

Assignment - 3

i) $230V, 50Hz$ ac is supplied to a coil of $0.06H$ inductor, 2.5Ω resistance & $6.8\mu F$ capacitor in series. Calculate the current & phase angle of the circuit. Also calculate the voltage across each of the circuit components.
 \Rightarrow Soln.

$$V = 230V$$

$$f = 50Hz$$

$$L = 0.06H$$

$$R = 2.5\Omega$$

$$C = 6.8\mu F = 6.8 \times 10^{-6}F$$

we know,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 6.8 \times 10^{-6}} \\ = 468.10\Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.06 = 18.85\Omega$$

now,

$$Z = R \angle 0^\circ + X_L \angle 90^\circ + X_C \angle -90^\circ \\ = 2.5\Omega + j18.85 - j468.10\Omega \\ = 2.5 - 449.25j\Omega \\ = 449.5 \angle -89.68^\circ$$

$$\text{i) The current } I = \frac{V}{Z} = \frac{230 \angle 0^\circ}{449.5 \angle -89.68^\circ} \\ = 0.5119 \angle 89.68^\circ$$

$$\text{ii) } V_R = I Z_R = (0.5119 \angle 89.68^\circ) \times (2.5\Omega \angle 0^\circ) \\ = 1.278V \angle 89.68^\circ$$

$$\text{iii) } V_L = I X_L = (0.5119 \angle 89.68^\circ) (18.85 \angle 90^\circ) \\ = 9.649V \angle 179.68^\circ$$

iv) $V_C = I_Z C = (0.5119 \angle 89.68^\circ)(468.10 \angle -90^\circ)$
 $= 239.2 V \angle -0.32^\circ.$

2) The following loads are connected to common supply.

⇒ Soln

Load A) $S = 150 \text{ kVA} \text{ at } 0.707 \text{ lag}$

Load B) $P_1 = 100 \text{ kW} \text{ at unity}$

Load C) $P_2 = 75 \text{ kW} \text{ at } 0.8 \text{ lag}$

Load D) $P_3 = 50 \text{ kW} \text{ at } 0.6 \text{ lag}$

We know,

$$P_{f_p} = \frac{P_0}{S_0}$$

$$\Rightarrow P_0 = P_{f_p} \times S_0$$

$$= 0.707 \times 150$$

$$= 106.05$$

$$\begin{aligned} \text{Total active Power (D)} &= P_0 + P_1 + P_2 + P_3 \\ &= 106.05 + 100 + 75 + 50 \\ &= 331.05 \text{ kW.} \end{aligned}$$

$$\text{Apparent Power} = S_0 + S_1 + S_2 + S_3$$

$$= S_0 + \frac{P_1}{PF_1} + \frac{P_2}{PF_2} + \frac{P_3}{PF_3}$$

$$= 150 + \frac{100}{0.8} + \frac{75}{0.6} + 50$$

$$= 427.08 \text{ kVA}$$

$$\text{Overall power factor (PF)} = \frac{\text{overall active power (P)}}{\text{overall apparent power (S)}}$$

$$= \frac{331.05}{427.08}$$

$$= 0.775$$

Now,

$$\text{PF} = \cos \phi$$

$$\phi = \cos^{-1}(0.775)$$

$$= 39.19^\circ$$

$$\frac{\text{Reactive power}}{\text{active power}} = \frac{V I \sin \phi}{V I \cos \phi} = \tan \phi$$

$$R = \tan(39.19^\circ) \times 331.05$$

$$= 264.82 \text{ kVAR}$$

3)

\Rightarrow Soln,

$$V = 240 \text{ V}$$

$$Z = 10 \Omega < 60^\circ$$

$$Q_1 = 1250 \text{ VAR}$$

ii) we know,

$$I = \frac{V_B}{Z}$$

$$= \frac{240 < 0^\circ}{10 < 60^\circ}$$

$$= 24 \angle -60^\circ$$

$$\phi = 60^\circ$$

$$P_f = VI \cos \phi = \cos 60^\circ$$

$$= \frac{1}{2} = 0.5$$

$$P = VI \cos \phi$$

$$= 240 \times 24 \times \cos (-60)$$

$$= 2880 \text{ W}$$

$$\delta = VI \sin \phi$$

$$= 240 \times 24 \times \sin (-60)$$

$$= 4980.3 \text{ VAR}$$

$$\frac{R}{Z} = \cos \phi$$

$$R = Z \cos \phi$$

$$= 10 \cos (-60)$$

$$= 5 \Omega$$

$$Z = \sqrt{r^2 + x^2}$$

$$Z^2 - R^2 = x^2$$

$$x^2 = \sqrt{Z^2 - R^2}$$

$$= \sqrt{10^2 - 5^2}$$

$$= 8.66 \Omega$$

ii) Now, when capacitor is connected

$$\text{Reactive power } (Q) = \delta - P$$

$$= (4980.3 - 1250) \text{ VAR}$$

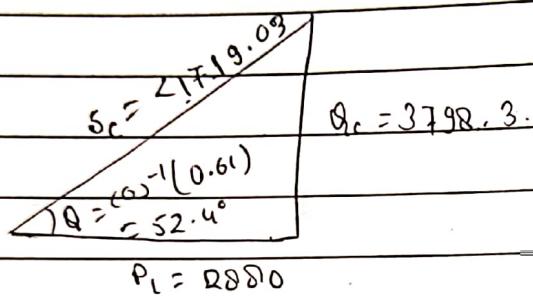
$$= 3738.3 \text{ VAR}$$

Active power remain same
 $P_L = 2280 \text{ W.}$

$$\text{Resultant power factor (PF)} = \frac{P_c}{S_c} = \frac{P_L}{\sqrt{P_L^2 + Q^2}}$$

$$= \frac{2280}{\sqrt{2280^2 + (3798.3)^2}}$$

$$= 0.61$$



power triangle.

- (1) A voltage source $E_{an} = 120 \angle 30^\circ$ & the current through it is, $I_{an} = 10 \angle 60^\circ \text{ A}$. find the value of P & Q & state whether the source is delivering or receiving $\rightarrow 501^n$

$$E_{an} = 120 \angle 30^\circ$$

$$I_{an} = 10 \angle 60^\circ \text{ A}$$

$$P = E_{an} I_{an}$$

$$= 1200 \angle -30^\circ$$

$$P = VI \cos \phi$$

$$= 1200 \cos(30^\circ)$$

$$= 1039.23 \text{ W.}$$

$$\theta_s = \sqrt{P} \sin \phi$$

$$= 1200 \sin(30)$$

$$= 600 \text{ VAR}$$

Here, current lead the voltage, so the load is supplying reactive power & the source is receiving reactive power.

