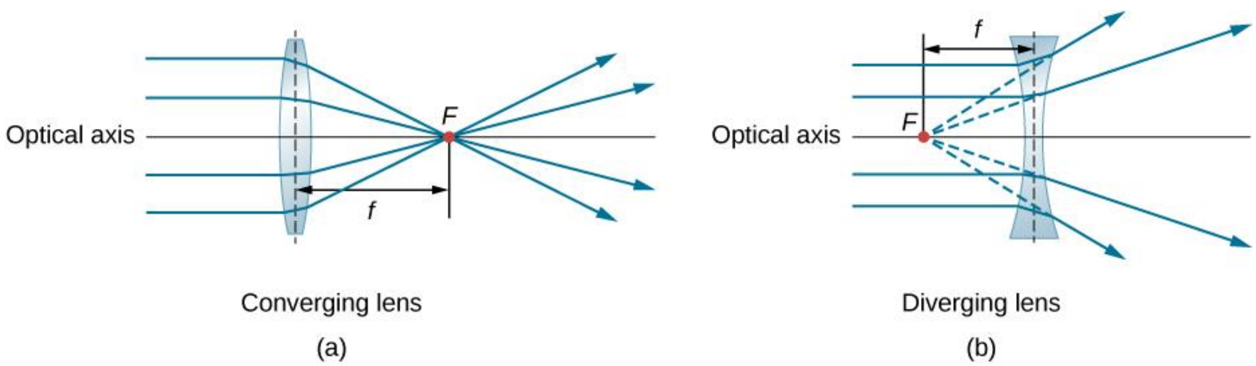
Geometrical Optics

Refraction through Lens:-

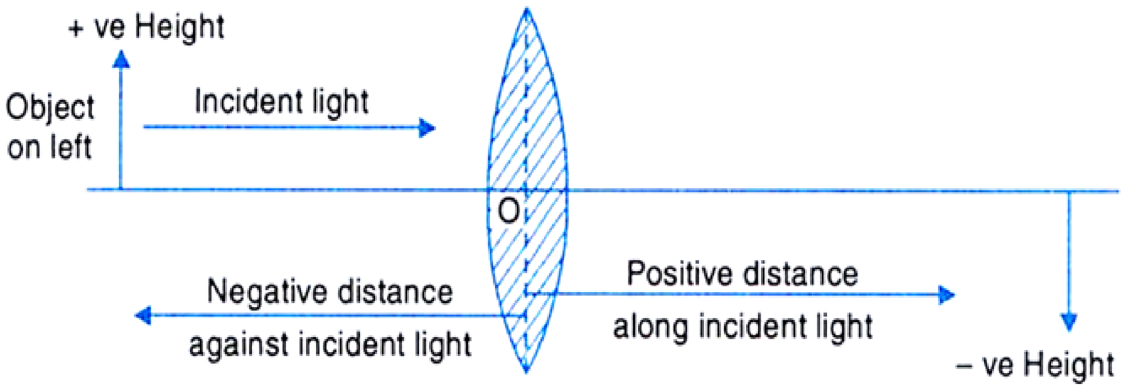
A lens is a portion of a transparent refracting medium bounded by two spherical surfaces. A lens is built up from a series of truncated prism, whose refracting angle change continuously.



Sign Convention:-

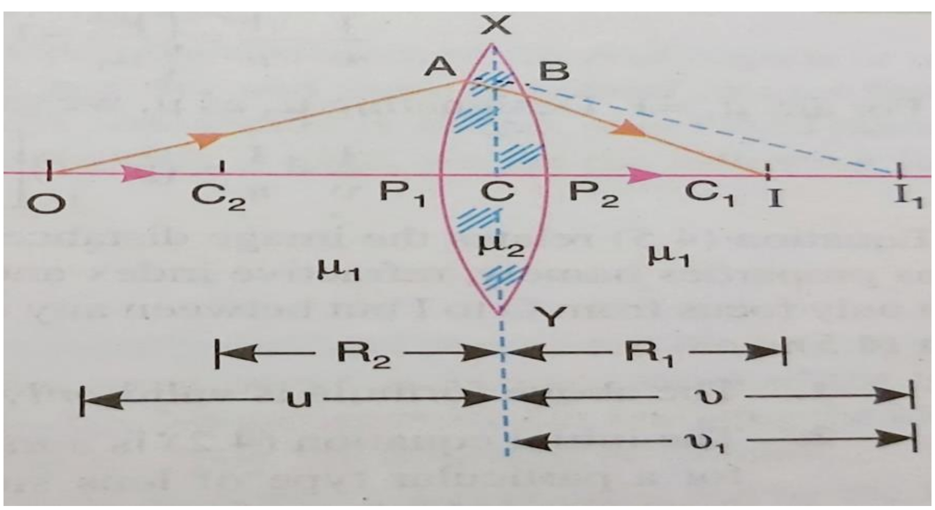
The following convention of the sign is adopted for obtaining the relation between these quantities.

1. All the distances are measured from the optical center of the lens.
2. The distances are positive and negative as shown in the following figure.
3. The focal length of a convex lens is positive and that of a concave lens.



Refraction through a Lens (Lens Maker's formula):-

A lens forms an image by refraction of light at its two bounding surfaces. Each surface acts as an image forming component, and contributes to the final image formed by the lens. If we know the focal length of a lens and the position of an object, the position of the image can be determined either by using ray diagram or mathematical relation. It is not always convenient to draw a ray diagram. We study here how the position of the image is determined using mathematical equation.



Consider a thin lens of material of refractive index μ_2 separates a medium of refractive index μ_1 on its two sides. The radius of curvature of the coaxial refracting surface is R_1 and R_2. Consider a point object *O*, be kept on the principal axis at distance *u* from the first refracting surface as shown in figure.

