DS & AI
CS & IT

Probability & Statistics

Lecture No. 08

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Recap of previous lecture

Topic

BASICS of PROBABILITY (Part-3)

Topics to be Covered

Topic

PROBABILITY (Part-4)

→ Practice Q. of BAYE'S Theorem

→ Concept of with & W/o Replacement

→ Concept of TREE Diagram.

Thumb Rule of this chapter → Try to avoid making Question by using following words;

" If , what if , AGAR YADI , TON , ... "

OR

Don't Try to develop Question by your little mind until you have a complete understanding of the chapter & try to solve the Quest.
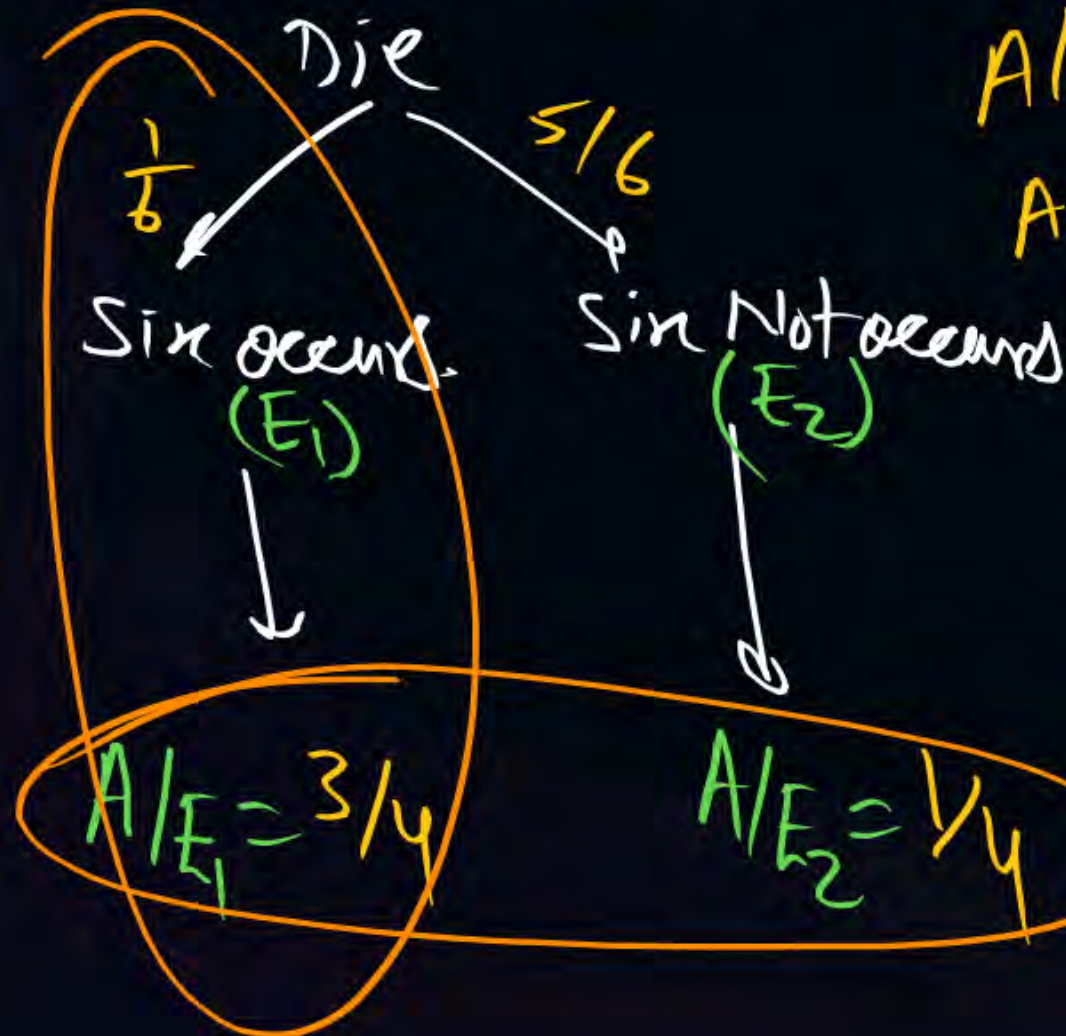
Q. A Person is known to speak Truth 3 out of 4 times. He throw a die & [Reports that it is six] then (Find) the prob that it is Actually six?

