STREAMLINES AND SKETCHES OF FIELDS

It is difficult to visualize the nature of electric fields due to various charg distributions from the mathematical equations. A pictorial representation c electric fields, on the other hand, gives clear understanding of the nature of th fields.

The continuous lines which are used to represent the electric field around charge are called streamlines. These are also called flux lines or direction lines The streamlines have basically two properties:

i. These are the continuous lines from the charge which only show the direction of $\vec{E}$ and are everywhere tangent to $\vec{E}$.

ii. The magnitude of the field can be shown to be inversely proportional to the spacing of the streamlines.

The streamlines are associated with the arrows which are used to show the direction of $\vec{E}$. If a small positive test charge is placed at any point in the field and is free to move, then the direction in which it will accelerate is indicated by the streamline at that point.

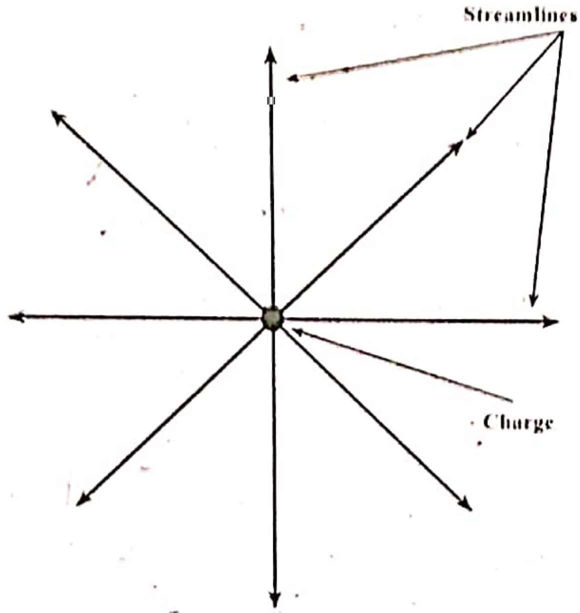

Figure 2.8 The streamlines for a line charge (the cross-section of the line charge is shown).

The three dimensional sketching of streamlines is very difficult. Hence, in practice, only two dimensional sketching is used. For such sketching, the z-component of $\vec{E}$ is assumed to be zero.

Equation of streamlines

Consider the streamlines shown in the figure.

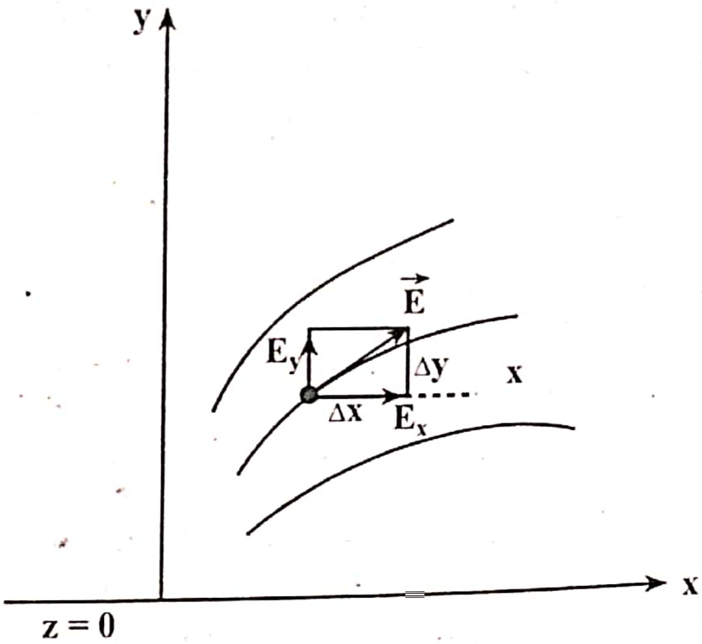

Figure 2.9 The equation of a streamline is obtained by solving the differential equation $E_y/E_x = dy/dx$.

Consider a point P on any of the streamlines. The $\vec{E}$ is tangential to the streamline at that point P. The $\vec{E}$ can be resolved into two components E_x in x direction and E_y in y direction.