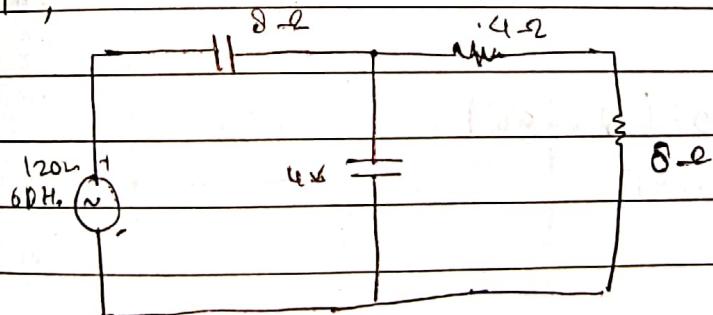
5. For the circuit shown

a) Find the effective value of the current delivered by the source. Write down the expression for the instantaneous values of the current.

b) Draw a carefully labelled phasor diagram showing the source voltage, source current, voltage V_{AB} & $\mathbf{J}_{V_{BC}}$

c) Compute the power supplied by the source & the line pf.

$$\Rightarrow 501^\circ$$



$$\Rightarrow 561^\circ$$

We have,

$$V = 120 \angle 0^\circ$$

$$f = 60\text{Hz}$$

$$\text{a) } Z_1 = -j8 = 8 \angle -90^\circ \Omega$$

$$Z_2 = -j4 = 4 \angle -90^\circ \Omega$$

$$Z_3 = 12 \Omega \angle 0^\circ$$

So,

$$Z = Z_1 + (Z_2 / Z_3)$$

$$= Z_1 + \frac{Z_2 \times Z_3}{Z_2 + Z_3}$$

$$= (8 < -90) + \frac{(4 < -90) \times (12 < 0)}{(4 < -90) + (12 < 0)}$$

$$= 11.68 < -84.09^\circ$$

Now,

$$I_{rms} = \frac{V}{Z} = \frac{120\sqrt{2}}{11.68} < -84.09^\circ$$

$$= 10.29 < 84.09^\circ$$

$$I_m = I_{rms} \times \sqrt{2}$$

$$= 10.29 \times \sqrt{2}$$

$$= 14.55$$

$$i = im \sin(\omega t + \phi)$$

$$= (14.55 \sin(\omega t + 84.09^\circ))$$

$$V_1 = I_{rms} Z_1$$

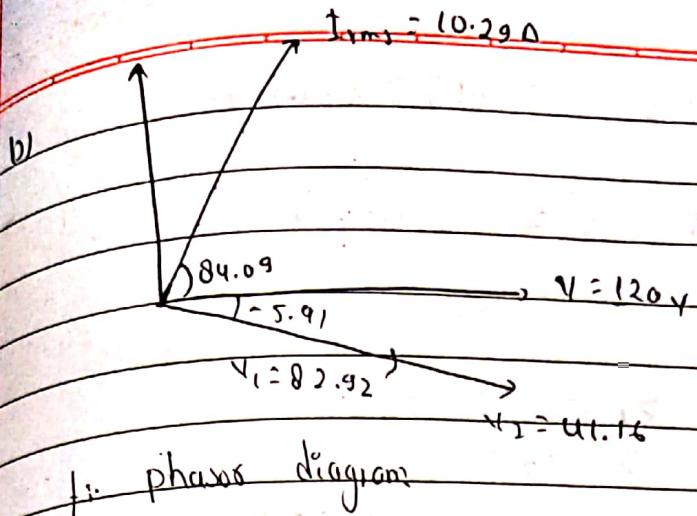
$$= (10.29 < 84.09^\circ) (8 < -90^\circ)$$

$$= 89.32 < -5.91^\circ$$

$$V_2 = I_{rms} Z_2$$

$$= (10.29 < 84.09^\circ) (4 < -90^\circ)$$

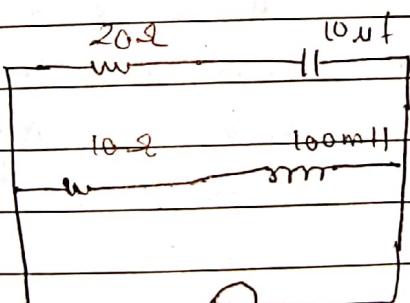
$$= 41.16 < -5.91^\circ$$



$$\begin{aligned}
 (a) P &= VI \cos \phi \\
 &= 120 \times 10.29 \times \cos(84.09^\circ) \\
 &= 127.15 \text{ W}
 \end{aligned}$$

$$\begin{aligned}
 P_f &= I \cos \phi = \cos(84.09^\circ) \\
 &= 0.1029 \text{ (lead)}
 \end{aligned}$$

6) calculate the magnitude & phase of the current from the source in the following AC parallel circuit.



200V, 50Hz, 0°

$\Rightarrow 50\Omega$,

We have,

$$V = 200 \angle 0^\circ$$

$$f = 50 \text{ Hz}$$

$$Y_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10^{-6}} = 318.3 \text{ S}$$

$$X = 2\pi f L = 2\pi \times 50 \times 100 \times 10^{-3} = 31.4 \text{ S}$$

$$Z_1 = 20 - j31.8 \cdot 3.0 = 318.9 \angle -86.45^\circ$$

$$Z_2 = 10 + j31.41 = 32.96 \angle 72.34^\circ$$

$$Z = Z_1 // Z_2 = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$$

$$= (318.9 \angle -86.45^\circ)(32.96 \angle 72.34^\circ) \\ (318.9 \angle -86.45^\circ) + (32.96 \angle 72.34^\circ)$$

$$= 36.44 \angle 69.96^\circ$$

$$I = \frac{V}{Z} = 200 \angle 0^\circ$$

$$= 5.48 \angle -69.96^\circ$$

7) Two coil A & B are connected in series across a 240V 50Hz supply. The resistance of A is 5Ω & the inductance of B is 0.015H. If the input from the supply is 3kV & 2kVAR. Find the inductance of A & the resistance of B. Calculate the voltage across each other.

