

CS & IT ENGINEERING

THEORY OF COMPUTATION



✓ Grammar

Lecture - 03



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Recap of Previous Lecture



Topic

????

Grammar Construction

↙ Grammar \Rightarrow language

↙ Types of Grammar

\Rightarrow F.A \leftrightarrow Regular grammar \leftrightarrow Regular Expression

\Rightarrow Ambiguous Grammar

Topics to be Covered



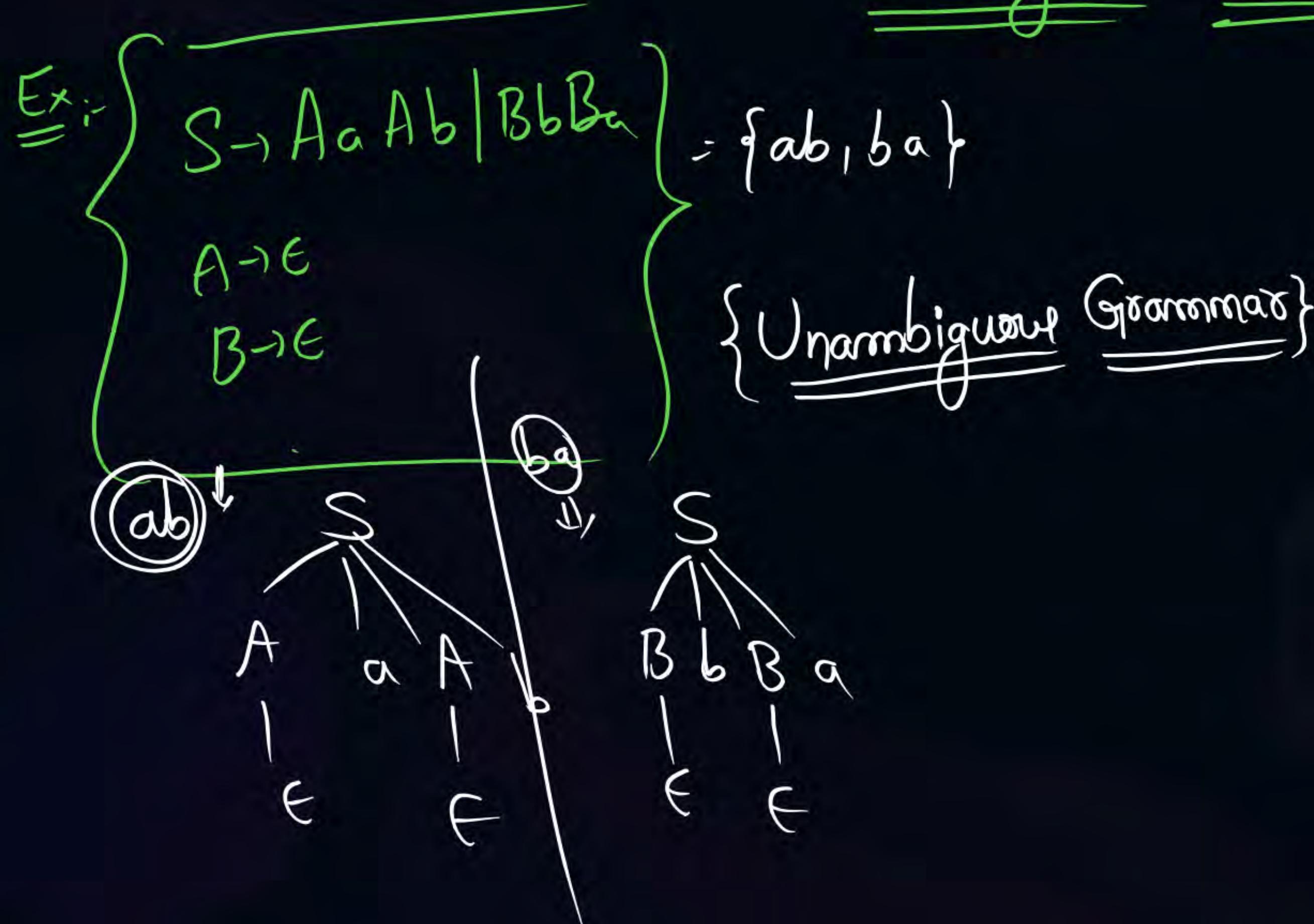
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Ambiguous Grammar

