CS & IT ENGINEERING

Algorithms

Analysis of Algorithms

Lecture No.- 05

By- Aditya sir

Topics to be Covered

Topic

Topic

Asymptotic Notations Practice Properties

About Aditya Jain sir

1. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
2. Represented college as the first Google DSC Ambassador.
3. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored 12,000+ students & working professions in field of Data Science and Analytics
11. Have been mentoring & teaching GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

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Topic : Adding Functions

The sum of two functions is governed by the dominant one, namely:

$O(f(n)) + O(g(n)) \rightarrow O\ (\max(f(n), g(n)))$

$\Omega(f(n)) + \Omega(g(n)) \rightarrow \Omega(\max(f(n), g(n)))$

$\theta(f(n)) + \theta(g(n)) \rightarrow \theta(\max(f(n), g(n)))$

$d_1 \quad d_2$

$f + g =$

$d_1 > d_2$

Topic : Adding Functions

Example:

$f(n) = 5n^2 + 2 \quad \rightarrow \quad O(n^2)$

$g(n) = 10n^3 \quad \rightarrow \quad O(n^3)$

$O(f(n)) = n^2$ and $O(g(n)) = n^3$

$n^2 + n^3 = O(\max(5n^2 + 2, 10n^3))$

$= O(10n^3)$

$= O(n^3)$

Topic : Multiplying Functions

$O(f(n)) * O(g(n)) \rightarrow O(f(n) * g(n))$

$\Omega(f(n)) * \Omega(g(n)) \rightarrow \Omega(f(n) * g(n))$

$\theta(f(n)) * \theta(g(n)) \rightarrow \theta(f(n) * g(n))$

Topic : General Properties of Big Oh Notation

Imp. Practice Question (T/F):

(1) $n^2 = O(2^{(2 \log n)})$

$a^{\log_c b} = b^{\log_c a}$

$n^2 \quad 2^{2\log n}$

$n^2 \quad n^{2\log_2 2}$

$n^2 = n^2$

True

Topic : General Properties of Big Oh Notation

Imp. Practice Question (T/F):

(2) $(\log n)^{1/2} = O(\log(\log n))$ —> False

Let $\log n = x$

$(\log n)^{1/2} > \log(\log n)$

$x^{1/2}$ $\log(x)$

$\sqrt{x} > \log(x)$

Topic : General Properties of Big Oh Notation

Imp. Practice Question (T/F): (T)

(3) $a^n \neq O(n^x)$, for $a > 1, x > 0$

$a^n \longrightarrow$ expo

$n^x \longrightarrow$ poly

poly = O(expo)

expo $\neq$ O(poly)

expo $\leq$ poly

expo > poly $\longrightarrow$ True

Topic : Trichotomy Property

#Q. Does Asymptotic notations follow Trichotomy Property?

Topic : Trichotomy Property

Trichotomy Property in Real Numbers:

- Given any two real numbers x & y (fixed).

  Then, x and y follow exactly one of the below relation:

  1) $x = y$ or,

  2) $x > y$ or,

  3) $x < y$

Topic : Trichotomy Property

Trichotomy Property in Asymptotic Comparison of two function:

- Given two functions f(n) & g(n), does f(n) & g(n) always follow exactly one of the below?

1) $f(n) >_A g(n) \quad \Rightarrow \quad f = \Omega(g), \omega(g)$

or, 2) $f(n) =_A g(n) \quad \Rightarrow \quad f = \theta(g)$

or, 3) $f(n) <_A g(n) \quad \Rightarrow \quad f = O(g), o(g)$

Topic : Trichotomy Property

Example1:

$f(n) = 5n^2$

$g(n) = 7n^3$

$f < g$

$f = O(g)$

Topic : Trichotomy Property

Example2:

$f(n) = 5n^2$

$g(n) = 1/n$

$f > g$

$f = \Omega(g)$

Topic : Trichotomy Property

Example3:

$f(n) = 10n^3 + 7$

$g(n) = 5n^3 + 15$

$f = \Theta(g)$

$f = g$?

$\times$

$f \overset{A}{=} g$

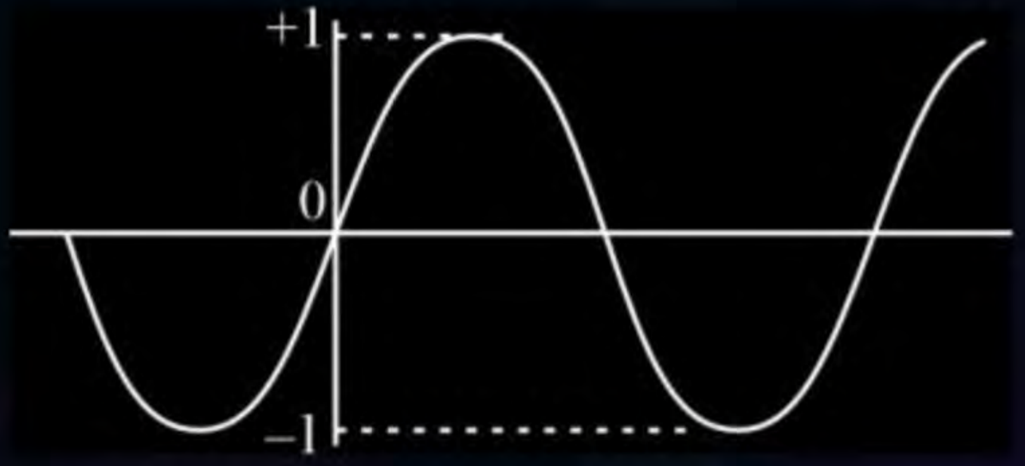
Topic : Trichotomy Property

Example4:

$f(n) = n$

$g(n) = n^{(1+\sin x)}$

Sin x plot (wave) [-1, +1]



$f = n, g = n^{(1+\sin x)}$

Let $\sin x = -1$

$F = n, \; g = n^{(1+(-1))}$

$F = n, \; g = n^{0} = 1$

$F = \Omega(g)$

Let $\sin x = 1$

$F = n, \; g = n^{(1+1)}$

$f = n, \; g = n^2$

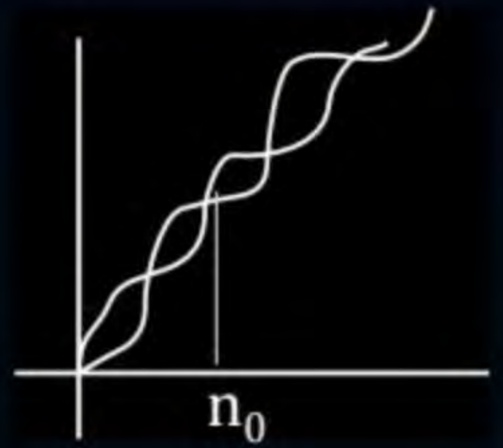
$F = O(g)$

Topic : Trichotomy Property

Observation from previous equation:

$$f(n) = n$$

$$g(n) = n^{(1+\sin x)}$$



- f(n) & g(n) are does not hold the trichotomy property as we can't get any clear asymptotic comparison between both the functions for all $n \geq n_0$.

Topic : Trichotomy Property

Conclusion:

- Asymptotic notations may or may not follow Trichotomy Property (holds sometimes)
- Hence, property does not hold

  (As property holds if its always satisfied for all cases)

$2 < 3$ & $3 < 5 \Rightarrow 2 < 5$

Topic : Analysis of Algorithms

Discrete Properties of Asymptotic Notations:

	Reflexive	Symmetric	Transitive	Transpose Symmetry
O	✓	×	✓	✓
Ω	✓	×	✓	✓
θ	✓	✓	✓	×
o	×	×	✓	✓
ω	×	×	✓	✓

$f = O(g)$

$g = O(f)$?

$f = O(g) \rightarrow g = O(f)$

$a < b \& b < c \rightarrow a < c$

Topic : Analysis of Algorithms

Let a, b be two real numbers and f, g be two functions of n.

(1) If $f(n) = O(g(n)) \Rightarrow a \leq b$

(2) If $f(n) = \Omega(g(n)) \Rightarrow a \geq b$

(3) If $f(n) = \theta(g(n)) \Rightarrow a = b$

(4) If $f(n) = o(g(n)) \Rightarrow a < b$

(5) If $f(n) = \omega(g(n)) \Rightarrow a > b$

Topic : Analysis of Algorithms

Adv Ques

#Q. f(n), g(n) are two functions [MCQ]

Given that f(n) = O(g(n))

- (a) $f(n) = O(f(n)^2)$ → False
- (b) $2^{f(n)} = O(2^{g(n)})$ → False
- Which of the following option is true ?

16 %

D

A Only (a) is true ✗

B Only (b) is true ✗

C Both (a) & (b) are true ✗

D Both (a) & (b) are false ✓

① $f(n) = n$

$(f(n))^2 = n^2$

$f(n) = O(f(n)^2)$

$n \leq n^2$

② $f(n) = \frac{1}{n^2}$

$(f(n))^2 = \left(\frac{1}{n^2}\right)^2 = \frac{1}{n^4}$

$\frac{1}{n^2} > \frac{1}{n^4}$

$f(n) > (f(n))^2$

(b)

(1) $f(n) = n$
$g(n) = 10$

?

No

(2) $f(n) = n$
$g(n) = n^2$

$2^n = O(2^{n^2})$

True

(3) $f(n) = 3n$

$g(n) = n$

$2^{3n} = O(2^n)$?

↳ False

Topic : Analysis of Algorithms

#Q. Which one of the following statements is True for all positive functions f(n)?