\Rightarrow Soln,

$$R = 5\Omega$$

$$L = 0.015H$$

$$P = 3kV$$

$$\theta = 2kVAR$$

$$V = 240V$$

$$f = 50Hz$$

$$Z_1 = 5 + jX_L = 5 + j4.71 = 6.86 \angle 43.28^\circ$$

$$Z_2 = R + jX_L = 10.97 + j = 11.93 \angle 23.23^\circ$$

we know,

$$\frac{Q}{P} = \frac{\sqrt{I} \sin \phi}{\sqrt{I} \cos \phi}$$

$$\frac{Q}{P} = \tan \phi$$

$$\phi = 33.7^\circ$$

$$P = \sqrt{I} \cos \phi$$

$$I = P = \frac{3 \times 1000}{240 \times \cos 33.7^\circ}$$

$$= 15.024 \angle 33.7^\circ$$

$$\text{In series } I = I_1 = I_2$$

$$Z = Z_1 + Z_2 = (5 + jX_L) + (R + jX_L)$$

$$Z = (5 + R) + j(X_L + 4.71)$$

we know,

$$I = \frac{\sqrt{I}}{Z}$$

$$Z = \frac{\sqrt{I}}{I} = \frac{240}{15.024} = 15.97$$

we have

$$15.97 = (5 + R) + j(X_L + 4.71)$$

comparing, we get,

$$15.97 = 5 + R$$

$$R = 10.97$$

$$0 = X_L + Y_{\parallel}$$

$$X_L = 4.71$$

$$2\pi f L' = 4.71$$

$$L' = \frac{4.71}{2\pi \times 50} = 0.015 \text{ H.}$$

Now,

$$V_1 = IZ_1$$

$$= (15.024 \angle 53.7^\circ) (6.86 \angle 43.28^\circ)$$

$$= 103.06 \angle 76.98^\circ$$

$$V_2 = IZ_2$$

$$= (15.024 \angle 53.7^\circ) \times (11.93 \angle 23.23^\circ)$$

$$= 179.23 \angle 56.93^\circ$$

8) A circuit of three branches A, B, & C. A & B are in parallel while C is in series with this parallel combination of A & B. The impedances of branches are $Z_A = (4 + j3) \Omega$, $Z_B = (10 - j7) \Omega$ & $Z_C = (6 + j5) \Omega$. If the voltage $230V$ at 50 Hz is applied to the circuit - calculate

a) Total power of the whole circuit.

b) Draw the complete phasor diagram.

SOL:

According to the question,

$(A \parallel B) + C \Rightarrow$ A parallel B whole series with C.

We have

$$Z_A = (4 + j3) \Omega = 5 \angle 36.36^\circ$$

$$Z_B = (10 - j7) \Omega = 12.2 \angle -34.99^\circ$$

$$Z_C = (6 + j5) \Omega = 7.81 \angle 39.8^\circ$$

$$V = 230V$$

$$f = 50Hz$$

we know,

$$Z = (Z_A || Z_B) + Z_C$$

$$= \frac{Z_A Z_B}{Z_A + Z_B} + Z_C$$

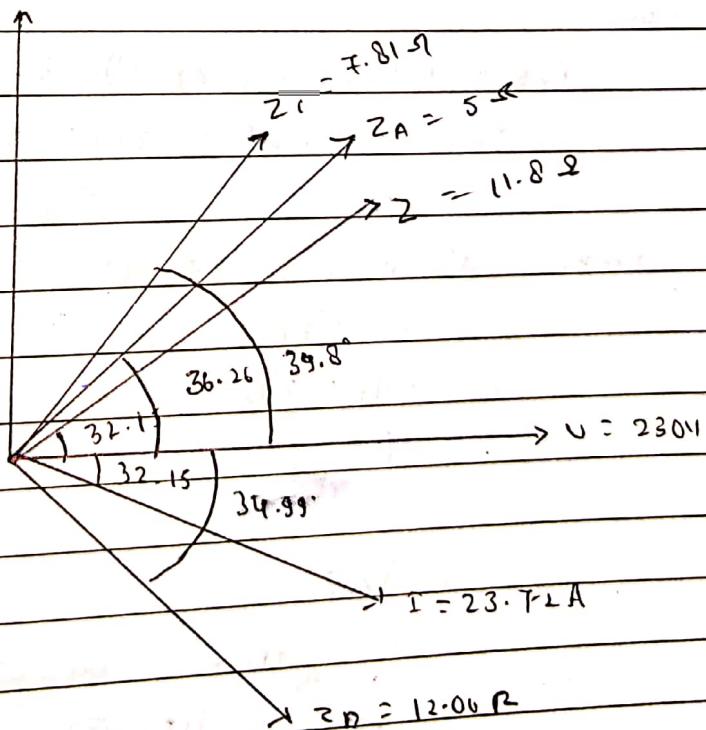
$$= \frac{(5 \angle 36.86^\circ)(12.20 \angle -34.99^\circ)}{(5 \angle 36.86^\circ) + (12.20 \angle -34.99^\circ)} + (7.81 \angle 39.8^\circ)$$

$$= 9.99 + 6.28j$$

$$= 11.8 \angle 32.15^\circ$$

Here,

$$\text{Power factor} = \cos \theta = \cos(-32.15^\circ)$$



9) A circuit is composed of a resistor of $9\ \Omega$ & series connected inductive reactance of $12\ \Omega$. When a voltage is applied to the circuit, the resulting steady current is found to be $i = 28.3 \sin 377t$. What is the value of complex impedance? Determine time expression for applied voltage. Find the value of inductance in Henry $\Rightarrow 50\text{H}$.

$$Z = 9 + j12 = 15 \angle 53.13^\circ$$

$$i = 28.3 \sin 377t$$

we know

$$I_{\text{rms}} = \frac{i_m}{\sqrt{2}}$$

$$I_{\text{rms}} = \frac{28.3}{\sqrt{2}} = 20 \angle 0^\circ$$

now,

$$i \times R = (28.3 \sin 377t) (15 \angle 53.13^\circ)$$

$$V = 424.5 (\sin 377t + 53.13^\circ)$$

where, $\sin \omega t = \sin 377t$

$$\omega = 377$$

$$2\pi f = 377$$

$$f = \frac{377}{2\pi}$$

we have

$$\sin \omega t = \sin 377t$$

$$\omega = 377$$

$$X_L = 12 \Omega$$

$$\therefore 2\pi f L = 12$$

$$\therefore L = \frac{12}{2\pi f} = \frac{12}{2\pi \times \frac{377}{2\pi}} = 0.0318 \text{ H}$$

- ii) A 50Hz, 220V is applied to a 0.637 inductance.
- iii) Write the eqn for the applied voltage & current through the inductor.
- iv) Plot the voltage & current wave forms.

