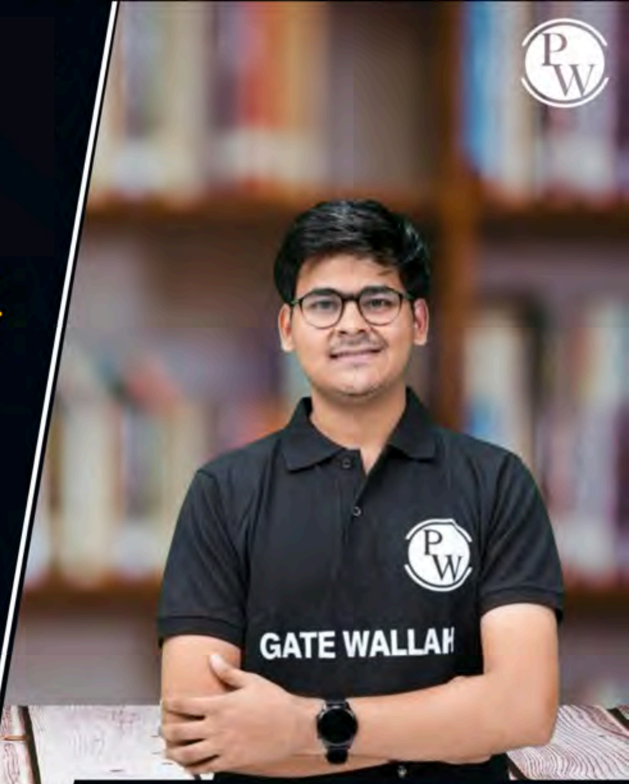
CS & IT ENGINEERING

Algorithms

Analysis of Algorithms

DPP 01 Discussion Notes



By- Aditya sir

[MSQ] $\rightarrow$ multiple

#Q. Which of the following natation is/are transitive but not reflexive

A Big oh (O) ✗

B Big omega (Ω) ✗

C Small oh (O) ✓

D Small omega (ω) ✓

Transitive ✓ Reflexive ✗

Ans: C, D

	O	Ω	Θ	o	ω
Reflexive	✓	✓	✓	×	×
Transitive	✓	✓	✓	✓	✓

[MSQ] $\longrightarrow$ multiple

Ans:- A, B, D

#Q. If $f(n) = \sum_{i=1}^{n} i^3$

Then which of the following choices is/are true for f(n)?

A $\theta(n^4)$ — $O(n^4) \rightarrow$ Tight UB, $\Omega(n^4) \rightarrow$ Tight LB

B $\Omega(n^4)$ — $= O(n^4)$ ✓, $= O(n^5)$ ✓, $= O(n^6)$ ✓ ⋮

C $\theta(n^5)$ — $O(n^5)$ ✓, $\Omega(n^5)$ ✗

D $\Omega(n^3)$ — $\Omega(n^5)$ ✗, $\Omega(n^4)$, $\Omega(n^3)$ ✓, $\Omega(n^2)$ ⋮

Soln :-

$$f(n) = \sum_{i=1}^{n} i^3$$

$$= 1^3 + 2^3 + 3^3 \ldots\ldots n^3$$

$$= \left[\frac{n(n+1)}{2}\right]^2$$

$(a+b)^2 = \underline{\underline{a^2+2ab+b^2}}$

$$= \left(\frac{n^2+n}{2}\right)^2$$

$$= n^4 + \ldots\ldots$$

$$= \underline{\underline{\Theta(n^4)}}$$

[MCQ]

#Q. Consider the following program:

```
main ( )
{
    P = n!
    for (i = 1; i ≤ n ; ++i)
   for (j = 1 ; j ≤ P ; 2*j)
        C = C + 1;
}
```

j = j * 2

$\log_2 P$

What is the time complexity of above code?

A. $O(n^2)$

B. $O(n^2 \log n)$

C. $O(n \log n)$

D. $O(n)$

Ans :- B

Soln :-

For (i=1; i<=n; i++) $\longrightarrow$ O(n) $\rightarrow$ runs n times

{

For (j=1; j<=P ; j=j*2) $\longrightarrow$ runs $\underline{\log_2 P}$ times

}

Overall TC $\Rightarrow$ $n * \log_2 P$

given $P = n! \approx \underline{n^n}$

$n * \log_2 n^n$

$n * n * \log n$

$= \underline{O(n^2 \log n)}$

[MCQ]

#Q. Consider the following code:

```
main ( )
{
    i = 1; j = 1
    while (j ≤ n )
    {
        ++ i;
        j = j + i;
    }
```

What is the time complexity of above code?

