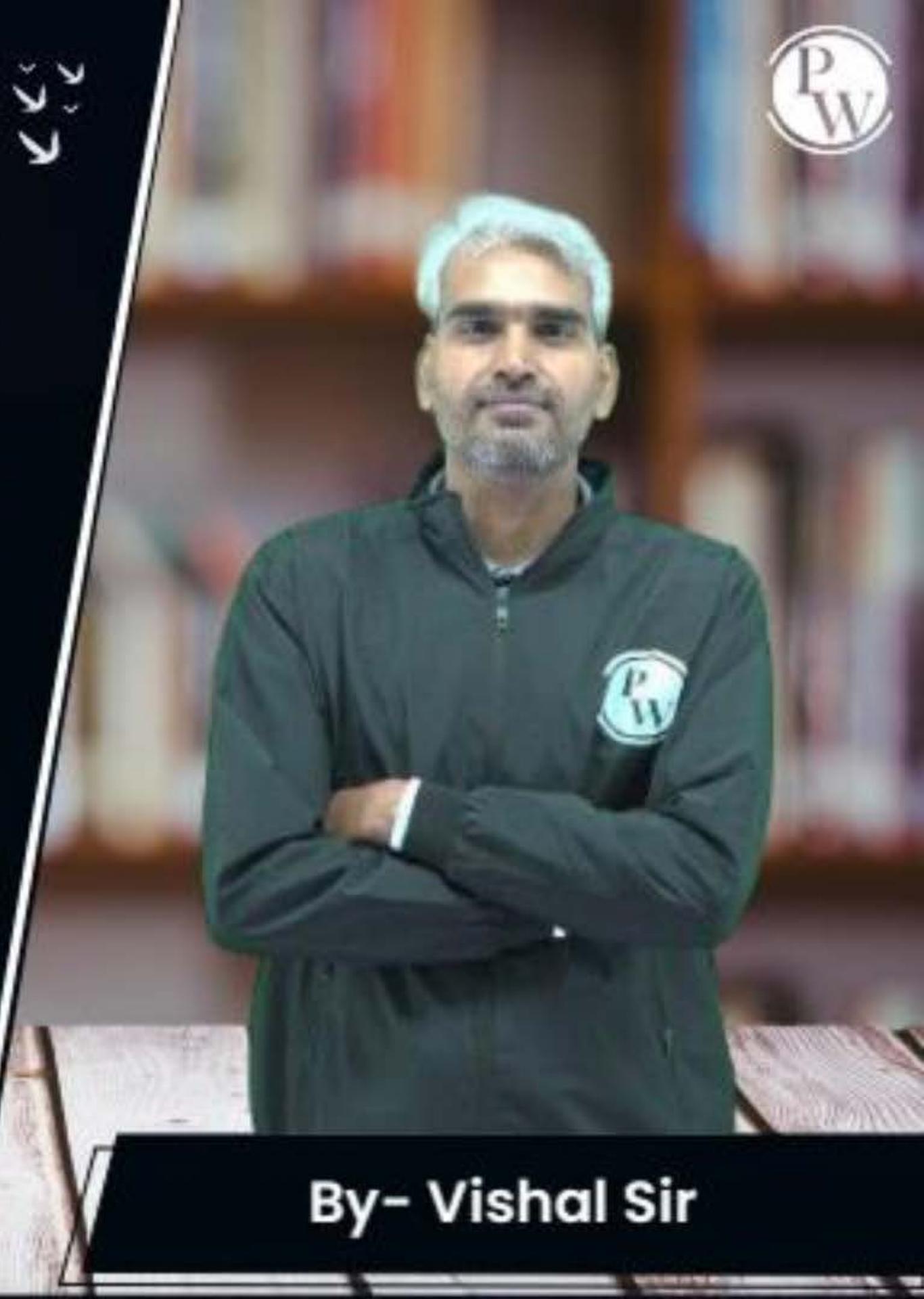


Computer Science & IT

Database Management System

Relational Model & Normal Forms

Lecture No. 06



By- Vishal Sir

Recap of Previous Lecture



- * **Topic** Identification of candidate keys in a relation
- * **Topic** Membership test
- * **Topic** Relationship between two FD sets
- * **Topic** FD set of a sub-relation



Topics to be Covered



- * **Topic** FD set of a subrelation
- * **Topic** Minimal cover (Canonical cover)
- * **Topic** Number of superkeys in a relation

FD set of a Sub-relation

- + Let R be the relation with FD set F , and R_1 is any sub-relation of R .
Concept of membership test can be used to identify the FDs of sub-relation
- Let $R(A, B, C, D, E)$ is a relation, then $R_1(A, B, E)$ can be called a subrelation of relation R

Q: Let $R(A, B, C, D, E)$ is a relation with FD set F.

$$F = \{ A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E \}$$

And let $R_1(A, B, E)$ is a sub-relation of relation $R(A, B, C, D, E)$

Find the Candidate keys of sub-relation $R_1(A, B, E)$

Soln. To find the candidate keys of any relation
we need the set of functional dependencies
w.r.t. that relation

- a. First we need to identify the FDs
that exists in sub-relation $R_1(A, B, E)$

