# Computer Science & Information Technology

## Theory Of Computation

DPP: 1

## Grammar

**Q1** Consider alphabet Σ = {a, b}, the empty string ∈ and the set of strings S, P, Q and R generated by the corresponding non-terminals of a regular grammar. S, P, Q and R related as follows (S is a start symbol):

S → aP | bQ | ∈

P → bR | aS

Q → aR | bS

R → aQ | bP

(A) L = {w:$n_a$(w) and $n_b$(w) both are even}.

(B) L = {w:$n_a$(w) and $n_b$(w) both are odd}.

(C) L = {w:$n_a$(w) or $n_b$(w) are even}.

(D) None of these.

**Q2** Consider the following language L on alphabet eod reΣ = {a, b}

$L = \{wxw^R \mid w, x \in \{a, b\}^+\}$

The correct regular grammar of above language is/are possible?

(A) S → aAa | bAb

A → aA | bA | a | b

B → aA | bA | a | b

(B) S → aAa | bAb | ∈

A → ab

(C) S → aA | bB

A → aA | bA | a

B → bB | aB | b

(D) S → Aa | Bb

A → Aa | Ab | a

B → Bb | Ba | b

**Q3** Consider the following grammar G:

**G:**

S → A B C

A → aA | a

B → bc

C → cC | ∈

The language generated by above grammar is?

(A) L = {$a^*$ bc $c^*$}

(B) L = {$a^+$ b $c^+$}

(C) L = {$a^*$ b $c^*$}

(D) None of these

**Q4** Consider the following two language $L_1$ and $L_2$.

$L_1 = \{www \mid w \in \{a\}^*\}$

$L_2 = \{\{a^{n^n}\}^* | n \geq 1\}$

Which of the following is correct?

(A) $L_1$ is regular.

(B) $L_2$ is regular.

(C) Both $L_1$ and $L_2$ are regular.

(D) None of these.

**Q5** Which of the following language is non-regular?

(A) $L = \{wxw^R \mid x, w \in \{a, b\}^*\}$.

(B) $L = \{wxw \mid w, x \in \{a, b\}^*\}$.

(C) L = {wxwx | w, x ∈ {a, b}*}.

(D) None of these

**Q6** Consider the following grammars $G_1$ and $G_2$:

**$G_1$:**

S → aAb

A → aB |∈

B → Ab

**$G_2$:**

S → aABb

A → aA |∈

B → bB | ∈

Which of the following grammar is/are regular?

(A) $G_1$ only

(B) $G_2$ only

(C) Both $G_1$ only $G_2$

(D) None of these

**Q7** Consider the following three languages:

(1) $L = \{a^{n^n} | n \geq 1\}$

(2) $L = \{a^{m^n} | m = n^2, n \geq 1\}$

(3) $L = \{a^{m^n} | n \geq 1, m > n\}$

Total number of regular languages is/are_____.

**Q8** Which of the following language is non-regular?

(A) $L = \{a^{2m} b^n b^n \mid m, n \geq 1\}$

(B) $L = \{a^m b^n X \mid m, n \geq 1, X \in \{a,b\}^*\}$

(C) $L = \{\{a^{n^2}\}^* | n \geq 0\}$

(D) None of these

**Q9** Consider following statements:

$S_1$: Kleene Closure (*)of infinite set is always finite.

$S_2$: Kleene Closure (*) of finite set is always infinite.

Which of the following is correct?

(A) $S_1$ only (B) $S_2$ only

(C) Both $S_1$ and $S_2$ (D) None of these

**Q10** Consider the following statements:

[I] If L is regular, then $\overline{L}$ is regular.

[II] If $\overline{L}$ is regular, then L is regular.

[III] Union of L and its complement is $\Sigma^*$

Number of correct statement is/are_____.

**Q11** Consider a regular language L, which of the following statements are true regarding L.

(A) Prefix(L) = $\{w \mid ww_1 \in L, w_1 \in \Sigma^*\}$ is regular.

(B) Suffix(L) = $\{w \mid w_1 w \in L, w_1 \in \Sigma^*\}$ is regular.

(C) Quotient(L) is regular.

(D) L is closed under infinite intersection.

**Q12** Consider a regular language L over the alphabet

$\Sigma = \{a, b\}$. L is defined as L= (a + b*) (bab*).

