

# DS & AI CS & IT

## Probability & Statistics

Lecture No. 14



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# Recap of previous lecture



Topic

EXPONENTIAL DISTRIBUTION





# Topics to be Covered



Topic

UNIFORM DISTRIBUTION  
&  
Normal Distribution (Part-1)



Vehicles arriving at an intersection from one of the approach roads follow the Poisson distribution. The mean rate of arrival is 900 vehicles per hour. If a gap is defined as the time difference between two successive vehicle arrivals (with vehicles assumed to be points), the probability (up to four decimal places) that the gap is greater than 8 seconds is \_\_\_\_\_.

HW

Arrival Rate ( $\lambda$ ) = 900 veh/hr =  $\frac{900}{60} \text{ veh/min} = \frac{900}{60 \times 60} \text{ veh/sec} = \frac{1}{4} \text{ veh/sec}$   
 i.e. Average time Gap b/w two successive vehicles =  $\frac{1}{\mu} = 4 \text{ sec} \Rightarrow \mu = \frac{1}{4} \text{ per sec}$

C.D.F:  $F(t) = \begin{cases} 1 - e^{-\mu t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$F(8) = 1 - e^{-\mu(8)} = 1 - e^{-2}$

$P(t > 8) = \int_8^{\infty} f(t) dt = \dots = \frac{1}{e^2} = 0.135$

(M-Q)  $P(t > 8) = 1 - P(t \leq 8) = 1 - P(0 \leq t \leq 8) = 1 - F(8) = e^{-2} = \frac{1}{e^2}$



The life of a bulb (in hours) is a random variable with an exponential distribution  $f(t) = \alpha e^{-\alpha t}$ ,  $0 \leq t \leq \infty$ . The probability that its value lies between 100 and 200 hours is

(a)  $e^{-100\alpha} - e^{-200\alpha}$

(b)  $e^{-100} - e^{-200}$

(c)  $e^{-100\alpha} + e^{-200\alpha}$

(d)  $e^{-200\alpha} - e^{-100\alpha}$

$t = \{\text{Life of Bulb in hrs}\}$ , pdf is  $f(t) = \alpha e^{-\alpha t}$

ie  $\mu = \alpha$  so c.d.f is  $F(t) = 1 - e^{-\alpha t}$

$$F(100) = 1 - e^{-100\alpha}$$

$$F(200) = 1 - e^{-200\alpha}$$

(M-I)  $P(100 < t < 200)$

$$= \int_{100}^{200} f(t) dt = \int_{100}^{200} \alpha e^{-\alpha t} dt$$

$$= \alpha \left[ \frac{e^{-\alpha t}}{-\alpha} \right]_{100}^{200} = - (e^{-200\alpha} - e^{-100\alpha}) = e^{-100\alpha} - e^{-200\alpha} = \text{(a)}$$

(M-II)  $P(100 < t < 200)$

$$= F(200) - F(100)$$

$$= e^{-100\alpha} - e^{-200\alpha}$$



For a single server with Poisson arrival and exponential service time, the arrival rate is 12 per hour. Which one of the following service rates will provide a steady state finite queue length?

- (a) 6 per hour
- (b) 10 per hour
- (c) 12 per hour
- (d) 24 per hour



Arrival Rate ( $\lambda$ ) = 12  
Service Rate ( $\mu$ ) = ?

Traffic Intensity in Queue is

$$\rho = \frac{\lambda}{\mu} = \frac{\text{Arrival Rate}}{\text{Service Rate}}$$

for finite Queue length  $\rho < 1$  otherwise Queue will keep on Increasing

ie  $\text{Arrival Rate } (\lambda) < \text{Service Rate } (\mu)$

$\therefore \lambda = 12$  so  $\mu > 12$  Hence only option is (d) ie  $\mu = 24$  per hr.



# UNIFORM DIST (General points)

D.R.V

$X = \{ \text{outcome of Die} \}$   
 $= \{ 1, 2, 3, 4, 5, 6 \}$

$X :$	1	2	3	4	5	6
$P(X) :$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Here  $P(X) = \text{const} \quad \forall X$

So  $X$  is uniformly distributed D.R.V

C.R.V Let  $x \in (0, l)$



$P(\text{choosing any Random Point } x) = ?$

$$\approx \frac{1}{\infty} \rightarrow 0$$

Another way of thinking :-

we can assume that we have  $l$  points.

So  $P(x) = \frac{1}{l}$  where  $l \rightarrow \infty$

ie  $P(x) = \text{const}$  So  $x$  is U.R.V b/w 0 &  $l$ .



# UNIFORM DISTRIBUTION

“ If probabilities are Distributed uniformly for given set of values of  $x$  then we should follow U-Dist ”

for eg if a Die is thrown once

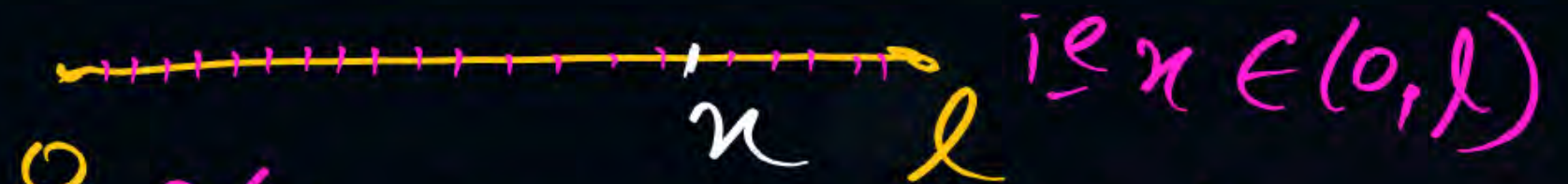
then possible outcomes are

$$X = \{1, 2, 3, 4, 5, 6\}$$

$x:$	1	2	3	4	5	6
$P(x):$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

i.e for  $1 \leq X \leq 6 \Rightarrow P(x) = \frac{1}{6} = \text{const} \Rightarrow X$  is U.R.V of Discrete type.

eg: let  $x = \{\text{Random point of stick of length } l\}$



$P(\text{Choosing any Random point } x) =$   
i.e  $P(x) = \frac{1}{l} = \text{const}$  So  $x$  is U.R.V in  $(0, l)$



Def<sup>n</sup>: Let  $x \in \mathbb{C}, \mathbb{R}, \mathbb{V}$  defined in  $[a, b]$

s.t it's p.d.f is given as

$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otw} \end{cases}$$

$f(x)$

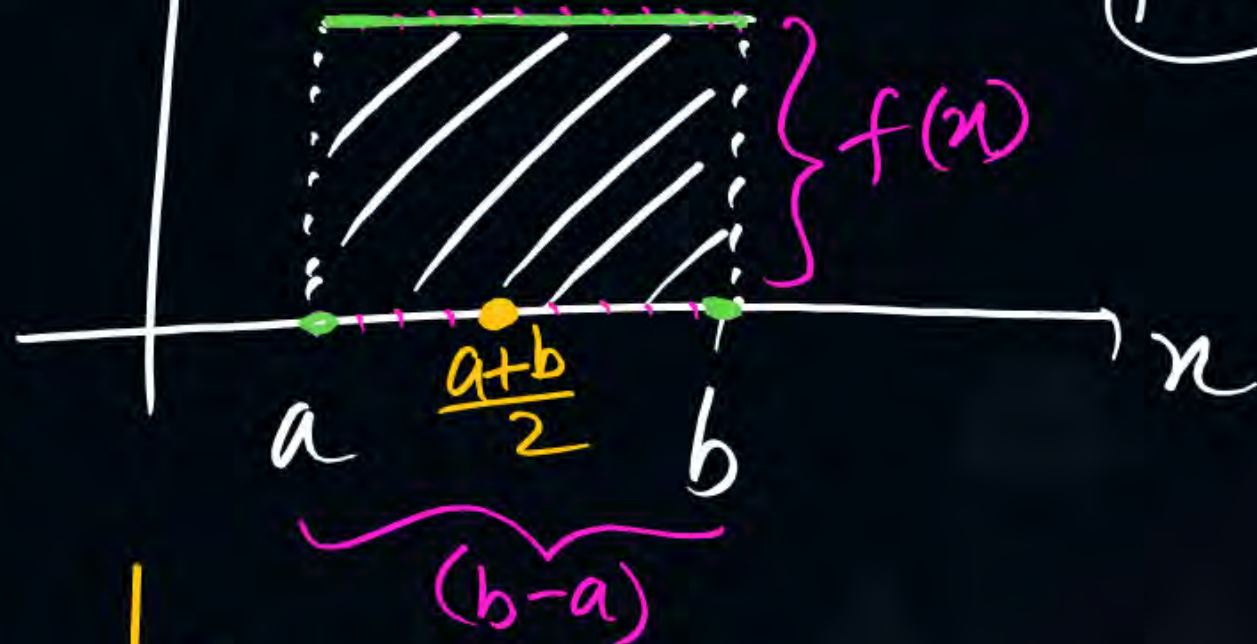
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then  $x$  is called U.R.V

