DS & AI
CS & IT

Probability & Statistics

Lecture No. 10

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Recap of previous lecture

Topic

1. DISCRETE RANDOM VARIABLE
2. Basics of PROBABILITY DISTRIBUTION

Topics to be Covered

Topic

① Probability Distribution (Continued)
② Geometric Distribution
③ Binomial Distribution

Probability Dist^n – The Table representing Distribution of probabilities is called Prob Distribution.

eg A coin is tossed until H appears then find Prob Dist of No. of tosses.

Sol: $X = \{\text{Number of tosses}\} = \{1, 2, 3, 4, \ldots\}$

$S = \{\overset{=1}{H}, \overset{=2}{TH}, \overset{=3}{TTH}, \overset{=4}{TTTH}, \ldots\}$

Prob Dist:

X :	1	2	3	4	5	$\ldots\ \infty$
$P(X)$:	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\ldots$

CROSS Check :-

$$\sum p_i = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\ldots\infty$$

$$= \frac{1/2}{1-\frac{1}{2}} = \frac{1/2}{1/2} = 1$$

Hence verified.

Q. A coin is tossed until Head appears then find the Average No. of tosses Req.?

Sol: $X = \{\text{No. of tosses}\} = \{1, 2, 3, 4 \ldots\ldots\}$

X :	1	2	3	4	-------
$P(X)$:	1/2	1/4	1/8	1/16	-------

$$E(X) = \sum p_i x_i = p_1 x_1 + p_2 x_2 + p_3 x_3 + \ldots\ldots$$

$$= \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \ldots\ldots$$

$$= \frac{1}{2}\left[1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots\ldots\right]$$

$$\boxed{1 + 2x + 3x^2 + 4x^3 + \ldots\ldots = (1-x)^{-2}}$$

$$= \frac{1}{2}\left[\left(1-\frac{1}{2}\right)^{-2}\right] = \frac{1}{2}\left[\left(\frac{1}{2}\right)^{-2}\right] = \frac{2^2}{2} = 2$$

Average, No. of tosses $= 2$

Note: Minimum Tosses Required $= 1$

Maximum " " $=$ No idea (∞)

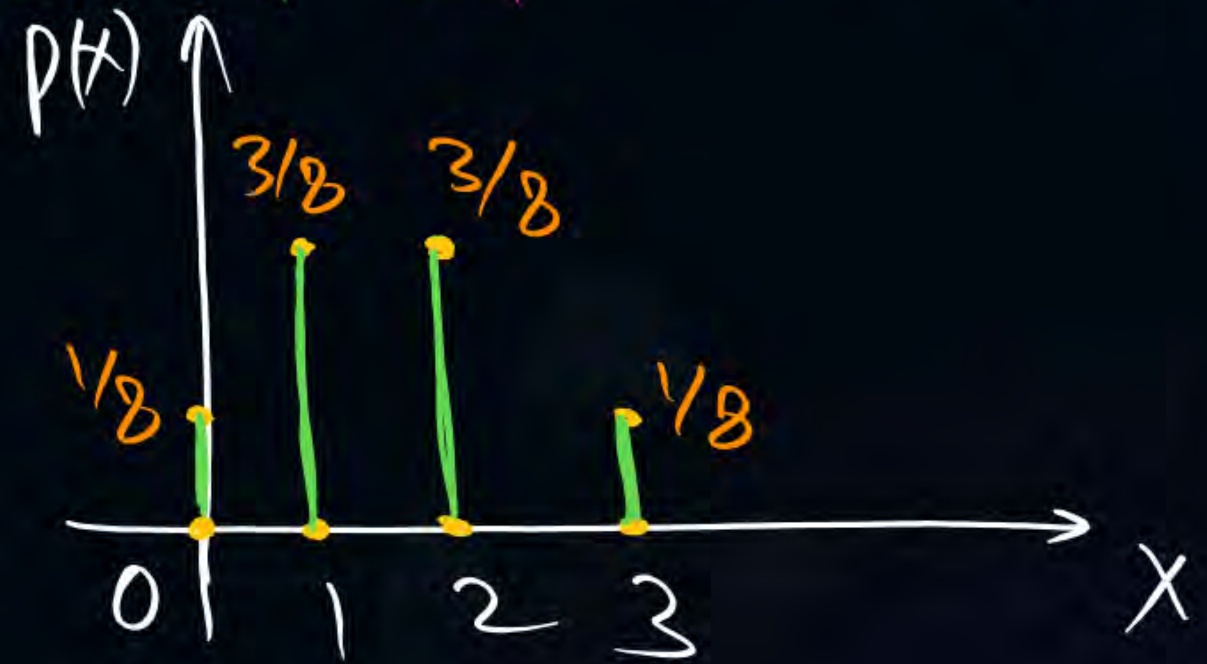
eg: A coin is tossed thrice then find the prob Dist of Number of Heads?