\therefore

$\omega = 2\pi f$

$$f = 50 \text{ Hz}$$

$$V = 220 \text{ V}$$

$$L = 0.637 \text{ H}$$

we know

$$\begin{aligned} i) \quad V &= V_m \sin(\omega t + \phi) \\ &= V_m \sin \omega t \\ &= (V_{m1} \times \sqrt{2}) \sin \left(\frac{\omega t}{\sqrt{2}} \right) + \\ &= 311 \sin \left(\frac{2\pi \times 50 \times t}{\sqrt{2}} \right) + \end{aligned}$$

$$V(t) = 311 \sin(314t)$$

l

$$V = V_m \sin(\omega t + \phi)$$

$$V = \frac{V_m}{\sqrt{2}} \sin(\omega t + \phi)$$

$$i = \frac{220}{2\pi \times 50 \times L} \sin \left(314t - \frac{\pi}{2} \right) \quad \left[\phi = \frac{\pi}{2} \right]$$

$$i(t) = 1.55 \sin(314t + \phi)$$

iii)

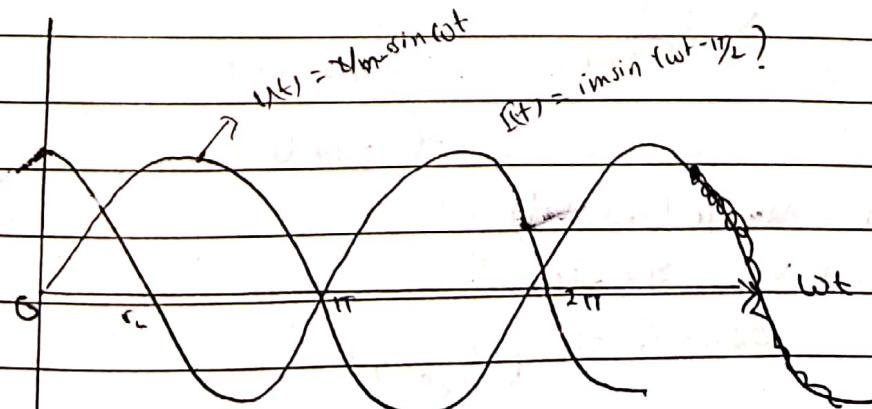


fig: Voltage & current waveform

iii) Draw the phasor diagram for voltage & current

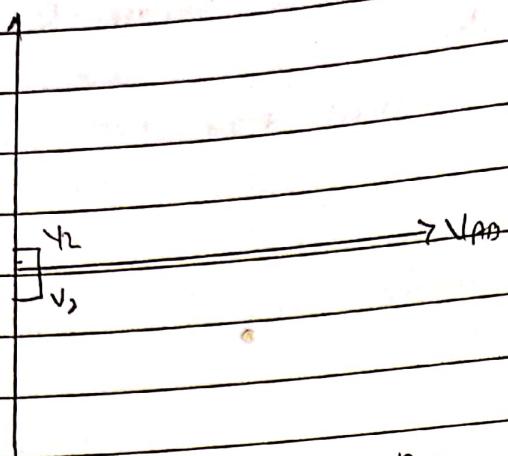


fig: phasor Diagram

12. A 50Hz, 220V voltage is applied to a capacitor of value 0.636 μF.

i) write eqn for instantaneous voltage & current as function of time t. Assume voltage at time $t=0$.

ii) to be zero & going positive

iii) Plot V & I against angle ωt .

iv) Draw the phasor diagram for voltage & current

\Rightarrow Soln.

$$f = 50 \text{ Hz}$$

$$V_m = 220 \text{ V}$$

$$C = 0.636 \mu\text{F} = 0.636 \times 10^{-6} \text{ F}$$

$$V_i = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 0.636 \times 10^{-6}}$$

$$= 5004.87$$

$$i) V(t) = V_m \sin(\omega t + \phi)$$

$$= V_m \times \sqrt{2} \sin \omega t$$

$$= 220 \times \sqrt{2} \sin \left(\frac{1}{5004.87} \right) t$$

$$= 311. \sin \left(\frac{1}{5004.87 \times 0.636 \times 10^{-6}} \right) t$$

$$= 311 \sin 314t$$

$$\frac{V(t)}{R} = \frac{V_m}{X_C} \sin (\omega t + \phi)$$

$$i(t) = \frac{V_{rms} \times \sqrt{2}}{X_C} \times \sin \left(\omega t + \frac{\pi}{2} \right)$$

$$= \frac{220 \times \sqrt{2}}{5004.87} \sin \left(\omega t + \frac{\pi}{2} \right)$$