Verify:

$$\int_{-\infty}^{\infty} f(x) dx = \int_a^b f(x) dx = \int_a^b \frac{1}{b-a} dx$$
$$= \frac{1}{b-a} (b-a) = 1 \quad \checkmark$$

②  $x = \{ \text{which is Required} \}$



Area of this Rectangle = 1

length  $\times$  height = 1

$$(b-a) \times f(x) = 1 \Rightarrow f(x) = \frac{1}{b-a}$$



PW

$$(3) \text{Mean}(x) = E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_a^b x \left( \frac{1}{b-a} \right) dx = \frac{1}{b-a} \left( \frac{x^2}{2} \right)_a^b = \frac{a+b}{2}$$

Sp. Note: whenever in a question,  $x$  is defined in any interval then  $x$  is U-R-V.

$$(4) \text{Var}(x) = E(x^2) - E^2(x) = \int_{-\infty}^{\infty} x^2 f(x) dx - \left( \frac{a+b}{2} \right)^2 = \int_a^b x^2 f(x) dx - \left( \frac{a+b}{2} \right)^2 = \dots = \frac{(b-a)^2}{12}$$

$$(5) \text{SD}(x) = \frac{b-a}{\sqrt{12}}$$

$$(6) P(\alpha \leq x \leq \beta) = \int_{\alpha}^{\beta} f(x) dx = \text{Area under } f(x) \text{ b/w } \alpha \text{ \& } \beta = \frac{\beta - \alpha}{b-a}$$

$$(7) \text{C.D.f at 'x'} = F(x) = \int_{-\infty}^x f(x) dx = \int_a^x \left( \frac{1}{b-a} \right) dx = \frac{x-a}{b-a}$$



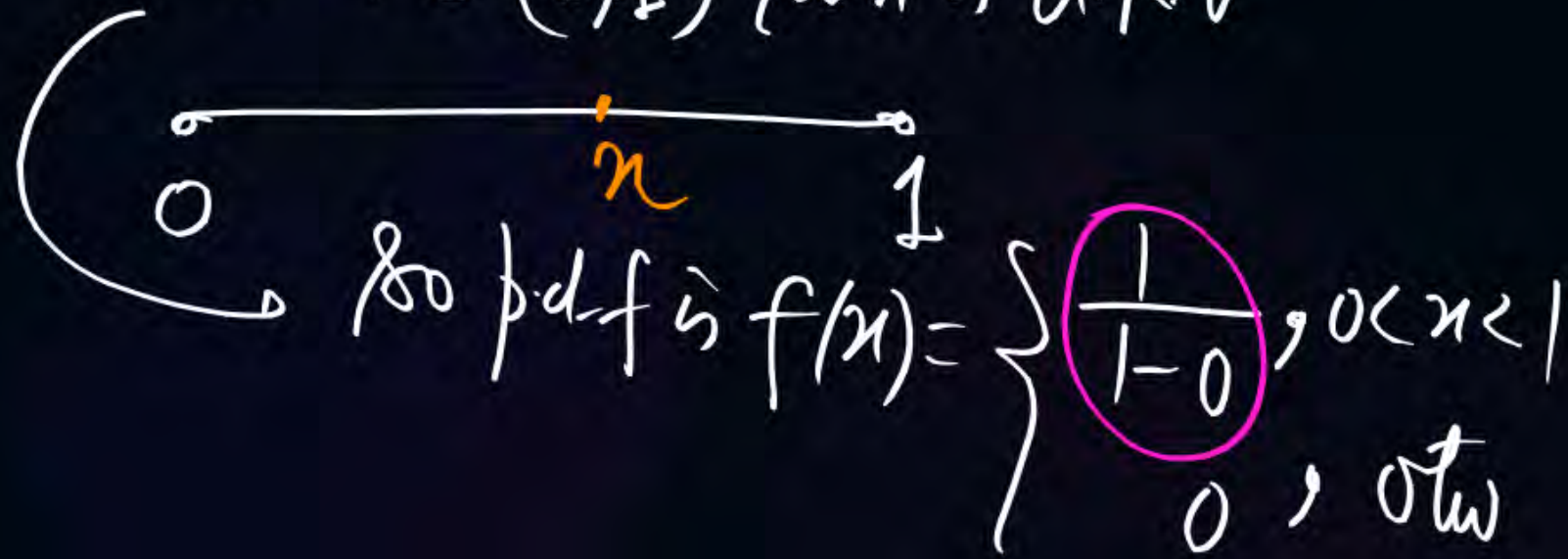
Q Suppose you break a stick of unit length at a uniformly chosen Random point then find Expected

(1) length of broken stick, (2) length of shorter stick

(3) length of larger stick. Gate

Sol: (1)  $x = \{\text{length of broken stick}\}$

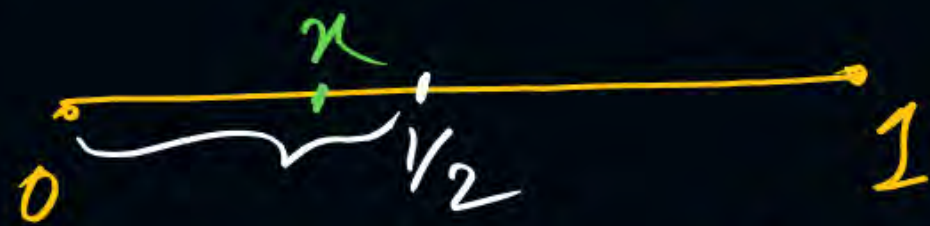
$\because x \in (0, 1)$  so  $x$  is U.R.V



$$E(x) = ? \rightarrow \begin{aligned} &= \frac{a+b}{2} = \frac{0+1}{2} = 0.5 \text{ units} \\ &= \int_{-\infty}^{\infty} x f(x) dx = \int_0^1 x(1) dx = 0.5 \end{aligned}$$



(ii)  $x = \{\text{length of shorter stick}\}$



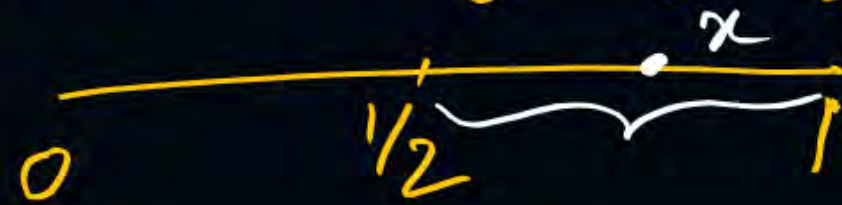
ie  $x \in (0, \frac{1}{2})$  is  $x$  is U.R.V.

so p.d.f is  $f(x) = \begin{cases} \frac{1}{\frac{1}{2} - 0} & 0 < x < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

$$E(x) = \frac{a+b}{2} = \frac{0 + \frac{1}{2}}{2} = \frac{1}{4} = 0.25 \text{ units.}$$

Av length of shorter stick = 0.25 units.

(iii)  $x = \{\text{length of larger stick}\}$



so  $x \in (\frac{1}{2}, 1)$  is  $x$  is U.R.V.

so p.d.f is  $f(x) = \begin{cases} \frac{1}{1 - \frac{1}{2}} & \frac{1}{2} < x < 1 \\ 0 & \text{otherwise} \end{cases}$

$$E(x) = \frac{a+b}{2} = \frac{\frac{1}{2} + 1}{2} = \frac{3}{4} = 0.75$$

so Av length of larger stick = 0.75



Sp. observation: In part (ii)

p.d.f is  $f(x) = \begin{cases} \textcircled{2}, & 0 < x < \frac{1}{2} \\ 0, & \text{otw} \end{cases}$

? ?

that's why I told you,  $f(x)$  is not probability at  $x$

& C.D.F,  $F(x)$  is the representation of prob. i.e.  $0 \leq F(x) \leq 1$



$X$  is a uniformly distributed random variable that takes values between 0 and 1. The value of  $E(X^3)$  will be

