Engineering Economics

Lecture 9

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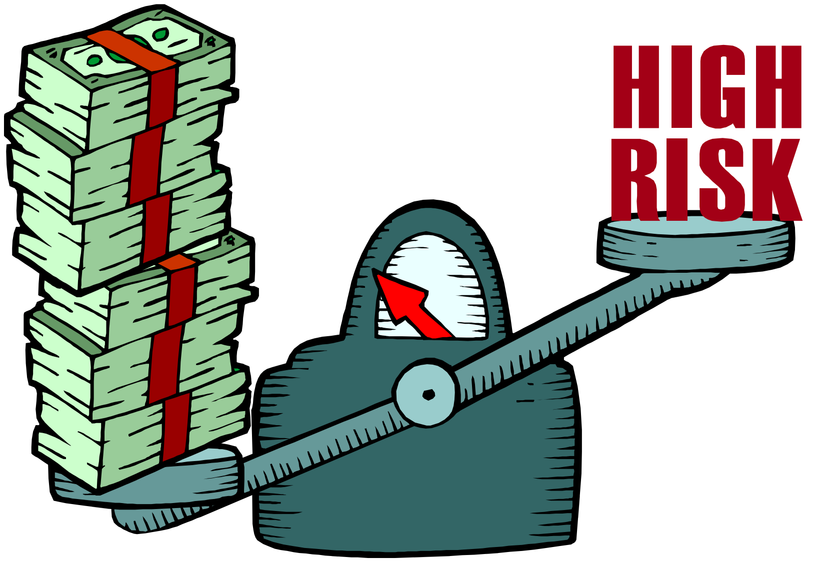
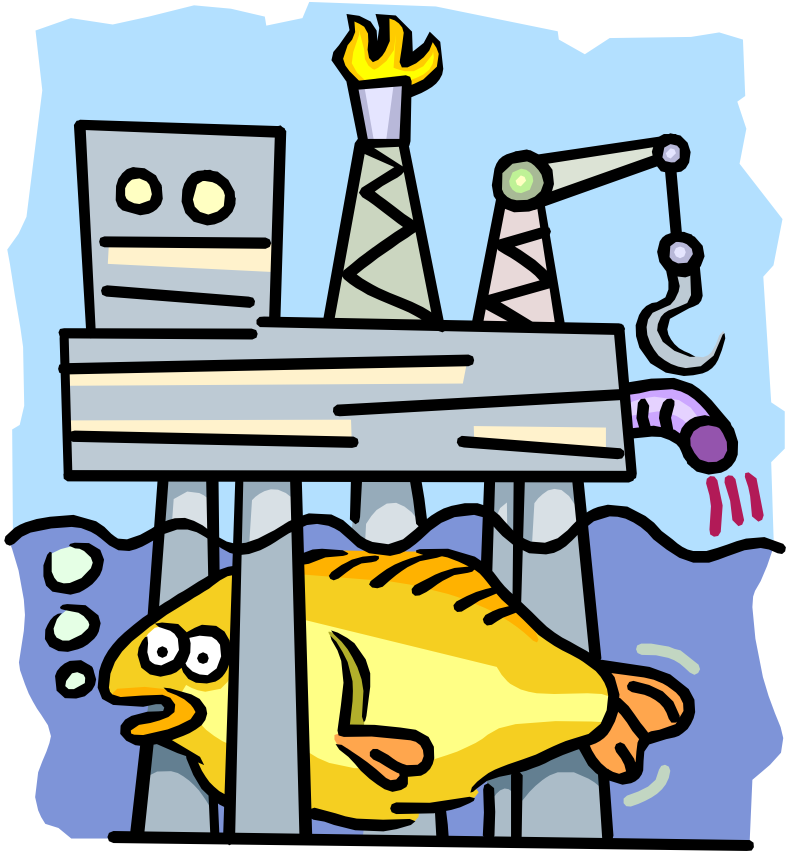
Chapter 14
Project Risk and Uncertainty

- Origin of Project Risk
- Methods of Describing Project Risk
- Probability Concepts fro Investment Decisions
- Probability Distribution of NPW
- Decision Trees and Sequential Investment Decisions



Origins of Project Risk

- Risk: the potential for loss
- Project Risk: variability in a project’s NPW
- Risk Analysis: The assignment of probabilities to the various outcomes of an investment project



Example 14.1 Improving the Odds – All It Takes is $7 Million and a Dream

Number of Prizes	Prize Category	Total Amount
1	First Prize	$27,007,364
228	Second Prizes ($899 each)	204,972
10,552	Third Prizes ($51 each)	538,152
168,073	Fourth Prizes ($1 each)	168,073
	Total Winnings	$27,918,561

15 26 3 34 1 39

Number of Prizes	Prize Category	Winning Odds
1	First Prize	0.0000001416
228	Second Prizes	0.0000323
10,552	Third Prizes	0.00149
168,073	Fourth Prizes	0.02381

$$C(44,6) = \frac{44!}{6!(44-6)!} = 7{,}059{,}052$$

Odds for First Prizes:

$$\frac{1}{7{,}059{,}052} = 0.0000001416$$

Methods of Describing Project Risk

- Sensitivity Analysis: a means of identifying the project variables which, when varied, have the greatest effect on project acceptability.
- Break-Even Analysis: a means of identifying the value of a particular project variable that causes the project to exactly break even.
- Scenario Analysis: a means of comparing a “base case” to one or more additional scenarios, such as best and worst case, to identify the extreme and most likely project outcomes.

Example 14.2 - After-tax Cash Flow for BMC's Transmission-Housings Project – "Base Case"

	0	1	2	3	4	5
Revenues:						
Unit Price		50	50	50	50	50
Demand (units)		2,000	2,000	2,000	2,000	2,000
Sales revenue		$100,000	$100,000	$100,000	$100,000	$100,000
Expenses:						
Unit variable cost		$15	$15	$15	$15	$15
Variable cost		30,000	30,000	30,000	30,000	30,000
Fixed cost		10,000	10,000	10,000	10,000	10,000
Depreciation		17,863	30,613	21,863	15,613	5,575
Taxable Income		$42,137	$29,387	$38,137	$44,387	$54,425
Income taxes (40%)		16,855	11,755	15,255	17,755	21,770
Net Income		$25,282	$17,632	$22,882	$26,632	$32,655

(Example 14.2, Continued)

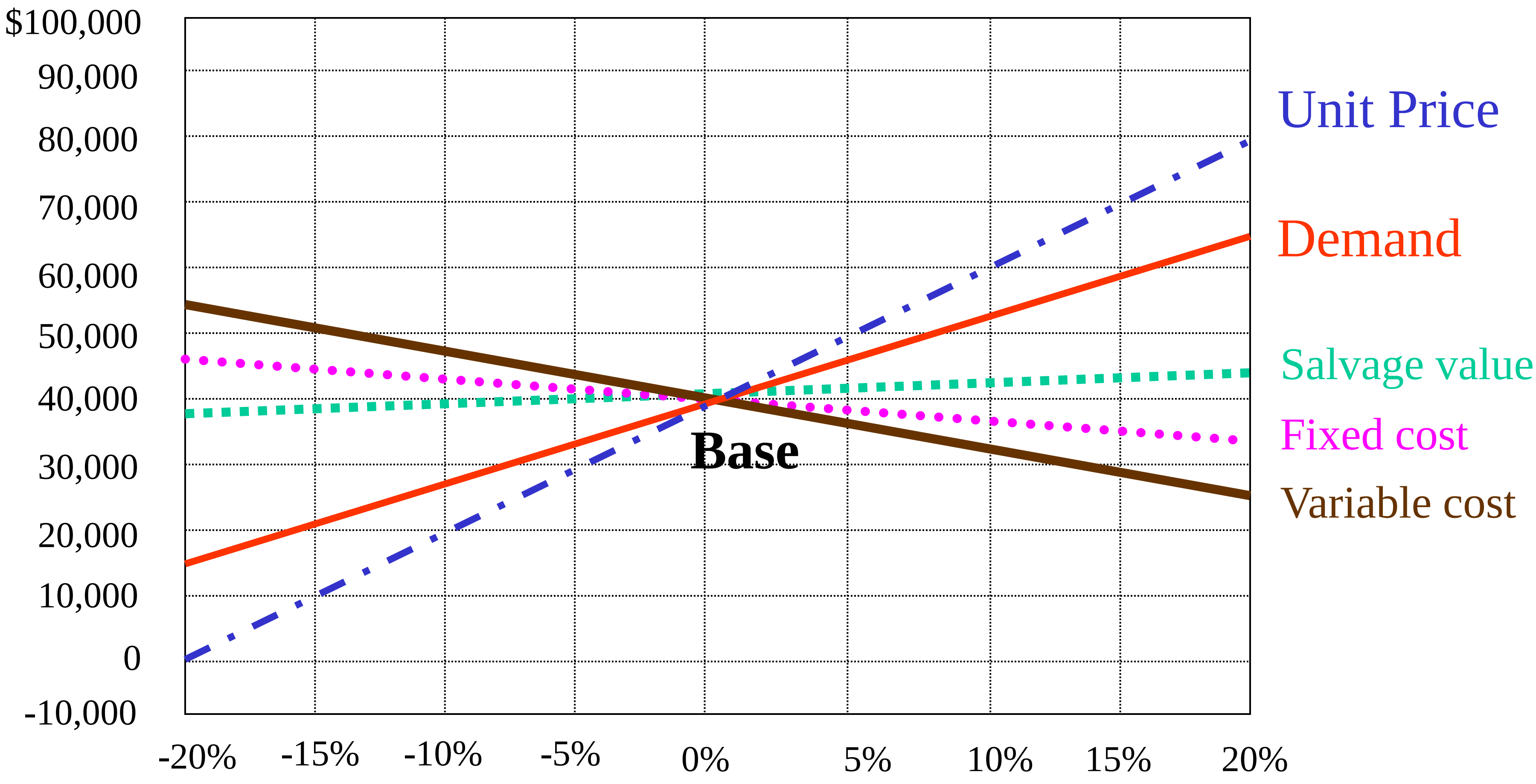
Cash Flow Statement	0	1	2	3	4	5
Operating activities						
Net income		25,282	17,632	22,882	26,632	32,655
Depreciation		17,863	30,613	21,863	15,613	5,575
Investment activities						
Investment	125,000					
Salvage						40,000
Gains tax						2,611
Net cash flow	$125,500	$43,145	$48,245	$44,745	$42,245	$75,619

