CS & IT ENGINEERING

Algorithm

Miscellaneous

Lecture No. [redacted] 03

By- Aditya Jain sir

Recap of Previous Lecture

Topic

Topic

Topic

Heaps

PYQ + Practice

Topics to be Covered

Topic

Topic TC analysis using Recurrence Tree

Topic

About Aditya Jain sir

1. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
2. Represented college as the first Google DSC Ambassador.
3. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored 12,000+ students & working professions in field of Data Science and Analytics
11. Have been mentoring & teaching GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

@ADITYASIR_PW

Telegram

Telegram Link for Aditya Jain sir: https://t.me/AdityaSir_PW

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Time Complexity Analysis : (Recurrence Relations)

min-max : $\left[\frac{3n}{2} - 2\right]$

Approaches	We can find
1) Back-substitution	Value of Recurrence * TC: notation
Master Method / Theorem	TC: notation
Recurrence / Recursion Tree Approach	TC: notation

$O(n)$

Symm

Asymm

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Types of TC Recurrences :

1. Symmetric : $T(n) = a \times T(n/b) + f(n), a \geq 1,\ b > 1,\ f(n) = +ve$

1. Asymmetric

   (a) Case 1 : $T(n) = T(\alpha n) + T((1-\alpha)n) + f(n)$

   (b) Case 2 : $T(n) = T(\alpha n) + T(\beta n) + T(\delta n) \ldots + f(n)$

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Symmetric Recurrence:

$$T(n)=a\times T(n/b)+f(n), a\geq 1,\ b>1,\ f(n) = +ve$$

Size of each sub-problem

Number of ub-problem

Can be solved by:

1. Back substitution
2. Master method
3. Recursion tree approaches

Equation:

$\cancel{T(n) = 2T(n/2) + 2}$ $T(n) = 2T(n/2) + n$

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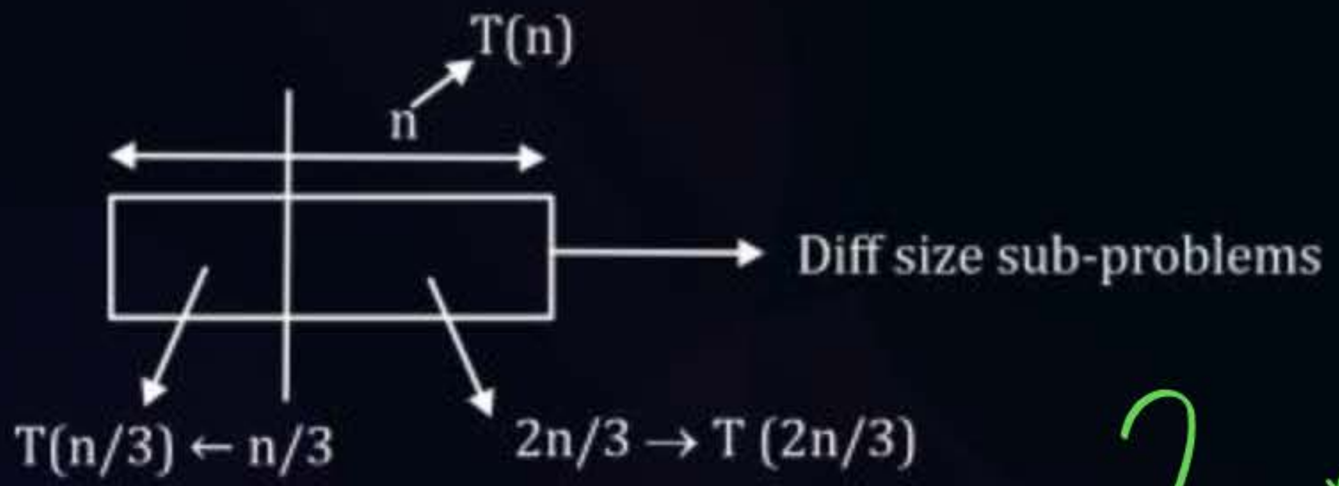
Asymmetric:

Case-1 $T(n)=T(\alpha n)+T((1-\alpha)n)+f(n)$

T(n) 0 ≤ & ≤ 1

Equation:



1. $T(n) = T(n/3) + T(2n/3) + n$
2. $T(n) = T(n/4) + T(3n/4) + n^2$

Rec Tree

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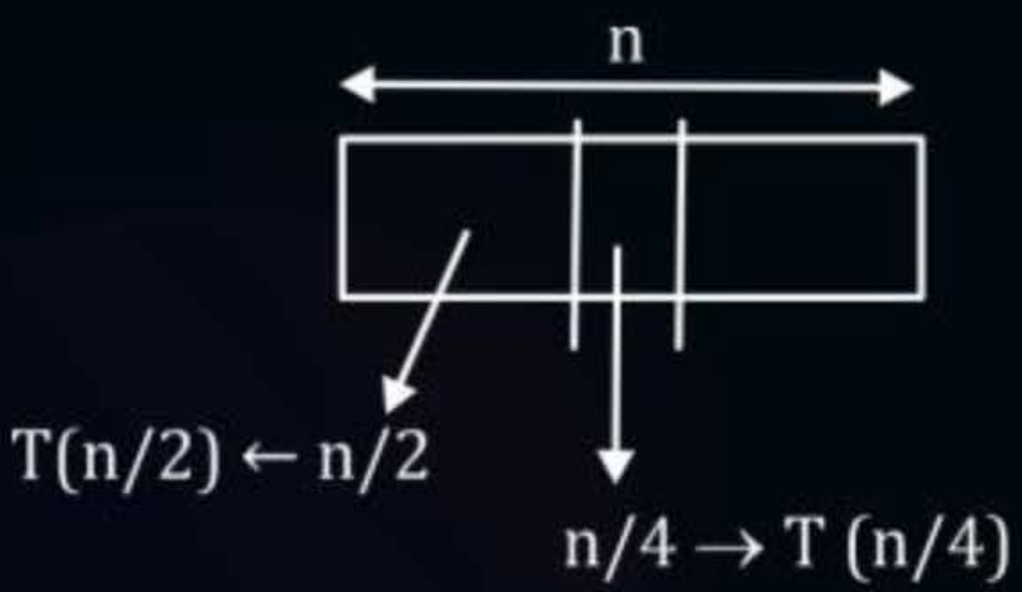
Asymmetric:

Case-2



$$T(n) = T(n/2) + T(n/4) + n^2$$

Recursion tree approach.

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Equation:

$$\boxed{T(n) = 2T(n/2) + n} \longrightarrow \text{Merge sort}$$

1. Back → substitution → value of recurrence? → $O(n\log n)$
2. Master method.

→ $O(n\log n)$

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Solution:

Back- substitution:

$T(n) = 2T(n/2) + n$ ________ (1)

$T(n/2) = 2T(n/2^2) + n/2$

$T(n) = 2T(n/2^2) + n/2) + n$

$T(n) = 2^2 (n/2^2) + 2n$ ______ (2)

$T(n) = 2^3 (n/2^3) + 3n$ ______ (3)

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General Term

$T(n) = 2^k T(n/2^k) + k * n$ ________ (4)

for B-C , $\frac{n}{2^k} = 1 \Rightarrow n = 2^k$

$\boxed{k = \log_2^n}$

$T(n) = n * T(1) + n * k$

$T(n) = n * c + n \log_2^n$ → value of recurrence

$\boxed{T(n) = O(\log_2^n)}$

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Master Method:

$T(n) = 2T\,(n/2) + n$ (given)

$T(n) = a^*\,T\,(n/b) + f(n)$

$a = 2$

$b = 2$

$F(n) = 2$

Valid

$\log_b^a = \log_2^2 = 1$

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Case-1:

Is $f(n) = O(n^{\log_b^a - \varepsilon})$

Is $\cancel{n = \theta(n^{1-\varepsilon})}$, Sone $\varepsilon > 0$? Invalid

$n = O(n^{1-\varepsilon})$?

