

Computer Science & IT

Database Management System



Relational Model & Normal Forms

Lecture No. 12



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Recap of Previous Lecture



Topic

Decomposition of relation up to BCNF



Topic

Multi-valued dependency and 4NF



Topics to be Covered



Topic

Properties of multi-valued dependency



Topic

4NF



Multi-valued dependency

eg:

Non-trivial FDs that can be defined w.o. 3 attributes

Sid	Cid	Mob_No
S ₁	C ₁	M ₁
S ₁	C ₁	M ₂
S ₁	C ₂	M ₁
S ₁	C ₂	M ₂
S ₂	C ₂	M ₂

~~Sid → Cid~~

~~Sid → Mob_No~~

~~Cid → Sid~~

~~Cid → Mob_No~~

~~Mob_No → Sid~~

~~Mob_No → Cid~~

~~Sid, Cid → Mob_No~~

~~Sid, Mob_No → Cid~~

~~Cid, Mob_No → Sid~~

None of this non-trivial FDs exist in the relation

∴ Relation is in BCNF

↓
But redundancy is still present in the relation



Topic : Multivalued dependency

"X multivalued determines Y" is denoted by $X \twoheadrightarrow Y$

- ⊗ If there exist two or more independent attributes which are dependent on some other set of attributes then multi-valued dependency will (may) exist in the relation

★ Formal definition of MVD:

Let R is a relation, and X and Y are two sets of attributes from relation R { Let us define $Z = \underbrace{\text{Attributes of } R - \text{Attributes in 'X U Y'}}_{\text{ie, } Z \text{ is the set of remaining attributes}}$

If there exist '4' tuples $t_1, t_2, t_3, t_4 \in R$

s.t. $\left\{ \begin{array}{l} t_1.X = t_2.X = t_3.X = t_4.X \end{array} \right.$

and $\left\{ \begin{array}{l} t_1.Y = t_2.Y \text{ and } t_3.Y = t_4.Y \end{array} \right.$

and $\left\{ \begin{array}{l} t_1.Z = t_3.Z \text{ and } t_2.Z = t_4.Z \end{array} \right.$

then Multi-valued dependency $X \twoheadrightarrow Y$ exist in R

← "if" cond is true
then "then" condition is
definitely true.

But if "if" Condⁿ is
false, then also "then"
Condⁿ may be true

X	Z	Y
X	Y	Z
Sid	Cid	Mob_No
t ₁	C ₁	M ₁
t ₃	C ₁	M ₂
t ₂	C ₂	M ₁
t ₄	C ₂	M ₂
	C ₂	M ₂

Wrt this numbering of tuples
 ✓ Sid \rightarrow Mob-no. will also exist in relation R.

Wrt this numbering of tuples
 ✓ Sid \rightarrow Cid exist in the relation

* Another definition w.r.t. MVD :-

Whenever we swap the values of attribute set Y in two tuples $\{t_1, t_2\}$ which agree on the value of attribute set X {i.e., $t_1.X = t_2.X$ } and if the resulting tuple was already a member of the relation, then $X \twoheadrightarrow Y$ exists in the relation.

Sid	Cid	Mob_No
S ₁	C ₁	M ₁
S ₁	C ₁	M ₂
S ₁	C ₂	M ₁
S ₁	C ₂	M ₂
S ₂	C ₂	M ₂

Same
Cid

Swap
Mob-No

Q. Check whether $Cid \twoheadrightarrow Mob_No$ exist in the relation or not

$\Rightarrow S_1 \ C_2 \ M_2$

$\Rightarrow S_2 \ C_2 \ M_1$

it was not present
in original relation

\Downarrow
Hence, $Cid \twoheadrightarrow Mob_no$
does not exist in
the relation



Topic : Properties of Multivalued dependency

- ✓ ① Complementation Rule :- Let R be the relational schema and X & Y be some set of attributes over relation R ,
If $X \twoheadrightarrow Y$ exist in relation R .
then $X \twoheadrightarrow (\text{Attributes}_{\text{of } R} - (X \cup Y))$ also exist in relation R

