Chapter-3
Basic Methodologies of Engineering Economics Analysis.

MARR (Minimum Attractive Rate of Return)

A minimum return the company'll accept on the money it invest is called MARR.

* Factors influencing the determination of MARR

1. The amount of fund available for investment.
2. The nature of investment alternatives.
3. Types of organisation involved.

I. Equivalent worth method:

i) Present Worth method.

Decision: PW > 0 (Accept)

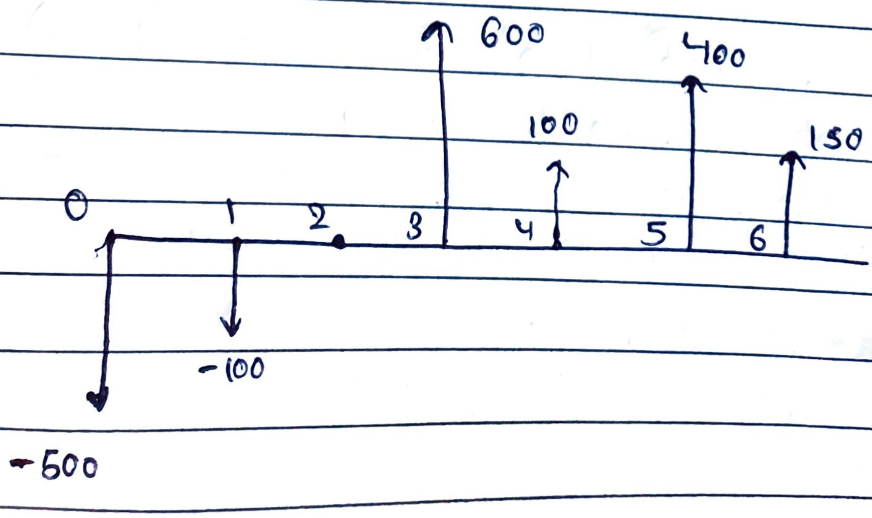
PW = 0 (Remain indifferent)

PW < 0 (Reject)

Q. Find the net present value (NPV) of the cash ~~received~~ receipts shown below when effective interest is 10% compounded anually:

EOY	0	1	2	3	4	5	6
Cash Flow	-500	-100	0	600	100	400	150

Here,



EOY	Cash-flow	Present $P = F(1+i)^{-n}$
0	-500	-500
1	-100	-90.90
2	0	~~545.45~~ 0
3	600	~~495.87~~ 450.78
4	100	68.30
5	400	248.36
6	150	84.67

∴ Net present value = 261.21 . >0. ~~So, the p~~

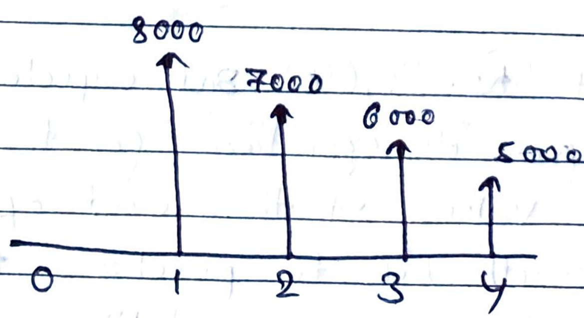
ii) Future worth method:

Decision : FW > 0 (Accept)
FW = 0 (Remain indifferent)
FW < 0 (Reject)

Q. Calculate the equivalent future worth of the project having the following information of cash flow. Take MARR = 10%.

EOY	1	2	3	4
Cash flow	8000	7000	6000	5000

Here,



$F = P(1+i)^n$

EOY	Cash flow	Future value
1	8000	10648
2	7000	8470
3	6000	~~7200~~ 6600
4	5000	5000

$\therefore$ Equivalent future worth = 30718.

ii) Annual Worth method.

