DS & AI
CS & IT

Linear Algebra

Lecture No. 09

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Recap of previous lecture

Topic

EIGEN VALUES & their properties

Topics to be Covered

Topic

(1) CAYLEY-HAMILTON THEOREM

(2) Methods of finding Eigen Vectors

Cayley-Hamilton Theorem → "Every square Mat satisfies it's own C.Eq."

i.e. we can replace $\lambda \to A$ in C.Equ^n.

Consider $A_{n\times n}$ then it's C.Equ^n is $|A-\lambda I|=0$

$$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots\dots + a_{n-1}\lambda + a_n = 0 \longrightarrow \text{Algebraic Equ}^n$$

using C.H.T, $\lambda \to A$

$$1.A^n + a_1A^{n-1} + a_2A^{n-2} + \dots\dots + a_{n-1}A + a_nI = O \longrightarrow \text{Matrix Equ}^n$$

Application: with the help of C.H.T we can calculate $Tr(A)$, $|A|$, A^{-1} & $A^m = ?$

① $Tr(A) = -(a_1)$

② $|A| = (-1)^n a_n$

③ A^{-1} = will be discussed in Questions

④ A^m = " " " " "

Take Care :- Above Results are valid when coeff of $A^n = 1$

Some More Conclusions from C.H.T. :-

(1) Constant term in the C.Eq of $A_{n\times n} = \boxed{a_n = \begin{cases} -|A|, & n = \text{odd} \\ +|A|, & n = \text{even} \end{cases}}$

Proof: we know that (by C.H.T)

$$|A| = (-1)^n a_n$$

$$(-1)^n |A| = (-1)^n \cdot (-1)^n a_n$$

$$(-1)^n |A| = (-1)^{2n} \cdot a_n$$

$$(-1)^{2n} a_n = (-1)^n |A|$$

i.e. $a_n = (-1)^n |A| = \begin{cases} -|A|, & n = \text{odd} \\ +|A|, & n = \text{even} \end{cases}$

(2) **Shortcut Method of finding C.Eq. of $A_{2\times 2}$ →**

C.Equ^n is $|A - \lambda I| = 0$

$$\lambda^2 + a_1 \lambda + a_2 = 0$$

$$\lambda^2 - (-a_1)\lambda + \{(-1)^2 a_2\} = 0$$

$$\boxed{\lambda^2 - (\text{Tr}A)\lambda + (|A|) = 0}$$

Q. for the Matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ Evaluate $Tr(A)$, $|A|$, & A^{-1}? using C.H.Th.

Sol: C. Equ^n of A is $|A-\lambda I|=0$

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$(\lambda-3)(\lambda-1)^2=0$

$\lambda^3-5\lambda^2+7\lambda-3=0$

By C.H.Th, $\lambda \rightarrow A$

$\boxed{A^3-5A^2+7A-3I=O}$ — ①

on Comparision with,

$A^3+a_1A^2+a_2A+a_3I=O$.

① $Tr(A)=-(a_1)=-(-5)=+5$ ✓

② $|A|=(-1)^3a_3=-(-3)=+3$ ✓

③ $\because |A|\neq 0$ ie A^{-1} exist. So By ①,

$A^{-1}(A^3-5A^2+7A-3I)=A^{-1}\cdot O$

$A^2-5A+7I-3A^{-1}\cdot I=O$

$-3A^{-1}=-(A^2-5A+7I)$

$A^{-1}=\frac{1}{3}(A^2-5A+7I)$ Ans

(iv) Evaluate $B = \underbrace{A^8 - 5A^7 + 7A^6 - 3A^5} + \underbrace{A^4 - 5A^3 + 7A^2 + 4A} + 2I = ?$

