DS & AI
CS & IT

Probability & Statistics

Lecture No. 03

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Recap of previous lecture

Topic

PERMUTATION - COMBINATION

(Part-2)

Topics to be Covered

Topic

"PERMUTATION & COMBINATION"

(Part-3)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

" If, what if, AGAR, YADI, TOH, ... "

OR

Don't Try to develop Question (by your $\boxed{\text{little mind}}$ until you have a complete understanding of the chapter) & try to solve the Quest.

COUNTING PRINCIPLE

Fundamental Principle of Addition → If we have to perform only one of the job at a time out of n jobs then use this principle.

RECAP

Keywords: "Either or / only one / Anyone"

Fundamental Principle of Multiplication → If we have to perform all the jobs at a time out of n jobs then use this principle.

Keywords: "AND / BOTH / ALL / Every"

GAZAB KA Conclusion →

① if $n > r$ & RNA, then Multi Rule $\equiv$ Perm Rule

② if $n = r$ & RNA, Then Multi Rule $\equiv$ Perm Rule $\equiv$ Factorial Rule

③ if RA, then only use Multi Rule

ie the Concept of ${}^nC_r$, ${}^nP_r$ & $r!$ is applicable only when RNA

RECAP

Some Useful Information (Based on Experience) → POSTER

1. Always together / Not separated ⟶ Assume them as one unit with in Bracket.
2. All Never together / All do not come together ⟶ Total − Always together.
3. No two Girls are together ⟶ First arrange Boys.
4. Alternately (Linear Case) < Two Cases will arise.
5. Alternately (Circular Case) ⟶ only one Case will arise.
6. Particular / fix ⟶ No Need to select & No Need to arrange
7. At least one = Total − None.
8. At least = Go upto last Point (using Common Sense)
9. At Most = Include None also (if Possible)

The number of words of four letters containing equal number of vowels and consonants (~~Repetition allowed~~)

(RNA)

(a) 60×210 (b) 210×243

(c) 210×315 (d) 630 (e) None.

1.

Total four letter words = $^{5}C_2 \times {}^{21}C_2 \times 4!$
(2 vowels & 2 Consonants)
RNA

$= 10 \times 210 \times 24$

$= 210 \times 240$

$= 50400$

CIRCULAR PERMUTATION →

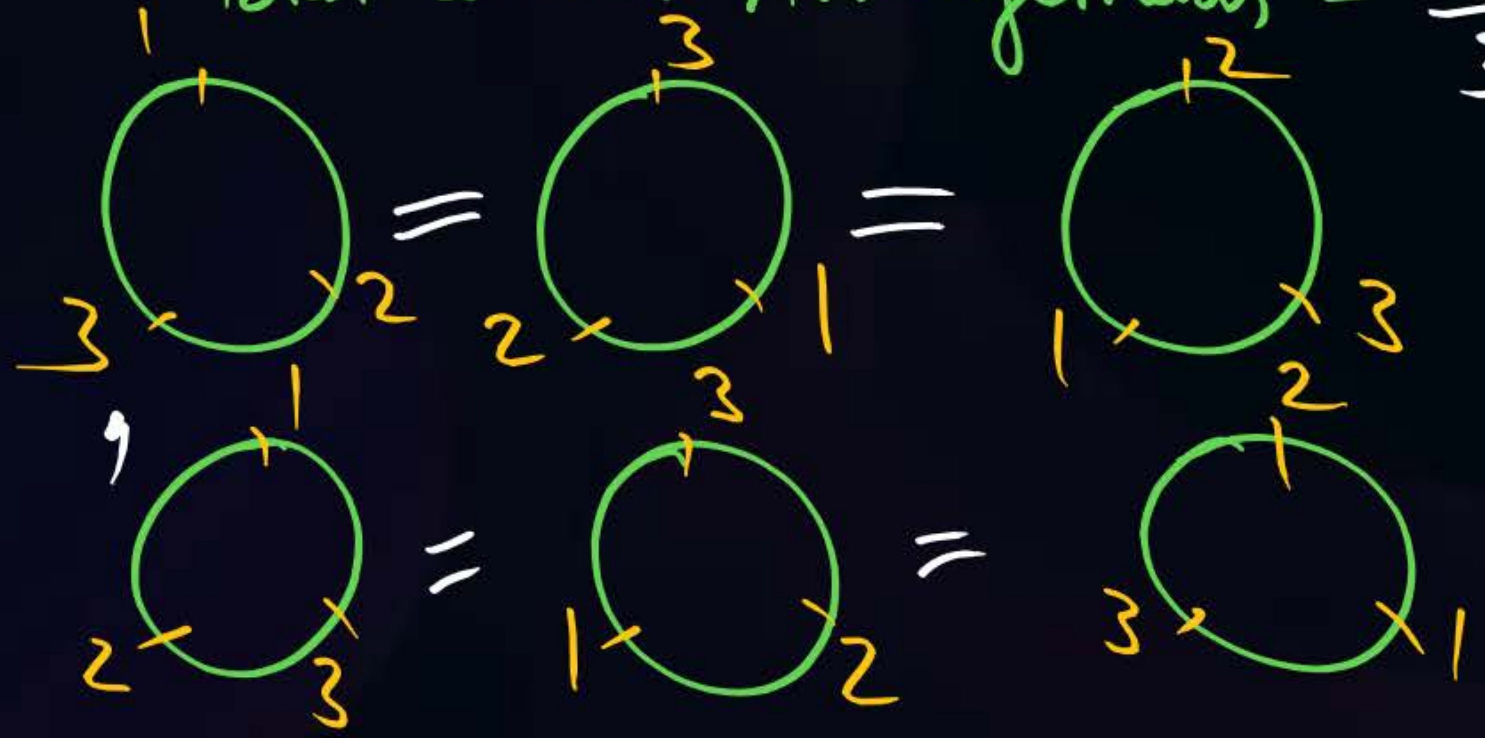
Number of Linear Arrangements of n different things $= n!$