(a) 0

(b)  $1/8$

☒ (c)  $1/4$

(d)  $1/2$



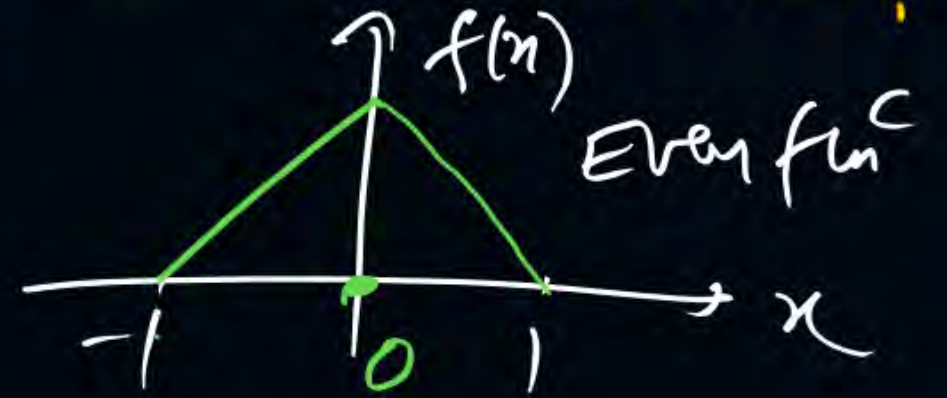
$x \in (0,1)$  so p.d.f is  $f(x) = \begin{cases} \frac{1}{1-0} & , 0 < x < 1 \\ 0 & , \text{otw} \end{cases}$

3<sup>rd</sup> Moment  $E(x^3) = \int_{-\infty}^{\infty} x^3 f(x) dx = \int_0^1 x^3 (1) dx = \left( \frac{x^4}{4} \right)_0^1 = \frac{1}{4}$



Q If  $f(x) = \begin{cases} 1+x & , -1 \leq x \leq 0 \\ 1-x & , 0 \leq x \leq 1 \end{cases}$  is valid p.d.f for R.V 'x' then  $E(x) = ?$

(M-I) Already solved:  $f(x) = 1 - |x|$ ,  $-1 \leq x \leq 1$



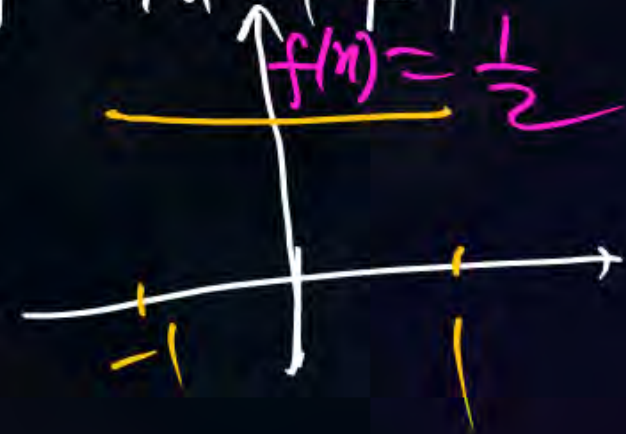
$$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_{-1}^1 \underbrace{x f(x)}_{\text{odd}} dx = 0$$

(M-II) Using the concept of U. Dist

$\therefore x$  is defined in  $-1$  to  $1$  i.e.  $x \in [-1, 1]$  & p.d.f is  $f(x) = \begin{cases} \frac{1}{1-(-1)} & , -1 \leq x \leq 1 \\ 0 & , \text{otw} \end{cases}$

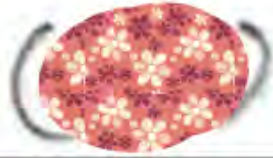
so we can take  $x$  is uniformly distributed b/w  $-1$  &  $1$

$$\therefore E(x) = \frac{a+b}{2} = \frac{-1+1}{2} = 0. \underline{\underline{Ans}}$$





Assume that in a traffic junction, the cycle of the traffic signal lights is 2 minutes of green (vehicle does not stop) and 3 minutes of red (vehicle stops). Consider that the arrival time of vehicles at the junction is uniformly distributed over 5 minute cycle. The expected waiting time (in minutes) for the vehicle at the junction is \_\_\_\_\_



(a) 0.5 min (b) 0.3 min

(c) 0.9 min (d) Data Inadequate

M-I w.k. that in Exp Dist, Av. waiting time =  $\frac{1}{\mu}$  (but  $\mu$  is not given)

So how to find  $E(x) = ??$



Av waiting time (if vehicle is arriving with in 2 mins) = 0 min =  $\bar{w}_R$

Av " " ( " " " " in next 3 mins) =  $\frac{0+3}{2} = 1.5 \text{ min} = \bar{w}_G \because x \in (0, 3)$



$$\text{overall waiting time} = \frac{\sum X}{N} = \frac{\sum X_1 + \sum X_2}{l_1 + l_2} = \frac{\bar{w}_p \times l_1 + \bar{w}_q \times l_2}{l_1 + l_2}$$

$$\bar{w}_p = \frac{\sum X_1}{l_1}$$

$$\bar{w}_q = \frac{\sum X_2}{l_2}$$

$$= \frac{0 \times 2 + 1.5 \times 3}{2 + 3} = \frac{4.5}{5} = 0.9 \text{ mins}$$

$$= 0.9 \times 60 \text{ sec}$$

$$= 54 \text{ sec}$$

Note: if  $C_1$  &  $C_2$  are Quantities, whose weights are  $w_1$  &  $w_2$  then weighted Average =  $\frac{C_1 w_1 + C_2 w_2}{w_1 + w_2}$  where  $C_1$  &  $C_2$  are Quantities, whose Av is being found.

Q If Amit Borrowed Rs 6000 at 4% ROI & Rs 4000 at 6% ROI then find the Average ROI, amit has to pay at the end of one year = ?

$$= \frac{4\% \times 6000 + 6\% \times 4000}{6000 + 4000} = 4.8\%$$



Q II Interest paid on Rs 4000 = 240 Rs

" " " Rs 6000 = 240 Rs.

---

Total Interest, Amit has to pay on Rs 10000 = ? = 480 Rs.

As 480 is how much % of Rs 10000

$$\Rightarrow 480 = \frac{?}{100} \times 10000$$

$$\Rightarrow ? = \frac{480 \times 100}{10000} = 4.8\% \text{ R.O.I.}$$



(M-II)

$x = \{ \text{Arrival time of vehicle at junction} \}$

i.e.  $x \in (0, 5)$  so  $x$  is U.R.V. so  $f(x) = \begin{cases} \frac{1}{5-0}, & 0 < x < 5 \\ 0, & \text{otw} \end{cases}$



i.e. waiting time is a func<sup>n</sup> of 'x'

Let us take it as  $g(x)$

then waiting time =  $g(x) = \begin{cases} 0, & 0 < x < 2 \\ 5-x, & 2 < x < 5 \end{cases}$

$$E\{g(x)\} = \int_{-\infty}^{\infty} g(x) f(x) dx = \int_0^5 g(x) \left(\frac{1}{5}\right) dx = \int_0^2 0 + \int_2^5 (5-x) \frac{1}{5} dx$$

$$= \frac{1}{5} \left( 5x - \frac{x^2}{2} \right) \Big|_2^5 = \dots = 0.9 \text{ min} = 54 \text{ sec.}$$



Q If a die is thrown Large number of times then find the Expected Value of the outcome?

Ans:  $X = \{ \text{Value of outcome in single throw} \} = \{ 1, 2, 3, 4, 5, 6 \}$

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$  i.e.  $X$  is U.R.V of Discrete Type.

Each  $p_i = \frac{1}{6} \because$  Die is thrown large number of times (ie using concept of history)

$$\begin{aligned} E(X) &= \sum p_i X_i = p_1 X_1 + p_2 X_2 + \dots + p_6 X_6 \\ &= \frac{1}{6} (1 + 2 + 3 + 4 + 5 + 6) = \boxed{3.5} \end{aligned}$$

ie Min outcome = 1, Max outcome = 6, Av outcome = 3.5  
M-II  $E(X) = \frac{a+b}{2} = \frac{1+6}{2} = 3.5$



A random variable is uniformly distributed over the interval 2 to 10. Its variance will be

