Assignment

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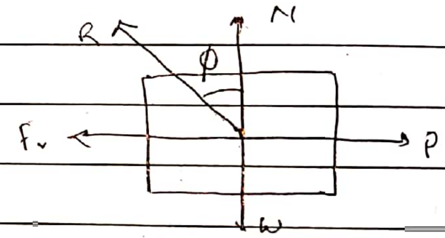
[BCT-A 019]

Friction

1. Define limiting friction, Angle of Friction & coefficient of static friction & dynamic friction.

⇒ The max^n value of friction force which comes into play along the common surface of contact bet^n the two bodies when a body is ~~function~~ just on the point of motion over the other is known as limiting friction.

Angle of friction is defined as the angle which the resultant of normal reaction & limiting frictional forces makes with the normal reactor.



Ø is angle of friction.

The coefficient of statice friction is the ratio of maximum static force (F_s) bet^n the surface in contact before movement to the Normal force (N).

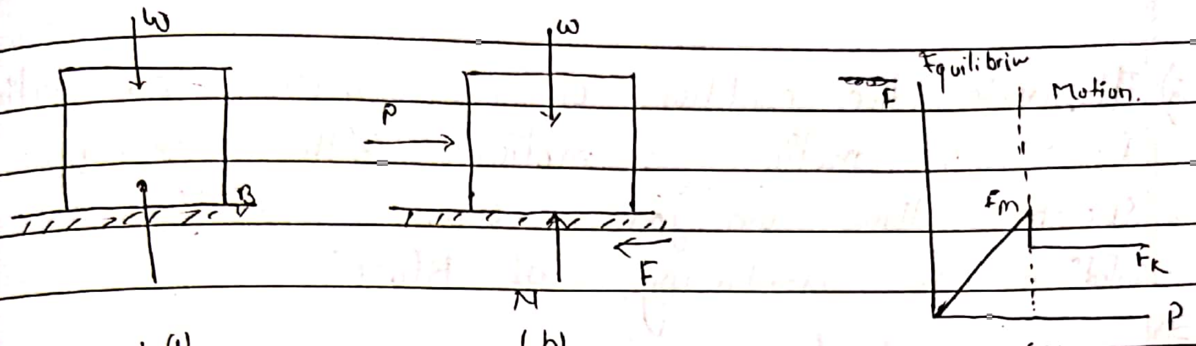
$$\mu_s = \frac{F_s}{N}.$$

The coefficient of dynamic friction is the ratio of the kinetic friction force (F_k) bet^n the surface in contact during movement to normal force (N).

ie. $\mu_k = \frac{F_k}{N}.$

2. State the law of static & dynamic friction (or Dry friction).

Laws of Dry friction.



(a) (b) (c)

a) When no horizontal force acts, No frictional force is developed

b) If P is small, the block will not move; some other horizontal force must therefore exist which balance P. This other force is the static friction force F.

c) If the force P is increased, the friction force F also increase, continuing to oppose P, until its magnitude reaches a certain max^m value F_m.

d) If P is further increased, the friction force cannot balance it any more & the block start sliding. As soon as block has been set in motion the magnitude F drops from F_m to a lower value F_k.

- From then on the block keep sliding with increasing velocity while the friction force, denoted by F_k & called the kinetic friction force remains approximately constant

2) Illustrate the conditions of no friction no motion impending motion & motion with necessary sketches. How can you assure condition of sliding or overturning of block?

=>

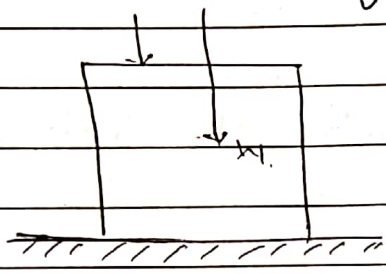


fig (a)

it represents the case of no friction Here the applied force is vertical & there is no horizontal component of force so, there is no friction.

