Chapter - 4
Comparative Analysis of Alternatives.

[12 marks]

(A) Comparing mutually exclusive alternative having same useful life by:

a) Payback period method and Equivalent worth method.
b) Rate of Return method and B/C ratio method.

Example no. 1

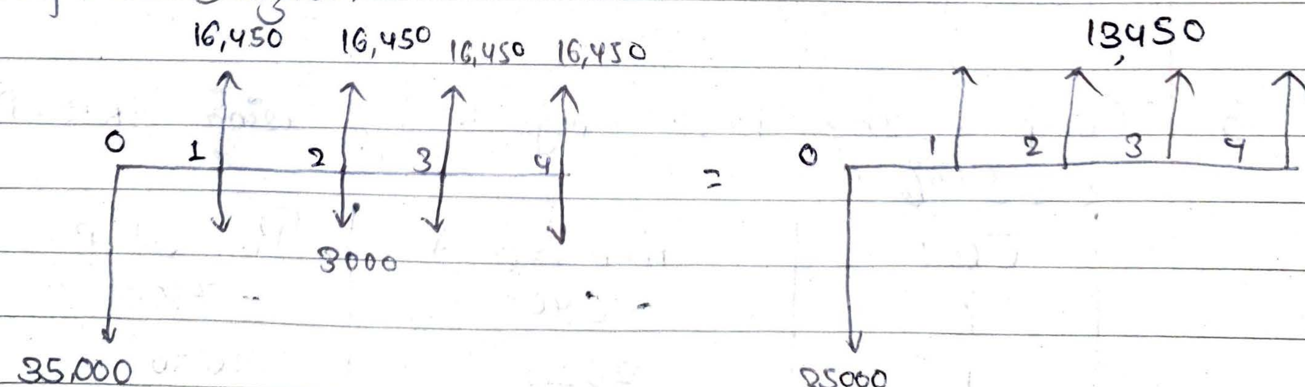
From the following information, select the best project by using FW method and useful life is 4 years.

Project	Project A	Project B
Initial Investment	35,000	50,000
Annual Revenue	16,450	25,000
Annual Cost	3,000	13,830
Useful life	4 years	4 years
Salvage value	0	0
MARR	10 %	10 %

Solution:

For Project A,

Cash-flow diagram

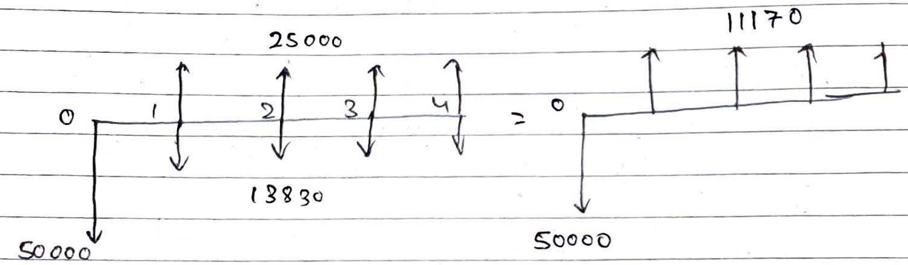


$$FW_A = A\left[\frac{(1+i)^n - 1}{i}\right] - 35000(1+i)^{+4}$$

$$= 13450\left[\frac{(1+0.1)^4 - 1}{0.1}\right] - 35000(1+0.1)^{+4}$$

$= 11177.95$

For Project B,



Here

$FW_B = -50000(1+i)^4 + 11170\left(\frac{(1+i)^4-1}{0.1}\right)$

$= -50000 \times (1+0.1)^4 + 11170\left(\frac{1.1^4-1}{0.1}\right)$

≈ -21365.03

Since, $F.W._A > F.W._B$. So, project A is the best project.

