Engineering Economics

Lecture 4

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Chapter 5
Understanding Money and Its Management

- Nominal and Effective Interest Rates
- Equivalence Calculations
- Changing Interest Rates
- Debt Management

Focus

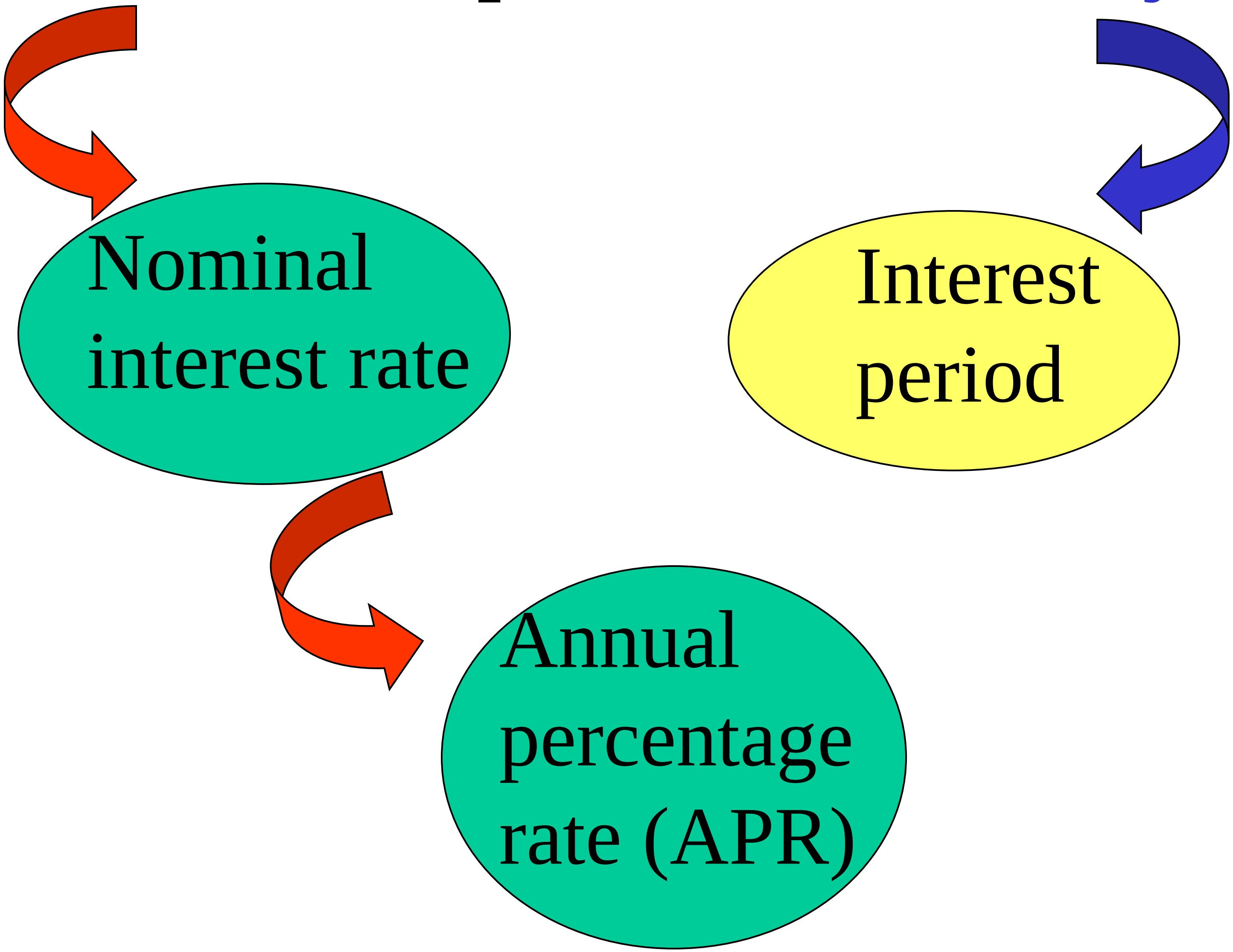
1. If payments occur more frequently than annual, how do we calculate economic equivalence?
2. If interest period is other than annual, how do we calculate economic equivalence?
3. How are commercial loans structured?
4. How should you manage your debt?

Nominal Versus Effective Interest Rates

Nominal Interest Rate:
Interest rate quoted based on an annual period

Effective Interest Rate:
Actual interest earned or paid in a year or some other time period

18% Compounded Monthly



Effective Annual Interest Rate

$$i_a = (1 + r / M)^M - 1$$

r = nominal interest rate per year
i_a = effective annual interest rate
M = number of interest periods per year

