DS & AI
CS & IT

Linear Algebra

Lecture No. 03

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Recap of previous lecture

Topic Algebra of MATRICES

Topics to be Covered

Topic

TYPES OF MATRICES

MAJEDAAR QUESTION $\rightarrow$ Consider $A_{2\times3}$, $B_{3\times4}$ & $C_{4\times2}$ then find the minimum number of Multiplications & Additions that will be Required to find Matrix Product (ABC) ?

Sol: ABC $\rightarrow$ Case I: $(AB)C$
$\rightarrow$ Case II: $A(BC)$

Case II:

$$B_{3\times4}\,C_{4\times2} = (BC)_{3\times2}$$

No. of Multi $= 3\times4\times2 = 24$

No. of Additions $= 3(4-1)2 = 18$

Now

$$A_{2\times3}(BC)_{3\times2} = (ABC)_{2\times2}$$

No. of Multi $= 2\times3\times2 = 12$

No. of Additions $= 2(3-1)2 = 8$

Overall to find ABC using Case II, Multi $= 24+12 = 36$ & Additions $= 18+8 = 26$

PQ: if $E=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}_{3\times3}$ & $G=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ s.t multi of E & F is G then $F=?$

Sol: ATQ, $EF=G \Rightarrow EF=I$

$\to E^{-1}=F$ i.e $F=E^{-1}=$? (Ans)

$\to F^{-1}=E$

$\because |E| = \ldots\ldots = 1$ i.e E is Non sing so E^{-1} will exist.

$\cos\theta$	$-\sin\theta$	0	$\cos\theta$	$-\sin\theta$
$\sin\theta$	$\cos\theta$	0	$\sin\theta$	$\cos\theta$
0	0	1	0	0
$\cos\theta$	$-\sin\theta$	0	$\cos\theta$	$-\sin\theta$
$\sin\theta$	$\cos\theta$	0	$\sin\theta$	$\cos\theta$

$$E^{-1}=\frac{\operatorname{adj}E}{|E|}=\frac{1}{(1)}\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}=E^{T}$$

i.e $F=E^{-1}=E^{T}=$ Ans

RECAP

1. $A_{m\times n}$ = R.Mat (Diag DNE)
2. $A_{n\times n}$ = Sq. Mat (Diag exist)
3. for upper diag elements $i < j$
4. for lower " " $i > j$
5. for diag elements $i = j$
6. Addition & Subtraction are possible if A & B are of same order
7. $A_{m\times n} B_{n\times p}$ = Possible

8. $A_{m\times n} B_{n\times p}$ → Multi = $m \cdot n \cdot p$; Add = $m(n-1)p$
9. Commutative law:

$A+B = B+A$ But $AB \neq BA$

10. Associative law:-

$A+(B+C) = (A+B)+C$

& $A(BC) = (AB)C$

(*) In Matrix Algebra, Concept of Division is not defined.

i.e If $AB = C$ then $B = \cancel{\frac{C}{A}}$ Blunder

But in Number System

$ab = c \Rightarrow b = \frac{c}{a}$ (Valid if $a \neq 0$)

if A is Nonsingular i.e A^{-1} exist then we can proceed as follows;

$$A^{-1}(AB) = A^{-1}C$$

$$IB = A^{-1}C$$

or $B = A^{-1}C$ ✓

Note: In Number System

Cancellation Law Holds when $\boxed{\text{No} \neq 0}$

i.e if $a^2 = a$

$\Rightarrow \cancel{a}a = \cancel{a} \Rightarrow a = 1, (a \neq 0)$

Similarly in Matrix Algebra,

Cancellation Law Holds only when $\boxed{A \text{ is Non Sing.}}$

if $|A| \neq 0$ & $A^2 = A \Rightarrow A = I$

(*) In Matrix Algebra,

"Cancellation Law Holds only when Matrix is Non Singular."

eg if A is a Non Sing Mat s.t

$\cancel{A}B = \cancel{A}$ then $\boxed{B = I}$

Proof: $\because$ A is Non Sing so A^{-1} exist.

so $AB = A$

$A^{-1}(AB) = A^{-1}A$

$IB = I$ ie $B = I$

eg if $A\cancel{B} = \cancel{A}$ then $B = I$ (T/F)?

