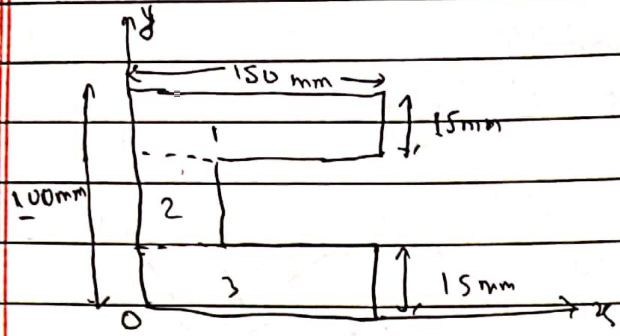
Assignment Ashwani kr. Chaudhary.

Centroid & Moment of Inertia

1)



let areas of 1, 2, 3. be :- a_1, a_2, a_3

$a_1 = 2250\ mm^2$

$a_2 = 100-(2\times15)\times15 = 1050\ mm^2$

$a_3 = 15\times50 = 2250\ mm^2$

Centroid of splitted respective parts be x_1, x_2, x_3 & y_1, y_2, y_3. Then,

$x_1 = 75mm$ $y_1 = (100-7.5) = 92.5mm$

$x_2 = 7.5\ mm$ $y_2 = (15+35) = 50mm$

$x_3 = 7.5\ mm$ $y_3 = 7.5\ mm$

first moment of area about x-axis,

$\Sigma a_i y_i$

$= y_1 a_1 + y_2 a_2 + y_3 a_3$

$= 92.5\times2250 + 50\times1050 + 7.5\times2250$

$= 277500\ mm^3$

first moment of area about y-axis.

$= \Sigma a_i x_i$

$= x_1 a_1 + x_2 a_2 + x_3 a_3$

$= 75\times2250 + 7.5\times1050 + 75\times2250$

$= 345375\ mm^3$

Then

$$\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i}$$

$$= \frac{345375}{2250 + 1050 + 2250}$$

$$= 62.23 \text{ mm}$$

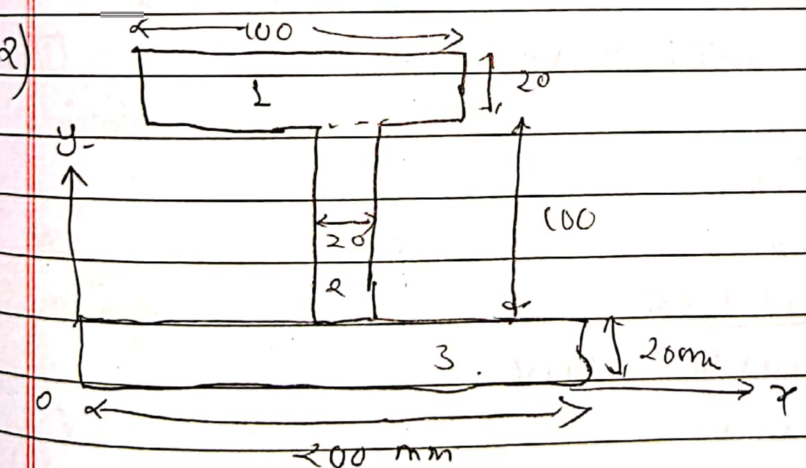
$$\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$$

$$= \frac{277500}{2250 + 1050 + 2250}$$

$$= 50 \text{ mm}$$

$\therefore$ Centroid of plane $(\bar{x}, \bar{y}) = (62.23, 50)$.

2)



Let area of 1, 2, 3 be a_1, a_2, a_3 lly.

Centroid of respective shapes be represented by x_1, x_2, x_3 & y_1, y_2, y_3

now,

$$a_1 = 100 \times 20 = 2000 \text{ mm}$$

$$a_2 = 100 \times 20 = 2000 \text{ mm}.$$

$a_3 = 200 \times 20 = 4000$ mm

$x_1 = 100$ mm $(50+50)$. $y_1 = 20 + 100 + 20/2 = 130$ mm

$x_2 = 100$ mm $(90 + 20/2)$ $y_2 = 20 + 100/2 = 50$ mm

$x_3 = 100$ mm $y_3 = 10$ mm.

first moment of area about x-axis.

$\Sigma a_i y_i$

$= y_1 a_1 + y_2 a_2 + y_3 a_3$

$= 130 \times 2000 + 50 \times 2000 + 10 \times 4000$

$= 400000 \text{ mm}^3$

IIly,

first MOA about y-axis

$\Sigma a_i x_i$

$= a_1 x_1 + a_2 x_2 + a_3 x_3$

$= 2000 \times 100 + 2000 \times 100 + 4000 \times 100$

$= 800000 \text{ mm}^3.$

Then,

$\bar{x} = \dfrac{\Sigma a_i x_i}{\Sigma x_i}$

$= \dfrac{800000}{2000 + 2000 + 4000}$

$= 100$ mm

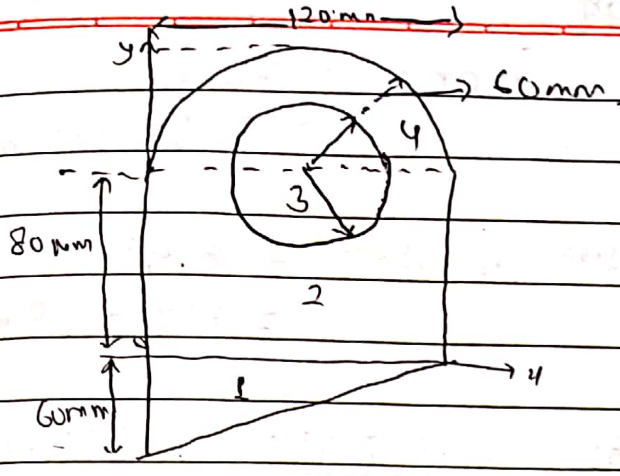
$\bar{y} = \dfrac{\Sigma a_i y_i}{\Sigma a_i}$

$= \dfrac{400000}{2000 + 2000 + 4000}$

$= 50$ mm.

Centroid of plane $(\bar{x}, \bar{y}) = (100, 50)$

3. sol^n,



Here,

1 → triangle
2 → rectangle
3 → circle
4 → semicircle

Comp.	x_i mm	y_i mm	a_i mm^2	$a_i x_i$ mm^3	$a_i y_i$ mm^3
(1) triangle	$120/3 = 40$	$-60/3 = -20$	$\frac{1}{2} \times 60 \times 120 = 3600$	144 000	-72000
(2) rectangle	$120/2 = 60$	$80/2 = 40$	$80 \times 120 = 9600$	576 000	384000
3 circle	$120/2 = 60$	80	$-\pi . 40^2$	-301592.4	-402123.2
4 Semi-circle	$120/2 = 60$	$80 + \frac{4 \times 60}{3\pi} = 105.46$	$(\pi \times 60^2)/2 = 5654.87$	339292.2	596362.6
	X	X	$\Sigma a_i = 13828.33$	$\Sigma a_i x_i = 757699.8$	$\Sigma a_i y_i = 5062$[illegible]4

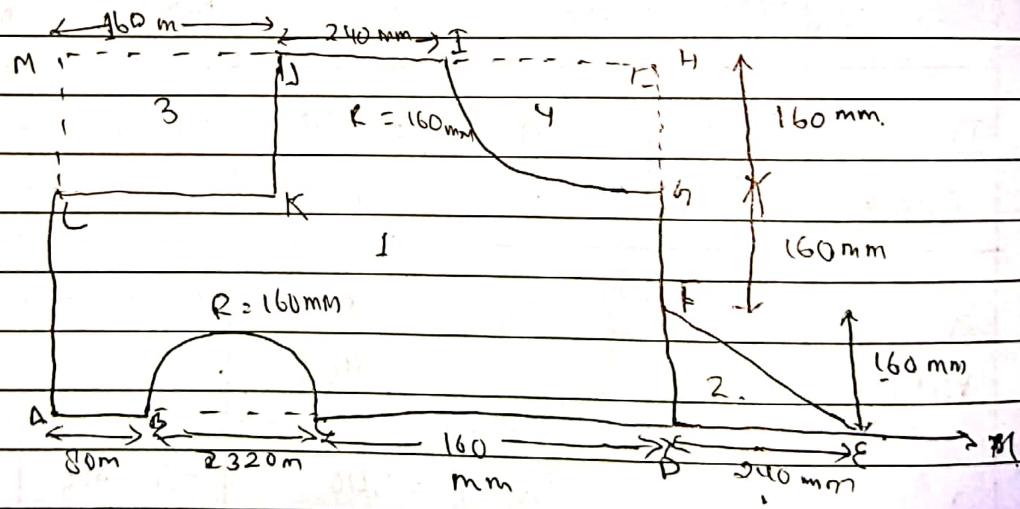
first MOA about x-axis $= \Sigma a_i y_i$
$= 506239.4\ mm^3$

first MOA about y-axis $= \Sigma a_i x_i$
$= 757699.8\ mm^3$

$$\bar{x} = \frac{\Sigma x_i a_i}{\Sigma a_i} = \frac{757699.9}{13828.33} = 54.79\ mm$$

$$\bar{y} = \frac{\Sigma y_i a_i}{\Sigma a_i} = \frac{506239.4}{13828.33} = 36.54\ mm$$

4) sol^n. (Replacing x in question with 40 mm)



1 ⇒ Rectangle ADHM
2 ⇒ Triangle DEF
3 ⇒ Rectangle JKLM
4 ⇒ Quarter circle
5 ⇒ Semi-circle

comp	x_i mm.	y_i mm.	a_i mm^2	$a_i x_i$ mm^3	$a_i y_i$ mm^3
1 Rect.	$560/2 = 280$	$480/2 = 240$	$560 \times 480 = 268800$	75264000	64512000
2 triangle	$560 + \frac{240}{3} = 80$	$160/3 = 53.33$	$\frac{1}{2} \times 240 \times 160 = 19200$	1536000	1024000
3 Rectangle	$160/2 = 80$	$320 + 160/2 = 400$	$160 \times 160 = 25600$	-2048000	-1024000
4 Q. circle	$-4 \times 160/3\pi + 560 = 492.1$	$80 - 4 \times \frac{160}{3\pi} = 412.1$	-20106.2	-9894257.56	-828576.128
5 semi circle	$160 + 80 = 240$	$4 \times 160/3\pi = 67.9$	$-\pi \times \frac{160^2}{2} = -40212.38$	-9650972.063	-2730421.001
			$\Sigma a_i = 202081.42$	$\Sigma a_i x_i = 55206769.81$	$\Sigma a_i y_i = 53495811.87$

first MOA about x-axis

$= \Sigma a_i y_i$

$= 53495816.87$

first MOA about y-axis

$= \Sigma a_i x_i$

$= 55206769.81$

$$\bar{x} = \frac{\Sigma x_i a_i}{\Sigma a_i}$$

$$= \frac{55206769.81}{202081.42}$$

$$= 273.2 \text{ mm}$$

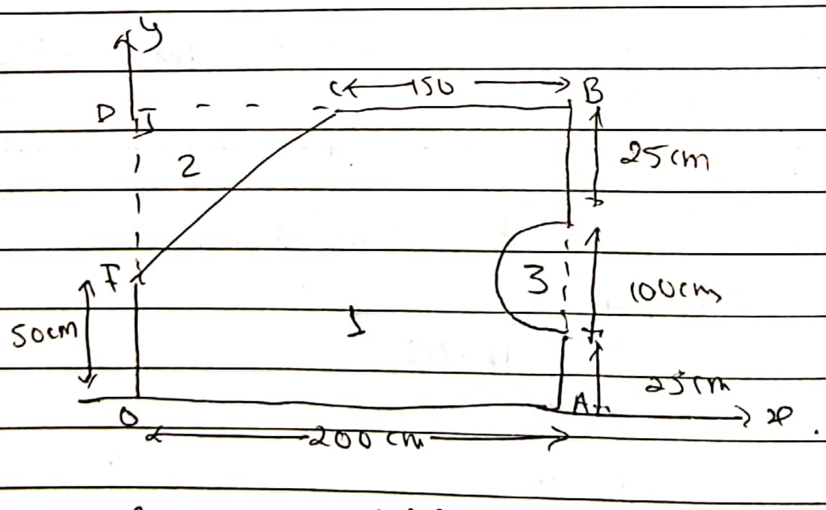
$$\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$$

$$= \frac{53495816.87}{202081.42}$$

$$= 264.72 \text{ mm.}$$

Centroid of plane $(\bar{x}, \bar{y}) = (273.2, 264.72)$

5) Sol^n.



1 => Rectangle OABD.

2 => Triangle CDF

3 => semi circle

Comp	x_i cm.	y_i cm.	a_i cm^2	$x_i a_i$ cm^3	$y_i a_i$ cm^3
(1) Rect. ABDO	100	75	$200 \times 150 = 30000$	3×10^6	2.25×10^6
(2) Triangle	$50/3 = 16.67$	$50 + \frac{100 \times 2}{3} = 116.67$	$-\frac{1}{2} \times 100 \times 50$	-41666.67	-291675
(3) semi-circle	$200 - \frac{4 \times 50}{3\pi} = 178.78$	$150/2 = 75$	$-\frac{\pi \times 50^2}{2} = -3927$	-702069.06	-294525
			$\Sigma a_i = 23573$	$\Sigma x_i a_i = 2256264.27$	$\Sigma y_i a_i = 1663800$

first MOA about x-axis

$= \Sigma a_i y_i$

$= 1663800 \ cm^3$

first MOA about y-axis

$= \Sigma a_i x_i$

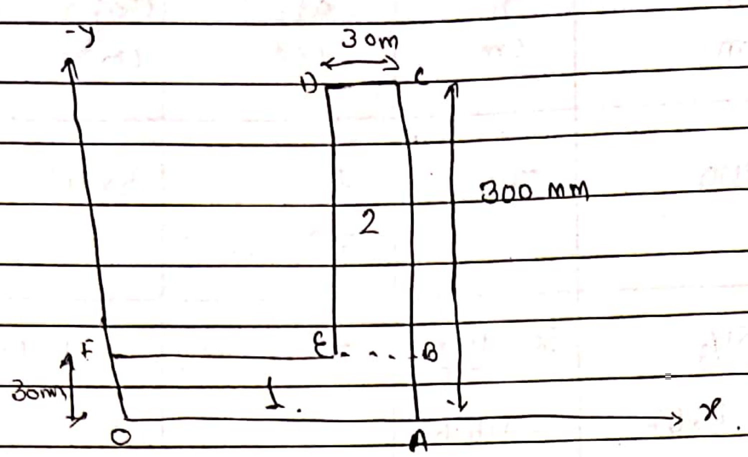
$= 2256264.27 \ cm^3$

$$\bar{x} = \frac{\Sigma x_i a_i}{\Sigma a_i} = \frac{2256264.27}{23573} = 95.71 \text{ cm}$$

$$\bar{y} = \frac{\Sigma y_i a_i}{\Sigma a_i} = \frac{1663800}{23573} = 70.58 \text{ cm.}$$

Centroid of plate $(\bar{x}, \bar{y}) = (95.71, 70.58)$.