?? Simplification of grammar

?? Normal form of grammar

?? Decision Properties of Grammar

Ambiguous Grammar

Ambiguous Gram

②

$$S \rightarrow AB | BC$$

$$A \rightarrow BA | a$$

$$B \rightarrow CD | b$$

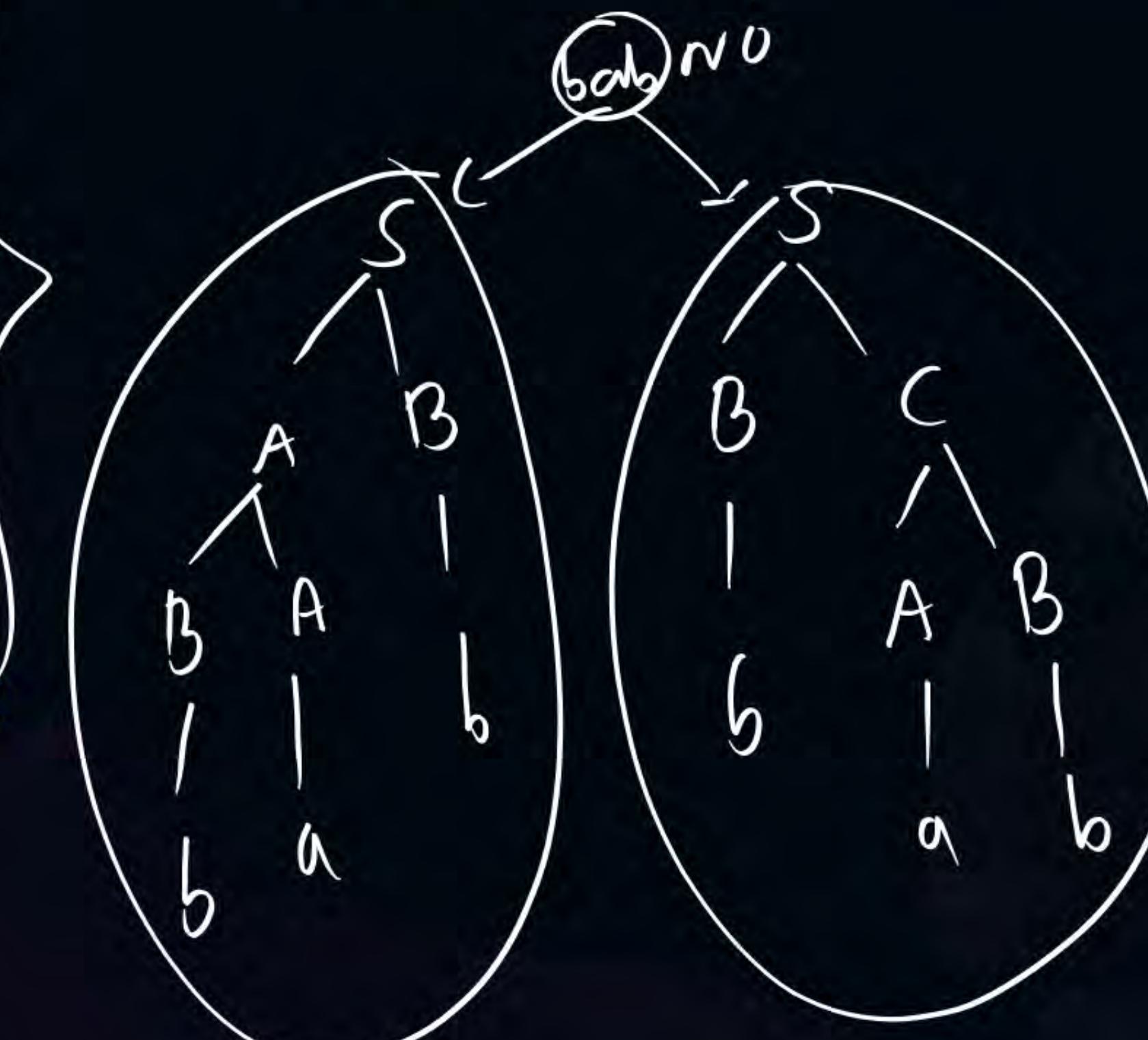
$$C \rightarrow a | AB$$

③

Ambiguous or not?