False $\because$ it is not given that A is Non Singular

Doubt: If $(ABC = AC) \not\Rightarrow (B = I)$

Q. If $\boxed{XY = Y}$ —(1) & $\boxed{YX = X}$ —(2) then $X^2 + Y^2 = ?$

MSQ

(a) $X+Y$

(b) $2I$

(c) 0

(d) I

Case I: if X & Y are Non Singular Matrices i.e. X^{-1} = exist, Y^{-1} = exist.

i.e. $X\cancel{Y} = \cancel{Y} \Rightarrow X = I$

& $Y\cancel{X} = \cancel{X} \Rightarrow Y = I$

So $X^2 + Y^2 = I^2 + I^2$

$= I + I$

$= 2I$

Case II: if X & Y are singular then

$X^2 + Y^2 = XX + YY$

$= X(YX) + Y(XY)$

$= (XY)X + (YX)Y$ (By Associative law)

$= (Y)X + (X)Y$

$= X + Y$

If X and Y are two singular matrices such that $XY = Y$ and $YX = X$ then $X^2 + Y^2$ equals

(a) $X + Y$ (b) XY

(c) YX (d) $2(X + Y)$

Complex Conjugate : $z = x + iy$ then $\bar{z} = x - iy$ eg $z_1 = 3 - 2i$, $\bar{z}_1 = 3 + 2i$

$z_2 = 4i$, $\bar{z}_2 = -4i$

$z_3 = 2$, $\bar{z}_2 = 2$

Conjugate Mat if $A = \begin{bmatrix} 2 & 4-i & 2i \\ -2+i & 0 & -5 \end{bmatrix}_{2\times 3}$

then $A^T = \begin{bmatrix} 2 & -2+i \\ 4-i & 0 \\ 2i & -5 \end{bmatrix}_{3\times 2}$ $\quad \bar{A} = \begin{bmatrix} 2 & 4+i & -2i \\ -2-i & 0 & -5 \end{bmatrix}_{2\times 3}$

Tranjugate Mat (Transposed Conjugate of Mat)

$A^\theta = \overline{(A^T)}$ or $(\bar{A})^T$ So $A^\theta = \overline{(A^T)} = \begin{bmatrix} 2 & -2-i \\ 4+i & 0 \\ -2i & -5 \end{bmatrix}$

(*) The Concept of Trace, Det, Inverse are defined only for Square Matrices while the " of Transpose, Conjugate & Tranjugate & Rank are also defined for Rectangular Matrices

Matrix

Real Mat: If in a Mat all the elements are Real numbers then Mat is Called Real Matrix

i.e. if A is a Real Mat then $\boxed{\bar{A} = A}$ or $A^{\theta} = A^{T}$ Learn.

Complex Mat: If in a Mat, at least one element is Complex Number then Mat is Called Complex Mat i.e. $\boxed{\bar{A} \neq A}$

eg $A = \begin{bmatrix} 2 & -5 & 4 \\ 1 & 2 & 0 \end{bmatrix}$ = Real Mat, $B = \begin{bmatrix} 2 & -5 & 0 \\ 1 & 2 & 4i \end{bmatrix}$ = Complex Mat

Sp. Points:

1. $\text{Tr}(A+B+C) = \text{Tr}(A) + \text{Tr}(B) + \text{Tr}(C)$
2. $(A+B+C)^{\theta} = A^{\theta} + B^{\theta} + C^{\theta}$
3. $(A+B+C)^{T} = A^{T} + B^{T} + C^{T}$
4. $|A+B+C| \leq |A| + |B| + |C|$
5. $(A+B+C)^{-1} \neq A^{-1} + B^{-1} + C^{-1}$

But $(A+B+C)^{-1} = \dfrac{\text{adj}(A+B+C)}{|A+B+C|}$

6. $|ABC| = |A| \cdot |B| \cdot |C|$

REVERSAL LAW:

$$(ABC)^{T} = C^{T} B^{T} A^{T}$$

$$(ABC)^{\theta} = C^{\theta} B^{\theta} A^{\theta}$$

$$(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$$

* we know that, $AB \neq BA$ in general

But $\text{Tr}(AB) = \text{Tr}(BA)$ always.