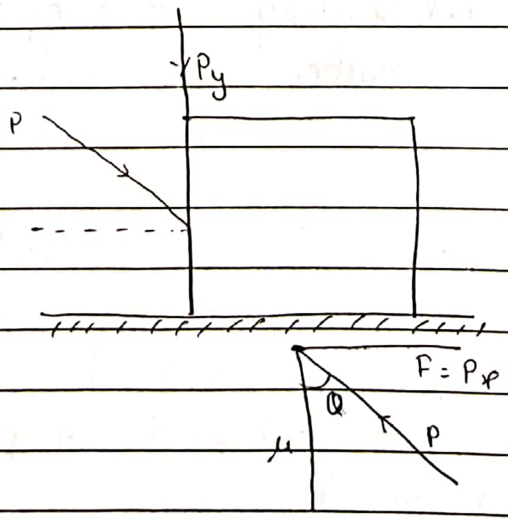
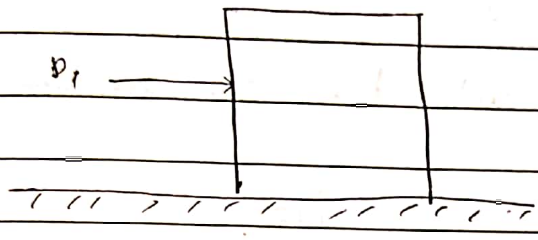


fig (b)

In fig (b) the horizontal force (P_x) is equal to the frictional force. Also, the angle betn the Normal & resultant force (R) ie. $\theta < \phi$ (Angle of friction). this is the condition of no motion.

Impending motion



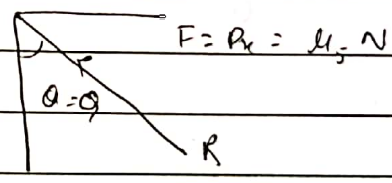


fig (c)

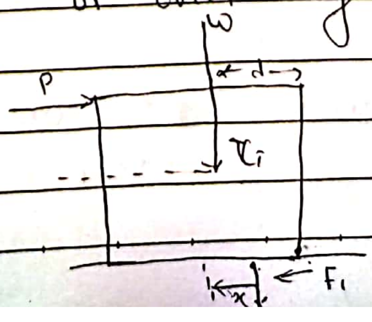
fig (c) represents the case of impending motion. Here the resultant angle (θ) is equal to the angle of friction (ϕ) &

$F = P_x = \mu_s N$

Again,

If $P_x > \mu_s N$ this represents the case of motion of block. In this case there is kinetic friction ie $\boxed{F_k = \mu_k N}$

Condition of sliding or overturning



→ check for static equilibrium condition.

$\Sigma F_y = 0 \Rightarrow N.$

$F_s = \mu_s N$

If $P > P_s \Rightarrow$ Dynamic

Determine Moment about c.g. to determine line of action of N.

$\Sigma M_g = 0 \Rightarrow$ Find x

so,

i) if $x > d$, body will tent to overturn

ii) If $x < d$; body will side.

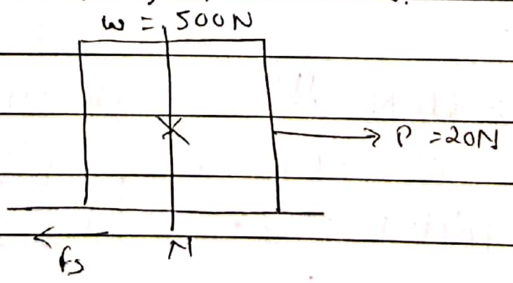
4) For the block of 500N weight resting on a surface; 20N of force in applied horizontally as shown. In the figure below. Determine the frictional force developed if:

a) $\mu_s = 0.25$.

b) $\mu_s = 0.01$ & $\mu_s = 0.008$.

⇒ soln,

Case (a)



applied force.

$P = 20N$

$N = W = 500N.$

$F_s = \mu_s N = 0.25 \times 500$

$= 125N$

Since : applied force $P(20N) < F_s(125N)$
$\therefore$ Frictional force developed is 20 N.

Case ib.

