Engineering Economics

Lecture 7

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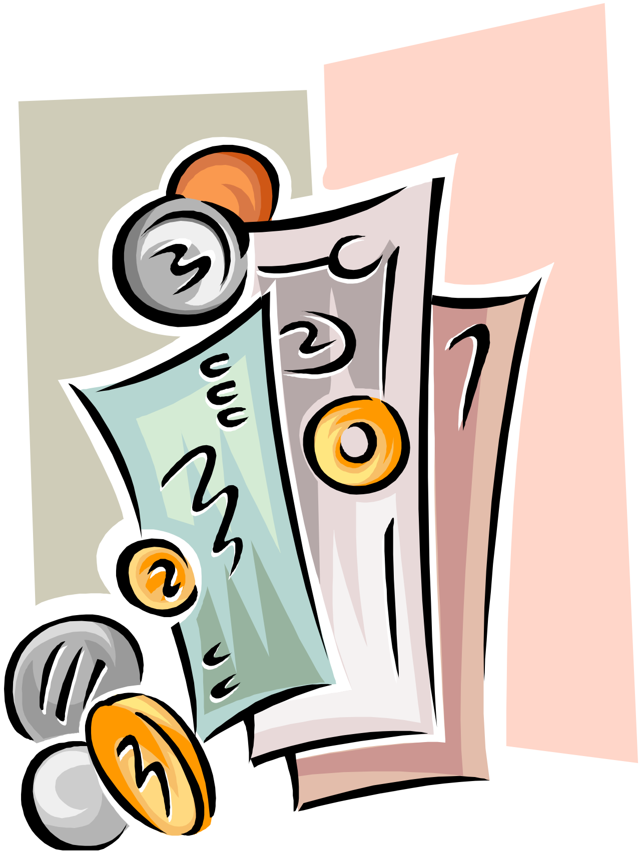
Chapter 9
Rate of Return Analysis

- Rate of Return
- Methods for Finding ROR
- Internal Rate of Return (IRR) Criterion
- Incremental Analysis
- Mutually Exclusive Alternatives



Rate of Return

- Definition: A relative percentage method which measures the yield as a percentage of investment over the life of a project

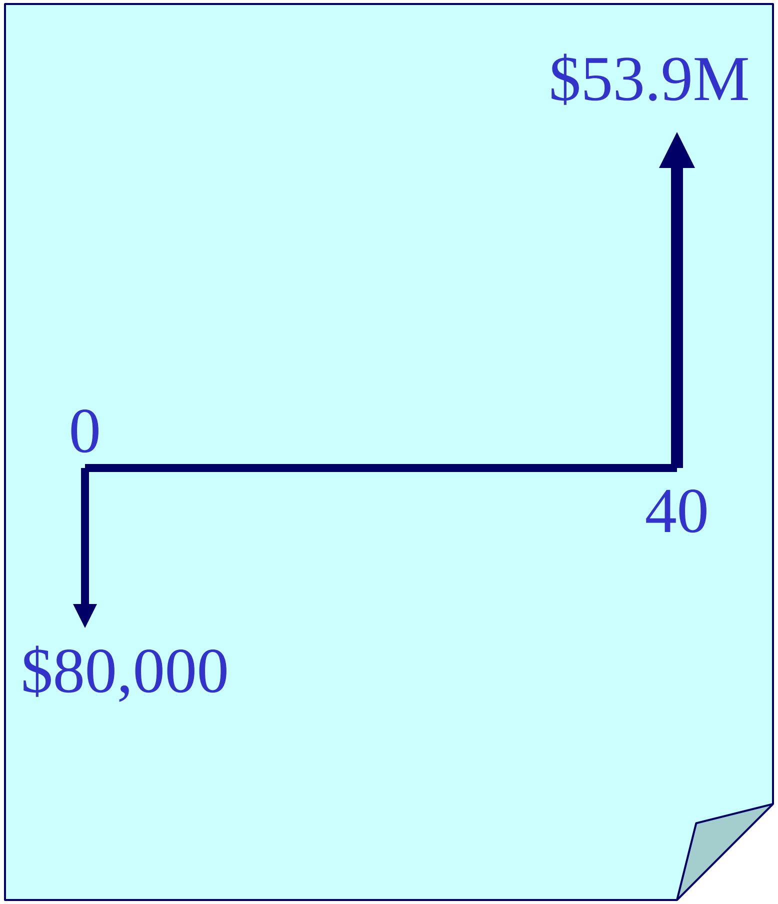
- Example: Vincent Gogh's painting "Irises"
  - John Whitney Payson bought the art at $80,000.
  - John sold the art at $53.9 million in 40 years.
  - What is the rate of return on John's investment?

Rate of Return

- Given: P =\$80,000, F = \$53.9M, and N = 40 years
- Find: i
- Solution:

