

# Computer Science & IT

## Database Management System



Relational Model & Normal Forms

Lecture No. 05



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# Recap of Previous Lecture

Topic

Closure of attribute Set

Topic

Candidate Key





# Topics to be Covered



Topic

Identification of candidate keys in a relation



Topic

Membership test





H.W.



#Q. Assume a relation R (A, B, C, D, E) that has the following functional dependencies:

$AB \rightarrow C$ ,

$B \rightarrow E$ ,

$C \rightarrow D$

$E \rightarrow A$

Find the Candidate key of R.

Essential attribute = B

$(B)^+ = \{B, E, A, C, D\}$

all  $\therefore B$  is S.K

as well as C.K

B is the only C.K

P.A = {B}, N.P.A = {A, C, D, E}

$(AB)^+ = \{A, B, C, D, E\}$

all  $\therefore AB$  is S.K

check for C.K

Proper subsets  
of {A, B}

$\{A\}^+ = \{A\} \therefore A$  is not a S.K

$\{B\}^+ = \{B, E, A, C, D\}$

all  $\therefore B$  is a S.K

Proper subset is a S.K.  
 $\{A, B\}$  is not minimal  
Hence  $\{A, B\}$  is just a S.K.  
but not a C.K

$\{B\}$  is a S.K

as well as C.K.

B is the only C.K.

$\therefore P.A = \{B\}$

N.P.A = {A, C, D, E}





## Topic : Note



If there exists any FD in the set of FDs  
of the type

X  $\longrightarrow$  Any of the prime attribute

then we can replace that prime  
attribute in the corresponding  
Candidate Key by 'X' {i.e. L.H.S of FD}  
and by doing so we may  
get a new Super Key of relation

$\hookrightarrow$  then we will check  
for candidate key



#Q. Assume a relation  $R(A, B, C, D)$  that has the following functional

dependencies:

$AB \rightarrow CD$ ,  $\equiv$   $AB \rightarrow C$   
 $AB \rightarrow D$

$D \rightarrow A$

Find all the Candidate keys of  $R$ .

$$(AB)^+ = \{A, B, C, D\}$$

all attributes  $\therefore AB$  is a S.K.  
 as well as a C.K.

$AB$

is a C.K.

Prime attribute =  $\{A, B\}$

$AB \rightarrow D$   
 $\therefore$  Replace  
 $D$  by  $AB$

$D \rightarrow A$  is present &.  
 $\therefore$  Replace  $A$  by  $D$

RHS contains  
 Prime attribute 'A'

$DB$

is a S.K.  
 as well as a C.K.

$$(D)^+ = \{D, A\}$$

$$(B)^+ = \{B\}$$

$DB$

is a C.K.

$AB$  &  $DB$  are the only  
 Candidate keys of relation  
 $\therefore$  Prime Attributes =  $\{A, B, D\}$   
 Non-prime attribute =  $\{C\}$



#Q. Assume a relation R (M, N, O, P, Q) that has the following functional dependencies:

$MNO \rightarrow PQ$  and  $MNO \rightarrow Q$   
 $P \rightarrow MN$

Find the Candidate keys of R.

'O' is Essential,  $(O)^+ = \{O\}$

$(MNO)^+ = \{M, N, O, P, Q\}$

all attributes i.e. SK.

Check for Candidate Key

$(MO)^+ = \{M, O\}$

$(NO)^+ = \{N, O\}$

No proper subset is a SK.

MNO

is a C.K.

Prime attributes =  $\{M, N, O\}$

$MNO \rightarrow P$

$\therefore$  Replace 'P' by 'MNO'

$P \rightarrow MN$

Prime attributes  $\therefore$  Replace 'MN' by P

PO

is a SK as well as a C.K.

PO

is a C.K.

'MNO' & 'PO' are the only C.Ks of the relation

$\therefore$  Prime Attributes =  $\{M, N, O, P\}$

Non-Prime attributes =  $\{Q\}$



#Q. Assume a relation  $R(A, B, C, D)$  that has the following functional dependencies:

$AB \rightarrow CD$   $\equiv$   $AB \rightarrow C$   
 $AB \rightarrow D$

$C \rightarrow A$

$D \rightarrow B$

Find the Candidate keys of  $R$ .

$AB, CB, AD, CD$   
 are the Candidate keys  
 of the relation  
 Prime Attribute =  $\{A, B, C, D\}$   
 Non-prime =  $\{ \}$

$(AB)^+ = \{A, B, C, D\}$   
 all  $\therefore$  S.K.

