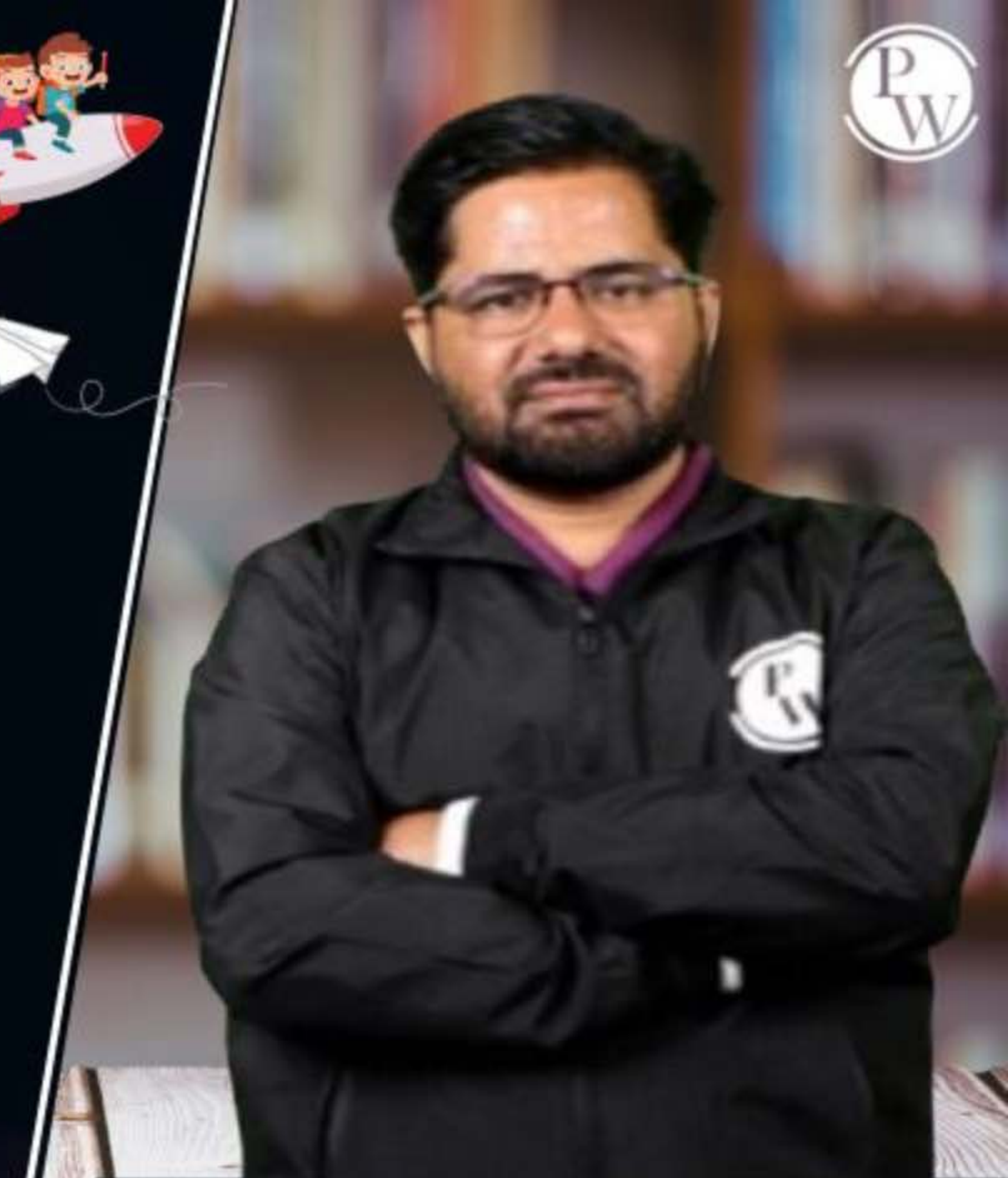
DS & AI
CS & IT

Probability & Statistics

Lecture No. 02



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Recap of previous lecture

Topic

PERMUTATION - COMBINATION

(Part-1)

Topics to be Covered

Topic

“PERMUTATION & COMBINATION”

(Part-2)

Thumb Rule of this Chapter → Try to avoid making Question by using following words;

" If, what if, AGAR, YADI, TOH, ..."