When light from the object *O* falls on the first refracting surface of the lens the image I_1 is formed at a distance v_1 from the first refracting surface. So from **Gauss formula**;

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \ldots\ldots\ldots (i)$$

This image I_1 acts as virtual object in medium of refracting index μ_2 of the second refracting surface and forms the final image *I* at distance *v* from the second refracting surface in medium of refracting index μ_1. Again

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} \ldots\ldots\ldots (ii)$$

Adding equations (i) and (ii), we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$or, \qquad \frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

When the lens is in air,

$$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \ldots\ldots\ldots (iii)$$

Where μ is the refractive index of the material of the lens with respect to air. The equation (iii) so obtained is lens maker

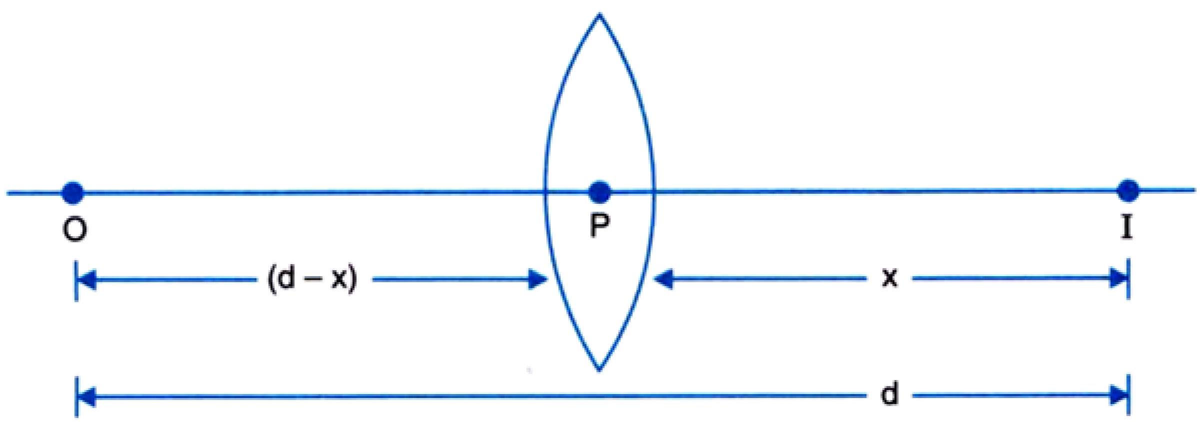
formula. This relation holds true for paraxial rays for which the angle made by the rays are small. However, the relation is true for thin lenses where the thickness is negligibly small compared to the object distance, image distance and radii of curvatures.
As explained earlier in case of a lens also, we can obtain the two principal foci with focal lengths f_2 and f_1 as,

$$\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$-\frac{1}{f_1} = -(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For lens placed in air, it has air on both sides. In such case $f_1 = f_2$ but of opposite signs. The above relation is also called lens makers formula.

Least possible distance between an object and its real image in a convex lens:-



Consider a thin convex lens. Let the focal length of the lens be f, O is the point object on the principal axis and after refraction the image I is formed. Let the distance between O and I be d and distance of image from the lens be x. Here objective distance is negative and image distance is positive. Thus;

$$v = x$$

$$u = -(d - x)$$

We know,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$or, \qquad \frac{1}{x} - \frac{1}{-(d-x)} = \frac{1}{f}$$

$$or, \qquad \frac{d - x + x}{x(d-x)} = \frac{1}{f}$$

$$or, \qquad xd - x^2 = fd$$

$$or, \qquad x^2 - xd + fd = 0$$

This is the quadratic equation and its solution is given by;

$$x = \frac{-d \pm \sqrt{d^2 - 4fd}}{2}$$

For image to be real, the distance x should be real. This condition is satisfied if

$$d^2 - 4fd \geq 0$$

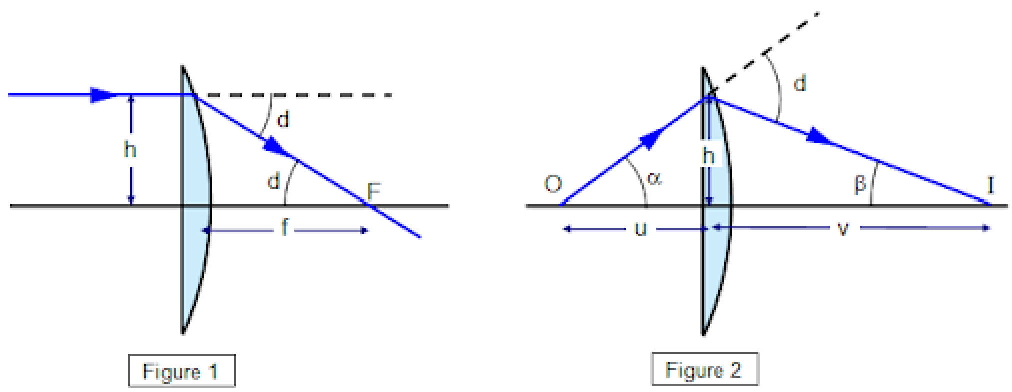
$$or, \qquad d^2 \geq 4fd$$

$$\therefore \quad d \geq 4f$$

Thus *d* should be greater than *4f*, or at least equal to *4f* for the image to be real. Therefore the minimum distance between the real object and real image is *4f*.

Deviation by a thin lens:-

A lens may be considered to be made up of a large number of prisms placed one above the other. As the action of the lens is to deviated the incident rays of light, it is necessary to find the deviation produced by a particular section of the lens. Let a ray of monochromatic light parallel to the principal axis be incident on a thin lens at a height 'h' above the axis and let 'f' be the focal length of the lens.



