

CS & IT ENGINEERING



THEORY OF COMPUTATION

Regular Language

Lecture No. - 02



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Recap of Previous Lecture



Topic

?????

Regular Language Detection



Topics to be Covered



Topic

✓ Regular Language Detection

Topic

Pumping Lemma



✓ { closure properties of Regular Languages }

$$L_1 = \{x \$ y \mid \eta_a(x) = \eta_b(y)\} \Rightarrow \underline{\underline{\text{Non regular}}}$$

$x, y \in (a+b)^*$

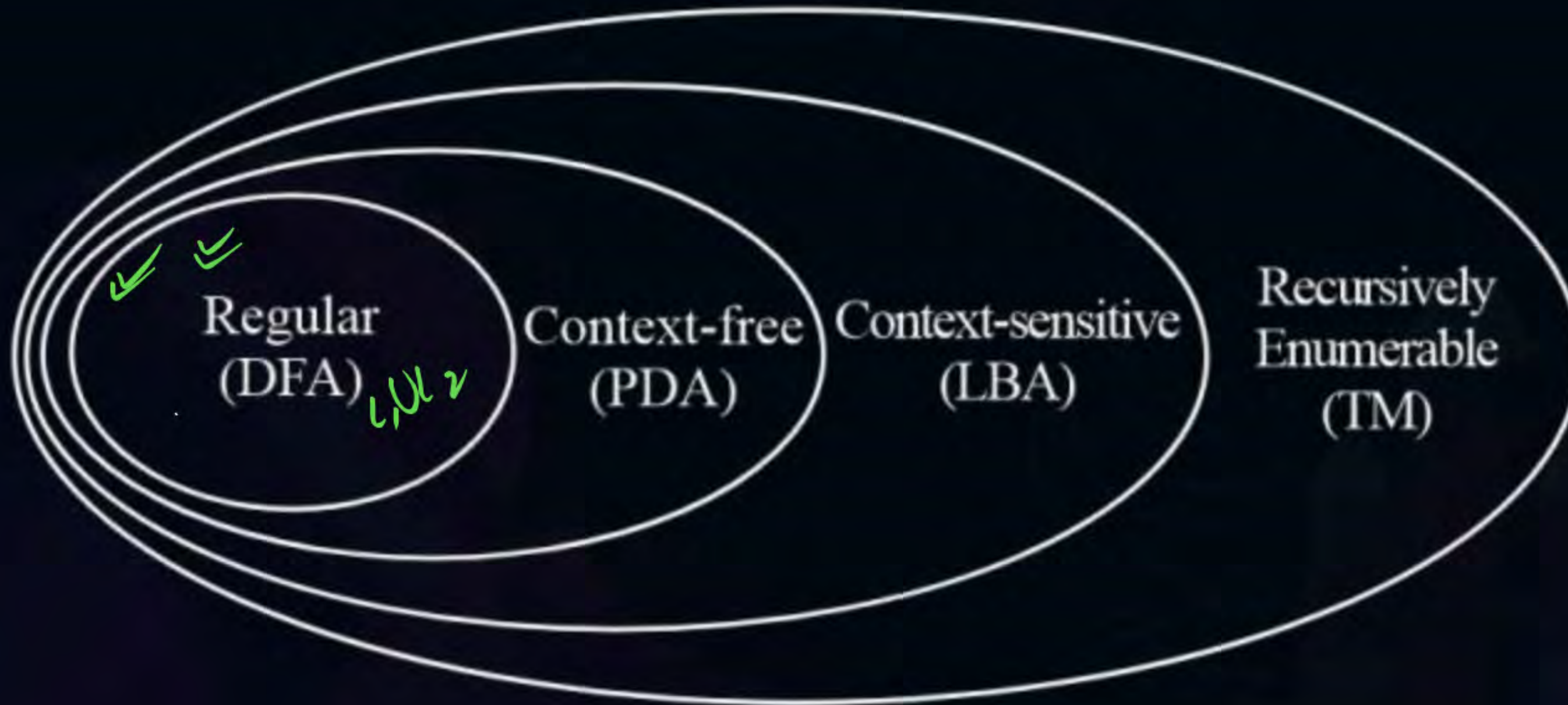
$$L_2 = \{xy \mid \eta_a(x) = \eta_b(y)\} \Rightarrow (a+b)^* \Rightarrow \underline{\underline{\text{regular}}}$$

