1) $x + y\frac{dy}{dx} = 2y.$

$$\frac{dy}{dx} = \frac{2y - x}{y}$$

put $y = vx$

$$\frac{dy}{dx} = \frac{d(vx)}{dx}$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

then,

$$v + x\frac{dv}{dx} = \frac{2vx - x}{vx}$$

$$x\frac{dv}{dx} = \cancel{\frac{2vx - x}{vx}} - v \quad \frac{\cancel{x}(2v-1)}{v\cancel{x}}$$

$$x\frac{dv}{dx} = \frac{2vx - x - v^2x}{vx} \quad \frac{2v - 1 - v^2}{v}$$

$$\frac{dx}{x} = \frac{v\,dv}{2v - v^2 - 1}$$

Integrating we got.

v^2-2v+1
$2v-2$

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$$\int\left(\frac{dx}{x}\right)=\int\left(\frac{v}{2v-v^2-1}\right)dv$$

$$\log x+\log c=\log v+\log x$$

log

(2) $(x^2-y^2)dx+2xy\,dy=0$

Soln.

$$(x^2-y^2)\,dx=-2xy\,dy$$

$$\frac{dy}{dx}=-\frac{x^2-y^2}{2xy}$$

put $y=vx$

the.

$$v+x\frac{dv}{dx}=-\frac{x^2-v^2x^2}{2xvx}$$

$$v+x\frac{dv}{dx}=-\frac{\cancel{x^2}(1-v^2)}{\cancel{x^2}(2v)}$$

$$v+x\frac{dv}{dx}=-\frac{1-v^2}{2v}$$

$$x\frac{dv}{dx}=-\left(\frac{1-v^2+2v^2}{2v}\right)$$

$$\frac{x}{dx}=-\left(\frac{1-v^2+2v^2}{2v}\right)dv$$

$$\frac{v^2x^2-x^2}{2vx^2}$$

$$\frac{x^2(v^2-1)}{x^2 2v}$$

$$\frac{v^2-1}{2v}-v$$

$$=\left(\frac{v^2-1-2v^2}{2v}\right)dv$$

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$$\frac{dx}{x} = \left(\frac{2v}{1-v^2}\right) dv = \frac{2v}{(1+v^2)}$$

Integrating we get,

$$\int \frac{dx}{x} = \int \frac{2v}{1+v^2} dv$$

$$\log x + \log c = \log(1+v^2)$$

$$\log x + \log c = \log\left(1+\frac{y^2}{x^2}\right)$$

$$\log x + \log c = \log\left(\frac{x^2+y^2}{x^2}\right)$$

$$\log(xc) = \log\frac{x^2+y^2}{x^2}$$

$$xc = \frac{x^2+y^2}{x^2}$$

(13). $\frac{dy}{dx} + y = \cos x.$

$\Rightarrow$ Sol^n $\frac{dy}{dx} + y = \cos x.$

$\cancel{\frac{dy}{dx} = \cos x - y.}$

$P = y. \quad \& \quad Q = \cos x.$

$If = e^{\int P dy} \quad = e^{x}.$

Multipling eq^n.

$y \times I.F = \int (Q \times If) + c$

$y \times e^{x} = \int e^{x} \cos x \, dx + c$

$y \cdot e^{x} = \frac{e^{x} (1 \cdot \cos x + 1 \sin x)}{1^2 + 1^2} + c$ // using $\int e^{ax} \cos bx \, dx = \frac{e^{ax} (a \cos bx + b \sin bx)}{a^2 + b^2}$

$y e^{x} = \frac{e^{x} (\cos x + \sin x)}{2}$ // $\int e^{ax} \sin bx \, dx = \frac{e^{ax} (a \sin bx - b \cos bx)}{a^2 + b^2}$

$2y = \cos x + \sin x + 2c e^{-x}$

$2y = \cos x + \sin x + k e^{-x}$

wher $k = 2c$

which is reqd. sol^n.

