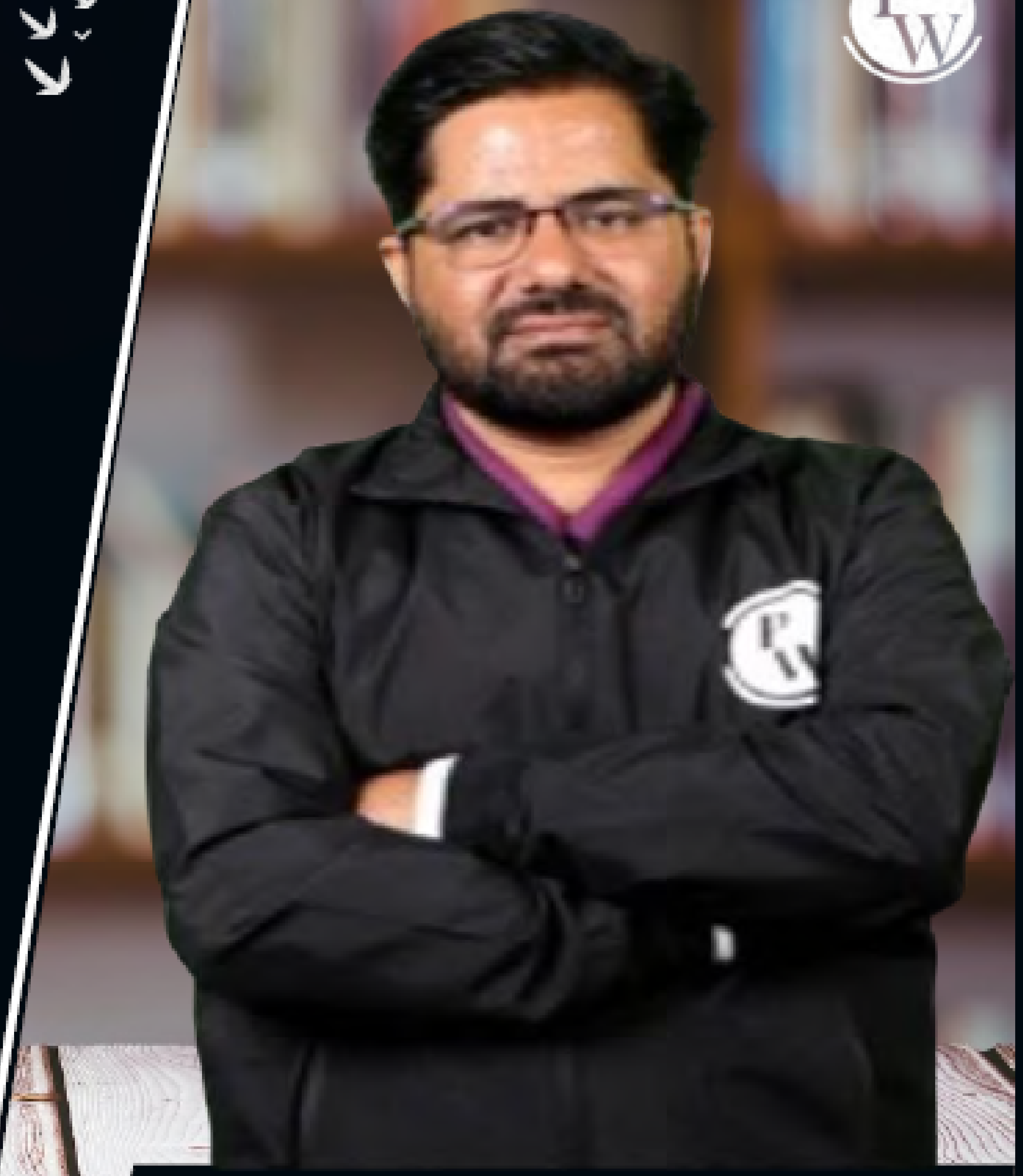
Computer Science & DA

Linear Algebra

DPP 02 Discussion Notes



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[NAT]

#Q. Find the rank of the matrix $A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow R_1} \begin{bmatrix} 2 & 1 & 0 & 1 \\ 6 & 3 & 4 & 7 \\ 4 & 2 & 1 & 3 \end{bmatrix}$