Case-2:

Is $F(n) = \theta\left(n^{\log_b^a} * (\log n)^k\right)$, for some

(a) $k \geq 0$

For k =n, $n\ \theta(n^1 * (\log n)^0)$

$n = \theta(n)$

Valid to for $k = 0$

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Then,

$$T(n) = \theta\left(n^{\log_b^a} * (\log n)^{k+1}\right)$$

$$k = 0 \Rightarrow T(n)\ \theta\left(n^1 * (\log n)^{k+1}\right)$$

$$\boxed{T(n) = \theta\ (n \log n)}$$

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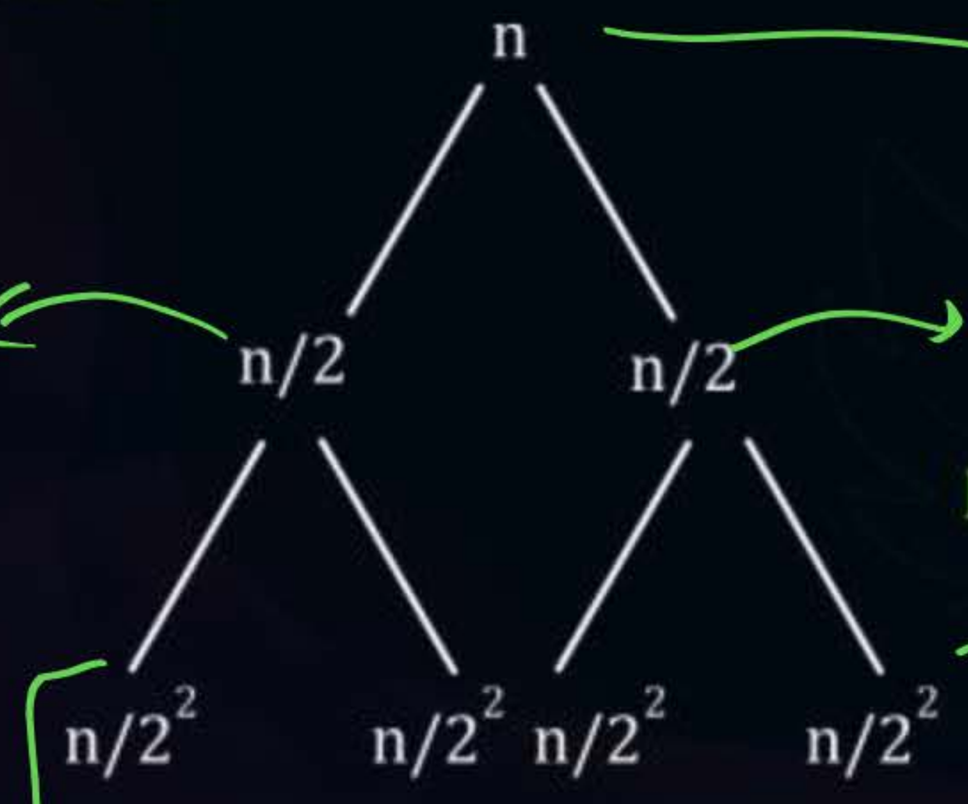
Recurrence tree Approach:

$T(n) = 2T(n/2) + n$

Cost/fee of recursion



$n \rightarrow n$

$n/2 \qquad n/2 \rightarrow 2 \times n/2 = n$

Every level has cost of 'n'.

$4 \times \frac{n}{2^2} = n$

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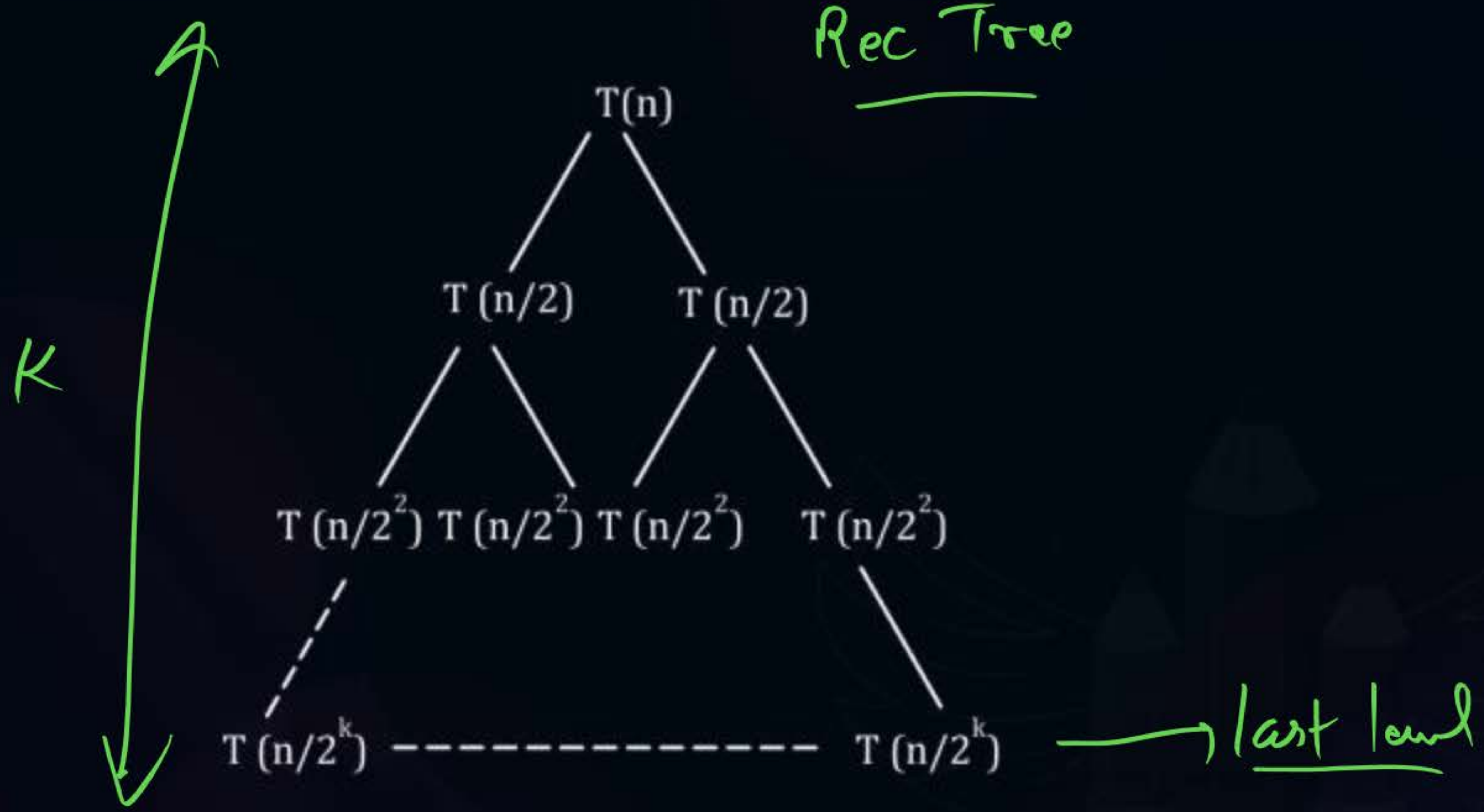
K levels



Assume this is last level (Base condtion)

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At last level, it should be base condition

$T(n/2^k)$

$$\boxed{\frac{n}{2^k} = 1}$$

$n = 2^k$

$$\boxed{k = \log_2^{n}}$$

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Level -1 $\rightarrow T(n)$	:	cost $= n$
Level -2 $\rightarrow T(n/2)$	:	cost $= n$
Level -3 $\rightarrow T(n/2^2)$	:	cost $= n$
Level –(k+1) $\rightarrow T(n/2^k)$	:	cost $= n$

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Total number of levels = (k+1)

The cost/ time of every level = n

Hence, total cost/ time = n* (k+1)

