DPP 01

CS & IT

Algorithms

Analysis of Algorithms

Q1 Which of the following natation is/are transitive but not reflexive ?
(A) Big oh (O)
(B) Big omega (Ω)
(C) Small oh (o)
(D) Small omega (ω)

Q2 If f(n) =$\sum_{i=1}^{n} i^3$
Then which of the following choices is/are true for f(n)?
(A) $\theta(n^4)$ (B) $\Omega(n^4)$
(C) $\theta(n^5)$ (D) $\Omega(n^3)$

Q3 Consider the following program:

```
main ( )
{
 P = n!;
 for (i = 1; i <= n ; ++i)
     for (j = 1 ; j <= P ; j=2*j )
      C = C + 1;
 }
```

What is the time complexity of above code?
(A) $O(n^2)$ (B) $O(n^2 \log n)$
(C) O (nlogn) (D) O(n)

Q4 Consider the following code:

```
main ( )
{
i = 1, j = 1;
while (j <= n )
{
  ++ i;
  j  = j + i;
}
```

What is the time complexity of above code?
(A) $\theta(n)$
(B) $\theta(\sqrt{n})$
(C) $\theta(\log n)$
(D) $\theta(n\log(\log n))$

Q5 Consider the following code:

```
Algorithm T(n)
{
if (n = =1) return;
else
{
T(n/2);
}
}
```

What is the space complexity of above code?
(A) $\theta(\log n)$ (B) $\theta(n)$
(C) $\theta(n\log\log n)$ (D) $\theta(\sqrt{n})$

Q6 $f(n)= 2^{n^2}$, $g(n)= n!$, $h(n)= 2^{\log n^2}$
Which of the following is/are correct?
(A) f(n) =Ω(g(n))
(B) h(n) =Ω(g(n))
(C) h(n) =O(g(n))
(D) g(n)=Ω(f(n))

Q7 Consider the following rotations:
1. $\sqrt{\log n} = O(\log\log n)$
2. $\log n = \Omega(\frac{1}{n})$
3. $n^2 = \theta(2^{2\log n})$
4. $(0.061)^n = \theta(1.02)^n$

How many notations is/are correct?_____.

Q8 Consider the following functions:

$f_1 = 2^n$
$f_2 = n!$
$f_3 = n^n$
$f_4 = e^n$

What is the correct increasing order of above function?

(A) f1 f4 f2 f3
(B) f2 f1 f4 f3
(C) f2 f4 f1 f3
(D) f2 f1 f4 f3

Q9 Sort the functions in ascending order of asymptotic(big-O) complexity.

$f_1(n) = n,\ f_2(n) = 80,\ f_3(n) = n^{\log n},$
$f_4(n) = \log\log^2 n\ f_5(n) = (\log n)^{\log n}$

(A) f2(n), f4(n), f1(n), f5(n), f3(n)
(B) f2(n), f1(n), f4(n), f5(n), f3(n)
(C) f2(n), f1(n), f4(n), f3(n), f5(n)
(D) f1(n), f5(n), f4(n), f3(n), f2(n)

Q10 Consider two function f(n) =10n+2logn and $g(n) = 5n + 2(\log n)^2$, then which of the following is correct option?

(A) f(n) = θ(g(n))
(B) f(n) = O(g(n))
(C) f(n) = ω(g(n2))
(D) None of the above

Q11 Consider two function $f(n) = \sqrt{n}$ and g(n) = n logn + n then f(n) /g(n) is equivalent to how many of the following given below? _____.

(a) $o(n^{-1/2})$
(b) $O(n^{-1/2})$
(c) $\Omega(1/\log n)$
(d) $\theta(n^{-1/2})$

Q12 Consider the following C-code

```
void foo (int x)
  {
     int a = 1;
     if (n = = 1)
     return;
for (a=1; a ≤ n; a++)
     {
        printf("GATEWALLAH");
        break;
     }
  }
```

What is the worst time complexity of above program?