Yes

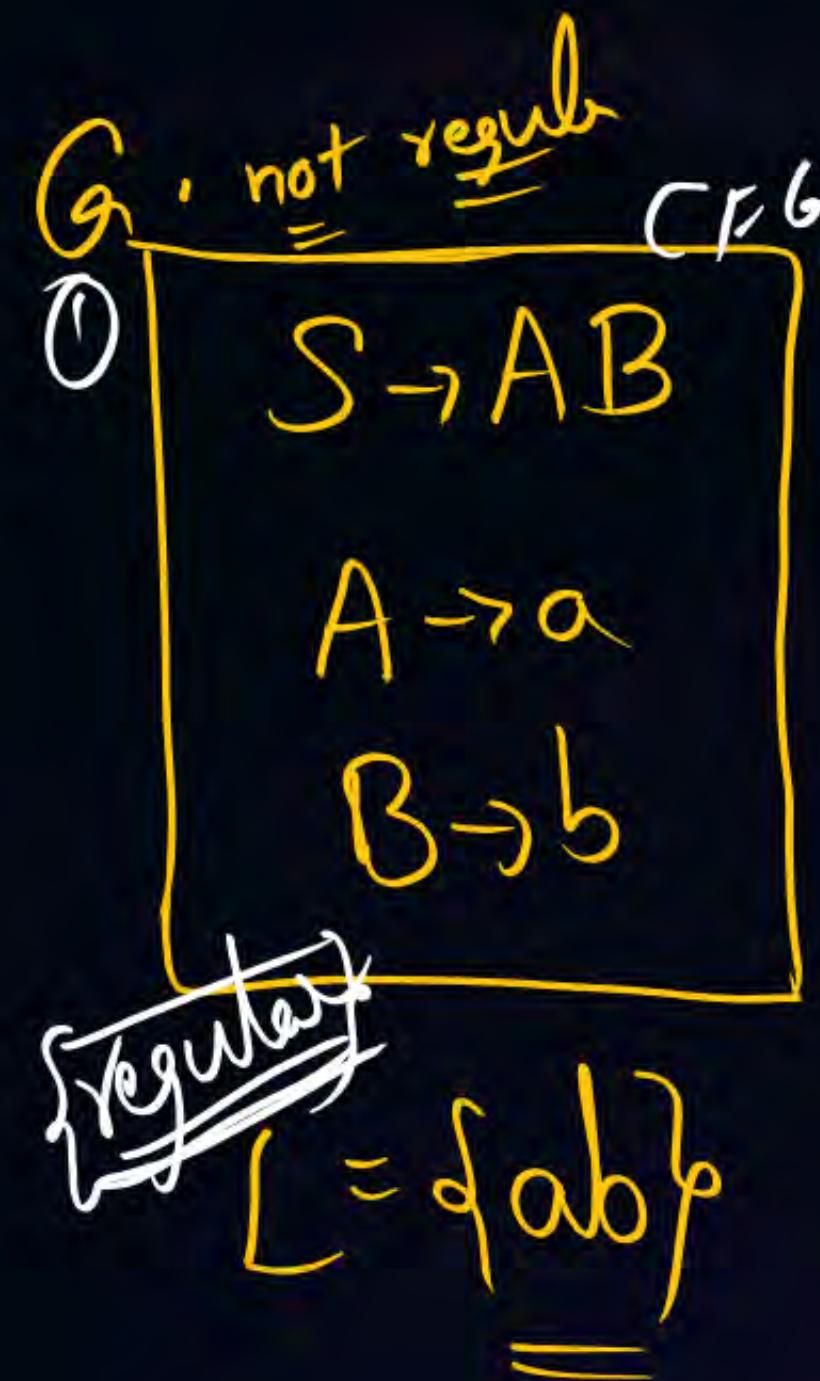
No



① { Ambiguity Checking is Undecidable Problem }

② Eliminating ambiguity is also Undecidable Problem

$$\underline{\underline{C \cdot F \cdot G}} \quad (\underline{\underline{A \rightarrow \omega}}) \quad \omega \in (v + T)^*$$



② $S \rightarrow aSb/ab$

$L = \{a^n b^n \mid n \geq 1\}$

non regular

③ $\{CFG\}$

$S \rightarrow AB/BC$

$A \rightarrow B A | a$

$B \rightarrow C C | b$

$C \rightarrow A B | a$

language = regular or not?

Regularity Problem

Undecidable Problem

Simplification of CFG

~~Removal of~~

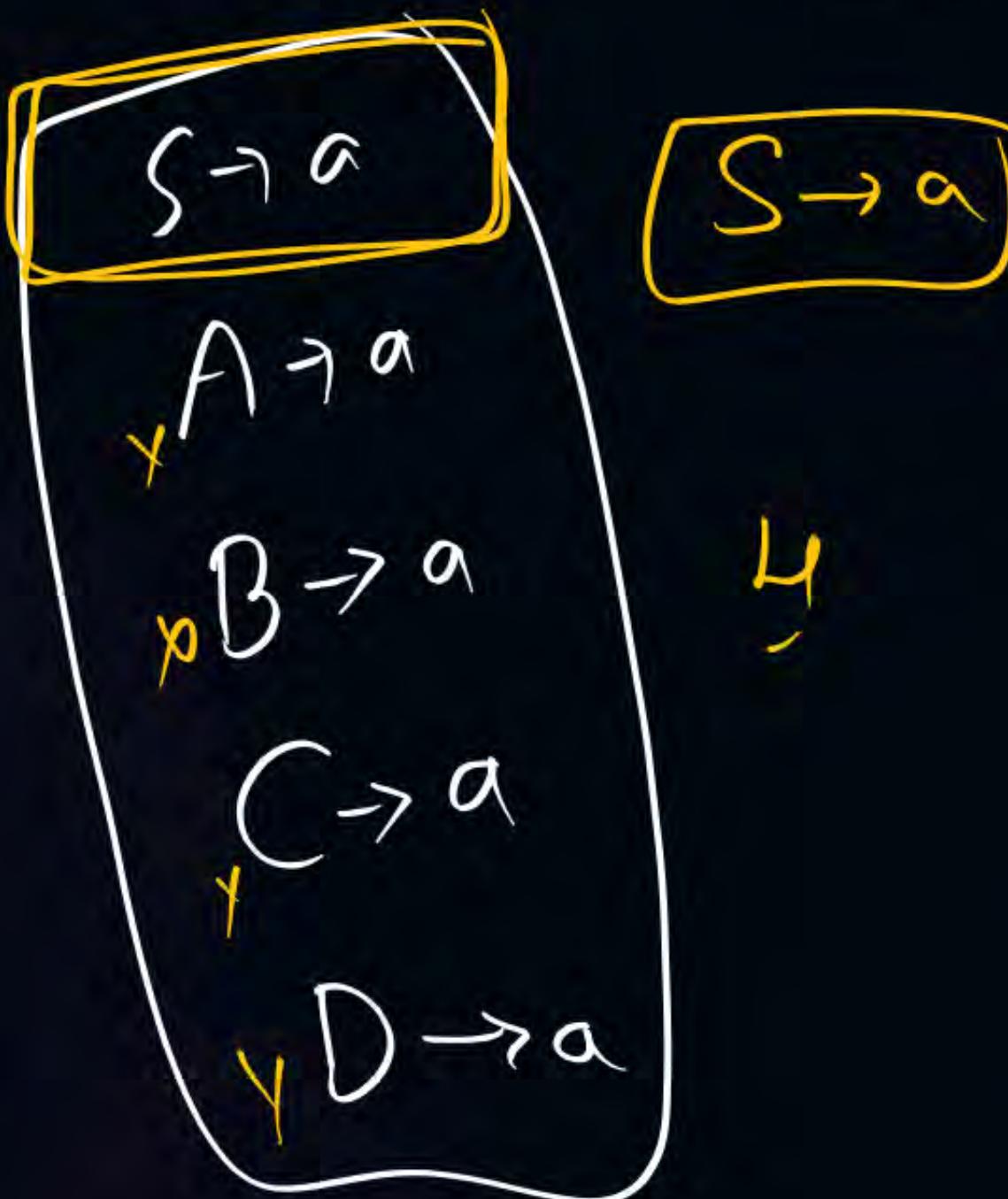
① Useless Variables (N·T)

② Unit Productions ($A \rightarrow B$)