$O(k)$

$\Rightarrow \underline{\underline{O(\sqrt{n})}}$

$\underline{\underline{\text{Ans :- B}}}$

A $\theta(n)$

B $\theta\left(\sqrt{n}\right)$

C $\theta(\log)$

D $\theta(n\log(\log n)$

Soln:- $i=1, j=1$

```
while(j <= n)
{
    i++
    j = j + i
}
```

$i=1 \longrightarrow i=2 \longrightarrow i=3$

$j=1 \quad j=1+2 \longrightarrow j=(1+2)+3$

$i=4 \quad \cdots$

$j=1 \rightarrow 4$

Let us assume that loop runs for k times.

$i=k$

$\downarrow$

$j = 1+2+3 \cdots + k \Longrightarrow n$

$$\sum_{i=1}^{k} i = n$$

$$\boxed{\frac{k(k+1)}{2}} = n \approx k^2 = n$$

$$\boxed{k = \sqrt{n}}$$

[MCQ]

#Q. Consider the following code:

```
Algorithm T(n)
{
    if (n = 1) return;
    else
{
    T(n/2);
    }
}
```

$\rightarrow TC(n)$

$\rightarrow O(1) \rightarrow$ B.C

$\rightarrow TC(n/2)$

Ans:- A

What is the space complexity of above code?

A $\theta(\log n)$

B $\theta(n)$

C $\theta(n\log(\log n)$

D $\theta(\sqrt{n})$

Soln :- Using Back - Substitution logic.

Step1 :- Algo $\longrightarrow$ Time Complexity Recurrence

$$T(n) = a, \quad n = 1$$
$$T(n) = T(n/2) + b ; \quad n > 1$$

Step2 :- Solve Recurrence using Back Substitution.

$$T(n) = T(n/2) + b \longrightarrow (1)$$
$$T(n/2) = T(n/2^2) + b$$
$$T(n) = T(n/2^2) + b + b$$

$$T(n) = T(n/2^2) + 2b$$

$$T(n/2^2) = T(n/2^3) + b$$

$$T(n) = T(n/2^3) + 3b \longrightarrow 2$$

$\vdots$

General Term $\quad T(n) = T(n/2^k) + k*b$

For Base Condition, $\quad n/2^k = 1$

$$n = 2^k \longrightarrow k = \log_2 n$$

$$T(n) = T(1) + k*b$$

$$= a + \log_2 n * b$$

$$T(n) \longrightarrow \boxed{O(\log_2 n)}$$

[MSQ] $\rightarrow$ multiple

#Q. $f(n) = 2^{n^2}$, $g(n) = n!$ $h(n) = 2^{\log n^2}$ Ans :- A, D

Which of the following is/are correct?

A $f(n) = \Omega\ (g(n))$ ✓

B $h(n) = \Omega\ (g(n))$ ✗

C $h(n) = O\ (g(n))$ ✓

D $g(n) = \Omega\ (f(n))$ ✗

Asymptotic Comparisons

$h(n) = O\ (g(n))$

$h(n) \leq c * g(n)$ ✓ $\longrightarrow$ True.

$g(n) = \Omega\ (f(n))$

$g(n) \geq f(n) \longrightarrow$ False

Soln :- $f(n) = 2^{n^2}$

$g(n) = n! \approx n^n$

$h(n) = 2^{\log n^2} = 2^{2\log n}$

$\underline{\underline{2^{n^2} > n^n}}$

2^{n^2} vs n^n

$\log(2^{n^2})$ $\quad$ $\log(n^n)$

n^2 $\quad$ $n \log n$

$n \times \cancel{n}$ $\quad$ $\cancel{n} \times \log n$

$n > \log n$

$2^{2\log n} < n^n$
vs

Take log

$\log(2^{2\log n})$ vs $\log(n^n)$

$2 \cancel{\log n}$ $\quad$ $n \cancel{\log n}$

$2 < n$

Conclusion :-

$$2^{n^2} > n! > 2^{\log n^2}$$

$$\boxed{f(n) > g(n) > h(n)}$$

Check option A) :- $f(n) = \Omega(g(n))$

$f \geq c * g \longrightarrow$ True.

Check option B) :- $h(n) = \Omega(g(n))$

$h(n) \geq c * g \longrightarrow$ False

[NAT] → Numerical

#Q. Consider the following notations:

1. $\sqrt{\log n} = O(\log \log n)$ ✗

2. $\log n = \Omega\left(\frac{1}{n}\right)$ ✓

3. $n^2 = \theta\left(2^{2 \log n}\right)$ ✓

4. $(0.061)^n = \theta(1.02)^n$ ✗

How many notations is/are correct? $\boxed{2}$ ________.

