DS & AI
CS & IT

Probability & Statistics

Lecture No. 01

By- Dr. Puneet Sharma Sir

Topics to be Covered

Topic

" PERMUTATION & COMBINATION "

(Part 1)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

"If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your $\boxed{\text{little mind}}$ until you have a complete understanding of the chapter) & try to solve the Quest.

COUNTING PRINCIPLE

Fundamental Principle of Addition → If we have to perform only one of the job at a time out of n jobs then use this principle.

Keywords : " Either or / only one / Anyone "

Fundamental Principle of Multiplication → If we have to perform all the jobs at a time out of n jobs then use this principle.

Keywords: " AND / BOTH / ALL / Every "

eg: There are 10 Boys & 8 Girls in a class then in how many ways we can select

(i) Either a Boy or a Girl = ? $= {}^{18}C_1 = 18$ M-II ${}^{10}C_1 + {}^{8}C_1 = 10+8 = 18.$

(ii) A Boy & a Girl = ? $= {}^{10}C_1 \times {}^{8}C_1 = 10 \times 8 = 80$ ways

eg (B_1, G_1) (B_1, G_2) (B_1, G_3) - - - - (B_1, G_8)

$(B_2 G_1)$ $(B_2 G_2)$ $(B_2 G_3)$ - - - $(B_2 G_8)$

$(B_3 G_1)$ $(B_3 G_2)$ - - - $(B_3 G_8)$

$(B_4 G_1)$ - - - - - - $(B_4 G_8)$

⋮ - - - - - ⋮

$(B_{10} G_1)$ - - - - - - $(B_{10} G_8)$

$\Bigg\}$ Total possibilities = 80 ways.

eg: there are 3 students appearing for Maths scholarship test, 4 for Physics test, & 5 for Chemistry scholarship test. then in how many ways ?

(i) these scholarships can be awarded = ?

$$= {}^3C_1 \times {}^4C_1 \times {}^5C_1 = 3 \times 4 \times 5 = 60 \text{ ways}$$

(ii) one of these scholarships can be awarded = ?

$$= {}^3C_1 + {}^4C_1 + {}^5C_1 = 3 + 4 + 5 = 12 \text{ ways}$$

Explanation: $(M_1 P_1 C_1), (M_1 P_2 C_1), \ldots\ldots (M_1 P_4 C_1)$

$(M_1 P_1 C_2), (M_1 P_2 C_2) \ldots (M_1 P_4 C_2)$

$(M_1 P_1 C_3), (M_1 P_2 C_3) \ldots (M_1 P_4 C_3)$

$(M_1 P_1 C_4), (M_1 P_2 C_4) \ldots (M_1 P_4 C_4)$

$(M_1 P_1 C_5), (M_1 P_2 C_5) \ldots (M_1 P_4 C_5)$

$(M_2 P_1 C_1), (M_2 P_2 C_1) \ldots$

$(M_2 P_1 C_2), M_2 P_2 C_2) \ldots$

$(M_2 P_1 C_3), (M_2 P_2 C_3) \ldots$

$(M_2 P_1 C_4), (M_2 P_2 C_4) \ldots$

$(M_2 P_1 C_5), (M_2 P_2 C_5) \ldots (M_3 P_4 C_5)$

$\left.\right\}$ 60 cases.

eg In a Restaurant there are 8 veg dishes & 5 Non veg dishes then in how many ways you can order a dish ?

Sol: Total ways of ordering Dish $= {}^8C_1 + {}^5C_1 = 8 + 5 = 13$ ways

eg If there are 15 NIT'S & 20 IIT'S in INDIA & you are selected in JEE then in how many ways student can choose a college ?

Sol: Total ways of Choosing college $= \underset{\text{(Job-1)}}{15 \text{ ways}} + \underset{\text{(Job-2)}}{20 \text{ ways}} = 35$ ways.

Q: If there are 20 IITS, each having 7 Branches then in how many ways Topper can take addmission?

Sol: Total ways of taking addmission by Topper = 20 ways (Job-1) × 7 ways (Job-2)

= 140 ways.

Q: there are two friends sharing a Room in a hostel in IIT DELHI & they have common wardrobe in which there are only 5 jeans & 4 shirts. If they want to prepare for a party then in how many ways they can dress up?

Sol: Number of way of dressing up $= \left(\underset{J}{\underline{5}} \times \underset{S}{\underline{4}}\right)_{f_1} \times \left(\underset{J}{\underline{4}} \times \underset{S}{\underline{3}}\right)_{f_2}$

$= 240$ ways.

Q: There are 4 letters & 5 letter Boxes. In how many ways we can post these letters ?

