CS & IT ENGINEERING

2024

Algorithms

Divide and Conquer

DPP – 01 Discussion Notes

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[MCQ]

#Q. Consider an array containing the following elements in unsorted order (placed randomly) but 120 as first elements

120 160 30 190 14 24 70 180 110

Quick sort partitioning algorithm is applied by choosing first elements as pivot element. Then what is the total number of arrangements of array integers are possible preserving the effect of first pass of partitioning algorithm.

A 680

B 700

C 720

D 740

Ans :- C

pivot = 120

Sorted order :- 14 24 30 70 110 120 160 180 190

0 1 2 3 4 5 6 7 8

$$\left(\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}\right)\left(\frac{1}{6!}+\frac{1}{7!}+\frac{1}{8!}\right)$$

5 $\quad$ 3

$5!$ $\quad$ $3!$

Total orderings $= 5! \times 3!$

$= (5 \times 4 \times 3 \times 2 \times 1) \times 6$

$= 120 \times 6$

$= \boxed{720}$

[MCQ]

#Q. Let $T(n) = [n(\log(n^3) - \log n) + \log n]n + \log n$, complexity of $T(n)$ is

A $O(n^2)$

B $O(n^3)$

C $O(n \log n)$

D $O(n^2 \log n)$

$T(n) = [n(\log n^3 - \log n) + \log n]n + \log n$

$\{\log A - \log B = \log(A/B)\}$

Ans : D

$$T(n) = \left(n\left(\log\left(\frac{n^3}{n}\right) + \log n\right)\right) n + \log n$$

$$= \left(n * \log(n^2) + \log n\right) n + \log n$$

$$= \left(2n \log(n) + \log n\right) n + \log n$$

$$= \underbrace{2n^2 \log n} + n \log n + \log n$$

↓ dominating term

$$T(n) = O(2n^2 \log n)$$

$$\boxed{T(n) = O(n^2 \log n)}$$

[MCQ]

#Q. Assume that there are 4 sorted lists of $\frac{n}{4}$ elements each, if these lists are merged into a single sorted list of 'n' elements then how many key comparisons are required in the worst case using an efficient algorithm?