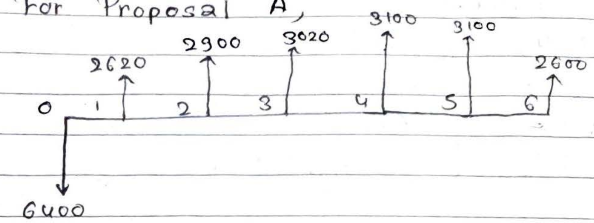
Compute the best project by using ERR. MARR = 20%

$\varepsilon = 14\%$

EOY	Proposal A	Proposal B
0	−6400	−7500
1	2620	2050
2	2900	4046
3	3020	4000
4	3100	3900
5	3100	3900
6	2600	3400

Solution,

For Proposal A,



$P.W_A = -6400$

$F.W._A = 2600 + 3100(1+\varepsilon)^1 + 3100(1+\varepsilon)^2 + 3020(1+$
$+ 2900(1+\varepsilon)^4 + 2620(1+\varepsilon)^5$

$= \cancel{21482.69}\ 24579.59$

Now

$FW_A = PW_A(1+i)^6$

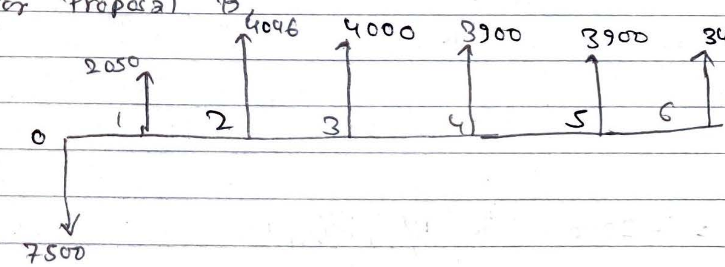
or $24579.59 = 6400(1+i)^6$

or, $1+i = 1.2514$ $\quad ERR_A = 25.14\%$

$\therefore i = 0.2514 = 25.14\% > MARR$

So, proposal A is accepted.

For Proposal B,



$PW_B = -7500$

$FW_B = 3400 + 3900 \times 1.14 + 3900 \times 1.14^2 + 4000 \times 1.14^3$
$4046 \times 1.14^4 + 2050 \times 1.14^5$

$= 29621.24$

So, project C is accepted.

Since, $IRR_C > IRR_A > IRR_B$. So, project C is the best project //

Incremental Analysis:

The estimated capital investment & the annual expenses of four alternative designs of a diesel powered air compressor are shown: The study period is 5 years & the MARR is 20% per year. Based on these infos. determine the preferred design alternative using IRR on incremental investment:

Design Alternative	D_1	D_2	D_3	D_4
Capital Investment	-100000	-140600	-148000	-122000
Annual Expenses	-29000	-16900	-14800	-22100
Useful Life	5	5	5	5
Market Value	10000	14000	25600	14000

Arranging the alternatives according to their initial investment cost,

D_1, D_4, D_2 & D_3

D_1 is taken base.

Alternative	$D_4 - D_1$	$D_2 - D_4$	$D_3 - D_4$
C.I.	-22000	-18600	-26000
A.E.	6900	+ ~~7800~~ 5200	+7300
U.L	5	5	5
M.V	4000	0	11600

For $IRR_{D_4-D_1}$,

$$PW = -22000 + 6900\left(\frac{P}{A}, i\%, n\right) + 4000\left(\frac{P}{F}, i\%, n\right)$$

$$0 = -22000 + 6900\left[\frac{(1+i)^n - 1}{(1+i)^n \times i}\right] + 4000\,(1+i)^{-n}$$

$$0 = \cancel{-22000 + 6900\left[\frac{1.2^5 - 1}{1.2^n \times 0.2}\right]} + 4000\,(1.2)^{-n}$$

$$0 = -22000 + 6900\left[\frac{(1+i)^5 - 1}{(1+i)^5 \times i}\right] + 4000\,(1+i)^{-5}$$

or $i = 0.2047 = 20.47\% > MARR$

So, D_4 is accepted, D_1 is rejected.

For $IRR_{D_2-D_4}$

$$0 = -18600 + 5200\left[\frac{(1+i)^5 - 1}{(1+i)^5 \times i}\right] + 0$$

$i = 0.123 = 12.3\% < MARR$

So, D_2 is rejected, D_4 is accepted.

