\int \sin x \, dx = -\cos x + C$

(6) $\int \cos x \, dx = \sin x + C$

(7) $\int \tan x \, dx = \log \sec x + C$

(8) $\int \cot x \, dx = \log \sin x + C$

(9) $\int \sec x \, dx = \log(\sec x + \tan x) + C$

(10) $\int (\operatorname{cosec} x) dx = \log(\operatorname{cosec} x - \cot x) + C$

(11) $\int \sec^2 x \, dx = \tan x + C$

(12) $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

(13) $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$

(14) $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx)$

(15) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

(16) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + C$

(17) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right) + C$

(18) $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left(x+\sqrt{x^2+a^2}\right) + C$

(19) $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left(x+\sqrt{x^2-a^2}\right) + C$

(20) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$

(21) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right) + C$

(22) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right) + C$

(23) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

(24) $\int \sec x \tan x\,dx = \sec x + C$, (25) $\int (\operatorname{cosec} x \cot x)\,dx = -\operatorname{cosec} x + C$

(26) $\boxed{\int \frac{f'(x)}{f(x)}dx = \log f(x) + C}$; (27) $\boxed{\int e^x\{f(x)+f'(x)\}dx = e^x f(x) + C}$

eg $\int \frac{dx}{x} = ? = \ln x + C$

eg $\int \tan x\, dx = ?$

$= \int \frac{\sin x}{\cos x}dx = -\int \frac{(-\sin x)}{\cos x}dx$

$= -\log(\cos x) + C$

$= \log(\sec x) + C$

eg $\int e^x(\sin x + \cos x)dx = ? = e^x \cdot \sin x + C$

eg: $\int [\sin(\log x) + \cos(\log x)]\underline{dx} = ?$ Ans: $x\sin(\log x) + C$

Put $\log_e x = t \Rightarrow x = e^t$

$dx = e^t dt$

$= \int (\sin t + \cos t)e^t dt$

$= \int e^t(\sin t + \cos t)dt = e^t \cdot \sin t$

$= x \cdot \sin(\log_e x)$ //

(*) $\boxed{\int x^a\,dx = \frac{x^{a+1}}{a+1}}$, $a \neq -1$; eg $\int k\,dx = ? = k\int x^0\,dx = k\left(\frac{x^{0+1}}{0+1}\right) = kx + c$

eg $\int (x^2)\,dx = ? = \frac{x^{2+1}}{2+1} = \frac{x^3}{3} + c$

eg $\int \left(\frac{1}{x^2}\right) dx = ? = \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} = \frac{-1}{x}$

eg $\int (\sqrt{x})\,dx = ? = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2}$

eg $\int \left(\frac{1}{\sqrt{x}}\right) dx = ? = \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = 2\sqrt{x}$

eg $\int (x)\,dx = ? = x^2/2$

eg $\int \left(\frac{1}{x}\right) dx = ? = \log_e x$

Q. $\int \left(\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}\right) dx = ?$

$= \int \left(\frac{e^{\log_e x^5} - e^{\log_e x^4}}{e^{\log_e x^3} - e^{\log_e x^2}}\right) dx$

$= \int \left(\frac{x^5 - x^4}{x^3 - x^2}\right) dx = \int \frac{x^4(x-1)}{x^2(x-1)} dx$

$= \int x^2 dx = \frac{x^3}{3} + C$

Q. $I = \int \frac{x^4}{x^2+1} dx = ?$

$= \int \frac{x^4 - 1 + 1}{x^2+1} dx = \int \frac{x^4 - 1}{x^2+1} dx + \int \frac{1}{x^2+1} dx$

$= \int \frac{(x^2-1)(x^2+1)}{x^2+1} dx + \tan^{-1} x$

$= \int (x^2 - 1) dx + \tan^{-1} x$

$= \frac{x^3}{3} - x + \tan^{-1} x + C$

(PQ) $I = \int \frac{x^2+x+1}{(x-1)^3} dx = ?$

(M-I) $I = \int \left\{ \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} \right\} dx$

= Lengthy Approach ....

Sol: Put $x-1 = t \Rightarrow x = t+1$

$dx = dt$

$I = \int \left(\frac{(t+1)^2 + (t+1) + 1}{t^3} \right) dt$

$= \int \left(\frac{t^2+1+2t+t+1+1}{t^3} \right) dt$

$= \int \left(\frac{t^2+3t+3}{t^3} \right) dt$

$I = \int \left(\frac{1}{t} + \frac{3}{t^2} + \frac{3}{t^3} \right) dt$

$= \log t + 3\left(\frac{-1}{t}\right) + 3\left(\frac{t^{-3+1}}{-3+1}\right)$

$= \log(x-1) - \frac{3}{x-1} - \frac{3}{2(x-1)^2} + C$

Questions Based on (M-II)

Q. $\int x^2 \cdot \sin(x^3)\, dx = ?$

Put $x^3 = t$

$3x^2\, dx = dt$

$x^2\, dx = dt/3$

$I = \int \sin(t) \cdot \frac{dt}{3}$

$= -\frac{1}{3}\cos t + C$

$= -\frac{1}{3}\cos(x^3) + C$

Q. $I = \int \sec^3 x \cdot \tan x\, dx = ?$

~~Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$~~

~~So $I = \int (\sec x) \cdot t \cdot dt$~~ = Not good.

