CS & IT ENGINEERING

Algorithms

Miscellaneous

Lecture No.- 05

By- Aditya Jain sir

Recap of Previous Lecture

Topic

Topic

Set

Topics to be Covered

Topic

Topic

Backtracking

Branch-Bound

Guidance.

About Aditya Jain sir

1. Appeared for GATE during BTech and secured AIR 60 in GATE in very first attempt - City topper
2. Represented college as the first Google DSC Ambassador.
3. The only student from the batch to secure an internship at Amazon. (9+ CGPA)
4. Had offer from IIT Bombay and IISc Bangalore to join the Masters program
5. Joined IIT Bombay for my 2 year Masters program, specialization in Data Science
6. Published multiple research papers in well known conferences along with the team
7. Received the prestigious excellence in Research award from IIT Bombay for my Masters thesis
8. Completed my Masters with an overall GPA of 9.36/10
9. Joined Dream11 as a Data Scientist
10. Have mentored 12,000+ students & working professions in field of Data Science and Analytics
11. Have been mentoring & teaching GATE aspirants to secure a great rank in limited time
12. Have got around 27.5K followers on Linkedin where I share my insights and guide students and professionals.

Telegram

Telegram Link for Aditya Jain sir: https://t.me/AdityaSir_PW

Topic : Back Tracking

- Represents one of the most general techniques.
- Many problems which deal with searching for a set of solutions or which ask for an optimal solution satisfying some constraints can be solved using the backtracking formulation.
- In order to apply the backtrack method, the desired solution must be expressible as an n-tuple (x_1, x) where the x_i are chosen from some finite set S_i. Often the problem to be solved calls for finding one vector which maximizes (or minimizes or satisfies) a criterion function $P(x_1, ..., x_n)$. Sometimes it seeks all such vectors which satisfy P.
- For example

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E.g.: n-Queens Problem:

- Given n Queens $(q_1, q_2, \ldots, q_n)$

  $X = [x_1 \rightarrow x_n] \rightarrow$ feasible solution

  $X = [x_1, x_2, \ldots, x_n]$

  X_i = Position of Queen Q_i in the i^{th} row of n × n matrix (chess board)

  $1 \leq X_i \leq n$

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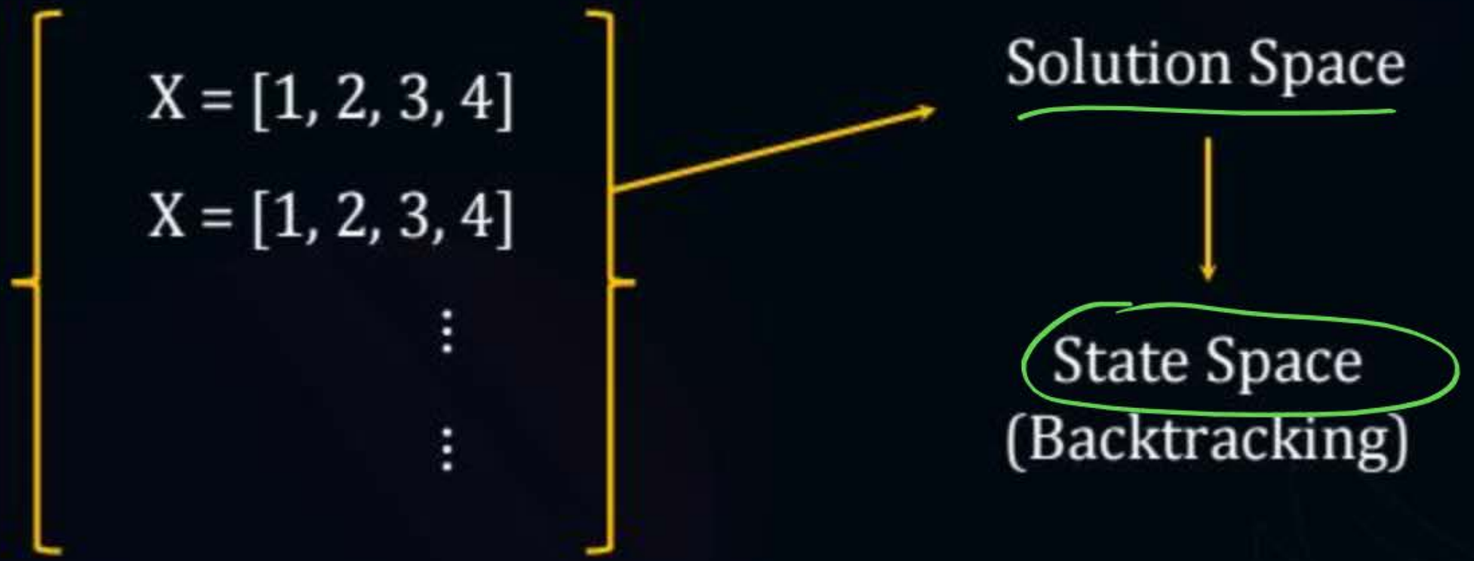