$R(A_1, A_2, A_3, A_4, A_5, A_6)$

let $X = \{A_1, A_3\}$

$Y = \{A_2, A_5\}$

} Attributes of $R - (X \cup Y) = \{A_4, A_6\}$

Complementation Rule : -

if $X \rightarrow Y$

i.e. if $\{A_1, A_3\} \rightarrow \{A_2, A_5\}$

then $\{A_1, A_3\} \rightarrow \{A_4, A_6\}$



Topic : Properties of Multivalued dependency

② Trivial MVD :-

- Let $X \twoheadrightarrow Y$ exist in relation R,

$X \twoheadrightarrow Y$ is called a trivial MVD

if $X \supseteq Y$

or

$$X \cup Y = R$$

{ i.e., Attributes of $X \cup Y$ = Attributes of R }

$$\begin{array}{ccccccc}
 X & \longrightarrow & Y & \neq & Y & \longrightarrow & Z & \text{ then } & X & \longrightarrow & Z \\
 \downarrow & & \downarrow & & & & & & & & \\
 \{A_1\} & & \{A_2, A_3\} & & \{A_2, A_3\} & & \{A_3, A_4\} & & & &
 \end{array}$$

$$\equiv \boxed{A_1 \longrightarrow \{A_3, A_4\}}$$

Splitting \therefore

$$\begin{array}{l}
 A_1 \longrightarrow \{A_3\} \\
 A_1 \longrightarrow \{A_4\}
 \end{array}$$



Topic : Properties of Multivalued dependency

③ Splitting

Multivalued dependencies are not allowed to split.

i.e. if $X \twoheadrightarrow YW$ exist in relation, then, $\begin{cases} X \twoheadrightarrow Y \text{ and } X \twoheadrightarrow W \\ \text{need not hold true.} \end{cases}$

R
eg.

X	Y	W
X_1	Y_1	W_1
X_1	Y_2	W_2

But FDs are allowed to split.
i.e. if $X \rightarrow YW$, then $\begin{matrix} X \rightarrow Y \\ X \rightarrow W \end{matrix}$

$X \twoheadrightarrow YW$ holds true, and it is trivial
but neither $X \twoheadrightarrow Y$ holds true
nor $X \twoheadrightarrow W$ holds true



Topic : Properties of Multivalued dependency

④ Transitivity:-

If $X \twoheadrightarrow Y$ and $Y \twoheadrightarrow W$ exists in relation R

then $X \twoheadrightarrow (W - Y)$ will also exist in the relation.

By Transitivity of MVD

If $X \twoheadrightarrow Y$ and $Y \twoheadrightarrow Z$, then $X \twoheadrightarrow Z - Y$

\therefore if $\{A_1\} \twoheadrightarrow \{A_2, A_3\}$ & $\{A_2, A_3\} \twoheadrightarrow \{A_3, A_4\}$

then

$A_1 \twoheadrightarrow \{A_3, A_4\} - \{A_2, A_3\}$

i.e. $A_1 \twoheadrightarrow A_4$





Topic : Properties of Multivalued dependency

⑤ Union: If $X \twoheadrightarrow Y$ and $X \twoheadrightarrow W$ holds in relation R ,
then, $X \twoheadrightarrow YW$ will also hold true in relation R .