Reversal Law $\to$ $\boxed{(ABC)^T = C^T B^T A^T}$, $\boxed{(ABC)^\theta = C^\theta B^\theta A^\theta}$, $\boxed{(ABC)^{-1} = C^{-1} B^{-1} A^{-1}}$

& these Results can also be generalized.

Qs if A, B, C, D are Non Singular Matrices of same order s.t $\boxed{ABCD = I}$ then evaluate B & $B^{-1} = ?$

Sol: ~~$ABCD = I \Rightarrow B = \frac{I}{ACD}$~~ $= A^{-1}C^{-1}D^{-1}$

Blunder

Correct App:

$$ABCD = I$$

$$A^{-1}(ABCD)D^{-1} = A^{-1}.I D^{-1}$$

$$I BC I = A^{-1}.I.D^{-1}$$

$$BC = A^{-1}D^{-1}$$

$$(BC)C^{-1} = A^{-1}D^{-1}C^{-1} \Rightarrow B = A^{-1}D^{-1}C^{-1}$$

(ii) $B^{-1} = (A^{-1}D^{-1}C^{-1})^{-1}$

$$= (C^{-1})^{-1}(D^{-1})^{-1}(A^{-1})^{-1}$$

$$B^{-1} = C.D.A$$ Ans

Some More Special Matrices

$\Rightarrow |A| = 0$

eg

$$\begin{bmatrix} a & b & c \\ b & e & d \\ c & d & f \end{bmatrix}, \quad \begin{bmatrix} 0 & b & -c \\ -b & 0 & d \\ c & -d & 0 \end{bmatrix}_{3\times3}, \quad \begin{bmatrix} 2 & 5+i & -7 \\ 5-i & 0 & 2i \\ -7 & -2i & -3 \end{bmatrix}, \quad \begin{bmatrix} 2i & 5+i & -7 \\ -5+i & 0 & 2i \\ 7 & 2i & -3i \end{bmatrix}$$

Symmetric Mat. $(\because A^T = A)$

Skew Symm Mat (Anti Symm Mat) $(\because A^T = -A)$

Hermitian Mat $(\because A^\theta = A)$

Skew Herm. Mat. $(\because A^\theta = -A)$

(*) Imp. if $A_{n\times n}$ s.t A is skew symm. then $|A| = \begin{cases} 0 & , n = \text{odd order} \\ \text{Perfect sq.} & , n = \text{even order} \end{cases}$

i.e Det of skew symm Mat of odd order is always Zero.

Note ① Symm Mat is symmetrical about leading diagonal i.e Corresponding elements are same & $A^T = A$

② Skew symm Mat : Diagonal elements are all zero. and corresponding elements are of opposite sign & $A^T = -A$

③ Hermitian Mat : Diagonal elements are either zero or purely real Nos & corresponding elements are conjugates of each other & $A^\theta = A$.

④ Skew Hermitian Mat : Diagonal elements are either zero or purely Imaginary numbers and corresponding elements are −ve conjugates of each other. & $A^\theta = -A$

PODCAST

Consider Sq Mat $A_{n\times n}$ then we can write (using common sense).

① $A = \left(\frac{A+A^T}{2}\right) + \left(\frac{A-A^T}{2}\right)$

$A = P + Q$

where $P = \frac{A+A^T}{2}$, Here $P^T = P$ i.e. P is symm.

& $Q = \frac{A-A^T}{2}$, Here $Q^T = -Q$ i.e. Q is skew symm

i.e. $\boxed{\text{Sq Mat} = \text{Symm} + \text{Skew Symm}}$

② $A = \left(\frac{A+A^\theta}{2}\right) + \left(\frac{A-A^\theta}{2}\right)$

$A = R + S$

$R = \frac{A+A^\theta}{2}$, Here $R^\theta = R$ i.e. R is Herm.

$S = \frac{A-A^\theta}{2}$, Here $S^\theta = -S$ i.e. S is Skew Hermitian.

$\boxed{\text{Sq Mat} = \text{Herm} + \text{Skew Herm.}}$

(5) Every sq. Mat can be expressed as the sum of symm & skew symm Mat. as follows; consider sq Mat $A_{n\times n}$ then by common sense we can write.

$$\boxed{A = \left(\frac{A+A^T}{2}\right) + \left(\frac{A-A^T}{2}\right)}$$

ie $A = P + Q$

where $P = \frac{A+A^T}{2}$ = Symm Mat

& $Q = \boxed{\frac{A-A^T}{2}}$ = skew symm Mat

for, $P^T = \left(\frac{A+A^T}{2}\right)^T = \frac{A^T+(A^T)^T}{2} = \frac{A^T+A}{2} = P$ Hence symm.