Q: Let $R(A, B, C, D, E)$ is a relation with FD set F .

$$F = \{A \rightarrow B, AB \rightarrow C, D \rightarrow AC, D \rightarrow E\}$$

and let $R_1(A, B, E)$ is a sub-relation of relation $R(A, B, C, D, E)$

We need to find FD set F_1 w.r.t. sub-relation R_1
i.e., we need identify the relationship b/w $A, B \nmid E$

$$(A)^+ \text{ wrt } F = \{A, B, C\}$$

$$\text{i.e. } A \rightarrow \cancel{A} \quad \cancel{B} \quad \cancel{C}$$

trivial

Not in relation R_1

$$\therefore \boxed{A \rightarrow B}$$

$$(B)^+ \text{ wrt } F = \{\cancel{B}\} \quad \therefore \text{No useful FD}$$

$$(E)^+ \text{ wrt } F = \{\cancel{E}\} \quad \therefore \text{No useful FD}$$

$$(AB)^+ \text{ wrt } F = \{\cancel{A}, \cancel{B}, \cancel{C}\} \quad \therefore \text{No useful FD}$$

$$(AE)^+ \text{ wrt } F = \{\cancel{A}, \cancel{E}, \cancel{B}, \cancel{C}\} \quad \therefore \boxed{AE \rightarrow B}$$

$$(BE)^+ \text{ wrt } F = \{\cancel{B}, \cancel{E}\} \quad \therefore \text{No useful FD}$$

Hence FD set F_1 w.r.t
sub relation $R_1(A, B, E)$ is

$$F_1 = \{A \rightarrow B\}$$

o/c of R_1 is (AE)

H.W.

Q: Let $R(A, B, C, D, E, F)$ is a relation with FD set F .
 $F = \{AB \rightarrow C, B \rightarrow D, BC \rightarrow A, D \rightarrow EF\}$

and let $R_1(A, B, C, D)$ is a sub-relation of relation $R(A, B, C, D, E, F)$

Find the Candidate keys of sub-relation $R_1(A, B, C, D)$

$$(A)^+ = \{A\}$$

$$(B)^+ = \{B, D, E, F\}$$

$\boxed{B \rightarrow D}$

$$(C)^+ = \{C\}$$

$$(D)^+ = \{D, E, F\}$$

$$(AB)^+ = \{A, B, C, D, E, F\}$$

$\boxed{AB \rightarrow CD}$

$$(AC)^+ = \{A, C\}$$

$$(AD)^+ = \{A, D, E, F\}$$

$$(BC)^+ = \{B, C, D, A, E, F\}$$

$\boxed{BC \rightarrow AD}$

$$(BD)^+ = \{B, D, E, F\}$$

$$(CD)^+ = \{C, D, E, F\}$$

$$(ABC)^+ = \{A, B, C, D, E, F\}$$

$\boxed{ABC \rightarrow D}$

$$(ABD)^+ = \{A, B, C, D, E, F\}$$

$\boxed{ABD \rightarrow C}$

$$(ACD)^+ = \{A, C, D, E, F\}$$

$$(BCD)^+ = \{B, C, D, A, E, F\}$$

$\boxed{BCD \rightarrow A}$

FD set F_1 of $R_1(A, B, C, D)$ is
 $F_1 = \{B \rightarrow D, AB \rightarrow CD\}$ ∵ CKs are
 AB, BC



Topic : Minimal cover (Canonical cover)

Irreducible set
of functional dependencies

- Minimal cover or canonical cover of FD set F is a set of functional dependencies (F_m) such that,

- $F_m = F$ and
- F_m does not contain any redundant FD, and F_m must not contain any extraneous attribute at either side of any of its FD

$$A \rightarrow B \cancel{C}, \quad B \rightarrow C \Rightarrow A \rightarrow B \wedge B \rightarrow C$$

III

$$\overbrace{A \rightarrow B \wedge B \rightarrow C}^{\Rightarrow A \rightarrow C}$$

Redundant

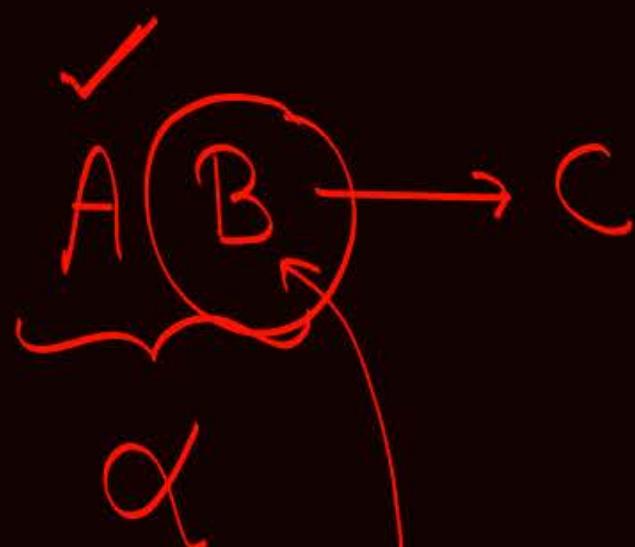


Topic : Procedure to obtain minimal cover of FD set

- 1. Simplify RHS of all FDs (i.e., split the FDs such that RHS contain exactly one attribute)
- * 2. For all FDs find redundant (extraneous) attribute in LHS.
- 3. Eliminate all redundant FDs ← { As soon as any redundant FD is identified, remove it from the set & check for other redundant FDs }
- 4. Apply Union if needed
(it is not mandatory)
- 5. The result is minimal Cover