OR

Don't Try to develop Question (by your $\boxed{\text{little mind}}$ until you have a complete understanding of the chapter) & try to solve the Quest.

COUNTING PRINCIPLE

Fundamental Principle of Addition → If we have to perform only one of the job at a time out of n jobs then use this principle.

RECAP

Keywords : "Either or / only one / Anyone"

Fundamental Principle of Multiplication → If we have to perform all the jobs at a time out of n jobs then use this principle.

Keywords : "AND / BOTH / ALL / Every"

Combination → (when Counting is based on Selection only then use this Rule)

$$\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}} \quad \& \quad \boxed{{}^nC_r = {}^nC_{n-r}}, \quad {}^nC_0 = {}^nC_n = 1$$

$${}^nC_1 = {}^nC_{n-1} = n$$

eg $${}^{11}C_3 = \frac{11\times10\times9}{3\times2\times1}, \quad {}^{22}C_4 = \frac{22\times21\times20\times19}{4\times3\times2\times1}$$

$${}^{22}C_{18} = ? = {}^{22}C_{22-18} = {}^{22}C_4 =$$

$$\boxed{{}^nC_3 = \frac{n(n-1)(n-2)}{3\times2\times1}}$$

RECAP

$$\boxed{{}^nC_2 = \frac{n(n-1)}{2}},$$

eg $${}^{11}C_2 = \frac{11\times10}{2}$$

PLAYING CARDS →

RECAP

FACECARDS: K, Q, J (12)

Honour Cards: K, Q, J, A (16)

- Cards (52)
  - Red suits (26)
    - Diamond (13)
    - Heart (13)
  - Black suits (26)
    - Spade
    - Club (13)

Permutation :→ (Selection & Arrangement both) ⟶

If in a Question, Counting is Based on, Selection $\underbrace{\text{as well as}}$ on Arrangement also then use this Rule.

$$ {}^nP_r = \frac{n!}{(n-r)!} = \boxed{{}^nC_r \times r!} $$

$${}^{11}P_3 = 11\times 10\times 9, \quad {}^{11}C_3 = \frac{11\times10\times9}{3\times2\times1}$$

$${}^{22}P_4 = 22\times21\times20\times19$$

$${}^{22}C_4 = \frac{22\times21\times20\times19}{4\times3\times2\times1}$$

$y \quad {}^{22}C_{18} = ? = {}^{22}C_4 = ✓$

$${}^{22}P_{18} = ? = 22\times21\times20\times\ldots\times3\times2\times1$$

$${}^nP_0 = 1$$

$${}^nP_1 = n$$

$${}^nP_2 = n(n-1)$$

$$\vdots$$

$${}^nP_n = n!$$

RECAP

Ex of Combination :

- formation of team,
- " of Committee
- No of Handshakes.
- No of St. lines & Δ's
- No. of $||^{gms}$.

- - - - - -

Ex of Perm :- If in a Question there is a feeling of interchanging things then use ${}^nP_r$.

- formation of words.
- " of Numbers.
- Seating arrangement.
- Formations of Photographs
- " of signals.

RECAP

GAZAB KA Conclusion →

① if $n > r$ & RNA, then Multi Rule ≅ Perm Rule

② if $n = r$ & RNA, Then Multi Rule ≅ Perm Rule ≅ Factorial Rule

③ if RA, then only use Multi Rule

ie the Concept of ${}^nC_r$, ${}^nP_r$ & $r!$ is applicable only when RNA

RECAP

F.Q 1: out of 6 Gents & 4 ladies, a committee of 5 persons is to be formed. then find the number of committees if

(1) there is No Restriction = ? $= {}^{10}C_5 = 252$ committees (Max Ans).