For $IRR_{D_3-D_4}$

$$0 = -26000 + 7300\left[\frac{(1+i)^5 - 1}{(1+i)^5 \times i}\right] + 11600\,(1+i)^{-5}$$

$\therefore i = 0.2074 = 20.74\%$

So, D_3 is accepted.

Finally, alternative D_3 is accepted.

v.Imp (B) Mutually exclusive alternatives having different useful life

Imp (a) Repeatability Assumption (L.C.M.)

Two or more alternatives having different ~~use~~ useful life are changed into the project having same useful life by expanding their life upto atleast common year.

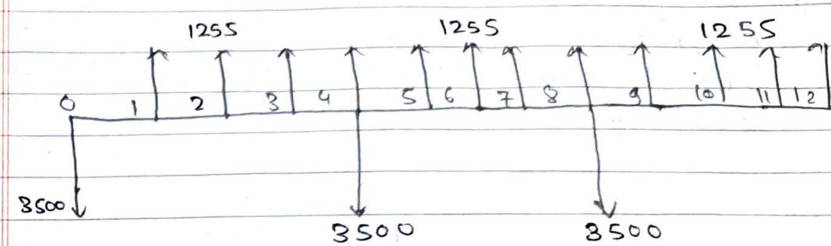
E.g. no.1

The following data have been estimated for two feasible investment X & Y having different useful life. If minimum attractive rate of return (MARR) is 10%, choose the best project using PW method. Use repeatability assumptions.

Alternative	X	Y
Capital Investment	-3500	-5000
Annual Expenses	-645	-1020
Annual Revenue	1900	2500
Useful life	4	6
Market value	0	0

L.C.M. of useful life = 12 years.

For project X,



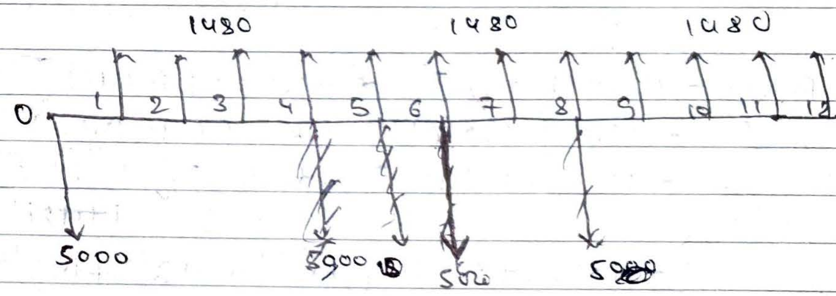
Now,

Present Worth $= -3500 - 3500(1+0.1)^{-4} - 3500(1+0.1$

$+ 1255 \left(\frac{(1+0.1)^{12} - 1}{(1+0.1)^{12} \times 0.1} \right)$

$PW_x = 1027.86$.

For project Y,



Present Worth $(PW_y) = -5000 - 5000(1+0.1)^{-6}$

$+ 1480 \left(\frac{1.1^{12} - 1}{1.1^{12} \times 0.1} \right)$

$= 2261.89$

Here, $P.W_Y > P.W_X$. So, project Y is selected

Q. Use repeatability assumption to select the best project.

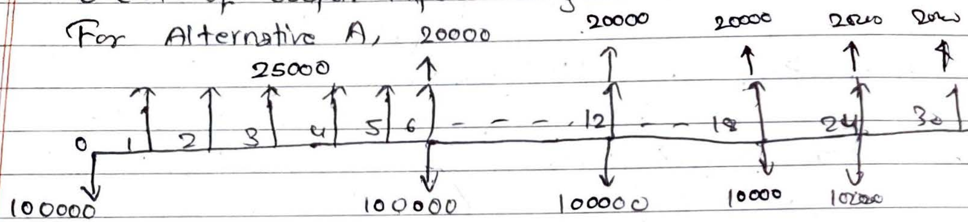
Alternative	A	B	C
Capital Investment	100000	200000	800000
~~Annual Exp.~~			
Annual Revenue	25000	30000	45000
Useful life	6	10	15
Market Value	20000	50000	7000

MARR = 12% using A.W.

