[Transformation of Axis.]

Assignment - 7.

Date: ....

Page: ....

1) If the axes be turned through an angle $\tan\theta = 2$, what does the eq^n.

$4xy - 3x^2 = a^2$ become.

=> Sol^n,

eq^n is.

$$4xy - 3x^2 = a^2 \quad -(i)$$

Since the axes be turned through angle $\tan\theta = 2$

$$\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \frac{2}{\sqrt{\cancel{1}+4}} = \frac{2}{\sqrt{5}}$$

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+4}} = \frac{1}{\sqrt{5}}$$

then, put,

$$x = x\cos\theta - y\sin\theta = x\frac{1}{\sqrt{5}} - y\frac{2}{\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}(x-2y)$$

$$y = y\cos\theta + x\sin\theta = y\frac{1}{\sqrt{5}} + x\frac{2}{\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}(y+2x) \text{ in eq}^n \text{ (i)}$$

or $4\frac{1}{\sqrt{5}}(x-2y)\frac{1}{\sqrt{5}}(y+2x) - 3\cdot\frac{1}{5}(x-2y)^2 = a^2$

or, $\frac{4}{5}(x-2y)(2x+y) - \frac{3}{5}(x-2y)^2 = a^2$

or $4(2x^2 - 3xy - 2y^2) - 3(x^2 - 4xy + 4y^2) = 5a^2$

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$\Rightarrow 8x^2 - 12xy - 8y^2 - 3x^2 + 12xy - 12y^2 = 5a^2$

$\Rightarrow 5x^2 - 20y^2 = 5a^2$

$\therefore x^2 - 4y^2 = a^2$

reqd. eq^n.

Q.2 What does the eq^n $3x^2+3y^2+2xy=2$ become when the axes are turned through an angle 45° to the original axes.

$\Rightarrow$ sol^n,

eq^n is

$$3x^2+3y^2+2xy=2. \quad \text{—(i)}$$

since the axis be turned angle $\theta = 45°$ then put

$$x = x\cos\theta - y\sin\theta = x\cos 45° - y\sin 45°$$

$$= x\frac{1}{\sqrt{2}} - y\frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(x-y)$$

$$y = y\cos\theta + x\sin\theta$$

$$= y\cos 45° + x\sin 45°$$

$$= y\frac{1}{\sqrt{2}} + x\frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}(x+y)$$

Narayani

Date: ....
Page: ....

put x & y in eqⁿ (ii) we get

$$\frac{3}{2}(x-y)^2 + \frac{3}{2}(x+y)^2 + 2\cdot\frac{1}{\sqrt{2}}(x-y)\frac{1}{\sqrt{2}}(x+y) = 2$$

$$3(x^2-2xy+y^2) + 3(x^2+2xy+y^2) + 2(x^2-y^2) = 4$$

$$3x^2+3y^2+3x^2+3y^2+2x^2-2y^2 = 4.$$

$$8x^2+4y^2 = 4$$

$$2x^2+y^2 = 1$$

is reqd. eqⁿ.

Q.3) Through what a angle should the axes be rotated so that the eqⁿ $9x^2 - 2\sqrt{3}xy + 7y^2 = 10$ may be changed to $3x^2+5y^2=5$.

⇒ Solⁿ.

eqⁿ is

$$9x^2 - 2\sqrt{3}xy + 7y^2 = 10 \quad \text{— (i)}$$

let the axes be turned through an angle θ, then the eqⁿ become

$$3x^2 + 5y^2 = 5$$

$$\frac{3}{5} + y^2 = 1. \quad \text{— (2)}$$

put $x = x\cos\theta - y\sin\theta$

$y = x\cos\theta + y\sin\theta$

putting the value in eqⁿ (i)

Narayani

Date: ....
Page: ....

