Assignment - I.

Q1. Construct the truth table for each of the following compound proposition:

1. $(p \leftrightarrow q) \vee (\neg p \rightarrow \neg r)$

Truth table is:

p	q	r	$\neg p$	$\neg r$	$p \leftrightarrow q$	$\neg q \rightarrow \neg r$	$(p \leftrightarrow q) \vee (\neg p \rightarrow \neg r)$
T	T	T	F	F	T	T	T
T	F	F	F	T	F	T	T
F	T	T	T	F	F	F	F
F	F	F	T	T	T	T	T

2. $(p \vee \neg q)' \rightarrow q$

Truth table is:

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q)'$	$(p \vee \neg q)' \rightarrow q$
F	F	T	T	F	T
F	T	F	F	T	T
T	F	T	T	F	T
T	T	F	T	F	T

Q 2. State the converse, contrapositive and inverse of each of these conditional statements.

a) If it snows at tonight then I will stay home.

let p denote " it snows at tonight"

q denote " I will stay home".

The above statement can be expressed as:

$p \rightarrow q$ then:

Converse:

$q \rightarrow p$

If I will stay home then it snows at tonight.

Contrapositive:

$\neg p \rightarrow \neg q$

If it doesnot snows at tonight then I will not stay home.

Inverse:

$\neg q \rightarrow \neg p$

If I will not stay hom then it doesnot snow at tonight.

b)

b) Indian team wins whenever match is played in kolkata, home town of Ganguly.

let P : "Match is played in kolkata, home town of Ganguly"

Q : "Indian team wins"

The above statement can be expressed as :

Q whenever P

i.e $P \to Q$

Converse:

$Q \to P$

If Indian team wins than match is played in kolkata, home town of Ganguly.

Contropositive:

$\neg P \to \neg Q$

If match is not played in kolkata, home town of Ganguly, then indian team doesnot win.

Inverse:

$\neg Q \to \neg P$

If indian team doesnot win then match is not played in kolkata, home town of Ganguly.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

let
P : " A positive integer is a prime"
Q : " It has no divisors other than 1 and itself".
The above statement can be expressed as:

$P \rightarrow Q$

Converse:

$Q \rightarrow P$

If it has no divisors other than 1 and itself then a positive integer is a prime.

Contrapositive:

$\neg P \rightarrow \neg Q$

If a positive integer is not a prime then it has divisors other than 1 and itself.

Inverse:

$\neg Q \rightarrow \neg P$

If it has divisors other than 1 and itself then ~~it is~~ a positive integer is not a prime.

d) If flood destroys my house or the fire destroys my house then my insurance company will pay me.

Let:

P: "Flood destroys my house"

Q: "Fire destroys my house"

R: "My insurance company will pay me"

The above statement can be written as:

$$(P \vee Q) \rightarrow R$$

Converse

$$R \rightarrow (P \vee Q)$$

If my insurance company will pay me then flood destroys my house or fire destroys my house.

Contrapositive

$$\neg(P \vee Q) \rightarrow \neg R \Rightarrow (\neg P \wedge \neg Q) \rightarrow \neg R$$

If flood doesnot destroy my house and fire doesnot destroy my house then my insurance company willnot pay me.

Inverse

$$\neg R \rightarrow (\neg P \wedge \neg Q)$$

If my insurance company willnot pay me then flood doesnot destroy my house and fire doesnot destroy my house.

Q3. Show that the following argument is valid. "If today is Tuesday, I have a test in Mathematics or Economics. If my economics professor is sick, I will not have a test in Economics. Today is Tuesday and my Economics professor is sick. Therefore I have a test in Mathematics" by using rules of inference."

Let: P : "Today is Tuesday"

Q : " I have test in Mathematics"

R : " I have test in Economics"

S : " My economics professor is sick"

Hypothesis	Conclusion
$P \to (Q \vee R)$	Q
$S \to \neg R$	
$P \wedge S$	

	Statement	Reasons
i)	$P \wedge S$	Hypothesis
ii)	P	Using simplification on ①
iii)	$P \to (Q \vee R)$	Using hypothesis
iv)	$Q \vee R$	Using ⓘⓘ, ⓘⓘⓘ on modus Ponens.
v)	S	Using simplification on ①
vi)	$S \to \neg R$	Using hypothesis.

vi) ¬R Using modus ponens on (v), (vi)

vii) Q Using Disjunctive syllogism on (iv), (vi).