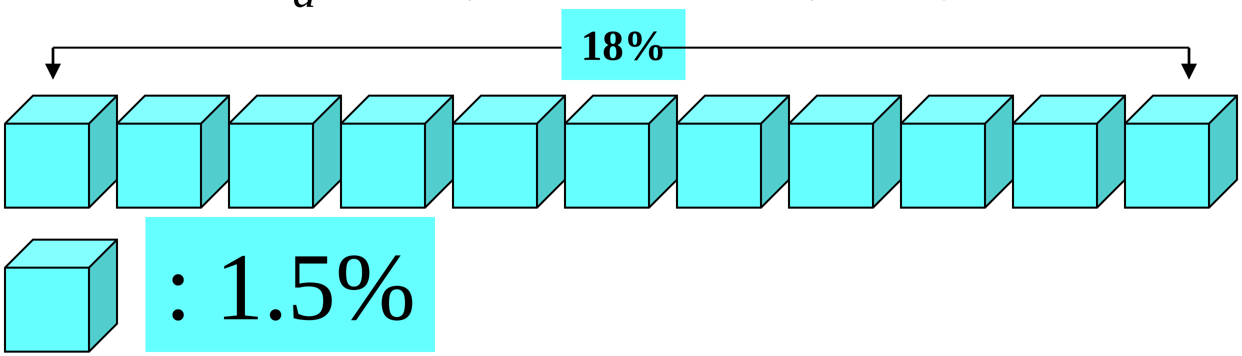
18% compounded monthly

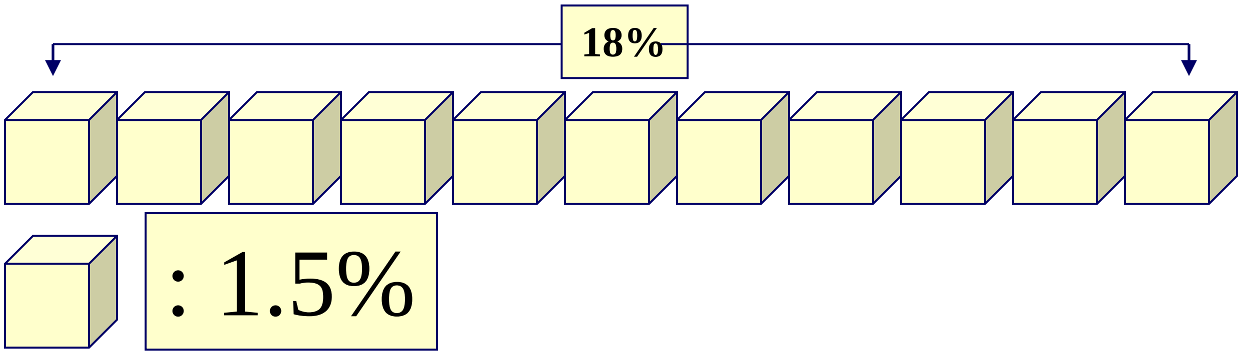
- Question: Suppose that you invest \$1 for 1 year at 18% compounded monthly. How much interest would you earn?
- Solution:

$$F = \$1(1+i)^{12} = \$1(1+0.015)^{12}$$

$$= \$1.1956$$

$$i_a = 0.1956 \text{ or } 19.56\%$$





18% compounded **monthly**

or

1.5% per month for **12 months**

=

19.56 % compounded **annually**

Nominal and Effective Interest Rates with Different Compounding Periods

Effective Rates					
Nominal Rate	**Compounding Annually**	**Compounding Semi-annually**	**Compounding Quarterly**	**Compounding Monthly**	**Compounding Daily**
4%	**4.00%**	**4.04%**	**4.06%**	**4.07%**	**4.08%**
5	**5.00**	**5.06**	**5.09**	**5.12**	**5.13**
6	**6.00**	**6.09**	**6.14**	**6.17**	**6.18**
7	**7.00**	**7.12**	**7.19**	**7.23**	**7.25**
8	**8.00**	**8.16**	**8.24**	**8.30**	**8.33**
9	**9.00**	**9.20**	**9.31**	**9.38**	**9.42**
10	**10.00**	**10.25**	**10.38**	**10.47**	**10.52**
11	**11.00**	**11.30**	**11.46**	**11.57**	**11.62**
12	**12.00**	**12.36**	**12.55**	**12.68**	**12.74**

Effective Annual Interest Rates (9% compounded quarterly)

First quarter	**Base amount** **+ Interest (2.25%)**	**$10,000** **+ $225**
Second quarter	**= New base amount** **+ Interest (2.25%)**	**= $10,225** **+$230.06**
Third quarter	**= New base amount** **+ Interest (2.25%)**	**= $10,455.06** **+$235.24**
Fourth quarter	**= New base amount** **+ Interest (2.25 %)** **= Value after one year**	**= $10,690.30** **+ $240.53** **= $10,930.83**

Effective Interest Rate per Payment Period (i)