= n* k+ n

We know

$\boxed{k = \log_2^n}$

$= n * \log_2^n + n$

$\boxed{T(n) = \theta(n\log n_2^n)}$

$$T(n) = T(n/3) + T\left(\frac{2n}{3}\right) + n.$$

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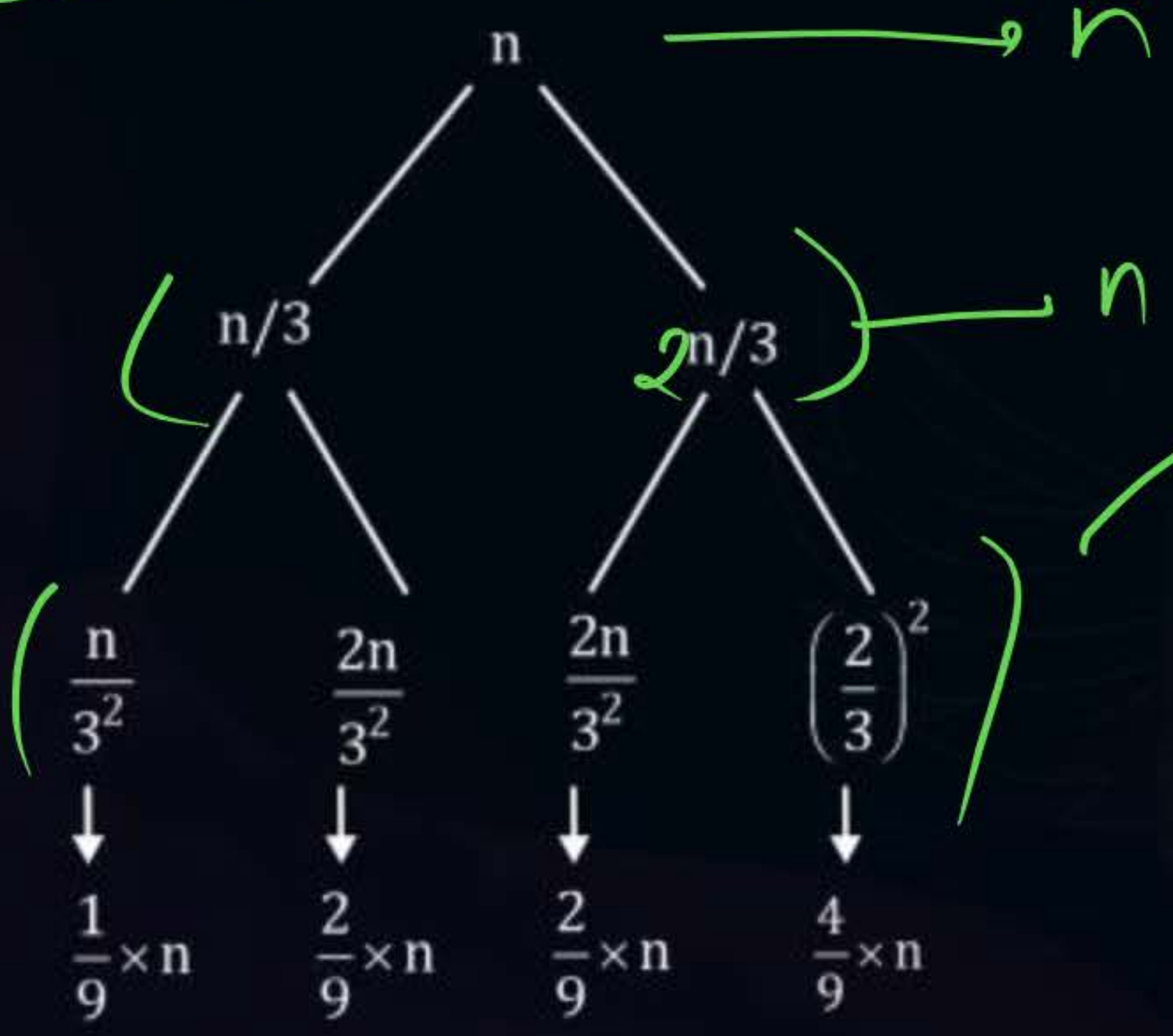
Equation:

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n \rightarrow \text{cost}$$

Asymmetric case -1



$$n \rightarrow \left(\frac{1}{9} + \frac{2}{9} + \frac{2}{9} + \frac{4}{9}\right)n$$

$$= \frac{1+2+2+4}{9} \times n$$

$$= \frac{9}{9} \times n$$

$$= 1 \times n$$

Cost at every level = n

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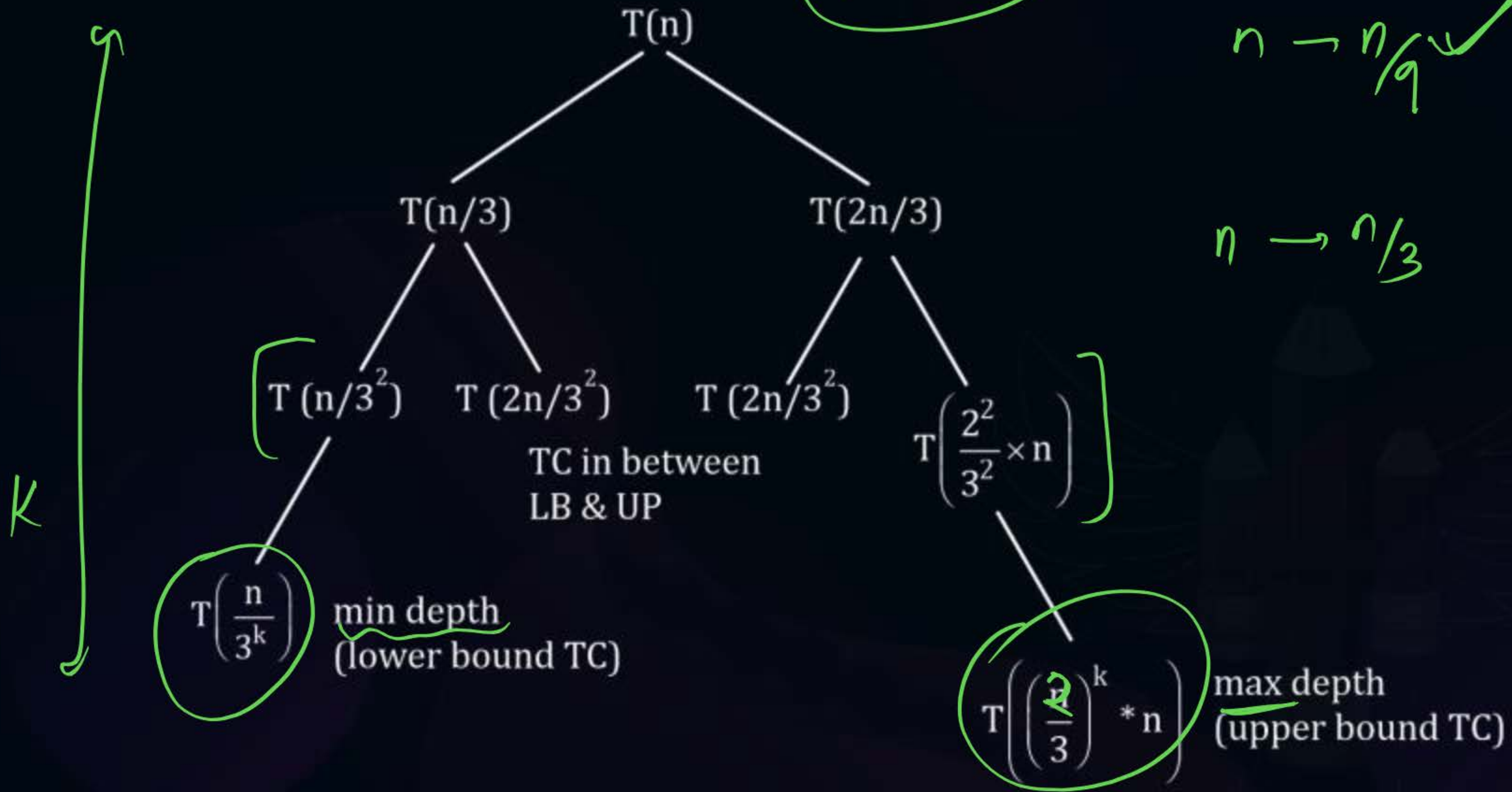
V.V.Imp



