

# Computer Science & IT

## Database Management System



**Relational Model & Normal Forms**

**Lecture No. 06**



**By- Vishal Sir**

# Recap of Previous Lecture



Topic

Identification of candidate keys in a relation



Topic

Membership test



Topic

Relationship between two FD sets



Topic

FD set of a sub-relation





# Topics to be Covered



Topic

FD set of a subrelation



Topic

Minimal cover (Canonical cover)



Topic

Number of superkeys in a relation



## FD set of a Sub-relation

- Let  $R$  be the relation with FD set  $F$ , and  $R_1$  is any sub-relation of  $R$ .

Concept of membership test can be used to identify the FDs of sub-relation

- Let  $R(A, B, C, D, E)$  is a relation, then  $R_1(A, B, E)$  can be called a subrelation of relation  $R$



Q: Let  $R(A, B, C, D, E)$  is a relation with FD set  $F$ .

$F = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$

And let  $R_1(A, B, E)$  is a sub-relation of relation  $R(A, B, C, D, E)$

Find the Candidate keys of sub-relation  $R_1(A, B, E)$

Soln. To find the candidate keys of any relation  
we need the set of functional dependencies  
w.r.t. that relation

∴ First we need to identify the FDs  
that exists in sub-relation  $R_1(A, B, E)$



Q: Let  $R(A, B, C, D, E)$  is a relation with FD set  $F$ .

$F = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$

And let  $R_1(A, B, E)$  is a sub-relation of relation  $R(A, B, C, D, E)$

We need to find FD set  $F_1$  w.r.t. sub-relation  $R_1$   
i.e., we need identify the relationship b/w  $A, B$  &  $E$

$(A)^+$  w.r.t.  $F = \{A, B, C\}$

i.e.  $A \rightarrow \cancel{A} \cancel{B} \cancel{C}$

trivial

Not in relation  $R_1$

$\therefore \boxed{A \rightarrow B}$

$(B)^+$  w.r.t.  $F = \{\cancel{B}\} \therefore$  No useful FD

$(E)^+$  w.r.t.  $F = \{\cancel{E}\} \therefore$  No useful FD

$(AB)^+$  w.r.t.  $F = \{\cancel{A}, \cancel{B}, \cancel{C}\} \therefore$  No useful FD

$(AE)^+$  w.r.t.  $F = \{\cancel{A}, \cancel{E}, \cancel{B}, \cancel{C}\} \therefore \boxed{AE \rightarrow B}$

$(BE)^+$  w.r.t.  $F = \{\cancel{B}, \cancel{E}\} \therefore$  No useful FD

Hence FD set  $F_1$  w.r.t. sub-relation  $R_1(A, B, E)$  is

$F_1 = \left\{ \begin{array}{l} A \rightarrow B \\ AE \rightarrow B \end{array} \right\}$

$\therefore$  CK of  $R_1$  is  $(AE)$



H.W.

Q: Let  $R(A, B, C, D, E, F)$  is a relation with FD set  $F$ .

$F = \{AB \rightarrow C, B \rightarrow D, BC \rightarrow A, D \rightarrow EF\}$

And let  $R_1(A, B, C, D)$  is a sub-relation of relation  $R(A, B, C, D, E, F)$

Find the Candidate keys of sub-relation  $R_1(A, B, C, D)$

$$(A)^+ = \{A\}$$

$$(B)^+ = \{B, D, E, F\}$$

$$(C)^+ = \{C\}$$

$$(D)^+ = \{D, E, F\}$$

$$(AB)^+ = \{A, B, C, D, E, F\}$$

$$(AC)^+ = \{A, C\}$$

$$(AD)^+ = \{A, D, E, F\}$$

$$(BC)^+ = \{B, C, D, A, E, F\}$$

$$(BD)^+ = \{B, D, E, F\}$$

$$(CD)^+ = \{C, D, E, F\}$$

$$(ABC)^+ = \{A, B, C, D, E, F\}$$

$$(ABD)^+ = \{A, B, C, D, E, F\}$$

$$(ACD)^+ = \{A, C, D, E, F\}$$

$$(BCD)^+ = \{B, C, D, A, E, F\}$$

FD set  $F_1$  of  $R_1(A, B, C, D)$  is  
 $F_1 = \{B \rightarrow D, AB \rightarrow C, BC \rightarrow A\}$   $\therefore$  Cks are  $AB, BC$