$$= 62.2 \times 10^{-3} \sin \left(314t + \frac{\pi}{2} \right).$$

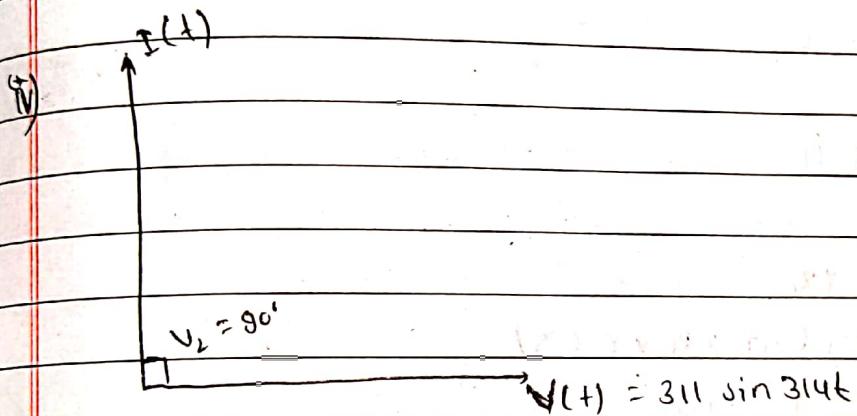


Fig: phasor diagram

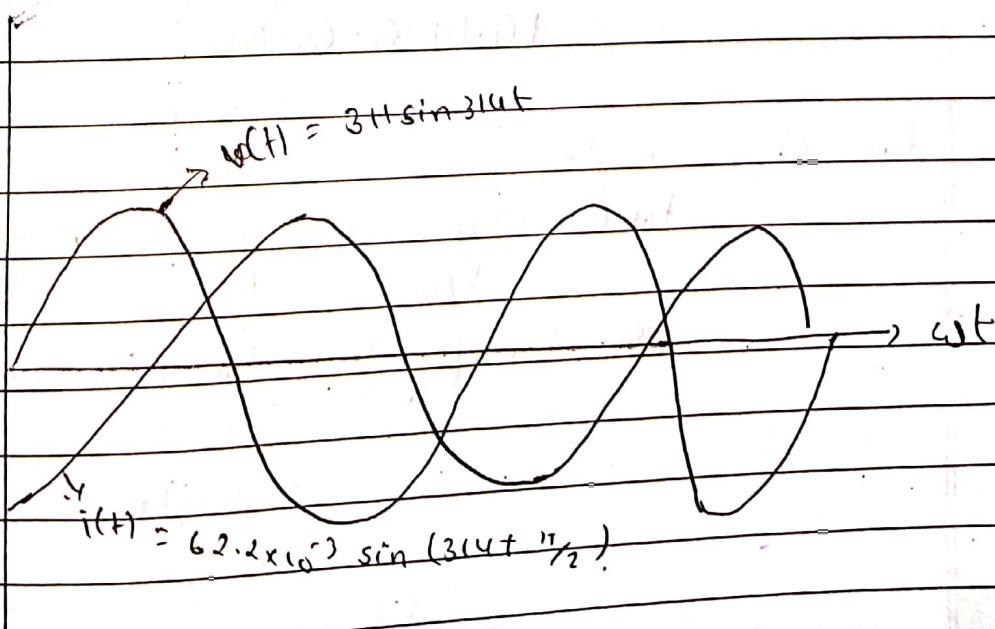


fig: V & i , vs. ωt .

- Q3. A sinusoidal voltage $V = 230 \angle 15^\circ$ of frequency 50Hz is applied to a series R-L circuit consisting of $R = 4\Omega$ & $L = 0.05H$. Calculate
- The rms current & its phase angle
 - Power factor.
 - Average power drawn by the circuit
 - Reactive power & apparent power
 - ϕ -factor of the coil.
- \Rightarrow Soln.

$$V = 230 \angle 15^\circ$$

$$f = 50\text{Hz}$$

$$R = 4\Omega$$

$$L = 0.05\text{H}$$

We know

$$\begin{aligned} Z &= R + jX_L \\ &= 4 + j(2\pi \times 50 \times 0.05) \\ &= 4 + j15.70 \\ &= 16.20 \angle 75.75^\circ \end{aligned}$$

Now

$$\begin{aligned} I &= \frac{V}{Z} = \frac{230 \angle 15^\circ}{16.20 \angle 75.75^\circ} \\ &= 14.19 \angle -60.70^\circ \end{aligned}$$

for phase angle,

$$\tan \theta = \frac{I/X_L}{I/R} = \frac{jX_L}{R} = \frac{X_L}{R}$$

$$\theta = \tan^{-1} \left(\frac{X_L}{R} \right) = \tan^{-1} \left(\frac{2\pi \times 50 \times 0.05}{4} \right)$$

$$= 75.75^\circ$$

ii) Power factor = $\cos \theta$

$$= \cos (75.71)$$

$$= 0.246$$

iii) Average Power (P) = $VI \cos \theta$

$$= 230 \times 14.19 \times \cos (75.71)$$

$$= 805 \text{ W}$$

iv) Reactive Powers (Q) = $VI \sin \theta$

$$= 230 \times 14.19 \times \sin (75.71)$$

$$= 3162.71 \text{ or } 3163 \text{ VAQ}$$

Apparent power (S) = VI

$$= 230 \times 14.19$$

$$= 3263.7$$

$$\approx 3204 \text{ VAT}$$

Q) A 200V, 50Hz supply is applied to a coil or resistance $R = 10\Omega$ & inductance 0.2H . Calculate

i) the reactance & impedance of the coil.

ii) current & its phase angle relative to applied voltage

Draw the phasor diagram.

\Rightarrow Soln,

$$V = 200 \angle 0^\circ$$

$$f = 50\text{Hz}$$

$$R = 10\Omega$$

$$L = 0.2\text{H}$$

We know

$$\begin{aligned}
 i) \quad Z &= R + jX_L \\
 &= 10 + j(2\pi f L) \\
 &= 10 + j(62.83) \\
 &= 63.62 \angle 80.95^\circ \Omega
 \end{aligned}$$

$$\begin{aligned}
 X_L &= 2\pi f L \\
 &= 2\pi \times 50 \times 0.2 \\
 &= 62.83 \Omega \angle 90^\circ
 \end{aligned}$$

we know,

$$\begin{aligned}
 iii) \quad I &= \frac{V}{Z} = \frac{200 \angle 0^\circ}{63.62 \angle 80.95^\circ} \\
 &= 3.14 \angle -90.95^\circ
 \end{aligned}$$

for phase angle

$$\begin{aligned}
 \tan \phi &= \frac{I/X_L}{I/R} = \frac{I}{R} \times \frac{X_L}{I} = \frac{X_L}{R} \\
 \phi &= \tan^{-1} \left(\frac{X_L}{R} \right) = \tan^{-1} \left(\frac{62.83}{10} \right)
 \end{aligned}$$

$$= 80.95^\circ$$

now

$$\begin{aligned}
 V_R &= I Z_R \\
 &= (3.14 \angle -80.95^\circ) (10 \angle 0^\circ) \\
 &= 31.4 \angle -80.95^\circ
 \end{aligned}$$

$$\begin{aligned}
 V_L &= I Z_L \\
 &= (3.14 \angle -80.95^\circ) (62.83 \angle 0^\circ) \\
 &= 197.28 \angle 90.95^\circ
 \end{aligned}$$

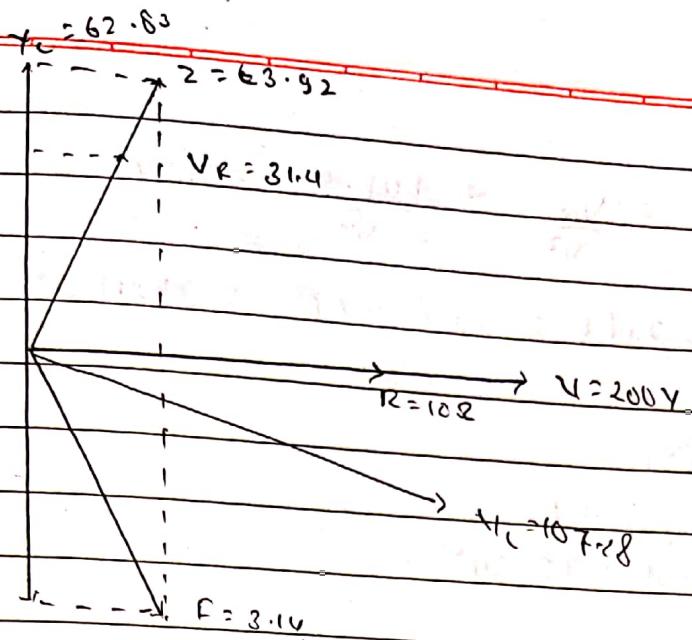


fig.: - phasor Diagram

Q5. A voltage $V = 141.4 \sin 314t$ is applied to a coil having inductive ohm 0.1H & resistance 10Ω to find:

i) expression for instantaneous current

ii) rms value of the voltage & current

iii) Power factor

iv) Power consumed in the coil.