Example 14.2 - Sensitivity Analysis for Five Key Input Variables

Deviation	-20%	-15%	-10%	-5%	0%	5%	10%	15%	20%
Unit price	$57	$9,999	$20,055	$30,111	$40,169	$50,225	$60,281	$70,337	$80,393
Demand	12,010	19,049	26,088	33,130	40,169	47,208	54,247	61,286	68,325
Variable cost	52,236	49,219	46,202	43,186	40,169	37,152	34,135	31,118	28,101
Fixed cost	44,191	43,185	42,179	41,175	40,169	39,163	38,157	37,151	36,145
Salvage value	37,782	38,378	38,974	39,573	40,169	40,765	41,361	41,957	42,553

Base

Sensitivity graph – BMC's transmission-housings project (Example 14.2)



Example 14.3 - Sensitivity Analysis for Mutually Exclusive Alternatives

	Electrical Power	LPG	Gasoline	Diesel Fuel
Life expectancy	7 year	7 years	7 years	7 years
Initial cost	$29,739	$21,200	$20,107	$22,263
Salvage value	$3,000	$2,000	$2,000	$2,200
Maximum shifts per year	260	260	260	260
Fuel consumption/shift	31.25 kWh	11 gal	11.1 gal	7.2 gal
Fuel cost/unit	$0.05/kWh	$1.02/gal	$1.20/gal	$1.13/gal
Fuel cost/shift	$1.56	$11.22	$13.32	$8.14
Annual maintenance cost				
Fixed cost	$500	$1,000	$1,000	$1,000
Variable cost/shift	$4.5	$7	$7	$7

a) Ownership cost (capital cost):

Electrical power: CR(10%) = (\$29,739 - \$3,000)(*A/P*, 10%, 7)
+ (0.10)\$3,000 = <u>\$5,792</u>

LPG: CR(10%) = (\$21,200 - \$2,000)(*A/P*, 10%, 7)
+ (0.10)\$2,000 = <u>\$4,144</u>

Gasoline: CR(10%) = (\$20,107 - \$2,000)(*A/P*,10%,7)
+ (0.10)\$2,000 = <u>\$3,919</u>

Diesel fuel: CR(10%) = (\$22,263 - \$2,200)(*A/P*, 10%,7)
+(0.1)\$2,200 = <u>\$4,341</u>

b) Annual operating cost as a function of number of shifts per year (M):

Electric:

$$\$500+(1.56+4.5)M = \$500+5.06M$$

LPG:

$$\$1,000+(11.22+7)M = \$1,000+18.22M$$

Gasoline:

$$\$1,000+(13.32+7)M = \$1,000+20.32M$$

Diesel fuel:

$$\$1,000+(8.14+7)M = \$1,000+15.14M$$

c) Total equivalent annual cost:

Electrical power:

$$AE(10\%) = 6{,}292 + 5.06M$$

LPG:

$$AE(10\%) = 5{,}144 + 18.22M$$

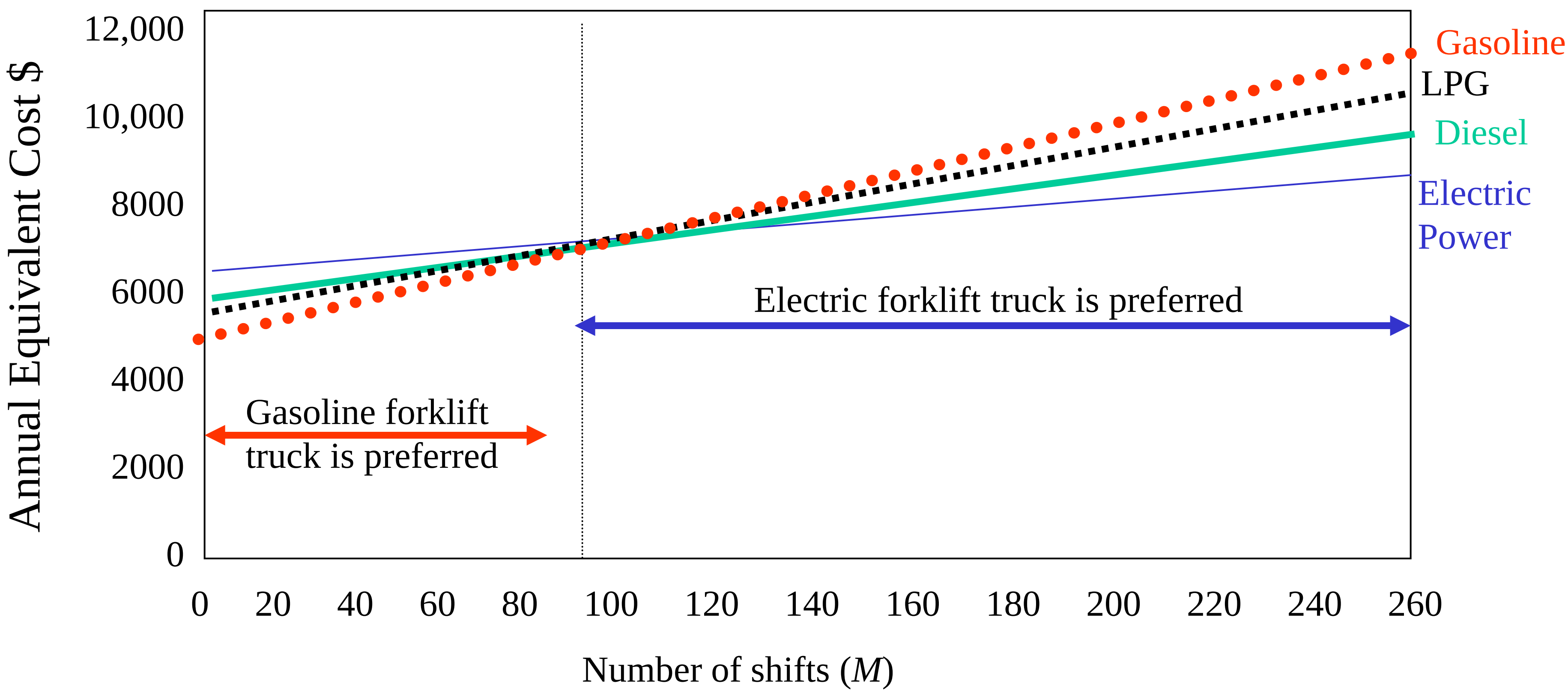
Gasoline:

$$AE(10\%) = 4{,}919 + 20.32M$$

Diesel fuel:

$$AE(10\%) = 5{,}341 + 15.14M$$

Sensitivity Graph for Multiple Alternatives



Break-Even Analysis with unknown Sales Units (X)

	0	1	2	3	4	5
Cash Inflows:						
Net salvage						37,389
X(1-0.4)($50)		30X	30X	30X	30X	30X
0.4 (dep)		7,145	12,245	8,745	6,245	2,230
Cash outflows:						
Investment	-125,000					
-X(1-0.4)($15)		-9X	-9X	-9X	-9X	-9X
-(0.6)($10,000)		-6,000	-6,000	-6,000	-6,000	-6,000
Net Cash Flow	-125,000	21X + 1,145	21X + 6,245	21X + 2,745	21X + 245	21X + 33,617

Break-Even Analysis

1. PW of cash inflows

$PW(15\%)_{\text{Inflow}}$ = (PW of after-tax net revenue)

+ (PW of net salvage value)

+ (PW of tax savings from depreciation

$= 30X(P/A, 15\%, 5) + \$37{,}389(P/F, 15\%, 5)$

$+ \$7{,}145(P/F, 15\%,1) + \$12{,}245(P/F, 15\%, 2)$

$+ \$8{,}745(P/F, 15\%, 3) + \$6{,}245(P/F, 15\%, 4)$

$+ \$2{,}230(P/F, 15\%,5)$

$= 30X(P/A, 15\%, 5) + \$44{,}490$

$= 100.5650X + \$44{,}490$

2. PW of cash outflows:

$PW(15\%)_{\text{Outflow}}$ = (PW of capital expenditure_
+ (PW) of after-tax expenses
= \$125,000 + (9X+\$6,000)(P/A, 15%, 5)
= 30.1694X + \$145,113