Sol: wild Ans: 5^4, 4^5, 5×4, $5+4$, ${}^5P_4$, ${}^5C_4$, $4!$, $5!-4!$, $5!$

Total ways of posting letters (R.A) $= \underset{L_1}{\underline{5\text{ ways}}} \times \underset{L_2}{\underline{5\text{ ways}}} \times \underset{L_3}{\underline{5\text{ ways}}} \times \underset{L_4}{\underline{5\text{ ways}}} = 5^4$ ways

WRONG APP :- Req ways $= \cancel{\underset{B_1}{\underline{4}} \times \underset{B_2}{\underline{4}} \times \underset{B_3}{\underline{4}} \times \underset{B_4}{\underline{4}} \times \underset{B_5}{\underline{4}}} = 4^5$ ways

~~In this situation, you are moving letter Boxes~~ (Not possible)

Q: How many 3 letter words (with or w/o meaning) can be formed using vowels if a, e, i, o, u.

(i) there is No Restriction = ? $= \underset{P_1}{\underline{5\text{ ways}}} \times \underset{P_2}{\underline{5\text{ ways}}} \times \underset{P_3}{\underline{5\text{ ways}}} = 5^3$ ways

(RA)

(ii) A letter can come at most once = ? $= \underset{P_1}{\underline{5\text{ ways}}} \times \underset{P_2}{\underline{4\text{ ways}}} \times \underset{P_3}{\underline{3\text{ ways}}} = 120$ ways

(RNA)

Q: How many 3 letter words can be formed (with/w/o meaning) using Eng. alphabets.

① if Repitition is Not allowed = ? = $\underline{26} \times \underline{25} \times \underline{24}$

② if " is allowed = ? = $\underset{P_1}{\underline{26}} \times \underset{P_2}{\underline{26}} \times \underset{P_3}{\underline{26}}$

Q How many 4 digit Nos can be formed using 1, 3, 5, 7, 9

① RNA = ? = $\underline{5 \text{ ways}} \times \underline{4 \text{ ways}} \times \underline{3 \text{ ways}} \times \underline{2 \text{ ways}}$ = 120 ways

② RA = ? = $\underset{P_1}{\underline{5}} \times \underset{P_2}{\underline{5}} \times \underset{P_3}{\underline{5}} \times \underset{P_4}{\underline{5}}$ = 625 ways eg: How many 4 digit Nos Can be formed with $\underbrace{\text{distinct digits}}_{\text{RNA}}$

Various digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Total 4 digit Nos (RNA) $= \underset{P_1}{\underline{9\text{ways}}} \times \underset{P_2}{\underline{9\text{ways}}} \times \underset{P_3}{\underline{8\text{ways}}} \times \underset{P_4}{\underline{7\text{ways}}} = ?$

(ii) if RA then Total 4 digit Nos = ?

$$\underset{P_1}{\underline{9\text{ways}}} \times \underset{P_2}{\underline{10\text{ways}}} \times \underset{P_3}{\underline{10\text{ways}}} \times \underset{P_4}{\underline{10\text{ways}}} = ?$$

The number of five digits odd numbers greater than 40000 that can be made using the digits 0, 1, 2, 4, 5, 7 if digits can be repeated in the same number, is

(a) $3^2 \times 6^3$
(b) $2^3 \times 3^6$
(c) $2^3 + 3^6$
(d) $3^2 + 6^3$

0, 1, 2, 4, 5, 7

Total 5 digit odd Nos $(> 40000) = ?$
(RA)

$$= \underset{\substack{P_1 \\ (4 \text{ or } 5 \text{ or } 7)}}{\underline{3 \text{ ways}}} \times \underset{P_2}{\underline{6 \text{ ways}}} \times \underset{P_3}{\underline{6 \text{ ways}}} \times \underset{P_4}{\underline{6 \text{ ways}}} \times \underset{\substack{P_5 \\ (1 \text{ or } 5 \text{ or } 7)}}{\underline{3 \text{ ways}}} = 3^2 \times 6^3$$

The number of all possible selections which a student can make for answering one or more questions out of eight given questions in a paper, when each question has an alternative is

(a) 3^8 (b) $2^8 - 1$ (c) $3^8 - 1$ (d) 2^8

Sol. (c)

$Q1 <$? OR ?
$Q2 <$ OR
$Q3 <$ OR
⋮
$Q8 <$ OR

Total ways to deal with $\boxed{\text{each}}$ Question = 3 ways

" " " " all Questions $= \underset{Q_1}{3} \times \underset{Q_2}{3} \times \underset{Q_3}{3} \times \underset{Q_4}{3} \times \underset{Q_5}{3} \times \underset{Q_6}{3} \times \underset{Q_7}{3} \times \underset{Q_8}{3}$

$= 3^8$ ways.

