

[Numerical]

1

Solution

Given, Minimum volume (V_{min}) = 1 m^3

Maximum volume (V_{max}) = 2 m^3

Initial state

$$P_1 = 1 \text{ MPa} = 1000 \text{ kPa}$$

$$T_1 = 400^\circ\text{C}$$

Final state:

$$T_{final} = 100^\circ\text{C}$$

Temperature of the surrounding (T_s) = 25°C

$$= 25 + 273$$

$$= 293 \text{ K}$$

Pressure required to support the piston ($P_{support}$) = 400 kPa

Referring to the table,

$$T_{sat}(1000 \text{ kPa}) = 179.92^\circ\text{C}$$

Here, $T > T_{sat}$, hence it is a superheated vapor. Then again referring to the table,

$$u_1 = 0.3066 \text{ m}^3/\text{kg}, u_2 = 2957.2 \text{ kJ/kg}, s_1 = 7.4648 \text{ kJ/kg}$$

Mass of H_2O is given as

$$m = \frac{V_1}{u_1} = \frac{1}{0.3066} = 6.5232 \text{ kg}$$

Minimum specific volume of H_2O is given by

$$v_{min} = \frac{V_{min}}{m} = \frac{2}{6.5232} = 0.3066 \text{ m}^3/\text{kg}$$

Initial pressure of the system is 1000 kPa but the pressure required to support the piston is 400 kPa. Hence, during initial state of cooling piston remains stationary although heat is removed from the system, so the process is constant volume cooling (Process 1-2). During constant volume cooling, pressure of the system decreases from 1000 kPa to 400 kPa. Hence, we can define state 2 as,

$$\text{State 2: } P_2 = 400 \text{ kPa} ; v_2 = 0.3066 \text{ m}^3/\text{kg}$$

Referring to the table, v_f (400 kPa) = 0.001084 m³/kg, v_g (400 kPa) = 0.4625 m³/kg. Here $v_f < v < v_g$; hence it is a two phase mixture.

$$\therefore \text{Temperature at state 2, } T_2 = T_{\text{sat}}(400 \text{ kPa}) = 143.64^\circ\text{C}$$

But the final required temp. is 100°C, hence it should be further cooled to decrease the temp. & the process occurs at constant pressure of 400 kPa (Process 2-3) until the piston reaches to the lower stops. Hence, we can define state 3 as

$$\text{State 3: } P_3 = 400 \text{ kPa}, v_3 = 0.1533 \text{ m}^3/\text{kg}$$

Here, $v_f < v_3 < v_g$, hence it is a two phase mixture

$$\therefore \text{Temp. at state 3, } T_3 = T_{\text{sat}}(400 \text{ kPa}) = 143.64^\circ\text{C}$$

If it is further cooled to decrease temp. from 143.6°C to 100°C and the process occurs at constant volume (Process 3-4). Hence, we can define state 4 as,

State 4: $u_4 = 0.1533 \text{ m}^3/\text{kg}$, $T_4 = 100^\circ\text{C}$

Referring to the table, $u_{fg}(100^\circ\text{C}) = 0.001043 \text{ m}^3/\text{kg}$, $u_g(100^\circ\text{C})$
 $= 1.6943 \text{ m}^3/\text{kg}$, $v_{fg}(100^\circ\text{C}) = 1.6933 \text{ m}^3/\text{kg}$, $u_x(100^\circ\text{C}) =$
 417.41 kJ/kg ; $u_{lg}(100^\circ\text{C}) = 2088.3 \text{ kJ/kg}$, $s_x(100^\circ\text{C}) =$
 1.3027 kJ/kg K

$s_{lg}(100^\circ\text{C}) = 6.0562 \text{ kJ/kg K}$. Here, $u_x < u_4 < u_g$, hence it is a two phase mixture.