Decision: AW > 0 (Accept)

AW = 0 (Remain indifferent)

AW < 0 (Reject)

$$AW = F(A/F, i\%, n) = F\left[\frac{i}{(1+i)^n - 1}\right]$$

$$AW = P(A/P, i\%, n) = P\left[\frac{(1+i)^n \times i}{(1+i)^n - 1}\right]$$

Capital Recovery:

$$CR = I\left(\frac{A}{P}, i\%, n\right) - S\left(\frac{A}{F}, i\%, n\right)$$

Investment

2. Consider a machine cost Rs. 20,000 and expected to earn Rs. 50,000 per year from its operation for the period of 5 years. The salvage value at the end of 5th year is Rs 10,000. Is it worthwhile to purchase the machine? Justify by using annual worth formulation. (Take MARR = 10%) Also calculate the CR cost of this machine.

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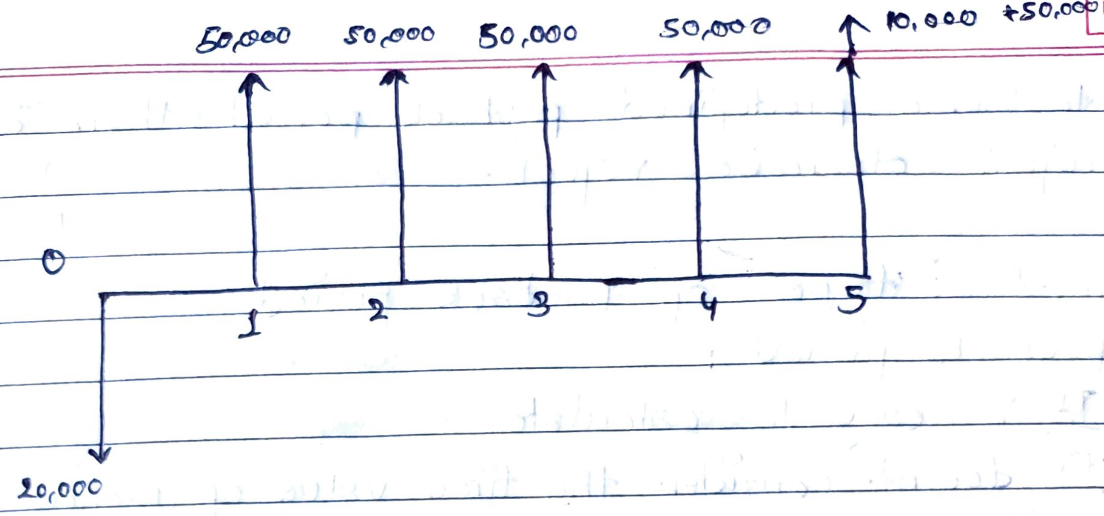

$$AW = -20{,}000\left(\frac{A}{P}, i\%, n\right) + 50{,}000 + 10{,}000\left(\frac{A}{F}, i\%, n\right)$$

$$= -20000\left[\frac{(1.1)^5 \times 0.1}{1.1^5 - 1}\right] + 50000 + 10000\left(\frac{0.1}{(1.1)^5 - 1}\right)$$

$$= -5275.94 + 50000 + 1637.97$$

$$= 46362.03$$

Since, $AW > 0$, it is worthwhile to purchase the machine

$$CR = I\left(\frac{A}{P}, i\%, n\right) - S\left(\frac{A}{F}, i\%, n\right)$$

$$= 20{,}000\left(\frac{1.1^5 \times 0.1}{1.1^5 - 1}\right) - 10000\left(\frac{0.1}{1.1^5 - 1}\right)$$

$$= 5275.94 - 1637.97$$

$$= 3637.97$$

II. Payback period method.

It is the length of time in which an investment pays back its original cost.

Rule:

If the calculated payback period is less than or equal to some predefined payback period then accept the project otherwise reject.

There are two types of payback period

(a) Simple payback period :

- i. It is easy to calculate.
- ii. It doesn't consider the time value of money.
- iii. It is interpreted in year.

Simple payback period = $\frac{\text{Initial Investment}}{\text{Annual saving}}$

Q. Calculate the simple payback period for the given cash flow of the project

Period	0	1	2	3	4	5
~~Net Cashflow~~	-25,000	8,000	8,000	8,000	8,000	13,000

EOY	Net cash flow	Cummulative cash flow
0	-25,000	-25,000
1	8000	-17,000
2	8000	- 9000
3	8000	-1000
4	8000	7000 > 0
5	13000	

Payback period $= 3 + \frac{1000}{8000}$

$= 3.125$

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(b) Discounted payback period:

- It is the time period that the project takes ~~place~~ to pay back its initial investment taking the time value of money in account.
- By this time you have discounted the cash flow and calculated the NPV.

