CHAPTER :→ 1

Introduction :→

Engineering economics is the application of economic techniques to evaluati of design and engineering alternatives.

1.1> Origin of Engineering Economy :→

The perspective that ultimately economy is a concern to the engineer and the availability of sound techniques to address this concern differentiate this aspect of modern engineering practice from that of the past.

Adam smith, also known as father of economics publish "The Wealth of Nations" in 1778. Two important concepts were derived from this:

i> **Theory of specialization :→**

It is most important to engineers as it focuses on concept of mass production.

ii> **Principle of an Invisible Hand :→**

Every individual has self-interest which is guided by invisible hand.

In 1930, Eugene L. Grant published "Principles of Engineering Economy" and discussed about the importance of determing factors, short-term and long term investment in capital assets based on compound interest calculations. He is known as father of Engineering Economics.

In latter part of nineteenth century, Arthur M. Wellington (Civil Engineer) addressed role of economic analysis in engineering projects. Then followed by other contributions which emphasized techniques depending on financial and actuarial mathematics.

1.2> Principles of Engineering Economy:→

The development, study and application of any subject begins with a basic foundation. The foundation of Engineering economy is a set of principles that provides a sound basis for the development methodology.

The following seven principles are crucial to decipline and accomplish a good engineering economy study:→

P1:- Develop the alternatives:-

The choice is among the alternatives. The alternatives need to be identified and then defined for subsequent (various) analysis.

P2:- Focus on the differences:-

The differences in expected outcomes among alternatives are compared and decision is made.

P3:- Use a consistent View point:→

The prospective outcomes of the alternatives should be consistently developed from a defined view-point.

P4:- Use a common units of measure:-

The two alternatives having different units of measurement can't be compared so that they have to be convert in same unit as far as possible.

P5: Consider all relevant criteria :-

All relevant social, environmental and other criteria should be considered.

P6: Make risk and Uncertainity Explicit :→

Risk and Uncertainty are inherent in estimating the future outcomes of the alternatives and should be recognized in their analysis and comparision.

P7: Revisit your decision:-

Improved decision making results from an adaptive process to the extent practicable. The initial projected outcomes should be compared with actual results achieved.

1.3> Role of Engineers in Decision Making:→

Q.> "Engineers play the important role in Economic decision". Do you agree with this? Discuss [2068 Bhadra]

An engineering economist have the knowledge of engineering and economics to identify alternatives, use of resource efficiently & to select preferred option in course of action. Hence, it is devoted to problem solving & decision making at all levels.

A decision is simply the selection from two or more options whether it takes place in construction or production or service sector. The techniques and models of engineering economy assist people in making decisions. The decisions are made by engineers to choose one project over another and the decision often reflect to choice of how to make best use of capital funds.

Hence, the knowledge of engineering economy can assist engineers in making decision from following grounds.

i> Understand the problem.
ii> Define the objective.
iii> Collect relevent information.
iv> Develop the alternatives.
v> Identify decision making criteria.
vi> Evaluate the alternatives.
vii> Select best alternatives.
viii> Effective implementation.

In decision making process role of engineers in different situation for strategic economic decisions are in:-
(i) Equipment & process selection.
(ii) Equipment replacement.
(iii) New product & product expansion.
(iv) Cost reduction and
(v) Service improvement.

1.4) Cash flow and CFD

Cash flow is the statement which shows inflows and outflows of cash and cash equivalents during life of projects. i.e. actual rupees coming into or goes out in different time periods.

Cash flow is the main basis for evaluation of different alternatives.

The graphical presentation of different cash flow streams is known as cash flow diagram (CFD).

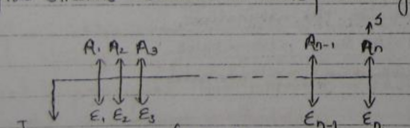


fig:- CFD

where,
I = initial investment
R = Revenue
E = Expenses
S = Salvage value.

Drawing Cash flow Diagram:→

↳ The horizontal line is time scale with progression of time moving from left to right.

↳ The period (years) lables are applied to interval of time, i.e., end of period 1st is coinciding with beginning of period 2nd.

↳ The arrows signifies cash flows, i.e., downward arrow represent cash out flows [investment; annual maintainance cost; expences] and upward arrow represents cash inflows [annual revenue; salvage value; income]

Example 1 :→

A man borrowed Rs 1000 from a bank at 8% interest. Two end of year payments at end of first year, he will repay half of Rs1000 principal plus the Interest that is due. At the end of the second year, he will repay remaining half plus the interest for second year.

Draw CFD.

⇒ Soln:

End of year (Horizontal line)	Cash flow (Vertical line)
0	+ Rs 1000
1	− Rs500 − 80 = −Rs580
2	− Rs 500 − 40 = −Rs540

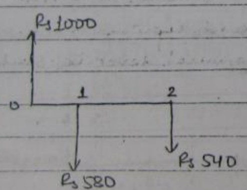


fig:- CFD

Note: at 0^{th} year :→

man borrowed money, so cash inflow of Rs 1000.

at 1^{st} year :→

man pay money, so cash outflow of half of principle $\left[\frac{Rs\,1000}{2} = Rs\,500 \right]$ plus interest

$$\left[\text{Interest} = \frac{\text{Principle} * \text{Time} * \text{Rate}}{100} = 1000 * 1 * 8 / 100 \right]$$

$= Rs\ 80$

$= -(Rs\ 500 + 80) = -Rs\ 580$

−ve for out-flow

at 2^{nd} year :→

man pay remaining half plus interest for second year, so cash outflow.

$$= -\left(Rs\ 500 + \frac{Rs\,500 * 1\,year * 8}{100}\right)$$

$$= -(Rs\ 500 + 40)$$

$$= -Rs\ 540$$ #

Example 2 :→

Initial investment (I) = Rs 10,000

Annual income (A) = Rs 5310

Annual Expenses (E) = Rs 3000.

Salvage value (S) = Rs 2000,

Life (N) = 5 years.

Draw CFD?

⇒ Soln.

End of year	Cash flow (Rs)	
	Outflow	Inflow
0	10000	—
1	3000	5310
2	3000	5310
3	3000	5310
4	3000	5310
5	3000	5310 + 2000

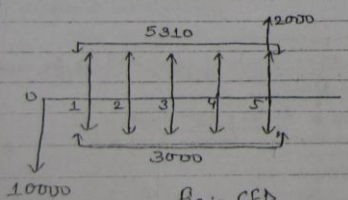


fig :- CFD

CHAPTER :→ 2

2.1> Introduction to time value of Money :→

Time value of money is fundamental concept in comparision of economic performance between alternatives. Money has a time value, value of given sum of money depends on when money is received.

Time value of money is defined as time dependent value of money because of changes in purchasing power (inflation or deflation) and real earning capacity of capital over time. Since, money has ability to earn interest, it's values increases with time. Hence, it is the relationship between interest and time.

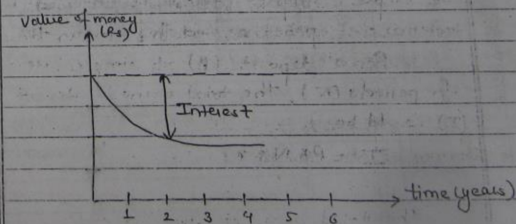


fig :→ Time Vs value of money.

Interest Calculation :→

Interest is price of money services or it is the fee that is charged for use of money. Interest depends upon total amount of money and length of time over which it is borrowed. Normally, it is calculated as an annual rate and interest payments are made by borrowers to lenders.

There are 2 methods on interest calculation:-

a) Simple Interest.

b) Compound Interest.

2.2) Simple Interest :→

When total amount of interest earned is directly proportional to initial principle amount, then the interest is ~~not used~~ said to be simple. Simple Interest is not used in commercial practice at this modern time.

For a deposit (P) at simple interest rate(r) for periods (N), the total earned interest amount (I) would be,

$$I = P * N * r$$

The total amount available at end of N periods. i.e, total future amount (F) would be,

$$F = P + I$$

$$\therefore F = P(1 + Nr)$$

2.3 Compound Interest :→

Whenever interest charge for any interest period or year is based on principal loan amount plus any accumulated interest charges upto beginning of that period, the interest is said to be compounded. It is mostly used in practice.

Following notation is utilized for compounding interest calculations:-

i = effective interest rate per interest period.

N = Number of compounding periods,

P = present sum of money,

F = Future sum of money,

A = End of period cash flow in a uniform series.

Future amount (F) $= P(1+i)^N$

2.3.1 Nominal Interest Rate:→

The nominal interest rate is periodic interest rate times the number of periods per year. For example, a nominal annual interest rate of 12% based on monthly compounding means 1% interest rate per month (compounded).

If a financial institution rate per month uses a unit of time other than a year - a month or quarter (e.g: when calculating interest payments), the institution usually quotes the interest rate on annual basis. Commonly, this rate is stated as

$r\%$ Compounded M-ly

where,

r = nominal interest rate per year

M = compounding frequency or number of interest periods per year.

r/M = interest rate per compounding period.

For example, if interest rate is 12% per year and compounded semiannually [or if interest rate is 6% per interest period and interest period is six month], in this sitution nominal interest rate is 12%, (i.e, $r = 12\%$). But actual rate of interest would be greater than 12% because of compounding twice during a year.

2.3.1: Effective Interest Rate:→

The actual rate of interest earned during one year is known as effective rate and it is also expressed on the annual basis unless specifically stated otherwise. Effective interest rate is represented by 'i'.

Relation between effective (i) & nominal (r) rate,

$$\boxed{i = (1 + r/M)^m - 1}$$

where,

m = compounding period per year.

Example:

As from above formula, the effective interest rate for 12% compounded semi-annually,

$$i = \left(1 + \frac{0.12}{2}\right)^2 - 1 = 12.36\%$$

Note:

In this case,

$r = 12\%$ & $m = 2$

2.3.3: Continuous Compounding :→

Continuous compounding can be thought of as making compounding period infinitesimally small, therefore achieved by taking limit of m [no. of compounding periods in year] to Infinity.

In continuous compounding,

$$i = \lim_{m \to \infty} \left[\left(1 - \frac{r}{m} \right)^m - 1 \right]$$

$$\therefore \boxed{i = e^r - 1}$$

2.4: Economic Equivalence :→

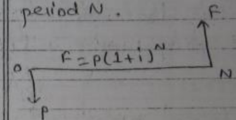
In engineering economy, 2 things are said to be equivalent when they have the same effect.

Economic equivalence exists between cash flows that have the same economic effect & could therefore be traded for one another.

Even though the amounts and timing of the cash flows may differ, the appropriate interest rate makes them equal in economic sense.

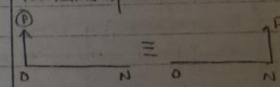
* Equivalence from Personal Financing Point of View :→

↳ If you deposit P dollars today for N periods at 'i', you will have F dollars at end of period N.



* Alternate Way of Defining Equivalence :→

↳ F dollars at end of period N is equal to a single sum P dollars now, if your earning power is measured in terms of interest rate i.



$$P = F(1+i)^{-N}$$

2.5: Development of Interest Formulas :→

As we begin to compare series of cash flows instead of single payments, the required analysis becomes more complicated. However, when patterns in cash flow transactions can be identified, we can take advantage of these patterns by developing concise expression for computing either present or future worth of the series. We will classify 5 major categories of cash flows transactions; develop interest formulas for them and present several working examples for them.

2.5.1. The 5 types of Cash flows :→

(Described in 2.5.2 to 2.5.6
① to ⑤)

(1) Single Cash Flow :→

The simplest case involves equivalence of a single present amount and it's future worth. The single cash flow formulas deal with only 2 amounts: a single present amount P, and its future worth F.

e.g.

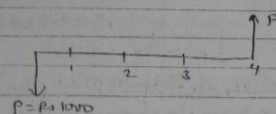


fig:- Single Cashflow.

(2) Uneven (Equal or Uniform) Payment Series:→

In this type, transactions are arranged as a series of equal cash flows at regular intervals, known as an equal payment series (Uniform series). This describes the cash flows of common installment loan contract, which arranges repayment of the loan loan in equal periodic installments.

e.g.-

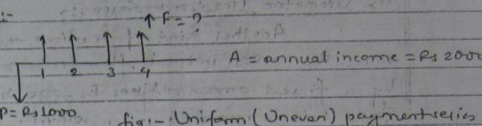


fig:- Uniform (Uneven) payment series.

(3) Linear Gradient Series:→

While many transactions involve series of cash flows, the amounts are not always uniform, they may vary in some regular way. One common pattern of variation occurs when each cash flow in a series increases (or decreases) by a fixed amount. For example, A 10 year loan repayment plan might specify series of annual payments that increase by Rs 1000 each year. This type of cash flow pattern is called linear gradient series.

E.g:

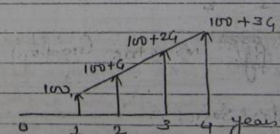


fig:- Linear gradient series.

4) Geometric Gradient Series :→

Another kind of gradient series is formed when the series in cash flow is determined not by a fixed amount like Rs 1000, but by some fixed rate, expressed as a percentage.

For e.g :-

In a 10 year financial plan for a project, the cost of particular raw material might be budgeted to increase at a rate of 4% per year. The curving gradient in the diagram of such a series suggests its name which is geometric gradient series. However, we dont need formulas for geometric gradient series in detail.

e.g :-

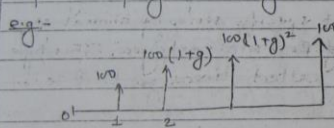


fig: Geometric Gradient series.

5) Irregular Series :→

A series of cash flow may be irregular. It doesn't exhibit a overall regular pattern

e.g

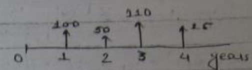


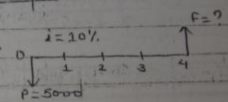
fig Irregular series.

2.5.2) Single Cash Flow Formulas :→

	To find	Given	Factor	Factor Name	Functional symbol
i)	F	P	$(1+i)^N$	single payment compound amount	(F/P, i%, N)
ii)	P	F	$(1+i)^{-N}$	single payment present worth	(P/F, i%, N)

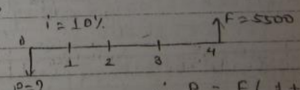
Examples:

i)



$$\therefore F = P(1+i)^N$$
$$= 5000\left(1+\frac{10}{100}\right)^4$$
$$= 5500$$

ii)



$$\therefore P = F(1+i)^{-N}$$
$$= 5500\left(1+\frac{10}{100}\right)^{-4}$$
$$= 5000$$

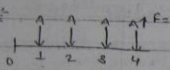
P.14

2.5.3> Uneven Payment Series $\equiv$ Uniform Payment Series.

2.5.4> Uniform (Equal) Payment Series:

To-find	Given	Factor	Factor Name	Functional symbol.
F	A	$\frac{(1+i)^N-1}{i}$	Uniform series compound amount	$(F/A, i\%, N)$

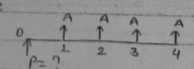
e.g.:-



$$F = A\left\{\frac{(1+i)^N-1}{i}\right\}$$

P	A	$\frac{(1+i)^N-1}{i(1+i)^N}$	Uniform series Present worth	$(P/A, i\%, N)$

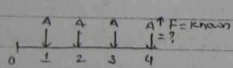
e.g.:



$$P = A\left\{\frac{(1+i)^N-1}{i(1+i)^N}\right\}$$

A	F	$\frac{i}{(1+i)^N-1}$	Sinking Fund	$(A/F, i\%, N)$

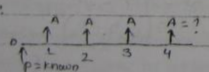
e.g.:



$$A = F\left\{\frac{i}{(1+i)^N-1}\right\}$$

A	P	$\frac{i(1+i)^N}{(1+i)^N-1}$	Capital Recovery	$(A/P, i\%, N)$

e.g.:

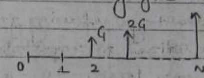


$$A = P\left\{\frac{i(1+i)^N}{(1+i)^N-1}\right\}$$

2.5.5> Linear Gradient Series [Uniform Gradient Series]

An arithmetical progression of cash flow service which begins to increase or decrease from end of second period onwards is called linear gradient series & it is generally known as gradient (G).

e.g.:- Increasing gradient series.



G = gradient amount

* Finding F when given G [F/G, i%, N]

$$F = \frac{G}{i}\left[\frac{(1+i)^N-1}{i}\right] - \frac{NG}{i}$$

$$= \frac{G}{i}\left[F/A, i\%, N\right] - \frac{NG}{i}$$

Term in bracket is called gradient to future worth conversion factor.

* Finding A when given G [A/G, i%, N]

$$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N-1}\right]$$

Term in bracket is called gradient to uniform series conversion factor.

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* finding P when given G [P/G, i%, N]

$$P = G\left[\frac{(1+i)^N - 1 - Ni}{i^2(1+i)^N}\right]$$

Term in bracket is called gradient to present worth conversion factor.

A major advantage of using gradient conversion factors is realized on computational time saving when N becomes large.

2.5.6> Geometric Gradient Series:→

Suppose that there is a series of 'N' payements uniformely spaced but differing from one period to next by a constant multiple.

The change or "gradient" multiple from one period to next is denoted as "g".

* finding P when given g & A [P/A, g, i, N]

$$P = \begin{cases} \dfrac{A}{i-g}\left[1-\left(\dfrac{1+g}{1+i}\right)^N\right. & \text{if } i \neq g \\[2ex] \dfrac{NA}{1+i} & \text{if } i = g \end{cases}$$

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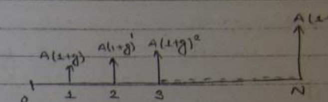


* finding F,

$$F = A(P/A, g, i, N)\ (F/P, i, N)$$

Examples

1> Given,

A = Rs 54600

g = 7% ('A' gets increased by 7% per year of previous year value)

N = 5 yrs

i = 12% per year

Solⁿ:

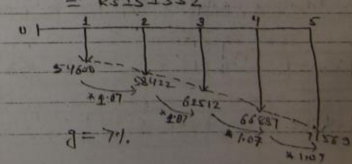
$$P_{old} = 54600\left[\frac{1-(1+0.07)^5\,(1+0.12)^{-5}}{0.12-0.07}\right]$$

= Rs 222937

$$P_{new} = 54600(1-0.23)(P/A, 12\%, 5)$$

= 42042 (3.6048)

= Rs 151552



P.15

2) Given,

$F = Rs\ 1000000$,

$g = 6\%$

$i = 8\%$

$N = 20$

Find A.

Sol^n:

$F = A(P/A, 6\%, 8\%, 20)\ (F/P, 8\%, 20)$

$= \frac{A}{0.08-0.06}\left[1-\left(\frac{1+0.06}{1+0.08}\right)^{20}\right][F/P, 8\%, 20]$

$= A(72.6911)$

$\therefore A = \frac{1000000}{72.6911} = Rs\ 13757$

Note:

[5 types of Cash Flows Formulas in Continuous Case]

i.e., Continuous Compounding and Formulas:-

(A.) Continuous Compounding & Discrete Cash Flows,

imp

$\boxed{e^r = (1+i)}$ $\left[\because i = e^r - 1, \text{ (see 2.3.3)}\right]$

Ex:-

$F = P(1+i)^N$ becomes, $\boxed{F = Pe^{rN}}$

$F = A\left[\frac{(1+i)^N - 1}{i}\right]$ becomes, $\boxed{F = \frac{e^{rN}-1}{e^r - 1}}$

and so on - - -.

(B) Continuous Compounding & Continuous Cash flows:→

imp

$\boxed{e^r = 1+i \quad \text{and; } i = r}$

Ex:-

$F = A\left[\frac{(1+i)^N-1}{i}\right]$ becomes $\boxed{F = \bar{A}\left[\frac{e^{rN}-1}{r}\right]}$

$P = A\left[\frac{(1+i)^N-1}{i(1+i)^N}\right]$ becomes $\boxed{P = \bar{A}\left[\frac{e^{rN}-1}{re^{rN}}\right]}$

and so on - - - -.

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Examples :→ [For Chapter - 2]

1> Suppose you borrow Rs 8000 from a bank, that charges interest at a rate of 10% compounded annually. How much would you owe at end of year 4?

Sol^n :→

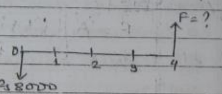
Given, P = Rs 8000, N = 4yr, i = 10%, F = ?

We know,

$F = P(1+i)^N$

$= 8000(1+0.1)^4$

$=$ Rs 11,712.8 #



2> Suppose you have invested Rs 1000 at present. How long does it take for your investment to double if interest rate is 8% compounded annually?

Sol^n :→

Let investment P will become 2P after N years

Then, F = 2P

We know that,

$F = P(1+i)^N$

or, $2P = P(1+i)^N$

or, $2 = (1+0.08)^N$

or, $1.08^N = 2$

Taking 'log' both sides,

$N \log 1.08 = \log 2$

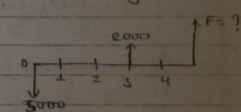
$N = \dfrac{\log 2}{\log 1.08}$

$\therefore N = 9$ years #

3> Ram deposite Rs 5000 now in a bank which gives 10% interest per year. He draws Rs 2000 at end of 3rd year. What will be remaining amount at end of 5th year?

Sol^n :→

At end of 3rd year, accumulated amount will be,



$F = P(1+i)^N$

$= 5000(1+0.1)^3$

$=$ Rs 6655

After drawing Rs 2000 remaining deposit amount at end of 3rd year will be,

Rs 6655 − Rs 2000 = Rs 4655

At end of 5th year, total accumulated amount will be, $F = 4655(1+0.1)^2$

$=$ Rs 5632.5

4> Find effective interest rate when nominal rate of interest is 18% per year & compounding is (i) Monthly (ii) Daily (iii) Hourly & (iv) Continuously.

Soln :→

(i) Monthly, $i = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 19.561\%$.

(ii) Daily,

$$i = \left(1 + \frac{0.18}{365}\right)^{365} - 1 = 19.716\%.$$

(iii) Hourly,

$$i = \left(1 + \frac{0.18}{365*24}\right)^{365*24} - 1 = 19.7215\%.$$

(iv) Continuously,

$$i = e^{r} - 1 = e^{0.18} - 1 = 19.7217\%.$$

5> A person deposits a sum of Rs 5000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find maturity of deposit after 10 years.

Soln :→

$P = Rs\ 5000$

$N = 10$ yrs,

$r = 12\%$ Compounded quartely

$$i_{effective} = \left(1 + \frac{0.12}{4}\right)^{4} = 12.5508\% \text{ per year}$$

$$\therefore F = 5000\,(F/P,\ 12.55\%,\ 10)$$

$$= 5000\,(1 + 0.1255)^{10}$$

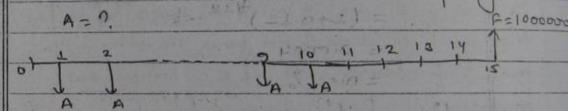
$$= Rs\ 16310 \quad //$$

6> Mr Ramesh wants to have Rs 1000000 for the studies of his daughter after period of 15 years. How much rupees does he has to deposit each year for 10 continuous years in a saving account that earns 8% interest annually. [IOE, 2061]

Soln :-

$F = Rs\ 1000000$, $N = 15$, $i = 8\%$ per year

$A = ?$



First, discounting Rs 1000000 [F] to year 10,

$$P = F\,(P/F,\ 8\%,\ 5)$$

$$= 1000000\,(1 + 0.08)^{-5}$$

$$= Rs\ 680,583.197$$

Now using sinking fund factor,

$$A = F\,(A/F,\ 8\%,\ 10)$$

$$A = 680,583.197 \left[\frac{0.08}{(1.08)^{10} - 1}\right]$$

$$\therefore A = Rs\ 46,980.31$$

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Imp 7> How many deposits of Rs 25000 each should Dr. Thakur make each month so that the final accumulated amount will be Rs 10,00,000 if bank interest rate is 12% per year?
[IOE 2063]

Soln :→

Given,

$A = 25000$ per month

$F = 1000000$

$i = 25\%$ per year

Monthly interest rate,

Imp $\boxed{i_m = (1 + i_{year})^{1/12} - 1}$

$= (1+0.12)^{1/12} - 1$

$= 0.0094$

$= 0.94\%$

Using uniform series compound amount factor,

$F = A(F/A, 0.94\%, N)$

or, $1000000 = 25000\left[\dfrac{(1+0.0094)^N - 1}{0.0094}\right]$

or, $1.376 = (1.0094)^N$

Taking 'log' both sides & solving,

$N = 34$

Thus, Dr. Thakur should make 34 deposites.

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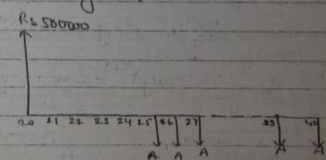
8> A man aged 40 years now had borrowed Rs 500000 from a bank for his further studies at the age of 20 yrs. Interest was charged at 11% per year compounded quarterly. He wished to pay loan in semi-annual equal installments with the first installement beginning 5 years after receiving loan. He has just cleared the loan now. What amount did he pay in each installement?
[IOE, 2062]

Soln :→

Given,

$P = Rs\ 500000$, $i = 11\%$ per year & compounded quarterly.

$N = 20$ years, $A = ?$



quarterly interest rate,

$i_q = \dfrac{11\%}{4} = 2.75\%$

semi-annual interest rate,

$i_{semi} = (1 + i_q)^2 - 1$

$= (1 + 0.0275)^2 - 1 = 5.57\%$

Contd.

Note

Trick ठुलो [year > semi > quarter > monthly] सानो

[For compounding] **Note :-**

Rule 1:- ठुलो compounding period लाई सानोमा ल्याइने,

e.g.:- $i_{quarter} = \frac{i_{year}}{4}$ ← rate per quarter, $i_{semi} = \frac{i_{year}}{2}$ ← rate per semi.

$i_{quarter} = \frac{i_{semi}}{2}$ ← rate per quarter

Rule 2:→ तर, सानो compounding period लाई ठुलो बनाउदा,

e.g.:- $i_{semi} = (1 + i_{quarter})^2 - 1$

$i_{year} = (1 + i_{month})^{12} - 1$

P.20 Imp

[For withdraw] * withdraw

Rule 1:→ if i = interest rate per year and withdrawal is monthly.

$\therefore i_{monthly} = (1 + i_{yearly})^{1/12} - 1$ ← rate per month.

* * * *

(above Q.8 is contd.)

Now,

Using single payment compound amount factor,

$F = 500000 (F/P, 5.57\%, 40) \longrightarrow (1)$

Using uniform series compound amount factor,

$F = A(F/A, 5.57\%, 30) \longrightarrow (2)$

Equating eqn (1) & (2)

$$500000(1+0.057)^{40} = A\left[\frac{(1+0.057)^{30}-1}{0.057}\right]$$

$\therefore A = Rs\ 61217.3$ is semi annual payment.

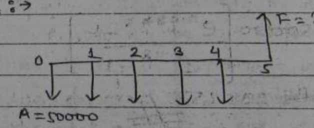
9> Calculate the future worth of following cash flows deposited at 8% compounded continuously for 5 years.

imp (i) Rs 50,000 at beginning of each year,

(ii) Rs 50000, at end of each year. [IOE, 2064]

Soln :→

(i)



Using continuous compounding compound amount factor,

$$F = A[F/A, r\%, N]\, e^r$$

$$= A\left[\frac{e^{rN}-1}{e^r - 1}\right] e^r$$

$$= 50000\left[\frac{e^{0.08 \times 5} - 1}{e^{0.08} - 1}\right] e^{0.08}$$

$= Rs\ 319850$ #

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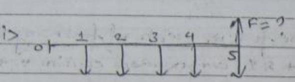
Note:→ If annuaity starts at begining.

$$F = A[F/A, i\%, N] * (1+i)$$

where,

$(1+i) = e^r$ for continuous compounding

<ii>



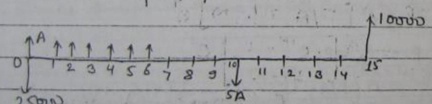
$$F = A[F/A, r\%, N] = 50000\left[\frac{e^{rN}-1}{e^r-1}\right]$$

$$= 50000\left[\frac{e^{0.08*5}-1}{e^{0.08}-1}\right]$$

$$= Rs\ 295258$$

#

10) Find value of A if $i = 15\%$ [IOE, 2065]



Soln :→

Converting all cash flows into ~~present~~ present value [using concept of equivalence].

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$$25000 + 5A(P/F, 15\%, 10) = A + A(P/A, 15\%, 6) - 10000(P/F, 15\%, 15)$$

or, $25000 + 5A(1+0.15)^{-10} = A + A\left[\frac{(1+0.15)^6-1}{0.15*(1+0.15)^6}\right] + 10000(1+0.15)^{-15}$

or, $25000 - 1.229 = A + 3.785A - 1.24A$

or, $23771 = 3.5445A$

$\therefore A = 6705.5$ #

11) A person is planning for his retired life and has 10 more years of service. He would like to deposit 30% of his salary, which is Rs 5000 at the end of first year and thereafter he wishes to deposit the amount with an annual increase of Rs 1000 for next 9 years with an interest rate of 15%. Find the total amount at end of 10^{th} year with the above series.