$n \to n/9$ ✓

$n \to n/3$

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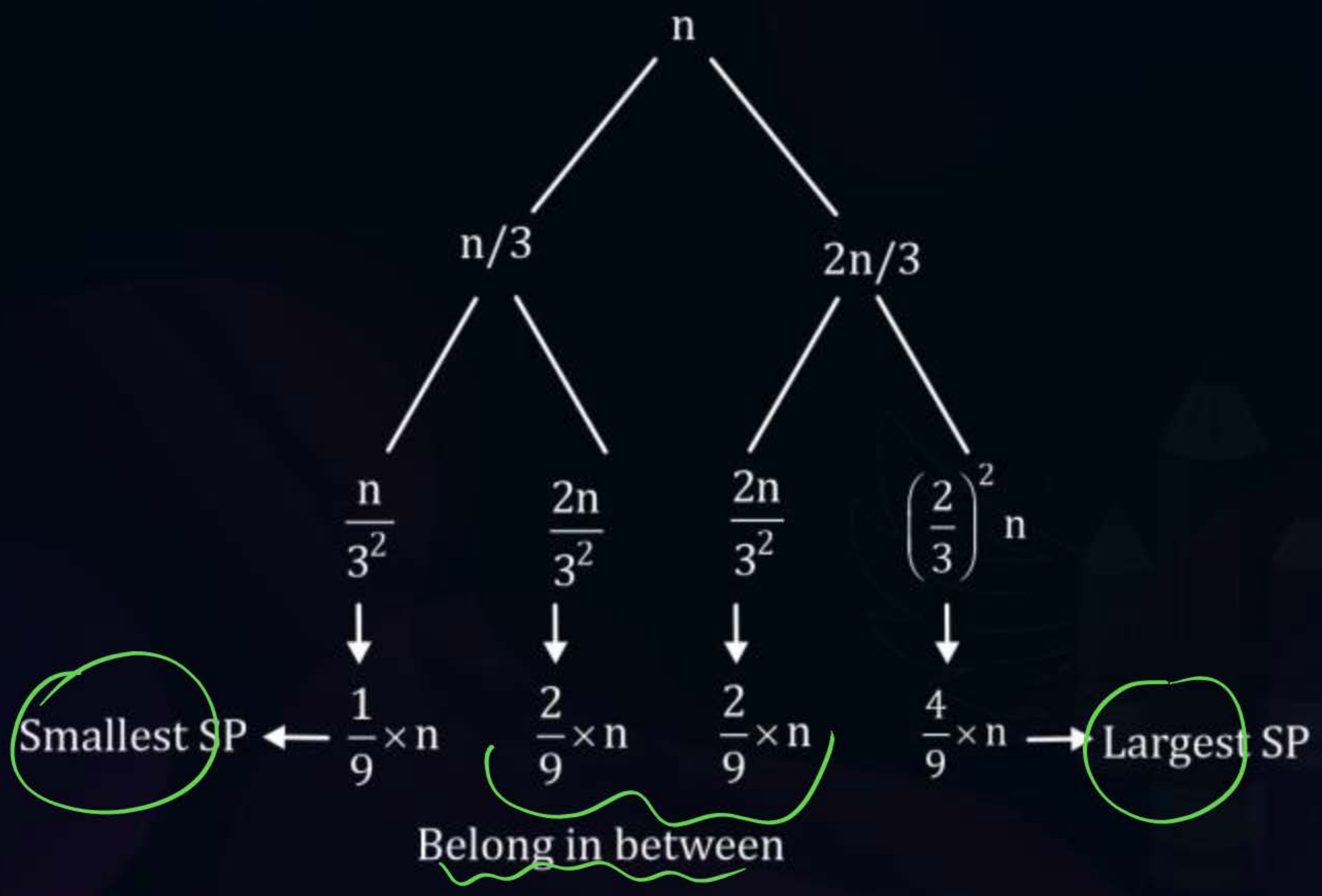

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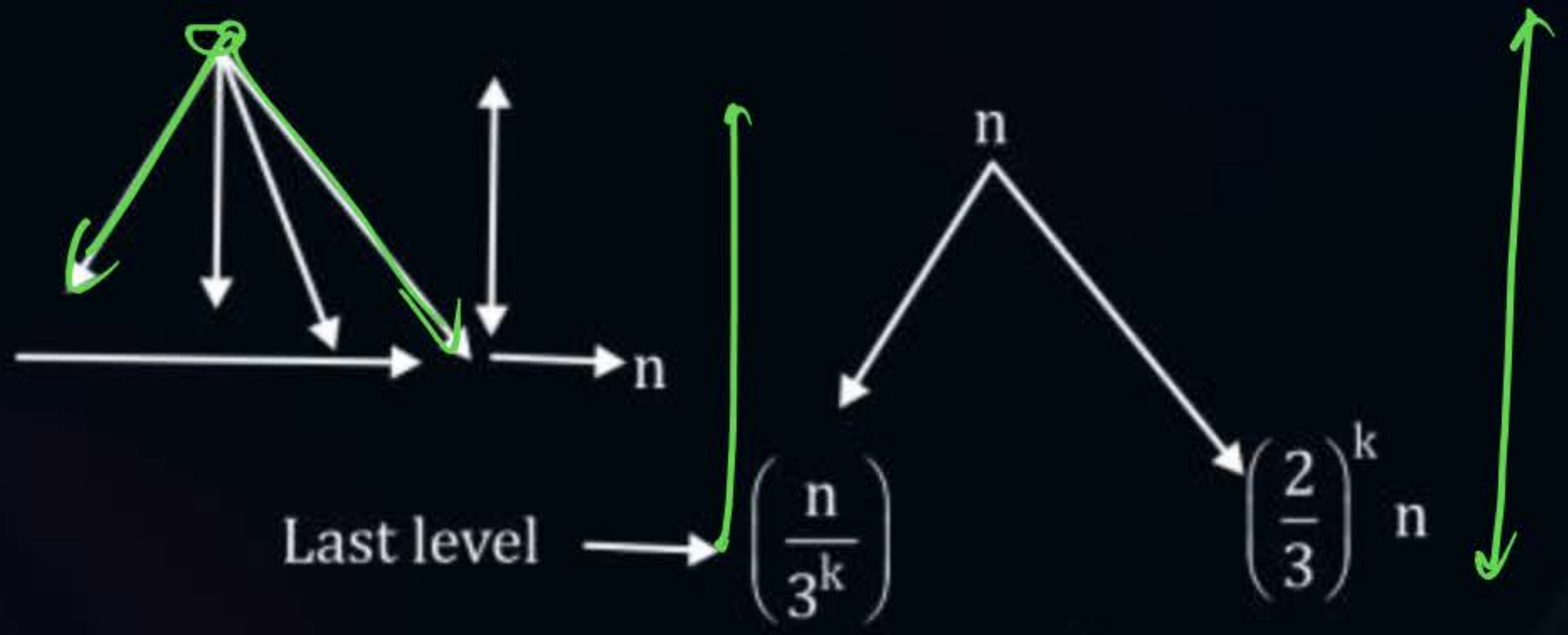

1. For smallest/ mint depth (LB of TC)

   At least level k, $\frac{n}{3^k} = 1$ (BC)

   $n = 3^k$

   $\boxed{k = \log_3^n}$

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Assuming there are k levels and cost time at each level = n

Hence, $T(n) \geq k * n$

$T(n) \geq \log_3^n * n$

$\boxed{T(n) \geq n * \log_3^n}$ ——————— (i)

$T(n) = \Omega\ (\log_3^n * n)$

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For largest depth → (UB of TC)

Let last level be k

For last level $\left(\frac{2}{3}\right)^k * n = 1$

level $\left(\frac{3}{2}\right)^k = n$

$\log n = k * \log\left(\frac{3}{2}\right)$

$k = \frac{\log n}{\log\left(\frac{3}{2}\right)} = \log_{3/2}^{n}$

$k = \log_{3/2}^{n}$

$\frac{\log^a}{\log^b} = \log_b^a$

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Cost at every level $= \underline{\underline{n}}$

Total cost $\leq k^*n$

$T(n) \leq k^*n \cdot \log_{3/2}^{n} * n$

$T(n) \leq n^* \log_{3/2}^{n}$ ________ (ii)

~~$T(n) = O(\log_{3/2}^{n})$~~

$T(n) = O(n * \log_{3/2} n)$

From (i) & (ii)

