Electromagnetic Waves

Maxwell's Equations (Integral Form):-

Maxwell discovered that the basic principle of electromagnetism can be expressed in term of four equations, though he developed a concept of displacement current. Maxwell did not discover all of these equation himself but he put them together and recognize their significance particularly in predicting the existence of electromagnetic wave. These equations are called Maxwell equations which are described below.

1. Gauss's law for electrostatics:-

It States that "The total flux through a closed surface inclosing a charge q is equal to $\frac{1}{\varepsilon_0}$ times the magnitude of charge enclosed.

$$i.e. \oint \vec{E}.\,\vec{dA} = \frac{q}{\varepsilon_0}$$

This relates the electric field and charge distribution. It conform the existence of single charge.

2. Gauss's law for magnetism:-

It states that "The total magnetic flux through a closed surface is zero".

$$i.e. \oint \vec{B}.\,\vec{dA} = 0$$

It conforms that the magnetic monopole doesn't exist.

3. Faraday's law of induction:-

It states that "The induced emf in the circuit is equal to the rate of change of magnetic field."

$$i.e. \oint \vec{E}.\,\vec{dl} = \frac{-d\phi_B}{dt}$$

it says that changing magnetic field with time produces an electric field.

4. Ampere Maxwell law:-

It is the modification of Ampere's law by Maxwell. It describe the there are at least two ways of setting a magnetic field:

i).By means of steady current (from Ampere's law):-

$$i.e. \oint \vec{B}.\,\vec{dl} = \mu_0\, I$$

ii).By means of changing electric field (from Maxwell law of induction):-

$$i.e. \oint \vec{B}.\,\vec{dl} = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

The combined form of these two equations is Ampere Maxwell law.

$$i.e. \oint \vec{B}.\,\vec{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

$$\oint \vec{B}.\,\vec{dl} = \mu_0 (I + \varepsilon_0 \frac{d\emptyset_E}{dt})$$

$$\oint \vec{B}.\,\vec{dl} = \mu_0 (I + I_d)$$

Where $I_d = \varepsilon_0 \frac{d\emptyset_E}{dt}$ is called displacement current.

Maxwell's equation in differential Form:-

1. Maxwell's first equation:-

From gauss law of electrostatic we have

$$i.e. \oint \vec{E}.\,\vec{dA} = \frac{q}{\varepsilon_0}$$

$$i.e. \oint \vec{E}.\,\vec{dA} = \frac{1}{\varepsilon_0} \oint \rho dV \qquad (\because \rho = \frac{q}{dV})$$

Now using Gauss- Divergence theorem

$$\oint (\nabla.\vec{E}) dV = \frac{1}{\varepsilon_0} \oint \rho dV \quad \text{Where, } Gradient(\nabla) = \frac{\partial}{\partial \mathrm{x}} + \frac{\partial}{\partial \mathrm{y}} + \frac{\partial}{\partial \mathrm{z}}$$

$$\therefore \nabla.\vec{E} = \frac{\rho}{\varepsilon_0}$$

Where ρ is charge density.

2. Maxwell's second equation:-

From Gauss law of magnetism we have

$$\oint \overrightarrow{B}.\,\overrightarrow{dA} = 0$$

Now using Gauss- Divergence theorem

$$\oint (\nabla.\vec{B})dV = 0$$

$$\therefore \nabla.\overrightarrow{B} = 0$$

3. Maxwell's third equation:-

According to Faraday's law of electromagnetic induction we have;

$$\oint \overrightarrow{E}.\,\overrightarrow{dl} = \frac{-d\emptyset_B}{dt}$$

$$\oint \overrightarrow{E}.\,\overrightarrow{dl} = \frac{-d}{dt}\oint \overrightarrow{B}.\,\overrightarrow{dA}$$

Now using Curl Stokes theorem;

$$\oint (\nabla \times \vec{E})\overrightarrow{dA} = \frac{-d}{dt}\oint \overrightarrow{B}.\,\overrightarrow{dA}$$

$$\therefore \nabla \times \overrightarrow{E} = \frac{-d\vec{B}}{dt}$$

4. Maxwell's fourth equation:-

According to Ampere Maxwell law we have

$$\oint \overrightarrow{B}.\,\overrightarrow{dl} = \mu_0(I + I_d)$$

Where displacement current $I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt} \oint \vec{E}. \overrightarrow{dA}$