[MCQ]

A. $f(n^2) = \theta(f(n)^2)$, when f(n) is a polynomial

B. $f(n^2) = O(f(n)^2)$ → Small oh → False

C. $f(n^2) = O(f(n)^2)$, when f(n) is an exponential function → False

D. $f(n^2) = \Omega(f(n^2))$ → False

(A) $f(n^2) = O(f(n)^2)$, $F(n) \to \text{Poly}$

① $f(n) = n$

$f(n^2) = n^2$

$(f(n))^2 = n^2$ } ✓

② $f(n) = n^2$

$F(n^2) = (n^2)^2 = n^4$

$(f(n))^2 = (n^2)^2 = n^4$ }

(B) $F(n^2) = o(F(n)^2)$

$F(n) = n$

$F(n^2) = n^2$

$(F(n))^2 = n^2$

$n^2 = o(n^2)$ ✗

$n^2 > 2n$

(c) $f(n) = 2^n$ False

$f(n^2) = 2^{(n^2)}$

$(f(n))^2 = (2^n)^2$

$= 2^{2n}$

$2^{n^2} = O(2^{2n})$?

$2^{n^2} \not\leq 2^{2n}$

(D)

$f(n) = n^3$

$f(n^2) = (n^2)^3 = n^{2\times 3} = n^6$

$(F(n))^2 = (n^3)^2 = n^{3\times 2} = n^6$ ✓

Eg2 :

$f(n) = \log n$

$f(n^2) = \log(n^2)$

$= 2\log(n)$

$(f(n))^2 = (\log n)^2$

$f(n^2) < (f(n))^2$

$2\log n$ $\log n \times \log n$

$2 < \log n$

Topic : Analysis of Algorithms

$f < g = h$

#Q. Consider the following statements [MCQ]

1. f(n) is O(g(n))
2. g(n) is NOT O(f(n))
3. g(n) is O(h(n))
4. h(n) is O(g(n))

v. good

AJ Sir Conclusion Method

Which of the following option is TRUE ?

A. f(n) is O(h(n)) → $f \leq h$

B. h(n) ≠O(f(n)) → $h \leq f \Rightarrow h > f$

C. f(n) + h(n) is O(g(n)) + h(n) → $f + h \leq g + h \Rightarrow f \leq g$

D. f(n).g(n) ≠ O(g(n)). h(n)) → $f \times g \leq g \times h \Rightarrow f > h$

ans :- A, B, C

1) $f(n) = O(g(n)) \longrightarrow f \leq g$

2) $g(n) \neq O(f(n)) \longrightarrow g \not\leq f \longrightarrow g > f$

3) $g(n) = O(h(n)) \longrightarrow g \leq h$

4) $h(n) = O(g(n)) \longrightarrow h \leq g$

$\longrightarrow g = h$

$\longrightarrow g > f$

Conclusion $\Rightarrow$ $\boxed{f < g = h}$

Topic : Analysis of Algorithms

[MCQ]

#Q. $f(n) = 2^n$, $g(n) = n^n$

$2^n < n^n$

$2 < n$

$f < g$

A $f(n) = O(g(n))$

B $f(n) = \theta(g(n))$

C $f(n) = \Omega(g(n))$

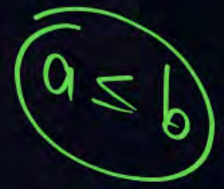
D None of these

Topic : Analysis of Algorithms

V. imp

(1) Every small oh(o) is also Big Oh(O), but every Big Oh(O), may or may not be small oh (o)

(2) Every small omega(ω) is also Big Omega(Ω), but every Big Omega(Ω), may or may not be small omega (ω).



Topic : Analysis of Algorithms

#Q. $f(n) = n.2^n$, $g(n) = 4^n$ [MSQ]

A $f(n) = O(g(n))$

B $f(n) = \theta(g(n))$

C $f(n) = \Omega(g(n))$

D None of these

$f = n \times 2^n < g = 4^n \longrightarrow f = O(g)$

$n \times 2^n$ $\quad 4^n = (2^2)^n = (2^n)^2$

$n \times 2^n$ $\quad 2^n \times 2^n$

$n < 2^n$

$$n \times 2^n < 4^n$$

Take $\log_2(\ )$

$\log_2(n \times 2^n)$ $\log_2(4^n)$

$\log n + n \times \log 2$ $n \times \log_2 4$

$n + \log n$ $2 * n$

~~n~~ $+ \log n$ ~~$n + n$~~

$\log n < n$

Topic : Analysis of Algorithms

#Q. Let w(n) and A(n) represent respectively, the worst case and average case running time of an algorithm with input size of n, Which is always TRUE? **[MCQ]**

$B \leq A \leq W$

A. $A(n) = o(w(n))$ $A < W$

B. $A(n) = \theta(w(n))$ $A = W$

C. $A(n) = \Omega(w(n))$ $A \geq W$

D. $A(n) = O(w(n))$ $A \leq W$

(D)

Topic : Analysis of Algorithms

#Q. Asymptotic Comparison of 2 functions: [MSQ]

$f(n) = n$

$g(n) = n \log n$

HW1

A $f = O(g)$

B $f = o(g)$

C $f = \Omega(g)$

D $f = \omega(g)$

Topic : Analysis of Algorithms

#Q. Asymptotic Comparison of 2 functions: [MSQ]

$f(n) = n^2 (\log n)$

$g(n) = n (\log n)^{10}$

Hw2

A $f = O(g)$

B $f = \Omega(g)$

C $f = \omega(g)$

D $f = o(g)$

THANK - YOU