Direct current

Current:-

The time rate of change of electric charge is called current. $i.e. I = q/t$.

Current density:-

The current density is equal to the current per unit area.

$$i.e.\ J = \frac{I}{A}$$

Drift speed(v_d):-

When a conductor does not have a current through it, its conduction electron move randomly, since, there is no net flow of charge so, there is no current. When a potential difference is applied across it, the free electron tends to move in the direction opposite to that of applied electric field with a speed called drift speed (v_d).

Relation between drift speed and current:-

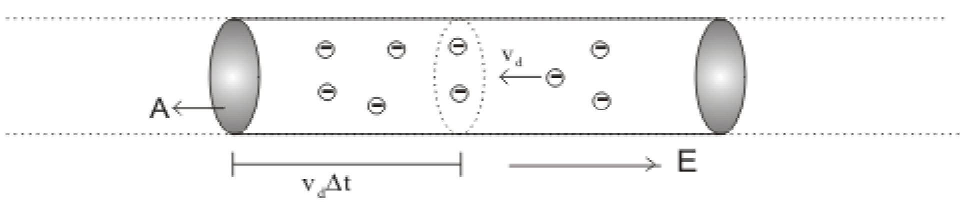


Figure 1:- Electrons moving opposite to electric field in a conductor

Consider a section of a conductor of length L and cross-sectional area A. Let n be the number of electrons per unit volume of conductor and e be the charge of each electron.

Then, volume of conductor $= A.L$

Total number of electron in this volume $= nAL$

$\therefore$ Total charge in this volume $= nALe$

When the electric field is applied across the conductor, the charges flow through it. Let all the charge carrier move with same drift speed v_d, then the time in which all the charge cross the given length of conductor is given by;

$$t = \frac{L}{v_d}$$

$$\therefore \text{current } (I) = \frac{\text{Charge}}{\text{time}} = \frac{nALe}{\frac{L}{v_d}}$$

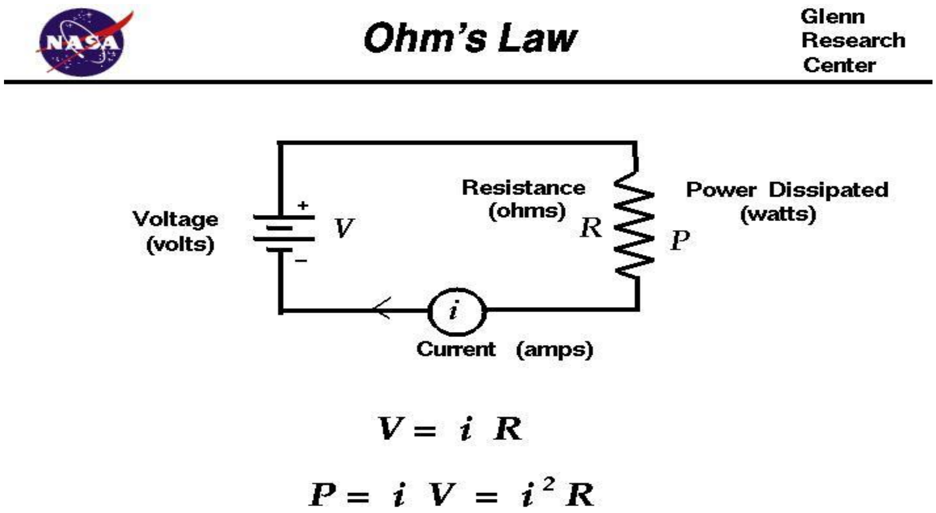
$$\therefore I = v_d enA$$

$$\text{Now, Current density } (J) = \frac{I}{A}$$

$$or, J = \frac{v_d enA}{A}$$

$$\therefore J = v_d en$$

Ohm’s Law:-



Ohm’s law states that; $V = IR$ … … … (1)

$$V = IR = \frac{I\rho l}{A}$$

$$Now, \qquad E = \frac{V}{l} = \frac{I\rho}{A}$$

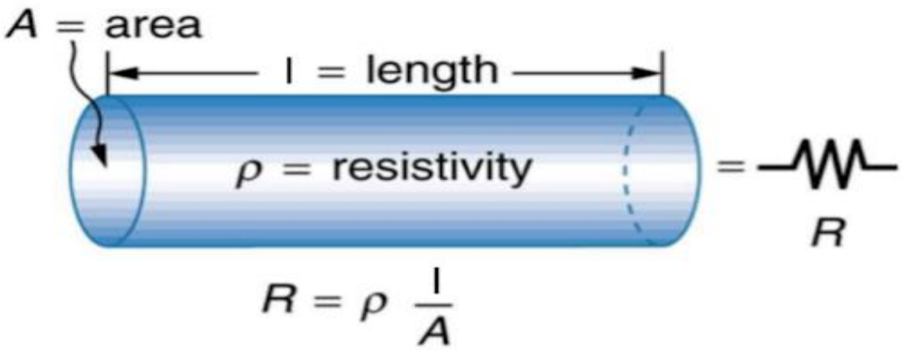
Therefore, Equation (1) can be expressed in terms of electric field, current density and resistivity as;