Put $\sec x = t \Rightarrow \sec x \tan x\, dx = dt$

$I = \int \sec^2 x \cdot \sec x \tan x\, dx$

$= \int t^2 \cdot dt = \frac{t^3}{3} = \frac{\sec^3 x}{3} + C$

Q. $I = \int \sin(\log x)\, dx = ?$

Put $\log x = t \Rightarrow x = e^t$

$dx = e^t\, dt$

$I = \int \sin t \cdot e^t\, dt$

$= \int e^t \sin t\, dt$

(By Formula 13), $a = 1$, $b = 1$

$= \frac{e^t}{1^2+1^2}\left[1 \cdot \sin t - 1 \cdot \cos t\right]$

$= \frac{x}{2}\left[\sin(\log x) - \cos(\log x)\right]$

Q. $I = \int \frac{e^x}{\sqrt{4 - e^{2x}}}\, dx = ?$

Put $e^x = t \Rightarrow e^x dx = dt$

$I = \int \frac{dt}{\sqrt{4 - t^2}} = \int \frac{1}{\sqrt{2^2 - t^2}}\, dt$

$= \sin^{-1}\left(\frac{t}{2}\right) + c$ (By Formula 20)

$= \sin^{-1}\left(\frac{e^x}{2}\right) + c$

(2) Integration by parts → $\boxed{\int \underset{I}{u} \cdot \underset{II}{v}\, dx = u\int v\, dx - \int \left\{ \frac{du}{dx} \cdot \int v\, dx \right\} dx + c}$

Log func^n — Trig func^n

I L A T E

Inverse Trig func^n — Algebraic func^n — Exp. func^n

the letter which is coming 1st in the word ILATE should be assumed as 1st func^n 'u'

Shortcut - $\int uv\, dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$

where Dash denotes diff & suffix denotes integration.

Q. $\int x \sin x \, dx = ?$

M-I $= x(-\cos x) - \int (1)(-\cos x)\, dx$

$= -x\cos x + \sin x + C$

M-II $\int x \sin x \, dx = x(-\cos x) - 1(-\sin x) + 0$

M-II

Q. $\int x^3 \sin x \, dx = ?$

M-I use u-v formula 3 times (very lengthy)

M-II $\int x^3 \sin x \, dx = x^3(-\cos x) - 3x^2(-\sin x) + 6x(\cos x) - 6(\sin x)$

$+0$

Q $\int x^2 e^x \, dx = ? = x^2(e^x) - 2x(e^x) + 2(e^x) - 0$

Take Care: this method is applicable only when u is polynomial

Q. $I=\int(x\log x)\,dx=?=\int(\log x)\cdot x\,dx$

$u=\log x$, $v=x$

$=\log x\left(\frac{x^2}{2}\right)-\int\left\{\frac{1}{x}\left(\frac{x^2}{2}\right)\right\}dx$

$=\frac{x^2}{2}\log x-\frac{1}{2}\left(\frac{x^2}{2}\right)+C$

Q. $I=\int(\log x)\,dx=?=\int(\log x)\cdot 1\,dx$

$u=\log x$, $v=1$

$=\log x\,(x)-\int\left\{\frac{1}{x}(x)\right\}dx$

$=\boxed{x\log x-x}$ Learn as Standard Result.

Q. $I=\int(\log x)^2\,dx=?$

$=\int(\log x)^2\cdot 1\,dx$

$u=(\log x)^2$, $v=1$

$=(\log x)^2\cdot(x)-\int\left(\frac{2\log x}{x}\right)\cdot x\,dx$

$=x(\log x)^2-2\left[x\log x-x\right]$

Note: $\int x^3 dx = \frac{x^4}{4} \Rightarrow \int (2x+5)^3 dx = ? = \frac{(2x+5)^4}{4} \cdot \left(\frac{1}{2}\right)$ Put $2x+5 = t$

$\int \sin x = -\cos x \Rightarrow \int \sin(5x)\, dx = ? = -\frac{\cos(5x)}{5}$ Put $5x = t$