It can be seen that E_y is proportional to a small component Δy in y direction while E_x is proportional to the small component Δx in x direction. Thus, we can write

$$\boxed{\frac{E_y}{E_x} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}}$$

SOME IMPORTANT FORMULAE:

$$\vec{E}_P = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\vec{E}_L = \frac{\rho_L}{2\pi\varepsilon_0 R}\hat{a}_R$$

$$\vec{E}_S = \frac{\rho_S}{2\varepsilon_0}\hat{a}_N.$$

Additional Questions

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Q. Obtain the equation of the streamline that passes through the point $P(-2, 7, 10)$ in the field:

$$\vec{E} = 2(y-1)\hat{a}_x + 2x\hat{a}_y$$

⇒ Sol^n,

Here,

$$\vec{E} = \underbrace{2(y-1)}_{\Downarrow}\hat{a}_x + \underbrace{2x}_{\Downarrow}\hat{a}_y$$

$$= E_x\hat{a}_x + E_y\hat{a}_y$$

At any point, the equation of streamline satisfies:

$$\frac{dy}{dx} = \frac{E_y}{E_x}$$

$$\text{or, } \frac{dy}{dx} = \frac{2x}{2(y-1)}$$

$$\text{or, } 2(y-1)dy = 2x\,dx$$

Integrating on both sides,

$$\int 2y\,dy - 2\int dy = \int 2x\,dx$$

$$\text{or, } \cancel{2}\frac{y^2}{\cancel{2}} - 2y + c_1 = \cancel{2}\frac{x^2}{\cancel{2}} + c_2$$

$$\text{or, } y^2 - 2y - x^2 + c = 0$$

At point $(-2, 7, 10)$,

$$7^2 - 2\times 7 - 4 + c = 0$$

$$\text{or, } 49 - 14 - 4 + c = 0$$

$$\therefore c = -31$$

$$\therefore y^2 - 2y - x^2 - 31 = 0$$

$$\text{or } \boxed{y^2 - x^2 - 2(y+31) = 0}$$

is the req. eq^n A.

$$y^2 - 2y + 1 - x^2 - 31 - 1 = 0$$

$$\text{or, } (y-1)^2 - x^2 = 32$$

versa.

STANDING WAVE RATIO (SWR):

We have,

$$E^{+}_{xs1} = E^{+}_{xo1}\, e^{-j\beta_1 z}$$

$$E^{-}_{xs1} = E^{-}_{xo1}\, e^{+j\beta_1 z}$$

$$= \Gamma E^{+}_{xo_1}\, e^{+j\beta_1 z} \quad \left(\because \Gamma = \frac{E^{-}_{xo_1}}{E^{+}_{xo_1}} \right) \quad = |\Gamma| e^{j\phi} E^{+}_{xo1}\, e^{+j\beta_1 z}$$

$$\text{Now, } E_{xs1} = E^{+}_{xs1} + E^{-}_{xs1}$$

$$= E^{+}_{xo1}\, e^{-j\beta_1 z} + |\Gamma| e^{j\phi} E^{+}_{xo1}\, e^{+j\beta_1 z} = E^{+}_{xo1} (e^{-j\beta_1 z} + |\Gamma|\, e^{j\phi} e^{+j\beta_1 z})$$

The E_{xs1} will be maximum when the phase angles of the terms in the larger parenthesis are equal (the phase difference is zero) because they will be directed in the same direction, and their magnitudes are therefore added constructively. Hence,

$E_{xs1,max} = (1 + |\Gamma|)\, E^{+}_{xo1}$ which is true for $-\beta_1 z - (\beta_1 z + \phi) = 0 + 2n\pi$ ($n = 0, \pm 1, \pm 2, \ldots\ldots$)

$$\text{or, } -2\beta_1 z = 2n\pi + \phi$$

$$\text{or, } -2 \times \frac{2\pi}{\lambda_1} z = 2n\pi + \phi$$

$$\text{or, } z_{max} = \frac{-\lambda_1}{2} \left(n + \frac{\phi}{2\pi} \right)$$

$$\boxed{\text{or, } z_{max} = \frac{-1}{2\beta_1} (\phi + 2n\pi)}$$

Similarly, the E_{xs1} will be minimum when the phase angles of the terms in the larger parenthesis differ by 180°, because they will be directed in the opposite direction, and their magnitudes are therefore added destructively. Hence,

$$E_{xs1,min} = (1 - |\Gamma|)\, E_{x01}^{+}$$

and this occurs when

$$-\beta_1 z - (\beta_1 z + \phi) = \pi + 2n\pi \quad (n = 0, \pm 1, \pm 2, \ldots\ldots\ldots)$$

$$\text{or,} \quad -2\beta_1 z = 2n\pi + \phi + \pi$$

$$\text{or,} \quad z_{min} = -\frac{\lambda_1}{2}\left(n + \frac{\phi}{2\pi} + \frac{1}{2}\right)$$

$$\boxed{\text{or,} \quad z_{min} = \frac{-1}{2\beta_1}\left[\phi + (2n+1)\pi\right]}$$

