Computer Science & DA

Linear Algebra

DPP 01 Discussion Notes

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#Q. Consider the following two statements with respect to the matrices $A_{m \times n}$, $B_{n \times m}$, $C_{n \times n}$ and $D_{n \times n}$.

Statement 1: $tr(AB) = tr(BA)$

Statement 2: $tr(CD) = tr(DC)$

Where tr() represents the trace of a matrix. Which one of the following holds?

A Statement 1 is correct and Statement 2 is wrong.

B Statement 1 is wrong and Statement 2 is correct.

C Both Statement 1 and Statement 2 are correct.

D Both Statement 1 and Statement 2 are wrong.

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}_{3 \times 2}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 3 \end{bmatrix}_{2 \times 3} \quad A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & -2 & -1 \\ -1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}_{3 \times 3}$$

$$BA = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}_{2 \times 2}$$

$$Tr(AB) = Tr(BA) = 6$$

#Q.   Calculate the determinant of the following matrix-

$$|A| = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 7 & 3 \\ 4 & 10 & 14 & 6 \\ 3 & 4 & 2 & 7 \end{vmatrix}_{4 \times 4} = \xrightarrow{R_3 \to R_3 - 2R_2} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 7 & 3 \\ 0 & 0 & 0 & 0 \\ 3 & 4 & 2 & 7 \end{vmatrix} = 0$$

A 4

B 5

C 0

D 7

[MCQ]

#Q. The determinant of the matrix

$$A = \begin{bmatrix} x & 4 & y+z \\ y & 4 & z+x \\ z & 4 & x+y \end{bmatrix}$$ is equal to

$$C_1 \rightarrow C_1 + C_3$$

$$\begin{vmatrix} x+y+z & 4 & y+z \\ y+z+x & 4 & z+x \\ z+x+y & 4 & x+y \end{vmatrix} = \begin{matrix} (x+y+z) \times 4 \\ \begin{vmatrix} 1 & 1 & y+z \\ 1 & 1 & z+x \\ 1 & 1 & x+y \end{vmatrix} \end{matrix}$$

A 4x

B x + y + z

C xyz

D 0

$$= 4(x+y+z) \times 0$$
$$= 0$$

#Q. Find the area of triangle in (determinant form) whose vertices are A(0, 0), B(0, −5), and C(8,0).

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 0 & -5 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \frac{1}{2}\left[1\{0-(-40)\}\right]$$

$$= \frac{40}{2} = 20$$

A 20

B 22

C 23

D 24

#Q. Let $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $|2A|$ is equal to.

$|A| = \cos^2\theta - (-\sin^2\theta)$

$= 1$

w.k.that if $A_{n\times n}$ & k is any constant

$$\boxed{|kA| = k^n \cdot |A|}$$

More $A_{2\times2}$ then

$|2A| = 2^2 |A|$

$= 4 \times 1$

A $4\cos 2\theta$

B 1

C 2

D 4

#Q. If A, B, C are non-singular $n \times n$ matrices, then $(ABC)^{-1} = $ _____ .

$$|A| \neq 0$$
$$|B| \neq 0$$
$$|C| \neq 0$$

$$(ABC)^{\theta} = C^{\theta} B^{\theta} A^{\theta}, \quad (ABC)^{T} = C^{T} B^{T} A^{T}, \quad (ABC)^{-1} = C^{-1} B^{-1} A^{-1}$$

A $A^{-1}C^{-1}B^{-1}$

B $C^{-1}B^{-1}A^{-1}$

C $C^{-1}A^{-1}B^{-1}$

D $B^{-1}C^{-1}A^{-1}$

#Q. Let A, B, C, D be n × n matrices, each with non zero determinant and ABCD = I then B =

$\Rightarrow$ A, B, C, D are Non Sing.

| A | $A^{-1}D^{-1}C^{-1}$ | B | CDA |
| C | ABC | D | Does not exist |

$$ABCD = I$$

$$A^{-1} \cdot (ABCD) D^{-1} = A^{-1} \cdot I \cdot D^{-1}$$

$$I \, BC \, I = A^{-1} D^{-1}$$

$$BC = A^{-1} D^{-1}$$

$$(BC) C^{-1} = A^{-1} D^{-1} C^{-1}$$

$$B = A^{-1} D^{-1} C^{-1}$$

$(z^2+x^2+zx) - (y^2+x^2+yx) = z^2-y^2+x(z-y) = (z-y)(z+y+x)$

#Q. The value of the determinant of the matrix $A = \begin{bmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{bmatrix}$ is equal to.