2 & 3 are correct

Ans :- 2

Check 1) $\sqrt{\log n} = O(\log(\log n))$

$(\log n)^{1/2} \leq c * \log(\log n)$

Let $\log n = x$

$\sqrt{x} > \log x$

$\sqrt{x} \leq c * \log(x)$ ✗

↳ False

Check 2 :- $\log n = \Omega(1/n)$

$\log n \geq c * \frac{1}{n}$ ✓

log ↑

Decr

$\frac{1}{n} \rightarrow \left(\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5}\right)$

Check 3) $n^2 = \Theta(2^{2\log n})$ ✓ equal rate of growth

n^2 vs $2^{2\log n}$

Taking $\log_2()$ both sides

$\log_2(n^2)$ $\log_2(2^{2\log n})$

$2\log n$ $2\log n * \log_2 2$

$$\boxed{2 * \log n = 2 * \log n}$$

Check 4)

$(0.061)^n = O(1.02)^n$ ✗

c_1^n $\qquad$ c_2^n ✗ $\qquad$ >1

$1.02 > 1$

$0.061 < 1$

$\frac{1}{2} = 0.5$

$(>1)^n \rightarrow$ Incr

$2^2 < 2^3 < 2^4$

$\left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$

Decr functn

$\downarrow$

Expo Decr

$\downarrow$

Expo incr.

$\left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0.125$

[MCQ]

#Q. Consider the following functions:

$f_1 = 2^n$

$f_2 = n!$

$f_3 = n^n$

$f_4 = e^n$

What is the correct increasing order of above function?

Asymptotic Comparison

A $f_1\ f_4\ f_2\ f_3$ ✓

B $f_2\ f_1\ f_4\ f_3$ ✗

C $f_2\ f_4\ f_1\ f_3$ ✗

D ~~$f_2\ f_1\ f_4\ f_3$~~ ✗

$f_2\ f_1\ f_3\ f_4$

Ans :- A

Soln :-

$f_1 = 2^n \longrightarrow$ Expo

$f_2 = n! \longrightarrow$ Expo

$f_3 = n^n \longrightarrow$ Expo

$f_4 = e^n \longrightarrow$ Expo

$n! \approx n^n$

$n! < n^n$

$n(n-1) \cdots 1 < n \times n \times n \cdots$

$e \simeq 2.71$

$e > 2$

2^n vs e^n

$f_3 > f_2$

$2^n < e^n$

$f_3 > f_2$ and $f_4 > f_1$

$f_3 > f_2 > f_4 > f_1$

$f_1 < f_4 < f_2 < f_3$

way 1

$n!$ vs e^n

$\Rightarrow \underline{\underline{n^n}}$ $\underline{\underline{e^n}}$

$n\uparrow$

$n^n > e^n$

way 2

n^n e^n

Taking $\log_e()$

$\log_e(n^n)$ $\log_e(e^n)$

$n * \log n \gg n$

[MCQ]

increasing

#Q. Sort the functions in ascending order of asymptotic(big-O) complexity.

$f_1(n) = n$, $f_2(n) = 80$, $f_3(n) = n^{\log n}$, $f_4(n) = \log\log^2 n$, $f_5(n) = (\log n)^{\log n}$

A $f_2(n), f_4(n), f_1(n), f_5(n), f_3(n)$

A

B $f_2(n), f_1(n), f_4(n), f_5(n), f_3(n)$

C $f_2(n), f_1(n), f_4(n), f_3(n), f_5(n)$

D $f_2(n), f_1(n), f_4(n), f_3(n), f_2(n)$

Rate of growth: decr < Const < Poly < Expo

Soln:

$f_1(n) = n \longrightarrow$ Poly

$f_2(n) = 80 \longrightarrow$ Const

$f_3(n) = n^{(\log n)} \longrightarrow$ Expo

$f_4(n) = \log((\log n)^2) \longrightarrow$ Poly

$f_5(n) = \log n^{(\log n)} \longrightarrow$ Expo

$F_2 < F_4 < F_1 < F_5 < F_3$

$n \qquad \log((\log n)^2)$

$n > 2\log(\log n)$

$$n^{\log n} > \log n^{\log n}$$

$$\cancel{\log n} * \log n \qquad \cancel{\log n} * \log(\log n)$$

Let $\log n \to x$

$$\log n \qquad \log(\log n)$$

$$x > \log(x)$$

[MSQ] → multiple soln can be correct

#Q. Consider two function $f(n) = 10n + 2\log n$ and $g(n) = 5n + 2(\log n)^2$, then which of the following is correct option?