The deviation suffered by the ray is given by;

$$\tan d = \frac{h}{f}$$

In the paraxial region d being small, $\tan d \approx d$

$$\therefore d = \frac{h}{f}$$

Next, consider a luminous point O and its corresponding image I. The deviation suffered by the incident ray is given by;

$$d = \alpha + \beta$$

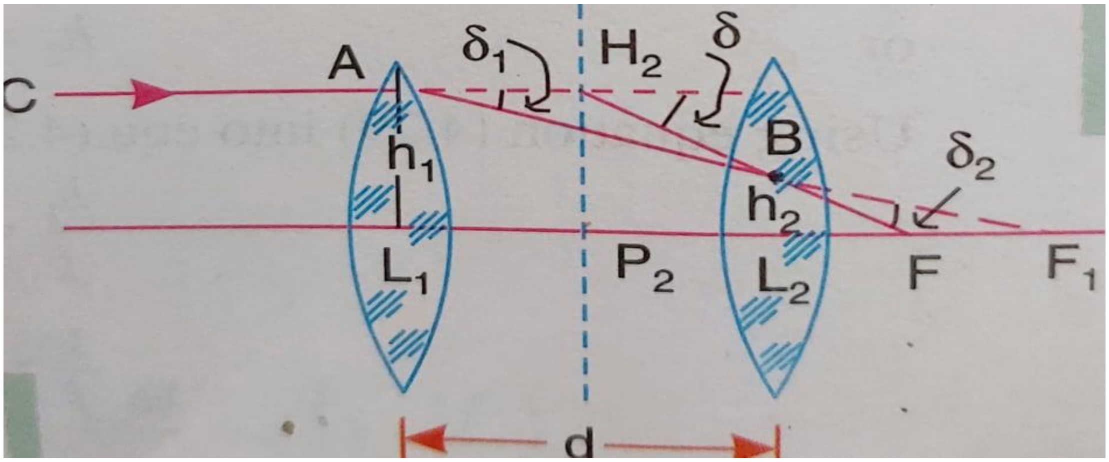
$$d = \frac{h}{-u} + \frac{h}{+v} = h\left(\frac{1}{v} - \frac{1}{u}\right) = h\left(\frac{1}{f}\right)$$

$$\therefore d = \frac{h}{f}$$

This shows that the deviation produced by a lens is independent of the position of the object.

Combination of two thin Lenses separated by a finite distance:-

Consider two thin lenses L_1 & L_2 of focal lengths f_1 and f_2 respectively. These are placed coaxially and separated by a finite distance d in air. δ_1 and δ_2 are the angle of deviation produced by the first and second lens respectively. The phenomenon of refraction is as shown in figure. The lens placed at H_1P_1 and H_2P_2 is called equivalent lens and $P_1F = P_2F = f$ is called equivalent focal length. The planes at H_1P_1 & H_2P_2 are the first and second principal planes and the points P_1 and P_2 on the principal axis are the first and second principal points respectively.



From figure, the total deviation produced by the lens system is $\delta = \delta_1 + \delta_2$

For small δ, $\tan\delta \cong \tan\delta_1 + \tan\delta_2$

$$\text{i.e.} \frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \dots\dots\dots (i)$$

Since, Δ A L_1F_1 and BL_2F_1 are similar in figure (a),

$$\text{i.e.} \ \frac{A\,L_1}{L_1F_1} = \frac{BL_2}{L_2F_1}$$

$$\text{or,} \qquad \frac{h_1}{f_1} = \frac{h_2}{(f_1 - d)} \dots\dots\dots (ii)$$

$$\text{or,}\, h_2 = \frac{(f_1 - d)h_1}{f_1} \dots\dots\dots (iii)$$

Now from equation (i) and (iii), we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{(f_1 - d)h_1}{f_1 f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Therefore equivalent focal length is given by;

$$f = \frac{f_1 f_2}{f_1 + f_2 - d}$$

$$\text{or,} \qquad f = \frac{-f_1 f_2}{\Delta}$$

Where $\Delta = d - (f_1 + f_2)$ and is known as the optical interval between the two lenses. It is numerically equal to the distance between the second principal focus of the first lens and the first principal focus of the second lens.

Further let the distance between the imaginary equivalent lens and the second lens be $\beta(P_2L_2)$. Now from the similar Δ H_2P_2F and BL_2F we get;

$$\frac{P_2F}{L_2F} = \frac{H_2P_2}{BL_2}$$

$$\frac{f}{f-\beta} = \frac{h_1}{h_2}$$

Sine distance β is the left side of L_2, following the sign convention we can write;

$$\frac{f}{f+\beta} = \frac{h_1}{h_2} = \frac{h_1 f_1}{(f_1-d)h_1} \quad \text{(from equation iii)}$$

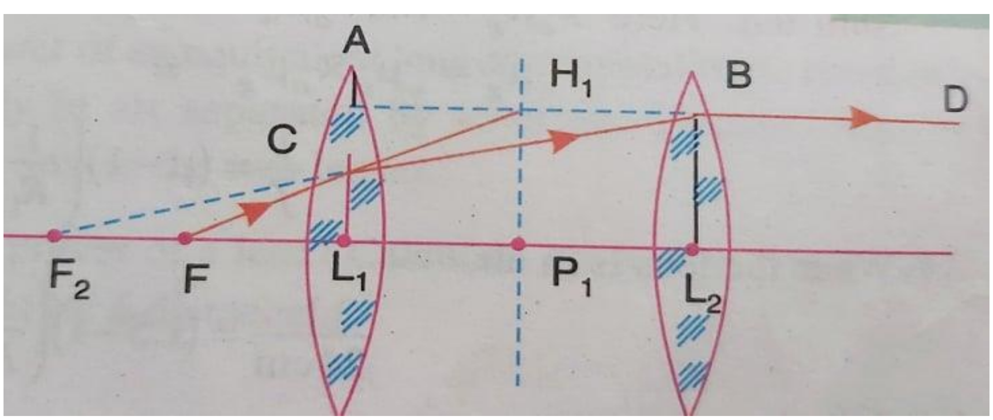
$$\text{or,} \quad \frac{f}{f+\beta} = \frac{f_1}{(f_1-d)}$$

$$\text{or,} \quad ff_1 - fd = ff_1 + f_1\beta$$

$$\text{or,} \quad \beta = \frac{-fd}{f_1} \ldots\ldots\ldots.(iv)$$

Further let the distance between the imaginary equivalent lens and the first lens be $\alpha\,(P_1L_1)$. From similar triangles H_1P_1F and CL_1F;

$$\frac{H_1P_1}{P_1F} = \frac{CL_1}{L_1F}$$



$$\text{or,} \qquad \frac{h_2}{f} = \frac{h_1}{f - \alpha} \ldots \ldots \ldots (v)$$