The ratio of the maximum to the minimum electric field is known as the standing wave ratio, SWR.

$$SWR = \frac{E_{xs1,max}}{E_{xs1,min}} = \frac{(1 + |\Gamma|)\, E_{x01}^{+}}{(1 - |\Gamma|)\, E_{x01}^{+}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Since, $|\Gamma| \leq 1$, we have $1 \leq SWR < \infty$. The higher the SWR, the greater the portions of the standing wave in the wave comprising of both the travelling and the standing waves.

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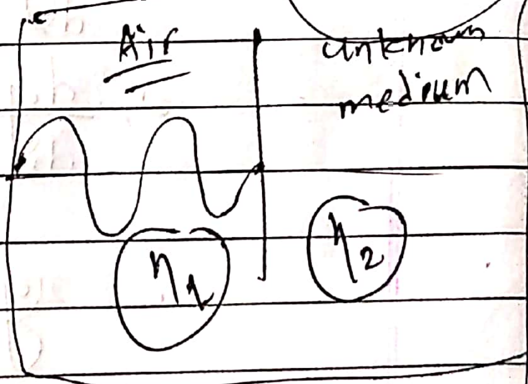
Q. A uniform plane wave in air partially reflects from the surface of a meterial whose properties are unknown. Measurements of the electric field in the region in front of the interface yield a 1.5m spacing between maxima, with the first maximum occuring 0.75m from the interface.
A SWR of 5 is measured. Determine the intrinsic impedance of the unknown material.

⟹ We know,

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} \quad ; \; |\Gamma| \rightarrow \text{magnitude of reflection coefficient.}$$



$$\text{or, } 5 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\text{or, } 5-5|\Gamma| = 1+|\Gamma|$$

$$\text{or, } 6|\Gamma| = +4$$

$$\therefore |\Gamma| = \frac{\cancel{+4}^{\,2}}{\cancel{6}_{\,3}} = +0.67 = \frac{2}{3}.$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} \quad ; \; \eta_1 = 377\,\Omega$$
(being air medium)

$$\text{or, } \Gamma = \frac{\eta_2-377}{\eta_2+377} \quad \text{—— (1)}$$

$$\Gamma = |\Gamma| e^{j\phi}$$

We know, In case of standing wave,

$$Z_{max} = \frac{-1}{2\beta_1}(\phi + 2n\pi) \quad \text{—— (11)}$$

For $n = 1$, $Z_{max} = 0.75\,m$

$\beta_1 = \frac{2\pi}{\lambda_1}$; Here, $\lambda_1 = 1.5\,m$ (spacing betn maxima)

$$= \frac{2\pi}{1.5} = 4.189$$

So, (11) becomes:

or, $0.75 = \frac{-1}{2 \times \frac{2\pi}{1.5}}(\phi + 2\pi)$

or, $0.75 = \frac{-1}{8.378}(\phi + 2\pi)$

or, $2\pi = -\phi - 2\pi$

or, $\phi = -2\pi - 2\pi = -4\pi$

$\therefore \phi = -4\pi.$

$$\therefore \Gamma = \left(\frac{2}{3}\right) e^{j(-4\pi)}$$

$$= \frac{2}{3}\left[\cos(-4\pi) + j\sin(-4\pi)\right]$$

$$= \frac{2}{3}\left[1 - j0\right] = \frac{2}{3}.$$

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from (1),

or, $\frac{2}{3} = \frac{\eta_2 - 377}{\eta_2 + 377}$

or, $2\eta_2 + 377 \times 2 = 3\eta_2 - 3 \times 377$

or, $\eta_2 = 5 \times 377$

$\therefore \eta_2 = \boxed{1885\ \Omega}$ is the required intrinsic impedance of the unknown material.