VIJAY

# DATA SCIENCE & ARTIFICIAL INTELLIGENCE & CS

## Calculus and Optimization

Lecture No. 01



By- Dr. Puneet Sharma Sir

# Topics to be Covered

**Topic**

FUNCTIONS & GRAPHS - 1

# STRATEGY

1. Live Class
2. Class Notes — (3 times Revision)
3. DPP
4. W. Test
5. O.T.S
6. P.Y.Q

} Judge yourself

- PARACHUTE LANDING
  - → Doubts Not allowed for 1 or 2 days.
- Doubts Engine → ✓

- Book: No Book Required.
- Tel: drpuneet sir pw

## CALCULUS

300-400 Q Class

70-80 Q DPP

26 Q WT

300+ Q PYQ

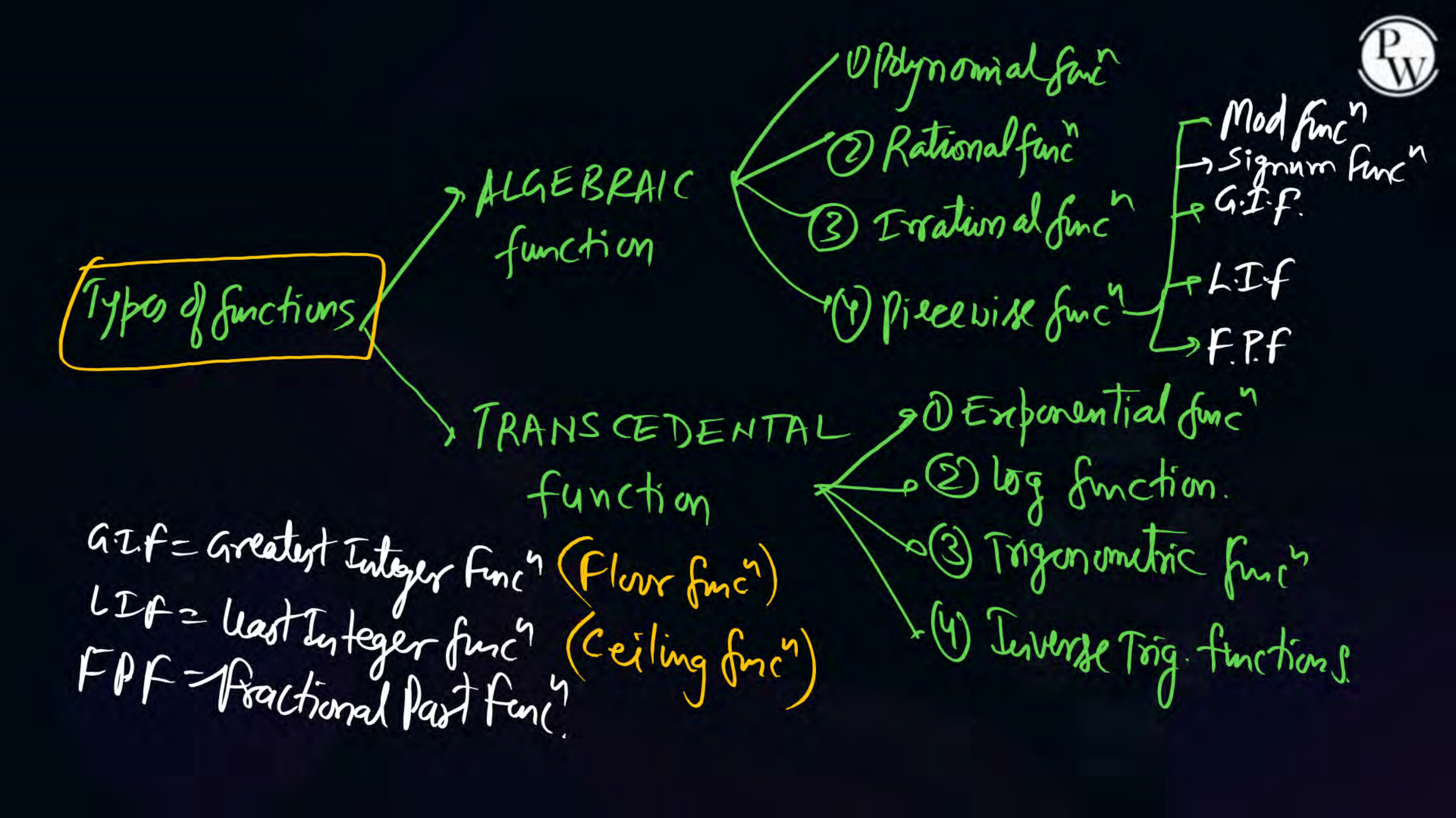
1000 Q

CS/IT: 7-8 Marks.

Maths

DA → 43 Marks (2025)

1. Linear Algebra
   - Part 1 (for Both DA & CS)
   - Part 2 (only for DA)
2. Calculus
3. Prob & Stats
   - Part 1 (for Both DA & CS)
   - Part 2 (only for DA)


Types of functions
ALGEBRAIC function
① Polynomial func^n
② Rational func^n
③ Irrational func^n
④ Piecewise func^n
Mod func^n
Signum func^n
G.I.f.
L.I.f
F.P.f
TRANSCEDENTAL function
① Exponential func^n
② log function.
③ Trigonometric func^n
④ Inverse Trig. functions
G.I.f = Greatest Integer Func^n (Floor func^n)
L.I.f = Least Integer func^n (Ceiling func^n)
F.P.F = Fractional Part Func^n

**Polynomial:** It's Domain is $(-\infty, \infty)$ & Degree $= 0, 1, 2, 3, 4, 5 :-$ ____.

& it's Definition is same at all points in the Domain of $y = f(x)$

eg $y = k$ (Constant poly) $\simeq$ degree $= 0$

$y = ax + b$ (Linear poly) $\simeq$ Deg $= 1$

$y = ax^2 + bx + c$ (Quad. poly) $\simeq$ Deg $= 2$

$y = ax^3 + bx^2 + cx + d$ (Cubic poly) $\simeq$ Dg $= 3$

⋮

Sp Note- $y = |x| = \begin{cases} -x, & x < 0 \\ +x, & x \geq 0 \end{cases}$, $D_f = (-\infty, \infty)$. It's not a poly bcoz it's Def$^n$ is not unique at all points in the Domain.

**Even func$^n$:** if $f(-x)=f(x) \Rightarrow f(x)$ is called an Even func$^n$
& it's Graph is symmetrical about Y axis

**odd func$^n$** if $f(-x)=-f(x) \Rightarrow f(x)$ is called an odd func$^n$.
& it's Graph is symmetrical about origin i.e ( I $\leftrightarrow$ III & II $\leftrightarrow$ IV )

**NENO func$^n$** if $f(-x) \neq f(x)$ & $f(-x) \neq -f(x)$ } then $f(x)$ is called NENO func$^n$.

it's Graph is neither symmetrical about Y axis, nor about origin.