The other pairs of similar triangles BL_2F_2 and CL_1F_2 yields

$$\frac{BL_2}{L_2F_2} = \frac{CL_1}{L_1F_2}$$

$$\text{or,} \qquad \frac{h_2}{f_2} = \frac{h_1}{f_2 - d} \ldots \ldots \ldots (vi)$$

Solving equation (v) and (vi) we get;

$$\frac{h_1 f_2}{f(f_2 - d)} = \frac{h_1}{f - \alpha}$$

$$\text{or,} \qquad ff_2 - f_2\alpha = ff_2 - fd$$

$$\text{or,} \qquad \alpha = \frac{fd}{f_2} \ldots \ldots \ldots (vii)$$

In case if the two lenses are in contact, distance between them d = 0 which will provide the equivalent focal length of the lenses in contact as;

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Power of a Lens:-

The power of a lens is the measure of its ability to produce convergence of a parallel beam of light. A convex lens of large focal length produces a small converging effect and a convex lens of small focal length produces a large converging effect. Due to this reason, the power of a convex lens is taken as positive and convex lens of small focal length has high power. On the other hand, a concave lens produces divergence. Therefore its power is taken as negative.

The unit in which the power of a lens is measured is called a diopter (D).

$$\text{Mathematically,} \qquad \text{Power} = \frac{1}{\text{Focal length in meters}}$$

The power of a pair of lenses, of focal lengths f_1 and f_2, placed in contact is simply the sum of their individual powers.

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2$$

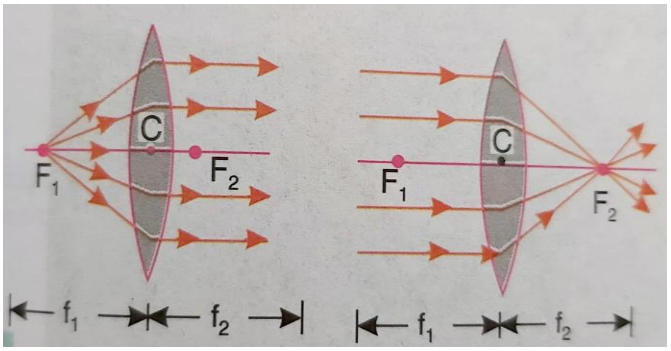
Where P_1 and P_2 are the powers of the lenses and P is the equivalent power.

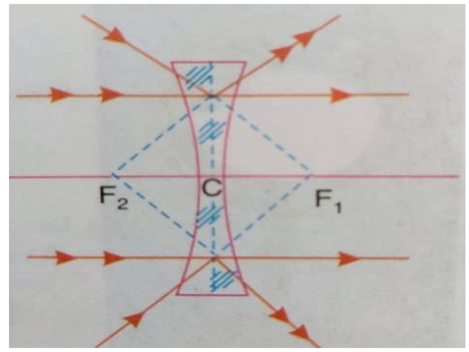
Cardinal points:-

There are six major points in an optical system, which are two principal foci, two principal points, and two nodal points called as cardinal points. The planes passing through these points and perpendicular to the principal axis are called cardinal planes.

1. **Principal foci and focal planes:-**
   Consider an optical system consisting of a thick lens or number of coaxial lenses either in contact or separated by a finite distance. In case of converging or diverging system, the points F_1 and F_2 are two principal foci and the planes

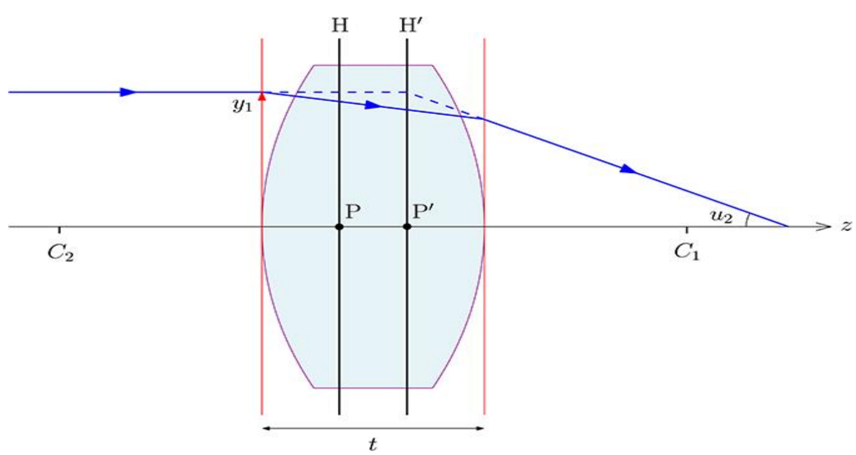
passing through them and perpendicular to the principal axis are called focal planes.





2. **Principal points and principal planes:-**

Consider a thick lens or a co-axial refracting system having its principal foci F_1 and F_2. The locus of points of intersection of the incident ray and emergent ray when they produced to each other are called principal planes. In the figure, The points P and P^I are first and second principal points which are intersection of the planes HP and H^IP^I respectively, on the principal axis.