$$\boxed{i = [1 + r / CK]^C - 1}$$

C = number of interest periods per payment period

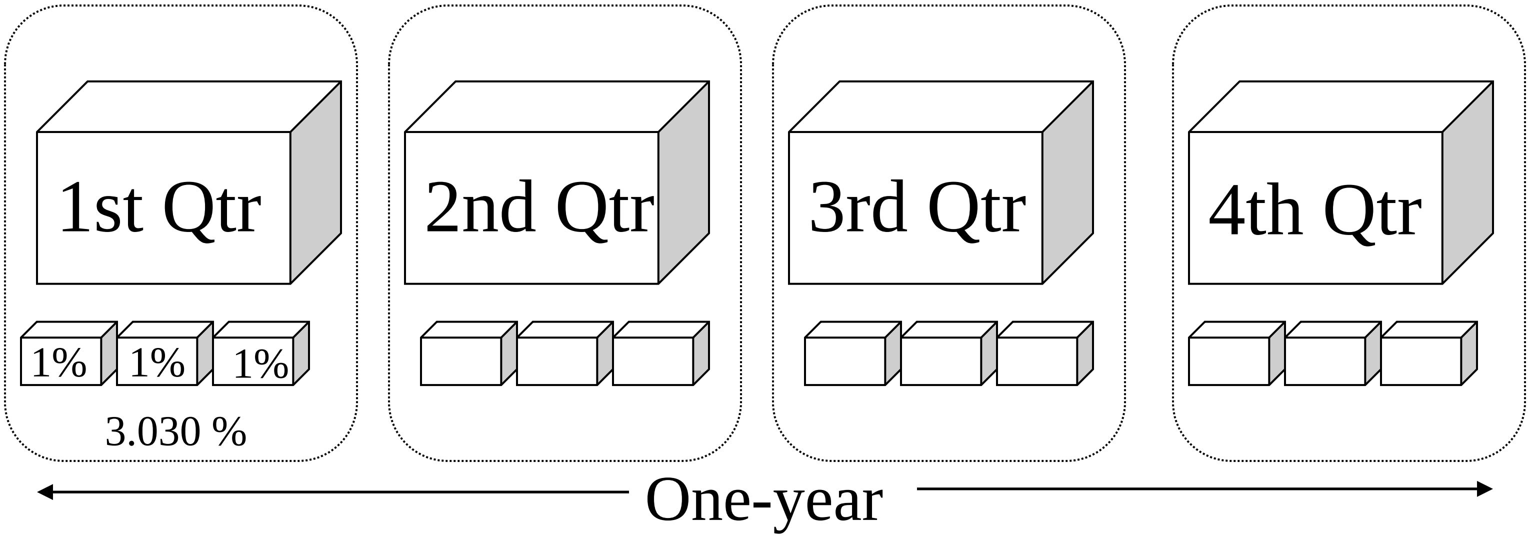
K = number of payment periods per year

r/K = nominal interest rate per payment period

12% compounded monthly

Payment Period = Quarter

Compounding Period = Month



- **Effective interest rate per quarter**

$$i = (1 + 0.01)^3 - 1 = 3.030\%$$

- **Effective annual interest rate**

$$i_a = (1 + 0.01)^{12} - 1 = 12.68\%$$

$$i_a = (1 + 0.03030)^4 - 1 = 12.68\%$$

Effective Interest Rate per Payment Period with Continuous Compounding

$$\boxed{i = [1 + r/CK]^{C} - 1}$$

where CK = number of compounding periods per year

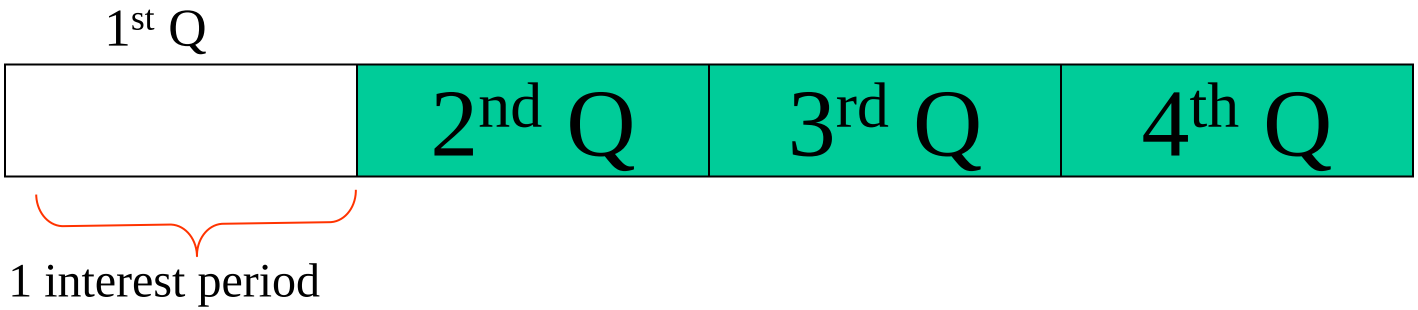
continuous compounding =>

$$\begin{aligned} i &= \lim_{C \to \infty} [(1 + r/CK)^{C} - 1] \\ &= (e^{r})^{1/K} - 1 \end{aligned}$$

Case 0: 8% compounded quarterly

Payment Period = Quarter

Interest Period = Quarterly



Given r = 8%,

K = 4 payments per year

C = 1 interest periods per quarter

M = 4 interest periods per year

$$i = [1 + r / CK]^C - 1$$

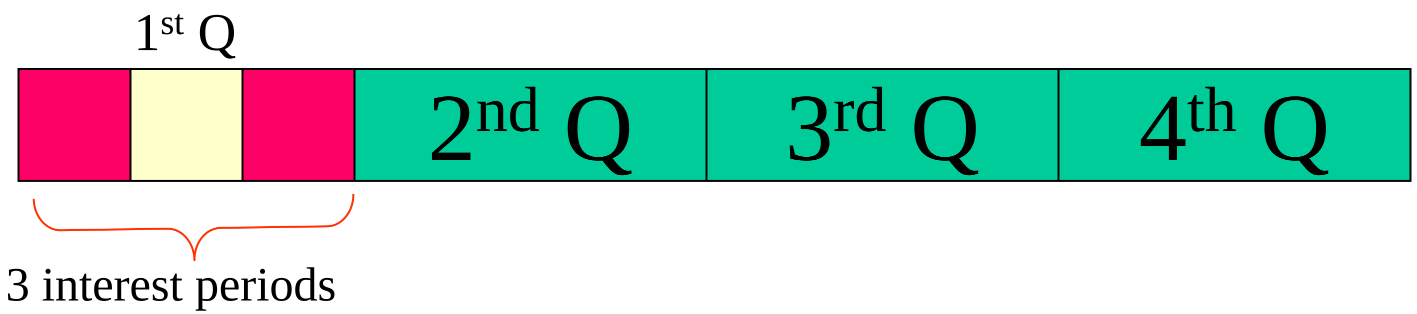
$$= [1 + 0.08/(1)(4)]^1 - 1$$

$$= 2.000\% \text{ per quarter}$$

Case 1: 8% compounded monthly

Payment Period = Quarter

Interest Period = Monthly



Given r = 8%,

K = 4 payments per year

C = 3 interest periods per quarter

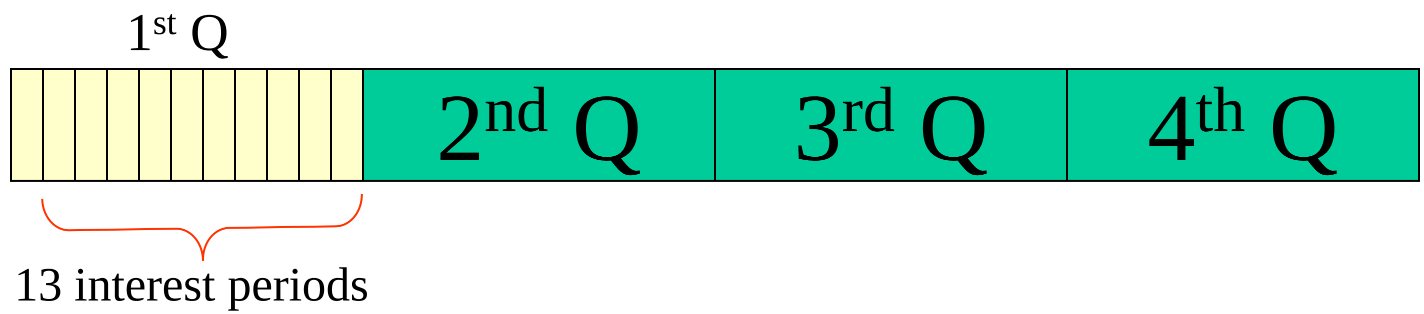
M = 12 interest periods per year

$$\begin{aligned} i &= [1 + r / CK]^{C} - 1 \\ &= [1 + 0.08 / (3)(4)]^{3} - 1 \\ &= 2.013\% \text{ per quarter} \end{aligned}$$