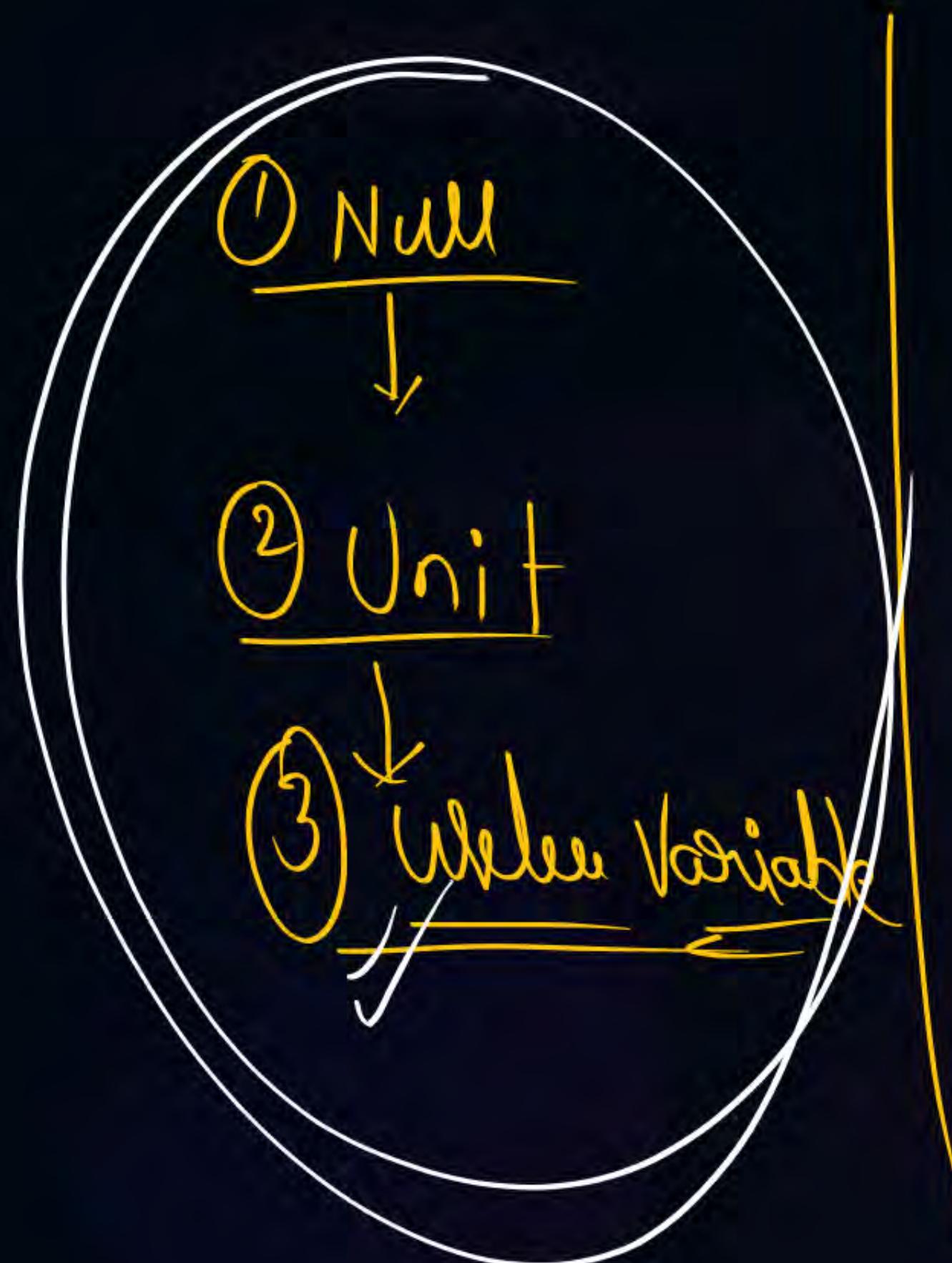
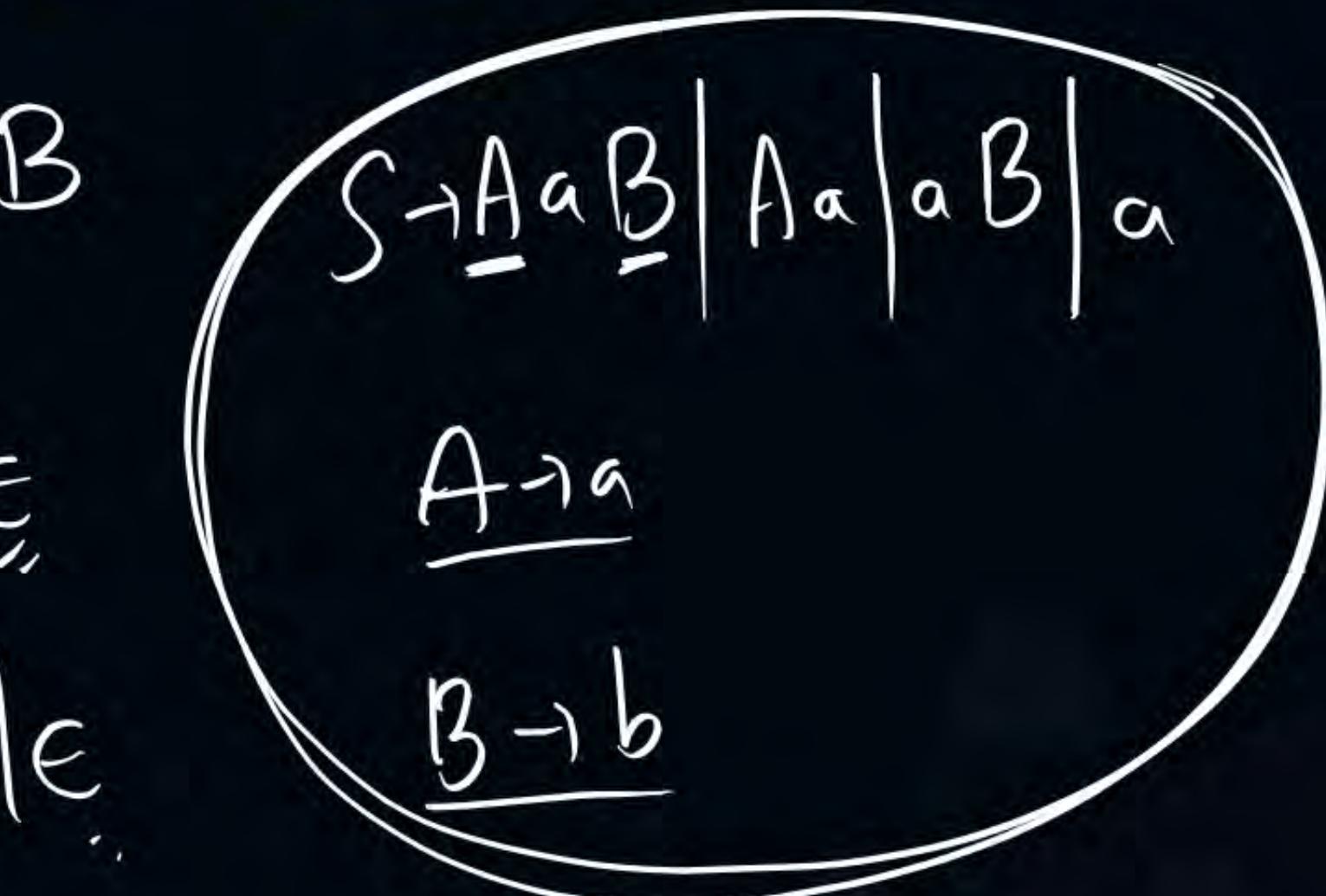
③ Null Productions ($A \rightarrow \epsilon$)