Sol: $X = \{\text{Number of Heads}\} = \{0, 1, 2, 3\}$

$$S = \{\underbrace{(HHH)}_{3H}, \underbrace{(HHT)}_{2H}, \underbrace{(HTH)}_{2H}, \underbrace{(HTT)}_{1H}, \underbrace{(THH)}_{2H}, \underbrace{(THT)}_{1H}, \underbrace{(TTH)}_{1H}, \underbrace{(TTT)}_{0H}\} = 8 \text{ Triplets}$$

Prob Dist:

X:	0	1	2	3
$p(x)$:	$1/8$	$3/8$	$3/8$	$1/8$



Combined Height of these Bars $= \sum p_i = 1$

Q: A coin is tossed thrice then Find Mean, Variance & S.D of Numbers of Heads.

Sol: $X = \{\text{No. of tosses}\} = \{0, 1, 2, 3\}$

X:	0	1	2	3
$P(X)$:	$\frac{1}{8}$	$3/8$	$3/8$	$1/8$

$E(X) = \sum p_i x_i$

$= p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$

$= \frac{1}{8}(0) + \frac{3}{8}(1) + \frac{3}{8}(2) + \frac{1}{8}(3)$

$= \frac{12}{8} = \frac{3}{2} = 1.5$

Average No. of Heads $= 1.5$

Now, $E(X^2) = \sum p_i x_i^2$

$= p_1x_1^2 + p_2x_2^2 + p_3x_3^2 + p_4x_4^2$

$= \frac{1}{8}(0)^2 + \frac{3}{8}(1)^2 + \frac{3}{8}(2)^2 + \frac{1}{8}(3)^2$

$= 3$

So $Var(X) = E(X^2) - (E(X))^2$

$= 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4} = 0.75$

$SD(\sigma) = +\sqrt{Var(X)} = +\frac{\sqrt{3}}{2} = 0.86$

ANALYSIS: (1) Average w.r.to 1 = Prob & Av w.r.to 100 = %

A coin is tossed thrice then Average No. of Heads = $\boxed{1.5}$, Variance = 0.75, SD = $\frac{\sqrt{3}}{2}$

" " once, then Average No. of Heads = 0.5, Var = 0.25, SD = $\frac{\sqrt{3}/2}{\sqrt{3}}$ = 0.5

" " 10000 times, Av No. of Heads = 5000, Var = 2500, SD = $0.5 \times \sqrt{10000}$ = 50

ie $\mu = 5000,\ \sigma = 50$

$\mu - 3\sigma = 5000 - 150 = 4850$

$\mu + 3\sigma = 5000 + 150 = 5150$

or we have very good chance (99.7% chance) of having No. of Heads lying in b/n (4850, 5150) ie $\boxed{4850 \le \text{No of Heads} \le 5150}$

Further Analysis: $X = \{0, 1, 2, 3\}$ then $\bar{X} = \frac{\sum X}{N} = \frac{0+1+2+3}{4} = 1.5$ (So Easy)

then what is the need of MATURE method ??

Let us try to calculate Variance also using childhood Method.

$$Var(X) = \frac{\sum (X-\bar{X})^2}{N} = \frac{(0-1.5)^2+(1-1.5)^2+(2-1.5)^2+(3-1.5)^2}{4} = \frac{5}{4} = 1.25$$

Hmmmmmmm.... what is this ?? why Variance is not coming To 0.75 ??

ie we have flaw in childhood Method

For Correct APPROACH See Next Slide.