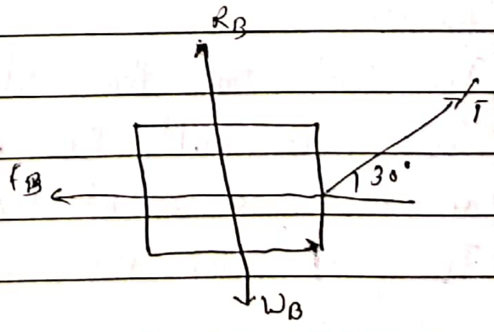
$F_s = \mu_s N$
$= 0.01 \times 500$
$= 5N$

Here applied force (20N) > 5N.
So the body is moving
Frictional force $= \mu_k N$
$= 0.008 \times 500$
$= 4N.$

5) Two blocks A & B of 40 N & 20N respectively are in equilibrium position as shown in figure below calculate the force P required to move block Take $\mu = 0.3$ for all surface.

=> Sol^n.

Step 1: Draw FBD of block A & block B.
considering block B.



$\Sigma F_y = 0$ (+ve)
$R_B + T\sin 30 - W_B = 0$
$R_B + 0.5T = 20$
$R_B = 20 - 0.5T$ —(i)

$\Sigma F_x = 0$ ($\rightarrow$ +ve)
$T\cos 30 - F_B = 0$
$T\cos 30 - \mu_s R_B = 0$
$T\cos 30 - 0.3(20 - 0.5T) = 0$ [from eq^n i]
$0.866T - 6 + 0.15T = 0$
$T = 5.905$ N.

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$R_B = 20 - 0.5 \times 5.905$

$= 17.047$ N

Then,

Considering Block A

$\Sigma f_y = 0 \ (\uparrow +ve)$

$R_A - W_A - R_B = 0$

$R_A = 40 + 11.047$

$= 57.047$ N

$\Sigma f_y = 0 \ (\rightarrow +ve)$

$F_A + F_B - P = 0$

$\mu_s R_A + \mu_s R_B - P = 0$

$\mu_s (R_A + R_B) = P$

$P = 0.3 (57.047 + 17.047)$

$= 22.22$ N

$P = 22.22$ N

6) A block of 200 kg rest on a rough horizontal plane. The force required to each of the following case

a) To just pull the block by a horizontal force P.

b) To just pull the block by inclined force P inclined at 30 at the horizontal

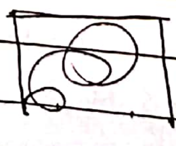
c) To just push the block by a horizontal force P.

⇒ Solⁿ,

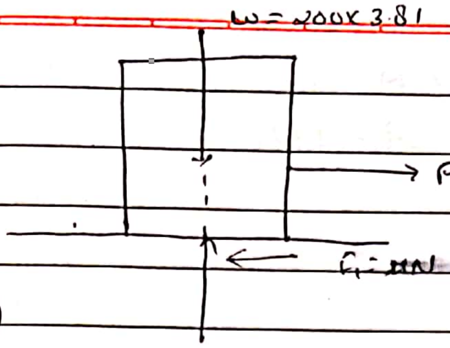
$\mu_s = 0.3$

mass of block (m) = 200 kg

$\theta = 30°$



a)



$\Sigma F_y = 0$ ($\uparrow$ +ve)

$N - W = 0$

$N = 200 \times 9.81 = 1962$ N

$\Sigma F_x = 0$ ($\rightarrow$ +ve)

$P - F_s = 0$

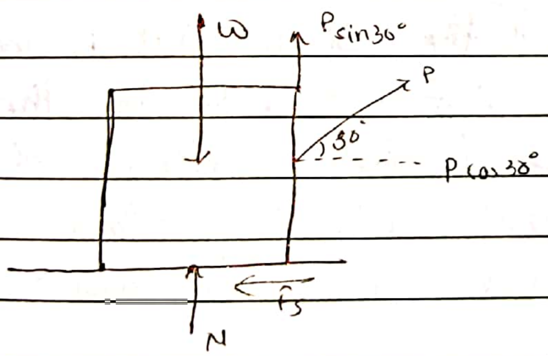
$P = \mu N = 0$

$P = 0.3 \times 1962$

$P = 588.6$ N

Required force (P) = 588.6 N

b)



$\Sigma F_y = 0$ ($\uparrow$ +ve)

$N + P\sin 30 - W = 0$

$N = W - P\sin 30°$

$N = W - 0.5P$

$\Sigma F_x = 0$ ($\rightarrow$ +ve)