REXp.

Condition

$= 5/6$.

(a) 3/8   A = { Man Reports that it is six }, $P(six\ occurs) = \frac{1}{6}$ & $P(six\ Not\ occurs)$

(b) 1/3   (M.I)

(c) 1/8

(d) 2/3

Die

$\frac{1}{6}$ 5/6

Six occurs.   Six Not occurs
(E₁)   (E₂)

A: $A/E_1 = 3/4$ $A/E_2 = 1/4$

$A/E_1 = $ Man telling truth $= 3/4$
$A/E_2 = $ " " lie $= 1/4$

$P(A) = \frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4} = 8/24$

P(Actually six)
$= P(E_1/A) = \dfrac{\frac{1}{6} \times \frac{3}{4}}{8/24} = \dfrac{3}{8}$.

$\boxed{M-II}$ $A = \{$ Man Reports that it is Six $\}$, $P(E_1) = \dfrac{3}{4}$, $P(E_2) = \dfrac{1}{4}$.

DIE

$\dfrac{3}{4}$ $\dfrac{1}{4}$.

Telling Truth   Telling Lie

E_1 (E_2)

$A/E_1 = $ Six occurs $= \dfrac{1}{6}$

$A/E_2 = $ Six Not occurs $= \dfrac{5}{6}$

$A:$ $A/E_1 = \dfrac{1}{6}$ $A/E_2 = \dfrac{5}{6}$

$P(A) = \dfrac{3}{4} \times \dfrac{1}{6} + \dfrac{1}{4} \times \dfrac{5}{6} = 8/24$

$P(\text{Actually Six})$

$= P(E_1/A) = \dfrac{\dfrac{3}{4} \times \dfrac{1}{6}}{8/24} = \boxed{\dfrac{3}{8}}$

Q. In a town there are (equal) number of Man & Woman in which 80% M & 50% W are (employed). (A person is selected at Random) then (find) the prob that person is an unemployed person