Number of Circular Arrangements of n different things $= \frac{n!}{n} = (n-1)!$

Eg $n=3$, Total Linear Arrangements $= 3! = 6$

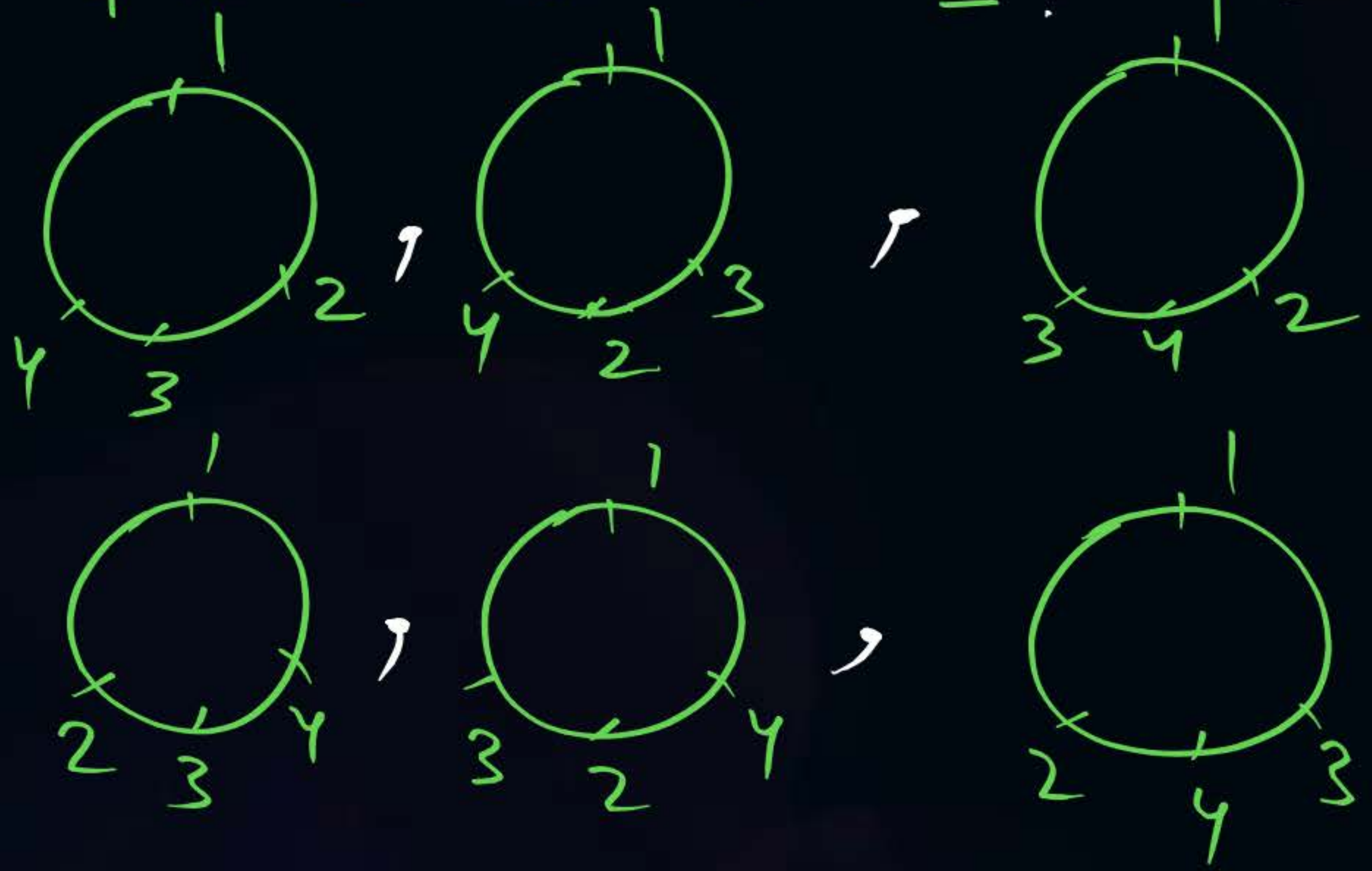
(123), (132), (213), (231), (312), (321)