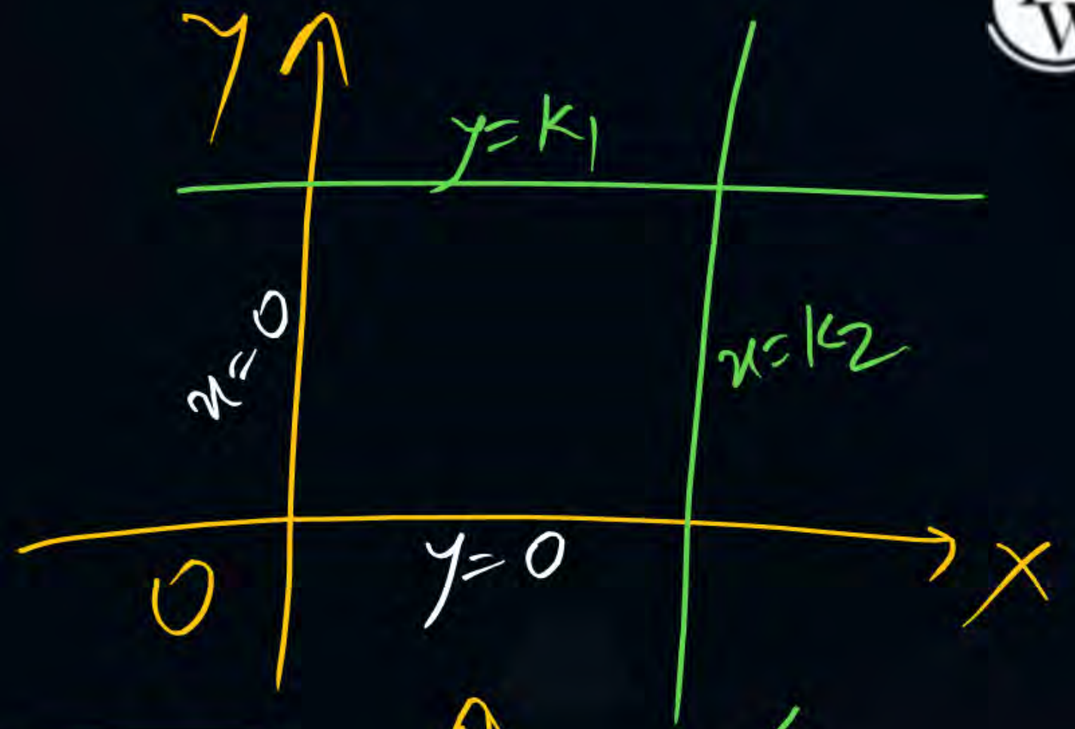
## BASIC GRAPHS :→

① Equⁿ of x axis is $y = 0$

② " of line ∥ᵍ to x axis $y = k_1$

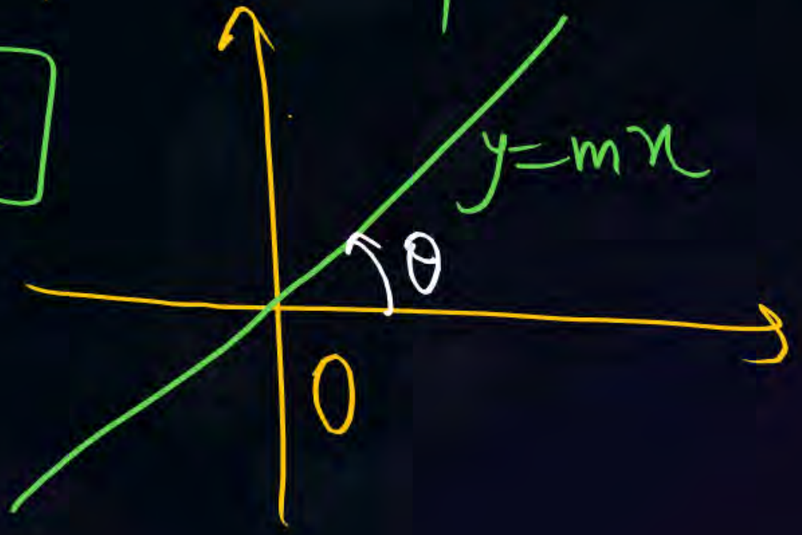
③ Equⁿ of y axis $x = 0$

④ Equⁿ of line ∥ᵍ to y axis $x = k_2$




⑤ Equⁿ of line passing through origin is $\boxed{y = mx}$

where $m$ = slope of line = $\tan\theta$.



## ⑥ Slope Intercept Form of line :

$y = mx + c$


$m = \tan\theta = -ve$

$c = y$ Intercept.

