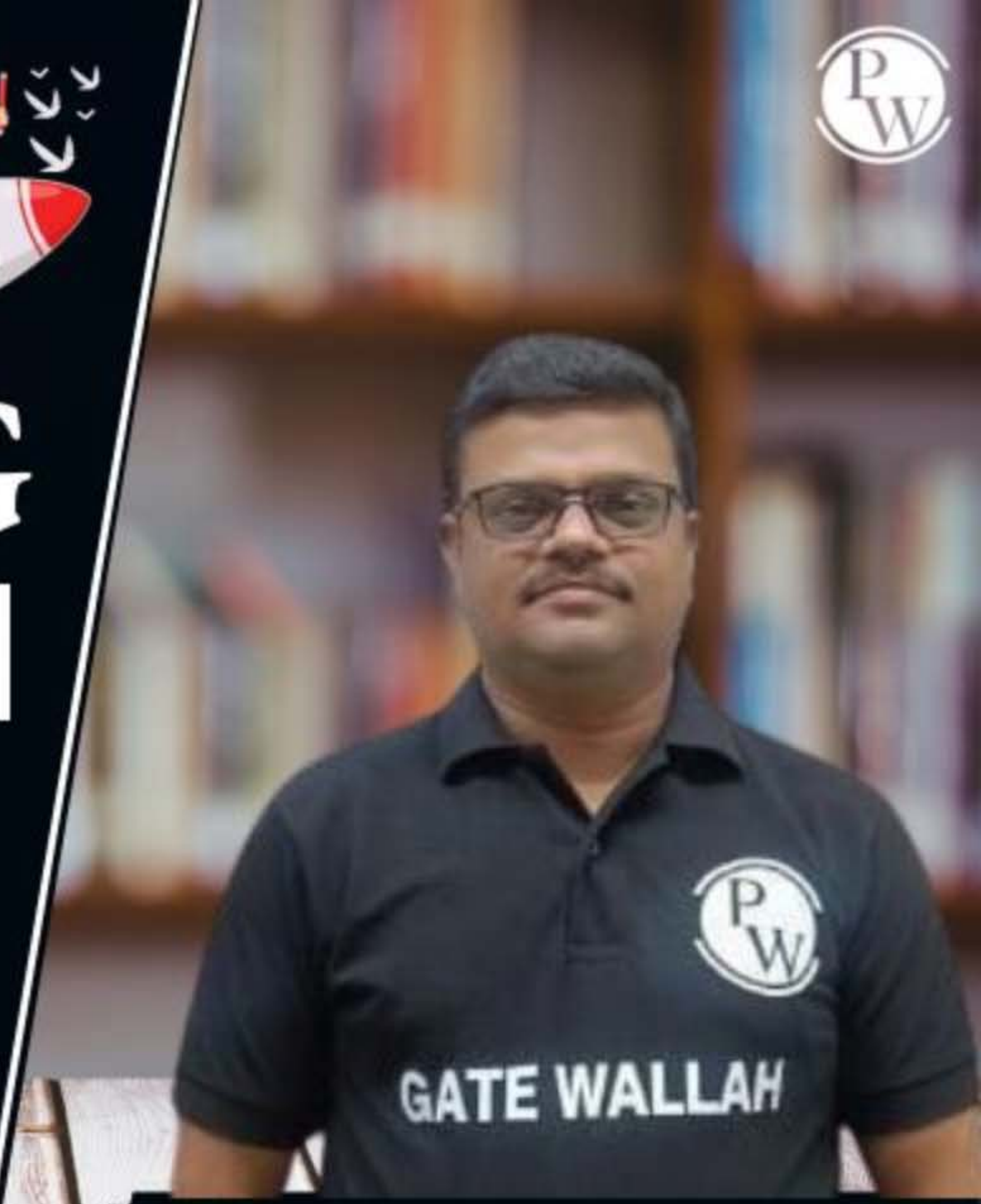
CS & IT ENGINEERING

THEORY OF COMPUTATION

REGULAR EXPRESSION

Lecture No.- 02



By- Venkat sir

Recap of Previous Lecture

Topic ?????

Regular Expression Construction

Mealy machine

Moore machine

Topics to be Covered

Topic — Regular Expression

Topic ?? Properties of Regular Expression

Topic ?? Finite Automata ⇒ Regular Expression

Topic ?? Regular Expression ⇒ Finite Automata

Topic : Regular Expression

$L = \{\epsilon, a, b, aa, ab, ba, bb \ldots\}$

$(a+b)^*$

- The simplest way of representing a regular language is known as Regular expression.