3. **Nodal points:-**

Nodal points are defined as the pair of conjugates points on the principal axis having unit positive angular magnification. This means a ray of light directed towards one of these points, after refracting through the optical system, appears to produced from the second point in the parallel direction.

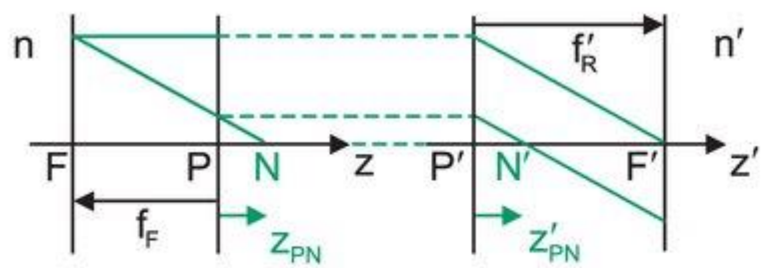
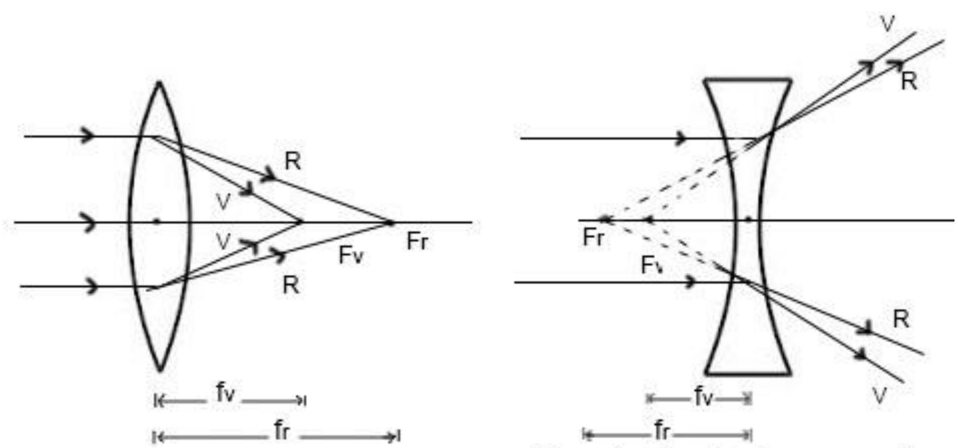


Figure shows all the cardinal points. Also PF = $-f_1$, and $P^IF^I = +f_2$. Implies that PN = P^IN^I = (f1+f2). If the medium on both sides of the system is optically similar, $-f_1 = f_2$. Hence PN = P^IN^I = 0 i.e. the nodal points are coincident with principal points. The planes passing through N and N^I are nodal planes.

Chromatic Aberration:-

The refractive index of the material of a lens is different for different wavelengths of light. Hence the focal length of a lens is different for different wavelengths. Furthermore, as the magnification of the image is dependent on the focal length of a lens, the size of the image is different for different wavelengths (colors). The variation of the image distance from the lens with refractive index measures axial or longitudinal chromatic aberration and the variation in size of the image measures lateral aberration. Thus, a single lens produces a colored image of an object illuminated by white light and this defect is called chromatic aberration.



Chromatic aberration in a convex lens. Chromatic aberration in a concave lens.

Consider a chromatic object O in the principal axis of a lens as shown in figure. The rays after refracting through the lens disperse in to the constituent colors. Let F_r be the focus for red and F_v be that for violet color with focal length f_r and f_v, $f_r - f_v$ is a measure of longitudinal chromatic aberration.

The focal length of a lens is given by;

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$or, \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f(\mu - 1)}$$

Similarly

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_v} = \frac{(\mu_v - 1)}{f(\mu - 1)} \dots\dots\dots (i)$$

And

$$\frac{1}{f_r} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_r} = \frac{(\mu_r - 1)}{f(\mu - 1)} \dots\dots\dots (ii)$$

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{(\mu_v - 1 - \mu_r + 1)}{f(\mu - 1)}$$

$$\frac{f_r - f_v}{f_v f_r} = \frac{(\mu_v - \mu_r)}{f(\mu - 1)}$$

Taking $f_v f_r = f^2$ (where f is the mean focal length), we can write

$$\frac{f_r - f_v}{f^2} = \frac{(\mu_v - \mu_r)}{f(\mu - 1)}$$

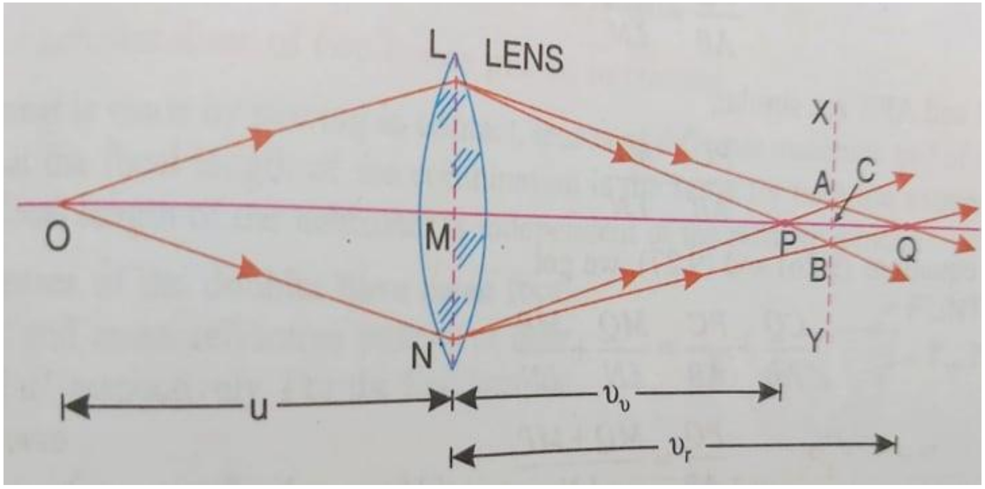
$$or, \qquad f_r - f_v = \frac{(\mu_v - \mu_r)f}{(\mu - 1)}$$

$$f_r - f_v = \omega f \dots\dots\dots (iii)$$

Where, ω

$= \dfrac{(\mu_v - \mu_r)}{(\mu - 1)}$ is known as the dispersive power of the material.