6) Solⁿ,



Here

1 => Rectangle OABF

2 => Rectangle BCDE

Comp	x_i mm	y_i mm	a_i mm^2	$x_i a_i$ mm^3	$y_i a_i$ mm^3
(1) Rectangle OABF	240/2 = 120	30/2 = 15	240×30 = 7200	864000	108000
(2) Rectangle	240-15 = 225	30+270/2 = 165	270×30 = 8100	1822500	1336500
			Σa_i = 15300	$\Sigma x_i a_i$ = 2686500	$\Sigma y_i a_i$ = 1444500

first Mom about x-axis. $= \Sigma y_i a_i = 1444500 \ mm^3$

first MOA about y-axis $= \Sigma x_i a_i = 2686500 \ mm^3$

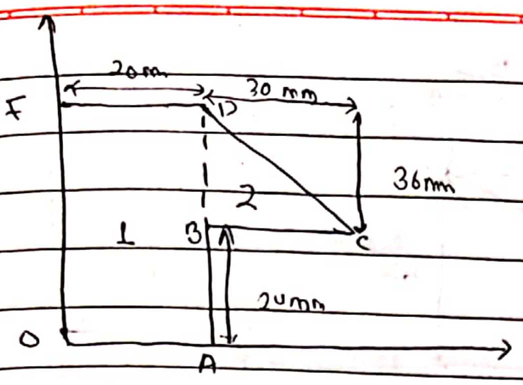
And

$\bar{x} = \frac{\Sigma x_i a_i}{\Sigma a_i} = \frac{2686500}{15300}$ $\qquad$ $\bar{y} = \frac{\Sigma y_i a_i}{\Sigma a_i} = \frac{1444500}{15300}$

$= 175.58 mm$ $\qquad$ $= 94.41 m$

∴ centroid of plane $(\bar{x}, \bar{y}) = (175.58, 94.41)$

7) Sol^n,



1 ⇒ rectangle OADF
2 ⇒ Triangle BCD.

Comp.	x_i mm	y_i mm	a_i mm^2	$x_i a_i$ mm^3	$y_i a_i$ mm^3
(1) Rectangle OADF	$20/2 = 10$	$60/2 = 30$	$20 \times 60 = 1200$	12000	36000
2 Triangle	$20 + 30/3 = 30$	$24 + 36/3 = 36$	$\frac{1}{2} \times 36 \times 30 = 540$	16200	19440
			$\Sigma a_i = 1740$	$\Sigma x_i a_i = 28200$	$\Sigma y_i a_i = 55440.$

Now,

first MOA about x-axis. $= \Sigma y_i a_i = 55440 \ mm^3$

first MOA about y-axis $= \Sigma x_i a_i = 28200 \ mm^3.$

Ans

$$\bar{x} = \frac{\Sigma x_i a_i}{\Sigma a_i} = \frac{28200}{1740} = 16.20 mm$$

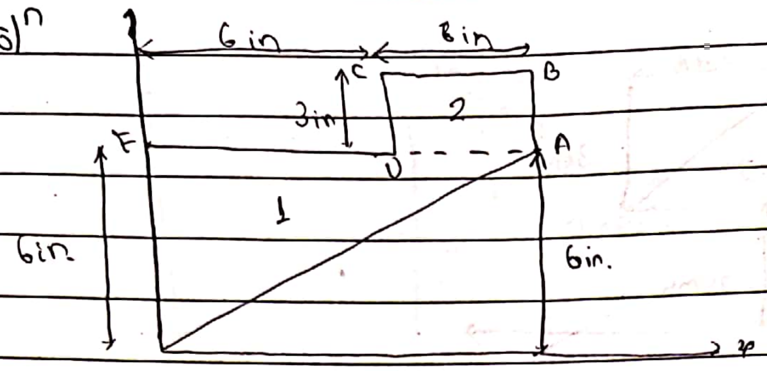
$$\bar{y} = \frac{\Sigma y_i a_i}{\Sigma a_i} = \frac{55440}{1740} = 31.86 \ mm$$

$\therefore$ Centroid of plane $(\bar{x}, \bar{y}) = (16.20, 31.86)$

classmate

Date
Page

8) Soln



1 => Triangle OAE

2 => Rectangle ABCD.

Comp.	x_i in	y_i in	a_i in^2	$x_i a_i$ in^3	$y_i a_i$ in^3
(1) Triangle	12/3 = 4	2×6/3 = 4	½×6×12 = 36	144	144
(2) Rectangle	6+3 = 9	6+1.5 = 7.5	3×6 = 18	162	135
			Σa_i = 54	$\Sigma x_i a_i$ = 306	$\Sigma y_i a_i$ = 279

first MOA about x-axis = $\Sigma y_i a_i$ = $279 \, in^3$

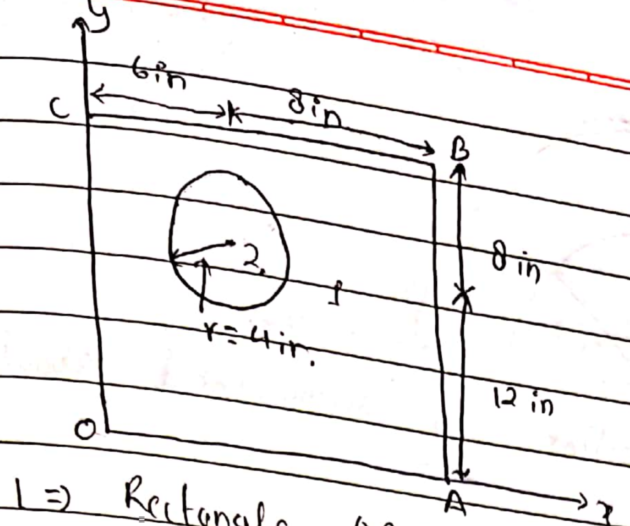
first MOA about y-axis = $\Sigma x_i a_i$ = $306 \, in^3$.

$$\bar{x} = \frac{\Sigma x_i a_i}{\Sigma a_i} = \frac{306}{54} = 5.66 \text{ in}$$

$$\bar{y} = \frac{\Sigma y_i a_i}{\Sigma a_i} = \frac{279}{54} = 5.16 \text{ in}$$

Centroid of plane $(\bar{x}, \bar{y}) = (5.66, 5.16)$

9) Sol^n.



1 ⇒ Rectangle OABC
2 ⇒ Circle

Comp.	x_i	y_i	a_i	$x_i a_i$	$y_i a_i$
(1) rectangle	$(6+8)/2 = 7$	$(12+8)/2 = 10$	$14 \times 20 = 280$	1960	2800
(2) Circle	6	12	$-\pi \times 4^2$ -50.26	-301.6	-603.18
			$\Sigma a_i = 229.74$	$\Sigma x_i a_i = 1658.4$	$\Sigma y_i a_i = 2196.82$

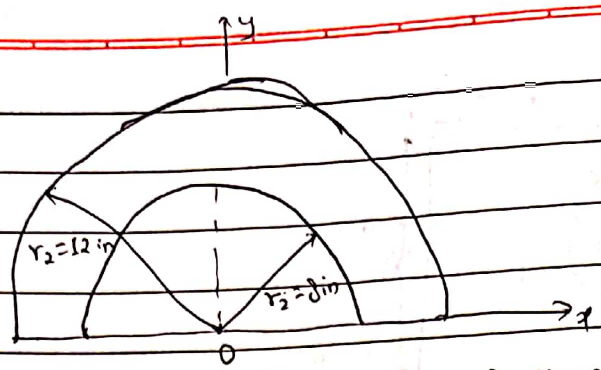
first MOA about x-(axis) $= \Sigma y_i a_i = 2196.82 \text{ in}^3$

first MOA about y-axis $= \Sigma x_i a_i = 1658.4 \text{ in}^3$

$$\bar{x} = \frac{\Sigma x_i a_i}{\Sigma a_i} = \frac{1658.4}{229.74} = 7.21 \text{ in} \qquad \bar{y} = \frac{\Sigma y_i a_i}{\Sigma a_i} = \frac{2196.82}{229.74} = 9.56 \text{ in.}$$

Centroid of plane $(\bar{x}, \bar{y}) = (7.21, 9.56)$.

10) sol^n,



Since the given section is symmetrical about y-axis

So,

$\bar{x} = 0.$

Then,

$Q_1 = (\pi \times 12^2)/2 = 226.2 \text{ in}^2.$ $\quad y_1 = 4 \times 12 / 3\pi$
$= 5.09$

$Q_2 = (-\pi \times 8^2)/2 = -100.53 \text{ in}^2.$ $\quad y_2 = 4 \times 8 / 3\pi$
$= 3.39.$

So,

$$\bar{y} = \frac{\Sigma y_i a_i}{\Sigma a_i}$$

$$= \frac{y_1 a_1 + y_2 a_2}{a_1 + a_2}$$

$$= \frac{5.09 \times 226.2 + 3.39 \times -100.53}{226.2 - 100.53}$$

$$= 6.4 \text{ in}$$

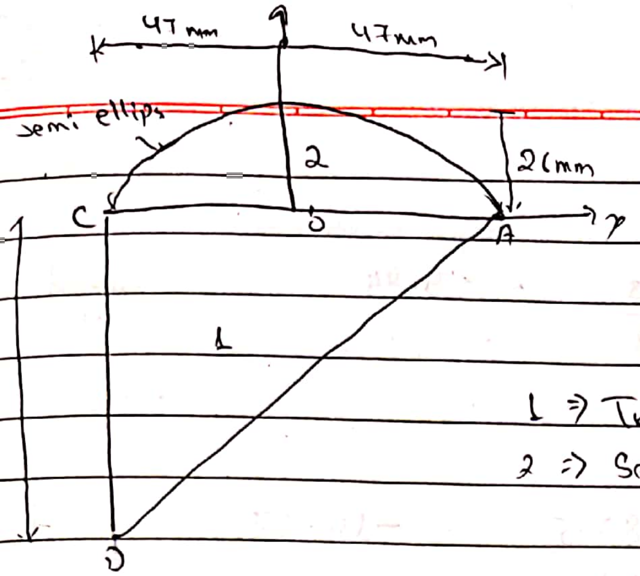
$\therefore$ Centroid of plane $(\bar{x}, \bar{y}) = (0, 6.4).$

First MOA about x-axis.
$= \Sigma y_i a_i$
$= 810.56 \text{ in}^3.$

first MOA about y-axis
$= \Sigma x_i a_i$
$= 0 \text{ in}^3$

Scanned with CamScanner

11) Sol^n



1 ⇒ Triangle ABC

2 ⇒ Semi ellipse

for triangle ABC,

$x_1 = -(47 - 94/3)$

$= -15.67$

$y_1 = -70/3 = -23.33$

$A_1 = 1/2 \times 70 \times 94 = 3290\ mm^2$

for semi-ellipse

$x_2 = 0$ (y-axis is the axis of symmetry)

$y_2 = \frac{4 \times 26}{3\pi} = 11.03$

$A_2 = \frac{\pi \times 26 \times 47}{2} = 1919.51\ mm^2$

Now,

first MOA about x-axis is $\Sigma y_i A_i$

$= y_1 A_1 + y_2 A_2$

$= -23.33 \times 3290 + 11.03 \times 1919.51$

$= -55583.5\ mm^3$.

first MOA about y-axis is $\Sigma x_i A_i$

$= x_1 A_1 + x_2 A_2$

$= -15.67 \times 3290 + 0 \times 1919.51$

$= -51554.3\ mm^3$.

$$\bar{x} = \frac{\Sigma x_i a_i}{\Sigma a_i}$$

$$= \frac{-51554.3}{5209.3} = -9.89$$

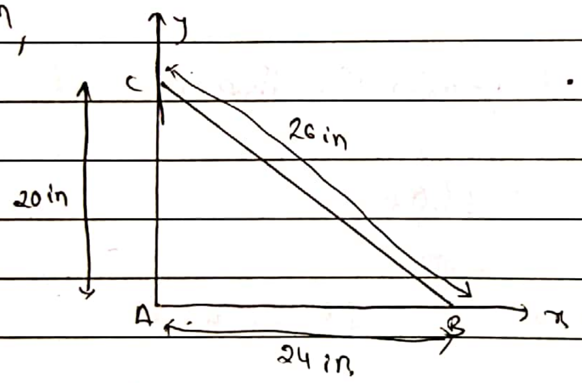
$$\bar{y} = \frac{\Sigma y_i a_i}{\Sigma a_i}$$

$$= \frac{-55583.5}{5209.3} = -10.67$$

Centroid of plane $(\bar{x}, \bar{y}) = (-9.89, -10.67)$

2) Determine the centroid of given lines.

1) Sol^n,



1 ⇒ segment AB
2 ⇒ segment BC
3 ⇒ segment AC.

Segments	l_i in	x_i in	y_i in	$l_i x_i$ in	$l_i y_i$ in
AB	24	12	0	288	0
BC	26	24/2 = 12	10/2 = 5	312	130
AC	20	0	5	0	50
				$\Sigma l_i x_i = 600$	$\Sigma l_i y_i = 180$

$$\bar{x} = \frac{\Sigma x_i y_i}{\Sigma l_i}$$

$$= \frac{600}{60}$$

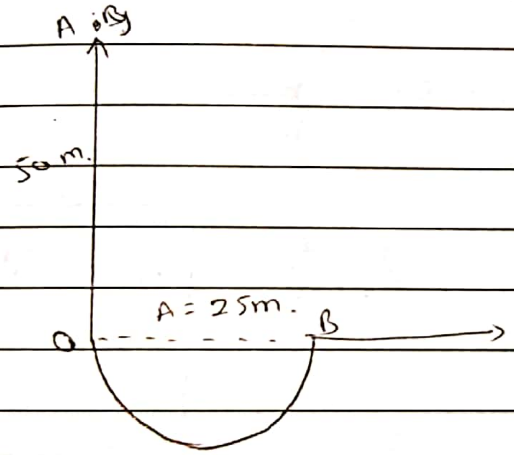
$$= 10 \text{ in}$$

$$\bar{y} = \frac{\Sigma l_i y_i}{\Sigma l_i}$$

$$= \frac{18\cancel{0}}{6\cancel{0}} = 3 \text{ in.}$$

$\therefore$ centroid of line $(\bar{x}, \bar{y}) = (10, 3)$

2) Sol^n,



1 => Segment OA

2 => semi circular Segment (OB).