or $9(x\cos\theta - y\sin\theta)^2 - 2\sqrt{3}(x\cos\theta - y\sin\theta)(y\cos\theta + x\sin\theta)$
$+ 7(y\cos\theta + x\sin\theta)^2 = 10$

or ~~$9\cos^2\theta - 2\sqrt{3}$~~

or $(9\cos^2\theta - 2\sqrt{3}\cos\theta . \sin\theta + 7\sin^2\theta)x^2 + (-18\cos\theta\sin\theta -$
$2\sqrt{3}\cos^2\theta + 2\sqrt{3}\sin^2\theta + 14\cos\theta\sin\theta)xy +$
$(9\sin^2\theta + 2\sqrt{3}\sin\theta . \cos\theta + 7\cos^2\theta)y^2 = 1$

or $\frac{1}{10}(9\cos^2\theta - 2\sqrt{3}\cos\theta . \sin\theta + 7\sin^2\theta)x^2 + \frac{1}{10}$
$(-14\cos\theta\sin\theta - 2\sqrt{3}\cos^2\theta + 2\sqrt{3}\sin^2\theta)xy +$
$\frac{1}{10}(9\sin^2\theta + 2\sqrt{3}\sin\theta . \cos\theta + 7\cos^2\theta)y^2 = 1$

This is eq. to 2 if.

or $\frac{1}{10}(9\cos^2\theta - 2\sqrt{3}\cos\theta\sin\theta + 7\sin^2\theta) = \frac{3}{5}$ —(3)

or $\frac{1}{10}(-4\cos\theta\sin\theta - 2\sqrt{3}\cos^2\theta + 2\sqrt{3}\sin^2\theta) = 0$ —(4)

& $\frac{1}{10}(9\sin^2\theta + 2\sqrt{3}\sin\theta\cos\theta + 7\cos^2\theta) = 1$ —(5)

from (4)

$-2\sin 2\theta - 2\sqrt{3}\cos 2\theta = 0$

$\sin 2\theta = -\sqrt{3}\cos 2\theta$

$\tan 2\theta = -\sqrt{3} = \tan 120$

$2\theta = 120°$

$\theta = 60°.$

Narayani

Date: ....

Page: ....

Q.4. Transform the eq^n $3x^2 - 2xy + 4y^2 + 8x - 10y + 8 = 0$ by translating the axes into an equation with linear term missing.

=> Sol^n.

the eq^n of a curve is

$$3x^2 - 2xy + 4y^2 + 8x - 10y + 8 = 0. \quad \text{---(i)}$$

Let the origin be transferred into (h, k) so put $x = x + h$, $y = y + k$ in (i) we get.

or $3(x+h)^2 - 2(x+h)(y+k) + 4(y+k)^2 + 8(x+h) - 10(y+k) + 8 = 0$

or, $3x^2 - 2xy + 4y^2 + 2(3h - k + 4)x + 2(4k - h - 5)y + (3h^2 - 2hk + 4k^2 + 8h - 10k + 8) = 0$ ---(2)

To transform the eq^n containing only term of the second degree i.e. missing of linear term we have

$$3h - k + 4 = 0 \quad \text{---(3)}$$

& $$4k - h - 5 = 0 \quad \text{---(4)}$$

Solving (2) & (3) we get $h = -1$, $k = 1$.

putting the value of h & k in (2) we get

$$3x^2 - 2xy + 4y^2 = 1$$

is req. eq^n

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Q.5. Transform the eq^n $x^2+3y^2+3x-40=0$ to parallel axis through $(4,-1)$.

⇒ sol^n.

the eq^n is $x^2+3y^2+3x-40=0$ —(i)

the origin is transferred to $(4,-1)$ so putting

$x = x+4$, $y = y-1$ in eq^n (i) we get.

or $(x+4)^2+3(y-1)^2+3(x+4)-40=0$.

or $x^2+8x+16+3y^2-6y+3+3x+12-40=0$.

or $x^2+3y^2+11x-6y-9=0$

is req^d sol^n

31
16
3
12
40

[The Ellipse + The Hyperbola]

Assignment - 8

in back

Date: ....

Page: ....

Q. If e_1 & e_2 be the ecentricities of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, then show that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$.