By (1) we know that

$$\boxed{A^3 - 5A^2 + 7A - 3I = 0} \quad (1)$$

(a) $A + 2I$ (b) $2I$

(c) $7A + 2I$ (d) O

$$B = A^5(A^3 - 5A^2 + 7A - 3I) + A(A^3 - 5A^2 + 7A - 3I) + 7A + 2I$$

$$= A^5(O) + A(O) + 7A + 2I$$

$$B = 7A + 2I$$

(ii) Also Evaluate $|B| = ? = |7A + 2I| = \cancel{7|A| + 2|I| = 7(3) + 2(1) = 23}$

$$B = 7A + 2I = 7\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 16 & 7 & 7 \\ 7 & 16 & 7 \\ 0 & 0 & 9 \end{bmatrix}$$

$$|B| = 9[256 - 49] = 9 \times 207 = 1863$$

Q. If $P_{3\times3}$ Mat s.t. it's C.Eq is $a(\lambda) = |\lambda I - P| = \boxed{\lambda^3 + \lambda^2 + \lambda + 1 = 0}$

then evaluate $Tr(P)$, $|P|$, P^{-1} $f(\lambda)$

Sol: C.Equ^n of P is $|P - \lambda I|_{3\times3} = 0$

$$(-1)^3 |\lambda I - P| = 0$$

$$|\lambda I - P| = 0$$

$$\lambda^3 + \lambda^2 + \lambda + 1 = 0$$

By C-H-T, $\lambda \to P$

$$P^3 + 1P^2 + 1P + 1I = 0 \quad \text{--- (1)}$$

on Comparision with, $(P^3 + a_1 P^2 + a_2 P + a_3 I = 0)$

(i) $Tr(P) = -(a_1) = -(1) = -1$

(ii) $|P| = (-1)^3 a_3 = -(1) = -1$

(iii) By (1), $P^{-1}(P^3 + P^2 + P + I) = P^{-1} \cdot 0$

$$P^2 + P + I + P^{-1} = 0$$

$$P^{-1} = -(P^2 + P + I)$$

(iv) if $B = \underbrace{P^8 + P^7 + P^6 + P^5} + \underbrace{P^4 + P^3 + P^2 + P} + 4I$ then $|B| = ?$

(a) 1

(b) -1

(c) 4

(d) 64

$$= P^5(P^3 + P^2 + P + I) + P(P^3 + P^2 + P + I) + 4I$$

$$B = P^5(0) + P(0) + 4I$$

$$\boxed{B = 4I} \Rightarrow |B| = \underset{3\times 3}{|4I|} = 4^3 |I| = 64(1) = 64$$

Q. Evaluate the constant term in the C.Equ^n of $A = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}_{4\times 4}$

(M-I) C.Equ^n is $|A-\lambda I| = 0$

$$\begin{vmatrix} (2-\lambda) & 1 & 1 & 1 \\ 1 & (2-\lambda) & 1 & 1 \\ 1 & 1 & (2-\lambda) & 1 \\ 1 & 1 & 1 & (2-\lambda) \end{vmatrix} = 0$$

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$$\lambda^4 - 8\lambda^3 + a_2\lambda^2 + a_3\lambda + (5) = 0$$

So constant term $= 5$ Ans

(M-II) $|A| = \text{.........} = 5$

$\because$ A is of even order so

$$a_n = (-1)^4 |A| = +|A| = +5 \quad \text{Ans}$$

Q. Constant term in the C.Eq of $\begin{bmatrix} 0 & 1 & 2 & -3 & 4 \\ -1 & 0 & 4 & -1 & 2 \\ -2 & -4 & 0 & 3 & 0 \\ 3 & 1 & -3 & 0 & 2 \\ -4 & -2 & 0 & -2 & 0 \end{bmatrix}_{5\times 5}$

$\because$ A is Skew Symm Mat of odd order

so $|A| = 0 \Rightarrow$ const term $a_5 = -|A| = 0$

Q. if $A = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$ then A will satisfy ?

(a) $A^2 - 2A - 3I$

(b) $(A+I)(A-3I) = 0$

(c) $A - 2I - 3A^{-1} = 0$

(d) $A^2 - 3A + 2I = 0$

C.Equ^n of A is $|A - \lambda I| = 0$

$\lambda^2 - (Tr(A))\lambda + |A| = 0$

$\lambda^2 - (2)\lambda + (-3) = 0$

By C.H.T, $\boxed{A^2 - 2A - 3I = 0}$ Ans

A^{-1} factor

$\boxed{A - 2I - 3A^{-1} = 0}$ Ans

$\boxed{(A+I)(A-3I) = 0}$ Ans

TAG DAA QUEST :-

Qs if $A = \begin{bmatrix} -1 & -2 \\ 0 & 3 \end{bmatrix}$ then which is/are True