$x \in (a+b)^*$



Topic : Theory of Computation

L_1 L_2



① Subset op

X

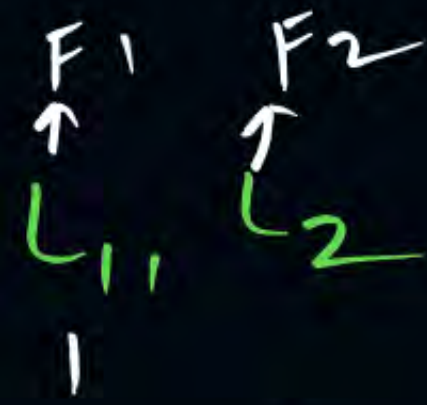
$$\underbrace{\{a^n b^n \mid n \geq 1\}}_{a^* b^*} \subseteq \underbrace{(a+b)^*}$$

closed?

Subset of regular lang is may (or) may not regular.

Hence regular language not closed Under Subset.

finite
 ② Union op



$L_1 \cup L_2 =$ always Regular / closed

(OR)

$\boxed{F_1 \times F_2}$

closed

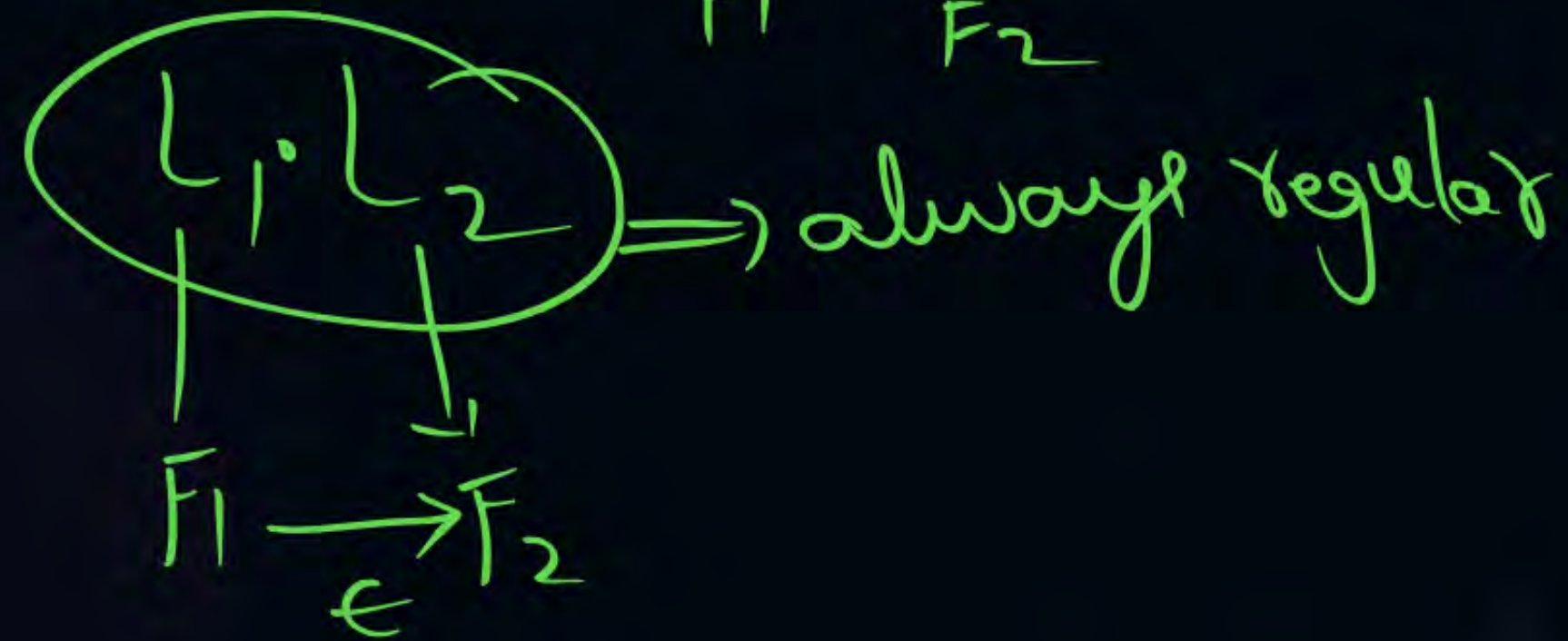
③ Concatenation op

closed

$\{a^n\}$ $\{b^n\}$

L_1
 \downarrow
 F_1

L_2
 \downarrow
 F_2



④ Intersection of $\begin{matrix} F_1 & F_2 \\ \uparrow & \uparrow \\ L_1 & L_2 \end{matrix}$