⇒ Soln,

the eqn of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

If e_1 be the ecentricity of this hyperbola then

$$b^2 = a^2(e_1^2 - 1)$$

or, $\frac{b^2}{a^2} (e_1^2 - 1)$

or, $1 + \frac{b^2}{a^2} = e_1^2$

$$e_1^2 = \frac{a^2 + b^2}{a^2}$$

$$\frac{1}{e_1^2} = \frac{a^2}{a^2+b^2} \quad \text{---}(i)$$

& the eqn of conjugate hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

Date:

Page:

If e_2 be the ecentricity of this hyperbola

the,

$$a^2 = b^2(e_2^2 - 1)$$

$$\frac{a^2}{b^2} = e_2^2 - 1$$

$$\frac{a^2}{b^2} + 1 = e_2^2$$

$$\frac{a^2 + b^2}{b^2} = e_2^2$$

$$\frac{1}{e_2^2} = \frac{b^2}{a^2+b^2} \quad \text{--- (2)}$$

On addition (1) & (2)

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = \frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2}$$

$$= \frac{a^2+b^2}{a^2+b^2}$$

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$$

Date: ....

Page: ....

6. Show that the eccentricity of hyperbola with transverse axis is $2a$ & axes are the axes of coordinate which passes through the point (h,k) is given by $\left(\frac{h^2+k^2-a^2}{h^2-a^2}\right)^{1/2}$

⇒ Sol^n,

the eq^n of hyperbola with transverse axis $2a$ is.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad \text{—(i)}$$

If (1) passes through the point (h,k) then

$$\frac{h^2}{a^2}-\frac{k^2}{b^2}=1$$

$$\frac{k^2}{b^2}=\frac{h^2}{a^2}-1 = \frac{h^2-a^2}{a^2}$$

$$b^2=\frac{a^2k^2}{h^2-a^2}$$

If e be the eccentricity of the hyperbola (1)

$$b^2=a^2(e^2-1)$$

$$\text{or } \frac{a^2k^2}{h^2-a^2}=a^2(e^2-1)$$

$$\text{or } \frac{k^2}{h^2-a^2}=e^2-1$$

Narayani

Date:
Page:

$$\frac{k^2}{h^2-a^2}+1=e^2,$$

$$e^2=\frac{k^2+h^2-a^2}{h^2-a^2}$$

$$e=\left(\frac{k^2+h^2-a^2}{h^2-a^2}\right)^{1/2}$$

Q ~~Find the eq^n of the hyperbola referred to its axis as axes coordinate~~

Q Find the center, vertice, foci, eccentricity directrix & latus rectum of hyperbola.

i) $3x^2-4y^2=36.$

=> sol^n:

eq^n of hyperbola is

$$3x^2-4y^2=36$$

$$\frac{x^2}{12}-\frac{y^2}{9}=1$$

comparing it with eq^n

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

find $a^2=12$, $b^2=9$

$a=2\sqrt{3}$ $b=3$

Narayani

Date: ....
Page: ....

center $(0,0)$

transverse axis $(\pm a, 0) = (\pm 2\sqrt{3}, 0)$

we have,

$$b^2 = a^2 (e^2 - 1)$$

$$9 = 12 (e^2 - 1)$$

$$9 + 12 = 12 e^2$$

$$21 = 12 e^2$$

$$e^2 = \frac{21}{12}$$

$$e = \frac{\sqrt{7}}{2}$$

foci $= (\pm ae, 0) = (\pm \sqrt{21}, 0)$

Directrix $x = \pm \frac{a}{e} = \pm 4\sqrt{\frac{3}{7}}$

latus rectum $= \frac{2b^2}{a} = 2 \times \frac{9}{2\sqrt{3}} = 3\sqrt{3}$.

Date: ....
Page: ....

iii. $\frac{x^2}{9} - \frac{y^2}{16} = -1$

⇒ Sol^n.

eq^n of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = -1$$

comparing with the conjugate hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

we find

$a^2 = 9$ $\qquad$ $b^2 = 16$

$a = 3$ $\qquad$ $b = 4$

Center $(0,0)$

vertic ~~are~~ $(0, \pm b) = (0, \pm 4)$

we have

$$a^2 = b^2(e^2-1)$$
$$9 = 16(e^2-1)$$
$$9 + 16 = 16e^2$$
$$e^2 = \frac{25}{16}$$
$$e = 5/4.$$

eccentricity $= 5/4.$

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Assignment-8

(7) Q. Show the pair of tangent drawn from the centre of a hyperbola & its asymptote.