MSQ

(a) $A^2 - 2A - 3I$

(b) $(A-3I)(A+I) = 0$

(c) $A - 2I - 3A^{-1}$

(d) $A^2 - 3A - 2I$

Q. If E-Values of $A_{4\times4}$ are ±1 & $\pm i$ then $A^4 = ?$

(a) A

(b) O

(c) I_3

(d) I_4 ✓

$\because \lambda = 1, -1, i, -i$

So C-Equ^n is

$$(\lambda-1)(\lambda+1)(\lambda-i)(\lambda+i) = 0$$

$$(\lambda^2-1^2)(\lambda^2-i^2) = 0$$

$$(\lambda^2-1)(\lambda^2+1) = 0$$

$$(\lambda^4-1^2) = 0$$

$$\lambda^4-1 = 0$$

By C-H-Th, $\lambda \to A$

$$A^4 - I = O$$

$$A^4 = I = I_{4\times4} = I_4$$

Q: if CE of $A_{4\times4}$ is $2\lambda^4-6\lambda^3+4\lambda^2-8\lambda+12=0$ then $|A|=$?

Sol: C.Equ^n is $\lambda^4-3\lambda^3+2\lambda^2-4\lambda+6=0$

$$\text{So } |A|=(-1)^4 a_4=(+1)(6)=6$$

Q. Constant term in the CEq of $A=\begin{bmatrix}0&1&2&3\\1&0&3&0\\2&3&0&1\\3&0&1&2\end{bmatrix}$ will be ?

HW

$\because |A|=\ldots\ldots\ldots=88$

So constant term in C.Equ^n $=a_n=(-1)^4|A|=+(88)$

EIGEN VECTORS

Def^n: Consider Sq. Mat $A_{n\times n}$ then Non Zero Vector X is called Eigen vector, corresponding to Eigen value λ (Real/Complex/Zero) if we are able to find a relationship of the type,

$$\boxed{AX = \lambda . X}$$

λ = Eigen value
X = Eigen vector.

LHS is the Multi of Two Matrices = RHS is the Scalar Multi in a Mat
(Tough) (Easy)

(*) Here we are considering Homogeneous system as follows

$$AX = \lambda X \Rightarrow AX - \lambda X = 0 \Rightarrow \boxed{(A - \lambda I)X = 0}$$

So it will satisfy all the prop of Homog System.

PROPERTIES of E. VECTORS →

(1) If X is an E. Vector of A then (KX) will also be an E-Vector

$$\underbrace{\text{If } X \text{ is an E. Vector of } A}_{AX = \lambda X} \iff \underbrace{(KX) \text{ will also be an E-Vector}}_{A(KX) = \lambda(KX)}$$

$$AX = \lambda X$$

$$\boxed{(A - \lambda I)X = 0}$$

E. Vector

$$K(A - \lambda I)X = K \cdot 0$$

$$\boxed{(A - \lambda I)(KX) = 0}$$

E Vector

ie " we are free to multiply or divide with any constant in case of E Vector.

(2) E-Vectors Corresponding to different E-Values of Symm Mat are orthogonal

Q (PYQ) if $A_{3\times3}$ s.t $A^T = A$ = Symm. and a, b are the E-Values & Corresponding E-Vectors are $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ & $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ then for $a \neq b$ different E-Values the value of

$x_1y_1 + x_2y_2 + x_3y_3 = \underline{\ ?\ }$

By above property $X \cdot Y = 0 \Rightarrow X^T Y = 0$

$\Rightarrow x_1y_1 + x_2y_2 + x_3y_3 = 0$

(3) For Different E-Values, Corresponding E-Vectors are also LI

(4) If E-Value Repeats (then Headache will start) then Corresponding E Vectors may be LI or may be LD.

(5) MODAL MATRIX : → Matrix formed by E-Vectors is Called Modal Mat

Let $A_{3\times 3}$ is the given Mat & X_1, X_2, X_3 are the E-Vectors then

Modal Mat is $P = [X_1 \; X_2 \; X_3]$

Eg: for $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

$\rightarrow \lambda = 6,\ X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \because AX_1 = 6X_1$

$\rightarrow \lambda = 2,\ X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad \because AX_2 = 2X_2$

Now we try to understand the Procedure of finding E. vector.