Case 2: 8% compounded weekly

Payment Period = Quarter

Interest Period = Weekly



Given r = 8%,

K = 4 payments per year

C = 13 interest periods per quarter

M = 52 interest periods per year

$$i = [1 + r / CK]^C - 1$$

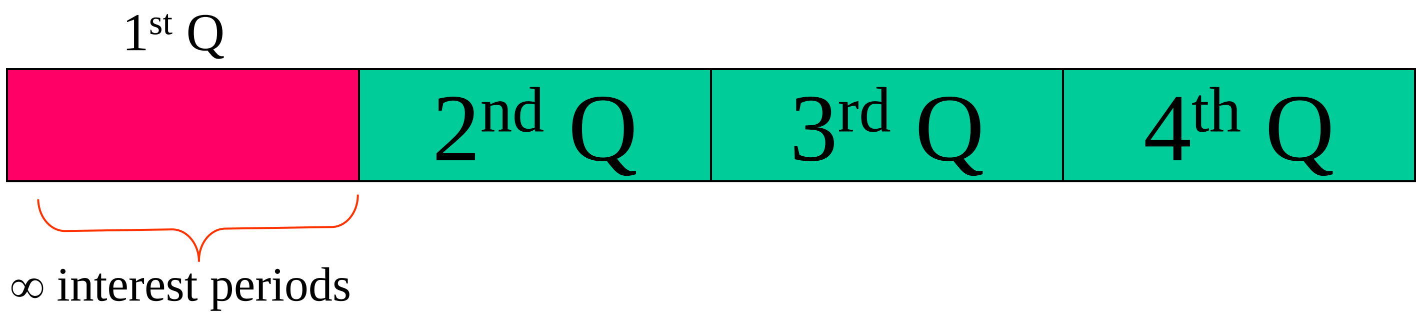
$$= [1 + 0.08 / (13)(4)]^{13} - 1$$

$$= 2.0186\% \text{ per quarter}$$

Case 3: 8% compounded continuously

Payment Period = Quarter

Interest Period = Continuously



Given $r = 8\%$,

$K = 4$ payments per year

$$
\begin{aligned}
i &= e^{r/K} - 1 \\
&= e^{0.02} - 1 \\
&= 2.0201\% \text{ per quarter}
\end{aligned}
$$

Summary: Effective interest rate per quarter

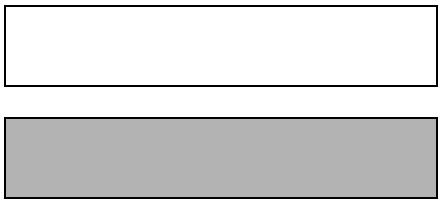
Case 0	Case 1	Case 2	Case 3
8% compounded quarterly	8% compounded monthly	8% compounded weekly	8% compounded continuously
Payments occur quarterly	Payments occur quarterly	Payments occur quarterly	Payments occur quarterly
2.000% per quarter	2.013% per quarter	2.0186% per quarter	2.0201% per quarter

Equivalence Analysis using Effective Interest Rate

- Step 1: Identify the payment period (e.g., annual, quarter, month, week, etc)
- Step 2: Identify the interest period (e.g., annually, quarterly, monthly, etc)
- Step 3: Find the effective interest rate that covers the payment period.

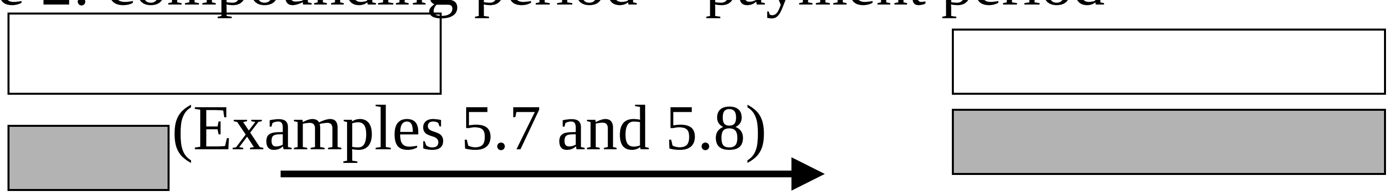
Principle: Find the effective interest rate that covers the payment period

Case 1: compounding period = payment period



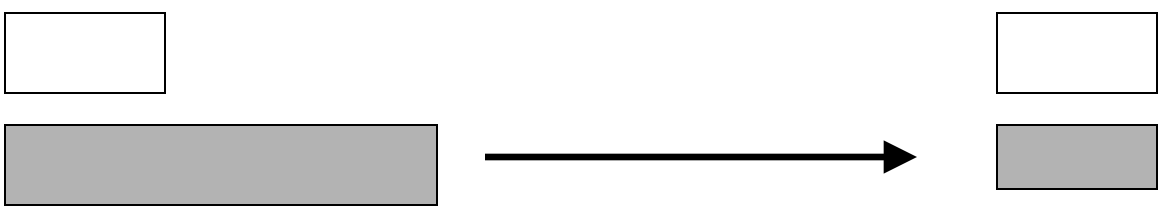
(Example 5.5)

Case 2: compounding period < payment period



(Examples 5.7 and 5.8)

Case 3: compounding period > payment period



(Example 5.9)

Case I: When Payment Periods and Compounding periods coincide

Step 1: Identify the number of compounding periods (M) per year

Step 2: Compute the effective interest rate per payment period (i)

$$i = r/M$$

Step 3: Determine the total number of payment periods (N)

$$N = M \text{ (number of years)}$$

Step 4: Use the appropriate interest formula using i and N above

Example 5.5: Calculating Auto Loan Payments

Given:

- Invoice Price = $21,599
- Sales tax at 4% = $21,599 (0.04) = $863.96
- Dealer's freight = $21,599 (0.01) = $215.99
- Total purchase price = $22,678.95
- Down payment = $2,678.95
- Dealer's interest rate = 8.5% APR
- Length of financing = 48 months
- Find: the monthly payment

Example 5.5: Payment Period = Interest Period

Given: P = \$20,000, r = 8.5% per year

K = 12 payments per year

N = 48 payment periods

Find A

- Step 1: $M = 12$
- Step 2: $i = r/M = 8.5\%/12 = 0.7083\%$ per month
- Step 3: $N = (12)(4) = 48$ months
- Step 4: $A = \$20{,}000(A/P, 0.7083\%, 48) = \$492.97$

Dollars Up in Smoke

What three levels of smokers who bought cigarettes every day for 50 years at $1.75 a pack would have if they had instead banked that money each week:

Level of smoker	Would have had
1 pack a day	$169,325
2 packs a day	$339,650
3 packs a day	$507,976

Note: Assumes constant price per pack, the money banked weekly and an annual interest rate of 5.5%

Source: USA Today, Feb. 20, 1997

Sample Calculation: One Pack per Day

Step 1: Determine the effective interest rate per payment period.