And real current $I = \oint \vec{J}. \overrightarrow{dA}$

$$\therefore \oint \vec{B}. \vec{dl} = \mu_0(\oint \vec{J}. \overrightarrow{dA} + \varepsilon_0 \frac{d}{dt} \oint \vec{E}. \overrightarrow{dA})$$

$$\oint \vec{B}. \vec{dl} = \mu_0[\oint (\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt})] . \overrightarrow{dA}$$

Now using Curl Stokes theorem;

$$\oint (\nabla \times \vec{B}) \overrightarrow{dA} = \mu_0[\oint (\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt})] . \overrightarrow{dA}$$

$$\therefore (\nabla \times \vec{B}) = \mu_0[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}]$$

Electromagnetic wave equation in free space:-

In free space the charge density ρ and current density J are zero. Therefore the Maxwell's equation becomes

$$\nabla . \vec{E} = 0 \ldots \ldots \ldots (1)$$

$$\nabla . \vec{B} = 0 \ldots \ldots \ldots (2)$$

$$\nabla \times \vec{E} = \frac{-d\vec{B}}{dt} \ldots \ldots \ldots (3)$$

$$\left(\nabla \times \vec{B}\right) = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} \dots \dots \dots (4)$$

Taking the curl of equation (3)

$$\nabla \times \nabla \times \overrightarrow{E} = \frac{-d(\nabla \times \vec{B})}{dt}$$

$$\nabla\left(\nabla . \vec{E}\right) - \vec{E}(\nabla . \nabla) = -\frac{d}{dt}(\mu_0 \varepsilon_0 \frac{d\vec{E}}{dt})$$

$$0 - \nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{d^2 \vec{E}}{dt^2} \dots \dots \dots (5)$$

Similarly taking the curl of equation (4) and proceeding in similar ways, we get;

$$\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{d^2 \vec{B}}{dt^2} \dots \dots \dots (6)$$

Equation (5) and (6) are the equation of electromagnetic wave .now comparing this equation with general wave equation

$$i.e. \nabla^2 \vec{Y} = \frac{1}{v^2} \frac{d^2 \vec{Y}}{dt^2}$$

$$\therefore v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\therefore v = 3 \times 10^8 \, m/sec$$

This is the same as the velocity of light in vacuum.

Electromagnetic wave equation in non- conducting (dielectric) medium:-

In non-conducting medium the charge density ρ and current density J are zero. If the medium have permittivity ε and permeability μ then the Maxwell's equation becomes

$$\nabla.\overrightarrow{E} = 0 \ldots\ldots\ldots(1)$$

$$\nabla.\overrightarrow{B} = 0 \ldots\ldots\ldots(2)$$

$$\nabla \times \overrightarrow{E} = \frac{-d\vec{B}}{dt} \ldots\ldots\ldots(3)$$

$$\left(\nabla \times \vec{B}\right) = \mu\varepsilon\frac{d\vec{E}}{dt} \ldots\ldots\ldots(4)$$

Taking the curl of equation (3)

$$\nabla \times \nabla \times \overrightarrow{E} = \frac{-d(\nabla \times \vec{B})}{dt}$$

$$\nabla\left(\nabla.\vec{E}\right) - \vec{E}(\nabla.\nabla) = -\frac{d}{dt}(\mu\varepsilon\frac{d\vec{E}}{dt})$$

$$0 - \nabla^2\vec{E} = -\mu\varepsilon\frac{d^2\vec{E}}{dt^2}$$

$$\nabla^2 \vec{E} = \mu\varepsilon \frac{d^2\vec{E}}{dt^2} \dots\dots\dots(5)$$

Similarly taking the curl of equation (4) and proceeding in similar ways, we get;

$$\nabla^2 \vec{B} = \mu\varepsilon \frac{d^2\vec{B}}{dt^2} \dots\dots\dots(6)$$

Equation (5) and (6) are the equation of electromagnetic wave. Now comparing this equation with general wave equation

$$i.e. \nabla^2 \vec{Y} = \frac{1}{v^2}\frac{d^2\vec{Y}}{dt^2}$$

$$\therefore v = \frac{1}{\sqrt{\mu\varepsilon}}$$

This is required relation for velocity of wave in non- conducting medium.

