CS & DA

Probability and Statistics

DPP- 03 Discussion Notes

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[NAT]

$\Rightarrow$ Boys = 20

#Q. A class consists of 50 students, out of which 30 are girls. The mean of marks scored by girls in a test is 73 (out of 100), and that of boys is 71. Determine the mean score of the whole class.

$$\bar{X} = \frac{\sum x}{N}$$

$$73 = \frac{\sum x_G}{30} \Rightarrow \sum x_G = 73 \times 30$$

$$71 = \frac{\sum x_B}{20} \Rightarrow \sum x_B = 71 \times 20$$

$$\bar{X} = \frac{\text{Total Marks}}{\text{Total No. of Students}} = \frac{73 \times 30 + 71 \times 20}{30 + 20} = ? = 72.2 \text{ Marks}$$

[NAT]

#Q. Find the median for the data 8, 5, 7, 10, 15, 21.

Sol: 5, 7, 8, 10, 15, 21 $\Rightarrow N = 6$

3^{rd} 4^{th}

$$\text{So } Md = \frac{\left(\frac{N}{2}\right)^{th} + \left(\frac{N}{2}+1\right)^{th}}{2} = \frac{3^{rd} + 4^{th}}{2} = \frac{8+10}{2} = 9$$

[NAT]

#Q. For a moderately skewed distribution, mean = 12 and mode = 6. Using these values, find the value of the median.

$$\boxed{\text{Mode} = 3\,\text{Md} - 2\,\text{Mean}}$$

$$6 = 3\,\text{Md} - 2(12)$$

$$3\,\text{Md} = 6 + 24$$

$$= 30$$

$$\text{Md} = 10$$

[NAT]

#Q. The marks scored by a student in different subjects are 45, 91, 62,71, 55. Find the median of the given data using the median formula.

$$\underbrace{45, 55, 62, 71, 91}_{N = 5 \text{ (odd)}}$$

$$M_d = \left(\frac{N+1}{2}\right)^{th} = \left(\frac{5+1}{2}\right)^{th} = 3^{rd} = 62 \text{ Ans}$$

[NAT]

#Q. For any given data, the mean is 45.5, and the median is 43. Find the modal value.

$$\text{Mode} = 3\,Md - 2\,Me$$

$$= 3(43) - 2(45.5)$$

$$= 129 - 91$$

$$= 38$$

[MCQ]

#Q. Let X be random variable with mean μ_x and variance $\sigma_x^2 > 0$, then Var (aX +b) is

A $a\sigma_x^2$

B $a^2\sigma_x^2$

C $a\sigma_x^2 + b$

D $a^2\sigma_x^2 + b$

$$Var(x) = \frac{\Sigma (x-\bar{x})^2}{n}$$

$$Var(ax+b) = a^2 Var(x) + Var(b)$$

$$= a^2 \sigma_x^2 + 0$$

[MCQ]

#Q. If X and Y are tow random variables with respective expected values E(X) and E(Y), then Cov (X,Y)is-

A E(XY) – E (X) E(Y)

B E(X/Y) = E (XY)

C E(XY) – E (X/Y)

D E(XY) – E (X) /E(Y)

$$Cov(x,y) = E(x-\bar{x})(y-\bar{y})$$

$$= \frac{\sum(x-\bar{x})(y-\bar{y})}{N}$$

$$= \text{---} \_\_\_\_\_\_\_\_$$

$$= E(xy) - E(x).E(y)$$

[MCQ]

#Q. The covariance of two independent variate is equal to-

A zero

B unity

C sum of their expectation

D The product of their expectations

Cov (Ht, Age) = 0, after the age of 20 yrs

Cov (X, Y) = 0 if X & Y are Ind

[MCQ]

$\Rightarrow Cov(X_1, X_2) = 0$

#Q. If X_1 and X_2 are independent, then $V(X_1 - X_2)$ is equal to

$Var(ax - by) = a^2 Var(X) + (-b)^2 Var(Y) - 2ab\,Cov(X, Y)$

$Var(X_1 - X_2) = Var(X_1) + 1\,Var(X_2) - 0$

A $V(X_1) + V(X_2)$

B $V(X_1) - V(X_2) - 2\,Cov(X_1, X_2)$

C $V(X_1) - V(X_2)$

D $V(X_1) + V(X_2) - 2\,Cov(X_1, X_2)$

[NAT]

#Q. A random variable X has the following probability function:

x	0	1	2	3	4	5	6
$f(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

(i) Find k,

(ii) Find P (X ≥ 5), P(3 < X ≤6), P(X < 4)