A. $2n - 3$

B. $\frac{7}{4}n - 3$

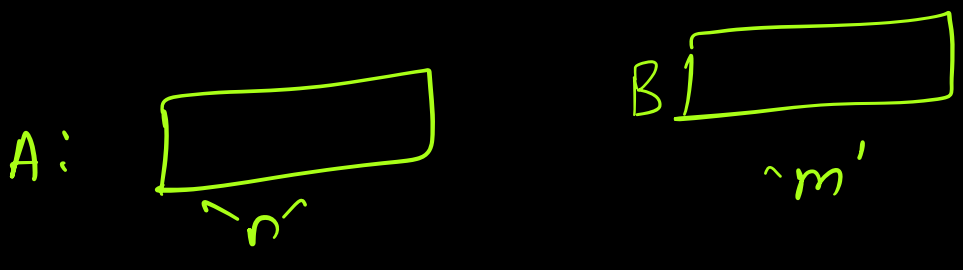
C. $\frac{9}{4}n - 3$

D. $\frac{6}{4}n - 3$

Merging Concept

Ans: A

★ Merging Concept :-



$n < m$

Best Case :- # no. of Comparisons

$= O(n) \longrightarrow O(\min(n, m))$

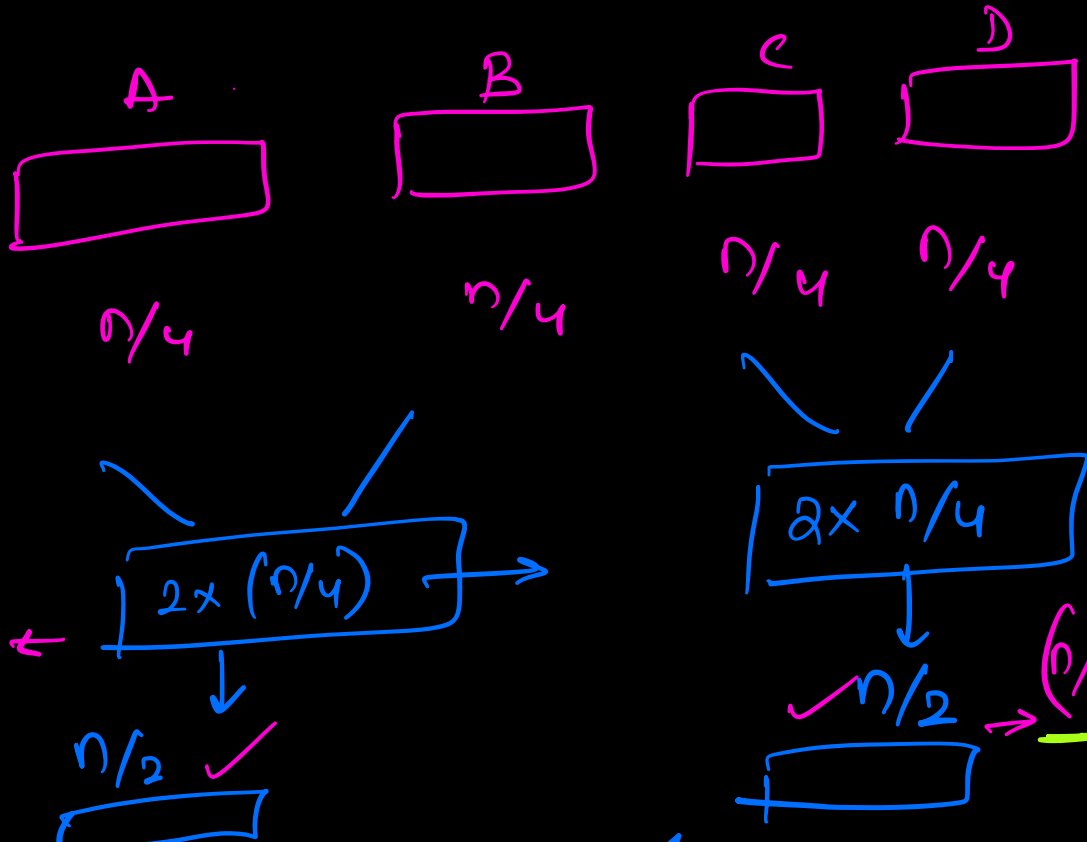
Worst Case # no. of Comparisons

$= (n + m - 1) \longrightarrow$ exact value

$\rightarrow O(n + m - 1)$

Total elements $= 4 \times \frac{n}{4} = n$

Soln :-



No. of Comparisons in worst Case :-

final ans

$$\Rightarrow \left(\frac{n}{2}-1\right) + \left(\frac{n}{2}-1\right) + (n-1)$$

$$= 2 \times \frac{n}{2} - 2 + (n-1)$$

$$= n - 2 + n - 1$$

$$= \underline{(2n-3)}$$

[NAT]

#Q. Consider the number in the sequence
2 5 11 17 19 21 26 33 39 40 51 65 79 88 99
Using binary search, the number of comparisons required to search elements '2' is___

Ans : 4

key = 2

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2	5	11	17	19	21	26	33	39	40	51	65	79	88	99

L → 0, H → 14

Pass 1: Low = 0, High = 14

$$mid = \left\lfloor \frac{low + high}{2} \right\rfloor = \left\lfloor \frac{0+14}{2} \right\rfloor = 7$$

$A[mid] = A[7] = 33$

key = 2 ⇒ key < A[mid]

↳ explore left

High = mid − 1

= 7 − 1 = 6

Pass 2: Low = 0, High = 6

$$mid = \left\lfloor \frac{low + high}{2} \right\rfloor = \left\lfloor \frac{0+6}{2} \right\rfloor = 3$$

$A[mid] = A[3] = 17$

A[3] > key

↳ explore left.

High = mid − 1

= 3 − 1 = 2

Pass 3 :- Low = 0, high = 2

$$mid = \left\lfloor \frac{low + high}{2} \right\rfloor = \left\lfloor \frac{0+2}{2} \right\rfloor = 1$$

A[mid] = A[1] = 5 > key

↳ explore left

High = mid - 1

= 1 - 1 = 0

Pass 4 :-

Low = 0, High = 0

$$mid = \left\lfloor \frac{low + high}{2} \right\rfloor = \left\lfloor \frac{0+0}{2} \right\rfloor = 0$$

A[0] = 2 == key (stop and exit.

Total passes to search for 2

= (4)

[MCQ]

#Q. Merging 4 sorted files having 400, 100, 250, 50 records will take O (___) time?

