

**DS & AI  
CS & IT**

## **Linear Algebra**

**Lecture No. 07**



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# Recap of previous lecture



Topic

Non Homogeneous System of Equ<sup>n</sup>





# Topics to be Covered



## Topic

- Homogeneous System of Equ<sup>n</sup>
- Basics of Eigen Values.





# Homogeneous system of linear Equ<sup>n</sup>

$$AX=0$$



eg 
$$\begin{cases} 2x - y + 4z = 0 \\ x + 2y - 2z = 0 \\ -x - y + 3z = 0 \\ x + 2y - 4z = 0 \end{cases} \Rightarrow \begin{bmatrix} 2 & -1 & 4 \\ 1 & 2 & -2 \\ -1 & -1 & 3 \\ 1 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \boxed{A_{4 \times 3} X_{3 \times 1} = O_{4 \times 1}}$$

By observation, we can see that  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$  is a solution of above system.  
because these values of  $x, y, z$  satisfying all 4 equ<sup>n</sup> at a time.

& this type of solution always exist & this sol. is known as TRIVIAL sol.

⊛ Homog system is always consistent OR Homog system Never inconsistent.  
because Reason 1: Trivial sol always exist.

Reason 2:  $P(A) = P(A: 0)$  always. Hence there is No Need to write Aug Mat further.



Why zero vector is treated as LD vector  $\rightarrow$

Vectors  $x_1, x_2, x_3, \dots, x_r$  are called LD if  $\exists$  a relationship of the type  $k_1 x_1 + k_2 x_2 + \dots + k_r x_r = 0$  ——— (1)  
where  $k_1, k_2, k_3, \dots, k_r$  should not all zero simultaneously.

eg(i) Let us consider  $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$  then we have

$$2x_1 + x_2 + x_3 = 0 \quad \text{i.e. } x_1, x_2, x_3 \text{ are LD}$$

eg(ii) Let us consider  $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$  then we have

$$0x_1 + 0x_2 + kx_3 = 0 \quad \text{i.e. relationship exist where not all constants are zero}$$

eg(iii) Let us consider  $x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$  then we can write

$$kx = 0 \quad \text{where } k \text{ is non zero} \quad \text{Hence relationship again exist so } x \text{ is LD}$$

Note: ① underdetermined, homog system always consist  $\infty$  solutions

unique sol  $\subseteq$  DNE

No sol  $\subseteq$  DNE

② unique sol  $\subseteq$  Trivial sol  $\subseteq$  ZERO solution (always exist)

③  $\infty$  sol  $\subseteq$  Non Trivial sol also exist  $\subseteq$  Non zero sol also exist.

④ ZERO sol  $\neq$  No sol.

$$\therefore X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$



# Methods of Solving Homog. system

$(A_{m \times n} X_{n \times 1} = O_{m \times 1})$

## RANK Method

$(m > n, m = n, m < n)$

- ① If  $\rho(A) = \text{No. of Variables} \Rightarrow$  unique sol.
- ② If  $\rho(A) < \text{ " " } \Rightarrow \infty$  sol.

## MATRIX Method

$(\text{only for } m = n)$

- ① if  $|A| \neq 0 \Rightarrow$  unique sol exist
- ② if  $|A| = 0 \Rightarrow \infty$  sol exist

Note → ① unique sol = Trivial sol = Zero sol. always exist.  
 ②  $\infty$  sol = Non Trivial sol = Non Zero sol also exist



Qr Find k for which

msq  $(3k-8)x + 3y + 3z = 0$

$3x + (3k-8)y + 3z = 0$

$3x + 3y + (3k-8)z = 0$

has Non Trivial sol.

- (a)  $\frac{2}{3}$  (b)  $\frac{11}{3}$  (c)  $\frac{4}{3}$  (d)  $\frac{8}{3}$

for Non Trivial sol ( $\infty$  sol)  $\rightarrow |A| = 0$

$$\begin{vmatrix} (3k-8) & 3 & 3 \\ 3 & (3k-8) & 3 \\ 3 & 3 & (3k-8) \end{vmatrix} = 0 \rightarrow \begin{vmatrix} (3k-2) & 3 & 3 \\ (3k-2) & (3k-8) & 3 \\ (3k-2) & 3 & (3k-8) \end{vmatrix} = 0$$

$$(3k-2) \begin{vmatrix} 1 & 3 & 3 \\ 1 & (3k-8) & 3 \\ 1 & 3 & (3k-8) \end{vmatrix} = 0$$

$$(3k-2) \begin{vmatrix} 1 & 3 & 3 \\ 0 & (3k-11) & 0 \\ 0 & 0 & (3k-11) \end{vmatrix} = 0 \Rightarrow (3k-2)(3k-11)^2 = 0$$