Soln :→

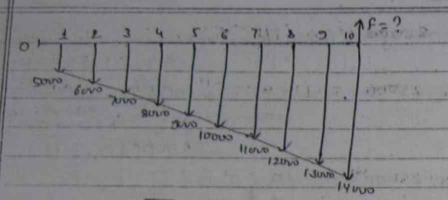
Given,

G = Rs 1000,

A = Rs 5000,

N = 10 years

$i = 15\%$, $F = ?$



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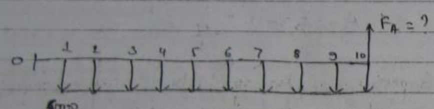

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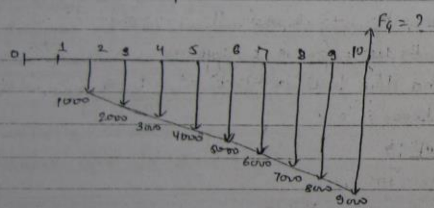


Using uniform series & gradient to compute future equivalent,

$F = F_A + F_G$

$F = (F/A, 15\%, 10) + G(F/G, i\%, 10)$

or, $F = A\left[\frac{(1+i)^N - 1}{i}\right] + \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$

$= 5000\left[\frac{(1+0.15)^{10} - 1}{0.15}\right] + \frac{1000}{0.15}\left[\frac{(1+0.15)^{10} - 1}{0.15}\right] - \frac{10 \times 1000}{0.15}$

$=$ Rs 170209.33 #

12) Suppose a cash flow schedule is given as follows:

End of year : 1 2 3 4

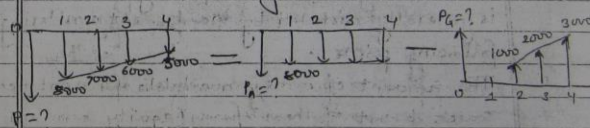
Payment (Rs) : 8000 7000 6000 5000

Calculate the equivalent present worth at $i = 15\%$, using gradient interest formulas.

Soln:-

Given,

$G = 1000$, $N = 4$ years, $i = 15\%$.



$P = P_A - P_G$

$= A(P/A, 15\%, 4) - G(P/G, 15\%, 4)$

$= 8000(2.855) - 1000(3.79)$

$=$ Rs 19050 #

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CHAPTER :- 3

(16 marks)

Basic Methodologies of Engineering Economic Analysis :→

3.1> Determining minm attractive (acceptable) Rate of Return [MARR] :→

(न्यूनतम आकर्षक प्रतिफल दर)

The minimum attractive rate of return (MARR) is the interest rate at which a firm can always earn or borrow money. MARR is the interest rate used in time value of money calculation. It is generally dictated by management considering the following points :-

↳ The amount of money available for investment source & cost of these funds (equity, borrowed funds) etc.

↳ The number of good project available for investment

↳ The amount of perceived risk associated with investment.

↳ The type of organization involved (government, public, private).

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↳ MARR is determined from the opportunity cost viewpoint, which results from capital rationing phenomenon.

Capital rationing refers to situation where funds available for capital investment are not sufficient to cover potentially acceptable projects. Opportunity cost is the best - rejected project or the worst accepted is the best opportunity foregone and its value is called opportunity cost.

* Example for determination of MARR :→

\# Suppose that you invested that amount (Rs 1650) in a saving account at 6% per year. Then you could have only Rs 10648 on January, 2002. What is the meaning of this 6% interest here? P.23

Soln :→

This is your opportunity cost if putting money in saving account was the best you can do at that time! So, in 1970 as long as you earn more than 6% interest in another investment, you will take that investment. Therefore, that 6% is viewed as a minimum attractive rate of return (or required rate of return).

P.24

3.2> Payback Period Method :→

The payback period method calculates the number of years required for positive cash flows to just equal the total initial investment (I).

Hence, this method screens projects on basis of how long it takes for net receipts to equal investment. But it does not account for savings after payback period generated by the project. A low valued payback period is considered for better project when choosing among mutually exclusive projects.

On the basis of way to compute, payback period method can be classified into 2 types:-

(a) Simple Payback Period :→

Simple payback indicates the required time period to break even on an investment without considering time value of money. It is computed as follows,

$$\text{Simple Payback Period (SP)} = \frac{\text{Initial Investment}}{\text{Net cash inflow per period}}$$

Mathematically,

$$\text{Simple payback period } (\theta) = \sum_{K=1}^{\theta} (R_K - E_K) - I \geq 0$$

When $\theta = N$, salvage value is included in the determination of payback period.

Advantages :→

(i) Easy to calculate.
(ii) It is interpreted in terms of time.
(iii) It does not require any assumption about the project in terms of timing like useful life, interest rate etc.

Disadvantages :→

(i) Fails to measure profitability.
(ii) Ignores time value of money.
(iii) It does not account for savings after payback period.

(b) Discounted Payback Period :→

This method includes time value of money for determining payback period. Hence, it is defined as the number of years required to recover the investment from discounting cash flow, i.e., considering time value of money.

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Steps :→

i> Discount each of future cash flows into present.

ii> Calculate the required number of years to recapture initial investment.

It is computed as,

Discounted payback period $(\theta') = \sum_{K=1}^{\theta'} (R_K - E_K)(P/F, i\%, K) - I \geq 0$

Advantages

↳ considers time value of money.

↳ Considers riskiness of projects cash flow (through cost of capital).

Disadvantage :→

↳ No concrete decision criteria that indicate whether the investment increases the firm's value.

↳ Requires an estimate of the cost of capital in order to calculate the payback.

↳ Ignores cash flows beyond the discounted payback period.

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Examples :→ (Ef 3.2)

1> Calculate simple and discounted payback periods from the given cash flows of a project when MARR is 20%.

End of period	Net cash flow (Rs)
0	− 25000
1	+ 8000
2	+ 8000
3	+ 8000
4	+ 8000
5	+ 13000

Soln

i> Simple Payback Period:

End of period	Net cash flows (Rs)	Cumulative Cash flow (Rs)
0	− 25000	− 25000
1	+ 8000	− 17000
2	+ 8000	− 9000
3	+ 8000	− 1000
4	+ 8000	+ 7000
5	+ 13000	+ 20000

Here, cumulative cash flow turns to positive in period 4. Therefore, payback period lies between period 3 & 4.

By interpolating we get payback period.

$$\theta = 3 + \frac{1000}{8000} = 3.125 \text{ periods.}$$

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(ii) Discounted Payback Period:

End of Period	Net Cash flow (Rs)	Discounted C.F into Present @ 20%. (Rs) = (net C.F) * $(1+20\%)^{-N}$	Cumulative Cash flow (Rs)
0	−25000	$-25000 \times (1+0.2)^0 = -25000$	−25000
1	+8000	$8000 * (1+0.2)^{-1} = 6667$	−18333
2	+8000	$8000 * (1+0.2)^{-2} = 5556$	−12777
3	+8000	$8000 * (1+0.2)^{-3} = 4630$	−8147
4	+8000	$8000 * (1+0.2)^{-4} = 3858$	−4289
5	+13000	$13000 * (1+0.2)^{-5} = 5224$	+935

Here, cumulative cash flow turns to positive in period 5. Therefore, payback period lies betn period 4 & 5. By interpolating, we get the required payback period.

$$\therefore \theta' = 4 + \frac{4289}{5224} = 4.82 \text{ periods.}$$

3.3> Equivalent Worth Methods (Profitability)

Equivalent worth methods convert all cash flows into equivalent worth at some points of time (present or annual or future) by using an interest rate equal to MARR.

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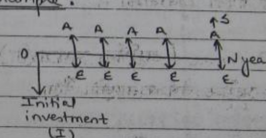
3.3.1> Present Worth Method (PWM):→

Net present worth or Net Present Value (NPV) of a given series of cash flow is the equivalent values of cash flows at the end of year zero (i.e. beginning of year 1). In other words, how much money we have to set aside to provide for future cash flows.

NPW = Equivalent Present Worth of future cash flow — Initial Investment.

Example:



$$\therefore PW = A_{\text{at present}} - E_{\text{at present}} - I_{\text{at present}} + S_{\text{at present}}$$

["सबै Cash flow लाई 'P' मा ल्याउने i.e. P = ?"]

annual revenue / also annual but expenses / as final value

$$\therefore PW = A(P/A, i\%, N) - E(P/A, i\%, N) - I + S(P/F, i\%, N)$$

already at present

$$\therefore PW(i\%) = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] - E\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] - I + S\left[(1+i)^{-N}\right]$$

OR

$$\therefore PW(i\%) = (A-E)\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] - I + S\left[(1+i)^{-N}\right]$$ #

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Decission Rule :

If $PW(i\%) > 0$, accept the project.
If $PW(i\%) = 0$, remain indifferent.
If $PW(i\%) < 0$, reject the project.

Note :→

I = सुरुमा गरेको लगानी
e.g: नयाँ गाडी किनेको

A = Annual Revenue (वार्षिक आम्दानी)
eg:- गाडीले प्रत्येक वर्ष कमाएको पैसा

E = Annual Expenses (वार्षिक खर्च)
eg:- गाडी बनाउदा लागेको खर्चहरु

S = Salvage value (कबाडी मुल्य)
e.g: गाडी बिग्रेर बेच्दा आएको मुल्य

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3.3.2) Annual Worth Method (AWM) :-

Annual worth of a project is a uniform series of amount for a stated period. It provides the basis for measuring investment worth into a series of equal payements at end of each period. AW of a project is its annual equivalent receipts (A) minus annual equivalent expences (E) less its annual equivalent capital recovery (CR) with given MARR.

$$\therefore AW(i\%) = A_{\text{at annual}} - E_{\text{at annual}} - CR_{\text{at annual}}$$

where, $C.R = I - S$

$$\therefore AW(i\%) = A_{\text{at annual}} - E_{\text{at annual}} - I_{\text{at annual}} + S_{\text{at annual}}$$

$$AW(i\%) = A - E - I(A/P, i\%, N) + S(A/F, i\%, N)$$

already in Annual form — present मा E annual मा लाने — future मा E annual मा लाने

$$\therefore AW(i\%) = A - E - I\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] + S\left[\frac{i}{(1+i)^N - 1}\right]$$

Decision Rule :-

If $AW(i\%) > 0$, accept the project.
If $AW(i\%) = 0$, remain indifferent.
If $AW(i\%) < 0$, reject project.

Note :-

Capital Recovery (C.R)

When only cost are involved, the AW method is sometimes called annual equivalent cost method. In this case, two types of costs are involved i.e. operating and capital cost. Operating cost (labours & raw materials etc) is recurred over life of project and they are estimated on annual basis where as capital costs (purchasing assets or establishing company etc) tend to be one time cost. So, for purpose of annual equivalent cost analysis this onetime cost (capital cost) must be translated into its annual equivalent over life of project. This annual equivalent of capital cost is given a special name: Capital Recovery Cost designated as CR(i%). It covers depreciation & interest on invested capital.

Two general monetary transactions are associated with purchase and retirement of capital asset. Initial investment cost (I) & its salvage value (S).

$$\therefore CR(i) = I(A/P, I\%, N)$$

3.3.3> Future Worth Method (FWM):-

Future worth of a project is equivalent worth of all cash flows at end of study period. Hence according to this method all cash inflows and outflows are compounded forward to a reference point in time called future with a given MARR to obtain the future worth because of primary objective of all time value of money methods is to maximize future wealth of owner, FW criterian has become more popular in recent years.

$$\therefore FW(i\%) = \underset{\text{future}}{A_{at}} - \underset{\text{future}}{E_{at}} - \underset{\text{future}}{I_{at}} + \underset{\substack{\uparrow \\ \text{already at future.}}}{S}$$

$$= A(F/A, i\%, N) - E(F/A, i\%, N) - I(F/P, i\%, N) + S$$

$$= (A-E)(F/A, i\%, N) - I(F/P, i\%, N) + S$$

$$= (A-E)\left[\frac{(1+i)^N - 1}{i}\right] - I\left[(1+i)^N\right] + S$$

#

Decision Rule:→

If FW(i%) > 0, accept project

If FW(i%) = 0, indifferent.

If FW(i%) < 0, reject project.

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Examples :→ (cf. 3.3)

1> A new computer costing Rs 100000 is estimated to have life of 10 years and expected annual revenue Rs 20,000 with annual cost Rs 5000. Determine investment decision to this computer if salvage value is Rs 25000 and MARR is 10% per year. Also make CFD. Use (i) PW, (ii) AW, (iii) FW methods.

Soln :→

Given,

$I = 100000$

$A = 20000$

$E = 5000$

$S = 25000$

$N = 10$ years

MARR = 10% per year.

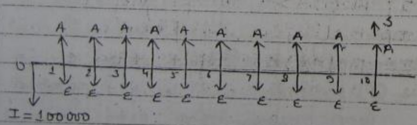

Fig :- CFD

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(i) Using PWM,

$$PW(10\%) = -100000 + (20000 - 5000)[P/A, 10\%, 10] + 25000[P/F, 10\%, 10]$$

$$= -100000 + 15000\left[\frac{(1+0.1)^{10} - 1}{0.1(1+0.1)^{10}}\right] + 25000(1+0.1)^{-10}$$

$$= \text{Rs } 1807$$

Since, PW(10%) > 0, Project is acceptable.

(ii) Using Annual Worth formulation (AWM):

$$AW(10\%) = -100000[A/P, 10\%, 10] + (20000 - 5000) + 25000[A/F, 10\%, 10]$$

$$= -100000\left[\frac{0.1(1+0.1)^{10}}{(1+0.1)^{10} - 1}\right] + 15000 + 25000\left[\frac{0.1}{(1+0.1)^{10} - 1}\right]$$

$=$

Since,

(iii) Using FWM,

$$FW(10\%) = -100000[F/P, 10\%, 10] + (20000 - 5000)[F/A, 10\%, 10] + 25000$$

$$= -100000\left[(1+i)^N\right] + 15000\left[\frac{(1+i)^N - 1}{i}\right]$$

$$= -100000\left[(1+0.1)^{10}\right] + 15000\left[\frac{(1+0.1)^{10} - 1}{0.2}\right] + 25000$$

$=$

Since,

P.29

P.30

Imp 2) National College needs an equipment costing of Rs 500000 and its salvage value will be Rs 50000 at end of 5 years. But the supplier gives another option to provide same equipment on hire Rs 60,000 per year for 5 years. Determine which option is economically beneficial for the college when MARR = 10%.

Soln ⇒

PW for purchasing case:

$PW(10\%) = -500000 + 50000\,[P/F, 10\%, 5)$

$= -500000 + 31046$

$= -Rs\ 468954$

∴ PW of purchasing cost = Rs 468954.

PW for hiring case:-

$PW(10\%) = -60000\,[P/A, 10\%, 5]$

$= -60000\left[\dfrac{(1+i)^n - 1}{i(1+i)^n}\right]$

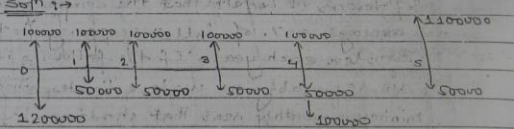
$= -Rs\ 227400$

∴ PW of hiring cost = Rs 227400

Since, PW of hiring cost is less than PW of purchasing cost ∴ Equipment hire option is beneficial for the college.

3) Suppose you have purchased a new car 5 years ago at Rs 1200000. The car needed Rs 50000 to maintain annual basis. At the end of 3rd year Rs 100000 was extra spent for air condition set-up. Now, suppose you wants to sell it for Rs 1100000 & you are receiving rent amount of Rs 100000 per year at beginning of each year from car. Evaluate your investment on car by AW method when MARR is 12%.

Soln :→



Imp

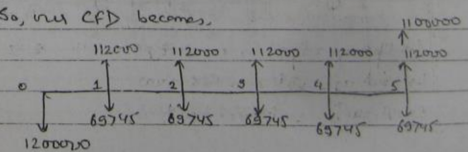
Annual Revenue (A) = 100000 (1 + i) ⇐ ∵ series start from 0th year

∴ A = Rs 112000

Annual expences (E) = [50000 + {100000 (P/F, 12%, 3)}] (A/P, 12%, 5)

∴ E = Rs 69745

So, our CFD becomes,



$\therefore AW(12\%) = -1200000\,(A/P, 12\%, 5] + 112000$
$\quad - 69745 + 1100000\,[A/F, 12\%, 5]$
$\quad = -Rs\ 117485.$

Since, $AW(12\%) < 0$, investment on car is not beneficial.

4> An investment company is considering building with a 25-unit apartment complex in growing town. Because of long term growth potential of town, it is felt that the company could average 90% of full occupancy for the complex each year. If the following items are reasonably accurate estimates, what is minimum monthly rent that should be charged if a 12% MARR is desired ?

Use AW method.

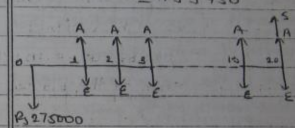
↳ Land investment cost = Rs 50000.
↳ Building investment cost = Rs 225000.
↳ Project Life = 20 years.
↳ Property tax & insurance per year = 10% of total investment.
↳ Upkeep expenses per unit per month = Rs 35
↳ Salvage value = Rs 50000.
↳ Rent per unit per month = ?

Soln :→

Total initial investment cost.
$= Rs\ 50000 + Rs\ 225000 = Rs\ 275000$

Taxes & insurance per year,
$= 10\% * Rs\ 275000$
$= Rs\ 27500$

Upkeep expenses per year.
$= Rs\ 35 * 25 * 12 * 0.9$
$= Rs\ 9450$



A = (reqd) revenue per yr.
E = annual expenses
$= Rs\ 27500 + Rs\ 9450$
$= Rs\ 36950$
S = Rs 50000

We know,

$AW(12\%) = -275000\,[A/P, 12\%, 20] + A - E + 50000\,[A/F, 12\%, 20]$

$= -275000\left[\dfrac{(1+0.12)^{20} * 0.12}{(1+0.12)^{20} - 1}\right] + A - 36950 + 50000\left[\dfrac{0.12}{(1+0.12)^{20} - 1}\right]$

$AW(12\%) = A - 36950 - 36122.72$

For no loss & no gain condn, i.e, to accept project marginally,

$AW = 0 \Rightarrow 0 = A - 36950 - 36122.72$

$\therefore A = Rs\ 73072.72$

The minm monthly rent = 73072.72 / (25 * 0.9 * 12)
$= Rs\ 270.639$ #

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3.4> Rate of Return Methods :→

Rate of return is defined as an annual rate of profit resulting from an investment.

It is especially relevant to evaluate mutually exclusive alternatives.

3.4.1> Internal Rate of Return (IRR) :→

IRR is that interest rate (return rate) which equates the equivalent worth of an alternative's cash inflows to equivalent worth of cash outflows.

In other words, IRR is the break-even interest rate at which equivalent worth of the project's cash flows is zero.

According to this, we can compute equivalent worth with different ways [PW or AW or FW]. IRR is term used for rate of return that stresses interest earned by project that is internally invested.

* **Methods of calculating IRR :→**

Step 1 :→ Develop an equation for equivalent worth of any point of time [present or future or annual] indicating rate of interest by i%. This value is to be found out.

Step 2 :- Equate developed equation as to zero.

Step 3 :- Solve it to get value of i% which would be IRR.

Imp **Note :→**

3 ways to calculate IRR
- 1> Direct Solution methods
- 2> Trial & Error Method
- 3> Computer Solution Method.

Decision Rule :→

If IRR > MARR, accept project

If IRR = MARR, remain indifferent.

If IRR < MARR, reject project.

* **Drawbacks of IRR :→** (**Note :-** Read after doing Examples for better understand).

(1) The IRR method is based on assumption that the recovered funds, if not consumed in each time period, are reinvested at i% rather than at MARR. This is not always practical.

<2> IRR method involves linear interpolation of non-linear function & when solved manually by trial and error method, may not give accurate result and it is more time consuming.

<3> There are situations in which its ~~no~~ iterative calculation process fails to produce a solution.

<4> When algebric sign of cash flow changes more than one in series, it is possible to obtain the multiple rates of return.

e.g.:-

End of year	Net cash flow
0	−1000
1	+2300
2	−1320

From the above cash flow pattern, we find IRR = 10% & 20%, but actually both of them may not be correct.

<5> IRR method doesnot consider scale of investment. Therefore, it can be misleading when choosing between mutually enclusive projects that have substantially different outlays.

3.4.2> External Rate of Return:-

ERR (i'%) is unique rate of return for a project that assumes that net postive cash flows, which represents money not immediately needed by project are reinvested at reinvestment rate %. The investment rate depends upon the market rate available for investments.

* Steps of ERR calculation :→

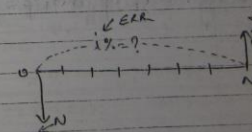
Steps 1 :→ All cash outflows are discounted to period zero (the present) at ∈% per compounding period.

Step 2 :→ All cash inflows are compounded to period N at ∈%.

Step 3 :→ ERR is interest rate that equivalence betn two quantities.

i.e.,

$$\sum_{k=0}^{N} A_k[F/P, \in\%, N-k]$$



$$\sum_{k=0}^{N} E_k(P/F, \in\%, k)$$

A_k = Revenues for k^{th} year,

E_k = Expenses for k^{th} year

∈ = External reinvestment rate per period.

Mathematically,

ERR is $i'\%$ at which,

$$\left[\sum_{k=0}^{N} E_k(P/F, \epsilon\%, k)\right][F/P, i'\%, N] = \sum_{k=0}^{N} A_k[F/P, \epsilon\%, N-k]$$

Decision

If ERR > MARR, accept project.

If ERR = MARR, remain indifferent.

If ERR < MARR, reject the project.

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* **Advantages of ERR over IRR :→**

↳ It does not need trail and error process to solve for $i'\%$,

↳ There is no possibility of multiple rates of return

Examples : (of 3.4)

1) "IRR using direct Solution Method".

Q.) Consider 2 investment projects with following cash flow transactions. Compute rate of return for each project.

Period (n)	Project 1	Project 2
0	−Rs 1000	−Rs 2000
1	Rs 0	Rs 1300
2	Rs 0	Rs 1500
3	Rs 0	
4	Rs 1500	

Soln :→

Project 1

$1000 = 1500(P/F, i\%, 4)$

$1000 = 1500(1+i)^{-4}$

$0.6667 = (1+i)^{-4}$

$\frac{\ln 0.6667}{-4} = \ln(1+i)$

$0.101365 = \ln(1+i)$

$\therefore i = 10.67\%$

Project 2

$0 = PW(i) = -2000 + \frac{1300}{(1+i)} + \frac{1500}{(1+i)^2}$

let, $x = \frac{1}{1+i}$, then

$0 = PW(i) = -2000 + 1300x + 1500x^2$

solve for x,

$x = 0.8$ or -1.667

Solving for i yields,

$0.8 = \frac{1}{1+i} \Rightarrow i = 25\%$

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or, $-1.667 = \frac{1}{1+i} \Rightarrow i = -160\%$

Since, $-100\% < i < \infty$

project's $i = 25\%$ #

Note:
We don't use this method generally if question is of 8 marks or greater.

2) "IRR using Trial & Error Method" + "Drawing unrecovered balance":

(i) Compute IRR for following project.
(ii) Also show unrecovered investment balance in graphical and tabular form.

Initial investment (I) = Rs 25000.
Net annual Revenue (A − E) = Rs 8000
Salvage Value (S) = Rs 5000
Useful life = 5 years
MARR = 20%

Soln :→

(i) By writting an equation with given information for net PW and setting as to zero,

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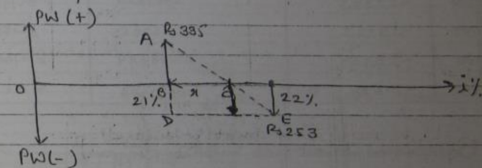
$-25000 + 8000[P/A, i'\%, 5] + 5000[P/F, i'\%, 5] = 0$

Put $i'\% = 0$, $PW(0\%) = Rs\ 20,000$
Put $i'\% = 10$, $PW(10\%) = Rs\ 8431$
Put $i'\% = 20$, $PW(20\%) = Rs\ 934$
Put $i'\% = 21$, $PW(21\%) = Rs\ 335$
Put $i'\% = 22$, $PW(22\%) = Rs\ -253$

Since, we have both +ve & −ve PW, we can solve for $i'\%$ as,



Using equilateral triangle ABC & ADE,

$$\frac{BC}{DE} = \frac{AB}{AD}$$

or, $$\frac{x\%}{22\% - 21\%} = \frac{335}{335 + 253}$$

$\therefore x\% = 0.57\%$

Then, $i'\% = 21\% + x\% = 21.57\%$

Since, IRR [$i'\% = 21.57\%$] > MARR [20%]

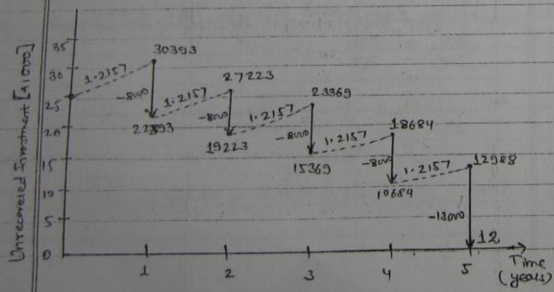
Project is accepted.

(ii) ⇒

Unrecovered investment calculation [in tabular-form]

Year	Cash flow	Unrecovered Investment Beginning of year	Unrecovered Investment End of year	Unrecovered Investment -(A-E) at end of years
0	-25000	—	$-25000(1+0.2157)^0 = -25000$	-25000
1	8000	-25000	= -30393	(-30393+8000) = -22393
2	8000	-22393	= -27223	= -19223
3	8000	-19223	= -23369	= -15369
4	8000	-15369	= -18684	= -10684
5	13000	-10684	= -12988	= 12

Unrecovered Investment balance diagram,



3> "IRR using Computer Solution Method".

#.

I = Rs 25000, S = Rs 5000 after 5 years,

Net revenue = Rs 8000 per year, MARR = 20%.

Is investment good.

Soln :→

By writing an eqn. with given information for net PW and setting at to zero.

i.e, $PW(i'\%) = 0$

or, $\left[8000(P/A, i'\%, N)\right] + \left[5000(P/F, i\%, N)\right] - 25000 = 0$

or, $8000\left[\frac{(1+i')^5 - 1}{(1+i')^5 i'}\right] + 5000(1+i')^5 - 25000 = 0$

or, $8\left[\frac{(1+i')^5 - 1}{(1+i')^5 i'}\right] + 5(1+i')^5 - 25 = 0$

Write equation in calculator as,

$8\left[\frac{(1+x)^5 - 1}{(1+x)^5 * x}\right] + 5(1+x)^5 - 25 = 0$

Press SHIFT → CALC

It may take some-time to calculate. The value in the calculator gives the value of 'x' & th is the IRR.

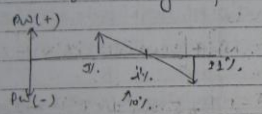
Trick ⇒

Note :→

In exam first use computer solution method to calculate actual IRR ⇒ eg:- $i' = 10\%$ [answer], but for obtaining full marks you should do Trial & error method [only for questions of 8marks or above], so take trial points one step below and one step above the IRR,

e.g: PW(11%) & PW(9%)

[when using compute method $i' = 10\%$]



(4) Consider following cash flow of project

Initial investment (I) = Rs 25000.

Net annual revenue (A−E) = Rs 8000.

Salvage value (S) = Rs 5000

Useful life (N) = 5 years

Suppose, ϵ = MARR = 20% per year, what is projects ERR and is project accepted?

Sol^n :→

Discounting all cash outflows to time zero at 20%. = Rs 25000

Compounding all cash inflows to year 5 at 20%.

= Rs [8000 (F/A, 20%, 5) + 5000]

= Rs 64532.8

Establishing the equivalence between 2 quantities,

$25000\,(F/P, i'\%, 5) = 64532.8$

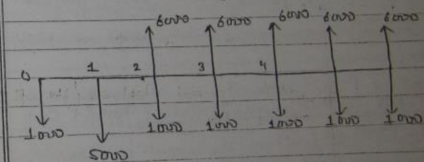
$(1+i')^5 = 2.5813$

$\therefore\ i' = 20.88\%$.

i.e, ERR = 20.88%.

Since, ERR (20.88%) > MARR (20%), project is accepted.

(5) Consider the following cash flow of project.



Calculate ERR of project if MARR = 20% and reinvestment rate $\epsilon\%$ = 15%. Is project accepted?

Sol^n :-

Discounting all cash outflows to time zero at 15%.

= 1000 + 5000 (P/F, 15%, 1)

$= 1000 + 5000\left[(1+0.15)^{1}\right]$ = Rs 5347.82

Compounding all cash inflows to year 6 at 15%,

$= 5000\,(F/A, 15\%, 5)$

$= 5000\left[\dfrac{(1.15)^2 - 1}{0.15}\right]$

$=$ Rs 33711.90

Establishing equivalence between two eqn.

$5347.82\,[F/P, i', 6] = 33711.90$

or, $5347.82\,(1+i')^6 = 33711.90$

$\therefore i' = 1.359 - 1$

$= 35.91\%$.

i.e, ERR $= 35.91\%$.

Since, ERR $(35.91\%) >$ MARR (20%), project is accepted.

3.5> Public Sector Economic Analysis [Benefit Cost Ratio Method] :→

Public sector project are owned, used & financed by citizenry of any government level. Public sector projects have a primary purpose to provide services to citizenry for public good at no profit. Areas such as health, safety, economic welfare, and utilities comprise a of alternatives that require engineering economic analysis.

In public sector benefit cost ratio method is used for economic analysis. Benefit - cost relation is a special tool of cost benefit analysis (CBA) and it is used to evaluate public sector projects, although it can be extended to any private sector projects. As name implies B-C relation, it involves the calculation of a ratio of benefits to costs [B/C ratio].