3. The NPW:

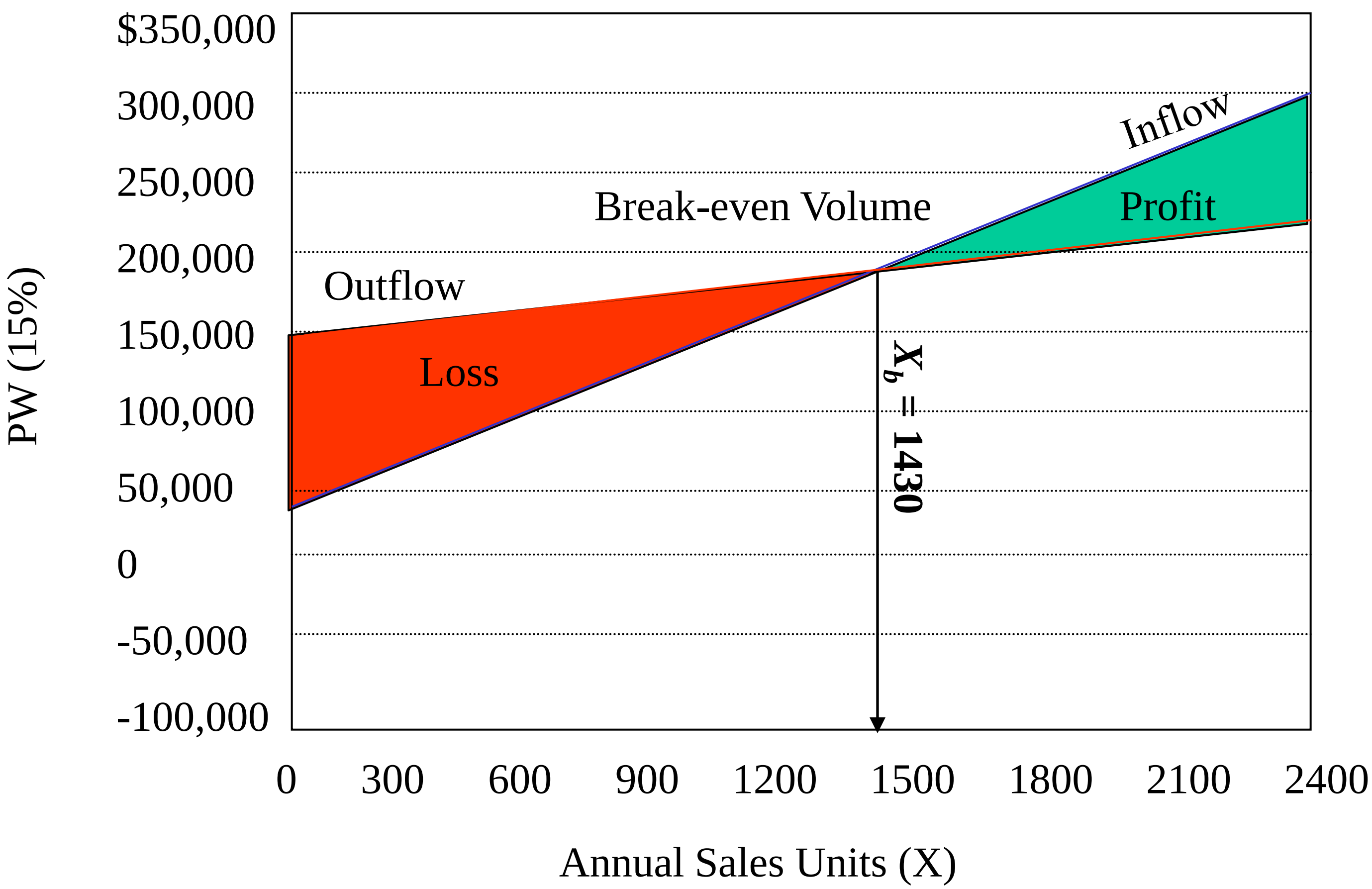
PW (15%) = 100.5650X + \$44,490
- (30.1694X + \$145,113)
=70.3956X - \$100,623.

4. Breakeven volume:

PW (15%) = 70.3956X - \$100,623 = 0
X_b =1,430 units.

Demand	**PW of inflow**	**PW of Outflow**	**NPW**
X	**100.5650*X* - $44,490**	**30.1694*X* + $145,113**	**70.3956*X* -$100,623**
0	$44,490	$145,113	100,623
500	94,773	160,198	65,425
1000	145,055	175,282	30,227
1429	188,197	188,225	28
1430	188,298	188,255	43
1500	195,338	190,367	4,970
2000	245,620	205,452	40,168
2500	295,903	220,537	75,366

Break-Even Analysis Chart



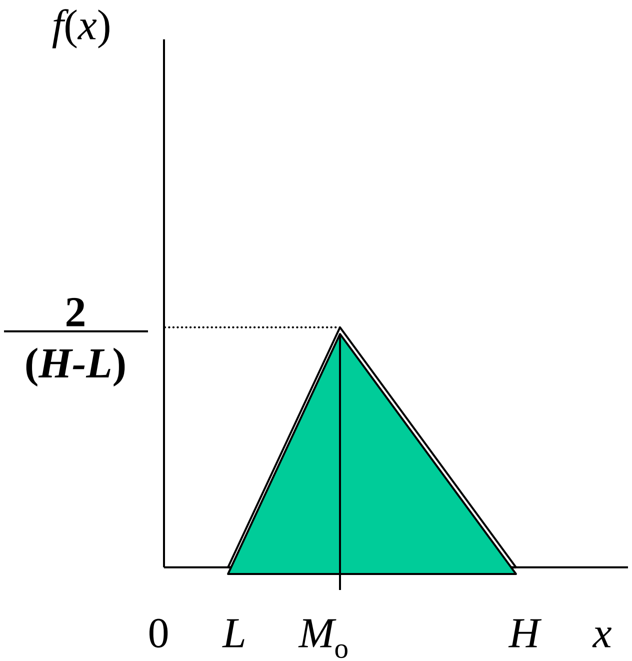
Scenario Analysis

Variable Considered	Worst-Case Scenario	Most-Likely-Case Scenario	Best-Case Scenario
Unit demand	1,600	2,000	2,400
Unit price ($)	48	50	53
Variable cost ($)	17	15	12
Fixed Cost ($)	11,000	10,000	8,000
Salvage value ($)	30,000	40,000	50,000
PW (15%)	-$5,856	$40,169	$104,295

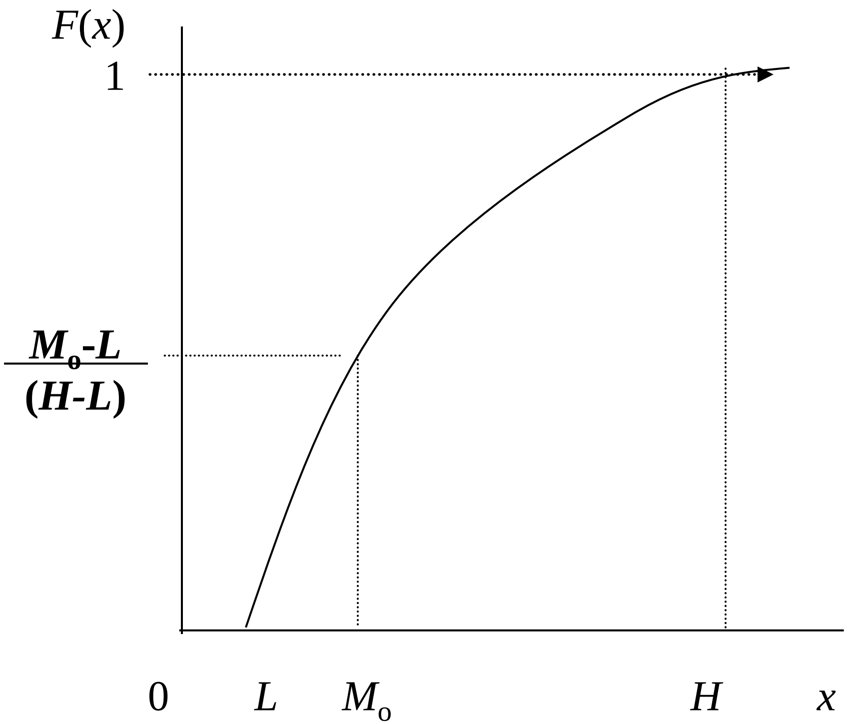
Probability Concepts for Investment Decisions

- Random variable: variable that can have more than one possible value
- Discrete random variables: Any random variables that take on only isolated values
- Continuous random variables: any random variables can have any value in a certain interval
- Probability distribution: the assessment of probability for each random event