(2) Atleast 2L are there = ? we can make committee either by selecting

$= (2L \& 3G)$ or $(3L \& 2G)$ or $(4L \& 1G)$ or $(5L \cancel{\&} 0G)$

$= ({}^4C_2 \times {}^6C_3) + ({}^4C_3 \times {}^6C_2) + ({}^4C_4 \times {}^6C_1) = 186$ committees

WRONG APP:- At least 2L are there $= {}^4C_2 \times {}^8C_3 = 336 > 252$

(3) there are exactly 2L = ? $= {}^4C_2 \times {}^6C_3 = 6 \times 20 = 120$

(3) At Most 2L are there = ? = (0L & 5G) or (1L & 4G) or (2L & 3G)

$$= \left({}^4C_0 \times {}^6C_5\right) + \left({}^4C_1 \times {}^6C_4\right) + \left({}^4C_2 \times {}^6C_3\right) = 186$$

(4) Gents are in Mazority = ? = same as above = 186

(5) Ladies are in Mazority = ? = (4L & 1G) or (3L & 2G)

$$= \left({}^4C_4 \times {}^6C_1\right) + \left({}^4C_3 \times {}^6C_2\right) = 66$$

(6) there are exactly 3L in that Committee = ? = (3L & 2G) = ${}^4C_3 \times {}^6C_2 = 60$

(7) At least one L is there = ?

(M-I) $= (1L \,\&\, 4G)$ or $(2L \,\&\, 3G)$ or $(3L \,\&\, 2G)$ or $(4L \,\&\, 1G)$

$$= \left({}^4C_1 \times {}^6C_4\right) + \left({}^4C_2 \times {}^6C_3\right) + \left({}^4C_3 \times {}^6C_2\right) + \left({}^4C_4 \times {}^6C_1\right) = 246$$

(M-II) At least one 'L' $= \boxed{\text{Total} - \text{None}}$

$= \text{Total Committees} - (\text{No Lady})$

$$= {}^{10}C_5 - (\text{All } G) = {}^{10}C_5 - {}^6C_5 = 252 - 6 = 246$$

PODCAST :→ Various possible cases are as follows;

$$\underbrace{(0L\,\&\,5G)\text{ or }(1L\,\&\,4G)\text{ or }(2L\,\&\,3G)}_{\text{At Most 2L}}\text{ or }(3L\,\&\,2G)\text{ or }(4L\,\&\,1G) = \text{Total Cases.}$$

Exactly 3L

$$({}^4C_0 \times {}^6C_5) + ({}^4C_1 \times {}^6C_4) + ({}^4C_2 \times {}^6C_3) + ({}^4C_3 \times {}^6C_2) + ({}^4C_4 \times {}^6C_1) = {}^{10}C_5$$

$$(1\times 6) + \underbrace{(4\times 15) + \underbrace{(6\times 20) + (4\times 15) + (1\times 6)}_{\text{At least 2L}}}_{\text{At least one Lady}} = 252$$

QUESTIONS BASED on PERMUTATIONS

Q. How many 4 letter words (w or w/o meaning) can be formed using the letters of the word 'EQUATION' = ?

(RNA)/Toys

$$= \underset{P_1}{\underline{8}} \times \underset{P_2}{\underline{7}} \times \underset{P_3}{\underline{6}} \times \underset{P_4}{\underline{5}} = {}^{8}C_4 \times 4! = {}^{8}P_4$$

Q. How many 5 letter words can be formed using the letters of the word 'LOGARITHMS'

Total 5 letter words (RNA) $= \underset{P_1}{\underline{10}} \times \underset{P_2}{\underline{9}} \times \underset{P_3}{\underline{8}} \times \underset{P_4}{\underline{7}} \times \underset{P_5}{\underline{6}} = {}^{10}C_5 \times 5! = {}^{10}P_5$

Q. How many four letters words can be formed using the letters of the word "FAILURE" if (1) there is no Restriction = ? $= 7\times6\times5\times4 = {}^7C_4\times4!\times{}^7P_4$

$\underbrace{\text{FAILURE}}_{\text{TOYS}}$ (RNA)

(2) f is included in each word = ? $= {}^1C_1\times{}^6C_3\times4!$

(3) f is not included in any word = ? $= \underline{6}\times\underline{5}\times\underline{4}\times\underline{3} = {}^6C_4\times4! = {}^6P_4$

Q. out of 7 Consonants and 5 Vowels, how many 5 letter words can be formed including 3 Consonants & 2 Vowels = ? $= {}^7C_3\times{}^5C_2\times5!$

Q. There are 5 flags of different colours then how many different signals can be formed using

(1) 3 flags at a time = ?