(and) $\checkmark \underbrace{L_1 \cap L_2}_{F_1 \times F_2} \Rightarrow \text{always regular}$

{closed}

⑥ Infinite Intersection op (not closed)

Yes
No

$$L_1 \cap L_2 \cap L_3 \cap L_4 \cap \dots = \overline{L_1 \cup L_2 \cup L_3 \cup L_4 \cup \dots}$$

$$F_1 \times F_2 \times F_3 \dots$$

$$L_1 \cap L_2 = \overline{L_1 \cup L_2}$$

not closed

done
 ⑦ Complement op

$$L = \text{regular} = F$$

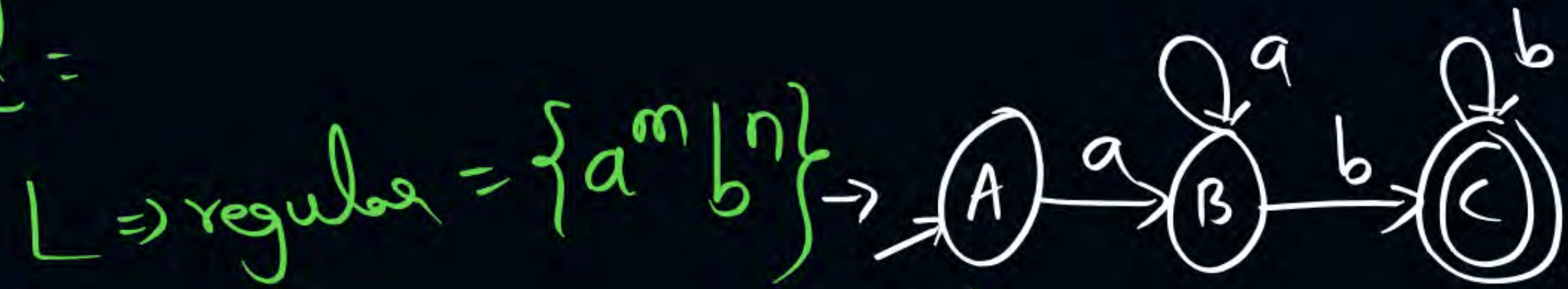
$$\Sigma^* - L$$

is also regular

$$L^c = (\Sigma^* - L) = F^c$$

Interchange final and Non final states

⑧ Reversal = closed



Interchange Initial state and Final state
Reverse transition Direction

closed

⑨ Kleene closure

$L \rightarrow \text{regular} \Rightarrow L^*$

$L^* \Rightarrow \text{always regular}$

⑩ Positive closure

$L^+ \Rightarrow \text{always regular}$

① Difference op

closed

L_1, L_2 regular

$$L_1 - L_2 = L_1 \cap L_2^c$$

$$= L_1 \cap L_2$$

reg \cap reg

⑫ Prefix op (closed)

Prefix of a string } Sequence of leading symbols over given string

toc

ε
T
To
Toc

gate

ε
g
ga
gat
gate

delhi

ε
d
de
del
delh
delhi

--- abc...n

How many prefixes?
(n+1) prefixes

⑬ Suffixes (used)

Suffix of a string:

Sequence of trailing symbols over given string

I
T O C

e
c
OC
TOC

gate

e

e

te

ate

gate

abc...n

(n+1) Suffixes

⑭ Substitution op

done

$$\Sigma \xrightarrow{\quad} \Delta$$

$$\{a\} \xrightarrow{\quad} \{0,1\}$$

$$\downarrow$$

$$L = aa$$

$$S(L) = \underline{S(a)} \underline{S(a)}$$

$$S(L) = \underline{(0+1)(0+1)} \underline{(0+1)(0+1)}$$

$$S(a) = \{00, 01, 10, 11\}$$

$$S(a) = \underline{(0+1)(0+1)}$$

$$L = a^*$$

$$\underline{S(L)} = \underline{(S(a))^*}$$

$$= \underline{((0+1)(0+1))^*}$$

⑮ Homomorphism op } It is a substitution in which
symbol is replaced by single string

ex

$$L = a^* b^*$$

$$h(L) = \underbrace{h(a)}_{(01)^*} \underbrace{h(b)}_{(00)^*}$$

$$\left\{ \begin{array}{l} h(a) = 01 \\ h(b) = 00 \end{array} \right\}$$

⑩ Inverse Homomorphism up { Applying Homomorphism in reverse)
 string replaced by symbol.