$k = \frac{2}{3}, \frac{11}{3}, \frac{11}{3}$



The value of  $\alpha$  for which the system of equation

PYQ

$$x + y + z = 0$$

$$y + 2z = 0$$

$$\alpha x + z = 0$$

has more than one solution is

(a) -1

(b) 0

(c)  $\frac{1}{2}$

(d) 1

$$|A| = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ \alpha & 0 & 1 \end{vmatrix} = 0$$

$$1[1-0] - 0 + \alpha[2-1] = 0$$

$$1 + \alpha = 0$$

$$\alpha = -1$$

M-II Expanding along  $R_3$

$$\alpha[2-1] - 0 + 1[1-0] = 0$$

$$\alpha + 1 = 0 \Rightarrow \alpha = -1$$



NW8 for the System to have Infinite Sol, which of the following is/are True

MS8  $px + qy + rz = 0$

$qx + ry + pz = 0$

$rx + py + qz = 0$

$|A| = 0 \Rightarrow \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix} = 0$

$\dots \Rightarrow \boxed{p^3 + q^3 + r^3 - 3pqr = 0}$  Ans

☒ (a)  $p = q = r$  (By observation)

☒ (b)  $p + q + r = 0$

☒ (c)  $p^3 + q^3 + r^3 - 3pqr = 0$

☐ (d) None

$(p+q+r) \cdot [(p-q)^2 + (q-r)^2 + (r-p)^2] = 0$


is  $\boxed{p+q+r=0}$  or  $\boxed{p=q=r}$  Ans



Q Consider  $A_{m \times n} X_{n \times 1} = B_{m \times 1}$  then which one is false ?

- (a) if  $m > n$ ,  $B \neq 0$ ,  $\rho(A) < \rho(A:B)$  then system has NO sol. (T)
- (b) if  $m = n$ ,  $B = 0$ ,  $|A| \neq 0$  then system has only Trivial sol. (T)
- (c) if  $m = 3$ ,  $n = 5$ ,  $B = 0$  then system has also Non Zero sol. (T)
- (d) if  $m = 5$ ,  $n = 3$ ,  $B = 0$  &  $\rho(A) = 3$  then system has only zero sol. (T)
- (e) if  $m = n$ ,  $B = 0$ , then system has sol. (T) (obviously True)
- (f) if  $m > n$ ,  $B = 0$  then system has Multiple sol. (false)
- (g) if  $m > n$ ,  $B = 0$ ,  $\rho(A) < n$  then system has Multiple sol. (True)
- (h) Sun Rises from east (T) (obviously True)



Analysis (PODCAST) → Consider  $x+y+z=0$  — (1)  $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}_{1 \times 3} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3 \times 1} = \begin{bmatrix} 0 \end{bmatrix}_{1 \times 1}$  

ie  $A_{1 \times 3} X_{3 \times 1} = O_{1 \times 1}$  then it's  $\mathbb{R}^n$  are;

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\text{Trivial sol.}}, \underbrace{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ 0 \end{bmatrix}, \begin{bmatrix} -7/2 \\ 0 \\ 7/2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}, \dots}_{\text{Non Trivial sol's}} \dots \infty \mathbb{R}^n$$

Null space

$X$  branches into:

- $X_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/3 \\ 1/3 \\ 0 \end{bmatrix}, \begin{bmatrix} -0.2 \\ +0.2 \\ 0 \end{bmatrix} \dots$  But all are (L.D) on  $X_1$
- $X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} -\sqrt{5} \\ 0 \\ \sqrt{5} \end{bmatrix}, \dots$  But all are (L.D) on  $X_2$
- $X_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = X_1 + X_2, X_4 = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} = 2X_1 + X_2 \dots$  ie again all are (L.D)



ie out of  $\infty \mathbb{R}^n$  only  $x_1$  &  $x_2$  are LI & Rest are LD on them.



Nullity (Dimension of Nullspace)  $\rightarrow$

"the Counting of LI  $\mathbb{R}^n$  in any Homog system  $A_{m \times n} X_{n \times 1} = O_{m \times 1}$  is called Nullity" & it is given as  $N(A)$ .