Q. A company has decided to purchase a new machine cost of Rs. 10,000 & is likely to bring in after tax cash inflow of Rs. 4,000 in the 1st year, Rs. 4,500 in 2nd year, Rs. 10,000 in 3rd year & Rs. 8,000 in 4th year. The company has policy of buying the equipment only if the payback period is 3 years or less. Calculate the ~~Here,~~ discounted payback period of the machine by using the discount rate of 10%.

EOY	Net Cash Flow	Net Present Worth (NPV)	Cummulative cash flo
0	-10,000	-10,000	-10,000
1	4,000	3636.36	-6363.64
2	4500	3719.00	-2644.64
3	10,000	7513.14	4868.5
4	8000	5464.10	—

Here, Payback period = $2 + \frac{2644.64}{7513.14}$

$= 2.35$

Here, Calculated payback period is less than predefined payback period. So, the project is accepted

III. Rate of Return Method:

2) ~~Internal Rate of Return (IRR):~~

Rate of Return (ROR):

It is defined as the interest rate ~~on~~ earned on the unpaid balance of an ~~investment~~ installment loan.

a) Internal Rate of Return (IRR):

It is the rate of interest earned on the unrecovered balance of the investment. IRR is that interest than makes the NPW 0.

Advantage:

- Easy to understand & communicate.

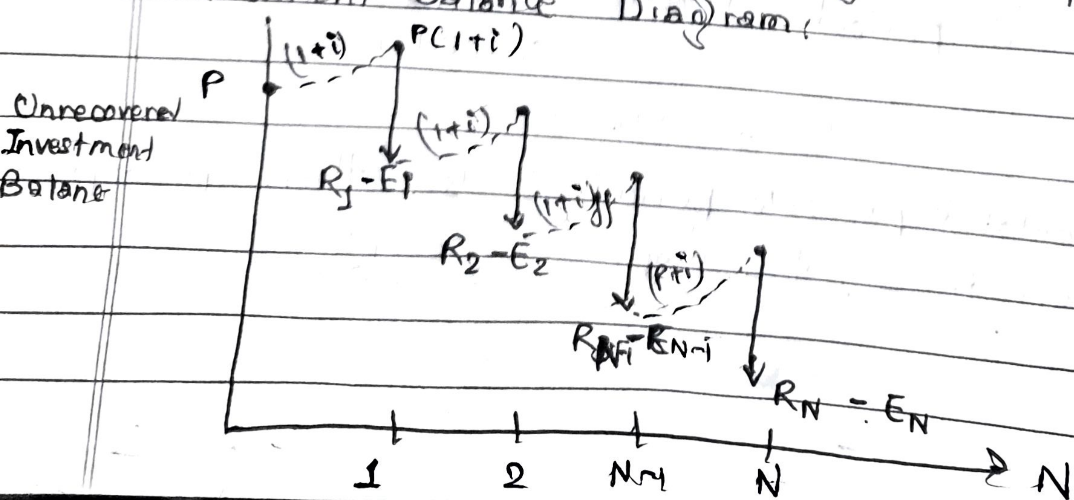
Q. Write any two disadvantage of IRR

- IRR may not exist or there may be multiple IRR.
- Doesn't distinguish between investing or borrowing

Decision Rule:

IRR > MARR : Accept the project
IRR = MARR : Remains indifferent
IRR < MARR : Reject the project

Investment Balance Diagram:



i) Direct Solution Method:

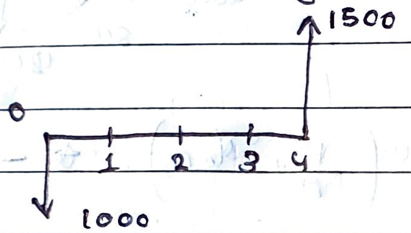
Example:

Period	0	1	2	3	4
Project 1	-1000	0	0	0	1500
Project 2	-2000	1300	1500	0	0

Calculate of the IRR of these two project.