Two commonly used formulation B/C ratio are:→

(1) Conventional B/C ratio:→

It is the ratio of gross benefits to costs and expressed as,

(a) with PW formulation

$$B/C \text{ ratio} = \frac{PW(B)}{PW(I) - PW(S) + PW(O\&M)}$$

(b) with FW formulation

$$B/C \text{ ratio} = \frac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$$

(c) with AW formulation

$$B/C \text{ ratio} = \frac{AW(B)}{AW(I) - AW(S) + AW(O\&M)} = \frac{AW(B)}{CR + AW(O\&M)}$$

(2) Modified B/C ratio :→

It is ratio of net benefits to costs and expressed as,

(a) with PW formulation

$$B/C \text{ ratio} = \frac{PW(B) - PW(O\&M)}{PW(I) - PW(S)}$$

(b) with FW formulation

$$B/C \text{ ratio} = \frac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$$

(c) with AW formulation

$$B/C \text{ ratio} = \frac{AW(B) - AW(O\&M)}{AW(I) - AW(S)} = \frac{AW(B) - AW(O\&M)}{CR}$$

where,

B = Benefits of proposed project.

O&M = Operation & maintenance cost of proposal project.

S = Salvage value of proposed project.

I = initial investment of proposed project.

CR = Capital Recovery amount.

Decision :→

If B/C ratio > 1, accept the project.

If B/C ratio = 1, remain indifferent.

If B/C ratio < 1, reject the project.

Examples :→ (of 3.5)

(1) Find both types of B/C ratio using (i) PW (ii) AW method

Initial investment = Rs 20000

Revenue / year = Rs 10000

Expenses / year = Rs 4400

Salvage value = Rs 4000

Useful life = 5 years.

MARR = 8%.

Soln :→

(i) Using PW method :-

$I = PW(I) = 20000$

$$PW(B) = 10000(P/A, 8\%, 5) = 10000\left[\frac{(1.08)^5 - 1}{(1.08)^5 \times 0.08}\right] = 39927$$

$$PW(S) = 4000(P/F, 8\%, 5) = 4000\left[\frac{1}{(1.08)^5}\right] = 2722.33$$

$$PW(O\&M) = 4400(P/A, 8\%, 5) = 4400\left[\frac{(1.08)^5 - 1}{(1.08)^5 \times 0.08}\right] = 17567.9$$

$$\text{Conventional } B/C \text{ ratio} = \frac{PW(B)}{I - PW(S) + PW(O\&M)}$$

$$= \frac{39927}{20000 - 2722.33 + 17567.9}$$

$= 1.146 > 1$ [project accepted]

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Modified B/C ratio $= \dfrac{PW(B) - PW(O\&M)}{I - PW(S)}$

$= \dfrac{39927 - 17587.9}{20000 - 2722.33}$

$= 1.294 > 1$ [project accepted]

(ii) Using AW method:→

$CR = AW(I) - AW(S) = 20000[A/P, 8\%, 5] - 4400[A/F, 8\%, 5]$

$= Rs\ 4327$

$B = AW(B) = Rs\ 10000.$

$O\&M = AW(O\&M) = 4400$

Conventional B/C ratio $= \dfrac{AW(B)}{CR + AW(O\&M)}$

$= \dfrac{10000}{4327 + 4400}$

$= 1.146 > 1$ [project accepted]

Modified B/C ratio $= \dfrac{AW(B) - AW(O\&M)}{CR}$

$= \dfrac{10000 - 4400}{4327}$

$= 1.294 > 1$ [project accepted]

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(2) Find both types of B/C ratio using FW formulation.

Initial investment $(I) = Rs\ 200000$

Project life $(N) = 5$ years.

Salvage value $(S) = Rs\ 40000$

Annual O&M costs $= Rs\ 20000$

MARR $= 17\%$

Annual benefits $= Rs\ 50000$ at end of year 1 & increasing by Rs 25000 each year for 5 years.

Soln:→

$FW(I) = 200000(F/P, 17\%, 5) = Rs\ 438489.6$

$FW(S) = Rs\ 40000 = S$

$FW(O\&M) = Rs[20000(F/A, 17\%, 5)] = Rs\ 140288$

$FW(B) = 50000(F/A, 17\%, 5) + 25000(F/G, 17\%, 5)$

$= Rs\ 646955.3$

(Use gradient formula section: 2.5.5)

Conventional B/C ratio $= \dfrac{FW(B)}{FW(I) - FW(S) + FW(O\&M)}$

$= \dfrac{646955.3}{438489.6 - 40000 + 140288}$

$= 1.20 > 1$ project is accepted

Modified B/C ratio $= \dfrac{FW(B) - FW(O\&M)}{FW(I) - FW(S)}$

$= \dfrac{646955.3 - 140288}{438489.6 - 40000}$

$= 1.27 > 1$ (project accepted)

3> Compare below 2 projects by B/C ratio. Assume that the projects are independent investment opportunities and MARR is 10%.

	Project A	Project B
Initial Investment (Rs)	3500	5000
Annual benefits (Rs)	1900	2500
Annual O&M cost (Rs)	645	1383
Useful life	4 years	8 years
Salvage Value (Rs)	0	0

Soln :→

Using modified B/C ratio and AW formulation

	Project A	Project B
AW(B) − AW(O&M):		
A = 1900 − 645	= 1255	
B = 2500 − 1383		= 1117
CR costs		
A = 3500 (A/P, 10%, 4)	= 1104	
B = 5000 (A/P, 10%, 8)		= 937
Modified B/C ratio		
A = 1255/1104	= 1.14	
B = 1117/937		= 1.19

Project B is better of two, but both are satisfactory because B/C ratios are > 1.

3.6> Introduction to Life Cycle Costing :→

The technique used to estimate the total life cycle cost of a procurement is called life cycle costing. Alternatively, life cycle costing may simply be described as a procurement process which considers overall total cost (i.e., acquisition cost plus life cycle ownership cost) of an item.

Examples of an items on which life cycle cost analysis could be quite useful are aircrafts, computers, military systems, heavy industrial equipment, automobiles, ships, appliances, hospital facilities and buildings.

The application of life cycle costing is following an increasing trend in recent years. There could be several factors for this upward trend.

i> Rising inflation
ii> Budget limitations.
iii> Competition.
iv> Costly products.
v> Increasing maintenance cost.

Life cycle cost analysis are important in establishing, reducing and controlling cost and ha

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applications as,

(i) Choosing most beneficial procurement strategy.

(ii) Determining cost drivers.

(iii) Selecting among options.

(iv) Making source selection.

(v) Assessing new technology application

(vi) Providing objectives for program control.

(vii) Formulating contractor incentives.

(viii) Optimizing training needs.

(ix) Forecasting future budget needs.

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Thus, life cycle costing [LCC] of an asset is defined as, "the total cost throughout its life including planning, design, acquisition and support costs and any other costs directly attributable to owning or using the asset".

3.7> Introduction to financial & Economic Analysis :→

Let, market price of any object = Rs 40000.
Real price [shadow price] = Rs 35000
∴ Profit = Rs 5000

In financial analysis, $\underbrace{\text{market price}}_{\text{(देखीने/तिर्ने, लिने मूल्य)}}$ is used in calculation!

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But, In economic analysis, $\underbrace{\text{shadow price}}_{\text{(नदेखिने मूल्य)}}$ is used for calculation.

Since while calculating B/C Ratio, market price is used, it is financial method.

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CHAPTER :→ 4

Comparision of Alternatives

Most engineering projects can be accomplished by more than one feasible design alternatives when selection of one of these alternatives excludes the choice of any of others, the alternatives are called mutually exclusive. Typically, alternatives being considered require investment of different amounts of capital, and their annual revenues & costs may vary because different levels of investment normally produce varying economic outcomes, we must perform an analysis to determine which one of mutually exclusive alternatives is preferred & consequently how much capital should be invested.

Five of basic methods discussed in chapter-3 for analysing cash flows are used in analyses for comparing cash flows are used in analyses for comparing alternatives [PW, FW, AW, IRR & ERR methods].

The alternative that requires minimum investment of capital and produce satisfactory functional results will be choosen unless investmental (additional) capital associated with an alternative having a

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larger investment can be justified with respect to its incremental benefits.

* Analysis Period :→

The analysis period is a time span over which economic effects of an investment will be evaluated. The analysis period may also be called the study period or planning horizon. The length of the analysis period is determined by company policy, the service period, the useful life of shorter lived alternatives, the useful life of longer lived alternatives etc.

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* Useful life :→

The useful life of an asset is time period during which it is kept in productive use in a trade or business.

The useful lives of alternatives being compared, relative to selected study period, can involve two situations :-

4.1 → Case 1 : Useful lives are same for all alternatives & equal to study period.

4.2 → Case 2 : Useful lives are different among the alternatives and at least one does not matches the study period.

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4.1> Mutually Exclusive Project With Equal Life:→

4.1.1> Payback Period Method & Equal Worth Method:→

* Payback Period Method :-

In this method while comparing two or more projects for given study period, the project with least payback period is selected as best alternative.

Example:

From following four mutually exclusive projects. Recommend the best project using payback period method. The study period is 5 years and MARR = $\epsilon = 15\%$.

Projects	A	B	C	D
Initial Investment	500000	400000	700000	600000
Net annual Revenue	125000	110000	170000	135000

Salvage value is 20% of initial investment.

⇒ Sol^n :-

For project A,

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Period	Cash flow	PW of net cash flow ($i=15\%$)	Cumulative cash flow
0	−500000	−500000	−500000
1	125000	108700	−391300
2	125000	94512.5	−296787.5
3	125000	82187.5	−214600
4	125000	71475	−143125
5	125000 + 100000 (salvage value)	111870	−31225

Here, the cumulative balance doesn't changes into positive in 5 years & payback period is more than 5 years.

For project B,

Period	Cash flow	PW of net cash flow ($i=20\%$)	Cumulative cash flow
0	−400000	−400000	−400000
1	110000	95656	−304344
2	110000	83171	−221173
3	110000	72325	−148848
4	110000	62898	−85950
5	110000 + 80000	94468	18518

Here, cumulative balance turns positive in the year 5. So, payback period lies between year 4 & 5. By inter-polation we get,

$$= 4 + \frac{85950}{94468}$$

$$= 4.90 \text{ years.}$$

For Project C

Period	Cash flow	PW of net cash flow (i = 20%)	Cumulative cash flow
0	−700000	−700000	−700000
1	170000	147832	−552168
2	170000	128537	−423631
3	170000	111775	−311856
4	170000	97206	−214650
5	170000+140000	154	−60518

Here, payback period is more than 5 year

For project D

Period	Cash flow	PW of net cash flow (i = 20%)	Cumulative cash flow
0	−600000	−600000	−600000
1	135000	117396	−482604
2	135000	102073.5	−380530.5
3	130000	88762.5	−291768
4	135000	77193	−214575
5	135000+120000	126786	−37789

Here, payback period is more than 5 year.

Hence, project B is choosen as best alternatives which gives payback period of 4.90 years (min^{m}.)

* Equivalent Worth Method :→

In this method while comparing two or more projects for given study period, project with highest equivalent worth [PW or AW or FW] is selected as best alternatives.

Example:

Consider the mutually exclusive alternatives:

	Option 1	Option 2	Option 3
Investment cost (Rs)	269000	319000	330000
Annual net savings (Rs)	81500	88500	98300
Useful life (years)	5	5	5
Salvage value (Rs)	100000	120000	120000

Which option should be selected based on equivalent worth method at $i = 12\%$.

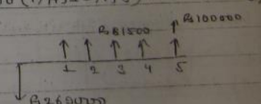
Soln :→

Since useful life of each alternatives is 5 years, simply calculate equivalent worth of each option. Using PW formulation:

Option 1

$PW_1 = -269000 + 81500\,(P/A, 12\%, 5) + 100000\,(P/F, 12\%, 5)$

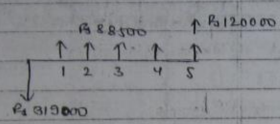
$= Rs\ 81532$



Option 2

$PW_2 = -319800 + 88500(P/A, 12\%, 5) + 120000(P/F, 12\%, 5)$

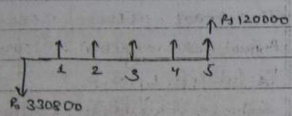
$= Rs\ 67314$



Option 3

$PW_3 = -330800 + 98300(P/A, 12\%, 5) + 120000(P/F, 12\%, 5)$

$= Rs\ 91640$



Here, $PW_3 > PW_1 > PW_2$

So, option 3 is most economical project.

Using FW formulation :-

$FW_1 = -269000(F/P, 12\%, 5) + 81500(F/A, 12\%, 5) + 100000$

$= -474070 + 517757 + 100000$

$= Rs\ 143687$

$FW_2 = -319000(F/P, 12\%, 5) + 88500(F/A, 12\%, 5) + 120000$

$= -562187 + 562227 + 120000$

$= Rs\ 120040$

$FW_3 = -330000(F/P, 12\%, 5) + 98300(F/A, 12\%, 5) + 120000$

$= -581559 + 624480 + 120000$

$= Rs\ 162921$

Here, $FW_3 > FW_1 > FW_2$

So, option 3 is most economical.

Using AW formulation :

$AW_1 = -269000(A/P, 12\%, 5) + 81500 + 100000(A/F, 12\%, 5)$

$= Rs\ 22620$

$AW_2 = -319000(A/P, 12\%, 5) + 88500 + 120000(A/F, 12\%, 5)$

$= Rs\ 18898$

$AW_3 = -330000(A/P, 12\%, 5) + 98300 + 120000(A/F, 12\%, 5)$

$= Rs\ 25646$

Here,

$AW_3 > AW_2 > AW_2$

So,

option 3 is most economical

4.1.2> Rate of Return Method & BC ratio method :→

Under equivalent worth method, mutually exclusive project with the highest worth figure was preferred as in 4.1.1.

Unfortunately, the same principle cannot be applied to IRR, ERR & BCR analysis. The project with highest IRR, ERR & BCR may not be the preferred alternative.

Examples :→

Let us consider two mutually exclusive alternatives with 1 year of service life.

Year (N)	Alternatives 1 (A_1)	Alternatives 2 (A_2)
0	- Rs 1000	- Rs 5000
1	Rs 2000	Rs 7000
IRR	100%	40%
BCR	1.82	1.27
PW (10%)	Rs 818	Rs 1364

We can see that, A_2 is preferred over A_1 by PW method.

A_1 is preferred over A_2 by IRR & BCR methods. This inconsistency is because equivalent worth method (AW, PW, FW) are absolute method where as IRR and ERR is relative (percentage) measure and cannot be applied in same way. For this purpose "Incremental Analysis" should be done.

* Incremental Analysis :→

It evaluates difference, or the 'increment' between two or more mutually exclusive alternatives. As we have learned, the PW and AW techniques can be used to do incremental analysis. When independent projects are evaluated, no incremental analysis is necessary between projects. Each project is evaluated separately from others and more than one can be selected.

* Steps for Incremental Analysis :→

1> Identify all the alternatives.

2> Compute the IRR / ERR / BCR of each alternatives. Any alternative with the IRR < MARR / ERR < MARR / BCR < 1 should be rejected.

Imp [Note: Use computer method to find IRR]

3> Order alternatives in increasing order of investment cost to ensure that the increments have cash flow

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corresponding to investments.

4> Establish a base alternatives:-

Alternative having least capital investment is established as the base alternative and should have been pre-qualified.

i.e, IRR > MARR / ERR > MARR / BCR > 1.

5> Perform an incremental analysis between base alternative and alternative with next higher initial cost. If the incremental IRR MARR / ERR MARR / BCR 1, reject base alternative and accept higher cost alternative and retain it as base alternative.

6> Select the next higher cost alternative and perform the incremental analysis until all the alternatives have been evaluated.

Note:-> Evaluation should always be done based on same study period for all alternatives.

Decision Rule:->

If, $IRR_{B-A} > MARR \,/\, ERR_{B-A} > MARR \,/\, BCR_{B-A} > 1$

select B [higher first cost alternatives]

If, $IRR_{B-A} = MARR \,|\, ERR_{B-A} = MARR \,|\, BCR_{B-A} = 1$

select either one

If, $IRR_{B-A} < MARR \,/\, ERR_{B-A} < MARR \,/\, BCR_{B-A} < 1$

select A [lower first cost alternatives]

From above (previous) example,

Year N	$A_2 - A_1$	Result
0	-Rs 4000	
1	Rs 5000	
IRR	25% > MARR	select A_2
BCR	1.14 > 1	select A_2

#

Examples:->

(1) The cash flows for 2 mutually exclusive alternatives are given as follows:

EOY	A	B
0	-Rs 3000	-Rs 12000
1	Rs 1350	Rs 4200
2	Rs 1800	Rs 6225
3	Rs 1500	Rs 6330

Which project would be selected based on IRR criterion, at MARR = 10%.

Soln:->

Calculating IRR of each alternatives,

$IRR_A = 25\% > 10\%$. [MARR]; accepted

$IRR_B = 17.43\% > 10\%$. [MARR]; accepted.

Performing incremental Analysis :→

Let us consider mutually exclusive alternatives as below :-

EOY	A	B	B − A
0	−Rs 3000	−Rs 12000	−Rs 9000 ← should be negative
1	Rs 1350	Rs 4200	Rs 2850
2	Rs 1800	Rs 6225	Rs 4425
3	Rs 1500	Rs 6330	Rs 4830
IRR	25%	17.43%	?

V^2 Imp

Note :- We always want first flow of the incremental cash flow series to be negative i.e at EOY = 0, difference both A & B should be −ve.

$$PW(i'\%)_{B-A} = 0$$

$$-9000 + 2850(P/F, i', 1) + 4425(P/F, i', 2) + 4830(P/F, i', 3) = 0$$

$$\therefore (i'\%)_{B-A} = 15\%$$

Here, $(i'\%)_{B-A} > 10\%$ (MARR)

Hence, option B is selected.

(2) Consider the following 3 set of mutually exclusive alternatives :-

End of year	Alternatives D_1	D_2	D_3
0	−Rs 2000	−Rs 1000	−Rs 3000
1	Rs 1500	Rs 800	Rs 1500
2	Rs 1000	Rs 500	Rs 2000
3	Rs 800	Rs 500	Rs 1000

Which project would you select based on IRR, ERR & BCR methods on incremental investment assuming that MARR = ε = 15% ?

Soln :→

(a) Calculating IRR of each project & compared with MARR.

(i) IRR of D_1,

$$PW = 0$$

$$-2000 + 1500(P/F, i', 1) + 1000(P/F, i', 2) + 800(P/F, i', 3) = 0$$

$$\Rightarrow -2000 + 1500 * \frac{1}{(1+i')^1} + 1000 * \frac{1}{(1+i')^2} + 800 * \frac{1}{(1+i')^3} = 0$$

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Note:→
Use computer method. Type below in calculator

i.e, $-2000 + \frac{1500}{(1+x)} + \frac{1000}{(1+x)^2} + \frac{800}{(1+x)^3} = 0$

$\boxed{\text{shift}}$ $\boxed{\text{Calc}}$ $\therefore x = 34.37\%$

$\therefore i' = 34.37\% > 15\%$ [MARR]; accepted.

ii> IRR of D_2,

PW = 0

$-1000 + 800(P/F, i', 1) + 500(P/F, i', 2) + 500(P/F, i', 3) = 0$

$\therefore i' = 40.76\% > 15\%$ [MARR]; accepted

iii> IRR of D_3,

PW = 0

$-3000 + 1500(P/F, i', 1) + 2000(P/F, i', 2) + 1000(P/F, i', 3) = 0$

$\therefore i' = 24.81\% > 15\%$ [MARR]; accepted.

Performing incremental IRR,

↳ select D_2 as base alternatives because it has lower initial cost (Rs 1000)

↳ Compare D_1 with base alternative D_2 which is next highest initial cost (Rs 2000).

↳ Calculate the incremental cost and incremental

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benefits $(D_1 - D_2)$.

↳ Calculate IRR of incremental cash flow $[D_1 - D_2]$

$-1000 + 700(P/F, i', 1) + 500(P/F, i', 2) + 300(P/F, i', 3) = 0$

$\therefore i' = 27.61\% > 15\%$ [MARR]

↳ Eliminate D_2 from consideration because alternative D_1 gives higher return. Again, (after eliminating D_2)

↳ Select D_1 as base alternative which initial cost is lower than next remaining alternative D_3.

↳ Compare D_1 with D_3 and compute incremental cash flow $[D_3 - D_1]$

↳ Calculating IRR of $D_3 - D_1$ [incremental cash flow]

$-1000 + 0(P/F, i', 1) + 1000(P/F, i', 2) + 200(P/F, i', 3) = 0$

$\therefore i' = 8.8\% < 15\%$ [MARR] [not accepted]

So, select D_1 #

OR In tabular form:-

EOY	D_2	$D_1 - D_2$	$D_3 - D_1$
0	-1000	-1000	-1000
1	800	700	0
2	500	500	1000
3	500	800	200
Incremental IRR	40.76%	27.61%	8.8%
Is increment accepted [Justified		Yes	No
Decision		Select D_1 Reject D_2	Reject D_3 Select D_1

Select D_1 as best alternative.

(b) Calculate ERR of each project using PW formulation

(i) ERR of D_1,

Compounding all cash inflow to future value,

$3000(F/P, 15\%, 2) + 1500(F/P, 15\%, 1) + 800$

$= Rs\ 3933.75$

Discounting all cash outflow to present value,

$= Rs\ 2000$

Making equivalence of two equation

$$2000(1+i')^3 = 3933.75$$

$\therefore i' = 25.28\% > MARR$ [acceptable]

(ii) ERR of D_2,

Compounding all cash inflow to future value,

$800(F/P, 15\%, 2) + 500(F/P, 15\%, 1) + 500$

$= Rs\ 2133$

Discounting all cash outflow to present value

$= Rs\ 1000$

Making equivalence of two equation

$$1000(1+i')^3 = 2133$$

$\therefore i' = 28.72\% > MARR$ (acceptable)

(iii) ERR of D_3,

Compounding all cash inflow to future value,

$1500(F,P,15\%,2) + 2000(F,P,15\%,1) + 1000$

$= Rs\ 5283.75$

Discounting all cash outflow to present value,

$= Rs\ 3000$

Making equivalence of two equation

$$3000(1+i')^3 = 5283.75$$

or, $(1+i') = 1.76$

$\therefore i' = 20.76\% > MARR$ (acceptable)

Performing the Incremental Analysis:

Incremental ERR

↳ select D_2 as base alternative (Lower initial cost)

↳ Compare D_2 with D_1 [next higher initial cost)

↳ calculate incremental cost & incremental benefits,

[$D_1 - D_2$]

↳ Calculate ERR of incremental cash flow,

$700(F/P, 15\%, 2) + 500(F,P,15\%,1) + 300 = 1000(1+i')^3$

$(1+i')^3 = 1.80075$

$\therefore i' = 21.66\% > MARR$ [acceptable)

Here, D_2 is being eliminated from consideration and again D_1 is taken as base alternative and compared with alternative D_3.

Calculate incremental cost & incremental benefits $D_3 - D_1$ and, calculating ERR of incremental cash flow,

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$0(F|P, 15\%, 2) + 1000(F|P, 15\%, 1) + 200 = 1000(1+i')^3$

$\therefore i' = 9.13\% < MARR$ [not acceptable]

Thus,

D_1 is selected as best alternative.

OR In tabular form,

EOY	$D_1 - D_2$ [D_2 as base]	$D_3 - D_1$ [D_1 as base]
0	−1000	−1000
1	700	0
2	500	1000
3	300	200
Incremental ERR	21.66%	9.13%
Is increment acceptable	Yes	NO
Decision	Select D_1 Reject D_2	Reject D_3 select D_1

So, select D_1 as best alternatives.

(c) Calculating BCR of each project using PW method,

(i) BCR of D_1 − PW (Benefits)/ PW(costs)

PW (Benefits) = $1500(P|F, 15\%, 1) + 1000(P|F, 15\%, 2) + 800(P|F, 15\%, 3)$

= Rs 2586.53

PW (costs) = Rs 2000

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$\therefore$ BCR of $D_1 = \frac{2586.5}{2000} = 1.293 > 1$ (acceptable)

(ii) BCR of D_2,

PW (benefits) = $800(P|F, 15\%, 1) + 500(P|F, 15\%, 2) + 500(P|F, 15\%, 3)$

= Rs 1402.48

PW(costs) = Rs 1000

$\therefore$ BCR of $D_2 = \frac{1402.48}{1000} = 1.402 > 1$ (acceptable)

(iii) BCR of D_3,

PW (Benefits) = $1500(P|F, 15\%, 1) + 2000(P|F, 15\%, 2) + 1000(P|F, 15\%, 3)$

= Rs 3474.1

PW (costs) = Rs 3000

$\therefore$ BCR of $D_3 = \frac{3474.1}{3000} = 1.15 > 1$ (acceptable)

Performing incremental BCR

Calculate the BCR of incremental cash flow [selecting D_2 as base alternative (having lower initial cost) & comparing D_2 with D_1 (having next initial cost)]

BCR of $D_1 - D_2 = \frac{\text{PW (benefits) of } D_1 - D_2}{\text{PW (cost) of } D_1 - D_2}$

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$$= \frac{700(P/F, 15\%, 1) + 500(P/F, 15\%, 2) + 300(P/F, 15\%, 3)}{1000}$$

$$= 1.18 > 1 \text{ (acceptable)}$$

So, D_1 is accepted and D_2 is rejected. Thus D_2 is eliminated from consideration and D_1 is taken as base which is now compared with alternative D_3.

$$\text{BCR of } D_3 - D_1 = \frac{0 + 1000(P/F, 15\%, 2) + 200(P/F, 15\%, 3)}{1000}$$

$$= 0.88 < 1 \text{ (reject)}$$

So, D_3 is rejected & D_1 is selected as best alternative.

OR In tabular form:

EOY	$D_1 - D_2$	$D_3 - D_1$
0	-Rs 1000	-Rs 1000
1	700	0
2	500	1000
3	300	200
Incremental BCR	1.18	0.88
Is increment justified	Yes	NO
Decision	select D_1 Reject D_2	Reject D_3 select D_1

Select D_1 as best alternative.

(3) An engineering firm is considering the following exclusive alternatives.

	Projects			
EOY	Alternative 1 (A_1)	A_2	A_3	A_4
0	- Rs 2500	-Rs 1200	-3600	-2000
1	1200	400	1700	800
2	1400	800	2000	700
3	1500	1000	1600	850

Which project would you select based on IRR method. Assuming MARR = 20% per year. P.53

Soln :→

Step 1 :- Calculating the IRR of each project

For A_1,

$$PW(i') = -2500 + 1200(P/F, i', 1) + 1400(P/F, i', 2) + 1500(P/F, i', 3)$$

Using Trial & Error [Note in exam used computer method]

$$i' = 28.19\%$$

For A_2,

$$PW(i') = -1200 + 400(P/F, i', 1) + 800(P/F, i', 2) + 1000(P/F, i', 3)$$

By trial & error,

$$i' = 31.84\%$$

For A_3,

$$PW(i') = -3600 + 1700(P/F, i', 1) + 2000(P/F, i', 2) + 1600(P/F, i', 3)$$

Solving, $PW(i') = 0$

$\therefore i' = 22.33\%$

For A_4,

$$PW(i') = -2000 + 800(P/F, i', 1) + 700(P/F, i', 2) + 850(P/F, i', 3)$$

Solving, $PW(i') = 0$

$\therefore i' = 8.43\%$

Step 2 :- Comparing with MARR = 20%

A_1, $i' = 28.19\% > MARR(20\%)$; accepted

A_2; $i' = 31.84\% > MARR(20\%)$; "

A_3, $i' = 22.33\% > MARR(20\%)$; "

A_4, $i' = 8.43\% > MARR(20\%)$; Rejected.

So, A_4 will be removed from consideration.

Step 3 :→ Taking A_2 as base alternative [having lowest investment]

EOY	$A_1 - A_2$	$A_3 - A_1$
0	−1300	−1100
1	800	500
2	600	600
3	500	100
Incremental IRR	23.87%	5.30%
Is increment justified	Yes	No
Decision	Select A_1 Reject A_2	Reject A_3 select A_1

Thus, A_1 is selected as best alternative.

4.2> Comparing Mutually Exclusive Projects With different Useful life :→

In 4.1 we assumed the simplest scenario possible when analyzing the mutually exclusive projects. The project had useful lives equal to each other and to required service period.

But in practice this is seldom case often projects lives do not match required analysis period and/or donot match each other. For example, two machines may perform exactly same function, but one lasts longer than the analysis period for which they are being considered. In this sections and examples, we will develop some techniques for dealing with those complications

4.2.1 ⟶ Repeatability Assumption

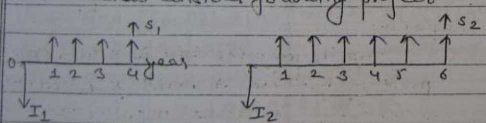
↳ Two alternatives having different useful life are changed into projects having same useful life by expanding their life upto least common year

↳ The study period is equal to least common multiple [LCM] of lives of alternatives.

↳ The economic consequences that are estimated to happen in an alternatives' initial life span will also happen in all succeeding life spans.

Example :→

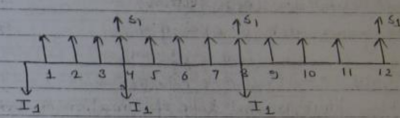
Let us consider following project.



Project A　　　　project B

Here, LCM of 4 & 6 is 12 years, so we assume study period as 12 years.

Project A is repeated 3 times,



Project B is repeated 2 times,

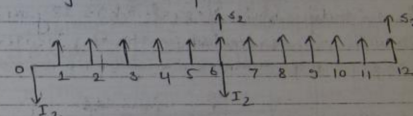


fig:- CFD for repeatability assumption.

Example:- (of 4.2.1)

(1) A project engineer with environment care is assigned to startup a new office in a city. Two lease options are available, each with a first cost, annual lease cost, and deposite return estimates shown below. Determine which lease option should be selected on the basis of a present worth, FW & AW comparision, if MARR = 15% per year.

