CS & IT ENGINEERING

THEORY OF COMPUTATION

Regular Language

Lecture No.- 1

By- Venkat sir

Recap of Previous Lecture

Topic ?????

{ Regular Expressions }

Topics to be Covered

Topic — Regular Language Detection

Topic — Pumping Lemma

→ {Closure properties of Regular languages}

(Q) How many states in min DFA for

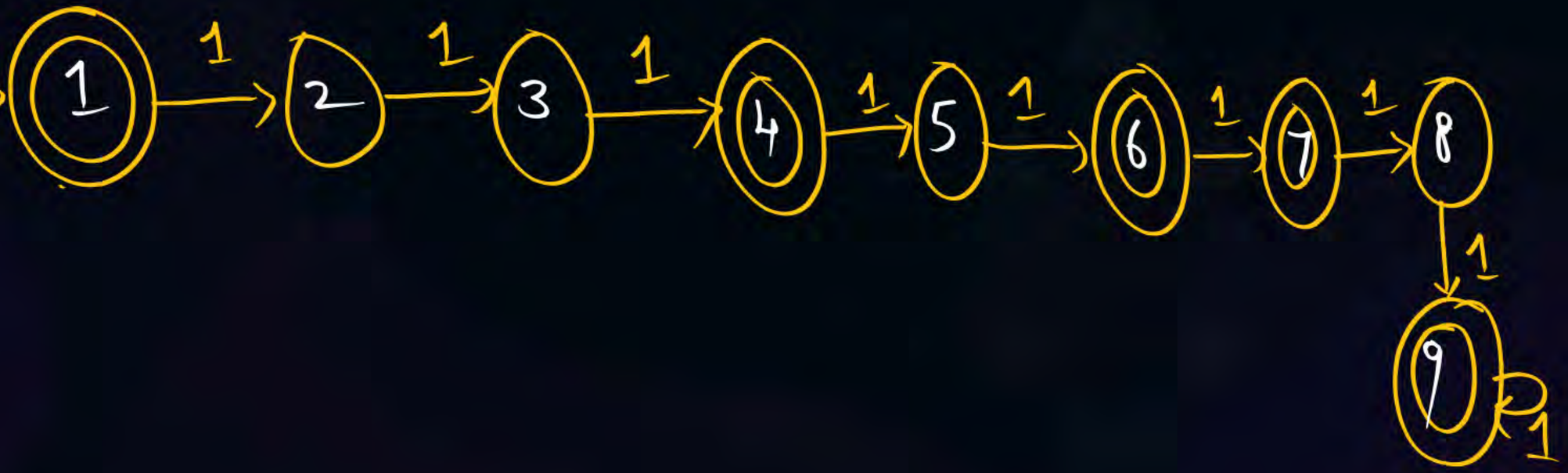
$r = (111 + 11111)^*$

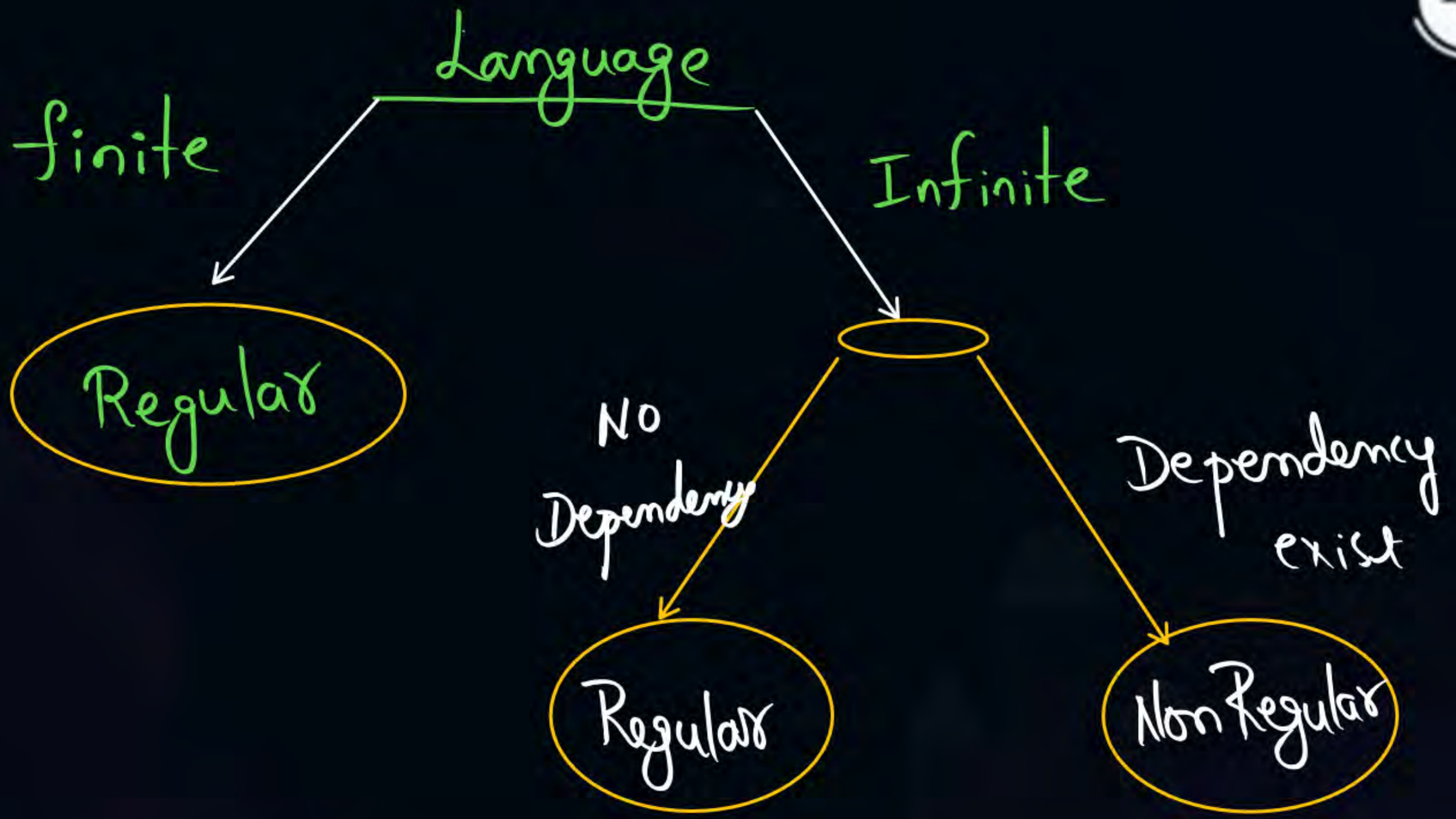
{ 9 states }

{ 1,2, 3, 4, 5, 6, 7, 8, 9, 10, 11 ---- }

GATE

{ 2 Marks }




Language
finite
Infinite
Regular
No Dependency
Dependency exist
Regular
Non Regular


Regular Langs
$\{a^n b^m / n, m \geq 1\}$
finite Lang
$\{a^n b^n / n \leq 10\}$

Topic : Regular Language Detection

Yes ✓
NO ✓

Which of these languages are Regular?