~~(a) 16/3~~

PyQ

(b) 6

(c) 256/9

(d) 36

ATQ,  $x \in (2, 10)$

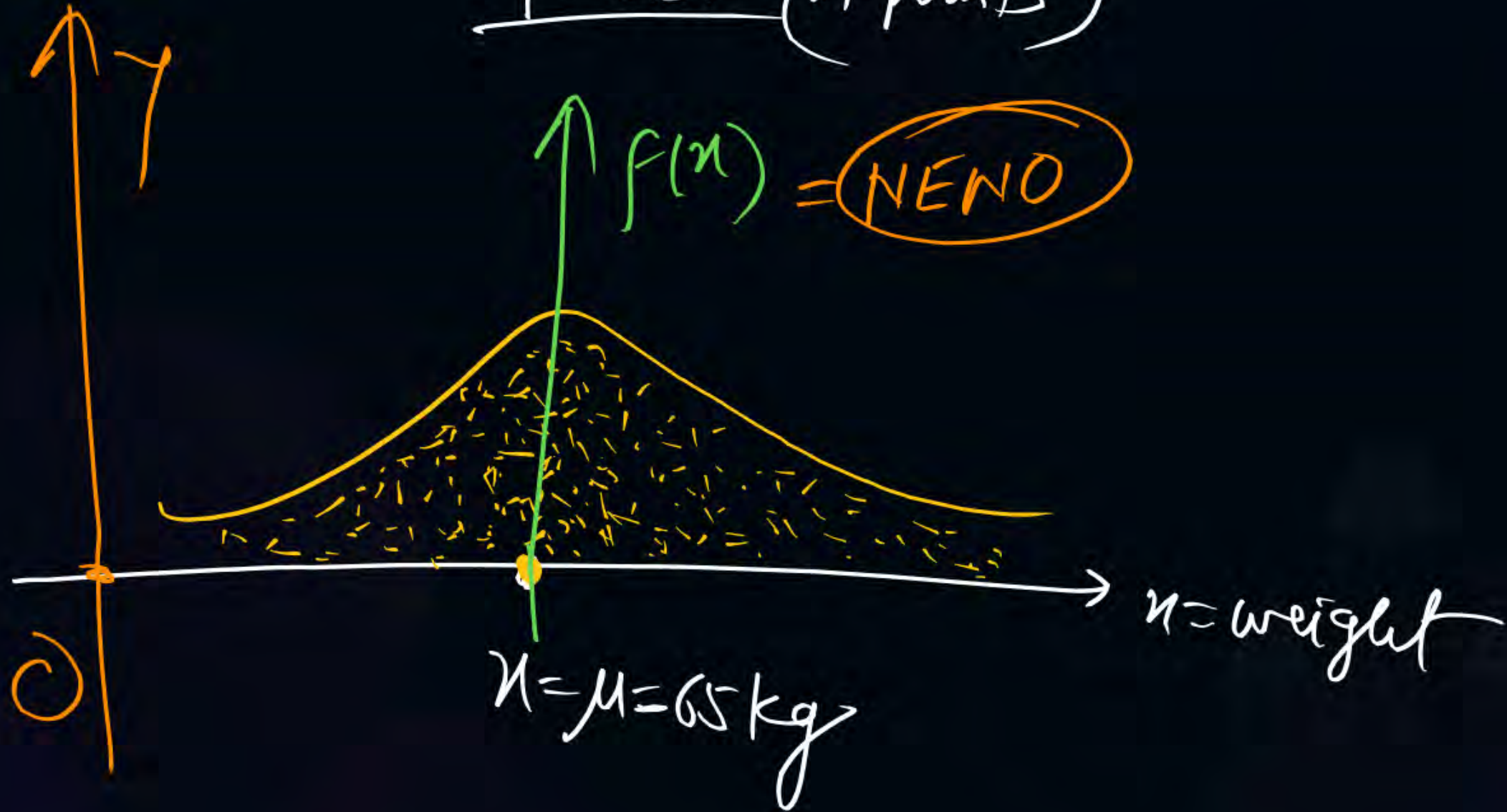
$$\text{Var} = \frac{(b-a)^2}{12} = \frac{(10-2)^2}{12} = \frac{64}{12} = \frac{16}{3}$$

M-II

$$\text{Var}(x) = E(x^2) - (E(x))^2 = \int_2^{10} x^2 f(x) dx - \left( \int_2^{10} x f(x) dx \right)^2 = \dots = \frac{16}{3}$$



N Dist (G-points)

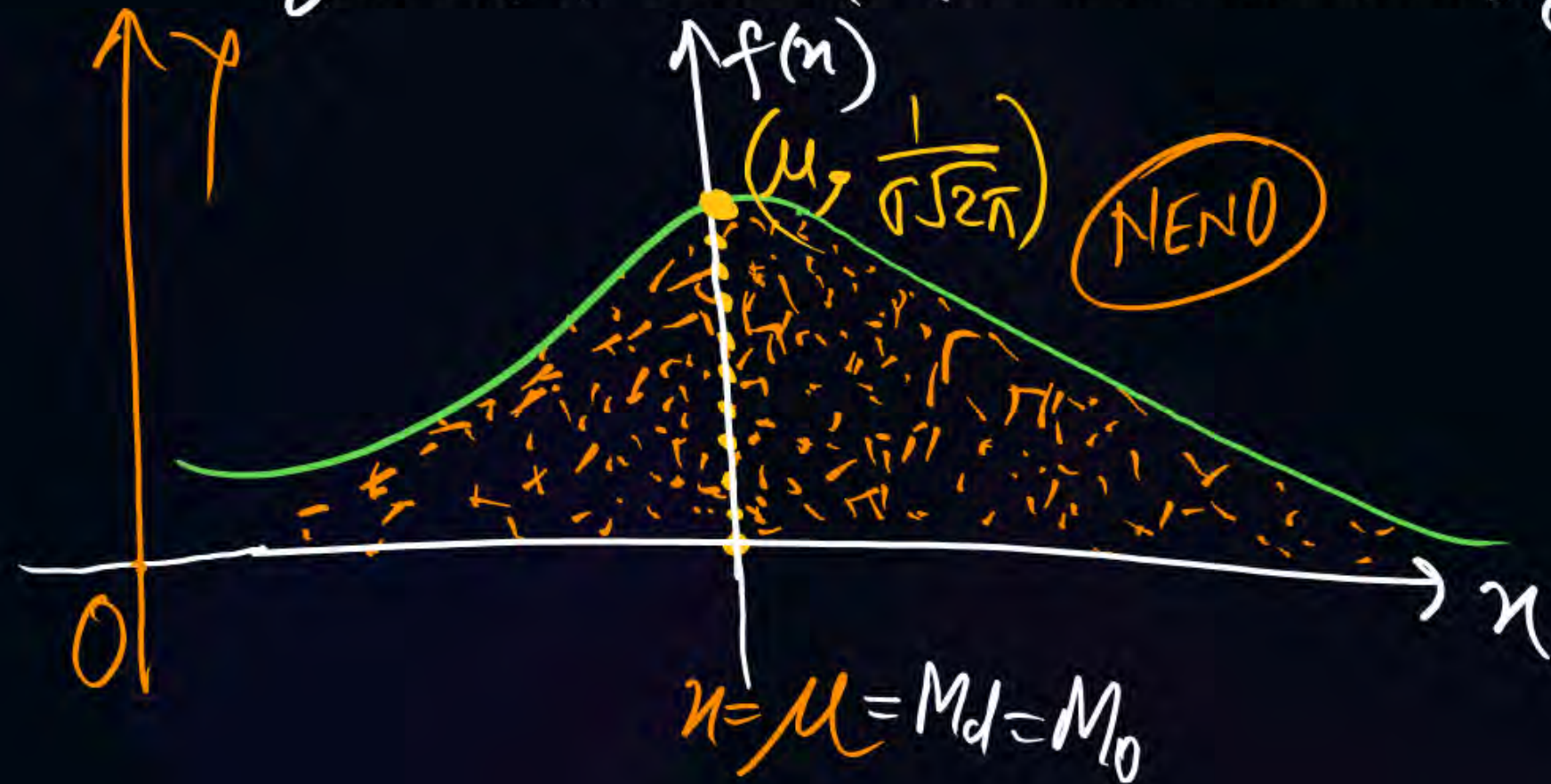




# NORMAL DIST (GAUSSIAN DIST.) / Bell Curve

(100% they will ask quest from this topic)

“Whenever Random Variable  $x$  has a tendency to accumulate about its average value ( $\mu$ ) then  $x$  is called Normal R.V and the distribution formed is called N. Distribution for eg Height, weight, Age etc.