	Location A	Location B
First cost (Rs)	-15000	-18000
Annual Operating Cost (Rs)	-3500	-3100
Deposit Returns (Rs)	1000	2000
Life (years)	6	9

<u>Sol^n</u> :→

Here, useful life of location A and B is 6 and 9 respectively.

Study period = LCM of 6 & 9 = 18 years

Location A is repeated for 3 times & location B is repeated for 2 times.

For location A :→ ≡

0 – 6	6 – 12	12 – 18
A_1	A_1	A_1

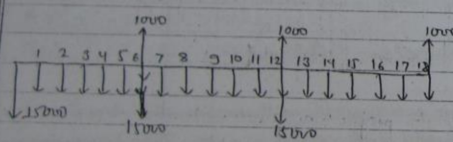


fig: CFD for location A

Using PW formulation:

$$PW_A(15\%) = -15000\left[1 + (P/F, 15\%, 6) + (P/F, 15\%, 12)\right]$$
$$+ 1000\left[(P/F, 15\%, 6) + (P/F, 15\%, 12) + (P/F, 15\%, 18)\right]$$
$$- 3500\left[(P/A, 15\%, 18)\right]$$
$$= -15000\left[1 + (1+0.15)^6 + (1+0.15)^{12}\right]$$
$$+ 1000\left[(1+0.15)^6 + (1+0.15)^{12} + (1+0.15)^{18}\right]$$
$$- 3500\left[\frac{(1+0.15)^{18} - 1}{(1+0.15)^{18} * 0.15}\right]$$
$$= -\text{Rs } 45036$$

Using FW formulation

$$FW_A(15\%) = -15000\left[(F/P, 15\%, 18) + (F/P, 15\%, 12) + (F/P, 15\%, 6)\right]$$
$$+ 1000\left[(F/P, 15\%, 6) + (F/P, 15\%, 12) + 1\right]$$
$$- 3500\left[(F/A, 15\%, 18)\right]$$
$$= \text{Rs } 557345.40$$

Using AW formulation

$$AW_A(15\%) = -15000\left[(A/P, 15\%, 6)\right] - 3500 + 1000\left[(A/F, 15\%, 6)\right]$$
$$= -15000 * 0.2642 - 3500 + 1000 * 0.1142$$
$$= -\text{Rs } 7349.80$$

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OR

$AW_A(15\%) = PW(15\%)\,[(A/P, 15\%, 18)]$

$= -45036 * 0.1632$

$= -Rs\ 7349.80$

For location B:- ≡

0	9	18
B_1	B_2	

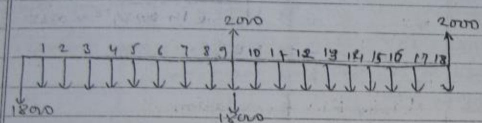

fig: CFD for location B

Using PW formulation:

$PW_B(15\%) = -18000 - 18000(P/F, 15\%, 9)$
$+ 2000(P/F, 15\%, 9) + 2000(P/F, 15\%, 18)$
$- 3100(P/A, 15\%, 18)$
$= -Rs\ 41384$

Using FW formulation.

$FW_B(15\%) = -18000\,(F/P, 15\%, 18) - 18000(F/P, 15\%, 9)$
$+ 2000(F/P, 15\%, 9) + 2000$
$- 3100\,(F/A, 15\%, 18)$
$= -Rs\ 512138.24$

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Using AW formulation,

$AW_B(15\%) = PW_B(15\%)\,(A/P, 15\%, 18)$
$= 41384 * 0.1632$
$= -Rs\ 6753.86$

Now,

$PW_B(15\%) > PW_A(15\%)$

$FW_B(15\%) > FW_A(15\%)$

$AW_B(15\%) > AW_A(15\%)$

So, select location B. #

4.2.2> Co-terminated Assumption:→

The co-terminated uses a finite and identical study period for all alternatives. This planning horizon combined with appropriate adjustments to the estimated cash flows plus the alternatives on a common and comparable basis. The planning horizon chuosen could be,

↳ Life of shorter lived alternative.
↳ Life of longer lived alternative.
↳ Less than shorter lived alternative.
↳ Greater than longer lived alternative.
↳ In between shortest and longest lived alternatives.

Two cases are involved in this assumption:→

(a) Useful life greater than study period.
(b) Useful life shorter than study period.

(a) Useful Life Greater than study Period:→

Projects lives rarely conveniently coincide with a firm's predetermined required analysis period; they are often too long or too short. The case of project lives that are too long is the easier one to address. A common instance of project lives that are longer than analysis period occurs in the construction industry, where a building project may have a relatively short completion time, but the equipment purchased has a much longer useful life.

Imp Two assumptions are considered:→

(i) Cash flow accumulated at end of useful life will be reinvested for extended periods.

(ii) Replacement / Re-investment is necessary for remaining period (study period - useful life) and economic consequences that are estimated to happen in an alternatives initial life span will also happen in all succeeding life spans. (As in repeatability assumption)

Examples

Consider the following mutually exclusive projects,

	A	B
Investment (Rs)	350000	500000
Annual Revenue (Rs)	190000	250000
Annual cost	64500	138300
Useful life	4 years	8 years = study period ⇓ ist question मा नदिए मो नै लिने
Salvage value	0	0

no adj

MARR = 10%. which alternative is more desirable based on co-terminated assumption?

Solⁿ:→

Taking analysis period as 8 years (the value should be taken in such a way that the study period is either equal to or greater than useful lives of all alternatives) If lesser useful life is taken than we have to shorten down the cash flow to end of study period and suitable market value should be assigned to the alternatives.

just equ

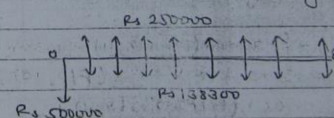

fig:- CFD of project B.

There is no adjustment required for alternative B. The adjustment is required in case of A which study period is 8 years greater than its useful life (4 yrs).

1st way :- [Considering assumption (i)]

no adjustment

$$FW_B(10\%) = -500000(F/P, 10\%, 8) + (2500000 - 198300)(F/A, 10\%, 8)$$

$$= -500000\left[(1+0.1)^8\right] + 111700\left[\frac{(1+0.1)^8 - 1}{0.1}\right]$$

$$= Rs\ 205594.2958$$

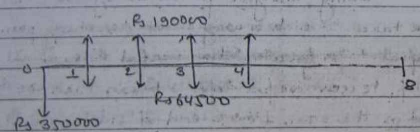

fig:- CFD of project A.

adjustment required

$$\Rightarrow FW_A(10\%) = \{-350000(F/P, 10\%, 4) + (190000 - 64500)(F/A, 10\%, 4)\}[F/P, 10\%, (8-4)]$$

$$= \left[-350000(0.1+1)^4 + 125500\left(\frac{(0.1+1)^4 - 1}{0.1}\right)\right] * (1+0.1)^4$$

$$= Rs\ 102502.37$$

Here, $FW_B(10\%) > FW_A(10\%)$

So, select alternative B. #

OR

2nd way :- [Considering assumption (ii)]

no adjustment

$$FW_B(10\%) = (\text{same as previous})$$

$$= Rs\ 205594.2958$$

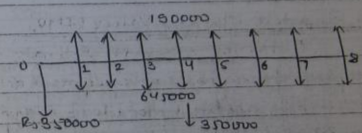

fig:- CFD of project A.

adjustment required.

$$FW_A(10\%) = -350000(F/P, 10\%, 8) - 350000(F/P, 10\%, 4) + (190000 - 64500)(F/A, 10\%, 8)$$

$$= Rs\ 172510$$

Here, $FW_B(10\%) > FW_A(10\%)$

So, select alternative B.

#

(b) Useful Life shorter than Study Period :→

When the project lives are shorter than required service period, we must consider how at the end of project lives, we will satisfy rest of required service period. The most common technique is to truncate the alternative at the end of study period using an estimated market value. This assumes that disposable assets will be sold at the end of study period at that value.

* The Imputed Market Value (IMV)

Obtaining a current estimate from the market place for a piece of equipment or another type of asset is preferred procedure in engineering practice when a market value at time T < useful life is required. This approach, however may not be feasible in some cases. For example, a type of asset may have low turnover in the market place and information for recent transactions is not available. Hence, it is sometimes necessary to estimate the market value for an asset without current and representative historical data.

The imputed market value technique which is sometimes called the implied market value can be used for this purpose as well as for comparision with market place values when current data are available. If an imputed market value is needed for a piece of equipment, say at end of the T < useful life, the estimate is calculated based on sum of two parts as follows:-

MV_T = [PW at end of year T of remaining capital recovery amounts] + [PW at end of year T of original market value at the end of useful life]

Examples:

(1) Use the imputed market value at end of year five if the useful life of alternative is 9 yrs, capital investment is Rs 47600, market value at end of useful life is Rs 5000 and MARR = 20%.

Soln :→

$PW(20\%)_{CR}$ = [47600 (A/P, 20%, 9) − 5000 (A/F, 20%, 9)] * (P/A, 20%, 4)

= Rs 29949.

Computing the PW at end of year five, based on original MV at end of useful life (9 years),

PW(20%) = 5000 (P/F, 20%, 4)

= Rs 2412

Then, the estimated market value at end of year five (T = 5) is,

$$MV_5 = PW(20\%)_{CR} + PW(20\%)_{MV}$$
$$= 20949 + 2412$$
$$= Rs\ 32361$$

Imp (2) Using co-terminated assumption recommend the best project taking study period as 5 years. [TU, IOE, 2063]

Projects	A	B
Initial investment (Rs)	350000	500000
Annual Revenue (Rs)	130000	175000
Annual cost (Rs)	15000	25000
Salvage value (Rs)	35000	50000
Useful life	5 yrs ← study period	8 yrs.
MARR	10%	

Soln:→

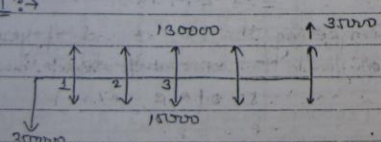

fig:- CFD of project A.

Using FW formulation,

$$FW_A(10\%) = -350000(F/P, 10\%, 5) + (130000 - 15000)(F/A, 10\%, 5) + 35000$$
$$= -563678.5 + 702086.5 + 35000$$
$$= Rs\ 173408$$

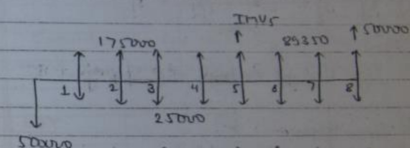

fig:- CFD of project B.

Applying imputed market value (IMV) calculation,

$$CR(10\%) = 500000(A/P, 10\%, 8) - 50000(A/F, 10\%, 8)$$
$$= 93722 - 4372$$
$$= Rs\ 89350$$

PW (at year 5) of CR for remaining 3 years,

$$PW_{CR}(10\%) = 89350(P/A, 10\%, 3)$$
$$= Rs\ 222200$$

Present Worth (at year 5) of Market value MV] for remaining 3 years.

$$PW_{MV}(10\%) = 50000(P/F, 10\%, 3)$$
$$= Rs\ 37565$$

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$IMV_5 = PW_{CR}(10\%) + PW_{MV}(10\%)$
$= Rs\ 259765$

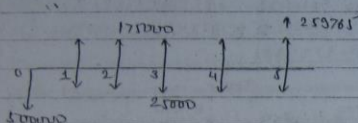

Fig:- Revised CFD of project B

Using FW formulation,

$FW_B(10\%) = -500000(F/P, 10\%, 5) + (175000 - 25000)(F/A, 10\%, 5) + 259765$
$= Rs\ 370275$

Here,

$FW_B(10\%) > FW_A(10\%)$;

So, recommended project is B.

#

** IRR Method For Unequal Project Lives:→*

The IRR method can also be used to compare projects with unequal lives as long as we establish a common analysis period. This can be performed by using Repeatability as well as co-terminated assumptions.

Examples:

1> Consider the following mutually exclusive investment projects A & B, recommend the best project using IRR method.

	A	B
Investment (Rs)	350000	500000
Annual Revenue (Rs)	190000	250000
Annual cost (Rs)	64500	138300
Useful life	4 years	8 years
MARR	10%	

Soln :→

Study period = LCM of 4 & 8 = 8 years.

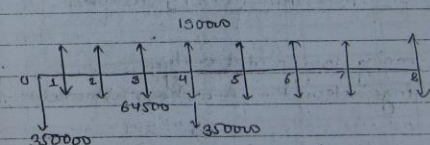

Fig: CFD of project A

$PW_A(i'\%) = 0$

or, $-350000 + (190000 - 64500)(P/A, i', 8) - 350000(P/F, i', 4) = 0$

Solving,

$i' = 16.2\%$

Here,

$i' = 16.2\% > MARR(10\%) \therefore$ accepted.

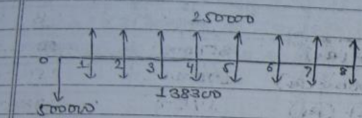

fig: CFD of project B.

$PW_B(i'\%) = 0$

$-500000 + (250000 - 138300)(P/A, i'\%, 8) = 0$

Solving,

$i' = 15.10\% > MARR\ (10\%)$; accepted.

Performing incremental analysis,
taking alternative A as base alternative [having lower investment]

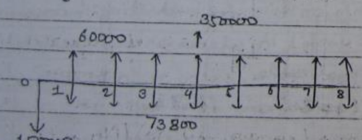

fig: CFD of B-A

$PW_{B-A}(i'\%) = 0$

$-150000 + (60000 - 73800)(P/A, i', 8) + 350000(P/F, i', 4) = 0$

Solving,

$i'\% = 12.70\% > MARR(10\%) \therefore$ accepted

So, select project B.

(2) Consider the two mutually exclusive projects. Recommend best project using IRR method. MARR = 15%.

EOY	A	B
0	−Rs 12500	−Rs 15000
1	−Rs 5000	−Rs 4000
2	−Rs 5000	−Rs 4000
3	−Rs 5000 + Rs 2000	−Rs 4000
4		−Rs 4000 + Rs 1500

Soln :→

Since the study period is LCM of 3 and 4, i.e. 12 years. We may compute the incremental cash flow over this 12 year period. As shown in figure below, we subtract cash flows from A to cash flows from B to form increment of the investment.

[Note :- First cash flow difference should be -ve]

A is repeated 4 times & B is repeated 3 times.

criteria,

$$PW(15\%)_{B-A} = -2500 + 1000(P/F, 15\%, 1) + \dots + 500(P/F, 15\%, 12)$$
$$= Rs\ 5123 > 0$$

This indicates $PW(15\%)_B > PW(15\%)_A$

So, select project B

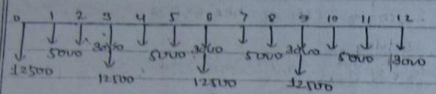

fig:→ CFD of project A.

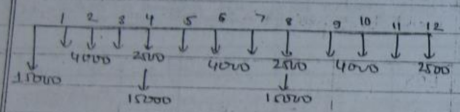

fig:- CFD of project B.

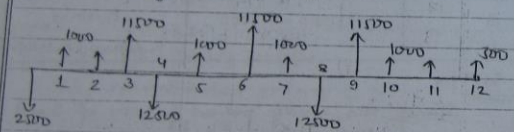

fig:- CFD of B-A

v.imp Here, five sign changes in incremental cash flow, indicating non-simple incremental investment. This results in multiple rate of return. Therefore, we abandon rate of return analysis and use PW

4.2.3) Capitalized Worth Method :→

Capitalized cost is the PW of an alternative that will last "forever". It is the special case of PW criterion which is useful when the life of proposed project is perpetual or the planning horizon is extremely long (say, 40 years or more). Many public sector projects such as bridges, waterway construction, irrigation systems and hydroelectric dams are expected to generate benefits over an extended period of time (or forever). This criterion for evaluating & comparing alternatives is useful in places where repeatability assumption is applicable.

Let us consider cash flow as,

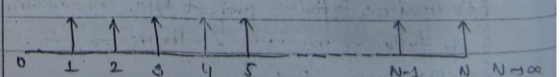

fig:- Equivalent present worth of infinite cash flow series.

The capitalized cost represents the amount of money that must be invested today to yield a certain return A at end of each and every period forever, assuming interest rate i.

Formula to calculate CW is derived from relation, $P = A(P/A, i, n)$, where $N = \infty$

The equation of P using P/A factor formula is,

$$PW(i\%) = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

$$= A\left[\frac{1}{i} - \frac{1}{(1+i)^N}\right]$$

As $N \to \infty$, $(1+i)^N \to \infty$ & PW changes to CW

$$\therefore CW(i\%) = A\left[\frac{1}{i} - \frac{1}{\infty}\right]$$

$$\boxed{\therefore CW(i\%) = \frac{A}{i}}$$

Examples:- (of 4.2.3)

(1) Assume infinite project life, recommend one of following mutually exclusive projects. [TU, IOE, 2064].

Project	A	B
Initial Investment (Rs)	50000	120000
Salvage value (Rs)	10000	10000
Annual cost (Rs)	9000	6000
Useful life (years)	10	25
MARR	15%	

Soln :→

Calculating AW of both alternatives.

$AW_A(15\%) = -50000(A/P, 15\%, 10) - 9000 + 10000(A/F, 15\%, 10)$

$= -9962.60 - 9000 + 492.52$

$\therefore AW_A(15\%) = -Rs\ 18470.08 \longrightarrow (1)$

[Note: $AC_A = Rs\ 18470.08$
annual cost]

$AW_B(15\%) = -120000(A/P, 15\%, 25) - 6000 + 10000(A/F, 15\%, 25)$

$= -18563.92 - 6000 + 4699$

$AW_B(15\%) = -Rs\ 24516.93 \longrightarrow (2)$

[Note: $AC_B = Rs\ 24516.93$]

Dividing AW of eq^n ① & ② by rate 'i',

$$CW_A(15\%) = \frac{AW_A(15\%)}{i} = \frac{-18470.08}{0.15} = -Rs\,123133.86$$

$$CW_B(15\%) = \frac{AW_B(15\%)}{i} = \frac{-Rs\,24516.93}{0.15} = -Rs\,164446.2$$

Here,

$CW_A(15\%) < CW_B(15\%)$; so, project A is selected. #

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(2) How much Rupee should you deposite now in a saving account which gives 10% interest per year if you wish to draw Rs 10000 per month for:-

(i) 50 years. (ii) Continuously (infinite period).

Soln:-

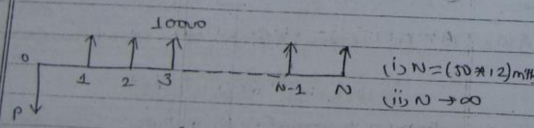

fig:- CFD

Here,

i = 10% per year & withdrawal is monthly. So,

we have to convert yearly rate to monthly interest rate.

Imp. $i_{monthly} = (1 + i_{year})^{1/12} - 1$ ⇐ Note:- Formula of छोटो तरीका during withdrawal i.e. Rule 1^

$= 0.797\%$

We know,

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]$$

(i) If $N = [50\text{ years} * 12] = 600$ months.

$$P = 1000\left[\frac{(1+0.00797)^{600} - 1}{0.00797(1+0.00797)^{600}}\right]$$

$= Rs\,1247086.35$

(ii) If $N \to \infty$

$$P = \frac{A}{i} = \frac{10000}{0.00797}$$

$= Rs\,1254705.144$ #

(3) How much Rupees should you deposit now in a saving account which gives 8% interest per year to draw Rs 250000 at the end of 3rd year each?

Soln:-

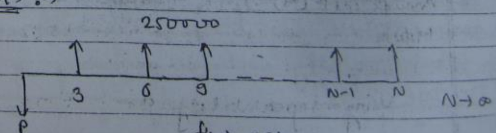

fig:- CFD

If $N \to \infty$

$$P = \frac{A}{i}$$

A = Rs 250000

Interest rate at the end of 3rd year,

$$i = (1+0.08)^3 - 1$$

$$= 0.2597 = 25.97\%$$

$$\therefore P = \frac{250000}{0.2597} = Rs\ 962604.73$$

4) A selection is to be made between two structural designs. Because revenues do not exist (or can be assumed to be equal) only negative cash flow amounts (costs) and the market value at end of useful life are estimated as follows:→

Designs	Structure M	Structure N
Initial investment (Rs)	−12000	−40000
Salvage value (Rs)	0	10000
Annual cost (Rs)	−2200	−1000
MARR	15%	

Using repeatability assumption and CW method of analysis, determine which of structure is better if usefullife of structure M is 10 years and structure N is 25 years.

Soln :→

Calculating AW of both alternatives,

$$AW_M(15\%) = -12000(A/P, 15\%, 10) - 2200 + 0$$

$$= -Rs\ 4592$$

$$AW_N(15\%) = -40000(A/P, 15\%, 25) - 1000 + 10000(A/F, 15\%, 25)$$

$$= -Rs\ 7141$$

Divide AW of both alternatives by interest rate 'i'

$$CW_M(15\%) = \frac{AW_M(15\%)}{0.15} = -Rs\ 30613$$

$$CW_N(15\%) = \frac{AW_N(15\%)}{0.15} = -Rs\ 47607$$

Here, $CW_M(15\%) < CW_N(15\%)$

So, select project M. #

4.3> Comparing Mutually Exclusive, Contingent & Independent Projects in Combination :→

For a single project, we have two investment alternatives: to accept or reject the project. But, for independent projects, we can have four investment

alternatives :-
(i) to accept both projects,
(ii) to reject both projects.
(iii) to accept only first project. &
(iv) to accept only second project.

To perform a proper analysis a firm must group all projects under consideration into decision alternatives. This grouping requires firm to distinguish between projects that are independent of one another & those that are dependent on one another to formulate alternatives correctly.

* Independent Project:- [Note:- to be read here]
An independent project is one that may be accepted or rejected without influencing the accept-reject decision of another independent project.
For example, the purchase of machine, office furniture and truck constitutes 3 independent projects. Only projects that are economically independent of one another can be evaluated separately.

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* Dependent Project :-
In many decision problems, several investment projects are related to one another such that the acceptance or rejection of one project influences acceptance of others.
There are 2 types of dependencies as,
(i) Contingent :- (depends on other)
Two or more projects are said to be contingent if the acceptance of one requires the acceptance of another. For example, the purchase of a computer printer is dependent upon purchase of a computer, but computer may be purchased without considering purchase of printer.

(ii) Mutually Exclusive :-
When there are several alternatives to achieve the same objectives and we can choose only one of them then the alternatives are called mutually exclusive project. Example :- selecting best one from project A & project B.

Note :→ 0 ≡ — and 1 ≡ ✓

Formulation of Mutually Exclusive Alternatives

1) If A, B are two independent projects the mutually exclusive combinations is :

Mutually Exclusive Combination	A	B	Remarks
1	—	—	Do nothing
2	✓	—	Accept A
3	—	✓	Accept B
4	✓	✓	Accept both A & B

2) If A, B, C is three mutually alternatives then we can make following combination :

Mutually exclusive combination	A	B	C	Remarks
1	—	—	—	Do nothing
2	✓	—	—	Accept A
3	—	✓	—	Accept B
4	—	—	✓	Accept C

3) If A, B, C are three projects where C is contingent on acceptance of B and acceptance of B is contingent of acceptance of A, we can make following combination :

Mutually exclusive combination	A	B	C	Remarks
1	—	—	—	Do nothing
2	✓	—	—	Accept A
3	✓	✓	—	Accept A & B
4	✓	✓	✓	Accept All

* Do Nothing Option (DN) :→

Selection of DN alternative means that current approach is maintained ; nothing new is initiated. If there is certainly that one of defined alternatives be selected, do nothing is not considered an option.

Examples :→

1) Engineering projects B1, B2, C1, C2 and D are being considered with cash flows estimated over four years are as shown in table below. Using PW method & MARR = 10% per year, determine what combination of projects is best if capital to be invested is, (a) unlimited & (b) limited to Rs 48000

The combination of project is,

Project B1 & project B2 (mutually exclusive & indep—

endent of C set)

Project C_1 & Project C_2 (mutually exclusive & contigent on acceptance of B2]

Project D [contingent on acceptance of C_1].

Project	Cash flows for End of Year (Rs) 0	1	2	3	4
B_1	-50000	20000	20000	20000	20000
B_2	-30000	12000	12000	12000	12000
C_1	-14000	4000	4000	4000	4000
C_2	-15000	5000	5000	5000	5000
D	-10000	6000	6000	6000	6000

Soln :→

Calculating PW of all alternative,

$PW_{B1} = -50000 + 20000\,(P/A, 10\%, 4) = Rs\ 13400$

$PW_{B2} = -30000 + 12000\,(P/A, 10\%, 4) = Rs\ 8000$

$PW_{C1} = -14000 + 4000\,(P/A, 10\%, 4) = -Rs1300$

[Not been eliminated from considelation because project D is contingent on it].

$PW_{C2} = -15000 + 5000\,(P/A, 10\%, 4) = -Rs\ 800$

$PW_D = -10000 + 6000\,(P/A, 10\%, 4) = -Rs\ 9000$

Mutually exclusive project combination

Mutually exclusive combination	Project B1	B2	C1	C2	D
1	0	0	0	0	0
2	1	0	0	0	0
3	0	1	0	0	0
4	0	1	1	0	0
5	0	1	0	1	0
6	0	1	1	0	1

Mutually exclusive combination	Cash flows for End of Years (Rs) 0	1	2	3	4	Invested capital (Rs)	PW(10%) (Rs)
1. (None)	0	0	0	0	0	0	0
2. (B_1)	-50000	20000	20000	20000	20000	-50000	13400
3. (B_2)	-30000	12000	12000	12000	12000	-30000	8000
4. (B_2C_1)	-44000	16000	16000	16000	16000	-14000	6700
5. (B_2C_2)	-45000	17000	17000	17000	17000	-45000	8900
6. (B_2C_1D)	-54000	22000	22000	22000	2200	-54000	15700

Decision:→

1) Mutually exclusive combination 6 is the best combination which PW is Rs15700 when capital is unlimited.

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2) Mutually exclusive combination no. 6, 2 are not feasible if capital is limited to Rs 48000, of the remaining combinations, combination 5 has the highest present worth of Rs 8900.

(2) Government of Nepal has started four projects A, B, C & D for extension of information technology in the country. The estimated cash flows over 10 years as shown in table below. The capital investment capacity is limited to Rs 80000 & MARR is 10% per year.

Project	A	B	C	D
Initial Investment (Rs)	82000	25000	72000	80000
Annual Revenue (Rs)	7000	5000	12000	16000
Salvage Value (Rs)	4000	3000	5000	6000
A	Contingent on acceptance of B			
B and C	Mutually exclusive			
D	Contingent on acceptance of C			

Recommend which investment alternative should be best? Use FW formulation?

Soln :→

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Calculating FW of each alternative,

$FW(10\%)$ of $A = -32000(F/P, 10\%, 10) + 7000(F/A, 10\%, 10) + 4000$

$= -83000 + 111562 + 4000$

$=$ Rs 32562

$FW(10\%)$ of $B = -25000(F/P, 10\%, 10) + 5000(F/A, 10\%, 10) + 3000$

$= -64844 + 79687 + 3000$

$=$ Rs 17843.

$FW(10\%)$ of $C = -72000(F/P, 10\%, 10) + 5000(F/A, 10\%, 10) + 5000$

$= -186750 + 191249 + 5000$

$=$ Rs 9500

$FW(10\%)$ of $D = -80000(F/P, 10\%, 10) + 12000(F/A, 10\%, 10) + 6000$

$= -207500 + 255000 + 6000$

$=$ Rs 53500

Combinations of project (alternative)

Combination	A	B	C	D	Explanation
1	0	0	0	0	Do nothing
2	1	1	0	0	Select A & B
3	0	1	0	0	select B
4.	0	0	1	0	select C
5	0	0	1	1	select C & D.

Combined Project Investment & Future Worth

Combination	Invested Capital (Rs)	PW (Rs) @ MARR = 10%
1. [None]	0	0
2. [A, B]	32000 + 25000 = 57000	32562 + 17843 = 50405
3. [B]	25000	17843
4. [C]	72000	9500
5. [C, D]	72000 + 80000 152000	9500 + 53500 = 63000

Decision :→

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(1) If invested capital is limited to Rs 80000, only combination 5 is rejected.

(2) Among remaining combinations, combination 2 is best because it has highest future worth of Rs 50405.

CHAPTER :→ 5

Replacement Analysis

5.1> Fundamentals of Replacement Analysis :→

Every business firm must frequently decide whether the existing assets should be continued in service or exchange by new assets to meet current & future needs more economically. These decision are commonly used in worldwide competition & careful engineering economy studies. Hence, the concept of replacement refers to selection of similar but new assets to replace existing assets. For example, old trucks could be replaced with new models that operate similarly but have advanced features that improve performance.

Replacement analysis is a choice between present asset (called defender) and new alternatives (called challenger).

The question here is not only whether but when should existing equipment be replaced with more efficient equipment.

The terms defender and challenger are commonly used in boxing world in which the current defending champion is constantly faced with a new challenger. In replacement analysis, defender is existing machine or system and challenger is best available replacement equipment.

* Reasons for replacement :→

(1) Physical Impairment [Wear & Tear] :→

Physical assets will be worn-out due to normal use or accident unless the extensive repairs are made.

(2) Inadequacy :→

Existing asset doesnot have sufficient capacity to fill the current & expected demand of market.

(3) Obsolescence :→

It may be functional & economic due to change in technology. In case of functional obsolescence, there has been decrease in demand for output of asset and results in loss of profit.

For example :→

Typewriter & computer.