(RNA)

$$= \underset{P_1}{\underline{5}} \times \underset{P_2}{\underline{4}} \times \underset{P_3}{\underline{3}} = \underbrace{{}^5C_3}_{\text{selection of 3 flags}} \times \underbrace{3!}_{\text{Arrangement of 3 selected flags.}} = {}^5P_3$$

$\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$ $\uparrow$

f_1 f_2 f_3 f_4 f_5

Q. there are 5 flags of different colours then how many different signals can be formed using

(2) Any Number of flags at a time? (RNA)

we can make signal either by taking

$= 1f$ or $2f$ or $3f$ or $4f$ or $5f$

M-I $= (5 \text{ ways}) + (5\times4)\text{ ways} + (5\times4\times3)\text{ ways} + (5\times4\times3\times2)\text{ ways} + (5\times4\times3\times2\times1)\text{ ways}$

M-II $= ({}^5C_1\times1!) + ({}^5C_2\times2!) + ({}^5C_3\times3!) + ({}^5C_4\times4!) + ({}^5C_5\times5!)$

M-III $= {}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5 = 325$ ways

(3) using at least 3 f at a time = ? $= {}^5P_3 + {}^5P_4 + {}^5P_5 = 60 + 120 + 120$

$= 300$

(4) " at Most 2 flags at a time = ? $= {}^5P_0 + {}^5P_1 + {}^5P_2$

$=$ (Not possible) $+ 5 + 20 = 25$

The number of ways in which ten candidates A_1, A_2, ... A_{10} can be ranked so that A_1 is always above A_2, is

(a) $2 \times 9!$ (b) $9!$

(c) $2 \times 10!$ (d) $\frac{10!}{2}$

Sol. (d)

$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{10} \end{pmatrix}, \begin{pmatrix} A_1 \\ A_3 \\ A_2 \\ \vdots \\ A_{10} \end{pmatrix}, \begin{pmatrix} A_1 \\ A_3 \\ A_4 \\ \vdots \\ A_2 \end{pmatrix}, \ldots \begin{pmatrix} A_3 \\ A_4 \\ A_5 \\ \vdots \\ A_1 \\ A_2 \end{pmatrix}$

Total Cases $= 10!$ (R∩A)

Sometimes A_1 has good Rank as A_2 & " A_2 " " " A_1

∴ Req cases in which A_1 is above A_2 $= \frac{10!}{2}$

M-I fav cases (in which A_1 has Better Rank as compare to A_2) $\rightarrow$

Case I: A_1 _ _ _ _ _ _ _ _ _ = 9 ways × 8!

OR

Case II: _ A_1 _ _ _ _ _ _ _ _ = 8 ways × 8!

OR

Case III: _ _ A_1 _ _ _ _ _ _ _ = 7 ways × 8!

OR

⋮

_ _ _ _ _ _ _ _ A_1 _ = 2 ways × 8!

OR

Case IX: _ _ _ _ _ _ _ _ _ A_1 _ = 1 way × 8!

So Total Fav. Cases $= (9+8+7+6+5+4+3+2+1) \times 8!$

$$= \frac{9 \times (9+1)}{2} \times 8! = \frac{10 \times 9 \times 8!}{2} = \frac{10!}{2} \text{ Ans}$$

Some Useful Information (Based on Experience) → POSTER

1. Always together / Not separated → Assume them as one unit with in Bracket.
2. All Never together / All do not come together → Total − Always together.
3. No two Girls are together → First arrange Boys.
4. Alternately (Linear Case) < Two Cases will arise.
5. Alternately (Circular Case) → only one Case will arise.
6. Particular / fix → No Need to select & No Need to arrange
7. At least one = Total − None.
8. At least = Go upto last Point (using Common Sense)
9. At Most = Include None also (if Possible)

EQ 2:- (4) Boys & (4) Girls are to be seated in a Row in which there are exactly 2 sisters & 1 Brother. Then find the number of seating arrangements if

(1) there is No Restriction = ? $= 8!$ (Max Arr)
(RNA)

(2) All Boys are together & All Girls are together = ?