Segment	x_i m	y_i m	l_i m	$l_i x_i$ m^2	$l_i y_i$ m^2
OA	0	25	50	0	1250
OB	25	$-2r/\pi$ => -15.91	$2\pi r/2$ = 78.54	1963.5	-1249.57.
	×	×	Σl_i = 128.54	$\Sigma l_i x_i$ = 1963.5	$\Sigma l_i y_i$ = 0.43.

Now,

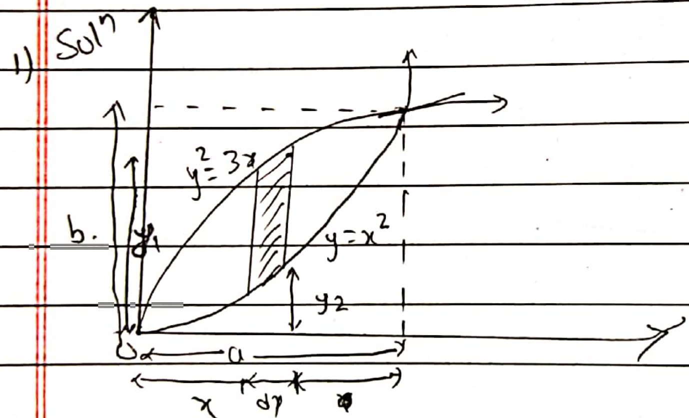
$$\bar{x} = \frac{\Sigma l_i x_i}{\Sigma l_i} = \frac{1963.5}{128.54} = 15.27$$

$$\bar{y} = \frac{\Sigma l_i y_i}{\Sigma l_i} = \frac{0.43}{128.54} = 0.0034 \approx 0$$

Centroid of line $(\bar{x}, \bar{y}) = (15.27, 0)$.

3) Determine (a) centroid & (b) first MOA w.r.t x & y-axis for the following area by method of Direct integration

1) Soln



take vertical strip for $\bar{x}$
take horizontal strip for $\bar{y}$

Here,

$y_1^2 = 3x$

$\Rightarrow y_1 = \sqrt{3x}$ —(i)

& $y_2 = x^2$ —(2)

We have,

$$dA = (y_1 - y_2)\, dx$$
$$= (\sqrt{3x} - x^2)\, dx$$

total area $A = \int_0^a dA = \int_0^a (\sqrt{3x} - x^2)\, dx$

$$= \int_0^a \sqrt{3x}\, dx - \int_0^a x^2\, dx$$

$$= \sqrt{3} \times \int_0^a x^{1/2}\, dx - \left[\frac{x^3}{3}\right]_0^a$$

$$= \sqrt{3} \times \left[\frac{x^{3/2}}{\left(\frac{3}{2}\right)}\right]_0^a - \frac{a^3}{3}.$$

$$= \frac{\sqrt{3}}{3/2} \times a^{3/2} - \frac{a^3}{3}$$

$$= \frac{2\sqrt{3}}{3} a^{3/2} - \frac{a^3}{3}$$

$$= \frac{2\sqrt{3}\, a^{3/2} - a^3}{3}.$$

Then

$$\bar{x} = \frac{1}{A} \int x_c\, dA$$

$$= \frac{3}{2\sqrt{3}\, a^{3/2} - a^3} \times \int x \cdot x(\sqrt{3x} - x^2)\, dx$$

$$= \frac{3}{2\sqrt{3}\, a^{3/2} - a^3} \times \int_0^a (\sqrt{3}\, x^{3/2} - x^3)\, dx$$

$$= \frac{3}{2\sqrt{3}\, a^{3/2} - a^3} \times \left[\sqrt{3} \int_0^a x^{3/2} dx - \int_0^a x^3 dx\right]$$

$$= \frac{3}{2\sqrt{3}\, a^{3/2} - a^3} \times \left[\sqrt{3} \left[\frac{x^{5/2}}{5/2}\right]_0^a - \left[\frac{x^4}{4}\right]_0^a\right]$$

$$= \frac{3}{2\sqrt{3}\, a^{3/2} - a^3} \times \left[\sqrt{3}\, \frac{a^{5/2}}{5/2} - \frac{a^4}{4}\right]$$

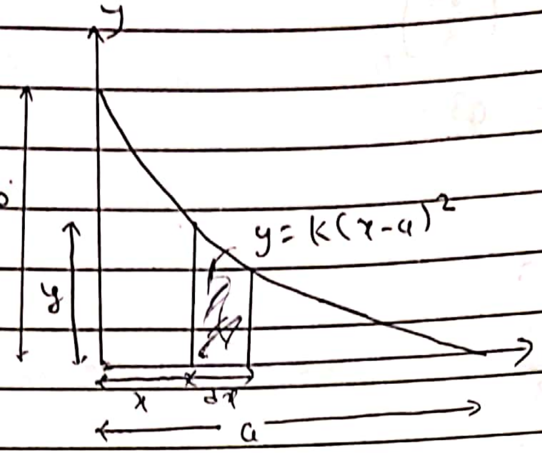
$$= \frac{3}{2\sqrt{3}\, a^{3/2} - a^3} \times \frac{2\sqrt{3}\, a^{5/2}}{5} - \frac{a^4}{4}$$

$$= \frac{3}{2\sqrt{3}\, a^{3/2} - a^3} \times \frac{2\sqrt{3}\, a^{5/2} - 5a^4}{20}$$

$$= \frac{3}{2\sqrt{3}\, a^{3/2} - a^3} \times \frac{8\sqrt{3}\, a^{5/2} - 5a^4}{20}$$

$$= \frac{24\sqrt{3}\, a^{5/2} - 15a^4}{40\sqrt{3}\, a^{3/2} - 2a^3}. \quad \#$$

2) Sol^n



we have

$$dA = y\,dx \qquad y = k(x-a)^2 \quad \text{---(i)}$$

then,

$$A = \int_0^a dA$$

$$= \int_0^a y\,dx$$

$$= \int_0^a k(x-a)^2 dx$$

$$= k\int_0^a (x^2 - 2ax + a^2)\,dx$$

$$= k\left\{\left[\frac{x^3}{3}\right]_0^a - 2a\left[\frac{x^2}{2}\right]_0^a + a^2[x]_0^a\right\}$$

$$= k\left[\frac{a^3}{3} - \cancel{2}a \cdot \frac{a^2}{\cancel{2}} + a^2 \times a\right]$$

$$= k\left[\frac{a^3}{3} - a^3 + a^3\right]$$

$$= \frac{ka^3}{3}$$

then,

$$\bar{x} = \frac{1}{A}\int x_c\,dA$$

$$= \frac{3}{ka^3}\int_0^a x\left[k(x-a)^2\right]dx$$

$$= \frac{3 \times k}{ka^3}\int_0^a x(x^2 - 2ax + a^2)\,dx$$

$$= \frac{3}{a^3}\int (x^3 - 2ax^2 + a^2x)\, dx$$

$$= \frac{3}{a^3}\left\{\left[\frac{x^4}{4}\right]_0^a - 2a\left[\frac{x^3}{3}\right]_0^a + a^2\left[\frac{x^2}{2}\right]_0^a\right\}$$

$$= \frac{3}{a^3} \times \left[\frac{a^4}{4} - \frac{2a.a^3}{3} + \frac{a^2.a^2}{2}\right]$$

$$= \frac{\cancel{3}}{a^3} \times \left[\frac{3a^4 - 8a^4 + 6a^4}{\cancel{12}\,4}\right]$$

$$= \frac{1}{\cancel{a^3}} \times \frac{a^4}{4} = \frac{a}{4}.$$

again.

$$\bar{y} = \frac{1}{A}\int y_c\, dA$$

$$= \frac{3}{ka^3}\int_0^a \frac{y}{2a}\, y\, dx$$

$$= \frac{3}{2ka^3}\int_0^a y^2\, dx$$

$$= \frac{3}{2ka^3}\int_0^a \{k(x-a)^2\}^2\, dx$$

$$= \frac{3}{2ka^3} \times k^2 \times \int_0^a (x^2 - 2ax + a^2)^2\, dx$$

$$= \frac{3}{2ka^3} \times k^2 \times \int_0^a (x^4 - 4ax^3 + 6a^2x^2 - 4a^3x + a^4)\, dx$$

$$= \frac{3k}{2a^3} \times \left[\left[\frac{x^5}{5}\right]_0^a - 4a \times \left[\frac{x^4}{4}\right]_0^a + 6a^2\left[\frac{x^3}{3}\right]_0^a - 4a^3\left[\frac{x^2}{2}\right]_0^a + a\right.$$

$$= \frac{3k}{2a^3} \times \frac{a^5}{5} - \cancel{4}a\frac{a^4}{4} + 6a^2\frac{a^3}{3} - 4a^3\frac{a^2}{2} + a.$$

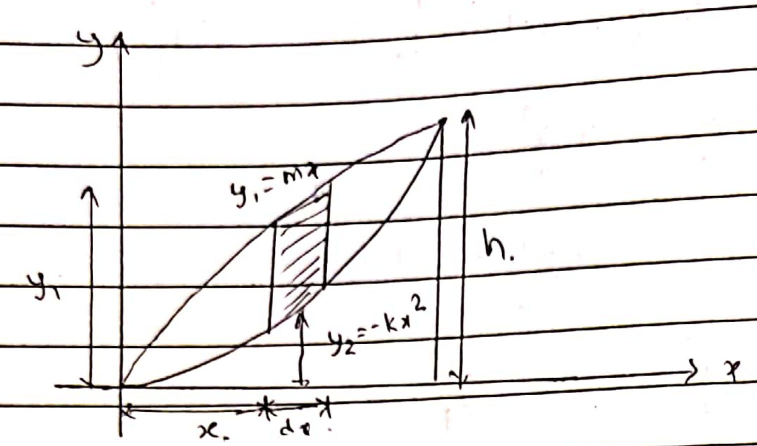
$$= \frac{3k}{2a^3} \times \left[\frac{a^5}{5} - a^5 + 2a^5 - 2a^5 + a^5\right]$$

$$= \frac{2k}{2a^3} \times \frac{a^5}{5} = \frac{2ka^2}{10}$$ #

Scanned with CamScanner

3) Sol^n,

for vertical strip.



we have $y = mx$ —(i)
$y = kx^2$ —(ii)

for $x = a, y = h$. these eq^n a become
$h = ma$ & $h = ka^2$
$\Rightarrow m = \frac{h}{a}$ & $k = \frac{h}{a^2}$

$dA = (y_1 - y_2)\,dx = (mx - kx^2)\,dx$

so, total Area is

$$A = \int_0^a dA$$

$$= \int_0^a (mx - kx^2)\,dx$$

$$= m\int_0^a x\,dx - k\int_0^a x^2\,dx$$

$$= m\left[\frac{x^2}{2}\right]_0^a - k\left[\frac{x^3}{3}\right]_0^a$$

$$= \frac{h}{a} \times \frac{a^2}{2} - \frac{h}{a^2} \times \frac{a^3}{3} = \frac{ah}{6}$$

now

$$\bar{x} = \frac{1}{A}\int x_c\,dA$$

$$= \frac{6}{ah}\int_0^a x \cdot (mx - kx^2)\,dx$$

$$= \frac{6}{ah}\int_0^a (mx^2 - kx^3)\,dx$$

Scanned with CamScanner

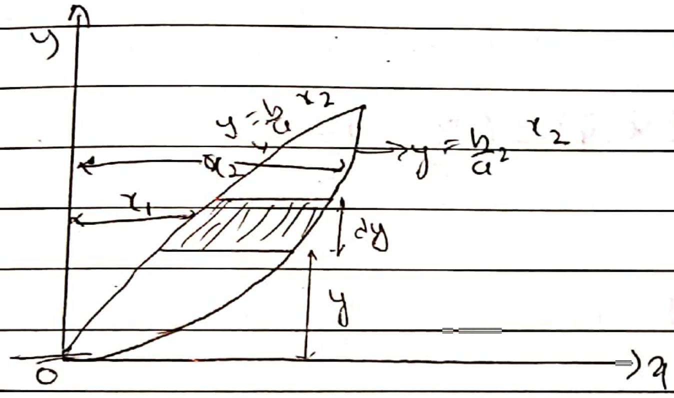
$$= \frac{6}{ah}\left[\frac{h}{a}\cdot\frac{x^3}{3} - \frac{h}{a^2}\cdot\frac{x^4}{4}\right]_0^a$$

$$= \frac{6}{ah}\left[\frac{h}{a}\cdot\frac{a^3}{3} - \frac{h}{a^2}\cdot\frac{a^4}{4}\right]_0^a$$

$$= \frac{6}{ah}\left[\frac{4ha^2 - 3ha^2}{12}\right]$$

$$= \frac{6}{ah} \times \frac{a^2h}{12} = \frac{a}{2}$$

for horizontal strip.



$$dA = (x_2 - x_1)\,dy$$

$$\bar{y} = \int \frac{y_c\,dA}{A} = \frac{1}{A}\int y_c\,dA$$

$$= \frac{6}{ah}\int y\cdot(x_2 - x_1)\,dy \quad \text{—} (*)$$

we have,

$$y = \frac{h}{a^2} \times x_2^2 \qquad\qquad y = \frac{h}{a}x_1$$

$$\Rightarrow x_2 = \frac{a}{\sqrt{h}}y^{1/2} \qquad\qquad \Rightarrow x_1 = \left(\frac{a}{h}\right)y.$$

from eq^n $(*)$ we have

$$\bar{y} = \frac{6}{ah}\int_0^h y\left(\frac{a}{\sqrt{h}}y^{1/2} - \frac{ay}{h}\right)dy$$

$$= \frac{6}{ah}\int_0^h \frac{a}{\sqrt{h}}y^{3/2} - \frac{a}{h}y^2\,dy.$$

$$= \frac{6}{ah}\left[\frac{a}{\sqrt{h}} \cdot \frac{y^{5/2}}{5/2} - \frac{a}{h} \cdot \frac{y^3}{3}\right]_0^h$$

$$= \frac{6}{ah}\left[\frac{a}{h^{1/2}} \cdot \frac{h^{5/2}}{5/2} - \frac{a}{h} \cdot \frac{h^3}{3}\right]$$

$$= \frac{6}{ah}\left[\frac{2ah^2}{5} - \frac{ah^2}{3}\right]$$

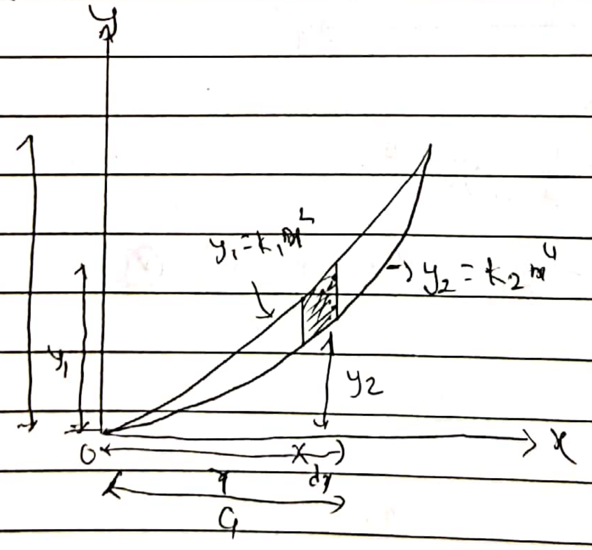
$$= \frac{6}{ah} \times \left[\frac{6ah^2 - 5ah^2}{15}\right]$$

$$= \frac{\cancel{6}^{2}}{a\cancel{h}} \times \frac{h^{\cancel{2}}}{\cancel{15}_{5}}$$

$$= \frac{2b}{5}$$

$$\text{centroid } (\bar{x}, \bar{y}) = \left(\frac{a}{2}, \frac{2b}{5}\right) \underline{A}$$