↳ No. of Comparisons :-

exact value

A. 800

B. ~~400~~ 799

C. ~~2000~~ 1597

D. ~~1000~~ 1347 ✓

Ans: D

1347

Soln:-

400 100 250 50

2 way merging pattern:-

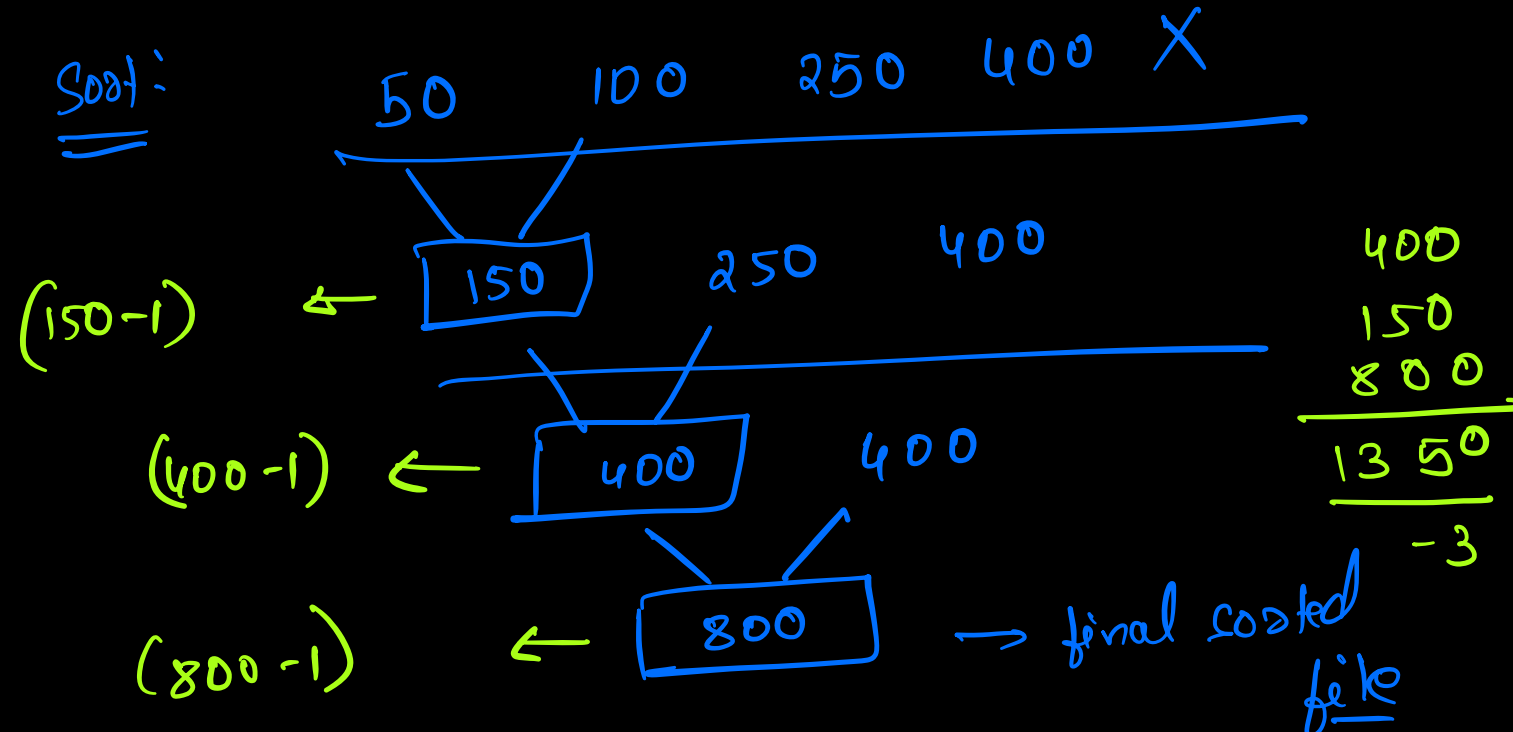
Logic: Always pick 2 smallest files

Resultant size: (n+m)

↳ put it back to list

and remove the individual 'n' & 'm' size files.

Soln:



Total: 149 + 399 + 799 = 1347

[NAT]

#Q. Consider a machine which needs a minimum of 50 seconds to sort 500 names by quick sort, then what is the minimum time required to sort 50 names (approximately)is ____ (round off to 2 decimal)

Ans: 3.14 sec

Quick Sort

min Time: 50 sec → Sort 500 elements

50 elems → Time = ?