$R_2 \leftarrow R_2 - 3R_1$

$R_3 \leftarrow R_3 - 2R_1$ i.e. $A \sim \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 4 \end{bmatrix}$

$R_3 \leftarrow R_3 - R_2$ then $A \sim \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = \text{Two}$ Ans

[NAT]

#Q. Find the rank of the matrix

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftrightarrow R_6} \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = \text{three}$$

[NAT]

#Q. Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ a & a & a \\ a^3 & a^3 & a^3 \end{bmatrix}$ $R_2 \to \frac{1}{a} R_2$

$R_3 \to \frac{1}{a^3} R_3$

$$A \sim \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow[R_2 \to R_2 - R_1]{R_3 \to R_3 - R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \rho(A) = 1$$

[MCQ]

#Q. Let $M = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$. Then, the rank of M is-

$$\xrightarrow[\substack{R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 + R_1}]{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_4 \to R_4 - \frac{1}{2}R_2$ so $A \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \rho(A) = \text{two}$

A 3

B 4

C 2

D 1

[MCQ]

#Q. If P and Q are non- singular matrices, then for matrix M, which of the following is correct?

let $P_{4\times4} \Rightarrow \rho(P)=4$, let $Q_{3\times3} \Rightarrow \rho(Q)=3$

A Rank (PMQ) > Rank M

B Rank (PMQ) = Rank M

C Rank (PMQ) < Rank M

D Rank (PMQ) = Rank M + Rank (PQ)

$$PMQ = P_{4\times4} M_{4\times3} Q_{3\times3}$$

$$\leq \min\{4, \rho(M), 3\}$$

$$= \rho(M)$$

$$\leq \rho(M)$$

[MCQ]

#Q. Rank of singular matrix of order 4 can be at most

$|A| = 0$ where $A_{4\times4} \Rightarrow \rho(A) \neq 4$

A 1

B 2

C 3

D 4

[MCQ]

#Q. The rank of $(m \times n)$ matrix (where $m < n$) cannot be more than

$\rho(A_{m \times n}) \leq m$

A. m

B. n

C. mn

D. Non

[MCQ]

#Q. If for a matrix, rank equals both the number of row and number of columns, then the matrix is called.

$A_{n\times n}$ s.t $\rho(A)=n$ → No. of LI Row vectors $= n$

" " Column " $= n$

$|A| \neq 0$

A. Non- singular

B. singular

C. transpose

D. minor

~~[NAT]~~ MSQ

#Q. Determine whether each of the following sets of vectors is a linearly independent subset of V.

(a) $V = R^2, \{(1, 0,), (-1, -1)\}$ $x_1 \neq k x_2$ LI

(b) $V = R^2, \{(1, -1,)\}, (1, 1), (2, 1)\}$ $V = [x_1 x_2 x_3] = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & 1 \end{bmatrix}_{2\times 3} \Rightarrow \rho(V) \leq 2$ < No of Vectors LD

(c) $V = R^3, \{(1,1,0,)\}, (-1, 1,1)\}$ $\because x_1 \neq k x_2$ so LI

(d) $V = R^3, \{(1,0,0,)\}, (1, 1,0), (1, 1, 1)\}$ $V = (x_1 x_2 x_3) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |V| \neq 0 \Rightarrow$ LI

~~$V = R^3, \{(1,0,0,)\}, (1, 1,0), (1, 1, 1)\}$~~

$V = \{x_1 x_2 x_3 \ldots x_r\}$ — if $\rho(V) = r$ LI; if $\rho(V) < r$ LD

or $|A| \neq 0 \Rightarrow$ L.I; $|A| = 0 \Rightarrow$ L.D

[MCQ]

#Q. A set of r, n dimensional vector x_1, x_2, x_3 ...... $\cancel{xr}$ x_r is said to be linearly independenly, if every relation of the type $k_1x_1 + k_2\,x_2 + k_3\,x_3 + \ldots\ldots\ldots + k_r\,x_r = 0$ implies.

$$K_1X_1 + K_2X_2 + K_3X_3 + \ldots\ldots + K_rX_r = 0$$

$$\Rightarrow K_1 = K_2 = K_3 = \ldots\ldots = K_r = 0$$

A. $k_1 + k_2 + k_3 + \ldots\ldots\ldots\ldots + k_r = 0$

B. $k_1 = k_2 = k_3 = \ldots\ldots\ldots\ldots = 0$ ✓

C. $k_1 + k_2 + k_3 + \ldots\ldots\ldots\ldots + k_r = 0$

D. None

[MCQ]