Triangular Probability Distribution

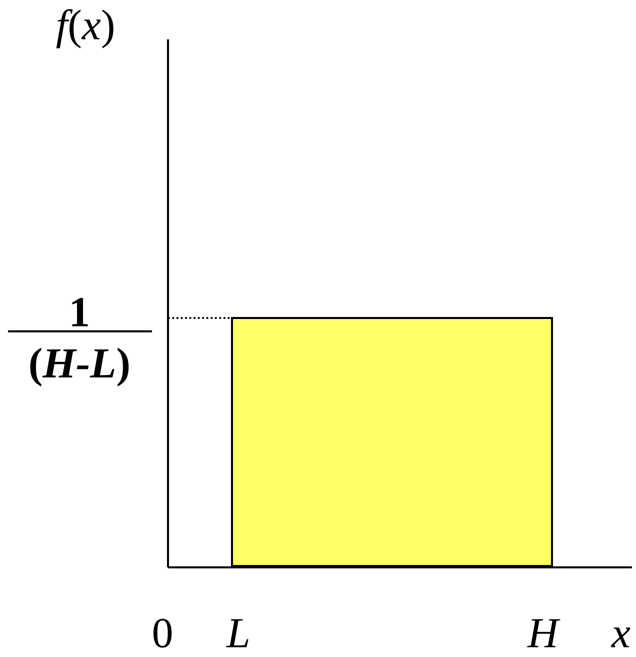


(a) Probability function

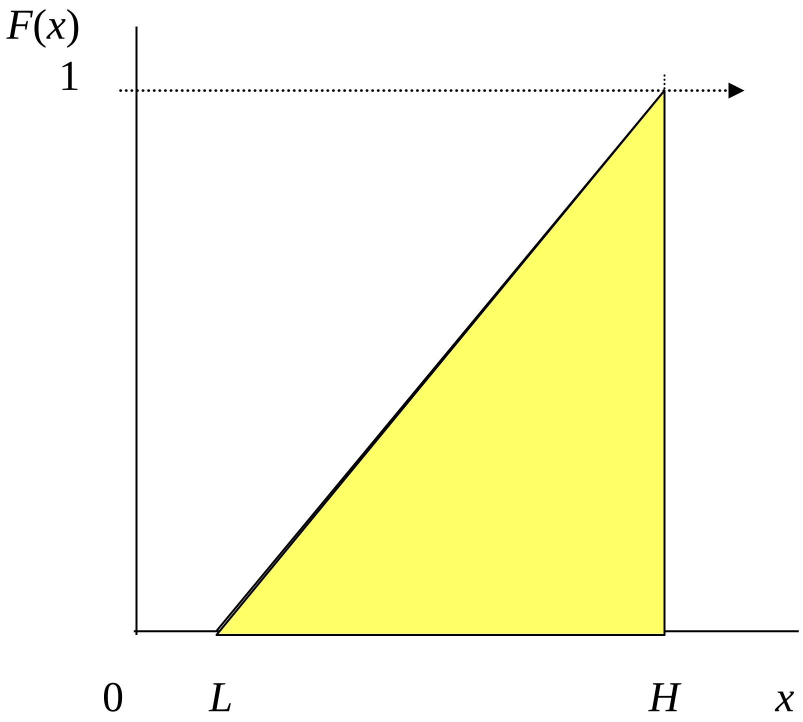


(b) Cumulative probability distribution

Uniform Probability Distribution



(a) Probability function



(b) Cumulative probability distribution

Probability Distributions for Unit Demand (*X*) and Unit Price (*Y*) for BMC's Project

Product Demand (*X*)		Unit Sale Price (*Y*)	
Units (x)	**P($X = x$)**	**Unit price (y)**	**P($Y = y$)**
1,600	0.20	$48	0.30
2,000	0.60	50	0.50
2,400	0.20	53	0.20

Cumulative Distribution

$$F(x) = P(X \leq x) = \sum_{j=1}^{j} p_j$$

(for a discrete random variable)

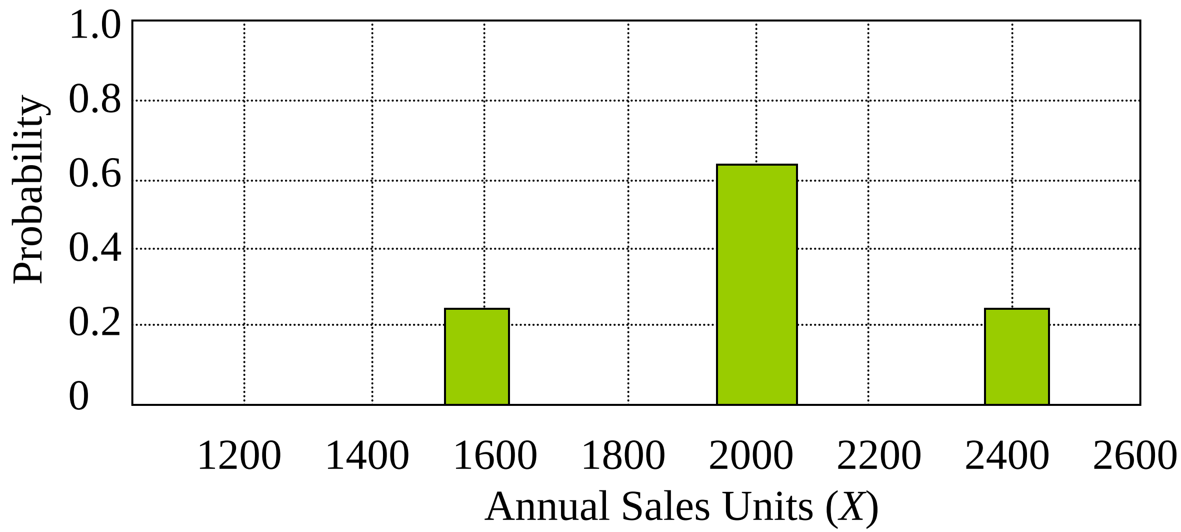
$$\int f(x)dx$$

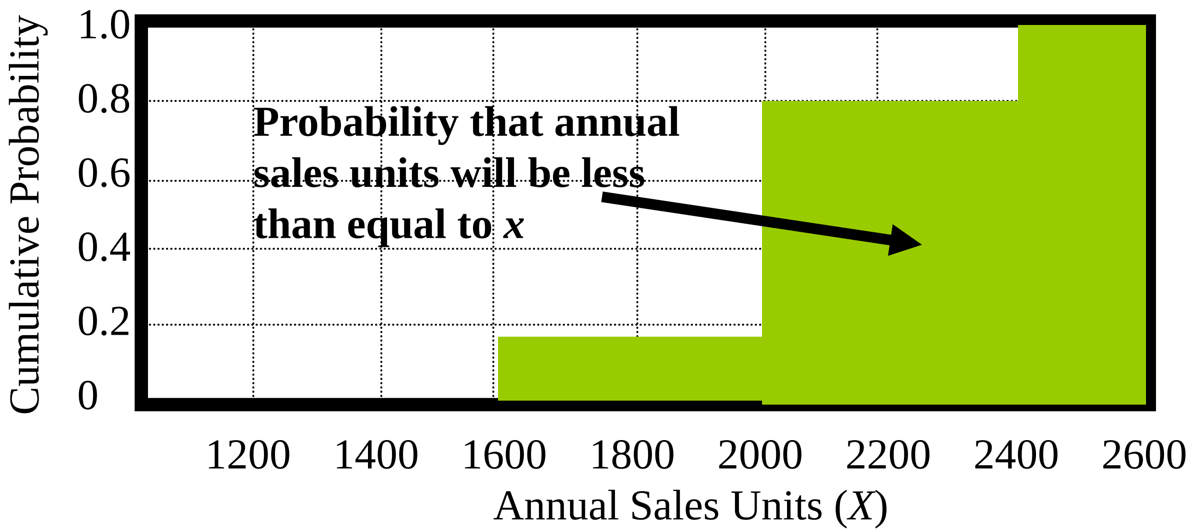
(for a continuous random variable)

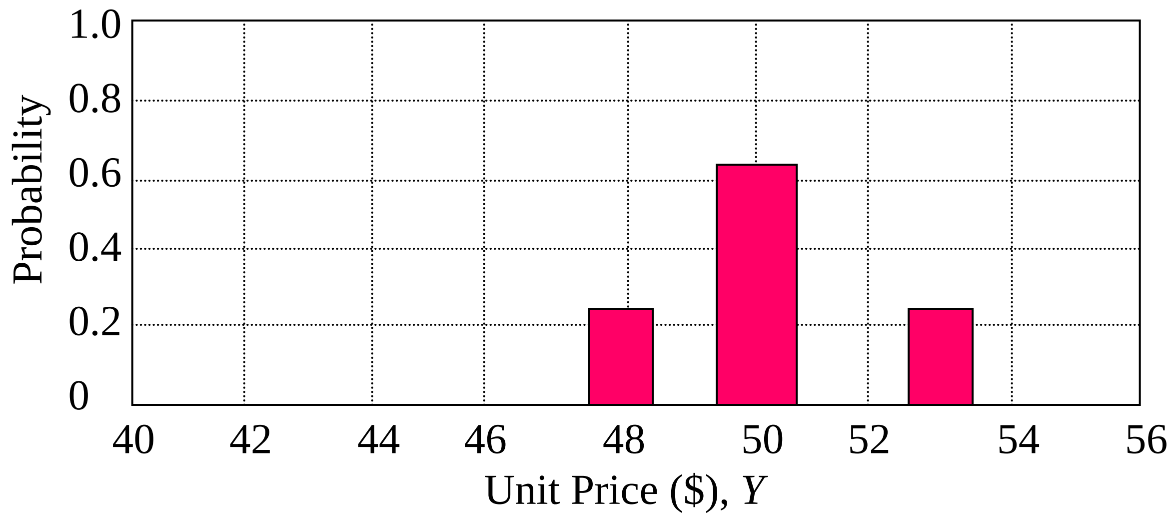
Cumulative Probability Distribution for *X*