$\Sigma p_i = 1$

$k+3k+5k+7k+9k+11k+13k = 1$

$49k = 1 \Rightarrow k = \frac{1}{49}$

$P(X \geq 5) = P(X = 5 \text{ or } 6)$

$= 11k + 13k = 24k = \frac{24}{49}$

$P(3 < X \leq 6) = P(4 \leq x \leq 6)$

$= P(X=4) + P(X=5) + P(X=6)$

$= 9k + 11k + 13k = 33k = \frac{33}{49}$

$P(X < 4) = P(X \leq 3) = P(X = 0, 1, 2, 3)$

$= k + 3k + 5k + 7k = 16k = \frac{16}{49}$

[NAT]

#Q. If X is the number of points rolled with a balanced die, find the expected value of $g(X) = 2X^2 + 1$

$X = \{\text{Number of points on die}\} = \{1, 2, 3, 4, 5, 6\}$

X :	1	2	3	4	5	6
$p(x)$:	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$E(x^2) = \sum p_i x_i^2 = p_1 x_1^2 + p_2 x_2^2 + \dots + p_6 x_6^2$

$= \frac{1}{6}(1)^2 + \frac{1}{6}(2)^2 + \frac{1}{6}(3)^2 + \dots + \frac{1}{6}(6)^2$

$= \frac{1}{6}\left[\frac{6(7)(13)}{6}\right] = \frac{91}{6}$

$E\{g(x)\} = E\{2x^2 + 1\}$

$= 2E\{x^2\} + E\{1\}$

$= 2\left(\frac{91}{6}\right) + 1 = \frac{91}{3} + 1 = \frac{94}{3}$

[NAT]

#Q. A discrete random variable X has the following probability distribution:

X	1	2	3	4	5
P(X = x)	0.05	0.15	0.35	0.4	0.05

Calculate: (i) F(3) $= P(X \le 3) = P(X = 1 \text{ or } 2 \text{ or } 3) = 0.55$ Ans

(ii) E[X]

(iii) var [X]

$E(X) = \sum p_i x_i = p_1 x_1 + p_2 x_2 + \cdots + p_5 x_5$

$= 0.05(1) + 0.15(2) + \cdots 0.05(5)$

$= 3.25$

$E(X^2) = \sum p_i x_i^2 = p_1 x_1^2 + p_2 x_2^2 + \cdots + p_5 x_5^2$

$= 0.05(1)^2 + 0.15(2)^2 + \cdots + 0.05(5)^2$

$= 11.45$

$Var(X) = E(X^2) - E^2(X)$

$= 0.88$

[MCQ]

#Q. If random variable X assumes only positive integral values, with the probability $P(X=x)=\frac{2}{3}\left(\frac{1}{3}\right)^{x-1}$, $x=1,2,3,\ldots\ldots$, then E(X) is

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}\cdot x^2+\frac{n(n-1)(n-2)}{3!}x^3+\cdots$$

$$(1-x)^{-2} = 1+2x+3x^2+4x^3+\cdots$$

$$E(x)=\sum p_i x_i = p_1x_1+p_2x_2+p_3x_3+\cdots\infty$$

$$=\frac{2}{3}(1)+\frac{2}{3}\cdot\frac{1}{3}(2)+\frac{2}{3}\left(\frac{1}{3}\right)^2(3)+\cdots$$

$$=\frac{2}{3}\left[1+2\left(\frac{1}{3}\right)+3\left(\frac{1}{3}\right)^2+4\left(\frac{1}{3}\right)^3+\cdots\right]=\frac{2}{3}\left[1-\frac{1}{3}\right]^{-2}$$

$$\frac{2}{3}\left(\frac{2}{3}\right)^{-2}$$

$$\frac{2}{3}\times\left(\frac{3}{2}\right)^2=\frac{3}{2}$$

$$=1.5$$

A. 2/9

B. 2/3

C. 1

D. 3/2

[MCQ]

$\mu_x = E(x) = \sum p_i x_i = 0.1(10) + 0.1(20) + 0.4(30) + 0.3(40) + 0.1(50) = 32$

#Q. A discrete random variable X has the following probability function

x	: 10	20	30	40	50
p :	: 0.1	0.1	0.4	0.3	0.1

Denote by μ_x and σ_x the mean and the standard deviation of X. Find $P(|X - \mu_x| \le \sigma_x^2|)$

$E(x^2) = \sum p_i x_i^2 = 0.1(10)^2 + 0.1(20)^2 + 0.4(30)^3 + 0.3(40)^2 + 0.1(50)^2 = 1140$