4) Sol^n,



here

$y_1 = k_1 x^2$ —(i)

$y_2 = k_2 x^4$ —(2)

$y_1 = y_2 = b$

$x = a$

so &

$b = k_1 a^2$ $b = k_2 a^4$

classmate

Date

Page

$$\Rightarrow k_1 = \frac{b}{a^2} \qquad \Rightarrow k_2 = \frac{b}{a^4}.$$

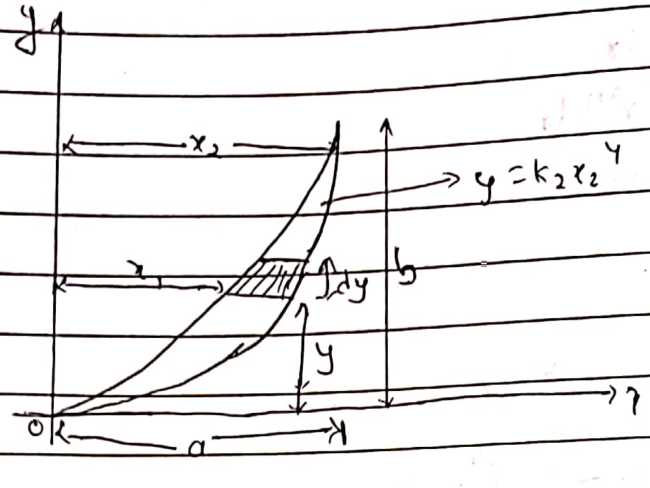
$$dA = (y_1 - y_2)\,dx$$
$$= (k_1x^2 - k_2x^4)\,dx$$

Thus, required area is

$$A = \int_0^a dA$$
$$= \int_0^a (k_1x^2 - k_2x^4)\,dx$$
$$= k_1\int_0^a x^2 dx - k_2\int_0^a x^4 dx$$
$$= k_1 \times \left[\frac{x^3}{3}\right]_0^a - k_2 \times \left[\frac{x^5}{5}\right]_0^a$$
$$= k_1 \times \frac{a^3}{3} - k_2 \times \frac{a^5}{5}$$
$$= \frac{b}{\cancel{a^2}} \times \frac{a^{\cancel{3}}}{3} - \frac{b}{\cancel{a^4}} \times \frac{a^{\cancel{5}}}{5}$$
$$= \frac{ab}{3} - \frac{ab}{5} \qquad = \frac{5ab - 3ab}{15} \qquad = \frac{2ab}{15}.$$

$$\bar{x} = \frac{1}{A}\int x_c\,dA$$
$$= \frac{1}{2ab/15}\int_0^a x \times (k_1x^2 - k_2x^4)\,dx$$
$$= \frac{15}{2ab}\left\{k_1\int_0^a x^3 dx - k_2\int_0^a x^5 dx\right\}$$
$$= \frac{15}{2ab}\left\{k_1\left[\frac{x^4}{4}\right]_0^a - k_2\left[\frac{x^6}{6}\right]_0^a\right\}$$
$$= \frac{15}{2ab} \times \left[\frac{b}{a^2} \times \frac{a^4}{4} - \frac{b}{a^4} \times \frac{a^6}{6}\right]$$
$$= \frac{15}{2ab}\left[\frac{a^2b}{4} - \frac{a^2b}{6}\right]$$
$$= \frac{15}{2ab} \times \frac{a^2b}{12}$$
$$= \frac{5a}{8}$$

Horizontal strip:-



$\bar{y} = \frac{1}{A} \int y_c \, dA.$

$= \frac{1}{A} \int y_c (x_2 - x_1) \, dy$ —— (x)

now,

$y = k_1 x_1^2$ &

$y = \frac{b}{a^2} \times x_1^2$

$\Rightarrow x_1^2 = \left(\frac{a^2}{b}\right) y.$

$\Rightarrow x_1 = \frac{a}{b^{1/2}} y^{1/2}$

$y = k_2 x_2^4$

$y = \frac{b}{a^4} \times x_2^4$

$\Rightarrow x_2^4 = \left(\frac{a^4}{b}\right) y.$

$\Rightarrow x_2 = \frac{a}{b^{1/4}} y^{1/4}$

so, eqn (x) become

$$\bar{y} = \frac{15}{2ab} \int_0^b y \times \left(\frac{a}{b^{1/4}} y^{1/4} - \frac{a}{b^{1/2}} \cdot y^{1/2} \right) dy.$$

$$= \frac{15}{2ab} \int_0^b \left(\frac{a y^{5/4}}{b^{1/4}} - \frac{a y^{3/2}}{b^{1/2}} \right) dy$$

$$= \frac{15}{2ab} \left[\int_0^b \frac{a y^{5/4}}{b^{1/4}} dy - \int_0^b \frac{a y^{3/2}}{b^{1/2}} dy \right]$$

$$= \frac{15}{2ab} \left[\frac{a}{b^{1/4}} \left[\frac{y^{9/4}}{9/4} \right]_0^b - \frac{a}{b^{1/2}} \left[\frac{y^{5/2}}{5/2} \right]_0^b \right]$$

$$= \frac{15}{2ab} \left[\frac{a}{b^{1/4}} \times \frac{b^{3/4}}{9/4} - \frac{a}{b^{1/2}} \times \frac{b^{5/2}}{5/2} \right]$$

$$= \frac{15}{2ab}\left[\frac{4}{9}ab^2 - \frac{2ab^2}{5}\right]$$

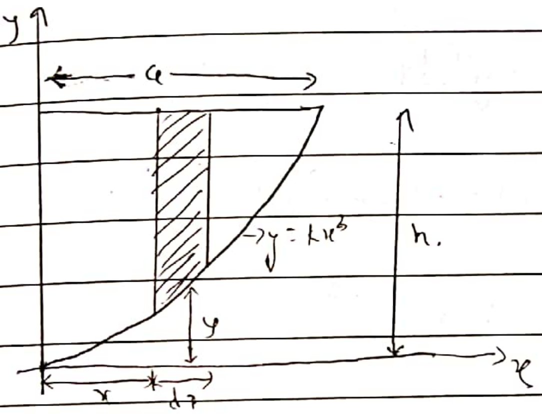
$$= \frac{15}{2ab}\left[\frac{20ab - 18ab^2}{45}\right]$$

$$= \frac{b}{3}$$

centroid $(\bar{x}, \bar{y}) = \left(\frac{5a}{8}, \frac{b}{3}\right)$

5) Sol^n,

For vertical strip.



here,

At $x=a$, $y=h$

$y = kx^3$

$\Rightarrow h = ka^3$

$\Rightarrow k = \frac{h}{a^3}$

Then,

$dA = (h-y)\,dx$

So,

total area is.

$$A = \int_0^a dA$$

$$= \int_0^a (h-y)\,dx$$

$$= \int_0^a \left(h - \frac{h}{a^3}x^3\right)dx$$

$$= \int_0^a h\,dx - \int_0^a \frac{h}{a^3} x^3\,dx$$

$$= h\int_0^a dx - \frac{h}{a^3}\int_0^a x^3\,dx$$

$$= ha - \frac{h}{a^3} \times \frac{a^4}{4}$$

$$= ha - \frac{ha}{4} = \frac{3ha}{4}$$

$$\bar{x} = \frac{1}{A}\int_0^a x_c\,dA$$

$$= \frac{4}{3ha}\int_0^a x(h-y)\,dx$$

$$= \frac{4}{3ha}\int_0^a (xh - xy)\,dx$$

$$= \frac{4}{3ha}\int_0^a \left(x \times h - x \times \frac{h}{a^3} \times x^3\right) dx$$

$$= \frac{4}{3ha}\left[\int_0^a xh\,dx - \int_0^a \frac{h}{a^3} x x^4\,dx\right]$$

$$= \frac{4}{3ha}\left[h\int_0^a x\,dx - \frac{h}{a^3}\int_0^a x^4\,dx\right]$$

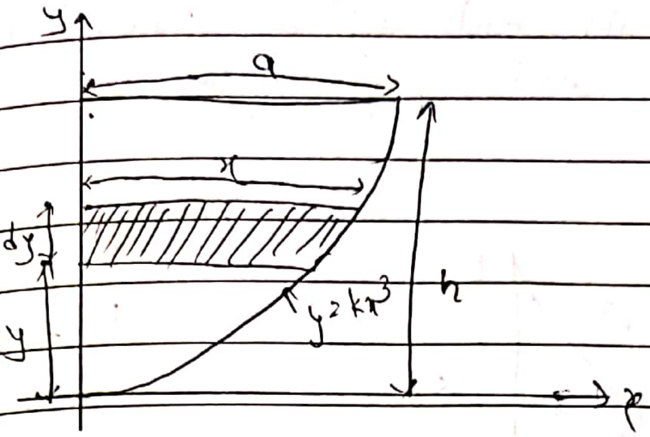
$$= \frac{4}{3ha}\left[\frac{ha^2}{2} - \frac{h}{a^3} \times \frac{a^5}{5}\right]$$

$$= \frac{4}{3ha}\left[\frac{ha^2}{2} - \frac{ha^2}{5}\right]$$

$$= \frac{4}{3ha} \times \frac{3ha^2}{20}$$

$$= \frac{2a}{5}$$

For horizontal strip



here,

$y = kx^3$

$\Rightarrow x^3 = y/k$

$\Rightarrow x = (y/k)^{1/3}$

$dA = x\,dy$

$\bar{y} = \frac{1}{A}\int y_c\,dA$

$= \frac{4}{3ha}\int_0^h y \cdot x\,dy$

$= \frac{4}{3ha}\int_0^h y \times \frac{y^{1/3}}{k^{1/3}}\,dy$

$= \frac{4}{3ha}\int_0^h \frac{y^{4/3}}{k^{1/3}}\,dy$

$= \frac{4}{3ha \times \left(\frac{h}{a^3}\right)^{1/3}} \times \int_0^h y^{4/3}\,dy$

$= \frac{4}{3ha} \times \frac{1}{\frac{h^{1/3}}{a}} \left[\frac{y^{7/3}}{7/3}\right]_0^h$

$= \frac{4}{3ha} \times \frac{1}{1} \times \frac{a}{h^{1/3}} \times \frac{h^{7/3}}{7/3}$

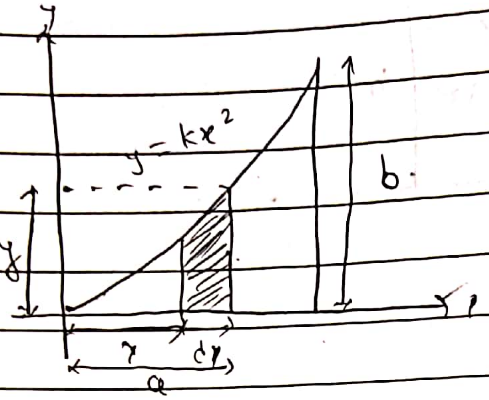
$= \frac{4}{3ha} \times \frac{a}{h^{1/3}} \times \frac{3}{7} \times h^{7/3}$

$= \frac{4h^{7/3}}{h^{1/3} \times 7} = \frac{4h}{7}$ $\therefore$ Centroid $(\bar{x}, \bar{y}) = \left(\frac{2a}{5}, \frac{4h}{7}\right)$

Scanned with CamScanner

4) Determine MOI of the shaded area about co-ordinate axes & using its result determine radius of gyration of shaded area w.r.t co-ordinate axes

1) Soln.



For every question
dI_y = Vertical strip
dI_x = Horizontal strip

$y = kx^2$

At $x = a$, $y = b$

$\Rightarrow k = \frac{b}{a^2}$

$\therefore \left(y = \frac{b}{a^2} \cdot x^2 \right)$

MOI, I_y.