If homomorphism h is defined over T = {c, d, e} and

h(a) = cd

h(b) = cddec

Then the regular language h(L) is given as

(A) (cd + cddec) (cddec cd cddec)

(B) (cddec) (cd + cddec*)

(C) (cd + (cddec)*) ((cddec) (cd) (cddec)*)

(D) None of these

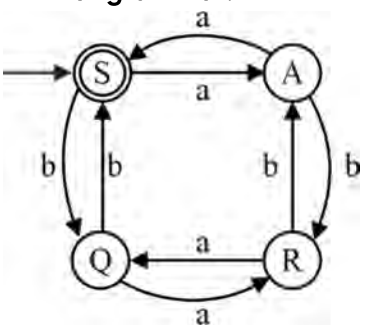
# Answer Key

| | |
|---|---|
| Q1 | (A) |
| Q2 | (A) |
| Q3 | (B) |
| Q4 | (C) |
| Q5 | (D) |
| Q6 | (B) |
| Q7 | 1 |
| Q8 | (D) |
| Q9 | (D) |
| Q10 | 3 |
| Q11 | (A, B, C) |
| Q12 | (C) |

# Hints & Solutions

**Q1 Text Solution:**

**DFA for grammar:**




$L = (aa + ab + ba + bb)^*$

Hence, option (a) is correct.

**Q2 Text Solution:**

$L = \{wxw^R \mid w, x \in \{a, b\}^+\}$

$a(a + b)^+ a \mid b(a + b)^+ b$

↓ ↓

$ab(a + b)^+ba$ $ba(a + b)^+ ba$

$aa(a + b)^+aa$ $bb(a + b)^+ bb$

L = Regular

Regular expression = $a(a + b)^+ a + b(a + b)^+b$

S → aAa | bAb

A → aA | bA | a | b

B → aA | bA | a | b

Regular expression for above grammar is $a(a + b)^+ a + b(a + b)^+b$

Hence, only (a) is correct.

**Q3 Text Solution:**

S → ABC = $aa^*bcc^*$

A → aA | a = $aa^*$

B → bc = bc

C → cC | ∈ = $c^*$

Regular expression = $aa^*bcc^*$

$= a^+ bc^+$

Hence, option (b) is correct.

**Q4 Text Solution:**

$L_1 = \{wxw \mid w \in \{a\}^*\}$

$L_1 = (aaa)^*$

Regular language

$L_2 = \{\{a^{n^n}\}^* \mid n \geq 1\}$

$L_2 = \{a^1\}^*$

$= a^*$

= Regular.

**Q5 Text Solution:**

(a) $L = \{wxw^R \mid x, w \in \{a, b\}^*\}$

Minimal string = $\in \cdot (a + b)^* \in$

$= (a + b)^*$

Regular

(b) $L = \{wx\, w \mid w, x \{a, b\}^*\}$

$L = \in \cdot (a + b)^* \cdot \in$

$= (a + b)^*$

Regular

(c) $L = \{wxwx \mid w, x \in \{a, b\}^*\}$

regular

Hence option (d) is correct.

**Q6 Text Solution:**

Only G2 is regular.

**Q7 Text Solution:**

(1) $L = \{a^{n^n} \mid n \geq 1\}$

$L = \{a, a^4, a^{27}, ....\}$ Non-regular

(2) $L = \{a^{m^n} \mid m = n^2, n \geq 1\}$

$L = \{a^{1^1}, a^{4^2}, a^{9^3} ....\}$

$= \{a, a^{16}, a^{43} ....\}$

Non-regular

(3) $L = \{a^{m^n} \mid n \geq 1, m > n\}$

$L = \{a^{2^1}, a^{3^1}, a^{4^1} ....\}$

$= \{a^2, a^3, a^4 ....\}$

$= aa(a)^*$

Regular

**Q8 Text Solution:**

(a) $L = \{a^{2m} b^n b^n \mid m, n \geq 1\}$

$= (aa)^+ b^{2n}$

$= (aa)^+ (bb)^+$ Regular

(b) $L = \{a^m b^n X \mid X \in \{a, b\}^* m, n \geq 1\}$

$= (a)^+ (b)^+ (a + b)^*$

= Regular

(c) $L = \left\{\left\{a^{n^2}\right\}^* \middle| n \geq 0\right\}$

$L = \{\in, a, aa, aaa \ldots\}$

$= a^*$

= Regular

Hence, option (d) is correct.

**Q9 Text Solution:**

**$S_1$:** False

Set = $\{\in\} = \{\in\}^* = \in$ only (Finite)

**$S_2$:** Set = $\{a\} = \{a\}^* = \in, a, aa, aaa, \ldots = (a^*)$ (Infinite)

So, both statements are false.

Hence, option (d) is correct.

**Q10 Text Solution:**

- L is regular if and only if Complement of L is regular.
- $L \cup \overline{L} = \Sigma^*$

Hence, all are correct statements.

**Q11 Text Solution:**

Regular language is closed under Prefix, Suffix and quotient of the language. But regular language are not closed under infinite intersection.

So, a, b, c are correct.

**Q12 Text Solution:**

Homomorphism is a function from strings to string which is based on concatenation.

for any a and b

L is defined as

$x = (a + b)^* (bab^*)$

then,

$h(L) = (h(a) + h(b)^* (h(b)h(a)h(b)^*)$

$= (cd + (cddec)^*)((cddec)(cd)(cddec)^*)$.