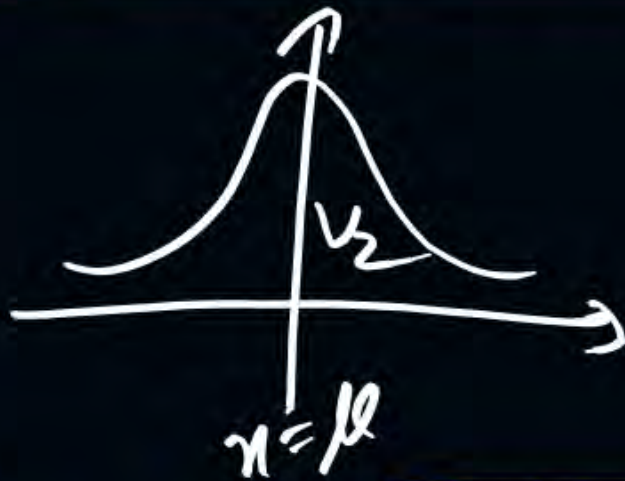
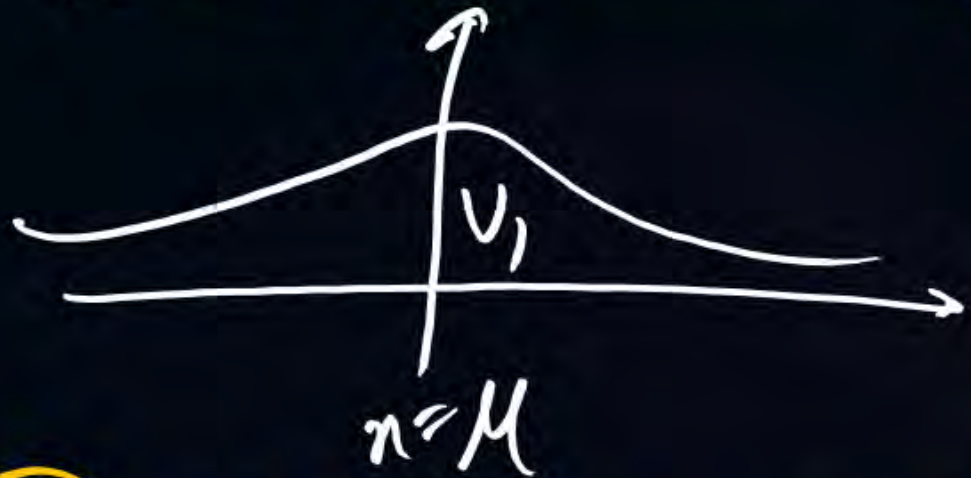


$$f(x) = \frac{1}{\sigma \times \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2} = \text{NEND}$$

$$\text{So } f(\mu) = \frac{1}{\sigma \sqrt{2\pi}}$$



- Note ① N-Curve is a symmetrical curve with symmetry about Mean is about  $x = \mu$
- ② N-Curve is a Bell shaped curve where variance is represented by width of the Bell is More Flat Curve has more Variance.



$$V_1 > V_2$$

- ③ N-Curve is a uniModal Curve i.e.  $M_e = M_d = M_o$
- ④ Highest Point of Normal Curve occurs at  $x = \mu$   
 & highest Value is  $f(\mu) = \frac{1}{\sigma\sqrt{2\pi}}$
- Point  $\approx x$   
 Value  $\approx y$



Def<sup>n</sup>: Let  $x$  is C.R.V & it's p.d.f is given as  $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$



then  $x$  is called Normal Random Variable with parameters  $\underline{\mu}$  &  $\underline{\sigma^2}$   
& it is denoted as  $x \sim N\{\mu, \sigma^2\}$

Note ①  $x = \{ \text{which is required} \}$

② Mean ( $x$ ) =  $\int_{-\infty}^{\infty} x \cdot f(x) dx = \text{wastage of time} = \underline{\mu}$

③ Var ( $x$ ) =  $E(x^2) - E^2(x) = \int_{-\infty}^{\infty} x^2 f(x) dx - (\mu)^2 = \text{wastage of time} = \underline{\sigma^2}$

④ Total area under  $f(x) = \int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \cdot dx = \text{using Gamma func!} = \underline{1}$  😊



④  $P(a \leq x \leq b) = \int_a^b f(x) dx = \text{Not easy to Integrate} = \text{☹}$



(M-II)

= Area under  $f(x)$  b/w  $x=a$  &  $x=b$

= still Not easy to find area ☹

Hence we have to develop some New Concepts to find above prob.



Note if  $x$  is N.R.V. st  $x \sim N\{\mu, \sigma^2\}$  &  $Z = \frac{x - \mu}{\sigma}$  then find  
Mean & Variance of  $Z$ ?

Sol: Mean( $Z$ ) =  $E(Z) = E\left\{\frac{x - \mu}{\sigma}\right\} = \frac{1}{\sigma} \{E(x) - E(\mu)\} = \frac{1}{\sigma} \{\mu - \mu\} = 0$

$$\begin{aligned} \text{Var}(Z) &= \text{Var}\left(\frac{x - \mu}{\sigma}\right) = \frac{1}{\sigma^2} \text{Var}(x - \mu) = \frac{1}{\sigma^2} \{ \text{Var}(x) - \text{Var}(\mu) \} \\ &= \frac{1}{\sigma^2} \{ \sigma^2 - 0 \} = 1. \end{aligned}$$

ie if  $Z = \frac{x - \mu}{\sigma}$  then  $\mu_Z = 0$   
 $\sigma_Z = 1$ .

$$\begin{aligned} \text{Cov}(x, k) &= E(x \cdot k) - E(x) \cdot E(k) \\ &= k E(x) - k E(x) \\ &= 0 \end{aligned}$$



Standard Normal Variable (Z)  $\rightarrow$  if  $x \sim N\{\mu, \sigma^2\}$  &  $Z = \frac{x - \mu}{\sigma}$  then



Mean(Z) = 0 & Var(Z) = 1 & in that case Z is called Standard Normal Variable with p.d.f is defined as.

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \xrightarrow[\substack{\mu=0, \sigma=1 \\ Z = \frac{x-\mu}{\sigma}}]{\text{NEHO}} f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

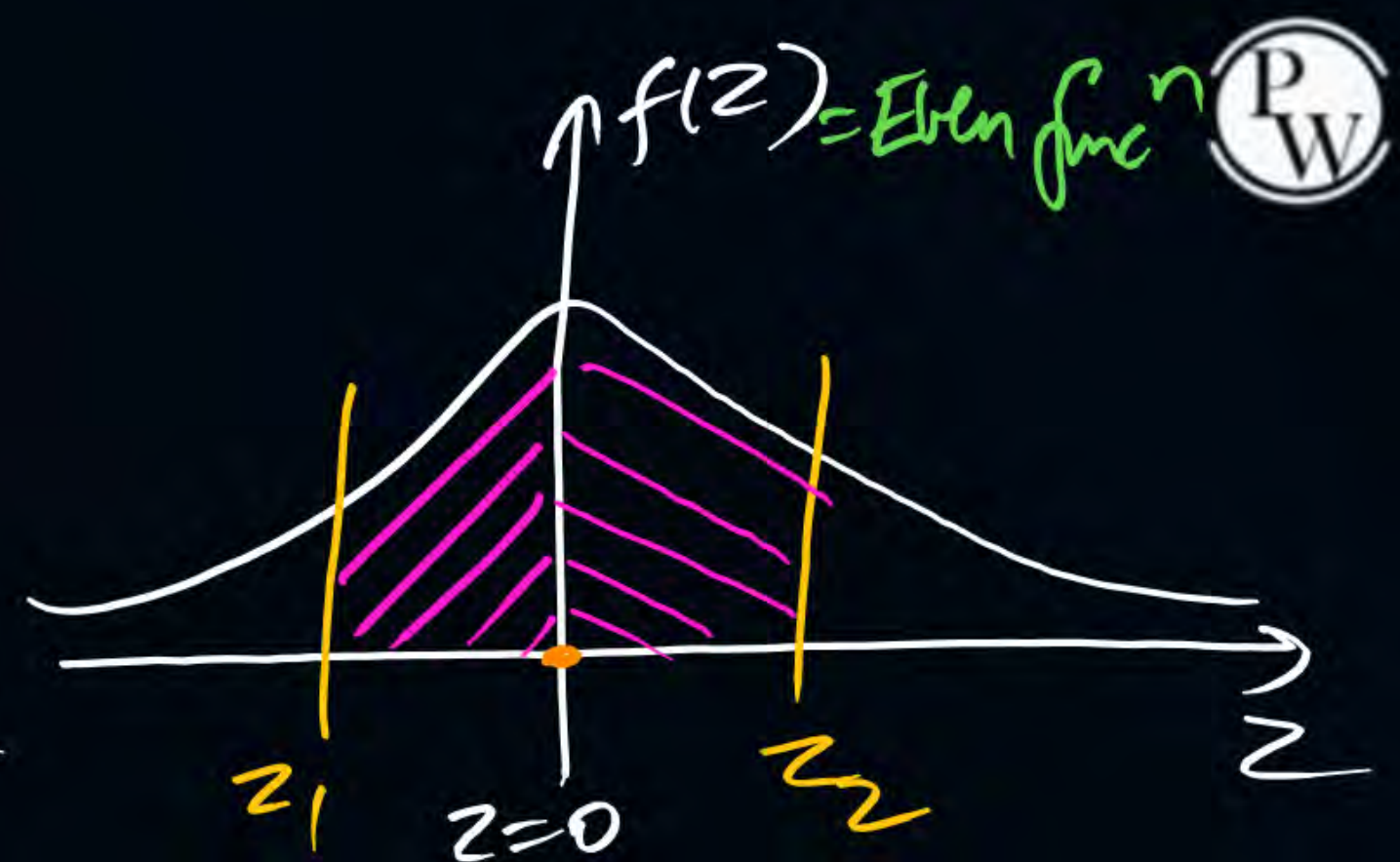
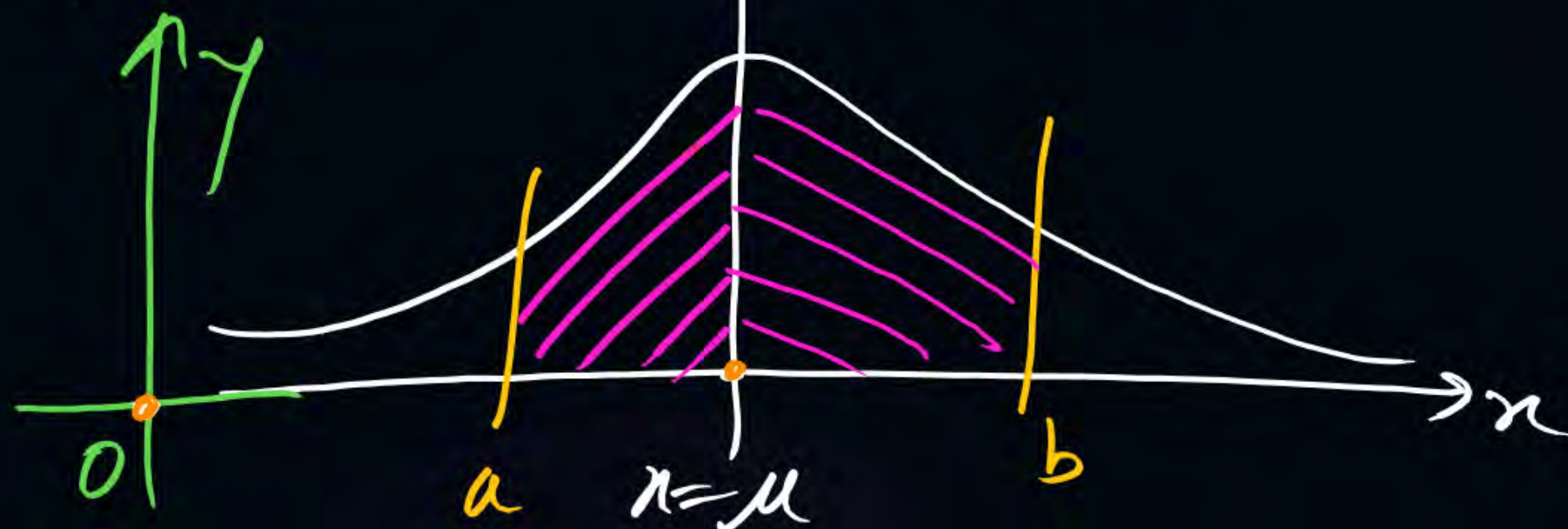
ie  $x \sim N\{\mu, \sigma^2\}$