## Topic : Minimal cover (Canonical cover)

Irreducible set  
of functional dependencies

- Minimal cover or canonical cover of FD set  $F$  is a set of functional dependencies ( $F_m$ ) such that,
- $F_m = F$  and
  - $F_m$  does not contain any redundant FD, and  $F_m$  must not contain any extraneous attribute at either side of any of its FD



$$A \rightarrow B \text{ (C)}, B \rightarrow C \Rightarrow A \rightarrow B \text{ \& B} \rightarrow C$$

|||

$$\{A \rightarrow B \text{ \& B} \rightarrow C\} \Rightarrow A \rightarrow B \text{ \& B} \rightarrow C$$

~~$A \rightarrow C$   
Redundant~~





## Topic : Procedure to obtain minimal cover of FD set

1. Simplify RHS of all FDs (i.e., split the FDs such that RHS contain exactly one attribute)
2. For all FDs find redundant (extraneous) attribute in LHS.
3. Eliminate all redundant FDs
4. Apply Union if needed  
(it is not mandatory)
5. The result is minimal Cover

As soon as any redundant FD is identified, remove it from the set & check for other redundant FDs

As soon as any extraneous attribute is identified in the LHS, remove it and check for other extraneous attributes



$$A \rightarrow B$$

✓

$$\underbrace{A \ B} \rightarrow C$$

$\alpha$

$$\hookrightarrow (\alpha - A)^+ = (B)^+$$

$$(A)^+ = \{A, B\}$$

$$B \in (A)^+ \therefore B \text{ is extraneous w.r.t. } A$$

$A \ B \rightarrow C$

$$(B)^+ = \{B\}$$

$$A \notin (B)^+ \therefore A \text{ is extraneous w.r.t. } B$$





## Topic : Testing if an Attribute is Extraneous in LHS

Consider a set  $F$  of functional dependencies and functional dependency  $\alpha \rightarrow \beta$  in  $F$ .

let ' $A$ ' is an attribute belonging to set ' $\alpha$ '

$\alpha$  is a set of attributes that contain two or more attributes  
After simplification  $|\beta| = 1$

To test if attribute  $A \in \alpha$  is extraneous in  $\alpha$  (i.e., Any extraneous attribute in LHS of FD)

1. compute  $(\{\alpha\} - \{A\})^+$  using the dependencies in  $F'$ , where  $F' = (F - \{\alpha \rightarrow \beta\})$
2. check if  $(\{\alpha\} - A)^+$  contains  $A$ ; if it does then,  $A$  is extraneous.

