Maths Assignment (1)

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(1) $\int \frac{\cos x}{\sin^2 x + 4\sin x + 5} \, dx$

$\int \frac{\cos x}{\sin^2 x + 2 \cdot \sin x \cdot 2 + 4 - 4 + 5} \, dx$

$\int \frac{\cos x \, dx}{(\sin x + 2)^2 + 1}$

$\int \frac{\cos x \, dx}{(\sin x + 2)^2 + 1^2}$

let $\sin x + 2 = y$

diff w.r.t. x

$\Rightarrow \cos x \, dx = dy$

$= \int \frac{dy}{y^2 + 1}$

$= \tan^{-1} y + C$

$= \tan^{-1}(\sin x + 2) + C$ #

DATE

2. $\int \frac{\log x \, dx}{(1+\log x)^2}$

$= \int \frac{1+\log x - 1}{(1+\log x)^2} dx$

$= \int \left[\frac{1+\log x}{(1+\log x)^2} dx - \frac{1}{(1+\log x)^2} dx \right]$

$= \int \frac{dx}{1+\log x} - \int \frac{dx}{(1+\log x)^2}$

$= \int 1 \cdot (1+\log x)^{-1} dx - \int (1+\log x)^{-2} dx$

= Integrating by parts.

$= (1+\log x)^{-1} \int 1 \, dx + \int \frac{1}{(1+\log x)^2} \cdot \frac{1}{x} \int 1 \cdot dx - \int (1+\log x)^{-2} dx$

$= (1+\log x)^{-1} x + \int \frac{1}{(1+\log x)^2} \cdot \frac{1}{x} \cdot x - \int \frac{dx}{(1+\log x)^2}$

$= (1+\log x)^{-1} x + C$

$= \frac{x}{1+\log x} + C$ //

DATE

3) $\int \frac{dx}{(x^2+a^2)(x^2+b^2)}$

$= \frac{1}{b^2-a^2} \int \left[\frac{1}{x^2+a^2} - \frac{1}{(x^2+b^2)} \right] dx$

$= \frac{1}{b^2-a^2} \left[\frac{1}{a} \tan^{-1} \frac{x}{a} - \frac{1}{b} \tan^{-1} \frac{x}{b} \right] + C$

$= \frac{1}{b^2-b^2} \left[\frac{1}{b} \tan^{-1} \frac{x}{b} - \frac{1}{a} \tan^{-1} \frac{x}{a} \right] + C$ //

4) $\int \frac{x e^x}{(x+1)^2} dx$

$= \int \left[\frac{x+1-1}{(x+1)^2} \right] e^x dx$

$= \int \left[\frac{x+1}{(x+1)^2} - \frac{1}{(x+1)^2} \right] e^x dx$

$= \int \left[\frac{1}{x+1} - \frac{1}{(x+1)^2} \right] e^x dx$

$= \frac{e^x}{(x+1)} + C$ $\quad \because e^{f(x)} \left[f(x) + f'(x) \right]$

$= e^{f(x)} \cdot f(x) + C$ //

5) $\int \frac{x+2}{\sqrt{4x-x^2}}\,dx$

$= \frac{1}{2}\int \frac{2x+4}{\sqrt{4x-x^2}}\,dx$

$= \frac{1}{2}\int \frac{2x+4-4+4}{\sqrt{4x-x^2}}\,dx$

$= \frac{1}{2}\int \frac{2x+4+8}{\sqrt{4x-x^2}}\,dx$

$= \frac{1}{2}\int \frac{2x-4}{\sqrt{4x-x^2}}\,dx + \frac{1}{2}\int \frac{8}{\sqrt{4x-x^2}}\,dx$

$= -\frac{1}{2}\int \frac{4-2x}{\sqrt{4x-x^2}}\,dx + 4\int \frac{dx}{\sqrt{-(x^2-4x+4-4)}}$

$= -\frac{1}{2}\times 2\sqrt{4x-x^2} + 4\int \frac{dx}{\sqrt{4-(x-2)^2}}$

$= -\sqrt{4x-x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right) + C$

$= 4\sin^{-1}\left(\frac{x-2}{2}\right) - \sqrt{4x-x^2} + C$ //