Payment period = weekly

"5.5% interest compounded weekly"

$i = 5.5\%/52 = \boxed{0.10577\% \text{ per week}}$

Step 2: Compute the equivalence value.

Weekly deposit amount

$A = \$1.75 \text{ x } 7 = \boxed{\$12.25 \text{ per week}}$

Total number of deposit periods

$N = (52 \text{ weeks/yr.})(50 \text{ years})$

$= \boxed{2600 \text{ weeks}}$

$F = \$12.25\ (F/A,\ 0.10577\%,\ 2600) = \boxed{\$169{,}325}$

Case II: When Payment Periods Differ from Compounding Periods

Step 1: Identify the following parameters

M = No. of compounding periods

K = No. of payment

C = No. of interest periods per payment period

Step 2: Compute the effective interest rate per payment period

- For discrete compounding

$$i = [1 + r / CK]^C - 1$$

- For continuous compounding

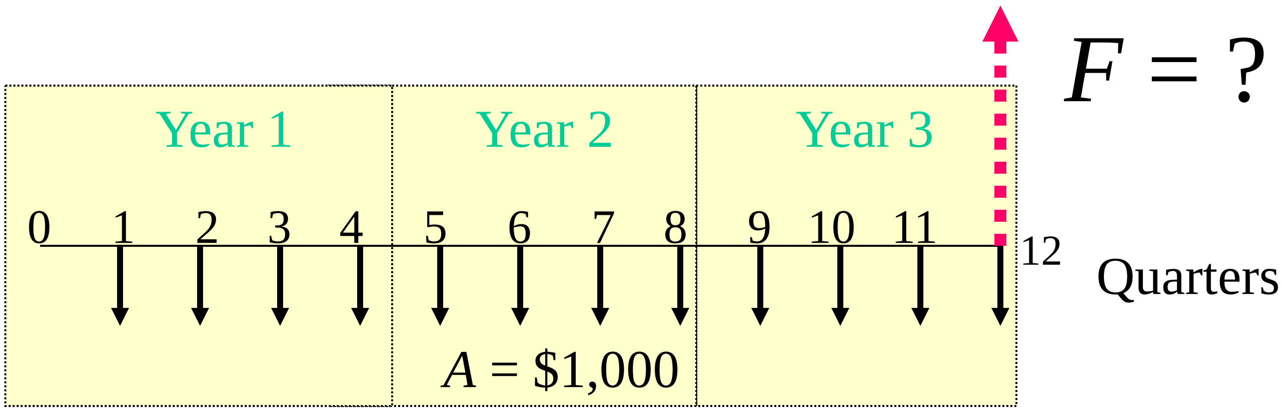
$$i = e^{r/K} - 1$$

Step 3: Find the total no. of payment periods

$N = K$ (no. of years)

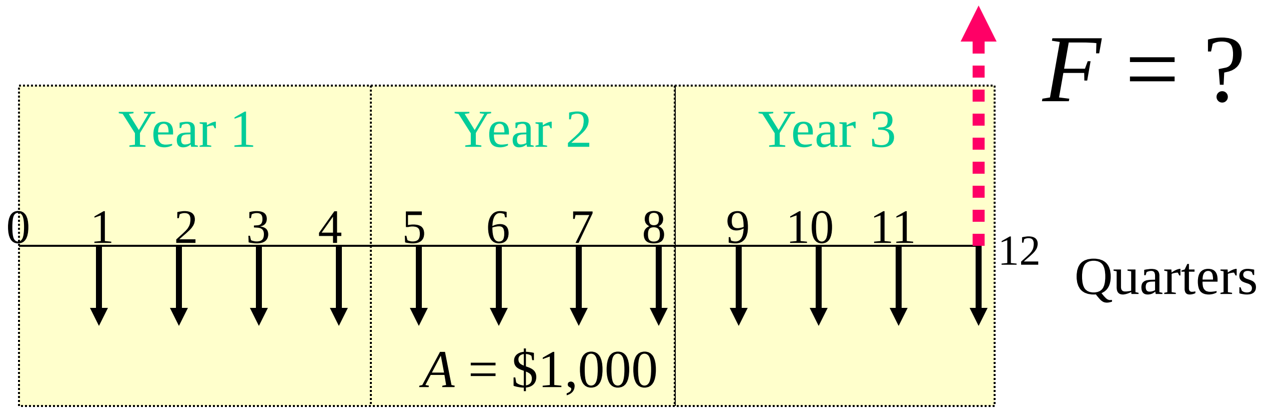
Step 4: Use i and N in the appropriate equivalence formula

Discrete Case: Quarterly deposits with Monthly compounding



- Step 1: M = 12 compounding periods/year
  K = 4 payment periods/year
  C = 3 interest periods per quarter
- Step 2: $i = [1 + 0.12/(3)(4)]^3 - 1$
  $= 3.030\%$
- Step 3: N = 4(3) = 12
- Step 4: F = $1,000 ($F/A$, 3.030%, 12)
  = $14,216.24