$dI_y = dA\, x^2$

$= x^2 dA$

$= x^2 (1 y dx)$

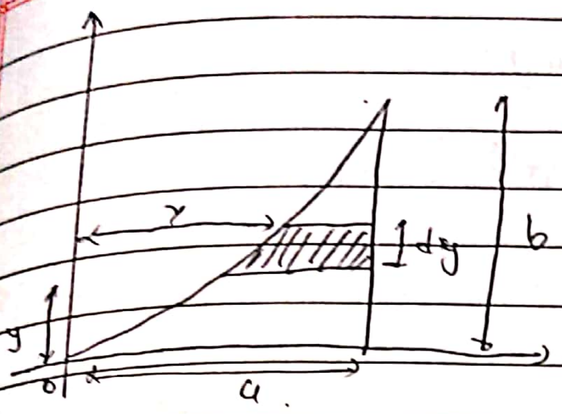
$= x^2 \times \frac{b}{a^2} x^2 dx$

$I_y = \int_0^a dI_y = \int_0^a \frac{b}{a^2} x^4 dx$

$= \frac{b}{a^2} \left[\frac{x^5}{5} \right]_0^a$

$= \frac{b}{a^2} \times \frac{a^5}{5}$

$= \frac{ba^3}{5}$



$dA = (a-x)\,dy$

$y = \frac{b}{a^2} x^2$

$x^2 = \frac{a^2 y}{b}$

$\Rightarrow x^2 = \frac{a y^{1/2}}{b^{1/2}}$

For MoI, I_x:

$dI_x = y^2 dA$

$= y^2 (a-x)\,dy$

$= y^2 \left(a - \frac{a y^{1/2}}{b^{1/2}} \right) dy$

$I_x = \int_a^b dI_x$

$= \int_0^b y^2 \left(a - \frac{a y^{1/2}}{b^{1/2}} \right) dy$

$= \int_0^b \left(a y^2 - \frac{a y^{5/2}}{b^{1/2}} \right) dy$

$= \left[\frac{a y^3}{3} - \frac{a y^{7/2}}{(7/2)\, b^{1/2}} \right]_0^b$

$= \frac{a b^3}{3} - \frac{a b^{7/2}}{\frac{7}{2} b^{1/2}}$

$= \frac{a b^3}{3} - \frac{2 a b^3}{7}$

$= \frac{7ab^3 - 6ab^3}{21} \quad = \frac{ab^3}{21}$

Scanned with CamScanner

so,

$$I_x = \frac{ab^3}{21} \quad , \quad I_y = \frac{a^3b}{5}$$

we know, radius of gyration as,

also,

$$A = \int dA = \int_0^a y\,dx = \int_0^a \frac{b}{a^2} x^2\,dx = \frac{b}{a^2}\left[\frac{x^3}{3}\right]_0^a$$

$$= \frac{ab}{3}.$$

And,

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$= \sqrt{\frac{ab^3}{21} \Big/ \left(\frac{ab}{3}\right)}$$

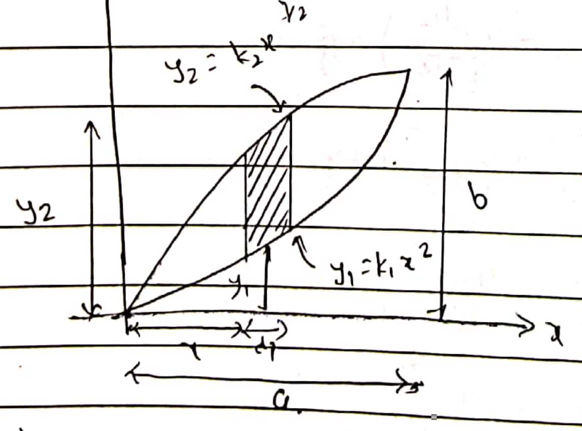
$$= \sqrt{\frac{1 \cdot b}{7}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

$$= \sqrt{\frac{a^3b}{5} \Big/ \frac{ab}{3}}$$

$$= \sqrt{\frac{3}{5}}\, a.$$

2) Sol^n.



We have,

$x = a$

$y_1 = y_2 = b$

so for:

$y_2 = k_2 x^{1/2}$

$\Rightarrow b = k_2 a^{1/2}$

$k_2 = \frac{b}{a^{1/2}}$

&

$y_1 = k_1 x^2$

$b = k_1 a^2$

$k_1 = \frac{b}{a^2}$

Now

for MOI I_y,

$dI_y = x^2 \times (y_2 - y_1)\,dx$

$= x^2 \times (k_2 x^{1/2} - k_1 x^4)\,dx$

$I_y = \int_0^a dI_y$

$= \int_0^a (k_2 x^{5/2} - k_1 x^4)\,dx$

$= k_2 \left[\frac{x^{7/2}}{7/2}\right]_0^a - k_1 \left[\frac{x^5}{5}\right]_0^a$

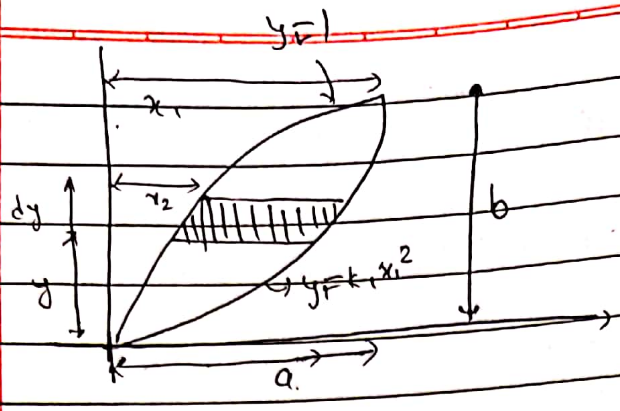
$= k_2 \frac{a^{7/2}}{7/2} - k_1 \frac{a^5}{5}$

$= \frac{b}{a^{1/2}} \times \frac{2a^{7/2}}{7} - \frac{b}{a^2} \times \frac{a^5}{5}$

$= \frac{2ba^3}{7} - \frac{ba^3}{5}$

$= \frac{10ba^3 - 7ba^3}{35}$

$= \frac{3a^3 b}{35}.$



For MOI I_x

$$dI_x = y^2\, dA$$
$$= y^2 \times (x_1 - x_2)\, dy \quad \text{---} \circledast$$

Also,

$$y = k_2 x_2^{1/2} \qquad\qquad\qquad \& \quad y =$$
$$\Rightarrow x_2^{1/2} = \frac{y}{k_2} \qquad\qquad \Rightarrow x_1^2 = \frac{y}{k_1}$$
$$\Rightarrow x_2 = \frac{y^2}{k_2^2} \qquad\qquad \Rightarrow x_1 = \frac{y^{1/2}}{k_1^{1/2}}$$

putting these value in eq^n $\circledast$

$$I_x = \int_0^b dI_x$$
$$= \int_0^b y^2 \cdot \left(\frac{y^{1/2}}{k_1^{1/2}} - \frac{y^2}{k_2^2} \right) dy$$
$$= \int_0^b \left(\frac{y^{5/2}}{k_1^{1/2}} - \frac{y^4}{k_2^2} \right) dy$$
$$= \frac{1}{k_1^{1/2}} \int_0^b y^{5/2}\, dy - \frac{1}{k_2^2} \int_0^b y^4\, dy$$
$$= \frac{1}{k_1^{1/2}} \left[\frac{y^{7/2}}{7/2} \right]_0^b - \frac{1}{k_2^2} \left[\frac{y^5}{5} \right]_0^b$$
$$= \frac{1}{\left(\frac{b}{a^2}\right)^{1/2}} \times \frac{b^{7/2}}{7/2} - \frac{1}{\left(\frac{b}{a^{1/2}}\right)^2} \times \frac{b^5}{5}.$$
$$= \frac{1}{\frac{b^{1/2}}{a}} \times \frac{2 b^{7/2}}{7} - \frac{1}{\frac{b^2}{a}} \times \frac{b^5}{5}$$

$$= \frac{a}{b^{1/2}} \times \frac{2b^{7/2}}{7} - \frac{a}{b^2} \times \frac{b^{5}}{5}$$

$$= \frac{2ab^3}{7} - \frac{ab^3}{5}$$

$$= \frac{10ab^3 - 7ab^3}{35}$$

$$= \frac{3ab^3}{35}$$

$$I_x = \frac{3ab^3}{35}$$

$$I_y = \frac{3a^3b}{35}$$

now,

$$A = \int_0^a dA$$

$$= \int_0^a (y_2 - y_1)\, dx$$

$$= \int_0^a (k_2 x^{1/2} - k_1 x^2)\, dx$$

$$= k_2 \int_0^a x^{1/2} dx - k_1 \int_0^a x^2 dx$$

$$= k_2 \times \frac{a^{3/2}}{3/2} - k_1 \frac{a^3}{3}$$

$$= \frac{b}{a^{1/2}} \times a^{3/2} \times \frac{3}{2} - \frac{b}{a^2} \times \frac{a^3}{3}$$

$$= \frac{2ab}{3} - \frac{ab}{3} = \frac{ab}{3}$$

so,

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$= \sqrt{\frac{3ab^3}{35} \times \frac{3}{ab}}$$

$$= \sqrt{\frac{9b^2}{35}}$$

$$= \frac{3b}{\sqrt{35}}$$

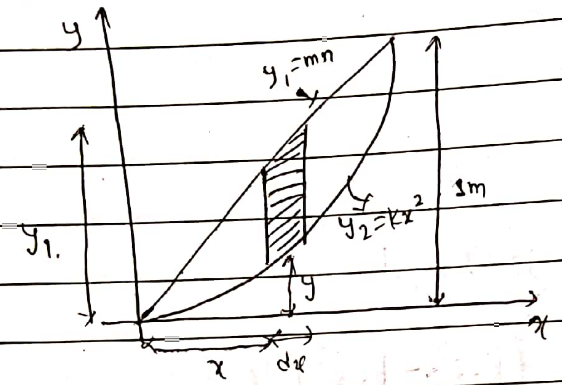
$$k_y = \sqrt{\frac{I_y}{A}}$$

$$= \sqrt{\frac{3a^3b}{35} \times \frac{3}{ab}}$$

$$= \sqrt{\frac{9a^2}{35}}$$

$$= \frac{3a}{\sqrt{35}}$$

4) Soln,



For $y_1 = mx$,

$x = 1,\ y_1 = 1$

$\Rightarrow 1 = m \times 1$

$\therefore m = 1$

For $y_2 = kx^2$

$x = 1,\ y_2 = 1$

$\Rightarrow 1 = k.1$

$\Rightarrow k = 1$

Now,

For MOI, I_y,

$$dI_y = x^2\, dA$$

$$= x^2 \times (y_1 - y_2)\, dx$$

$$= x^2 \times (x - x^2)\, dx$$

$$= (x^3 - x^4)\, dx$$

$$I_y = \int_0^1 dI_y$$

$$= \int_0^1 (x^3 - x^4)\, dx$$

$$= \int_0^1 x^3\, dx - \int_0^1 x^4\, dx$$

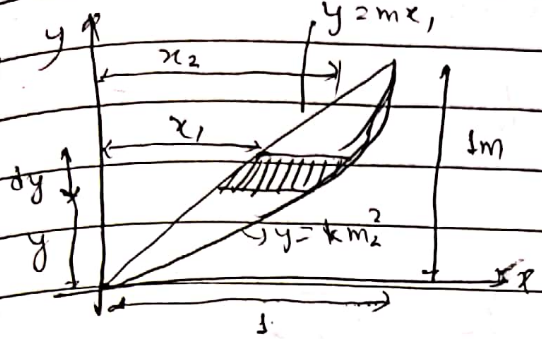
Scanned with CamScanner

$= \left[\frac{x^4}{4}\right]_0^1 - \left[\frac{x^5}{5}\right]_0^1$

$= \frac{1}{4} - \frac{1}{5}$

$= \frac{1}{20}$

Then,



$y = mx_1$

$y = kx_2^2$

$\Rightarrow x_2^2 = \frac{y}{k}$

we got, $m = k = 1$,

$\Rightarrow x_2 = \frac{y^{1/2}}{k^{1/2}}$

now,

$dI_x = y^2 \, dA$

$= y^2 (x_2 - x_1) \, dy$

$= y^2 (y^{1/2} - y) \, dy$

$= (y^{5/2} - y^3) \, dy.$

Then,

$I_x = \int_0^1 dI_m$

$= \int_0^1 (y^{5/3} - y^3) \, dy$

$= \int_0^1 y^{5/2} \, dy - \int_0^1 y^3 \, dy.$

$= \left[\frac{y^{7/2}}{7/2}\right]_0^1 - \left[\frac{y^4}{4}\right]_0^1$

$= \frac{2}{7} - \frac{1}{4} = \frac{1}{28}$

Thus.

$$A\ dA = \int_0^1 (y_1 - y_2)\,dx$$

$$= \int_0^1 (x - x^2)\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$k_x = \sqrt{\frac{I_x}{A}}$$

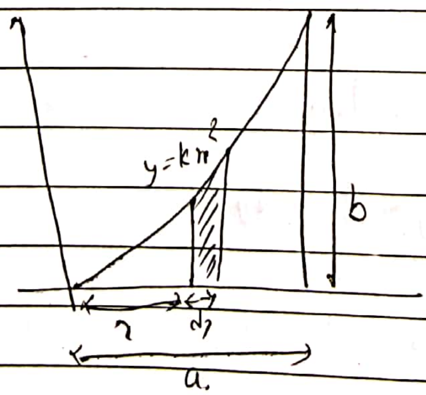
$$= \sqrt{\frac{1}{28} \times \frac{6}{1}} = \sqrt{\frac{6}{28}} = \sqrt{\frac{3}{14}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

$$= \sqrt{\frac{1}{20} \times \frac{6}{1}} = \sqrt{\frac{3}{10}}$$

5). For 4(1) & (2) determine MOI about centroidal axes & polar MOI about centroidal axes using parallel axes theorem

=> sol^n



we get,

$$A = \frac{ab}{3}$$

$$I_x = \frac{ab^3}{21}$$

$$I_y = \frac{a^3 b}{5}$$

$$\bar{x} = \frac{1}{A} \int_0^a x_c \, dA$$

$$= \frac{3}{ab} \int_0^a x \times \frac{b}{a^2} x^2 \, dx$$

$$= \frac{3}{ab} \int_0^a x^3 \frac{b}{a^2} \, dx$$

$$= \frac{3}{ab} \times \frac{b}{a^2} \times \frac{a^4}{4}$$

$$= \frac{3a}{4}$$

$$\bar{y} = \frac{1}{A} \int y_c \, dA$$

$$= \frac{3}{ab} \int_0^a \frac{y}{2} y \, dx$$

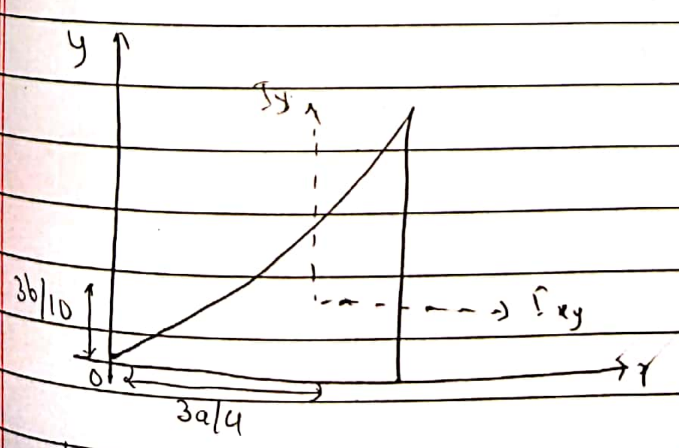
$$= \frac{3}{ab} \times \frac{1}{2} \int_0^a y^2 \, dx$$

$$= \frac{3}{2ab} \int_0^a k^2 x^4 \, dx$$

$$= \frac{3}{2ab} \times k^2 \times \frac{a^5}{5}$$

$$= \frac{3}{2ab} \times \frac{b^2}{a^4} \times \frac{a^5}{5}$$

$$= \frac{3b}{10}$$



MOI about centroidal x-axes : I_{xg}

$$I_x = I_{xg} + A(\bar{y})^2$$

$$I_{xg} = I_x - A(\bar{y})^2$$

$$= \frac{ab^3}{21} - \frac{ab}{3}\left(\frac{3b}{10}\right)^2$$

$$= \frac{ab^3}{21} - \frac{ab}{3} \times \frac{9}{100} b^2$$

$$= \frac{ab^3}{21} - \frac{3ab^3}{100}$$

$$= \frac{100ab^2 - 63ab^3}{2100}$$

$$= \frac{37ab^3}{2100}$$

MOI about centroid y-axis : I_{yg}.