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E.g.: n = 4 (q_1, q_2, q_3, q_4)

X = [1, 3, _, _]

X = [1, _, _, _]

X = [2, 4, 1, 3] → feasible solution

X = [3, 1, 4, 2] ✓

	1	2	3	4	
q_1	✗	✓			3
q_2	✗	✗	✗	✓	1
q_3	✓				4
q_4	✗	✗	✓		2

X = [2, 4, 1, 3]

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State Space Tree:

(1) Fixed Size Tuple:

E.g.: n-Queens

(2) Variable Size Tuple:

E.g.: Sum of Subsets (SOS)

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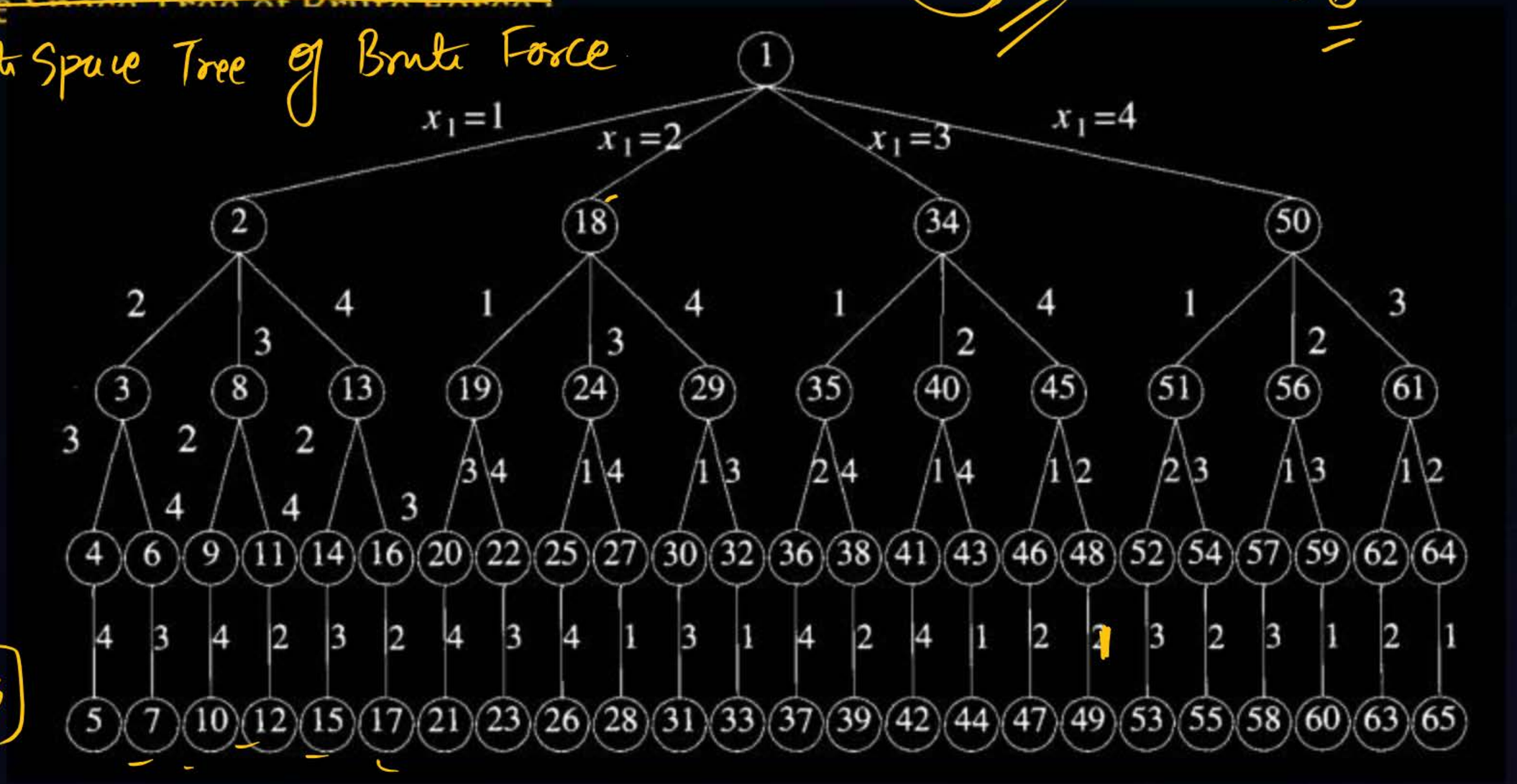
DFS