$$\boxed{n * \log_3^n \leq T(n) \leq n * \log_{3/2}^n}$$

$T(n) = \Omega(n * \log_3^n)$

$T(n) = O(n * \log_{3/2}^n)$

Approximation $\Rightarrow T(n) = \theta\,(n \log n)$

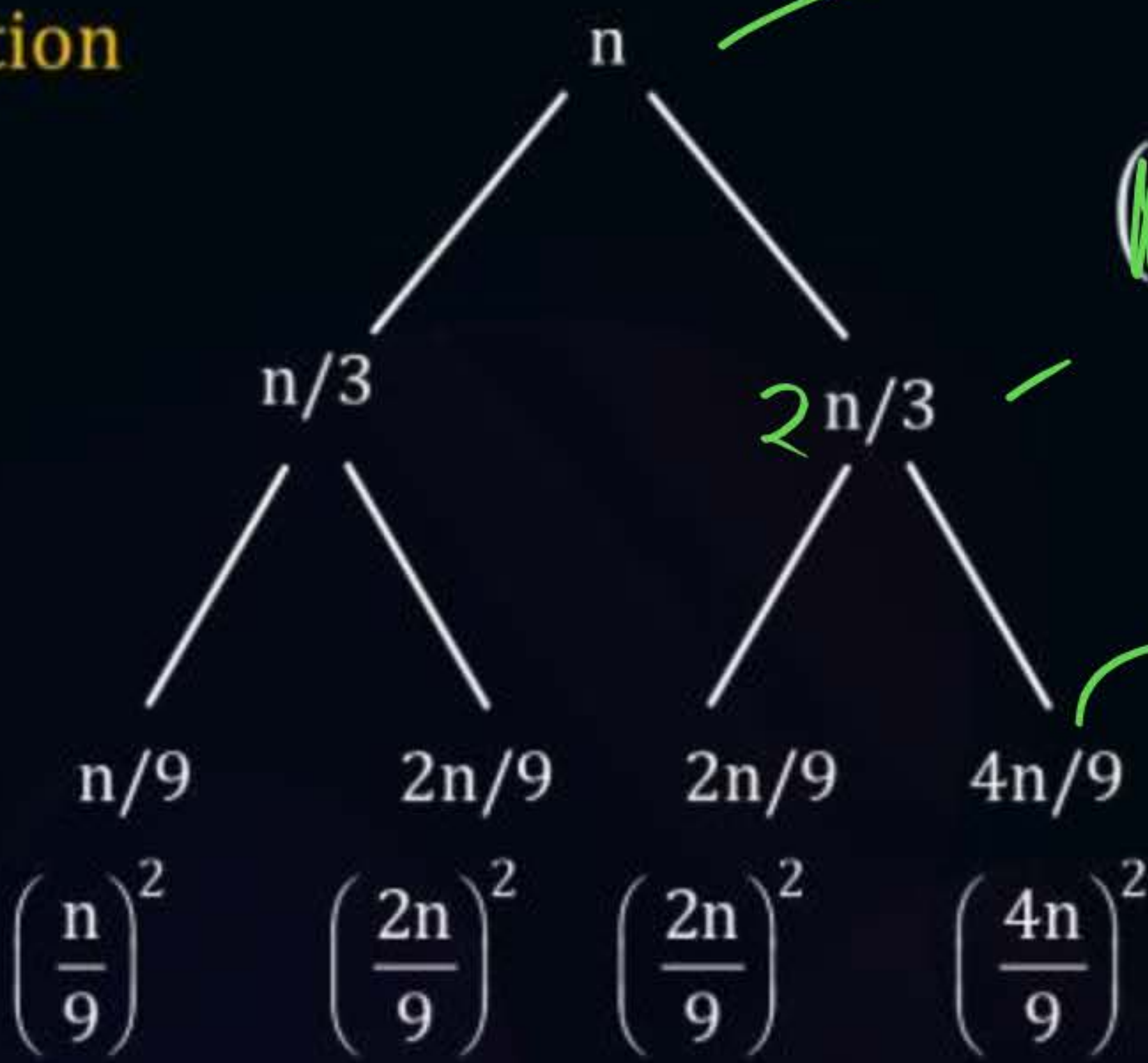
[NAT]

#Q. $T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n^2$

solve

cost/fee

Solution



cost

$n^2 \longrightarrow \left(\frac{5}{9}\right)^0 n^2$

$\left[\left(\frac{n}{3}\right)^2 + \left(\frac{2n}{3}\right)^2\right] = \frac{n^2}{9} + \frac{4n^2}{9} = \frac{5n^2}{9} \rightarrow \left(\frac{5}{9}\right)^1 n^2$

$\frac{n^2}{9^2} + \frac{4n^2}{9^2} + \frac{4n^2}{9^2} + \frac{16n^2}{9^2} = \frac{5^2}{9^2} n^2 \longrightarrow \left(\frac{5}{9}\right)^2 n^2$

$\longrightarrow \left(\frac{5}{9}\right)^2 n^2$

$k^{th} \longrightarrow \left(\frac{5}{9}\right)^{k-1} n^2$

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Let there be k levels

Total at each level

1^{st} Level	$\rightarrow n^2$	$\rightarrow$	$\left(\frac{5}{9}\right)^0 n^2$
2^{nd} Level	$\rightarrow \frac{5}{9} n^2$	$\rightarrow$	$\left(\frac{5}{9}\right)^1 n^2$
3^{rd} Level	$\rightarrow$		$\left(\frac{5}{9}\right)^2 n^2$
k^{th} Level	$\rightarrow$		$\left(\frac{5}{9}\right)^{k-1} n^2$

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Hence, Total lost (T(n))

Sum of cost at each level.

$$T(n) = \left(\frac{5}{9}\right)^0 n^2 + \left(\frac{5}{9}\right)^2 n^2 \ldots\ldots \left(\frac{5}{9}\right)^{k-1} n^2$$

$$= \sum_{i=0}^{k-1} \left(\frac{5}{9}\right)^i n^2$$

$$= \frac{1}{\left(1-\frac{5}{9}\right)} * n^2$$

$$\boxed{\text{Total cost} \approx \frac{9}{4} n^2}$$

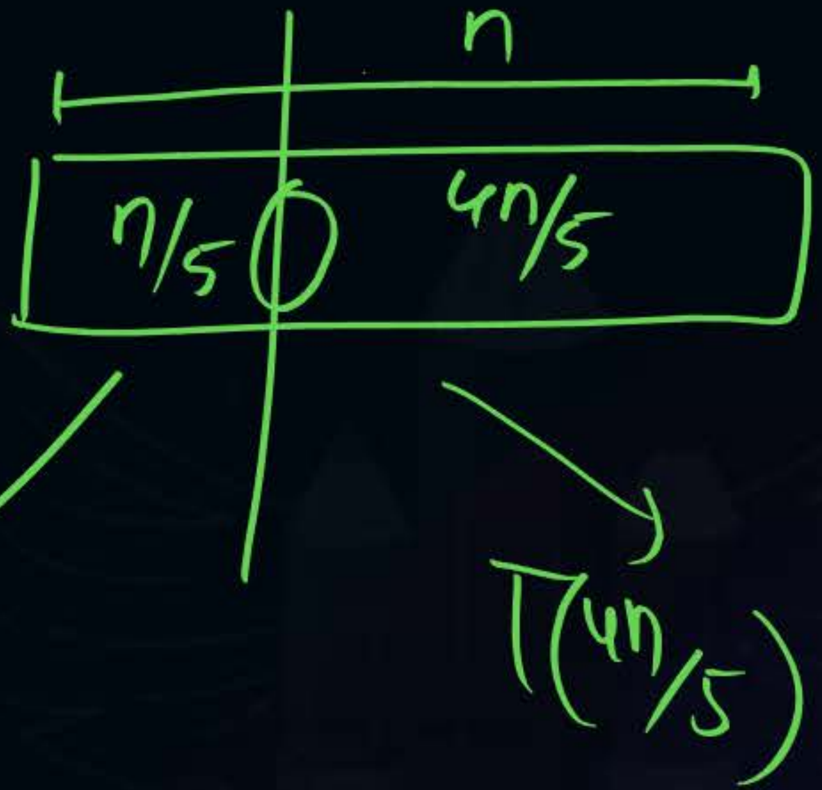
$\Rightarrow \underline{\underline{O(n^2)}}$

[NAT]

#Q. $T(n) = T(n/5) + T(4n/5) + n$

Solve

Q.S. $T(n) = 2T(n/2) + n$



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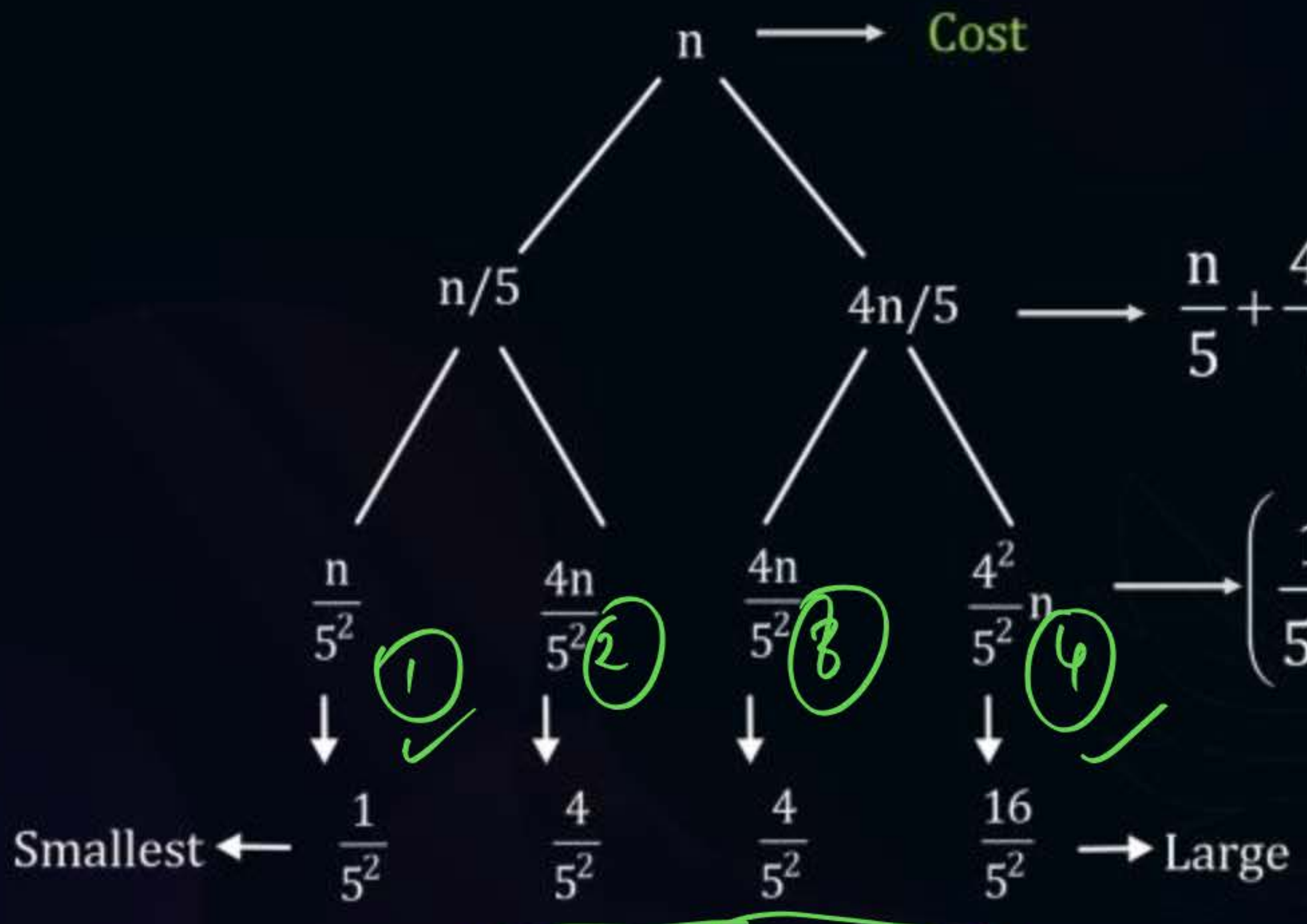