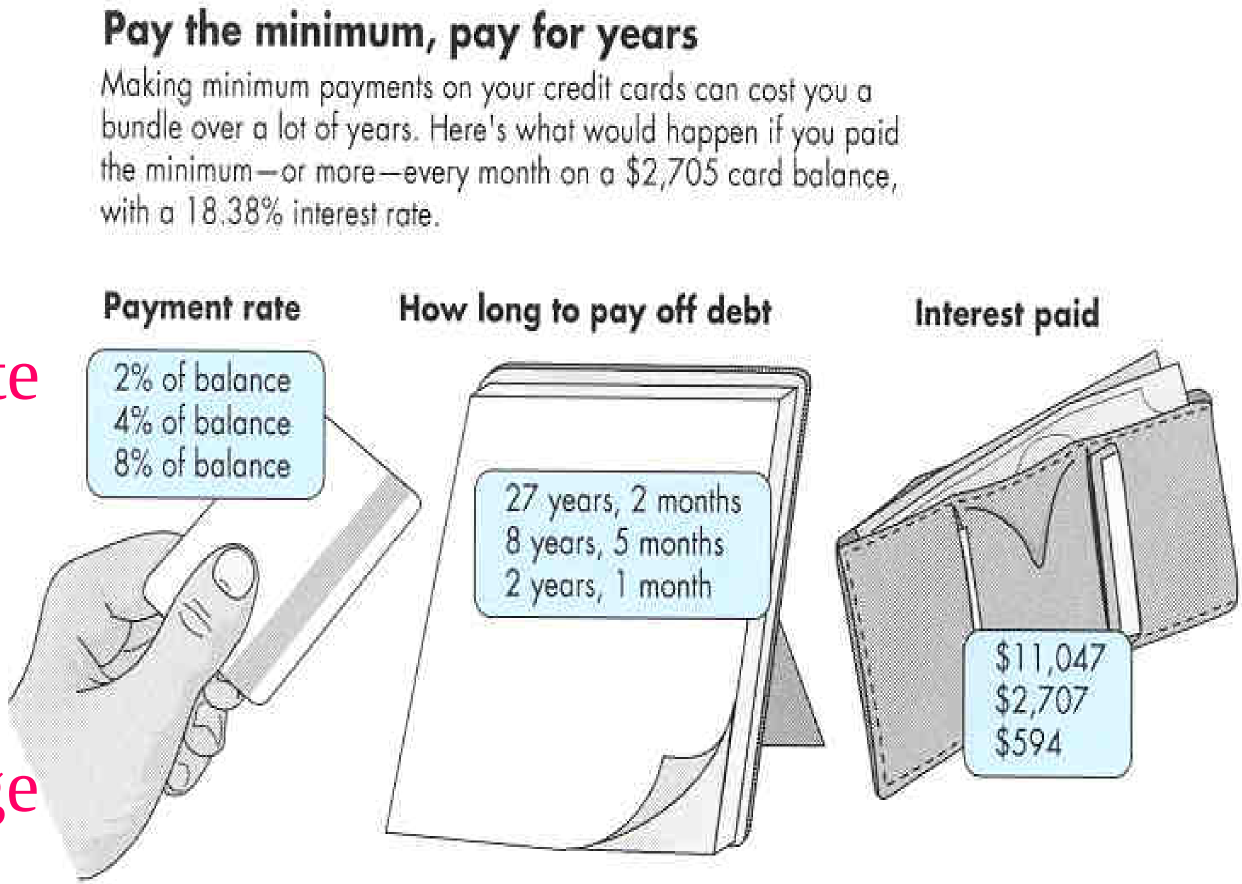
Continuous Case: Quarterly deposits with Continuous compounding



- Step 1: $K = 4$ payment periods/year
  $C = \infty$ interest periods per quarter
- Step 2: $i = e^{0.12/4} - 1$
  $= 3.045\%$ per quarter
- Step 3: $N = 4(3) = 12$
- Step 4: $F = \$1,000\ (F/A, 3.045\%, 12)$
  $= \$14,228.37$

Credit Card Debt

- Annual fees
- Annual percentage rate
- Grace period
- Minimum payment
- Finance charge



(Source: *USA Today*, April 21, 1998, © *USA Today*, used with permission)

Methods of Calculating Interests on your Credit Card

Method	Description	Interest You Owe
Adjusted Balance	The bank subtracts the amount of your payment from the beginning balance and charges you interest on the remainder. This method costs you the least.	Your beginning balance is $3,000. With the $1,000 payment, your new balance will be $2,000. You pay 1.5% on this new balance, which will be $30.
Average Daily Balance	The bank charges you interest on the average of the amount you owe each day during the period. So the larger the payment you make, the lower the interest you pay.	Your beginning balance is $3,000. With your $1,000 payment at the 15th day, your balance will be reduced to $2,000. Therefore, your average balance will be (1.5%)($3,000+$2,000)/2=$37.50.
Previous Balance	The bank does not subtract any payments you make from your previous balance. You pay interest on the total amount you owe at the beginning of the period. This method costs you the most.	Regardless of your payment size, the bank will charge 1.5% on your beginning balance $3,000: (1.5%)($3,000)=$45.

Commercial Loans

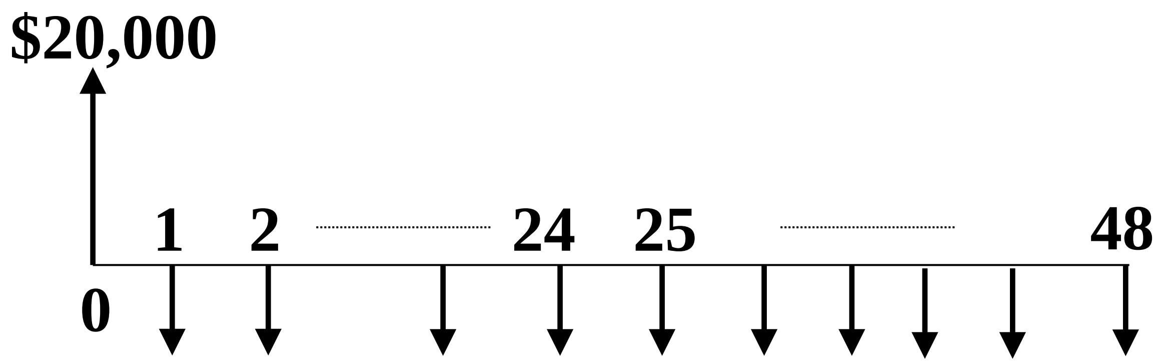
Amortized Loans

- Effective interest rate specified
- Paid off in installments over time
- Examples: Auto-loans, home mortgage loans, most business loans

Add-on Loans

- Simple interest rate specified to pre-calculate the total interest
- $$A = \frac{\text{Principal} + \text{Total simple interest}}{\text{Number of payment periods}}$$
- Examples: financing furniture and appliances