It is valid. #

Q4. For the set of premises "If I play hockey then I am sore the next day". "I use the whirlpool if I am sore". "I didnot use the whirlpool". What relavant condusion can be draw? Explain the rules of inference used to draw the conclusion.

let P : I play hockey.

Q : I am sore the next day.

R : I use the whirlpool.

Hypothesis.

$P \rightarrow Q$

$Q \rightarrow R$

$\neg R$

	Statement	Reasons.
i)	$Q \rightarrow R$	hypothesis
ii)	$\neg R$	hypothesis.
iii)	$P \rightarrow Q$	hypothesis
iv)	$P \rightarrow R$	Using hypothesis syllogism on (i), (i)

v) $\neg P$ using Modus Tollens on (iv), (ii)

$\therefore$ conclusion is $\neg P$
ie " I didnot play hockey".

Q5. Use the rules of inference to prove the following. If a baby is hungry, then the baby cries. If the baby is not mad then he doesnot cry. If a baby is mad then he has a red face. Therefore if a baby is hungry then he has a red face.

Let P: A baby is hungry.
Q: The baby cries.
R: The baby is mad.
S: He has a red face.

Hypothesis

$P \rightarrow Q$

$\neg R \rightarrow \neg Q$

$R \rightarrow S$

Conclusion

$P \rightarrow S$

	Statement	Reasons.
i)	$P \to Q$	Hypothesis
ii)	$\neg R \to \neg Q$	hypothesis
iii)	$Q \to R$	Inverse of (ii)
iv)	$P \to R$	Hypothetical syllogism of (i), (iii)
v)	$R \to S$	Hypothesis.
vi)	$P \to S$	Hypothetical syllogism of (v), (iv).

Hence, The given hypothesis are valid.

Q6. Construct an argument using rules of inference to show that the hypothesis "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on", "If the sailing race is held, then the trophy will be awarded" and "The trophy was not awarded" imply the conclusion "It rained".

Let P: It rains.

Q: It is foggy

R: The sailing race will be held.

S: The lifesaving demonstration will go on.

T: The trophy will be awarded.

Hypothesis	Conclusion.
$(\neg P \vee \neg Q) \rightarrow (R \wedge S)$	P
$R \rightarrow T$	
$\neg T$	

	Statement	Reason
i)	$R \rightarrow T$	Hypothesis
ii)	$\neg T$	Hypothesis.
iii)	$\neg R$	Using Modus Tollens (i), (ii)
iv)	$(\neg P \vee \neg Q) \rightarrow (R \wedge S)$	Hypothesis.
v)	$(\neg P \vee \neg Q) \rightarrow R$	Using simplification on (iv).
vi)	$\neg(\neg P \vee \neg Q)$ $P \wedge Q$	Using Modus Tollens on (iii), (v)
vii)	P	Using simplification on (vi)

Hence, the given hypothesis is valid #

Q7. Contruct an argument using rules of inference to show that the hypothesis "All integeres are irrational numbers." "Some integers are power of 2" imply the conclusion "some irrational numbes is a power of 2."

let $P(x)$: " x is irrational number"
$Q(x)$: " x is power of 2"

Hypothesis
$\forall x\, P(x)$
$\exists x\, Q(x)$

Conclusion
$\exists x\, (P(x) \wedge Q(x))$

	Statement	Reason
i)	$\forall x\, P(x)$	Hypothesis
ii)	$\exists x\, Q(x)$	Hypothesis
iii)	$P(a)$	Universal Instantiation on (i)
iv)	$Q(a)$	Existential instantiation on (ii)
v)	$P(a) \wedge Q(a)$	Using conjuction on (iii), (iv)
vi)	$\exists x\, (P(x) \wedge Q(x))$	using Existential generalization on (v).

Hence, the hypothesis is valid #.

Q8. Using resolution principle show that "If Mohan is a lawyer, then he is ambitious." "If Mohan is an early riser, then he doesnot like idlies." "If Mohan is ambitious, then he is an early riser". "Then if Mohan is a lawyer, then he doesnot like idlies."