Electromagnetic wave equation in conducting (isotropic) medium:-

For a conducting medium the charge density ρ and current density J, the Maxwell's equation becomes

$$\nabla . \vec{E} = \frac{\rho}{\varepsilon} \dots\dots\dots(1)$$

$$\nabla . \overrightarrow{B} = 0 \dots\dots\dots (2)$$

$$\nabla \times \overrightarrow{E} = \frac{-d\vec{B}}{dt} \dots\dots\dots (3)$$

$$\left(\nabla \times \vec{B}\right) = \mu(\vec{J} + \varepsilon \frac{d\vec{E}}{dt}) \dots\dots\dots (4)$$

Taking the curl of equation (3)

$$\nabla \times \nabla \times \overrightarrow{E} = \frac{-d(\nabla \times \vec{B})}{dt}$$

$$\nabla\left(\nabla . \vec{E}\right) - \vec{E}(\nabla . \nabla) = -\frac{d}{dt}(\nabla \times \vec{B})$$

$$\nabla\left(\nabla . \vec{E}\right) - \nabla^2 \vec{E} = -\frac{d}{dt}(\nabla \times \vec{B})$$

For the medium of constant charge density $\nabla\left(\nabla . \vec{E}\right) = 0$

$$\therefore 0 - \nabla^2 \vec{E} = -\frac{d}{dt}[\mu(\vec{J} + \varepsilon \frac{\overrightarrow{dE}}{dt})]$$

$$or,\ \nabla^2 \vec{E} = \frac{d}{dt}[\mu(\sigma\vec{E} + \varepsilon \frac{\overrightarrow{dE}}{dt})] \qquad \text{Where, } \vec{J} = \sigma\vec{E}$$

$$\therefore \nabla^2 \vec{E} = \mu\sigma \frac{\overrightarrow{dE}}{dt} + \mu\varepsilon \frac{d^2\vec{E}}{dt^2} \dots\dots\dots (5)$$

Similarly taking the curl of equation (4) and proceeding in similar ways, we get;

$$\therefore \nabla^2 \vec{B} = \mu\sigma \frac{\overrightarrow{dB}}{dt} + \mu\varepsilon \frac{d^2\vec{B}}{dt^2} \dots\dots\dots (6)$$

Equation (5) and (6) are electromagnetic waves in conducting medium.

Displacement current:-

The Ampere Maxwell law is

$$\oint \vec{B}.\ \vec{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\emptyset_E}{dt} \ldots\ldots\ldots (1)$$

If we compare the two terms on right side of this equation it is seen that the product $\varepsilon_0 \frac{d\emptyset_E}{dt}$ must have the dimension of current. This product is considered as fictitious current associated with changing electric field E between the plates of capacitor is called the displacement current (I_d).

We can now rewrite Ampere- Maxwell law as;

$$\oint \vec{B}.\ \vec{dl} = \mu_0 (I + I_d) \ldots\ldots\ldots (2)$$

$$\text{where}, I_d = \varepsilon_0 \frac{d\emptyset_E}{dt}$$

The charge stored in parallel plate capacitor is $q = CV$

$$or, q = \frac{\varepsilon_0 A}{d}.V \quad \text{where}, C = \frac{\varepsilon_0 A}{d}$$

Where, A is the area of each plate and d is the distance between them.

$$\therefore q = \varepsilon_0 A.\frac{V}{d} = \varepsilon_0 AE$$

$$\therefore \text{real current } (I) = \frac{dq}{dt} = \varepsilon_0 A\frac{dE}{dt} \ldots\ldots\ldots (3)$$

Here displacement current is;

$$I_d = \varepsilon_0 \frac{d\emptyset_E}{dt} = \varepsilon_0 \frac{d}{dt}(EA)$$

$$\therefore \quad I_d = \varepsilon_0 A\frac{dE}{dt} \ldots\ldots\ldots (4)$$

From equation (3) and (4), it is seen that, the real current I during charging and discharging of the capacitor is equal to displacement current I_d between plates.