$$\frac{n}{5}+\frac{4n}{5}=n$$

$$\left(\frac{1}{5^2}+\frac{4}{5^2}+\frac{4}{5^2}+\frac{4^2}{5^2}\right)n=n$$

$$\frac{25}{25}\times n=n$$

Cost at every level = n

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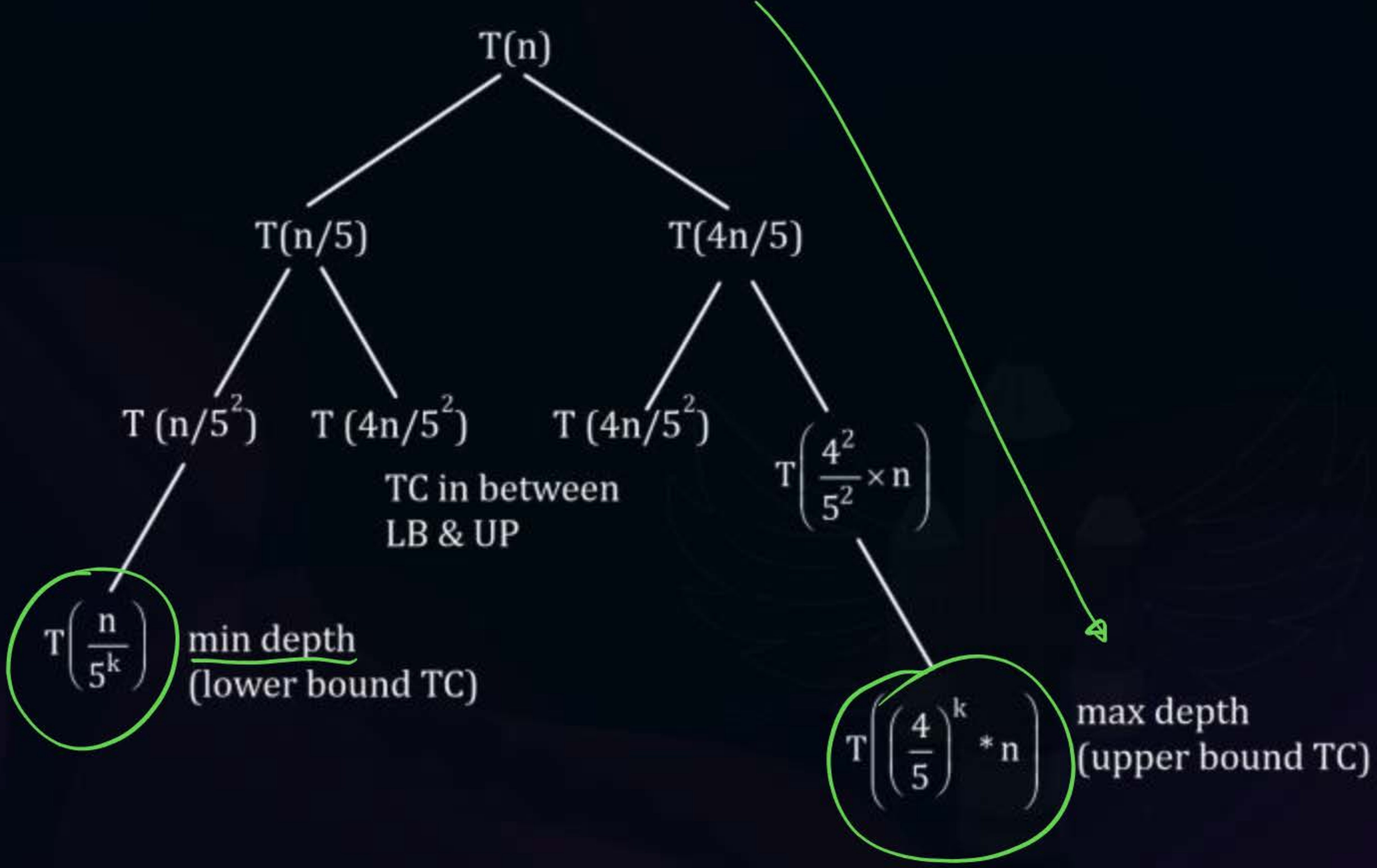

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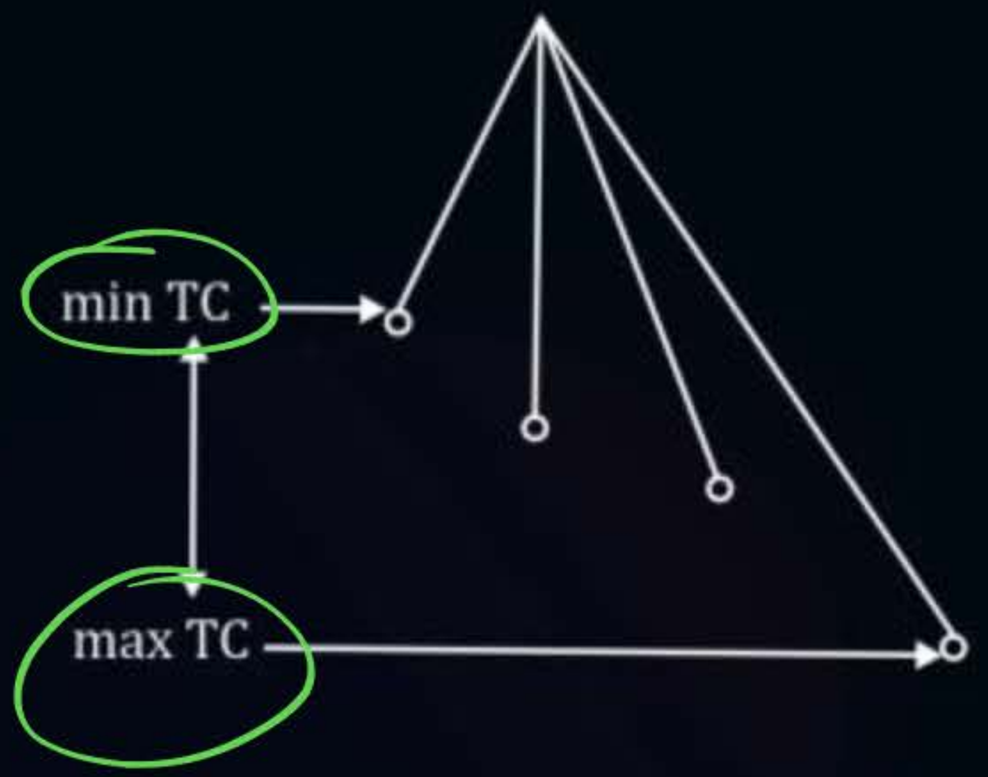