Let P : Mohan is a lawyer.

Q : He is ambitious.

R : Mohan is an early riser.

S : He like idlies.

Hypothesis	Conclusion.
$P \rightarrow Q$	$P \rightarrow \neg S$
$R \rightarrow \neg S$	
$Q \rightarrow R$	

Writing above expressions into clause form we get.

$C1$: $\neg P \vee Q$

$C2$: $\neg R \vee \neg S$

$C3$: $\neg Q \vee R$

$C4$: $\neg(\neg P \vee \neg S)$

using clause C2 and C3 and using resolution principle we get,

$C_5 : \neg Q \vee \neg S$

$C_6 : \neg P \vee \neg S$ Taking C_1 and C_5

Finally taking C_4 and C_6.

$C_7 : \square$

Here an empty clause is reached so, by using resolution principle we can say that the given argument is valid. #

Q9. Use mathematical induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive integer.

Let $P(n)$: "$n^3 - n$ is divisible by 3"

Basic step:

for $n = 0$

$P(0) = 0 - 0$ is divisible by 3 is true.

for $n = 1$

$P(1) = 1^3 - 1$ is divisible by 3 is true.

Inductive step:
let $P(K)$ holds true for arbitary integer k.
i.e $k^3 - k$ is divisible by 3.

We show that $P(K+1)$ holds true.
i.e $(K+1)^3 - (K+1)$ is divisible by 3.

$$(K+1)^3 - (K+1) = K^3 + 3K^2 + 3K + 1 - K - 1$$
$$= (K^3 - K) + 3(K^2 + K)$$

which is divisible by 3 is true.

Here $P(K+1)$ is true whenever $P(K)$ is true.
So, the given mathematical statement $n^3 - n$ is divisible by 3 is true. #

Q 10. Use mathematical induction to prove that $2^n < n!$ for every positive integer n with $n >= 4$.

let $P(n) : 2^n < n!$

Basic step:
For $n = 4$
$2^4 < 4!$
$\Rightarrow 16 < 24$ holds true.

For $n=5$

$2^5 < 5!$

$\Rightarrow 32 < 120$ is true

Inductive step:

let $P(K)$ holds true for arbitary integer K.

i.e $2^K < K!$

We show that $P(K+1)$ holds true.

i.e $2^{K+1} < (K+1)!$

We know,

$2^K < K!$

Multiplying by $(K+1)$

$2^K (K+1) < K! (K+1)$

$2^K (K+1) < (K+1)!$

Also, $2 < (K+1)$ for +ve integer $K >= 4$.

So,

$2^K . 2 < (K+1)!$

$2^{K+1} < (K+1)!$

which is true.

So, $2^{K+1} < (K+1)!$ is true whenever $2^K < K!$ is true.

i.e $2^n < n!$ is true for every +ve integer n with $n >= 4$.

Q11) Using mathematical induction to prove that.

a) $3 + 3 \cdot 5 + 3 \cdot 5^2 + \dots + 3 \cdot 5^n = 3(5^{n+1} - 1)/4$

Let $P(n)$: $3 + 3 \cdot 5 + 3 \cdot 5^2 + \dots + 3 \cdot 5^n = 3(5^{n+1} - 1)/4$

Basic step:

For $n = 0$,

$3 = 3(5 - 1)/4$

$= 3$

$\therefore$ LHS = RHS

For $n = 1$,

$LHS = 3 + 3 \cdot 5$

$= 18$

$RHS = 3(5^2 - 1)/4$

$= 18$

$\therefore$ LHS = RHS

Inductive Step:

Let $P(K)$ holds true for arbitary integer K.

i.e $3 + 3 \cdot 5 + 3 \cdot 5^2 + \dots + 3 \cdot 3^K = 3(5^{K+1} - 1)/4$

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We show that P(k+1) is true.