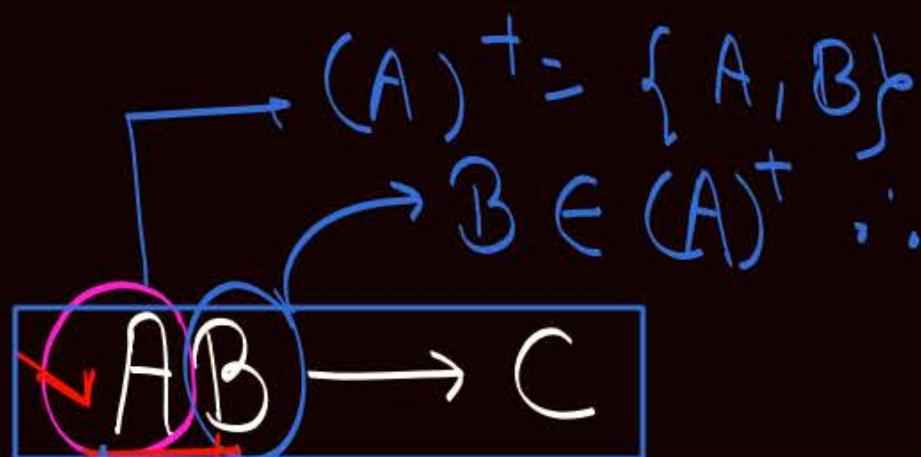
As soon as any extraneous attribute is identified in the LHS, remove it and check for other extraneous attributes.

$$A \rightarrow B$$



α

$$\begin{aligned} &\hookrightarrow (\alpha - A)^+ \\ &= (B)^+ \end{aligned}$$



$$(B)^+ = \{B\}$$

$$A \notin (B)^+ \therefore A \text{ is extraneous w.r.t. } B$$

$B \in (A)^+ \therefore B$ is extraneous w.r.t. A



Topic : Testing if an Attribute is Extraneous in LHS

α is a set of attributes that contain two or more attributes.

Consider a set F of functional dependencies and functional dependency $\alpha \rightarrow \beta$ in F .
let 'A' is an attribute belonging to set ' α '

After simplification $|\beta|=1$

To test if attribute $A \in \alpha$ is extraneous in α (i.e., Any extraneous attribute in LHS of FD)

1. compute $(\{\alpha\} - \{A\})^+$ using the dependencies in F' , where $F' = \underbrace{(F - \{\alpha \rightarrow \beta\})}_{\text{all FDs of } F \text{ except } \alpha \rightarrow \beta}$
2. check if $(\{\alpha\} - A)^+$ contains A; if it does then, A is extraneous.
& if $A \notin (\alpha - A)^+$ then
A is not Extraneous

#e.g., Consider the FD set

$$F = \{AB \rightarrow C, B \rightarrow A, A \rightarrow B\}$$

Find minimal cover of F

$$F: \left\{ \begin{array}{l} AB \rightarrow C \\ B \rightarrow A \\ A \rightarrow B \end{array} \right\}$$

"Step-1"

$\xrightarrow{\text{Simplify w.r.t. R.H.S.}}$

Single attribute
in LHS can
never be
extraneous

Step-2 Check for extraneous attribute in $AB \rightarrow C$

① Check if 'A' is extraneous when B is present

$(B)^+ = \{A, B\}$ wrt $\{F - AB \rightarrow C\}$

$A \in (B)^+ \therefore A$ is extra when B is present

∴ Remove it

$$\left\{ \begin{array}{l} B \rightarrow C \\ B \rightarrow A \\ A \rightarrow B \end{array} \right\}$$

FD set after removing A

Step-2 Check for extraneous attribute in $AB \rightarrow C$

② Check if 'B' is extraneous when 'A' is present

$(A)^+ = \{A, B\}$ wrt $\{F - AB \rightarrow C\}$

$B \in (A)^+ \therefore B$ is extra when A is present

∴ Remove it

$$\left\{ \begin{array}{l} A \rightarrow C \\ B \rightarrow A \\ A \rightarrow B \end{array} \right\}$$

FD set after removing B



Topic : Testing if any FD is redundant

Consider a set F of functional dependencies and functional dependency $\alpha \rightarrow \beta$ in F .