$$\$53.9M = \$80{,}000(1+i)^{40}$$

$$\boxed{i = 17.68\%}$$

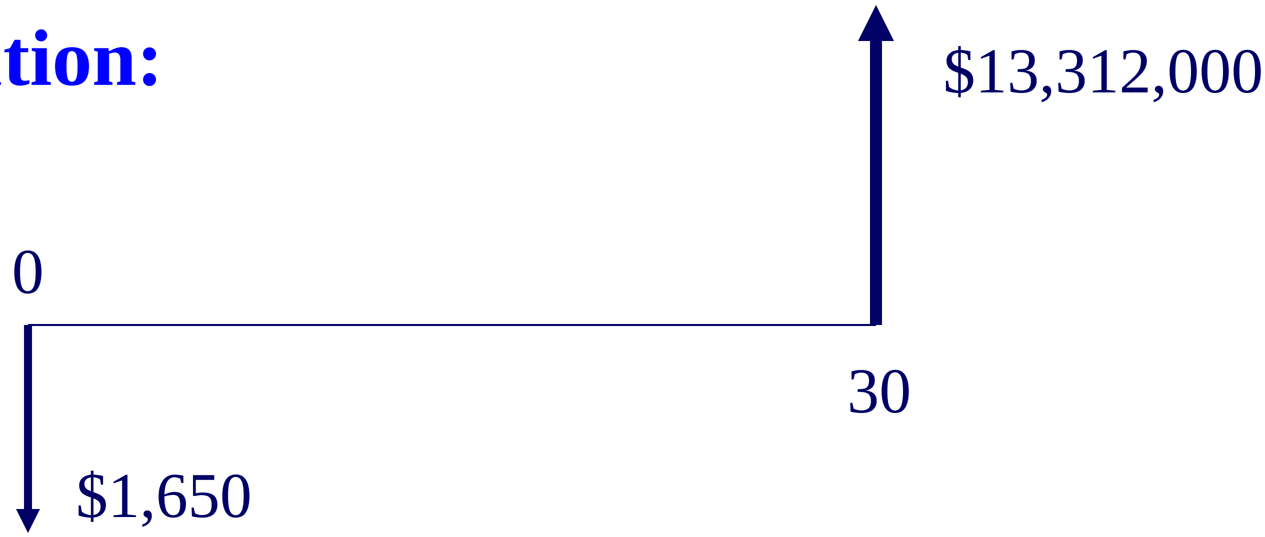


Meaning of Rate of Return

In 1970, when Wal-Mart Stores, Inc. went public, an investment of 100 shares cost $1,650. That investment would have been worth $13,312,000 on January 31, 2000.

What is the <u>rate of return</u> on that investment?

Solution:



Given: P = \$1,650

F = \$13,312,000

N = 30

Find i:

$$F = P(1+i)^N$$

$$\$13{,}312{,}000 = \$1{,}650\,(1+i)^{30}$$

$$i = 34.97\%$$ Rate of Return

Suppose that you invested that amount ($1,650) in a savings account at 6% per year. Then, you could have only $9,477 on January, 2000.

What is the meaning of this 6% interest here?

This is your opportunity cost if putting money in savings account was the best you can do at that time!

So, in 1970, as long as you earn more than 6% interest in another investment, you will take that investment.

Therefore, that 6% is viewed as a minimum attractive rate of return (or required rate of return).

So, you can apply the following decision rule, to see if the proposed investment is a good one.

$$ROR > MARR$$

Why ROR measure is so popular?

- This project will bring in a 15% rate of return on investment.
- This project will result in a net surplus of $10,000 in NPW.

Which statement is easier to understand?

Return on Investment

- Definition 1: Rate of return (ROR) is defined as the interest rate earned on the *unpaid balance* of an installment loan.

- Example: A bank lends $10,000 and receives annual payment of $4,021 over 3 years. The bank is said to earn a *return of 10%* on its loan of $10,000.

Loan Balance Calculation:

$$A = \$10{,}000\ (A/P,\ 10\%,\ 3) = \$4{,}021$$

Year	Unpaid balance at beg. of year	Return on unpaid balance (10%)	Payment received	Unpaid balance at the end of year
0	-$10,000			-$10,000
1	-$10,000	-$1,000	+$4,021	-$6,979
2	-$6,979	-$698	+$4,021	-$3,656
3	-$3,656	-$366	+$4,021	0

A return of 10% on the amount still outstanding at the beginning of each year

Rate of Return:

- Definition 2: Rate of return (ROR) is the break-even interest rate, i^*, which equates the present worth of a project's cash outflows to the present worth of its cash inflows.

- Mathematical Relation:

$$PW(i^*) = PW(i^*)_{\text{cash inflows}} - PW(i^*)_{\text{cash outflows}}$$
$$= 0$$

Return on Invested Capital

- Definition 3: Return on invested capital is defined as the interest rate earned on the *unrecovered project balance* of an investment project. It is commonly known as internal rate of return (IRR).

- Example: A company invests $10,000 in a computer and results in equivalent annual labor savings of $4,021 over 3 years. The company is said to earn a *return of 10%* on its investment of $10,000.

Project Balance Calculation:

	0	1	2	3
Beginning project balance		-$10,000	-$6,979	-$3,656
Return on invested capital		-$1,000	-$697	-$365
Payment received	-$10,000	+$4,021	+$4,021	+$4,021
Ending project balance	-$10,000	-$6,979	-$3,656	0

The firm earns a 10% rate of return on funds that remain internally invested in the project. Since the return is internal to the project, we call it internal rate of return.

Methods for Finding Rate of Return

- Investment Classification
  - Simple Investment
  - Nonsimple Investment
- Computational Methods
  - Direct Solution Method
  - Trial-and-Error Method
  - Computer Solution Method