$$E = J\rho$$

$$J = \frac{E}{\rho} = \sigma E$$

$$\therefore J = \sigma E$$

Relation Between Resistivity And Resistance:-



Resistance is the physical property of a substance, it opposes the flow of current and resistivity is the physical property of the particular substance which have particular dimensions. Consider a wire of uniform cross-section area A and length l, then the potential difference V due to the electric field E across the conductor is given by;

$$E = \frac{V}{l} \quad and\ current\ density\ J = \frac{I}{A}$$

$$Since, \qquad J = \sigma E$$

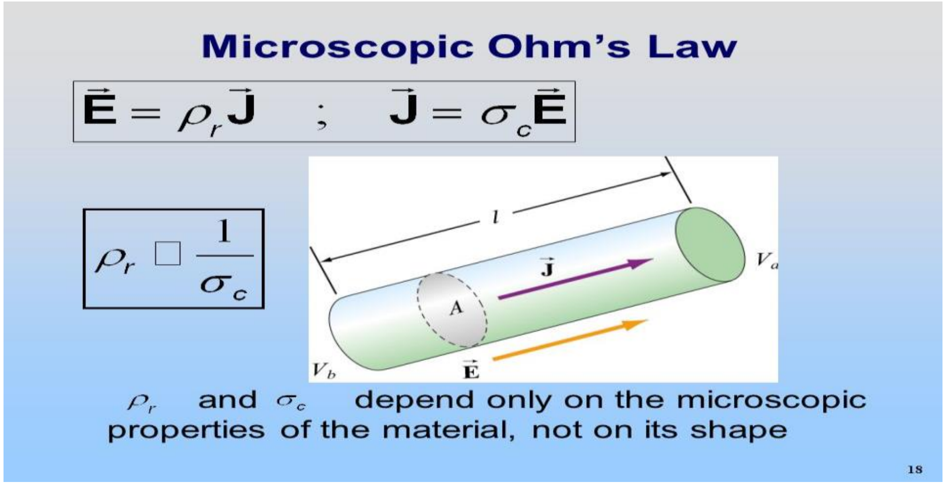
$$or, \qquad \frac{I}{A} = \frac{1}{\rho}\frac{V}{l}$$

$$or, \qquad \rho = \frac{V}{I}\frac{A}{l} = \frac{RA}{l}$$

$$\therefore \rho = \frac{RA}{l}$$

Which is the relation between resistivity and resistance.

Microscopic view (Atomic view) of Ohm's law:-



In metal the conduction electrons are free to move throughout. When an electric field is applied the electrons tends to drift along the electric field. The random motion of electron is average to zero and make no contribution to drift speed. Thus the drift speed is only due to effect of electric field on the electrons.

Let an electron of mass m and charge e placed in electric field E. The electric force experienced by it is

$$F = eE$$

Also from Newton's second law;

$$F = ma$$

$$\therefore ma = eE$$

$$\therefore a = \frac{eE}{m}$$

The average drift speed of electron is $v_d = a.\tau$

Where, τ is the average time between collisions.

$$v_d = \frac{eE}{m}.\tau$$

$$Since, \quad J = v_d en$$

$$or, \quad J = \frac{eE}{m}.\tau.e.n$$

$$\therefore J = \frac{ne^2\tau}{m}.E$$

$$Also, \quad J = \sigma E$$

$$or, \quad \frac{ne^2\tau}{m}.E = \sigma E$$

$$\therefore \sigma = \frac{ne^2\tau}{m}$$

$$\therefore \rho = \frac{m}{ne^2\tau}$$

This relation shows that, resistivity and conductivity are independent of applied field and depends upon nature of material.