M-II

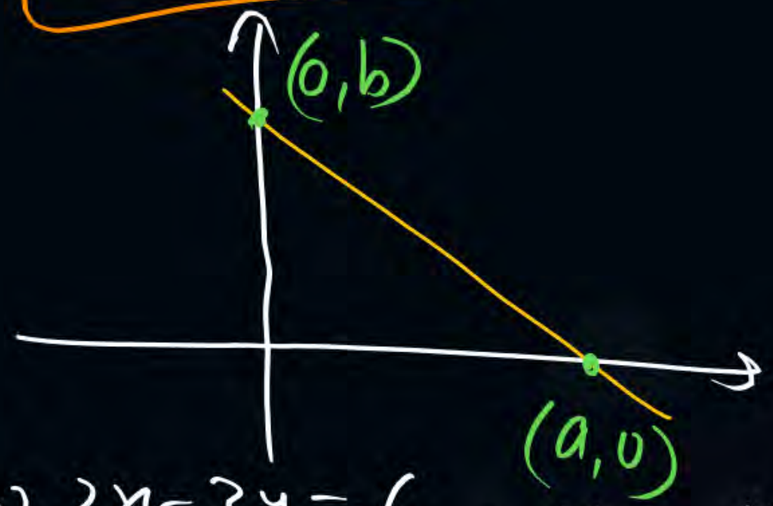
$-mx + y = c$

$-\frac{mx}{c} + \frac{y}{c} = 1$

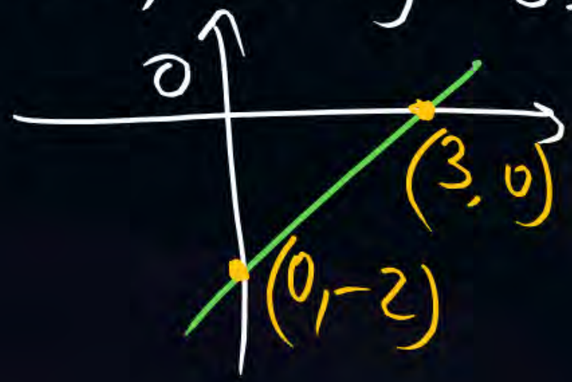
$\frac{x}{-c/m} + \frac{y}{c} = 1$

## ⑦ Intercept Form of Line

$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$




eg Draw, $2x - 3y = 6 \Rightarrow \frac{x}{3} + \frac{y}{-2} = 1$



**PQ**: Draw the graph of $y = \begin{cases} 1+x, & -1 \le x \le 0 \\ 1-x, & 0 \le x \le 1 \end{cases}$ = Even func$^n$

**Case I**: $-1 \le x \le 0$

$y = 1 + x$

$-x + y = 1$

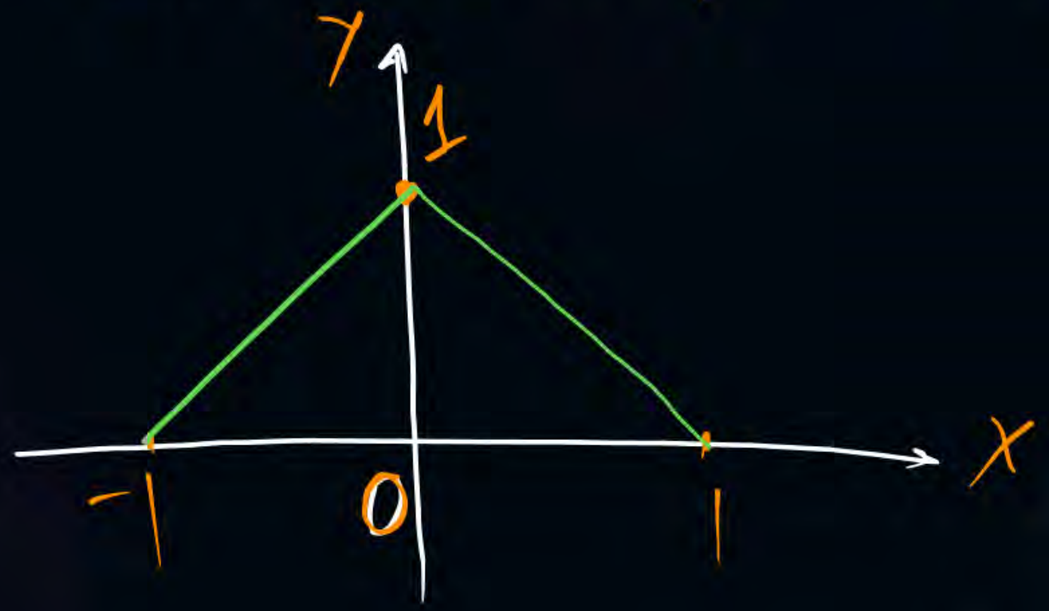
or $\boxed{\frac{x}{-1} + \frac{y}{1} = 1}$

**Case II**: $0 \le x \le 1$

$y = 1 - x$

$x + y = 1$

or $\boxed{\frac{x}{1} + \frac{y}{1} = 1}$

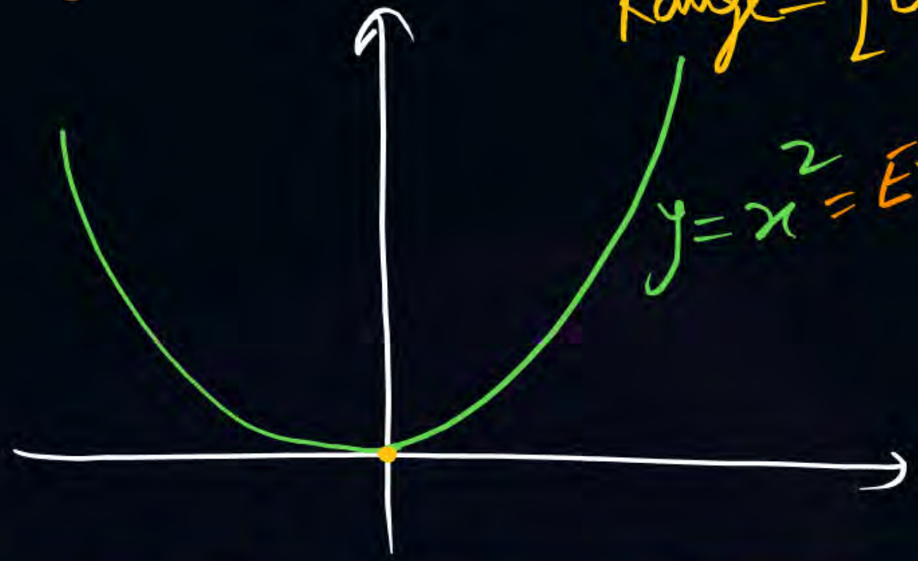


## ⊛ Poly of Even Degree

$y = x^2, x^4, x^6 \ldots$ Dom $= (-\infty, \infty)$

Range $= [0, \infty)$

$y = x^2 =$ Even func$^n$



| $x$: | $\ldots -3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3 \ldots$ |
|---|---|---|---|---|---|---|---|
| $y$: | $\ldots 9$ | $4$ | $1$ | $0$ | $1$ | $4$ | $9 \ldots$ |

$y = x^2$

## ⊛ Poly of Odd Degree

Dom $= (-\infty, \infty)$

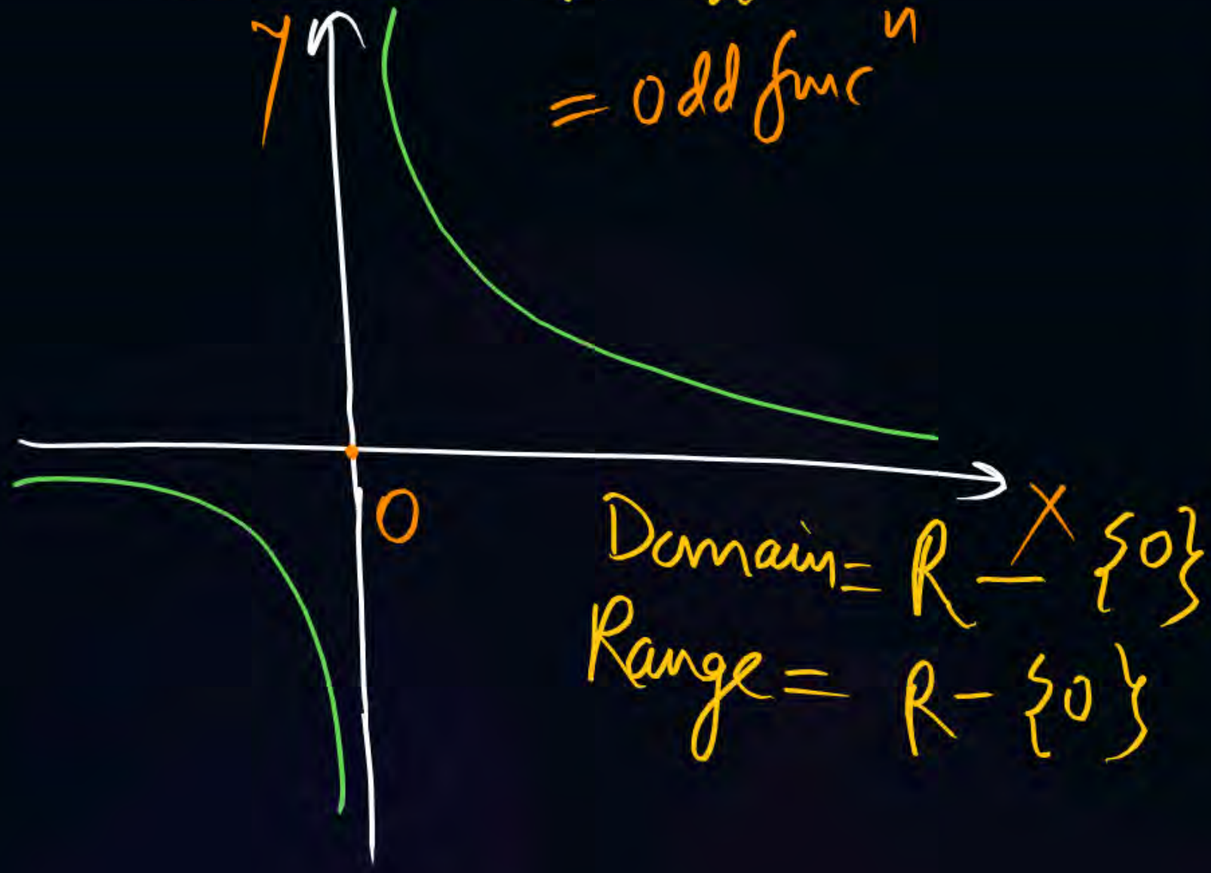
$y = x^3, x^5, x^7, \ldots$ Range $= (-\infty, \infty)$