Investment Classification

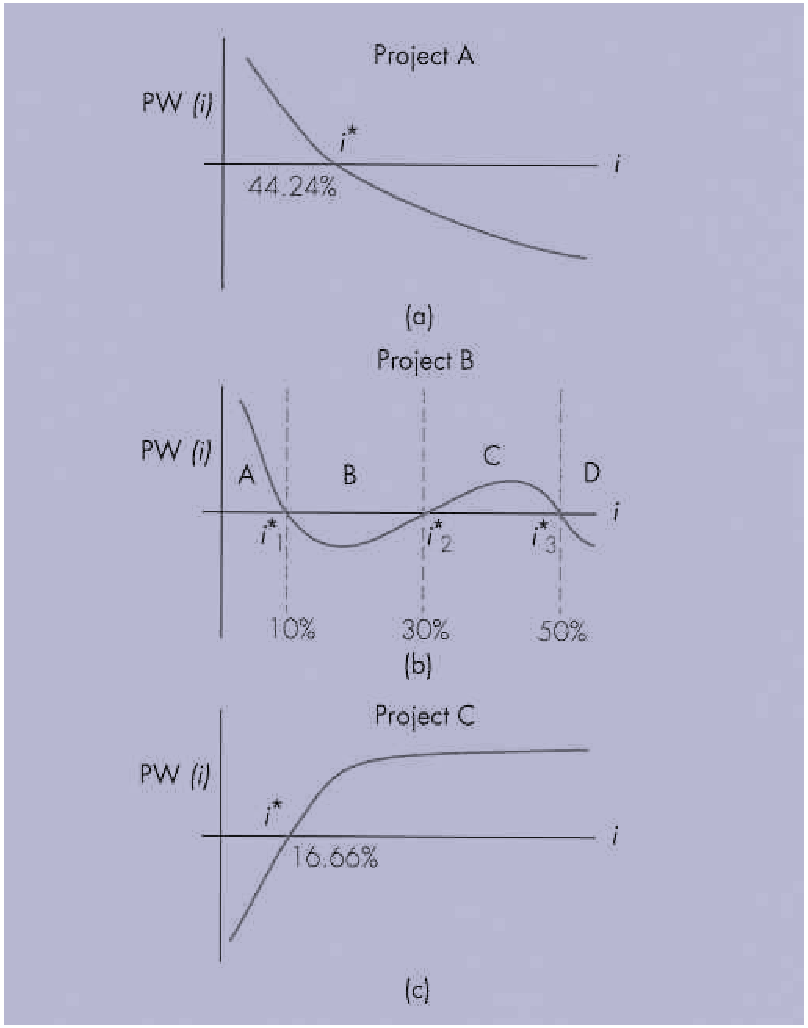
Simple Investment

- Def: Initial cash flows are negative, and only one sign change occurs in the net cash flows series.
- Example: -\$100, \$250, \$300 ⇨ (-, +, +)
- ROR: A unique ROR

Nonsimple Investment

- Def: Initial cash flows are negative, but more than one sign changes in the remaining cash flow series.
- Example: -\$100, \$300, -\$120 ⇨ (-, +, -)
- ROR: A possibility of multiple RORs

Period (N)	Project A	Project B	Project C
0	-$1,000	-$1,000	+$1,000
1	-500	3,900	-450
2	800	-5,030	-450
3	1,500	2,145	-450
4	2,000		



Project A is a simple investment.
Project B is a nonsimple investment.
Project C is a simple borrowing.

Computational Methods

	Direct Solution Log	Direct Solution Quadratic	Trial & Error Method	Computer Solution Method
n	Project A	Project B	Project C	Project D
0	-$1,000	-$2,000	-$75,000	-$10,000
1	0	1,300	24,400	20,000
2	0	1,500	27,340	20,000
3	0		55,760	25,000
4	1,500			

Direct Solution Methods

- Project A

$$\$1{,}000 = \$1{,}500(P/F, i, 4)$$

$$\$1{,}000 = \$1{,}500(1+i)^{-4}$$

$$0.6667 = (1+i)^{-4}$$

$$\frac{\ln 0.6667}{-4} = \ln(1+i)$$

$$0.101365 = \ln(1+i)$$

$$e^{0.101365} = 1+i$$

$$i = e^{0.101365} - 1$$

$$= \boxed{10.67\%}$$

- Project B

$$PW(i) = -\$2{,}000 + \frac{\$1{,}300}{(1+i)} + \frac{\$1{,}500}{(1+i)^2} = 0$$

Let $x = \frac{1}{1+i}$, then

$$PW(i) = -2{,}000 + 1{,}300x + 1{,}500x^2$$

Solve for x:

$x = 0.8$ or -1.667

Solving for i yields

$$0.8 = \frac{1}{1+i} \to i = 25\%, \quad -1.667 = \frac{1}{1+i} \to i = -160\%$$

Since $-100\% < i < \infty$, the project's $i^* = \boxed{25\%.}$

Trial and Error Method – Project C

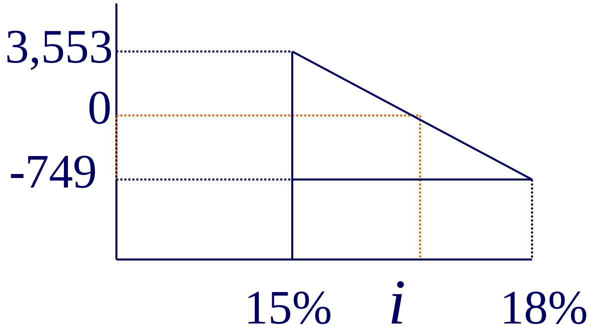
- Step 1: Guess an interest rate, say, $i = 15\%$
- Step 2: Compute PW(i) at the guessed i value.

PW (15%) = \$3,553

- Step 3: If PW(i) > 0, then increase i. If PW(i) < 0, then decrease i.

PW(18%) = -\$749

- Step 4: If you bracket the solution, you use a linear interpolation to approximate the solution

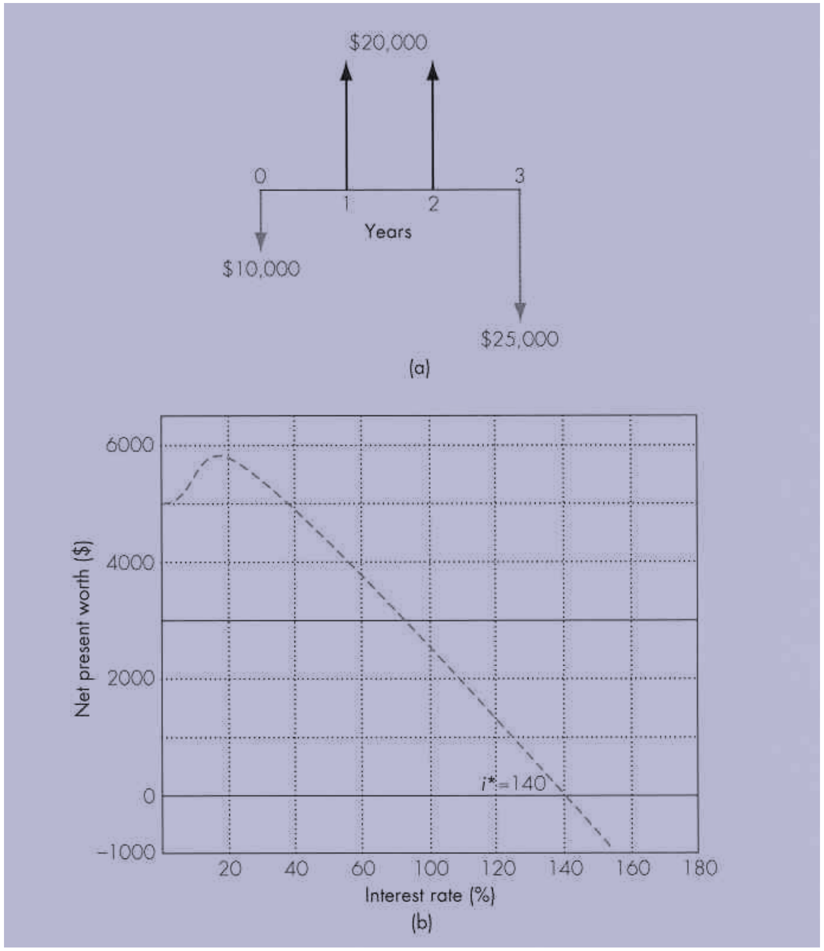


$$i = 15\% + 3\%\left[\frac{3{,}553}{3{,}553 + 749}\right]$$

$$= 17.45\%$$

Graphical Solution – Project D

- Step 1: Create a NPW plot using Excel.
- Step 2: Identify the point at which the curve crosses the horizontal axis closely approximates the *i**.
- Note: This method is particularly useful for projects with multiple rates of return, as most financial softwares would fail to find all the multiple *i**s.