ie  $Z \sim N\{0, 1\}$

& Standard Normal Variable is free from any parameter



Sp. Conclusion:



$$P(a \leq x \leq b) = \text{Area b/w } a \text{ \& } b = P(z_1 \leq z \leq z_2) = \text{Area b/w } z_1 \text{ \& } z_2$$

= use Normal Table = 😊  
😊

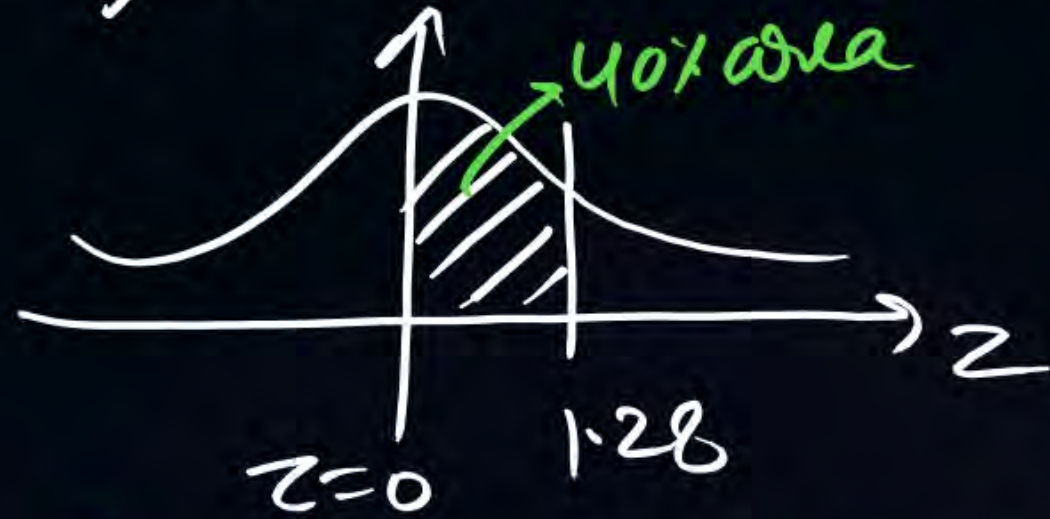
$$z = \frac{x - \mu}{\sigma}$$

- At  $x = \mu$ ,  $z = \frac{\mu - \mu}{\sigma} = 0$
- At  $x = a$ ,  $z_1 = \frac{a - \mu}{\sigma}$
- At  $x = b$ ,  $z_2 = \frac{b - \mu}{\sigma}$



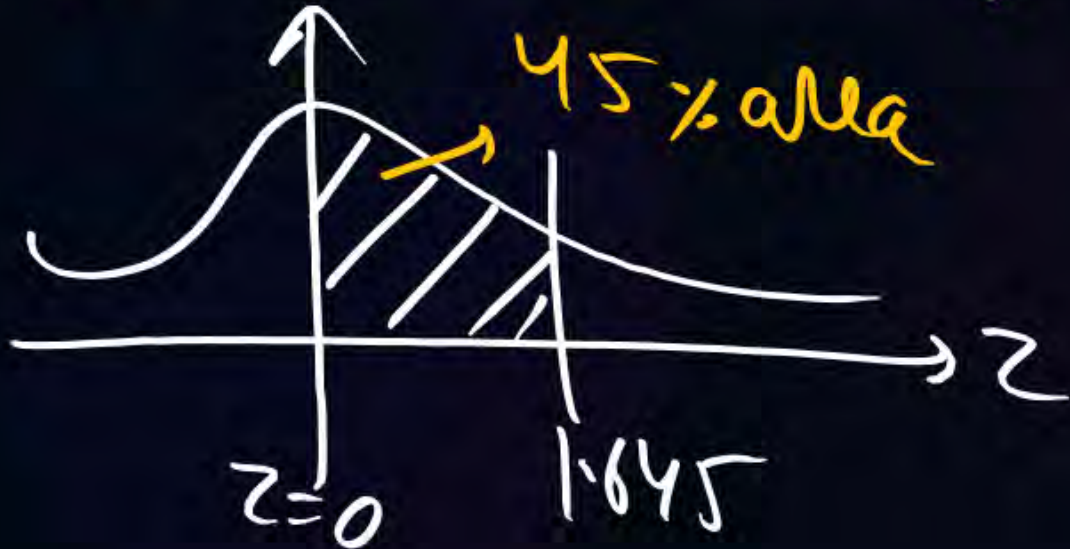
① H. Table starts from  $Z=0$  and is defined for true values of  $Z$ .

eg  $P(0 \leq Z \leq 1.28) = ? = \text{Area under the S.N. Curve b/w } Z=0 \text{ to } 1.28$



$$= \int_0^{1.28} f(z) dz = \int_{z=0}^{1.28} \left( \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \right) dz = 0.4$$

eg  $P(0 < Z \leq 1.645) = \text{Area under the S.N. Curve b/w } 0 \text{ to } 1.645$



$$= \int_0^{1.645} f(z) dz = \int_{z=0}^{1.645} \left( \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \right) dz = 0.45$$