Rank-Nullity Th: Nullity = Number of Columns - Rank(A)

$$\text{ie } N(A) = N(\text{Columns}) - \rho(A)$$

9 Find Nullity of  $x+y+z=0$ ? Here  $A = [1 \ 1 \ 1]_{1 \times 3}$  so  $\rho(A) = 1$ .

$$\text{Hence Nullity}(A) = 3 - 1 = 2$$

ie above system has two LI  $\mathbb{R}^n$  & Rest are LD on them.



PODCAST: Consider  $A_{m \times n} X_{n \times 1} = O_{m \times 1}$



Row Space: set of all linear combinations of Row vectors of  $A$  is called Row space

& Dimension of Row Space = Man No. of LI Row vector =  $\rho(A)$

Column Space set of all linear combinations of Column vectors of  $A$  is called Column<sup>Space</sup>  
(Range space)

& Dimension of Column space = Man No. of LI Column vector =  $\rho(A)$

Null Space set of all solutions of  $Ax = 0$  is called Null Space

& Dimension of Null Space = Nullity = No. of LI solutions in terms of  $X$ .



⑧  $N(A) \perp R(A)$  i.e. Row space is  $\perp$  to Null space (or  $N$  space) 

"Every vector in Null space is orthogonal to every vector in Row space"

$$AX = 0 \Rightarrow \begin{pmatrix} a_1 & a_2 & a_3 & \dots & a_n \\ b_1 & b_2 & b_3 & \dots & b_n \\ c_1 & c_2 & c_3 & \dots & c_n \\ \vdots & \vdots & \vdots & & \vdots \\ \alpha_1 & \alpha_2 & \alpha_3 & \dots & \alpha_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{m \times 1}$$

$m \times n$                        $n \times 1$                        $m \times 1$

$a_1x_1 + a_2x_2 + \dots + a_nx_n = 0$  i.e.  $R_1 \perp X$  are orthogonal  
again  $b_1x_1 + b_2x_2 + \dots + b_nx_n = 0$  i.e.  $R_2 \perp X$  "  
& so on ... i.e. Hence proved.



Q If every sol of system  $A_{4 \times 5} X = 0$  is a scalar Multiplication of  $\begin{bmatrix} 2 \\ -3 \\ 4 \\ -1 \\ 2 \end{bmatrix}$  then  $\rho(A) = ?$



Ans, No. of LI sol<sup>n</sup> of (1) = one =  $N(A)$

By Rank-Nullity Th:  $N(A) = N(C) - \rho(A)$   
 $1 = 5 - \rho(A) \Rightarrow \rho(A) = 4$  Ans

Solution set  $X = \begin{bmatrix} 2 \\ -3 \\ 4 \\ -1 \\ 2 \end{bmatrix}$  —  $\begin{bmatrix} 4 \\ -6 \\ 8 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -4 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1.5 \\ 2 \\ 0.5 \\ 1 \end{bmatrix} \dots \infty$  sol<sup>n</sup>.

(LI) (LD)



The nullity of system of equations:

$$x_1 + x_2 - x_3 + x_4 = 0$$

$$2x_1 + 3x_2 + x_3 + 4x_4 = 0$$

$$3x_1 + 2x_2 - 6x_3 + x_4 = 0$$

(a) 1

✓ (b) 2

(c) 3

(d) 4

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 3 & 1 & 4 \\ 3 & 2 & -6 & 1 \end{bmatrix} \xrightarrow[\substack{R_2 - 2R_1 \\ R_3 - 3R_1}]{}$$

$3 \times 4$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & -1 & -3 & -2 \end{bmatrix} \xrightarrow{R_3 + R_2} \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2 \quad \text{N(Columns)} = 4$$

$$\text{So } N(A) = N(C) - \rho(A) = 4 - 2 = 2$$

$$A_{3 \times 4} X_{4 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3 \times 1} \Rightarrow X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \text{only two are LI} \quad \text{Rest are LD on them.}$$



The number of linearly independent solutions of the system of equations

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \text{ is equal to}$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}$$