if  $A \notin (\alpha - A)^+$  then  
 $A$  is not Extraneous

all FDs of Set  $F$  Except  $\alpha \rightarrow \beta$



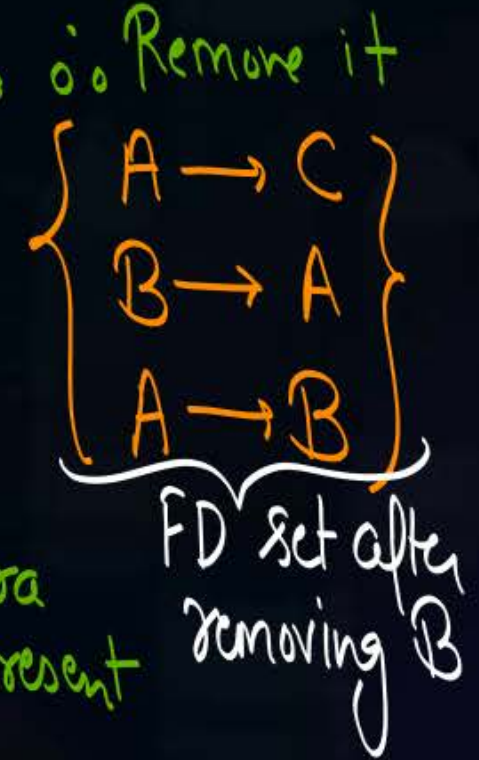
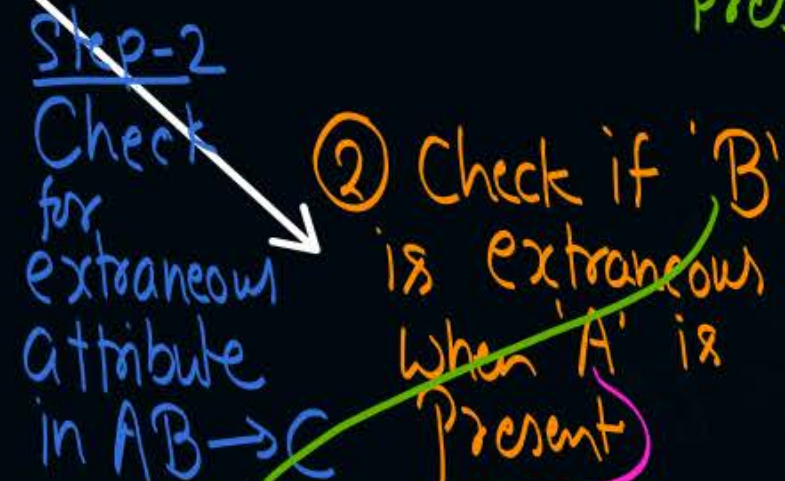
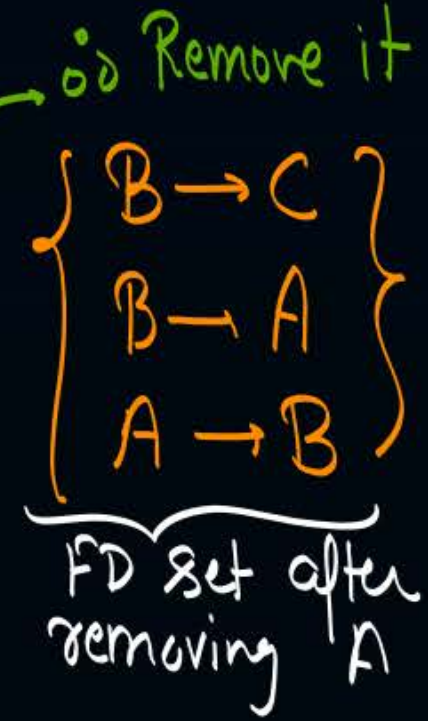
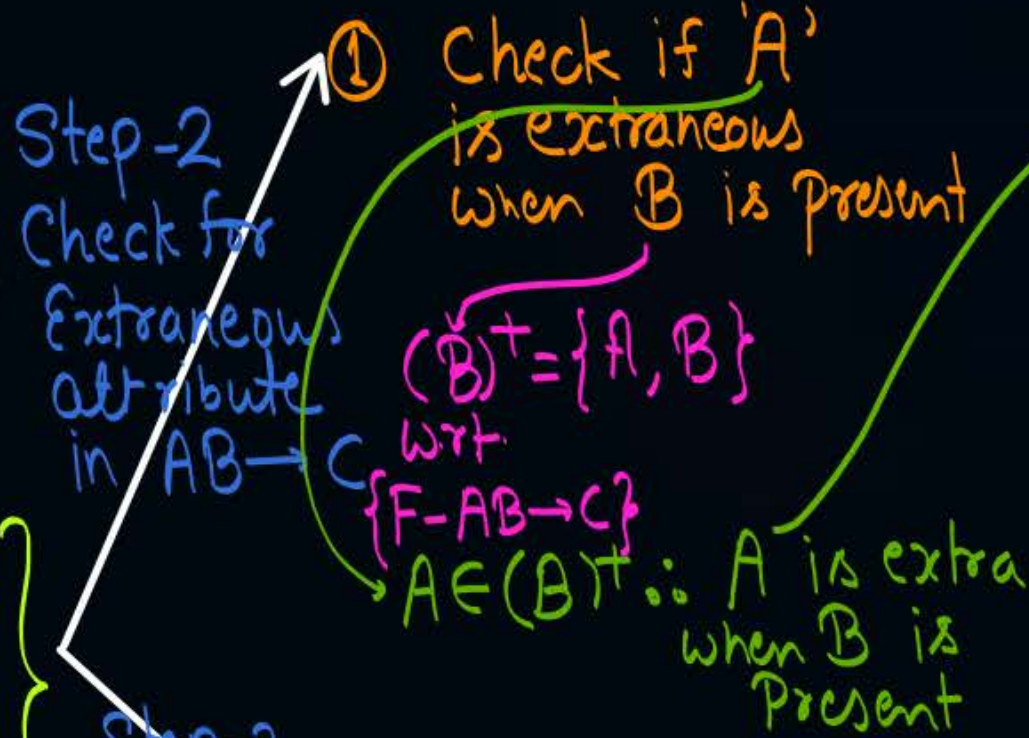
#e.g., Consider the FD set

$F = \{AB \rightarrow C, B \rightarrow A, A \rightarrow B\}$

Find minimal cover of  $F$



Single attribute in LHS can never be extraneous







## Topic : Testing if any FD is redundant

Consider a set  $F$  of functional dependencies and functional dependency  $\alpha \rightarrow \beta$  in  $F$ .