(1, 3×2×1 = 6)

State ~~Space Tree of Brute Force~~

State Space Tree of Brute Force



[1,2,3,6]

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X = [1, 2, 3, 4]

X = [1, 2, 3, 4]

⋮

⋮

Total branches = 6 × 4 = 24

n! = Size of solution space

n = 4

4! = 24

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Important Points:

(1) Backtracking uses DFS.

(2) DFS is used to find the feasible solution among all the solutions in the state space.

(3) It applies a bounding function to terminate/stop those nodes that can never lead to an optimal/feasible solution.

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Backtracking:

- ~~DFS~~ along with some Bounding Function.

DFS

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Potential Question:

- Portion of the State Space Tree that gets generated due to backtracking process for n = 4 Queens Problem.
- To get 1st feasible solution: 16 states

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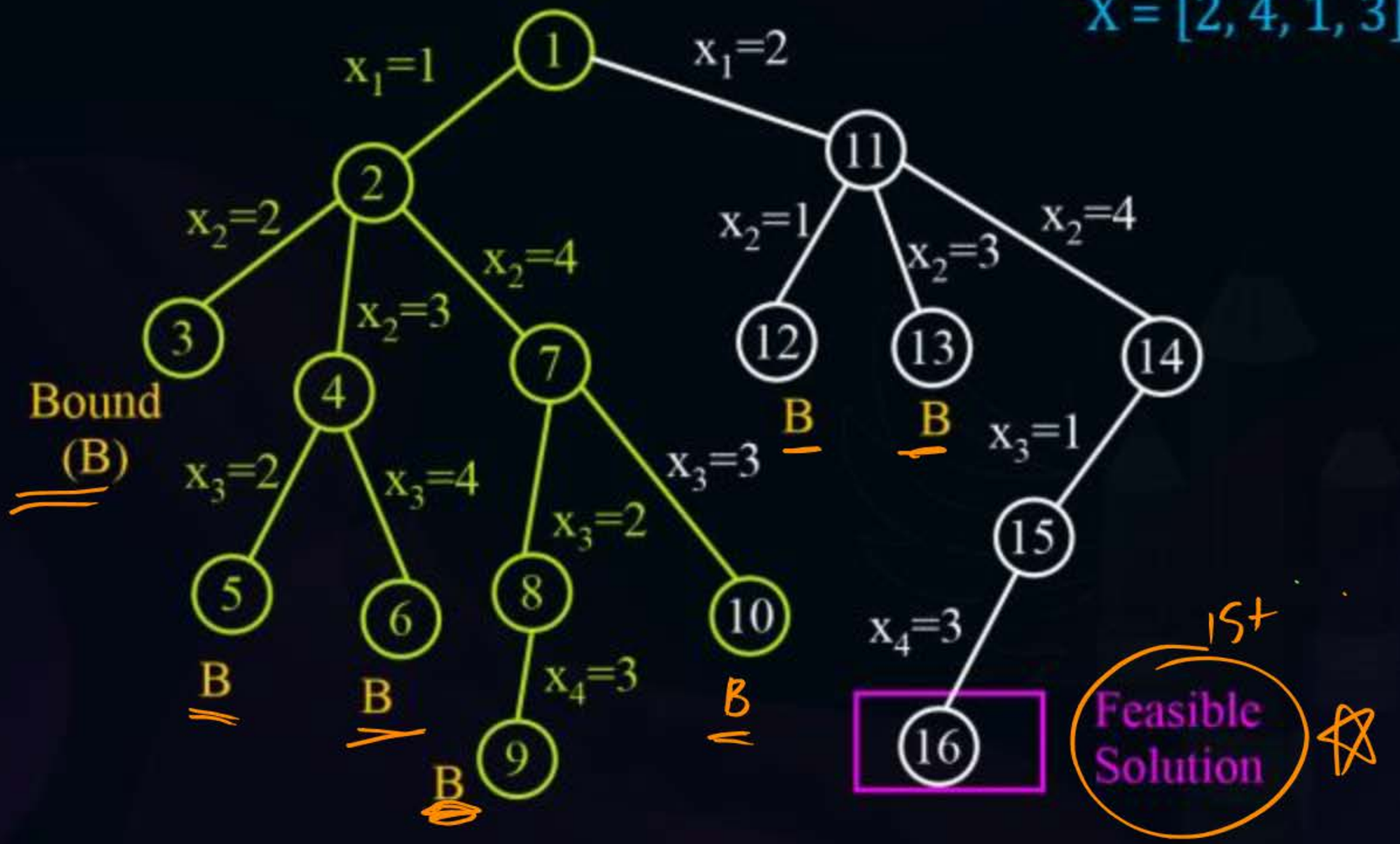
State Space Tree using Back-tracking:

X = [2, 4, 1, 3]



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	1	1	1	1
q_1		✓		
q_2				✓
q_3	✓			
q_4		✓		

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Example of Variable Sized Tuple Problem and Solution using Backtracking

(State Space Tree):

Sum of Subset (SOS):

- A[n] : n elements A_1, ..., A_n ; M = Target sum

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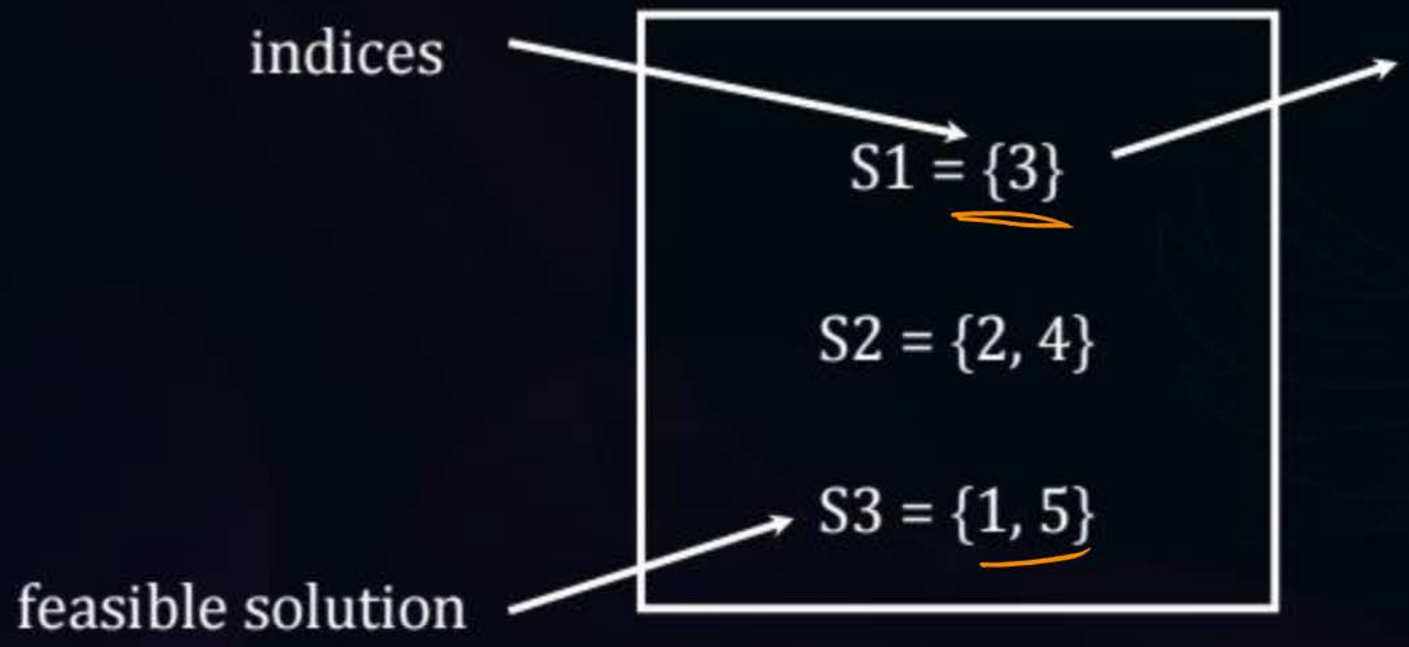
E.g.:

n = 6, M = 50 (Target Sum)

A = [20, 40, 50, 10, 30, 5]



Variable sized tuple as the feasible solutions are not at the same level (size) in the state space Tree.