Total Circular Arrangements $= \frac{3!}{3} = (3-1)! = 2! = 6$



eg $n=4$, Total linear arrangements $= 4! = 24$

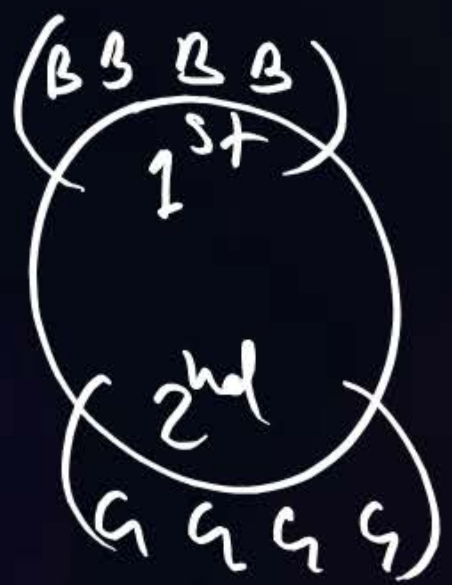
& Total circular " $= ? = \frac{4!}{4} = (4-1)! = 3! = 6$



FQ 3 : 4 Boys & 4 Girls are to be seated around circular Table for Tea Party in which there are two sisters & 1 Brother. then find the number of possible arrangements if.

① there is No Restriction = ? = $(8-1)! = 7!$ (Man Am)
(RNA)

② All Boys are together & All Girls are together — ?



$= (2-1)! \times 4! \times 4!$

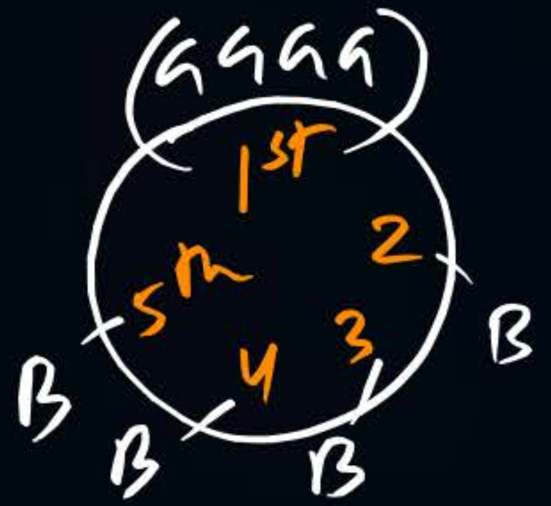
(P) (B) (G)

(=CA) (=LA) (=LA)

(3) Girls are not separated = ?

OR

Girls are always together = ?

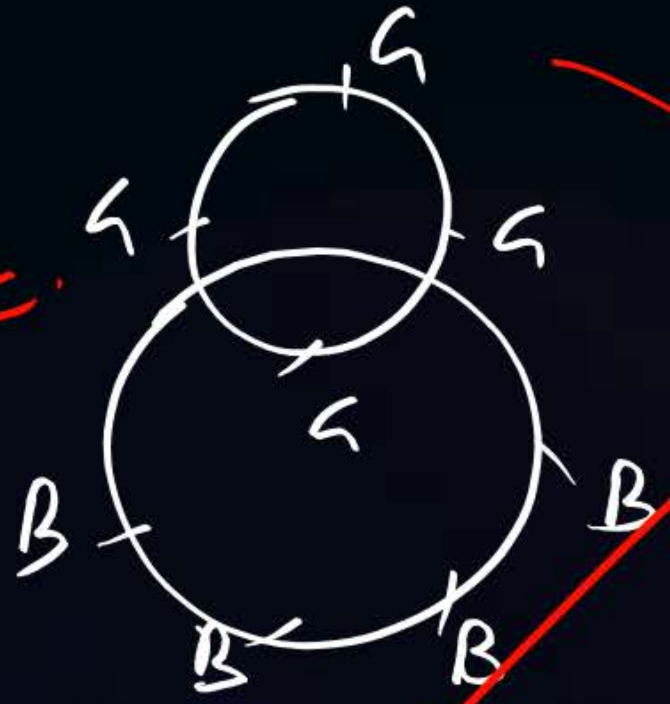


$= (5-1)! \times 4!$

(P) $(= CA)$ (G) $(= L.A)$

(MI)

WRONG



~~Req Arr = $(5-1)! \times (4-1)!$~~

$= P$ $(= CA)$ $= G$ $(= CA)$

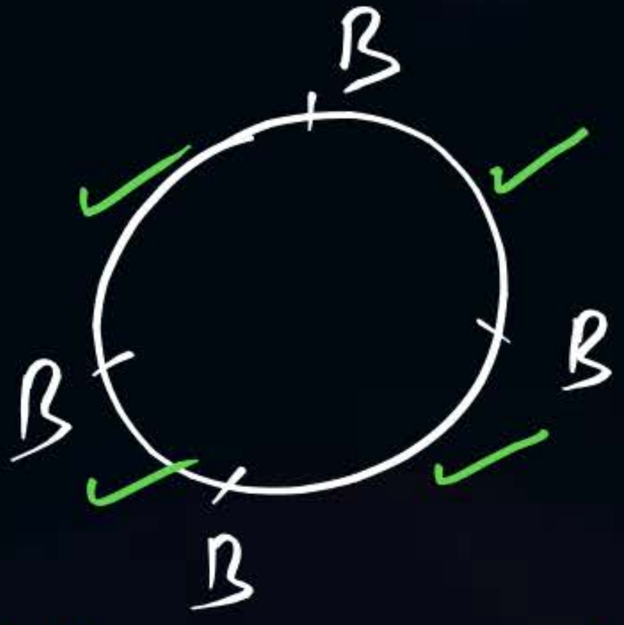
∵ they are enjoying Tea party so it is looking WIERED.