So we can consider the fictitious displacement current simply to be a continuation of the real current through the capacitor.

Energy transport and Poynting vector (S):-

The magnitude of poynting vector is defined as the rate of energy transport per unit area in plane electromagnetic wave.

$$i.e.\ S = \frac{1}{A}\frac{dU}{dt}$$

Consider a propagation of electromagnetic wave in a box of area A and thickness dx at any instant; the energy stored in box is given by;

$$dU = dU_E + dU_B$$

$$= \mu_E A dx + \mu_B A dx$$

Where, $\mu_E = \frac{1}{2}\varepsilon_0 E^2$ is the energy density in electric field and $\mu_B = \frac{1}{2}\frac{B^2}{\mu_0}$ is the energy density in magnetic field.

$$\therefore dU = (\mu_E + \mu_B)Adx$$

$$dU = (\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0})Adx$$

$$= (\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{E^2}{C^2\mu_0})Adx$$

$$= (\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{\mu_0\varepsilon_0 E^2}{\mu_0})Adx$$

$$\therefore dU = \varepsilon_0 E^2 Adx$$

$$\therefore S = \varepsilon_0 E^2 \ \frac{\mathrm{dx}}{\mathrm{dt}}$$

$$\therefore S = C\varepsilon_0 E^2 \ldots\ldots\ldots (1)$$

$$S = C\varepsilon_0 E.BC = c^2\epsilon_0 E.B$$

$$or\ S = \frac{1}{\mu_0\varepsilon_0}\varepsilon_0 E.B$$

$$\therefore S = \frac{E.B}{\mu_0}$$

In vestor form above relation can be written as, $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$

The intensity of electromagnetic wave is defined as the average of Poynting vector.

$$\therefore I = \frac{E_0 B_0}{2\mu_0}$$

Radiation pressure (P_r):-

Electromagnetic waves have linear momentum as well as energy. This means electromagnetic waves can exert a pressure when incident on an object. The force per unit area on an object due to electromagnetic radiation is called radiation pressure.

Let a beam of electromagnetic radiation of intensity I is incident on an object of area A perpendicular to the path of the radiation. The object is free to move and the energy (ΔU) of radiation is entirely absorb by the object in time interval(Δt). The energy absorb by area A is;

$$\Delta U = IA\Delta t$$

The change in momentum (ΔP) of the object related to the change in energy (ΔU) is given by;

$$\Delta P = \frac{\Delta U}{C} = \frac{IA\Delta t}{C}$$

Also from Newton's second law a change in momentum is related to a force as;

$$F = \frac{\Delta P}{\Delta t} = \frac{IA}{C}$$

$$\therefore \frac{F}{A} = \frac{I}{C}$$

Therefore radiation pressure $(P_r) = \frac{I}{C}$

Instead of being absorbed if the radiation is entirely reflected back along its original path, the change in momentum is given by;

$$\Delta P = \frac{2\Delta U}{C}$$

Proceeding in similar way, the radiation pressure in this case can be calculated as;

$$(P_r) = \frac{2I}{C}$$

If the radiation is partially reflected and partially absorbed, the radiation pressure is in between the value of $\frac{I}{C}$ and $\frac{2I}{C}$.

Relation between E_0 and B_0 $i.e. C = \frac{E_0}{B_0}$:-

We have third Maxwell equation as;

$$\nabla \times \overrightarrow{E} = \frac{-d\vec{B}}{dt}$$

In one dimension it can be expressed as

$$\frac{d\vec{E}}{dx} = \frac{-d\vec{B}}{dt} \dots \dots \dots (1)$$

Science, $E = E_0 \sin(kx - wt)$

Now differentiating this equation with respect to x

$$\frac{dE}{dx} = E_0 \, k \cos(kx - wt) \dots \dots \dots (2)$$

And, $B = B_0 \sin(kx - wt)$

Now differentiating this equation with respect to t

$$\frac{dB}{dt} = (-w)B_0 \cos(kx - wt) \dots \dots \dots (3)$$

Now from equation (1), (2) and (3);

$$E_0 \, k \cos(kx - wt) = wB_0 \cos(kx - wt)$$

$$\therefore \frac{E_0}{B_0} = \frac{w}{k} = C$$

Charge conservation theorem (Continuity- equation):-

or, $\boldsymbol{\nabla.\vec{J} + \frac{d\rho}{dt} = 0}$

We have fourth Maxwell equation as;

$$(\nabla \times \vec{B}) = \mu_0[\vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt}]$$