Correct App of Childhood Method:

$$S = \{\underset{3}{(HHH)}, \underset{2}{(HHT)}, \underset{2}{(HTH)}, \underset{1}{(HTT)}, \underset{2}{(THH)}, \underset{1}{(THT)}, \underset{1}{(TTH)}, \underset{0}{(TTT)}\}$$

$$X = \{\text{No of Heads}\} = \{0, 1, 1, 1, 2, 2, 2, 3\}$$

So $\text{Mean}(X) = \bar{X} = \dfrac{\sum X}{N} = \dfrac{0+1+1+1+2+2+2+3}{8} = 1.5$

& $\text{Var}(X) = \dfrac{\sum (X-\bar{X})^2}{N} = \dfrac{(0-1.5)^2 + 3\times(1-1.5)^2 + 3\times(2-1.5)^2 + (3-1.5)^2}{8} = 0.75$

App II $E(X) = 0\times\frac{1}{8} + 1\times\frac{3}{8} + 2\times\frac{3}{8} + 3\times\frac{1}{8} = 1.5$

Now feeling Good.

Advice: Try to Follow MATURE method.

Q: A Coin is tossed until Head appears or Tail appears 4 times in succession, then Find the Average Number of tosses Required

(a) 2.50

(b) 1.87

(c) 4

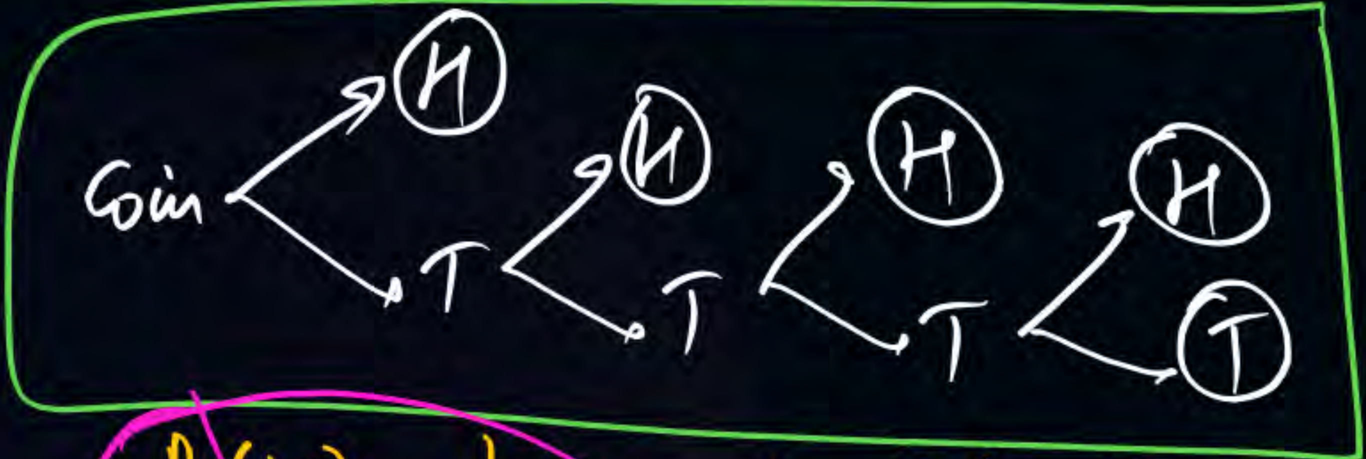
(d) 3.89

$X = \{\text{Number of tosses}\} = \{1, 2, 3, 4\}$

Min tosses Req = 1

Max " " = 4

Av " " $= E(X) = ??$



$S = \{\underset{=1}{H}, \underset{=2}{TH}, \underset{=3}{TTH}, \underset{=4}{TTTH}, \underset{=4}{TTTT}\}$

$P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$, Tosses are Ind

$\cancel{P(H) = \frac{1}{5}}$,

$\cancel{P(TH) = 1/5}$.