$R_2 - R_1 \ \& \ R_3 - R_1$

$|A| = \begin{vmatrix} 1 & x & x^3 \\ 0 & y-x & y^3-x^3 \\ 0 & z-x & z^3-x^3 \end{vmatrix} = \begin{vmatrix} 1 & x & x^3 \\ 0 & (y-x) & (y-x)(y^2+x^2+yx) \\ 0 & (z-x) & (z-x)(z^2+x^2+zx) \end{vmatrix} = (y-x)(z-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & ① & (y^2+x^2+yx) \\ 0 & 1 & (z^2+x^2+zx) \end{vmatrix}$

A $(x - y)(y - z)(z - x)$

B $(x - y)(y - z)(z - x)(x + y + z)$

C $(x + y + z)$

D $(x - y)(y - z)(z - x)(xy + yz + zx)$

$R_3 - R_2 \ then \ |A| = (y-x)(z-x)\begin{vmatrix} 1 & x & x^3 \\ 0 & 1 & y^2+x^2+yx \\ 0 & 0 & (z-y)(z+y+x) \end{vmatrix} = (y-x)(z-x)(z-y)(z+y+x)$

$= (x-y)(y-z)(z-x)(x+y+z)$

#Q. If A is 3×3 matrix and $|A| = 4$ then $|A^{-1}|$ is equal to-

$$\det A^{-1} = \frac{1}{\det A}$$

$$|A^{-1}| = \frac{1}{|A|}$$

$$= \frac{1}{4}$$

A $\frac{1}{4}$ ✓

B $\frac{1}{16}$

C 4

D 2

#Q.   If $|A| = 0$ where A is defined as the matrix $\begin{bmatrix} 4 & -4 & 0 \\ a & b+4 & c \\ a & b & c+4 \end{bmatrix}$, , then $a+b+c$ is equal to.

$C_1 \to C_1 + C_2 + C_3$ then $|A| = 0$

$|A| = 0$

$\Rightarrow \begin{vmatrix} 0 & -4 & 0 \\ a+b+c+4 & b+4 & c \\ a+b+c+4 & b & c+4 \end{vmatrix} = 0$

A 41

B 116

C 628

D -4

$(a+b+c+4) \begin{vmatrix} 0 & -4 & 0 \\ 1 & b+4 & c \\ 1 & b & c+4 \end{vmatrix} = 0 \Rightarrow a+b+c+4 = 0$

$\text{or } a+b+c = -4$

#Q. If I_3 is the identity matrix of orders 3×3, the value of $(I_3)^{-1}$ is :

$$I_3 = I_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \implies I_3 = \frac{adj\, I}{|I|} = \frac{(cof\, I)^T}{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|I| = 1$$

$$= I_3$$

$$\boxed{(I_n)^{-1} = I_n}$$

A   0

B $3I_3$

C I_3

D   Does not exist.

#Q. If A is any square matrix, then

$$A = \left(\frac{A + A^T}{2}\right) + \left(\frac{A - A^T}{2}\right)$$

$$A = P + Q \begin{cases} P^T = P \text{ ie } P \text{ is Symm} \\ Q^T = Q \text{ ie } Q \text{ is skewsymm.} \end{cases}$$

$$\left(AA^T\right)^T = \left(A^T\right)^T \left(A^T\right) = \left(AA^T\right) \Rightarrow AA^T \text{ is symm Mat}$$