→ For Project 1

Cash-Flow diagram.



$PW = -1000 + F\left(\frac{P}{F}, i\%, n\right)$

$0 = -1000 + 1500(1+i)^{-4}$

or $\frac{1000}{1500} = (1+i)^{-4}$

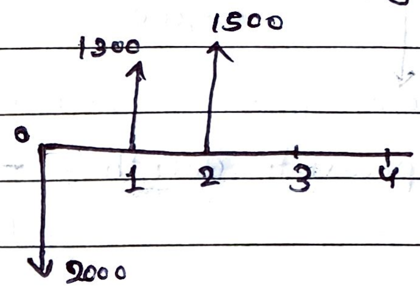
or, $(1+i)^4 = 1.5$

or $1+i = 1.106$

or $i = 0.1066 = 10.66\%$.

For Project 2

Cash Flow diagram



$PW = -2000 + 1300(1+i)^{-1} + 1500(1+i)^{-2}$

$0 = -2000 + 1300(1+i)^{-1} + 1500(1+i)^{-2}$

or $i = 0.25$

$= 25\%$

ii) Trial and Error method:

Q. Initial investment = Rs. 1,00,000

Salvage value = 0

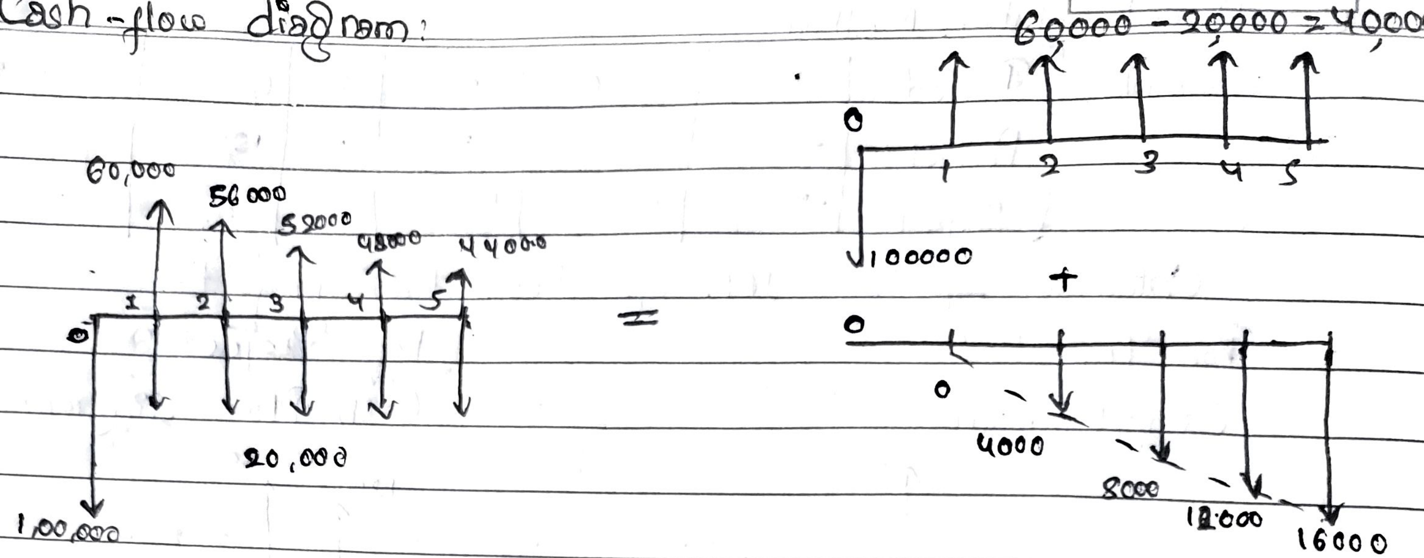
Annual Operation and Maintaince Cost = Rs. 20,000

Useful life = 5 years.

Annual benefit, Rs 60,000 at the end of 1st year thereafter decreases by Rs. 4000 each year for the remaining years.

