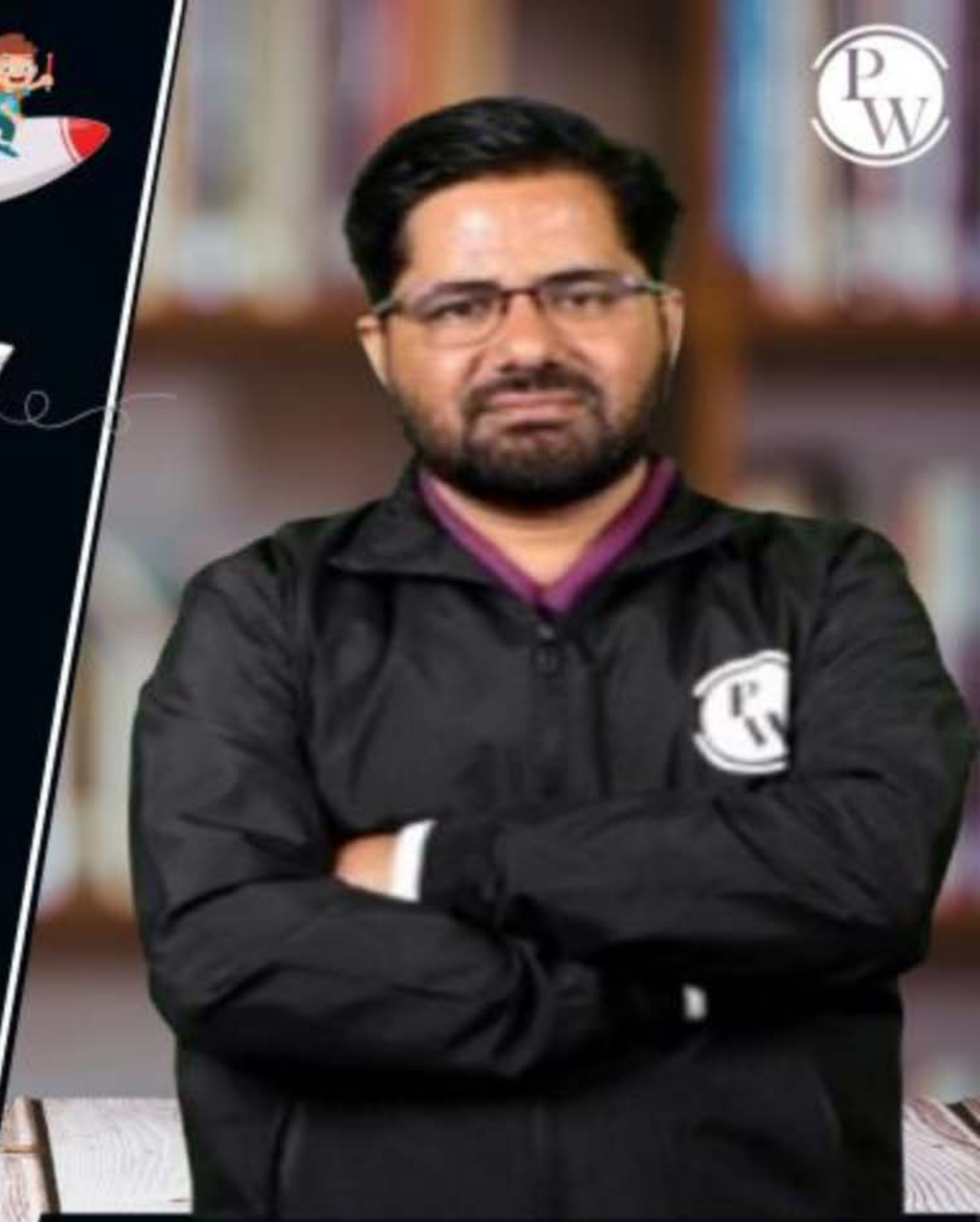
DS & AI
CS & IT

Linear Algebra

Lecture No. 04



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Recap of previous lecture

Topic TYPES of MATRICES

Topics to be Covered

Topic

→ RANK OF MATRIX

→ Nature of Vectors.