To test if  $\alpha \rightarrow \beta$  is redundant

1. compute  $\alpha^+$  using only the dependencies in  $F'$ , where  $F' = (F - \{\alpha \rightarrow \beta\})$
2. check that  $\alpha^+$  contains  $\beta$ ; if it does then,  $\alpha \rightarrow \beta$  is redundant

otherwise  $\alpha \rightarrow \beta$  is required

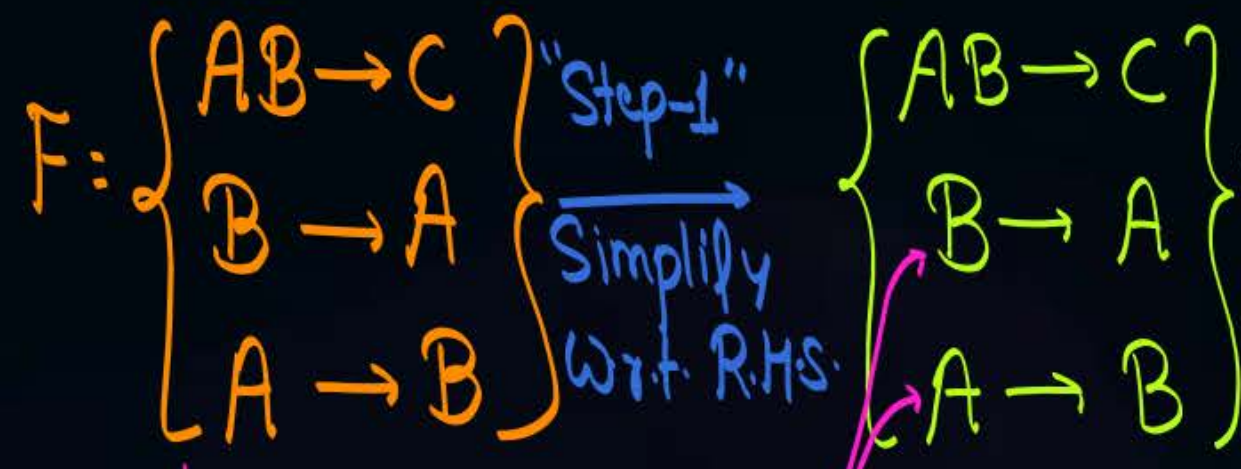
↑  
All FD except  
 $\alpha \rightarrow \beta$



#e.g., Consider the FD set

$F = \{AB \rightarrow C, B \rightarrow A, A \rightarrow B\}$

Find minimal cover of  $F$



Single attribute in LHS can never be extraneous

Step-2 Check for extraneous attribute in  $AB \rightarrow C$

① Check if 'A' is extraneous when B is present

$(B)^+ = \{A, B\}$  w.r.t.  $\{F - AB \rightarrow C\}$

$A \in (B)^+ \therefore A$  is extra when B is present

Step-2 Check for extraneous attribute in  $AB \rightarrow C$

② Check if 'B' is extraneous when 'A' is present

$(A)^+ = \{A, B\}$  w.r.t.  $\{F - AB \rightarrow C\}$

$B \in (A)^+ \therefore B$  is extra when A is present

Remove it

Step-3

Remove redundant FDs

$\{B \rightarrow C, B \rightarrow A, A \rightarrow B\}$

FD set after removing A

Step-4 = Union

$\{B \rightarrow A, A \rightarrow B\}$

Both are Minimal cover of FD set  $F$

Required FDs

Remove it

Step-3

Remove redundant FDs

$\{A \rightarrow C, A \rightarrow B\}$

FD set after removing B

Step-4 = Union

$\{A \rightarrow B, B \rightarrow A\}$

Required FDs







## Topic : NOTE



Minimal cover of FD set  $F$  need not be unique, but all minimal cover are logically equivalent.

If  $F_{m_1}$  &  $F_{m_2}$  are two different minimal covers  
of FD set  $F$ , then  $F_{m_1} = F_{m_2} = F$



#e.g., Consider the FD set

$$F = \{A \rightarrow BC, B \rightarrow C\}$$

Find all minimal cover of F

~~Step-2~~  
Eliminate  
Extraneous  
attribute  
from LHS

$$F = \left\{ \begin{array}{l} A \rightarrow BC \\ B \rightarrow C \end{array} \right\} \xrightarrow[\text{Simplify w.r.t. RHS}]{\text{Step-1}} \left\{ \begin{array}{l} A \rightarrow B \\ A \rightarrow C \\ B \rightarrow C \end{array} \right\}$$

$$\left\{ \begin{array}{l} A \rightarrow B \\ A \rightarrow C \\ B \rightarrow C \end{array} \right\}$$

Step-3  
Eliminate  
redundant FDs

Redundant  
∴ remove

$$\boxed{\left\{ \begin{array}{l} A \rightarrow B \\ B \rightarrow C \end{array} \right\}}$$

$$\left\{ \begin{array}{l} A \rightarrow B \\ \text{[Redacted]} \\ B \rightarrow C \end{array} \right\}$$

↑  
it is the only  
Minimal cover of  
FD set F.

