First order differential equations Reducible to homogeneous form

An equation of the form $\frac{dy}{dx} = \frac{ax+by+c}{Ax+By+C}$,then it can be solved by the following ways:

1. If $\frac{a}{A} = \frac{b}{B} = \frac{1}{l}$, then the equation can be easily reduced to the form

$$\frac{dy}{dx} = \frac{ax+by+c}{l(ax+by)+C} \qquad (1)$$

And put $ax + by = v$,

Diff. w.r.t. x we get;

$$a + b\frac{dy}{dx} = \frac{dv}{dx}$$

Putting the values in (1) and separating the variables after integrating we get the required solution.

2. If $\frac{a}{A} \neq \frac{b}{B}$ then the equation can be reduced to the homogeneous form by putting

$$x = X + h, y = Y + k,$$

Where $h\ and\ k$ are constants and the terms of the equation $h\ and\ k$ can be chosen in such a way that the differential equation should be homogeneous.

<u>Exercise -22</u>

Solve the differential equations;

1. $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$

Solution:

Given differential equation is

$$\frac{dy}{dx} = \frac{x+y+1}{x+y-1} \qquad (1)$$

Comparing it with $\frac{dy}{dx} = \frac{ax+by+c}{Ax+By+C}$ we get

$$a = 1, b = 1, c = 1, A = 1, B = 1, C = -1$$

Here $\frac{a}{A} = \frac{b}{B} = \frac{1}{l}$

$$i.e\ \frac{a}{A} = \frac{b}{B}$$

$$i.e\ \frac{1}{1} = \frac{1}{1}$$

So put $x + y = v,$

Diff. w.r.t. x we get;

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

Now equation (1) becomes

$$\frac{dv}{dx} - 1 = \frac{v+1}{v-1}$$

Or, $\frac{dv}{dx} = \frac{v+1}{v-1} + 1$

Or, $\frac{dv}{dx} = \frac{v+1+v-1}{v-1}$

Or, $\frac{dv}{dx} = \frac{2v}{v-1}$

Or, $\frac{v-1}{v}\ dv = 2dx$; Integrating on both sides

Or, $\int \frac{v-1}{v}\ dv = 2\int dx$

Or, $\int \frac{v}{v}\ dv - \int \frac{1}{v}\ dv = 2\int dx$

Or, $\int dv - \int \frac{1}{v}\ dv = 2\int dx$

Or, $v - logv = 2x + c$

Since $x + y = v$

Or, $x + y - log(x + y) = 2x + c$

Or, $y - log(x + y) = x + c$

$$y - x - c = log(x + y)$$

Which is the required solution.