Qs (MSQ) (HW) if $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ then A is

(a) orthogonal

(b) Involutary

(c) unitary

(d) Hermitian

By observation, A is Hermitian ($\because A^{\theta} = A$)

Now, $A^2 = A \cdot A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} \cdot \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$= \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

i.e. A is Involutary also.

Again, $AA^{\theta} = A(A) = A^2 = I$ i.e. A is unitary also.

QUICK RECAP:-

1. Singular Mat if $|A| = 0$
2. Non Sing Mat if $|A| \neq 0$
3. Invertible Mat if A^{-1} exist & $A^{-1} = \frac{adj A}{|A|}$
4. Real Mat if $\bar{A} = A$ or $A^{\theta} = A^{T}$
5. Complex Mat if $\bar{A} \neq A$
6. Symm Mat if $A^{T} = A$
7. Skew Symm Mat if $A^{T} = -A$
8. Hermitian Mat if $A^{\theta} = A$
9. Skew Herm Mat if $A^{\theta} = -A$
10. Idempotent if $A^{2} = A$
11. Involutary if $A^{2} = I$
12. Nilpotent if $A^{K} = 0$
13. orthogonal Mat if $AA^{T} = I$ or $A^{-1} = A^{T}$
14. unitary Mat if $AA^{\theta} = I$ or $A^{-1} = A^{\theta}$
15. U.T.M $A = [a_{ij}]_{n \times n}$ where $a_{ij} = 0 \ \forall i > j$
16. L.T.M $A = [a_{ij}]_{n \times n}$ where $a_{ij} = 0 \ \forall i < j$
17. Diag Mat $A = [a_{ij}]_{n \times n}$ where

$$a_{ij} = \begin{cases} 0 & , i \neq j \\ \text{atleast one element is Non Zero} & , i = j \end{cases}$$

VECTORS & their Properties

Ordered n-tuple (n-dim vector) → Any ordered set of n numbers is called Ordered n-tuple. Generally it is represented in the form of Column Matrix (But we can also represent it in the form of Row Mat).

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} \quad \text{or} \quad \begin{bmatrix} x_1 & x_2 & x_3 & \cdots & x_n \end{bmatrix}_{1\times n}$$

for eg
$$A = \begin{array}{c} \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}_{3\times 4} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \\ \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \end{array}$$
has 3 Row vectors & 4 Column vectors

ordered pair $= (x_1, x_2) = x_1\hat{i} + x_2\hat{j} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2$ dim vector

(Point in 2D)

ordered Triplet $= (x_1, x_2, x_3) = x_1\hat{i} + x_2\hat{j} + x_3\hat{k} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 3$-dim vector

(Point in 3D)

ordered Quadruple $= (x_1, x_2, x_3, x_4) = ?? = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 4$ dim vector

$\vdots$

ordered n-tuple $= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = n$ dim vector.

(Q) Consider $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$ & $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}$ as two given vectors then

Dot product :→ $\boxed{X \cdot Y = X^T Y} = (x_1y_1 + x_2y_2 + x_3y_3 + \dots\dots + x_ny_n)$

Proof: $X \cdot Y = X^T Y = [x_1\, x_2\, x_3 \dots x_n]_{1\times n} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}_{n\times 1} = \left[(x_1y_1 + x_2y_2 + \dots + x_ny_n)\right]_{1\times 1}$

Also Calculate $XY = ? = [\;]_{n\times 1}[\;]_{n\times 1} = \text{N.D}$

Norm of vector →
(Length)
$\boxed{\|X\| = \sqrt{X^T X}} = \sqrt{x_1^2 + x_2^2 + x_3^2 + \dots\dots + x_n^2}$

Normalised Vector (unit vector) $\rightarrow$ if $\|X\| = 1$ then X is called Normalised vector.

orthogonal Vectors $\rightarrow$ if $[X \cdot Y = 0]$ then $\Longleftrightarrow$ [X & Y are called orthogonal vectors]

(while if $AA^T = I$ then A is called orthogonal Matrix)

three vectors X, Y, Z are called orthogonal if they are pairwise orthogonal.

ie $X \cdot Y = Y \cdot Z = Z \cdot X = 0$

orthonormal vectors $\rightarrow$ if $[X \cdot Y = 0,\ \|X\| = 1,\ \|Y\| = 1] \Longleftrightarrow$ then [X & Y are called orthonormal vectors.]