Q. Find the E-Values & E-Vectors of $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

Sol: C-Eqⁿ is $|A - \lambda I| = 0 \Rightarrow \lambda^2 - (8)\lambda + (12) = 0 \Rightarrow (\lambda - 6)(\lambda - 2) = 0 \Rightarrow \lambda = 6 \,\&\, 2$

Eigen Vector for ($\lambda = 6$) →

Consider $AX = \lambda X$

$$(A - \lambda I)X = 0$$

$$(A - 6I)X = 0$$

$$\begin{bmatrix} (4-6) & 2 \\ 2 & (4-6) \end{bmatrix} X = 0$$

$$\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} X = 0$$

$R_2 \to R_2 + R_1$

$$\underset{2\times 2}{\begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} \Rightarrow \begin{bmatrix} -2x_1 + 2x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

ie $\left.\begin{aligned} -2x_1 + 2x_2 &= 0 \\ \& \quad 0 &= 0 \end{aligned}\right\} \Rightarrow x_1 = x_2 = K$ (let)

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} K \\ K \end{bmatrix} = K \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

E.Vector for $\lambda=2$ →

Consider $AX=\lambda X$

$$(A-\lambda I)X=0$$

$$(A-2I)X=0$$

$$\begin{bmatrix} (4-2) & 2 \\ 2 & (4-2) \end{bmatrix} X = 0$$

$$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} X = 0$$

$R_2 \to R_2 - R_1$ $$\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (2x_1+2x_2) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left.\begin{matrix} 2x_1+2x_2=0 \\ 0=0 \end{matrix}\right\} \Rightarrow x_1=-x_2$$

Let $x_2=K$ then $x_1=-K$ so

$$X=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -K \\ K \end{bmatrix} = K\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Finally, $A=\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ → $\lambda=6,\ X=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

→ $\lambda=2,\ X=\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Analysis (PODCAST) →

① $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

$\lambda = 6,\ X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 1/4 \\ 1/4 \end{bmatrix}$ ----- ∞ Evectors exist.

But all are LD on X_1

$\lambda = 2,\ X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1/5 \\ -1/5 \end{bmatrix}$ --- ∞ Evectors exist.

But all are LD on X_2

② Here X_1 & X_2 are LI (By observation)

Actually we are getting two Ind families of E.vectors. & if we are taking one member from F_1 & one member from F_2 then these are LI

i.e for Mat A, ∃ two LI eigenvectors at a time

(3) $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$, $\begin{cases} \lambda = 6, \; X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ \lambda = 2, \; X_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \end{cases}$ $\because X_1 \cdot X_2 = (1)(-1) + (1)(1)$
$= 0$

= Symm Mat.

i.e X_1 & X_2 are orthogonal vectors $\Longrightarrow$ Hence they are LI also.

i.e for different eigen values of symm Mat, corresponding E.vectors are orthogonal as well as LI

(4) In this Mat, $X_1 \cdot X_2 = \begin{bmatrix}-1\\1\end{bmatrix} \cdot \begin{bmatrix}1\\1\end{bmatrix} = (-1)(1) + (1)(1) = 0$

i.e. X_1 & X_2 are orthogonal Vectors.

(5) one pair of L-I E vectors of $A = \begin{bmatrix}4 & 2\\2 & 4\end{bmatrix}$ will be

HW

(a) $\begin{bmatrix}-3\\3\end{bmatrix}, \begin{bmatrix}4\\-4\end{bmatrix}$

(b) $\begin{bmatrix}3\\3\end{bmatrix}, \begin{bmatrix}-1\\-1\end{bmatrix}$

(c) $\begin{bmatrix}-1\\1\end{bmatrix}, \begin{bmatrix}2\\3\end{bmatrix}$

(d) $\begin{bmatrix}3\\-3\end{bmatrix}, \begin{bmatrix}0.5\\0.5\end{bmatrix}$

Q. find E-Values & E-Vectors of $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

Sol: $\because$ A is U.T.M so $\lambda = 1 \& 2$

E-Vector for $\lambda = 1$ $\longrightarrow$

$$AX = \lambda X$$

$$(A - \lambda I)X = 0$$

$$(A - I)X = 0$$

$$\begin{bmatrix} (1-1) & 2 \\ 0 & (2-1) \end{bmatrix} X = 0$$

$$\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} X = 0$$

$R_2 \to R_2 - \frac{1}{2}R_1$; $\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 2x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_2 = 0$, Let $x_1 = K$.