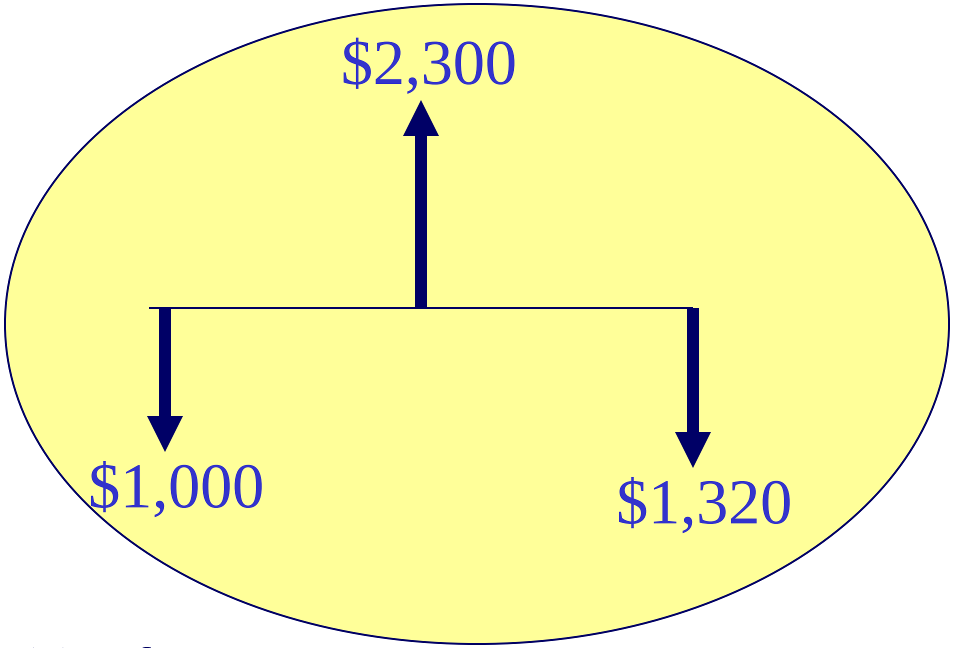


Basic Decision Rule:

If ROR > MARR, Accept

This rule does not work for a situation where an investment has multiple rates of return

Multiple Rates of Return Problem



- Find the rate(s) of return:

$$PW(i) = -\$1{,}000 + \frac{\$2{,}300}{1+i} - \frac{-\$1{,}320}{(1+i)^2}$$

$$= 0$$

Let $x = \frac{1}{1+i}$. Then,

$$PW(i) = -\$1{,}000 + \frac{\$2{,}300}{(1+i)} - \frac{\$1{,}320}{(1+i)^2}$$

$$= -\$1{,}000 + \$2{,}300x - \$1{,}320x^2$$

$$= 0$$

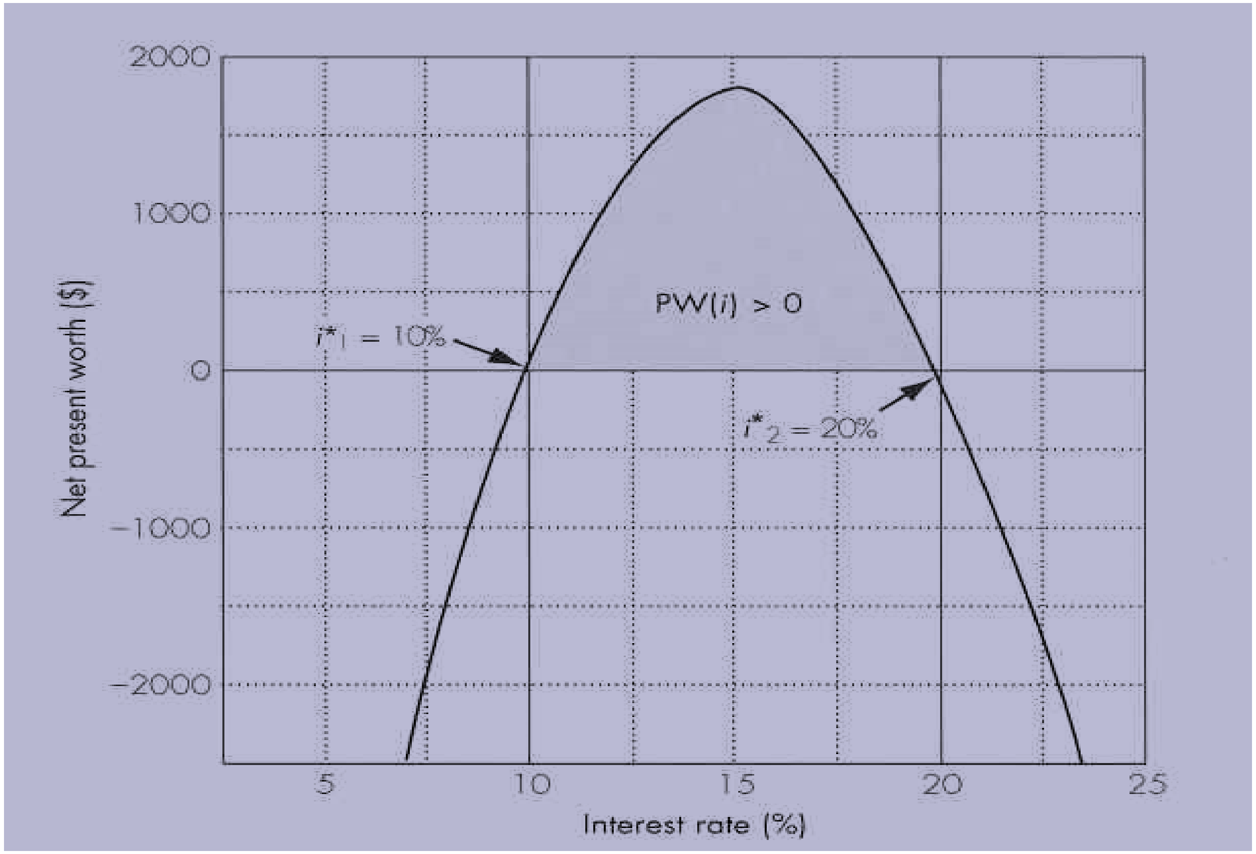
Solving for x yields,

$$x = 10/11 \text{ or } x = 10/12$$

Solving for i yields

$$i = 10\% \text{ or } 20\%$$

NPW Plot for a Nonsimple Investment with Multiple Rates of Return



Project Balance Calculation

$i^* = 20\%$

	$n = 0$	$n = 1$	$n = 2$
Beg. Balance		-$1,000	+$1,100
Interest		-$200	+$220
Payment	-$1,000	+$2,300	-$1,320
Ending Balance	-$1,000	+$1,100	$0

Cash borrowed (released) from the project is assumed to earn the same interest rate through external investment as money that remains internally invested.

Critical Issue: Can the company be able to invest the money released from the project at 20% externally in Period 1?

If your MARR is exactly 20%, the answer is "yes", because it represents the rate at which the firm can always invest the money in its investment pool. Then, the 20% is also true IRR for the project.

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Suppose your MARR is 15% instead of 20%. The assumption used in calculating *i** is no longer valid.

Therefore, neither 10% nor 20% is a true IRR.

How to Proceed: If you encounter multiple rates of return, abandon the IRR analysis and use the NPW criterion (or use the procedures outlined in Appendix A).

- **If NPW criterion is used at MARR = 15%**

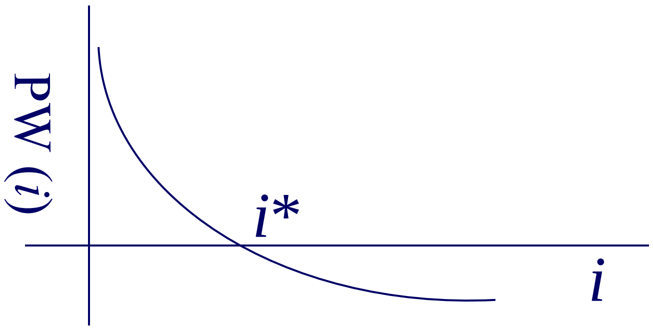
$$\begin{aligned} \mathbf{PW(15\%)} = \; & \mathbf{-\$1{,}000} \\ & \mathbf{+\ \$2{,}300\ (\textit{P/F}, 15\%, 1)} \\ & \mathbf{-\ \$1{,}320\ (\textit{P/F}, 15\%, 2\ )} \\ & \mathbf{=\ \$1.89 > 0} \end{aligned}$$