Economic obsolescence is result of existence of new asset that will produce lower cost than current asset.

4) Rental or lease possibility :→

There may be possibility of economic advantage from leasing or rental service rather than purchase option.

* Opportunity Cost approach to comparing defender and Challenger :→

We call the sum of various cost items related to the operation of an asset like; repair and maintenance, wages for operators, energy consumption & cost of materials as operating costs. (variable costs). Increase in any one or a combination of these cost items over a period of time may push us to find a replacement for the existing asset. The challenger is usually newer than defender and often incorporates improvements in design and newer technology. As a result, some or all of variable cost items for challenger are likely to be less expensive than those for defender.

Generally defender involves lower initial cost than purchasing the challenger but it requires more

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annual operating costs specially repairing & maintenance purpose due to old technology & asset ages (old).

We consider a basic approach to analysing replacement problem as opportunity cost approach Replacement studies are [generally used as] annual equivalent cost [AEC] calculation approach to take advantage of annual data collection.

With annual equivalent cost [AEC] approach, salvage value is best estimate of defender's worth which is market value at time of study. This cost (salvage value) is called an opportunity cost since by keeping defender we are forgoing the opportunity cost since by keeping defender we are forgoing the opportunity to receive its current value when we replace it by a challenger. In another worlds, we consider the salvage value as a cash outflow for defender (or an investment required in order to keep the defender).

Example:→

1) Consider the choice between a defender that has a current market value of Rs 5000 and challenger that can be purchased for Rs 7500. Both have a service life of 3 years with no salvage value at the end of life. (MARR = 12%). Their operating costs are as follows:→

Year	Defender (D)	Challenger (C)
0	5000 ⇐ opportunity cost	7500 ⇐ investment
1	1700	500
2	2000	1100
3	2500	1300

Soln:→

From above cash flow schedule, operating cost has increased as an asset is getting order. Since defender has already provided service, its annual costs are usually higher than the challengers.

If market values of defender is considered to be capital cost of its continued service with MARR at 12%, then annual equivalent cost (AEC) of defener will be,

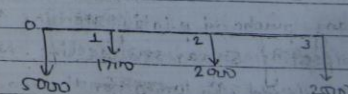

fig: CFD of defender.

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$\therefore AEC(12\%)_D = [5000 + 1700(P/F, 12\%, 1) + 2000(P/F, 12\%, 2) + 2500(P/F, 12\%, 3)](A/P, 12\%, 3)$
$= (5000 + 4892) * 0.41635$
$= Rs\ 4119$.

The annual equivalent cost (AEC) of challenger with MARR at 12% will be,

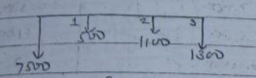

fig: CFD of challenger

$\therefore AEC(12\%)_C = [7500 + 500(P/F, 12\%, 1) + 1100(P/F, 12\%, 2) + 1300(P/F, 12\%, 3)](A/P, 12\%, 3)$
$= (7500 + 2249)(0.41635)$
$= Rs\ 4059$

Here, $AEC(12\%)_C < AEC(12\%)_D$.

So, replacement should be justified now or replacement of defender by challenger is accepted.

(2) A company purchased printing machine two years ago by expecting 5 year service life. Now, the company has found Rs 10000 for the market value of that machine, the anticipated salvage value of the machine will be Rs 2500 at the end of remaining useful life & operating costs are running at the rate of Rs 8000 per year. On the other hand the company has offered a chance to purchase another new printing machine for Rs 15000 & over its 3 year useful life the operating cost will be Rs 6000 per year & it is also estimated that the new machine can be sold for Rs 5500 at end of life. Decide whether replacement is justified now by assuming MARR is 12% per year?

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Soln:→

There may be two options:-

Option 1:→ Keep the defender.

If decision is to keep defender, the opportunity cost approach treats the Rs 10000 current salvage value of defender as an investment cost. The annual operating cost for next three years will be Rs 8000 per year and the defender's salvage value for 3 years from today will be Rs 2500. We calculate net annual equivalent cost (AEC) for defender as,

$AEC(12\%)_D = 8000 + 10000(A/P, 12\%, 3) - 2500(A/F, 12\%, 3)$
$= 8000 + 10000(0.4163) - 2500(0.2963)$
$= Rs\ 11423$

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Option 2 :→ Replace defender with challenger.

The new printing machine i.e., investment cost for challenger is Rs 15000. The operating cost of challenger is Rs 6000 per year & its salvage value will be Rs 5500 at end of year 3. With this information, calculating net annual equivalent cost (AEC) of challenger is,

$$AEC(12\%)_c = 6000 + 15000(A/P, 12\%, 3) - 5500(A/F, 12\%, 3)$$
$$= 6000 + 15000(0.4163) - 5500(0.2963)$$
$$= Rs\ 10615$$

Here,

$$AEC(12\%)_c < AEC(12\%)_D$$

So, replacement is justified now.

5.2) Economic Service Life of Challenger and Defender:→

Determination of an economic life for the challenger & defender is important in a replacement analysis because of fundamental principle that new & existing assets should be compared over their economic lives.

In all engineering economy studies upto this point, we have made the analyses with assumed

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useful lives for all alternatives under consideration for any new asset (challenger), an economic life can be computed if various operation and maintenance costs are known (or can be estimated) in addition to its year by year salvage values. The economic life minimizes the annual equivalent cost of owning & operating an asset and it is often shorter than useful or physical life. Economic data regarding defenders and challengers are periodically updated (often annually) & replacement studies are then repeated to ensure (an ongoing evaluation of improvement opportunities) the selection of challengers.

Imp ✓ The total annual equivalent costs of owning & operating an asset AEC (i) is the summation of the capital recovery cost & annual equivalent of operating cost of the asset.

$$\therefore AEC(i) = CR(i) + OC(i) \longrightarrow (1)$$

v^2 Imp The economic service life of an asset is defined as the period of useful life that minimizes the AEC of owning & operating the asset. To get exact economic service life, we need to find the value of N (years) that minimizes AEC as expressed in equation (i).

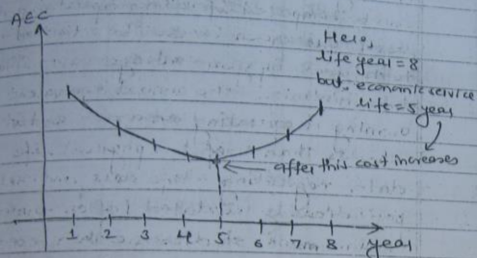


fig: AEC Vs years

Example: [Also see 2069 Bhadra, Q.No. 5. → solved at last] Imp

Suppose a company has decided to purchase a new machine having initial cost of Rs 18000 by expecting 8 years of useful life and having operating cost of Rs 3000 in first year. For the remaining years, operating cost increase each year by 15% over previous years operating cost and salvage value previous years operating cost and salvage value declines each year by 20% from previous year's salvage value. Find the economic service life of this new machine when MARR is 12% per year."

Soln :→

The question gives information about initial investments, operating & maintenance (O&M) cost, salvage value of the asset over the eight years & $i = 12\%$.

To determine required economic service life, we can use following equation that could make minimum annual equivalent cost (AEC)

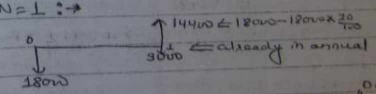
$$AEC(i\%) = \underset{\uparrow \text{ capital recovery}}{CR(i\%)} + \underset{\uparrow \text{ expenses annual}}{OC(i\%)}$$

where,

$$CR(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N)$$

$$OC(i\%) = \left[\sum_{n=1}^{N} OC_n (P/F, i\%, n)\right](A/P, i\%, N)$$

let, N = 1 :→



$I = 18000$, $S = 18000 - 18000 \times \frac{20}{10} = 14400$, $\overset{OC}{E} = 3000/\text{yr}$

$\therefore CR(12\%) = 18000(A/P, 12\%, 1) - 14400(A/F, 12\%, 1)$

$= 5760$

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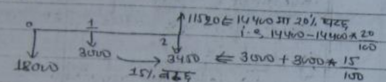
$OC(12\%) = 3000(A/F, 12\%, 1) = 3000 * \frac{0.12}{1+0.12-1} = 3000$

$\therefore AEC_1(12\%) = 5760 + 3000 = 8760$

Let $N = 2$:→



$I = 18000,\ S = 14400 - 14400 * \frac{20}{100} = 11520.$

$\therefore CR(12\%) = 18000(A/P, 12\%, 2) - 11520(A/F, 12\%, 2)$
$= 5217$

$OC(12\%) = [3000(P/F, 12\%, 1)](A/P, 12\%, 2) + 3450(A/F, 12\%, 2)$
$= [3000(0.8929)](0.5917) + 3450(0.4717)$
$= 1585 + 1627$
$= 3212$

$\therefore AEC_2(12\%) = 5217 + 3212 =$ Rs $8429.$

Let $N = 3$:→

$CR(12\%) = 18000(A/P, 12\%, 3) - 9216(A/F, 12\%, 3)$
$= 18000(0.4163) - 9216(0.2963)$
$= 4763$

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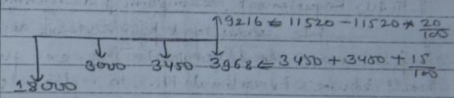
$OC(12\%) = [3000(P/F, 12\%, 1)][A/P, 12\%, 3) + [3450(P/F, 12\%, 2)](A/P, 12\%, 3)] + [(3450 + 3450 * \frac{20}{100})](A/F, 12\%, 3)]$
$= [3000(0.8929)](0.4163) + [3450(0.7972)](0.4163) + 3968(0.2963)$
$= 1115 + 1145 + 1176$
$= 3436$



$\therefore AEC_3(12\%) = 4763 + 3436 = 8159$

Similarly,

Let $N = 4$;

$AEC_4(12\%) = 8055$

Let $N = 5$;

$AEC_6(12) = 7985$

Let $N = 6$;

$AEC_6(12\%) = 7977$

Let $N = 7$;

$AEC_7(12\%) = 8024$

From the preceeding calculated values for $N = 1, 2, ---- 7$, we find that $AEC(12\%)$

is smallest when $N=6$. Hence, we conclude that economic service life of machine is 6 years eventhough its useful life is 8 years.

5.3> Replacement Analysis when required Service Life is Long:→

5.3.1> Requirement Assumptions & decision framework:→

We understand how economic service life of an asset is determined. The next question will be when is optimal time to replace defender? For it we need to consider following 3 factors:-

a) Planning Horizon (study period)
b) Technology
c) Relevant cash flow information

} ∈ Assumptions or elements of replacement decision.

Assumptions and replacement decision framework -

a) Planning Horizon | (b) Anticipated technology improvement | (c) Relevant cash flow information

(a) Planning Horizon:→

It refers required economic service period by the defender and a sequence of future challengers. Infinite planning horizon is used when we are unable to predict activity under consideration will be terminated. When the project will have definite & predictable duration, the replacement policy should be formulated more realistically on the basis of finite planning horizon.

(b) Technology:→

It refers to development of types of challengers that may replace under study. If we assume that all future machines will be same as those now in use (service), we are saying that no technological progress will occur. In other ~~words~~ cases, we may express possibility of machine becoming in future more efficient, reliable or productive than those are available in market currently, which leads to recognition of technological changes & obsolescence. If available machine gets better & better over time, we should investigate possibility of delaying asset's replacement for a couple of years.

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(c) Relevant Cash Flows over life of an asset:→

Many predictions can be used to estimate patterns of revenue, cost and salvage value over life of an asset. We will focus whether replacement analysis is directed towards cost minimization or profit maximization. We formulate a replacement policy for an asset whose salvage value does not increase with age.

Decision Framework

Decision framework is developed by indicating a replacement sequence of an assets with notation $(J_0, n_0), (J_1, n_1), (J_2, n_2), \ldots, (J_K, n_K)$. Each pair of numbers (J, n) indicates a types of asset & life time over which that asset will be retained.

For example; defender asset (J_0) replaced now $(n_0 = 0)$ can be written as $(J_0, 0)$

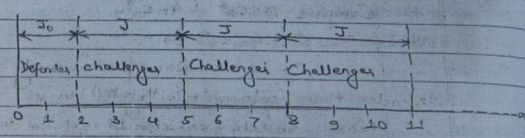
defender लाई 2 वर्ष use गर्ने — challenger 1 लाई 5 वर्ष use गर्ने — challenger 2 लाई 3 वर्ष use गर्ने

The sequence $(J_0, 2), (J_1, 5), (J_2, 3)$ indicates retaining the defender ~~with an asset type of~~ for two years & then replacing defender with an asset type of J_1 (challenger 1) which is used for 5 years & then replacing J_1 with an asset type J_2 (challenger 2) & using it for three years. Hence, total planning horizon covers $2+5+3 = 10$ yrs.

Example:- of Inifite planning horizon with repeated identical replacements.

$(J_0, 2)$, $(J, 3)$ ∞

defender लाई 2 वर्ष use गर्ने — अब प्रत्येक challenger लाई 3-3 वर्ष use गर्ने



→ planning horizon (year)

5.3.2> Replacement Analysis Under Infinite Planning Horizon :→

Under infinite planning horizon, service is required for a very long time, either we continue to use defender to provide service or we replace defender with best available challenger for the same service. In this case, we may apply the following procedure in

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replacement analysis :→

Step 1 :→

Compute economic (life lives of both defender and challenger. Let, N_D^* & N_C^* is used to indicate the economic lives of the defender & challenger. The annual equivalent costs (AEC) for both defender & challenger at their respective economic lives are AEC_D^* & AEC_C^*.

Step 2 :→

Compare AEC_D^* & AEC_C^*

(a) If $AEC_D^* > AEC_C^*$:→

It is situation of more costly to keep defender than replace it with challenger.

Hence, challenger should replace defender now.

(b) If $AEC_D^* < AEC_C^*$:→

It costs less to keep defender than to replace it with challenger.

Hence, challenger is not used now, i.e. defender should not be replaced now.

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Step 3 :→

For step 2 (b), if defender should not be replaced now, when should it be replaced.

↳ First we need to continue to use it until its economic life is over.

↳ Then, we should calculate cost of running defender for one more year after its economic life, this approach is called marginal analysis, i.e. we calculate the incremental cost of operating defender for just one more year.

↳ In other words, we want to see whether cost of extending the use of defender for an additional year exceeds the savings resulting from delaying the purchase of the challenger. In this case, P.81

↳ If marginal cost of defender is greater than AEC_C^*, the defender should be replaced at end of its economic life. Otherwise, we should calculate the cost of running the defender for second year (i.e. marginal cost for second year) after its economic life.

↳ If marginal cost of defender for the second year is greater than AEC_C^*, the defender should be replaced one year after its economic life. This process should be continued until we find optimal replacement time. Here, we have assume that best available challenger does not change.

Example :→

An electrical insulator company is considering replacing an old machine with newer one, for it,

Option 1 :→

If repair old machines, that can be used for another 6 years, that will require Rs 1200 to overhaul repair. The operating cost is estimated Rs 2000 for first year and expected to increase by Rs 1500 per year thereafter. However, the company can sell it now for Rs 5000 & future market values are expected to decline by 25% each year over previous year's value.

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Option 2 :→

The new machine cost is 10000 & will have operating cost of 2200 in first year & expected to increase by 20% per year thereafter. The expected salvage value is 6000 after one year & will decline by 15% each year for 5 years. Find the economic life for each option & determine when defender should be replaced at MARR = 15%.

Soln :→

First finding Economic Service life of defender

The opportunity cost of machine is Rs 5000. Also, overhauling cost of Rs 1200 is needed to machine operation. The total initial cost of machine is Rs 5000 + Rs 1200 = Rs 6200.

We calculate annual equivalent cost (AEC) if defender is to be kept for one year, two year, three year & so on as following equation & diagram:

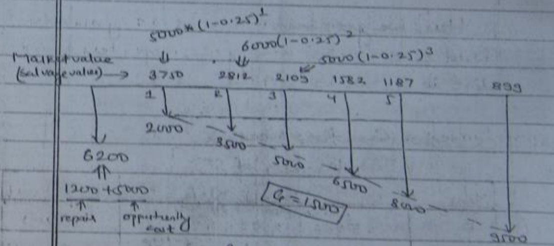

fig: CFD of defender

$$\therefore AEC(15\%)_N = \overset{\text{opportunity cost}}{6200}(A/P, 15\%, N) + 2000 + 1500(A/G, 15\%, N) - \Big[\underset{\text{salvage value}}{5000(1-0.25)^N}\Big](A/F, 15\%, N)$$

At $N_D^* = 1$

$$AEC_D^* = 6200(A/P, 15\%, 1) + 2000 + 1500(A/G, 15\%, 1) - [5000(1-0.25)^1](A/F, 15\%, 1)$$

$$= 6200 * 1.15 + 2000 + 1500 * 0 - 5000 * 0.75 * 1$$

$$= 5380$$

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at $N_D^* = 2$

$$AEC_D^* = 6200(A/P, 15\%, 2) + 2000 + 1500(A/G, 15\%, 2) - [5000(1-0.25)^2](A/F, 15\%, 2)$$
$$= 5203$$

at $N_D^* = 3$

$$AEC_D^* = 6200(A/P, 15\%, 3) + 2000 + 1500(A/G, 15\%, 3) - [5000(1-0.25)^3](A/F, 15\%, 3)$$
$$= 5468$$

Here, we get lowest AEC at N = 2 years after ward cost start increasing. So, defender's economic life is 2 years with $AEC_D^* = 5203$

Note :→ Cash flow schedule
[Tabular form of CFD]

Year	Market Value	O&M cost
0	$\underbrace{5000}_{\text{salvage value}} + \underbrace{1200}_{\text{opportunity cost}} = 6200$	1200
1	$5000(1-0.25)^1 = \underbrace{3750}_{\text{salvage value}}$	2000
2	$5000(1-0.25)^2 = 2812$	3500
3	$500(1-025)^3 = 2019$	5000
4	= 1582	6500
5	= 1187	8000
6	= 890	9500

fig:- Cash flow schedule for defender

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Again, find economic service life of challenger.

Year	Market value	O&M cost
0	(opportunity cost) 10000	0
1	$\underbrace{6000}_{\text{salvage value}}$	2200
2	$6000 - \frac{15}{100} * 6000 = 5100$	$2200 * 1.15 = 2640$
3	$5100 - 15/100 * 5100 = 4335$	3802
4	$4333 * 0.85 = 3685$	3802
5	$3685 * 0.85 = 3132$	4562
6	= 2662	5474
7	= 2263	6569
8	= 1926	7883

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Note :→ General eqn is.

$$AEC_c^*(15\%)_N = 10000(A/P, 15\%, N) - [6000(1-0.15)^N][A/F, 15\%, N] + [2200(P/A, 20\%, 15\%, N)](A/F, 15\%, N)$$

at $N_c^* = 1$,

$$AEC_c^* = \underbrace{10000}_{\text{opportunity cost}}(A/P, 15\%, 1) - \underbrace{6000}_{\text{salvage value}}(A/F, 15\%, 1) + \underbrace{2200}_{\text{O\&M cost for year 1}}(A/F, 15\%, 1)$$

$$= 10000 * 1.15 - 6000 * 1 + 2200 * 1$$
$$= 7700$$

at $N_c^* = 2$,

$$AEC_c^* = \underbrace{10000}_{\text{O.C}}(A/P, 15\%, 2) - \underbrace{5100}_{\text{S.V}}(A/F, 15\%, 2) + [\underbrace{2200}_{\text{O\&M}}(P/F, 15\%, 1)](A/P, 15\%, 2) + \underbrace{2640}_{\text{O\&M cost for year 2}}(A/F, 15\%, 2)$$

$$= 6184$$

at $N_c^* = 3$,

$AEC_c^* = 5756$

at $N_c^* = 4$,

$AEC_c^* = 5625$

at $N_c^* = 5$,

$AEC_c^* = 5631$

So, economic life of challenger is four years with $AEC_c^* = 5625$

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Now, $AEC_D^* = 5203 < AEC_c^* = 5625$, the defender should not be replaced for now. If there are no technological advances in the few years, the defender should be used for at least 2 more years. It depends on what type of challenger would be available in future. When should the defender be replaced?

If we need to find answer to this question, at present, we have to calculate the cost of keeping & using defender for the third year from today. For it,

[opportunity cost] i.e, (Market value) salvage value of defender at year 2 end,

$= 5000(1-0.25)^2 = 2813$.

operating cost for third year [i.e. O&M cost]

= cost for 1st year + cost for 2nd year + cost for 3rd yr.

$= 2000 + 1500 + 1500$

$= 5000$

Salvage value of defender at end of year 3,

$= 2813(1-0.25)$ OR $5000(1-0.25)^3$

$= 2109$

Hence, the cost of using defender for one more year from end of its economic life is,

$$\underset{[3^{rd}\ yrs]}{AEC_D^*} = \overset{\text{or } 5000(A/P,15\%,3)}{2813(F/P,15\%,1)} + \overset{\text{O\&M cost}}{5000} - \overset{SV}{2109}$$

$= 6126 > 5625$ i.e. AEC_c^*

Since, $AEC_c^* = 5625$, it is more expensive to keep the defender for third year than to replace it with challenger. #

Note :→ Accordingly, we conclude that we should replace the defender at end of year 2. If the third year cost of defender is still smaller than AEC_c^* we need to calculate the cost of using defender for the fourth year & then compare it with AEC_c^* of challenger.

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(N−1) i.e. अघिल्लो yearको S.V अहिलेको yearमा ल्याउने

V. Imp **Note :→**

AEC for year N = opportunity cost + operating & maintain cost − salvage value of year N

· imp.

O & M cost upto year N

5.3.3 > Replacement Analysis under finite Planning Horizon :→

If the planning period is finite, i.e. fixed & specified, a comparison with AEC method over economic service life does not generally apply. In such cases there is to establish all reasonable replacement patterns and then use PW value for planning period to select the most economical pattern.

Example :

By example in 5.3.2, suppose that the company has contract to perform same service using defender or challenger for the next eight years.

After the contract time neither defender nor challenger will be retained.

What is best replacement strategy, i.e. what is most economical replacement scenario?

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Soln :→

First, calculating AEC of both defender & challenger as, (already calculated in 5.3.1)

$$AEC(15\%)_D = 6200(A|P,15\%,N) + 2000 + 1500(A|G,15\%,N) - 5000(1.025)^N(A|F,15\%,N)$$

$$\& \; AEC(15\%)_C = 10000(A|P,15\%,N) + 2200(P|A,20\%,15\%,N)(A|P,15\%,N) - 6000(1-0.15)^N(A|F,15\%,N)$$

Year N	Defender: $AEC(15\%)_D$	Challenger: $AEC(15\%)_C$
1.	5380	7700
2.	5203 ⇐ S.L = 2	6184
3.	5468	5766
4.	5845	5625 ⇐ S.L = 4
5.	6258	5631

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Many replacement scenario options would fulfill during 8-year planning horizon. Some likely replacement patterns under a finite planning horizon of 8 years be:

Option 1 :− $(J_0, 0)$ $(J, 4)$ $(J, 4)$
" 2 :− $(J_0, 1)$ $(J, 4)$ $(J, 3)$
" 3 :− $(J_0, 2)$, $(J, 4)$ $(J, 2)$
" 4 :− $(J_0, 3)$ $(J, 4)$ $(J, 1)$
" 5 :− $(J_0, 3)$ $(J, 5)$
" 6 :→ $(J_0, 4)$ $(J, 4)$

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[**Note** :→ option बनाउदा economic service life लाई ध्यान राख्ने.]

For option 1,

$PW(15\%)_1 = 5825\,(P/A, 15\%, 8)$

$= 5625 \left[\frac{(1.15)^8 - 1}{0.15\,(1.15)^8} \right]$

$= 25241$

(OR)

$PW(15\%)_1 = 5825\,(P/A, 15\%, 4) + [5825\,(P/A, 15\%, 4)]\,(P/F, 15\%, 4)$

$= 25241$

For option 2,

$PW(15\%)_2 = 5380\,(P/F, 15\%, 1) + [5825\,(P/A, 15\%, 4)]\,(P/F, 15\%, 1) + [5758\,(P/A, 15\%, 3)]\,(P/F, 15\%, 5)$

$= 5380 * 0.869 + 5825\,(2.855)(0.8696) + 5758\,(2.2832)(0.4972)$

$= 25177$

For option 3,

$PW(15\%)_3 = 5203\,(P/A, 15\%, 2) + [5825\,(P/A, 15\%, 4)]\,(P/F, 15\%, 2) + [6184\,(P/A, 15\%, 2)]\,(P/F, 15\%, 6)$

$= 24948$

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For option 4;

$PW(15\%)_4 = 5468\,(P/A, 15\%, 3) + [5831\,(P/A, 15\%, 5)]\,(P/F, 15\%, 3)$

$= 24896 \Leftarrow$ Minimum cost

For option 5;

$PW(15\%)_5 = 5468\,(P/A, 15\%, 3) + [5825\,(P/A, 15\%, 4)]\,(P/F, 15\%, 3) + 7700\,(P/F, 15\%, 8)$

$= 25580$

For option 6;

$PW(15\%)_6 = 5845\,(P/A, 15\%, 4) + [5825\,(P/A, 15\%, 4)]\,(P/F, 15\%, 4)$

$= 25870$

Here, an economic evaluation of the present equivalent cost of planning horizon for eight years indicates that least-cost solution among the options appears to be option 5.

It reveals that the defender retains for 3 years & then purchase the challenger for 5 years.

#

CHAPTER :→ 6

Risk Analysis

6.1> Origin / Sources of Project Risks :→

* Risk :-

When there may be two or more observable values for a parameter and it is possible to estimate the chance that each value may occur, risk is present.

* Uncertainty :→

Decision making under uncertainty means there are two or more values observable but the chance of their occuring cannot be estimated or no one is willing to assign the chances. The observable value in uncertainty analysis is often referred to as states of natures.

* Decision making under Certainty :→

Deterministic estimates are made & entered into measure of worth relations - PW, FW, AW, ROR, B/C and decision making is based on results. The value estimated can be considered the most likely to occur with all chance placed on the single value estimate.

* Decision Making Under Risk :→

Decision situation where several states are possible & probabilities of their occurrence are explicitly stated.

Risk & Uncertainity are different, though complementary concepts. Risk refers to a situation where a project has a number of possible alternative outcomes, but probability of each occuring is known. Uncertainty refers to a situation in which these probabilities are not known. In practice, the terms are often used interchangeably. To insist on the separateness of meaning of these terms would perhaps be slightly legalistic. When we call an investment risky, we mean that we are uncertain about future outcome of investment in practice.

Imp * Sources or Origin of Risk :→

It is useful to consider some of factors that affect uncertainty involved in the analysis of future economic consequences of an engineering project. It would be almost impossible to list & discuss all of potential factors. There are major 4 sources of risk

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(uncertainty) in engineering economy study.

(1) Possible inaccuracy of cash flow estimates used in the study.

(2) Type of business involved in relation to the future health of economy.

(3) Type of physical plant & Equipment involved.

(4) Length of study period used in analysis.

6.2> Methods of Describing Projects Risks:→

We may begin analyzing project risk by first determining the uncertainty inherent in a project's cash flow. We can do this analysis in a number of ways, which range from making informal judgements to calculating complex economic & statistical analysis.

To make a decision on this type, we adopt following methods:→

1.> Sensitivity analysis,
2> Break even analysis,
3> Scenario Analysis.
4> Risk adjusted MARR.
5> Reduction of useful life.

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6.2.1> Sensitivity Analysis:→

It refers how much NPW of a project will change in response to a given change in one input variable or parameter.

Hence, it is response on economic merit (value) due to change in one parameter of a project. This is also called "What If" analysis.

Example:→

Perform sensitivity analysis of the following project over a range of ±40% in
(a) initial investment (b) Annual net revenue,
(c) Salvage value (d) Useful life.

$I = 11500$, $A \cdot R = 3000$, $S = 1000$, $N = 6$ years, $MARR = 10\%$. Draw the sensitivity diagram also.

Soln:→

From given information our original equation will be,

$$PW(10\%) = -11500 + 3000(P/A, 10\%, 6) + 1000(P/F, 10\%, 6)$$

NPW

$$= -11500 + 3000\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] + 1000\left[\frac{1}{(1+i)^N}\right]$$

$$= -11500 + 3000(4.3552) + 1000(0.5644)$$

$$= 2130$$

ⓐ When initial investment (I) varies ± 40%.,
PW be, at I = + 40%.

$$PW = -\underbrace{11500\,(1.4)}_{11500 + 11500 \times \frac{40}{100}} + \underbrace{13065.6 + 584.4}_{\text{as it is}}$$

= −24470

At I = −40%.

PW = −11500 (0.6) + 13065.6 + 584.4
= 6730

ⓑ When AR varies ± 40%., PW be

At AR = +40%.

$$PW = -11500 + \underbrace{13065.6}_{3000\,(P/A,\,10\%,\,6)}(1.4) + 584.4$$

= 7356

At AR = −40%.

PW = −11500 + 13065.6 (0.6) + 584.4
= −3096

ⓒ When salvage value varies ± 40%., PW be
at S = +40%.

$$PW = -11500 + 13065.6 + \underbrace{584.4}_{1000\,(F/P,\,10\%,\,6)}(1.4)$$

= 2356

At S = −40%.

PW = −11500 + 13065.6 + 584.4 (0.6)
= 1904

ⓓ When N varies ± 40%., PW would be,
At N = +40%. i.e., N = 6 * 1.4 = 8.4 years.