i.e $3 + 3\cdot5 + 3\cdot5^2 + \dots + 3\cdot5^k + 3\cdot5^{k+1} = 3(5^{(k+1)+1} - 1)/4$

$$
\begin{aligned}
LHS &= 3 + 3\cdot5 + 3\cdot5^2 + \dots + 3\cdot5^k + 3\cdot5^{k+1} \\
&= \frac{3(5^{k+1} - 1)}{4} + 3\cdot5^{k+1} \\
&= \frac{3}{4}(5^{k+1} - 1 + 4\cdot5^{k+1}) \\
&= \frac{3}{4}(5^{k+1}(1+4) - 1) \\
&= \frac{3}{4}(5^{(k+1)+1} - 1) \\
&= RHS \text{ proved } \#
\end{aligned}
$$

So, $3 + 3\cdot5 + 3\cdot5^2 + \dots + 3\cdot5^k + 3\cdot5^{k+1} = 3(5^{(k+1)+1} - 1)/4$ is true if P(K) holds true.

So, $3 + 3\cdot5 + 3\cdot5^2 + \dots + 3\cdot5^n = 3(5^{n+1} - 1)/4$ is true using mathematical induction.

#

b) $1.2.3 + 2.3.4 + 3.4.5 + \dots + n(n+1)(n+2) = \dfrac{n(n+1)(n+2)(n+3)}{4}$

let $p(n) = 1.2.3 + 2.3.4 + 3.4.5 + \dots + n(n+1)(n+2)$

$= \dfrac{n(n+1)(n+2)(n+3)}{4}$.

Basic step:

for $n=0$

LHS = RHS = 0

for $n=1$

LHS = $1.2.3 = 6$

RHS = $\dfrac{1.2.3.4}{4} = 6$

LHS = RHS.

Inductive step.

let $p(K)$ holds true.

i.e $1.2.3 + 2.3.4 + \dots + K(K+1)(K+2) = \dfrac{K(K+1)(K+2)(K+3)}{4}$

Now, we need to show $P(K+1)$ is true.

ie $1.2.3 + 2.3.4 + \dots + (K+1)((K+1)+1)((K+1)+2)$

$= \dfrac{(K+1)((K+1)+1)((K+1)+2)((K+1)+3)}{4}$

$$LHS = 1.2.3 + 2.3.4 + \dots + K(K+1)(K+2) + (K+1)((K+1)+1)((K+1)+2)$$

$$= \frac{K(K+1)(K+2)(K+3)}{4} + (K+1)(K+2)(K+3)$$

$$= (K+1)(K+2)(K+3)\left(\frac{K}{4}+1\right)$$

$$= \frac{(K+1)(K+2)(K+3)(K+4)}{4}$$

$$= \frac{(K+1)((K+1)+1)((K+1)+2)((K+1)+3)}{4}$$

$= RHS$ proved.

So, $P(K+1)$ is true if $P(K)$ is true.

$\therefore$ $1\cdot2\cdot3 + 2\cdot3\cdot4 + \dots\dots + n(n+1)(n+2)$

$$= \frac{n(n+1)(n+2)(n+3)}{4}$$

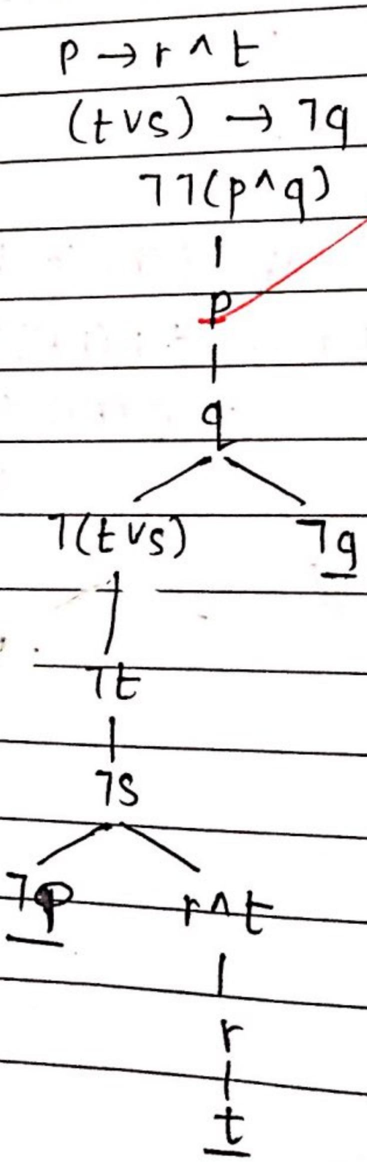
is true #

Q 12. Define satisfiable and unsatisfiable formula. Draw the tableau for the formula set.