Accept the investment

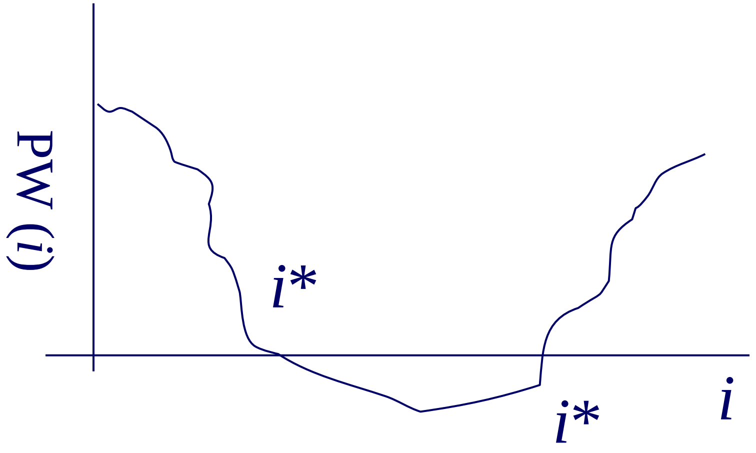
Decision Rules for Nonsimple Investment

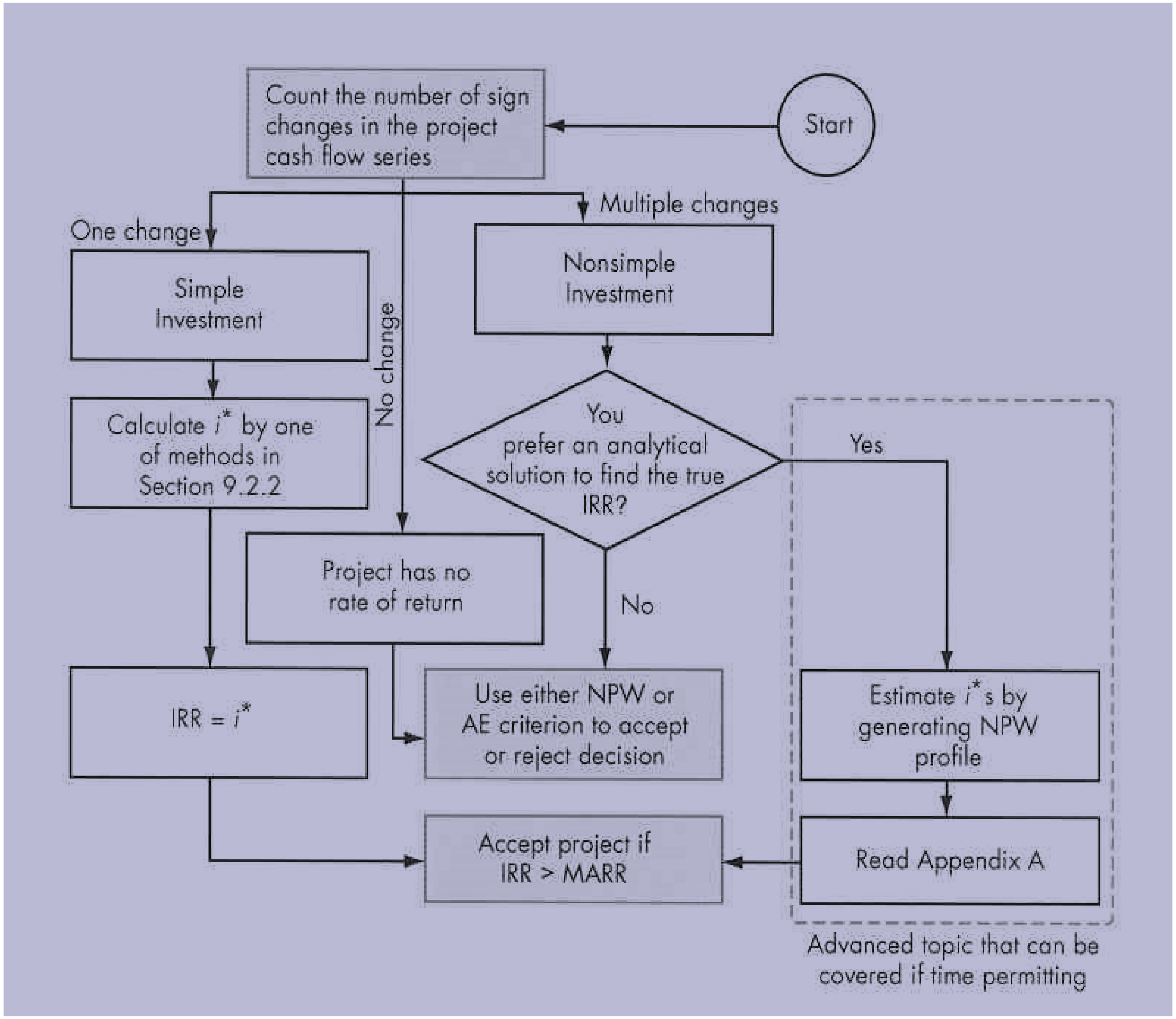
- A possibility of multiple RORs.
- If PW (i) plot looks like this, then, IRR = ROR.

  If IRR > MARR, Accept



- If PW(i) plot looks like this, Then, IRR ≠ ROR (i*).
  - Find the true IRR by using the procedures in Appendix A or,
  - Abandon the IRR method and use the PW method.





Count the number of sign changes in the project cash flow series
Start
One change
No change
Multiple changes
Simple Investment
Nonsimple Investment
Calculate i^* by one of methods in Section 9.2.2
You prefer an analytical solution to find the true IRR?
Yes
No
Project has no rate of return
IRR = i^*
Use either NPW or AE criterion to accept or reject decision
Estimate i^*s by generating NPW profile
Read Appendix A
Accept project if IRR > MARR
Advanced topic that can be covered if time permitting

Comparing Mutually Exclusive Alternatives Based on IRR

- Issue: Can we rank the mutually exclusive projects by the magnitude of IRR?

n	A1		A2
0	-\$1,000		-\$5,000
1	\$2,000		\$7,000
IRR	100%	>	40%
PW (10%)	\$818	<	\$1,364

Incremental Investment

n	Project A1	Project A2	Incremental Investment (A2 – A1)
0	-$1,000	-$5,000	-$4,000
1	$2,000	$7,000	$5,000
ROR	100%	40%	25%
PW(10%)	$818	$1,364	$546

- Assuming MARR of 10%, you can always earn that rate from other investment source, i.e., $4,400 at the end of one year for $4,000 investment.

- By investing the additional $4,000 in A2, you would make additional $5,000, which is equivalent to earning at the rate of 25%. Therefore, the incremental investment in A2 is justified.

Incremental Analysis (Procedure)

Step 1: Compute the cash flows for the difference between the projects (A,B) by subtracting the cash flows for the lower investment cost project (A) from those of the higher investment cost project (B).

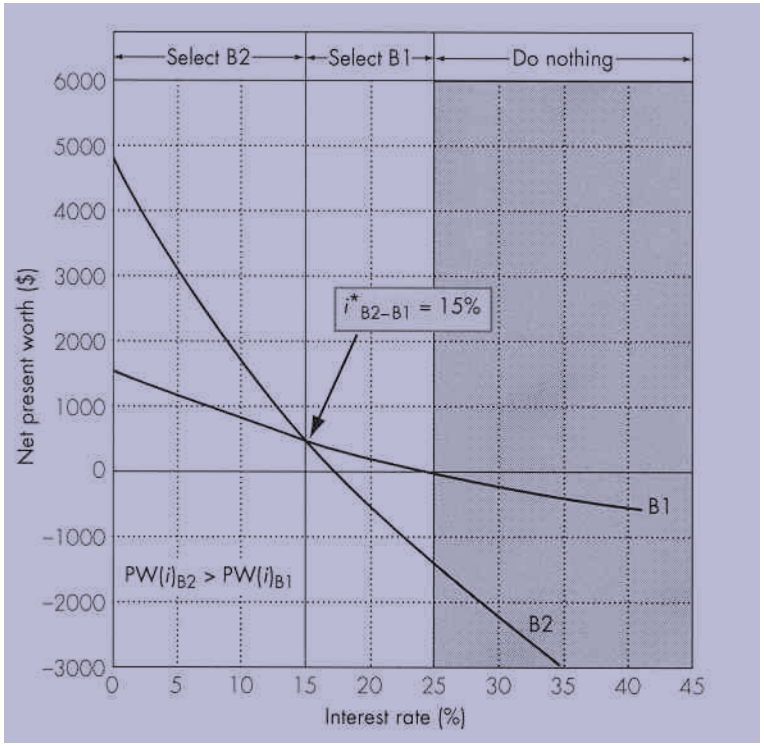
Step 2: Compute the IRR on this incremental investment (IRR_{B-A}).