A. 1

B. 0.8

C. 0.7

D. 0.5

$Var(x) = E(x^2) - E^2(x)$

$= 116$

$|x-a| \le l \Rightarrow -l \le (x-a) \le l$

$|x-a| > l \Rightarrow (x-a) < -l \text{ or } (x-a) > l$

$P(|x-\mu| \le \sigma^2) = P(|x-32| \le 116)$

$= P[-116 \le (x-32) \le 116]$

$= P[-84 \le x \le 148]$

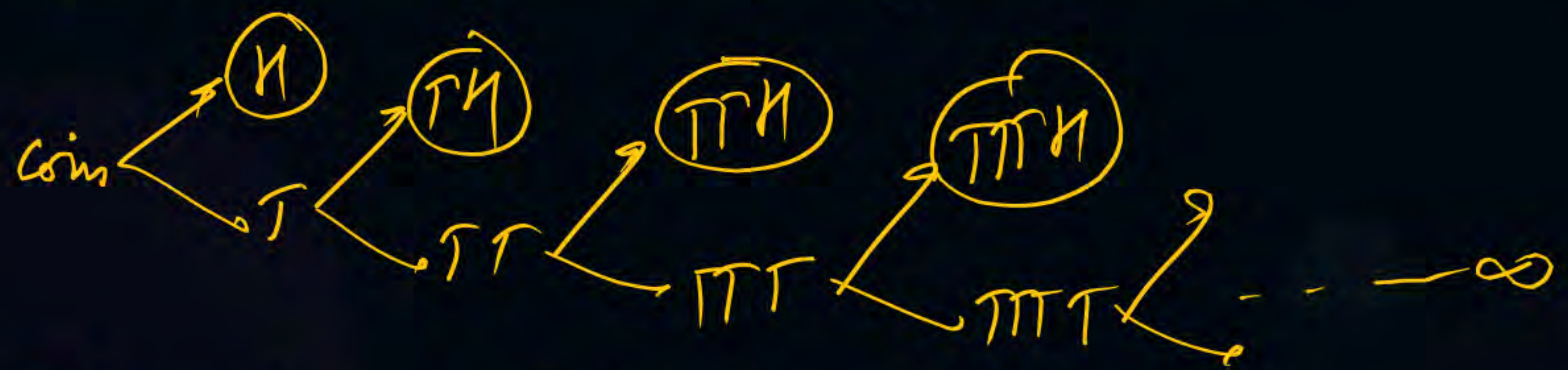
$= P(0 \le x \le 148) = P(10 \le x \le 50)$

$= 1$

[MCQ]

#Q. An unbiased coin is tossed until a head is obtained. If N denotes the number of tosses required, what is $P(N > 1)$?

$N = \{\text{Number of tosses Required}\} = \{1, 2, 3, 4, 5 \ldots\}$



A. 1/2

B. 1

C. 1/4

D. 1/8

$$P(N>1) = 1 - P(N \le 1)$$
$$= 1 - P(N=1)$$
$$= 1 - P(H) = 1 - \frac{1}{2} = \frac{1}{2}$$

[NAT]

#Q. Probability mass function of a discrete random variables are given below.

Xi	= 0	1	2	3	4	5	6
P(Xi)	= k	3k	5k	7k	9k	11k	13k

(A) Find K

(B) Evaluate P (X < 4)

(C) Evaluate P(3 < x ≤ 6)

$\Sigma p_i = 1$

$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

$k = \frac{1}{49}$

$P(x<4) = P(x = 0,1,2,3) = 16k = \frac{16}{49}$

$P(3 < x \le 6) = P(4 \le x \le 6)$

$= P(4 \text{ or } 5 \text{ or } 6)$

$= 33k = \frac{33}{49}$

[MCQ]

#Q. Which one of the following is not possible for a binomail distribution ?

$\Downarrow$

Meam > Variance

(np) (npq)

A. Mean = 2, variance = 3/2

B. Mean = 5, variance = 9

C. Mean = 10, variance = 5

D. Mean = 4, variance = 8/3

[MCQ]

#Q. A coin is twice as likely to turn up tails as heads. If the coin is tossed independently, what is the probability that the third head occurs on the fifth toss?