$$\phi = \{ p \to (r \wedge t), (t \vee s) \to \neg q, \neg\neg(p \wedge q) \}$$

If a completed tableau for a formula ϕ is closed then formula ϕ is said to be 'unsatisfiable formula' else the formula ϕ is said to be satisfiable formula.

$$\phi = \{ p \to (r \wedge t), (t \vee s) \to \neg q, \neg\neg(p \wedge q) \}$$



Here, all the branches are closed. So, given set of formulae is unsatisfiable.

Q 13. Why tableau method is important in propositional logic? Draw the tableau for the formula:

$$\phi = (p \wedge \neg q) \rightarrow s$$

⇒ Tableau method is important in propositional logic because:

a) It is an easy process.

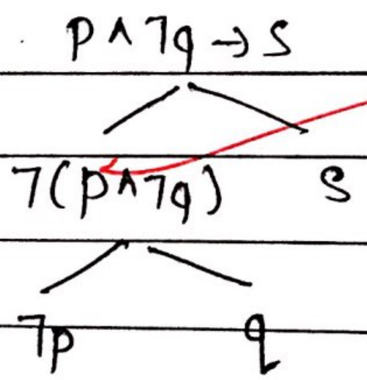
b) It only uses tree diagram.

c) No formulas are used.

d) Only by find the branch is closed or opened, we can know the consistency of formula.

Now,

$$\phi = (p \wedge \neg q) \rightarrow s$$



Here all the branches are opened, it is satisfiable formula.

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Tutorial - 2

1. Find all solutions of the recurrence ~~sol~~ relation.

$a_n = 2a_{n-1} + 3^n$ with initial condition $a_1 = 5$.

First consider the associated linear homogeneous recurrence relation,

$$a_n = 2a_{n-1}$$

Put $a_n = r^n$

$\Rightarrow r^n = 2r^{n-1}$

Dividing by r^{n-1}

$\Rightarrow r = 2$

Its homogenous solution is.

$$a_n^{(h)} = \alpha_1 (2)^n$$

For calculating the particular sol^n, we consider the Term $f(n)$ Here,

$$f(n) = 3^n = n^0 \cdot 3^n$$

Now, compare $f(n)$ with $P(n) s^n$ we get.

$s = 3$

$P(n) = P_0 n^0 = P_0$ ~~where s = r = 2~~

Now, its particular sol^n is:

$$\cancel{f(n)}\ a_n^{(p)} = P(n) s^n$$

$$= P_0 3^n$$

The required solution is

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$\therefore a_n = \alpha_1 (2)^n + P_0 3^n$$

We calculate one more initial condition as:

$$a_n = 2a_{n-1} + 3^n$$

$$\text{or, } a_2 = 2a_1 + 3^2$$

$$\text{or, } a_2 = 2 \times 5 + 9$$

$$\therefore a_2 = 19$$

Now,

$$a_1 = \alpha_1 2 + P_0 3$$

$$\text{or, } 5 = 2\alpha_1 + 3 P_0$$

$$a_2 = \alpha_1 2^2 + P_0 3^2$$

$$\therefore 19 = 4\alpha_1 + 9 P_0$$

$$\therefore \alpha_1 = -2$$

$$P_0 = 3$$

The final required solution is:

$$a_n = -2(2)^n + 3 \cdot 3^n$$

2. Find all solutions of the recurrence relation: $a_n = 3a_{n-1} + 2^n$ with initial cond^n $a_0 = 5$.

First consider the associated linear homogeneous recurrence relation,

$$a_n = 3a_{n-1}$$

Put $a_n = r^n$

$$r^n = 3r^{n-1}$$

or, $r = 3$ Dividing by r^{n-1}

Its homogeneous sol^n is:

$$a_n^{(h)} = \alpha_1 3^n$$

For calulating the particular solution we consider the term $F(n)$

Here, $f(n) = 2^n$

Comparing $f(n)$ with $P(n) s^n$ we get,

$$S = 2$$

$$P(n) = P_0 n^0 = P_0$$

Its particular solution is:

$$a_n^{(P)} = P(n) s^n$$

$$= P_0 2^n$$

The required solution is:

$$a_n = a_n^{(h)} + a_n^{(P)}$$

$$\therefore a_n = \alpha_1 3^n + P_0 2^n$$

We calculate one more initial condition as:

$$a_n = 3a_{n-1} + 2^n$$

Now, $n=1$,

$$a_1 = 3a_0 + 2^1$$
$$= 3 \times 5 + 2$$
$$= 17$$

Now,

$$a_0 = \alpha_1 3^0 + P_0 2^0$$

$$\therefore 5 = \alpha_1 + P_0$$

$$a_1 = \alpha_1 3^1 + P_0 2^1$$

$$\therefore 17 = 3\alpha_1 + 2P_0$$

On solving,

$$\alpha_1 = 7$$

$$P_0 = -2$$

The final required soln is:

$$\therefore a_n = 7\,3^n - 2 \cdot 2^n$$

#

3. Find all solutions of the recurrence soluti relation

$$a_n = 2a_{n-1} + 2^n$$

with initial condition $a_0 = 2$.

First consider the associated linear homogeneous recurrence relation,

$$a_n = 2a_{n-1}$$

Put $a_n = r^n$

$$\Rightarrow r^n = 2r^{n-1}$$

Dividing by r^{n-1}

$$\Rightarrow r = 2$$

Its homogenous solution is:

$$a_n(h) = \alpha_1 2^n$$

For calculating the particular solution we consider term $F(n)$. Here $F(n) = 2^n$

Comparing $F(n)$ with $P(n)s^n$ we get.

$$S = 2$$

$$P(n) = P_0 n^0 = P \qquad \text{Here } s = r = 2$$

Its particular solution is:

$$a_n(P) = P_0 2^n n^{m \to 1} = n P_0 2^n$$

The required solution is:

$$a_n = a_n(h) + a_n(P)$$

$$\therefore a_n = \alpha_1 2^n + n P_0 2^n$$

We need to calculate one more initial conditions.

$$a_1 = 2a_0 + 2^1$$
$$= 2 \times 2 + 2$$
$$= 6$$

Now,

$$a_0 = \alpha_1 2^0 + 0 \times P_0 2^0$$
$$\therefore 2 = \alpha_1$$

$$a_1 = \alpha_1 2^1 + 1 \times P_0 \times 2^1$$
or, $6 = 2\alpha_1 + 2P_0$
or, $6 = 4 + 2P_0$
$$\therefore P_0 = 1$$

The required sol^n is:

$$\Rightarrow a_n = 2 \cdot 2^n + n 2^n$$
$$\therefore a_n = (2+n)\, 2^n$$

#

$r^2 - 2r + r - 2 = 0$
$r(r-2) + 1(r-2) = 0$
$(r-2)(r+1) = 0$

Q. Find general sol^n of $\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$ with initial condition $a_0 = a_1 = 1$

given,

$$\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$$

let $\sqrt{a_n} = b_n$

so,

$$b_n = b_{n-1} + 2 b_{n-2}$$

Put $b_n = r^n$

$$\Rightarrow r^n = r^{n-1} + 2 r^{n-2}$$

Dividing by r^{n-2}

$$\Rightarrow r^2 = r + 2$$

$$\Rightarrow r^2 - r - 2 = 0$$

$$\therefore r = 2, -1$$

$a_0 = 1 = b_0^2$

$\therefore b_0 = 1$

$a_1 = 1 = b_1^2$

$\therefore b_1 = 1$

The required solution is:

$$b_n = \alpha_1 2^n + \alpha_2 (-1)^n$$

Using initial condition,

$$b_0 = \alpha_1 2^0 + \alpha_2 (-1)^0$$

$$1 = \alpha_1 + \alpha_2$$

and,

$$b_1 = \alpha_1 2^1 + \alpha_2 (-1)^1$$

$$1 = 2\alpha_1 - \alpha_2$$

On solving,

$$\alpha_1 = \frac{2}{3}$$

$$\alpha_2 = \frac{1}{3}$$

So, the final solution is:

$$b_n = \frac{2}{3}(2)^n + \frac{1}{3}(-1)^n$$

So,

$$\sqrt{a_n} = \frac{2}{3}(2)^n + \frac{1}{3}(-1)^n$$

$$\therefore \ a_n = \left(\frac{2}{3}(2)^n + \frac{1}{3}(-1)^n\right)^2$$

#

5. Find all the solutions of the recurrence relation: $a_n = 5a_{n-1} - 6a_{n-2} + 2^n$ with initial conditions $a_0 = 1$ and $a_1 = 4$