$$I_y = I_{yg} + A(\bar{x})^2$$

$$\Rightarrow I_{yg} = I_y - A(\bar{x})^2$$

$$= \frac{a^3b}{5} - \frac{ab}{3}\left(\frac{3a}{4}\right)^2$$

$$= \frac{a^3b}{80}$$

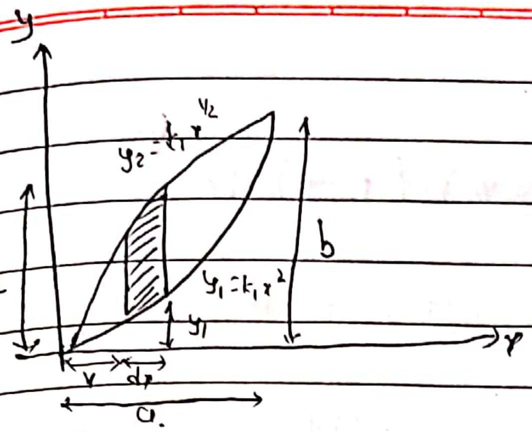
Polar, MOI about centroidal axes.

$$I_{zg} = I_{xg} + I_{yg}$$

$$= \frac{37ab^3}{2100} + \frac{a^3b}{80}$$

$$= \frac{148ab^2 + 105a^3b}{8400}.$$

2) Sol^n,



Here,

$$A = \frac{ab}{3}$$

$$I_x = \frac{3ab^3}{35}$$

$$I_y = \frac{3a^3b}{35}$$

Now,

$$\bar{x} = \frac{1}{A}\int_0^a x_c \, dA$$

$$= \frac{3}{ab}\int_0^a x(y_2 - y_1)\, dx$$

$$= \frac{3}{ab}\int_0^a x\left(k_2 x^{1/2} - k_1 x^2\right) dx$$

$$= \frac{3}{ab}\int_0^a \left(k_2 x^{3/2} - k_1 x^3\right) dx$$

$$= \frac{3}{ab}\left[k_2 \times \frac{a^{5/2}}{5/2} - \frac{k_1 a^4}{4}\right]$$

$$= \frac{3}{ab}\left[\frac{b}{a^{1/2}} \times a^{5/2} \times \frac{2}{5} - \frac{b}{a^2} \times \frac{a^4}{4}\right]$$

$$= \frac{3}{ab} \times \left[\frac{2a^2b}{5} - \frac{a^2b}{4}\right]$$

$$= \frac{3}{ab} \times \left(\frac{8a^2b - 5a^2b}{20}\right)$$

$$= \frac{3}{ab} \times \frac{3a^2b}{20} = \frac{9a}{20}$$

Again

$$\bar{y} = \frac{1}{A}\int y_c\, dA$$

$$= \frac{3}{ab}\int_0^a \left(y_1 + \frac{y_2 - y_1}{2}\right)\cdot(y_2 - y_1)\, dx$$

$$= \frac{3}{ab}\int_0^a \left(\frac{y_1 + y_2}{2}\right)(y_2 - y_1)\, dx$$

$$= \frac{3}{2ab}\int_0^a \left(k_2^2 x - k_1^2 x^4\right) dx$$

$$= \frac{3}{2ab}\left[\int_0^a k_2^2 x \cdot \int_0^a k_1 x^4 dx\right]$$

$$= \frac{3}{2ab}\left[k_2^2 \times \frac{a^2}{2} - k_1^2 \frac{a^5}{5}\right]$$

$$= \frac{3}{2ab}\left[\frac{b^2}{a} \times \frac{a^2}{2} - \frac{b^2}{a^4} \times \frac{a^5}{5}\right]$$

$$= \frac{3}{2ab}\left[\frac{ab^2}{2} - \frac{ab^2}{5}\right]$$

$$= \frac{3}{2ab} \times \frac{3ab^2}{10}$$

$$= \frac{9b}{20}$$

Then, MoI about centroidal x-axis :- I_{xg}

$$I_x = I_{xg} + A(\bar{y})^2$$

$$\Rightarrow I_{xg} = I_x - A\bar{y}^2$$

$$= \frac{3ab^3}{35} - \frac{ab}{3} \times \frac{81b^2}{400}$$

$$= \frac{3ab^3}{35} - \frac{27ab^3}{400}$$

$$= \frac{240ab^3 - 189ab^3}{2800}$$

$$= \frac{51ab^3}{2800}$$

MoI about centroidal y-axis :- I_{yg}

$I_y = I_{yg} + A\bar{x}^2$

$\Rightarrow I_{yg} = \frac{3a^3b}{35} - \frac{ab}{3} \times \frac{81a^2}{400}$

$= \frac{3a^3b}{35} - \frac{27a^3b}{400}$

$= \frac{240a^3b - 189a^3b}{2800}$

$= \frac{51a^3b}{2800}$

Polar MoI about centroidal axes

$I_{zg} = I_{xg} + I_{yg}$

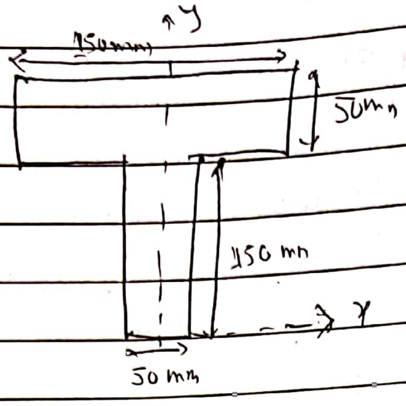
$= \frac{51ab^3}{2800} + \frac{51a^3b}{2800}$

$= \frac{51ab^3 + 51a^3b}{2800}$

$= \frac{51ab(b^2+a^2)}{2800}$

6) Determine MOI of following area about centroidal axes.

Soln,



given section being symmetrical about y-axis.

$\bar{x} = 0$

$\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$

$= \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$

$= \frac{(150 \times 50) \times 175 + (150 \times 50) \times 75}{150 \times 50 + 150 \times 50}$

$\bar{y} = 125$ mm

$\therefore$ centroid $(\bar{x}, \bar{y}) = (0, 125)$

now

MOI of section about centroidal axis

$I_{xy} = \Sigma (I_{xi} + Ad^2)$

$= (I_{x_1} + A_1 d_1^2) + (I_{x_2} + A_2 d_2^2)$

$= 50^3 \times 150 = I_{x_1} + d_1^2 + \frac{1}{2}$

$= 53.125 \times 10^6 mm^4$

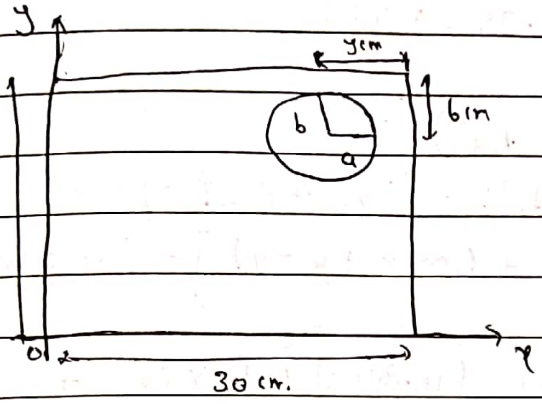
$I_{xy} = 53.125 \times 10^6 mm^4$

$I_y = \Sigma (I_y + Ad^2)$

$$I_{yg} = 53.125 \times 10^6 mm^4$$

$$I_{yg} = 15.625 \times 10^6 mm^4$$

(2) Sol^n,



we have,

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$$

$$= \frac{(30 \times 20) \times 15 + (-\pi ab) \times 22}{(30 \times 20) - \pi ab}$$

$$= \frac{600 \times 15 - 22 \pi ab}{600 - \pi ab}$$

$$= \frac{9000 - 22 \pi ab}{600 - \pi ab}$$

(Taking major & minor axis of ellipse as 6 cm & 4 cm (not mentioned)) we get

$$\bar{x} = \frac{9000 - 1658.76}{600 - 75.4}$$

$$\simeq 13.99 \text{ cm.}$$

Also,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{600 \times 10 + (-\pi \times 6 \times 4) \times 14}{600 - \pi \times 6 \times 4}$$

$$= 9.43 \text{ cm}$$

Centroid $(\bar{x}, \bar{y}) = (13.99, 9.43)$.

now,

$$I_{xg} = \Sigma (I_{xi} + Ad^2)$$

$$= (I_{x_1} + A_1 d_1^2) - (I_{x_2} + A_2 d_2^2)$$

$$= \left[\frac{20^3 \times 30}{12} + 600 \times (y_1 - \bar{y})\right] - \left[\frac{\pi \times 6 \times 4^3}{4} + 75.39 \times (y_2 - \bar{y})^2\right]$$

$$= \left[\frac{20^3 \times 30}{12} + 600 \times (10 - 9.43)^2\right] - \left[\frac{\pi \times 6 \times 4^3}{4} + 75.39 \times (10 - 9.43)^2\right]$$

$$= 18318.66 \text{ cm}^4$$

$$I_{yg} = \Sigma (I_{yi} + Ad^2)$$

$$= (I_{y_1} + A_1 d_1^2) - (I_{y_2} + A_2 d_2^2)$$

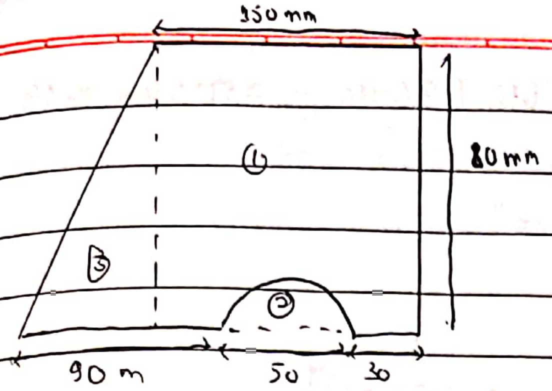
$$= \left[\frac{20 \times 30^3}{12} + 600 \times (x_1 - \bar{x})^2\right] - \left[\frac{\pi \times 6^3 \times 4}{4} + 75.398 \times (x_2 - \bar{x})^2\right]$$

$$= \left[\frac{20 \times 30^3}{12} + 600 \times (15 - 13.99)^2\right] - \left[\frac{\pi \times 6^3 \times 4}{4} + 75.398 \times (22 - 13.99)^2\right]$$

$$= 40095.93 \text{ cm}^4$$

$$I_{xg} = 18318.66 \text{ cm}^4$$
$$I_{yg} = 40095.93 \text{ cm}^4$$

3)



Considering origin & dividing into simpler components

$$\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i}$$

$$\bar{x} = \frac{(150\times80)\times\frac{(20+150)}{2} + \left(\frac{1}{2}\times20\times80\right)\left(\frac{2}{3}\times20\right) - \left(\frac{\pi\times25^2}{2}\right)(90+25)}{(150\times80) + \left(\frac{1}{2}\times20\times80\right) - \left(\frac{\pi\times25^2}{2}\right)}$$

$$\bar{x} = \frac{12000\times95 + 800\times13.33 - 981.74\times115}{12000+800-981.74}$$

$$\bar{x} = 87.81 \text{ mm}$$

llly,

$$\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$$

$$= \frac{12000\left(\frac{80}{2}\right) + (800)\left(\frac{80}{3}\right) - (981.74)\left(\frac{4\times25}{3\pi}\right)}{12000+800-981.74}$$

$$= 41.54 \text{ mm}$$

Now

$$I_{xg} = \sum_{i=1}^{3}\left[I_{x_i} + A_i(\bar{y}-y_i)^2\right]$$

$$= \left[\frac{150\times80^3}{12} + 12000(41.54-40)^2\right] + \left[\frac{20(80)^3}{36} + 800\left(41.54-\frac{80}{3}\right)^2\right]$$

$$- \left[0.11(25)^4 + 981.74\left(41.54 - \frac{4\times25}{3\pi}\right)^2\right]$$

$$I_{xg} = 6428459.2 + 461337.8644 - 982164.9489$$

$$I_{xg} = 5.907 \times 10^6 \ mm^4.$$

Now,

$$I_{yg} = \sum_{i=1}^{n} \left[I_{y_i} + A_i(\bar{x} - x)^2\right]$$

$$= \left[\frac{150^3 \times 80}{12} + 12000 \cdot (87.81 - 95)^2\right] + \left[\frac{20^3 \times 80}{36} + 800\left(87.81 - \frac{2}{3} \times 20\right)^2\right]$$

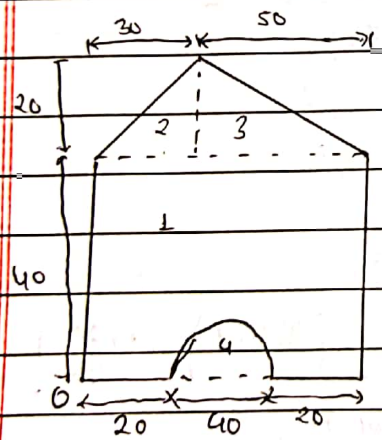
$$- \left[\pi\left(\frac{25}{8}\right)^4 + 981.74(87.81 - 115)^2\right]$$

$$= 23120353.2 + 4455196.88 - 879194.632$$

$$= 26696355.45$$

$$= 2.66 \times 10^7 \ mm^4.$$

Q)