$P(T) = 2P(H) \Rightarrow P(T) = \frac{2}{3},\ P(H) = \frac{1}{3}$

$H =$ Success

A. 8/81

B. 40/243

C. 16/81

D. 80/243

$P(3^{rd}\ H \text{ in } 5^{th} \text{ toss}) = P(\text{exactly } 2H \text{ in } 1^{st}\ 4 \text{ trials}) \times P(H \text{ in } 5^{th} \text{ toss})$

$= \left({}^4C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{4-2}\right) \times \frac{1}{3} = 6 \times \frac{2^2}{3^5} = \frac{2^3}{3^4} = \frac{8}{81}$

[MCQ]

#Q. If X follows binomial distribution with parameters n and p, the variance of X/n is-

$Var\left(\frac{X}{n}\right) = \frac{1}{n^2} Var(X) = \frac{npq}{n^2} = \frac{1}{n} p(1-p)$

(A) $\frac{p(1-p)}{n}$

(B) np (1–p)

(C) p(1–p)

(D) $\frac{p(1-p)}{n^2}$

[NAT]

#Q. In a class of Pathology, the professor decided to conduct a test of swine flu on all the students. Test result says that one student in every ten is having swine flue. What is the probability that out of 5 students expected to attend a class, at least 4 will not have swine flu?

$X = \{\text{Number of students not having swine flu}\} \rightarrow \text{success}$

$n = 5,\ p = \frac{9}{10},\ q = \frac{1}{10},\ P(X = r \text{ success}) = {}^nC_r\, p^r q^{n-r}$

$P(X \geq 4) = P(X=4) + P(X=5)$

$= {}^5C_4 \left(\frac{9}{10}\right)^4 \left(\frac{1}{10}\right)^{5-4} + {}^5C_5 \left(\frac{9}{10}\right)^5 \left(\frac{1}{10}\right)^{5-5} = \left(\frac{9}{10}\right)^4 \times \frac{7}{5}$

[NAT]

#Q. If the probability of a defective item is 0.1, find (i) the mean (ii) standard deviation for the distribution of items in a total of 400.

$n = 400$, $X = \{\text{Number of defective items}\} \to$ success

$p = P(\text{Def items}) = 0.1$, $q = P(\text{Non Def}) = 0.9$

$Av = np = 400 \times 0.1 = 40$ //

$SD = \sqrt{npq} = \sqrt{400 \times 0.1 \times 0.9} = ? = 6$ //

[NAT]

#Q. The mean and variance of a binomial distribution are 2.5 and 1.875 respectively. Obtain the binomial probability distribution.

ATQ, $np = 2.5$

$npq = 1.875$

$p+q=1$

$\Rightarrow \frac{npq}{np} = \frac{1.875}{2.5}$

$q = 0.75, \quad p = 1-q = 0.25, \quad n = 10$

So Complete B. Dist $= (q+p)^n = (0.75+0.25)^{10}$ Ans

$\neq (p+q)^n$, $\sum_{r=0}^{10} {}^{10}C_r\, p^r q^{n-r}$

[NAT]

#Q. 100 dice are thrown. How many are expected to fall 6. What is the variance in the number of 6's ?

$X = \{\text{Number of times 6 occurs}\} = \{0, 1, 2, 3, \ldots, 99, 100\}$

X :	0	1	2	3	...	100
$P(x)$:	?	?	?	?		?

Each throw → Either six occurs = success
→ or six Not occurs = fail

$p = \frac{1}{6}$, $q = \frac{5}{6}$, $n = 100$

$E(X) = \sum p_i x_i = \ldots = np = 100 \times \frac{1}{6} = 16.67$ times

$\text{Var} = npq = 100 \times \frac{1}{6} \times \frac{5}{6} = ? = \frac{500}{36} = 13.9$

[MCQ]

#Q. Let X be a poison random variable and $\boxed{P(X = 1) + 2\,P(X = 0) = 12\,P(X = 2)}$. Which one of the following statements is TRUE ?

$$\frac{e^{-\lambda}\lambda^1}{1!} + 2\frac{e^{-\lambda}\lambda^0}{0!} = 12\frac{e^{-\lambda}\lambda^2}{2!} \Rightarrow \lambda + 2 = 6\lambda^2$$

A $0.40 < P(X = 0) \le 0.45$

B $0.40 < P(X = 0) \le 0.50$

C $0.50 < P(X = 0) \le 0.55$ ✓

D $0.55 < P(X = 0) \le 0.60$

$$6\lambda^2 - \lambda - 2 = 0$$

$$6\lambda^2 - 4\lambda + 3\lambda - 2 = 0$$

$$2\lambda(3\lambda - 2) + 1(3\lambda - 2) = 0$$

$$(3\lambda - 2)(2\lambda + 1) = 0$$

$$\lambda = \frac{2}{3} \text{ ✓} \quad \& \quad \lambda = -\frac{1}{2} \text{ ✗}$$