Here,

L.C.M. of Useful life = 30 year

For Alternative A,



$$F.W_A = -100000(1+0.12)^{+30} - 100000(1+0.12)^{+24} - 100000(1+0.12)^{+18} - 100000(1+0.12)^{+12} - 100000(1+0.12)^{+6} + 25000\left(\frac{(1+0.12)^{30}-1}{\cancel{(1+0.12)^{30}}\,0.12}\right)$$

$$+20000 + 20000(1.12^{24} + 1.12^{18} + 1.12^{12} + 1.12^{6})$$

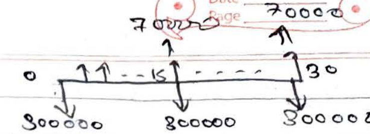
$$= 758253.42$$

For Alternative B,

$$F.W_B = -200000(1.12^{30} + 1.12^{20} + 1.12^{10}) + 30000\left(\frac{1.12^{30}-1}{0.12}\right) + 50000(1 + 1.12^{10} + 1.12^{20})$$

$$= -614825.089$$

For Alternative C



$$F.W_C = -300000(1.12^{15} + 1.12^{30}) + 450000\left(\frac{1.12^{30}-1}{0.12}\right) + 70000(1.12^{15}+1)$$

$$= 683074.03$$

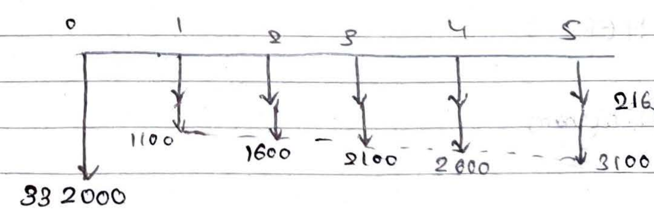
Since, $F.W_A > F.W_C > F.W_B$. So, project A is selected

Q. Two pumps of equal capacity

	Pump SP240	model HEP59
Capital Investment	-332000	-47600
Annual Expenses	-2165	~~-1760~~ -1720
Maintainence cost	-1100 in 1 year & increasing -500 per year	-500 in year 4 & increasing -100 per year
Useful life	5 years	9 years
Salvage value	0	5000

Interest rate = 20%.

For pump SP240:



$$AW_{SPF240} = -332000\left(\frac{A}{P}, i\%, n\right) - 2165 - 1100$$
$$- 500\left(\frac{A}{G}, i\%, n\right)$$

$$AW_{Investment} = -332000\left(\frac{(1+i)^n \times i}{(1+i)^n - 1}\right)$$
$$= -332000\left(\frac{1.2^5 \times 0.2}{1.2^5 - 1}\right)$$
$$= -111014.06$$

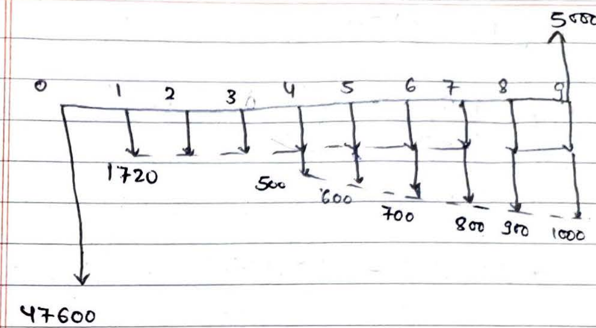
$$PW_{gradient} = \frac{G}{i^2}\left[\frac{(1+i)^n - 1 - ni}{(1+i)^n}\right]$$
$$= \frac{-500}{0.2^2}\left[\frac{1.2^5 - 1 - 0.2\times 5}{1.2^5}\right]$$
$$= -\cancel{2813.14}\ 2453.06$$