#Q. If A is matrix of order ~~$n \times m$~~ $m \times m$ such that A in singular then column vectors are

$|A| = 0$

$\Downarrow$

$\rho(A) < m$

$\Downarrow$

column vectors are LD

A. LD

B. LI

C. orthogonal

D. orthonormal

[MCQ]

#Q. If there exist $\underbrace{\text{no relationship be}}$tween the column vectors of $A_{m \times n}$ then

$LI \Rightarrow \rho(A) = n$

A $\rho(a) < n$

B $\rho(a) = n$

C $\rho(a) < m$

D $\rho(a) \leq n$

[MCQ]

#Q. Find λ for which there exits a linear relationship between the vector

$\hat{i}+2\hat{j}+3\hat{k}$;

$4\hat{i}+5\hat{j}+6\hat{k}$, $\lambda\hat{i}+8\hat{j}+9\hat{k}$.

$LD \Rightarrow |A|=0$ or $\rho(A)<3$

$A=[x_1\ x_2\ x_3]$

$= \begin{bmatrix} 1 & 4 & \lambda \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}_{3\times 3}$

$\begin{vmatrix} 1 & 4 & \lambda \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{vmatrix}=0$

$1[45-48]-4[18-24]+\lambda[12-15]=0$

$\lambda=7$

A $\lambda = 3$

B $\lambda = 7$

C $\lambda \pm 7$

D $\lambda = 0$

[NAT]

$(1,-4,2)$

#Q. (a) Show that (2, 1, 1) and ~~(1,)~~ are orthogonal.

(b) Determine which of the following vectors are orthogonal :

$v_1 = (-2, 6, 1)$, $V_2 = (9, 2, 6)$, $V_3 = (4, -15, -1)$.

$$X = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad Y = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$X \cdot Y = a_1 b_1 + a_2 b_2 + a_3 b_3$$

(1) $X = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad Y = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix} \Rightarrow X \cdot Y = 2 + (-4) + 2 = 0$

$V_1 \cdot V_2 = \begin{bmatrix} -2 \\ 6 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix} = -18 + 12 + 6 = 0$ i.e. V_1 & V_2 are O.V.

$V_2 \cdot V_3 = \begin{bmatrix} 9 \\ 2 \\ 6 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -15 \\ -1 \end{bmatrix} = 36 - 30 - 6 = 0$ i.e. V_2 & V_3 are O.V.

$V_3 \cdot V_1 = \begin{bmatrix} 4 \\ -15 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ +6 \\ 1 \end{bmatrix} = -8 - 90 - 1 \neq 0$ Not orthogonal

i.e. V_1 & V_3 are not O.

~~[MCQ]~~ MSQ

#Q. Among the following, the pair of the vector orthogonal to each other is

A [3, 4, 7] , [3, 4, 7] $\because x \cdot y \neq 0$

B [0, 0, 0], [1, 1, 0]

C [1, 0, 2], [0, 5, 0] $\Rightarrow x \cdot y = (1)(0) + (0)(5) + (2)(0) = 0$

D [1, 1, 1] , [–, –1, 1] $\because x \cdot y \neq 0$

[MCQ]

#Q. If $\overline{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\overline{b} = 4\hat{i} + 3\hat{j} - \lambda\hat{k}$ are orthogonal then $\lambda = ?$

$$= \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} \qquad = \begin{bmatrix} 4 \\ 3 \\ -\lambda \end{bmatrix}$$

$\because \vec{a} \cdot \vec{b} = 0$

$(3)(4) + (-2)(3) + (1)(-\lambda) = 0$

$12 - 6 - \lambda = 0$

$\lambda = 6$

A. 6

B. 12

C. -6

D. -12

~~[MCQ]~~ MSQ

#Q. The vector which is orthogonal is every column vector of $A = \begin{array}{ccc} \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \\ C_1 \quad C_2 \quad C_3 \end{array}$ will be?

A $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

B $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

C $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix} = C$

D $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$C \cdot C_1 = (-1)(1) + (-1)(1) + (1)(1) + (1)(1) = 0$

$C \cdot C_2 = 0$ & $C \cdot C_3 = 0$

[NAT]

#Q. Norm of vector $[8\ 4\ \ 1]^T$ is given as ______?