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Sum of the State Space for SOS problem with 'n' elements:

= Number of Subsets

$= 2^n$

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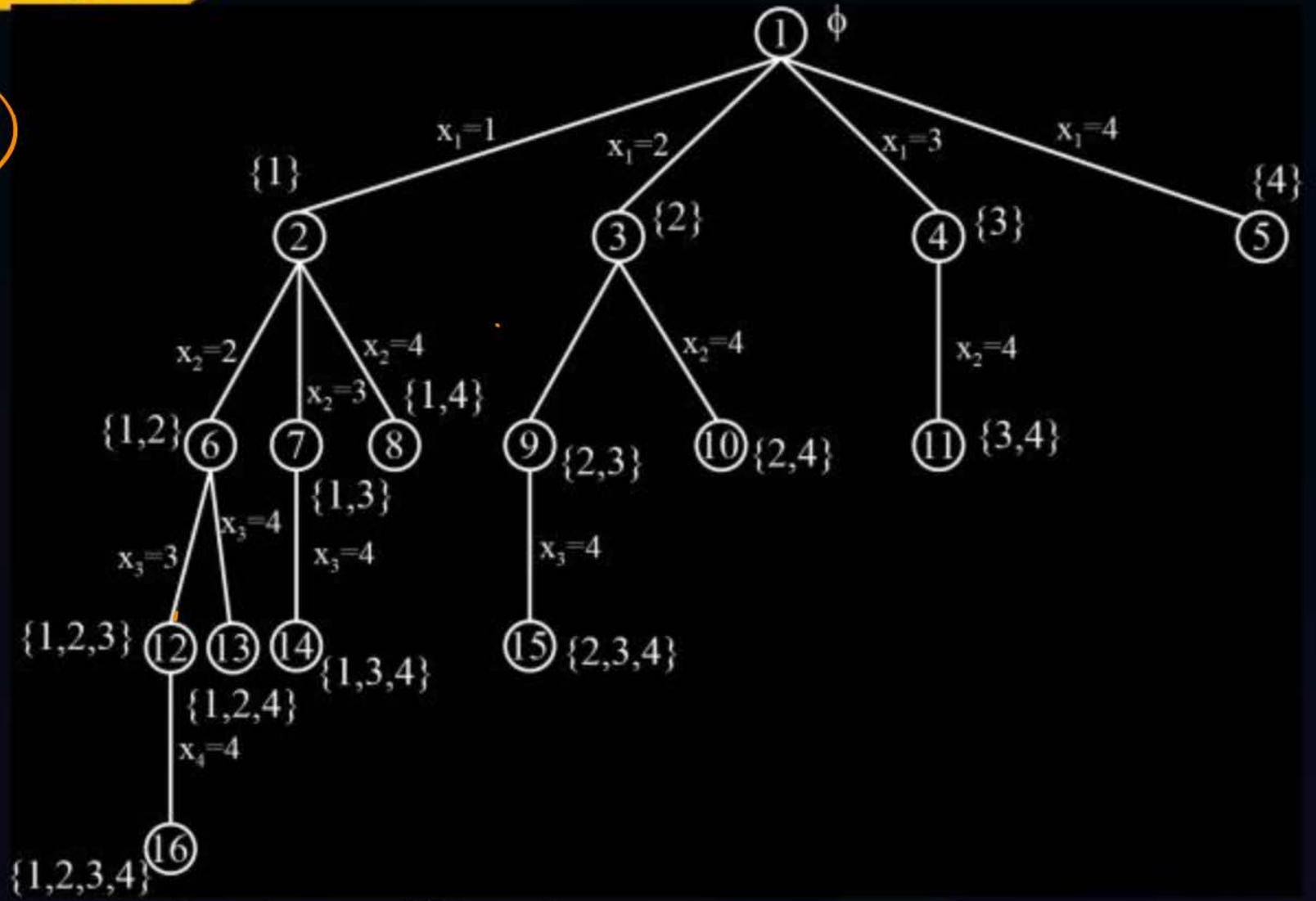
State Space Tree for n = 4



In this tree, each node is representing 1 possible subset.

