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Solve the following

(*) $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$

given differential eq^n is,

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$$

$$(x^2D^2 - xD + 2)y = x\log x \quad -(1)$$

which is homogeneous diff. eq^n.

$$x = e^z, \quad z = \log x$$

let $xDy = \delta y$

$$x^2D^2y = (\delta^2 - \delta)y$$

So,

eq^n (1) become

$$(\delta^2 - \delta - \delta + 2)y = e^z . z$$

$$(\delta^2 - 2\delta + 2)y = e^z . z$$

Its A E is

$$m^2 - 2m + 2 = 0$$

$$m = \frac{2 \pm \sqrt{4-8}}{2}$$

$$m = \frac{2 \pm \sqrt{4}}{2}$$

$$m = 1 + i$$

Now

$$CF = e^z (A\cos z + B\sin z)$$

$$= x(A\cos\log x + B\sin\log x)$$

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$$PI = \frac{1}{\delta^2 - 2\delta + 2} e^{2z}$$

$$= \frac{e^{2z}}{(\delta+1)^2 - 2(\delta+1) + 2}$$

$$= \frac{e^{2z}}{\delta^2 + 1}$$

$$= e^{2z}(1+\delta^2)^{-1}$$

$$= e^{2z}(1 - \delta^2 + \delta^4 - \dots)$$

$$= e^{2z}$$

$$= x \log x.$$

So, the general sol^n is

$$y = CF + PI$$

$$= Ax\cos(\log x) + Bx\sin(\log x) + x\log x$$

is req^n sol^n,

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4. $y = yp^2 + 2px$

given eq^n.

$$x = \frac{y}{2p} - \frac{yp^2}{2p}$$

$$x = \frac{1}{2}\frac{y}{p} - \frac{1}{2}yp \quad \text{—(i)}$$

Differentiating w.r. to y.

$$\Rightarrow \frac{dx}{dy} = \frac{-1}{2p^2}\left[p - y\cdot\frac{dp}{dy}\right] - \frac{1}{2}\left[y\cdot\frac{dp}{dy} + p\right]$$

$$\Rightarrow \frac{1}{p} = \frac{1}{2p} - \frac{1}{2}y\frac{dp}{dy}\left(\frac{1}{p^2}+1\right) - \frac{p}{2}$$

$$\Rightarrow \frac{1}{p} - \frac{1}{2p} + \frac{p}{2} = \frac{-y}{2}\frac{dp}{dy}\left(\frac{1}{p^2+1}\right)$$

$$\Rightarrow \frac{p^2+1}{2p} = \frac{-y}{2}\frac{dp}{dy}\frac{p^2+1}{p^2}$$

$$\Rightarrow \int\frac{dy}{y} + \int\frac{dp}{p} = 0$$

$$\log y + \log p = \log c$$

$$p = \frac{c}{y} \quad \text{—(ii)}$$

from (i) & (ii)

$$y = y\frac{c^2}{y^2} + 2\frac{c}{y}x$$

$$y = c^2 + 2cx \quad \text{Ans}$$

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3. $(xp - y)^2 = p^2 - 1$

given eq^n i)

$$xp - y = \sqrt{p^2-1}$$

$$y = -\sqrt{p^2-1} + xp. \quad \text{— i)}$$

Diff. eq^n (1) w.r.t to x.

$$\frac{dy}{dx} = p + x\frac{dp}{dx} - \frac{\cancel{2}p}{\cancel{2}\sqrt{p^2-1}}\frac{dp}{dx}$$

$$\cancel{p} = \cancel{p} + x\frac{dp}{dx} - \frac{p}{\sqrt{p^2-1}}\frac{dp}{dx}$$

$$0 = \frac{dp}{dx}\left(x - \frac{p}{\sqrt{p^2-1}}\right) \quad \text{—(iii)}$$

either,

$$\frac{dp}{dx} = 0$$

$$p = c \quad \text{—(ii)}$$

which gives $y = cx - \sqrt{c^2-1}$ from eq^n (i)

$$x - \frac{p}{\sqrt{p^2-1}} = 0$$

$$x = \frac{p}{\sqrt{p^2-1}} \quad \text{—(iv)}$$

eq^n (i) become (using eq^n (iv))

$$y = \frac{p}{\sqrt{p^2-1}} \times p - \sqrt{p^2-1}$$

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$$y = \frac{p^2}{\sqrt{p^2-1}} - \sqrt{p^2-1}$$

$$y = \frac{p^2 - (\sqrt{p^2-1})^2}{\sqrt{p^2-1}}$$

$$y = \frac{\cancel{p^2} - \cancel{p^2}+1}{\sqrt{p^2-1}}$$

$$y^2 = \frac{1}{\sqrt{p^2-1}}$$

$$p^2 - 1 = \frac{1}{y^2}$$

$$p^2 = 1 + \frac{1}{y^2}$$

putting eq^n (iii)

$$x^2 = \frac{\frac{y^2+1}{y^2}}{\frac{1+y^2}{y^2} - 1}$$

$$x^2 = \frac{1+y^2}{y^2} \times \frac{y^2}{1}$$

$$x^2 = 1+y^2$$

$x^2 - y^2 = 1$ is req. sol^n

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5) $\frac{dy}{dx} = \frac{2x-y+1}{x+2y-3}$

$\Rightarrow$ given eq^n becomes.

$$xdy + 2ydy - 3dy = 2xdx + dx$$

Integrating both side

$$xy + \frac{2y^2}{2} - 3y = \frac{\cancel{2}x^2}{\cancel{2}} + x$$

$$xy + y^2 - 3y = x^2 + x$$

$$x^2 - y^2 + x - xy + 3y = 0$$ is req^n sol^n

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2. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 10\sin x$

$\Rightarrow$ Soln,

Eqn can be written as

$(D^2 - 2D + 5)y = 10\sin x$ $\left[\frac{d}{dx} = D\right]$

so its AE is

$m^2 - 2m + 5 = 0$

$m = \frac{2 \pm \sqrt{4-20}}{2}$

$m = 1 \pm 2i$

now,

$CF = e^x (A\cos 2x + B\sin 2x)$

again

$PI = \frac{1}{(D^2 - 2D + 5)} 10\sin x$

$= \frac{10\sin x}{(-1^2 - 2D + 5)}$

$= \frac{10\sin x}{4 - 2D}$

$= -\frac{10\sin x}{2(D-2)}$

$= -\frac{1}{2} \frac{D+2}{(D+2)(D-2)} 10\sin x$

$= \frac{-5}{-1-4} (D+2)\sin x.$

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$= (D+2)\sin x$

$= \cos x + 2\sin x$

Now,

$y = CF + PI$

$= e^x(A\cos 2x + B\sin 2x)$ is req^d sol^n.

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6)

=> sol^n,

Case I

$$\frac{dr}{dt} - \frac{20}{600} = 0.03 \quad \left[\begin{array}{l} r \text{ is temperature} \\ t \text{ is time} \end{array} \right]$$

Case II

$$\frac{dr}{dt} = \frac{60}{T}$$

$$0.03 = \frac{60}{T}$$

$$T = 1800 \text{ sec} \quad \text{or} \quad 30 \text{ min.}$$