Quality at state 4 is given as

$$x_4 = \frac{u_4 - u_x}{u_{lg}} = \frac{0.1533 - 0.001043}{1.6933} = 0.0899$$

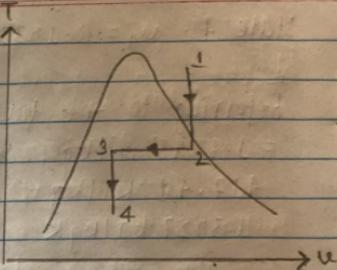
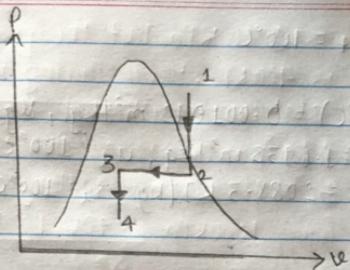
Then, specific internal energy and specific entropy are given as

$$u_4 = u_x + x_4 u_{lg} = 417.41 + 0.0899 \times 2088.3 \text{ kJ/kg} = 605.148 \text{ kJ/kg}$$

$$s_4 = s_x + x_4 s_{lg} = 1.3027 + 0.0899 \times 6.0562 = 1.8471 \text{ kJ/kg K}$$

Change in total internal energy is given by

$$\Delta U = m(u_4 - u_1) = 6.5232(605.148 - 2957.2) \\ = -15342.91 \text{ kJ}$$



Work transfer during the process is given by

$$\begin{aligned} W &= W_{12} + W_{23} + W_{34} \\ &= 0 + P_2(V_3 - V_2) + 0 \\ &= 400 \times (1-2) \\ &= -400 \text{ kJ} \end{aligned}$$

∴ Total heat transfer is given by

$$\begin{aligned} Q &= \Delta U + W \\ &= -15342.91 - 400 \\ &= -15742.92 \text{ kJ} \end{aligned}$$

Then, total entropy generated during the process is given by

$$\begin{aligned} S_{\text{gen}} &= (\Delta S)_{\text{cm}} - \sum \left(\frac{Q_i}{T_i} \right)_{\text{cm}} \\ &= m(s_4 - s_1) - \frac{Q}{T_i} \\ &= 6.5232(1.8472 - 7.4648) - \frac{(-15742.92)}{298} \\ &= 16.1838 \text{ kJ/K} \end{aligned}$$

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solution

(Given, Mass of N_2 (m) = 1 kg)

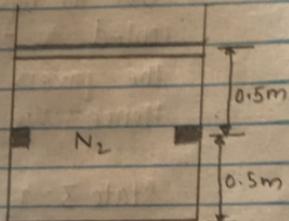
Initial state

$$P_1 = 250 \text{ kPa}$$

$$T_1 = 500^\circ\text{C} = 500 + 273 \\ = 773 \text{ K}$$

Final state

$$T_{\text{final}} = 90^\circ\text{C} = 90 + 273 \\ = 313 \text{ K}$$



Temperature of the surrounding (T_{sur}) = $20^\circ\text{C} = 20 + 273$
 $= 293 \text{ K}$

Volume of N_2 at initial state is given by

$$V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 293 \times 773}{250 \times 10^3} = 0.918324 \text{ m}^3$$

If heat is lost by the system, piston drops downward & process (Process 1-2) occurs at constant pressure of 250 kPa & volume decreases to half of the initial volume. Hence, the temperature of the system when the piston just hits the stop is calculated as

$$\begin{aligned} T_2 &= \frac{V_1}{V_2} \times T_1 \\ &= \frac{1}{2} \times 773 \\ &= 386.5 \text{ K} \\ &= 113.5^\circ\text{C} \end{aligned}$$

But the required final temp. is 40°C , hence it is further cooled to decrease the temp. from 113.5°C to 40°C and the process occurs at constant volume (Process 2-3). Hence, we can define state 3 as

State 3: $T_3 = 313\text{ K}$

$$V_3 = V_2 - \frac{V_1}{2} = \frac{0.918324}{2} = 0.459162 \text{ m}^3$$

$$\therefore \text{Pressure of } N_2 \text{ at final state } P_3 = \frac{MRT_3}{V_3}$$

$$= \frac{1 \times 297 \times 313}{0.459162}$$

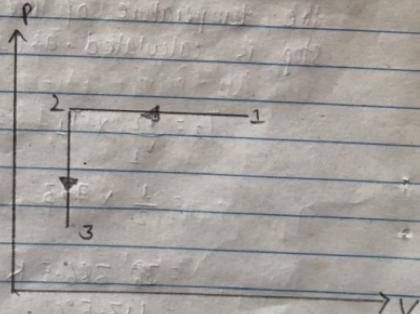
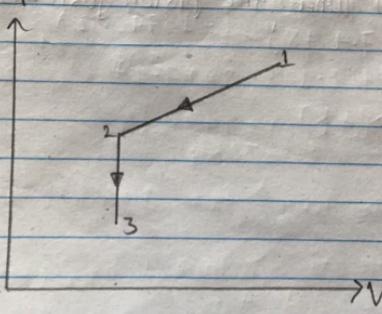
Then, change in total internal energy is given by