closed

$$L = \{ \underline{00}, 10 \}$$

$$\downarrow \quad \downarrow$$

$$h^{-1}(L) = \{ \underline{a}, \underline{b} \}$$

$$h(a) = 00$$

$$h(b) = \underline{10}$$

⑮ Homomorphism of

⑩ Inverse Homomorphism oP

⑤ Infinite Union of $\{L_1, L_2, L_3, \dots\} = \text{regular}$

$L_1 \cup L_2 \cup L_3 \cup L_4 \cup \dots$

$\{\underline{ab}\} \cup \{\underline{a^2b^2}\} \cup \{\underline{a^3b^3}\} \cup \{\underline{a^4b^4}\} \dots = \{\underline{a^n b^n}\}$
Nonregular

may (or) may not regular
 not closed

(Q) How ^{many} states in $L_1 \cap L_2$ min DFA where

$$\phi \begin{cases} L_1 = \underline{(a+b)^* a} \rightarrow F_1 \\ L_2 = \underline{(a+b)^* b} \rightarrow F_2 \end{cases}$$

$$\underline{\underline{L_1 \cap L_2}} \quad \} \rightarrow \text{DFA}_{a,b}$$

$$F_1 \times F_2 = \}$$

(Q) which of the following is true?

(a) Subset of any Infinite language is Regular

(b) Subset of any Non regular language is Regular

☒ (c) Subset of any finite language is regular

(d) none



Topic : Pumping Lemma



$$\{a^n b^n \mid n \geq 1\}$$



#Q. To Prove a Language L is Non-Regular

- ① Assume L is Regular ✓
 2. There exist F.A for L and n is no. of states in that F.A
 3. Select some string W from L such that $|W| > n$.
 4. Divide W into XYZ such that $|xy| \leq n$ and $|y| > 0$.
 5. Find a suitable integer i such that UV^iW is not belongs to L.
- Then L is not Regular.



If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

I. $L_1 \cdot L_2$ is a regular language $\checkmark \rightarrow \text{regul}$

II. $L_1 \cdot L_2 = \{a^n b^n \mid n \geq 0\} \rightarrow \text{false}$

Which one of the following is CORRECT?

[2014-Set2: 1 Mark]

- A** Only I $\checkmark \checkmark$ $a^* b^*$
- B** Only II
- C** Both I and II
- D** Neither I nor II



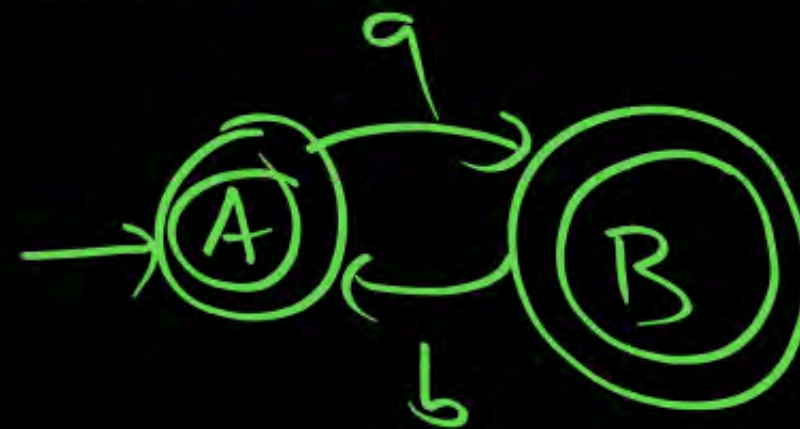
Consider the following two statements:

- I. If all states of an NFA are accepting states then the language accepted by the NFA is Σ^* \rightarrow false
- II. There exists a regular language A such that for all language B, $A \cap B$ is regular. \rightarrow true

[2016-Set2: 2 Marks]

Which one of the following is CORRECT

- ☐ A Only I is true
- ☒ B Only II is true
- ☐ C Both I and II are true
- ☐ D Both I and II are false



$A \cap B$
 $\rightarrow \{ \epsilon \}$



2 mins Summary



Topic

One

Topic

Two

Topic

Three

Topic

Four

Topic

Five

THANK - YOU