$\cancel{P(TTH) = 1/5}$,

$\cancel{P(TTTH) = 1/5}$,

$\cancel{P(TTTT) = 1/5}$,

$p_1 = P(X = 1\text{ toss}) = P(H) = 1/2$

$p_2 = P(X = 2\text{ tosses}) = P(TH) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$p_3 = P(X = 3\text{ tosses}) = P(TTH) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$$p_4 = P(X = 4 \text{ tosses}) = P(\overbrace{TTTH \text{ or } TTTT}^{ME})$$

$$= \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 = \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$$

Prob Dist:

X:	1	2	3	4
$P(X)$:	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

$\because \sum p_i = 1$

so all correct.

$$E(X) = \sum p_i x_i = p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4$$

$$= \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{8}(4)$$

$$= \frac{15}{8} = 1.87$$

Q In a Game a person is paid Rs 5 when all Heads or all Tail occurs and he will have to pay Rs 3 if either one or two Head occurs when three coins are tossed simultaneously then find the expected amount wins or looses by him on an average per game?

(ii) If above game is played by 150 persons, 200 times each then find the Expected amount wins or looses by game organiser

Sol: $N_1 = 150$ persons,
$N_2 = 200$ games,

For single person in single game; $X = \{\text{Amount Received}\} = \{5, -3\}$

$$S = \{\underbrace{(HHH)}_{+5}, \underbrace{\underbrace{(HHT)}_{-3}, \underbrace{(HTH)}_{-3}, \underbrace{(HTT)}_{-3}, \underbrace{(THH)}_{-3}, \underbrace{(THT)}_{-3}, \underbrace{(TTH)}_{-3}}, \underbrace{(TTT)}_{+5}\} \cong 8 \text{ Triplets}.$$

$$p_1 = P(X = 5\text{ Rs}) = P[\text{All H or All T}] = P(W) = \frac{2}{8} = \frac{1}{4}$$

$$p_2 = P(X = -3\text{ Rs}) = P(L) = \frac{6}{8} = 3/4.$$

X:	5	-3
$P(X)$:	$\frac{1}{4}$	$\frac{3}{4}$

$$E(X) = \Sigma p_i x_i = \frac{1}{4} \times 5 + \frac{3}{4} \times (-3) = -1\text{ Rs}.$$

Average Amount Received by single Person in single Game $= -1$ Rs.

ie He will loose on an Average 1 Rs per Game.

(ii) single Person in single Game will loose $= 1$ Rs.

So 150 " in 200 Games " " $= 1 \times 150 \times 200 = 30000$ Rs. Ans

Q: In a Game Man wins one Rupee for six and also looses one Rupee for any other number when a fair die is thrown.
Man decided to throw die three times but to quit as & when he gets six, then find the Expected amount wins or looses by him on an average per Game?
(ii) If above Game is played by 108 persons, 200 times each then find the Expected amount wins or looses by Game organize?

Sol: $N_1 = 108$ Persons, $N_2 = 200$ Games,

For single Person in single Game: $X = \{\text{Amount Received}\}$

$$P(W) = P(\text{six occurs}) = \frac{1}{6}$$

$$P(L) = P(\text{six Not occurs}) = \frac{5}{6}$$

$S=\{\underset{=1}{W}, \underset{=0}{LW}, \underset{=-1}{LLW}, \underset{=-3}{LLL}\}$ (so $X=\{1, 0, -1, -3\}$

$p_1 = P(X=1\text{ Rs}) = P(W) = \frac{1}{6}$

$p_2 = P(X=0\text{ Rs}) = P(LW) = \frac{5}{6}\cdot\frac{1}{6}$

$p_3 = P(X=-1\text{ Rs}) = P(LLW) = \frac{5}{6}\cdot\frac{5}{6}\cdot\frac{1}{6}$ so

$p_4 = P(X=-3\text{ Rs}) = P(LLL) = \left(\frac{5}{6}\right)^3$

X:	1	0	−1	−3
$P(x)$:	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{25}{216}$	$\frac{125}{216}$

So $E(X) = \sum p_i x_i = \frac{1}{6}(1) + \frac{5}{36}(0) + \frac{25}{216}(-1) + \frac{125}{216}(-3) = -\frac{91}{54}\text{ Rs} = -1.68$

(ii) Game organiser will earn $= ? = \left(\frac{91}{54}\times 200\right)\times 108 = 36400\text{ Rs.}$

$$E(x) = \sum p_i x_i = p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 x_4$$

$$= \frac{1}{6}(1) + \frac{5}{36}(0) + \frac{25}{216}(-1) + \frac{125}{216}(-3) = -\frac{91}{54} \text{ Rs.}$$

He will loose $(91/54)$ Rs on an Average per Game.