To test if $\underbrace{\alpha \rightarrow \beta}$ is redundant

1. compute $\underbrace{\alpha^+}$ using only the dependencies in F' , where $F' = \underbrace{(F - \{\alpha \rightarrow \beta\})}$
↑
All FD Except
 $\alpha \rightarrow \beta$
2. check that α^+ contains β ; if it does then, $\alpha \rightarrow \beta$ is redundant
Otherwise $\alpha \rightarrow \beta$ is required

#e.g., Consider the FD set

$$F = \{AB \rightarrow C, B \rightarrow A, A \rightarrow B\}$$

Find minimal cover of F

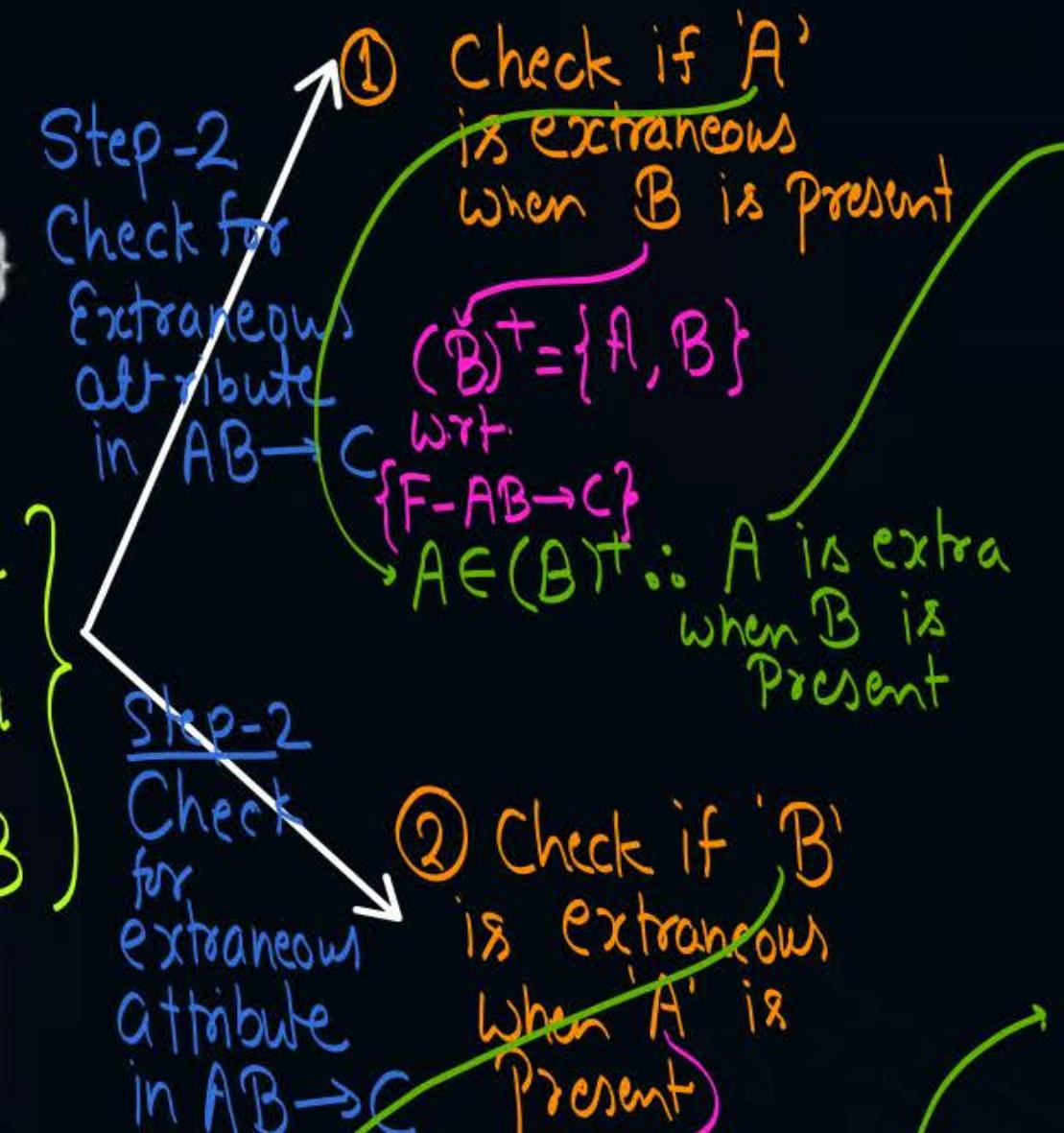
$$F: \left\{ \begin{array}{l} AB \rightarrow C \\ B \rightarrow A \\ A \rightarrow B \end{array} \right\}$$

"Step-1"

$\xrightarrow{\text{Simplify w.r.t. R.H.S.}}$

$$\left\{ \begin{array}{l} AB \rightarrow C \\ B \rightarrow A \\ A \rightarrow B \end{array} \right\}$$

Single attribute
in LHS can
never be
extraneous



Step-2

Check for extraneous attribute in $AB \rightarrow C$

② Check if 'B' is extraneous when 'A' is present

$(A)^+ = \{A, B\}$ wrt $\{F - AB \rightarrow C\}$

$B \in (A)^+ \therefore B$ is extra when A is present

∴ Remove it

$$\left\{ \begin{array}{l} B \rightarrow C \\ B \rightarrow A \\ A \rightarrow B \end{array} \right\}$$

FD set after removing A

$$\left\{ \begin{array}{l} B \rightarrow AC \\ A \rightarrow B \end{array} \right\}$$