>> Sol:

$$V = 141.4 \sin 314t \rightarrow i$$

$$L = 0.1\text{H}$$

$$R = 10 \Omega$$

Comparing (i) with $V = \sin \omega t$, we get

$$V_m = 141.4 \text{ V}$$

$$\omega = 314 \text{ rad/s}$$

$$\therefore 2\pi f = 314$$

$$\therefore f = \frac{314}{2\pi} \text{ Hz}$$

$$\therefore f = 50 \text{ Hz}$$

We know

$$V_{rm} = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100V$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.41 \Omega$$

$$\begin{aligned} Z &= R + jX_L \\ &= 10 + j31.41 \\ &= 32.96 \angle 72.34^\circ \end{aligned}$$

$$\frac{V}{Z} = \frac{V_m}{Z} \sin(314t)$$

$$\begin{aligned} i(t) &= \frac{141.4 \angle 0}{32.96 \angle 72.34} (\sin 314t) \\ &= 4.29 \sin(314t - 72.34^\circ) \end{aligned}$$

$$I_{rm} = I_m = \frac{4.29}{\sqrt{2}} = 3.03A$$

$$\begin{aligned} \text{Power factor} &= \cos \phi \\ &= \cos(-72.34^\circ) \\ &= \cos(72.34) \text{ (log)} \end{aligned}$$

$$\begin{aligned} \text{Power (P)} &= VI \cos \phi \\ &= 100 \times 3.03 \times \cos(72.34^\circ) \\ &= 91.92 W \end{aligned}$$

18) When connected to 200V, 50Hz supply a certain coil, draws current of 13A. If the frequency of the supply is changed to 60Hz, the current decrease to 11A. Calculate the resistance & inductive of coil.

$\Rightarrow \text{Soln}$

$$V = 200V$$

$$f_1 = 50\text{Hz}$$

$$I_1 = 13A$$

$$f_2 = 60\text{Hz}$$

$$I_2 = 11A$$

Now find

$$Z_1 = \frac{V}{I_1} = \frac{200}{13}$$

$$= 15.38 \Omega$$

$$Z_1 = \sqrt{R^2 + X_1^2}$$

$$\therefore (15.38)^2 = R^2 + 4\pi^2 \times f^2 \times L$$

$$R^2 = (15.38)^2 - 4\pi^2 \times 50^2 \times 1^2 \rightarrow i)$$

again

$$Z_2 = \frac{V}{I_2} = \frac{200}{11}$$

$$= 18.18 \Omega$$

again

$$Z_2 = \frac{V}{I_2} = \frac{200}{11}$$

$$= 18.18 \Omega$$

$$Z_2 = \sqrt{R^2 + X_{L2}^2}$$

$$(18.18)^2 = R^2 + 4\pi^2 \times 60^2 \times L^2$$

$$R^2 = (18.18)^2 - 4\pi^2 \times 60^2 \times L^2 \rightarrow ii)$$

Solving eqn (i) & (ii)

$$(15.38)^2 - 4\pi^2 \times 50^2 \times l = (18.18)^2 - 4\pi^2 \times 60^2 \times l^2$$

$$4\pi^2 (50^2 - 50^2) l^2 = (18.18)^2 - (15.38)^2$$

$$l^2 = \frac{(18.18)^2 - (15.38)^2}{4\pi^2 (60^2 - 50^2)}$$

$$l^2 = \sqrt{2.16 \times 10^{-3}}$$

$$= 0.04655$$

from (i)

$$R^2 = (15.38)^2 - 4\pi^2 \times 50^2 \times 0.04655^2$$

$$R = \sqrt{22.67}$$

$$= 4.76 \Omega$$

Ques. A 50Hz sinusoidal voltage $(40 + j30)$ Volt is applied to a series R-L circuit resulting in sinusoidal current $(4 + j1)$ Amperes. Calculate:

(i) Impedance of the circuit

(ii) Power consumed in the circuit

(iii) Power factor of the circuit.

Solution:

We have,

$$f = 50 \text{ Hz}$$

$$V = 40 + j30 \text{ V} = 50 \angle 36.86^\circ \text{ V}$$

$$I = 4 + j1 \text{ A} = 4.12 \angle 14.03^\circ \text{ A}$$

We know,

$$(i) I = \frac{V}{Z}$$

$$\text{Or, } Z = \frac{V}{I} = \frac{50 \angle 36.86^\circ}{4.12 \angle 14.03^\circ}$$

$$= 11.18 - 4.71j$$

$$= 12.135 \angle -22.83^\circ$$

$$(ii) \text{Power (P)} = VI \cos \phi$$

$$= 50 \times 4.12 \times \cos(-22.83^\circ)$$

$$= 190 \text{ W}$$

$$\begin{aligned}
 \text{(iii) Power factor} &= \cos\phi \\
 &= \cos(-22.83^\circ) \\
 &= \cos(22.83^\circ) \quad (\text{lag})
 \end{aligned}$$

An inductive coil having resistance 10Ω takes a current of 2.2A when connected to 110V , 60Hz supply. If the coil is connected to 50Hz supply. Calculate:

(i) Current (ii) Power (iii) Power factor

Draw phase diagram when 50Hz supply is connected.

Solution:

We have,

$$R = 10\Omega$$

$$I = 2.2\text{A}$$

$$V = 110\text{V}$$

$$f = 60\text{Hz}$$

$$f_1 = 50\text{Hz}$$

We know,

$$(1) Z = R + jX_L$$

$$20\pi fL$$

$$377\Omega$$

$$I = \frac{V}{Z}$$

$$\text{or, } Z = \frac{V}{I} = \frac{110}{2.2} = \cancel{45} - 45\Omega \quad 50\Omega$$