(a) 65%

(b) 50%

(c) 35%

(d) None

$$A$$

REM

Man ⟶ 0.5   Woman ⟶ 0.5
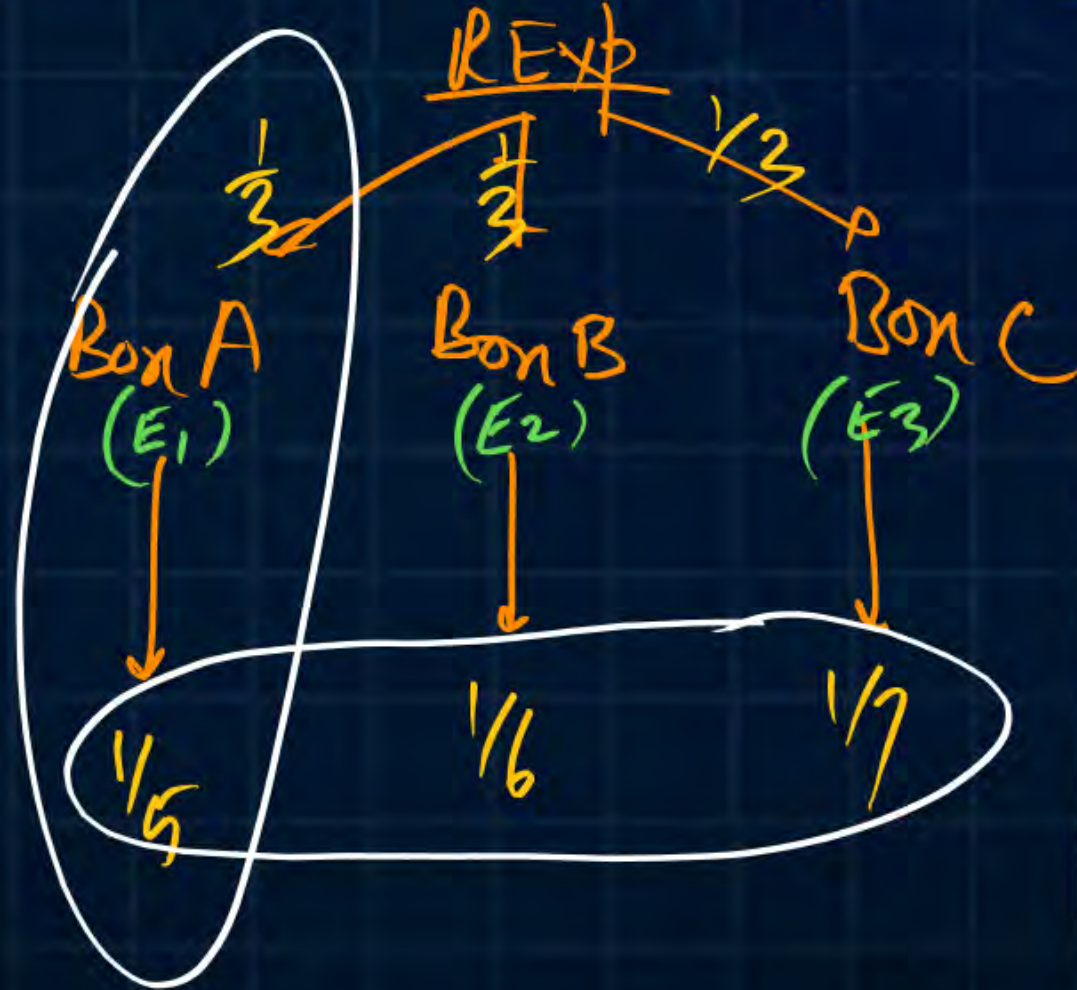
$$A : \quad 0.2 \qquad 0.5$$

$$P(A) = (0.5 \times 0.2) + (0.5 \times 0.5)$$

$$= 0.35$$

The chances of defective screws in three boxes A, B, C are $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$ respectively. A box is selected at random and screw drawn from it at random is found to be defective. Then the probability that it came from box A is

$= M$

(a) 0.0169

(b) 0.039

(c) 0.169

(d) 0.39

$M = \{$ Screw drawn is defective $\}$

PREXP

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$

Box A   Box B   Box C
(E_1) (E_2) (E_3)

Def. Screw
Mi

$\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{7}$

$P(m) = \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{7} =$

$= P(\text{condition}) = 0.169$

$P(E_1/m) = \dfrac{\frac{1}{3} \times \frac{1}{5}}{P(m)}$

$= 0.39$ Ans

Concept of with or w/o Replacement →

eg: ③ Cards are drawn from a pack of 52 Cards then find the number of ways if, Cards are drawn

① Simultaneously $= {}^{52}C_3 = \dfrac{52 \times 51 \times 50}{3 \times 2 \times 1}$

② one by one (with) Replacement $= {}^{52}C_1 \times {}^{52}C_1 \times {}^{52}C_1 = 52 \times 52 \times 52$

③ one by one w/o Replacement $= {}^{52}C_1 \times {}^{51}C_1 \times {}^{50}C_1 = 52 \times 51 \times 50$

Q:- From a pack of Regular Playing Cards, two Cards are drawn then find the Prob that (both) will be kings if 1st Card is not Replaced?

Sol: we are drawing Cards one by one w/o Replacement.

$$\text{Req Prob} = P(K \cap K) = \frac{^4C_1}{^{52}C_1} \times \frac{^3C_1}{^{51}C_1}$$

$$= \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

EGO is the Most FUTILE HUMAN EMOTION

(ii) Also find the ans if Cards are drawn one by one **with** Replacement?