(4) All G do not come together = ? = Total − G always together $= (8-1)! - (5-1)! \times 4!$

(All G never together)

$= 7! - 4! \times 4!$

(5) No two Girls are together = ? = First arrange Boys in circle.



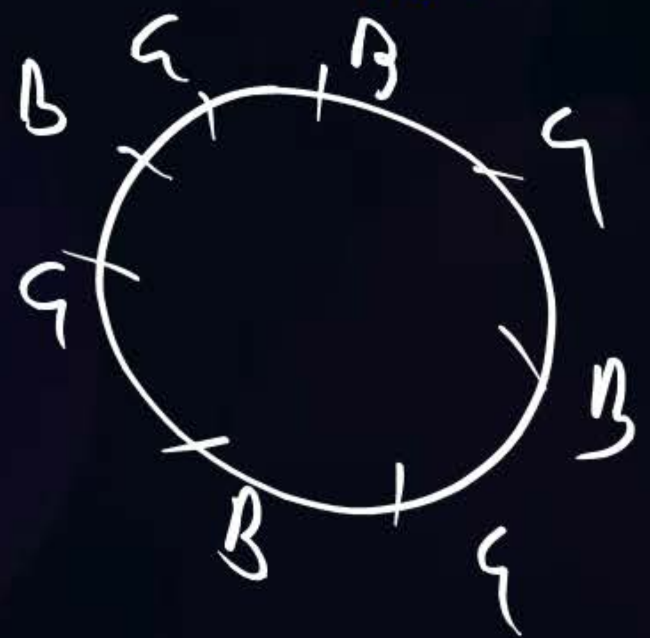
$= (4-1)! \times 4!$

(Boys) (Girls)

(= CA) (= LA)

Sp. Note: Form circle only once, in a given Question.

(6) Boys & Girls are seated Alternately = ? = same as above



$= (4-1)! \times 4!$

(7) Two sisters wants to be seated at the adjacent sides of Host?

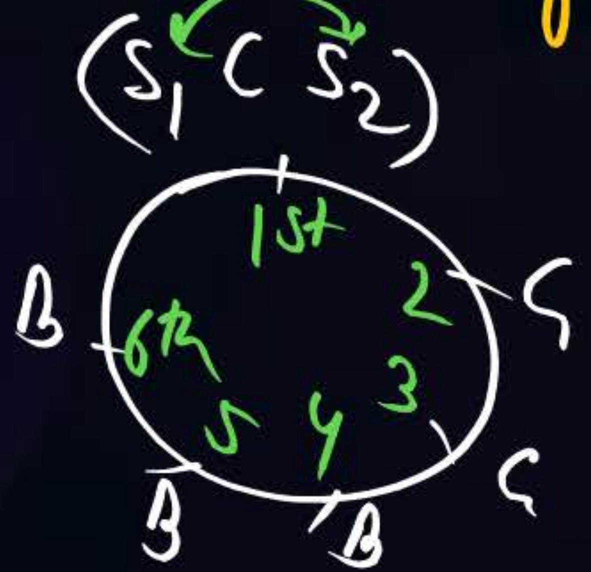
$= {}^{1}C_1 \times (6-1)! \times 2!$

No Need to select Host (∵ Host is particular person)

(8) there is exactly one G b/n two sisters = ? $= {}^{2}C_1 \times (6-1)! \times 2!$

(G) (P) (S)

(9) Two sisters are always separated by 'CHANDAN'



$= {}^{1}C_1 \times (6-1)! \times 2!$

Q. If there are 5B & 4G are to be seated around a Circular Table then Find No. of arrangements (1) in which B & G are seated alternately?

= WRONG DATA given in Question.

i.e Ans = 0

(2) No two Boys are together = ? = First arrange Girls in Circular fashion.

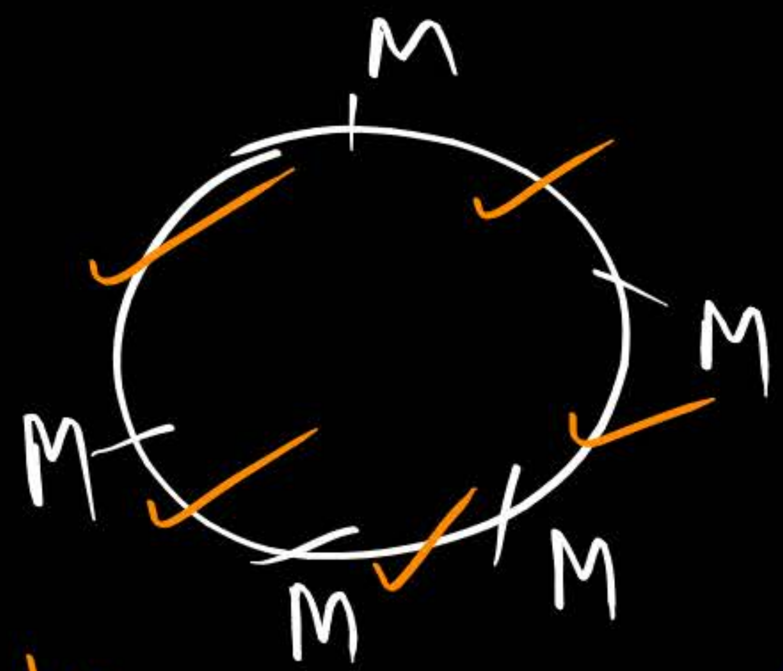
Now Vacant places are 4 But Boys are 5 so they Can't be seated Hence Ans = 0

WRONG DATA

There are 5 gentlemen and four ladies to dine at a round table. In how many ways can they seat themselves so that no two ladies are together?