MARR = 12%. Find the IRR and also draw VIB diagram.

Cash-flow diagram:



Net present worth $= -100000 + 40000\left(\frac{P}{A}, i\%, n\right) -$

$$\frac{G}{i^2}\left[\frac{(1+i)^n - 1 - ni}{(1+i)^n}\right]$$

$$0 = -100000 + 40000\left[\frac{(1+i)^n - 1}{(1+i)^n \times i}\right] - \frac{4000}{i^2}\left[\frac{(1+i)^5 - 1 - 5i}{(1+i)^5}\right]$$

$$0 = -100000 + 40000\left[\frac{(1+i)^5 - 1}{(1+i)^5 \times i}\right] - \frac{4000}{i^2}\left[\frac{(1+i)^5 - 1 - 5i}{(1+i)^5}\right]$$

Suppose, $i = 0.15$, $PW = 10985.63$

$i = 0.25$, $PW = -9242.88$

Now,

Using linear Interpolation,

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

or, $$\frac{-9242.88 - 10985.63}{0.25 - 0.15} = \frac{0 - 10985.63}{i - 0.15}$$

or $$\frac{-202285.1}{-10985.63} = \frac{1}{i - 0.15}$$

or, $$18.41 = \frac{1}{i - 0.15}$$

or $i - 0.15 = \frac{1}{18.41}$ $\therefore i = 0.20 = 20\%$

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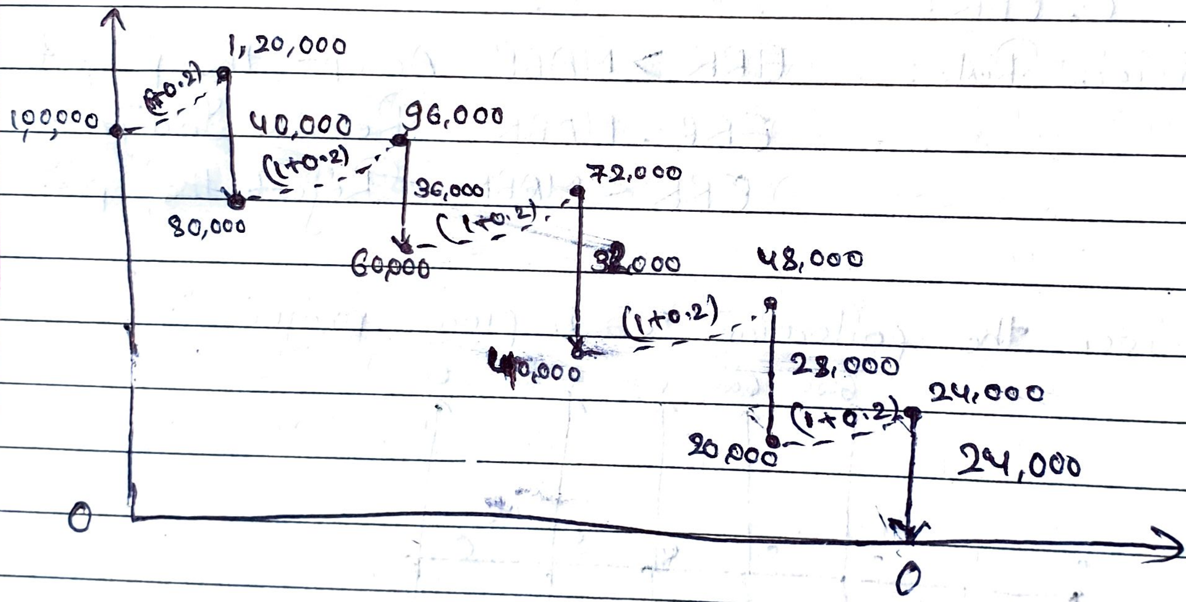
$$i = \frac{1}{18.41} + 0.15 = 0.20 = 20\%.$$

EOY	Unpaid Balance	Return of Unpaid	Payment Received	Unpaid balance at the end of year
0	-1,00,000	-	-	-1,00,000
1	-1,00,000	-20,000	40000	-80000
2	-80,000	-16,000	36000	-60,000
3	-60,000	-12,000	32,000	-40,000
4	-40,000	-8,000	28,000	-20,000
5	-20,000	-4,000	24,000	0.