$y = x^3 =$ Odd func$^n$

# Rational Func$^n$

$y = \frac{1}{x}, \frac{1}{x^3}, \frac{1}{x^5}, \ldots\ldots$

$=$ odd func$^n$

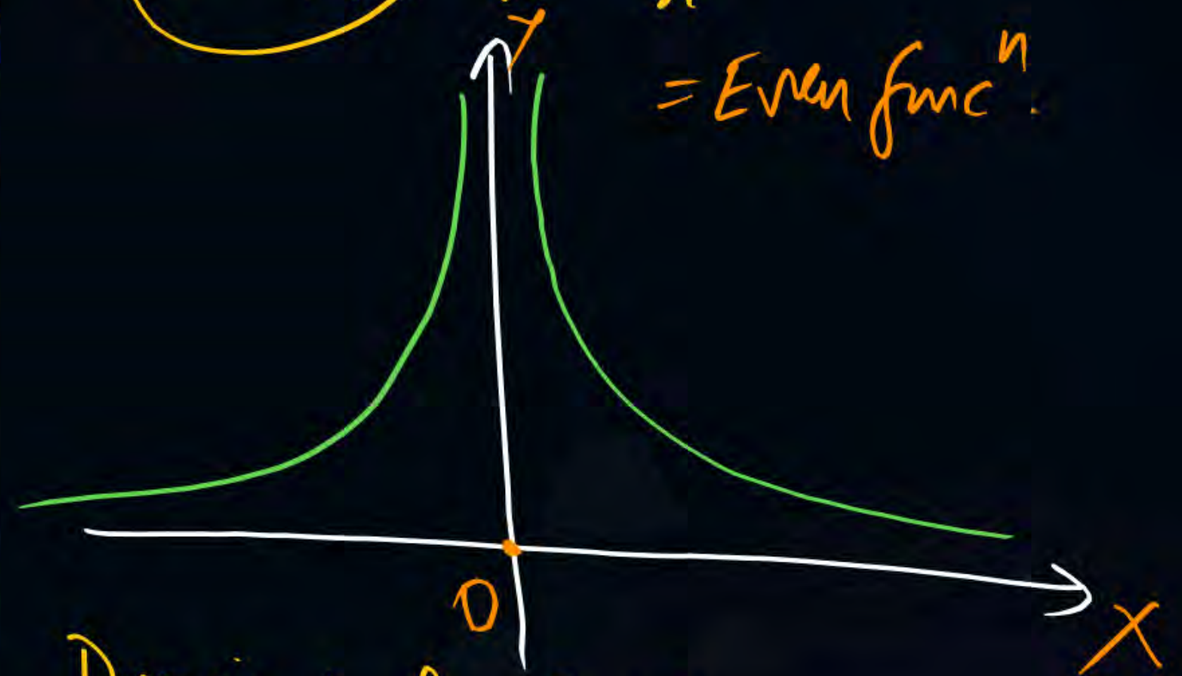



Domain $= R - \{0\}$

Range $= R - \{0\}$

$y = \frac{1}{x^2}, \frac{1}{x^4}, \frac{1}{x^6}, \ldots\ldots$

$=$ Even func$^n$.

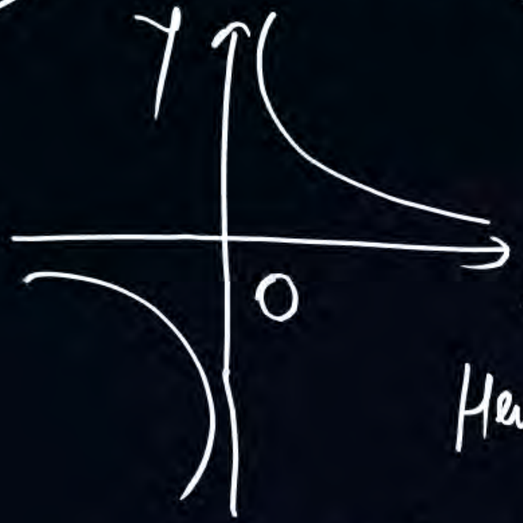



Domain : $R - \{0\}$

Range : $(0, \infty)$ or $y > 0$

ANALYSIS / PODCAST :→ $\frac{\text{Something}}{0} = \text{N.D.}$

(Ex-I) we know that, $y = \frac{1}{x}$



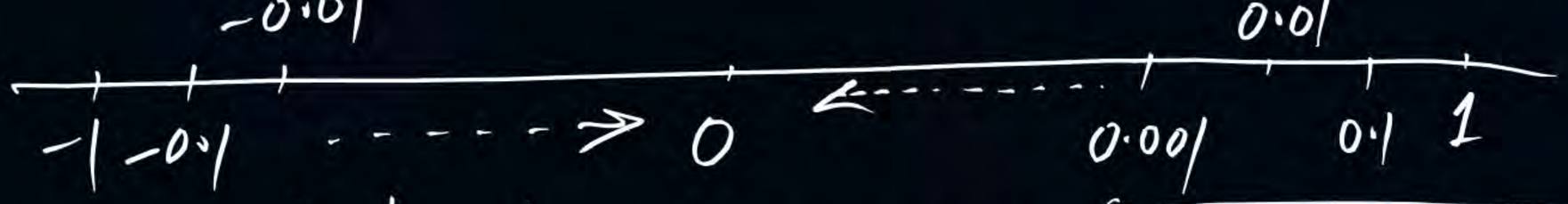

$y(0^-) = -\infty = \text{N.D}$

$y(0^+) = +\infty = \text{N.D}$

Hence $y(0) = \text{N.D}$.

Hence we have a liberty to choose $\frac{\text{Something}}{0} = \infty$ or $-\infty$ as Required.