∴ Remove it

$$\left\{ \begin{array}{l} A \rightarrow C \\ \rightarrow A \\ \rightarrow B \end{array} \right\}$$

FD set after removing B

$$\left\{ \begin{array}{l} A \rightarrow BC \\ B \rightarrow A \end{array} \right\}$$

P
W
Required FDs

$$\left\{ \begin{array}{l} B \rightarrow C \\ B \rightarrow A \\ A \rightarrow B \end{array} \right\}$$

Both are Minimal Cover of FD set F

Required FDs

P
W
Required FDs

$$\left\{ \begin{array}{l} A \rightarrow C \\ B \rightarrow A \\ A \rightarrow B \end{array} \right\}$$

$$\left\{ \begin{array}{l} A \rightarrow C \\ B \rightarrow A \\ A \rightarrow B \end{array} \right\}$$

$$\left\{ \begin{array}{l} A \rightarrow BC \\ B \rightarrow A \end{array} \right\}$$

$$\left\{ \begin{array}{l} A \rightarrow BC \\ B \rightarrow A \end{array} \right\}$$



Topic : NOTE

Minimal cover of FD set F need not be unique, but all minimal cover are logically equivalent.

If F_{m_1} & F_{m_2} are two different minimal Covers
of FD set F , then $F_{m_1} = F_{m_2} = F$

#e.g., Consider the FD set

$$F = \{A \rightarrow BC, B \rightarrow C\}$$

Find all minimal Cover of F

$$\begin{aligned} F &= \left\{ \begin{array}{l} A \rightarrow BC \\ B \rightarrow C \end{array} \right\} \xrightarrow{\text{Step-1}} \left\{ \begin{array}{l} A \rightarrow B \\ A \rightarrow C \\ B \rightarrow C \end{array} \right\} \xrightarrow{\text{Step-2}} \left\{ \begin{array}{l} A \rightarrow B \\ A \rightarrow C \\ B \rightarrow C \end{array} \right\} \\ &\quad \text{Simplify w.r.t. RHS} \qquad \text{Eliminate extraneous attribute from LHS} \end{aligned}$$

$$\left\{ \begin{array}{l} A \rightarrow B \\ A \rightarrow C \\ B \rightarrow C \end{array} \right\} \xrightarrow{\text{Step-3}} \left\{ \begin{array}{l} A \rightarrow B \\ B \rightarrow C \end{array} \right\} \quad \text{Eliminate redundant FDs}$$

$$\boxed{\left\{ \begin{array}{l} A \rightarrow B \\ B \rightarrow C \end{array} \right\}}$$

$$\left\{ \begin{array}{l} A \rightarrow B \\ B \rightarrow C \end{array} \right\}$$

it is the only
Minimal Cover of
FD set F.

Q. Find minimal cover of

$$F = \{ A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H \}$$

Step-1: $\{ A \rightarrow C, AC \rightarrow D, E \rightarrow A, E \rightarrow D, E \rightarrow H \}$

Step-2: $\{ A \rightarrow C, A \rightarrow D, E \rightarrow A, E \rightarrow D, E \rightarrow H \}$

Step-3: $\{ A \rightarrow C, A \rightarrow D, E \rightarrow A, E \rightarrow H \}$

↓ Union

$$\{ A \rightarrow C, D, E \rightarrow AH \}$$

~~H.W.~~
~~#c.g.,~~

Consider the following FD set

$$F = \{A \rightarrow BC$$

$$CD \rightarrow E$$

$$E \rightarrow C$$

$$D \rightarrow AEH$$

$$ABH \rightarrow BD$$

$$DH \rightarrow BC$$

}

Find minimal cover of F.

Simplify RHS.	Eliminate Extra Attribute from LHS.	Eliminate Redundant FD	<u>Union</u>
$A \rightarrow BC$			
$CD \rightarrow E$			
$E \rightarrow C$			
$D \rightarrow AEH$			
$ABH \rightarrow BD$			
$DH \rightarrow BC$			

#Q. The following functional dependencies hold true for the relational schema

$$\begin{array}{l}
 R\{V, W, X, Y, Z\} \\
 \begin{array}{c}
 (V)^+ = \{V, W\} \\
 (W)^+ = \{W\}
 \end{array}
 \quad = \quad
 \begin{array}{l}
 Y \rightarrow V \\
 Y \rightarrow X \\
 Y \rightarrow Z
 \end{array}
 \\
 [V \rightarrow W; \quad | \quad \underbrace{VW \rightarrow X; \quad Y \rightarrow VX; \quad Y \rightarrow X}_{W \text{ is extra}} \quad Y \rightarrow Z]
 \end{array}$$

Which of the following is irreducible equivalent for this set of functional dependencies?