PW = −11500 + 3000 [P/A, 10%, 8.4] + 1000 (P/F, 10%, 8.4)
= 5476

At N = −40%., i.e., N = 6 * 0.6 = 3.6 years.
PW = −11500 + 3000 (P/A, 10%, 3.6) + 1000 (P/F, 10%, 3.6)
= −2077

Calculation Table:-

	PW (10%)		
Parameter \ change	40%	0%	−40%
I	−2470	2130	6730
A	7356	2130	−3096
S	2356	2130	1904
N	5476	2130	−2077

Sensitivity Diagram:-

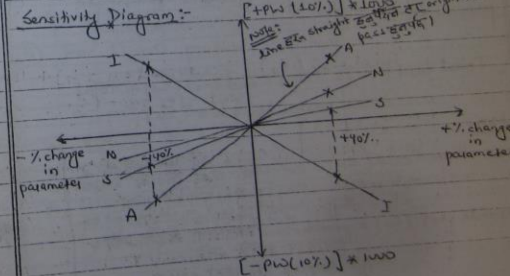

fig:→ Sensitivity diagram.

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On basis of above calculation & diagram annual revenue (A) is more sensitive compared to other parameter.

6.2.2) Break Even Analysis :-

It determines the value of a critical factor at which economic trade off [revenue & cost] are balanced. i.e, it is a situation at which there is neither profit nor loss.

It is more useful for economic evaluation of new investment project.

Unlike sensitivity analysis which was used when more factors are subjected to uncertain, break even analysis is done when selection among alternatives is heavily dependent on a single parameter.

Total cost (C_T) = Total fixed cost (C_F) + total variable cost (C_V).

Note :-

Fixed cost (C_F) = उत्पादन हुनु अगावै गरेको खर्च which is fixed.
e.g: building cost, machine cost etc.

Variable cost (C_V) = production बढाउदा बढ्दै जाने र घटाउदा घट्दै जाने cost.
e.g:- Laptop बनाउदा RAM, Harddisk को cost, घर बनाउदा इट्टा, cement को cost etc.

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$\therefore C_T = C_F + C_V$

$= C_F + V_C(x')$

where,

V_c = variable cost per unit.

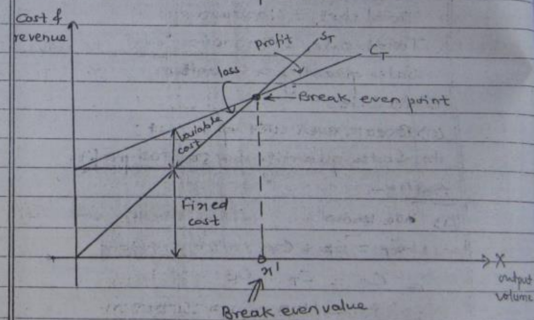
& x' = output volume

Total sales revenue = S_T

$= S_P(x')$

where, S_P = selling price per unit,

x' = (same) output volume.



For Break even output volume (x'),

$S_T = C_T$

or, $S_P(x') = C_F + C_V$

or, $S_P(x') = C_F + V_C(x')$

or, $C_F = S_p(x') - V_c(x')$

or, $C_F = x'(S_p - V_c)$

$$\therefore x' = \frac{C_F}{S_p - V_c}$$

Example [single project Case]

A company wants to produce new types of laptop by adopting following information.

fixed cost = 2000000.

Total cost = 4000000.

Total sales = 3000000.

Sales quantity = 50 units.

Calculate :-

(a) Break even unit of output :

(b) Sales quantity for 500000 profit.

Soln :→

(a) We know,

$C_T = C_F + C_V$

or, $C_V = C_T - C_F$

$= 4000000 - 2000000$

$\therefore C_V = 2000000$.

We know,

$C_V = V_c \cdot x'$

$$\Rightarrow V_c = \frac{C_V}{x'}$$

$$= \frac{2000000}{50} = 40000$$

$$S_p = \frac{3000000}{50} = 60000$$

So, break even output,

$$= \frac{C_F}{S_p - V_c}$$

$$= \frac{20000}{60000-40000}$$

$= 100$ units.

Note :→ Test :- for 100 units S_T should equal to C_T.

$S_T = 100 * 60000 = 6000000$

$C_T = C_F + C_V = 2000000 + 1000 * 40000$

$= 6000000$

$\therefore S_T = C_T$

(b) For 50000 profit, we have

profit = total revenue − total cost

or, profit = $S_T - C_T$

or, $500000 = S_p * x' - (C_F + C_V)$

or, $500000 = S_p * x' - C_F - V_c * x'$

or, $500000 = 60000 * x' - 2000000 - 40000 * x'$

$\therefore x' = 125$ units.

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Note :→ **Test :** at 125 output

$S_T = 125 * 60000 = 7500000$

$C_T = 2000000 + 125 * 40000 = 7000000$

$\therefore$ profit $= 7500000 - 7000000$

$= 500000$

* **Break Even Analysis for comparing two alternatives :-**

Example :→

Consider the two motors having 100hp o/p capacity :→

Items	Motors 'A'	Motor 'B'
Purchase cost	125000	160000
Efficiency	74%	92%
Life year	10	10
Maintainance cost/year	5000	2500
Tax	$1\frac{1}{2}$% of investment for each	
MARR	15% for each	

How many hours per year would motors have to be operated for annual cost to be equal when electricity cost is Rs 5KW/hr. Which motor should be selected when expecting operation is 60 hours per year?

Soln :→

For motor A,

Capital recovery (C_R) cost $= 125000 (A/P, 15\%, 10)$

$= 24906.5$

Maintainance cost = 5000

Tax = 1.5% of 125000 = 1875

Power cost :-

Let x be operating hour per year.

$\therefore$ Power cost = input * rate * hour.

$= \frac{\text{output}}{\text{efficiency}} *$ rate * hour.

Note :→ output $= 100hp = 100 * 746 W$

$= 100 * 0.746 KW$

rate $= 5 / Kwhr$

$\eta = 74\% = 0.74$

a, Power cost $= \frac{100 * 0.746}{0.74} * 5 * x$

$= 504.05x$

$\therefore$ Annual cost of motor A,

$= (24906.5 + 5000 + 1875) + 504.05x$

$= \underbrace{31781.5}_{\text{fixed cost}} + \underbrace{504.05x}_{\text{variable cost}}$

For motor B,

Capital recovery (C_R) cost

$= 160000 (A/P, 15\%, 10)$

$= 31880.32$

Maintainance = 2500

Tax = 15% of 160000 = 2400

Power cost :→

Let x be operating hour per year

$\therefore$ Power cost $= \frac{o/p}{efficiency} * rate * hour$

$= \frac{100 * 0.746}{0.92} * 5 * x$

$= 405.43x$

$\therefore$ Annual cost of motor B,

$= (31880.32 + 2500 + 2400) + 405.43x$

$= \underbrace{36780.32}_{fixed} + \underbrace{405.43x}_{variable}$

Now,

for breaks even on basis of annual cost between motors,

$31781.5 + 504.05x = 36780.32 + 405.43x$

Solving,

$\therefore x = 51$ hours.

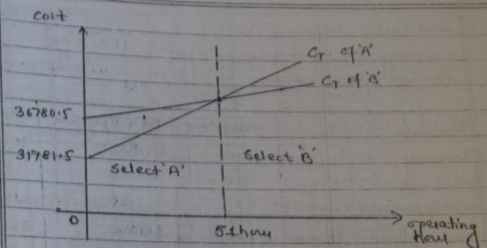

fig:- Break even analysis

6.2.3> Scenario Analysis :→

It is a technique that considers sensitivity of NPW (net present worth) due to change in key variable at a time. There may be two extreme cases in scenario analysis.

(a) Worst Case Scenario [Low sales | low price | high cost etc]

(b) Best Case Scenario [high scales | high price | low cost etc]

The NPW of each scenarios are calculated & then compared with normal or base-case scenario.

Example :-

From the following information of different variables

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and cases, calculate NPW for each scenario by assuming I = 125000, MARR = 15% and unit of demand, price, cost for five years are equal

Variable	Worst Case	Most-likely Case	Best Case
Demand	1600	2000	2400
price/unit	48	50	53
variable cost/unit	17	15	12
fixed cost/year	11000	10000	8000
salvage value	30000	40000	50000

Soln :→

Worst Case Scenario,

$$NPW(15\%) = -125000 + (1600*48 - 1600*17 - 11000)(P/A, 15\%, 5) + 30000(P/F, 15\%, 5)$$

$$= \underline{19311}$$

Most likely case scenario,

$$NPW(15\%) = -125000 + (2000*50 - 2000*15 - 10000)(P/A, 15\%, 5) + 40000(P/F, 15\%, 4)$$

$$= 96020$$

Best case scenario:-

$$NPW(15\%) = -125000 + (2400*53 - 2400*12 - 8000)(P/A, 15\%, 5) + 50000(P/F, 15\%, 5)$$

$$= 202899$$ #

6.3> Probability Concept of Economic Analysis :→

Probability information can provide the management following 2 things:

↳ A range of possible outcomes.

↳ Likelihood of achieving different goals.

Quantitative statements ~~different goals~~ about risk are given as numerical probabilities or likelihood of occurence.

Range varies from 0 to 1. Range tends to zero means very less chance of occurence & tends to one means sure for occurence.

Terms related to probability are:→

↳ Random Variable.

↳ Probability Distribution

↳ Cumulative probability Distribution.

Random Variable :-

Parameters that can have more than one possible value.

e.g:- In football game, 3 possible outcomes i.e, win, loss or draw

Two types of random variable,

↳(1) Discrete random variable [only countable values occur]

↳(2) Continuous random variable [with a certain range of interval]

Probability Distribution :→

A range of probabilities for each possible outcomes.

↳ ① Objective Probabilities [based on objective data]

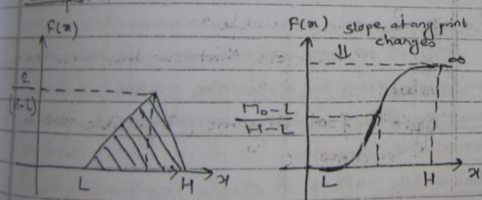
↳ ② Subjective probabilities [appropriate to possible states of nature]

* Variability of random Variable :→

a) Triangular distribution.

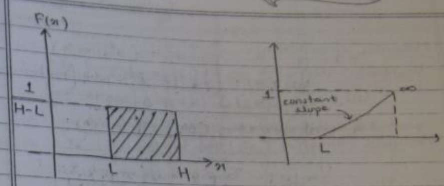
b) Uniform distribution.

Example :-



(a) Probability function (b) Cumulative probability distribution

fig:→ Triangular Probability distribution



(a) Probability function (b) Cumulative probability distribution.

fig:→ Uniform probability distribution

A common notation for cumulative distribution

$$F(x) = P(X \le x) = \begin{cases} \sum_{j=1}^{J} P_j & (\text{Discrete}) \\ \int_L^H f(x)\,dx & (\text{continuous}) \end{cases}$$

Measure of expectation:→

$$E(x) = \mu_x = \sum_{j=1} P_j X_j \quad [\text{Discrete}]$$

$$= \int_L^H x \cdot f(x)\,dx \quad [\text{continuous}]$$

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Measure of variation:→

Variance. $Var(x) = \sigma_x^2$ (Discrete)

$$Var[x] = \int_L^H (x-\mu)^2 f(x)\,dx \quad [\text{continuous}]$$

Standard deviation $(\sigma_x) = \sqrt{Var(x)}$

$$Var(x) = \sum p_j x_j^2 - \left(\sum p_j x_j\right)^2$$

$$= E(x^2) - [E(x)]^2$$

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Probability Distribution of NPW [steps]:→

↳ Express NPW as function of unknown random variables.
↳ Determine probability distribution of them
↳ Determine joint event and their probabilities.
↳ Evaluate the net present worth eqn at these joints events.

Rank NPW values in increasing order.

Probability concept in economic analysis is mainly needed in decision tree and sequential investment decisions.

6.4> Decision Tree & Sequential Investment Decision:⇒

Decision tree or decision flow diagram is a

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powerful means of facilitating the analysis of important problems, especially those that involve sequential decisions and variable outcomes over time.

A decision tree includes:→

↳ More than one stage of alternative selection.
↳ Selection of an alternatives at one stage that leads to another stage.
↳ Expected results from a decision at each stage.
↳ Probability estimates for each outcome.
↳ Estimate of economic value [cost or revenue] for each outcome.
↳ Measure of worth as the selection criterion such as E (PW).

Components of Decision Tree:→

↳ The decision tree is constructed left to right & includes each possible decision & outcome.
↳ Decision node:→
A square represents a decision node for making decision by a decision maker.
↳ Branch:→ It is a line connecting nodes from left to right of the diagram.
↳ Probability node:→ A circle represents probability node with the possible outcomes & estimated probabilities on the branches.

For evaluation & selection of alternative, following information is necessary.

- The probability that is estimated must sum to 1.0 for each set of outcomes (branches) that results from decision.
- Economic information for each decision alternative & possible outcome, such as initial investment & estimated cash-flows.

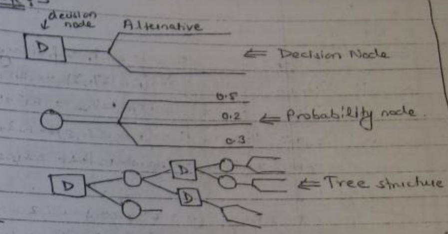
Procedures for Solving Decision Tree Using PW Analysis:→

- Start at top right of tree. Determine the PW value for each outcome branch.
- Calculate the expected value for each decision alternative

$$E(\text{decision}) = \Sigma\,(\text{outcome estimate}) \cdot P(\text{outcome})$$

- At each decision node, select best E(decision) value – minimum cost or maximum value (if both cost & revenues are estimated)
- Continue moving to the left of the tree to the root decision in order to select the best alternative.

Note:→



Examples:→

(1) A company is mechanizing its packaging plants by investing Rs 360000. Estimates for efficiency of design goals their probabilities & corresponding annual expenses saving are as follows: P.97

Design goal met (%)	Probability	Annual expenses saving (Rs)
85	0.35	40000
60	0.50	33000
50	0.15	26000

The company has another option to invest this money that earns 20% per year elsewhere. Based on Expected PW as the decision criterion determine whether the mechanizing is preferable or not? MARR = 12% & analysis period is

5 years.

Soln :→

The PW corresponding to each efficiency goals are:

$PW(12\%)_{85} = -360000 + 40000(P/A, 12\%, 5) = -Rs\ 215808$

$PW(12\%)_{60} = -360000 + 33000(P/A, 12\%, 5) = -Rs\ 241042$

$PW(12\%)_{50} = -360000 + 26000(P/A, 12\%, 5) = -Rs\ 266275$

For option 2 [Invest elsewhere that earns 20% per year]

Annual earning = $(0.2 * 360000) = Rs\ 72000$

$PW(12\%) = -360000 + 72000(P/A, 12\%, 5) = -Rs\ 100454$

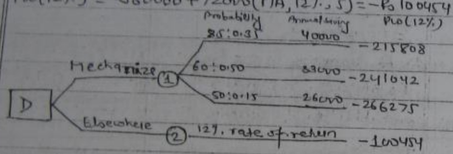


fig :- single state decision tree diagram.

Expected PW :

For option 1 :→

$E(PW) = -215808(0.35) - 241042(0.50) - 266275(0.15)$

$= -Rs\ 235995$

Option 2 :-

$E(PW) = -Rs\ 100454$

Here, analysis is indicates that none of the options are preferable & even if we make the decision, the option 2 should be selected.

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(2) A decision is needed to either market or sell a new invention. If the product is marketed, the next decision is to take it international or national. Assume details of outcomes branches in decision tree of given figure. The probability of each outcome & PW of CFBT are indicated. These payoffs are in millions of Rupees. Determine the best decision at decision node D1.

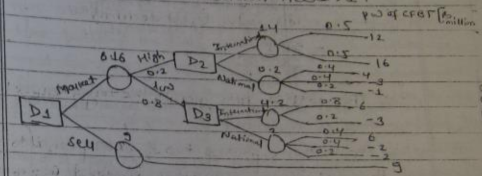

fig :- Two stage decision tree algorithm.

Soln :→

↳ Present Worth of CFBT is supplied.

↳ Calculate the expected PW values for alternatives from node D_2,

↳ Expected value (international) $= 12 * 0.5 + 16 * 0.5 = 14$

↳ Expected value (national) $= 4 * (0.4) - 3 * (0.4) - 1 * (0.2)$

$= 0.2$

↳ The expected PW values of 4.2 & 2 for D_3 are calculated in similar fashion.

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↳ select the larger expected value at each decision node. These are 14 (international) at D_2 4.4.2 (international) at D_3.

↳ Calculate expected PW for the two D_1 branches,

↳ Expected value (market decision)

$= 14 * 0.2 + 4.2 * 0.8$

$= 6.16$

↳ Expected value (sell decision)

$= 9 * 1$

$= 9$

↳ The sell decision yields the larger expected PW value of 9.

↳ The larger expected PW of CBFT path is to select the sell branch at D_1 for guaranteed Rs 900000.

(3) A business firm is considering to expand its branch in either Bhaktapur or Lalitpur. From the preliminary survey, following data have been obtained :-

	Lalitpur		Bhaktapur	
	Probability	Income (000)	Probability	Income (000)
Low success	0.2	2000	0.4	2000
Medium Success	0.4	3500	0.2	3000
High success	0.4	4000	0.4	4000

Determine which market should firm prefer when advertise expenses for

Lalitpur = 150000.

Bhaktapur = 200000

Soln :→

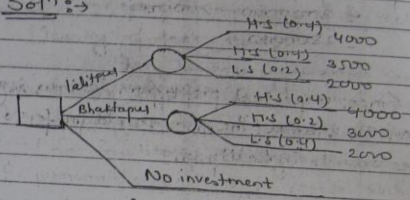

fig:- Decision tree

Note :→ □ = strategy

○ = state of nature

Analysis :→

Expected income from Lalitpur

$= 0.4 * 4000 + 0.4 * 3500 + 0.2 * 2000$

$= 3400000$

Expected profit from lalitpur

$= 3400000 - 150000$

$= 3250000$

Expected income from Bhaktapur
= 0.4 × 4000 + 0.2 * 3000 + 0.4 * 2000
= 3000000

Expected profit from Bhaktapur
= 3000000 − 200000
= 2800000

Since, expected profit from lalitpur is greater than Bhaktapur, the firm should prefer to expand the branch in Lalitpur.

CHAPTER :-> 7

Depreciation & Corporate Income Tax

7.1) Concept & Terminology of depreciation :->

Meaning of depreciation :->

Every business organization has long lived non-trading capital assets [all man made factors] like building, machine, vehicles etc. Such assets are used for facilitating trading activities and purchased for permanent use but not for resale. Such assets do not have infinite durability because on the process of use, the intrinsic value of such assets would decline & declining value of such assets is technically known as depreciation.

Hence, depreciation means fall in value of capital assets due to passage of time & physical wear & tear.

Reasons for depreciation,

1) Wear & tear.
2) Time expiration
3) Obsolescence (अप्रचलन)
4) Exhaustion or depletion
5) Accident.
6) Fall in market price.

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* Advantages (or Requirements) of Depreciation

1> To find net taxable income.

Note :→
Depreciable property is property for which depreciation is allowed under government income tax laws and regulations.

2> To replace old assets.

e.g.:→ When depreciation fund is made, that could be used to replace furniture when becomes old.

3> To find net cost of product.

4> To find net profit or loss from product.

5> To keep capital safe.

6> Fair financial positions.

* Factors affecting amount of Depreciation :→

1> Total cost of asset.

2> Estimated useful service life.

3> Estimated expenditure for repair.

4> Estimated salvage value.

5> Legal provision.

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7.2> Basic Methods For Depreciation :→

There are several methods of depreciation; the selection of method for providing depreciation depends upon types of asset, nature of asset use, management policy, legal provision etc.

7.2.1> Straight Line Method :→

According to this method, the fixed or constant amount is depreciated at end of year during life of any asset. It is also known as fixed installment depreciation. According to this method, the book value of asset will become zero at the end of life.

Note :→

$$\text{Book value} = \overset{\text{investment}}{\text{original cost}} - \text{accumulated depreciation} - \text{salvage value}$$

Salvage value = residual value [पुरानो सामान बेच्दा आएको पैसा i.e. कबाडी मूल्य]

Formula :→

Depreciation amount per period / year $[d_n]$

$$= \frac{\overset{\text{Investment}}{\text{or 'original initial cost'}}\ (I) - \text{Salvage cost}\ (S)}{\text{useful life of asset}\ (N)}$$

i.e. $\boxed{d_n = \frac{I - S}{N}}$ = constant in straight line method

depreciation for any year n

Rate of depreciation $\begin{bmatrix}\text{depreciation}\\ \text{rate}\end{bmatrix} = \frac{\text{amount of depreciation}}{\text{total depreciable value}} * 100\%$

i.e., $\boxed{Rate = \frac{1}{N} * 100\%}$

If rate is given,

$$\boxed{d_n = \text{total depreciable value} * \frac{Rate}{100}}$$

Book value at end of year is,

$$BV_K = I - D_K$$

where,

D_K = Cumulative depreciation (d_K) amount through year K.

Note: depreciable value = $\underset{\text{investment}}{I} - \underset{\text{salvage value}}{S} + \underset{\text{expenses}}{E}$

π,

Depreciation per year = annual depreciation
= depreciation amount
= d_K

Also,

Cumulative depreciation value for k^{th} year = D_K

* Advantages & Disadvantages of Straight Line Method :→

Advantages	Disadvantages
↳ simple to calculate.	↳ it is not proper to provide equal depreciation when asset is getting old.
↳ Easy to understand	↳ it ignores interest on capital invested in fixed assets.
↳ makes easy for valuation of assets.	↳ not recognized by income tax authorities.
Imp. ↳ it can write off total cost of asset completely at end of life.	

Example :→

1) A machine costing 11000 has a life of 10 years and its salvage value is estimated Rs 1000 at end of life. Calculate its depreciation amount for each year and rate of depreciation. Also find Book value for each year.

Soln :→

Original cost or investment cost (I) = 11000.

Salvage value or salvage cost (S) = 1000

Useful life (N) = 10 years.

$\therefore$ Depreciation amount (or Annual depreciation)

$$(d_K) = \frac{I - S}{N}$$

or, $$d_K = \frac{11000 - 1000}{10}$$

$\therefore$ $d_K = 1000$/year

$$\text{Rate} = \frac{1}{N} * 100\% = \frac{1}{10} * 100\% = 10\%.$$

OR

$$\text{Rate} = \frac{\text{Depreciation amount } (d_K)}{\text{Total depreciable value}} * 100\%$$

$$= \frac{1000}{1000 * 10} * 100\%$$

$$= 10\%.$$

We know,

Book value for year K $[BV_K] = I - D_K = I - (K * d_K)$

$\therefore$ BV for 1^{st} year $= 11000 - 1000 = 10000$

$\therefore$ BV for 2^{nd} year $= 11000 - (1000 * 2)$

$= 9000$

⋮

$\therefore$ BV for 10^{th} year $= 11000 - (1000 * 10)$

$= 1000$

↑

salvage value.

i.e.

Year (K)	Begining of year Book value (BV_K)	Amount of depreciation (d_K)	Cumulative d_K (D_K)
1	11000	1000	1000
2	10000	1000	2000
⋮		⋮	⋮
10	1000	1000	10000

imp (Q) 'A' has purchased a machine of Rs 200000. The installation expenses was Rs 75000 and salvage value at end of life will be Rs 25000. Find annual depreciation by using straight line method when

(a) Rate of depreciation is 20%.

(b) Life of machine is 5 years.

(c) Life is 5 years & machine was used only 3 months.

Soln :→

(a) When $R = 20\%$ per year

annual depreciation = total depreciable value $* \frac{R}{100}$

$= (\text{cost of machine} + \text{installation cost} - \text{salvage value}) * \frac{20}{100}$

$$= (200000 + 75000 - 25000) * \frac{20}{100}$$

$= $ Rs 50000

when life is 5 year,

annual depreciation = $\frac{\text{total depreciable value}}{\text{useful life}}$

$$= \frac{200000 + 75000 - 25000}{5}$$

$$= 50000$$

Depreciation for 3 months,

= annual depreciation * $\frac{\text{used period (in month)}}{\text{months in a year}}$

$$= 50000 * \frac{3}{12}$$

$$= Rs\ 12500$$

7.2.2> Declining Balance Method :→

It is also known as constant or uniform or average or fixed percentage method or Matheson method. This method assumes that the annual cost of depreciation will be fixed percentage of book value at begining of the year.

Note :→ In this method, Book value is multiplied by the fixed rate. The most commonly used multiplier is double straight line rate, for which it is known as double declining balance method.

To be studied late

To determine rate of depreciation (R),

$$\boxed{R = 1 - \sqrt[N]{\frac{S}{I}}}$$

where,

N = estimated useful life of asset,

S = Salvage value,

I = Original cost or Investment

Note :→ This formula cannot be applied if asset has no salvage value.

Formula for Matheson or declining balance method :→

$$d_K = I(1-R)^{K-1} * R$$

$$D_K = I\left[1-(1-R)^K\right]$$

$$BV_K = I(1-R)^K \qquad [\because BV_K = I - D_K]$$

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where,

d_k = annual depreciation deduction in year k,

D_k = cumulative depreciation through year k,

BV_k = Book value at end of year k.

* Advantages & disadvantages of Matheson Method:-

Advantages:→

(1) High amount of depreciation for earlier years of assets & as assets getting old, depreciation goes on diminishing is more scientific.

(2) It is suitable for assets having long life such as building, plant & machinery.

(3) It is recognized by income tax authorities.

Disadvantages:→

(1) It is difficult to decide the rate of depreciation if salvage value is not given.

(2) The book value of asset cannot be reduced to nil with this method.

(3) It is applicable only for long life assets.

(4) It ignores interest on capital invested in fixed assets.

* Double Rate Declining Balance Method:→

According to this method, depreciation rate will be calculated under straight line method & multiply it by 2. Then, the amount of depreciation is calculated by adopting declining balance method so as to bring down book value of asset to its salvage value.

Hence,

$$\text{Depreciation Rate}(R) = \frac{1}{N} * 100 * 2$$

Examples:→

(1) Use declining balance method to calculate depreciation amount for each year for following:→

A machine purchased for Rs 40000 & its salvage value is estimated Rs 10480 & useful life is 6 years.

Soln:→

$$R = 1 - \sqrt[6]{\frac{10480}{40000}} = 20\%$$

Amount of depreciation for 6 years.

Year	Book value at begining of year	Amount of depreciation @ 20%
1	40000	4000 * 20/100 = 8000
2	40000 − 8000 = 32000	32000 * 20/100 = 6400
3	32000 − 6400 = 25600	= 5120
4	= 20480	= 4096
5	= 16384	= 3276.8
6	= 13107.2	= 2521

Total depreciation = 29414

Note :- यो method मा 7th year को book value salvage value सँग equal हुनुपर्छ भन्ने छैन

A machine costing Rs 4000 is estimated to have life of 10 years. Find d_6, D_6 & BV_6 if R = 20% & there is no salvage value using Matheson method.

Soln :->

Given,

$I = Rs\ 4000$, $N = 10$ years, $R = 0.2$ & $S = 0$

$\therefore d_6 = 4000(1-0.2)^{6-1}(0.2) = Rs\ 262.144$

$\therefore D_6 = 4000[1-(1-0.2)^6] = Rs\ 2951.424$

$\therefore BV_6 = 4000(1-0.2)^6 = Rs\ 1048.576$ #

Consider the following accounting information for a computer system.

Initial cost of asset = Rs 10000.

Useful life = 5 years.

Compute the annual depreciation amount & resulting book value by using double rest declining balance method.

Soln :->

Given,

Original Cost (I) = Rs 10000.

Useful life (N) = 5 years.

Declining balance rate (R) $= \frac{1}{N} * 100 * 2$

$= \frac{1}{5} * 100 * 2$

$= 40\%$.

We can compute required annual depreciation amount & resulting book values with given information as follows :-

Year (k)	Annual depreciation amount in year k. $d_k = I(1-R)^{k-1} * R$	Book value of end of year k $BV_k = I(1-R)^k$
0	-	10000
1	4000	6000
2	2400	3600
3	1440	2160
4	864	1296
5	518.4	777.6

#

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7.2.3) Sinking fund Method :->

Sinking fund method of depreciation ensures that the full capital invested in a project is recovered at the end of project's life. This method provides fixed periodic charges (constant amount) at the end of each period that earns compound interest over the life of asset and would be equal to the original cost of asset. According to this, the book value of asset decreases at increasing rate with respect to life of asset.

Let,

I = Initial / Original Cost.

S = Salvage value.

N = Life of the asset.

i = Compound interest rate.

A = Annual equivalent amount.

BV_K = Book value at end of year 'K'.

d_K = depreciation charges in year K.

To find fixed annual equivalent amount.

$$\boxed{A = (I-S) * (A/F, i\%, N)} = (I-S) * \frac{i}{(1+i)^N - 1}$$

To find net depreciation charges in year K,

$$\boxed{\begin{aligned} d_K &= \left[(I-S) * (A/F, i\%, N)\right](F/P, i\%, K-1) \\ &= A(F/P, i\%, N) \end{aligned}}$$

To find book value at end of year / period K,

$$\boxed{\begin{aligned} BV_K &= I - A(F/A, i\%, K) \\ &= I - \left[(I-S)(A/F, i\%, N)\right](F/A, i\%, K) \end{aligned}}$$

Note :→

Sinking fund factor :→ $(A/F, i\%, N)$

$$\underset{\text{find } A}{\therefore A} = \underset{\text{Given}}{F} * \underbrace{\frac{i}{(1+i)^N - 1}}_{\text{multiply factor}}$$

Example :→

(1) A factory has purchased a machine for Rs 275000 & the expected salvage value of machine after 3 years is Rs 25000. The factory owner decided to invest the annual deprication in a bank that gives 5% interest per year.