But In these cases, one case is not possible in which we are Rejecting all the Questions Hence Req Ans $= 3^8 - 1$ ways.

Q: There are 5 T/F Questions then how many sequences of Ans are possible?

Sol: Total ways to deal with each Question = 2 way (either T or F)

" " " " all " = $\underset{Q_1}{2} \times \underset{Q_2}{2} \times \underset{Q_3}{2} \times \underset{Q_4}{2} \times \underset{Q_5}{2} = 2^5$ ways

= 32 ways.

Various sequences of Ans are as follows;

(TTTTT), (TTTTF), (TTTFF), (TTFFF) (TFFFF) (FFFFF)

${}^5C_0 = 1$ ${}^5C_1 = 5$ ${}^5C_2 = 10$ ${}^5C_3 = 10$ ${}^5C_4 = 5$ ${}^5C_5 = 1$ = 32 sequences

out of these 32 sequence, only one seq. has all correct Ans.

Let it be → (FTTFT)

Q: In a test each student has to solve 5 T/F Questions. No two students have given same sequence of answers and none of the student has given all correct ans then find the $\boxed{\text{max.}}$ number of students appearing in a test

(a) 4

(b) 31

(c) 32

(d) 63

out of 32 sequences of answers, only sequence has all correct answers

But this sequence is not picked by any student

Hence Max No. of students $= 32 - 1 = 31$

Combination $\rightarrow$ (when Counting is based on Selection only then use this Rule).

$$\boxed{{}^{n}C_r = \frac{n!}{r!\,(n-r)!}} \quad \& \quad \boxed{{}^{n}C_r = {}^{n}C_{n-r}}, \quad {}^{n}C_0 = {}^{n}C_n = 1$$

$${}^{n}C_1 = {}^{n}C_{n-1} = n$$

eg $${}^{11}C_3 = \frac{11\times10\times9}{3\times2\times1}, \quad {}^{22}C_4 = \frac{22\times21\times20\times19}{4\times3\times2\times1}$$

$${}^{22}C_{18} = ? = {}^{22}C_{22-18} = {}^{22}C_4 = \downarrow$$

$$\boxed{{}^{n}C_3 = \frac{n(n-1)(n-2)}{3\times2\times1}}$$

$$\boxed{{}^{n}C_2 = \frac{n(n-1)}{2}},$$

eg $${}^{11}C_2 = \frac{11\times10}{2}$$

Q. In a party there are 5 persons then find the number of handshakes in that party?

(M-I) Using Manual Calculation :-

Various Handshakes are

(P_1, P_2), (P_1, P_3), (P_1, P_4), (P_1, P_5)

$\cancel{(P_2 P_1)}$, $(P_2 P_3)$, $(P_2 P_4)$, $(P_2 P_5)$

$\cancel{(P_3 P_1)}$, $\cancel{(P_3 P_2)}$, $(P_3 P_4)$, $(P_3 P_5)$

$\cancel{(P_4 P_1)}$, $\cancel{(P_4 P_2)}$, $\cancel{(P_4 P_3)}$, $(P_4 P_5)$

$\cancel{(P_5 P_1)}$, $\cancel{(P_5 P_2)}$, $\cancel{(P_5 P_3)}$, $\cancel{(P_5 P_4)}$

(M-II) Total persons in a party = 5

Number of handshakes b/n 5 persons.

= No. of ways of selecting 2 P from 5 P.

$$= ? = {}^5C_2 = \frac{5\times 4}{2\times 1} = 10.$$

Total Handshakes = ? = 10

Q. In a party there are 66 Handshakes then find No. of persons in that party?

(a) 66

(b) 10

(c) 11

(d) 12

(e) 2145

M-I Total Handshakes = $\cancel{{}^{66}C_2} = 2145$ (BLUNDER)

M-II Let No. of persons in that party $= n$

then ATQ, No. of Handshakes $= 66$

$$^{n}C_2 = 66$$

$$\frac{n(n-1)}{2} = 66$$

$$n(n-1) = 132$$

$$n(n-1) = 12(12-1) \Rightarrow \boxed{n = 12}$$

Q In how many ways cricket team can be selected from Batch of 15 players if

(1) there is No Restriction = ? $= {}^{15}C_{11} = {}^{15}C_4 = \frac{15\times14\times13\times12}{4\times3\times2\times1}$

(2) A particular player is always selected = ? $= {}^{1}C_1 \times {}^{14}C_{10} = 1\times {}^{14}C_4$