~~A~~

$$\begin{array}{l}
 V \rightarrow W \quad \checkmark \\
 V \rightarrow X \quad \checkmark \\
 Y \rightarrow V \quad \checkmark \\
 Y \rightarrow Z \quad \checkmark
 \end{array}$$

~~B~~

$$\begin{array}{l}
 V \rightarrow W \quad \checkmark \\
 \times W \rightarrow X \\
 Y \rightarrow V \\
 Y \rightarrow Z
 \end{array}$$

~~C~~

$$\begin{array}{l}
 V \rightarrow W \quad \checkmark \\
 V \rightarrow X \quad \checkmark \\
 Y \rightarrow V \quad \checkmark \\
 \boxed{Y \rightarrow X} \quad \boxed{Y \rightarrow X} \quad \boxed{Y \rightarrow Z} \\
 (Y)^+ = \{Y, V, X, Z\} \quad \text{Redundant } \times
 \end{array}$$

~~D~~

$$\begin{array}{l}
 V \rightarrow W \quad \checkmark \\
 \times W \rightarrow X \\
 Y \rightarrow V \\
 Y \rightarrow X \\
 Y \rightarrow Z
 \end{array}$$

Number of Super keys

Q: Let $R(A, B, C, D, E)$ is a relation
with no non-trivial functional dependency {i.e. $F = \{\}$ }
then what will be the Candidate key of relⁿ R.

Solu'n No FD in the FD set,
Hence all attributes are essential attributes

$\therefore \underline{C.K}$ will be formed by combining
all attributes of the relation.

* In this case, we have only one Candidate key and
Only One Super key (it C.K itself)

H.W Q:- Consider the relational schema $R(A_1, A_2, A_3, \dots, A_n)$
Find the number of superkeys possible in relation R.
(i) When "A₁" is the only candidate key of relation R.

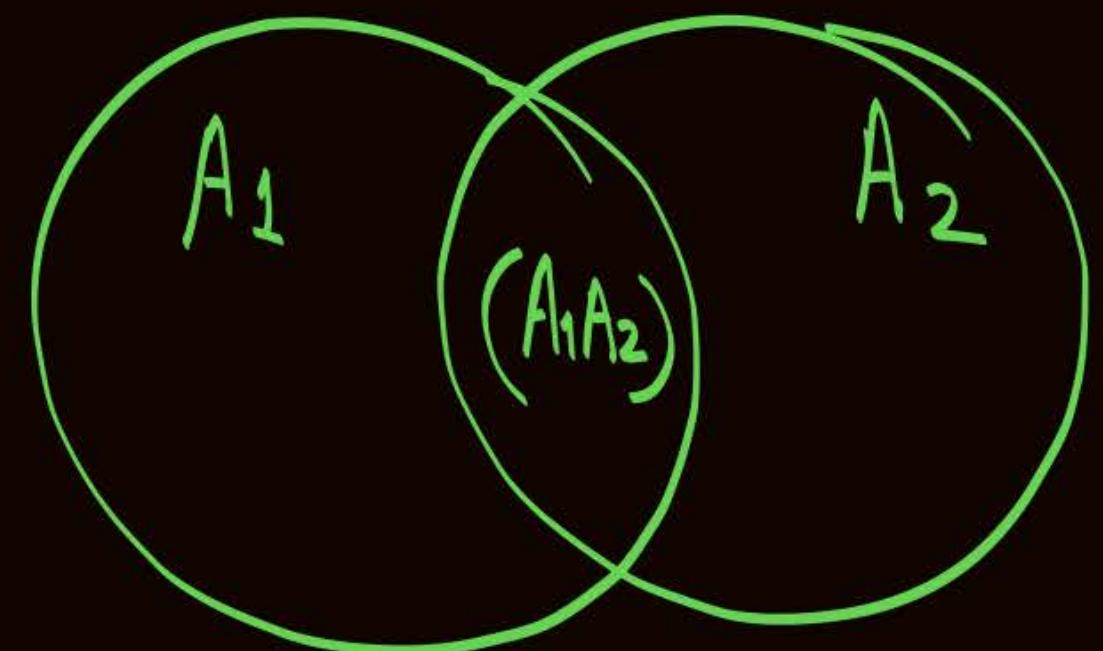
H.W. Q:- Consider the relational schema $R(A_1, A_2, A_3, \dots, A_n)$
Find the number of superkeys possible in relation R.
(ii) When (A_1A_2) is the only candidate key of relation R.

Q:- Consider the relational schema $R(A_1, A_2, A_3, \dots, A_n)$

H.W

Find the number of superkeys possible in relation R.