$(A)^+ = \{A\}$  No Proper Subset is a S.K.  
 $(B)^+ = \{B\}$

$AB$  is a C.K.

$C \rightarrow A$   
 $\therefore$  Replace 'A' by 'C'

$CB$  is a S.K.  
 $(C)^+ = \{C, A\}$   
 $(B)^+ = \{B\}$   
 $CB$  is a C.K.

$CB$  is a C.K.

$D \rightarrow B$   
 $\therefore$  Replace B by D

$C \rightarrow A$   
 $D \rightarrow B$   
 $\therefore$  Replace A by C & B by D

$CD$  is a S.K.

$CD$  is a C.K.

$D \rightarrow B$   
 $\therefore$  Replace B by D

$AD$  is a S.K.  
 $(A)^+ = \{A\}$   
 $(D)^+ = \{D, B\}$   
 $AD$  is a C.K.

$C \rightarrow A$   
 $\therefore$  Replace A by C

$(C)^+ = \{C, A\}$   
 $(D)^+ = \{D, B\}$



#Q. Assume a relation R (A, B, C, D, E, H) that has the following functional dependencies:

$A \rightarrow B$

$BC \rightarrow D$

$E \rightarrow C$

$D \rightarrow A$

Find the Candidate keys of R.

Essential attributes = E & H

$(AEH)^+ = \{A, E, H, B, C, D\}$

all attributes  $\therefore$  S.K. as well as a C.K.

$(EH)^+ = \{E, H, C\}$  it can never be a member of any C.K.

Prime Attributes =  $\{A, E, H\}$

$A \rightarrow B$   
 $\therefore$  Replace B by A

**AEH**

is a C.K.

$\downarrow$

$D \rightarrow A$   
 $\therefore$  Replace A by D

DEH is a S.K. as well as C.K.

**DEH**

is a C.K.

Prime attributes =  $\{A, E, H, D\}$

$\downarrow$

$BC \rightarrow D$   
 $\therefore$  Replace D by BC

BCEH is a S.K., but not a C.K.

**BEH**

is a C.K.

Prime Attributes =  $\{A, E, H, D, B\}$

if E & H are present then C can be removed

AEH, DEH & BEH are the Candidate keys.

$\therefore$  Prime Attribute =  $\{A, E, H, D, B\}$

Non-prime attribute =  $\{C\}$





## Topic : Membership test

➤ Membership test is used to check whether a given FD is a member of given FD set or not.

➤ To check whether  $X \rightarrow Y$  is a member of FD set  $F$  or not  
(i.e.,  $F \models X \rightarrow Y$  or not)

*infer/yields*

We first obtain  $X^+$  (closure of  $X$ ) w.r.t. FD set  $F$ .

If  $Y \in X^+$ , then  $X \rightarrow Y$  is a member of FD set  $F$   
otherwise not a member of FD set  $F$



\* If  $X \rightarrow Y$  is a member of FD set  $F$ ,  
then we can say that  
 $X \rightarrow Y$  can be inferred from  $F$



#Q. Let FD set  $F = \{ A \rightarrow B, B \rightarrow C \}$

Check whether  $A \rightarrow C$  is a member of  $F$  or not?

$(A)^+ = \{ A, B, C \}$   
 $C \in (A)^+ \therefore "A \rightarrow C"$  is a member of  $F$



#Q. Let FD set  $F = \{ A \rightarrow B, B \rightarrow C \}$

Check whether  $B \rightarrow A$  is a member of  $F$  or not?

$$(B)^+ \text{ w.r.t. } F = \{ B, C \}$$

$$A \notin (B)^+ \text{ w.r.t. } F$$

$\therefore B \rightarrow A$  is not a member of  $F$



#Q. Let FD set  $F = \{ AB \rightarrow C, BC \rightarrow D \}$

Check whether  $AB \rightarrow D$  is a member of  $F$  or not?

$(AB)^+$  w.r.t.  $F = \{ A, B, C, D \}$

$D \in (AB)^+$  w.r.t.  $F$ .

$\therefore AB \rightarrow D$  is a member of  $F$



#Q. Let FD set  $F = \{ AB \rightarrow C, C \rightarrow A \}$

Check whether  $C \rightarrow B$  is a member of  $F$  or not?