$$\Delta U = m(u_3 - u_1)$$

$$= m c_p (T_3 - T_1)$$

$$2 \cdot 1993 \cdot 49 = 1 \times 793 \times (313 - 773)$$

$$= -341.78 \text{ k}$$



Work transfer during the process is given by

$$W = W_{12} + W_{23}$$

$$= P(V_2 - V_1) + 0$$

$$= 250(0.959162 - 0.918324)$$

$$= -114.791 \text{ kJ}$$

Total heat transfer during the process is given by

$$Q = \Delta U + W$$

$$= -341.78 - 114.791$$

$$= -456.571 \text{ kJ}$$

Then, Change in entropy for the process is given by

$$\Delta S = S_2 - S_1$$

$$= mC_v \ln\left(\frac{T_3}{T_1}\right) + mR \ln\left(\frac{V_3}{V_1}\right)$$

$$= 1 \times 743 \times \ln\left(\frac{313}{293}\right) + 1 \times 297 \times \ln\left(\frac{0.959162}{0.918324}\right)$$

$$= -0.52323 \text{ kJ/K} - 0.877 \text{ kJ/K}$$

Therefore, entropy generation is given by

$$S_{gen} = (\Delta S)_{cm} - \sum_i \left(\frac{Q_i}{T_i} \right)_{cm}$$

$$= (S_2 - S_1) - \frac{Q}{T_{sur}}$$

$$= -0.52323 - \left(\frac{-456.571}{293} \right)$$

$$= -0.035033 \text{ kJ/K} / 0.68 \text{ kJ/K}$$

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Solution

$$\text{Mass of water } 1 (m_1) = 2 \text{ kg}$$

$$\text{Initial temp. of water } 1 (T_1) = 10^\circ\text{C}$$

$$= 10 + 273$$

$$= 283 \text{ K}$$

$$\text{Mass of water } 2 (m_2) = 4 \text{ kg}$$

$$\text{Initial temp. of water } 2 (T_2) = 20^\circ\text{C}$$

$$= 20 + 273$$

$$= 293 \text{ K}$$

Let T_3 be the equilibrium temp. then heat lost by water 1 is absorbed by the water 2, i.e.

$$m_1 c (T_1 - T_3) = m_2 c (T_3 - T_2)$$

$$\text{or, } 2 \times 4.18 \times (373 - T_3) = 4 \times 4.18 \times (T_3 - 293)$$

$$\text{or, } 8.36 \times (373 - T_3) = 16.72 \times (T_3 - 293)$$

$$\text{or, } 3118.28 - 8.36 T_3 = 16.72 T_3 - 4898.96$$

$$\text{or, } 8017.24 = 25.08 T_3$$

$$\therefore T_3 = 319.67 \text{ K}$$

$$= 46.67^\circ\text{C}$$

Then,

Change in entropy of the water 1 is given by

$$(\Delta S)_1 = m_1 c \ln \left(\frac{T_3}{T_1} \right)$$

$$= 2 \times 4.18 \times \ln \left(\frac{319.67}{373} \right)$$

$$= -1.2898 \text{ kJ/K}$$

Change in entropy of water 2 is given by

$$(\Delta S)_2 = m_2 c \ln \left(\frac{T_3}{T_2} \right)$$

$$= 4 \times 4.18 \times \ln \left(\frac{319.67}{293} \right)$$

$$= 1.4565 \text{ kJ/K}$$

Therefore, the net change in entropy is given by

$$(\Delta S)_{\text{net}} = (\Delta S)_1 + (\Delta S)_2$$

$$= -1.2898 + 1.4565$$

$$= 0.1667 \text{ kJ/K}$$

4

Solution

Properties of steam at inlet; $P_1 = 2 \text{ MPa} = 2000 \text{ kPa}$; $T_1 = 400^\circ\text{C}$; $\bar{V}_1 = 200 \text{ m/s}$

Properties of steam at outlet; $P_2 = 100 \text{ kPa}$, saturated vapor, $\bar{V}_2 = 80 \text{ m/s}$