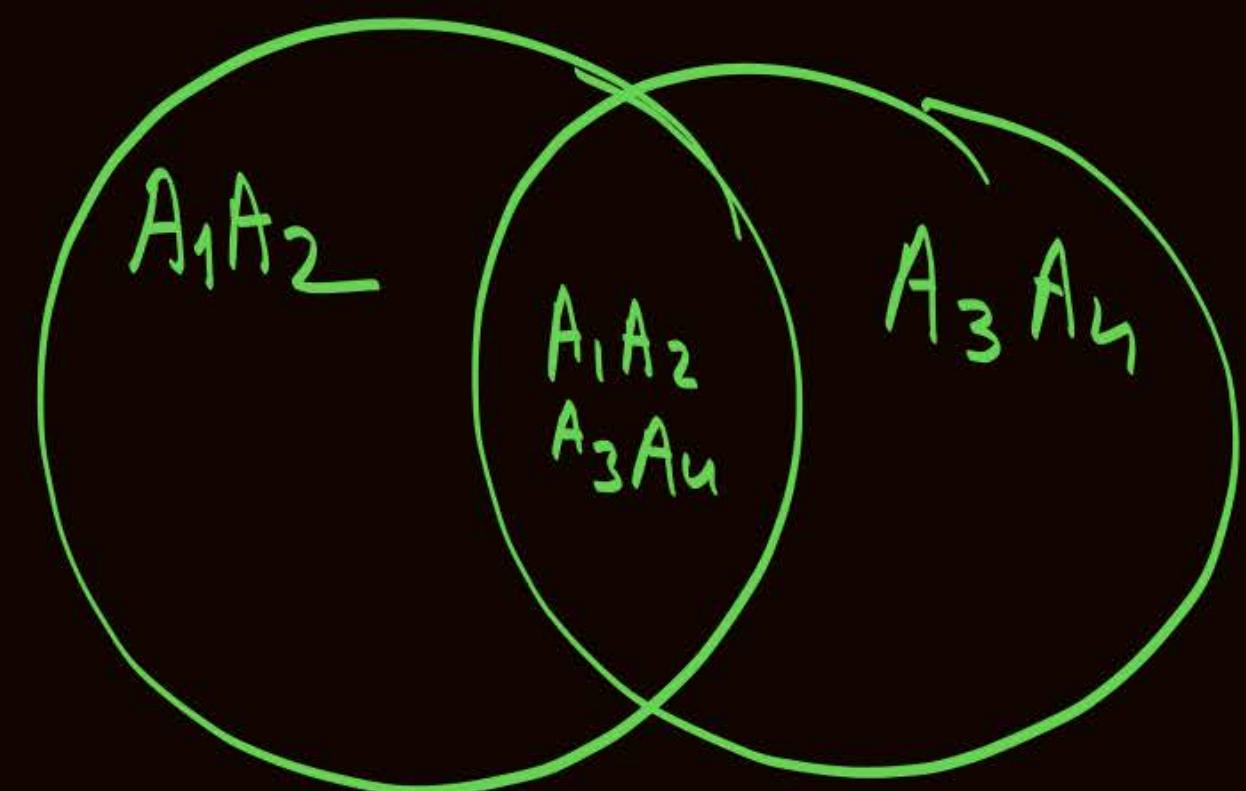
(iii) When " A_1 " & " A_2 " are the only two candidate keys of relation R.



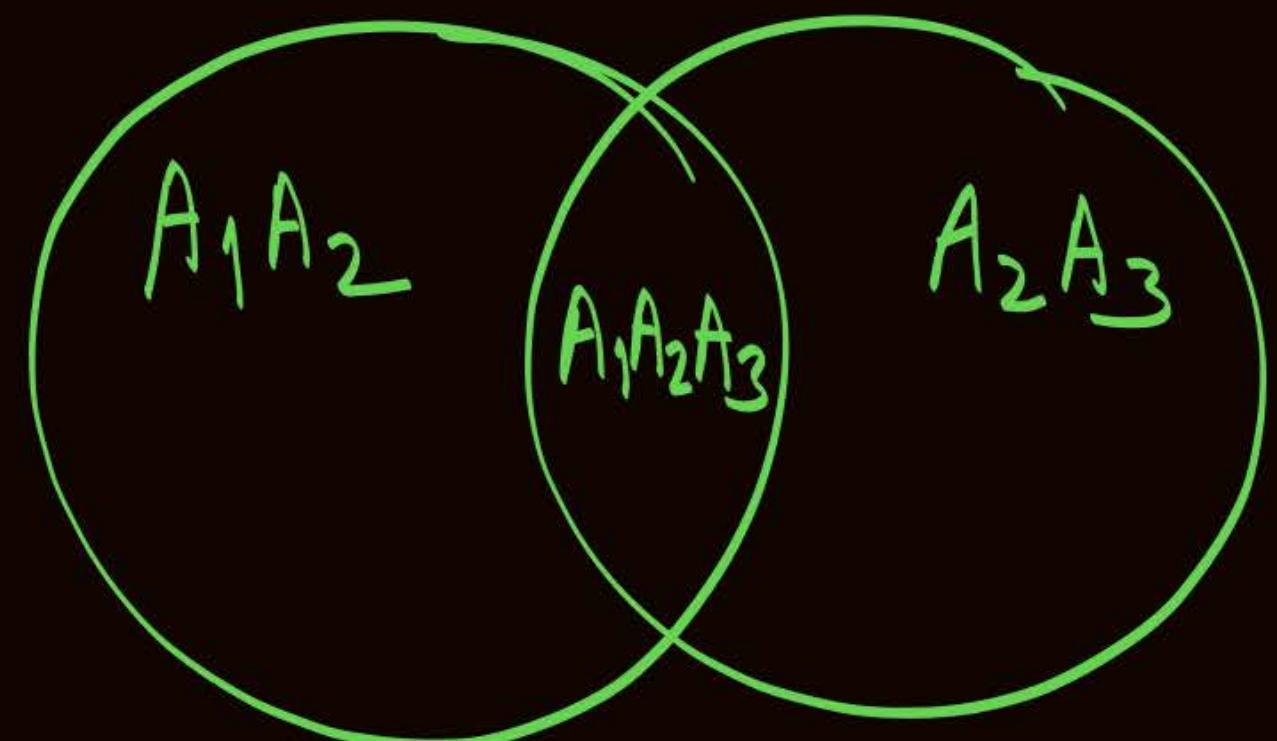
Q:- Consider the relational schema $R(A_1, A_2, A_3, \dots, A_n)$

W: Find the number of superkeys possible in relation R.