M-II




$\frac{1}{1} = 1$

$\frac{1}{0.1} = 10$

$\frac{1}{0.01} = 100$

$\frac{1}{0.001} = 1000$

$\frac{1}{0^+} = +\infty$

$\frac{1}{-1} = -1$

$\frac{1}{-0.1} = -10$

$\frac{1}{-0.01} = -100$

$\frac{1}{0^-} = -\infty$

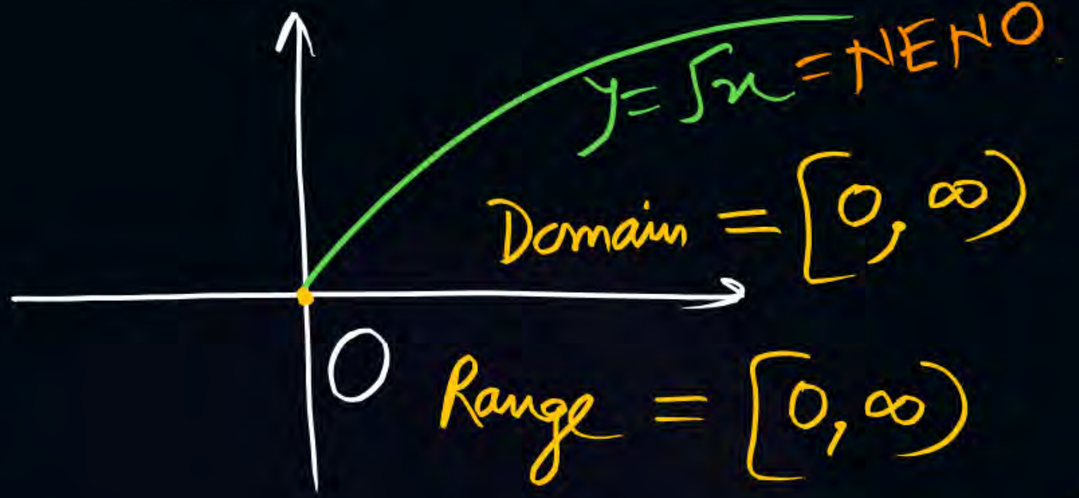
That is why $\frac{\text{Something}}{0} = \text{N.D}$

M-III $\frac{40}{8} = 5 \Rightarrow$ If we want to distribute 40 Mangoes among 8 children then each child will get 5 Mangoes

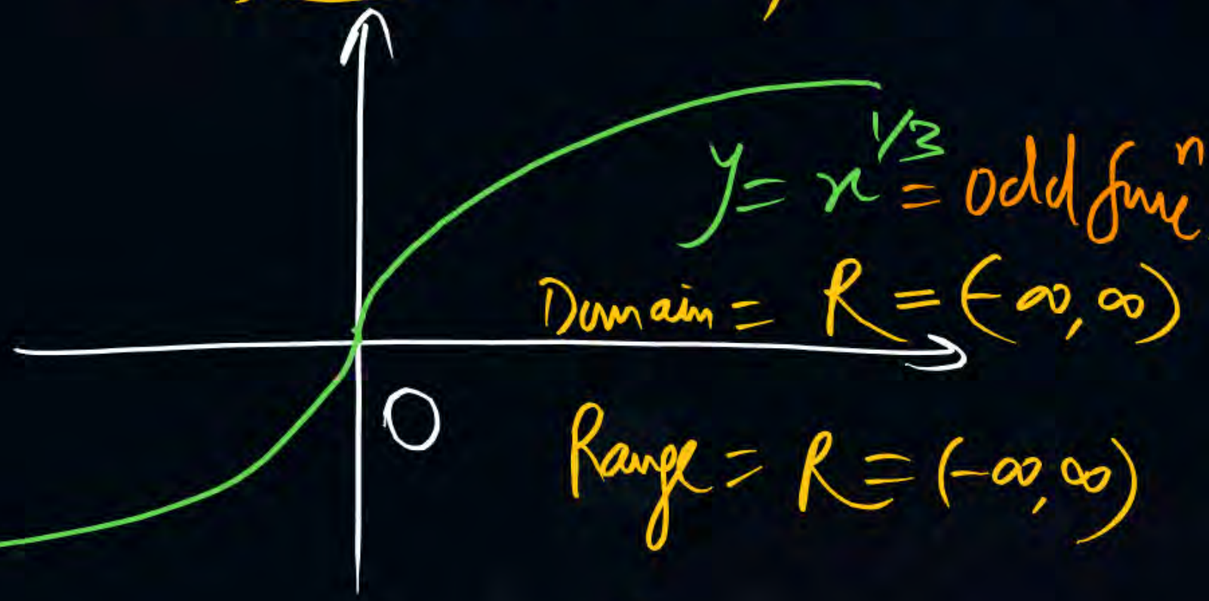
Similarly: $\frac{40}{0} = \text{N.D}$

# Irrational Func$^n$

(1) $y = x^{1/2}$, $x^{1/4}$, $x^{1/6}$ .....

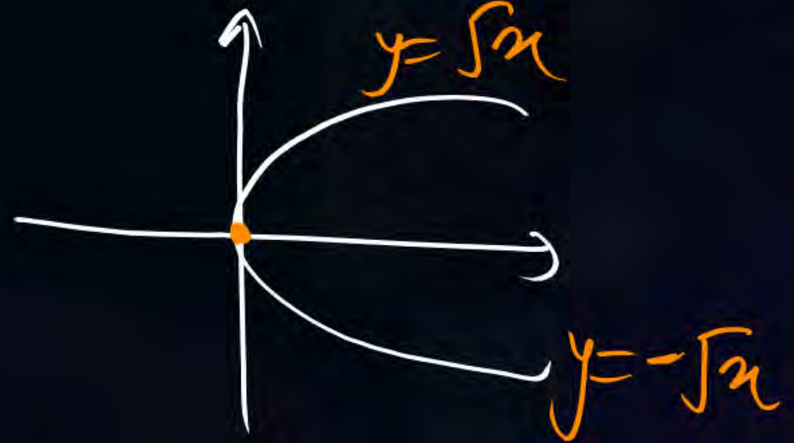



(2) $y = x^{1/3}$, $x^{1/5}$, $x^{1/7}$ .....