$(B_1 B_2 B_3 B_4), (G_1 G_2 G_3 G_4) = 2! \times 4! \times 4!$

1st 2nd (P) (B) (G)

M-II

Case I: $\underbrace{B_1 B_2 B_3 B_4}\ \underbrace{G_1 G_2 G_3 G_4} = 4! \times 4!$

or

Case II: $\underbrace{G_1 G_2 G_3 G_4}\ \underbrace{B_1 B_2 B_3 B_4} = 4! \times 4!$

$\Rightarrow$ Arr $= 2 \times 4! \times 4!$

③ All G are together = ? $= 5! \times 4!$

$$\underset{1^{st}}{\underline{B_1}},\ \underset{2^{nd}}{\underline{B_2}},\ \underset{3^{rd}}{\underline{(G_1G_2G_3G_4)}},\ \underset{4^{th}}{\underline{B_3}},\ \underset{5^{th}}{\underline{B_4}}$$

④ All Girls are Never together = ? = Total − 'G' always together

$= 8! - (5! \times 4!)$ = 37440

⑤ No two Girls are together = ? (First arrange Boys) $= \underset{(B)}{4!} \times \underset{(G)}{{}^5P_4}$ = 2880

$$\_\ B_1\ \_\ B_2\ \_\ B_3\ \_\ B_4\ \_$$

$= 4! \times ({}^5C_4 \times 4!)$

Deep Analysis: (No two Girls are together): Various cases are as follows;

Case I: $\left(\underset{1^{st}}{B_1}\, G_1\, \underset{3^{rd}}{B_2}\, G_2\, \underset{5^{th}}{B_3}\, G_3\, \underset{7^{th}}{B_4}\, G_4\right) = \underset{(B)}{4!} \times \underset{(G)}{4!} = 576$

OR

Case II: $\left(\underset{1^{st}}{G_1}\, B_1\, \underset{3^{rd}}{G_2}\, B_2\, \underset{5^{th}}{G_3}\, B_3\, \underset{7^{th}}{G_4}\, B_4\right) = \underset{(B)}{4!} \times \underset{(G)}{4!} = 576$

OR

Case III: $\left(G_1\, \underset{2}{B_1}\, \underset{3}{B_2}\, G_2\, \underset{5}{B_3}\, G_3\, \underset{7}{B_4}\, G_4\right) = 4! \times 4! = 576$

OR

Case IV: $\left(G_1\, B_1\, G_2\, B_2\, B_3\, G_3\, B_4\, G_4\right) = 4! \times 4! = 576$

OR

Case V: $\left(G_1\, B_1\, G_2\, B_2\, G_3\, B_3\, B_4\, G_4\right) = 4! \times 4! = 576$

$\left(G_1\, B_1\, G_2\, B_2\, B_3\, B_4\, (G_3\, G_4)\right)$

Not Possible.

Total cases $= 5 \times 4! \times 4! = 2880$ cases

⑥ Boys and Girls are seated alternately = ?

$B_1 G_1 B_2 G_2 B_3 G_3 B_4 G_4 \longrightarrow 4! \times 4!$

1^{st} 3^{rd} 5^{th} 7^{th}

OR

$G_1 B_1 G_2 B_2 G_3 B_3 G_4 B_4 \longrightarrow 4! \times 4!$

2^{nd} 4^{th} 6^{th} 8^{th}

So Ans $= 2 \times 4! \times 4! = 1152$

Part ⑥ is a particular case of Part ⑤

& " ⑤ " " " of " ④

⑦ Two sisters are not separated = ?