Here

$$\bar{x} = \frac{\sum a_i x_i}{\sum a_i}$$

$$= \frac{(80 \times 40)(40) + \left(\frac{1}{2} \times 30 \times 20\right)\left(\frac{2}{3} \times 30\right) + \left(\frac{1}{2} \times 50 \times 20\right)\left(30 + \frac{1}{3} \times 50\right) - \left(\pi\left(\frac{20}{2}\right)^2\right) \times (20 + 20)}{(80 \times 40) + \left(\frac{1}{2} \times 30 \times 20\right) + \left(\frac{1}{2} \times 50 \times 20\right) - \left(\frac{\pi 20^2}{2}\right)}$$

$$\bar{x} = \frac{3200\times40 + 300\times20 + 500\times4667 - 628.31(40)}{3200 + 300 + 500 - 628.31}$$

$$\bar{x} = \frac{132202.6}{337169}$$

$$\bar{x} = 39.20$$

now,

$$\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}.$$

$$\bar{y} = \frac{3200\times20 + 300\times(40 + \frac{1}{3}\times20) + 500(\frac{1}{3}\times20+40) - 628.3)(4\times20/3\pi)}{3371.69}$$

$$\bar{y} = \frac{96000.07}{3371.69}$$

$$\bar{y} = 28.47.$$

now,

$$I_{xg} = I_{xg_1} + I_{xg_2} + I_{xg_3} - I_{xg_4} \quad \text{— (iv)}$$

$$I_{xg_1} = \frac{80\times40^3}{12} + 3200(28.47-20)^2 = 656237.546.$$

$$I_{xg_2} = \frac{30\times20^3}{36} + 300\left[28.47 - \left(40+\frac{20}{3}\right)\right]^2 = 106002.77$$

$$I_{xg_3} = \frac{50\times20^3}{36} + 500\left[28.47 - \left(40+\frac{20}{3}\right)\right]^2 = 176670.45$$

$$I_{xg_4} = 0.11(20)^4 + 628.31\left(28.47 - \frac{4\times20}{3\pi}\right)^2 = 268465.2007$$

again

$$I_{xg} = 656237.546 + 106002.27 + 176670.45 - 268465.2007$$

$$= 670445.0653$$

now

$$I_{y-g} = I_{gg_1} + I_{yg_2} + I_{yg_3} - I_{yg_4} \quad -(3)$$

$$I_{yg_1} = \frac{40\times 80^3}{12} + 3200(39.20-40)^2 = 1708714.66$$

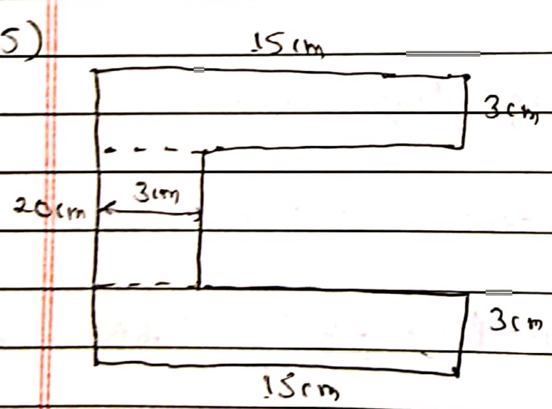
$$I_{gg_2} = \frac{30^3\times 20}{36} + 300(39.20-26)^2 = 125592$$

$$I_{yg_3} = \frac{50^3\times 20}{36} + 500\left(39.20-\left(30+\frac{50}{3}\right)\right)^2 = 97320$$

$$I_{yg_4} = \frac{\pi(20)^4}{8} + 628.31(39.20-40)^2 = 63233.97$$

$$I_{yg} = 1708714.66 + 125592 + 97320 - 63233.97$$
$$= 330492.6925$$

5)



=) Ans

$$\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i}$$

$$= \frac{(15\times 3)(15/2) + (14\times 3)(3/2) + (15\times 3)(15/2)}{(15\times 3) + (14\times 3) + (15\times 3)}$$

$$= \frac{45\times 7.5 + 42\times 1.5 + 45.75}{45+42+45}$$

$$= 5.590 \text{ cm}$$

$$\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$$

$$= \frac{45 \times (3/2) + 42 \times (3 + 14/2) + 45(17 + 3/2)}{45 + 42 + 45}$$

$$\bar{y} = 10 \text{cm}$$

Now,

$$I_{xg} = \left[\frac{15 \times 3^3}{12} + 45(10 - 1.5)^2\right] + \left[3 \times 14^3 (10-10)^2\right] + \left[\frac{15 \times 3^3}{12} + 45(10 - 18.5)^2\right]$$

$$= 3285 + 686 + 3285$$

$$= 7256 \text{ cm}^4.$$

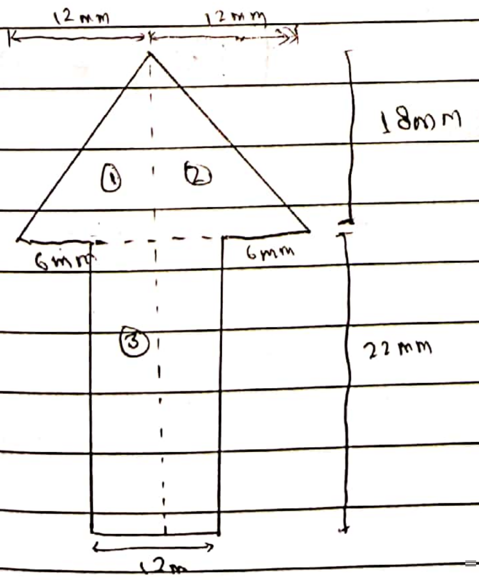
And,

$$I_{yg} = \left[\frac{3 \times 15^3}{12} + 45(5.59 - 7.5)^2\right] + \left[\frac{14 \times 3^3}{12} + 42(5.59 - 1.5)^2\right] + \left[\frac{3 \times 15^3}{12} + 45(10 - 7.5)^2\right]$$

$$= 1007.91 + 734.08 + 1007.91$$

$$= 2749.90 \text{ cm}^4.$$

6)



Soln

as origin is at axis of symetry,

$x = 0$

Now $\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i}$

$$\bar{y} = \frac{(\frac{1}{2}\times12\times18)(18/3) + (\frac{1}{2}\times12\times18)(18/3+22) + (12\times22)11}{(\frac{1}{2}\times12\times18) + (\frac{1}{2}\times12+18) + (12\times22)}$$

$$\bar{y} = \frac{108\times28 + 108\times28 + 264\times11}{108+108+264}$$

$\bar{y} = 18.65$ mm.

now,

$$I_{xy} = \left[\frac{12\times22^3}{12} + 264(18.65-11)^2\right] + \left[\frac{12\times18^3}{36} + 108(18.65-28)^2\right]$$
$$+\left[\frac{12\times18^3}{36} + 108\,(18.65-28)^2\right]$$

$= 26097.94 + 11385.63 + 11385.63.$

$= 48869.2\ mm^4.$

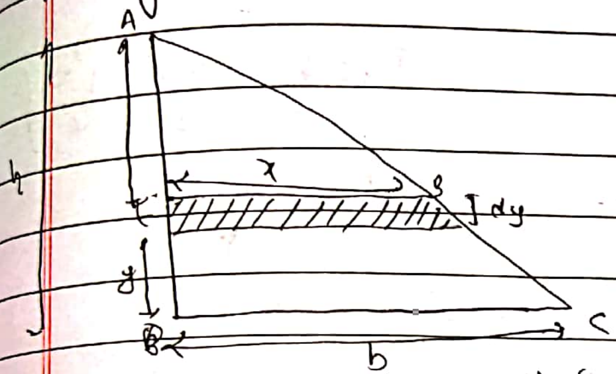
$$I_{xy} = \left[\frac{22\times12^3}{12} + 264(0-0)^2\right] + \left[\frac{18\times12^3}{36} + 108(0-0)^2\right] +$$
$$\left[\frac{18\times12^3}{36} + 108(0-0)^2\right]$$

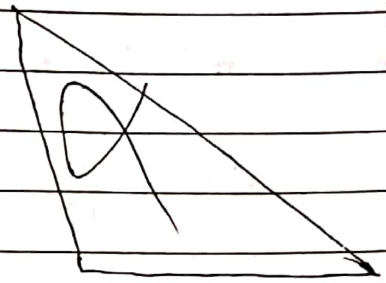
$= 3168 + 864 + 864.$

$= 4896\ mm^4.$

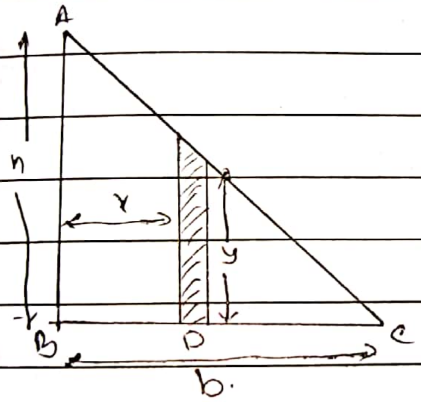
Q) Determine the ~~mom~~ MOI of the following regular shape about x & y axes & use parallel axis theorem to determine the MOI of these regular shapes about centroidal axes.

a) Triangle ~~centroidal axes~~ (Right angled triangle).





Horizontal strip



vertical strip

For the derivation of MOI we take small strip

In horizontal strip

$$\frac{x}{b} = \frac{h-y}{h}$$

$$x = \left(1 - \frac{y}{h}\right) b$$

In vertical strip

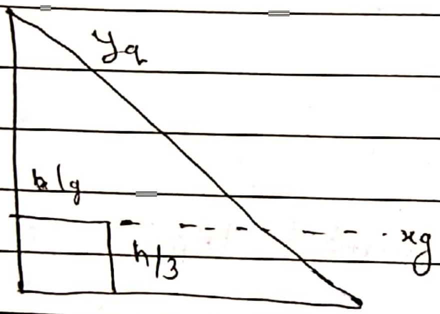
$$\frac{y}{h} = \frac{b-x}{b}$$

$$y = \frac{h}{b}(b-x)$$

$$dI_x = y^2 dA$$
$$I_x = \int y^2 \left(1-\frac{y}{h}\right) b\,dy$$
$$I_x = \int_0^h y^2 b - \frac{y^3}{h} b\,dy$$
$$I_x = b\left[\frac{y^3}{3} - \frac{1}{h} \times \frac{y^4}{4}\right]_0^h$$
$$I_x = b\left[\frac{h^3}{3} - \frac{h^3}{4}\right]$$
$$I_x = \frac{bh^3}{12}.$$

$$dI_y = x^2 dA$$
$$I_y = \int_0^b x^2 dA$$
$$I_y = \int_0^b x^2 \times \frac{h}{b}(b-x)\,dx$$
$$I_y = \frac{h}{b}\left[b\int_0^b x^2 dx - \int_0^b x^3 dx\right]$$
$$I_y = \frac{h}{b}\left[\frac{b^3}{3} - \frac{b^4}{4}\right]$$
$$I_y = \frac{b^3 h}{12}$$

using parallel axis theorem

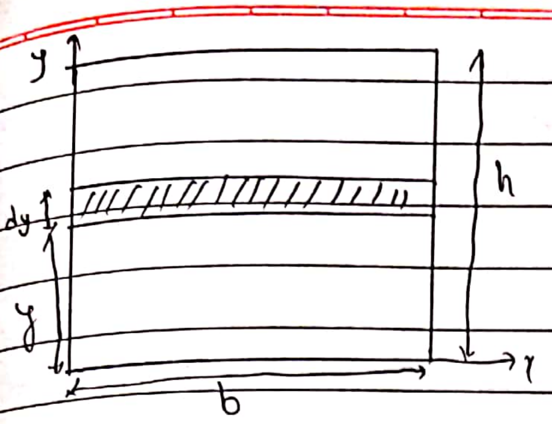


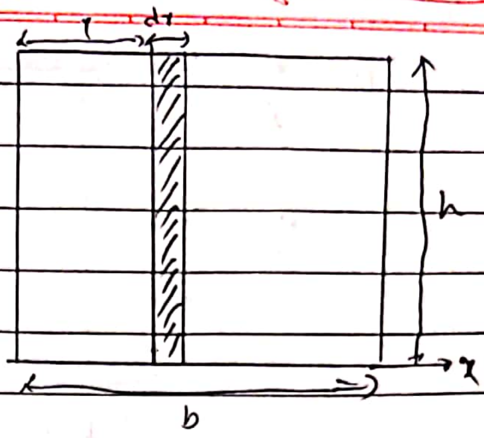
we know

$$I_x = I_{xg} + A\left(\frac{h}{3}\right)^2$$
$$I_x = \frac{bh^3}{12} - \frac{bh}{2} \times \frac{h^2}{9}$$
$$I_x = \frac{bh^3}{12} - \frac{bh^3}{18}$$
$$I_{xg} = \frac{bh^3}{36}$$

$$I_y = I_{yg} + A\left(\frac{b}{3}\right)^2$$
$$I_{yg} = \frac{b^3 h}{12} - \frac{bh}{2} \times \frac{b^2}{9}.$$
$$I_{yg} = \frac{b^3 h}{12} - \frac{b^3 h}{18}$$
$$I_{yg} = \frac{b^3 h}{36}.$$

b)





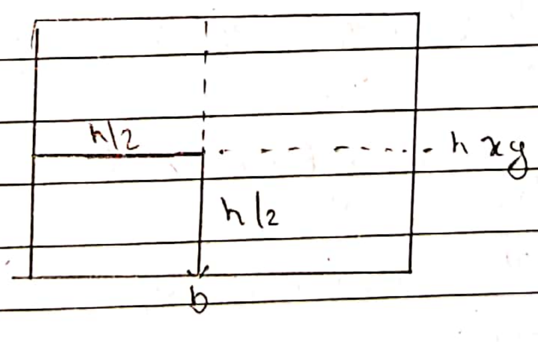
$$dA = b\,dy$$
$$dI_x = y^2\,dA$$
$$= y^2\,b\,dy$$

$$I_x = \int_0^h y^2 \cdot b\,dy$$
$$= b\left[\frac{y^3}{3}\right]_0^h$$
$$= \frac{bh^3}{3}$$

$$dA = x h\,dx$$
$$dI_y = x^2\,dA$$
$$= x^2 h\,dx$$

$$I_y = \int_0^b x^2 h\,dx$$
$$= h\left[\frac{x^3}{3}\right]_0^b$$
$$= \frac{b^3h}{3}.$$