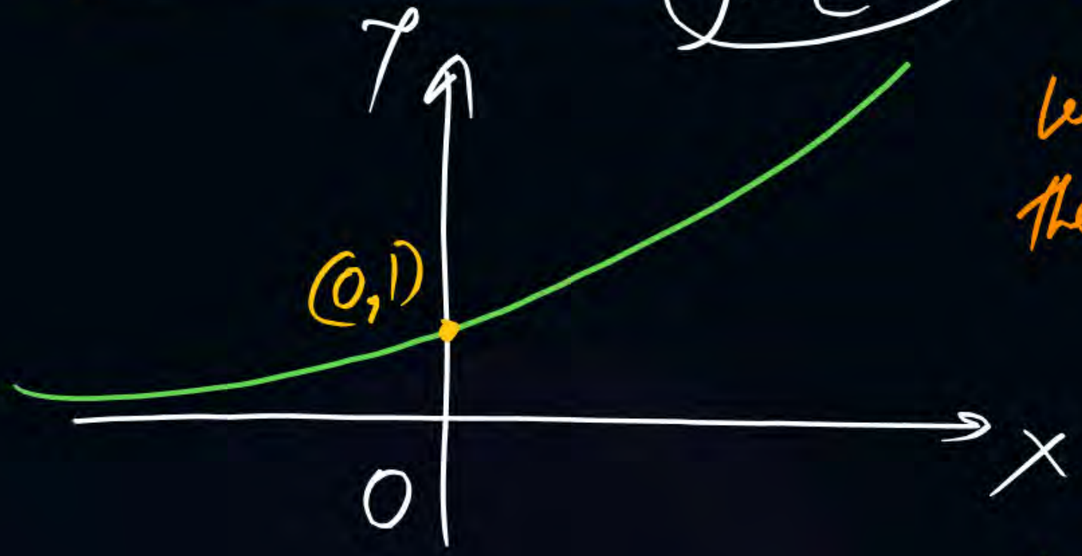



Analysis: Parabola: $y^2 = x$ $\Rightarrow$ $y = \sqrt{x}$, $y = -\sqrt{x}$

it is Not a func$^n$



## ⑧ Exponential func$^n$ $y = e^x$




$y(0) = e^0 = 1$

$y(-\infty) = e^{-\infty} = 0$

$y(\infty) = e^{\infty} = +\infty$

= NENO func$^n$

Let $f(x) = e^x$

Then $f(-x) = e^{-x} = \frac{1}{e^x} = \frac{1}{f(x)}$

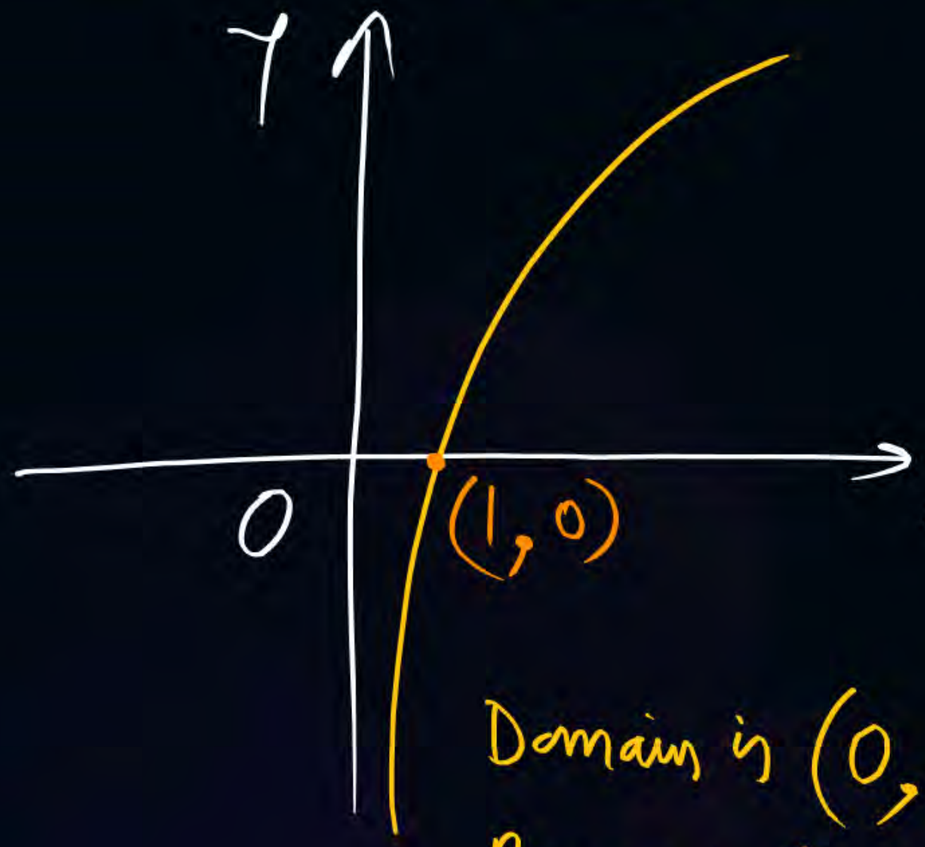
i.e $f(-x) \neq f(x)$ or $-f(x)$

So $f(x) = e^x$ is NENO func$^n$

Domain $= (-\infty, \infty)$ i.e $x \in (-\infty, \infty)$

Range $= (0, \infty)$ i.e $y > 0$

(x) log func$^n$ → $y = \log_e x$ or $y = \ln x$ = NEHO




$y(1) = \log_e 1 = 0$

$y(0) = \log_e 0 = -\infty$ = N.D

$y(\infty) = \log_e \infty = +\infty$ = N.D

$y(-\infty) = \log_e(-\infty)$ = N.D

Domain is $(0, \infty)$ i.e $y = \log_e x$ is defined for $x > 0$

Range is $(-\infty, \infty)$ i.e $y \in (-\infty, \infty)$

**PIECEWISE func$^n$** → If func$^n$ is defined by Multiple sub function

s.t, Domain of each subfunction is different

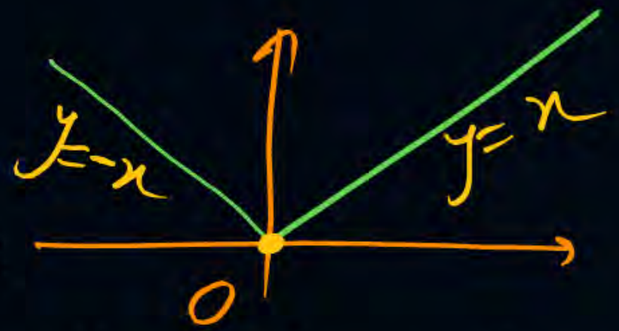
then function is called Piecewise func$^n$

for eg. Mod func$^n$, Signum func$^n$, G.I.F, L.I.F, Fractional Part func$^n$ etc

# MODULUS func$^n$ →

Def$^n$-1 | Def 2 | Def 3

$$y = |x| = \begin{cases} -x, & x < 0 \\ +x, & x \geq 0 \end{cases} = \max(x, -x) = \sqrt{x^2} =$$




= Even func$^n$ ← Domain $= (-\infty, \infty)$, Range $= [0, \infty)$ & it is not a polynomial.

eg $|3| = +(3) = +3$

$|-3| = -(-3) = +3$

eg $|3| = \max\{3, -3\} = +3$

$|-3| = \max\{-3, -(-3)\} = +3$

eg: $|3| = \sqrt{(3)^2} = \sqrt{9} = +3$

$|-3| = \sqrt{(-3)^2} = \sqrt{9} = +3$

$\because \sqrt{x^2} = |x| = +ve$

ie Sq. Root of any Real No is always +ve.