(3) " " " is $\boxed{\text{Never}}$ selected = ? $= {}^{14}C_{11} = {}^{14}C_3$

Q In a Batch of 10 Batsman, 8 Bowlers, 5 All rounders & 2 wicket keepers a cricket team has to be selected in which there must be 5 Batsman, 3 Bowlers, 2 all rounders and 1 W.K then in how many ways this can be done?

$$= \underset{(\text{Bat})}{{}^{10}C_5} \times \underset{(\text{Bow})}{{}^{8}C_3} \times \underset{(\text{AUR})}{{}^{5}C_2} \times \underset{(\text{W.K})}{{}^{2}C_1}$$

PLAYING CARDS →

FACECARDS: K, Q, J (12)

Honour Cards: K, Q, J, A (16)

Cards (52)

- **Red suits (26)**
  - Diamond (13)

  - Heart (13)

- **Black suits (26)**
  - Spade

  - Club (13)

Q. In how many ways we can select 4 cards out of 52 cards if

(1) there is No Restriction $= ? = {}^{52}C_4$

(2) four cards are of same suits

M-I $= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 = 4 \times {}^{13}C_4$

M-II $= {}^{4}C_1 \times {}^{13}C_4$

(3) " " " " different suits $= {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = ({}^{13}C_1)^4$

(4) " " " face cards $= {}^{12}C_4$

(5) " " " Honour cards $= {}^{16}C_4$

(6) Two are Black & Two are Red $= {}^{26}C_2 \times {}^{26}C_2$

(7) 4 cards are of same colour = Either all Black or all Red $= {}^{26}C_4 + {}^{26}C_4$

eg: A person has 6 friends. In how many ways he/she can organise B'day party?

Sol: (M-I) Number of ways to deal with each friend = 2 ways (either I or R)

Total No. of ways to deal with all friends = $\underset{f_1}{2} \times \underset{f_2}{2} \times \underset{f_3}{2} \times \underset{f_4}{2} \times \underset{f_5}{2} \times \underset{f_6}{2} = 2^6$ ways

$= 64$ ways.

Various dealings:

(IIIIII), (IIIIIR), (IIIIRR), (IIIRRR), (IIRRRR), (IRRRRR), (RRRRRR)

${}^6C_0 = 1$ ${}^6C_1 = 6$ ${}^6C_2 = 15$ ${}^6C_3 = 20$ ${}^6C_4 = 15$ ${}^6C_5 = 6$ ${}^6C_6 = 1$

But to organise party, All Rejections are not possible so Ans $= 64 - 1 = 63$ ways

M-II He/She can organise B'day party either by Inviting

good M.

$= 1f$ or $2f$ or $3f$ or $4f$ or $5f$ or $6f$

$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

$= 6 + 15 + 20 + 15 + 6 + 1 = 63$ ways.

Q. By selecting exactly 4f at a time = ? $= {}^6C_4 = 15$ ways

Q. " " at least 4f at a time = ? $= {}^6C_4 + {}^6C_5 + {}^6C_6 = 15 + 6 + 1 = 22$

Q. " " at least one f at a time = ? M-I $= 63$ ways

" M-II At least 1f $= \boxed{\text{Total Cases} - (\text{No } f)} = 2^6 - {}^6C_0 = 64 - 1 = 63$

In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls and in 190 games both the players were boy. The number of games in which one player was a boy and other was girl is

(a) 200 (b) 216 (c) 235 (d) 256

ATQ, $^{G}C_2 = 45$ & $^{B}C_2 = 190$

$\frac{G(G-1)}{2} = 45$ & $\frac{B(B-1)}{2} = 190$

$G(G-1) = 90$ & $B(B-1) = 380$

$= 10\times(10-1)$ & $= 20(20-1)$

$\boxed{G = 10}$ & $\boxed{B = 20}$

So Req Ans $= {}^{B}C_1 \times {}^{G}C_1$

$= {}^{20}C_1 \times {}^{10}C_1$

$= 20 \times 10$

$= 200$ games

Factorial → $\boxed{n! = n(n-1)(n-2)(n-3)\ldots\ldots 3.2.1}$ ⇒ $\boxed{n! = n(n-1)!}$

eg $\lfloor 5 = 5\times4\times3\times2\times1$, $\lfloor 4 = 4\times3\times2\times1$ & $\lfloor 1 = 1$ & $\lfloor 0 = 1$ Let.