~~Benifit Cost Ratio: Method~~

5.) External Rate of Return (ERR) : / Modified ROR

It equates the future worth of positive cash flow using MARR to the future worth of negative cash flow using ERR.

<u>Step 1:</u> Discounting all the cash outflow to the present at E% ~~for~~ per compounding period

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Step 2: Compounding all the cash inflow to the future at E %.

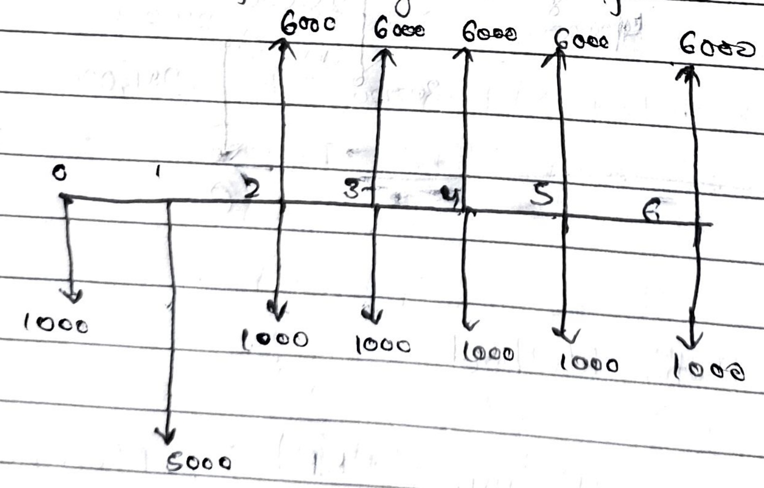
Step 3: ERR is the interest rate that equivalents be two quantities.

$$\sum_{t=0}^{n} R_t (1+r)^{n-t} = \sum_{t=0}^{n} C_t (1+i)^{n-t}$$

R_t = positive value
C_t = negative value
r = MARR
i = ERR.

Decision Rule:
ERR > MARR Accept the project
ERR = MARR Remains Indifferent
ERR < MARR Reject the project

Q. Consider the following cash flow project :



Calculate the ERR of the project if MARR = 20% and reinvestment rate (E) = 15% . Is the project accepted?

Step 1: PW = $-10{,}000 - 1000\left(\frac{P}{A}, i\%, n\right) - 4000\left(\frac{P}{F}, i\%, n\right)$

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$$PW = -1000 - 1000\left[\frac{(1+i)^n - 1}{(1+i)^n \times i}\right] - 4000\,(1+i)^{-n}$$

$$= -1000 - 1000\left[\frac{1.15^6 - 1}{1.15^6 \times 0.15}\right] - 4000 \times 1.15^{-1}$$

$$= -8262.74$$

Step 2:

$$FW = 6000\left(\frac{F}{A}, i\%, n\right)$$

$$= 6000\left(\frac{(1+i)^n - 1}{i}\right)$$

$$= 6000\left(\frac{1.15^5 - 1}{0.15}\right)$$

$$= 40454.2875$$

Step 3:

$$F = P(1+i)^n$$

or, $40454.2875 = 8262.74(1+i)^6$

or, $4.89 = (1+i)^6$

or $1.302 = 1+i$

$i = 0.302$

$= 30.2\%$

Since, ERR > MARR, Accept the project.

Benefit Cost Ratio Method: [ADT-2]

It is defined as the ratio of equivalent worth of benefit to the equivalent worth of cost.