$\downarrow$   
 $(C)^+ \text{ w.r.t. } F = \{ C, A \}$

$B \notin (C)^+ \text{ w.r.t. } F$

$\therefore C \rightarrow B$  is not a member of  $F$

Note:-

\* Let  $F$  is a non-empty set of FDs

The set of all functional dependencies that can be inferred from  $F$  can be denoted by  $F^+$

↪ Closure of FD set  $F$



#Q. Let FD set  $F = \{A \rightarrow B, B \rightarrow C\}$  over relation  $R(A, B, C)$

Find  $F^+$

$$\{A\}^+ = \{A, B, C\} = A \rightarrow BC$$

$$\{A, B\}^+ = \{A, B, C\}$$

$$\{A, B, C\}^+ = \{A, B, C\}$$

$$\{B\}^+ = \{B, C\}$$

$$\{A, C\}^+ = \{A, B, C\}$$

$$\{C\}^+ = \{C\}$$

$$\{B, C\}^+ = \{B, C\}$$

$$F^+ = \left\{ \begin{array}{l} A \rightarrow BC, \quad B \rightarrow C, \\ AB \rightarrow C, \quad AC \rightarrow B \end{array} \right\}$$

Let  $A$  &  $B$  are two sets then,

①  $A = B$  iff  $A \subseteq B$  &  $B \subseteq A$

② If  $A \subseteq B$  but  $B \not\subseteq A$ , then  $A \subset B$  &  $A \neq B$

③ If  $A \not\subseteq B$  but  $B \subseteq A$ , then  $B \subset A$  &  $A \neq B$

④ Neither  $A \subseteq B$  nor  $B \subseteq A$   $\left\{ \begin{array}{l} A \text{ \& } B \text{ are not} \\ \text{comparable} \end{array} \right\}$  &  $A \neq B$



## Relationship between two sets of FDs

Let  $F$  &  $G$  are two sets of functional dependencies:

- ① If all FDs of set  $F$  are member of FD set  $G$   
then  $F \subseteq G$ , { i.e., all FDs of  $F$  can be inferred from FD set  $G$   
(or) we can say  $G$  covers  $F$  }
- ② If all FDs of set  $G$  are member of FD set  $F$   
then  $G \subseteq F$ , { i.e., all FDs of  $G$  can be inferred from FD set  $F$   
(or) we can say  $F$  covers  $G$  }



Q: Consider two FD sets

$$F_1 = \{A \rightarrow B, B \rightarrow C, AB \rightarrow D\}$$

$$F_2 = \{A \rightarrow B, B \rightarrow C, A \rightarrow C, A \rightarrow D\}$$

} Find the relationship b/w  $F_1$  &  $F_2$

Check if  $F_1$  covers  $F_2$  or not  
i.e., check if  $F_2 \subseteq F_1$  or not

Check if  $F_2$  covers  $F_1$  or not  
i.e., check if  $F_1 \subseteq F_2$  or not

FDs of $F_2$	Closure w.r.t. $F_1$	Member or not	Relationship
$A \rightarrow B$	$(A)^+ \text{ w.r.t. } F_1 = \{A, B, C, D\}$	$B \in A^+ \checkmark$	All FDs of $F_2$ are member of $F_1$ $\therefore F_2 \subseteq F_1$
$B \rightarrow C$	$(B)^+ \text{ w.r.t. } F_1 = \{B, C\}$	$C \in B^+ \checkmark$	
$A \rightarrow C$	$(A)^+ \text{ w.r.t. } F_1 = \{A, B, C, D\}$	$C \in A^+ \checkmark$	
$A \rightarrow D$	$(A)^+ \text{ w.r.t. } F_1 = \{A, B, C, D\}$	$D \in A^+ \checkmark$	

FDs of $F_1$	Closure w.r.t. $F_2$	Member or not	Relationship
$A \rightarrow B$	$(A)^+ \text{ w.r.t. } F_2 = \{A, B, C, D\}$	$B \in A^+ \checkmark$	All FDs of $F_1$ are member of $F_2$ $\therefore F_1 \subseteq F_2$
$B \rightarrow C$	$(B)^+ \text{ w.r.t. } F_2 = \{B, C\}$	$C \in B^+ \checkmark$	
$AB \rightarrow D$	$(AB)^+ \text{ w.r.t. } F_2 = \{A, B, C, D\}$	$D \in (AB)^+ \checkmark$	

$$F_1 \subseteq F_2 \text{ \& } F_2 \subseteq F_1 \therefore \boxed{F_1 = F_2}$$



Q: Consider two FD sets  
 $F_1 = \{ A \rightarrow B, B \rightarrow C, AB \rightarrow D \}$

4  $F_2 = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C, A \rightarrow D \}$