Quest Based on M-IV

Q. $I=\int \frac{\cos\theta}{(2+\sin\theta)(3+4\sin\theta)}\,d\theta = ?$ $=\int \frac{1}{(2+t)(3+4t)}\,dt = \int\left(\frac{A}{2+t}+\frac{B}{3+4t}\right)dt$

(P.F: $A=-\frac{1}{5}$, $B=\frac{4}{5}$)

Put $\sin\theta = t$

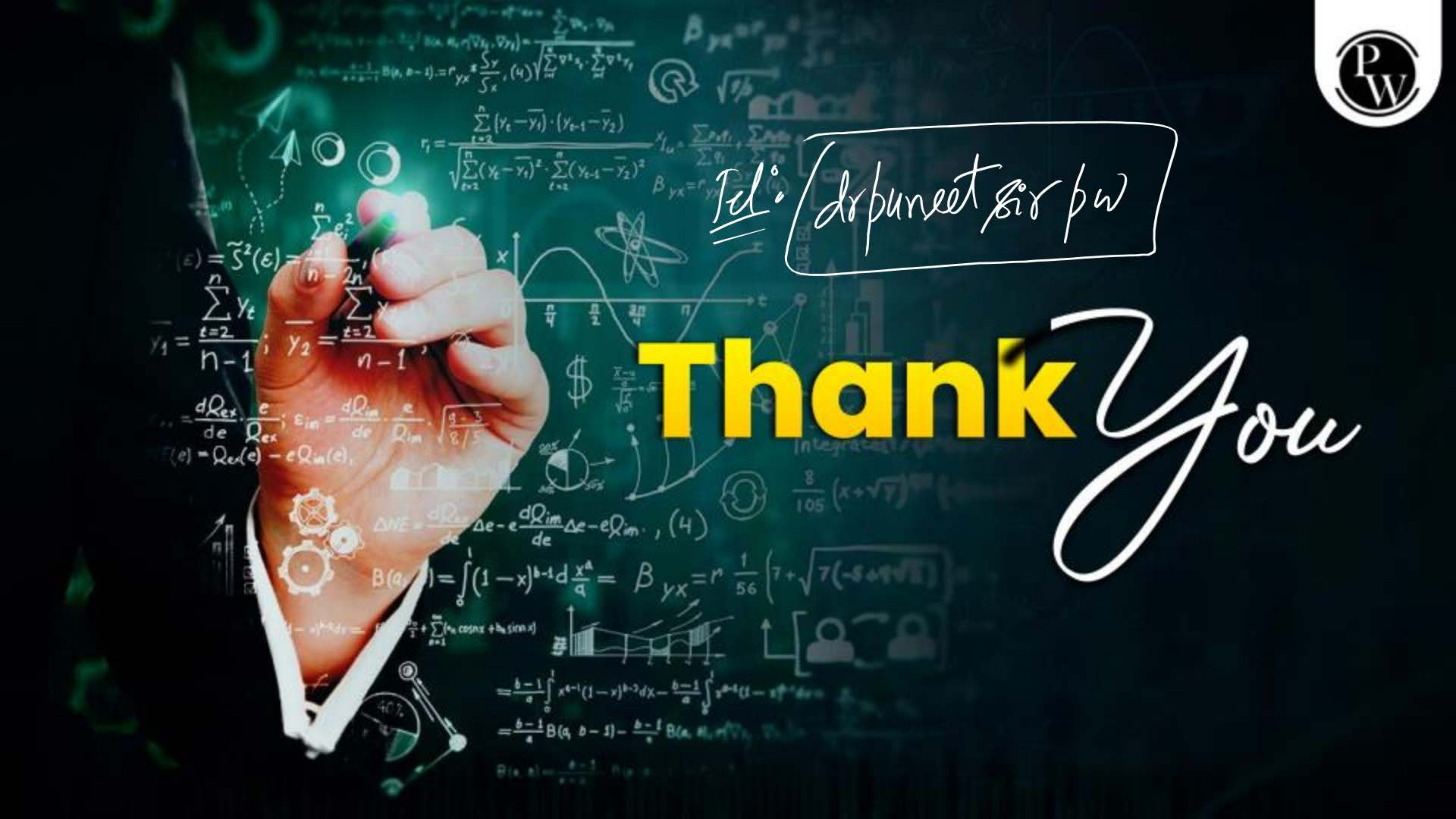
$\cos\theta\, d\theta = dt$

$= \int \frac{-1/5}{2+t}\,dt + \int \frac{+4/5}{3+4t}\,dt$

$= -\frac{1}{5}\log(2+t) + \frac{4}{5}\frac{\log(3+4t)}{4}$

$= -\frac{1}{5}\log(2+\sin\theta) + \frac{1}{5}\log(3+4\sin\theta)$

$= \frac{1}{5}\log\left(\frac{3+4\sin\theta}{2+\sin\theta}\right)$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Thank You