Amortized Loan - Auto Loan

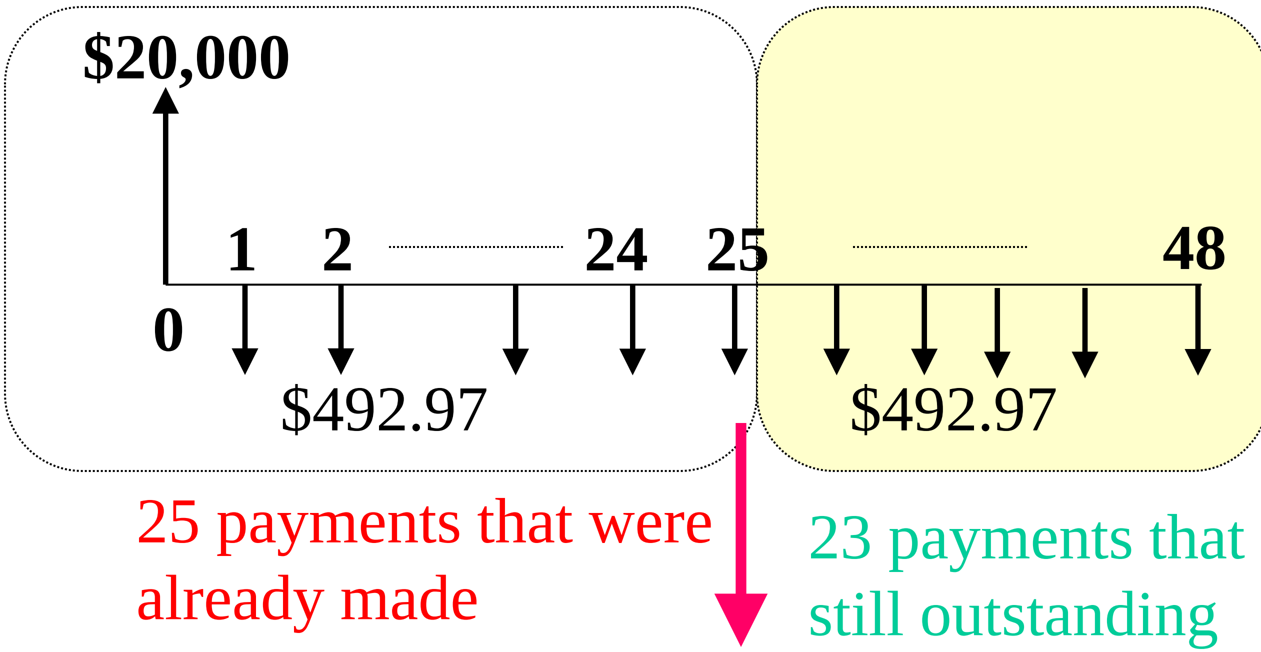


Given: *APR* = 8.5%, *N* = 48 months, and
P = \$20,000

Find: A

$$A = \$20{,}000(A/P, 8.5\%/12, 48)$$
$$= \boxed{\$492.97}$$

Suppose you want to pay off the remaining loan in lump sum right after making the 25th payment. How much would this lump be?



25 payments that were already made

23 payments that are still outstanding

$$P = \$492.97\,(P/A,\ 0.7083\%,\ 23)$$
$$= \$10{,}428.96$$

Add-on Loans

Given: You borrow \$5,000 for 2 years at an add-on rate of 12% with equal payments due at the end of each month.

Add-on Interest:

$(0.12)(\$5,000)(2) = \$1,200$

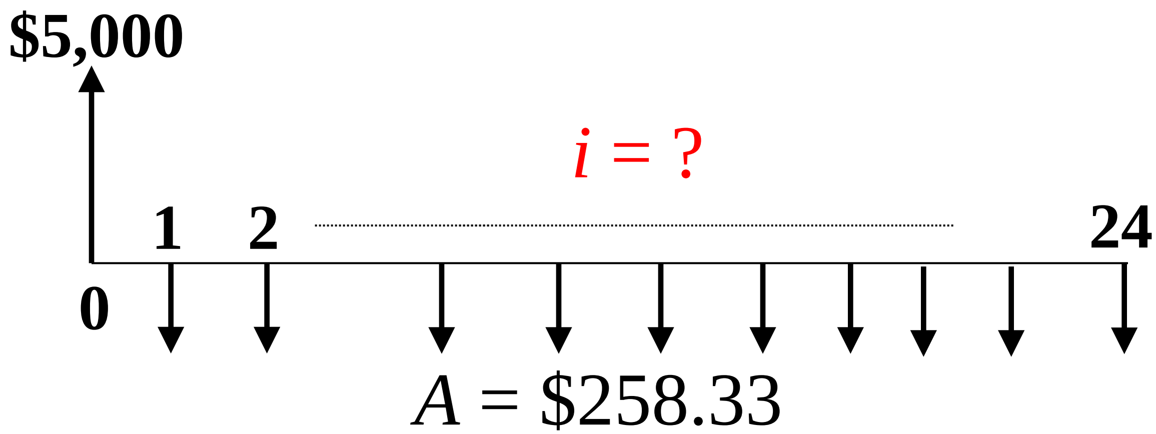
Principal + add-on interest

$\$5,000 + \$1,200 = \$6,200$

Monthly Installments

$A = \$6,200/24 = \$258.33$

Find: the effective interest rate for this add-on loan



$$\$5{,}000 = \$258.33\,(P/A, i, 24)$$

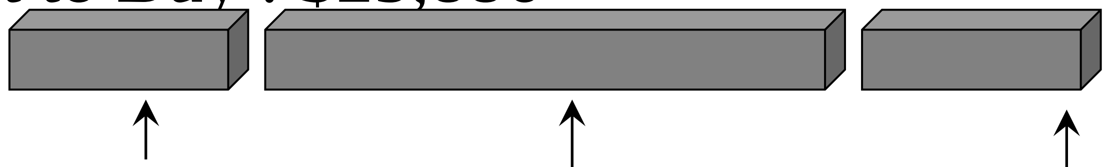
$$(P/A, i, 24) = 19.3551$$

By trial and error method, we find

- i = 1.7975% per month
- r = 1.7975% x 12 = 21.57% per year
- $i_a = (1 + 0.017975)^{12} - 1 = \boxed{23.84\% / Yr}.$