Circle of least chromatic aberration:-



Let us consider a point object illuminated by white light and situated on the axis of a lens. Colored images are formed along the axis. The violet image is nearest the lens and the red image is the farthest. In between these two images, if a screen is placed at the position XY, the image of least chromatic aberration is formed.

Let u be the distance of the object point and v_v and v_r be the distances of the violet and red images on the axis of the lens. If f_v and f_r are the focal lengths for the violet and red rays of light, then,

$$\frac{1}{v_v} - \frac{1}{u} = \frac{1}{f_v} \ldots\ldots\ldots (i)$$

And

$$\frac{1}{v_r} - \frac{1}{u} = \frac{1}{f_r} \ldots\ldots\ldots (ii)$$

Subtracting (i) from (ii), we get;

$$\frac{1}{v_v} - \frac{1}{v_r} = \frac{1}{f_v} - \frac{1}{f_r}$$

$$\frac{v_r - v_v}{v_r v_v} = \frac{f_r - f_v}{f_r f_v}$$

Taking $v_r v_v = v^2$ and $f_r f_v = f^2$

$$\frac{v_r - v_v}{v^2} = \frac{f_r - f_v}{f^2}$$

But we have $f_r - f_v = \omega f$

$$\frac{v_r - v_v}{v^2} = \frac{f_r - f_v}{f^2} = \frac{\omega}{f}$$

$$or, \qquad v_r - v_v = \frac{\omega v^2}{f} \ldots\ldots\ldots (iii)$$

It is clearly seen in this case that the longitudinal chromatic aberration depends on the distance of the image and hence on the distance of the object from the lens, in addition to its dependence on the dispersive power and focal length of the lens. Sometimes it becomes necessary to know the diameter of the circle of least chromatic aberration. It can be calculated as follows. Let O be the source of light on the axis. P is the image for violet and Q that for red rays. AB is the diameter of the circle of least chromatic aberration, d. Let LN be the aperture of the lens, D and *v* be the mean distance of the image.

Δ LQN and AQB are similar;

$$\frac{CQ}{AB} = \frac{MQ}{LN} \ldots\ldots\ldots (iv)$$

And, Δ LNP and ABP are similar;

$$\frac{PC}{AB} = \frac{MP}{LN} \dots \dots \dots (v)$$

Adding equation (iv) and (v), we get;

$$\frac{CQ}{AB} + \frac{PC}{AB} = \frac{MQ}{LN} + \frac{MP}{LN}$$

$$\frac{PQ}{AB} = \frac{MQ + MP}{LN} \dots \dots \dots . (vi)$$

But $PQ = v_r - v_v$ and $MQ + MP = v_r + v_v = 2v$

And substituting these values in equation (vi), we get;

$$\frac{v_r - v_v}{d} = \frac{2v}{D} \quad (\because AB = d \text{ and } LN = D)$$

$$\therefore d = D.\frac{v_r - v_v}{2v}$$

But we have from equation (iii), $v_r - v_v = \frac{\omega v^2}{f}$

$$d = D.\frac{\omega v^2}{f.2v} = \frac{1}{2} D\omega.\frac{v}{f}$$

If incident light is a parallel beam,

$$i.e. v = f$$

Then

$$d = \frac{1}{2} D\omega$$

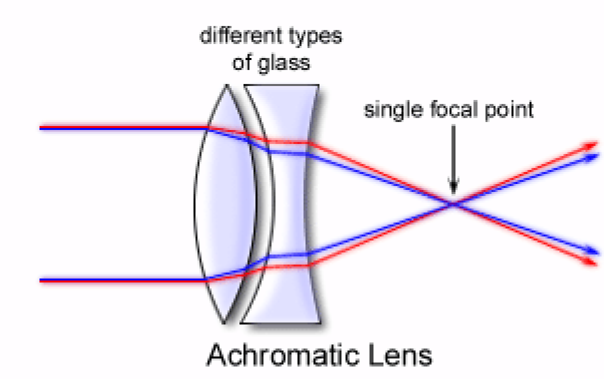
In this case, diameter of the circle of least chromatic aberration depends on the diameter of the lens aperture and the dispersive power of the material, but is independent of the focal length of the lens.

Achromatism:-

A single lens produces a colored image of an object illuminated by white light and this defect is called chromatic aberration. Elimination of this defect in a system of lenses is called achromatism. The lenses used for achromatism, in general, have different indices of refraction. Normally one lens is converging and other is diverging. In many cases they are cemented together, such type of combination is called an achromatic doublet or color corrected lens.

1. **Condition for Achromatism of two lenses placed in contact:-**

   An achromat is made by placing in contact, lenses of different materials and suitable focal lengths, such that the focal length of the combination is the same for both the extreme colors. It means that the focal length of the achromat is independent of the refractive index.



Let the lenses of the doublet have mean focal lengths f and f ´ and mean refractive indices of their materials μ and μ´ respectively.

We know the Lens maker's formula for a lens is;

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Differentiating above equation, we get

$$-\frac{df}{f^2} = d\mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$or, \quad -\frac{df}{f^2} = \frac{d\mu}{(\mu - 1)f}$$

If μ_v and μ_r are the refractive indices for the violet and red rays, then $d\mu$ is the change in refractive index $(\mu_v - \mu_r)$.