⇒ Soln,

let the eqn of a hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad -(i)$$

The eqn of tangent to the hyperbola (1) is

$$y = mx \pm \sqrt{a^2m^2 - b^2} \quad -(2)$$

But eqn (2) pass through point (0,0) ie centre.

$$0 = 0 \pm \sqrt{a^2m^2 - b^2}$$

$$0 = \pm\sqrt{a^2m^2 - b^2}$$

or $0 = a^2m^2 - b^2$

or $a^2m^2 = b^2$

or $m^2 = \frac{b^2}{a^2}$

$\therefore m = \pm\frac{b}{a}$.

So, eqn 2 become

$$y = \frac{b}{a}x \pm \sqrt{\frac{\cancel{a^2}b^2}{\cancel{a^2}} - b^2}$$

$$= \frac{b}{a}x \pm \sqrt{b^2 - b^2}$$

$y = \pm\frac{b}{a}x$. is req. eqn.

Date: ....

Page: ....

5) Find the eq^n of tangent to the hyperbola $3x^2 - 4y^2 = 1$ & perpendicular to the line $2x + 3y = 5$.

⇒ Sol^n

eq^n of hyperbola.

$$3x^2 - 4y^2 = 1$$

$$\frac{x^2}{\frac{1}{3}} - \frac{y^2}{\frac{1}{4}} = 1 \quad \text{--- (i)}$$

comparing with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$a^2 = \frac{1}{3}$$

$$b^2 = \frac{1}{4}$$

eq^n of tangent in slope form.

$$y = mx \pm \sqrt{a^2m^2 - b^2} \quad \text{--- (ii)}$$

the tangent line (2) in ⊥r to the l/p

$$2x + 3y = 5$$

$$3y = -2x + 5$$

$$y = -\frac{2}{3}x + \frac{5}{3}$$

& slope $= -\frac{2}{3}$

$$m \times -\frac{2}{3} = -1$$

$$m = \frac{3}{2}$$

Narayani

so eq^n (2) be come

$$y = \frac{3x}{2} \pm \sqrt{\frac{1}{\cancel{3}_1} \times \frac{\cancel{9}^3}{\cancel{12}_4} - \frac{1}{4}}$$

$$y = \frac{3x}{2} \pm \sqrt{\frac{3}{4} - \frac{1}{4}}$$

$$y = \frac{3x}{2} \pm \sqrt{\frac{3-1}{4}}$$

$$y = \frac{3x}{2} \pm \frac{\sqrt{2}}{2}$$

$$y = \frac{3x}{2} \pm \frac{\sqrt{2}}{2}$$

$$2y = 3x \pm \sqrt{2}$$

Assignment - 9.

~~Exam. a324~~

3. Find the position & length of the axes of the conic.

$$x^2 - 4xy - 2y^2 + 10x + 4y = 0.$$

$\Rightarrow$ soln,

given eqn

$$x^2 - 4xy - 2y^2 + 10x + 4y = 0$$

comparing with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ we

$a = 1$ $\quad$ $2f = 4$ $\Rightarrow$ $f = 2$

$b = -2$ $\quad$ $2g = 10$ $\Rightarrow$ $g = 5$

$2h = -4$ $\quad$ $c = 0.$

$h = -2$

now,

$$D = \cancel{abc + 2fgh - 2af}$$

$$= abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 1 \times -2 \times 0 + 2 \times 2 \times 5 \times (-2) - 1 \times 2^2 - (-2) \times 5^2 - 0$$

$$= 0 - 40 - 4 + 50 - 0$$

$$= 6 \neq 0$$

& $ab - h^2 = 1 \times (-2) - (-2)^2$

$$= -2 - 4 = -6 < 0.$$

$\therefore$ The given conic is hyperbola.

Narayani

Date: ....
Page: ....

ii. Centre of conic

let $\phi = x^2 - 4xy - 2y^2 + 10x + 4y$

The $\frac{\partial \phi}{\partial x} = 2x - 4y + 10$ & $\frac{\partial \phi}{\partial y} = -4x - 4y + 4$

for the centre $\frac{\partial \phi}{\partial x} = 0 \Rightarrow 2x - 4y + 10 = 0$

$\Rightarrow x - 2y + 5 = 0$ —— (i)

$\frac{\partial \phi}{\partial y} = 0 \Rightarrow -4x - 4y + 4 = 0$

$\Rightarrow x + y - 1 = 0$ —— (ii)