① $L = \{a^n b^n c^n \mid 1 \le n \le 1000\}$ ⟹ finite → {Regular}

② $L = \{a^n b^m \mid n + m = 10\}$ ⟹ $\{a^0b^{10}, a^1b^9, a^2b^8 \dots a^9b^1, a^{10}b^0\}$ → finite → Regular

③ $L = \{a^n b^m \mid n - m = 5\}$ ⟹ $\{a^{m+5} b^m\}$ = Infinite → Dependency → Non Regular
$n = m+5$

④ $L = \{a^n b^m \mid n \times m = 100\}$ ⟶ $\{a^1b^{100}, a^2b^{50}, a^4b^{25}, a^{25}b^4, a^{50}b^2, a^{100}b^1\}$ → finite → Regular

⑤ $L = \{a^n b^m \mid n = 2m + 1\}$ ⟶ $\{a^{2m+1} b^m\}$ → Infinite → Dependy ⇒ Non regular

⑥ $L = \{a^n b^m \mid n > m\}$ ⟹ non regular

⑦ $L = \{a^n b^m \mid n > m \text{ (and) } n < m\}$ ⟹ $\{\}$ → ϕ → regular

⑧ $L = \{a^n b^m \mid n \ge m \text{ (or) } n < m\} \Rightarrow \{a^n b^m \mid n \ne m\} \Rightarrow$ Non regular