$P\cos 30 - F_s = 0$

$P\cos 30 - \mu_s N = 0$

$P\cos 30 - 0.3(W - 0.5P) = 0$

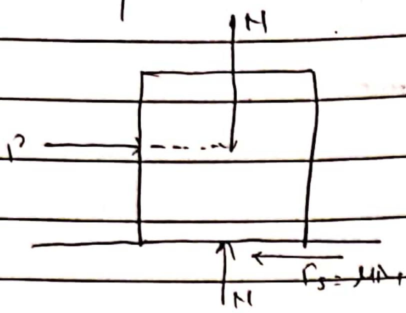
$P\cos 30 - 0.3W + 0.5P = 0$

$P = \frac{0.3 \times 1962}{1.01}$

$P = 582.77\,N$

Req. force $= 582.77N$

(c)



The process is same for question no. a, because of the transmissibility of force. P can be shifted along its line of action.

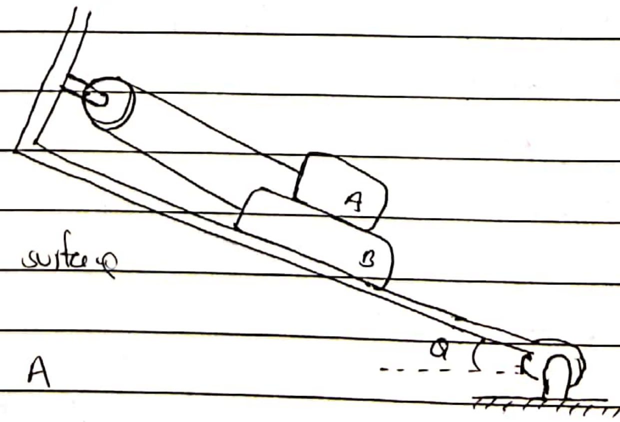
Req. forced $(P) = 588.6N$

7) The 20-lb block A & the 30lb block B are supported by an incline that is held in the position shown knowing that the coefficient of static friction is 0.15 between all the surface of contact, determine the value of θ for which the motion is impending.

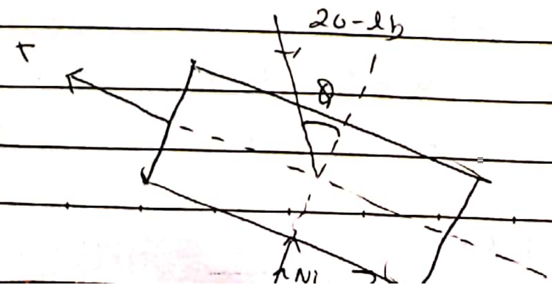
⇒ Soln,

since motion is impending

$F = \mu N$ at all surface



FBD for Block A



$\Sigma f_y = 0 \; (\uparrow +ve)$

$N_1 - 20\cos\theta = 0$

$N_1 = 20\cos\theta$

$\Sigma f_x = 0 \; (\rightarrow +ve)$

$\Rightarrow 20\sin\theta + F_1 - T = 0$

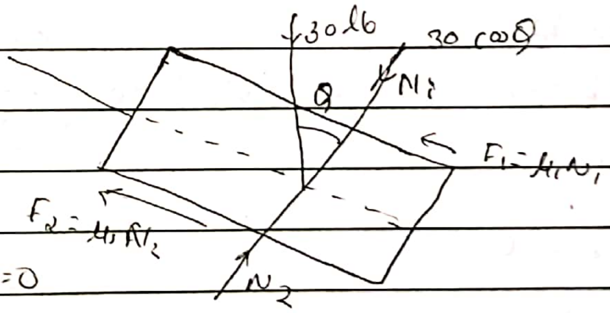
$\Rightarrow 20\sin\theta + \mu_s N_1 - T = 0$