eg $x^2 - 9 = 0$

$x^2 = 9$

$x = \sqrt{9}$ WRONG step

$x = \pm 3$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm\sqrt{9}$

$x = \pm(+3)$

$x = \pm 3$

$x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x = 3, -3$

Q: $\left[(-4)^2\right]^{\frac{1}{2}} = ?$

$\left[(-4)^{\frac{2}{1}}\right]^{\frac{1}{2}} = -4$ ✗

$\sqrt{(-4)^2} = \sqrt{16} = +4$ ✓

(a) 4

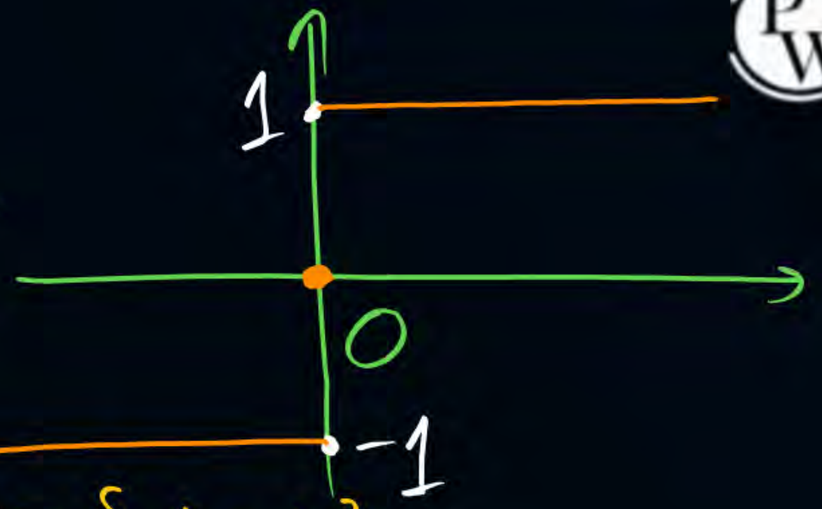
(b) $-4$

(c) $\pm 4$

(d) None

## ⊛ Signum func$^n$ →

$$y = \operatorname{sgn}(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} -1, & x < 0 \\ +1, & x > 0 \\ 0, & x = 0 \end{cases}$$

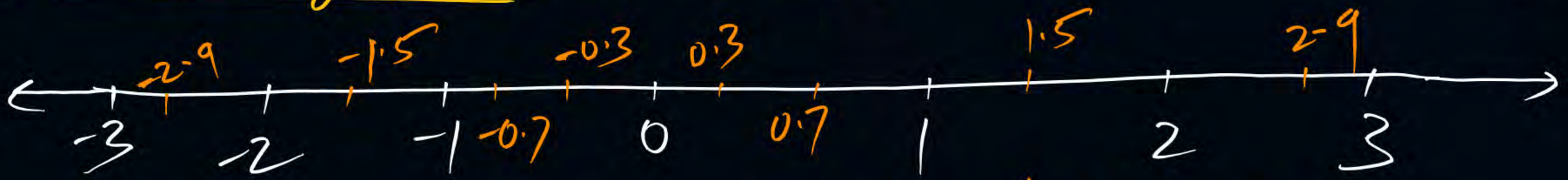



= Odd func$^n$, Domain $= (-\infty, \infty)$, Range $= \{-1, 0, 1\}$

eg $\operatorname{sgn}(0) = 0$, $\operatorname{sgn}(1.3) = 1$, $\operatorname{sgn}(1.9) = 1$, $\operatorname{sgn}(2.7) = 1$ .....

& $\operatorname{sgn}(-3.4) = -1$, $\operatorname{sgn}(-0.2) = -1$, $\operatorname{sgn}(-4.5) = -1$ .....

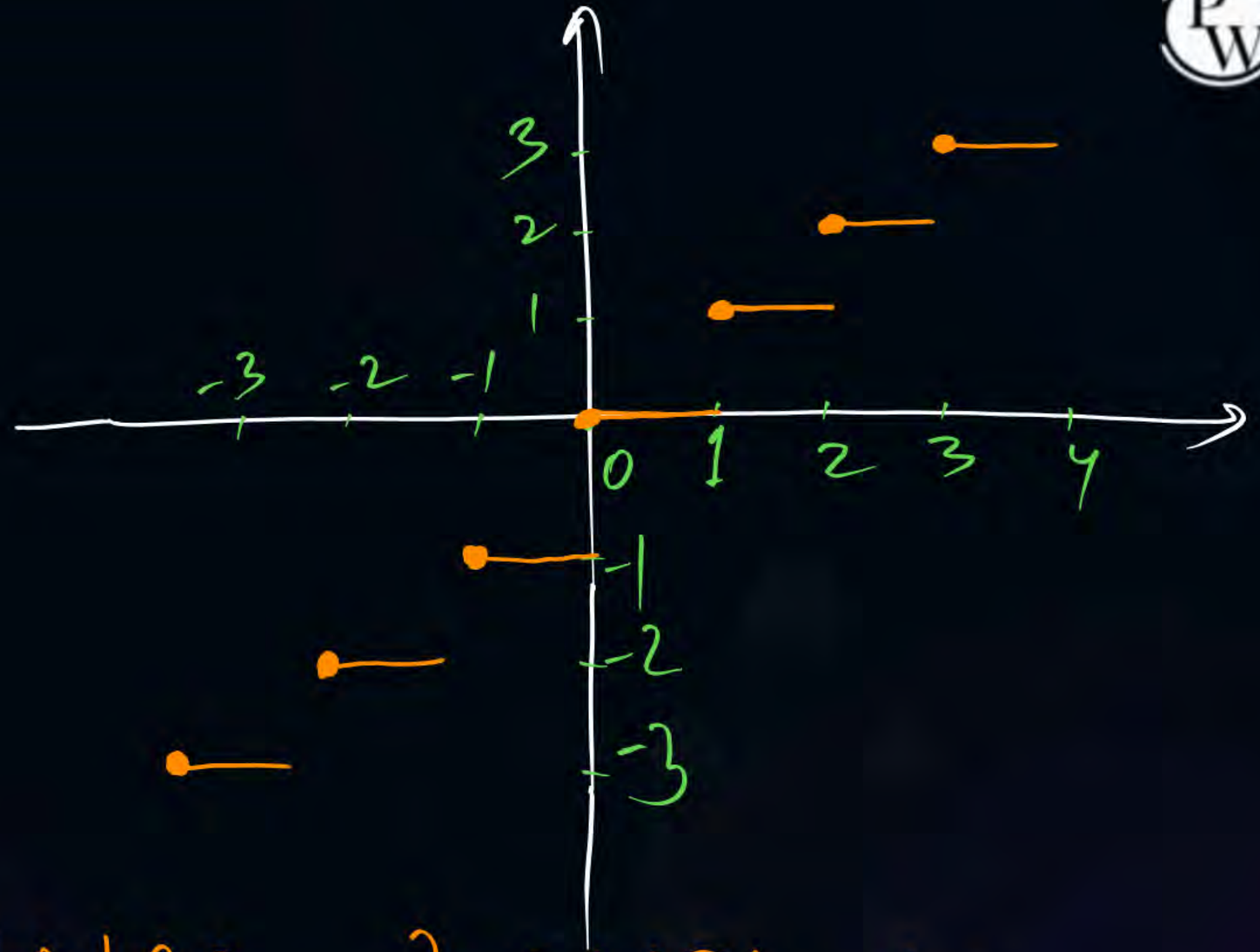
## ⊛ Greatest Integer func$^{n}$ (Floor function) →