$\underbrace{\text{are not separated}}_{\text{always together}}$

$= 7! \times 2!$

(P) (S)

$B\ B\ G\ (S_1 S_2)\ B\ G\ B$

1^{st} 2 3 4^{th} 5 6 7^{th}

(8) Two sisters do not come together = ? = Total − (Sisters always together)

or (Both the sisters are never together) $= 8! - (7! \times 2!)$

(9) There is exactly one Boy b/n two sisters = ?

$$\underset{1^{st}}{\underline{B}}\ \underset{2^{nd}}{\underline{G}}\ \underset{3^{rd}}{\underline{(S_1\,B\,S_2)}}\ \underset{4^{th}}{\underline{G}}\ \underset{5^{th}}{\underline{B}}\ \underset{6^{th}}{\underline{B}} = ({}^{4}C_1 \times 1) \times 6! \times 2!$$

(10) There is exactly one person b/n two sisters = ?

$$\underset{1^{st}}{\underline{P}}\ \underset{2^{nd}}{\underline{(S_1\,P\,S_2)}}\ \underset{3}{\underline{P}}\ \underset{4}{\underline{P}}\ \underset{5}{\underline{P}}\ \underset{6^{th}}{\underline{P}} = ({}^{6}C_1 \times 1) \times 6! \times 2!$$

(11) Two sisters are always separated by one particular persons

$$\underset{1^{st}}{\underline{P}}\ \underset{2}{\underline{P}}\ \underset{3^{rd}}{\underline{(S_1\,P\,S_2)}}\ \underset{4}{\underline{P}}\ \underset{5}{\underline{P}}\ \underset{6^{th}}{\underline{P}} = ({}^1C_1 \times 1) \times 6! \times 2!$$

(12) Sisters always want to be seated at the adjacent sides of their BRO

$$\underset{1^{st}}{\underline{B}}\ \underset{2}{\underline{B}}\ \underset{3}{\underline{G}}\ \underset{4}{\underline{G}}\ \underset{5^{th}}{\underline{(S_1\,B\,S_2)}}\ \underset{6^{th}}{\underline{B}} = ({}^1C_1 \times 1) \times 6! \times 2!$$

(13) Elder & younger sister wants to be seated at 1^{st} & last postion Resp.

$$\underset{1^{st}}{S_E}\ (\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{\text{6 Persons}})\ \underset{8^{th}}{S_Y} = 1 \times 6! \times 1$$

(14) Elder & younger sister wants to be seated at 1^{st} & last postion.

$$S_1\ (\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{\text{6 Persons}})\ S_2 = 6! \times 2!$$

(15) (Tough) Two sisters wants to be seated at extreme Positions but together.