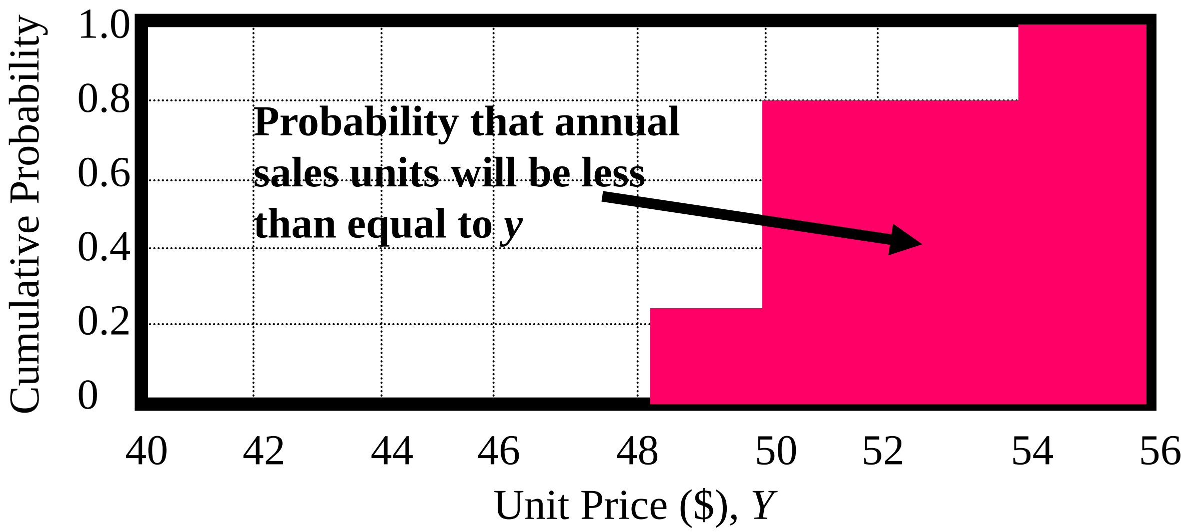
Unit Demand (X)	Probability $P(X = x)$
1,600	0.2
2,000	0.6
2,400	0.2

$$F(x) = P(X \le x) = \begin{cases} 0.2, & x \le 1{,}600 \\ 0.8, & x \le 2{,}000 \\ 1.0, & x \le 2{,}400 \end{cases}$$


1.0
0.8
0.6
0.4
0.2
0
Probability
1200
1400
1600
1800
2000
2200
2400
2600
Annual Sales Units (X)


1.0
0.8
0.6
0.4
0.2
0
Cumulative Probability
Probability that annual sales units will be less than equal to x
1200
1400
1600
1800
2000
2200
2400
2600
Annual Sales Units (X)


1.0
0.8
0.6
0.4
0.2
0
Probability
40
42
44
46
48
50
52
54
56
Unit Price ($), *Y*


1.0
0.8
0.6
0.4
0.2
0
Cumulative Probability
Probability that annual sales units will be less than equal to *y*
40
42
44
46
48
50
52
54
56
Unit Price ($), *Y*

Measure of Expectation

$$E[X] = \mu = \sum_{j=1}^{j} (p_j) x_j \qquad \text{(discrete case)}$$

$$\int xf(x)dx \qquad \text{(continuous case)}$$

Measure of Variation

$$Var[x] = \sigma_x^2 = \sum_{j=1}^{j} (x_j - \mu)^2 (p_j)$$

$$\sigma_x = \sqrt{Var[X]}$$

$$Var[X] = \sum p_j x_j^2 - (\sum p_j x_j)^2$$

$$= E[X^2] - (E[X])^2$$

X_j	P_j	$X_j(P_j)$	$(X_j\text{-E}[X])$	$(X_j\text{-E}[X])^2\ (P_j)$
1,600	0.20	320	$(-400)^2$	32,000
2,000	0.60	1,200	0	0
2,400	0.20	480	$(400)^2$	32,000
		E[X] = 2,000		Var[X] = 64,000
				σ = 252,98

Y_j	P_j	$Y_j(P_j)$	$[Y_j\text{-E}[Y]]^2$	$(Y_j\text{-E}[Y])^2\ (P_j)$
$48	0.30	$14.40	$(-2)^2$	1.20
50	0.50	25.00	(0)	0
53	0.20	10.60	$(3)^2$	1.80
		E[Y] = 50.00		Var[Y] = 3.00
				σ = 1.73

Joint and Conditional Probabilities

$$P(x,y) = P(X = x \mid Y = y)P(Y = y)$$

$$P(x,y) = P(x)P(y)$$

$$P(x,y) = P(1{,}600, \$48)$$

$$= P(x = 1{,}600 \mid y = \$48 \, P(y = \$48)$$

$$= (0.10)(0.30)$$

$$= 0.03$$

Assessments of Conditional and Joint Probabilities

Unit Price *Y*	Marginal Probability	Conditional Unit Sales *X*	Conditional Probability	Joint Probability
		1,600	0.10	0.03
$48	0.30	2,000	0.40	0.12
		2,400	0.50	0.15
		1,600	0.10	0.05
50	0.50	2,000	0.64	0.32
		2,400	0.26	0.13
		1,600	0.50	0.10
53	0.20	2,000	0.40	0.08
		2,400	0.10	0.02

Marginal Distribution for *X*

X_j	$P(x) = \sum_y P(x,y)$
1,600	P(1,600, \$48) + P(1,600, \$50) + P(1,600, \$53) = 0.18
2,000	P(2,000, \$48) + P(2,000, \$50) + P(2,000, \$53) = 0.52
2,400	P(2,400, \$48) + P(2,400, \$50) + P(2,400, \$53) = 0.30

After-Tax Cash Flow as a Function of Unknown Unit Demand (X) and Unit Price (Y)

Item	0	1	2	3	4	5
Cash inflow:						
Net salvage						
X(1-0.4)Y		0.6XY	0.6XY	0.6XY	0.6XY	0.6XY
0.4 (dep)		7,145	12,245	8,745	6,245	2,230
Cash outflow:						
Investment	-125,000					
-X(1-0.4)($15)		-9X	-9X	-9X	-9X	-9X
-(1-0.4)($10,000)		-6,000	-6,000	-6,000	-6,000	-6,000
Net Cash Flow	-125,000	0.6X(Y-15) +1,145	0.6X(Y-15) +6,245	0.6X(Y-15) +2,745	0.6X(Y-15) +245	0.6X(Y-15) +33,617

NPW Function

1. **Cash Inflow:**

$$\mathbf{PW(15\%) = 0.6}\mathbf{\mathit{XY}}\mathbf{(}\mathbf{\mathit{P/A}}\mathbf{, 15\%, 5) + \$44{,}490}$$
$$\mathbf{= 2.0113}\mathbf{\mathit{XY}}\mathbf{ + \$44{,}490}$$

2. **Cash Outflow:**

$$\mathbf{PW(15\%) = \$125{,}000 + (9}\mathbf{\mathit{X}}\mathbf{ + \$6{,}000)(}\mathbf{\mathit{P/A}}\mathbf{, 15\%, 5)}$$
$$\mathbf{= 30.1694}\mathbf{\mathit{X}}\mathbf{ + \$145{,}113.}$$

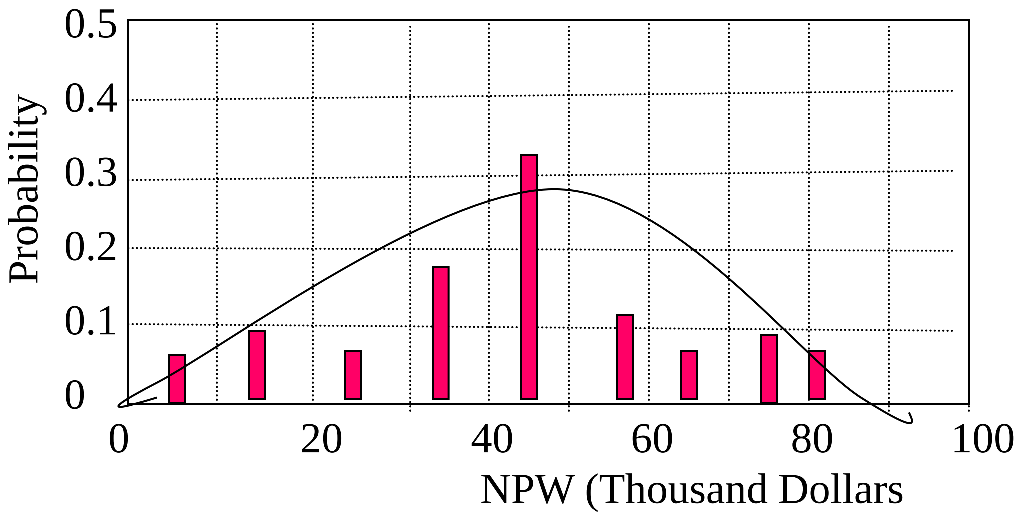
3. **Net Cash Flow:**

$$\mathbf{PW(15\%) = 2.0113}\mathbf{\mathit{X}}\mathbf{(}\mathbf{\mathit{Y}}\mathbf{ - \$15) - \$100{,}623}$$

The NPW Probability Distribution with Independent Random Variables

Event No.	*x*	*y*	P[*x*,*y*]	Cumulative Joint Probability	NPW
1	1,600	$48.00	0.06	0.06	$5,574
2	1,600	50.00	0.10	0.16	12,010
3	1,600	53.00	0.04	0.20	21,664
4	2,000	48.00	0.18	0.38	32,123
5	2,000	50.00	0.30	0.68	40,168
6	2,000	53.00	0.12	0.80	52,236
7	2,400	48.00	0.06	0.86	58,672
8	2,400	50.00	0.10	0.96	68,326
9	2,400	53.00	0.04	1.00	82,808