Sol: $\text{Req Prob} = P(K \cap K)$

$$= \frac{^4C_1}{^{52}C_1} \times \frac{^4C_1}{^{52}C_1}$$

$$= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

Qs A Box Contains 3R & 4B Marbles and 3 Marbles are drawn one by one (w/o) Replacement then Find the prob of drawing (1R) & 2 B Marbles?
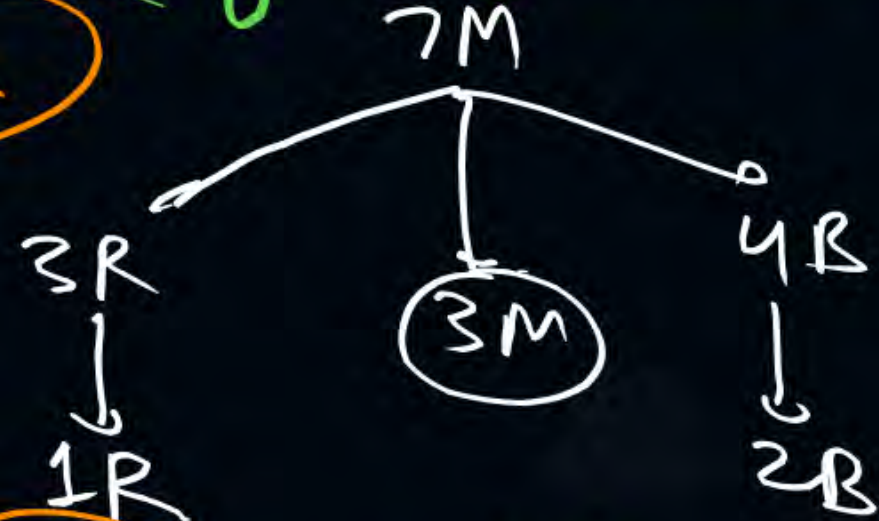
M-I (By Making Various Cases) →

$$\text{Req prob} = P\left[(RBB) \text{ or } (BRB) \text{ or } (BBR)\right]$$

$$= \left(\frac{3}{7} \cdot \frac{4}{6} \cdot \frac{3}{5}\right) + \left(\frac{4}{7} \cdot \frac{3}{6} \cdot \frac{3}{5}\right) + \left(\frac{4}{7} \cdot \frac{3}{6} \cdot \frac{3}{5}\right)$$

$$= \left(\frac{3}{7} \times \frac{4}{6} \times \frac{3}{5}\right) \times 3$$

M-II (using Hypergeometric Dist) →

TRICK

7M

3R — 3M — 4B

1R — 2B

$$\text{Req prob} = \frac{f}{t} = \frac{^3C_1 \times ^4C_2}{^7C_3} = \frac{3 \times \frac{4 \times 3}{2}}{\frac{7 \times 6 \times 5}{3 \times 2 \times 1}} = \frac{3 \times 4 \times 3}{7 \times 6 \times 5} \times 3$$

$$= \left(\frac{3}{7} \times \frac{4}{6} \times \frac{3}{5}\right) \times 3$$

$(3R \& 4B) \longrightarrow (1R \& 2B)$

(ii) Also find the Sol if Marbles are drawn one by one (with) Replacement.

Ⓜ① Req Prob = P[RBB or |BRB| or BBR]

$$= \left(\frac{3}{7} \times \frac{4}{7} \times \frac{4}{7}\right) + \left(\frac{4}{7} \times \frac{3}{7} \times \frac{4}{7}\right) + \left(\frac{4}{7} \times \frac{4}{7} \times \frac{3}{7}\right)$$

$$= \left(\frac{3}{7} \times \frac{4}{7} \times \frac{4}{7}\right) \times 3$$

Note: Find the Prob that these three Marbles are alternately in Colour.