Calculate depreciation amount by using sinking fund method.

Also compute depreciation charge & book value of each year.

Solⁿ :→

Total depreciable value at end of 3rd year,

$= I - S$

$= 275000 - 25000$

$= 250000$

Constant sinking fund depreciation amount,

= total depreciable value * sinking fund factor.

$$= 250000 * \frac{i}{(1+i)^N - 1}$$

$$= 250000 * \frac{0.05}{(1.05)^3 - 1}$$

$$= 79300$$

Now,

net depreciation for year 1,

$$d_1 = \left[(I-S)\,(A/F, i\%, N)\right](F/P, i\%, K-1)$$

$$= 79300$$

Net depreciation for year 2

$$= \left[(I-S)\,(A/F, i\%, N)\right](F/P, i\%, K-1)$$

$$= 79300 * (1+0.05)^{2-1}$$

$$= 83265$$

Net depreciation at year 3,

$$d_3 = \left[(I-S)\,(A/F, i\%, N)\right] * (F/P, i\%, K-1)$$

$$= 79300 * (1+0.05)^{3-1}$$

$$= 87428.25$$

Year (K)	Fixed annual depreciation (A)	Net depreciation (d_K)	Book value at end of year (BV_K) $= I - D_K$
0	—	—	275000
1	79300	79300	195700
2	79300	83265	112435
3	79300	87428.25	25006.75

Thus,

$BV_0 = 275000$, $BV_1 = 195700$, $BV_2 = 112435$

$BV_3 = 25006.75$ #

(2) Compute the depreciation charge and book value of each year by using sinking fund method with the following information.

Initial cost of asset = Rs 100000,

Salvage value = Rs 20000.

Life of asset = 8 years.

$i = 12\%$.

Soln :→

Fixed annual depreciation $(A) = (I-S)(A/F, i\%, N)$

$$= (100000 - 20000)\left\{\frac{i}{(1+i)^N - 1}\right\}$$

$$= \text{Rs } 6504.22$$

Net depreciation for year 1,

$$d_1 = \left[(I-S) * (A/F, i\%, N)\right] * (F/P, i\%, K-1) = A * (F/P, i\%, K-1)$$

$$= \text{Rs } 6504.22\,(1) = \text{Rs } 6504.22$$

Net depreciation for year 2,

$$d_2 = A * (F/P, i\%, K-1)$$

$$= 6504.22 * (1+0.12)^{2-1}$$

$$= \text{Rs } 7284.48$$

Book value at the end of year 1,

$$BV_1 = I - \left[(I-S) * (A/F, i\%, N)\right](F/A, i\%, K)$$

$$= I - \overset{6504.22}{A} * (F/A, i\%, K)$$

$$= 100000 - 6504.22\left[\frac{(1+0.12)^1 - 1}{0.12}\right]$$

$$= 100000 - 6504.22(1)$$

$$= \text{Rs } 93496$$

Book value at end of year 2,

$BV_2 = I - A(F/A, i\%, K)$

$= 100000 - 6504.22\left[\frac{(1+0.12)^2-1}{0.12}\right]$

$= 100000 - 6504.22\,(2.12)$

$= Rs\ 86211.52$

On continuing this process, we obtain the depreciation value & book value shown in following table:→

Year (K)	Fixed depreciation (A)	Net depreciation (d_k)	D_k	$BV_k = I - D_k$
0	—	—		100000
1	6504	6504	6504	93496
2	6504	7284.48	13788.48	86211.52
3	6504	8158.62	21947.1	78052.90
4	6504	9137.65	31084.75	68915.25
5	6504	10234.17	41318.92	58681.08
6	6504	11462.27	52781.19	47218.81
7	6504	12837.74	65618.93	34381.07
8	6504	14378.27	79997.2	20002.80

#

7.2.4 > Sum of Years Digit Method (SOYD)

Under this method, depreciation per annum is calculated on the basis of proportion of total number of estimated life years of an asset. This method results larger depreciation charges during beginning years of asset & smaller depreciation charges as asset gets old (i.e. as the asset nears the end of its estimated life). According to this method, per year depreciation charge is calculated from ratio of sum of years digit for total useful life & remaining useful life at beginning of particular year.

Formula:→

Depreciation for particular year = total depreciable value * respective proportion of year.

i.e., SOYD Depreciation $= (I-S) * \frac{\text{Remaining useful life at beginning of particular year}}{\text{SOYD for total useful life}}$

where,

I = Initial investment.

S = Salvage value.

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Example:→
A machine having a cost of Rs 36000 & estimated salvage value at the end of 5^{th} year is Rs 6000. Calculate depreciation charges per annum under sum of year digit method.

Solⁿ:→
Given,
I = Rs 36000, S = Rs 6000, N = 5 years
sum of year digit = 5+4+3+2+1 = 15
OR
$$= \frac{N(N+1)}{2} = \frac{5(5+1)}{2} = 15$$

Proportion of year = 5:4:3:2:1
Depreciation charges for particular year will be as follows.

Year	SOYD Depreciation
1	5/15 (36000 − 6000) = 10000
2	4/15 (36000 − 6000) = 8000
3	3/15 (36000 − 6000) = 6000
4	2/15 (36000 − 6000) = 4000
5	1/15 (36000 − 6000) = 2000
Total	Rs 30000

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7.2.5> Modified Accelerated Cost Recovery System [MACRS]
For better understand of MACRS method you should be familiar with below example of "Declining balance with switchover to straight line method":−

Example:→ I = 4000, N = 10 years, S = 0, R = 20% for declining balance. Calculate depreciation amount
Solⁿ:→

greater is selected ↓

Year (K)	Book Value	Declining balance method @ 20%	Comparision	Straight line method = I−S/N	Depreciation amount selected
1	4000	4000 * 20/100 = 800	>	$\frac{4000-0}{10}$ = 400	800
2	3200	3200 * 20/100 = 640	>	$\frac{3200-0}{9}$ = 355	640
3	2560	2560 * 20/100 = 512	>	$\frac{2560-0}{8}$ = 320	512
4	2048	= 409.6	>	$\frac{2048-0}{7}$ = 292	410
5	1638.4	= 327.68	>	= 273	327.68
6	1310.72	= 262.14	=	= 262.34	262.14
7	1048.58	= 209.72	<	"	" switch occurs
8	786.44	= 167.77	<	"	"
9	524.30	= 134.22	<	"	"
10	262.16	= 107.37	<	"	"

Note:→
annual depreciation amount is constant in straight line method.

#

IOE QUESTION SOLUTIONS :→

[New Course :→ After 2065 (i.e. 2066 Batch)]

2068 Bhadra (old Course)

1) (a) "Engineers play important role in economic decision". Do you agree? Discuss [6 marks]

⇒ Refer to chapter 1, section 1.3.

(b) ⇒ Not needed in new course.

2) (a) Mr. Kumar has inspected his yearly household expenses for last 10 years. Cost averages were steady at Rs 100000 per year for the first 5 years, but have increased consistently by Rs 15000 per year for each of least 5 years. Calculate total present worth in year zero. Use gradient formula. [8 marks]

Soln :→

The cash flow diagram (CFD) is,

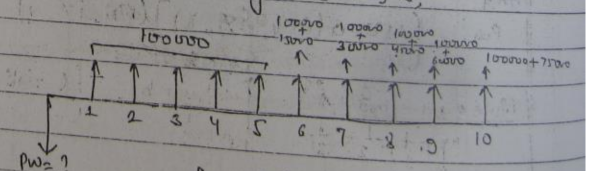
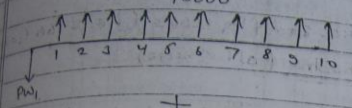


fig: CFD

For simplicity, above CFD can be written as,



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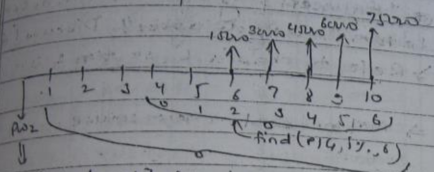


find [(P/G, i%., 6)] * (P/F, i%., 4)

Here,

$PW = PW_1 + PW_2$

Where,

find P, Given A, no. of years

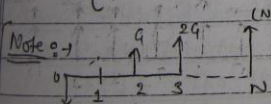
$PW_1 = 10000\,(P/A, i\%, 10)$

$= 10000\left(\frac{(1+i)^{10}-1}{i(1+i)^{10}}\right)$

And

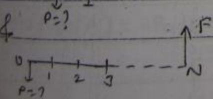
$PW_2 = \left[15000\,(P/G, i\%, 6)\right](P/F, i\%, 4)$

Note :→



$\therefore P = G\left[\frac{(1+i)^N - 1 - Ni}{i^2(1+i)^N}\right]$

&



$\therefore P = F(1+i)^{-N}$

$\therefore PW = PW_1 + PW_2$

$= 10000\left[\frac{(1+i)^{10}-1}{i(1+i)^{10}}\right] + \left[15000\left(\frac{(1+i)^6 - 1 - 6i}{i^2(1+i)^6}\right)\right] * (1+i)^{-4}$

Consider i% = 10%

i.e. i = 0.1

$PW = 10000\left[\frac{(1.1)^{10}-1}{0.1(1.1)^{10}}\right] + \left[15000\left(\frac{(1.1)^6 - 1 - 6*0.1}{(0.1)^2(1.1)^6}\right)\right] * (1.1)^{-4}$

$\therefore PW = Rs\ 206708.239$

Thus, total present worth in year zero is Rs 206708.239 #

2) (b) Use discounted payback period method to select the best option:→

	Initial Investment	Annual Income	Useful life	Salvage Value
Option A	Rs 1000000	Rs 15000	10 yrs	Rs 20000
Option B	Rs 150000	Rs 20000	12 yrs	Rs 40000

Soln :→

Say i% = 10%

Note :- How to solve?

↳ First : find payback period for option A.

↳ Second : find payback period for option B.

↳ Select option as best having least payback period

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For option A,

Period	Cash flow	PW = (cash flow)*(1+10%)-N PW of net cash flow @ 10%.	Cumulative cash flow
0	-1000000	$-1000000(1+0.1)^{-0} = -1000000$	-1000000
1	15000	$15000(1+0.1)^{-1} = 13636.363$	-9986363.637
2	15000	$15000(1+0.1)^{-2} = 12396.694$	-9973966.943
3	15000	= 11269.722	-9962697.221
4	15000	= 10245.201	-9952452.02
5	15000	= 9313.819	-9943138.20
6	15000	= 8467.108	-9934671.093
7	15000	= 7697.371	-9926973.722
8	15000	= 6997.610	-9919976.112
9	15000	= 6361.464	-9913614.848
10	15000 + 20000	= 5783.149	-9907831.5

Here, cumulative balance doesn't changes into positive in 10 years thus payback period is more than 10 years.

For option B,

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Period	cash flow	P.W. of cash flow @ 10%.	Cumulative Cash flows
0	-150000	$-150000(1.1)^{-0} = -150000$	-150000
1	20000	$20000(1.1)^{-1} = 18181.818$	-131818.182
2	20000	= 16528.925	-115289.257
3	20000	= 15026.296	-100262.961
4	20000	= 13660.269	-86602.692
5	20000	= 12418.426	-74184.266
6	20000	= 11289.478	-62894.788
7	20000	= 10263.162	-52631.626
8	20000	= 9330.147	-43301.479
9	20000	= 8481.952	-34819.527
10	20000	= 7710.865	-27108.662
11	20000	= 7009.877	-20098.785
12	20000 + 40000	= 6372.616	-13726.169

Here also cumulative balance does not changes into positive.

So, we should reject both the project (option).

But, if we have to select one option, option B is better since it has lower negative cumulative amount compared to option A when useful life will be finished.

#

3> (a) Find the IRR of the following cash-flow of a project. If MARR = 20%, comment on the acceptability of the project. Show investment balance diagram.

8 marks

End of year	Net cash-flow (in Rs)
0	−20000
1	+8000
2	+17000
3	+19000
4	+18000
5	−10000

Soln:→

By writing an eqn with given information for net PW & setting as to zero.

i.e. $PW[i'\%] = 0$

or, $-20000 + 8000[P/F, i'\%, 1] + 17000[P/F, i'\%, 2] + 19000[P/F, i'\%, 3] + 18000[P/F, i'\%, 4] - 10000[P/F, i'\%, 5] = 0$

or, $-20000 + 8000(1+i')^{-1} + 17000(1+i)^{-2} + 19000(1+i')^{-3} + 18000(1+i')^{-4} - 10000(1+i')^{-5} = 0$

Calculate this i' Rough ⟹

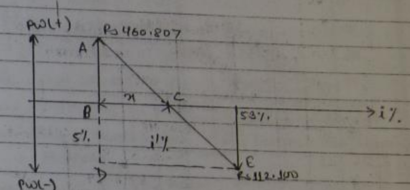
Note:→ First use computer method explained in section 3.4.2 [example 3]

i.e, $-20000 + 8000(1+X)^{-1} + 17000(1+X)^{-2} + 19000(1+X)^{-3} + 18000(1+X)^{-4} - 10000(1+X)^{-5} = 0$

Then, shift → CALC gives $X = 0.5260$, i.e. $X\% = i'\% = 52.60\%$

$\therefore PW[51\%] = Rs\ 460.807$

$PW[53\%] = -Rs\ 112.100$



Using Equilateral triangle ABC & ADE,

$$\frac{BC}{DE} = \frac{AB}{AD}$$

or, $$\frac{x\%}{53\% - 52\%} = \frac{460-807}{460.807+112.1}$$

$\therefore x\% = 0.804\%$

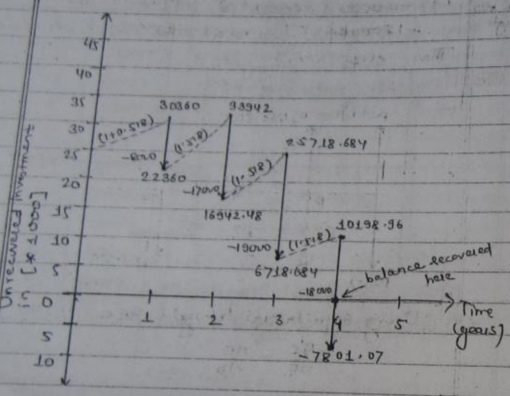
Then, $i'\% = 51\% + x\% = 51.804\%$

Here,

$MARR = 20\% < i'\% \, [51.804\%]$

So, project is accepted.

Investment balanced diagram is,



#

(b) Three mutually exclusive alternatives are to be compared by rate of return method & are describe below. MARR is 10%. Salvage value is 20% of first cost. Which option has highest IRR & what is it? Recommend the best alternative. [8 marks].

	X	Y	Z
First Cost (Rs)	70000	60000	100000
Annual Income (Rs)	15000	10000	18000
Economic life years	8	8	8

Soln :→

Calculating IRR of each project and comparing with MARR,

(i) IRR of X

$PW(i'\%) = 0$

salvage value

or, $-70000 + 15000\,(P/A, i'\%, 8) + \left[70000 + 70000 \times \frac{20}{100}\right](P/F, i'\%, 8) = 0$

or, $-70000 + 15000\left[\frac{(1+i')^8 - 1}{i'(1+i')^8}\right] + 84000\left[(1+i')^8\right] = 0$

Using Calculator,

$i' = 0.2253$, i.e, $i'\% = 22.53\%$.

Here,

$i' = 22.53\% > 10\%$ (MARR)

So, project is acceptable.

(ii) IRR of Y

$PW(i'\%) = 0$

or, $-60000 + 10000\left[\frac{(1+i')^8 - 1}{i'(1+i')^8}\right] + \left[60000 + 60000 \times \frac{20}{100}\right] \times (1+i')$ $= 0$

Using calculator,

$i'\% = 17.97\%$

Here,

$i'\% > MARR$

So, project Y is also acceptable.

(iii) IRR of Z

$PW(i'\%) = 0$

or, $-100000 + 18000\left[\frac{(1+i')^8-1}{i'(1+i')^8}\right] + \left(100000 + 100000 \times \frac{20}{100}\right) \times (1+i')^{-8} = 0$

Using Calculator,

$i'\% = 19.24\%$

Here, $i'\% > MARR$

So, project Z is also acceptable.

Performing Incremental Analysis,

↳ Select Y as base alternative since, it has lower initial or first cost.

↳ Compare Y and X (which has next higher initial cost)

↳ Calculate incremental cost & incremental benefits 'X − Y'.

↳ Calculate IRR of incremental cash flow,

i.e, $PW(i'\%)_{X-Y} = 0$

or, $-10000 + 5000(P/A, i', 8) + [(1.2 * 70000) - (1.2 * 60000)](P/F, i', 8) = 0$

or, $-10000 + 5000\left[\frac{(1+i')^8-1}{i'(1+i')^8}\right] + 12000(1+i')^{-8} = 0$

Using Calculator,

$i'\% = 50.4\%$

Here, $i'\% = 50.4\% > 10\%$ (MARR)

Thus, eliminate Y from consideration.

↳ Now, select X as base project and compare with Z.

↳ Calculating incremental cost, incremental benefits of Z − X.

i.e., incremental cost = 100000 − 70000

= Rs 30000

incremental annual income = 18000 − 15000

= Rs 3000

incremental salvage value = (1.2 * 100000) − (1.2 * 70000)

= Rs 36000

↳ Calculating IRR of incremental cash flow,

i.e, $PW(i'\%)_{Z-X} = 0$

or, $-30000 + 3000\left[\frac{(1+i')^8-1}{i'(1+i')^8}\right] + 36000(1+i')^{-8} = 0$

Using calculator,

$i'\% = 11.645\%$

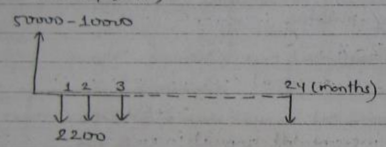
Here, $i\% = 11.645 > 10\%$ (MARR)

Thus, select Z & reject X & Y is our recommendation.

4)(a) The total purchase price of a 3 room set furniture is Rs 50000. However after a down payment of Rs 10000, two year series end of month payment of 2200 will have to be made. Determine the nominal & effective interest rate. [3+3 marks]

Soln:→

The CFD is,



Let, i_m = interest per month,

then $(50000 - 10000) = 2200 \, (P/A, i\%, 24)$

or, $40000 = 2200 * \left[\dfrac{(1+i)^{24} - 1}{i(1+i)^{24}} \right]$

Solving, $i_m = 0.0235$

$\therefore i_m\% = 2.35\%$ (per month)

So, nominal interest rate $(r\%) = 12 * 2.23\%$

$= 28.2\%$ per year

But, effective interest rate $(i\%)$ is,

$i\% = \left[(1 + i_{month})^{12} - 1 \right] * 100\%$

$= \left[(1 + 0.0235)^{12} - 1 \right] * 100\%$

$= 0.3214 * 100\%$

$= 32.14\%$ //

(b) Find acceptability of a project using both types of B/C ratio. [Use AW method]

Initial Investment = Rs 180000	Annual Benefits = 53000 at end of first year & decreases by 4000 each year
Annual Expenses = Rs 16000	
Useful life = 10 years	Salvage value = Rs 40000
	MARR = 10%

Soln:→

We know,

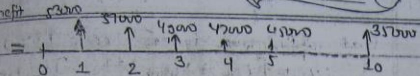
$C.R = AW(I) - AW(S)$

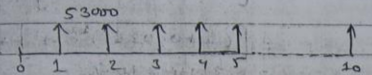
$= 180000 \, (A/P, 10\%, 10) - 40000 \, (A/F, 10\%, 10)$

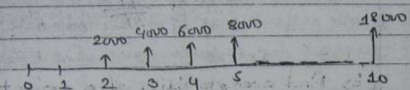
$= 180000 \left[\dfrac{0.1 \, (1+0.1)^{10}}{(1+0.1)^{10} - 1} \right] - 40000 \left[\dfrac{0.1}{(1+0.1)^{10} - 1} \right]$

$=$ Rs 26784.355

Benefit

$AW(B) =$ 

$=$ 

$-$ 

already in annual form — Gradient series with G=2000

$= 53000 - G\,[A/G, 10\%, 10]$

$= 53000 - 2000 * \left[\frac{1}{0.1} - \frac{10}{(1+0.1)^{10}-1}\right]$

$=$ Rs 45549.078

$AW(E) =$ Rs 16000
expenses

Conventional B/C ratio $= \frac{AW(B)}{C.R + AW(E)} = \frac{45549.078}{26784.355 + 16000}$

$= 1.064 > 1$

∴ Project is acceptable.

Modified B/C ratio $= \frac{AW(B) - AW(E)}{CR}$

$= \frac{45549.078 - 16000}{26784.355}$

$= 1.103 > 1$

∴ Project acceptable for both types of B/C ratio.

5>(a) Select the best project from the following 2 projects. [Use Repeatability & PW method]

	Project A	Project B
Initial Cost (Rs)	150000	180000
Annual Expenses (Rs)	35000	31000
Annual Revenues (Rs)	8500	10500
Salvage Value (Rs)	50000	80000
Useful life	6 years	9 years
MARR	10%	10%

Sol^n:

Study period = L.C.M of 6 & 9

$= 3 \,\underline{|\, 6, 9}$
$\quad\; 2, 3$

= 18 years.

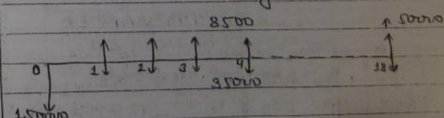


fig: CFD for project A

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$PW_A(10\%) = -150000 + (8500 - 35000)(P/A, 10\%, 18) + 50000\,(P/F, 10\%, 18)$

$= -150000 + (-26500) \times \dfrac{(1+0.1)^{18} - 1}{0.1\,(1+0.1)^{18}} + 50000 \times (1+0.1)^{-18}$

$= -Rs\,358344.4812 < 0$

∴ Project A is not acceptable.

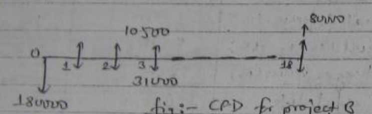


fig:- CFD for project B

$PW_B(10\%) = -180000 + (10500 - 31000)(P/A, 10\%, 18) + 80000\,(P/F, 10\%, 18)$

$= -180000 + (-20500)\left[\dfrac{(1+0.1)^{18} - 1}{0.1\,(1+0.1)^{18}}\right] + 80000 \times (1+0.1)^{-18}$

$= -Rs\,333740.2449 < 0$

∴ Project B is also not acceptable.

But, if we have to select one among project A & B, project B with less negative PW (present worth) is selected as best project.

N^2 3(b) Two types of power converters, alpha & beta are under construction for a specific application. An economic comparision is to be made at an interest rate of 12% and the following cost estimates have been obtained. Select best option by calculating PW of both projects if it will be operated if it will be operated for 4 years only. [8 marks]

	Alpha	Beta
Purchase price (Rs)	750000	2000000
Annual operating cost (Rs)	200000	100000
Estimated service life (yrs)	5	9
Salvage Value (Rs)	0	400000

Soln :-

For Alpha :- Study period [4 yrs] < useful life [5 yrs]

So, we need to calculate imputed market value at end of 4th year [IMV_4]

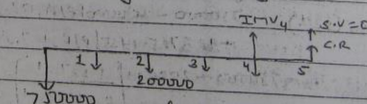


fig:- Initial CFD for alpha

We know,

$IMV_4 = PW_{CR}(12\%) + PW_{MV}(12\%)$

$= C.R.\,(P/A, 12\%, 1) + SV\,(P/F, 12\%, 1)$

5th year को C.R. लाई 4th year मा ल्याएको

5th year को S.V. लाई 4th year मा ल्याएको

where,

$$C.R = (Investment)_A - (SV)_A$$
$$= 750000(A|P, 12\%, 5) - 0(A, F, 12\%, 5)$$
$$= 750000 * \frac{0.12(1+0.12)^5}{(1+0.12)^5 - 1}$$
$$= Rs\ 208057.299$$

$$\therefore IMV_4 = 208057.299(P/A, 12\%, 1) - 0$$
$$= 208057.299 \left[\frac{1.12 - 1}{0.12(1.12)}\right]$$
$$= Rs\ 185765.4455$$

The revised CFD for alpha is,

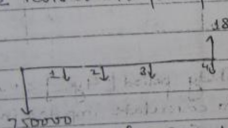
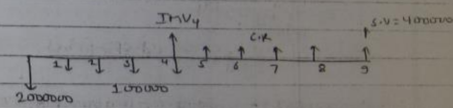


fig:- Revised CFD for alpha

$$\therefore PW_{alpha}(12\%) = -750000 - 200000(P/A, 12\%, 4) + 185765.4455(P/F, 12\%, 4)$$
$$= -750000 - 200000\left[\frac{1.12^4 - 1}{0.12(1.12)^4}\right] + \frac{185765.4455}{(1+0.12)^4}$$
$$= -Rs\ 1239412.57$$

Again,

For Beta, :- study period [4yrs] < useful life [9yrs]
so we need to calculate imputed market value at end of 4th year (IMV_4).



$$C.R = 2000000(A|P, 12\%, 9) - 400000(A, F, 12\%, 9)$$
$$= 2000000 * \frac{0.12(1+0.12)^9}{(1+0.12)^9 - 1} - 400000 * \left(\frac{0.12}{(1+0.12)^9 - 1}\right)$$ P.121
$$= Rs\ 348286.222$$

9th year को sv लाई 4th year मा ल्याएको

$$PW_{mv}(12\%) = 400000(P|F, 12\%, 5)$$
$$= 400000 * \frac{1}{(1+0.12)^5}$$
$$= Rs\ 226970.7423$$

Also,

"5,6,7,8,9" yrs को C.R. लाई 4th year मा ल्याएको

$$PW_{C.R.}(12\%) = C.R.(P|A, 12\%, 5)$$
$$= 348286.222 * \left[\frac{(1.12)^5 - 1}{0.12(1.12)^5}\right]$$
$$= Rs\ 1255493.885$$

$\therefore Inv_4 = PW_{c.R}(12\%) + PW_{M.V}(12\%)$

$= Rs\ 1482464.627$

The revised CFD for beta is,

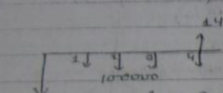


fig:- Revised CFD for beta

$\therefore PW_{beta}(12\%) = -2000000 - 100000(P/A, 12\%, 4)$
$+ 1482464.627 (P/F, 12\%, 4)$

$= -2000000 - 100000 * \left[\dfrac{(1.12)^4 - 1}{0.12(1.12)^4}\right] + 1482464.627 \left[\dfrac{1}{1.12^4}\right]$

$= -Rs\ 1361601.864$

Since, project alpha has less negative PW. Select alpha as best project among alpha and beta #

6) (a) ⇒ Not needed in new course.

(b) The purchase of a rental property is being considered in a neighbourhood where real estate prices are increasing rapidly. The following estimates have been developed for a preliminary before-tax analysis :-

First cost, Rs	Annual income from rent, Rs	Annual maintenance Rs	Investment period	Resale value	MARR
140000.	30000	7500	6 yrs	150000	10%

Construct sensitivity chart for variation within a ± 30% range of each annual income & MARR. Indicate acceptance & rejection zones. [10 marks]

Soln :→

Here,

$PW(10\%) = -140000 + (30000 - 7500)(P/A, 10\%, 6) + 150000 (P/F, 10\%, 6)$

$= -140000 + 22500 * \dfrac{(1+0.1)^6 - 1}{0.1(1+0.1)^6} + 150000 * \dfrac{1}{(1+0.1)^6}$

$= Rs\ 42664.455$

(i) Changing annual income by ± 30%.

$PW(10\%) = -140000 + 30000(1 \pm 30\%)(P/A, 10\%, 6) - 7500 (P/A, 10\%, 6) + 150000 (P/F, 10\%, 6)$

at +30% change,

$PW(10\%) = -140000 + 30000 * (1+0.3) * \dfrac{1.1^6 - 1}{.1(1.1)^6} - 7500 \times \dfrac{1.1^6 - 1}{.1(1.1)^6} + \dfrac{150000}{(1.1)^6}$

$= Rs\ 81861.801$

at -30%. change,

$PW(10\%) = -140000 + [30000 * 0.7 - 7500] (P/A, 10\%, 6) + 150000 (P/F, 10\%, 6)$

$= Rs\ 3467.108$

(ii) Changing MARR by ± 30%.

initial ~~cost~~ MARR = 10%.

at +30% change in MARR,

i.e, MARR = (10 * 1.3) % = 13%.

$$\therefore PW(13\%) = -140000 + (30000 - 7500)(P/A, 13\%, 6) + 150000(P/F, 13\%, 6)$$

$$= -140000 + 22500 * \frac{1.13^6 - 1}{0.13(1.13)^6} + \frac{150000}{(1.13)^6}$$

= Rs 21992.649

at −30% change in MARR,

i.e, MARR = (10 * 0.7%) = 7%.

$$\therefore PW(7\%) = -140000 + (30000 - 7500)(P/A, 7\%, 6) + 150000(P/F, 7\%, 6)$$

$$= -140000 + 22500 * \frac{1.07^6 - 1}{0.07(1.07)^6} + \frac{150000}{(1.07)^6}$$

= Rs 67198.475

Parameter \ Change %	+30%	0%	−30%
annual income (A)	81861.801	42664.455	3467.108
MARR	21952.649	42664.455	67198.475

Sensitivity Diagram

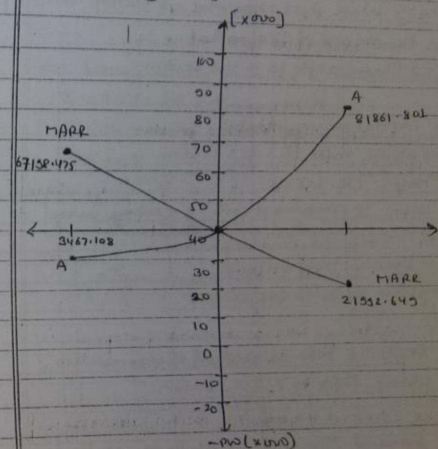


fig:- Sensitivity diagram

Note that project is accepted for variation of each annuan income (A) & MARR upto ± 30% because PW doesnot get to −ve value //

Q.No 7> Write short Notes on [each of 4 marks]

(a) Drawbacks of IRR method.