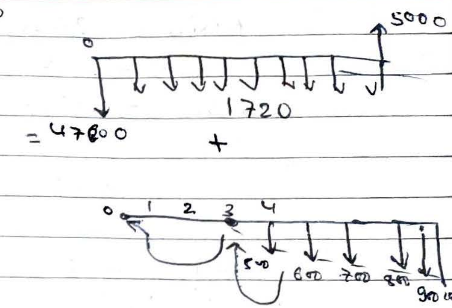
$$AW_{gradient} = \cancel{-2813.14}\ 2453.06\left(\frac{1.2^5 \times 0.2}{1.2^5 - 1}\right)$$
$$= \cancel{-940.65}\ -820.25$$

$$\therefore AW_{SPF240} = -111014.06 - 2165 - 1100 - 820.25$$
$$= \cancel{114999.31}\ -115099.31$$

For pump HEPS9

Cash Flow Diagram ⇒





$$AW = -47600\left(\frac{A}{P}, i\%, n\right) + 5000\left(\frac{A}{F}, i\%, n\right) \mp 1720\Bigg(- \Big\{\Big[500\left(\frac{P}{A}, i\%, n\right)$$
$$+ 1000\left(\frac{P}{G}, i\%, n\right)\Big]\left(\frac{P}{F}, i\%, n\right)\Big\}\left(\frac{A}{F}, i\%, n\right)\Bigg)$$

For Investment.

$$AW = -47600\left(\frac{(1+i)^n \times i}{(1+i)^n - 1}\right)$$
$$= -47600\left(\frac{1.2^9 \times 0.2}{1.2^9 - 1}\right) = -11808.58$$

$$AW_{S.V.} = 5000\left(\frac{\cancel{(1+i)}\ i}{(1+i)^n - 1}\right)$$
$$= 5000\left(\frac{0.2}{1.2^9 - 1}\right) = 240.39$$

500 + gradient in Year 3

$$PW_3 = -\left(500\left(\frac{P}{A}, i\%, n\right) + 100\left(\frac{P}{G}, i\%, n\right)\right)$$
$$= -\left(500\left(\frac{\cancel{100}\ 1.2^6 \times 0.2}{1.2^6 - 1}\right) + \frac{100}{0.2^2}\left(\frac{1.2^6 - 1 - 0.2\times}{1.2^6}\right.\right.$$
$$= \cancel{808.41}\ -2320.81$$

$PW_0 = -2320.81\,(1+0.2)^{-3}$

$= -1343.06$

$AW = -1343.06 \left(\frac{1.2^9 \times 0.2}{1.2^9 - 1} \right)$

$= -333.186$

$\therefore AW = -11808.59 + 240.39 - 333.186$

$=$

ii) Coterminated Assumption:

Case-A: Using co-terminated assumption, recommend the best project taking study period as 8 years.

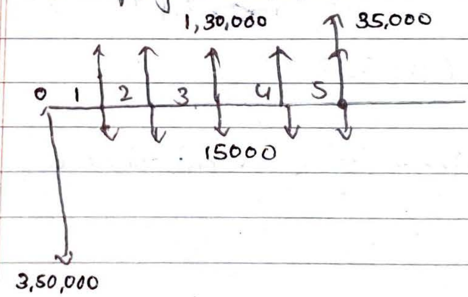
Project	A	B
Initial Investment	3,50,000	5,00,000
Annual Revenue	1,30,000	1,75,000
Annual Cost	15,000	25,000
Salvage Value	35,000	50,000
Useful life	5 years	8 years.

MARR = 10%

Here,

Study life > Useful life.