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$n = 3$

$\{a_1, a_2, a_3, a_4\} \Rightarrow 2^4 = 16$ Subsets

$\{a_1, a_2, a_3\}$

φ

a_1

a_2

a_3

$a_1\ a_2$

$a_1\ a_3$

$a_2\ a_3$

$a_1\ a_2\ a_3$

Subsets = $2^3 = 8$

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For 4 elements, how many subsets?

$= 2^4 = 16$

And no. of nodes/states in Tree also 16.

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Test: Graph Coloring

- You are given a graph having n vertices/nodes.
- You have to color this graph using minimum colors such that no two adjacent nodes have the same color.

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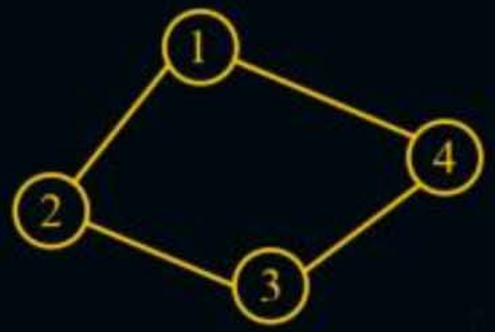
#Q. How many States gets generated for 1st optional solution.



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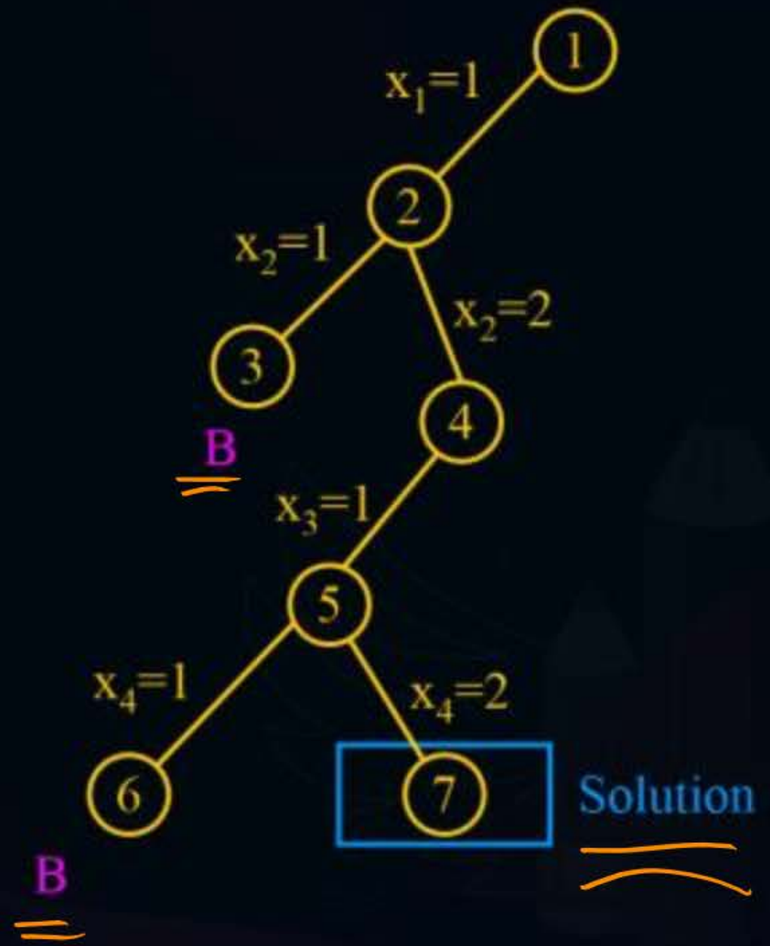
7.1.