Q. Find minimal cover of

$$F = \left\{ \begin{array}{l} A \rightarrow C \\ AC \rightarrow D \\ E \rightarrow AD \\ E \rightarrow H \end{array} \right\}$$

Step-1

$$\left\{ \begin{array}{l} A \rightarrow C \\ AC \rightarrow D \\ E \rightarrow A \\ E \rightarrow D \\ E \rightarrow H \end{array} \right\}$$

Step-2

$$\left\{ \begin{array}{l} A \rightarrow C \\ A \rightarrow D \\ E \rightarrow A \\ E \rightarrow D \\ E \rightarrow H \end{array} \right\}$$

Step-3

$$\left\{ \begin{array}{l} A \rightarrow C \\ A \rightarrow D \\ E \rightarrow A \\ E \rightarrow H \end{array} \right\}$$

Union

$$\left\{ \begin{array}{l} A \rightarrow C.D \\ E \rightarrow AH \end{array} \right\}$$



~~How.~~ #e.g., Consider the following FD set

$F = \{A \rightarrow BC$

$CD \rightarrow E$

$E \rightarrow C$

$D \rightarrow AEH$

$ABH \rightarrow BD$

$DH \rightarrow BC$

$\}$

Find minimal cover of F.

$A \rightarrow BC$

$CD \rightarrow E$

$E \rightarrow C$

$D \rightarrow AEH$

$ABH \rightarrow BD$

$DH \rightarrow BC$

Simplify RHS.

Eliminate Extra  
Attribute from LHS.

Eliminate Redundant  
FD

Union



#Q. The following functional dependencies hold true for the relational schema

$R\{V, W, X, Y, Z\}$   
 $(V)^+ = \{V, W\}$   
 $(W)^+ = \{W\}$

$w$  is extra  
 $= Y \rightarrow V$   
 $= Y \rightarrow X$

$[V \rightarrow W; \underline{VW} \rightarrow X; Y \rightarrow VX; Y \rightarrow Z]$

Which of the following is irreducible equivalent for this set of functional dependencies?

(A)  
 $V \rightarrow W$  ✓  
 $V \rightarrow X$  ✓  
 $Y \rightarrow V$  ✓  
 $Y \rightarrow Z$  ✓

(B)  
 $V \rightarrow W$  ✓  
 $\times W \rightarrow X$   
 $Y \rightarrow V$   
 $Y \rightarrow Z$

(C)  
 $V \rightarrow W$  ✓  
 $V \rightarrow X$  ✓  
 $Y \rightarrow V$  ✓  
 $\boxed{Y \rightarrow X}$  Redundant  $\times$   
 $Y \rightarrow Z$  ✓  
 $(Y)^+ = \{Y, V, X, Z\}$

(D)  
 $V \rightarrow W$  ✓  
 $\times W \rightarrow X$   
 $Y \rightarrow V$   
 $Y \rightarrow X$   
 $Y \rightarrow Z$

Number of Super keys



Q: Let  $R(A, B, C, D, E)$  is a relation  
with no non-trivial functional dependency. {i.e.  $F = \{\}$  }  
then what will be the Candidate Key of  $rel^h R$ .

Solu'n No FD in the FD set,  
Hence all attributes are essential attributes

$\therefore$  C.K will be formed by combining  
all attributes of the relation.