Unit Production
 $\equiv \underline{\underline{A \rightarrow B}}$

- ① $S \rightarrow A$
- ② $A \rightarrow B$
- ③ $B \rightarrow C$
- ④ $C \rightarrow D$
- ⑤ $D \rightarrow q$



Null Production ($A \rightarrow \epsilon$)



$S \rightarrow AaB$

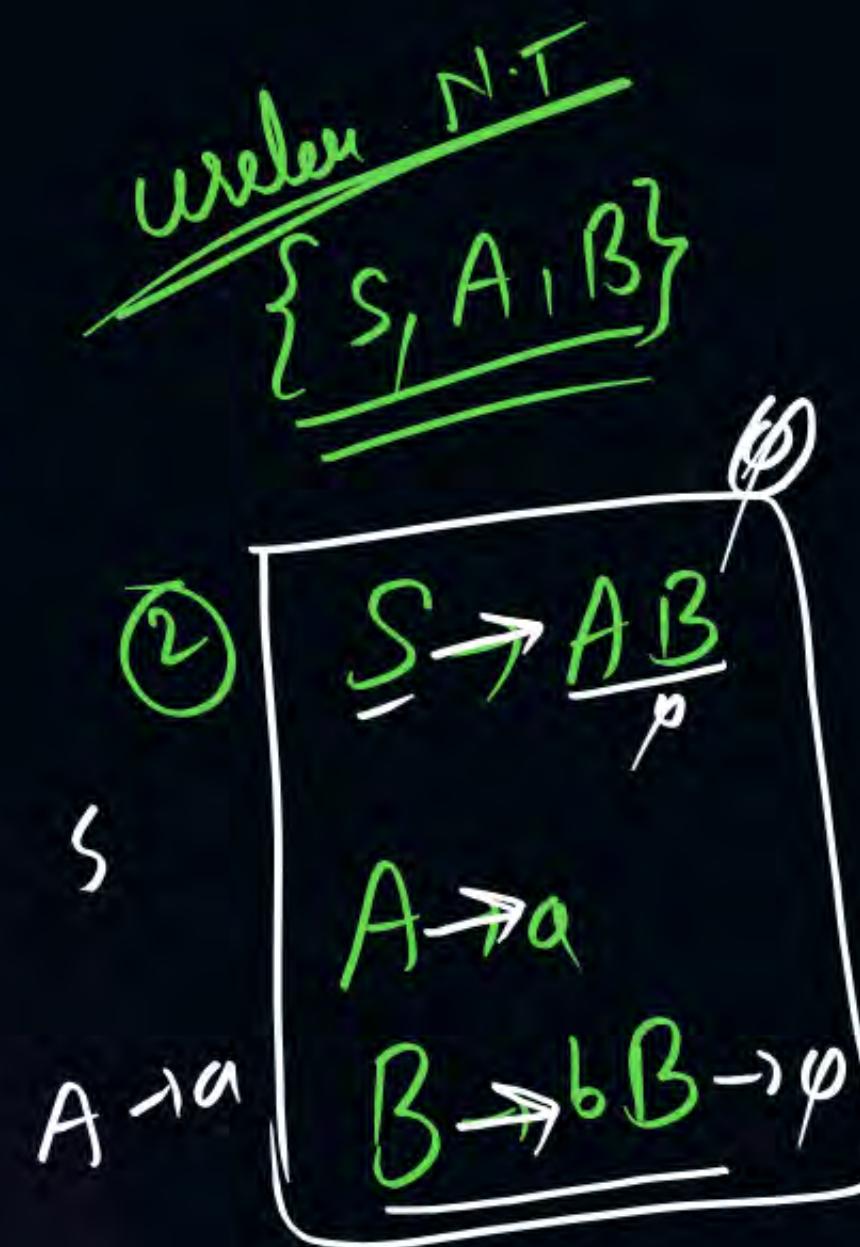
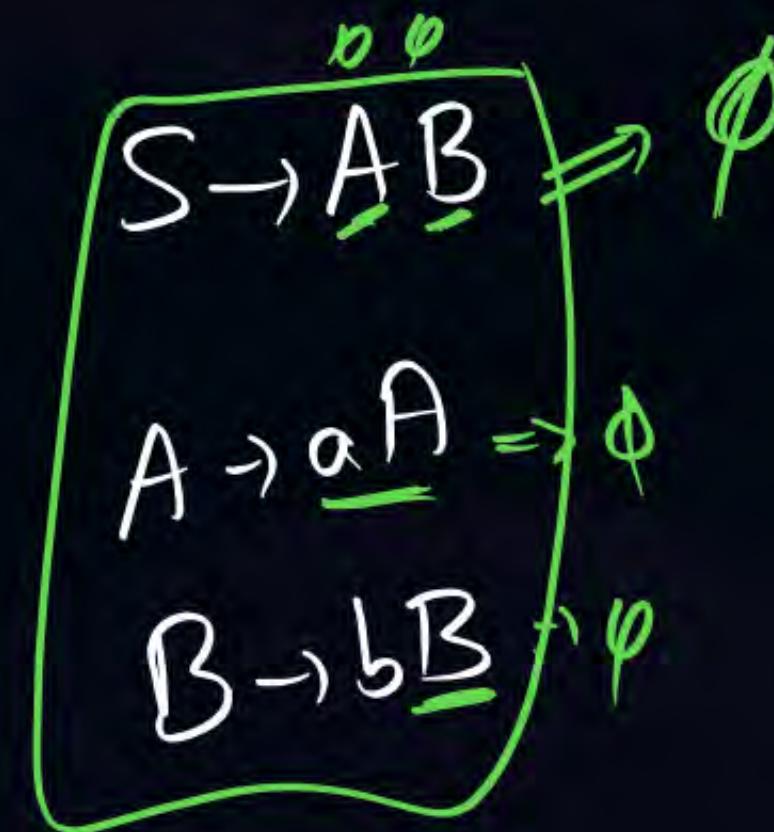
$A \rightarrow a \mid \epsilon$