$$(S_1 S_2)\ \_\ \_\ \_\ \_\ \_\ \_ = 6! \times 2!$$

OR

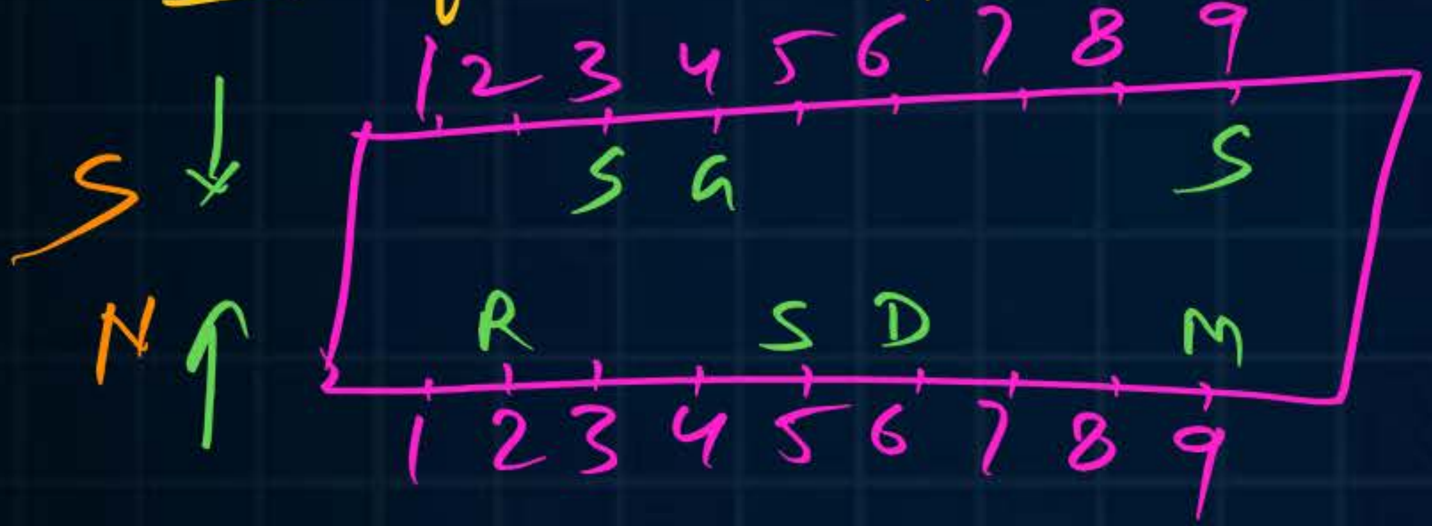
$$\_\ \_\ \_\ \_\ \_\ \_\ (S_1 S_2) = 6! \times 2!$$

So Ans $= 2 \times 6! \times 2!$

18 guests have to be seated, half on each side of a long table. Four particular guests desired to sit on one particular side and three others on the other side, then how many seating arrangements can be made?

(a) $^{18}C_4 . ^{14}C_3 . 9! . 9!$ (b) $^{2}C_1 . ^{9}P_4 . ^{9}P_3 . 11!$ (c) $^{9}P_4 . ^{9}P_3 . 11!$ (d) $^{2}C_1 . \frac{9!}{4!} . \frac{9!}{3!}$

Note: if we have No Restriction then Ans $= 18!$ (Max Ans).



Total Seating Arrangements (RNA)

$$= \left({}^{9}C_4 \times 4!\right) \times \left({}^{9}C_3 \times 3!\right) \times 11!$$

$$= {}^{9}P_4 \times {}^{9}P_3 \times 11!$$

(ii) If side was not particular then Ans $= ? = \left({}^{2}C_1 \times {}^{9}P_4\right) \times \left({}^{1}C_1 \times {}^{9}P_3\right) \times 11!$

There are n persons sitting in a row. Two of them are selected at random. Then how many selections are possible if two selected persons are not together?

(a) ${}^{n}C_2$ (b) ${}^{n-1}C_1$

(c) ${}^{n}C_2 - {}^{n-1}C_1$ (d) ${}^{n-1}C_2$

Sol. (c)

(ii) P(this Quest)

$= \frac{\text{fav}}{\text{Total}} = \frac{{}^{n}C_2 - {}^{n-1}C_1}{{}^{n}C_2}$

$= 1 - \frac{n-1}{\frac{n(n-1)}{2}} = 1 - \frac{2}{n}$

Total ways of selecting 2 persons at Random $= {}^{n}C_2$ ways.

No. of ways in which, two selected persons were together $= ? = {}^{n-1}C_1$

$P_1, P_2, P_3, P_4, P_5 - - - -, P_{n-2}, P_{n-1}, P_n$

i.e we have $(n-1)$ groups

So Two selected persons are not together

$= \text{Total} - \text{Always together} = {}^{n}C_2 - {}^{n-1}C_1$

The number of words of four letters containing equal number of vowels and consonants (Repetition allowed)

(a) 60×210 HW (b) 210×243

(c) 210×315 (d) 630

1. (c)

$V = 5$ (a, e, i, o, u)

$C = 21$

Total words = (All diff) or (Vowels alike & Cons diff)

M-I or (Vowels diff & Cons Alike) or (Vow alike & Cons alike)

= (?) + (?) + (?) + (?) = (C)

M-II Total words (RA) = ?. (use Multi Rule)

Thank you

Keep Hustling!