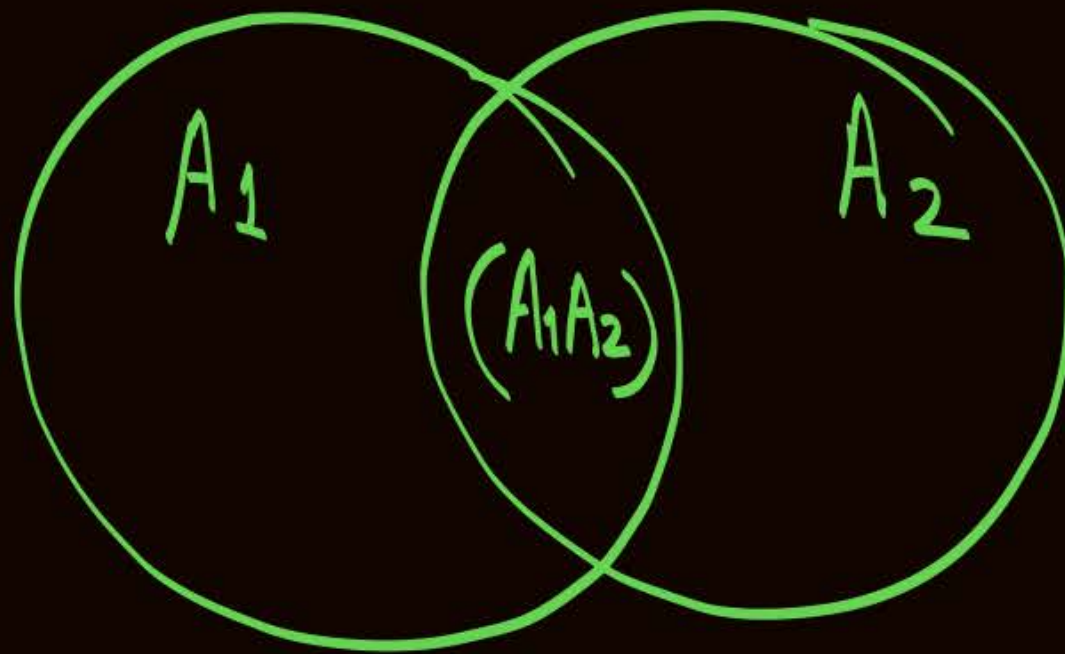
\* In this case, we have only one Candidate Key and  
Only one Super Key (it C.K itself)

H.W. Q:- Consider the relational schema  $R(A_1, A_2, A_3, \dots, A_n)$   
Find the number of superkeys possible in relation R.  
(i) When "A<sub>1</sub>" is the only candidate key of relation R.



H.W. Q:- Consider the relational schema  $R(A_1, A_2, A_3, \dots, A_n)$   
Find the number of superkeys possible in relation R.  
(ii) When  $(A_1A_2)$  is the only candidate key of relation R.

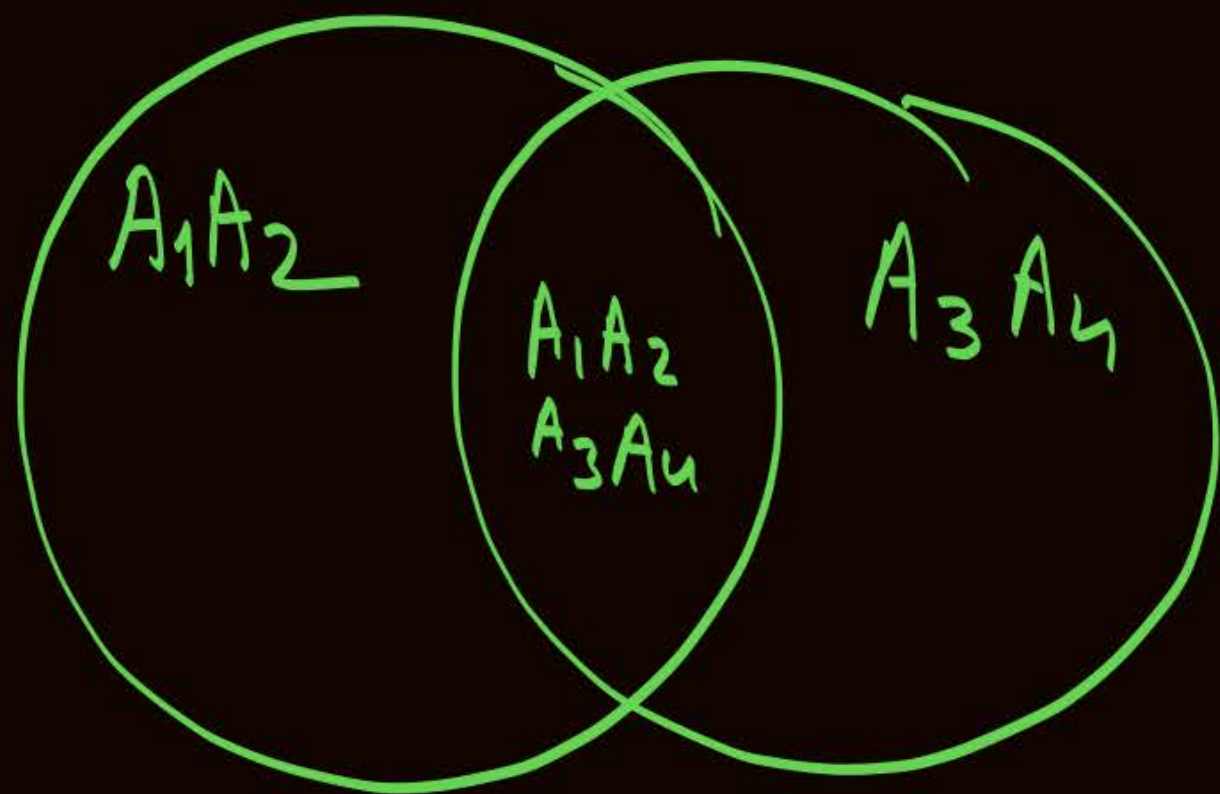
Q:- Consider the relational schema  $R(A_1, A_2, A_3, \dots, A_n)$   
H.W Find the number of superkeys possible in relation R.  
(iii) When " $A_1$ " & " $A_2$ " are the only two candidate keys of relation R.