Step 3: Accept the investment B if and only if

$$IRR_{B-A} > MARR$$

Example 9.7 - Incremental Rate of Return

n	B1	B2	B2 - B1
0	-$3,000	-$12,000	-$9,000
1	1,350	4,200	2,850
2	1,800	6,225	4,425
3	1,500	6,330	4,830
IRR	25%	17.43%	15%



Given MARR = 10%, which project is a better choice?
Since $IRR_{B2\text{-}B1}$=15% > 10%, and also IRR_{B2} > 10%, select B2.

IRR on Increment Investment: Three Alternatives

n	D1	D2	D3
0	-$2,000	-$1,000	-$3,000
1	1,500	800	1,500
2	1,000	500	2,000
3	800	500	1,000
IRR	34.37%	40.76%	24.81%

Step 1: Examine the IRR for each project to eliminate any project that fails to meet the MARR.

Step 2: Compare D1 and D2 in pairs.
IRR_{D1-D2}=27.61% > 15%,
so select D1.

Step 3: Compare D1 and D3.
IRR_{D3-D1}= 8.8% < 15%,
so select D1.

Here, we conclude that D1 is the best Alternative.

Incremental Borrowing Analysis

Principle:

- If the difference in flow (B-A) represents an increment of investment, then (A-B) is an increment of borrowing.
- When considering an increment of borrowing, the rate $i^*_{A\text{-}B}$ is the rate we paid to borrow money from the increment.

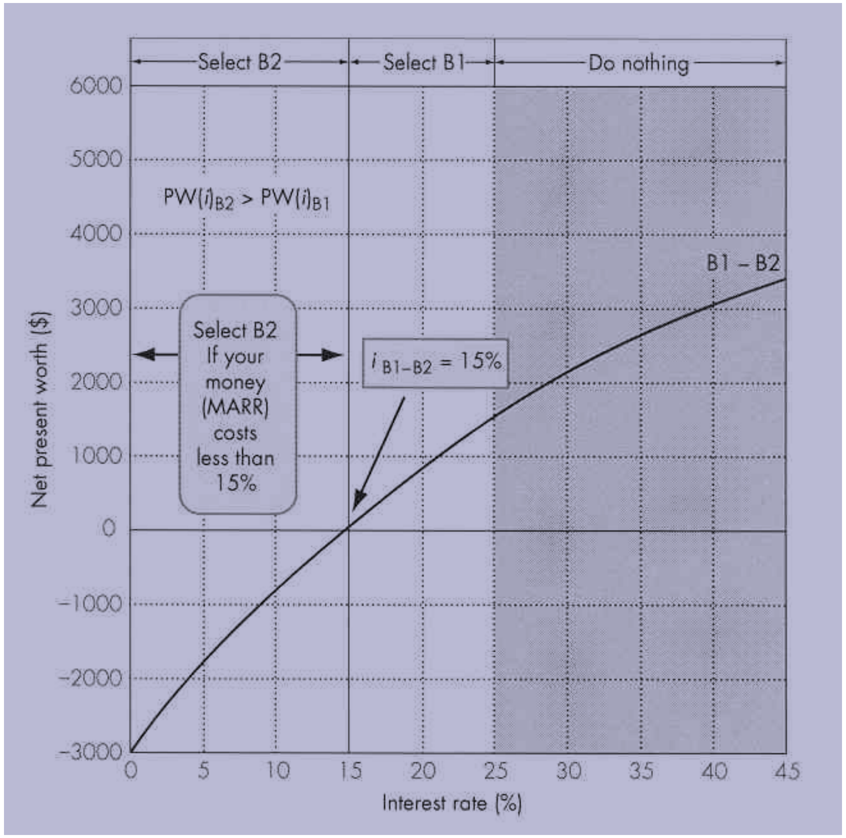
Decision Rule:

$$\boxed{i^*_{A-B} = \mathrm{BRR}_{A-B}}$$

- If $\mathrm{BRR}_{B\text{-}A} < \mathrm{MARR}$, select B.
- If $\mathrm{BRR}_{B\text{-}A} = \mathrm{MARR}$, select either one.
- If $\mathrm{BRR}_{B\text{-}A} > \mathrm{MARR}$, select A.

Borrowing Rate of Return

n	B1	B2	B1-B2
0	-$3,000	-$12,000	+$9,000
1	1,350	4,200	-2,850
2	1,800	6,225	-4,425
3	1,500	6,330	-4,830



Incremental Analysis for Cost-Only Projects

Items	CMS Option	FMS Option
Annual O&M costs:		
Annual labor cost	$1,169,600	$707,200
Annual material cost	832,320	598,400
Annual overhead cost	3,150,000	1,950,000
Annual tooling cost	470,000	300,000
Annual inventory cost	141,000	31,500
Annual income taxes	1,650,000	1,917,000
Total annual costs	$7,412,920	$5,504,100
Investment	$4,500,000	$12,500,000
Net salvage value	$500,000	$1,000,000

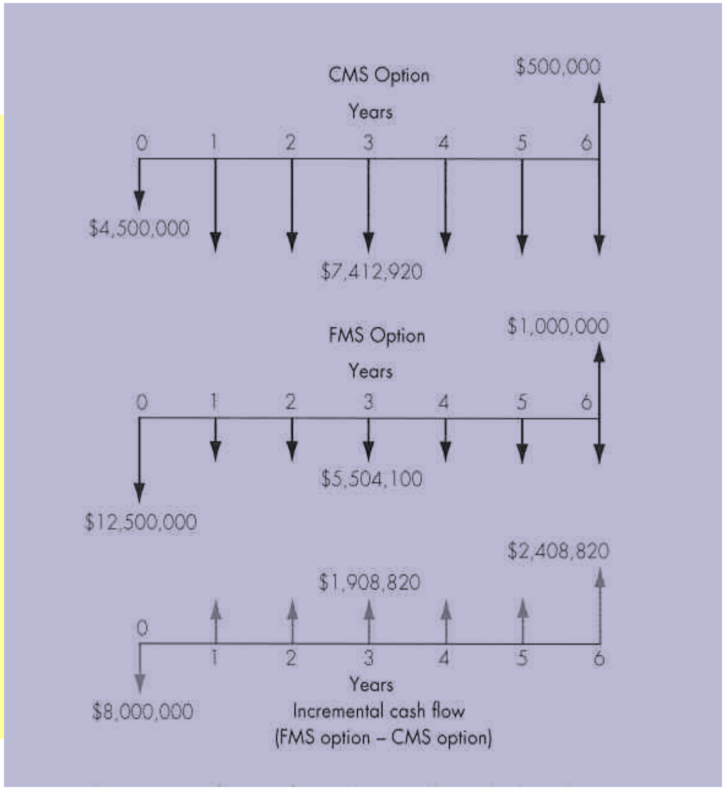
Incremental Cash Flow (FMS – CMS)

n	CMS Option	FMS Option	Incremental (FMS-CMS)
0	-$4,500,000	-$12,500,000	-$8,000,000
1	-7,412,920	-5,504,100	1,908,820
2	-7,412,920	-5,504,100	1,908,820
3	-7,412,920	-5,504,100	1,908,820
4	-7,412,920	-5,504,100	1,908,820
5	-7,412,920	-5,504,100	1,908,820
6	-7,412,920	-5,504,100	$2,408,820
Salvage	+ $500,000	+ $1,000,000	