$B \rightarrow b \mid \epsilon$

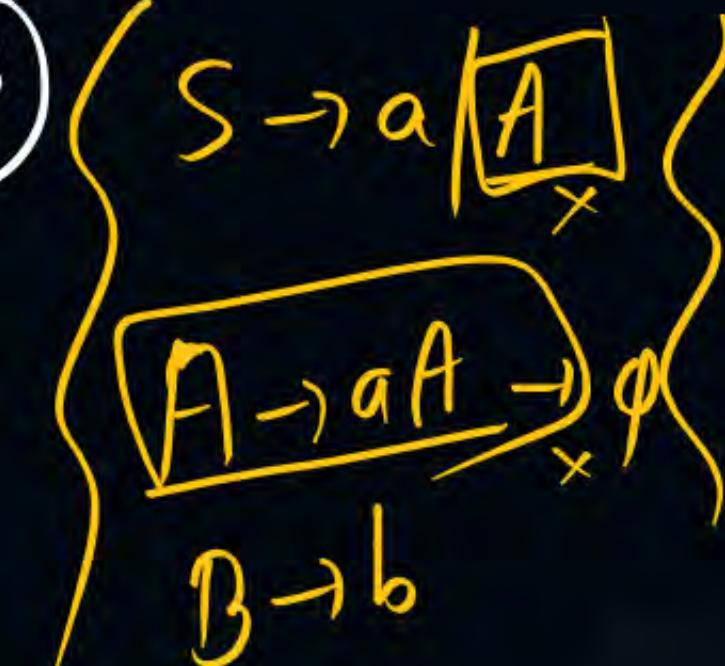
Elimination of useless Variables

- ~~Remove~~ (useless variables)
- ① Any Variable not generating ~~string~~
 - ② Any Variable not required for derivation