Firstly, consider the associated linear homogeneous recurrence relation,

$$a_n = 5a_{n-1} - 6a_{n-2}$$

let $a_n = r^n$

$$r^n = 5r^{n-1} - 6r^{n-2}$$

dividing by r^{n-2}

or, $r^2 = 5r - 6$

or, $r^2 - 5r + 6 = 0$

$\therefore r = 3, 2$

Its homogeneous solution is:

$$a_n^{(h)} = \alpha_1 3^n + \alpha_2 2^n$$

For calculating, the particular solution we consider term F(n). Here,

$$F(n) = 2^n$$

Comparing F(n) with $P(n) s^n$ we get

$$s = 2$$

$$P(n) = P_0 n^0 = P_0$$

Its particular sol^n is,

$$a_n^{(p)} = P_0 s^n . n^{m=1}$$

$$= n P_0 2^n$$

The required sol^n is:

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$\therefore a_n = \alpha_1 3^n + \alpha_2 2^n + n P_0 2^n$$

We have to calculate one more initial cond^n.

$$a_2 = 5a_1 - 6a_0 + 2^2$$

$$= 5 \times 4 - 6 \times 1 + 4$$

$$= 10$$

so,

$$a_0 = \alpha_1 3^0 + \alpha_2 2^0 + 0$$

$$\therefore 1 = \alpha_1 + \alpha_2$$

$$a_1 = \alpha_1 3^1 + \alpha_2 2^1 + 1 \cdot P_0 2^1$$

$$\therefore 4 = 3\alpha_1 + 2\alpha_2 + 2P_0$$

$$a_2 = \alpha_1 3^2 + \alpha_2 2^2 + 2 P_0 2^2$$

$$\therefore 10 = 9\alpha_1 + 4\alpha_2 + 8P_0$$

On solving,

$$\alpha_1 = -2$$

$$\alpha_2 = 3$$

$$b = 2$$

The final sol^{n} is:

$\therefore \quad a_n = -2\cdot 3^n + 3\cdot 2^n + 2n2^n$ #

6. Find all solutions of the recurrence relation.

$$a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3} + n4^n$$

with Initial condition $a_0 = -2$, $a_1 = 0$ and $a_2 = 5$

First ~~consider~~ the associated linear homogeneous recurrence relation,

$$a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3}$$

Put $a_n = r^n$

$$r^n = 7r^{n-1} - 16r^{n-2} + 12r^{n-3}$$

Dividing by r^{n-3}

$\Rightarrow r^3 = 7r^2 - 16r + 12$

$\Rightarrow r^3 - 7r^2 + 16r - 12 = 0$

So, $r = 2, 3, 2$

Its homogeneous solution is:

$$a_n^{(h)} = \alpha_1 (2)^n + \alpha_2 n (2)^n + \alpha_3 3^n$$

For particular solution, we consider term $F(n)$. then,

$$F(n) = n4^n$$

Comparing $F(n)$ with $P(n)S^n$ we get.

$$S = \cancel{8}4$$

$$P(n) = P_1 n^1 + P_0 n^0$$

$$= P_1 n + P_0$$

Its particular sol^n is,

$$a_n^{(P)} = (P_1 n + P_0) 4^n$$

The required sol^n is,

$$a_n = a_n^{(h)} + a_n^{(P)}$$

$$\therefore a_n = \alpha_1 2^n + n\alpha_2 2^n + \alpha_3 3^n + (P_1 n + P_0) 4^n$$