Req Prob = $P(BRB) = \frac{4}{7} \times \frac{3}{7} \times \frac{4}{7}$

Ⓜ-Ⅱ using Binomial Distribution
(TRAILOR)

$7M \longrightarrow 3m$

$p = P(R) = \frac{3}{7}, \quad q = P(B) = \frac{4}{7}$

$n = 3M, \quad X = \{No. \text{ of Red Marbles}\}$

$$P(X = r \text{ success}) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

$$P(X = 1 \, RM) = {}^{3}C_{1} \left(\frac{3}{7}\right)^{1} \left(\frac{4}{7}\right)^{3-1}$$

$$= 3 \times \frac{3}{7} \times \frac{4}{7} \times \frac{4}{7}$$

Doubt By Sonu Sinran → (GOOD Question)

Q. In a Box there are 3R & 4B Marbles. Marbles are drawn one by one w/o Replacement & put in a Row then find the prob that they are alternately in Colours ?

Sol: Here we have to draw all 7 Marbles.

So Prob of drawing all 7 Marbles = 1 (Sure Event)

Now Total Cases of putting them in a Row = $\dfrac{7!}{3! \, 4!}$ = 35 Cases.

Fav Cases = ?   So Fav Cases = 1 Case.

B R B R B R B → ✓

or

R B R B R B B → ✗

Hence Req Prob = $\dfrac{Fav}{Total}$ = $1 \times \dfrac{1}{35} = \dfrac{1}{35}$

Q8 there are ⑩ Markers on a Table in which 6 are Defective & 4 Non Defective. if 3 Markers are drawn one by one w/o Replacements then find the prob that there will be exactly one Defective?
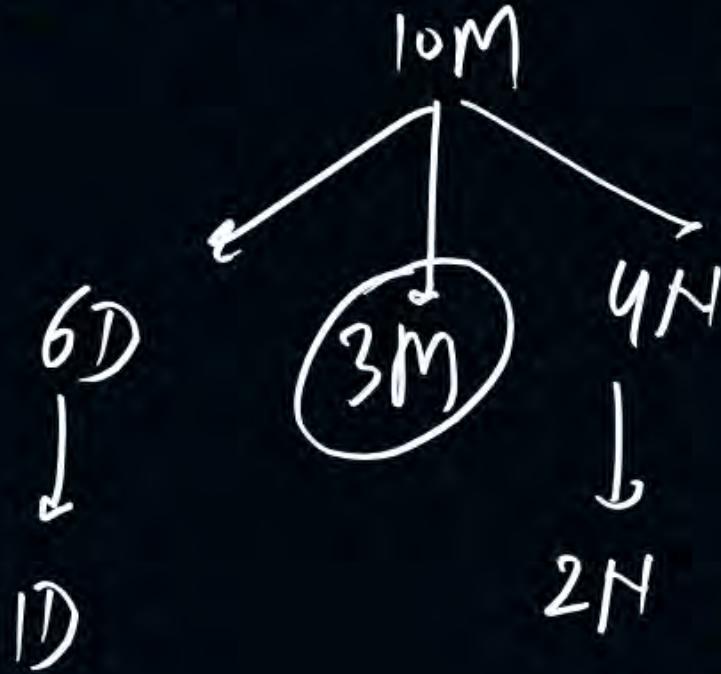
M-I (By Making cases) →

$$Req\ Prob = P[DNN\ or\ NDN\ or\ NND]$$

$$= \left(\frac{6}{10}\cdot\frac{4}{9}\cdot\frac{3}{8}\right) + \left(\frac{4}{10}\cdot\frac{6}{9}\cdot\frac{3}{8}\right) + \left(\frac{4}{10}\cdot\frac{3}{9}\cdot\frac{6}{8}\right)$$

$$= \left(\frac{6}{10}\times\frac{4}{9}\times\frac{3}{8}\right)\times 3$$

M-II (using Hypergeometric Dist) →

10M → 6D, 3M, 4N

6D
↓
1D

4N
↓
2N

$$Req\ Prob = \frac{f}{T} = \frac{{}^6C_1 \times {}^4C_2}{{}^{10}C_3}$$

$$= \frac{6\times\frac{4\times 3}{2}}{\frac{10\times 9\times 8}{3\times 2\times 1}}$$