$$\text{a, } (R + jX_L) = (20 + 35j)$$

Comparing corresponding terms we get,

$$j877L = \frac{35 - j540}{j377} \times 50$$

$$\text{on } L = \frac{35 - j540}{j377}$$

$$= 132 \text{ H}$$

Now, at $f_1 = 50 \text{ Hz}$

$$X_L = 2\pi f_1 L$$

$$= 2\pi \times 50 \times 132 \times 0.132 \angle -90^\circ$$

$$= 41.47 \angle -90^\circ$$

$$\text{Then, } Z = 10 + j29.53 - 41.47$$

$$= 42.65 \angle 76.44^\circ$$

we know,

$$I = \frac{V}{Z} = \frac{40 \angle 0^\circ}{347 \angle 3^\circ} = 3.5$$

$$= \frac{42.65}{7} \angle 102^\circ = 2.57 \angle 76.44^\circ$$

$$\therefore I = 2.57 \text{ A}$$

$$\text{Power factor} = \cos \phi$$

$$= \cos(-76.44)$$

$$= 0.234 \text{ (lag)}$$

$$\begin{aligned}\text{Power (P)} &= VI \cos \phi \\ &= 110 \times 2.57 \times 0.234 \\ &= 68 \text{ W}\end{aligned}$$

$$\begin{aligned}V_R &= IR \\ &= (2.57 \angle 76.44^\circ) (10 \angle 0^\circ) \\ &= 26.1 \angle 76.44^\circ\end{aligned}$$

$$\begin{aligned}V_L &= I \cancel{Z} x_L \\ &= (2.57 \angle 76.44^\circ) (41.47 \angle -90^\circ) \\ &= 106 \text{ V} \angle -13.56^\circ\end{aligned}$$

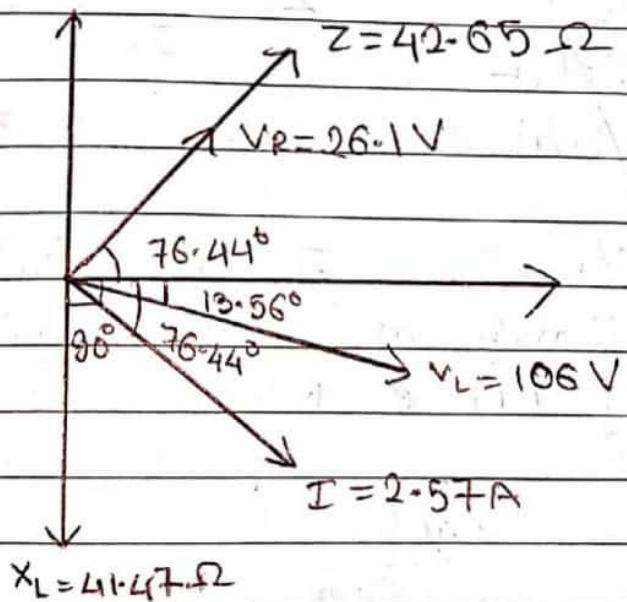


Fig: Phasor Diagram when $f = 50 \text{ Hz}$

- A. A coil having resistance of 10Ω and inductance of 0.2H is connected in series with coil B of resistance 30Ω and inductance of 0.1H . The two coils in series are fed from $200\text{V}, 50\text{Hz}$ supply. Determine:
- The voltage across each coil
 - Power dissipated in each coil
 - Power factor of the circuit as whole.

Solution:

$$R_1 = 10\Omega$$

$$L_1 = 0.2\text{H}$$

$$R_2 = 30\Omega$$

$$L_2 = 0.1\text{H}$$

$$V = 200\text{V}$$

$$f = 50\text{Hz}$$

We know,

$$\begin{aligned} Z_1 &= R_1 + jX_L \\ &= 10 + j2\pi f L_1 \\ &= 10 + j62.83 = 63.62 \angle 80.95^\circ \end{aligned}$$

$$\begin{aligned} Z_2 &= R_2 + jX_L \\ &= 30 + j2\pi f L_2 \\ &= 30 + j31.41 = 43.43 \angle 46.31^\circ \end{aligned}$$

In series, $I = I_1 = I_2$ so, and $V = V_1 + V_2$

$$I_1 = I_2$$

$$\text{or, } \frac{V_1}{Z_1} = \frac{V_2}{Z_2} \quad \text{or, } 200 \angle 0^\circ = I_1 Z_1 + I_2 Z_2$$

$$\text{or, } 200 \angle 0^\circ = I(Z_1 + Z_2)$$

$$\text{or, } \frac{200}{(10 + j62.83) + (30 + j31.41)} = I$$

$$\therefore I = 0.072 \angle -107.26^\circ \quad 1.95 \angle -66.99^\circ$$

Now,

$$(i) V_1 = IZ_1 \\ = (0.072 \angle 1.95^\circ) (33.02 \angle 80.95^\circ) \\ = 124.05 \angle 13.96^\circ$$

$$V_2 = IZ_2 \\ = (1.95 \angle -66.99^\circ) (43.43 \angle 46.31^\circ) \\ = 84.68 \angle -20.68^\circ$$

~~$$(ii) \text{ Power } (P_1) = V_1 I \cos \phi \\ = (124.05 \angle 13.96^\circ) (1.95 \angle -66.99^\circ) \\ = 124.05 \times 1.95 \times \cos (-66.99^\circ)$$~~

$$(ii) \tan \phi_1 = \left(\frac{X_L}{R_1} \right) = \left(\frac{62.83}{10} \right)$$

$$\therefore \phi_1 = 80.95^\circ$$

$$\tan \phi_2 = \left(\frac{X_L}{R_2} \right) = \left(\frac{31.41}{30} \right)$$

$$\therefore \phi_2 = 46.31^\circ$$

$$P_1 = V_1 I \cos \phi_1 \\ = 124.05 \times 1.95 \times \cos (80.95^\circ) \\ = 38 \text{ W}$$

$$\begin{aligned}P_s &= VI \cos \phi \\&= 84.68 \times 1.95 \times \cos(46.81) \\&= 114 \text{ W}\end{aligned}$$

$$\begin{aligned}\text{Overall pf} &= \cos \phi \\&= \cos 67^\circ \\&= \cos 67^\circ \text{ (lag)}\end{aligned}$$

20. A coil draws 11A and 800 W from 220V, 50 Hz supply. Determine:

- The impedance of the coil
- The power factor
- Reactive and apparent power.