Hence, dispersive power $(\omega) = \frac{d\mu}{(\mu-1)}$

$$\therefore \; -\frac{df}{f^2} = \frac{\omega}{f}$$

When two lenses of focal lengths f and f ´ are placed in contact to form an achromatic combination, then focal length F of the combination is given by,

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'}$$

Upon differentiating,

$$-\frac{dF}{F^2} = -\frac{df}{f^2} - \frac{df'}{f'^2}$$

To bring different colors in to focus at a point, dF, the change in focal length of the combination should be zero.

$$or, \qquad \frac{df}{f^2} + \frac{df'}{f'^2} = 0$$

$$or, \qquad \frac{\omega}{f} + \frac{\omega'}{f'} = 0$$

Where ω and ω' are the dispersive power of the materials of the two lenses and are positive quantity. Which gives condition for achromatism.

2. Condition for achromatism of two lenses separated by a finite distance:-

If the two lenses are of the same material, their achromatic combination is possible only if they are separated by a finite distance.

Let f_1 and f_2 be the focal lengths of two lenses placed coaxial and separated by a distance x. The mean focal length F of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2} \ldots\ldots\ldots.(i)$$

Differentiating the above equation, we obtain

$$-\frac{dF}{F^2} = -\frac{df_1}{{f_1}^2} - \frac{df_2}{{f_2}^2} - x\left(\frac{df_1}{{f_1}^2 f_2} - \frac{df_2}{{f_2}^2 f_1}\right) \ldots\ldots\ldots(ii)$$

But for achromatism;

$$or, \qquad dF = 0$$

Also for lenses of same materials;

$$-\frac{df_1}{f_1} = -\frac{df_2}{f_2} = \omega$$

Where, ω is the dispersie power of the lens material.

Now equation (ii) reduces to;

$$\frac{\omega}{f_1} + \frac{\omega}{f_2} - x\left(-\frac{\omega}{f_1 f_2} - \frac{\omega}{f_1 f_2}\right) = 0$$

$$or, \qquad \frac{1}{f_1} + \frac{1}{f_2} = \frac{2x}{f_1 f_2}$$

$$or, \qquad x = \frac{f_1 + f_2}{2}$$

Thus, the condition for achromatism of two thin co-axial lenses made of same material and separated by a distance is that the distance between the two lenses must be equal to the mean focal length of the two lenses.

Numerical Examples:-

1. **Two thin convex lenses having focal lengths 10 cm and 4 cm are coaxially separated by a distance 5 cm, find the equivalent focal length of the combination. Determine also the position of the principal points.**

Solution:-

Given: $f_1 = 10\ cm \qquad f_2 = 4\ cm \quad d = 5\ cm$

We know,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$or, \qquad \frac{1}{f} = \frac{1}{10} + \frac{1}{4} - \frac{5}{10 \times 4}$$

$$or, \qquad \frac{1}{f} = \frac{9}{40}$$

$$\therefore\ f = 4.4\ cm$$

Again,

$$\alpha = \frac{df}{f_2} = \frac{5 \times 4.4}{4} = 5.5\ cm$$

$$\beta = -\frac{df}{f_1} = -\frac{5 \times 4.4}{10} = -2.2\ cm$$

The first principal point is at 5.5 cm from first lens on its right and second principal point is at 2.2 cm from second lens on its left.

2. **Two thin lenses of focal lengths 8 cm each are identical and coaxially separated by 4 cm. Determine the equivalent focal length of this lens combination and illustrate the principal points in figure. If the image is formed at infinity at a particular position of the object, find the object distance.**

Solution:-

Given: $f_1 = f_2 = 8\ cm \quad d = 4\ cm$

We know,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$or, \qquad \frac{1}{f} = \frac{1}{8} + \frac{1}{8} - \frac{4}{8 \times 8}$$

$$\therefore f = \frac{64}{12} = 5.33\ cm$$

And,

$$\alpha = \frac{df}{f_2} = \frac{4 \times 5.33}{8} = 2.67\ cm$$

$$\beta = -\frac{df}{f_1} = -\frac{4 \times 5.33}{8} = -2.67\ cm$$

Again for the image to be formed at infinity, replace $v = \infty, u = ?$ and $f = 5.33\ cm$ in the following equation.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$or, \qquad -\frac{1}{u} = \frac{1}{5.33}$$

$$or, \qquad f = u = -5.33\ cm$$

This is the distance of object from the first principal point in the left side of the object is at 2.66 cm away on the left side of first lens or, 5.33 cm on the left side of first principal point.

3. **Dispersive powers of two different glass types are 0.0163 and 0.0243 respectively. How can you design an achromatic contact doublet of focal length 50 cm.**

Solution:-

For achromatic doublet,

$$\frac{\omega}{f} + \frac{\omega'}{f'} = 0$$

$$or, \qquad \frac{0.0163}{f} + \frac{0.0243}{f'} = 0 \dots\dots\dots (i)$$

Also,

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'}$$

$$or, \qquad \frac{1}{f} + \frac{1}{f'} = \frac{1}{50} \dots\dots\dots (ii)$$

Solving these two equations, we get,

$$f = \frac{4000}{243} = 16.5\ cm$$

$$f' = \frac{4000}{-163} = -24.5\ cm$$

So we can form an achromat by cementing two lenses of focal lengths 16.5 cm and 24.5 cm of those two materials. Also one of the lenses will be a convex lens and other a concave lens.

4. **Two thin converging lenses of focal lengths 3 cm and 4 cm are placed coaxially in air separated by distance of 2 cm. An object is placed 4 cm in front of the first lens. Find the position and nature of the images.**

Solution:-

Given: $f_1 = 3\ cm$ $f_2 = 4\ cm$ $d = 2\ cm$

We know,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$or, \qquad \frac{1}{f} = \frac{1}{3} + \frac{1}{4} - \frac{2}{3 \times 4}$$

$$\therefore f = 2.4\ cm$$

Distance of the first principal point from the first lens,

$$\alpha = \frac{df}{f_2} = \frac{2.4 \times 2}{4} = 1.2\ cm; \text{towards right}$$

Distance of the second principal point from the second lens,

$$\beta = \frac{df}{f_1} = \frac{2 \times 2.4}{3} = 1.6\ cm \text{ towards left}$$

Object distance $u = -(4 + 1.2) = -5.2\ cm$

Using the formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

We have,

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{-5.2} + \frac{1}{2.4}$$

$$or, \qquad v = 4.46\ cm$$

Distance of the image from the second lens = 4.46 – 1.6 = 2.86 cm. The image is real.