Sort

Quick sort :- To sort n elements

✓ Best Case : $O(n \log n)$ units of time

Worst Case : $O(n^2)$ "

Minimum time ⟶ Best Case.

$n = 500$ ⟶ Time 50 sec

↓

$500 \times \log_2(500)$ units = 50 sec

$$1 \text{ unit} = \frac{50}{500 \times \log_2 500} = \frac{1}{10 \times (\log_2 500)} \text{ sec}$$

Round : $n = 50$

Time read to sort 50 elements

$= 50 \times \log_2 50$ units

$= 50 \times \log_2 50 \times \frac{1}{10 \times \log_2 500}$ sec

$= 5 \times \frac{\log_2 50}{\log_2 500} = 3.14$ sec

[NAT]

#Q. What is the total number of comparisons that will be required in worst case to merge the following sorted files into a single sorted file into a single sorted file by merging together two files at a time___.

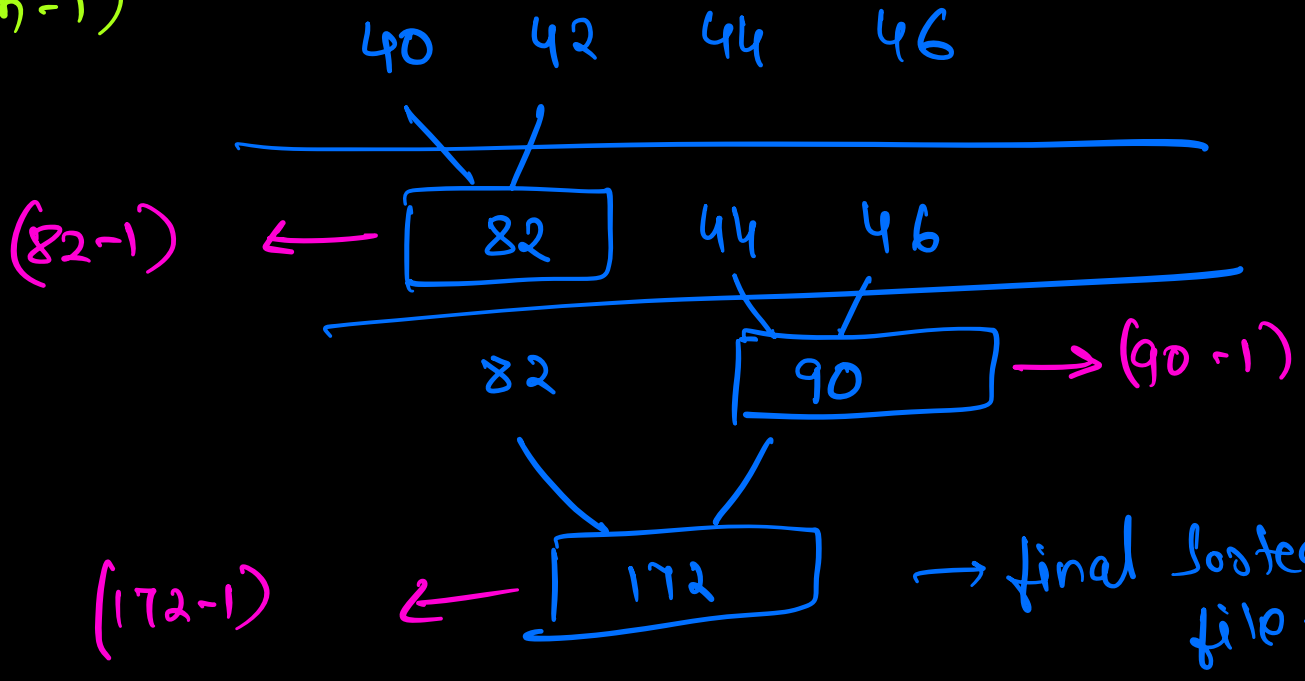
Files	F_1	F_2	F_3	F_4
Number of records	40	42	44	46

Ans: 341

→ Based on 2-way merging pattern.
(discussed earlier)

Sol:- given 40 42 44 46

$(n+m-1)$



Total no. of comparisons in WC

$= (82-1) + (90-1) + (172-1)$

$= 344 - 3$

$= 341$

```
  82
  90
2 172
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  344
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THANK - YOU