Types of BC Ratio:

a) Conventional Benefit Cost Ratio method
b) Modified BCR method

(a) Conventional BCR with PW formulation:

$$B/C \text{ Ratio} = \frac{PW(B)}{PW(I) - PW(R) + PW(O\&M)}$$

≥ 1 : Justified
< 1 : Not

(b) Modified B/C ratio:

$$B/C = \frac{PW(B) - PW(O\&M)}{PW(I) - PW(R)}$$

Q. Determine both type of B/C ratio from the following cash flow: Initial investment = 3,00,000
Annual revenue = 85,000
Annual Cost = 15,000
Salvage value = 20% of Initial Investment
Useful life = 6 years.
MARR = 10%

Use PW approach:

Here,

$$PW(B) = 85,000 \left(\frac{P}{A}, i\%, n\right)$$

$$= 85000 \times \left[\frac{(1+i)^n - 1}{(1+i)^n \times i}\right]$$

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$$= 85000 \times \left[\frac{(1+0.1)^6 - 1}{1.1^6 \times 0.1} \right]$$

$$= 3,70,197.15$$

$$PW(O\&M) = 15000 \left[\frac{(1+0.1)^6 - 1}{1.1^6 \times 0.1} \right]$$

$$= 65328.91$$

$$PW(F) = (20\% \times 300000) \times (1+i)^{-n}$$

$$= 60000 \times 1.1^{-6}$$

$$= 33868.43$$

$$PW(I) = 3,00,000$$

(a) Conventional BCR:

$$B/C = \frac{PW(B)}{PW(I) - PW(F) + PW(O\&M)}$$

$$= 1.11 > 1$$

(b) Modified B/C

$$B/C = \frac{POW(B) - PW(O\&M)}{PW(I) - PW(F)}$$

$$= 1.145 > 1$$

~~Base~~ : BCR with FW formulation

(a) Conventional B/C

$$B/C \text{ Ratio} = \frac{FW(B)}{FW(I) - SV + FW(O\&M)}$$

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(b) Modified B/C Ratio:

$$B/C = \frac{FW(B) - FW(O\&M)}{FW(I) - SV}$$

Q. Calculate both type of B/C Ratio using FW approach taking useful life = 10 years. Initial investment = Rs. 6,00[illegible] Annual benefit: 3000 ; Operating and Maintaince cost = Rs. 1,000. Salvage value = Rs. 15,000, MARR = 10%

Here, SV = 1500

$FW(I) = 6000(1+0.1)^{10}$

$= \cancel{10629}\ \cancel{10629.36}\ 15562.45$

$FW(B) = \cancel{3000\left[\frac{1.1^{10}-1}{1.1}\right]}\ 3000\left[\frac{1.1^{10}-1}{0.1}\right]$

$= 47812.27$

$FW(O\&M) = 1000 \times \left[\frac{1.1^{10}-1}{0.1}\right] = 15937.42$

(a) Conventional B/C Ratio $= \frac{FW(B)}{FW(I) - SV + FW(O\&C)}$

$= 1.59 > 1$; Justified

(b) Modified B/C Ratio $= \frac{FW(B) - FW(O\&M)}{FW(I) - SV}$

$= 2.266 > 1$; Justified.

B/C Ratio with AW Formulation:

Conventional B/C Ratio $= \frac{AW(B)}{CR + AW(O\&M)}$

Modified B/C Ratio $= \frac{AW(B) - AW(O\&M)}{CR}$

$CR = I\left(\frac{A}{P}, i\%, n\right) - S\left(\frac{A}{F}, i\%, n\right)$

Q. Find the benefit Cost Ratio by using AW approach where Investment : Rs. 9,00,000.

Annual Revenue : Rs. 5,00,000

Annual Cost : Rs. 2,000

Salvage value : Rs. 20,000

MARR = 12%

Useful life = 10 years.

Here,

AW (B) = Rs. 5,00,000

AW (O&M) = Rs. 2,000

$$CR = I\left(\frac{A}{P}, i\%, n\right) - S\left(\frac{A}{F}, i\%, n\right)$$

$$= 900000\left(\frac{1.12^{10} \times 0.12}{1.12^{10} - 1}\right) - 20000\left(\frac{0.12}{1.12^{10} - 1}\right)$$

$$= 158146.06$$

Now,

$$\text{Convention B/C Ratio} = \frac{AW(B)}{CR + AW(O\&M)} = \frac{500000}{158146.06 + 2000}$$

$$= 3.12 > 1$$

$$\text{Modified B/C Ratio} = \frac{AW(B) - AW(O\&M)}{CR}$$

$$= 3.14 > 1$$

Justified. //