For project A,



$F.W_A = -350000 \left(\frac{F}{P}, i\%, n\right) + 130000 \left(\frac{F}{A}, i\%, n\right) - 15000 \left(\frac{F}{A}, i\%, n\right) + 35000$

$= -350000\,(1+0.1)^5 + 130000 \left(\frac{1.1^5 - 1}{0.1} \right) - 15000 \left(\frac{1.1^5 - 1}{0.1} \right) + 35000$

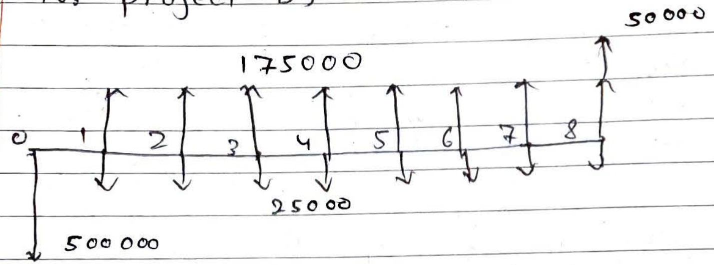
$= 173408$

$F.W_{A,8} = 173408(1+i)^3$

$= 173408 \times 1.1^3$

$= 230806.048$

For project B,



$F.W_B = -500000(1+0.1)^8 + 150000\left(\frac{(1+0.1)^8-1}{0.1}\right) + 50000$

$= \cancel{2837177.6}\ 693538.81$

Since, $FW_B > FW_A$, Project B is recommended.

Co-terminated Assumption

(b) Study life < Useful life

Imputed Market Value

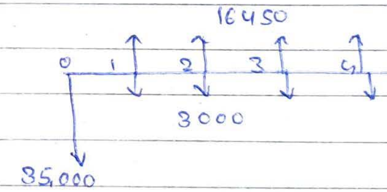
$M.V. = PW_{CR} + PW_{s.v}$

For the following information, select the best project by using F.W. method & ~~useful~~ study life is 4 year.

Project	A	B
Initial Investment	35,000	50,000
Annual Revenue	16,450	25,000
Annual Cost	3,000	13,830
Useful life	4	8
Salvage Value	0	0

MARR = 10%

For project A,

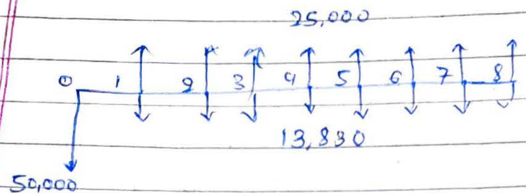


$F.W. = -35,000(F/P, i\%, n) + 13450(F/A, i\%, n)$

$= -35000(1+0.1)^4 + 13450\left(\frac{(1+0.1)^4-1}{0.1}\right)$

$= 11177.95$

For project B,



$MV = PW_{CR} + PW_{SV}$

$CR = I(A/P, i\%, n) - S\cancel{(A/F, i\%, n)}$ → 0

$= 50000 \left(\frac{(1+0.1)^8 \times 0.1}{(1+0.1)^8 - 1} \right)$

$= 9372.20$

Now, $n = 4$

$PW_{CR} = 9372.20 (P/A, i\%, n)$

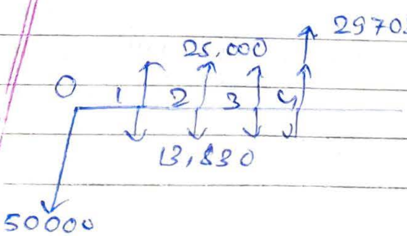
$= 9372.20 \left(\frac{(1+0.1)^4 - 1}{1.1^4 \times 0.1} \right)$

$= 29708.61$

$PW_{SV} = 0 (P/F, i\%, n)$; $n = 4$

$= 0$

$MV = 29708.61$



$FW_B = -50000(1+0.1)^4 + 11170 \left(\frac{1.1^4 - 1}{0.1} \right) + 29708.61$

$= 8343.58$

Since, $FW_A > FW_B$. So, Project A is accepted.

III. Capitalized Worth method

$C.W. = \frac{AW}{MARR}$

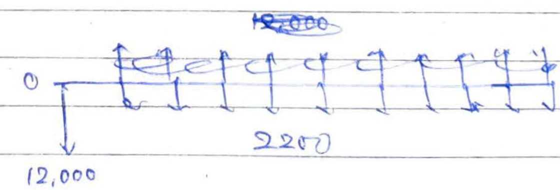
Select one among two structural design using capitalized worth method.