$$= \left(\frac{6\times 4\times 3}{10\times 9\times 8}\right)\times 3$$

$$6D \ \& \ 4N. \longrightarrow (1D \& 2N)$$

(ii) Also find the Ans if Markers are drawn one by one (with) Replacement

(M-I) By Making Cases :→

Req Prob $= P\left[DNN \ or \ NDN \ or \ NND\right]$

$$= \left(\frac{6}{10} \cdot \frac{4}{10} \cdot \frac{4}{10}\right) + \left(\frac{4}{10} \cdot \frac{6}{10} \cdot \frac{4}{10}\right) + \left(\frac{4}{10} \cdot \frac{4}{10} \cdot \frac{6}{10}\right)$$

$$= \left(\frac{6}{10} \times \frac{4}{10} \times \frac{4}{10}\right) \times 3$$

(M-II) By BINOMIAL DIST.← (TRAILOR)

$x = \{$ Number of Def. Markers $\} \rightarrow$ Success

$n = 3, \ p = P(Def) = \frac{6}{10}, \ q = P(Non \ Def) = \frac{4}{10}$

$$P(x = r \ Success) = {}^{n}C_{r} \ p^{r} \ q^{n-r}$$

$$P(x = 1 \ Def) = {}^{3}C_{1}\left(\frac{6}{10}\right)^{1}\left(\frac{4}{10}\right)^{3-1} = 3 \times \frac{6}{10} \times \left(\frac{4}{10}\right)^{2}$$
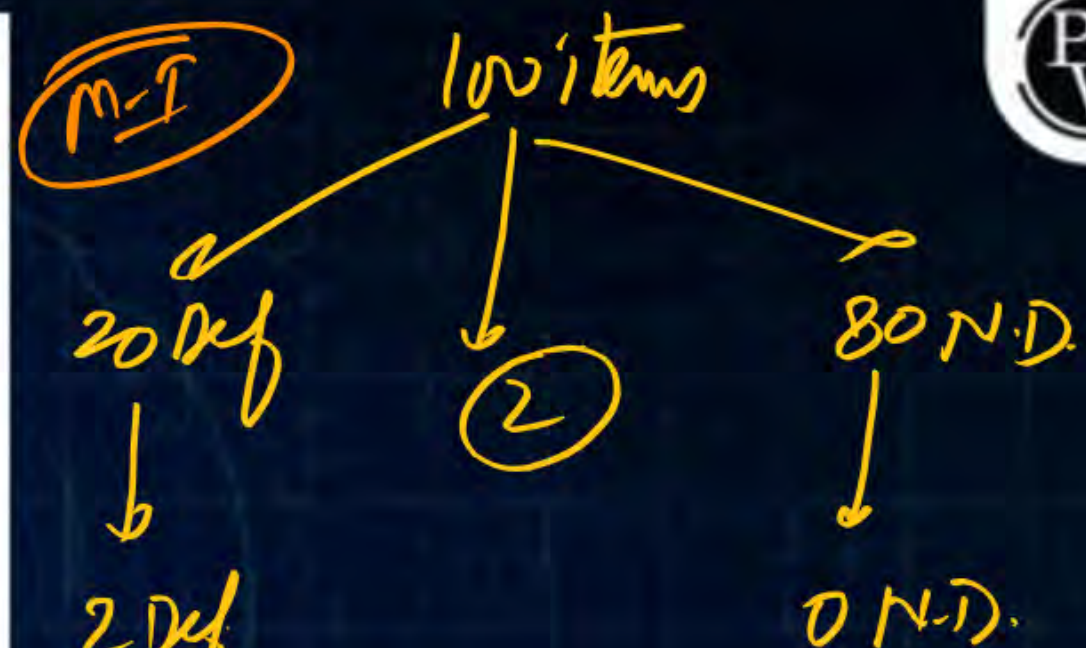
A box contains 20 defective items and 80 non-defective items. If (two items) are selected at random without replacement, what will be the probability that both items are defective?