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} K \\ 0 \end{bmatrix} = K \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ i.e. for $\lambda = 1$, $X = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

E. Vector for $\lambda = 2$ →

Consider $AX = \lambda X$

$$(A - \lambda I)X = 0$$

$$(A - 2I)X = 0$$

$$\begin{bmatrix} (1-2) & 2 \\ 0 & (2-2) \end{bmatrix} X = 0$$

$$\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -x_1 + 2x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{matrix} -x_1 + 2x_2 = 0 \\ 0 = 0 \end{matrix}\right\} \Rightarrow x_1 = 2x_2$$

Let $x_2 = K$ then $x_1 = 2K$ so

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2K \\ K \end{bmatrix} = K \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

so for $\lambda = 2$, $X = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

finally $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ $\begin{cases} \lambda = 1, \ X = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \lambda = 2, \ X = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \end{cases}$

Q: Find the E-Values and E-Vectors of $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Sol: $\because$ A is U.T.M so $\lambda = 2, 2, 3$

E-Vector for ($\lambda = 3$) $\rightarrow$

$$AX = \lambda X$$

$$(A - \lambda I)X = 0$$

$$(A - 3I)X = 0$$

$$\begin{bmatrix} (2-3) & 1 & 0 \\ 0 & (2-3) & 0 \\ 0 & 0 & (3-3) \end{bmatrix} X = 0$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} X = 0$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} (-x_1 + x_2) \\ -x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{array}{l} -x_1 + x_2 = 0 \\ -x_2 = 0 \\ 0 = 0 \end{array}$$

ie $x_2 = 0$, $x_1 = x_2 = 0$ so $x_1 = 0$

let $x_3 = K$ so

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} = K \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

ie for $\lambda = 3$, $X = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

E. Vector for $\lambda = 2$ →

Consider $AX = \lambda X$

$$(A - \lambda I)X = 0$$

$$(A - 2I)X = 0$$

$$\begin{bmatrix} (2-2) & 1 & 0 \\ 0 & (2-2) & 0 \\ 0 & 0 & (3-2) \end{bmatrix} X = 0$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = 0$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ x_3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow x_2 = 0,\ x_3 = 0$$

Let $x_1 = K$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} K \\ 0 \\ 0 \end{bmatrix} = K \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

So for $\lambda = 2$, $X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Finally:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$(\lambda = 3, 2, 2)$

$\lambda = 3,\ X = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = 2,\ X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

Qs Find E.Values & E.Vectors of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Sol: C.Eqn is $|A - \lambda I| = 0 \dashrightarrow (\lambda - 3)(\lambda - 1)^2 = 0 \Rightarrow \lambda = 3, 1, 1$

E.vector for $\lambda = 3$

$AX = \lambda X$

$(A - 3I)X_1 = 0$

⋮

HW

for $\lambda = 3$, $X_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

E.vector for $\lambda = 1$ →

$AX = \lambda X$

$(A - \lambda I)X = 0$

$(A - I)X = 0$

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} X = 0$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x_1 + x_2 + x_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow x_1 + x_2 + x_3 = 0$

Let $x_3 = K_1$, $x_2 = K_2$

$x_1 = -(K_2 + K_1)$

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -(K_2 + K_1) \\ K_2 \\ K_1 \end{bmatrix}$

$$X = \begin{bmatrix} -(k_2+k_1) \\ k_2 \\ k_1 \end{bmatrix} = \begin{bmatrix} -k_2 \\ k_2 \\ 0 \end{bmatrix} + \begin{bmatrix} -k_1 \\ 0 \\ k_1 \end{bmatrix} = k_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + k_1 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

ie for $\lambda = 1$ we are getting two E-vectors

$$X_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad \because AX_2 = 1 \cdot X_2$$

$$X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \because AX_3 = 1 \cdot X_3$$

THANK - YOU

Tel: $\boxed{\text{dr puneet sir pw}}$