$(0+1)^* 001 (0+1)^*$

- For every regular language regular expression can be constructed.

$L = \{a^n b^n \mid n \geq 1\}$

- To construct regular expression following 3 operators are used.
- + is known as union operator
- . is known as concatenation operator
- * is known as Kleene closure operator

$L_1 = \{\} \Rightarrow \phi$

$L_2 = \{ab, aa, ba, bb\} \Rightarrow$

$ab + aa + ba + bb$

$L_3 = \{a^n b^m \mid n > m \text{ (or) } n < m\}$ ✗

$n \neq m$

Topic : NOTE

- For one regular language many number of regular expressions can be possible.

- One regular expression can generate only one regular language.

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is exactly 4.

$$\underbrace{(a+b)}_{1}\ \underbrace{(a+b)}_{2}\ \underbrace{(a+b)}_{3}\ \underbrace{(a+b)}_{4} = 6$$

min DFA

$(a+b)^4$

min DFA

$$\left.\begin{array}{l}(a+b)\text{---}n\text{ times}\\(a+b)^n\end{array}\right\} \Rightarrow (n+2)$$

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is atleast 4

$(n+1)$

$(a+b)^4(a+b)^* \longrightarrow$ min DFA $\{5 \text{ States}\}$

$\{0,1,2,3,4\}$

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is atmost 4.

min DFA

$$(a+b+\epsilon)(a+b+\epsilon)(a+b+\epsilon)(a+b+\epsilon) = 6$$

$$(a+b+\epsilon)^n \longrightarrow (n+2)$$

$\{0, 4, 8, 12 \ldots\}$

#Q. Construct regular expression that generates set of all strings of a's and b's where length of each string is divisible by 4.

$$\left[(a+b)^4\right]^*$$

min DFA

$$\left[(a+b)^n\right]^* \longrightarrow \{n \text{ states}\}$$

$\Sigma = \{a, b\}$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are exactly 4.

$b^* a b^* a b^* a b^* a b^* \longrightarrow$ min DFA = 6 states

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atmost 3.

$$\boxed{b^*(a+\epsilon)b^*(a+\epsilon)b^*(a+\epsilon)b^*}$$

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are atleast 3.

#Q. Construct regular expression that generates set of all strings of a's and b's where number of a's are divisible by 3.

$$\boxed{(b^* a b^* a b^* a b^*)^* + b^*} \longrightarrow 3$$

min DFA

bbbbbb

#Q. Construct regular expression that generates set of all even length palindrome strings over {a}.

#Q. Construct regular expression that generates set of all odd length palindrome strings over {a}.

$$\{a, a^3, a^5, a^7 \ldots\} = \underline{\underline{a(aa)^*}}$$

#Q. Construct regular expression that generates set of all even length palindrome strings over $\{a, b\}$.

#Q. Construct regular expression that generates set of all odd length palindrome strings over {a, b}.

not possible

#Q. Construct regular expression that generates set of all odd length palindrome strings of English language. $\{a - - z\}$

not regular

not possible

$$L = \{ W W^R \mid w \in (a + - - + z)^* \}$$

Dependency

$\{W W^R\}$

W

WR

malayalam

liril

nitin

$$L_1 = \{ WW^R \mid W \in (a)^* \} = \{ \epsilon, aa, aaaa, aaaaaa \ldots \} = (aa)^*$$

$$L_2 = \{ WW^R \mid W \in (a+b)^* \} = \{ \epsilon, aa, bb, abba, baab, abbbba \ldots \}$$

not regular

$L_3 = \{ W C W^R \mid W \in (a+b)^* \}$ = odd length Palindrome

not regular

$(a+b)^*$

$$L_4 = \{ w\,x\,w^R \mid w \in (a+b)^*,\ x \in (a+b)^* \} = (a+b)^*$$

$$(a+b)^* + (a+b)^2 \Rightarrow (a+b)^*$$

$$(a+b)^* + a(a+b)^*a + b(a+b)^*b + ab(a+b)^*ba + \dots$$

$$\Downarrow$$

$$(a+b)^*$$

$(a+b)$

$L_5 = \left\{ \underline{\underline{w}} \, \underline{x} \, w^R \;\middle|\; \begin{array}{l} w \in (a+b)^* \\ x \in (a+b) \end{array} \right\}$ = odd length Palindrome