⇒ Please refer to 3.4.1 section.

(b) Capital recovery cost.

⇒ please refer to 3.3.2 section.

[Note: CR = I − S

where, I = investment cost, S = salvage value]

(c) Decision tree analysis.

⇒ please refer to 6.4 section

(d) Declining balance method of depreciation

⇒ please refer to 7.2.2 section

[(e) Demand Analysis :→ Not in new course]

2069 Bhadra [New Course]

1> Define Engineering economy. Enlist principles of engineering economy. [1+3 marks]

Soln :→

Engineering economics is the application of economic techniques to evaluation of design & engineering alternatives.

Following are the principles of engineering economics to accomplish a good engineering economy study :→

↳ Develop the alternatives.

↳ Focus on the difference.

↳ Use a consistent viewpoint.

↳ Use a common units of measure.

↳ Consider all relevant criteria.

↳ Make risk & uncertainty explicits.

↳ Revisit your decision.

(2) Ramesh, a civil engineer is planning to place a total of 20% of his salary, which is Rs 250000 per year now, each year in the mutual fund. He expects 7% salary increase each year for next 15 years. If the mutual fund will average 10% annual return,

what will be sum - amount at the end of 15 years? If salary increases by Rs 25000 per year. What will be amount? [4+4]

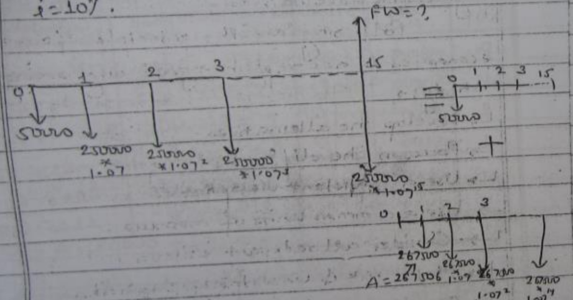
Soln :→

(a) When salary increases by 7% per year,

$I = 20\%$ of $250000 = \text{Rs } 50000$

$g = 7\%$ [Note:- g = Geometric Gradient Series]

$i = 10\%$.



$FW(10\%) = 50000\,(F/P, i\%, N) + A\,(P/A, g, i, N)\,(F/P, i, N)$

$= 50000\,(F/P, 10\%, 15) + 267500\,(P/A, 7\%, 10\%, 15)\,(F/P, 10\%, 15)$

$$= 50000 \times (1+0.1)^{15} + \frac{267500}{0.1-0.07} \times \left[1-\left(\frac{1+0.07}{1+0.1}\right)^{15}\right] \times (0.1+1)^{15}$$

$= 208862.4085 + 12645764.94$

$= \text{Rs } 12854627.35$

(b) When salary increases by Rs 25000 per year

$I = 20\%$ of $250000 = \text{Rs } 50000$

$G = \text{Rs } 25000$ [Note: G = gradient amount]

$i = 10\%$

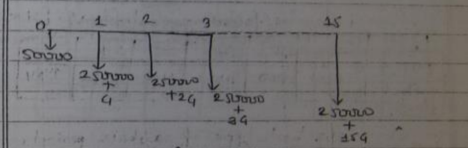


fig:- CFD

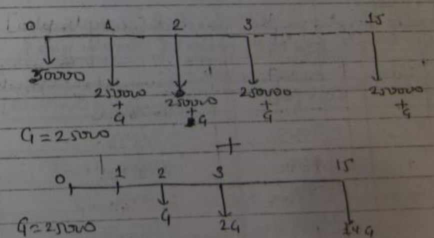


fig:- Equivalent CFD.

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$\therefore FW(10\%) = 500000(F/P, 10\%, 15) + (250000+G)(F/A, 10\%, 15) + G(F/G, 10\%, 15)$

$= 500000 * (1+0.1)^{15} + 275000 * \frac{(1+0.1)^{15}-1}{0.1}$

$+ \frac{25000}{0.1} * \left[\frac{(1+0.1)^{15}-1}{0.1}\right] - \frac{15 * 25000}{0.1}$

$=$ Rs 5990606.917 #

3) (a) From the following cash flow:- [4 marks]

EOY	0	1	2	3	4	5
cash flow	-3000	800	1000	1100	1210	1464

Calculate both type of payback period.

MARR = 10%.

Soln:-

(i) Simple payback period (simple pbp)

EOY	Cash flow	Cumulative Cash flow (Rs)
0	-3000	-3000
1	800	-2200
2	1000	-1200
3	1100	-100
4	1210	1110
5	1464	2574

Here, by interpolating period of 3 and 4 year,

Simple pbp (θ) $= 3 + \frac{100}{1210} = 3.08$ years.

pbp → payback period

(ii) Discounted pbp

EOY	Cash flow	Discounted Cash flow to present = cash flow * $(1+10\%)^{-N}$	Cumulative cash flow (Rs)
0	-3000	$-3000(1+0.1)^{-0} = -3000$	-3000
1	800	$800(1+0.1)^{-1} = 727.272$	-2272.728
2	1000	= 826.446	-1446.282
3	1100	= 826.446	-619.836
4	1210	= 826.446	206.61
5	1464	= 909.028	1115.638

Here, cumulative cash flows turns to positive in year 4. Therefore, payback period (pbp) lies in between year 3 & 4.

By interpolating,

Discounted pbp (θ') $= 3 + \frac{619.836}{826.446}$

$= 3.75$ years #

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(b) Equipment cost 250000 & has salvage value of 50000 at end of its expected life 5 years. Annual expenses will be 40000. It will produce a revenue of 120000 per year, MARR = 20% = E

[4+4+4] marks.

(i) Evaluate IRR using AW formulation.

(ii) Evaluate both types of B/C ratio with FW formulation

(iii) Find ERR.

Soln :-

Given, $I = 250000$, $S = 50000$, $N = 5$ years

$E = 40000$, $A = 120000$, MARR = 20%.

(i) Finding IRR, [using AW formulation]

$AW[i'\%] = 0$

or, $120000 - 40000 + 50000\,(A/F, i'\%, 5) - 250000\,(A/P, i'\%, 5) = 0$

or, $80000 + 50000 * \dfrac{x}{(1+x)^5 - 1} - 250000 * \dfrac{x(1+x)^5}{(1+x)^5 - 1} = 0 \quad [x = i']$

Solving,

$x = 3.2024 = i'$;

i.e., IRR $(i'\%) = 320.24\%$.

(ii) Finding B/C ratio [using FW formulation]

$FW(I) = 250000\,(F/P, 20\%, 5)$

$= 250000 * (1+0.2)^5$

$= 622080$

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$FW(S) = S = 50000$

$FW(A) = (120000)\,(F/A, 20\%, 5)$

$= 120000 * \left[\dfrac{(1+0.2)^5 - 1}{0.2}\right]$

$= 892992$

$FW(E) = 40000 * (F/A, 20\%, 5)$

$= 595328$

Using Conventional Method,

$$B/C \text{ ratio} = \frac{FW(A)}{FW(I) - FW(S) + FW(E)}$$

$$= \frac{892992}{622080 - 50000 + 595328}$$

$$= 0.764$$

Using modified method,

$$B/C \text{ ratio} = \frac{FW(A) - FW(E)}{FW(I) - FW(S)}$$

$$= \frac{892992 - 595328}{622080 - 50000}$$

$$= 0.52$$

(iii) Finding ERR,

Discounting all cash outflows to time zero @ 20%.

$= 250000 + 40000\,(P/A, 20\%, 5)$

$= 250000 + 40000 * \dfrac{(1+0.2)^5 - 1}{0.2(1+0.2)^5}$

$=$ Rs 369624.485

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Compounding all cash inflow to year 5 @ 20%.

$= 50000 + 120000 (F/A, 20\%, 5)$

$= 50000 + 120000 \left[\frac{(1+0.2)^5 - 1}{0.2} \right]$

$= Rs\ 942992$

Establishing equivalence betn 2 quantities,

$369624.485 (F/P, i'\%, 5) = 942992$

or, $(1+i')^5 = 2.5512$

or, $i' = 0.2060$

$\therefore i'\% = 20.60\%$

i.e. ERR = 20.60%.

#

4) From following information select best project:

	Project A	Project B
Initial Investment	35000	50000
Annual Revenue	16450	25000
Annual Costs	3000	13830
Useful life	4 years	8 years
Salvage value at end of useful life	0	0

MARR = 10%.

When services period required is,

(i) 4 years by FW method [4 marks]

(ii) 8 years by IRR method with PW formulation [8 marks]

Soln :→

(i) When study period (or service period) = 4 years

For project A,

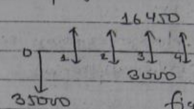

fig:- CFD of project A.

$FW_A(10\%) = -35000 (F/P, 10\%, 4) + (16450 - 3000) (F/A, 10\%, 4)$

$= -35000 * (1+0.1)^4 + 13450 * \frac{(1+0.1)^4 - 1}{0.1}$

$= Rs\ 11177.95$

For project B,

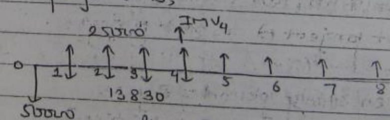

fig:- Initial CFD of project B.

Imputed market value $[IMV_4] = PW_{CR}(10\%) + PW_{MV}(10\%)$

$C.R.(10\%) = 50000 (A/P, 10\%, 8) - 0$

$= Rs\ 9372.2$

PW (at year 4) of CR for remaining 4 years,

$\therefore PW_{CR}(10\%) = 9372.2\ (P/A, 10\%, 4)$

$= Rs\ 29708.615$

present worth (at year 4) of market value

$PW_{MV}(10\%) = 0$ [$\because$ Salvage value $= 0$]

$\therefore IMV_4 = Rs\ 29708.615$

Then, revised cash flow diagram becomes.

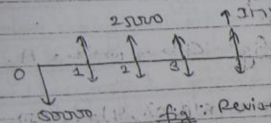

fig: Revised CFD for project B.

$FW_B(10\%) = -50000\ (F/P, 10\%, 4) + (25000 - 13830)\ (F/A, 10\%, 4) + 29708.615$

$= Rs\ 8343.585$

Since, $FW_A(10\%) > FW_B(10\%)$

Select project A as best project.

(ii) When study period = 8 years.

For project A [Repeated 2 times]

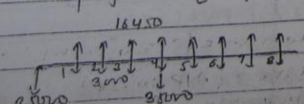

fig:- CFD of project A for 8 years.

$PW_A(i'\%) = 0$

or, $-35000 + (16450 - 3000)(P/A, i'\%, 8) - 35000\ (P/F, i'\%, 4) = 0$

(say $i' = x$),

or, $-35000 + 13450 \times \frac{(1+x)^8 - 1}{x(1+x)^8} - \frac{35000}{(1+x)^4} = 0$

Solving,

$x = 0.19$

i.e, $i'\% = x\% = 19\% > MARR(10\%)$

So, project A is acceptable.

For project B,

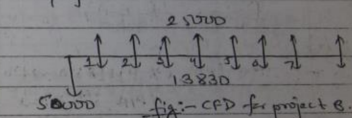

fig:- CFD for project B.

$PW_B(i'\%) = 0$

or, $-50000 + (25000 - 13830)(P/A, i'\%, 8) = 0$

(say $i' = x$),

or, $-50000 + 11170 \times \frac{(1+x)^8 - 1}{x(1+x)^8} = 0$

Solving,

$x = 0.1507$

i.e, $i'\% = x\% = 15.77\% > MARR(10\%)$

So, project B is also acceptable.

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Since both project A & B are acceptable in terms of IRR, we should perform incremental analysis for decision.

Taking project A as base alternative (having lower investment)

we find CFD for B-A,

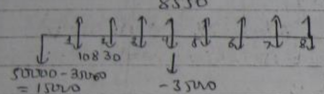

fig:- CFD for B-A.

$PW_{B-A}(i'\%) = 0$

or, $-15000 - (-35000)(P/F, i'\%, 4) + (8550 - 10830)(P/A, i'\%, 8) = 0$

(say $i' = x$),

or, $-15000 + 35000 * \frac{1}{(1+x)^4} + (-2280) * \frac{(1+x)^8 - 1}{x(1+x)^8} = 0$

Solving,

$x = 0.0317$

i.e., $IRR\% = i'\% = x\% = 3.17\% < MARR(10\%)$

So, reject project B.

Hence, project A is selected as best projects.

#

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5) What is the economic service life of an asset? Find the economic service life of a new electric lift truck which costs \$20000, have operating cost of \$1000 in first year & have salvage value of \$12000 at end of first year. For the remaining year operating costs increase each year by 10% over the previous year's operating costs. Similarly, the salvage value declines each year by 20% from the previous year's salvage value. The lift truck has a maximum life of 7 years. An overhaul costing of \$3000 & \$5000 will be required during the fifth & seventh year of service respectively. The firm's required rate of return is 15% per year. [10+2 marks]

Soln :- The economic service life of an asset is defined as period of useful life that minimizes the AEC of owning and operating the asset.

To determine economic service life (N), we need to find annual eq. cost [AEC].

$AEC(i\%) = CR(i\%) + OC(i\%)$

where,

$CR(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N)$

& $OC(i\%) = \left[\sum_{n=1}^{N} OC_n (P/F, i\%, n)\right](A/P, i\%, N)$

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Let N = 1,

$I = \$20000$, $E = \$1000$ per year, $S = \$12000$

$$CR(15\%) = 20000(A|P, 15\%, 1) - 12000(A|F, 15\%, 1)$$

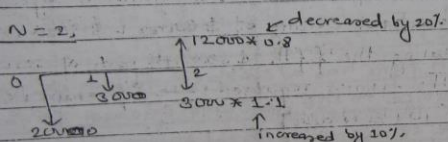
$$= 20000 * \frac{0.15*(1+0.15)^1}{(1+0.15)^1 - 1} - 12000 * \frac{0.15}{(1+0.15)^1 - 1}$$

$$= 11000$$

$OC(15\%) = 1000$

$\therefore AEC_1(15\%) = 11000 + 1000 = \$12000$.

Let N = 2,



$$CR(15\%) = 20000(A|P, 15\%, 2) - (12000 * 0.8)(A|F, 15\%, 2)$$

$$= 7837.209$$

$$OC(15\%) = [3000(P|F, 15\%, 1)](A|P, 15\%, 2) + (3000 * 1.1)(A|F, 15\%, 2)$$

OR

$$OC(15\%) = [3000(F|P, 15\%, 1) + (3000 * 1.1)](A|F, 15\%, 2)$$

$$= 3139.534$$

$\therefore AEC_2(15\%) = 10976.743$

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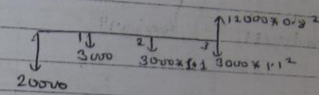
let N = 3,



$$CR(15\%) = 20000(A|P, 15\%, 3) - (12000 * 0.8^2)(A|F, 15\%, 3)$$

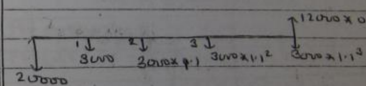
$$= 6547.876$$

$$OC(15\%) = [3000(F|P, 15\%, 2) + (3000 * 1.1)(F|P, 15\%, 1) + (3000 * 1.1^2)](A|F, 15\%, 3)$$

$$= 3280.777$$

$\therefore AEC_3(15\%) = 9828.653$

Let N = 4,



$$CR(15\%) = 20000(A|P, 15\%, 4) - (12000 * 0.8^3)(A|F, 15\%, 4)$$

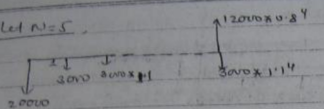
$$= 5774.876$$

$$OC(15\%) = [3000(F|P, 15\%, 3) + (3000 * 1.1)(F|P, 15\%, 2) + (3000 * 1.1^2)(F|P, 15\%, 1) + (3000 * 1.1^3)](A|F, 15\%, 4)$$

$$= 3423.411$$

$\therefore AEC_4(15\%) = 9198.287$

Let N=5,



$CR(15\%) = 20000(A|P, 15\%, 5) - (12000 * 0.8^4)(A|F, 15\%, 5)$

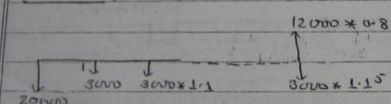
$= 5237.31$

$OC(15\%) = [3000 * 1.15^4 + 3000 * 1.1 * 1.15^3 + 3000 * 1.1^2 * 1.15^2 + 3000 * 1.1^3 * 1.15^1 + 3000 * 1.1^4](A|F, 15\%, 5)$

$= 3567.11$

$\therefore AEC_5 = 8804.422$

Let N=6,



$CR(15\%) = 20000(A|P, 15\%, 6) - (12000 * 0.8^5)(A|F, 15\%, 6)$

$= 4835.54$

$OC(15\%) = 3000[1.15^5 + 1.1 * 1.15^4 + 1.1^2 * 1.15^3 + 1.1^3 * 1.15^2 + 1.1^4 * 1.15 + 1.1^5](A|F, 15\%, 6)$

$= 4017.682$

$\therefore AEC_6 = 8853.222$

Among calculated AEC for N = 1, ---- 6, we find AEC(12%) is smallest when N=5 after which AEC starts increasing. So, we conclude that economic service life of electric lift truck is 5 years even though it's maximum life is 7 years.

5) (OR)

A firm has a contract to provide printing service to IOE for next 8 years. It can provide the service using its old printing machine (the current defender) or newly bought machine (the challenger). After the contract work neither old machines will be retained. Considering annual equivalent costs of old machine & new machines as follows, what are their economic service life? And what is the best replacement strategy?

Number of year (n)	Annual Equivalent Cost [AEC] (Rs)	
	Old Machine	New machine
1	515000	750000
2	510000	615000
3	550000	586000
4	596000	583000
5	644000	590000

[2+10] marks

Soln :→ Economic service life of old machine is year 2 & for new machine is year 4.

Many replacement scenario options would fulfill 8 year planning horizon. Some likely replacement patterns under a finite planning horizon of 8 years be :→

Option 1 :→ $(J_0, 0)$ $(J, 4)$ $(J, 4)$

" 2 = $(J_0, 1)$ $(J, 4)$ $(J, 3)$

" 3 = $(J_0, 2)$ $(J, 4)$ $(J, 2)$

" 4 = $(J_0, 3)$ $(J, 4)$ $(J, 1)$

" 5 = $(J_0, 3)$ $(J, 5)$

" 6 = $(J_0, 4)$ $(J, 4)$

Consider MARR = 15% per year.

For option 1

$$PW(15\%)_1 = 583000\,(P/A, 15\%, 8)$$

$$= 583000 \left[\frac{(1.15)^8 - 1}{0.15\,(1.15)^8}\right]$$

$$= Rs\ 2616108.439$$

For option 2

$$PW(15\%)_2 = 515000\,(P/F, 15\%, 1) + [583000\,(P/A, 15\%, 4)]\,(P/F, 15\%, 1) + [586000\,(P/A, 15\%, 3)]\,(P/F, 15\%, 5)$$

$$= 515000 * 0.869 + 583000 * 2.855 * 0.8696 + 586000 * 2.2832 * 0.4972$$

$$= Rs\ 2580185.089$$

For option 3

$$PW(15\%)_3 = 510000\,(P/A, 15\%, 2) + [583000\,(P/A, 15\%, 4)]\,(P/F, 15\%, 2) + [615000\,(P/A, 15\%, 2)]\,(P/F, 15\%, 6)$$

$$= Rs\ 2519922.532$$

For option 4,

$$PW(15\%)_4 = 550000\,(P/A, 15\%, 3) + [583000\,(P/A, 15\%, 4)]\,(P/F, 15\%, 3) + [750000\,(P/A, 15\%, 1)]\,(P/F, 15\%, 7)$$ P.133

$$= Rs\ 2555354.606$$

For option 5,

$$PW(15\%)_5 = 550000\,(P/A, 15\%, 3) + [530000\,(P/A, 15\%, 5)]\,(P/F, 15\%, 3)$$

$$= Rs\ 2556190.685$$

For option 6,

$$PW(15\%)_6 = 596000\,(P/A, 15\%, 4) + [583000\,(P/A, 15\%, 4)]\,(P/F, 15\%, 4)$$

$$= Rs\ 2653223.158$$

Here, an economic evaluation of present

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equivalent cost of planning horizon for 8 years indicates that least cost soln among the options appear to be option 3,
i.e., (J0,2), (J,4), (J,2) which means that use old machine (defender) for 2 years then replace it by challenger such that new machine (challenger) is used upto it's service life (4 years) & finally again new machine (challenger) is used for remaining 2 years.
#

P.134

6) (a) Calculate breakeven volume of a cable manufacturing company from the following data:
Total cost = Rs 1200000;
variable cost = Rs 400000
income from sales = 1500000 at production of 5000 units

Soln :->
Given,

C_T = Rs 1200000
C_V = Rs 400000
S_T = Rs 1500000

production quantity (x') = 5000 units
Breakeven volume = ?

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We know,

$$V_c = \frac{C_V}{x'}$$
$$= \frac{400000}{5000}$$
$$= \text{Rs } 80$$

Selling price per unit (S_P) = $\frac{S_T}{x'}$
$$= \frac{1500000}{5000}$$
$$= \text{Rs } 300.$$

We have,
$$C_T = C_V + C_F$$
or, C_F = 1200000 − 400000
$\therefore$ C_F = Rs 800000

$\therefore$ Break even volume, $x' = \frac{C_F}{S_P - V_C}$
$$= \frac{800000}{300-80}$$
$$= 3636.36$$
$\simeq$ 3637 units
#

(b) A proposal is described by the following estimates: P = \$ 20000; S = 0, N = 5 & net annual receipts = \$ 7000. A rate of return of 20 percent is described on such proposals. Construct a sensitivity graph of P, life, annual receipts & rate of return for deviations over a range of ± 20 percent. To which element is the decision most sensitive? [8 marks]

Soln :→

Given,

I = P = \$ 20000

A = \$ 7000

S = 0

N = 5 years

MARR = 20%.

The original eqn to find PW @ i = 20%. is,

$PW(20\%) = -20000 + 7000\,(P/A, 20\%, 5) \rightarrow (1)$

$= -20000 + 7000 * \left[\frac{(1+0.2)^5 - 1}{0.2\,(1+0.2)^5}\right]$

$= -20000 + 7000 * 2.99$

= Rs 934.284

(a) When I varies ± 20% PW will be,

At I = +20%. [i.e, $I = 20000 + 20000 \times \frac{20}{100} = 20000 \times 1.2$]

$PW(20\%) = -20000 \times 1.2 + 7000 \times 2.99$

= -Rs 3070

At I = -20%. [i.e, $I = 20000 - 20000 \times \frac{20}{100} = 20000 \times 0.8$]

$PW(10\%) = -20000 * 0.8 + 7000 * 2.99$ ← (P/A, 20%, 5)

= Rs 4930

(b) When A varies by ±20%, PW will be

At A = +20% change

$PW(10\%) = -20000 + (7000 * 1.2) * 2.99$

= Rs 5116

At A = -20% change

$PW(10\%) = -20000 + (7000 * 0.8) * 2.99$

= - Rs 3256

(c) When useful life N varies by ± 20%.

At N = + 20% change i.e, N = 5 * 1.2 = 6 years.

$PW(10\%) = -20000 + 7000\,(P/A, 20\%, 6)$

= Rs 3278.57

At N = -20% change, i.e, N = 5 * 0.8 = 4 years

$PW(10\%) = -20000 + 7000\,(P/A, 20\%, 4)$

= - Rs 1878.858

(d) When MARR change by ± 20%.

At MARR = + 20% change. i.e. MARR = (20 * 1.2)
= 24%.

$PW(24\%) = -20000 + 7000\,(P/A, 24\%, 5)$
$= -Rs\ 782.309$

At MARR = −20% change, i.e. MARR = 20×0.8 = 16%.

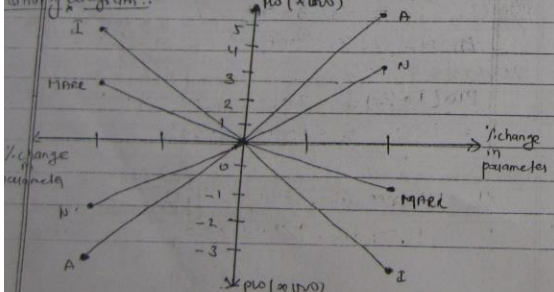
$$PW(16\%) = -20000 + 7000 \times \frac{(1+0.16)^5 - 1}{0.16(1+0.16)^5}$$

$= Rs\ 2920.055$

Calculation Table :→

Parameter \ % change	− 20%	0%	+ 20%
P or I	4930	934.284	−3070
A	−3256	934.284	5116
N	−1878.858	934.284	3278.57
MARR	2920.055	934.284	−782.309

...sitivity diagram :-



Note :→
How to find which is more sensitive ?

Tricks :→

1st way :- To find which is more sensitive?

sensitivity of I = |−3070 − 4930| = 8000

" " A = |5116 − (−3256)| = 8372

" " N = |3278.57 − (−1878.858)| = 5157.428

" " MARR = |−782.309 − 2920.055| = 3702.364

Here, highest value is of sensitivity of A i.e. A is more sensitive.

2nd way :-

Look at sensitivity graph, more stepper (ठाडो) is more sensitive.

In our case 'A' is more sensitive as it is more stepper in sensitivity diagram.

On the basis of above calculation & diagram, annual revenue (A) parameter of project is more sensitive as compared to other parameters.

7) (a) Define depreciation & list out important methods of calculating depreciation deductions [4 marks].

⇒ Soln :- Depreciation means fall in value of capital assets due to passage of time & physical wear & tear.

The list of important methods of calculating depreciations deductions are :→

(i) Straight line method.
(ii) Declining balance method.
(iii) Sinking fund method.
(iv) Sum of year digit method.
(v) Modified Accelerated Cost Recovery System [MACRS].

(b) A machine costs Rs 15000. Its useful life is 5 years & salvage value is Rs 900, Compute the annual depreciation allowances & resulting book values using double declining balance depreciation methods.

Soln :→

The depreciation rate is calculated as,

$$R = \frac{1}{N} * 100 * 2 = \frac{1}{5} * 100 * 2 = 40\%.$$

Now,

We can compute the required annual depreciation amount & resulting book values as,

Year (K)	Annual depreciation amount in year K $d_K = I(1-R)^{K-1} * R$	Book value of end of year K $BV_K = I(1-R)^K$
0	—	$15000(1-0.4)^2 = 15000$
1	$15000(1-0.4)^{1-1} * 0.4 = 6000$	$= 9000$
2	$= 3600$	$= 5400$
3	$= 2160$	$= 3240$
4	$= 1296$	$= 1944$
5	$= 777.6$	$= 1166.4$

8) (a) Define inflation. List out its effects. If the inflation rate is 15% per year & market interest rate is 13% per year. What is implied interest (inflation free) rate in inflationary economy? [1+1+2] P.137

Soln :→

Inflation is a state in which the value of money (purchasing power) is falling i.e, market prices are rising.

The effects of inflation are :→

↳ Inflation distorts the financial system of the country.

↳ Inflation cause decreased purchasing power of the dollar & its depreciation.

Given,

rate of inflation (f) = 15%.

market interest rate (MARR) = 13%.

implied or inflation free rate (i') = ?

We know

$MARR = (1+i')(1+f) - 1$

or, $0.13 = (1+i')(1+0.15) - 1$

or, $\frac{1.13}{1.15} = 1 + i'$

or, $0.9826 = 1 + i'$

or, $i' = -0.01739$

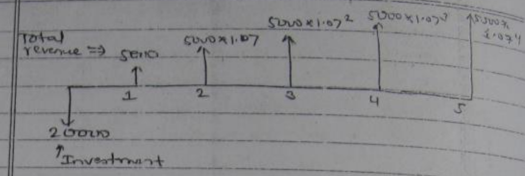
$\therefore i'\% = -1.739\%$ #

OR

(a) A series of five constant dollar (or real dollar) income (beginning with \$ 5000 at the end of the first year) are increasing at the rate of 7% per year for five years. Inflation free interest rate is 5% & inflation is 8%. Is it feasible investment if investment cost is \$ 20000? [4 marks]

⟹ Soln:-

Given problem can be formulated in a cash flow diagram as,



Inflation free rate (i') = 5%.

inflation rate (f) = 8%.

Then, we know,

$MARR = (1+f)(1+i') - 1$

$= (1+0.08)(1+0.05) - 1$

$= 0.134$

$\therefore MARR\% = 13.4\%$.

Total revenue in PW formulation will be,

$PW(13.4\%) = 5000(P/F, 13.4\%, 1) + (5000 * 1.07)(P/F, 13.4\%, 2) + 5000 * 1.07^2 (P/F, 13.4\%, 3) + 5000 * 1.07^3 (P/F, 13.4\%, 4) + 5000 * 1.07^4 (P/F, 13.4\%, 5)$

$=$ Rs 19693.956

Here,

investment (Rs 20000) > Total revenue (Rs 19693.956)

Thus, investment is not feasible. #