& $Q^T = \left(\frac{A-A^T}{2}\right)^T = \frac{A^T-(A^T)^T}{2} = \frac{A^T-A}{2} = -\left(\frac{A-A^T}{2}\right) = -Q$ Hence skew symm.

(5) Every sq. Mat can be expressed as the sum of Hermitian & Skew Herm Mat. as follows; consider sq. Mat $A_{n\times n}$ then by common sense we can write.

$$\boxed{A = \left(\frac{A+A^{\theta}}{2}\right) + \left(\frac{A-A^{\theta}}{2}\right)}$$

$$A = R + S$$

$$R = \frac{A+A^{\theta}}{2} = \text{Herm. Mat}$$

$$S = \frac{A-A^{\theta}}{2} = \text{Skew Herm. Mat.}$$

for, $R^{\theta} = \left(\frac{A+A^{\theta}}{2}\right)^{\theta} = \ldots\ldots = R$ Hence Proved

$S^{\theta} = \left(\frac{A-A^{\theta}}{2}\right)^{\theta} = \ldots\ldots = -S$ " ;

Doubt!

$(A+B)^{-1} \neq A^{-1} + B^{-1}$

$(A+B)^{-1} = \frac{\operatorname{adj}(A+B)}{|A+B|}$

If A and B are two symmetric matrices. Then consider the following staements.

(i) A + B is symmetric

(ii) AB is symmetric

(iii) AB + BA is symmetric

(a) Only (i) is true

(b) (i) and (ii) are true

(c) (i) and (iii) are true

(d) (i), (ii) and (iii) are true.

$A+B$, $A^T = A$

$B^T = B$

(i) $(A+B)^T = A^T + B^T = (A+B)$

i.e. $(A+B)$ is symmetric

(ii) $(AB)^T = B^T A^T = BA \neq AB$?

i.e. AB is Not symmetric

(iii) $(AB+BA)^T = (AB)^T + (BA)^T$

$= B^T A^T + A^T B^T$

$= BA + AB = (AB+BA)$ so $(AB+BA)$ is symm.

(*) If $A_{n\times n}$ s.t A is skew symm then $|A| = \begin{cases} 0 & , n = \text{odd} \\ \text{perfect sq} & , n = \text{even} \end{cases}$

eg $|A| = \begin{vmatrix} 0 & -1 & -2 & 0 & 4 \\ 1 & 0 & 5 & 0 & -7 \\ 2 & -5 & 0 & 2 & 1 \\ 0 & 0 & -2 & 0 & -8 \\ -4 & 7 & -1 & 8 & 0 \end{vmatrix}_{5\times 5} = ? = 0$, eg $|A| = \begin{vmatrix} 0 & 3 \\ -3 & 0 \end{vmatrix}_{2\times 2} = 9 = (3)^2$

$=$ Perfect sq.

(*) Determinant of skew symm Mat of odd order is always ZERO

eg $|A| = \begin{vmatrix} 0 & 1 & 2 & 0 \\ -1 & 0 & 4 & 0 \\ -2 & -4 & 0 & 6 \\ 0 & 0 & -6 & 0 \end{vmatrix}_{4\times 4} = ? = 36 = (6)^2 =$ Perfect sq.

(*) Max Number of Different elements that are required to construct a Symm Mat $A_{n\times n} = \frac{n(n+1)}{2}$

M-I $A = \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \\ x_2 & y_1 & y_2 & \cdots & y_{n-1} \\ x_3 & y_2 & z_1 & \cdots & z_{n-2} \\ \vdots & \vdots & \vdots & & \vdots \\ x_n & y_{n-1} & z_{n-2} & & \alpha \end{bmatrix}_{n\times n}$ = Symm.