Solution:

$$I = 11 \text{ A}$$

$$P = 800 \text{ W}$$

$$V = 220 \text{ V}$$

$$f = 50 \text{ Hz}$$

We know,

$$(i) Z = R + jX_L$$

and

$$I = \frac{V}{Z}$$

$$\text{or, } 11 = \frac{220}{R + jX_L}$$

$$\text{or, } R + jX_L = 20 \quad \text{--- (1)}$$

Also,

$$\tan \phi = \frac{x_L}{R} \quad \text{--- (ii)}$$

$$P = VI \cos \phi$$

$$\text{or, } \cos \phi = \frac{P}{VI}$$

$$\text{or, } \phi = \cos^{-1} \left(\frac{P}{VI} \right) = \cos^{-1} \left(\frac{800}{220 \times 11} \right) = 70.69^\circ$$

From (i),

$$\tan 70.69 = \frac{x_L}{12}$$

$$x_L = 2.85 R \quad \text{--- (iii)}$$

From (i) & (iii),

$$R + j2.85R = 20$$

$$\text{or, } R(1 + j2.85) = 20$$

$$\text{or, } R = \frac{20}{1 + j2.85}$$

$$= 6.621 \angle -70.66^\circ \Omega$$

Again,

$$\begin{aligned} x_L &= 2.85 \times 6.62 \\ &= 18.82 \end{aligned}$$

$$\therefore Z = (6.62 + j18.8) \Omega$$

$$T = 11 \angle -70.6^\circ$$

$$\text{Power factor} = \cos \phi = \cos(70.6^\circ) \text{ (lag)}$$

$$I_B = \frac{V}{Z_B}$$

$$\text{or, } Z_B = \frac{V}{I_B} = \frac{110}{11} = 10\Omega$$

since, Z_1 & B are in series,

$$\begin{aligned} Z_T &= Z_1 + Z_2 \\ &= 11\Omega + 10\Omega \\ &= 21\Omega \end{aligned}$$

Now,

$$I = \frac{V}{Z} = \frac{110}{21} = 5.23A$$

We have,

$$P_A = 200\omega$$

$$\text{or, } V I_A \cos \phi_A = 200$$

$$\text{or, } \cos \phi_A = \frac{200}{110 \times 10} = 0.181$$

$$P_B = 400\omega$$

$$\text{or, } V I_B \cos \phi_B = 400$$

$$\text{or, } \cos \phi_B = \frac{400}{110 \times 11} = 0.330$$

$$\cos \phi = \frac{\cos \phi_A + \cos \phi_B}{2}$$

$$= \frac{0.181 + 0.330}{2}$$

$$= 0.2555$$

$$(ii) Q = VI \sin \phi$$

$$= 220 \times 11 \times \sin(70.6)$$

$$= 2282 \text{ VAR}$$

$$S = VI$$

$$= 220 \times 11$$

$$= 2420 \text{ VA}$$

21. A coil A draws current of 10A and power of 2000W when driven by 110V, 60Hz supply. Another coil B draws current of 11A and power 400W when driven by the same source. What current and power will be drawn by from the source by coils A and B connected in series.

Solution:

$$I_A = 10A$$

$$P_A = 22200W$$

$$V = 110V$$

$$f = 60 \text{ Hz}$$

$$I_B = 11A$$

$$P_B = 400W$$

We know,

$$I_A = \frac{V}{Z_A}$$

$$\therefore Z_A = \frac{V}{I_A} = \frac{110}{10} = 11 \Omega$$

$$\begin{aligned} \text{Total Power } (P_T) &= VI \cos \phi \\ &= 110 \times 5.23 \times 0.2555 \\ &= 147.2 \text{ W} \end{aligned}$$

Power Factor Improvement in single phase AC Circuit.

22. An inductive load of 4kW at a lagging power factor of 0.8 is connected across a 220V, 50 Hz supply. Calculate the value of the capacitor to be connected in parallel with the load to bring the resultant power factor to 0.95 lagging.

Solution:

we have,

$$P = 4 \text{ kW} = 4000 \text{ W}$$

$$\text{PF}_L = 0.8 \text{ (lag)}$$

$$V = 220 \text{ V}$$

$$f = 50 \text{ Hz}$$

we know,

$$P = VI \cos \phi$$

$$\text{or, } 4000 = 220 \times I_L \times 0.8 \quad [\because \cos \phi = \text{PF}]$$

$$\text{or } I_L = \frac{4000}{220 \times 0.8}$$

$$\therefore I_L = 22.72 \text{ A}$$

$$I_L = \frac{V}{Z_L}$$

$$\text{or, } 22.72 = \frac{220}{Z_L}$$

$$Z_L = \frac{220}{22.72} = 9.68 \Omega$$

now

$$\cos \phi_2 = 0.95$$

$$\begin{aligned}\phi_2 &= \cos^{-1}(0.95) \\ &= 18.2^\circ\end{aligned}$$

$$(0) \quad \phi_1 = 0.8$$

$$\begin{aligned}\phi &= \cos^{-1}(0.8) \\ &= 36.86^\circ\end{aligned}$$

$$C = \frac{P (\tan \phi_1 - \tan \phi_2)}{\omega^2}$$

$$= \frac{4000 (\tan 36.86 - \tan 18.2)}{220^2 \times 2\pi \times 50}$$

$$= \frac{4000 (0.75 - 0.32)}{1.57 \times 10^7}$$

$$= 1.10 \times 10^{-4} F$$

$$= 110 \times 10^{-6} F$$

$$= 110 \mu F$$

23. A 100 kW load at 0.8 lagging power factor is being supplied by a 220V, 50Hz source. Calculate the reactive power drawn from the source. If a capacitor connected in parallel to the load improves its power factor to 0.9, find the capacitance of the capacitor. Also, calculate the current draw from the source before and after connecting the capacitor.

Solution:

We know,

$$P = 100 \text{ kW}$$

$$Pf_1 (\cos \phi_1) = 0.8 \text{ (lag)}$$

$$V = 220 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$Pf_2 (\cos \phi_2) = 0.9$$

We know,

$$P = VI \cos \phi$$

$$\therefore 100 = 220 \times I \times 0.8$$

$$\therefore I = \frac{100 \times 1000}{220 \times 0.8}$$

$$\therefore I_{\text{before}} = 0.568 \text{ A} \quad 568.18 \text{ A}$$

Now,

$$Q = VI_{\text{before}} \sin \phi$$

$$= 220 \times 568.18 \times \sin (36.87^\circ)$$

$$= 75000 \text{ VAR}$$

$$= 75 \text{ kVAR}$$

when capacitor is connected

$$\cos \phi_s = 0.9$$

$$\phi_s = \cos^{-1}(0.9) = 25.84^\circ$$

$$\cos \phi_1 = 0.8$$

$$\phi_1 = 36.86^\circ$$

we know

$$C = \frac{P(\tan \phi_1 - \tan \phi_s)}{V^2 \omega}$$

$$= \frac{100 \times 100 (\tan(36.86) - \tan(25.84))}{220^2 \times 2\pi \times 50}$$

$$= 1.74727 \times 10^{-3}$$

$$= 1.74727 \times 10^{-6} F$$

$$= 1.74727 \mu F$$

~~11~~