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Branch and Bound

- Mostly same idea as that of Backtracking but it uses different Searching approach.
- Backtracking → DFS + Bounding Function
- Branch & Bound → BFS + Bounding Function

DFS → Stack

BFS → Queue

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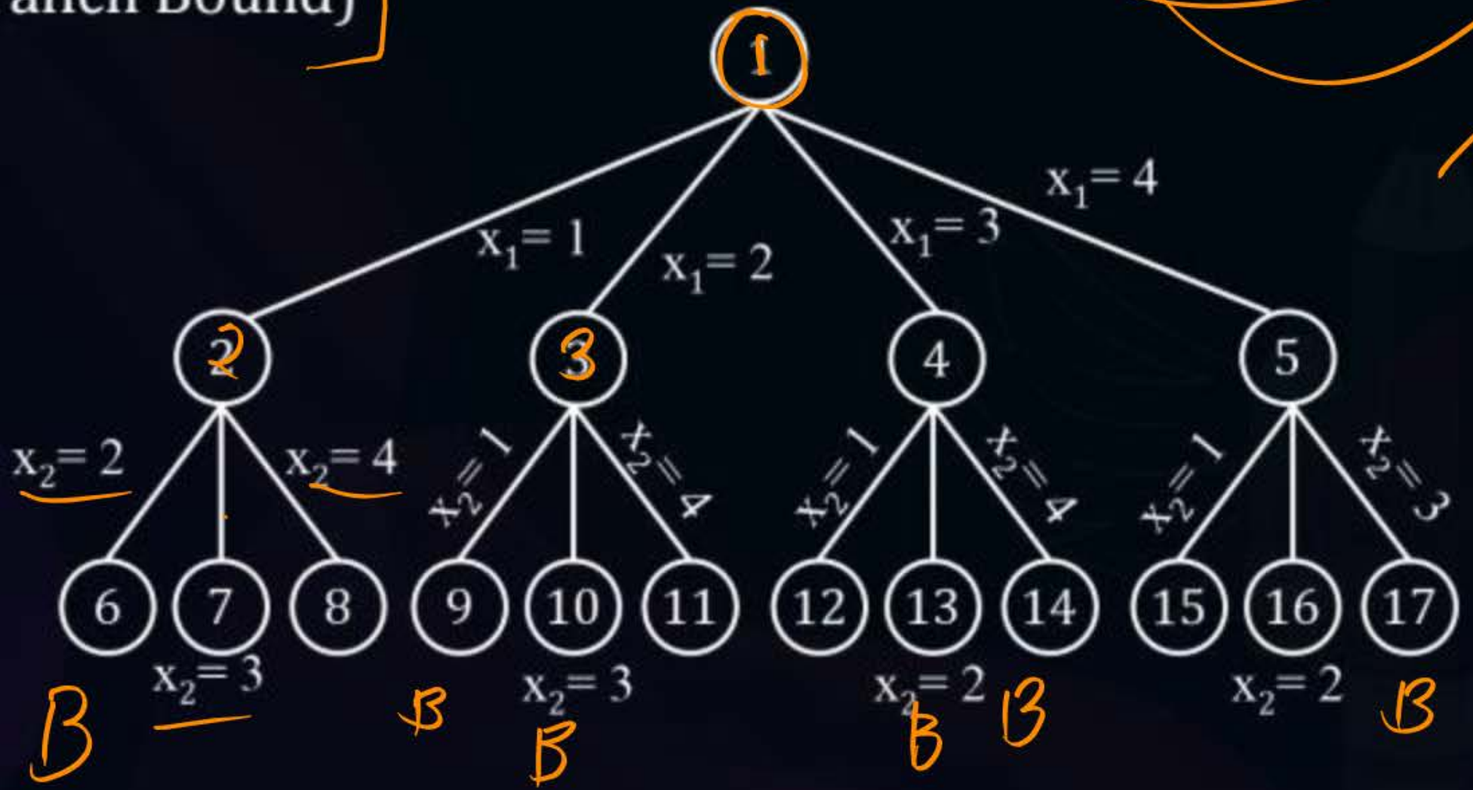
Queue (F1 F0)

2	3	4	5	7	8	11	12	15	16

Example:

n-Queens
(Using Branch Bound)



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	1	2	3	4
q_1				✓
q_2	✓	✓	×	×
q_3				
q_4				

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1 Problems that can be solved using Backtracking (Popular)

- N-Queens
- Graph Coloring
- Branch Bound
- O/I Knapsack
- Hamiltonian Cycle

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2. Popular Problems that can be solved using Branch - Bound:
   - N-Queens
   - Travelling Salesman Problem (TSP)
   - Job sequencing with deadline (JSD)
   - O/I Knapsack
   - 15 Puzzle Pattern

Summary

OS DBMS Algo OS

Dec / Jan → Revision → FLT → 3-4

Classes → 4 hrs

Self → 7-8 hrs

} 12-13 hr

THANK - YOU