(A) O(1) (B) O(n)
(C) O (log n) (D) $O\sqrt{n}$

Q13 Consider the following asymptotic functions :

$f1 = 2^n$
$f2 = 1.001^n$
$f3 = e^n$
f4 = n!

Which of the following is correct increasing order of above functions?

(A) f3, f4, f1, f2 (B) f2, f4, f1, f3
(C) f3, f2, f1, f4 (D) f2, f1, f3, f4

Q14 Consider the following functions

$f_1(n) = 4^{2^n}$
$f_2(n) = n!$
$f_3(n) = 4^{e^n}$
$f_4(n) = n^{n\ n}$

Which of the following is/are correct?

(A) f1(n) = O(f2(n))
(B) f1(n) = O(f4(n))
(C) f1(n) = O(f3(n))
(D) f2(n) = O(f3(n))

Q15 Consider two function $f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$ then which of the following is true.

(A) f1(n) =O(f2(n))
(B) f1(n) = θ(f2(n))
(C) f1(n) = ω(f2(n))
(D) None of these

Q16 $f(n) = \Sigma_{i=1}^{n}\ i^3 = x$, choices for x

I. $\theta(n^4)$ II. $\theta(n^5)$

III. $O(n^5)$ IV. $\Omega(n^3)$

(A) I, II, III

(B) II, III, IV

(C) I, II,. III, IV

(D) I, III, IV

Answer Key

Q1	C, D
Q2	A, B, D
Q3	B
Q4	B
Q5	A
Q6	A, C
Q7	2
Q8	A
Q9	A
Q10	A, B
Q11	2
Q12	A
Q13	D
Q14	B, C, D
Q15	C
Q16	D

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

	O	Ω	θ	o	ω
Reflective				x	x
Transitive					

Small oh (o) and small omega(ω) comes under transitive but not reflective.

Q2 Text Solution:

f(n) = $\sum_{i=1}^{n} x^3$

i = 1

= $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + ...n^3$

= $\left[\frac{n(n+1)}{2}\right]^2$

= O(n^4), θ(n^4), Ω(n^4)

A, B, D options are correct.

Q3 Text Solution:

Time complexity = n * log p

↓ ↓

first loop second loop

= n* log n!

= n * nlogn ,[logn!=θnlogn]

= $O(n^2 \log n)$

Q4 Text Solution:

i = 1, 2 , 3.....x

k =(1+2) (1+2) (1+2+3) (1+2+3+...x)

Time complexity = 1 + 2 + 3 + 4 +.....x $\leq$n

$\Rightarrow \frac{x(x+1)}{2} \leq n$

$\Rightarrow x^2 + x = 2n$

$\Rightarrow x^2 \cong 2n$

$\Rightarrow x = \sqrt{n}$

$\Rightarrow x = \theta(\sqrt{n})$

Q5 Text Solution:

Time complxity = $T\left(\frac{n}{2}\right)+1$

$= \theta(\log n)$

Space complixity = we are pushing k activation record for n = 2^k

$n = 2^k$

$\boxed{k = (\log_2 n)}$

Space = $\boxed{\theta(\log_2 n)}$

Q6 Text Solution:

$f(n) = 2^{n^2}$

$g(n) = n!$

$h(n) = 2^{\log_2 n^2}$

$= (n^2)^{\log_2 2}$

= n^2

- 2^{n^2} $n!$

taking both side log

$\log 2^{n^2}$ $\log n!$

n^2 $n\log n$

$n^2 = \Omega(n\log n)$

$f(n) = \Omega(g(n))$

- $h(n) = \Omega(f(n))$

$n^2 \leq n!$(True)

- $g(n) = \Omega(f(n))$

$g(n) \geq 2^{n^2}$

$n! \geq 2^{n^2}$ (False)

Q7 Text Solution:

1. $\sqrt{\log n} = 0(\log\log n)$

$(\log n)^{\frac{1}{2}} \leq \log\log n$

$\frac{1}{2}\log\log n \leq \log(\log\log n)$ (False)

2. $\log n = \Omega\left(\frac{1}{n}\right)$

$\log n \geq \frac{1}{n}$ (True)

3. $n^2 = \theta(2^{2\log n})$

$n^2 = 2^{2\log n}$

$n^2 = 2^{\log n^2}$

$n^2 = (n^2)^{\log_2^2}$

$n^2 = n^2$(True)

4. $(0.0161)^n = \theta(1.02)^n$ (False)

Q8 Text Solution:

- $2^n < e^n$
- $n! < n^n$
- $(2^n, e^n) < n!$
- $2^n < e^n < n! < n^n$

$f_1 < f_4 < f_2 < f_3$

Q9 Text Solution:

80 < n

log $\log^2 n$ < n

put n = 10^{100}

$\log(\log n)^2 = 10^{100}$

$\log(100)^2 < 10^{100}$

$4 < 10^{100}$

$n < n^{\log n}$

taking log on both side

log n < log n log n

we know that $(\log n)^2 > \log n$

now, $(\log n)^{\log n} < n^{\log n}$

as we can see that log n in LHS and n on RHS.

n $(\log n)^{\log n}$

taking log on both sides

log n < log n * log log n

From above we conclude that growth of log*log n is higher than 1.

option (a) is correct.

Q10 Text Solution:

f(n) = 10n + 2 log n = O(n)

g(n) = 5 n + 2 $(\log n)^2$ = O(n)

f(n) = O(g(n))

g(n) = O(f(n))

f(n) = g(n)

As we know that if θ is possible then O, Ω is possible.

Hence option a, b are correct.

Q11 Text Solution:

$f(n) = \sqrt{n}$

g (n) = n log n + n

$\frac{f(n)}{g(n)} = \frac{\sqrt{n}}{n(1+\log n)} = \frac{1}{\sqrt{n}(1+\log n)}$

$= \frac{1}{\sqrt{n}+\sqrt{n}\log n}$

Option (a)

f(n) < c.g (n) - o

$0\left(n^{\frac{-1}{2}}\right) = \frac{1}{\sqrt{n}}$

f(n) g(n)

$\frac{1}{\sqrt{n}+\sqrt{n}\log n} = o\left(\frac{1}{\sqrt{n}}\right)$ True

Option (b)

f(n) ≤ c.g (n) – O

- If o possible then O also possible, but vice versa is not possible.
- If w is possible then Ω also possible, but vice versa is not possible.

Option i, ii are true.

Hence (2) is the correct answer.

Q12 Text Solution:

If we see carefully, loop will execute only one time because of break statement, therefore time complexity will be O(1)

Q13 Text Solution:

By observing options and given options, we can conclude that-

$f_4 > f_3 > f_1 > f_2$ increasing order is $f_2\ f_1\ f_3\ f_4$.

Option (d) is correct.

Q14 Text Solution:

The increasing order of given functions is f2, f1, f3, f4.

Therefore (b,c,d) are correct.

Q15 Text Solution:

$f_1(n) = n^{2^n}$ and $f_2(n) = n^{n^2}$

n^{2^n} n^{n^2}

Taking log on both side

$2^n \log n$ $n^2 \log n$

as we can see that

2^n has more growth rate than n^2

we conclude

$f_2(n) < O(f_1(n))$ or $f_1(n) = \omega(f_2(n))$

(c) is correct.

Q16 Text Solution:

$$\sum_{i=1}^{n} = \left[\frac{n(n+1)}{2}\right]^2$$

$$= \frac{n^2(n+1)^2}{4}$$

$$= \frac{n^2[n^2+2n+1]}{4}$$

$f(n) = \frac{n^4+2n^3+n^2}{4}$

$O(n^5)$, $\Omega(n^3)$, $\theta(n^4)$ are correct