Total elements $= n^2$

Total diff elements $= \frac{(n^2-n)+2n}{2} = \frac{n^2+n}{2} = \frac{n(n+1)}{2}$

M-II $A = \begin{bmatrix} a & c \\ c & b \end{bmatrix} \Rightarrow$ max Diff elements $= 3$

$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} \Rightarrow$ " " " $= 6$

$A = \begin{bmatrix} a & b & c & d \\ b & e & f & g \\ c & f & h & i \\ d & g & i & j \end{bmatrix} \Rightarrow$ " " " $= 10$

& so on ...

(*) Max Number of Ind. entries that are required to construct skew symm. Mat $A_{n\times n} = \frac{n(n-1)}{2}$ Learn.

$$A = \begin{bmatrix} 0 & x_1 & x_2 & x_3 & \cdots & x_n \\ -x_1 & 0 & y_1 & y_2 & & y_{n-2} \\ -x_2 & -y_1 & 0 & z_1 & & z_{n-3} \\ \vdots & \vdots & \vdots & 0 & \ddots & \vdots \\ -x_n & -y_{n-2} & -z_{n-3} & \cdots & \cdots & 0 \end{bmatrix} = \text{skew symm.}$$

Lower Diag elements are Dependent
Diag elements are also Dependent.
i.e only upper diag elements are Ind

No. of Ind. entries
= No. of upper diag elements

$$= \frac{n^2 - n}{2} = \frac{n(n-1)}{2}$$

$$= \frac{\text{Total elements} - \text{Diag elements}}{2}$$

Q. If $A=[a_{ij}]_{n\times n}$ s.t. $a_{ij}=i^2-j^2$ $\forall i$ & j then for $n\geq 3$, $n=$ odd, $A^{-1}=?$

Sol: let $n=3$, $A=[a_{ij}]_{3\times 3}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}=\begin{pmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{pmatrix}_{3\times 3}$

= Skew symm of odd order.

$\because |A|=0$

$\Rightarrow A^{-1}=$ DNE

Q. If $A=[a_{ij}]_{n\times n}$ s.t. $\forall i$ & j; $a_{ij}=-a_{ji}$ then the value of

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij}) = \underline{\quad ? \quad}$$

Sol: Given $a_{ij}=-a_{ji}\ \forall i$ & j

For Diag elements: $i=j$ then

$$a_{ij}=-a_{ji}$$

$$\Rightarrow a_{ii}=-a_{ii} \Rightarrow 2a_{ii}=0$$

or $a_{ii}=0\ \forall i$ i.e. Diag elements are all zero.

For off Diag elements:

$a_{ij}=-a_{ji} \Rightarrow$ corresponding elements are of opposite sign

i.e. A is skew symm. so

$\boxed{\text{sum of all the elements in skew symm Mat} = 0}$

Some MORE SPECIAL MATRICES.

(*) if $A^2 = A$ then A is Idempotent Mat

$|A^2| = |A|$

$|A|^2 - |A| = 0$

$|A|(|A|-1) = 0$

$|A| = 0$ or 1

if [A is Idempotent] $\Longrightarrow$ $\not\Longleftarrow$ [$|A| = 0$ or 1.]

(*) if $A^2 = I$ then A is Involutary Mat

$|A^2| = |I|$

$|A|^2 = 1 \Rightarrow |A| = 1$ or -1

if [A is Involutary] $\Longrightarrow$ $\not\Longleftarrow$ [$|A| = 1$ or -1]

(*) if $A^k = O$ then A is Nilpotent Mat of Power K where K is least +ve int.

eg if A is s.t. $A^2 \neq 0, A^3 \neq 0$ But $A^4 = 0$ then $A^5 = A^6 = A^7 = \dots = O$

i.e A is Nilpotent Mat of index 4.

Idempotent Mat → If $\boxed{A^2 = A}$ then A is Called Idempotent Mat.

$|A^2| = |A| \Rightarrow |A|^2 = |A| \Rightarrow |A|.(|A|-1) = 0 \Rightarrow |A| = 0$ or $|A| = 1$.

i.e if A is an Idempotent Mat $\underset{\not\Longleftarrow}{\Longrightarrow}$ $|A| = 0$ or 1.