## ② Flowchart of solving questions →

Step (1) First Find  $x = \{ \text{which is Required} \}$

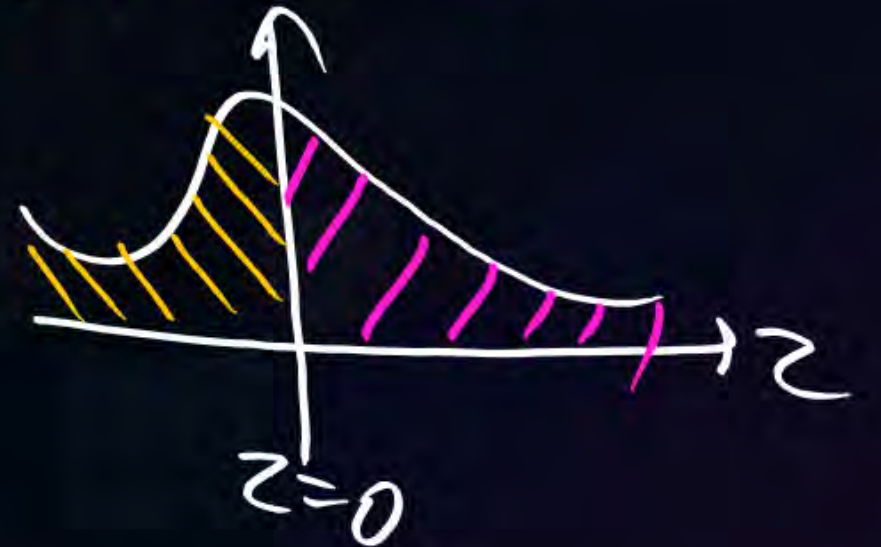
Step (2) Convert  $x$  into  $Z$  using  $Z = \frac{x - \mu}{\sigma}$

Step (3) Use Concept of Symmetry. (M. Imp)

Step (4) Use Normal Table.

Concept of Symmetry → ①  $P(-\infty < z < \infty) = \text{Total area under } f(z) = 1$

②  $P(-\infty < z \leq 0) = P(0 \leq z < \infty) = \frac{1}{2}$   
 (left side area) (Right side area)





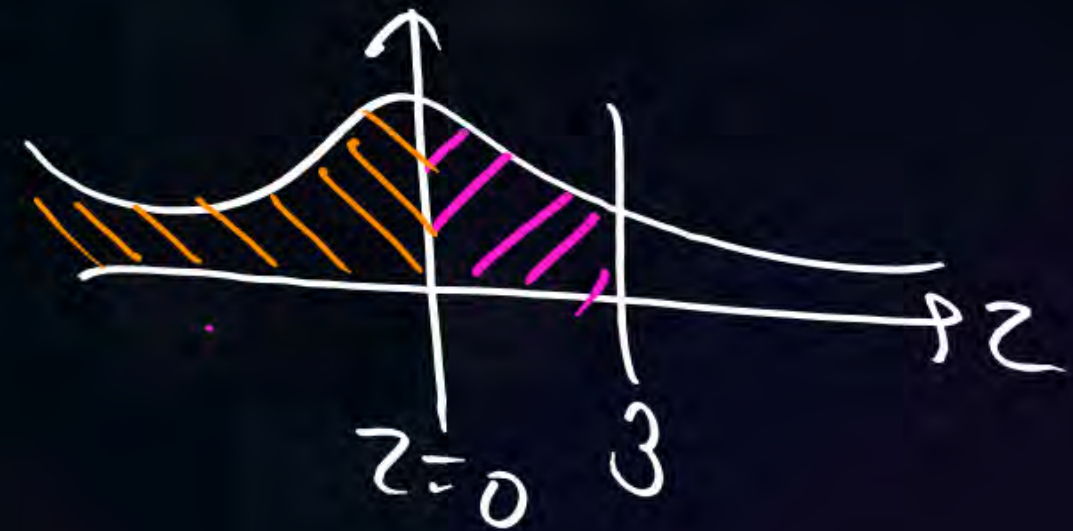
$$\textcircled{3} P(-1 \leq z \leq 1) = ? = 2P(0 \leq z \leq 1) \\ = 2(0.3413)$$



$$\textcircled{4} P(-1 \leq z \leq 2.33) = ? \\ = P(-1 \leq z \leq 0) + P(0 \leq z \leq 2.33) \\ = P(0 \leq z \leq 1) + P(0 \leq z \leq 2.33) \\ = (0.3413) + (0.4775)$$



$$\textcircled{5} P(-\infty < z \leq 3) = ? = \text{Left side area} + P(0 \leq z \leq 3) \\ = 0.5 + \left(\frac{0.997}{2}\right)$$





⑥  $P(Z > 2.58) = ?$  (MT)  $P(Z > 2.58) = \text{Right side area} - P(0 \leq Z < 2.58)$

$$= 0.5 - (0.4951)$$

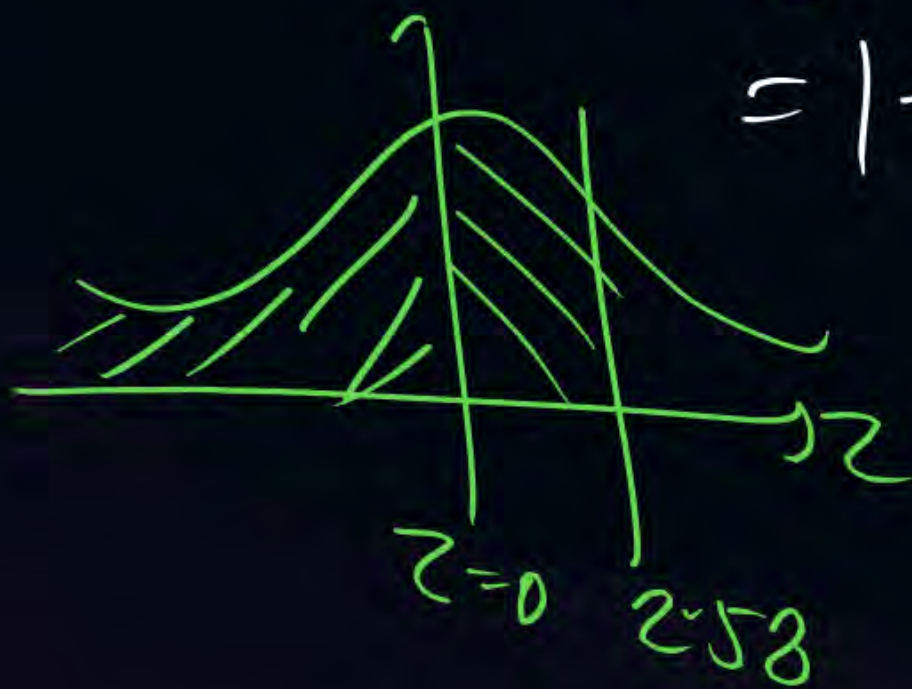
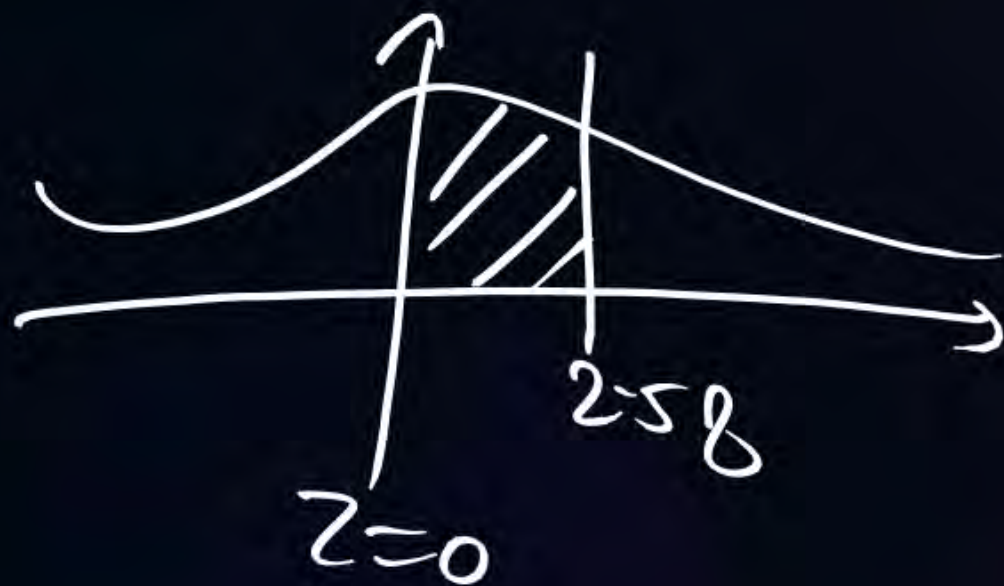