(a) 3280 (b) 2880 (c) 2080 (d) 2480

No two Ladies are together = First arrange Boys in circle



$= (5-1)! \times {}^5P_4$

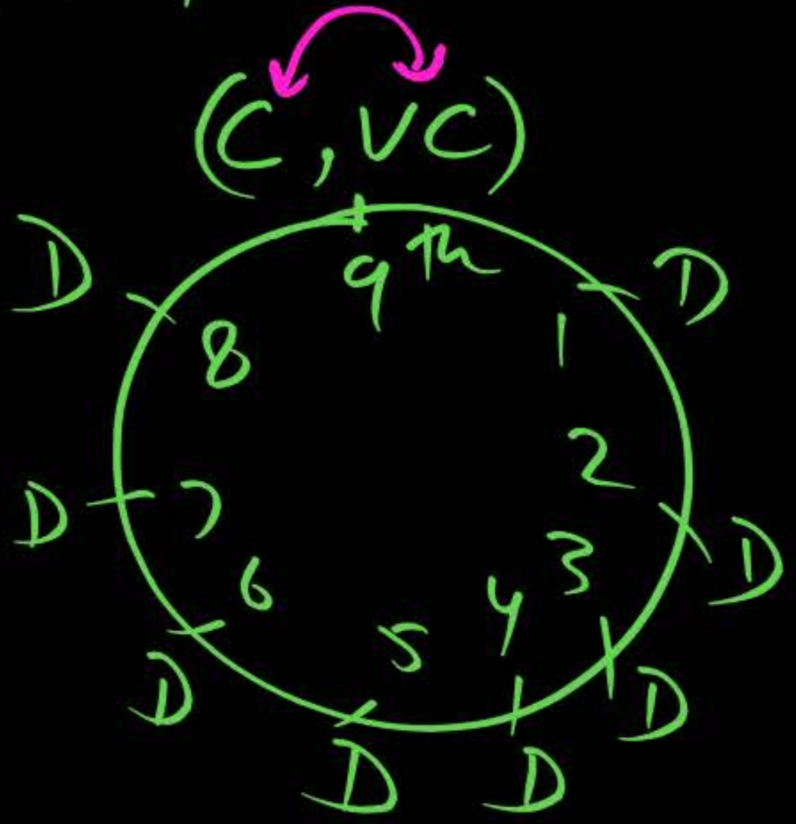
Boys (=LA) Ladies (=LA)

Vacant places = 5 & Ladies = 4.

In how many ways can 8 Directors, Vice-Chairman & Chairman of a firm be seated at a round table, if the Chairman has to sit between Vice-Chairman & Director

(a) $2 \times 9!$ (b) $2 \times 8!$ (c) $2 \times 7!$ (d) $3! \times 9!$

8D, 1C, 1VC



Total circular arrangements

$$= (9-1)! \times 2!$$

$$= 8! \times 2!$$

Permutation of Alike items :→

(i) No. of linear arrangements of n different things $= n!$

(ii) No of circular " " n different things $= (n-1)!$

(iii) No of Linear arrangements of n things in which $\underbrace{p \text{ are alike}}$, $\underbrace{q \text{ are alike}}$, $\underbrace{r \text{ are alike}}$ & rest are different $= \dfrac{n!}{p!\,q!\,r!} = \dfrac{{}^{n}P_{n}}{p!\,q!\,r!}$

where $p+q+r+\underset{(?)}{\text{Rest items}} = \text{Total items } (n)$

Meaning of RA → if it is possible to Repeat any given thing desired number of Times then we will say that RA.

Eg: How many 4 letter words can be formed using $\underbrace{a, a, a}$, b ?

Toys = RNA

(M-I) Total 4 letter words = 4 Ans.

(aaab), (aaba), (abaa), (baaa)

(M-II) Total wrong arrangements = $4! = 24$
(By Assuming all are diff)

So Total correct arrangements = $\frac{4!}{3!} = 4$ ✓

Q: How many 4 digit Number can be formed using 5, 5, 6, 6.

Toys (RNA)

(M-I) Total 4 digit Nos are as follows;

(5566), (5655), (5665), (6655), (6565), (6556)

i.e. Total Nos that can be formed = 6 Ans.