① \equiv



③



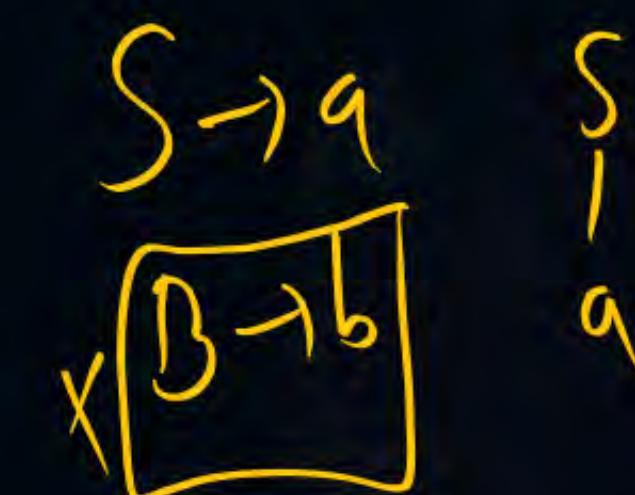
$S \xrightarrow{a} \square$

$S \xrightarrow{1-a} \square$

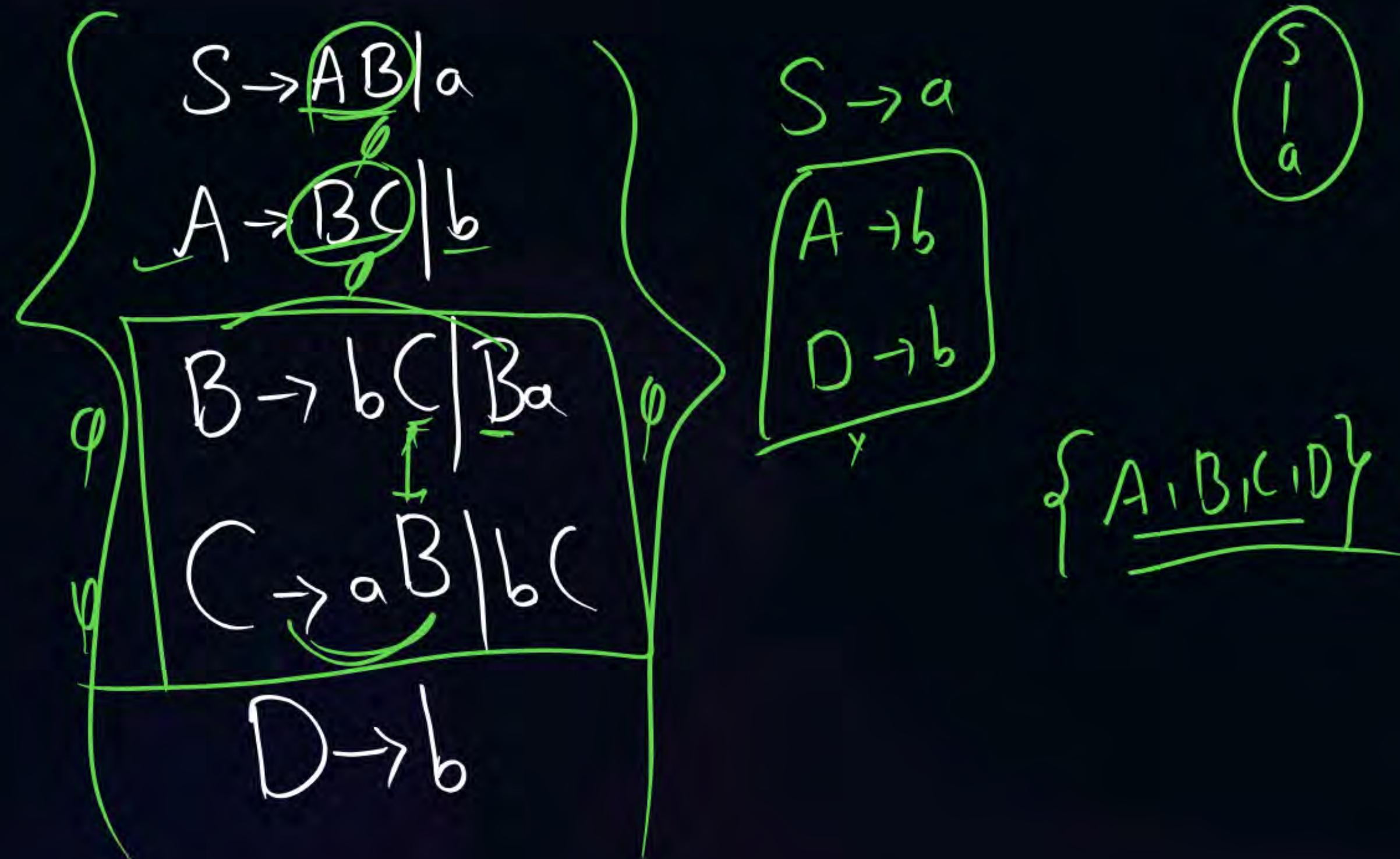
$\{A, B\}$



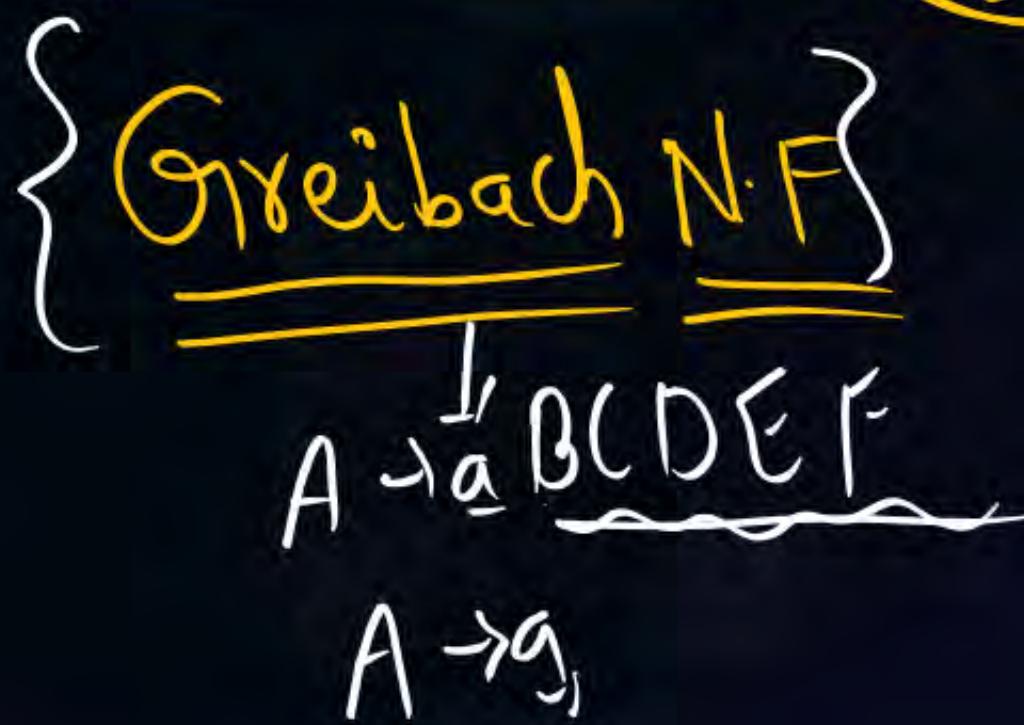
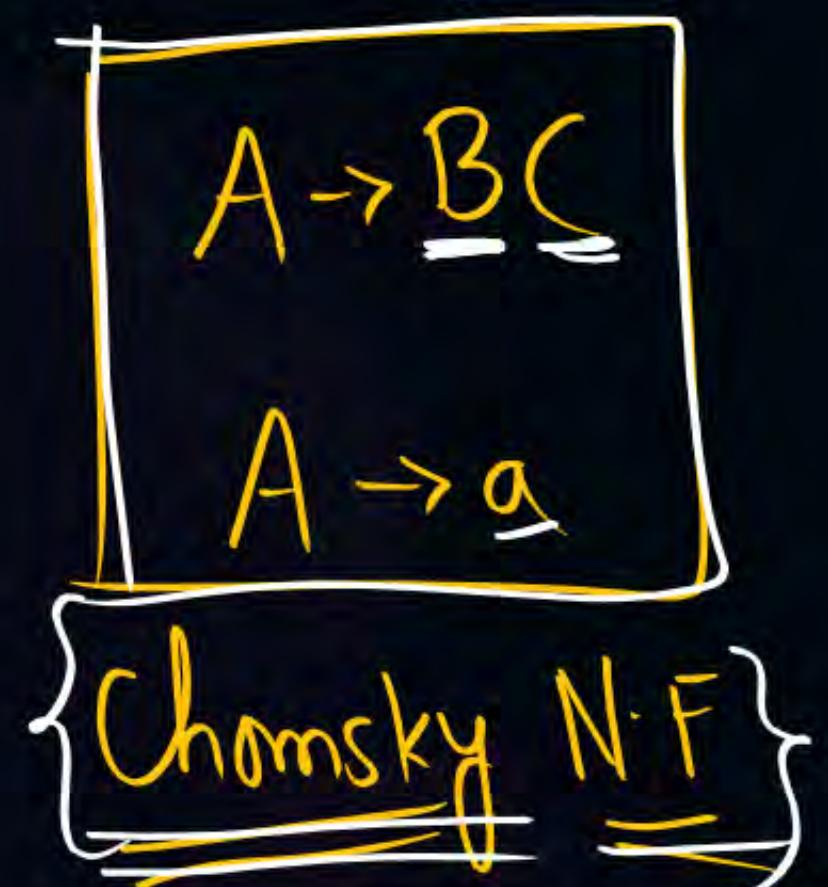
$\{S, B, A\}$



④ How many useless Variables Present in following grammar



Normal form of CFG ($A \rightarrow \underline{\alpha}$)



(Q) Convert following Grammar into CNF Grammar?

The logo consists of a stylized lowercase 'p' positioned above a lowercase 'w', all contained within a circular border.

$S \rightarrow aSb \mid ab$

①

```

graph TD
    S((S)) -- "S -> A S B" --> A_u((A))
    A_u -- "A -> a" --> B_u((B))
    B_u -- "B -> b" --> B_f((B))
  
```

$$\begin{array}{c}
 \textcircled{2} \\
 \begin{array}{c}
 \overline{S \rightarrow A S B} \\
 | \\
 S \rightarrow A X \mid A B \\
 | \\
 X \xrightarrow[A \rightarrow Y]{\quad} S B \\
 | \\
 B - V_0
 \end{array}
 \end{array}$$

① Convert total R.H.S into only N.T

by assigning terminals of new N.T

$$\begin{array}{l}
 S \rightarrow A \times | A B \\
 X \rightarrow S B \\
 A \rightarrow a \\
 B \rightarrow b
 \end{array}
 \quad
 \begin{array}{c}
 \text{S} \\
 / \quad \backslash \\
 A \quad B \\
 r \\
 a \\
 \boxed{2a \quad B} \\
 b \\
 \boxed{3a \quad b}
 \end{array}
 \quad
 \text{123}$$