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ then } \|\vec{X}\| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

$$\|\vec{X}\| = \sqrt{8^2 + 4^2 + 1^2} = \sqrt{81} = 9 \quad \underline{\underline{\text{Ans}}}$$

$$\sqrt{81} \begin{cases} +9 \ \checkmark \\ -9 \ \times \end{cases}$$

$$\left(\because \sqrt{x^2} = |x| = +ve\right)$$

[NAT]

#Q. The $\underbrace{\text{normalized vector}}_{\text{unit vector}}$ of $\begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} = X$ will be?

$$\hat{X} = \frac{\vec{X}}{\|\vec{X}\|} = \frac{1}{\sqrt{21}} \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{21} \\ 1/\sqrt{21} \\ -4/\sqrt{21} \end{bmatrix}$$

$$\|\vec{X}\| = \sqrt{4+1+16} = \sqrt{21}$$

[NAT]

#Q. For what values of α and β, the following simultaneous equations have an infinite number of solution?

$x + y + z = 5$

$x + 3y + 3z = 9$

$x + 2y + \alpha z = \beta$

$$[A \vdots B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 3 & 3 & 9 \\ 1 & 2 & \alpha & \beta \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 1 & \alpha-1 & \beta-5 \end{array}\right]$$

$$R_3 \to R_3 - \frac{1}{2}R_2 \quad \text{so} \quad [A \vdots B] \equiv \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & \alpha-2 & \beta-7 \end{array}\right]$$

for ∞ sol^{n}s:

$\rho(A) = \rho(A \vdots B) <$ No. of variables

$\rho(A) = \rho(A \vdots B) < 3$

let $\rho(A) = 2$ & $\rho(A \vdots B) = 2$

$\Downarrow \qquad\qquad \Downarrow$

$\alpha = 2 \qquad \beta = 7$

Ans $\alpha = 2$ & $\beta = 7$

[MCQ]

#Q. The following system of equations

$$x_1 + x_2 + 2x_2 = 1$$

$$x_1 + 2x_2 + 3x_3 = 2$$

$$x_1 + 4x_2 + ax_3 = 4$$

has a unique solution. The only possible value of a is/are

$|A| \neq 0$

For unique soln $|A| \neq 0$

$$\begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 4 & a \end{vmatrix} \neq 0$$

$R_2 - R_1$
$R_3 - R_1$

$$\begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 3 & a-2 \end{vmatrix} \neq 0 \Rightarrow 1[a-2-3] \neq 0$$

$$a \neq 5$$

A 0

B either 0 or 1

C one of 0, 1 and –1

D and real number other than 5

[MCQ]

#Q. The solution(s) to the equations $2x + 3y = 1$, $x - y = 4$, $4x - y = a$, will exists if a is equal to

consistent $\Downarrow$ $\rho(A) = \rho(A:B)$

A -33

B 0

C 9

D 59/5

$$[A:B] = \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 1 & -1 & 4 \\ 4 & -1 & a \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 2 & 3 & 1 \\ 4 & -1 & a \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 5 & -7 \\ 0 & 3 & a-16 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{5}R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 1 & -7/5 \\ 0 & 3 & a-16 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_2} \left[\begin{array}{cc|c} 1 & -1 & 4 \\ 0 & 1 & -7/5 \\ 0 & 0 & \frac{5a-59}{5} \end{array}\right]$$

$$a - 16 + \frac{21}{5}$$

$$\frac{5a - 80 + 21}{5} = \frac{5a-59}{5}$$

$$\rho(A) = 2 \Rightarrow \rho(A:B) = 2 \Rightarrow a = \frac{59}{5}$$

[MCQ]

#Q. For the following set of simultaneous equations:

$1.5x - 0.5y = 2$
$4x + 2y + 3z = 9$
$7x + 1y + 5z = 10$

$\Rightarrow |A| = \begin{vmatrix} 1.5 & -0.5 & 0 \\ 4 & 2 & 3 \\ 7 & 1 & 5 \end{vmatrix} \xrightarrow{C_1 \to C_1 + 3C_2} \begin{vmatrix} 0 & -0.5 & 0 \\ 10 & 2 & 3 \\ 10 & 1 & 5 \end{vmatrix}$

$= -(-0.5)[50 - 30] = \frac{20}{2} = 10$

$\because |A| \neq 0 \Rightarrow$ By Matrix Method $\Downarrow$ unique soln.