Mass flow rate of steam (\dot{m}) = 1.5 kg/s

Power output of the turbine (\dot{W}_{cv}) = 800 kW

Temperature of the surr. (T_{surr}) = 300 K

for the other properties of steam at inlet referring to table, $T_{\text{sat}}(2000 \text{ kPa}) = 212.42^\circ\text{C}$. Here $T > T_{\text{sat}}$, hence it is a superheated steam. Now, referring to the table,

$$h_1 = 3247.5 \text{ kJ/kg} \quad \text{and} \quad s_1 = 7.1269 \text{ kJ/kgK}$$

Similarly, for the other properties of steam outlet, referring to the table, $h_2 = h_g(100 \text{ kPa}) = 2675.1 \text{ kJ/kg}$ and $s_2 = s_g(100 \text{ kPa}) = 7.3589 \text{ kJ/kgK}$

Now, applying energy equation for the turbine,

$$\begin{aligned} Q_{cv} - w_{cv} &= m \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right] \\ \therefore Q_{cv} &= w_{cv} + m \left[(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1) \right] \\ &= 800 + 1.5 \left[(2675.1 - 3247.5) + \frac{1}{2000} (80^2 - 200^2) + 0 \right] \\ &= -83.8 \text{ kW} \end{aligned}$$

Then the rate of entropy generation during the steady operation of any control volume is given by

$$\begin{aligned} \dot{s}_{gen} &= (s_{out} - s_{in}) - \sum \left(\frac{q_i}{T_{surr}} \right)_{cv} \\ &= m(s_2 - s_1) - \frac{Q_{cv}}{T_{surr}} \\ &= 1.5 (7.3589 - 7.1269) - \frac{(-83.8)}{800} \\ &= 0.6273 \text{ kW/K} \end{aligned}$$

5

Solution

Given, Properties of air at inlet : $P_1 = 500 \text{ kPa}$

$$T_1 = 327^\circ\text{C}$$

$$= 327 + 273$$

$$= 600 \text{ K}$$

$$(P_1, V_1) = (500, \bar{V}_1) = 50 \text{ m/s}$$

Properties of air at exit : $P_2 = 100 \text{ kPa}$, $T_2 = 27^\circ\text{C}$

$$= 27 + 273$$

$$= 300 \text{ K}$$

$$\bar{V}_2 = 500 \text{ m/s}$$

Temperature of the surrounding ($T_{\text{sur}} = 20^\circ\text{C}$)

$$= 20 + 273$$

$$= 293 \text{ K}$$

$$\dot{q}_{cv} = \dot{m} [(h_2 - h_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + g(z_2 - z_1)]$$

For an ideal gas using $h_2 - h_1 = c_p(T_2 - T_1)$ and neglecting P.E.,

$$\frac{\dot{q}_{cv}}{\dot{m}} = q_{cv} = c_p(T_2 - T_1) + \frac{1}{2} (\bar{V}_2^2 - \bar{V}_1^2) + 0$$

$$\therefore q_{cv} = 1.005 (300 - 600) + \frac{1}{2000} (500^2 - 50^2) = -177.75 \text{ kJ/kg}$$

Then, Change in entropy per kg of air is given by

$$\Delta S = s_2 - s_1 = c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

$$= 1.005 \times \ln \left(\frac{300}{600} \right) - 0.287 \times \ln \left(\frac{100}{500} \right)$$

$$= -0.2347 \text{ kJ/kg K}$$

Therefore, the entropy generation per kg of air is given by

$$S_{gen} = (s_{out} - s_{in}) - \sum \left(\frac{q_i}{T_i} \right) \dot{v}$$

$$\therefore S_{gen} = (s_2 - s_1) - \frac{q_{cv}}{T_i}$$

$$= -0.2347 - \frac{(-177.75)}{293}$$

$$= 0.37195 \text{ kJ/kgK}$$

[8]

Solution,

Properties of steam at inlet : $P_1 = 10 \text{ MPa}$
 $= 10000 \text{ kPa}$

$$T_1 = 500^\circ\text{C}$$

Properties of steam at outlet : $P_2 = 0.06 \text{ MPa}$
 $= 60 \text{ kPa}$

$$x_{2r} = 96\% \\ = 0.96$$

Mass flow rate of steam (\dot{m}) = 20 kg/s

Process : isentropic (reversible and adiabatic)

For other properties of steam at inlet, referring to the table, $T_{sat}(10000 \text{ kPa}) = 311.803^\circ\text{C}$. Here, $T > T_{sat}$, hence the conditions of steam at turbine inlet is superheated vapor. Now, referring to the table,

$$h_1 = 3500.9 \text{ kJ/kg}, s_1 = 6.7561 \text{ kJ/kgK}$$

Since, entropy remains constant during isentropic process, specific entropy at turbine exit is $s_2 = 6.7561 \text{ kJ/kgK}$