Note: $\because X \cdot Y = Y \cdot X$ ie Dot product is commutative

Q: Check the Nature of following vectors given in following set

$$A = \{(1\ 2\ 1)', (2\ 1\ -4)', (3\ -2\ 1)'\} = \{X_1, X_2, X_3\}$$

$$\underbrace{X_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}, X_3 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}}_{\text{orthogonal vectors}}$$

$\because X_1 \cdot X_2 = 1\times 2 + 2\times 1 + 1\times(-4) = 0.$

$X_2 \cdot X_3 = 2\times 3 + 1\times(-2) + (-4)\times 1 = 0$

$X_3 \cdot X_1 = 3\times 1 + (-2)(2) + 1\times 1 = 0$

$\|X_1\| = \sqrt{1^2+2^2+1^2} = \sqrt{6}$

$\|X_2\| = \sqrt{2^2+1^2+(-4)^2} = \sqrt{21}$

$\|X_3\| = \sqrt{3^2+(-2)^2+1^2} = \sqrt{14}$

is Not orthonormal.

Note: $\because X_1, X_2, X_3$ are orthogonal vectors

$\Longrightarrow$ they are L.I also.

2009 CS

The vectors $(1,1,1)$ & $(1, a, a^2)$ (where $a = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$) are ?

MSQ

(a) Linearly Ind.

(b) L. Dep.

(c) orthogonal.

(d) orthonormal. ×

$X = \begin{bmatrix}1\\1\\1\end{bmatrix}, Y = \begin{bmatrix}1\\a\\a^2\end{bmatrix} = \begin{bmatrix}1\\\omega\\\omega^2\end{bmatrix}$

$= \omega =$ cube root of unity

(i.e. $\omega^3 = 1$, $1+\omega+\omega^2 = 0$)

$X \cdot Y = 1\times1 + 1\times\omega + 1\times\omega^2 = 1+\omega+\omega^2 = 0.$

Hence X & Y are orthogonal $\Longrightarrow$ LI also.

$\because \|X\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$ i.e. Not orthonormal.

& $\|Y\| = \sqrt{1^2+\omega^2+(\omega^2)^2} = \sqrt{1+\omega^2+\omega^4} = \sqrt{1+\omega^2+\omega} = 0$

$\because X \neq KY$ for any Non Zero Value of K

So vectors are LI

Note: $z^3 - 1 = 0 \Rightarrow \boxed{z = (1)^{1/3}}$ → 1, ω, ω^2 where $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

& $\omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

where $\omega^3 = 1$, $1 + \omega + \omega^2 = 0$, $\overline{\omega} = \omega^2$, $\overline{\omega^2} = \omega$, $\frac{1}{\omega} = \omega^2$

$\frac{1}{\omega} = ? = \frac{\omega^2}{\omega^3} = \frac{\omega^2}{1} = \omega^2$

Proof: $z^3 - 1 = 0$

$(z-1)(z^2 + z + 1) = 0$

$z = 1$ & $z^2 + z + 1 = 0$

$z = 1$ & $z = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$1, \omega$ $\quad = \omega^2$

TWO GAJAB KI PROPERTIES →

(1) "Column vectors of an orthogonal Matrix are Orthonormal Vectors"

eg $A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$ is an orthogonal Mat ($\because AA^T = I$)