$\Rightarrow 20\sin\theta + 0.15 \times N - T = 0$

$\Rightarrow 20\sin\theta + 0.15 \times 20\cos\theta = T$

$$T = 20\sin\theta + 3\cos\theta \longrightarrow i)$$

Free Body Diagram of Block B



$\Sigma f_y = 0 \; (\uparrow +ve)$

$N_2 - 30\cos\theta - N_1 = 0$

$N_2 - 30\cos\theta - 20\cos\theta = 0$

$N_2 = 50\cos\theta$

$\Sigma f_x = 0 \; (\xrightarrow{+ve})$

$30\sin\theta - T - f_2 - F_1 = 0$

$30\sin\theta - \mu_s N_2 - \mu_s N_1 = T$

$30\sin\theta - 0.15 \times 50\cos\theta - 0.15 \times 20\cos\theta = T$

$30\sin\theta - 7.5\cos\theta - 3\cos\theta = T$

$30\sin\theta - 10.5\cos\theta = T \quad T^{ii)}$

from eq^n (i) & (ii)

$20\sin\theta + 3\cos\theta = 30\sin\theta - 10.5\cos\theta$

$13.5\cos\theta = 10\sin\theta$

$\tan\theta = \dfrac{13.5}{10}$

$$\theta = \tan^{-1}\left(\frac{13.5}{10}\right)$$

$$\boxed{\theta = 53.47^{\circ}}$$

8) Two blocks A & B which are identical W are supported by a rod inclined as 45° to the horizontal as shown in the fig. If the block are in limiting equilibrium, find the coeff. of friction assuming it to be the same at both floor & the wall

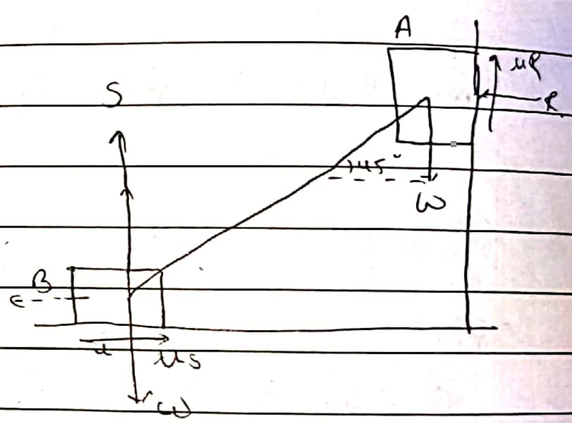
=> sol^n.

the blocks are in limiting eq^n.

we can use the eq^n. condition

i.e. $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_z = 0$

Then



$\xrightarrow{+}$

$$\Sigma F_x = 0$$

$$\mu S - R = 0$$

$$R = \mu S \quad \text{---(i)}$$

$$\Sigma F_y = 0 \quad (\uparrow + ve)$$

$$S - W - W + \mu R = 0$$

$$S = 2W - \mu R$$

$$S = 2W - \mu(\mu S)$$

$$S + \mu^2 S = 2W$$

$$S(1+\mu^2) = 2W$$

$$S = \frac{2W}{1+\mu^2} \quad \text{---(ii)}$$

From eq^n (i) ;

$R = \mu w.$

$R = \mu \times \frac{2w}{1+\mu^2} = \left(\frac{2\mu w}{1+\mu^2}\right)$ —(iii)

+ve ↺

$\Sigma M_B = 0$

from figure

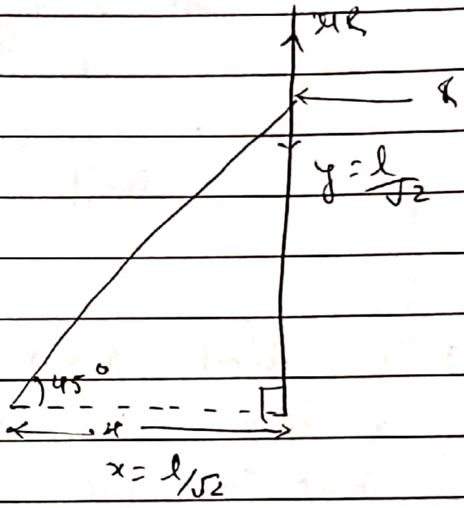
$\sin 45 = \frac{x}{l}$

$x = l \sin 45°$

$= l \times \frac{1}{\sqrt{2}} = \frac{l}{\sqrt{2}}$

$\cos 45 = \frac{y}{l}$

$y = l \cos 45 = \frac{l}{\sqrt{2}}$



$\cos 45 = \frac{y}{l}$

$y = l \cos 45° = \frac{l}{\sqrt{2}}$

Then,

$\Sigma M_B = 0$ ↺ +ve

$w \times \frac{l}{\sqrt{2}} - R \times \frac{l}{\sqrt{2}} - \mu R \times \frac{l}{\sqrt{2}} = 0$