Referring to the table, $s_1(60 \text{ kPa}) = 1.1454 \text{ kJ/kgK}$

$$s_g(60 \text{ kPa}) = 6.3856 \text{ kJ/kgK}$$

$$s_g(60 \text{ kPa}) = 7.5310 \text{ kJ/kgK}$$

$$h_1(60 \text{ kPa}) = 359.90 \text{ kJ/kg}$$

$$h_{fg}(60 \text{ kPa}) = 2293.1 \text{ kJ/kg}$$

Here, $s_1 < s_2 < s_g$, hence the condition of steam at turbine exit is given by

$$\frac{s_2 - s_1}{s_g} = \frac{6.7561 - 1.1454}{6.3856} = 0.8786$$

Then, specific enthalpy of steam at isentropic turbine exit is given by

$$h_2 = h_1 + x_2 h_{fg} = 359.90 + 0.8786 \times 2293.1 \\ = 2374.618 \text{ kJ/kg}$$

Now, applying steady state energy equation for an isentropic turbine,

$$\dot{W}_{cv} = m[(h_1 - h_2) + \frac{1}{2}(\bar{V}_1^2 - \bar{V}_2^2) + g(z_1 - z_2)]$$

Neglecting K.E. and P.E., we get

$$\dot{W}_{cv} = 10 \times (h_1 - h_2) + 0 + 0$$

$$= 10(3500.9 - 2374.618)$$

$$\therefore \dot{W}_{cv} = 11262.82 \text{ kW}$$

Again,
by

specific enthalpy of steam at turbine exit is given

$$h_{1r} = h_1 + x_{2r} h_{fg}$$

$$= 359.90 + 0.96 \times 2293.1 = 2561.276 \text{ kJ/kg}$$

Now, applying energy equation for an adiabatic turbine,

$$W_{\text{ev}} = \dot{m} [h_1 - h_2 + \frac{1}{2} (\bar{v}_1^2 - \bar{v}_2^2) + g(z_1 - z_2)]$$

Neglecting K.E. and P.E., we get

$$W_{\text{ev}} = \dot{m}(h_1 - h_2) + 0 + 0$$

$$\begin{aligned} W_{\text{ev}} &= 10 (3500 \cdot 9 - 2561 \cdot 276) \\ &= 9396.24 \text{ kW} \end{aligned}$$

Therefore, isentropic efficiency of the turbine is given by

$$\begin{aligned} \eta_{\text{IT}} &= \frac{W_{\text{actual}}}{W_{\text{isen}}} = \frac{9396.24}{11262.82} \\ &= 0.8343 \\ &= 83.43\% \end{aligned}$$

Actual work output per kg of steam is given as

$$\begin{aligned} W_{\text{actual}} &= \frac{W_{\text{actual}}}{\dot{m}} = \frac{9396.24}{10} \\ &= 939.624 \text{ kJ/kg} \end{aligned}$$

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Solution

$$\text{Given, } T_H - T_L = 200^\circ\text{C} (= 200\text{K})$$

$$W = 0.6 Q_L$$

$$\begin{aligned} Q_H &= W + Q_L \\ &= 0.6 Q_L + Q_L \\ &= 1.6 Q_L \end{aligned}$$

Then,

efficiency of Carnot engine is given as

$$\begin{aligned} \eta_{\text{rev}} &= \frac{W}{Q_H} = \frac{0.6 Q_L}{1.6 Q_L} \\ &= 0.375 = 37.5\% \end{aligned}$$

Efficiency of a Carnot engine operating between two reservoirs at temperature T_H and T_L is given by

$$\begin{aligned} \eta_{\text{rev}} &= 1 - \frac{T_L}{T_H} \\ &= \frac{T_H - T_L}{T_H} \\ &= \frac{200}{T_H} \end{aligned}$$

$$\therefore T_H = 533.33\text{K}$$

And,

$$\begin{aligned} T_L &= 533.33 - 200 \\ &= 333.33\text{K} \end{aligned}$$

1

10

Solution,

Given, COP of heat pump $(COP)_{HP} = 5$

Higher temperature (T_H) = $24^\circ C = 24 + 273 = 297 K$

Lower temperature (T_L) = $0^\circ C = 0 + 273 = 273 K$

$$\text{Heating rate } (\dot{Q}_H) = 72000 \text{ kJ/hr} = \frac{72000}{3600} = 20 \text{ kJ/s}$$