A The solutions is unique

B Infinite many solution exists

C The equations are incompatible

D Finite number of multiple solution exist.

[MCQ]

#Q. The condition for consistency of simultaneous equation $\boxed{AX = B}$ where $A : B = C$

$\Rightarrow \rho(A) = \rho(A \vdots B)$

$\rho(A) = \rho(C)$

A. Rank A = Rank C

B. Rank A ≠ Rank C

C. Rank A = Rank B

D. None of these

[MCQ]

$\rho(A) \neq \rho(C) \Rightarrow$ No soln.

$\rho(A) = \rho(C) =$ No. of unk $\Rightarrow$ unique soln.

#Q. In the system of equation AX = B and $[A \vdots B] = C$

s = (a) If the rank of A is not equal to rank of C (p) consistent with unique solution

p = (b) If the rank of A is ~~not~~ equal to rank of C = No. of unknowns (q) Infinite solutions ~~consistent with~~

q = (c) If the rank A = rank of C < No. of unknowns (r) have a solution

r = (d) The solution of AX = 0 is always (s) inconsistent

A ✓ $a \to s, b \to p, c \to q, d \to r$

B ✗ $a \to s, b \to q, c \to p, d \to p$

C ✗ $a \to s, b \to p, c \to q, d \to q$

D ✗ $a \to s, b \to p, c \to r, d \to q$

[MCQ]

#Q. The values of k for which equations $x + y + z = 1$, $x + 2y + 4z = k$, $x + 4y + 10z = k^2$ have a solution $\Rightarrow$ Consistency $\Rightarrow \rho(A) = \rho(A \vdots B)$

A. 1 or 2

B. 3 or 4

C. 5 or 6

D. Any values

$$[A \vdots B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & k \\ 1 & 4 & 10 & k^2 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - R_1} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & k-1 \\ 0 & 3 & 9 & k^2-1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_2} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & k-1 \\ 0 & 0 & 0 & (k^2-1-3k+3) \end{array}\right]$$

$\rho(A) = 2 \Rightarrow \rho(A \vdots B) = 2 \Rightarrow k^2 - 3k + 2 = 0$

$(k-1)(k-2) = 0$

$k = 1 \,\&\, 2$

[NAT]

#Q. For what value of b the following system of equations has Non- trivial solution?

$2x + y + 2z = 0$

$x + y + 3z = 0$

$4x + 3y + bz = 0$

$|A| = 0$

$$\begin{vmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & b \end{vmatrix} = 0$$

$2[b-9] - 1[b-12] + 2[3-4] = 0$

$2b - 18 - b + 12 - 2 = 0$

$b - 8 = 0$

$b = 8$

$\Downarrow$

$|A| = 0$

[MCQ]

m = No. of eqns & n = No. of variables.

#Q. Let $AX = B$ be a system of linear equations where A is an $m \times n$ matrix and B is an $n \times 1$ column matrix which of the following is false?

A. The system has a solution, it $\rho A) = r(A/B)$ (T)

B. If $m = n$ and B is non-zero vector then the system has a unique solution (F)

$\rho(A) = \rho(A \vdots B) = n$

C. If $m < n$ and B is a zero vector then the system has infinitely many solutions (T)

D. The system will have a trivial solution when $m = n$, B is the zero vector and rank of A is n. (T)

[MCQ]

#Q. Let A be a square matrix of order n, then nullity of A is

Let $A_{n\times n} \Rightarrow N(A) = \text{order} - \rho(A)$

$= n - r$

A. n – rank A

B. rank A – n

C. n + rank A

D. None of these

[MCQ]

$\Rightarrow A_{n\times n} X_{n\times 1} = O_{n\times 1}$

#Q. An n × n homogenous system of equations AX = 0 is given. The rank of A is r < n. Then the system has

$\Rightarrow \rho(A) = r < n$

A n – r independent solutions

i.e $\rho(A) <$ No. of Variables

B r independent solutions

$\Downarrow$

Non Zero Soln

C no solution

(or ∞ Solns)

D n – 2r independent solutions

No. of LI Soln of Homog System AX = 0 is Called N(A)

$N(A) = n - r$

[MCQ]

#Q. The simultaneous equation

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$r =$ (i) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ (p) no solution

$q =$ (ii) $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ (q) unique solution

$p =$ (iii) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ (r) infinitely many solutions

$e =$ (iv) $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ (s) None of these

$2x + 3y = 5$
$2x - 3y = 4$ $\Rightarrow \dfrac{a_1}{a_2} = 1$ & $\dfrac{b_1}{b_2} = -1$

is $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2} \Rightarrow$ unique sol.