We have calculate 2 more initial condition,

$$a_3 = 7a_2 - 16a_1 + 12a_0$$

$$= 7 \times 5 - 16 \times 0 + 12 \times (-2)$$

$$= 11$$

$$a_4 = 7a_3 - 16a_2 + 12a_1$$

$$= 7 \times 11 - 16 \times 5 + 12 \times 0$$

$$= -3$$

Now,

$$a_0 = \alpha_1 2^0 + 0.\alpha_2.2^0 + \alpha_3.3^0 + (P_1 \times 0 + P_0)4^0$$
$$-2 = \alpha_1 + \alpha_3 + P_0$$

$$a_1 = \alpha_1 2^1 + 1.\alpha_2.2^1 + \alpha_3.3^1 + (P_1 \times 1 + P_0)4^1$$
$$0 = 2\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4P_1 + 4P_0$$

$$a_2 = \alpha_1 2^2 + 2.\alpha_2.2^2 + \alpha_3.3^2 + (P_1 \times 2 + P_0)4^2$$
$$5 = 4\alpha_1 + 8\alpha_2 + 9\alpha_3 + 32P_1 + 16P_0$$

$$a_3 = \alpha_1 2^3 + 3.\alpha_2.2^3 + \alpha_3.3^3 + (P_1 \times 3 + P_0)4^3$$
$$11 = 8\alpha_1 + 24\alpha_2 + 27\alpha_3 + 192P_1 + 64P_0$$

$$a_4 = \alpha_1 2^4 + 4.\alpha_2.2^4 + \alpha_3.3^4 + (P_1 \times 4 + P_0)4^4$$
$$-3 = 16\alpha_1 + 64\alpha_2 + 81\alpha_3 + 1024P_1 + 256P_0$$

Now,

$$P_0 = -2 - \alpha_1 - \alpha_3 \quad — (*)$$
$$P_1 = -\frac{1}{4}\left(2\alpha_1 + 2\alpha_2 + 3\alpha_3 + 4(-2-\alpha_1-\alpha_3)\right)$$
$$= -\frac{1}{4}\left(-2\alpha_1 + 2\alpha_2 - \alpha_3 - 8\right) \quad — (**)$$

$$5 = 4\alpha_1 + 8\alpha_2 + 9\alpha_3 - 8(-2\alpha_1 + 2\alpha_2 - \alpha_3 - 8)$$
$$+ 16(-2 - \alpha_1 - \alpha_3)$$

or, $5 - 32 = 4\alpha_1 - 8\alpha_2 - \alpha_3$

or, $4\alpha_1 - 8\alpha_2 + \alpha_3 = -27$ —— (i)

$$11 = 8\alpha_1 + 24\alpha_2 + 27\alpha_3 + 192\left(\frac{-1}{4}\right)(-2\alpha_1 + 2\alpha_2 - \alpha_3 - 8)$$

$$+ 64(-2 - \alpha_1 - \alpha_3)$$

or, $11 = 8\alpha_1 + 24\alpha_2 + 27\alpha_3 + 96\alpha_1 - 96\alpha_2 + 48\alpha_3 + 384$
$- 128 - 64\alpha_1 - 64\alpha_3$

or, $-245 = 40\alpha_1 - 72\alpha_2 + 11\alpha_3$ —— (ii)

$$-3 = 16\alpha_1 + 64\alpha_2 + 81\alpha_3 - 256(-2\alpha_1 + 2\alpha_2 - \alpha_3 - 8)$$
$$+ 256(-2 - \alpha_1 - \alpha_3)$$

or, $-3 = 16\alpha_1 + 64\alpha_2 + 81\alpha_3 + 512\alpha_1 - 512\alpha_2 + 256\alpha_3$
$+ 2048 - 512 - 256\alpha_1 - 256\alpha_3$

or, $-1539 = 272\alpha_1 - 448\alpha_2 + 81\alpha_3$ —— (iii)

Solving (i), (ii), (iii) we get,

$\alpha_1 = 1$

$\alpha_2 = 7/2$

$\alpha_3 = -3$

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Eq^n ✱ becomes,

$P_0 = -2-1+3 = 0$

$P_1 = -\frac{1}{4}\left(-2\times1 + 2\times\frac{7}{2} + 3 - 8\right)$

$= 0$

The required sol^n is:

$a_n = 1\cdot 2^n + \frac{7}{2} n\, 2^n - 3\, 3^n$ #

checked

2074/4/17