⑨ $L = \{a^n b^m \mid n \geq m \text{ (and) } (n \leq m)\} = \{a^n b^n\} \Rightarrow$ Non regular

⑩ $L = \{a^n b^m \mid (n+m) \text{ is odd}\} \Rightarrow$ regular

$a(aa)^*(bb)^* + (aa)^* b(bb)^*$

⑪ $L = \{a^n b^m \mid (n+m) \text{ is even}\} \Rightarrow$ regular

$(aa)^*(bb)^* + a(aa)^* b(bb)^*$

(12) $L = \{1, 2, 4, 8 \dots 2^n \dots\} = 0^*|0^*$

all these numbers written in binary

(13) $L = \{1, 2, 4, 8, \dots 2^n \dots\} = \{1^{2^n} \mid n \geq 0\}$ = Non Regular

all these numbers written in Unary

Topic : Regular Language Detection

1. Every finite language is regular.

2. Every regular language need not be finite.

- All palindrome languages over more than one symbol are non regular.

- Any infinite language require infinite memory then it is non regular.

Topic : Regular Language Detection

~~$L = \{a^n b^m \mid n > m \text{ (or) } n < m\}$~~

(14) $L = \{a^n b^m c^{n+m} \mid n, m \geq 1\}$ → Non regular

(15) $L = \{a^n b^{n+m} \mid n, m \geq 1\}$ → Non regular

(16) $L = \{a^n b^{2m} c^{3k} \mid n, m\, k \geq 0\}$ → regular

(17) $L = \{a^n b^{m^2} c^{k^3} \mid n, m \geq 1\}$

$L = \{a^{2^n} \mid n \geq 0\}$

$L = \{a^{n^2} \mid n \geq 1\}$

Topic : Regular Language Detection

(18) $L = \{a2^n \mid n \geq 1\}$ ⟶ non regular

(19) $L = \{a^{n^n} \mid n \geq 1\}$ ⟶ non regular

(20) $L = \{a^{100^{100^{100}}} \mid \}$ ⟶ 1 ⟶ finite regular

(21) $L = \{(a^p)^* \mid p \text{ is prime number}\}$ ⟶ regular

(22) $L = \{a^p \mid p \text{ is prime number}\}$ ⟶ $\{a^2, a^3, a^5, a^7, a^{11}, a^{13} \cdots\}$ = non regular

(23) $L = \{a^k \mid k \text{ is even number}\}$ ⟶ regular

(24) $L = \{ww^R \mid w \in \{a, b\}^*\}$ ⟶ non regular

(25) $L = \{ww^R \mid w \in (a)^*\}$ ⟶ regular

(26) $L = \{ W \underline{b} W^R \mid W \in (a)^* \} = \{ \underbrace{a^n}_{n} b \underbrace{a^n}_{n} \mid n \geq 0 \}$ = non regular

(27) $L = \{ W \overset{\text{1 or 0}}{X} W^R \mid W \in (a+b)^* \}$ $X \in (a+b)$ $\Rightarrow$ non regular

(28) $L = \{ \overset{\epsilon}{W} \overset{\epsilon}{X} W^R \mid W, X \in (a+b)^* \} \Rightarrow (a+b)^*$ (W: $(a+b)^*$) $\rightarrow$ regular

(29) $L = \{ W X W^R \mid W, X \in (a+b)^+ \} \Rightarrow$ regular

$a(a+b)^+ a + b(a+b)^+ b$

(30) $L = \{ ww^R x \mid w, x \in (a+b)^+ \} \Rightarrow$ Non Regular

(31) $L = \{ \underbrace{w}\underbrace{w} \mid w \in (a+b)^* \} = \{\epsilon, aa, bb, abab \cdots\} \Rightarrow$ Non Regular

(32) $L = \{ ww \mid w \in (a)^* \} = \{\epsilon, aa, a^2a^2 \cdots\} = (aa)^* \} \Rightarrow$ Regular

(33) $L = \{ wbw \mid w \in (a)^* \} \Rightarrow \{b, aba, a^2ba^2 \cdots\} = \{a^n b a^n\} \Rightarrow$ Non Reg

(Q) which of the following is Regular?