} Find the  
relationship b/w  
 $F_1$  &  $F_2$

Q: Consider two FD sets

$$F_1 = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C \}$$

$$F_2 = \{ A \rightarrow B, B \rightarrow C, A \rightarrow D \}$$

} Find the relationship b/w  $F_1$  &  $F_2$

$$F_1 = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C \}$$

$$\therefore F_1 \subseteq F_2$$

$$(A)^+ \text{ wrt } F_2 = \{ A, B, C, D \}$$

$$F_2 = \{ A \rightarrow B, B \rightarrow C, A \rightarrow D \}$$

$$\therefore F_2 \not\subseteq F_1$$

$(A)^+ \text{ wrt } F_1 = \{ A, B, C \}$   
 $D \notin (A)^+$   
 $\therefore A \rightarrow D$  is not inferred by  $F_1$

$F_1 \subseteq F_2$  but  $F_2 \not\subseteq F_1$ ,  $\therefore F_1 \neq F_2$  but  $F_1 \subset F_2$



Q: Consider two FD sets

$$F_1 = \{ A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E \}$$

$$F_2 = \{ A \rightarrow BC, D \rightarrow AE \}$$

} Find the relationship b/w  $F_1$  &  $F_2$

$$F_1 = \left\{ \begin{array}{l} A \rightarrow B \checkmark \\ AB \rightarrow C \checkmark \\ D \rightarrow AC \checkmark \\ D \rightarrow E \checkmark \end{array} \right\} \therefore F_1 \subseteq F_2$$

$$F_2 = \left\{ \begin{array}{l} A \rightarrow BC \checkmark \\ D \rightarrow AE \checkmark \end{array} \right\} \therefore F_2 \subseteq F_1$$

Hence  $F_1 = F_2$

## FD set of a Sub-relation

- Let  $R$  be the relation with FD set  $F$ , and  $R_1$  is any sub-relation of  $R$ .

Concept of membership test can be used to identify the FDs of sub-relation

- Let  $R(A, B, C, D, E)$  is a relation, then  $R_1(A, B, E)$  can be called a subrelation of relation  $R$



Q: Let  $R(A, B, C, D, E)$  is a relation with FD set  $F$ .

$F = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$

And let  $R_1(A, B, E)$  is a sub-relation of relation  $R(A, B, C, D, E)$

Find the Candidate keys of sub-relation  $R_1(A, B, E)$

Soln. To find the candidate keys of any relation we need the set of functional dependencies w.r.t. that relation

∴ First we need to identify the FDs that exists in sub-relation  $R_1(A, B, E)$



Q: Let  $R(A, B, C, D, E)$  is a relation with FD set  $F$ .

$F = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$

And let  $R_1(A, B, E)$  is a sub-relation of relation  $R(A, B, C, D, E)$

We need to find FD set  $F_1$  w.r.t. sub-relation  $R_1$   
i.e., we need identify the relationship b/w  $A, B$  &  $E$

$(A)^+$  w.r.t.  $F = \{A, B, C\}$

i.e.  $A \rightarrow \cancel{A} \cancel{B} \cancel{C}$

trivial

Not in relation  $R_1$

$\therefore \boxed{A \rightarrow B}$

$(B)^+$  w.r.t.  $F = \{\cancel{B}\}$   $\therefore$  No useful FD

$(E)^+$  w.r.t.  $F = \{\cancel{E}\}$   $\therefore$  No useful FD

$(AB)^+$  w.r.t.  $F = \{\cancel{A}, \cancel{B}, \cancel{C}\}$   $\therefore$  No useful FD

$(AE)^+$  w.r.t.  $F = \{\cancel{A}, \cancel{E}, \cancel{B}, \cancel{C}\}$   $\therefore \boxed{AE \rightarrow B}$

$(BE)^+$  w.r.t.  $F = \{\cancel{B}, \cancel{E}\}$   $\therefore$  No useful FD

Hence FD set  $F_1$  w.r.t. sub-relation  $R_1(A, B, E)$  is

$F_1 = \left\{ \begin{array}{l} A \rightarrow B \\ AE \rightarrow B \end{array} \right\}$

$\therefore$  CK of  $R_1$  is  $(AE)$



H.W.

Q: Let  $R(A, B, C, D, E, F)$  is a relation with FD set  $F$ .

$F = \{AB \rightarrow C, B \rightarrow D, BC \rightarrow A, D \rightarrow EF\}$

And let  $R_1(A, B, C, D)$  is a sub-relation of relation  $R(A, B, C, D, E, F)$

Find the Candidate keys of sub-relation  $R_1(A, B, C, D)$



## 2 mins Summary



✓  
Topic

Identification of candidate keys in a relation

✓  
Topic

Membership test



**THANK - YOU**