☒ (a) 1

(b) 2

(c) 3

(d) 0

$$\because |A| = 0 \Rightarrow \rho(A) \neq 3$$

$$\text{ie } \rho(A) = 2$$

$$N(A) = N(C) - \rho(A)$$

$$= 3 - 2 = 1$$



Q Solve:

$$\begin{cases} 2x+y+z=0 \\ x+y=0 \\ y+z=0 \end{cases}$$

$\Downarrow$

$$A_{3 \times 3} X_{3 \times 1} = O_{3 \times 1}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\approx \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (x+y) \\ (-y+z) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < \text{No of Variables}$   
 $\Rightarrow \infty \text{ sol exist.}$

$$\& N(A) = 3 - \rho(A) = 3 - 2 = 1$$

$$x+y=0 \Rightarrow x=-y$$

$$-y+z=0 \Rightarrow y=z \text{ let } z=k, \text{ then } y=k$$

$$\& X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k \\ k \\ k \end{bmatrix} = k \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, k = \text{arbitrary const.}$$

if  $\infty$  sol exist in which only  $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$  is LI



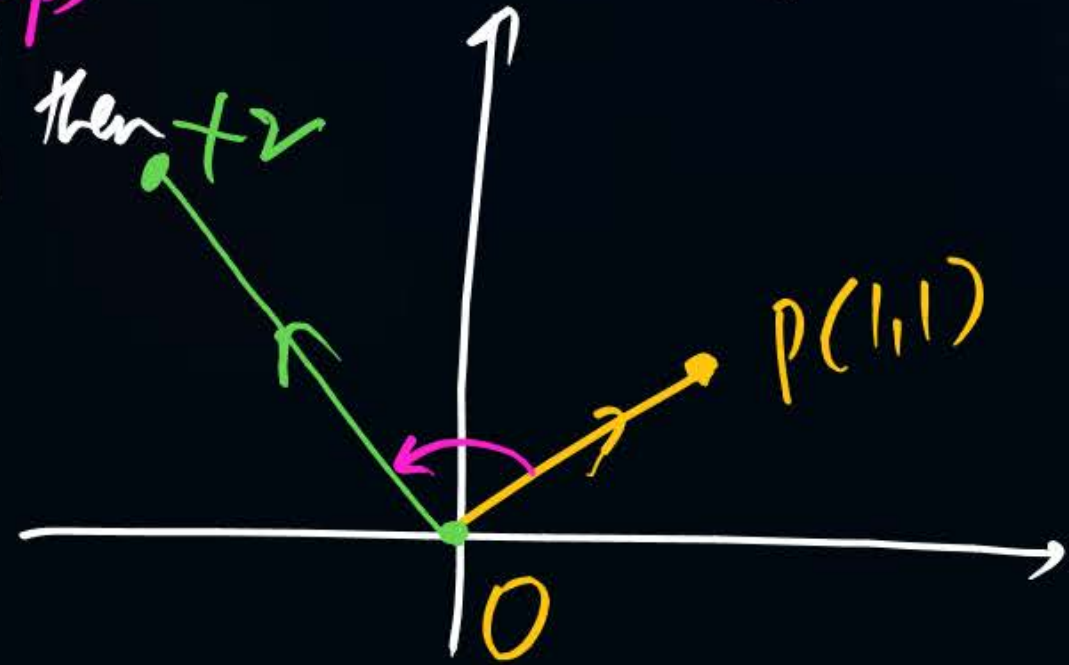


# Eigen Values / Eigen Vectors (for 1. they will ask 8.)



PODCAST: Consider  $A = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$  &  $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  then  $+2$