Taking divergence on both sides;

$$\nabla.(\nabla \times \vec{B}) = \mu_0[\nabla.\vec{J} + \varepsilon_0 \frac{d(\nabla.\overrightarrow{E})}{dt}]$$

Since divergence of curl of any vector is zero

$i.e. \nabla.(\nabla \times \vec{B}) = 0$

and we have; $\nabla.\overrightarrow{E} = \frac{\rho}{\varepsilon_0}$

$$so, \qquad 0 = \nabla.\vec{J} + \varepsilon_0 \frac{d\frac{\rho}{\varepsilon_0}}{dt}$$

$$\therefore \nabla.\vec{J} + \frac{d\rho}{dt} = 0$$

Numerical Examples:-

1. **Calculate the magnitude of the Poynting vector and the amplitude of the electric and magnetic fields at a distance of 10 cm from a radio station which is radiating power of 10^5 watt uniformly over a hemisphere with radio station as center.**

Solution:-

Power of the source $(\mathrm{P}) = 10^5$ watt

Radius of the hemisphere $(\mathrm{r}) = 10$ cm $= 0.01$ m

Magnitude of Poynting vector $(\mathrm{S}) = ?$, $\mathrm{E_0} = ?$ and $\mathrm{B_0} = ?$

$$\mathrm{I} = \frac{\mathrm{P}}{2\pi r^2} = \frac{10^5}{2 \times \pi \times 0.1^2} = \frac{10^7}{2\pi}\ \mathrm{W/m^2}$$

$$\text{We have, } \mathrm{I} = \frac{\mathrm{E_0^2}}{2\mu_0 \mathrm{C}}$$

$$\therefore\ \mathrm{E_0} = \sqrt{2 \times 4\pi \times 10^{-7} \times 3 \times 10^8 \times \frac{10^7}{2\pi}}$$

$$= 3.5 \times 10^4\ \mathrm{V/m}$$

$$\text{Similary, } \mathrm{B_0} = \frac{\mathrm{E_0}}{\mathrm{C}} = \frac{3.5 \times 10^4}{3 \times 10^8} = 1.2 \times 10^{-4}\ \mathrm{T} = 120\ \mu\mathrm{T}$$

$$\text{And, magnetude of Poynting vector } (\mathrm{S}) = \frac{\mathrm{E_0 B_o}}{\mu_0}$$

$$= \frac{3.5 \times 10^4 \times 1.2 \times 10^{-4}}{4\pi \times 10^{-7}}$$

$$= 3.34\ \times 10^6\ \mathrm{W/m^2}$$

2. Using the Pyonting vector calculate the maximum electric and magnetic fields for sun-light if the solar constant is 1.4 KW/m².

Solution:-

$$\text{Given ; } I = 1.4\ \text{KW/m}^2 = 1400\text{W/m}^2$$

$$\text{we have, } I = \frac{E_0 B_o}{\mu_0} \text{ and } B_0 = \frac{E_0}{C}$$

$$\therefore I = \frac{CB_0^2}{2\mu_0}$$

$$\text{or, } B_0 = \sqrt{\frac{2I\mu_0}{C}} = \sqrt{\frac{2 \times 1400 \times 4\pi \times 10^{-7}}{3 \times 10^8}}$$

$$\therefore, B_0 = 3.4 \times 10^{-6}\ \text{T} = 3.4\ \mu\text{T}$$

$$\text{And, } E_0 = CB_0 = 3 \times 10^8 \times 3.4 \times 10^{-6}$$

$$= 1020\ V/m$$

3. Calculate the displacement current between the square plates of capacitor having one side 1 cm if the electric field between the plate is changing at the rate of 3×10^6 V/meter-sec.