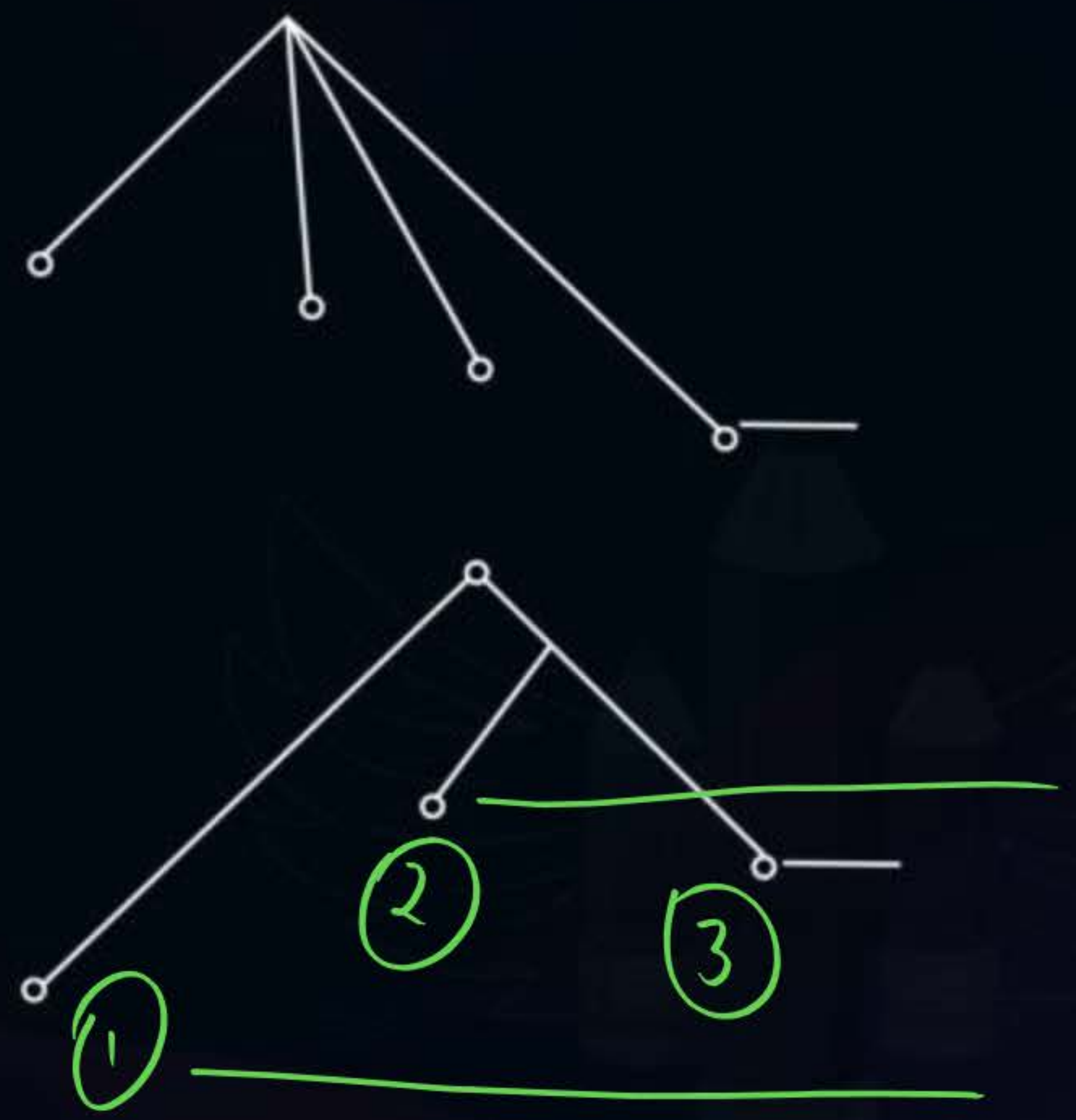

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For

Lowest depth (LB) TC

$$\frac{n}{5^k} = 1$$

$$5^k = n$$

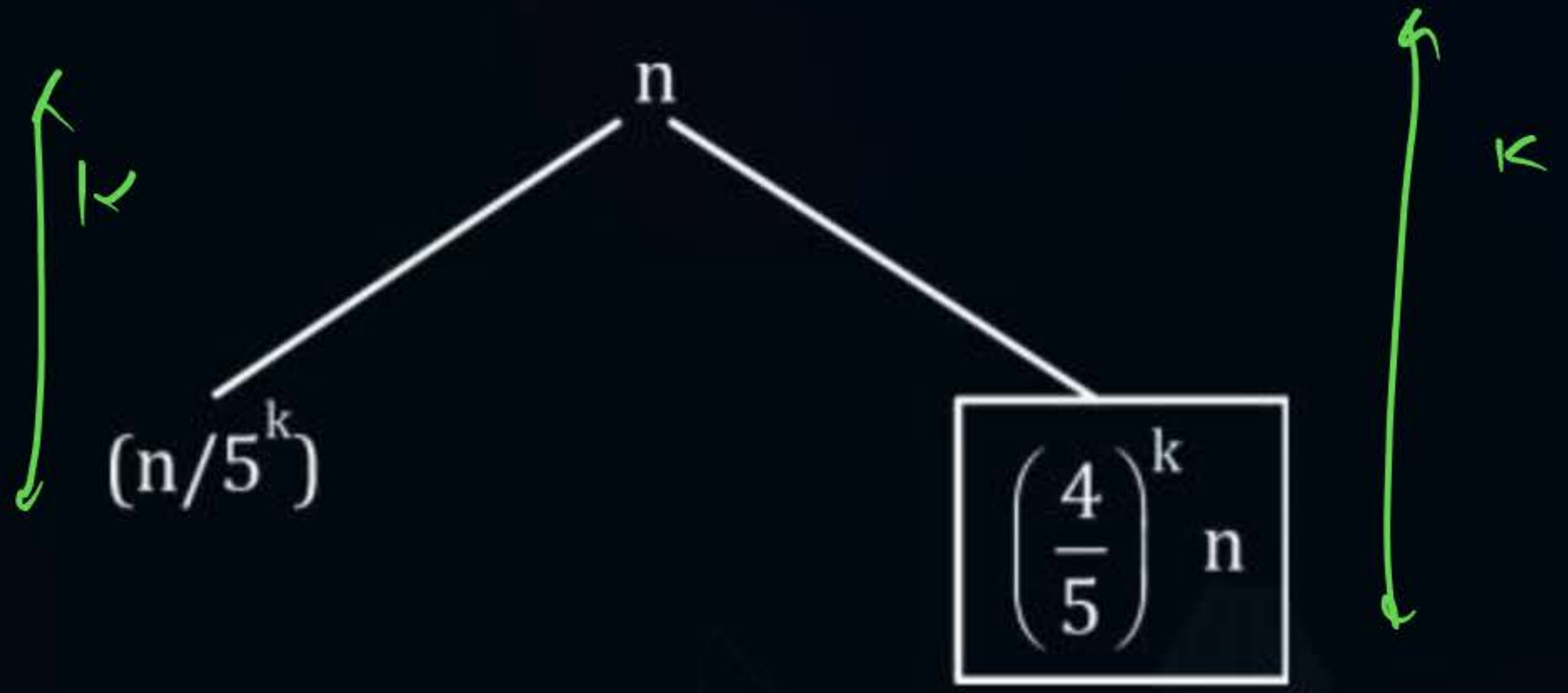
$$k = \log_5^n$$

Cost at ~~only~~ every level = n

$$T(n) \geq n^*k$$

$$T(n) \geq n^* \log_5^n$$

$$T(n) = \Omega(n\log_5^n)$$



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For largest depth:

$$\left(\frac{4}{5}\right)^k n = 1$$

$$\left(\frac{5}{4}\right)^k = n$$

$$k = \log_{5/4}^{n}$$

T(n) $\leq$ n*k

$$\boxed{T(n) \leq n * \log_{5/4}^{n}}$$

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$$n * \log_5^n < T(n) < n * \log_{5/4}^n$$