$\quad C_1 \quad C_2 \quad C_3$

$C_1 \cdot C_2 = C_2 \cdot C_3 = C_3 \cdot C_1 = 0$

& $\|C_1\| = \|C_2\| = \|C_3\| = 1$ Hence verified

(2) If [vectors are orthogonal] $\Longrightarrow$ then [these are L.I also.] (True)

$\not\Longleftarrow$

i.e LI set of vectors are also orthogonal vectors (F)

Q $A = \begin{bmatrix} 1/9 & -4/9 & 8/9 \\ 8/9 & 4/9 & 1/9 \\ \alpha/9 & -7/9 & \beta/9 \end{bmatrix}$ is an orthogonal Matrix then $\alpha+\beta =$? ____

$\quad C_1 \quad C_2 \quad C_3$

$\Downarrow$

Column vectors are orthogonal vectors as well as of unit Norm.

(a) 0

(b) 4

(c) -4

(d) 8

i.e $C_1 \cdot C_2 = 0$ & $C_2 \cdot C_3 = 0$

$\frac{1}{81}[-4+32-7\alpha]=0$ & $\frac{1}{81}[-32+4-7\beta]=0$

$28-7\alpha=0$

$-28-7\beta=0$

$\alpha=4$

$\beta=-4$

so $\alpha+4=0$

$\|c_1\| = 1 \quad , \quad \|c_2\| = 1 \quad , \quad \underbrace{\|c_3\| = 1}$

$\frac{1}{9}\sqrt{1^2+8^2+\alpha^2} = 1$

$65+\alpha^2 = 81$

$\alpha^2 = 16$

$\alpha = \pm 4$

$\frac{1}{9}\sqrt{64+1+\beta^2} = 1$

$65+\beta^2 = 81$

$\beta^2 = 16$

$\beta = \pm 4$

But $c_1 \cdot c_2 = 0 \Rightarrow \alpha = 4$

$c_2 \cdot c_3 = 0 \Rightarrow \beta = -4$

is $\alpha + \beta = 0$.

RANK

* Submatrix → By deleting some Rows or Some Columns or Both, the Matrix obtained is called Submatrix.

Def^n of Rank: "It is the order of Non singular Submatrix, of Highest order that can exist in a given Mat"

Def^n In Books:

if $\boxed{\rho(A_{6\times7})=4}$ then

→ ∃ atleast one Non Singular submatrix of order 4×4

→ Every square submatrix of order 5×5 & 6×6 are Singular

Eg: $A=\begin{bmatrix}1&2&4\\1&5&25\\1&3&9\end{bmatrix}_{3\times3}$ $\because |A|=(2-5)(5-3)(3-2)=(-3)(2)(1)=-6$

i.e. $|A|\neq 0 \Rightarrow$ A is Non singular of $3\times3 \Rightarrow \rho(A)=$ three

Eg $A=\begin{bmatrix}2&3&5\\3&5&2\\-5&-8&-7\end{bmatrix}_{3\times3}$ $\because |A|=0$ i.e. A is singular then $\rho(A)\neq$ three

Consider a submatrix of the type

$A_1=\begin{bmatrix}2&3\\3&5\end{bmatrix}_{2\times2}$ (By deleting R_3 & C_3)

$\because |A_1|=+1$ i.e. $|A_1|\neq 0$ or A_1 is $(\text{Nonsing})_{2\times2} \Rightarrow \rho(A)=$ order of $A_1=$ Two

Eg $A=\begin{bmatrix}2&-3&4\\-4&6&-8\\4&-6&7\end{bmatrix}_{3\times3}$ $\because |A|=0$ i.e. A is singular, $\rho(A)\neq 3$

Consider $A_1=\begin{bmatrix}6&-8\\-6&7\end{bmatrix}$ By deleting R_1 & C_1

$\because A_1$ is $(\text{Nonsing})_{2\times2} \Rightarrow \rho(A)=$ order of $A_1=$ Two

Q(4) $A = \begin{bmatrix} 1 & 2 & 4 & 8 & 16 \\ 1 & 2 & 4 & 8 & 16 \\ 1 & 2 & 4 & 8 & 16 \\ 1 & 2 & 4 & 8 & 16 \end{bmatrix}_{4\times 5}$

$\rho(A) \neq 5$ $\because$ it is not possible to find any submatrix of 5×5. ☹

$\rho(A) \neq 4$ or 3 or 2 $\because$ all the submatrices of 4×4, 3×3, 2×2 are sing.