Solution:-

Here length of one side of plate $(a) = 1$ cm

$\therefore$ Area of the plate $(A) = a^2 = 1\ \text{cm}^2 = 1 \times 10^{-4}\ \text{m}^2$

And $\frac{dE}{dt} = 3 \times 10^6$ V/ms

We have, $I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(EA) = \varepsilon_0 A \frac{dE}{dt}$

$$= 8.85 \times 10^{-12} \times 1 \times 10^{-4} \times 3 \times 10^{6}$$

$\therefore \quad I_d = 2.655 \times 10^{-9}$ A

4. **A parallel plate capacitor has capacitance 20 μF. At what rate the potential difference between the plates must be changed to produce a displacement current of 1.5 A?**

   Solution:-

   Here, $C = 20\ \mu F = 20 \times 10^{-6}\ F$

   $I_d = 1.5$ A

   $\frac{dE}{dt} =?$

   We have,

$$I_d = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\varepsilon_0 A}{d} \frac{dV}{dt}$$

$$or,\ I_d = C \frac{dV}{dt}$$

$$\therefore\ \frac{dV}{dt} = \frac{I_d}{C} = \frac{1.5}{20 \times 10^{-6}} = 7.4 \times 10^{4}\ \text{Volts/sec}$$

5. **What is the displacement current for a capacitor having radius 5 cm with variable electric field $\mathbf{8.9 \times 10^{12}} \boldsymbol{Volts/ms}$?**

Solution:

Here, $r = 5cm = 5 \times 10^{-2}m,\ \frac{dE}{dt} = 8.9 \times 10^{12} Volts/ms$

We have, $I_d = \varepsilon_0 A \frac{\mathrm{dE}}{\mathrm{dt}} = \varepsilon_0 \pi r^2 \frac{\mathrm{dE}}{\mathrm{dt}}$

$$I_d = 8.85 \times 10^{-12} \times \pi \times (5 \times 10^{-2})^2 \times 8.9 \times 10^{12}$$
$$= 0.62\ A$$

6. **An observer is 1.8 m from an isotropic point light source whose power is 250 watt. Calculate the *rms* values of the electric and magnetic fields due to the source at the position of observer.**

Solution:

Here, $r = 1.8\ m, P = 250\ watt, E_{rms} = ?\,, B_{rms} = ?$

We have, Intensity, $I = \frac{P}{A}$

Also for e.m. wave, $I = \frac{E_0 B_0}{2\mu_0}$

$$\frac{E_0 B_0}{2\mu_0} = \frac{P}{4\pi r^2}$$

$$\frac{CB_0^2}{2\mu_0} = \frac{P}{4\pi r^2}$$

$$B_0 = \sqrt{\frac{2\mu_0 P}{4\pi r^2 C}} = \sqrt{\frac{2 \times 4\pi \times 10^{-7} \times 250}{4\pi \times 1.8^2 \times 3 \times 10^8}}$$

$$= 2.27 \times 10^{-7}\ T$$

$$E_0 = CB_0 = 3 \times 10^8 \times 2.27 \times 10^{-7} = 68.04\ V/m$$

$$E_{rms} = \frac{E_0}{\sqrt{2}} = 48.11\ V/m$$

$$B_{rms} = \frac{B_0}{\sqrt{2}} = 1.61 \times 10^{-7}\ T$$

7. A parallel plate capacitor with circular plates of 10 cm radius is charged producing uniform displacement current of magnitude 20 A/m². Calculate (i) Calculate the curl of magnetic flux density between the plates (ii) Calculate the rate of change of electric field in this region.

Solution:

$$\text{Here, } Radius(r) = 10\ cm \Rightarrow r = 0.1\ m$$

$$\text{Current density } (J_d) = 20\ A/m^2$$

$$\therefore \text{ Displacement urrent } (I_d) = J_d \times \pi r^2 = 0.62\ A$$

$$(i)\text{We have, } \left(\nabla \times \vec{B}\right) = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt}$$

$$\nabla \times \vec{B} = \frac{\mu_0}{A} A\varepsilon_0 \frac{d\vec{E}}{dt} = \frac{\mu_0 I_d}{A} = \mu_0 J_d$$

$$= 4\pi \times 10^{-7} \times 20$$

$$= 2.512 \times 10^{-5}\ Wb/m^2$$

$$(ii)\ Again,\ I_d = \varepsilon_0 \mathrm{A} \frac{\mathrm{dE}}{\mathrm{dt}}$$

$$\frac{\mathrm{dE}}{\mathrm{dt}} = \frac{\mathrm{I_d}}{\varepsilon_o \mathrm{A}} = \frac{\mathrm{J_d}}{\varepsilon_o} = \frac{20}{8.85 \times 10^{-12}} = 2.26 \times 10^{12}\ \mathrm{V/ms}$$