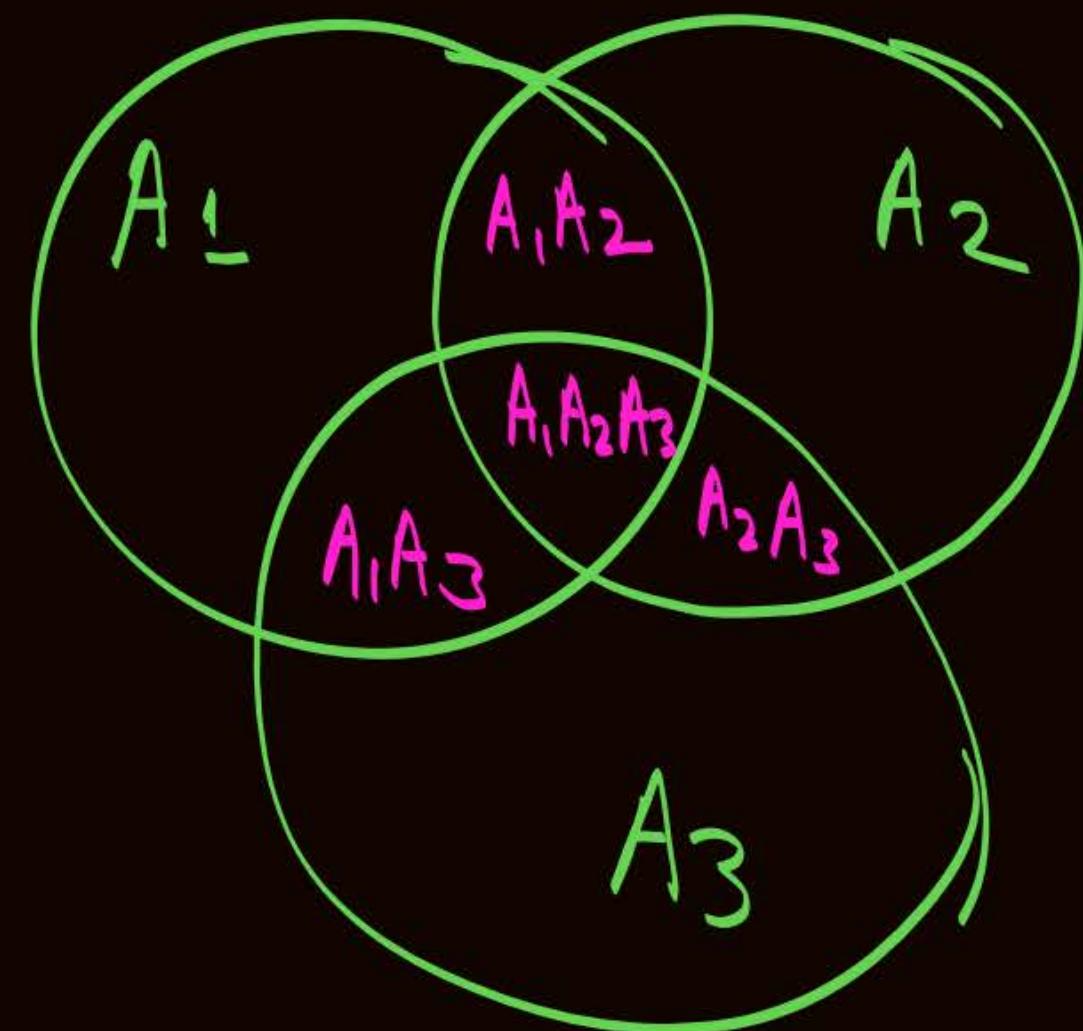
(iv) When (A_1A_2) and (A_3A_4) are two candidate keys of relation R.



Q:- Consider the relational schema $R(A_1, A_2, A_3, \dots, A_n)$
Find the number of superkeys possible in relation R.
(V) When (A_1A_2) & (A_2A_3) are only two candidate keys of relation R.



H.W.
Q:- Consider the relational schema $R(A_1, A_2, A_3, \dots, A_n)$
Find the number of superkeys possible in relation R.
(vi) When $(A_1), (A_2)$ and (A_3) are three candidate keys of relation R.



H.W.

Q:- Consider the relational schema $R(A_1, A_2, A_3, \dots, A_n)$

Find the number of superkeys possible in relation R.

When each attribute of relation R itself is a candidate key



2 mins Summary



Topic

FD set of a subrelation

Topic

Minimal cover (Canonical cover)

Topic

Number of superkeys in a relation

THANK - YOU