Solution:

$$PW(i)_{FMS-CMS} = -\$8{,}000{,}000$$
$$+\$1{,}908{,}820(P/A,i,5)$$
$$+\$2{,}408{,}820(P/F,i,6)$$
$$=0$$
$$IRR_{FMS-CMS} = 12.43\% < 15\%,$$

select CMS.



Ultimate Decision Rule:

If IRR > MARR, Accept

- **This rule works for any investment situations.**

- **In many situations,**

  IRR = ROR

  but this relationship does not hold for an investment with multiple RORs.

Predicting Multiple RORs

$-100\% < i^* < \text{infinity}$

- **Net Cash Flow Rule of Signs**

No. of real RORs (i*s)

$\leq$

No. of sign changes in the project cash flows

Example

n	Net Cash flow	Sign Change
0	-$100	
1	-$20	
2	$50	1
3	0	
4	$60	
5	-$30	1
6	$100	1

- No. of real *i**s = 3
- This implies that the project could have (0, 1, 2, or 3) *i**s but NOT more than 3.

Accumulated Cash Flow Sign Test

Find the accounting sum of net cash flows at the end of each period over the life of the project

Period ($\boldsymbol{n}$)	**Cash Flow** ($\boldsymbol{A_n}$)	**Sum** $\boldsymbol{S_n}$
0	A_0	$S_0 = A_0$
1	A_1	$S_1 = S_0 + A_1$
2	A_2	$S_2 = S_1 + A_2$
$\vdots$	$\vdots$	$\vdots$
N	A_N	$S_N = S_{N-1} + A_N$

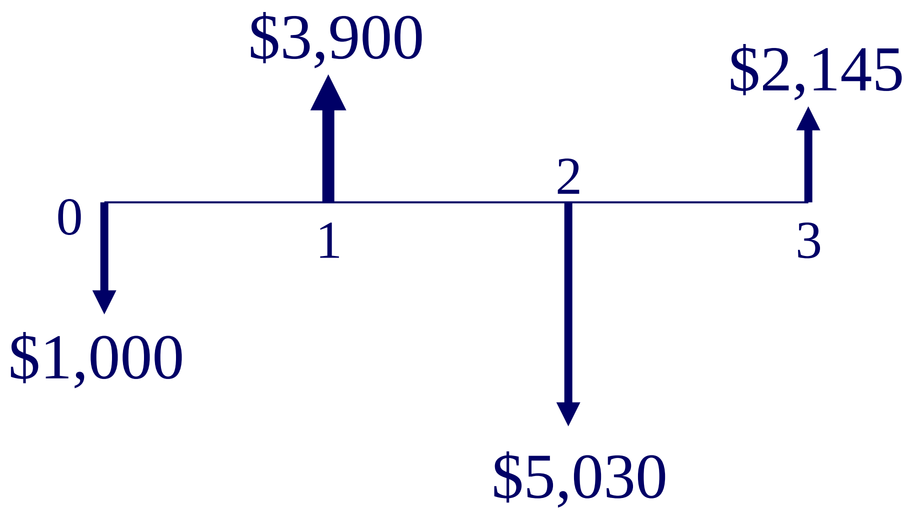
If the series S starts negatively and changes sign ONLY ONCE, there exists a unique positive i^*.

Example

n	A_n	S_n	**Sign change**
0	-$100	-$100	
1	-$20	-$120	
2	$50	-$70	
3	0	-$70	
4	$60	-$10	
5	-$30	-$40	
6	$100	$60	1

- No of sign change = 1, indicating a unique *i**.
- *i** = 10.46%

Example A.2



- Is this a simple investment?
- How many RORs (i*s) can you expect from examining the cash flows?
- Can you tell if this investment has a unique rate of return?

Summary

- Rate of return (ROR) is the interest rate earned on unrecovered project balances such that an investment's cash receipts make the terminal project balance equal to zero.
- Rate of return is an intuitively familiar and understandable measure of project profitability that many managers prefer to NPW or other equivalence measures.
- Mathematically we can determine the rate of return for a given project cash flow series by locating an interest rate that equates the net present worth of its cash flows to zero. This break-even interest rate is denoted by the symbol i^*.

- Internal rate of return (IRR) is another term for ROR that stresses the fact that we are concerned with the interest earned on the portion of the project that is internally invested, not those portions that are released by (borrowed from) the project.
- To apply rate of return analysis correctly, we need to classify an investment into either a <u>simple</u> or a <u>nonsimple</u> investment.
- A simple investment is defined as one in which the initial cash flows are negative and only one sign change in the net cash flow occurs, whereas a nonsimple investment is one for which more than one sign change in the cash flow series occurs.
- Multiple i^*s occur only in nonsimple investments. However, not all nonsimple investments will have multiple i^*s,

- For a simple investment, the solving rate of return (*i**) is the rate of return internal to the project; so the decision rule is:

  If IRR > MARR, accept the project.

  If IRR = MARR, remain indifferent.

  If IRR < MARR, reject the project.

  IRR analysis yields results consistent with NPW and other equivalence methods.
- For a nonsimple investment, because of the possibility of having multiple rates of return, it is recommended the IRR analysis be abandoned and either the NPW or AE analysis be used to make an accept/reject decision.
- When properly selecting among alternative projects by IRR analysis, incremental investment must be used.

End of Lecture 7