$$\frac{dy}{dx} = \frac{2x-y+1}{x+2y-3}$$

Sol^n,

$\Rightarrow (x+2y-3)\,dy = (2x-y+1)\,dx$

~~$M = x+2y-3$ & $N = 2x-y+1$~~

$\Rightarrow x\,dy + 2y\,dy - 3dy = 2x\,dx - y\,dx + 1dx$

or, $x\,dy + y\,dx + 2y\,dy - 3dy = 2x\,dx + dx$

or, $d(xy) + 2y\,dy - 3dy = 2x\,dx + dx$

Integrating,

$\Rightarrow \int d(xy) + 2\int y\,dy - 3\int dy = 2\int x\,dx + \int dx$

$\Rightarrow xy + \cancel{2}\cdot\frac{y^2}{\cancel{2}} - 3y = \cancel{2}\frac{x^2}{\cancel{2}} + x + c$

$\Rightarrow xy + y^2 - 3y - x^2 - x - c = 0$ #

$p^2 + 2px - 3x^2 = 0$

Sol^n, diff. eq^n

$p^2 + 2px - 3x^2 = 0$

$p^2 + 3px - px - 3x^2 = 0$

$p(p+3x) - x(p+3x) = 0$

$(p+3x)(p-x) = 0$

$\frac{dy}{dx} + 3x = 0 \Rightarrow dy + 3x\,dx = 0$

$\Rightarrow y + \frac{3x^2}{2} = c$

$\Rightarrow y + \frac{3x^2}{2} - c = 0$

& $p - x = 0 \Rightarrow dy - x\,dx = 0$

$\Rightarrow y - \frac{x^2}{2} = c$

$\Rightarrow y - \frac{x^2}{2} - c = 0$

$\left(y + \frac{3x^2}{2} - c\right)\left(y - \frac{x^2}{2} - c\right)$ Ans

Q(1) $p^2 - 2p - 3 = 0$

sol^n, diff. eq^n.

$$p^2 - 2p - 3 = 0$$
$$(p-3)(p+1) = 0$$

$$\frac{dy}{dx} - 3 = 0 \Rightarrow dy - 3dx = 0$$
$$\Rightarrow y - 3x = C$$
$$\Rightarrow y - 3x - C = 0$$

$$p + 1 = 0 \Rightarrow \frac{dy}{dx} + 1 = 0$$

$$\Rightarrow dy + dx = 0$$
$$\Rightarrow y + x = C$$
$$\Rightarrow y + x - C = 0$$ #

$(y - 3x - C)(y + x - C)$ are req sol^n

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3) $p^2 - p(e^x + e^{-x}) + 1 = 0$

Sol^n diff. eq^n

$p^2 - p(e^x + e^{-x}) + 1 = 0$

$p^2 - pe^x - pe^{-x} + 1 = 0$

$p(p - e^x) - e^{-x}(p - e^x) = 0$

$(p - e^{-x})(p - e^x) = 0$

either,

$p - e^x = 0$ or $p - e^{-x} = 0$

$\frac{dy}{dx} = e^x$ $\qquad$ $\frac{dy}{dx} = e^{-x}$

$\Rightarrow \int dy = \int e^x dx$ $\qquad$ $\Rightarrow \int dy = \int e^{-x} dx$

$\Rightarrow y = e^x + c$ $\qquad$ $\Rightarrow y = -e^{-x} + c$

$\Rightarrow y - e^x - c = 0$ $\qquad$ $\Rightarrow y + e^{-x} - c = 0$

$(y - e^x - c = 0)$ & $(y + e^{-x} - c)$ are req^d.