5. **The object glass of a telescope is an achromat of focal length 90 cm, If the magnitude of the dispersive power of the two lenses are 0.024 and 0.036, Calculate their focal lengths.**

Solution:-

Here, $\omega_1 = 0.024,\ \omega_2 = 0.036,\ F = 90\ cm$

We have, for achromatism,

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$or, \qquad \frac{1}{f_2} = -\frac{\omega_1}{\omega_2 f_1} = -\frac{0.024}{0.036 f_1} = -\frac{2}{3f_1}$$

Also,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$or, \qquad \frac{1}{90} = \frac{1}{f_1} - \frac{2}{3f_1} = \frac{1}{3f_1}$$

$$\therefore f_1 = 30\ cm \text{ and } f_2 = -\frac{3f_1}{2} = -45\ cm$$

6. **A converging achromat of 40 cm focal length is to be constructed out of a thin crown glass lens and a thin flint glass lens, The surface in contact having a common radius of curvature of 25 cm. Calculate the radius of curvature of second surface of each lens, given that the values of the dispersive power and mean refractive indices are 0.017 and 1.5 for crown glass, and 0.034 and 1.7 for flint glass.**

Solution:-

Here, $\omega_1 = 0.017,\ \omega_2 = 0.034,\ F = 40\ cm\ \ \mu_1 = 1.5,$ $\mu_2 = 1.7.$

We have, for achromatism,

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$or, \qquad \frac{1}{f_2} = -\frac{\omega_1}{\omega_2 f_1} = -\frac{1}{2f_1}$$

Also,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$or, \qquad \frac{1}{40} = \frac{1}{f_1} - \frac{1}{2f_1}$$

$$\therefore f_1 = 20\ cm \text{ and } f_2 = -2f_1 = -40\ cm$$

For the crown glass lens,

$$\frac{1}{\mathrm{f}_1} = (\mu_1 - 1)\left(\frac{1}{\mathrm{R}_1} - \frac{1}{\mathrm{R}_2}\right)$$

$$or, \quad \frac{1}{20} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{25}\right)$$

$$\therefore \; R_1 = \frac{50}{3} = 16.67\ cm$$

For the flint glass lens,

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\text{or,} \quad \frac{1}{40} = (1.7 - 1)\left(-\frac{1}{25} - \frac{1}{R_2}\right)$$

$$\therefore R_2 = -233.33\ cm$$

For crown glass lens,

$$R_1 = 16.67\ cm, \qquad R_2 = -25\ cm$$

For flint glass lens,

$$R`_1 = -25\ cm, \qquad R`_2 = -233.33\ cm$$

Exercise:-

1. Calculate the focal length of combination of two thin lenses of focal length f_1 and f_2 separated by a distance 'd'. Find the position of two principal points.
2. What is chromatic aberration? Show that a single lens is always accompanied with such aberration. Discuss in brief how can we minimize chromatic aberration in the combination of lenses.
3. Show that the diameter of circle of least confusion depends on the diameter of the lens aperture and dispersive power of the material of the lens but is independent of the focal length of the lens.
4. What are cardinal points of an optical system? Determine the equivalent focal length of a combination of two thin lenses separated by a finite distance.
5. Prove that the condition for achromatism for the combination of two lenses of focal length f_1 and f_2 having dispersive power ω_1 and ω_2 placed at a separate distance 'x'is $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = \left(\frac{x}{f_1 f_2}\right)(\omega_1 + \omega_2)$.
6. What is chromatic aberration? Show that longitudinal chromatic aberration is equal to (i) $\omega \times f$, when object is infinite and (ii) $\frac{\omega \times v^2}{f}$, when object is at finite. Where symbols have their usual meaning.
7. What are co-axial optical system and cardinal points? State their properties and show their position in a diagram.

Illustrate the use of these points in the formation of an image by a lens system.

8. Show that the least possible distance between an object and its real image in a convex lens is four times the focal length of the lens.
9. Two thin lenses of power P_1 and P_2 are separated by a distance d. Find an expression to show that equivalent power of the combination is given as $P = P_1 + P_2 - dP_1P_2$.
10. Define circle of least confusion and show that $d = \frac{1}{2}\omega D$, where d is diameter if circle of least confusion, ω is dispersive power and D is diameter of lens.
11. Two thin converging lenses of focal lengths 30 cm and 40 cm respectively are placed co-axially in air separated by a distance of 20 cm. An object is placed 40 cm in front of the first lens. Find the position and nature of the image.
12. In Ramsden's eyepiece a co-axially lens system is used. There are two lenses in air and are of equal focal length of separated by a distance 2f/3. Find position of the cardinal points.
13. Two thin lenses of focal lengths f_1 and f_2 separated by a distance having an equivalent focal length 50 cm. The combination satisfies the condition for no chromatic aberration and minimum spherical aberration. Find the separation between the two lenses if both lenses are same materials.

14. Two lenses of focal lengths 8 cm and 4 cm are placed at a certain distance apart. Calculate the position of principal points if they form an achromatic combination.

15. It is desire to make a converging achromatic lens of mean focal length 30 cm by using two lenses of materials A and B. If the dispersive power of A and B are in the ratio of 1:2, find the focal length of lens.

16. Two thin converging lenses of focal lengths 0.2 m and 0.3 m are placed co-axially 0.10 m apart in air. An object is located 0.6 m in front of the lens of smaller focal length. Find the position of two principal points and that of image.