RNA RNA

eg: How many 5 digit Nos can be formed using odd digits (if RNA)?
(1, 3, 5, 7, 9)

Total 5 digit Nos $= 5\times4\times3\times2\times1 = 5! = {}^5P_5$

Q How many 4 letter words can be form using the letters "ROSE" if each letter is coming exactly once = ? $4\times3\times2\times1 = 4! = {}^4P_4$
(RNA)

Q In how many ways 6 persons can be seated on 6 chairs ?

Total seating arrangements $= \underset{P_1}{\underline{6}} \times \underset{P_2}{\underline{5}} \times \underset{P_3}{\underline{4}} \times \underset{P_4}{\underline{3}} \times \underset{P_5}{\underline{2}} \times \underset{P_6}{\underline{1}} = 6! = {}^6P_6$

(RNA)

Q In how many ways we can arrange the letters of the word 'EQUATION' if

(TOYS)

E, Q, U, A, T, I, O, N

① there is No Restriction $= ? = \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 8! = {}^8P_8$.

(RNA)

② if words are starting with E = ? = E _ _ _ _ _ _ _ $= 7! = {}^7P_7$

③ words starts with E & ends with N = E _ _ _ _ _ _ N $= 6! = {}^6P_6$

Eg: In how many ways 5 persons can be seated on 8 chairs?

Total seating arrangements (RNA) $= \underset{P_1}{\underline{8}} \times \underset{P_2}{\underline{7}} \times \underset{P_3}{\underline{6}} \times \underset{P_4}{\underline{5}} \times \underset{P_5}{\underline{4}} = {}^8C_5 \times 5! = {}^8P_5$

M-II M-III

${}^8C_5 \Rightarrow$ selection of 5 chairs out of 8 chairs

$5! \Rightarrow$ Arrangement of 5 persons on 5 chairs.

Eg: In how many ways 8 persons can be seated on 8 chairs?

Total seating Arrangements (RNA) $= 8! = {}^8P_8$

Eg: In how many ways $\underbrace{\text{8 Persons can be seated on 5 Chairs}}_{\text{senseless data.}}$?

Permutation :→ (Selection & Arrangement both) →

If in a Question, Counting is Based on, Selection as well as on Arrangement also then use this Rule.

$$^nP_r = \frac{n!}{(n-r)!} = \boxed{^nC_r \times r!}$$

$$^nP_0 = 1$$

$$^nP_1 = n$$

$$^nP_2 = n(n-1)$$

$$\vdots$$

$$^nP_n = n!$$

$$^{11}P_3 = 11 \times 10 \times 9, \quad ^{11}C_3 = \frac{11 \times 10 \times 9}{3 \times 2 \times 1}$$

$$^{22}P_4 = 22 \times 21 \times 20 \times 19$$

$$^{22}C_4 = \frac{22 \times 21 \times 20 \times 19}{4 \times 3 \times 2 \times 1}$$

y $^{22}C_{18} = ? = {^{22}C_4} = ✓$

$$^{22}P_{18} = ? = 22 \times 21 \times 20 \times \dots \times 3 \times 2 \times 1$$

Eg: Consider three letters a, b, c. then

Various Combination : $(a\,b\,c) = {}^3C_3 = 1.$

" Permutation ; $(a\,b\,c), (a\,c\,b), (b\,a\,c), (b\,c\,a), (c\,a\,b), (c\,b\,a)$

M-I $= \underline{3} \times \underline{2} \times \underline{1} = 6$

M-II $= {}^3P_3 = 6$

M-III $= 3! = 6$

Ex of Combination :

- → formation of team,
- → " of Committee
- → No. of Handshakes.
- → No. of St. lines & Δ's
- → No. of ||gms.

- - - - - -

Ex of Perm :- If in a Question there is a feeling of interchanging things then use ${}^nP_r$.

- → formation of words.
- → " of Numbers.
- → Seating arrangement.
- → formations of Photographs
- → " of signals.

GAZAB KA Conclusion →

① if $n > r$ & RNA, then Multi Rule ≡ Perm Rule

② if $n = r$ & RNA, Then Multi Rule ≡ Perm Rule ≡ Factorial Rule

③ if RA, then only use Multi Rule

i.e the concept of ${}^nC_r$, ${}^nP_r$ & $r!$ is applicable only when RNA

The number of words of four letters containing equal number of vowels and consonants (Repetition allowed)

(a) 60 × 210 HW (b) 210 × 243

(c) 210 × 315 (d) 630

1. (c)

$V = 5$ (a, e, i, o, u)

$C = 21$

Total words = (All diff) or (vowels alike & cons diff)

M-I or (vowels diff & cons Alike) or (Vow alike & cons alike)

= (?) + (?) + (?) + (?) = C

M-II Total words (RA) = ?. (use Multi Rule)

Thank

YOU