(ii) on an Average, single person in single Game will loose $= \frac{91}{54}$ Rs.

so " " , 108 Persons in 200 Games will loose $= \frac{91}{54} \times 200 \times 108$

$= 36400$ Rs.

i.e Game organizer will earn 36400 Rs over the night.

The random variable X takes on the values 1, 2 (or) 3 with probabilities $p_1 = \frac{2+5P}{5}$, $p_2 = \boxed{\frac{1+3P}{5}}$ and $p_3 = \frac{1.5+2P}{5}$ respectively the values of P and E(X) are respectively

(a) ~~0.05, 1.87~~ (b) 1.90, 5.87

(c) 0.05, 1.10 (d) 0.25, 1.40

$\Sigma p_i = 1$

$$\frac{(2+5P)+(1+3P)+(1.5+2P)}{5} = 1$$

$10P + 4.5 = 5$

$P = \frac{0.5}{10} = \frac{1}{20} = 0.05$

$$\boxed{\begin{array}{lccc} X = & 1 & 2 & 3 \\ P(x) = & p_1 & p_2 & p_3 \end{array}}$$

So $E(X) = 1 \times p_1 + 2 \times p_2 + 3 p_3 = \cdots\cdots = 1.87$

Four bad oranges are mixed accidently with 16 good oranges. Find the probability distribution of the number of bad oranges in a draw of two oranges. and it's Expected Value is ____

(a) 0.3

(b) 0.4

(c) 1.37

(d) 1

20f → 4B, 2f, 16G

$X = \{\text{Number of Bad oranges}\} = \{0, 1, 2\}$

$p_1 = P(X = 0\text{ Bad}) = \dfrac{{}^4C_0 \times {}^{16}C_2}{{}^{20}C_2} = \dfrac{16\times15}{20\times19} = \dfrac{60}{95}$

$p_2 = P(X = 1\text{ Bad}) = \dfrac{{}^4C_1 \times {}^{16}C_1}{{}^{20}C_2} = \dfrac{4\times16}{20\times19}\times2 = \dfrac{32}{95}$

$p_3 = P(X = 2\text{ Bad}) = \dfrac{{}^4C_2}{{}^{20}C_2} = \dfrac{4\times3}{20\times19} = \dfrac{3}{95}$

X:	0	1	2
P(X):	$\frac{60}{95}$	$\frac{32}{95}$	$\frac{3}{95}$

$E(X) = \sum p_i x_i = \text{-----} = 0.4$

Doubt:- $S = \{BB, BG, GB, GG\}$, $P(B) = \frac{4}{20} = \frac{1}{5}$, $P(G) = \frac{16}{20} = \frac{4}{5}$

$\because P(B) + P(G) = 1$ i.e. B & G are ME & Exhaustive.

$X = \{2, 1, 1, 0\} = \{0, 1, 2\}$

$p_1 = P(X = 0 \text{ Bad}) = ? = P(GG) =$

$\rightarrow = \frac{4}{5} \times \frac{4}{5}$ (drawing one by one) (Not given in Quest)

$\rightarrow = \frac{^{16}C_2}{^{20}C_2}$ (drawing simultaneously)

The probability mass function $P(x)$ of a discrete random variable X is given by

$P(x) = \frac{1}{2^x}$, where $x = 1, 2, \ldots\ldots \infty$. The expected value of X is ______. [in integer]

X:	1	2	3	4	------
$P(X)$:	$\frac{1}{2}$	$\frac{1}{2^2}$	$\frac{1}{2^3}$	$\frac{1}{2^4}$	------

$$E(X) = \sum p_i X_i = p_1 X_1 + p_2 X_2 + p_3 X_3 + \cdots\cdots$$

$$= \frac{1}{2}(1) + \frac{1}{4}(2) + \frac{1}{8}(3) + \cdots\cdots$$

= Already solved YESTERDAY

$= 2$

$$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots\cdots$$

BERNOULLIE TRIAL → whenever in a Random Exp we have only two
(Bernoullie R Exp)

possible outcomes then such Types of Exp are called Bernoullie Trials.