Involutary Mat → If $\boxed{A^2 = I}$ then A is Called Involutary Mat

$|A^2| = |I| \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$.

i.e if A is an Involutary Mat $\underset{\not\Longleftarrow}{\Longrightarrow}$ $|A| = 1$ or -1

Nilpotent Mat → if $\boxed{A^K = O}$ then A is Called Nilpotent Mat of power K.
where K is least +ve integer.

if A is s.t $A \neq O, A^2 \neq O, A^3 \neq O$ But $A^4 = O$ then $A^5 = A^6 = A^7 = \dots\dots = O$

Here A is Nilpotent Mat of power 4. $|A^K| = |O| \Rightarrow |A|^K = 0 \Rightarrow |A| = 0$.

Q: if $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ then A is ________ Mat

$$A^2 = A \cdot A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \cdot \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$

$\because A^2 = O$ so A is Nilpotent Mat of index 2.

eg $A = \begin{bmatrix} 6 & 9 \\ -4 & -6 \end{bmatrix}$ then A is ? N-Mat of index 2

$(\because a = 2, b = 3)$

$(\because A^2 = O)$

If T is Idempdotent then $T^k = T$ for

(a) $k = 2$ MAG

$\Rightarrow k = 2, 3, 4, \ldots$

(b) All integer k

(c) All positive integer $k \geq 2$

(d) All of the above

W.K.that,

for an Idempotent Mat A

$A^2 = A$

ie $T^2 = T$

$(T^k = T) \Rightarrow \boxed{k = 2}$

But wait, wait, wait....

$A^3 = A^4 = A^5 = \cdots = A$

ie If T is Idempotent then $T^2 = T, T^3 = T, T^4 = T, \ldots$

(*) if $\boxed{A.A^T = I}$ then A is called $\boxed{\text{orthogonal Mat.}}$

or $A^{-1} = A^T$ i.e. orthogonal Mat is always Invertible

(*) if $\boxed{A.A^\theta = I}$ then A is called $\boxed{\text{unitary Mat}}$

or $A^{-1} = A^\theta$ i.e. unitary Mat is always invertible.

(*) $\underbrace{\text{unitary Mat}}_{AA^\theta = I \;\text{—}\; (1)}$ formed by $\underbrace{\text{Real Nos}}_{\text{Real Mat}}$ is orthogonal also.

i.e. $A^\theta = A^T$ — (2)

by (1) & (2), $A.A^T = I$

which is condition of orthogonal Mat

Orthogonal Mat $\rightarrow$ if $\boxed{A.A^T = I}$ then A is Called Orthogonal Mat

or $\left(A^{-1} = A^T\right)$

Unitary Mat $\rightarrow$ if $\boxed{AA^{\theta} = I}$ then A is Called Unitary Mat.

or $\left(A^{-1} = A^{\theta}\right)$

Note ① Det of O-Mat is either 1 or -1 (i.e. O-Mat can't be singular i.e always Invertible)

Proof: w.k.that $AA^T = I$

$$|AA^T| = |I|$$

$$|A|.|A^T| = 1$$

$$|A|.|A| = 1 \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$$

Sp Note:

if (A is an O-Mat) $\overset{\Longrightarrow}{\not\Leftarrow}$ $(|A| = 1 \text{ or } -1)$

If $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is an orthogonal Mat. ? True

Sol: $AA^T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

So A is orthogonal Mat & $A^{-1} = A^T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

For the given orthogonal matrix Q.

$$Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$$

The inverse is $Q^{-1} = ? = Q^{T} =$ (c)

(a) $\begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$

(b) $\begin{bmatrix} -3/7 & -2/7 & -6/7 \\ 6/7 & -3/7 & -2/7 \\ -2/7 & -6/7 & 3/7 \end{bmatrix}$

(c) $\begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$

(d) $\begin{bmatrix} -3/7 & -6/7 & -2/7 \\ -2/7 & -3/7 & -6/7 \\ -6/7 & -2/7 & 3/7 \end{bmatrix}$

Q. MSQ HW

if $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ then A is

(a) orthogonal

(b) Involutary

(c) unitary

(d) Hermitian

Q. if $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}_{4\times 4}$ s.t. A is an orthogonal mat then $(AA')^{-1} = ?$

MCQ

$\Downarrow$

$(AA^T = I)$

(a) $I_2 = I_{2\times 2}$

(b) $I_3 = I_{3\times 3}$

(c) $I_4 = I_{4\times 4}$

(d) $I = I_{n\times n}$

Now $(AA')^{-1} = (I)^{-1} = I = I_{4\times 4} = I_4$

THANK - YOU

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