$\{a, b, \underline{ab}a\underline{ba}, babab \ldots\}$ = not possible

$a+b+ababa+$

$$L_6 = \left\{ \underline{w}\, x\, \underline{w}^R \;\middle|\; \begin{array}{l} w \in (a+b) \\ x \in (a+b)^* \end{array} \right\}$$

$$\underline{\underline{a(a+b)^* a}} + b\, \underline{\underline{(a+b)^* b}}$$

$$L_7 = \{ \underline{\underline{w}}\, b\, w^R \mid w \in (a)^* \}$$

$$\{ b, aba, a^2ba^2, a^3ba^3 \ldots a^nba^n \ldots \}$$

not regular

X

$$L = \{ w x w^R \mid w, x \in (a+b)^* \} = (a+b)^*$$

$$L_8 = \{ \underset{\epsilon}{w} \underset{\epsilon}{w}^R x \mid w, x \in (a+b)^* \} = (a+b)^*$$

$$\underline{(a+b)^*} + abba(a+b)^* + baab(a+b)^* + \cdots = \underline{(a+b)^*}$$

$$L_9 = \{ x \underset{\epsilon}{\underline{w}} \overset{(a+b)^*}{} \underset{\epsilon}{\underline{w}}^R \mid x, w \in (a+b)^* \} = (a+b)^*$$

$$\underline{(a+b)^*} + (a+b)^* aa + (a+b)^* bb + \cdots = (a+b)^*$$

$L_{10} = \{ w x w^R \mid w, x \in (a+b)^+ \}$

$= a(a+b)^+a + b(a+b)^+b$

$a(a+b)^+a + b(a+b)^+b + ab(a+b)^+ba + ba(a+b)^+ab + \cdots$

$abb(a+b)^+bba$

aaabbababababbaaa

$aa(a+b)^+ + bb(a+b)^+ + abba(a+b)^+$

not possible

$L_{11} = \{ww^R x \mid w, x \in (a+b)^+\}$ $+ baab(a+b)^+ + \cdots\cdots$

$= \times$

$L_{13} = \{w x w^R \mid w, x \in (a+b)^+\} = a(a+b)^+a + b(a+b)^+b$

$L_{12} = \{x w w^R \mid w, x \in (a+b)^+\}$

$(a+b)^+aa + (a+b)^+bb + (a+b)^+abba + (a+b)^+baab + \cdots$

$\times$

$L_{14} = \{ w\,\overset{\frown}{w} \mid w \in (a+b)^* \}$ = not possible

$\{ \epsilon, aa, bb, abab, baba \ldots \} = X$

$L_{15} = \{ ww \mid w \in (a)^* \}$

$\{ \epsilon, aa, bb, a^2a^2, \ldots \} = (aa)^*$

$L_{16} = \{ \underline{w} \, c \, \underline{w} \mid w \in (a+b)^* \}$ = not regula

$\{ c, aca, bcb, abcab, bacba \ldots \}$

$L_{17} = \{ \overset{\epsilon}{W} \underline{X} \overset{\epsilon}{W} \mid w, x \in (a+b)^* \} = (a+b)^*$

$(a+b)^* + a(a+b)^* a + \ldots$

not regular

not possible

$ab(a+b)^+ab$

$$L_{18} = \{ wxw \mid w, x \in (a+b)^+ \} = \{ a(a+b)^+a + b(a+b)^+b \} \times$$

$$L_{19} = \{ wwx \mid w, x \in (a+b)^+ \}$$

$$L_{20} = \{ xww \mid w, x \in (a+b)^+ \}$$

Topic : NOTE

- Palindrome languages over more than one symbol are not regular .Hence regular expression not possible.

- Palindrome languages over one symbol are regular.

Regular

$L_1 = \{ a^n b^m \mid (n+m) \text{ is even} \}$

a^{10}

as bs

{even + even
odd + odd.}

{even}

$$\{ (aa)^* (bb)^* + a(aa)^* b(bb)^* \}$$

$L_2 = \{ a^n b^m \mid (n+m) \text{ is odd} \}$

$\left\{ \begin{matrix} \text{odd} + \text{even} \\ \text{even} + \text{odd} \end{matrix} \right\}^{\text{odd}}$

$a(aa)^*(bb)^* + (aa)^*(bb)^*b$

$L_3 = \{1, 2, 4, 8, \ldots 2^n \ldots\}$

all these numbers written in Unary

$\{1, 11, 1111, 1^8 \ldots\} \Rightarrow$ No common differen

X

not regula

$L_3 = \{1, 2, 4, 8, - - - 2^n - - - \}$

all these numbers written in binary.

✓ 01

✓ 010

000100

✓ 1000

$\{0^* 1 0^*\}$

2 mins Summary

Topic One

Topic Two

Topic Three

Topic Four

Topic Five

THANK - YOU