Here we will assume one outcome as success and another outcome as failure

i.e. R Exp < Success, Failure & p = Prob(success), q = Prob(failure) where $\boxed{q+p=1}$

Note: (1) Geometric Dist, Binomial Dist, Poisson Dist are based on Bernoullie Trials.

(2) Success = which is Required should be assumed as success.

eg A Die is thrown then success & failure can be assumed as
Success = {6}, failure = {1, 2, 3, 4, 5} i.e. $p=\frac{1}{6}$, $q=\frac{5}{6}$

Geometric Distribution (–ve Binomial Dist) →

→ this Distribution helps to find Number of Trials Required before getting 1st success

→ this Dist is also Based on Bernoullie Trials.

→ $$\boxed{(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots}$$

where $n = -ve$ integer or any fraction

eg $(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$

eg $\boxed{(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots}$

eg $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots$

eg $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \cdots$

eg $(1-x)^{1} = \boxed{1 - x + \cancel{x^2 - x^3 + \cdots}} = 1 - x$

Geometric Distribution → This Distribution helps to find Number of Trials required to get 1^{st} success.

Let $X = \{$ Number of Trials Required to get 1^{st} success $\}$

& $p = P(\text{getting success})$ & $q = P(\text{getting failure})$, where $q+p=1$

Prob Dist:

X :	1	2	3	4	5	.....	$(r+1)^{th}$	...... ∞
$P(X)$:	p	qp	q^2p	q^3p	q^4p	.....	(q^rp)	......

$\because \sum p_i = p + qp + q^2p + q^3p + \ldots \infty$

$= p\{1 + q + q^2 + q^3 + \ldots\} = p\left\{\frac{1}{1-q}\right\} = p\left(\frac{1}{p}\right) = 1$

ie $\boxed{P(X = r^{th}) = q^{r-1} \cdot p}$ & $\boxed{P(X = r+1^{th}) = q^r \cdot p}$

① $Mean(X) = E(X) = \sum p_i x_i = p_1 x_1 + p_2 x_2 + p_3 x_3 + \ldots\ldots$

$$= p(1) + qp(2) + q^2 p(3) + q^3 p(4) + \ldots\ldots$$

$$= p\{1 + 2q + 3q^2 + 4q^3 + \ldots\ldots\}$$

$$= p\{(1-q)^{-2}\} \quad \{\because 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots = (1-x)^{-2}\}$$

$$= p\left(\frac{1}{(1-q)^2}\right) = p\left(\frac{1}{p^2}\right) = \frac{1}{p}$$ learn.

② $Var(X) = E(X^2) - E^2(X) = (\text{Do yourself}) - \left(\frac{1}{p}\right)^2 = \ldots\ldots = \frac{q}{p^2}$ learn.

③ $SD(\sigma) = +\sqrt{\frac{q}{p^2}}$

Q. Suppose you are playing a Game of Dart and the probability of getting success is 0.4 then find the Prob that you will hit the Bull's Eye in 4th Trial ?

Sol $p = 0.4,\ q = 0.6$

Req Prob $= P(\text{getting success in } 4^{th} \text{ trial})$

$= P[FFFS]$

$= (0.6)^3 \times (0.4)$

(Here Each Trial is Ind)

M-II

$X = \{\text{No of Trials Req. to get } 1^{st} \text{ success}\}$

i.e $X = 4$ so

$$\boxed{P(X = r^{th}\ \text{Trial}) = q^{r-1} \cdot p}$$

$P(X = 4^{th}\ \text{Trial}) = (0.6)^{4-1}(0.4)$

A fair die with faces {1, 2, 3, 4, 5, 6} is thrown repeatedly till '3' is observed for the first time. Let X denote the number of times the die is thrown.

then $E(X) =$ ______

M-I $X = \{\text{No. of times die is thrown}\} = \{\text{No. of Trials Required to get 3 first time}\}$