NPW probability distributions: When *X* and *Y* are independent



Calculation of the Mean of NPW Distribution

Event No.	*x*	*y*	P[*x*,*y*]	Cumulative Joint Probability	NPW	Weighted NPW
1	1,600	$48.00	0.06	0.06	$5,574	$334
2	1,600	50.00	0.10	0.16	12,010	1,201
3	1,600	53.00	0.04	0.20	21,664	867
4	2,000	48.00	0.18	0.38	32,123	5,782
5	2,000	50.00	0.30	0.68	40,168	12,050
6	2,000	53.00	0.12	0.80	52,236	6,268
7	2,400	48.00	0.06	0.86	58,672	3,520
8	2,400	50.00	0.10	0.96	68,326	6,833
9	2,400	53.00	0.04	1.00	82,808	3,312
					E[NPW] =	$40,168

Calculation of the Variance of NPW Distribution

Event No.	*x*	*y*	P[*x*,*y*]	NPW	(NPW-E[NPW])^2	Weighted (NPW-E[NPW])
1	1,600	$48.00	0.06	$5,574	1,196,769,744	$71,806,185
2	1,600	50.00	0.10	12,010	792,884,227	79,228,423
3	1,600	53.00	0.04	21,664	342,396,536	13,695,861
4	2,000	48.00	0.18	32,123	64,725,243	11,650,544
5	2,000	50.00	0.30	40,168	0	0
6	2,000	53.00	0.12	52,236	145,631,797	17,475,816
7	2,400	48.00	0.06	58,672	342,396,536	20,543,792
8	2,400	50.00	0.10	68,326	792,884,227	79,288,423
9	2,400	53.00	0.04	82,808	1,818,132,077	72,725,283
					Var[NPW] =	366,474,326
						σ = $19,144

Comparing Risky Mutually Exclusive Projects

Event (NPW) (unit: thousands)	Probabilities			
	Model 1	Model 2	Model 3	Model 4
1,000	0.35	0.10	0.40	0.20
1,500	0	0.45	0	0.40
2,000	0.40	0	0.25	0
2,500	0	0.35	0	0.30
3,000	0.20	0	0.20	0
3,500	0	0	0	0
4,000	0.05	0	0.15	0
4,500	0	0.10	0	0.10

Comparison Rule

- If $E_A > E_B$ and $V_A \leq V_B$, select A.
- If $E_A = E_B$ and $V_A \leq V_B$, select A.
- If $E_A < E_B$ and $V_A \geq V_B$, select B.
- If $E_A > E_B$ and $V_A > V_B$, Not clear.

Model Type	E (NPW)	Var (NPW)
Model 1	$1,950	747,500
Model 2	2,100	915,000
Model 3	2,100	1,190,000
Model 4	2,000	1,000,000

Model 2 vs. Model 3 ⇒ Model 2 >>> Model 3
Model 2 vs. Model 4 ⇒ Model 2 >>> Model 4
Model 2 vs. Model 1 ⇒ Can't decide

$P\{NPW_1 > NPW_2\}$

Model 1	Model 2	Joint Probability
\$1,000	No event	(0.35)(0.00) = 0.000
2,000	1,000	(0.40)(0.10) = 0.040
	1,500	(0.40)(0.45) = 0.180
3,000	1,000	(0.20)(0.10) = 0.020
	1,500	(0.20)(0.45) = 0.190
	2,500	(0.20)(0.35) = 0.070
4,000	1,000	(0.05)(0.10) = 0.005
	1,500	(0.05)(0.45) = 0.230
	2,500	(0.05)(0.35) = 0.018
		0.445

Decision Tree Analysis

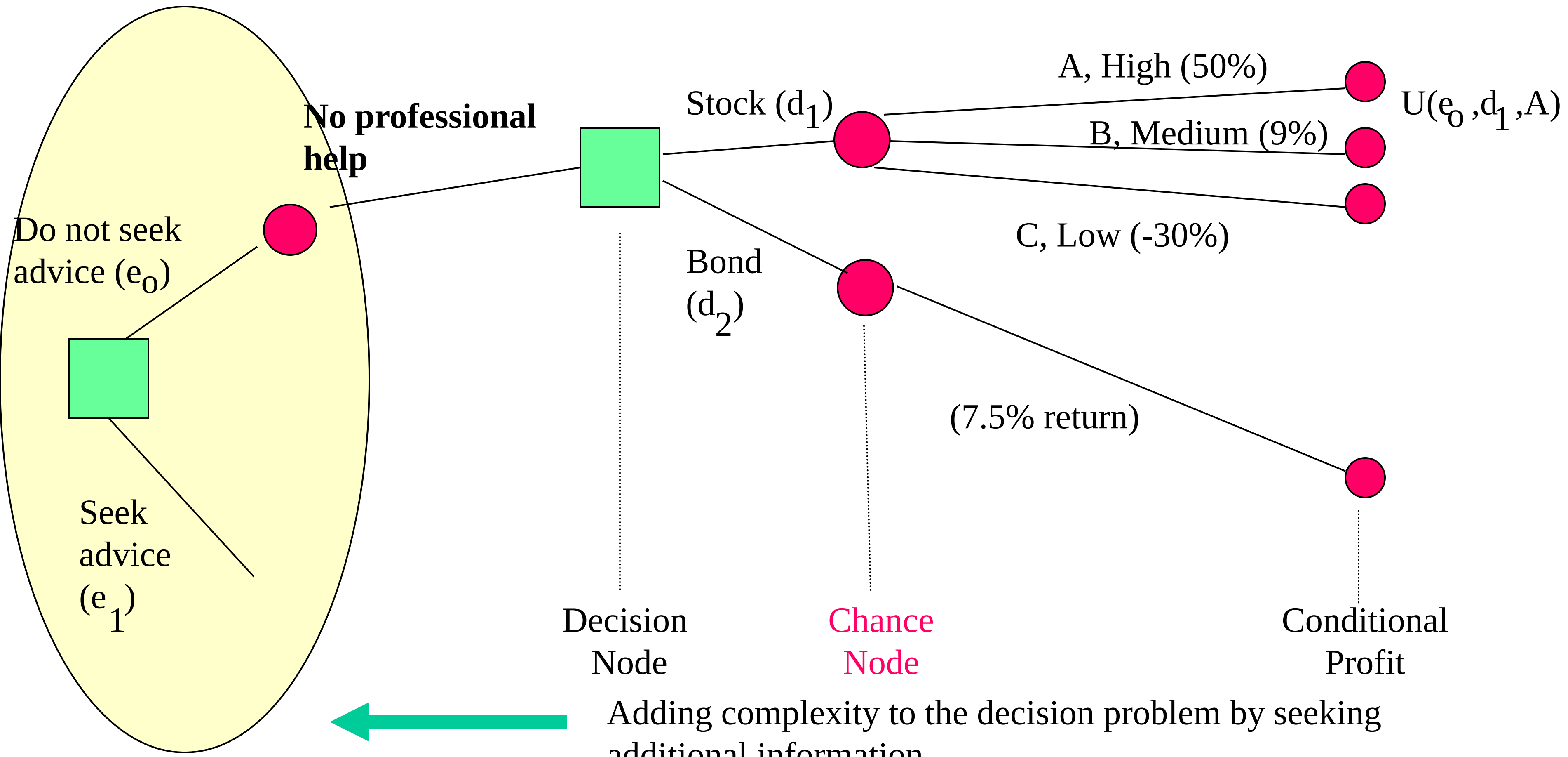
- A graphical tool for describing

  (1) the actions available to the decision-maker,

  (2) the events that can occur, and

  (3) the relationship between the actions and events.

Structure of a Typical Decision Tree



Adding complexity to the decision problem by seeking additional information

Bill's Investment Decision Problem

- Option 1:

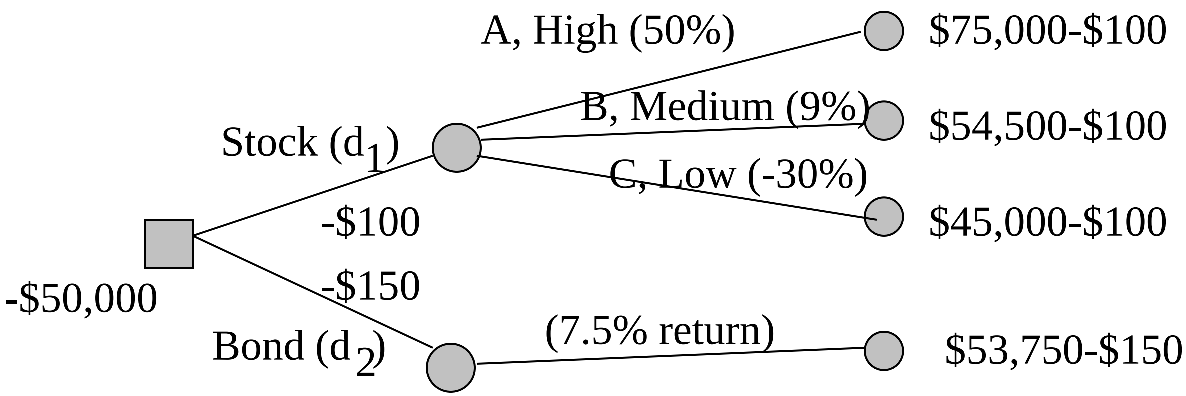
1) Period 0: (-\$50,000 - \$100) = -\$50,100
   Period 1: (+\$75,000 - \$100) = 0.20(\$24,800) =\$69,940
   PW(5%)=-\$50,100 + \$69,940 (*P/F*, 5%, 1) = \$16,510
2) Period 0: (-\$50,000 - \$100)= -\$50,100
   Period 1: (+\$54,000 - \$100)- (0.20)(\$4,200) = \$16,510
   PW(5%) = -\$50,100 + \$53,540 (*P/F*, 5%, 1) = \$890
3) Period 0: (-\$50,000 - \$100) = -\$50,100
   Period 1: (+\$35,000 - \$100) – (0.20)(-\$14,800) = \$37,940
   PW(5%)= - \$50,100 + \$37,940 (*P/F*, 5%, 1) = -\$13,967