$[0.3] = 0$, $[0.7] = 0$, $[1.5] = 1$, $[2.9] = 2$

$[-0.3] = -1$, $[-0.7] = -1$, $[-1.5] = -2$, $[-2.9] = -3$

$$y=[x]=\begin{cases} \vdots & \\ -2 & , -2\le x<-1 \\ -1 & , -1\le x<0 \\ 0 & , 0\le x<1 \\ 1 & , 1\le x<2 \\ 2 & , 2\le x<3 \\ 3 & , 3\le x<4 \\ \vdots & \end{cases}$$

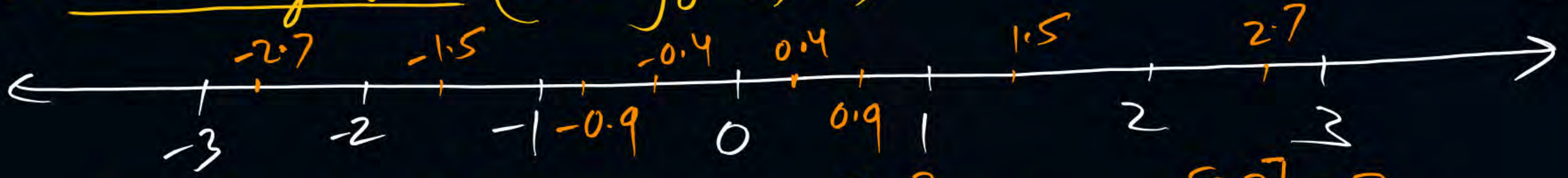



= NENO fun$^{n}$

Domain $= (-\infty, \infty)$

Range $= \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\} =$ Set of Integers $(Z)$

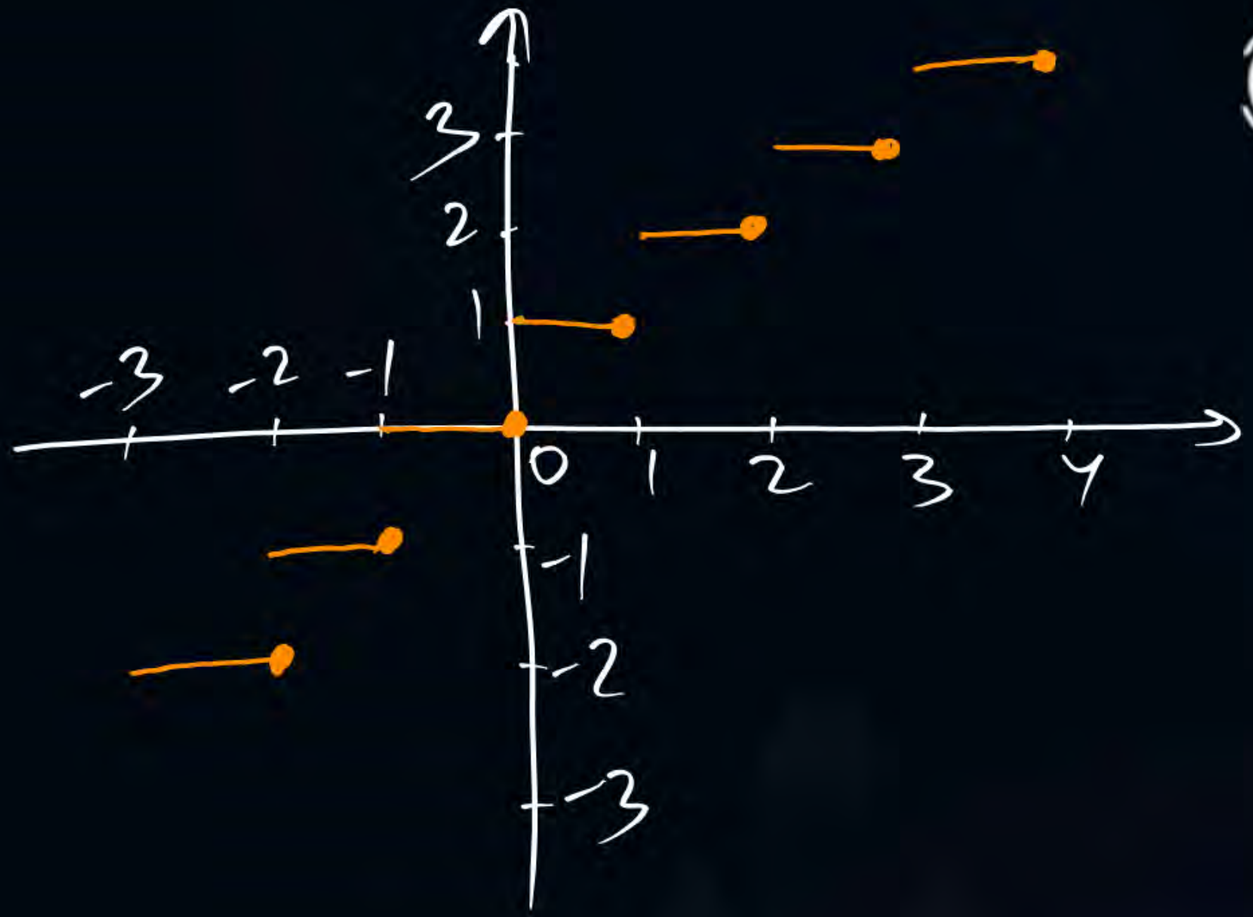
## (*) Least Integer func$^n$ (ceiling func$^n$) →




$\lceil 0.4 \rceil = 1$, $\lceil 0.9 \rceil = 1$, $\lceil 1.5 \rceil = 2$, $\lceil 2.7 \rceil = 3$

$\lceil -0.4 \rceil = 0$, $\lceil -0.9 \rceil = 0$, $\lceil -1.5 \rceil = -1$, $\lceil -2.7 \rceil = -2$

$$y = \lceil x \rceil = \begin{cases} \vdots & \vdots \\ -1 & -2 < x \le -1 \\ 0 & -1 < x \le 0 \\ 1, & 0 < x \le 1 \\ 2, & 1 < x \le 2 \\ 3, & 2 < x \le 3 \\ 4 & 3 < x \le 4 \\ \vdots & \vdots \end{cases}$$




= NENO func$^n$

Domain $= (-\infty, \infty)$, Range $= \{\ldots -2, -1, 0, 1, 2, 3 \ldots\} = \mathbb{Z}$.

Fractional Part func$^n$ : $y = \{x\} = \boxed{x - \lfloor x \rfloor}$

Fcl: dr puneet sir pw

# THANK you