By Transitivity of MVD

If $X \twoheadrightarrow Y$ and $Y \twoheadrightarrow Z$, then $X \twoheadrightarrow Z - Y$

\therefore if $\{A_1\} \twoheadrightarrow \{A_2, A_3\}$ & $\{A_2, A_3\} \twoheadrightarrow \{A_4, A_5\}$

then $A_1 \twoheadrightarrow \{A_4, A_5\} - \{A_2, A_3\}$

i.e. $A_1 \twoheadrightarrow A_4$

$\{A_1\} \twoheadrightarrow \{A_2, A_3\}$

$\{A_1\} \twoheadrightarrow \{A_4\}$

By Union

$\{A_1\} \twoheadrightarrow \{A_2, A_3, A_4\}$



Topic : Properties of Multivalued dependency

⑥ Augmentation :

If $\underline{x} \twoheadrightarrow \underline{y}$ exist in the relation R ,
And α, β are two set of attributes from
relation R , such that $\alpha \supseteq \beta$,

then $x\alpha \twoheadrightarrow y\beta$ will also hold true in relation R .



Topic : Properties of Multivalued dependency

⑦ Replication:-

- ⊕ Every functional dependency is also a M.V.D.
 { Every MVD need not be a functional dependency }

i.e., if $X \rightarrow Y$ exist in R , then $X \twoheadrightarrow Y$ will also exist in R

& if $X \twoheadrightarrow Y$ exist in R , then $X \rightarrow Y$ need not exist in R



Topic : Fourth normal form (4NF)

⊛ For a relation R to be in 4NF.

- ① Relation R must be in BCNF,
and ② Every non-trivial MVD $X \twoheadrightarrow y$.

must be with X as a super key of the relation

Candidate Keys and Superkeys
are always identified using Functional
dependencies.

FDs are key dependencies.

Whereas, MVDs are data dependency.

ie. Every non-trivial FD $X \rightarrow y$ must be with X as a Super Key

R

Sid	Cid	Mob_No
S ₁	C ₁	M ₁
S ₁	C ₁	M ₂
S ₁	C ₂	M ₁
S ₁	C ₂	M ₂
S ₂	C ₂	M ₂

following Non-trivial MVDs
exists in the relation

(i) Sid \twoheadrightarrow Cid
Not a SK

(ii) Sid \twoheadrightarrow Mob_No.
Not a SK

Non-trivial MVDs of
type $X \twoheadrightarrow Y$ exists
in the relation,
such that

'X' is not a Superkey
 \therefore Relation is not in 4NF.

No non-trivial FD exists
in the relation,

\therefore Relation is in BCNF,
and Candidate key of the
relation is = (Sid, Cid, Mob_No)

→ If the relation is in BCNF, but not in 4NF,

then, for 4NF decompose the relation w.r.t.

Every non-trivial MVD $X \twoheadrightarrow Y$, in which X is not a S.K.

Sid	Cid	Mob_No
S ₁	C ₁	M ₁
S ₁	C ₁	M ₂
S ₁	C ₂	M ₁
S ₁	C ₂	M ₂
S ₂	C ₂	M ₂

No non-trivial FD exists in the relation,
 ∴ Relation is in BCNF,
 and Candidate key of the relation is = {Sid, Cid, Mob_No}

Following Non-trivial MVDs exists in the relation

(i) Sid → → Cid
 Not a SK

(ii) Sid → → Mob_No
 Not a SK

Not in 4NF

For 4NF, decompose w.r.t. non-trivial MVDs in which LHS is not a S.K.

Sid → → Cid

Sid → → Mob.no

Sid	Cid
S ₁	C ₁
S ₁	C ₂
S ₂	C ₂

No. non-trivial FD
 ∴ Relation is in BCNF and C.K. = (Sid, Cid)

MVD "Sid → → Cid" exists in relation s.t. Sid is not a Super key, but it is trivial MVD

No non-trivial MVD
 ∴ Relation is in 4NF

Sid	Mob.no.
S ₁	M ₁
S ₁	M ₂
S ₂	M ₂

No. non-trivial FD
 ∴ Relation is in BCNF and C.K. = (Sid, Mob.no.)

MVD "Sid → → Mob.No" exists in relation s.t. Sid is not a Super key, but it is trivial MVD

No non-trivial MVD
 ∴ Relation is in 4NF



2 mins Summary



✓
Topic

Properties of multi-valued dependency

✓
Topic

4NF

THANK - YOU