$$T(n) = \Omega(n\log_5^n) \qquad\qquad T(n) = \Omega(n\log_{5/4}^n)$$

Approximation : $T(n) = \theta\,(n \log n)$

$$T(n) = T(n-1) + T(n-2) + 1$$

Cost/Fee

Solve

$$\begin{pmatrix} B.C \\ W.C \end{pmatrix} TC$$

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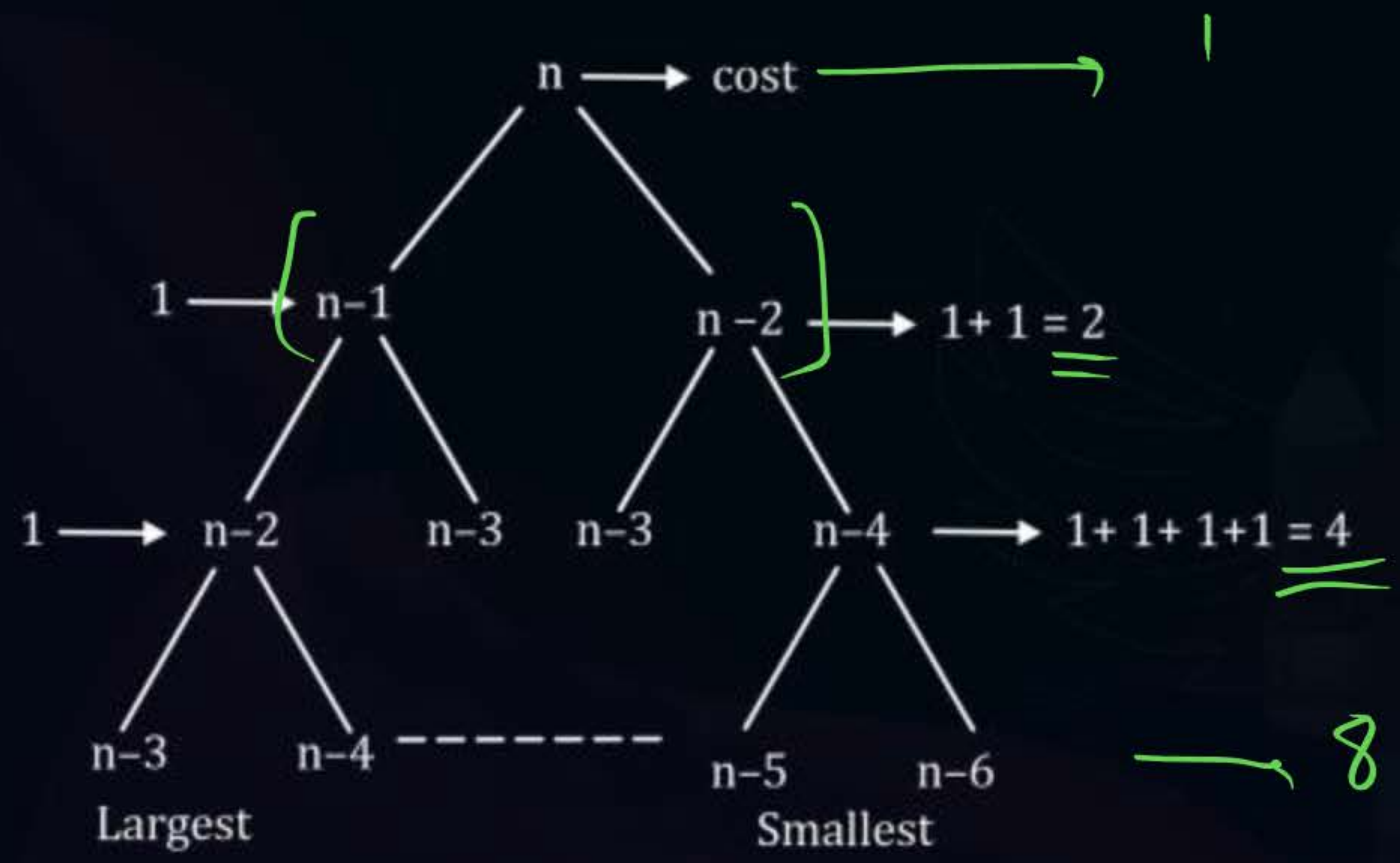
Fibonacci:

TC rearrange $$T(n) = T(n-1) + T(n-2) + 1$$



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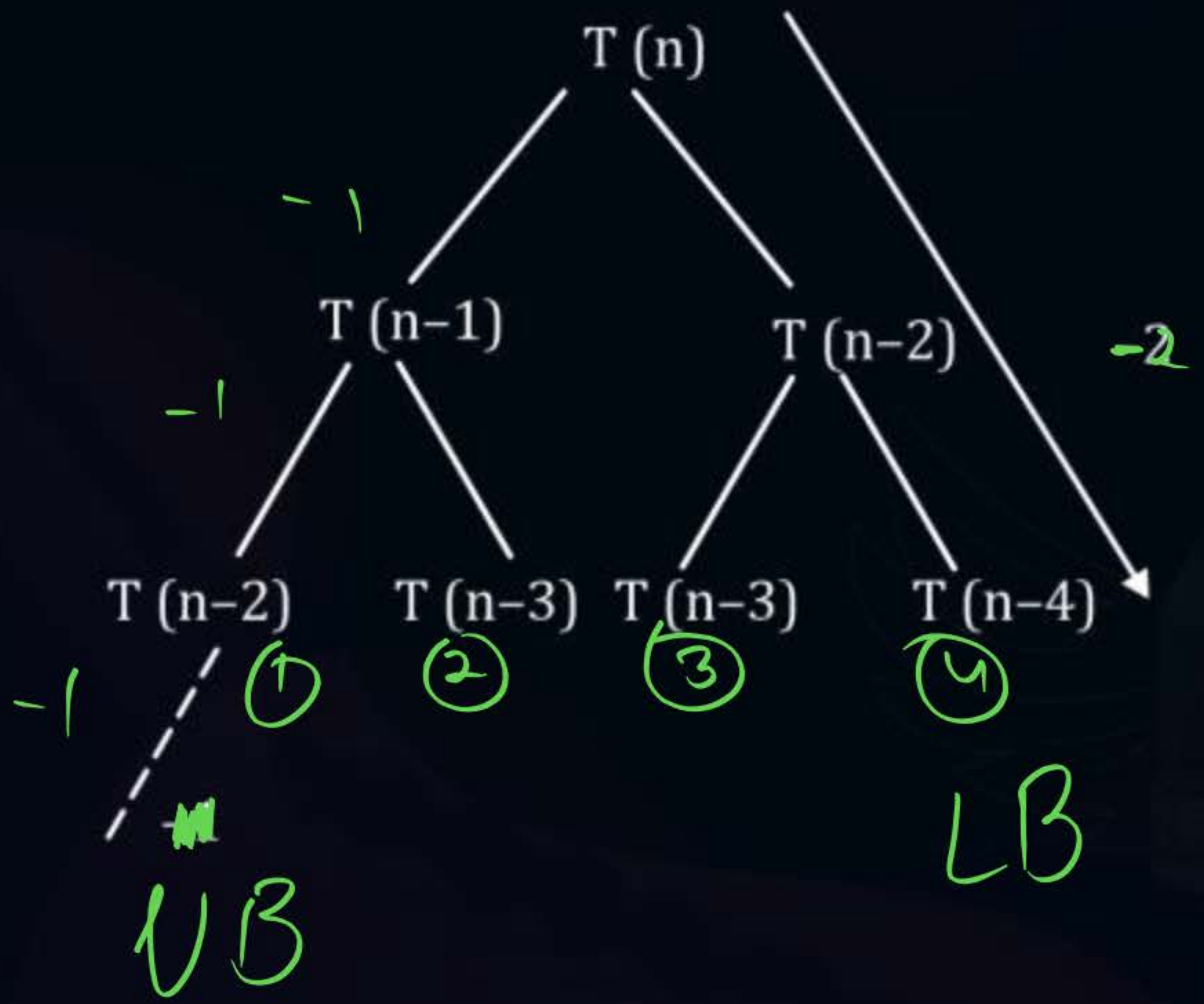

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Left –most branch

$n-1 \rightarrow n-2 \rightarrow n-3$ 1

Number of levels $K = n$ (UB TC)

Right –most branch

$n-2 \rightarrow n-4 \rightarrow n-6$ 1

Number of levels = $n/2$ (LB of TC)

$$8 \xrightarrow{-1} 0 \quad (8)$$

$$8 \xrightarrow{-2} 0 \quad (\cancel{8}) \quad (4)$$

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Cost of ~~answer~~ all level k

$= \cancel{20 + 21 + 22 \ldots\ldots\ldots 2k - 1} = 2^0 + 2^1 + 2^2 \cdots 2^{(k-1)} = 2^k - 1$

$= \cancel{(2k - 1)}$

1. Left most branch → k = n

$T(n) \leq 2^n - 1$

$\boxed{T(n) = O(2^n)}$

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2. Right most branch → k = n/2

$T(n) \geq 2^{n/2} - 1$

$$\boxed{T(n) = \Omega(2^{n/2})}$$

$T(n) = O(2^n)$

$T(n) = \Omega(2^{n/2})$

$\neq \theta(2^n)$

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Conclusion:

Fibonacci

$$T(n) = O(2^n)$$

$$\boxed{T(n) = \Omega(n/}2^n)$$

It is θ (2^n) ?

No

THANK - YOU