(a) $\dfrac{1}{5}$

(b) $\dfrac{1}{25}$

(c) $\dfrac{20}{99}$

(d) $\dfrac{19}{495}$

M-I   100 items

20 Def   80 N.D.
  ↓   (2)   ↓
2 Def   0 N.D.

$\text{Req Prob} = \dfrac{f}{T} = \dfrac{^{20}C_2}{^{100}C_2} = \dfrac{\frac{20\times19}{2\times1}}{\frac{100\times99}{2\times1}}$

$= \dfrac{20\times19}{100\times99} = \dfrac{19}{495}$

M-II Req Prob $= P[D \cap D] = \dfrac{20}{100} \times \dfrac{19}{99} = \dfrac{19}{495}$

A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its colour recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same colour is

Here in this Question, we are drawing Marbles one by one with Replacement

ie all three must be different Colour.

(a) $\dfrac{1}{36}$

(b) $\dfrac{1}{6}$

(c) $\dfrac{1}{4}$

(d) $\dfrac{1}{3}$

$Req\ Prob = P\left[BB'R\ or\ BRB'\ or\ RBB'\ or\ RB'B\ or\ B'RB\ or\ B'BR\right]$

OR

$Req\ Prob = P\left[BB'R\right] \times 3! = \left(\dfrac{10}{60} \times \dfrac{20}{60} \times \dfrac{30}{60}\right) \times 6 = \dfrac{1}{6}$ ie (b)

Questions Based on Tree Diagram : (when we have Tree of ∞ length).

i.e Tree may have Infinite Branches.

In that type of situations, Invidividual elements of S.Space of not of Same nature i.e these are not equally likely.

i.e their individual Probabilities are different

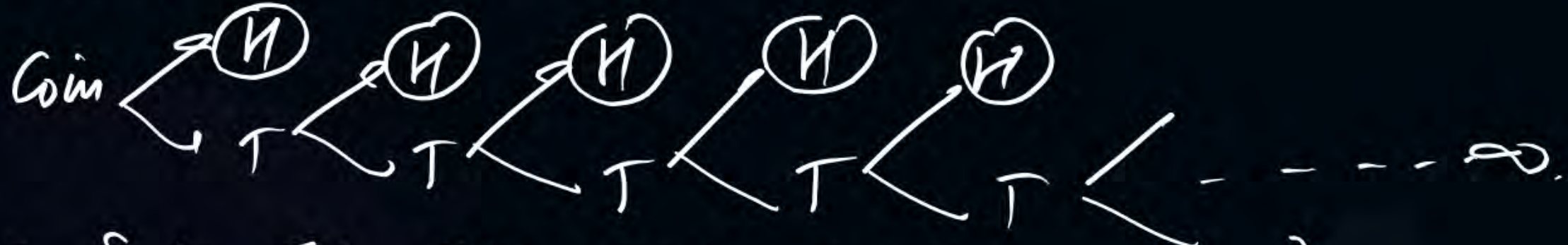so we will avoid App $\overline{II}$ i.e $Req\ Prob = \dfrac{fav\ Cases}{Total\ Cases}$

this formula should be avoided.

A coin is tossed (until) Head appears then Find the prob that

required no. of tosses to end such type of Game will be (odd)?