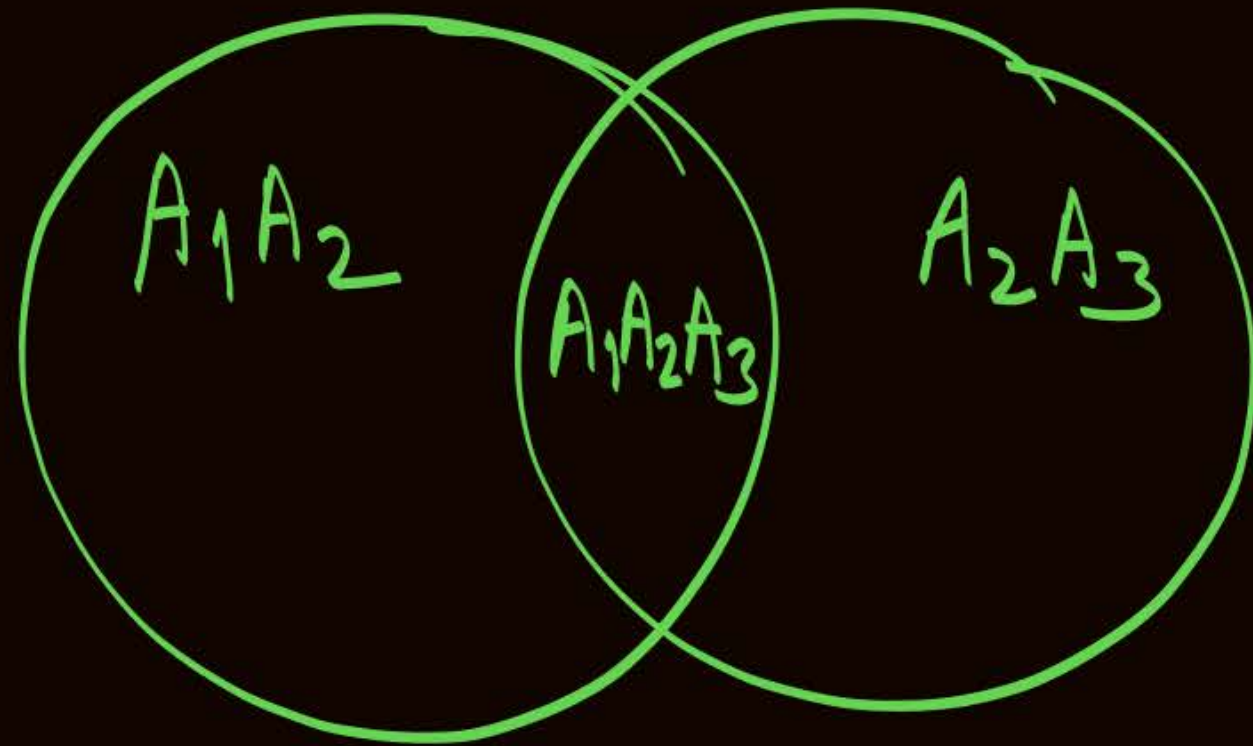


Q:- Consider the relational schema  $R(A_1, A_2, A_3, \dots, A_n)$

14W. Find the number of superkeys possible in relation R.  
(iv) When  $(A_1A_2)$  and  $(A_3A_4)$  are two candidate keys of relation R.