Project	A	B
Initial Investment	12,000	40,000
Salvage Value	0	10,000
Annual Expenses	2,200	1,000
Useful life	10	25

MARR = 15%

Solution,

For project A,

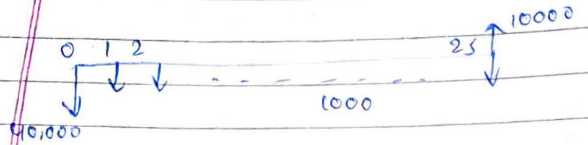


$AW_A = -12,000 \left(\frac{A}{P}, i\%, n \right) - \cancel{9800}\ 2200$

$= -12000 \left(\frac{1.15^{10} \times 0.15}{1.15^{10} - 1} \right) - \cancel{9800}\ 2200 = -4591.02$

$C.W_A = \frac{AW_A}{MARR} = -30606.83$

For project B,



$AW_B = -40000\left(\frac{1.15^{25} \times 0.15}{1.15^{25} - 1}\right) - 10000 + 10000\left(\frac{0.15}{1.15^{25} - 1}\right)$

$= -7140.98$

$CW_B = \frac{-7140.98}{MARR} = -47606.54$

Here, $CW_A > CW_B$. So, Project A is selected.

★ Mutually Exclusive Project, Independent Project, Contingent Project

Mutually Exclusive project:

Only one project can be chosen from a group of projects. E.g.:

~~Inde~~

Combination	A	B	C
Nothing	0	0	0
Accept A	1	0	0
Accept B	0	1	0
Accept C	0	0	1

Independent Project

Choice of one project is independent of the choice of any other independent project. All or none or some project can be accepted.

E.g.

Combination	A	B
Nothing	0	0
Accept A	1	0
Accept B	0	1
Accept All	1	1

Contingent Project:

The choice of project is conditional on the choice of one or more project.

E.g: Suppose there are three project & acceptance of project C and acceptence of project B is contingent upon acceptance of A.

E.g.

Combination	A	B	C
1.	0	0	0
2.	1	1	0
3.	1	0	1
4.	1	1	1
5.	1	0	0

Q. MARR = 12% and 10 years

Project	A	B1	B2	C
Capital Investment	80,000	22,000	70,000	82,000

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	A	B1	B2	C
Annual Revenue	8000	6,000	14,000	18,000
Market Value	3000	2,000	5,000	7,000

Note: B1 and B2 are mutually exclusive
C depends upon acceptance of B2
A depends upon acceptance of B1.
and, Capital Investment budget limited to 1,00,000
& use P.W.

Here,

$P.W_A = 16167.70$
$P.W_{B_1} = 12545.28$
$P.W_{B_2} = 10712.98$
$PW_C = 21957.82$

Combination	A	B1	B2	C
1.	0	0	0	0
2.	0	1	0	0
3.	0	0	1	0
4.	0	0	1	1
5.	1	1	0	0

	1	2	3	4	5
Capital Investment	0	22000	70000	152000	52000
Annual Revenue ~~Market Value~~	0	6000	14000	32000	14000
Market Value	0	2000	5000	12000	5000

X

Now,

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$P.W_1 = 0$

For combination 2,
$P.W_2 = P.W_{B_1} = 12545.28$

For combination 3,
$P.W_3 = P.W_{B_2} = 10712.98$

$\cancel{P.W_4 = P.W_{B_2}} + P.W_C = 32670.8$

$P.W_5 = \cancel{P.W_A} + P.W_{B_1}$
$= -52000 + 14000\left(\frac{1.12^{10} - 1}{1.12^{10} \times 0.12}\right) +$
$5000(1.12)^{-10}$
$= 28712.98$

Since, $P.W_5 > P.W_2 > P.W_3 > P.W_1$
So, Combination 5 i.e. Project A and B_1 are accepted //