(M-II) Total wrong arrangements = $4! = 24$

Total Correct " $= \frac{4!}{2!\,2!} = 6$ Ans

Eg: 4 letter words using $\underbrace{a, b, c, d}_{\text{RNA}} = ? = 4! = 24$ ✓

Q 4 letter words using $\underbrace{a, a}, \underbrace{b, b} = ?$ $= \frac{4!}{2!\,2!} = 6$ ✓

RNA (Toys)

M-II Various 4 letter words = (aabb), (abab), (abba)
(bbaa), (baba), (baab)

Q Five digit Nos using $\underbrace{4, 4, 4}, \underbrace{7, 7} = ?$ $= \frac{5!}{3!\,2!} = 20$

Toys (RNA)

Q. How many different 11 letter words can be formed using the letters of the word 'INEFFECTIVE'

Sol: $\underbrace{I, I}, \underbrace{E, E, E}, \underbrace{F, F}, \underbrace{N, T, C, V}_{\text{Rest items}}$

Total $(n) = 11$, $p = 2$, $q = 3$, $r = 2$

$$= \frac{11!}{2!\,3!\,2!}$$

Q. How many different 9 letter words can be formed using the letters of the word 'ALLAHABAD'

Sol: $\underbrace{A, A, A, A}_{p=4}\ \underbrace{L, L}_{q=2}\ \underbrace{H, B, D}_{\text{Rest.}}$

$n = 9$

$$\text{Ans} = \frac{9!}{4!\,2!}$$

Q. In how many ways 2 Maths Books, 3 Physics Books, 4 Chemistry Books 1 Hindi Book, 1 English Book & 1 Geography Book can be arranged in a shelf?

$$\underbrace{M, M,}_{p=2}\ \underbrace{P, P, P,}_{q=3}\ \underbrace{C, C, C, C,}_{r=4}\ \underbrace{H, E, G}_{\text{Rest}},\quad n = 12 \text{ Books.}$$

Total arrangements of Books in a shelf $= \dfrac{12!}{2!\,3!\,4!} = \dfrac{{}^{12}P_{12}}{2!\,3!\,4!}$

A library has 'a' copies of one book, 'b' copies of $\boxed{\text{each}}$ of two books, 'c' copies of $\boxed{\text{each}}$ of three books and single copies of 'd' books, then the total number of ways in which these books can be arranged are?

(a) $\frac{(a+b+c+d)!}{a!b!c!}$ (b) $\frac{(a+2b+3c+d)!}{a!(b!)^2(c!)^3}$ (c) $\frac{(a+2b+3c+d)!}{a!b!c!}$ (d) None

$\underbrace{B_1 B_1 B_1 \ldots B_1}_{a \text{ copies}}, \underbrace{B_2 B_2 \ldots B_2}_{b \text{ copies}}, \underbrace{B_3 B_3 \ldots B_3}_{b \text{ copies}}, \underbrace{B_4 B_4 \ldots B_4}_{c \text{ copies}}, \underbrace{B_5 B_5 \ldots B_5}_{c \text{ copies}}, \underbrace{B_6 B_6 \ldots B_6}_{c \text{ copies}}$

Total Books $n = (a+2b+3c+d)$

$\underbrace{B_\alpha, B_\beta, B_\gamma, \ldots \to B_\delta}_{\text{Rest items} = d.}$

Req Arrangements $= \frac{(a+2b+3c+d)!}{a!\,b!\,b!\,c!\,c!\,c!} = \frac{(a+2b+3c+d)!}{a!(b!)^2(c!)^3}$

FQ4 : How many 11 letter words can be formed using the letters of the words $\boxed{\text{MATHEMATICS}}$ if

① there is No Restriction = ? (RNA) $= \frac{11!}{2!2!2!} = \frac{{}^{11}P_{11}}{2!2!2!}$

$\underbrace{MM}_{p}, \underbrace{TT}_{q}, \underbrace{AA}_{r}, E, H, I, C, S$, Total $n = 11$

② Vowels are not separated = ? $= \frac{8!}{\underset{(M)}{2!}\,\underset{(T)}{2!}} \times \frac{4!}{\underset{(A)}{2!}}$

M, C, S, (A, A, E, I) T, T, M, H

1st 2 3 4th 5 6 7 8th

③ All vowels do not come together = ? = Total − Vowels always together

do not → Never

= ① − ② = Ans

M, M, T, T, H, C, S

(4) No two Consonants are together = ? = ~~first arrange vowels~~

_ A _ A _ E _ I _ $= \cancel{\frac{4!}{2!}} \times$ (Not possible to arrange Consonants)

(SENSELESS QUES)

$\because$ Places = 5 & Consonants = 7

(5) No two vowels are together = ? = first arrange Consonants

_ M _ T _ H _ M _ C _ S _ T _ $= \frac{{}^{7}P_{7}}{2!\,2!} \times \frac{{}^{8}P_{4}}{2!}$

(M) (T) (A)

Consonants Vowels

Ans

After arranging Consonants, Far places for vowels = 8

But Vowels are = 4

Explanation : After arranging consonants, there are 8 fav places for vowels (6 in b/n & 2 at the corners).

But vowels are only 4 so they can be arranged by ${}^{8}P_{4}$ ways.

Again it is wrong ∵ two 'A' are alike

So finally vowels can be arranged by $\frac{{}^{8}P_{4}}{2!}$ ways.

→ In this Ans, Most of the cases are of that types in which two or more consonants are together.

(6) Vowels & Consonants are alternately = ? WRONG Question

$(\because C = 7 \;\&\; V = 4)$??