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Q. ~~Soln~~

8. $(x^2+y^2)dx + 2xy\,dy = 0$

Soln

$(x^2+y^2)\,dx + 2xy\,dy = 0$

or $\frac{dy}{dx} = -\frac{x^2+y^2}{2xy}$

This is homogeneous Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

we get

or $v + \frac{x\,dv}{dx} = -\frac{x^2+v^2x^2}{2x \cdot vx}$

or $\frac{x\,dv}{dx} = -v - \frac{1+v^2}{2v}$

or $\frac{x\,dv}{dx} = -\frac{1+3v^2}{2v}$

or $\frac{dx}{dx} + \frac{2v\,dv}{1+3v^2} = 0$

or $3\frac{dx}{x} + \frac{6v\,dv}{1+3v^2} = 0$

Integrating

$3\log x + \log(1+3v^2) = \log c$ or

$x^3(1+3v^2) = c$

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5. $y^2 \cdot dx + (xy + x^2)dy = 0$

Sol^n,

$$y^2 dx + (xy + x^2)\, dy = 0$$

$$\frac{dy}{dx} = -\frac{y^2}{xy + x^2}$$

This is homogeneous. $\therefore$ Put $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

we get

$$\Rightarrow \quad v + x\frac{dv}{dx} = -\frac{v^2x^2}{x \cdot vx + x^2}$$

$$\Rightarrow \quad \frac{x\,dv}{dx} = -\frac{v^2}{v+1} - v$$

$$= \frac{-v^2 - v^2 - v}{v+1}$$

$$\Rightarrow \quad \frac{x\,dv}{dx} + \frac{2v^2 + v}{v+1} = 0$$

Separ the var

$$\frac{dx}{x} + \frac{v+1}{2v^2+v}\,dv = 0$$

$$\Rightarrow \quad \frac{2dx}{x} + 2\left(\frac{1}{v} - \frac{1}{2v+1}\right)dv = 0$$

Integrating.

$$\Rightarrow \quad 2\log x + 2\log v - \log(2v+1) = \log c.$$

$$\Rightarrow \quad \log x^2v^2 = \log c(2v+1)$$

$$\therefore \quad xy^2 = c(2y + x)$$

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Q $\frac{dy}{dx} + \frac{3xy + y^2}{x^2 + xy} = 0$

$\Rightarrow$ soln,

$$\frac{dy}{dx} + \frac{3xy + y^2}{x^2 + xy} = 0$$

$$\frac{dy}{dx} = -\frac{3xy + y^2}{x^2 + xy}.$$

This is homogenous.

Put $y = vx$,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

we get

$$v + x\frac{dv}{dx} = -\frac{3vx\,x + v^2x^2}{x^2 + x.vx}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{3v + v^2}{1+v} - v = \frac{-3v - v^2 - v - v^2}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{2v^2 + 4v}{1+v}.$$

Separating the variable

$$\Rightarrow \frac{dx}{x} + \frac{1+v}{2v(v+2)}dv = 0.$$

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$$\frac{dx}{x} + \frac{1}{2}\left(\frac{1}{2v} + \frac{1}{2(v+2)}\right) dv = 0$$

Integrating

or $\log x + \frac{1}{4}\log v + \frac{1}{4}\log(v+2) = \log c$

or $\log\{x v^{1/4}(v+2)^{1/4}\} = \log c$

$\{x v^{1/4}(v+2)^{1/4}\} = c$

or $x^4 v(v+2) = c^4$

$\therefore$ $x^2 y(y+2x) = c$ //

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Q. $x\frac{dy}{dx} + 2y = x^2 \log x$

=> Sol^n,

$$x\frac{dy}{dx} + 2y = x^2 \log x.$$

$$\frac{dy}{dx} + \frac{2}{x} dy = x\log x.$$

This is linear form, $P = \frac{2}{x}$

$Q = x\log x$

$$IF = e^{\int P dx} = \int \frac{2}{x} dx$$

$$= e^{2\log x} = x^2$$

Its general sol^n is.