Let Success = 3 occurs so $p = \frac{1}{6}$

failure = 3 Not occurs, $q = \frac{5}{6}$

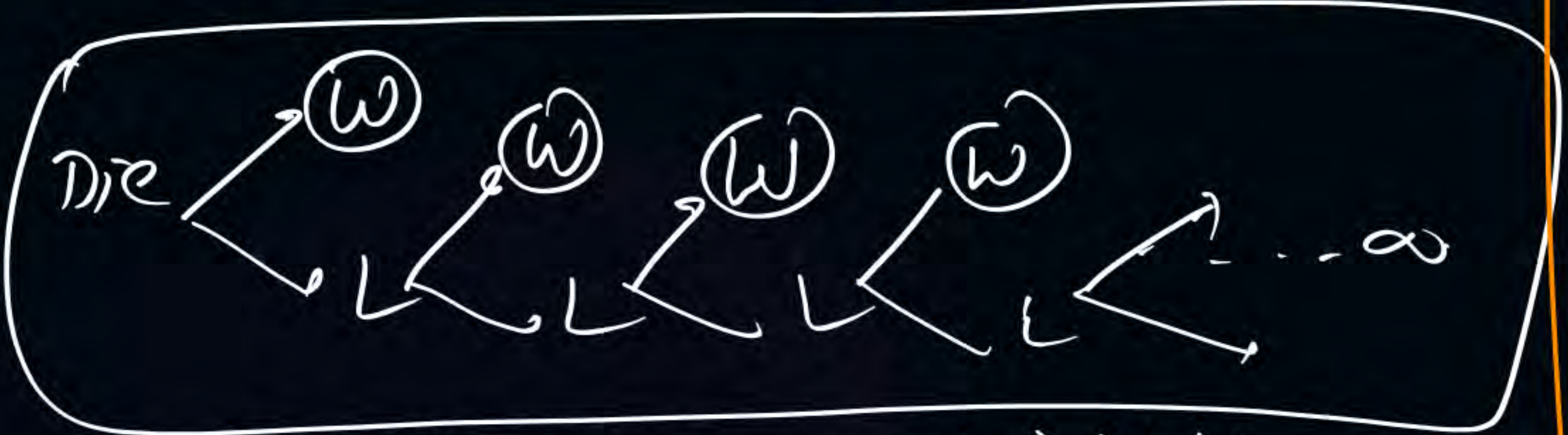
ie $X \sim$ Geometric Dist.

So $\boxed{E(X) = \frac{1}{p}} = \frac{1}{1/6} = 6$

M-II $X = \{\text{no. of times die is thrown}\} = \{1, 2, 3, 4, \ldots\}$.

let $W = \{3 \text{ occurs}\} \Rightarrow P(W) = \frac{1}{6}$

& $L = \{3 \text{ not occurs}\} \Rightarrow P(L) = \frac{5}{6}$



$S = \{\underset{=1}{W}, \underset{=2}{LW}, \underset{=3}{LLW}, \underset{=4}{LLLW}, \ldots\}$

X:	1	2	3	4	------
$P(x)$:	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{25}{216}$	$\frac{125}{1296}$	------

So $E(X) = \sum p_i x_i$

$= p_1 x_1 + p_2 x_2 + p_3 x_3 + \ldots\ldots$

$= \ldots\ldots$

$= 6$

Av No. of tosses Required to get 1st success $= 6$ Ans

Q: Consider a Company that produces on an Average 3 defective Bulbs out of 60 Bulbs. Then Find the prob that 1st def Bulb will be found when 6th one is tested?

Sol: $X = \{$Number of Trials Required to get 1st $\underbrace{\text{Defective Bulb}}_{\text{success}}\}$

$\because$ we have Bernoullie Trials

is $\boxed{X \sim G.Dist.}$ So $p = P(\text{success}) = P(\text{Def. Bulb}) = \frac{3}{60} = \frac{1}{20}$

& $q = P(\text{Failure}) = P(\text{Non Def Bulb}) = \frac{19}{20}$

$$P(X = r^{th}\text{ trial}) = q^{r-1} \cdot p$$

$$P(X = 6^{th}\text{ trial}) = q^5 \cdot p = \left(\frac{19}{20}\right)^5 \cdot \left(\frac{1}{20}\right)$$

eg: A coin is tossed 6 times then find the prob that only 1^{st} two outcomes are H?

Req Prob $= P[HHTTTT] = \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^4 = \frac{1}{64}$ fav case $= \{(HHTTTT)\}$

eg: A coin is tossed 6 times then Find the prob of getting exactly 2 Heads?