(7) Vowels occupy even places = ?

Vowels : $\underbrace{A, A}$, E, I

Consonants : $\underbrace{M, M}$, $\underbrace{T, T}$, H, C, S

$$= \underbrace{\frac{^5P_4}{\underset{(A)}{2!}}}_{\text{Vowels}} \times \underbrace{\frac{^7P_7}{\underset{(M)}{2!}\;\underset{(T)}{2!}}}_{\text{Consonants}}$$

✓ _ | _ 2nd | ✓ _ | _ 4th | ✓ _ | _ 6th | ✓ _ | _ 8th | ✓ _ | _ 10th | ✓ _

Now Fav places for consonants are $\underline{7}$
→ 6 odd places
→ 1 even place (vacant by Vowel)

Q. Six identical coins are arranged in a Row. The number of ways in which the Number of tails is equal to Number of Heads is?

(a) 20

(b) 120

(c) 9

(d) 40

Total arrangements $= \underset{C_1}{2} \times \underset{C_2}{2} \times \underset{C_3}{2} \times \underset{C_4}{2} \times \underset{C_5}{2} \times \underset{C_6}{2} = 64$

fav arrangements (i.e in which No. of Heads = No. of Tails)

$\left\{ \text{eg} \left(\underset{C_1}{H}\ \underset{C_2}{T}\ \underset{C_3}{T}\ \underset{C_4}{H}\ \underset{C_5}{T}\ \underset{C_6}{H} \right) \cdots\cdots \right\} = \dfrac{6!}{3!\,3!} = 20$

Analysis:- Total arrangement can be formed either by using

0H or 1H or 2H or 3H or 4H or 5H or 6H

${}^6C_0 = 1$ ${}^6C_1 = 6$ ${}^6C_2 = 15$ ${}^6C_3 = 20$ ${}^6C_4 = 15$ ${}^6C_5 = 6$ ${}^6C_6 = 1$ $= 64$

Analysis: Various Arrangements = (PODCAST)

(0H) or (1H) or (2H) or (3H) or (4H) or (5H) or (6H) = Total

$^6C_0 = 1$ $\quad$ $^6C_1 = 6$ $\quad$ $^6C_2 = 15$ $\quad$ $^6C_3 = 20$ $\quad$ $^6C_4 = 15$ $\quad$ $^6C_5 = 6$ $\quad$ $^6C_6 = 1$ $= 2^6$

① No. of Arrangements in which (No. of Heads = No. of Tails) $= {^6C_3} = \frac{6!}{3!\,3!} = 20$

② " " " " there are exactly 2 H $= {^6C_2} = \frac{6!}{2!\,4!} = 15$

③ " " " " there are at least 3 H = ? $= {^6C_3} + {^6C_4} + {^6C_5} + {^6C_6}$

④ " " " " there are at Most 2 H = ? $= {^6C_0} + {^6C_1} + {^6C_2}$

⑤ " " " " there is at least one H = ? = Total − No Head

$= 2^6 - {^6C_0} = 64 - 1 = 63$

Q. How many different 6 digit Nos can be formed using the digits $\underbrace{4, 5, 8, 9}$?

① If there are exactly 2 Triplets = ?

(444 555), (444888), (444 999)

(555888), (555999), (888999)

$$= 6 \times \frac{6!}{3!\,3!}$$

$$= {}^4C_2 \times \frac{6!}{3!\,3!}$$

eg (444999)

② If exactly one digit Repeats thrice & Remaining are different ?

$$= {}^4C_1 \times {}^3C_3 \times \frac{6!}{3!}$$

eg (555 489)

Digits = 4, 5, 8, 9.

(3) If there are exactly 3 pairs ? $= {}^4C_3 \times \frac{6!}{2!2!2!}$

(489489)

(4) with exactly 2 pair of digits & Remaining are different ?

$= {}^4C_2 \times {}^2C_2 \times \frac{6!}{2!2!}$

(eg 558849)

(5) one digit Repeats thrice & one digit Repeats twice ? $= {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times \frac{6!}{3!2!}$

V.V. Good

eg $(\underbrace{555}_{{}^4C_1}\ \underbrace{99}_{{}^3C_1}\ \underbrace{8}_{{}^2C_1})$

The number of words of four letters containing equal number of vowels and consonants (Repetition allowed)

(a) 60×210
(b) 210×243
(c) 210×315 ✓
(d) 630

RA

Vowels = a, e, i, o, u, Consonants = 21

We can make 4 letter words (in which there are 2 vowels & 2 consonants) either by using

Case I : Vow. Diff & Cons Diff $\rightarrow {}^5C_2 \times {}^{21}C_2 \times 4! = 50400$ (aelq)

OR

Case II : Vow Alike & Cons Diff $\rightarrow {}^5C_1 \times {}^{21}C_2 \times \frac{4!}{2!} = 12600$ (aahm)

OR

Case III : Vow Diff & Cons Alike $\rightarrow {}^5C_2 \times {}^{21}C_1 \times \frac{4!}{2!} = 2520$ (eupp)

OR

Case IV : Vow Alike & Cons Alike $\rightarrow {}^5C_1 \times {}^{21}C_1 \times \frac{4!}{2!2!} = 630$ (eehh)

$\left.\right\} 66150 = 210 \times 315$

(c) ✓

M-II :

Total 4 letter words (RA)

$$= \underset{V}{{}^5C_1} \times \underset{V}{{}^5C_1} \times \underset{C}{{}^{21}C_1} \times \underset{C}{{}^{21}C_1} \times \frac{4!}{2!2!}$$

Dearrangements → when No one goes at Right place assigned for him then such types of arrangements are called Dearrangements.