The average time between collision τ and the average distance between collisions λ are related as;

$$v_d = \frac{distance}{time}$$

$$or, \qquad v_d = \frac{\lambda}{\tau}$$

$$or, \qquad \tau = \frac{\lambda}{v_d}$$

$$\therefore \rho = \frac{m}{ne^2 \frac{\lambda}{v_d}}$$

$$or, \qquad \rho = \frac{mv_d}{\lambda ne^2} = \frac{mv_d}{\lambda ne^2}\frac{v_d}{v_d} = \frac{mv_d^2}{\lambda ne^2 v_d}$$

$$\therefore \quad \rho = \frac{3KT}{\lambda ne^2 v_d} \qquad where, \frac{1}{2}mv^2 = \frac{3}{2}KT$$

Numerical Examples:-

1. **A uniform copper wire of 1 m and cross-section area of $5 \times 10^{-7}\ m^2$ carries a current of 1 A assuming that there are 8×10^{28} free electrons per meter cube in copper. How long will an electron take to drift from one end of wire to other end?**

Solution:-

Length $(l) = 1\ m$

Cross-section area $(A) = 5 \times 10^{-7}\ m^2$

Current $(I) = 1\ A$

Then, we have;

$$I = v_d enA$$

$$or, \qquad v_d = \frac{I}{neA} = \frac{1}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-7}}$$

$$\therefore\ v_d = 1.56 \times 10^{-4}\ m/sec$$

And time period is;

$$t = \frac{l}{v_d} = \frac{1}{1.56 \times 10^{-4}}$$

$$\therefore t = 6400\ sec$$

2. A copper wire of length $50\ cm$ and area of cross-section $10^{-6}\ m^2$ carries a current of $1\ A$. If the resistivity of copper is $1.8 \times 10^{-8}\ \Omega m$, calculate the electric field across the wire?

Solution:-

Length $(l) = 50\ cm = 0.5\ m$

Area $(A) = 10^{-6}\ m^2$

Current $(I) = 1\ A$

Resistivity $(\rho) = 1.8 \times 10^{-8}\ \Omega m$

We have;

$$V = IR = \frac{I\rho l}{A}$$

$$Now, \qquad E = \frac{V}{l} = \frac{I\rho}{A}$$

$$= \frac{1 \times 1.8 \times 10^{-8}}{10^{-6}}$$

$$\therefore E = 0.018\ N/C$$

3. A copper wire is stretched to make it 0.2 % longer. What is the percentage change in resistance?

Solution:-

Given;

$$l_2 = l_1 + 0.2\%\ of\ l_1 = 1.002\ l_1$$

We know that;

$$R_1 = \frac{\rho l_1}{A_1} \quad and \quad R_2 = \frac{\rho l_2}{A_2}$$

$$\therefore \frac{R_2}{R_1} = \frac{l_2 A_1}{l_1 A_2}$$

But volume of wire remains unchanged so;

$$A_1 l_1 = A_2 l_2$$

$$i.e.\ \frac{A_1}{A_2} = \frac{l_1}{l_2}$$

$$\therefore\ \frac{R_2}{R_1} = \frac{l_2^2}{l_1^2} = (1.002)^2$$

$$or, \qquad R_2 = (1.002)^2 R_1$$

$$\therefore \text{ percentage change in resistance} = \frac{R_2 - R_1}{R_1} \times 100\ \%$$

$$= \frac{(1.002)^2 R_1 - R_1}{R_1} \times 100\ \%$$

$$= 0.4\ \%$$

4. A copper wire (Resistivity $\rho_1 = 1.7 \times 10^{-8}\ \Omega m$ and density $(d_1) = 8900\ kg/m^3$) and aluminum wire (Resistivity $\rho_2 = 2.8 \times 10^{-8}\ \Omega m$ and density $(d_2) = 2700\ kg/m^3$) have the same mass per unit length, find the ratio of the resistance per unit length of aluminum and copper wire.

Solution:-

$$R_1 = \frac{\rho_1 l_1}{A_1} \qquad\qquad R_2 = \frac{\rho_2 l_2}{A_2}$$

$$or,\ \frac{R_1}{l_1} = \frac{\rho_1}{A_1} \ldots\ldots (i) \quad and,\ \frac{R_2}{l_2} = \frac{\rho_2}{A_2} \ldots\ldots (ii)$$

Dividing (i) by (ii);

$$\frac{\frac{R_1}{l_1}}{\frac{R_2}{l_2}} = \frac{\frac{\rho_1}{A_1}}{\frac{\rho_2}{A_2}} = \frac{\rho_1 A_2}{\rho_2 A_1}$$

$$i.e.\ \frac{R_2/l_2}{R_1/l_1} = \frac{\rho_2 A_1}{\rho_1 A_2}\ \ldots\ldots\ldots(iii)$$

$$Mass\ (m) = density\ \times Area\ \times length$$

$$i.e. \frac{m}{l} = \ d_1 A_1 = \ d_2 A_2$$

$$\therefore\ \frac{A_1}{A_2} = \frac{d_2}{d_1} \ldots\ldots\ldots(iv)$$

Now from equation (iii) and (iv) we get;

