Aip

Date
Page

Assignment - 8.

Photon & Matter Waves.

1) A beam of electrons having energy of each 3eV. is incident on a potential barrier. of height 4ev. If the width of the barrier is 20A. calculate the transmision coefficient of the beam through the berrier.

⇒ solⁿ.

$$T = \frac{16E(V-E)}{V^2} \cdot e^{-2k_2L}.$$

then,

$$\hbar = \frac{h}{2\pi} = \frac{6.62\times10^{-34}}{2\pi} = 1.053\times10^{-34}\ \text{Jsec}$$

Then

$$k_2 = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

$$= \sqrt{\frac{2\times9.1\times10^{-31}\times(4-3)\times1.16\times10^{-19}}{(1.053\times10^{-34})^2}}$$

$$= 5.12\times10^9$$

And,

$$\frac{16E(V-E)}{V^2}$$

$$= \frac{16\times3\times1.6\times10^{-19}\times(4-3)\times1.6\times10^{-19}}{(4\times1.6\times10^{-19})^2}$$

$$= 3.$$

$$\therefore T = \frac{16E(V-E)}{V^2}\times e^{-2k_2L}$$

$$= 3\times e^{-2\times5.12\times10^9\times20\times10^{-10}}$$

$$= 3.83\times10^{-9}$$

2. A non realtivitive paritticle is moving three times as fast as electron. The ratio of de-Broglie wavelength of the particle to that of the electron is 1.813×10^{-4} calculate the mass of particle.

=> sol^n,

we know,

$$\lambda = \frac{h}{mv}$$

by the question,

$$\frac{\lambda_e}{\lambda_e} = 1.813 \times 10^{-4}$$

or, $$\frac{\frac{h}{m_p \times v_p}}{\frac{h}{m_e \times v_e}} = 1.813 \times 10^{-4}$$

or, $$\frac{m_e \times \cancel{v_e}}{m_p \times 3\cancel{v_e}} = 1.813 \times 10^{-4}$$

or, $$m_p = \frac{m_e}{1.813 \times 10^{-4} \times 3} = \frac{9.1 \times 10^{-31}}{1.813 \times 10^{-4} \times 3}$$

$$= 1.67 \times 10^{-27}\ kg.$$

$\therefore$ Mass of the particle $= 1.67 \times 10^{-27}$ kg.

3) Using the uncertainty principle, calculate the minimum uncertainty in velocity when an electron is confined to a length $\Delta x = 1nm$.

⇒ Soln.

Here,

$\Delta x = 1nm$

$= 1\times10^{-9}$ m.

Now,

$$\Delta x \times \Delta p \geq \frac{h}{4\pi}$$

$$1\times10^{-9} \times m \times \Delta v = \frac{h}{4\pi}$$

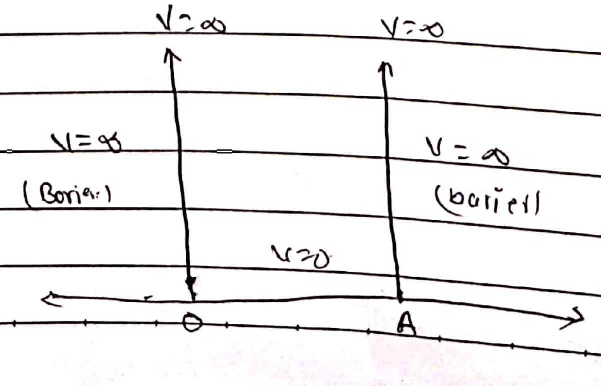
$$1\times10^{-9} \times 9.1\times10^{-31} \times \Delta v = \frac{h}{4\pi}$$

$$\Delta v = \frac{6.62\times10^{-34}}{4\pi\times10^{-9}\times9.1\times10^{-31}}$$

$$\Delta v = 5.78\times10^{4}\ m/sec.$$

4) An electron is confined in an one dimensional infinite potential well of width a the potential energy is $V=0$ for $0 \leq x \leq a$ & $V=\infty$ for $x<0$ & $x \geq a$. Find the Eigen function $\psi_n(x) = A\sin\left(\frac{n\pi x}{a}\right)$ & energy Eigen values $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$.

⇒ Soln,



Consider a particle restricted to move along $x=0$ & $x=a$. The potential (v) of the particle is zero inside the well but raise to ∞ on the outside

ie. $V=0$ for $0<x<a$

& $V=\infty$ for $x\leq 0$ & $x>a$

The such case the particle is said to be moving is an infinitely deep potential well. The schrodinger wave eq^n for the particle within the box is.

$$\frac{d^2\psi}{dx^2} + \frac{2m(E-V)}{\hbar}\psi = 0 \quad -(i)$$

$$\frac{d^2\psi}{dx^2} + \frac{2mE\psi}{\hbar^2} = 0 \quad -(2)$$

let $k^2 = \frac{2mE}{\hbar^2}$ $-(3)$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad -(4)$$

The sol^n of the eq^n is

$\psi(x) = A\sin kx + B\cos kx.$ $-(5)$

where A & B are constant to be determine using Boundary condition

Since the particle cannot exist outside the box. Therefore the wave function ψ must be zero, outside the box & at the walls;

Scanned with CamScanner

$x = 0$ & $x = a$.

for $x = 0$

$\psi(x) = 0$

$0 = 0 + B$.

$B = 0$.

lly, for $x = a$

$\psi(x) = 0$

$0 = A \sin ka + 0$

$\sin ka = 0$

$\sin ka = \sin n\pi$

$ka = n\pi$

$$k = \frac{n\pi}{a} \quad -(6)$$

from eq^n (3) & (6)

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\frac{n^2\pi^2}{a^2} = \frac{2mE}{\hbar^2}$$

$$E = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

This means the energy of the particle in potential well is quantized. Each value of energy given by above relation is called Eigen value & corresponding function is called Eigen function.

Now, substituting, $B = 0$ & $k = \frac{n\pi}{a}$ in eq^n(5) the allowed sol^n of schrodinger eq^n. are

$$\psi_n(x) = A \sin\left(\frac{n\pi x}{a}\right)$$

The coefficient A is called Normalizing constant & can be determined using Normalizing condition

$$\int_0^a \psi \psi^* dx = 1$$

$$A^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx = 1$$

$$A^2 \int_0^a \frac{1}{2}\left(1 - \cos\frac{2n\pi x}{a}\right) dx = 1$$

$$A^2 \left[\frac{1}{2}\int_0^a dx - \frac{1}{2}\int_0^a \cos\frac{2n\pi x}{a} \cdot dx\right] = 1$$

$$A^2 \times \frac{a}{2} = 1$$

$$A = \sqrt{\frac{2}{a}}$$

The normalized wave function of the particle are therefore.

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{l}\right).$$

Q) Calculate the permitted energy levels of an electron in one dimensional width 0.2 nm.

⇒ Sol^n,

$$E = \frac{n^2\pi^2\hbar^2}{2ml^2}$$

$$= \frac{1 \times \pi^2 \times \left(\frac{h}{2\pi}\right)^2}{2 \times 9.1 \times 10^{-31} \times (0.2 \times 10^{-9})^2}$$

$$= \frac{1 \times \pi^2 \times (6.62 \times 10^{-34})^2}{4\pi^2 \times 2 \times 9.1 \times 10^{-31} \times (0.2 \times 10^{-9})^2}$$

$= 1.5 \times 10^{-18} n^2 J$

permitted energy $(E) = 1.5 \times 10^{-18} n J$.