(a) $L = \{ WxW \mid W, x \in (a+b)^+ \}$

(b) $L = \{ WWx \mid W, x \in (a+b)^+ \}$

(c) $L = \{ xWW \mid W, x \in (a+b)^+ \}$

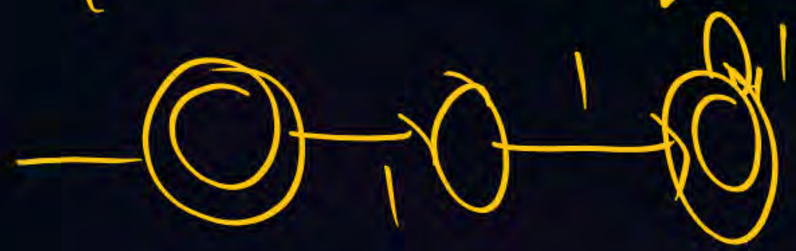
Non Regular

(d) none

$L = \{(a^P)^* \mid P \text{ is prime number}\} = \text{regular}$

$\{(a^2)^*, (a^3)^*, (a^5)^* \cdots\}$

$\{\epsilon, a^2, a^3, a^4, a^5, a^6, a^7 \cdots\}$



(Q) which of the following is non regular?

(a) $L = \{(a^n)^* \mid n \geq 1\}$ → regular $= a^*$

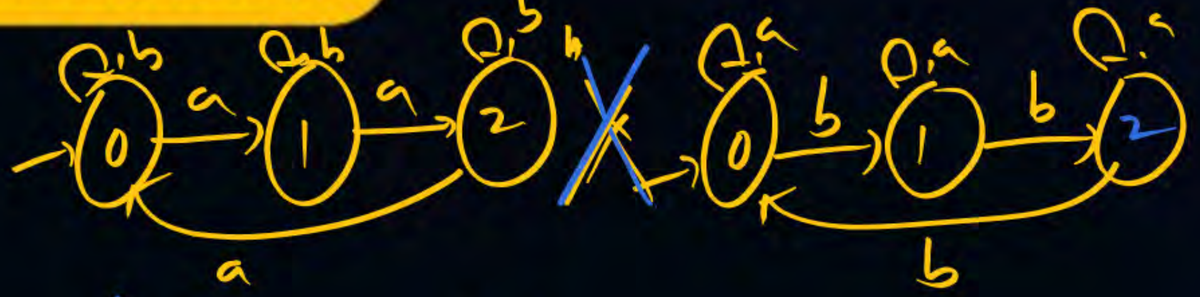
(b) $L = \{(a^2)^* \mid n \geq 0\} = a^*$

(c) $L = \{(a^{n!})^* \mid n \geq 1\} = a^*$

} regular

(d) none

Topic : Regular Language Detection



$L = \{wbw^R \mid w \in \{a\}^*\}$

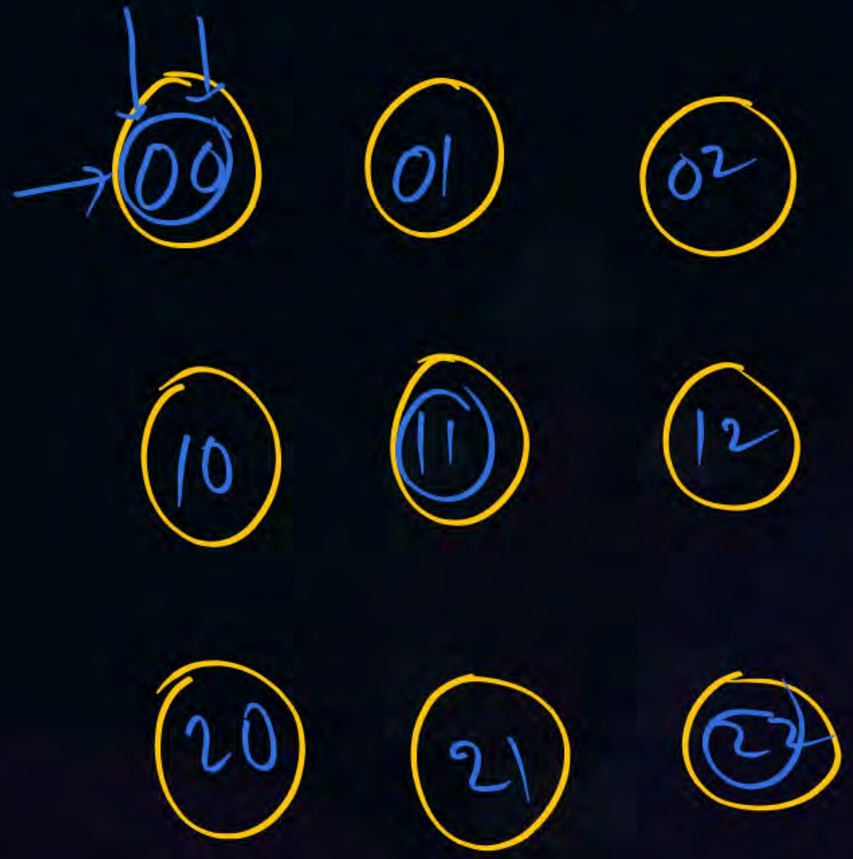
(36) $L = \left\{ x \mid x \in \{a, b\}^* \right.$
$\left. n_a(x) \bmod 3 = n_b(x) \bmod 3 \right\}$ = regular