$$y \times I.F = \int Q \times (IF)\, dx.$$

$$\Rightarrow y \cdot x^2 = \int x^2 \log x \, dx + c.$$

$$\Rightarrow y x^2 = \log x \cdot \frac{x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx + c$$

$$\Rightarrow yx^2 = \frac{x^4}{4} \log x - \frac{x^4}{16} + c$$

$$\therefore 16x^2 y = 4x^4 \log x - x^4 + c$$

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Q. $(x^3+1)\frac{dy}{dx} + 3x^2y = \sin^2 x$

→ Sol^n,

$$(x^3+1)\frac{dy}{dx} + 3x^2y = \sin^2 x$$

$$\frac{dy}{dx} + \frac{3x^2}{x^3+1}y = \frac{\sin^2 x}{x^3-1}$$

This is linear form,

$$P = \frac{3x^2}{x^3+1}, \quad Q = \frac{\sin^2 x}{x^3+1}$$

$$IF = e^{\int P dx} = e^{\int \frac{3x^2}{x^3+1} dx}$$

$$= e^{\log(x^3+1)}$$

$$= x^3+1,$$

The general sol^n.

$$y \times I.F = \int Q.(IF)\,dx.$$

$$y(x^3+1) = \int \frac{\sin^2 x}{x^3+1}(x^3+1)\,dx.$$

$$y(x^3+1) = \frac{x}{2} - \frac{\sin 2x}{4} + C.$$

$$4y(x^3+1) = 2x - \sin 2x + C$$

is req. sol^n,

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Q. $(x+y+1)\frac{dy}{dx} = 1$

=> sol^n

$$(x+y+1)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{x+y+1}$$

$$\frac{dx}{dy} = x+y+1$$

$$\frac{dx}{dy} - x = y+1$$

This linear form $P = -1,\ Q = y+1$

$$If = e^{\int P\,dy} = e^{-1\int dy} = e^{-y}$$

In general

$$x \times If = \int Q \times If\ dy$$

$$x e^{-y} = \int (e^{-y} y + e^{-y})\,dy + c$$

$$x e^{-y} = -y e^{-y} + \int e^{-y} dy + \int e^{-y} dy$$

$$x e^{-y} = -y e^{-y} - 2e^{-y} + c$$ is req^n soln.

$(x+y+2) = c e^{y}$ is reqd sol^n.

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Q. $(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$

=> Sol^n

$$(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{1+y^2}{x - e^{\tan^{-1}y}}.$$

$$\frac{dx}{dy} = -\frac{x - e^{\tan^{-1}y}}{1+y^2}$$

$$\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2}$$

$$If = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}}$$

$$= e^{\tan^{-1}y}.$$

Its general sol^n. is

$$x \times If = \int Q \times (If)\,dy.$$

$$x\,e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}}{1+y^2} \times e^{\tan^{-1}y}\,dy + C$$

put $\tan^{-1}y = t$, $\frac{1}{1+y^2}dy = dt.$

or, $x\,e^{\tan^{-1}y} = \int e^{2t}\,dt + C = \frac{e^{2t}}{2} + \frac{C}{2}.$

$2x\,e^{\tan^{-1}y} = e^{2\tan^{-1}y} + C.$

is req. sol^n.

1. $\frac{dy}{dx} = \frac{x-y+3}{2x-2y+5}$

$\Rightarrow$ Sol^n.

$$\frac{dy}{dx} = \frac{x-y+3}{2x-2y+5} = \frac{x-y+3}{2x(x-y)+5}$$

put $x-y = v$,

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

The eq^n become

$$1 - \frac{dv}{dx} = \frac{v+3}{2v+5}$$

$$\frac{dv}{dx} = 1 - \frac{v+3}{2v+5}$$

$$= \frac{2v+5-v-3}{2v+5}$$

$$= \frac{v+2}{2v+5}$$

Seperating the variable

$$dx = \frac{(2v+5)\,dv}{v+2} = \frac{2(v+2)+1}{v+2}\,dv$$

or $dx = \left(2 + \frac{1}{v+2}\right) dv.$

Integrating

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$x + c = 2v + \log(v+2)$

$x + c = 2x - 2y + \log(x - y + 2)$

$x - 2y + \log(x + y + 2) = c$ is req.d. solⁿ

Q. $(2x+y+1)\,dx + (4x+2y-1)\,dy = 0$

$\Rightarrow$ Solⁿ.