$f \le g$

A $f(n) = \theta(g(n))$

B $f(n) = O(g(n))$ Big oh

C $f(n) = \omega(g(n^2))$ small omega

$f(n) > g(n^2) * c$

D None of the above

A, B

Soln :- $f(n) = 10 * n + 2 * \log(n)$, $g(n) = 5n + 2(\log n)^2$

$\boxed{f(n) = \Theta(n)}$ $\quad$ $\boxed{g(n) = \Theta(n)}$

$O \quad \Omega \qquad\qquad O \quad \Omega$

$$\left.\begin{array}{l} f = \Theta(n) \\ g = \Theta(n) \end{array}\right] \quad \left.\begin{array}{l} f = \Theta(g(n)) \\ g = \Theta(f(n)) \end{array}\right\}$$

$$g(n) = 5n + 2(\log n)^2$$

$$g(n^2) = 5n^2 + 2(\log(n^2))^2$$

$$= 5n^2 + 2(2\log(n))^2$$

$$= \underline{5n^2} + 8\underline{(\log n)}^2 \longrightarrow \underline{\Theta(n^2)}$$

$$f(n) = \Theta(n), \quad g(n^2) = \Theta(n^2)$$

$$f(n) < \underline{\underline{g(n^2)}}$$

~~[MCQ]~~ NAT $\rightarrow$ Numerical Answer Type $\rightarrow$ Int

#Q. Consider two function $f(n) = \sqrt{n}$ and $g(n) = n \log n + n$ then $f(n)/g(n)$ is equivalent to how many of the following given below? _____.

small oh

Big oh

A $O(n^{-1/2})$ ✓

B $O(n^{-1/2})$ ✓

$$\frac{f(n)}{g(n)} < \frac{1}{\sqrt{n}}$$

C $\Omega(1/\log n)$ ✗

D $\theta(n^{-1/2})$ ✗

Ans: 2

Soln:-

$$f(n) = \sqrt{n} \quad , \quad g(n) = n\log n + n$$

$$\boxed{h(n)} = \frac{f(n)}{g(n)} = \frac{\sqrt{n}}{n\log n + n} = \frac{\cancel{\sqrt{n}}}{\cancel{n}(\log n + 1)} = \boxed{\frac{1}{\sqrt{n}(\log n + 1)}}$$

Check option A :- $h(n) = \dfrac{1}{\sqrt{n}\,(\log n+1)}$ is $O(n^{-1/2})$?

$\dfrac{1}{\log n+1} < 1$

$\dfrac{1}{\sqrt{n}(\log n+1)}$

$n^{-1/2} \rightarrow \dfrac{1}{\sqrt{n}}$

h(n) is not $\Omega(n^{-1/2})$

$\downarrow$

Hence not $O(n^{-1/2})$

$\dfrac{1}{\cancel{\sqrt{n}}(\log n+1)}$

$\dfrac{1}{\cancel{\sqrt{n}}}$

Hence A & B both are True

$\dfrac{1}{(\log n+1)} < 1$

Check ($\frac{1}{\sqrt{n}(\log n+1)}$ is $\Omega\left(\frac{1}{\log n}\right)$?

$$\frac{1}{\sqrt{n}\log n+\sqrt{n}} < \frac{1}{\log n}$$

$$h(n) = \Omega\left(\frac{1}{\log n}\right) \quad \times$$

$$\sqrt{n}\log n+\sqrt{n} > \log n$$

[MCQ]

Ans :- A

#Q. Consider the following C-code

void foo (int n)

{

int a = 1;

if (n = = 1)

return; ⟶ exit

~~for (; a ≤ n, a++)~~ for(a=1, a≤n ;a++)

{

printf("GATEWALLAH");

break;

}

}

What is the worst time complexity of above program?

- **A** O(1)
- **B** O(n)
- **C** O (log n)
- **D** $O\sqrt{n}$

[MCQ]

#Q.Consider the following asymptotic functions :

$f1 = 2^n$

$f2 = 1.001^n$

$f3 = e^n$

$f4 = n!$

Which of the following is correct increasing order of above functions?