(M-II)  $P(Z > 2.58) = 1 - P(Z \leq 2.58)$

$$= 1 - \{ \text{Left side area} + P(0 < Z < 2.58) \}$$

$$= 1 - \{ \frac{1}{2} + (N.\text{Table}) \}$$

$$= 0.5 - (0.4951)$$





Qe 2000 students are appearing in an Examination in which Distribution of Marks is assumed to be Normal with mean 30 Marks & S.D 6.25 Marks. Then Find the Number of Students getting marks b/w 20 & 40. It is given that area under the N-Curve b/w  $z=0$  &  $z=1.6$  is 0.4452 ? (a) 1780 (b) 1781 (c) 1782 (d) 890

Sol:  $N=2000$ , for single student:  $X = \{ \text{Marks obtained by this single student} \}$

$\mu=30, \sigma=6.25, Z = \frac{X-\mu}{\sigma}$

$z_1 = \frac{20-30}{6.25} = -1.6$

$z_2 = \frac{40-30}{6.25} = +1.6$

$P(20 < X < 40) = P(-1.6 < Z < 1.6) = 2P(0 < Z < 1.6) = 2 \times 0.4452 = 0.8904$

$= \frac{0.8904}{1} = \frac{890.4}{1000} = \frac{1780.8}{2000} \approx \frac{1781}{2000} = \frac{2671.2}{3000} \approx \frac{2671}{3000}$



A normal random variable  $X$  has the following probability density function

$$f_x(x) = \frac{1}{\sqrt{8\pi}} e^{-\left\{\frac{(x-1)^2}{8}\right\}}, -\infty < x < \infty = \frac{1}{2\sqrt{2\pi}} e^{-\frac{1}{2}\left\{\frac{x-1}{2}\right\}^2}$$

Then  $\int_1^{\infty} f_x(x) dx$  ?

(a) 0

☒ (b)  $\frac{1}{2}$

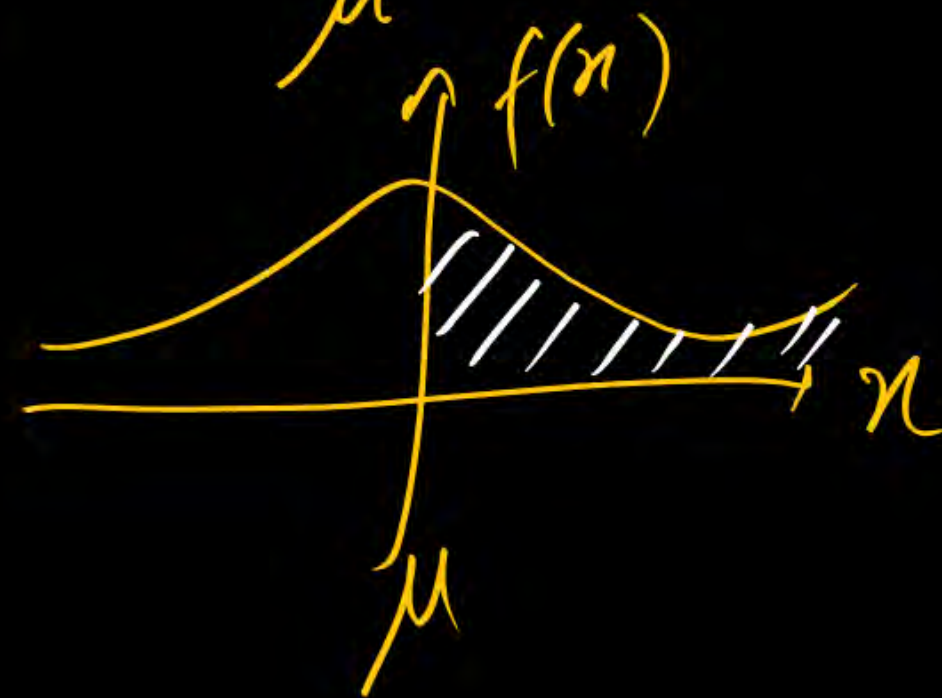
(c)  $1 - \frac{1}{e}$

(d) 1



$$\hat{\mu} \begin{cases} \mu = 1 \\ \sigma = 2 \end{cases}$$

$$I = \int_1^{\infty} f(x) dx = \int_{\mu}^{\infty} f(x) dx = 0.5$$





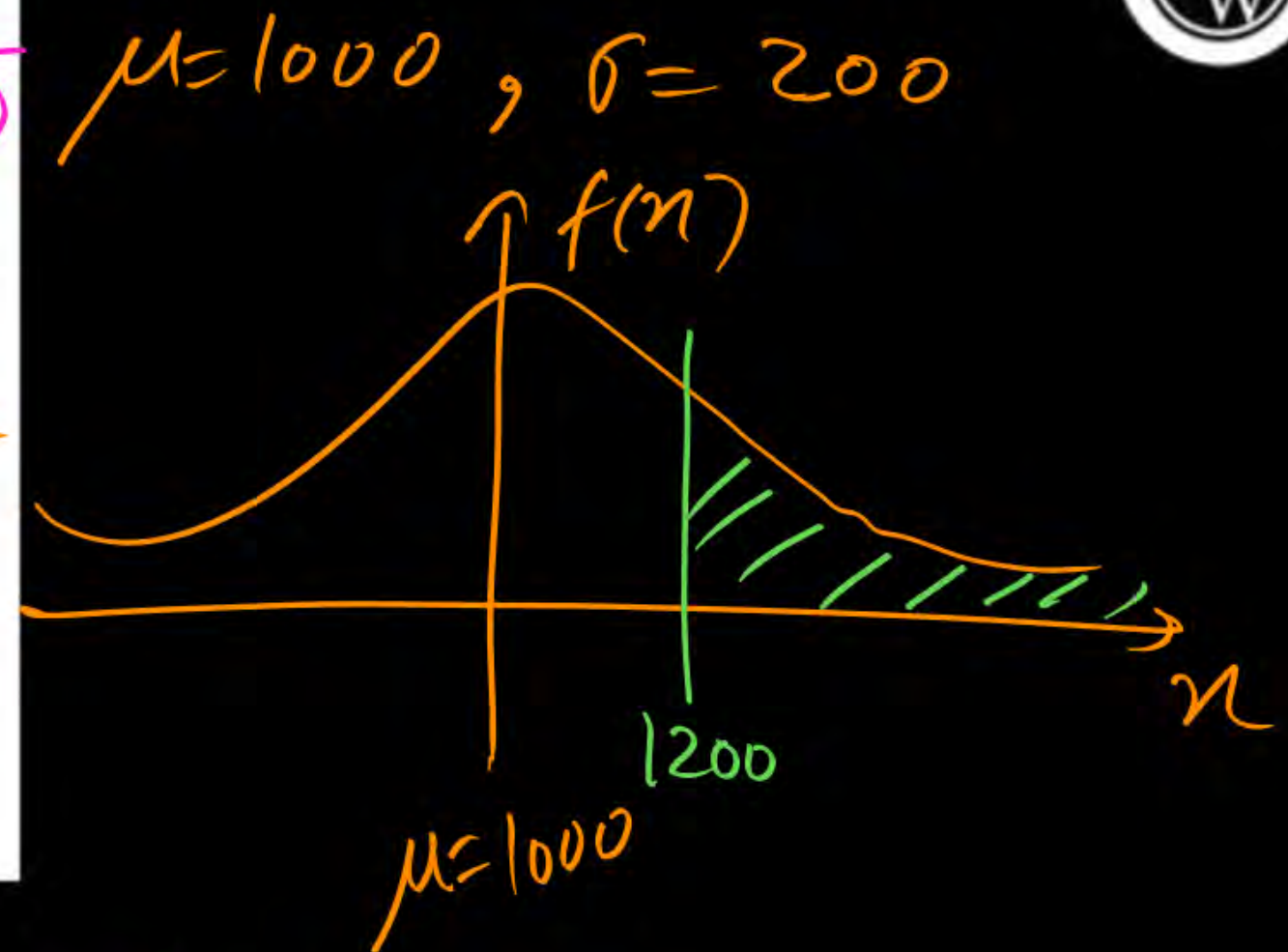
The annual precipitation data of a city is normally distributed with mean and standard deviation a 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is

(a) <50%

(b) 50%

(c) 75%

(d) 100%



$x = \{ \text{Annual Precipitation} \}, P(x > 1200) = \text{less than } 50\%$



Q2 In a school of 1000 students, Average height of student is 68.22 inches & Variance 10.8 (inches)<sup>2</sup> then Find the Number of students over 6 feet tall. If we have a Confidence of 37.46% with in the limits  $z=0$  to  $1.15$

Sol:

HW

- (a) 875
- ☒ (b) 125
- (c) 100.
- (d) 72



HW8 if  $x$  is Zero Mean unit Variance Gaussian Variable then  
Find  $E(|x|) = ?$

(a) 0

(b) 0.5

~~(c) 0.8~~

(d) 0.4



Thank  
YOU