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

$$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-2/3} = 0.5134$$

[MCQ]

#Q. In a Poisson distribution, the probability of observing 3 is 2/3 times that of observing 4. The mean of the distribution is

A 5

B 6

C 7

D 8

$$P(X=3) = \frac{2}{3} P(X=4)$$

Mean = Var = λ

$$\frac{e^{-\lambda} \cdot \lambda^3}{3!} = \frac{2}{3} \cdot \frac{e^{-\lambda} \cdot \lambda^4}{4!}$$

$$\lambda^3 = \frac{2}{3} \frac{\lambda^4}{4}$$

$$6\lambda^3 = \lambda^4$$

$$\lambda^4 - 6\lambda^3 = 0 \Rightarrow \lambda^3(\lambda - 6) = 0 \Rightarrow \lambda = 0 \text{ or } \lambda = 6$$

[MCQ]

#Q. The number of misprints per page of a book (X) follows the poison distribution such that P(X =1) = P(X =2). If the book contains 500 pages, the expected number of pages containing at most one misprint is-

$X = \{$Number of misprints per page$\}$ → success

A. ~~5005e^-2~~ $500e^{-2}$

B. ~~10005e^-2~~ $1000e^{-2}$

C. ~~15005e^-2~~ $1500e^{-2}$

D. $500(1 - 3e^{-2})$

$P(X=1) = P(X=2)$

$$\frac{e^{-\lambda}\lambda^1}{1!} = \frac{e^{-\lambda}\lambda^2}{2!} \Rightarrow \lambda^2 - 2\lambda = 0 \Rightarrow \lambda = 2$$

For single page :-

$P(X \le 1) = P(X=0) + P(X=1)$

$$= \frac{e^{-\lambda} \cdot 2^0}{0!} + \frac{e^{-\lambda} \cdot 2^1}{1!} = \frac{3}{e^2}$$

So Av No. of pages having at most one P. Error $= 500 \times \frac{3}{e^2}$

[NAT]

#Q. A manufacturing company supplies condensers with 1% defective pieces. Condensers are pack in boxes of 100. Find the probability that a box picked at random will have four or more faculty condensers

$X = \{\text{Number of faulty condensers}\}$

$p = P(\text{f. Cond}) = 1\% = \frac{1}{100}$

$n = 100$

$\lambda = np = 100 \times \frac{1}{100} = 1$

$P(X \geq 4) = ? = 1 - P(X \leq 3)$

$= 1 - P\{X = 0 \text{ or } 1 \text{ or } 2 \text{ or } 3\}$

$= 1 - e^{-\lambda}\left\{\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!}\right\}$

$= 1 - e^{-1}\left\{1 + 1 + \frac{1}{2} + \frac{1}{6}\right\}$

$= 0.019$ Ans

[NAT]

#Q. One fifth percent of the blades produced by a manufacturing factory turn out to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 1,00,000 packets. Given $e^{-0.02} = 0.9802$

$N = 100,000$ packets; For single packet;

$X = \{$Number of defective Blades in one packet$\}$

$n = 10$, $p = P(\text{Def Blade}) = \frac{1}{5}\% = 0.2\% = \frac{0.2}{100} = 0.002$

$\lambda = np = 10 \times 0.002 = 0.02$

(3) $P(X=2) = \frac{e^{-\lambda}\lambda^2}{2!} = \frac{(0.02)^2}{2} e^{-0.02}$ So $A = 10^5 \times \frac{(0.02)^2}{2} \times e^{-0.02}$

(1) $P(X=0) = \frac{e^{-\lambda}\lambda^0}{0!} = e^{-0.02}$

$= 0.9802 = \frac{9802}{10000}$

So Req. Ans $= 0.9802 \times 10^5$

(2) $P(X=1) = \frac{e^{-\lambda}\lambda^1}{1!} = e^{-\lambda}\cdot\lambda$

$= 0.02 \times e^{-0.02} = \frac{2}{100} \times 0.9802$

Ans $= \frac{2}{100} \times 0.9802 \times 10^5$

[NAT]

#Q. A certain company sells tractors which fail at a rate of 1 out of 1000. If 500 tractors are purchased form this company what is the probability of 2 of them failing with in first year ?

$X = \{$ Number of Tractors fails in 1^{st} yr $\}$ → Success

$\lambda = np = 500 \times \frac{1}{1000} = \frac{1}{2}$

$$P(X=2) = \frac{e^{-\lambda}\lambda^2}{2!} = \frac{e^{-0.5} \times (0.5)^2}{2!} = 0.07582$$

THANK - YOU