$(2x+y+1)\,dx + (4x+2y-1)\,dy = 0$

$$\frac{dy}{dx} = -\frac{(2x+y)+1}{2(2x+y)-1}$$

put $2x+y = v$

$$2 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 2$$

eqⁿ become

$$\frac{dv}{dx} - 2 = -\frac{v+1}{2v-1} = -\frac{v+1}{2v-1} + 2$$

$$= \frac{-v+1+4v-2}{2v-1}$$

$$= \frac{3v-3}{2v-1}$$

$$\frac{dv}{dx} = \frac{3(v-1)}{2v-1}$$

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seperating the variable

$$\left(\frac{2v-1}{v-1}\right) dv = 3dx$$

$$\left(2+\frac{1}{v-1}\right) dv = 3dx$$

Intergrating

$$2v + \log(v-1) = 3x + C$$

$$4x + 2y + \log(2x+y-1) = 3x + C$$

$$x + 2y + \log(2x+y-1) = c$$ is reqvid sol^n.

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Q. $(3y - 7x + 7)\,dx + (7y - 3x + 3)\,dy = 0$

⇒ Sol^n

$$(3y - 7x + 7)\,dx + (7y - 3x + 3)\,dy = 0$$

$$\frac{dy}{dx} = -\frac{3y - 7x + 7}{7y - 3x + 3}$$

put $x = X + h$, $y = Y + K$, where h, k are constant.

$$\frac{dY}{dX} = -\frac{3Y - 7X + (3k - 7h + 7)}{7Y - 3X + (7k - 3h + 3)}$$

choose h, k such that

$$3k - 7h + 7 = 0$$

$$\& \quad 7k - 3h + 3 = 0$$

solving these we get

$$k = 0$$

$$h = 1$$

$$\frac{dY}{dX} = -\frac{3Y - 7X}{7Y - 3X}$$

This is homogenous. Put $Y = vX$

$$\frac{dY}{dX} = v + X\frac{dv}{dX}$$

Then eq^n become

$$v + X\frac{dv}{dX} = -\frac{3vX - 7X}{7vX - 3X}$$

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$$\Rightarrow \quad x\cdot\frac{dv}{dx} = -\frac{3v-7}{7v-3} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = -\frac{(7v^2-7)}{7v-3}.$$

Seperating the variable

$$\Rightarrow \quad \frac{dx}{x} + \frac{(7v-3)}{7(v^2-1)}dv = 0$$

$$\Rightarrow \quad 2\frac{dx}{x} + \frac{2v\,dv}{v^2-1} - \frac{6}{7}\frac{1}{v^2-1}dv.$$

Integrating

$$\Rightarrow \quad 2\log x + \log(v^2-1) - \frac{6}{7}\cdot\frac{1}{2}\log\frac{v-1}{v+1} = \log c$$

$$\Rightarrow \quad \log\frac{x^2(v^2-1)}{(v-1)^{3/7}}(v+1)^{3/7} = \log c.$$

$$\Rightarrow \quad \frac{x^2(v+1)(v-1)(v+1)^{3/7}}{(v-1)^{3/7}} = c$$

$$\Rightarrow \quad x^2(v-1)^{4/7}(v+1)^{10/7} = c,$$

$$\Rightarrow \quad (Y-X)^4(Y+X)^{10} = c^7$$

$$\Rightarrow \quad (y-0-x+1)^4(y-0+x-1)^{10} = c^7$$

$$\therefore \quad (y-x+1)^2(x+y-1)^5 = c \quad \text{Ans}$$