- Option 2:

Period 0: (- \$50,000 - \$150) = -\$50,150
Period 1: (+\$53,750-\$150) = \$53,600
PW (5%)= -\$50,150 + \$53,600 (*P/F*, 5%, 1) = \$898

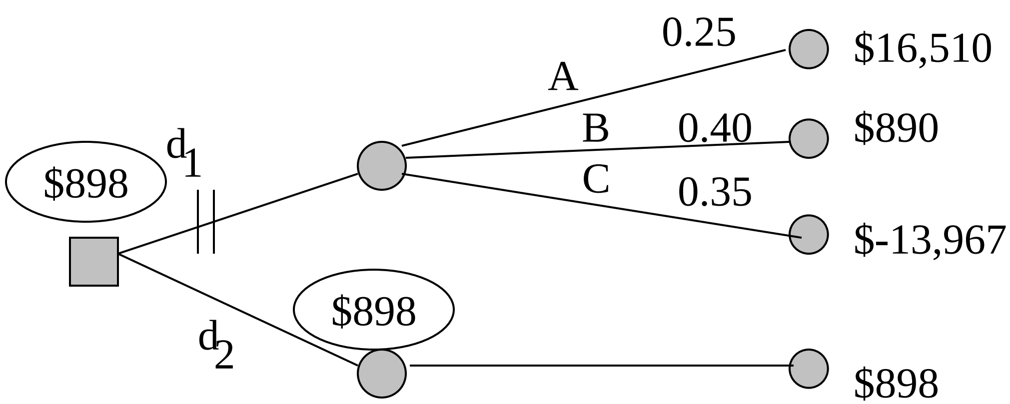
EMV = \$898
Or, prior optimal decision is Option 2

Decision tree for Bill's investment problem



(a)

(a) Relevant cash flows before taxes



(b)

(b) Net present worth for Each decision path

Expected Value of Perfect Information (EVPI)

- What is EVPI? This is equivalent to asking yourself how much you can improve your decision if you had perfect information.
- Mathematical Relationship:

$$EVPI = EPPI - EMV = EOL$$

where EPPI (Expected profit with perfect information) is the expected profit you could obtain if you had perfect information, and EMV (Expected monetary value) is the expected profit you could obtain based on your own judgment. This is equivalent to expected opportunity loss (EOL).

Expected Value of Perfect Information

Prior optimal decision, Option 2

Potential Return Level	Probability	Decision Option		Optimal Choice with Perfect Information	Opportunity Loss Associated with Investing in Bonds
		Option1: Invest in Stock	Option 2: Invest in Bonds		
High (A)	0.25	$16,510	$898	Stock	$15,612
Medium (B)	0.40	890	898	Bond	0
Low(C)	0.35	-13,967	898	Bond	0
	EMV	-$405	$898		$3,903

EPPI = (0.25)($16,510) + (0.40)($898)
+ (0.35)($898) = $4,801

EVPI = EPPI – EV
= $4,801 - $898
= $3,903

EOL = (0.25)($15,612)
+ (0.40)(0) + (0.35)(0)
= $3,903

Revised Cash flows after Paying Fee to Receive Advice (Fee = $200)

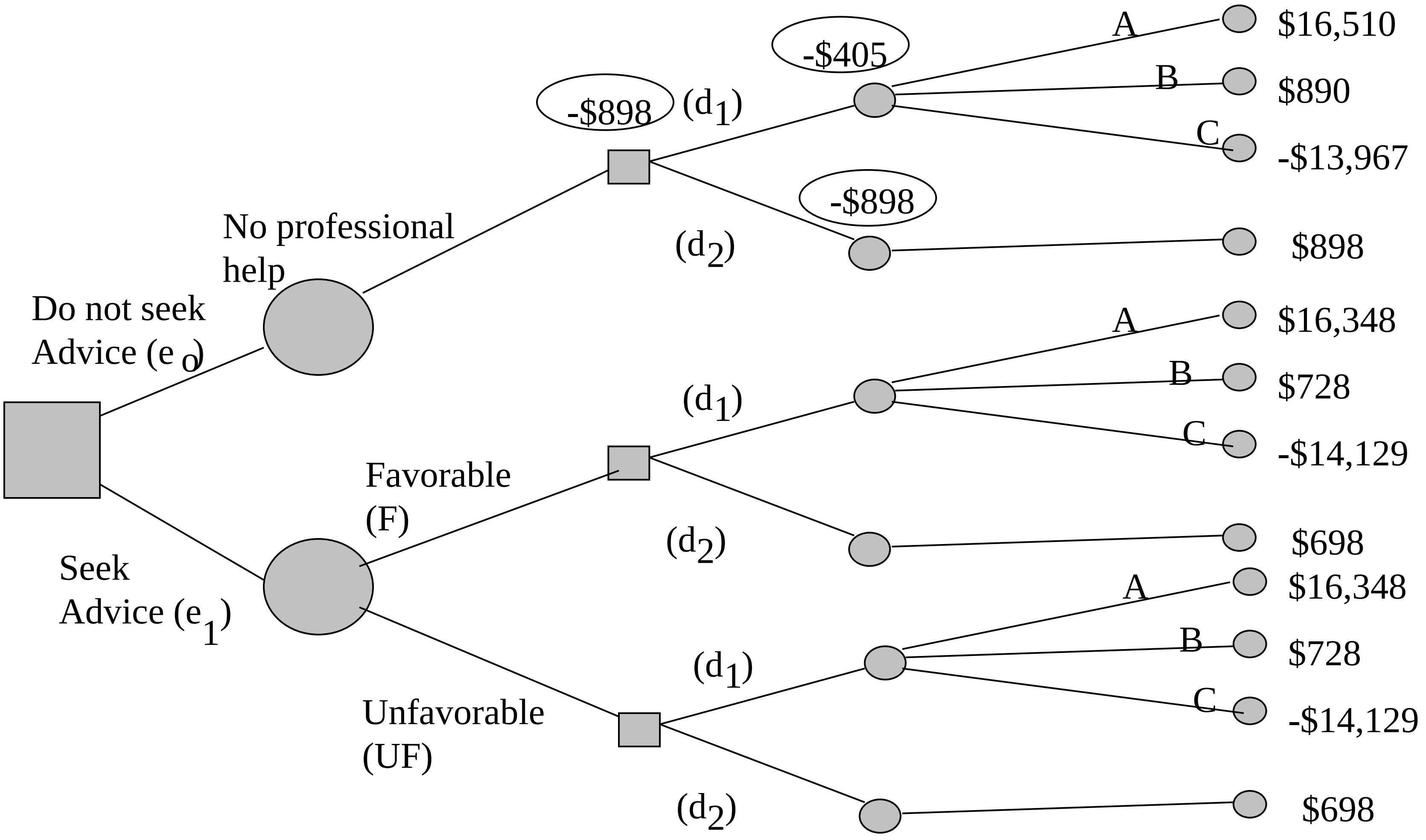
1. Stock Investment Option

Period 0: (- $50,000 - $100 - $200)= - $50,300
Period 1: (+$75,000 - $100) – (0.20)($25,000 - $400)= $69,980
PW(5%) = = $50,300 + $69,980(*P/F*, 5%, 1) = $\boxed{\$16{,}348}$

2. Bond Investment Option

Period 0 (-$50,000 - $150 -$200) = -$50,350
Period 1: (+$53,750 - $150) = $53,600
PW(5%)= -$50,350 + $53,600 (*P/F*, 5%, 1)= $\boxed{\$698}$

Decision tree for Bill's investment problem with an option having a professional advice