8. **Calculate the radiation pressure at the surface of the earth and sun assuming the solar constant has value of 2 cal/min cm^2 at the surface of the earth. Given, radius of sun is $7 \times 10^8\ m$ and the average distance between earth and sun is $1.5 \times 10^{11}\ m$.**

   Solution:-

   [$Hint;\ i)\ P_1 = \frac{I_1}{c} = 4.67 \times 10^{-6}\ \mathrm{N/m^2}$

   And $ii)\ \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$ gives I_2

   Then we can get, $P_2 = 0.214\ N/m^2$

   Where, $\mathrm{r_1} = 1.5 \times 10^{11}$ m and $\mathrm{r_2} = 7 \times 10^8$]

Exercise:-

1. What are Maxwell's equations? Using Maxwell's equations derive electromagnetic wave equation in dielectric medium. Prove that em wave travels with velocity less than velocity of light in such medium.
2. Write Maxwell's equation in differential form in free space. Derive electromagnetic equations in vacuum. Find their plane wave solution.
3. Define Poynting vector. Prove that $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$, where the symbols have their usual meaning.
4. State Maxwell's equation in integral form. Convert them in to differential form. Explain each of these equations.

5. Write and explain Ampere's law in magnetism. How Maxwell modified it. Based on this modified equation, explain the term displacement current. Prove displacement current is equal to conduction current.
6. Write Maxwell's equation in free space and dielectric medium. With the help of Maxwell's equations, Derive charge conservation theorem.
7. Obtain an expression for energy transfer rate by electromagnetic wave, from your result show that $I \propto E_{rms}^2$. Where I is the intensity of em wave and E_{rms} is root mean square value of electric field.
8. What is Poynting vector? Show that the intensity of an electromagnetic wave equals the average magnetic energy density times the speed of light.
9. Define Poynting vector and develop an expression of it in terms of electric and magnetic fields.
10. The maximum electric field 10 m from an isotropic point source of light is 2 V/m. What are (a) the maximum value of the magnetic field and (b) the average intensity of light there? (c) What is the power of the source?
11. A certain plane electromagnetic wave emitted by a microwave antenna has a wavelength of 3 cm and a maximum magnitude of electric field of $2 \times 10^{-4} V/cm$. i) What is frequency ii) What is the maximum magnetic field and iii) What is the Poynting vector?
12. The sun delivers about 10^3 W/m^2 of energy of the earth's surface through EM radiation calculate a) The total

power incident on a roof of dimension $8\text{m} \times 20\text{m}$, b) radiation pressure and force exerted on the roof , assuming roof is perfect absorber.

13. A radio wave transmits 25 W/ m^2 of power per unit area. The flat surface area is perpendicular to the direction of propagation of wave. Calculate the radiation pressure on it and maximum electric and magnetic field associated with the wave.

14. A parallel plate capacitor with circular plates of 20 cm radius is charged producing uniform displacement current of magnitude 20 A/m^2. Calculate (i) dE/dt in the region (ii) displacement current density and (iii) Induced magnetic field.

15. Calculate the displacement current between the capacitor plates of area $1.5 \times 10^{-2}\ \text{m}^2$ and rate of electric field change is $1.5 \times 10^{-12}\ \text{V/ms}$. Also calculate displacement current density.

16. If a parallel plate capacitor with circular plate be charged, prove that the induced magnetic field at a distance r in the region between the plates be $B = \frac{1}{2}\mu_0\epsilon_0 r\frac{dE}{dt}$ for $r \leq R$ and $B = \frac{1}{2}\frac{\mu_0\epsilon_0 R^2}{2r}\frac{dE}{dt}$ for $r \geq R$.