Ans → D

A f3, f4, f1, f2

B f2, f4, f1, f3

C f3, f2, f1, f4

D f2, f1, f3, f4

Soln :

$f_1 = 2^n \longrightarrow$ Expo

$f_2 = (1.001)^n \longrightarrow$ Expo

$f_3 = e^n \longrightarrow$ Expo

* $f_4 = n! \longrightarrow$ Expo (n^n)

$\hookrightarrow n(n-1) \times (n-2) \ldots 1 \longrightarrow \approx n^n$

$e \longrightarrow \approx 2.71$

$1.001 < 2 < 2.71(e)$

$(1.001)^n < 2^n < e^n < n^n$

$$\boxed{f_2 < f_1 < f_3 < f_4}$$

~~[MCQ]~~ MSQ $\rightarrow$ Multiple can be Correct options

#Q. Consider ~~two function~~ the following functions:

$f_1(n) = 4^{2^n}$

$f_2(n) = n!$

$f_3(n) = 4^{e^n}$

$f_4(n) = n^{n^n}$

Ans: B, C, D

$f_2 < f_1 < f_3 < f_4$

$f_1 = O(f_2)$

$f_1 \leq f_2$ ✗

Which of the following is/are correct?

A. $f_1(n) = O(f_2(n))$ ✗

B. $f_1(n) = O(f_4(n))$ ✓

$f_1 = O(f_4)$

$\downarrow$

$f_1 \leq f_4$

C. $f_1(n) = O(f_3(n))$ ✓

$f_1 \leq f_3$

D. $f_2(n) = O(f_3(n))$ ✓

$f_2 \leq f_3$

Soln :-

$f_1(n) = (4)^{2^n} \longrightarrow$ expo

$f_2(n) = n! \rightarrow (n)^n \longrightarrow$ expo

$f_3(n) = (4)^{e^n} \longrightarrow$ expo

$f_4(n) = (n)^{n^n} \longrightarrow$ expo

$$\boxed{f_2 < f_1 < f_3 < f_4}$$

$(4)^{2^n} < (4)^{e^n}$

$2^n < e^n \qquad e \approx 2.71$

$f_1 < f_3$

$(n)^n < (n)^{n^n}$

$f_2 < f_4$

$$(n)^n \quad < \quad (4)^{e^n}$$

$$\log(n^n) \qquad \log\left((4)^{e^n}\right)$$

$$n * \log n \qquad e^n * \log(4)$$

$$n\log n \quad < \quad e^n$$

Poly

expo

$$(n)^n < (4)^{2^n}$$

$$\log(n^n) \qquad \log\left((4)^{(2)^n}\right)$$

$$n * \log n \qquad 2^n * \log(4)$$

$$n \log n < 2^n$$

Poly

Expo

[MCQ]

Ans:- C

#Q. Consider two function $f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$ then which of the following is true.

$f_1 > f_2$

$f_1 \le f_2$

A $f_1(n) = (O f_2(n))$

B $f_1(n) = \theta(f_2(n))$

$f_1 \approx f_2$

small omega

C $f_1(n) = \omega(f_2(n))$

D None of these

(rate of growth)

$f_1 > f_2$

$2^n \neq \theta(n^2)$

Soln:

$$f_1(n) = n^{2^n} > f_2(n) = n^{n^2}$$

log on both sides

$$\log\left(n^{2^n}\right) \qquad \log\left(n^{n^2}\right)$$

$$2^n * \cancel{\log n} \qquad n^2 * \cancel{\log n}$$

$$2^n > n^2$$

expo   Poly

[MCQ]

$\theta \to O$, $\theta \to \Omega$

$x = \frac{1}{4}(n^4 + 2n^3 + n^2)$

#Q. $f(n) = \Sigma_{i=1}^{n} i^3 = x$, choices for x

I. $\theta(n^4)$ II. $\theta(n^5)$

III. $O(n^5)$ IV. $\Omega(n^3)$

$x \to O(n^4)$, $x \to \Omega(n^4)$ $\Big\}\ \theta(n^4)$

A I, II, III

B II, III, IV

C I, II, III, IV

D I, III, IV

(D)

$x \le n^4 \to$ Tight

$x \le n^5$

$\le n^6$ loose

$x \to O(n^5)$

$x \ge n^5$

$x = \Omega(n^3)$

$x \ge c * n^3$

Soln:

$$x = \sum_{i=1}^{n} i^3 = 1^3 + 2^3 + 3^3 \cdots n^3$$

$$= \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{n^2+n}{2}\right)^2$$

$$(n^2)^2 = n^{2\times 2} = \underline{\underline{n^4}}$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= \frac{1}{4}(n^2+n)^2 = \frac{1}{4}\left[(n^2)^2 + 2 * n^2 * n + n^2\right]$$

$$= \frac{1}{4}\left[n^4 + 2n^3 + n^2\right]$$

Dominating

$$\underline{\underline{\Theta(n^4)}}$$

THANK - YOU

Practice all the
Concepts
taught in
Class.