$4x = 9$ & $3y = 5 - \dfrac{9}{2} \Rightarrow y = \dfrac{1}{6}$

$x = \dfrac{9}{4}$

$2x + 3y = 5$
$4x + 6y = 10$ $\Rightarrow \dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2} = \dfrac{1}{2}$ ∞ sol^n.

$2x + 3y = 5$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1/3 \end{bmatrix} \ldots \infty$ sol^n.

A

$a \to r$

$b \to q$

$c \to p$

$d \to q$

B

$a \to p$

$b \to s$

$c \to q$

$d \to r$

C

$a \to s$

$b \to p$

$c \to r$

$d \to q$

D None of these

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (unique sol)

& $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (∞ soln)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (No soln)

[MCQ]

#Q. If $x + 2y - 2u = 0$, $2x - y - u = 0$, $x + 2z - u = 0$, $4x - y + 3z - u = 0$ is a system of equations, then it is

(A) consistent with trivial solution

(B) consistent without trivial solution

(C) inconsistent with trivial solution

(D) inconsistent without trivial solution

$$\left.\begin{aligned} x+2y+0z-2u&=0\\ 2x-y+0z-u&=0\\ x+0y+2z-u&=0\\ 4x-y+3z-u&=0 \end{aligned}\right\} \Rightarrow A=\begin{bmatrix} 1 & 2 & 0 & -2\\ 2 & -1 & 0 & -1\\ 1 & 0 & 2 & -1\\ 4 & -1 & 3 & -1 \end{bmatrix}$$

$$\begin{vmatrix} + & - & + & -\\ - & + & - & +\\ + & - & + & -\\ - & + & - & + \end{vmatrix}$$

ie $|A| \neq 0$

$$|A| = 0 - 0 + 2\begin{vmatrix} 1 & 2 & -2\\ 2 & -1 & -1\\ 4 & -1 & -1 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 & -2\\ 2 & -1 & -1\\ 1 & 0 & -1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 2 & -2\\ 2 & -1 & -1\\ 2 & 0 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 & -1\\ 2 & -1 & 1\\ 1 & 0 & 0 \end{vmatrix}$$

$$= 2(2)[-2-2] - 3[2-1] = -16-3 = -19$$

[MCQ]

#Q. The equations kx + y + z = 0, −x + ky + z = 0, −x − y + kz = 0 will have non-zero solution, when real k is

A 3

B Zero

C 1

D $\sqrt{3}$

for Non Zero Solⁿ of $AX = 0$ we have

$$|A| = 0$$

$$A = \begin{bmatrix} k & 1 & 1 \\ -1 & k & 1 \\ -1 & -1 & k \end{bmatrix}$$

$$\begin{vmatrix} k & 1 & 1 \\ -1 & k & 1 \\ -1 & -1 & k \end{vmatrix} = 0$$

$$\begin{vmatrix} k & 1 & 1 \\ -1 & k & 1 \\ 0 & -1-k & k-1 \end{vmatrix} = 0$$

$$+k\{k^2 - k + k + 1\} - (-1)\{k - 1 + k + 1\} = 0$$

$$k^3 + k + 2k = 0$$

$$k(k^2 + 3) = 0$$

$$k = 0$$

$$k = \pm i\sqrt{3}$$

[MCQ]

#Q. For the given set of equations:

$x + y = 1$

$y + z = 1$

$x + z = 1,$

Which one of the following statements is correct?

A. Equations are inconsistent ✗

B. Equations are consistent and a single nontrivial solution exists ✓

C. Equations are consistent and many solutions ✗

D. Equations are consistent and many solutions ✗

$x+y+0z=1$

$0x+y+z=1 \Rightarrow |A| = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$

$x+0y+z=1$

$= 1[1-0]-1[0-1]+0$

$= 1+1 = 2$

i.e. $|A| \neq 0$

$\Downarrow$

unique sol^n exist

THANK - YOU