(Q) Convert following Grammar into CNF Grammar?

$$\begin{array}{l} S \rightarrow bA | aB \\ A \rightarrow bAA | aS | a \\ B \rightarrow aBB | bS | b \end{array}$$

$$\begin{array}{l} ① \quad S \rightarrow YA | XB \\ A \rightarrow YAA | XS | a \end{array}$$

$$B \rightarrow XBB | YS | b$$

$$\begin{array}{l} X \rightarrow a \\ Y \rightarrow b \end{array}$$

$$② \quad S \rightarrow YA | XB$$

$$\begin{array}{l} A \rightarrow YP | XS | a \\ P \rightarrow AA \end{array}$$

$$\begin{array}{l} B \rightarrow XQ | YS | b \\ Q \rightarrow BB \end{array}$$

$$\begin{array}{l} X \rightarrow a \\ Y \rightarrow b \end{array}$$

③

NOTE

$$\frac{(2^k 4 - 1)}{(8 - 1)} \cdot 7 \quad \text{aabb}$$

① For generating n length string from CNF grammar total number of productions required is $\boxed{2n-1}$
(Derivation steps)

② For generating n length string from GNF grammar total number of productions required is $\boxed{n \text{ only.}}$

(Q) Convert following Grammar into CNF Grammar?

Decision Properties of CFG

- Decidable
- ① Emptyness Problem [$\text{if } L(\text{CFG}) = \emptyset?$]
 - ② Finiteness Problem [$\text{if } L(\text{CFG}) = \text{finite?}$]
 - ③ Membership Problem [$\text{if } w \in (\text{CFG})$]

① $\{ S \rightarrow A\overline{B} - \emptyset, A \rightarrow a, B \rightarrow b\overline{B} - \emptyset \}$ empty (a) not?



Thank You!