Using parallel axis therem



know

$$I_x = I_{xg} + A\left(\frac{h}{2}\right)^2$$

$$I_{xg} = \frac{bh^3}{3} - bh \times \frac{h^2}{4}$$

$$I_{xg} = \frac{bh^3}{3} - \frac{bh^3}{4}$$

$$I_{xg} = \frac{bh^3}{12}$$

$$I_y = I_{yg} + A\left(\frac{b}{2}\right)^2$$

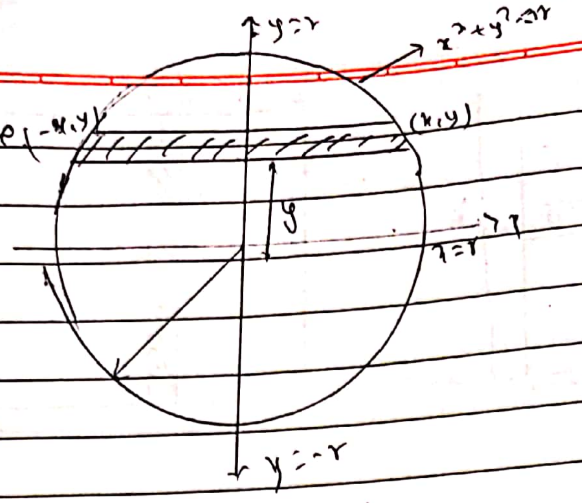
$$I_{yg} = \frac{b^3h}{3} - bh \times \frac{b^2}{4}$$

$$I_{y-g} = \frac{b^3h}{3} - \frac{b^3h}{4} \times \cancel{\frac{b^2}{4}}$$

$$I_{y-g} = \frac{b^3h}{12}$$

Scanned with CamScanner

c) Circle



considering the small strip

$$dA = 2x.dy$$

$$\int dI_x = y^2 dA$$

$$I_x = \int y^2 \, 2x \, dy$$

$$I_x = \int_{-r}^{r} y^2 \, 2\sqrt{r^2 - y^2} \, dy$$

put $y = r\sin\theta$

$$dy = r\cos\theta \, d\theta$$

$$I_x = \int_{-\pi/2}^{\pi/2} r^2 \sin^2\theta \times 2\sqrt{r^2(1-\sin^2\theta)} \cdot r\cos\theta \, d\theta$$

$$I_x = r^4 \int_{-\pi/2}^{\pi/2} 2\sin^2\theta . \cos^2\theta . d\theta$$

$$I_x = 2r^4 \int_{-\pi/2}^{\pi/2} \frac{4\sin^2\theta \cdot \cos^2\theta}{4} \, d\theta$$

$$I_x = \frac{2r^4}{2} \int_{-\pi/2}^{\pi/2} (2\sin\theta . \cos\theta)^2 \, d\theta$$

$$I_x = \frac{r^4}{2} \int_{-\pi/2}^{\pi/2} \frac{1-\cos 4\theta}{2} \, d\theta$$

$$I_x = \frac{r^4}{4} \left[\theta - \frac{\sin 4\theta}{4} \right]_{-\pi/2}^{\pi/2}$$

$$I_x = \frac{r^4}{4} \left[\frac{\pi}{2} + \frac{\pi}{2} - \frac{\sin 2\pi}{4} + \frac{\sin 2\pi}{4} \right]$$

$$I_x = \frac{\pi r^4}{4}$$

As circle is symmetrical, $I_x = I_y = \frac{\pi r^4}{4}$

As centroid of circle lies in center.

$$I_{xg} = I_{yg} = \frac{\pi r^4}{4}$$

9) Theory question

a) Define centroid Axis of symmetry, C.G. & radius of Gyration

⇒ The centroid of a body is the point where there are equal vol. on all sides.

CG (center of gravity) is a point where all the wt. of the body is concentrated. In regular shape, centroid & CG lies at same point.

Axis of symmetry is line that divides an object into two equal halves, thereby creating a mirror like reflection of either side of the object.

Radius of gyration is the perpendicular distance betn point mass to the axis of ~~rotation~~. Mathematically, the radius of gyration is the root mean sq. distance of the object parts from either it center of mass or a given axis.

Parallel axis Theorem

It state that if the moment of inertia of plane area 'A' about an axis thro its CG is $I_{xg\,xg}$, the moment of inertia of the area about any other axis xx' parallel to xx & at a distance 'h' from Cg is given by.

$$I_{x'x'} = (I_{xg\,xg} + Ah^2)$$

proof:
Let us consider a strip with elementary area dA at a distance of y from centroidal axis $x_g - x_g$.

Here
$$dI_{xg-xg} = y^2 \, dA.$$
MoI of entire area about x_g-axis
$$I_{xg\,xg} = \Sigma(y^2 dA) \quad \text{(i)}$$

Ily,
MoI of elementary area about $x'-x'$ axis
$$dI_{x'-x'} = dA\,(y+h)^2$$
where, h is the perpendicular distance betn $(x_g - x_g)$ & $(x'-x')$ axes.

MoI of entire area about $x'-x'$ axis
$$I_{x'-x'} = \Sigma dA\,(y+h)^2$$
$$= \Sigma dA\,(y^2 + 2hy + h^2)$$
$$= \Sigma dA y^2 + 2h \Sigma dA.y + h^2 \Sigma dA$$
$$= I_{xg-xg} + 0 + Ah^2$$
$[\because \Sigma dA y = \text{first moment of area about centroidal axis} = 0]$

$$\boxed{I_{x'-x'} = I_{xg-xg} + Ah^2}$$ proved

classmate

Date ______

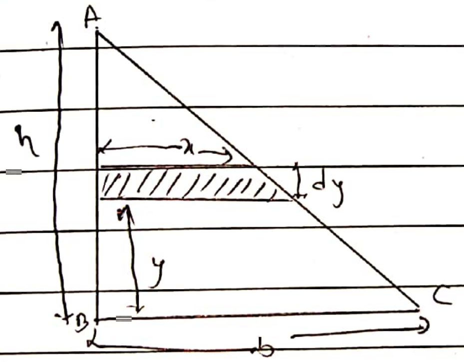
Page ______

c) Define Moment of Inertia & polar moment of inertia.

=> Moment of inertia is defined as the qualitative measure of the rotational inertia of body i.e. the opposition that the body exhibits to have its speed of rotation about an axis altered.

Polar moment of inertia is defined as the resisting to torsional deformation when torque is applied in a plane that is parallel to the cross section or in plane that is perpendicular to the objects central axis.

d) Determine centroid of triangle (right angle)



For determination of centroid lets take small horizontal strip. Then we have,

$$\frac{x}{b} = \frac{h-y}{h}$$

$$x = (h-y)\frac{b}{h}$$

We know, area of rectangle (A) $= \frac{h b}{2}$

now

If $\bar{y}$ be y-cordinate of centroid

Scanned with CamScanner

$$\bar{y} = \frac{1}{A}\int y\, dA$$

$$= \frac{2}{bh} x \int y \times (h-y)\frac{b}{h}\, dy$$

$$= \frac{2}{h^2}\int_0^h hy - y^2\, dy$$

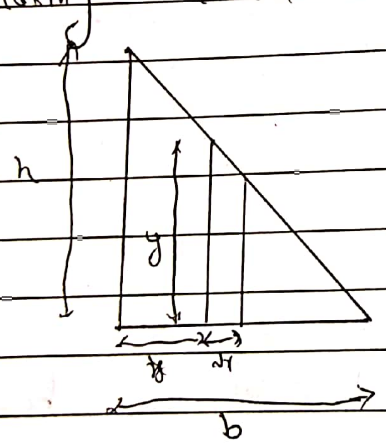
$$= \frac{2}{h^2}\left[\int_0^h hy\, dy - \int_0^h y^2\, dy\right]$$

$$= \frac{2}{h^2}\left[h \times \frac{h^2}{2} - \frac{h^3}{3}\right]$$

$$= 2\left[\frac{3h - 2h}{6}\right]$$

$$= \frac{h}{3}$$

taking verticle strip.



here,

$$\frac{h}{y} = \frac{b}{b-x}$$

$$y = (b-x)\frac{h}{b}$$

If $\bar{x}$ be x-coordinate of centroid

$$\bar{x} = \frac{1}{A}\int x\, dA$$

$$= \frac{2}{bh}\int_0^b x(b-x)\frac{h}{b}\, dx$$

$$\bar{x} = \frac{2}{b^2}\int_0^b x\, b - x^2\, dx$$

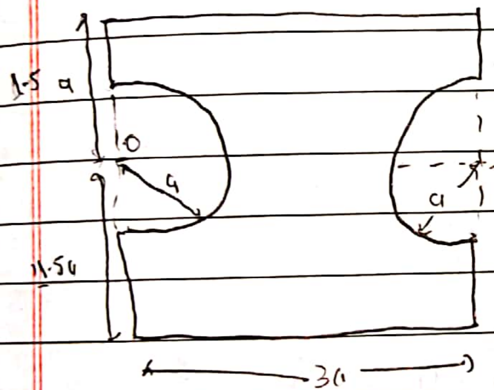
$$\bar{x} = \frac{2}{b^2}\left[\frac{b \times b^2}{2} - \frac{b^3}{3}\right]$$

$$\bar{x} = 2\left[\frac{3b - 2b}{6}\right]$$

$$\bar{x} = \frac{b}{3}$$

Hence, centroid $(\bar{x}, \bar{y}) = \left(\frac{b}{3}, \frac{b}{3}\right)$.

7) Determine MOI of the following figure about co-ordinate axes $(x-y)$ as shown in the figure. [Take $a = 20mm$].



Sol^n, As the given axis, the shape is symmetrical in

$\bar{y} = 0$

now,

$$\bar{x} = \frac{\Sigma a_i x_i}{\Sigma a_i}$$

$$= \frac{(3a \times 3a)(1.5a) - \left(\frac{\pi a^2}{2}\right)\left(\frac{4a}{3\pi}\right) - \left(\frac{\pi a^2}{2}\right)\left(3a - \frac{4a}{3\pi}\right)}{(3a \times 3a) - \left(\frac{\pi a^2}{2}\right) - \left(\frac{\pi a^2}{2}\right)}$$

$$= \frac{3600(30) - 628.31(8.48) - 628.31(60 - 8.48)}{3600 - 628.31 - 628.31}$$

$$\bar{x} = 30$$

Now,

$$I_{xx} = \Sigma\left(I_{xg_i} + A_i y_i^2\right)$$

$$= I_{xx_1} - I_{xx_2} - I_{xx_3}$$

$$= \left[\frac{60 \times 60^3}{12} + 3600 \times 6^2\right] - 2\left[\frac{\pi \times 20^4}{8} + 628.31 \times 6^2\right]$$

$$= 954336.29 mm^4$$

$$I_{yy} = \Sigma\left(I_{yg_i} + A_i x_i^2\right)$$

$$= I_{yy_1} - I_{yy_2} - I_{yy_3}$$

$$= \left[\frac{60\times60^3}{12} + 3600\times30^2\right] - \left[0.11\times20^4 + 628.31\times(8.48)^2\right]$$

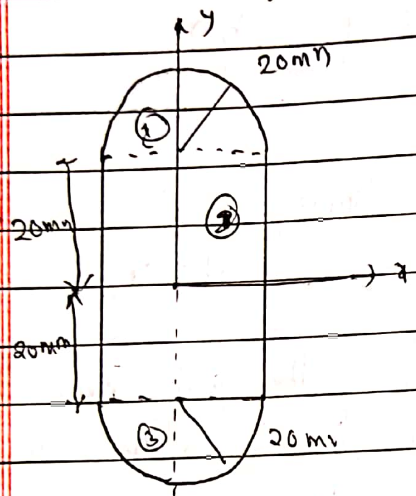
$$- \left[0.11\times20^4 + 628.31\times(51\times42)^2\right]$$

$$= \cancel{954336.29\ mm^4} = 2571888.209\ mm^4$$

$\cancel{I_{yy} = \Sigma(I_{yg_i} + A_i x_i^2)}$

$\cancel{\Rightarrow I_{yy_1} - I_{yy_2} - I_{yy_3}}$

2)



for rectangle

$\quad$ area $= 40\times40 = 1600$

$\quad$ area of semi-circle $= 2\times\left(\frac{\pi\times20^2}{2}\right)$

$$= 1256.63$$

now

for rectangle

$$I_x = I_y = \frac{40\times40^3}{12} = 213333.33$$

semi-circle

$$I_{x_g} = 0.11\times20^4 = 17600$$

$$I_x = I_{x_g} + Ah^2$$

$$= 17600 + 628.31\times\left(20 + \frac{4\times20}{3\pi}\right)^2$$

$$= 537524.56$$

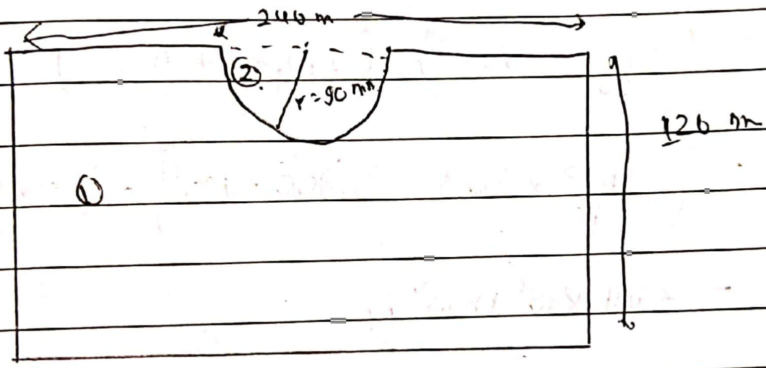
I_x of diagram $= 213333.33 + 2 \times (527524.50)$
$= 1268382.44 \ mm^4$

for semi-circle

$$I_y = \frac{\pi \times 20^4}{8} = 62831.85$$

I_y of diagram $= 213333.33 + 2(62831.85)$
$= 338997.03 \ mm^4$

7.B)



Here,

section ①

① area $= 240 \times 120$
$= 28800$

$x_1 = \frac{240}{2} = 120 \ mm$

$y_1 = \frac{120}{2} = 60 \ mm$

② semi-circle ②

$$\text{Area} = \frac{\pi r^2}{2} = \frac{\pi (90)^2}{2} = 12723.45 \ mm^2$$

$x_2 = 120 \ mm$

$$y_2 = 120 - \frac{4 \times 90}{3\pi} = 81.80 \ mm$$

Scanned with CamScanner

Then,

$$I_{xx} = \Sigma (I_{xgi} + A_i y_i^2)$$

$$= (I_{xg_1} + A_1 y_1^2) - (I_{xg_2} + A_2 y_2^2)$$

$$= \left\{\frac{240 \times 120^3}{12} + 28800 \times 60^2\right\} - \left[0.11 \times 90^4 + 12723.45 \times 81.80^2\right]$$

$$= 45.88 \times 10^6 \, mm^4$$

$$I_{yy} = \Sigma (I_{ygi} + A_i x_i^2)$$

$$= (I_{yg_1} + A_1 x_1^2) - (I_{yg_2} + A_2 x_2^2)$$

$$= \left[\frac{240^3}{12} \times 120 + 28800 \times 120^2\right] - \left[\frac{\pi \times 90^4}{8} + 12723.45 \times 120^2\right]$$

$$= 3.44 \times 10^8 \, mm^4$$

Scanned with CamScanner