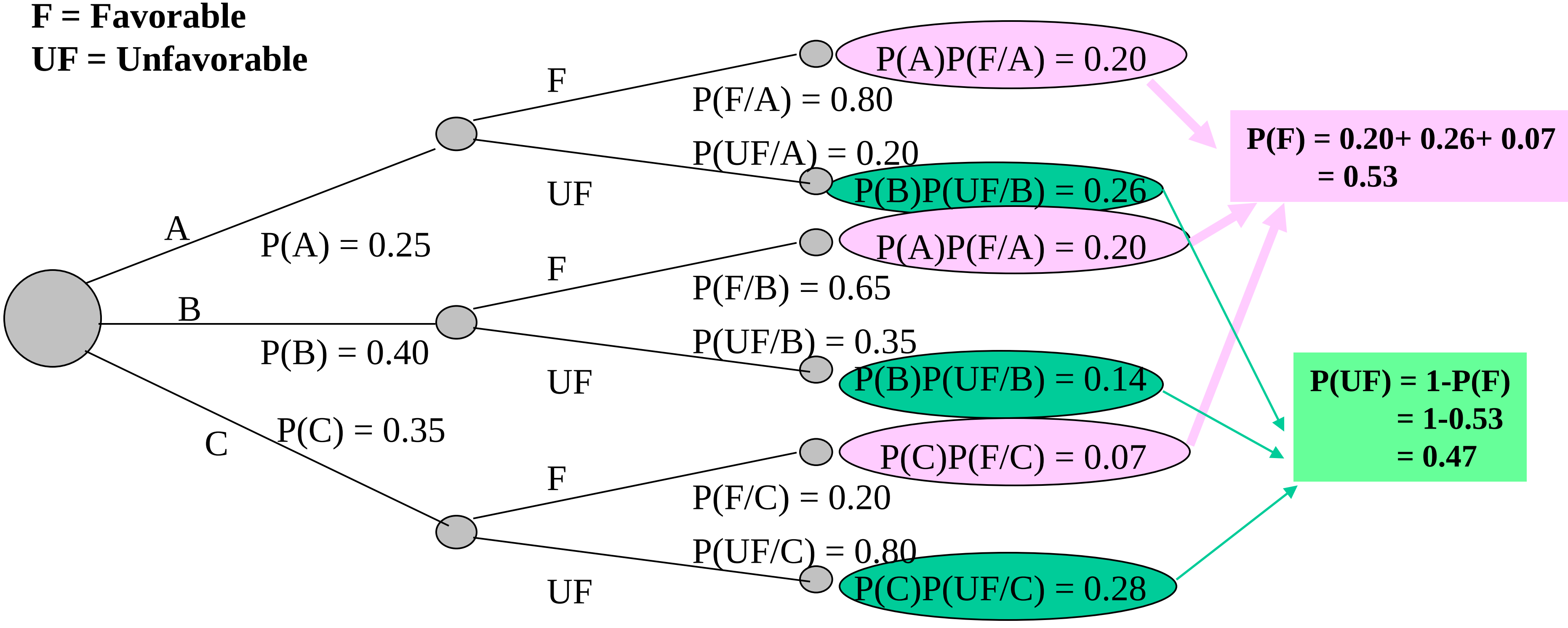


With Imperfect Sample Information

	Given Level of Stock Performance		
What the Report Will Say	High (A)	Medium (B)	Low (C)
Favorable (F)	0.80	0.65	0.20
Unfavorable (UF)	0.20	0.35	0.80

Nature’s Tree

F = Favorable
UF = Unfavorable



Joint and Marginal Probabilities

$$P(A,F) = P(F/A)P(A) = (0.80)(0.25) = 0.20$$

$$P(A,UF/A)P(A) = (0.20)(0.25) = 0.05$$

$$P(B,F) = P(F/B)P(B) = (0.65)(0.40) = 0.26$$

$$P(B,UF) = P(UF/B)P(B) = (0.35)(0.40) = 0.14$$

Joint as Well as Marginal Probabilities

	What Report Will Say Joint Probabilities		
When Potential Level of Return is Given	**Favorable (F)**	**Unfavorable (UF)**	**Marginal Probabilities of Return Level**
High (A)	0.20	0.05	0.25
Medium (B)	0.26	0.14	0.40
Low (C)	0.07	0.28	0.35
Marginal Probabilities	0.53	0.47	1.00

Determining Revised Probabilities

$P(A/F) = P(A,F)/P(F) = 0.20/0.53 = 0.38$

$P(B/F) = P(B,F)/P(F) = 0.26/0.53 = 0.49$

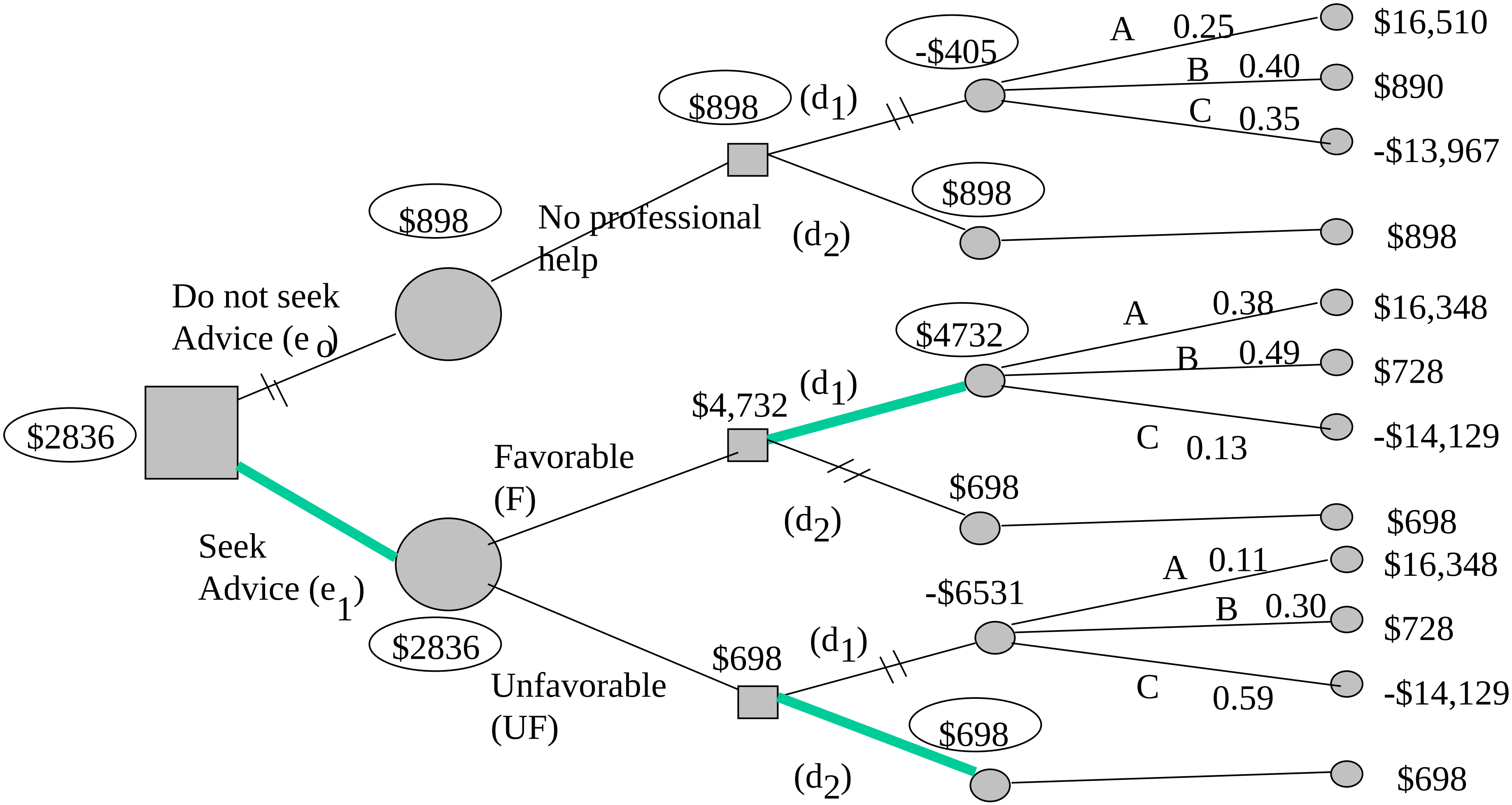
$P(C/F) = P(C,F)/P(F) = 0.07/0.53 = 0.13$

$P(A/UF) = P(A,UF)/P(UF) = 0.05/0.47 = 0.30$

$P(B/UF) + P(B,UF)/P(UF) = 0.14/0.47 = 0.30$

$P(C/UF) = P(C,UF)/P(UF) = 0.28/0.47 = 0.59$

Bill's investment problem after having professional advice



Summary

- Project risk—the possibility that in investment project will not meet our minimum requirements for acceptability and success.
- Our real task is not to try to find "risk-free" projects—they don't exist in real life. The challenge is to decide what level of risk we are willing to assume and then, having decided on your risk tolerance, to understand the implications of that choice.

•Three of the most basic tools for assessing project risk are as follows:

1. Sensitivity analysis– a means of identifying the project variables which, when varied, have the greatest effect on project acceptability.

2. Break-even analysis– a means of identifying the value of a particular project variable that causes the project to exactly break even.

3. Scenario analysis-- means of comparing a "base –case" or expected project measurement (such as NPW) to one or more additional scenarios, such as best and worst case, to identify the extreme and most likely project outcomes.

- Sensitivity, break-even, and scenario analyses are reasonably simple to apply, but also somewhat simplistic and imprecise in cases where we must deal with multifaceted project uncertainty.

- Probability concepts allow us to further refine the analysis of project risk by assigning numerical values to the likelihood that project variables will have certain values.

- The end goal of a probabilistic analysis of project variables is to produce a NPW distribution.

- From the NPW distribution, we can extract such useful information as the expected NPW value, the extent to which other NPW values vary from , or are clustered around, the expected value, (variance), and the best- and worst-case NPWs.
- The decision tree can facilitate investment decision making when uncertainty prevails, especially when the problem involves a sequence of decisions.

End of Lecture 9