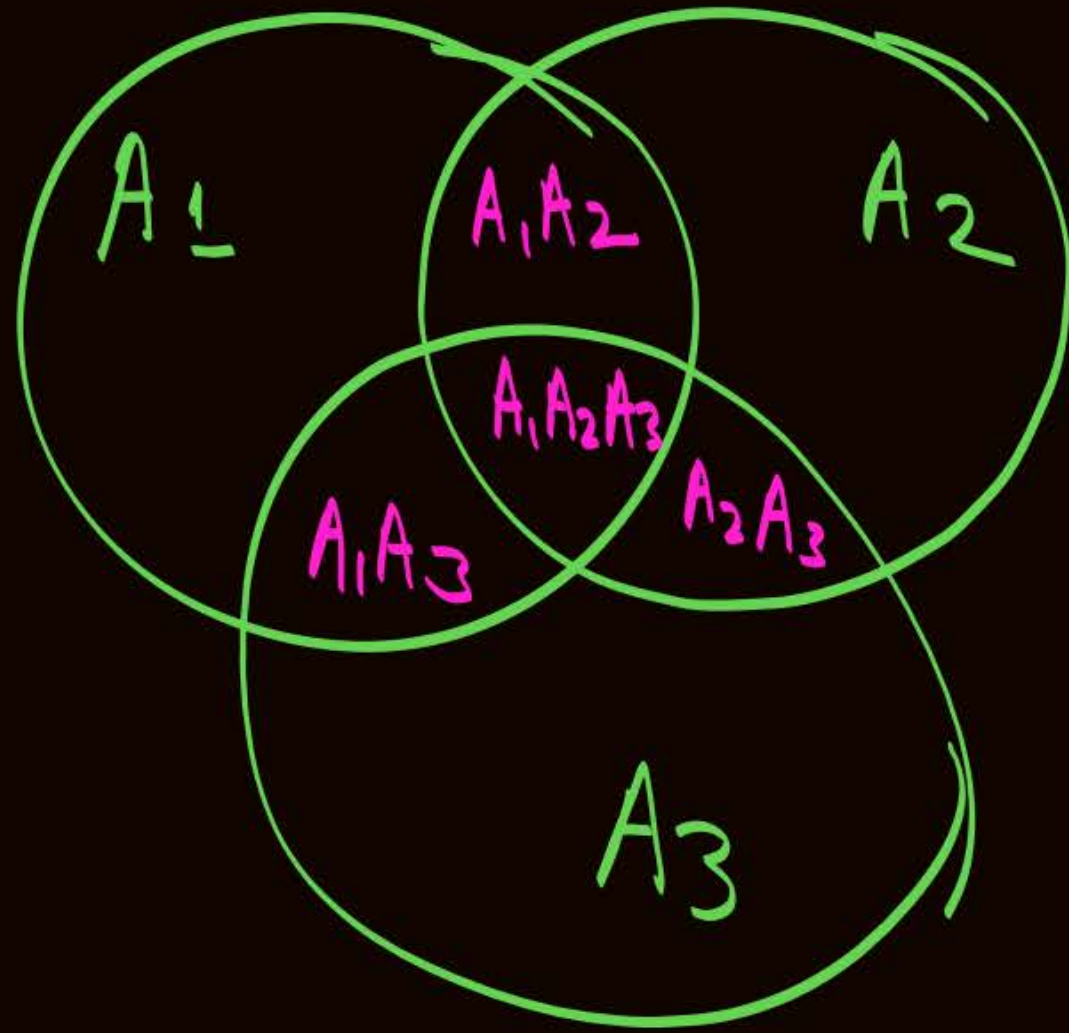


H.W. Q:- Consider the relational schema  $R(A_1, A_2, A_3, \dots, A_n)$   
Find the number of superkeys possible in relation R.  
(V) When  $(A_1A_2)$  &  $(A_2A_3)$  are only two candidate keys of relation R.





H.W. Q:- Consider the relational schema  $R(A_1, A_2, A_3, \dots, A_n)$   
Find the number of superkeys possible in relation R.  
(vi) When  $(A_1)$ ,  $(A_2)$  and  $(A_3)$  are three candidate keys of relation R.



H.W.

Q:-

Consider the relational schema  $R(A_1, A_2, A_3, \dots, A_n)$

Find the number of superkeys possible in relation R.

When each attribute of relation R itself is a Candidate Key





## 2 mins Summary



**Topic**

FD set of a subrelation

**Topic**

Minimal cover (Canonical cover)

**Topic**

Number of superkeys in a relation

**THANK - YOU**