A A + A^T is skew symmetric

B A − A^T is symmetric

C A A^T is symmetric

D AA^T is skew symmetric

#Q. Each diagonal element of a skew symmetric matrix is-

$A^T = -A$

$$A = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix} \cdots$$

ie $a_{ij} = -a_{ji}$

$\forall$ Diag elements $\bar{i} = \hat{g}$

$a_{ii} = -a_{ii}$

A	Zero	B	Positive and equal $2a_{ii} = 0$
C	Negative and equal	D	Any real number

$a_{ii} = 0 \forall i$

#Q. If A is a singular matrix, then adj A is

$$|A| = 0$$

w.k. that $\boxed{A(adj\,A) = |A|.\,I_n}$

$$A.(adj\,A) = \begin{bmatrix} |A| & 0 & 0 & \cdots & 0 \\ 0 & |A| & 0 & \cdots & 0 \\ 0 & 0 & |A| & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & |A| \end{bmatrix}$$

$$|A\,adj\,A| = |A|^n |I_n|$$

$$|A|.\,|adj\,A| = |A|^n.\,1$$

$$\boxed{|adj\,A| = |A|^{n-1}}$$

where $n = $ order of A

A Singular

B Non-singular

C Symmetric

D Non defined

Let $A_{3\times3}$ then

$$|adj\,A| = |A|^{3-1} = |A|^2 = 0$$

$A_{n\times n}$ then $\left|adj\,adj\,adj\cdots adj\,A\right| = |A|^{(n-1)^\gamma}$

γ times

#Q. If $A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$, then B is equal to :

$$\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$-3B = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \Rightarrow B = \frac{1}{3}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

A $\quad \dfrac{1}{3}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

B $\quad \dfrac{1}{3}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

C $\quad \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

D $\quad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

#Q.   If $x + \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then 'X' is equal to

$$X = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -6 & 0 \end{bmatrix}$$

A $\begin{bmatrix} 0 & 1 \\ 0 & 6 \end{bmatrix}$

B $\begin{bmatrix} 0 & -1 \\ 0 & -6 \end{bmatrix}$

C $\begin{bmatrix} -1 & 0 \\ -6 & 0 \end{bmatrix}$

D $\begin{bmatrix} 1 & 0 \\ 6 & 0 \end{bmatrix}$

#Q. If $\begin{bmatrix} x & 1 \\ -1 & -y \end{bmatrix} + \begin{bmatrix} y & 1 \\ 3 & x \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then

$$\begin{bmatrix} (x+y) & 2 \\ 2 & (-y+x) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$x + y = 1$

$x - y = 1 \implies y = 0$

$\overline{2x = 2 \implies x = 1}$

A   x = -1 , y = 0

B   x = 1 , y = 0

C   x = 0 , y = 1

D   x = 1 , y = 1

#Q. Let $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}_{3\times3}$ and $A + B - 4I = 0$, then B is equal to

$$B = 4I - A = 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

A $\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & -2 \\ -3 & -4 & -1 \end{bmatrix}$ ✓

B $\begin{bmatrix} 2 & 3 & 5 \\ 1 & -4 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & -2 \\ -3 & -4 & -1 \end{bmatrix}$

C Both of them

D None of them

$$a_{11} = 21 + 4 + 10 = 35 \quad \& \quad a_{21} = 27 + 8 + 5 = 40$$

#Q. $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ is equal to.

$$A_{2\times3} \; B_{3\times1} + 2 \, C_{2\times1} = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}_{2\times1} + \begin{bmatrix} 8 \\ 10 \end{bmatrix}_{2\times1} = \begin{bmatrix} 35 \\ 40 \end{bmatrix} + \begin{bmatrix} 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 43 \\ 50 \end{bmatrix}$$

A $\begin{bmatrix} 45 \\ 44 \end{bmatrix}$

B $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$

C $\begin{bmatrix} 44 \\ 43 \end{bmatrix}$

D $\begin{bmatrix} 43 \\ 50 \end{bmatrix}$

#Q. If $f(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, then $f(A)$ is equal to -

$A^2 = A \cdot A$

$$f(A) = A^2 + 4A - 5I$$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$$

$= \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$

$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}_{2 \times 2}$

A $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$

B $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

C $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

D $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$ ✓

$A^T = A$

$B^T = -B$

#Q. If A is a symmetric matrix and B is a skew-symmetrix matrix such that $A + B = ?$ $B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to.