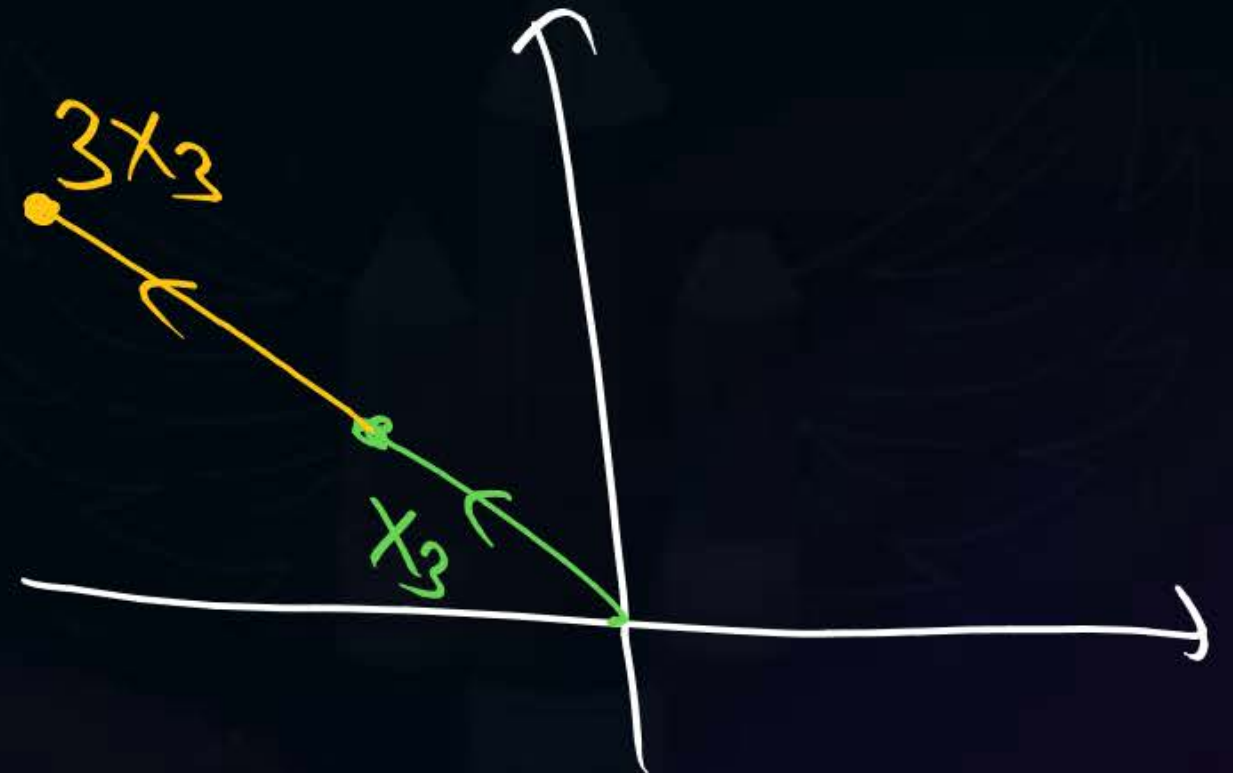
$$Ax_1 = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix} = x_2$$



again consider  $x_3 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ .

$$Ax_3 = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix} = 3 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = 3x_3$$

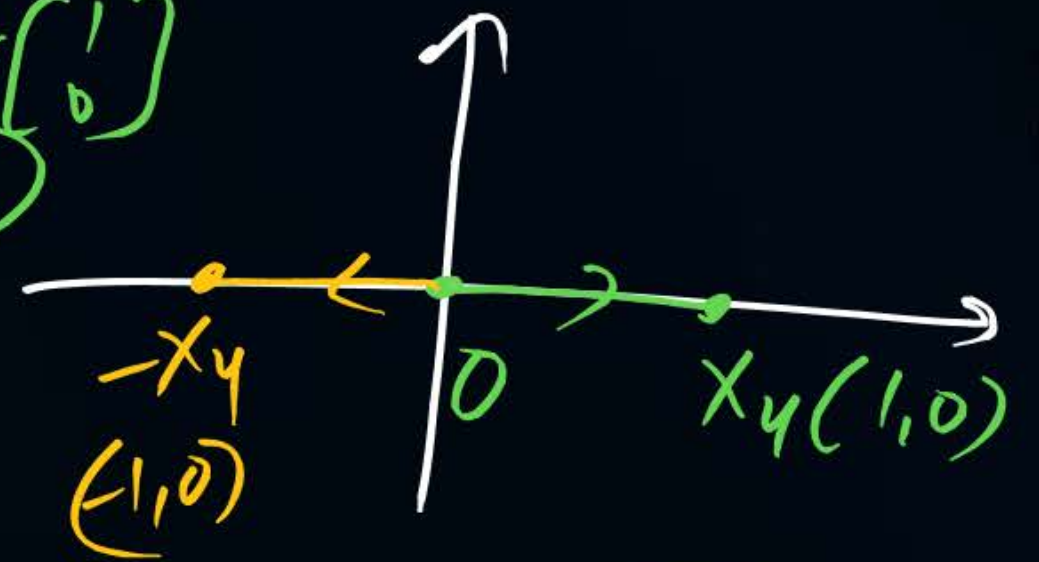
$x_3 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$  = Sp. vector & Sp Value = 3





Now consider  $A = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$ ,  $X_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  sp. vector =  $X_4 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$  & sp. value =  $-1$

$$AX_4 = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -X_4$$



Consider  $A = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$ ,  $X_5 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow AX_5 = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = X_5$

But  $X_5$  is not an eigen vector  $\therefore$  it is a zero vector.

ie E vector can not be zero vector.