{0,1,2,} {0,1,2,}

(37) $L = \left\{ x \mid x \in \{a, b\}^* \right.$
$\left. n_a(x) \bmod 3 > n_b(x) \bmod 3 \right\}$ = regular

(38) $L = \left\{ x \mid x \in \{a, b\}^* \right.$
$\left. n_a(x) \bmod 3 \neq n_b(x) \bmod 3 \right\}$ = regular

(39) $L = \left\{ x \mid x \in \{a, b, c\}^* \right.$
$\left. n_a(x) \neq n_b(x) \right\}$ } Non Regular



(34) $L = \{ x \mid \begin{matrix} x \in (a+b)^* \\ n_a(x) = n_b(x) \end{matrix} \} \Rightarrow$ {Non Regular}

(35) $L = \{ x \mid \begin{matrix} x \in (a+b)^* \\ n_a(x) > n_b(x) \end{matrix} \} \rightarrow$ {Non Regular}

Topic : Regular Language Detection

(((( ))) (( )) ( ))

40. Set of all balanced parentheses ⇒ Non Regular

41. Equal number of open and close parentheses ⇒ Non Regular

42. Even length palindrome strings of English Language. ⇒ Non Regular

43. Odd length palindrome strings of Hindi Language ⇒ Non Regul

(44) $L = \{ xCy \mid x, y \in (a+b)^* \}$ $(a+b)^*C(a+b)^*$ ⇒ regular

(45) $L = \{ xy \mid x, y \in (a+b)^* \} = (a+b)^*$ } regular

(46) $L = \{ xx \mid x \in (a+b)^* \}$ ⇒ Non regular

(47) $L = \{ x\$x \mid x \in (a+b)^* \}$ ⇒ non reg

a$a

$x \in (a+b)^*$

(48) $L = \{ x \$ y \mid n_a(x) = n_b(y) \}$ ⇒ Non reg

$x \in (a+b)^*$

(49) $L = \{ xy \mid n_a(x) = n_b(y) \}$ ⇒ reg $= (a+b)^*$

$\{ \epsilon, a, b, aa, bb, ab, ba, \ldots \}$

(50) Total population of world ⇒ regular

[MCQ] GATE 2025 [2 Mark]

#Q. Consider the following two languages over the alphabet $\{a,b\}$:

$L_1 = \{ \alpha\beta\alpha \mid \alpha \in \{a,b\}^+ \text{ AND } \beta \in \{a,b\}^+ \}$

$L_2 = \{ \alpha\beta\alpha \mid \alpha \in \{a\}^+ \text{ AND } \beta \in \{a,b\}^+ \}$

Which ONE of the following statements is CORRECT?

(A) Both L_1 and L_2 are regular languages.

(B) L_1 is a regular language but L_2 is not a regular language.

(C) L_1 is not a regular language but L_2 is a regular language.

(D) Neither L_1 nor L_2 is a regular language.

2 mins Summary

- **Topic** One
- **Topic** Two
- **Topic** Three
- **Topic** Four
- **Topic** Five

THANK - YOU