$w - \mu R - R = 0$

$w = R(1+\mu)$

$R = \frac{w}{1+\mu}$ —(iv)

Then from eq^n (iii) & (iv)

$\frac{2\mu \cancel{w}}{1+\mu^2} = \frac{\cancel{w}}{1+\mu}$

or $\dfrac{2\mu}{1+\mu^2} = \dfrac{1}{1+\mu}$

$2\mu(1+\mu) = 1+\mu^2$

$2\mu + 2\mu^2 = 1+\mu^2$

$2\mu + 2\mu^2 - \mu^2 - 1 = 0$

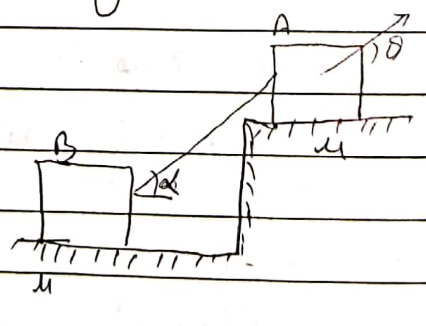
$\mu^2 + 2\mu - 1 = 0$

On solving we get,

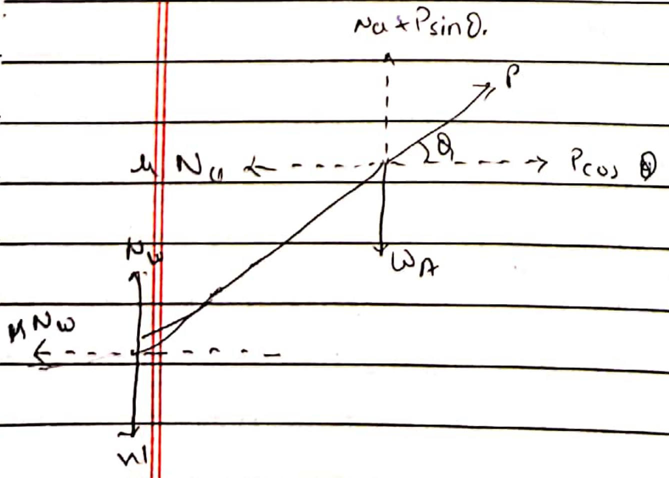
$\mu = \sqrt{2} - 1 = 0.41$

coeff. of friction $(\mu) = 0.41$ #

9) Two blocks A & B weighting W_a & W_b connected by a string rests on rough horizontal floors as shown in fig. find the magnitude & direction of the least force P that has to be applied on the upper block A, so as to make it slide toward right. the coeff. of friction at both supporting surface is. connecting string makes angle α with horizontal.



⇒ Sol^n.



$N_a + P\sin\theta = W_A$

$N_a = W_A - P\sin\theta$

Using eq^n conditions.

$$\Sigma F_x = 0 \quad (\xrightarrow{+ve})$$

$\Rightarrow P\cos\theta - \mu N_a - \mu N_b = 0$

$\Rightarrow P\cos\theta - \mu(W_a - P\sin\theta) - \mu W_b = 0$

$\Rightarrow P\cos\theta - \mu W_a - \mu W_b + \mu P\sin\theta = 0$

$\Rightarrow P(\cos\theta + \mu\sin\theta) = \mu(W_a + W_b)$

Let,

$\mu = \tan\phi$; where ϕ = angle of friction

Then,

$$P\left(\cos\theta + \frac{\sin\phi}{\cos\phi}\sin\theta\right) = \frac{\sin\phi}{\cos\phi}(W_a + W_b)$$

$$\Rightarrow P\left(\frac{\cos\theta\cos\phi + \sin\phi\sin\theta}{\cancel{\cos\phi}}\right) = \frac{\sin\phi}{\cancel{\cos\phi}}(W_a + W_b)$$

$$\Rightarrow P\cos(\theta - \phi) = \sin\phi(W_a + W_b)$$

$$P = \frac{\sin\phi(W_a + W_b)}{\cos(\theta - \phi)}$$

minimum value of P:

-> For P to be minimum

$\cos(\theta - \phi)$ is maximum

i.e. $\cos(\theta - \phi) = 1 = \cos 0°$

$\therefore \theta - \phi = 0$

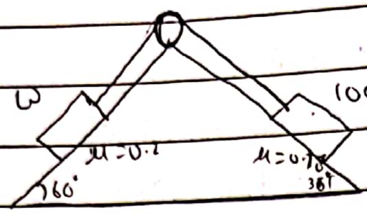
$\theta = \phi$

$P = \sin\phi\,(W_a + W_b)$

$\phi = \phi$ where

ϕ = angle of friction.

11. Determine the range of unknown wt. 'W' for which the system is in equilibrium.



⇒ Here

$\mu_1 = 0.2$

$\mu_2 = 0.28$

$W_2 = 1000N$

case I

upward motion of Block A

$\Sigma f_y = 0$ (↑ +ve)

$\Rightarrow N_2 - 866.02 = 0$

$\therefore N_2 = 866.02N$

$\Sigma f_x = 0$ ($\xrightarrow{+ve}$)

$\Rightarrow 500 - T - \mu_2 N_2 = 0$

$\Rightarrow 500 - 0.28 \times 866.02 = T$

$\therefore T = 257.5N$

$\Sigma f_y = 0$ (↑ +ve)

$\Rightarrow N_1 - W\cos 60° = 0$

$\therefore N_1 = W\cos 60$

$\Sigma f_x \xrightarrow{+} = 0.$

$T - (W\sin 60° + \mu_1 N_1) = 0$

$\Rightarrow 257.5 - (W\sin 60 + 0.2 \times W\cos 60°) = 0$

$\Rightarrow 257.5 - W\sin 60 - 0.2W\cos 60 = 0$

$\Rightarrow 257.5 - 0.866W - 0.1 \times W = 0$

$\therefore W = \dfrac{257.5}{0.866 + 0.1}$

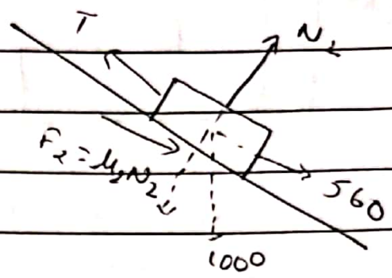
$= 266.0 + W$

Case II Downward motion of block A.

$\Rightarrow \Sigma F_y = 0 \; (\uparrow +ve)$

$N_2 - 866.02 = 0$

$\therefore N_2 = 866.02\,N$



FBD of 2nd block

$\Sigma F_x = 0 \; (\xrightarrow{+ve})$

$\Rightarrow 500 + \mu_1 N_2 - T = 0$

$\Rightarrow 500 + 0.28 \times 866.02 = T$

$\therefore T = 742.48\,N$

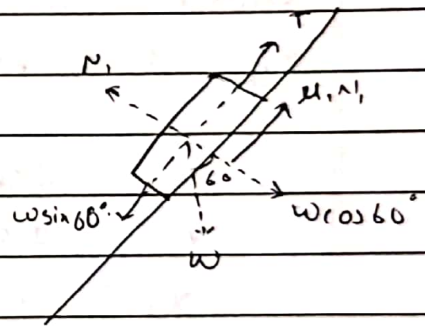
Now,

$\Sigma F_y = 0 \; (\uparrow +ve)$

$\Rightarrow N_1 - W\cos 60 = 0$

$\Rightarrow N_1 = W\cos 60$

$\therefore N_1 = 0.5\,W$



FBD of 1st block

$\Sigma F_x = 0 \; (\xrightarrow{+ve})$

$\Rightarrow T + \mu_1 N_1 - W\sin 60 = 0$

$\Rightarrow 742.48 + 0.2 \times 0.5W - W \times 0.866 = 0$

$\Rightarrow 742.48 + 0.1W - 0.866W = 0$

$\Rightarrow W = \dfrac{742.48}{0.766}$

$\therefore W = 969.29\,N$

$\boxed{\therefore \text{Range of } W : 266.04\,N \text{ to } 969.29\,N}$