$$\frac{R_2/l_2}{R_1/l_1} = \frac{\rho_2 d_2}{\rho_1 d_1}$$

$$= \ \frac{2.8 \times 10^{-8} \times 2700}{1.7 \times 10^{-8}\ \times 8900}$$

$$= 0.499$$

5. The area of cross-section, length and density of a piece of metal of atomic weight $60\ gm$ are $10^{-6}\ m^2$, $1\ m$ and $5 \times 10^{-3}\ kgm^{-3}$ respectively, If every atom

contributes one free electrons. Find drift velocity of electrons in metal, when a current of $\boldsymbol{16\,A}$ is passes through it. (Given; Avogadro number $\boldsymbol{N_A = 6 \times 10^{23}\ /mole}$).

Solution:-

$$n = \frac{N_A}{V} = \frac{N_A}{{}^{m}/_{\rho}}$$

$$i.e.\ n = \frac{\rho N_A}{m}$$

$$= \frac{5 \times 10^{-3} \times 6 \times 10^{23}}{60 \times 10^{-3}}$$

$$\therefore n = 5 \times 10^{22}\ m^3$$

$$we\ have;\ \ I = v_d enA$$

$$or, \qquad v_d = \frac{I}{neA}$$

$$= \frac{16}{5 \times 10^{22} \times 1.6 \times 10^{-19} \times 10^{-6}}$$

$$\therefore\ v_d = 2 \times 10^{3}\ m/sec$$

Exercise:-

1. Derive a relation of resistivity of conductor using microscopic view. From your result, explain why resistivity of a conductor increases with necessary with increasing temperature.
2. What are the current density and mobility? Explain the atomic view of resistivity and show that $\rho = \frac{m}{ne^2\tau}$, where symbols have their usual meanings.
3. Derive an equation $\vec{J} = \sigma \vec{E}$. Explain why resistivity of a conductor increases with increasing temperature ploat a graph between R_θ (Resistance in any temperature) and temperature.
4. What will be the conductivity of sodium metal having electron density $2.5 \times 10^{28}\ m^{-3}$ and relaxation time 3×10^{-14} sec?
5. Calculate the drift speed of electrons when 20 A current is supplied through a copper wire of cross-sectional area $1mm^2$ and electron density $10^{28}\ m^{-3}$.
6. The current density in a cylindrical wire of radius R = 2 mm and uniform cross-sectional area is given by $J = 2 \times 10^5\ Am^2$. What is the current through the outer portion of the wire between radial distance R/2 and R?
7. A cylindrical resistor of radius 6 mm and length 2.5 cm is made of material that has a resistivity of $4 \times 10^{-5} \Omega m$. What are (i) The magnitude of current density and (ii) The

potential difference between the energy dissipation rate in the resistor is 2 watt?

8. Calculate the (a) mean free time and (b) mean free path between collision for the conduction electron in copper having electron density $8.4 \times 10^{22}\ \mathrm{cm}^{-3}$ and resistivity $1.7 \times 10^{-8} \Omega m$. Charge of electron 1.6×10^{-19} C, mass of electron 9.1×10^{-31} Kg, effective speed of electron $1.6 \times 10^{6}\ \mathrm{m/s}$.
9. A copper wire of cross-sectional area $3 \times 10^{-6}\ \mathrm{m}^2$ carries a steady current of 60 A , assuming one electron per atom. Calculate (i) free electron density and (ii) average drift velocity. Given, density of $\mathrm{Cu} = 8.9 \times 10^{3}\ \mathrm{kg/m^3}$, molar mass of $\mathrm{Cu} = 64$ and $\mathrm{Avogadro's number} = 6.02 \times 10^{23}/\mathrm{mole}$.
10. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1 mm. Conductor B is a hollow tube of outside diameter 2 mm and inside diameter 1 mm. What is the resistance ratio R_A/R_B measured between their ends?
11. Calculate the Average time between collisions for the electrons of sodium atom. The number of atoms per cm^3 in sodium is 2.5×10^{22}, and the electrical conductivity is $1.9 \times 10^{7}\mathrm{mho/m}$.
12. What is the average time between collisions of free electrons in a copper wire? (At. Wt. = 63 g/mol, density = 9

gm/cc and resistivity $= 1.7 \times 10^{-4}$ Ωm, NA $= 6.02 \times 10^{23}$ mol^{-1})

13. A current of 1.2×10^{-10} A exist in a copper wire (At. Wt. = 63 g/mol, density = 9 gm/cc). Whose diameter is 2.5 mm and resistivity is1.7×10^{-8} Ωm. Assuming current to be uniform, calculate (a) current density, (b) Electrical conductivity and (c) mobility of electrons.