$$= 20 \text{ kW}$$

a) Actual power required to drive the heat pump is given by

$$W = \frac{\dot{Q}_H}{(COP)_{HP}} = \frac{20}{5} = 4 \text{ kW}$$

Maximum COP of the heat pump operating between the temp limits is given by

$$(COP)_{rev, HP} = \frac{T_H}{T_H - T_L} = \frac{297}{297 - 273}$$

$$= \frac{297}{24}$$

$$= 12.375$$

Therefore, theoretical power required to drive the heat pump is given as

$$W_{th} = \frac{\dot{Q}_H}{(COP)_{rev, HP}} = \frac{20}{12.375} = 1.6162 \text{ kW}$$

The actual cost per day is given by

$$C_{actual} = W \times 12 \times 10$$

$$= 4 \times 12 \times 10$$

$$= \text{Rs } 480$$

The minimum theoretical cost per day is given as

$$C_{th} = W_{th} \times 12 \times 10$$

$$= 1.6162 \times 12 \times 10$$

$$= \text{Rs } 193.944$$

b) The cost of direct electric resistance heating is given as

$$\begin{aligned} C_{direct} &= \dot{q}_h \times 12 \times 10 \\ &= 20 \times 12 \times 10 \\ &= \text{Rs } 2400. \end{aligned}$$

Solution

Given, lower temperature (T_L) = 20°C

$$= 20 + 273 = 293 \text{ K}$$

Rate at which heat is removed from a house (\dot{q}_L) = 0.8 $(T_h - T_L)$ kW

Power input (W) = 1.8 kW

Theoretical maximum COP of an air conditioning unit operating between the temp. limits is given by

$$(COP)_{rev,R} = \frac{T_L}{T_h - T_L}$$

Again, COP of an air conditioning unit is given by

$$(COP)_P = \frac{\dot{q}_L}{W} = \frac{0.8 \cdot (T_h - T_L)}{1.8}$$

According to the question, COP of an air conditioning unit is 50% of the theoretical maximum COP of an air conditioning unit i.e,

$$(\text{COP})_R = 50\% \text{ of } (\text{COP})_{\text{rev}}, R = 0.5 \times 1.0$$

$$\text{or}, \frac{0.8(T_n - T_L)}{1.8} = 0.5 \times \left(\frac{T_L}{T_n - T_L} \right)$$

$$\text{or}, \frac{0.8(T_n - 293)}{1.8} = 0.5 \times \left(\frac{293}{T_n - 293} \right)$$

$$\text{or}, (T_n - 293)^2 = \frac{0.5 \times 1.8 \times 293}{0.8}$$

$$= 329.625$$

Solving, we get

$$T_n = 293 + 18.156$$

$$= 311.156 \text{ K}$$

$$= 38.156^\circ\text{C}$$

[12]

Solution,

Power input (w) = 1.5 kW

COP of an air conditioning unit $(\text{COP})_R = 3$

Lower temperature (T_L) = 22°C

$$= 22 + 273 = 295 \text{ K}$$

Rate at which heat is removed from a hall (Q_1)
 $= 0.8(T_n - T_L) \text{ KW}$

COP of an air conditioning unit is given by

$$(\text{COP})_R = \frac{Q_1}{W}$$

$$\text{or, } 3 = \frac{0.8(T_n - T_L)}{1.5} - \frac{0.8(T_n - 295)}{1.5}$$

$$\text{or, } T_n - 295 = 5.625$$

$$\therefore T_n = 300.625 \text{ K}$$

$$= 27.625^\circ\text{C}$$

When an air conditioning unit is working as heating unit (heat pump) in winter:

COP of an air conditioning unit, $(\text{COP})_{NP} = 4$

Higher temperature (T_n) = $22^\circ\text{C} = 22 + 273 = 295 \text{ K}$

Rate at which heat is supplied in a hall (Q_n)

$$= 0.8(T_n - T_L) \text{ kW}$$

Then, COP of an air conditioning unit is given as

$$(\text{COP})_{NP} = \frac{Q_n}{W}$$

$$\text{pr, } 4 = \frac{0.8(T_n - T_L)}{1.5}$$

$$= \frac{0.8(295 - T_L)}{1.5}$$