So $\rho(A) = $ one Ans.

Q(5) $A = \begin{bmatrix} 2 & & & & \\ & -1 & & \text{zero} & \\ & & 0 & & \\ & \text{zero} & & 4 & \\ & & & & 3 \end{bmatrix}_{5\times 5}$ or diag$(2,-1,0,5,3)$ then $\rho(A) = ?$

$\because |A| = (2)(-1)(0)(5)(3) = 0$ i.e. $\rho(A) \neq 5$

Consider $A_1 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}_{4\times 4}$ By deleting R_3 & C_3

$\because A_1$ is Non-sing. So

$\rho(A) = $ order of $A_1 = $ Four

Eg: $A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}_{6\times 6}$

$\rho(A) \neq 6, 5, 4$ $\because$ all the submatrices of 6×6, 5×5, 4×4 are singular

Consider $A_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$ By deleting $R_2, R_4, R_5, C_4, C_5, C_6$

$\because A_1$ is $(\text{Non Sing})_{3\times 3} \Rightarrow \rho(A) = \text{three}$.

M-II

$A = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_6} \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{6\times 6} = A_1$

$\because (\text{Non Sing Mat})_{3\times 3}$ exist so $\rho(A_1) = 3$

$\because A \sim A_1 \Rightarrow \rho(A) = 3$.

Note i) $\boxed{\text{E-operations do not alter the Rank of Matrix}}$ i.e

we are free to apply all three E-operations while calculating Rank.

that's why RANK is called INVARIANT property of Mat.

(2) **Equivalent Matrices** :→ Matrix obtained by applying E-operations are equivalent to each other.

"Equivalent Matrices have same Rank"

PQ

PYQ

If $A = \begin{bmatrix} 1 & a & a^2 & a^3 & \cdots & a^n \\ 1 & a & a^2 & a^3 & \cdots & a^n \\ 1 & a & a^2 & a^3 & \cdots & a^n \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & a & a^2 & a^3 & \cdots & a^n \end{bmatrix}_{(n+1)\times(n+1)}$ then Rank of this $(n+1)\times(n+1)$ Matrix ?

$$\xrightarrow[\substack{R_3 \to R_3 - R_1 \\ \vdots \\ R_{n+1} \to R_{n+1} - R_1}]{R_2 \to R_2 - R_1} \begin{bmatrix} 1 & a & a^2 & \cdots & a^n \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{(n+1)\times(n+1)} \Rightarrow \rho(A) = \text{one}.$$

Q. 2015 if $A = [\alpha_{ij}]_{n\times n}$ s.t $\alpha_{ij} = i\cdot j$ $\forall i$ & j then for $n \geq 4$, $\rho(A) = ?$

Let $n = 4$, $A = [\alpha_{ij}]_{4\times 4} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}_{4\times 4}$

$$\xrightarrow[\substack{R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 4R_1}]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{4\times 4}$$

$\rho(A) =$ one

Q. If $A = [a_{ij}]_{n\times n}$; $a_{ij} = i - j$ $\forall$ i & j then for $n = 4$, $\rho(A) = ?$

$$A = [a_{ij}]_{4\times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & 0 \end{bmatrix}_{4\times 4}$$

$\because |A| = \cdots\cdots = 0$ i.e. $\rho(A) \neq 4$

& all the submatrices of order 3×3 are also singular. So $\rho(A) \neq 3$

So By observation, $\rho(A) = 2$

= skew symm Mat of even order

$\Rightarrow$ $|A|$ = Perfect Sq.

But 0 is also Treated as Perfect Sq.

So Directly we cannot say that $|A| \neq 0$.