If there are n persons & n directed places. then

Total No. of arrangements $= n!$ this Result is applicable when RNA

All Correct " $= 1$

All wrong " $= \boxed{n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\cdots\cdots+(-1)^n\frac{1}{n!}\right]}$

for eg $n=3$, then De Arrangements $= 3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right] = 2$

for $n=2$, No. of Dearrangements $= 2!\left[1-\frac{1}{1!}+\frac{1}{2!}\right] = 2!\left[0+\frac{1}{2!}\right] = 1$

for $n=3$, " " $= 3!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}\right] = \frac{3!}{2!}-\frac{3!}{3!} = 3-1 = 2$

for $n=4$, " " $= 4!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right] = \frac{4!}{2!}-\frac{4!}{3!}+\frac{4!}{4!} = 12-4+1 = 9$

for $n=5$, " " $= 5!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right]$

$= \frac{5!}{2!}-\frac{5!}{3!}+\frac{5!}{4!}-\frac{5!}{5!} = 60-20+5-1 = 44$

for $n=6$, " " $= 6!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}\right] = 265$

Q: If there are 3 letter & 3 addressed envelopes then find various arrangements?

Sol: Total arrangements $= 3! = 6$

All correct arrangements $= 1$

All wrong arrangements (De-Arrangements) $= 2$

$$\begin{pmatrix} E_1 & E_2 & E_3 \\ L_1 & L_2 & L_3 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_1 & L_3 & L_2 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_2 & L_1 & L_3 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_2 & L_3 & L_1 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_3 & L_1 & L_2 \end{pmatrix}, \begin{pmatrix} E_1 & E_2 & E_3 \\ L_3 & L_2 & L_1 \end{pmatrix}$$

All correct $= 1$

All wrong $= 2$

Q: there are six students appearing in a test in which there are 6 directed chairs. then find the number of arrangements in which

(1) None of the student is seated on his/her chair.

(a) 1

(b) 720

(c) 265

(d) 120

All wrong Arrangements (for $n=6$) $= 265$

Q2: Each student is seated on his/her assigned chair?

$\underbrace{\text{his/her assigned chair}}_{\text{All correct} = 1}$

(3) there are six students appearing in a test in which there are 6 directed chairs. then Find the number of arrangements in which at least two students are seated on wrong chair?

(a) 1

(b) 720

(c) 719

(d) 265

(e) 120

Total Arrangements $= 6! = 720$

All Correct Arrangements $= 1$

All wrong Arrangements $(n=6) = 6!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right] = 265$

No of Arrangements in which,

At least two students will be seated on wrong chair = Total − None

$=$ $\boxed{\text{Total} - \text{None}}$ will be seated on wrong Chair = Total − All C $= 720 - 1 = 719$

(4) No. of arrangements in which exactly 4 students are seated on wrong chair = ?

$$= \left({}^{6}C_{4} \times 9\right) \times \left({}^{2}C_{2} \times 1\right)$$

(5) No. of arrangements in which at least 4 students are seated on wrong chair = ?

$$= \left({}^{6}C_{4} \times 9\right) \times \left({}^{2}C_{2} \times 1\right) + \left({}^{6}C_{5} \times 44\right) \times \left({}^{1}C_{1} \times 1\right) + \left({}^{6}C_{6} \times 265\right)$$

PODCAST :- Various arrangements are as follows; (for $n=6$ persons)

$\underbrace{\text{(No wrong) or (Two wrong) or (Three wrong) or (Four wrong) or (Five wrong) or (All wrong)}}_{\text{Atleast 2 wrong}}$ = Total

(All correct) or (2 wrong) or (3 wrong) or (4 wrong) or (5 wrong) or (Dearrangents) = Total

$$ {}^6C_0 \times 1 + {}^6C_2 \times 1 + {}^6C_3 \times 2 + {}^6C_4 \times 9 + {}^6C_5 \times 44 + {}^6C_6 \times 265 = 720 $$

$$ = 1 + 15 \times 1 + 20 \times 2 + 15 \times 9 + 6 \times 44 + 1 \times 265 = 720 $$

$$ = 1 + ({}^6C_2 \times 1)({}^4C_4 \times 1) + ({}^6C_3 \times 2)({}^3C_3 \times 1) + ({}^6C_4 \times 9)({}^2C_2 \times 1) + ({}^6C_5 \times 44)({}^1C_1 \times 1) + ({}^6C_6 \times 265) = 720 $$

$f:\{1,2,3,4,5\}\to\{1,2,3,4,5\}$ that are ONTO and $f(i)\neq i\ \forall i$ the no. of such functions will be?

?

(a) 9 (b) 44 (c) 119 (d) 120

Sol.

HW

for $n=5$, De-Arrangement $= 44$

ONTO $\Rightarrow$ RNA

Thank you

Keep Hustling!