Consider  $A = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ,  $X_6 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  then  $AX_6 = \begin{bmatrix} 0 & 0 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 0 \cdot X_6$

So  $X_6$  is also special vector where sp. value = 0

ie E value can be zero also.



Def<sup>n</sup>! Consider Sq. Mat  $A_{n \times n}$  then Non Zero Vector  $X$  is called Eigen Vector, corresponding to Eigen value  $\lambda$  (Real/Complex/Zero) if we are able to find a relationship of the type,

$$\boxed{AX = \lambda X}$$

$\swarrow \lambda = \text{Eigen Value}$   
 $\searrow X = \text{Eigen Vector.}$

LHS is the Multi of Two Matrices = RHS is the Scalar Multi in a Mat

(Tough)
(Easy)

(\*) Here we are considering Homogeneous system as follows

$$AX = \lambda X \Rightarrow AX - \lambda X = 0 \Rightarrow \boxed{(A - \lambda I)X = 0}$$

So it will satisfy all the prop of Homog system.



(\*) Consider  $AX = \lambda X$   
 $(A - \lambda I)X = 0 \quad \text{--- (1)}$

$$MX = 0$$

Non Zero Eigen Vector  
Non Zero solution  
 $\Rightarrow \infty \text{ solns.}$

$$\rho(M) < n \text{ or } |M| = 0$$

$$\Rightarrow \rho(A - \lambda I) < n \text{ or } |A - \lambda I| = 0$$

So Necessary Condition for the existence of Non Zero Eigen Vector is

$$\rho(A - \lambda I) < n \text{ or } |A - \lambda I| = 0$$

Characteristic Eqn<sup>n</sup> of A  $\rightarrow$

eqn<sup>n</sup> (1) is called C.Eq of A & Roots of this eqn<sup>n</sup> i.e. values of  $\lambda$  are called  
 E. Values / E. Roots / Char Values  
 / Char Roots / Latent Roots / Sp. Values





eg: Find the E-Values of ①  $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  ②  $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  ③  $A = \begin{bmatrix} 8 & 6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$



$$\underline{\text{Sol:}} \text{ ① } A - \lambda I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (2-\lambda) & 1 & 1 \\ 1 & (2-\lambda) & 1 \\ 0 & 0 & (1-\lambda) \end{bmatrix}$$

$\neq$  C-Equ<sup>n</sup> But Char Mat

Now C-Equ<sup>n</sup>  $|A - \lambda I| = 0$

$$\begin{vmatrix} (2-\lambda) & 1 & 1 \\ 1 & (2-\lambda) & 1 \\ 0 & 0 & (1-\lambda) \end{vmatrix} = 0$$

$$(1-\lambda) [(2-\lambda)^2 - 1] = 0$$

$$(1-\lambda) [(2-\lambda+1)(2-\lambda-1)] = 0$$

$$\boxed{(\lambda-3)(\lambda-1)^2 = 0} \text{ — C-Equ<sup>n</sup> of } A$$

&  $\lambda = 3, 1, 1 \rightarrow$  Char Values.



②  $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  then

C. Equ<sup>n</sup> is  $|A - \lambda I| = 0$ .

$$\begin{vmatrix} (0-\lambda) & -1 \\ 1 & (0-\lambda) \end{vmatrix} = 0$$

$$\lambda^2 + 1 = 0 \rightarrow \text{C. Equ}^n$$

$$\lambda = i, -i \text{ E Values}$$

③  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

C. Equ<sup>n</sup> is  $|A - \lambda I| = 0$

$$\begin{vmatrix} (8-\lambda) & -6 & 2 \\ -6 & (7-\lambda) & -4 \\ 2 & -4 & (3-\lambda) \end{vmatrix} = 0$$

(HW)

$$\lambda(\lambda-3)(\lambda-15) = 0 \rightarrow \text{C. Equ}^n$$

$$\lambda = 0, 3, 15 \rightarrow \text{E Values}$$



**THANK - YOU**

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