(a) 6/11

(b) 2/3

(c) 1/3

(d) 1

$P(H) = \dfrac{1}{2} = P(T)$
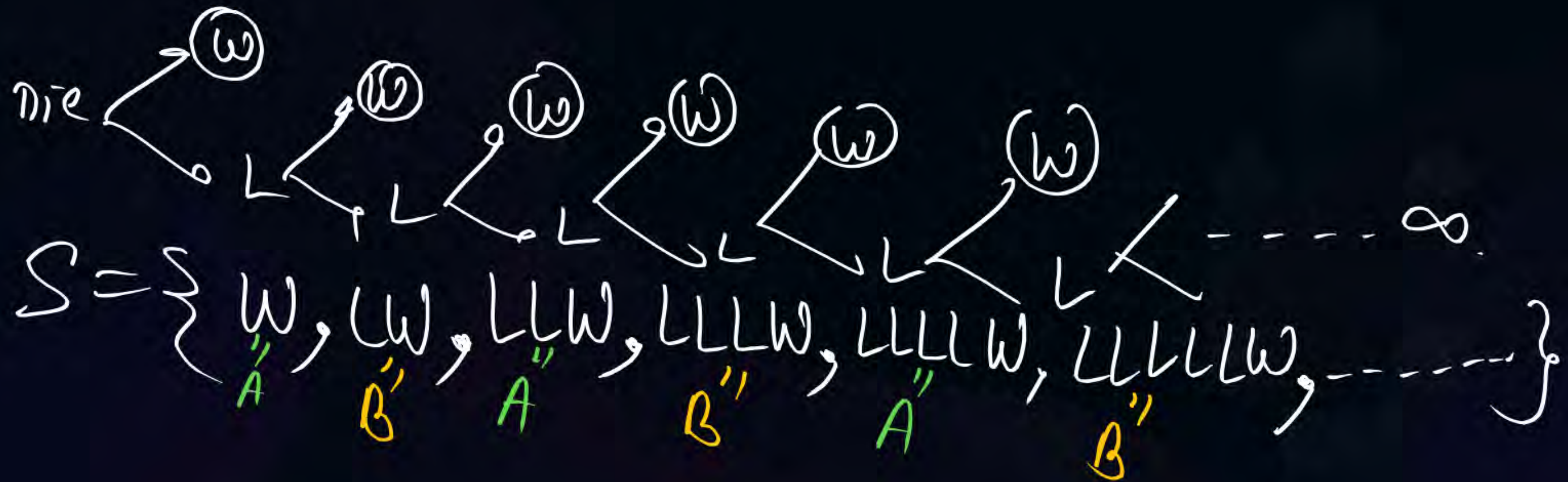
Coin

$S = \{ H, TH, TTH, TTTH, TTTTH, \dots \}$

$A = \{ odd\ tosses \} = \{ H, TTH, TTTTH, \dots \}$

$P(A) = P(H) + P(TTH) + P(TTTTH) + \dots$

$= \dfrac{1}{2} + \left(\dfrac{1}{2}\right)^3 + \left(\dfrac{1}{2}\right)^5 + \dots = \boxed{\dfrac{2}{3}}$

$$P(W) = P(\text{six occurs}) = \frac{1}{6}, \quad P(L) = \{\text{six not occurs}\} = \frac{5}{6}$$

Qs: Two persons A & B play a game of Dice (alternately) in which any one can win if six appears 1st time then find their resp (chances) of winning if A starts the game?

$$S = \{ \underset{A''}{W}, \underset{B''}{LW}, \underset{A''}{LLW}, \underset{B''}{LLLW}, \underset{A''}{LLLLW}, \underset{B''}{LLLLLW}, \text{------} \}$$

fav Cases for $A = \{ \overset{=1^{st}}{W}, \overset{=3^{rd}}{LLW}, \overset{=5^{th}}{LLLLW}, \ldots \ldots \}$

$P(A \text{ win}) = P(W) + P(LLW) + P(LLLLW) + \ldots \ldots$

$= \dfrac{1}{6} + \dfrac{5}{6} \cdot \dfrac{5}{6} \cdot \dfrac{1}{6} + \left(\dfrac{5}{6}\right)^4 \cdot \dfrac{1}{6} + \ldots \ldots$

$= \dfrac{1}{6}\left[1 + \left(\dfrac{5}{6}\right)^2 + \left(\dfrac{5}{6}\right)^4 + \ldots \right]$

$= \dfrac{1}{6}\left[\dfrac{1}{1 - \left(\dfrac{5}{6}\right)^2} \right] = \dfrac{6}{11}$

$\& \; P(B \text{ win}) = 1 - \dfrac{6}{11} = \dfrac{5}{11}$

$S_\infty = a + ar + ar^2 + ar^3 \ldots \ldots$

$= \boxed{\dfrac{a}{1-r}}$

$(M-II) \; \text{Req Prob} = \dfrac{f}{} = \dfrac{50\%}{100\%} = \dfrac{1}{2}$