Buying vs. Lease

- Cost to Buy : $25,886

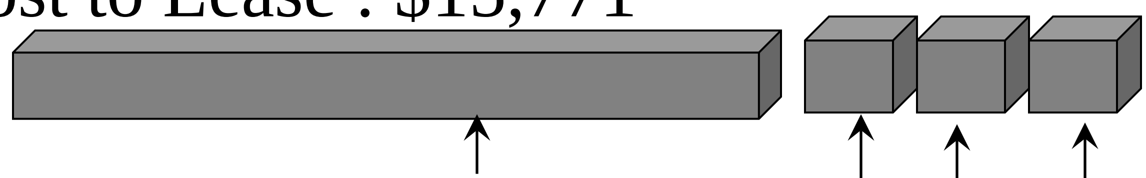


Down payment: $2,100

Car Loan at 8.5%
(48 payments of $466): $22,368

Sales tax (at 6.75%): $1,418

- Cost to Lease : $15,771



Lease (48 payments of $299) : $14,352

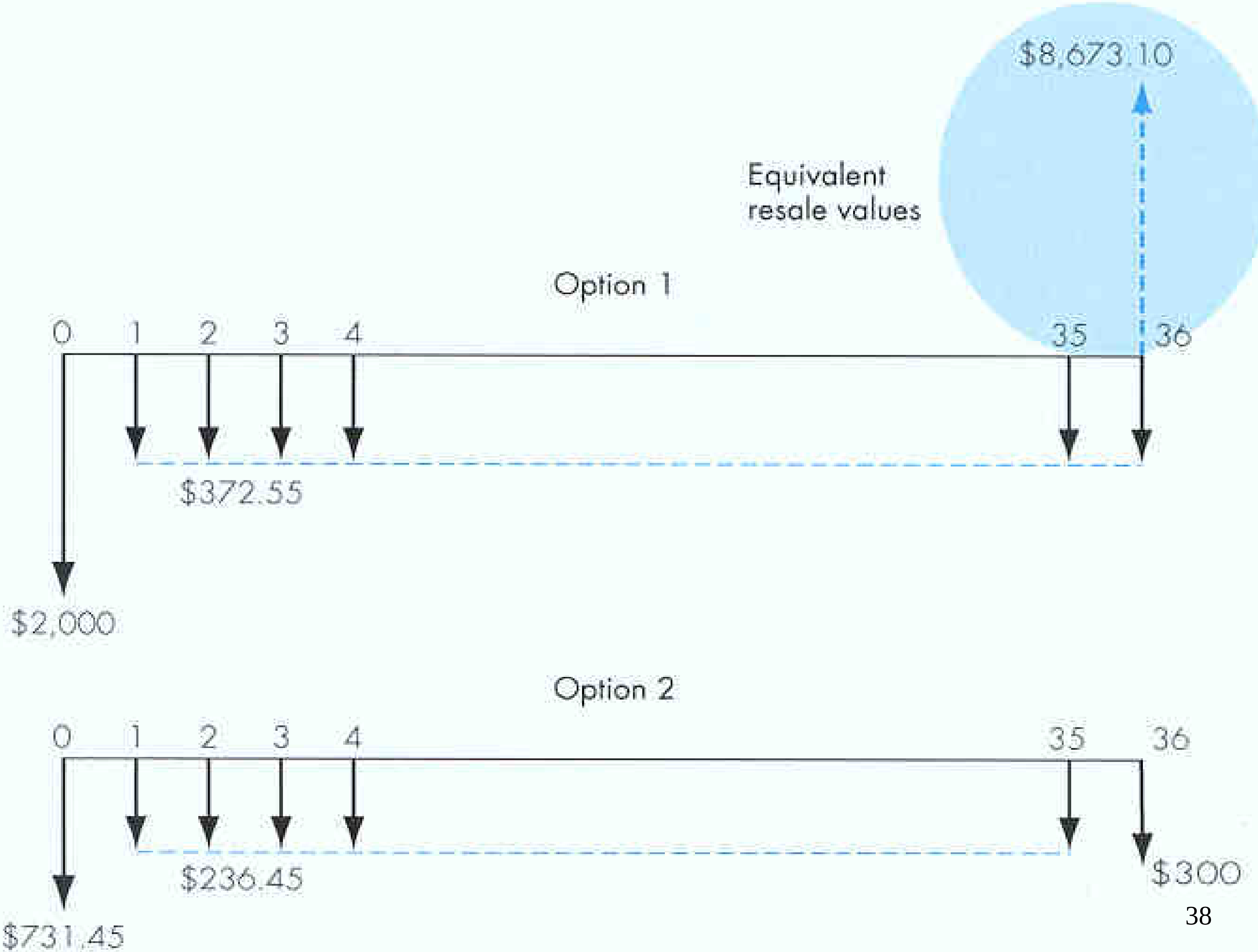
Sales tax (at 6.75%): $969

Document fee: $450

Refundable security deposit (not included in total) : $300

Buying versus Lease Decision

	Option 1 Debt Financing	Option 2 Lease Financing
Price	$14,695	$14,695
Down payment	$2,000	0
APR (%)	3.6%	
Monthly payment	$372.55	$236.45
Length	36 months	36 months
Fees		$495
Cash due at lease end		$300
Purchase option at lease end		$8.673.10
Cash due at signing	$2,000	$731.45


$8,673.10
Equivalent
resale values
Option 1
0 1 2 3 4 35 36
$372.55
$2,000
Option 2
0 1 2 3 4 35 36
$236.45
$300
$731.45

Which Option is Better?

- Debt Financing:

$$P_{\text{debt}} = \$2{,}000 + \$372.55(P/A,\ 0.5\%,\ 36)$$
$$-\ \$8{,}673.10(P/F,\ 0.5\%,\ 36)$$
$$= \$6{,}998.47$$

- Lease Financing:

$$P_{\text{lease}} = \$495 + \$236.45 + \$236.45(P/A,\ 0.5\%,\ 35)$$
$$+\ \$300(P/F,\ 0.5\%,\ 36)$$
$$= \$8{,}556.90$$

Summary

- Financial institutions often quote interest rate based on an APR.
- In all financial analysis, we need to convert the APR into an appropriate effective interest rate based on a payment period.
- When payment period and interest period differ, calculate an effective interest rate that covers the payment period.

End of Lecture 4