$A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ Now $(A+B)^T = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}^T$

ie $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$

①

$A^T + B^T = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$

$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$

A $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$

②

B $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

$2A = \begin{bmatrix} 4 & 8 \\ 8 & -2 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$

C $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

D $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

#Q. If A is involutory matrix and I is unit matrix of same order, then $(I - A)(I + A)$ is.

$$A^2 = I$$

$$(I - A)(I + A) = I^2 - A^2 = I - A^2 = I - I = 0$$

A Zero matrix

B A

C I

D 2A

#Q. If $A = \begin{bmatrix} 3 & -6 \\ a & -2 \end{bmatrix}$ is an idempotent matrix, then which of the following is/are TRUE

$$A^2 = A$$

$$A^2 = A$$

$$A = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -6 \\ a & -2 \end{bmatrix}\begin{bmatrix} 3 & -6 \\ a & -2 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ a & -2 \end{bmatrix}$$

$$|A| = -6 - (-6)$$
$$= 0$$

$$\begin{bmatrix} 9-6a & -6 \\ a & -6a+4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ a & -2 \end{bmatrix}$$

A   a = 4

B   a = 1

C |A| = 0

D |A| = 2

$$9 - 6a = 3 \Rightarrow a = 1$$
$$-6a + 4 = -2 \Rightarrow a = 1$$

[MCQ]

#Q. If $A = \begin{pmatrix} 2 & 4 \\ -1 & k \end{pmatrix}$ is ~~an idempotent matrix,~~ a (Nilpotent) matrix of index 2 then which of the following is/are TRUE for k?

A $\quad$ 2

C $\quad$ 4

B $\quad$ -3

D $\quad$ -2

$$A^2 = 0$$

$$\begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 8+4k \\ -2-k & -4+k^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$k = -2 \qquad k = -2 \quad ; \quad k = \pm 2$$

$$\boxed{k = -2}$$

$$AA^T = I \quad \& \quad BB^T = I$$

#Q. A square matrix A is said to be orthogonal if A 'A = AA '= I_n, A' is transpose of A

If A and B are orthogonal matrices, of the same order, then which one of the following is an orthogonal matrix

$$\text{let us check } (a) \quad (AB)(AB)^T = (AB)(B^T A^T) = A(BB^T)A^T$$
$$= A(I)A^T$$
$$= AA^T$$
$$= I \Rightarrow (AB) \text{ is an O.M.}$$

A AB

B A + B

C A + iB

D (A ÷ B)

$$(b) \quad (A+B)(A+B)^T = (A+B)(A^T+B^T) = I + AB^T + BA^T + I \neq I$$

#Q. Check the nature of the following matrices. $=$ orthogonal Mat

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

#Q. Check the Nature of the following matrices.

$$A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}.$$

$$C_1 \qquad C_2 \qquad C_3$$

$$C_1 \cdot C_2 = 0 \qquad \& \quad |C_1| = |C_2| = |C_3| = 1$$
$$C_2 \cdot C_3 = 0$$
$$C_3 \cdot C_1 = 0 \qquad ie \ A \ is \ an \ O-Mat.$$

M-II

$$A A^T = \begin{bmatrix} \cos\theta & 0 & +\sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ +\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I$$

So A is an O-Mat.

#Q. Check the Nature of the following matrices.

$$A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}.$$

$$A^T = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix}$$

$$A^\theta = \overline{(A^T)} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}$$

$-i^2 = i^2$

$-2i^2 = -2(-1)$

$= 2$

Let us Calculate

$$AA^\theta = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I \quad \text{So } A \text{ is Unitary Mat}$$

#Q. Check the Nature of the following matrices.

$$A = \begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix}$$

$A^T = A$ (Symm Mat)

$$A^\theta = \overline{(A^T)} = \begin{bmatrix} -i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & -i \end{bmatrix}$$

$$AA^\theta = \begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{bmatrix} \begin{bmatrix} -i & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & -i \end{bmatrix}$$

$$= \begin{bmatrix} -i^2 & 0 & 0 \\ 0 & -i^2 & 0 \\ 0 & 0 & -i^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I \implies A \text{ is Unitary Mat}$$

THANK - YOU