App II Total Cases $= \frac{\text{fav Cases}}{\text{Total Cases}} = \frac{\left(\frac{6!}{2!\,4!}\right)}{2^6} = \frac{15}{64}$

App III (Using Binomial Dist) Req Prob $= {}^6C_2 \cdot p^2 q^4 = 15 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^4 = \frac{15}{64}$

(ii) $P(\text{getting exactly } 3H) = {}^6C_3 \cdot p^3 q^3 = 20 \times \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 = \frac{20}{64}$

eg: A coin is tossed 6 times then find the prob that you will get 1^{st} Head in 6^{th} Trial?

$X = \{\text{No. of Heads}\}$, success so $X \sim$ G. Dist so Req Prob $= q^5 p = \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right) = \frac{1}{64}$

fav $= (TTTTTH)$

eg: 10 ships are going in an Atlantic ocean then find the prob that exactly 3 will come back if history suggest that out of 11000 ships only 10000 came back?

Sol: $X = \{\underbrace{\text{Number of ships Coming Back}}\}$ → success

$n = 10$ ships, $p = P(\text{success}) = \frac{10000}{11000} = \frac{10}{11}$, $q = P(\text{failure}) = \frac{1}{11}$

$$\boxed{P(X = r \text{ success}) = {}^{n}C_{r}\, p^{r} q^{n-r}}$$

$$P(X = 3 \text{ Ships}) = {}^{10}C_{3} \left(\frac{10}{11}\right)^{3} \left(\frac{1}{11}\right)^{7} \quad \underline{\text{Ans}}$$

Q: In a Box 10% items are Defective. If 10 items are choosen at Random then find the prob of getting exactly 2 Def items ?

Sol: $X = \{\text{Number of } \underbrace{\text{Def. items}}\}$ → success.

$n = 10$ items, $p = P(\text{Def. item}) = 10\% = 0.1$, $q = P(\text{Non Def.}) = \frac{9}{10}$.

$$\boxed{P(X = r \text{ success}) = {}^{n}C_{r} \cdot p^{r} q^{n-r}}$$

$$P(X = 2 \text{ Def. items}) = {}^{10}C_{2} (0.1)^{2} (0.9)^{8} = 0.194$$

(ii) $P(\text{getting at least one Defective item}) = ? = 1 - P(\text{No def item}) = 1 - P(X = 0 \text{ success}) = 1 - (0.9)^{10}$

A dice is thrown 8 times, find probability that 3 will show (i) exactly 2 times (ii) At least seven times (iii) at least once

$n = 8$, $X = \{\underbrace{\text{Number of times 3 is coming}}_{\text{success}}\} = \{0,1,2,3,4,5,6,7,8\}$

$p = P(\text{getting } 3) = \frac{1}{6}$ & $q = P(\text{Not getting}) = \frac{5}{6}$.

$$\boxed{P(X = r \text{ success}) = {}^nC_r\, p^r q^{n-r}}$$

(1) $P(X=2) = {}^8C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^6$

(2) $P(X \geq 7) = P(X = 7 \text{ or } 8) = {}^8C_7\, p^7 q^1 + {}^8C_8 . p^8 . q^0 = 8\left(\frac{1}{6}\right)^7\left(\frac{5}{6}\right) + 1\left(\frac{1}{6}\right)^8 = \frac{41}{6^8}$

(3) $P(X \geq 1) = 1 - P(X=0) = 1 - {}^8C_0\, p^0 q^8 = 1 - \left(\frac{5}{6}\right)^8$ Ans

Q6
DPP-2

$S = \{3, 5, 7, 9, 11, \ldots, 35, 37\} = \{\text{odd Nos.}\}$

$A = \{\text{Multiple of } 7\} = \{7, \cancel{14}, 21, 35\}$

$B = \{\text{Prime No}\} = \{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37\}$

Prime No $\Longrightarrow$ follows the pattern of $(6n \pm 1)$
$\not\Longleftarrow$

$A \cap B = \{7\}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18} = \frac{13}{18}$

$l = a + (n-1)d$

$37 = 3 + (n-1)(2)$

$n = 18$

Thank

YOU