& $|A| = \begin{vmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & 0 \end{vmatrix} \xrightarrow[R_4 \to R_4 - 3R_2]{R_3 \to R_3 - 2R_2} \begin{vmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 4 & 6 \end{vmatrix}$

$= -(1)\begin{vmatrix} -1 & -2 & -3 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{vmatrix} \xrightarrow{R_2 \to R_2 + R_1} -\begin{vmatrix} -1 & -2 & -3 \\ 0 & 0 & 0 \\ 2 & 4 & 6 \end{vmatrix} = 0 = (0)^2$

= Perfect Sq.

M-II Convert $A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$ into an Echelon form & Hence find Rank

HW

See Next Slide.

Sol:
$$A=\begin{bmatrix}0 & -1 & -2 & -3\\ 1 & 0 & -1 & -2\\ 2 & 1 & 0 & -1\\ 3 & 2 & 1 & 0\end{bmatrix}\xrightarrow{R_1\leftrightarrow R_2}\begin{bmatrix}1 & 0 & -1 & -2\\ 0 & -1 & -2 & -3\\ 2 & 1 & 0 & -1\\ 3 & 2 & 1 & 0\end{bmatrix}\xrightarrow[R_4-3R_1]{R_3-2R_1}\begin{bmatrix}1 & 0 & -1 & -2\\ 0 & -1 & -2 & -3\\ 0 & 1 & 2 & 3\\ 0 & 2 & 4 & +6\end{bmatrix}$$

$$\xrightarrow[R_4+2R_2]{R_3+R_2}\begin{bmatrix}1 & 0 & -1 & -2\\ 0 & -1 & -2 & -3\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} = \text{Echelon form}$$

So $\rho(A) = \text{Two}$.

Q: $A = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix}$ then $\rho(A) = \underline{?}$

$$\xrightarrow[\substack{R_3 \to R_3 - R_2 \\ R_2 \to R_2 - R_1}]{R_4 \to R_4 - R_3} \begin{bmatrix} 11 & 12 & 13 & 14 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_2]{R_4 \to R_4 - R_2} \underbrace{\begin{bmatrix} 11 & 12 & 13 & 14 \\ 10 & 10 & 10 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{4 \times 4} = A_1$$

By observation, $\rho(A_1) = $ two

$\therefore A \sim A_1 \Rightarrow \rho(A) = $ two $\underline{\text{Ans}}$

Echelon Form → (Triangular Form) →

Any Mat $A_{m \times n}$ is said to be in Echelon Form if

① Number of Zeros before the 1^{st} Non Zero element in a Row should be in an Increasing order in the subsequent Rows.

② Every Zero Row (if exist) should occur at the bottom of a Mat.

Note: ① $\boxed{\rho(\text{Echelon form}) = \text{Number of Non Zero Rows.}}$

② Any Mat can be converted into an E-form by using E-operations.

③ It is advisable to apply only E-Row operations while converting given Mat into an E-Form. (as per our syllabus)

Flowchart of Converting given Mat into an E-Form →

(1) Make a_{11} unity (Not compulsory but advisable)

(2) Make all the elements of C_1 (that lies below a_{11}) Zero by using E-Row Operation

(3) Make a_{22} unity (Not compulsory but advisable)

(4) Make all the elements of C_2 (that lies below a_{22}) Zero by " " "

(5) Make a_{33} unity & so on - - - - -

Note: Take care, In E-Form, $a_{21} = $ Zero.

Eg:

$$\begin{bmatrix} 2 & -1 & 4 & 0 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 4 & 7 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 2 & -3 & 1 & 4 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix},$$

✓ $\rho(A) = 4$ ✓ $\rho(A) = 3$ ✓ $\rho(A) = 4$

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 4 \\ 0 & 0 & 7 & 5 \end{bmatrix}, \quad \begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 2 & 4 & -1 & 3 \\ 0 & 2 & 1 & -3 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

✗ ✓ $\rho(A) = 3$ ✗

THANK - YOU

Tel:

$\boxed{\text{dr puneet sir pw}}$