Solving (i) & (ii)

$$\begin{array}{r} \cancel{x} - 2y + 5 = 0 \\ \cancel{x} + y - 1 = 0 \\ - \quad - \quad + \\ \hline -3y + 6 = 0 \end{array}$$

$6 = 3y$

$y = 2$

putting the value of y in eq^n (i)

$x - 2 \times 2 + 5 = 0$

$x - 4 + 5 = 0$

$x = -1$

Date:
Page:

centre $(\alpha,\beta) = (x,y) = (-1,2)$.

ii) Eqⁿ of centre refered to the centre (α,β)
Shifting the origin $(0,0)$ to the centre $(\alpha,\beta) = (-1,2)$

So, put $x = X - 1$,
$y = Y + 2$

Then by the formula

$$\frac{a(h^2-ab)}{\Delta} X^2 + 2h\left(\frac{h^2-ab}{\Delta}\right) XY + b\left(\frac{h^2-ab}{\Delta}\right) Y^2 = 1$$

$$1 \times \left(\frac{6}{6}\right) \times X^2 + 2\times(-2)\left(\frac{6}{6}\right) \cdot XY + (-2)\left(\frac{6}{6}\right) Y^2 = 1$$

$$X^2 - 4XY - 2Y^2 = 1$$

comparing with $AX^2 + 2HXY + BY^2 = 1$

$A = 1$

$2H = -4$

$H = -2$

$B = -2$

(3) length of axis.
by the formula

Narayani

Date: ....
Page: ....

$$\frac{1}{r^4} - \frac{1}{r^2}(A+B) + (AB - H^2) = 0.$$

$$\frac{1}{r^4} - \frac{1}{r^2}(1-2) + (-2-4) = 0$$

$$\frac{1}{r^4} + \frac{1}{r^2} - 6 = 0$$

$$\frac{1 + r^2 - 6r^4}{r^4} = 0$$

$$1 + r^2 - 6r^4 = 0$$

$$6r^4 - r^2 - 1 = 0$$

$$6r^4 - 3r^2 + 2r^2 - 1 = 0$$

$$3r^2(2r^2 - 1) + 1(2r^2 - 1) = 0$$

$$(2r^2 - 1)(3r^2 + 1) = 0$$

Either

$2r^2 - 1 = 0 \Rightarrow r^2 = 1/2$; $3r^2 + 1 = 0 \Rightarrow r^2 = -4/3$

Say $r_1^2 = \frac{1}{2}$ & $r_2^2 = -1/3$

Now, length of semi-transverse axis $= r_1 = \sqrt{1/2} = \frac{1}{\sqrt{2}}$

" " " -conjugate axis $= r_2 = \sqrt{|r_2^2|}$

$= \sqrt{|-1/3|}$

$= \sqrt{1/3}$

$= \frac{1}{\sqrt{3}}$

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length of transverse axis $= \frac{2}{\sqrt{2}} = \sqrt{2}$

" " conjugate axis $= \frac{2}{\sqrt{3}}$

4) Eq^n of Axis / position of Axis

The eq^n of transverse axis corresponding to the value of r_1^2 is

$$\left(A - \frac{1}{r_1^2}\right)X + HY = 0$$

$$\left(1 - \frac{1}{\cancel{4}_2}\right)X + (-2)Y = 0$$

$$(1-2)X - 2Y = 0$$

$$X + 2Y = 0$$

Shifting the origin basic to centre $(-1, 2)$

$X = x - 1$ $\Rightarrow x = X + 1$

$y = Y + 2$ $\Rightarrow Y = y - 2$

$$(x+1) + 2(y-2) = 0$$

$$x + 1 + 2y - 4 = 0$$

$$\boxed{x + 2y - 3 = 0}$$

which is eq^n of transverse

& the eq^n of conjugate axis corresponding to the value of r_2^2 is.

Date: ....

Page: ....

$$\left(A - \frac{1}{x_{1}2}\right) X + HY = 0$$

$$\left(1 - \frac{1}{-1/3}\right) X + (-2) Y = 0$$

$$(1 + 3) X - 2Y = 0$$

$$4X - 2Y = 0$$

$$\boxed{2X - Y = 0}$$

Shifting the origin basic from centre $(-1, 2)$

$x = X - 1 \Rightarrow X = x + 1$

$y = Y + 2 \Rightarrow Y = y - 2$

or $2(x+1) - (y-2) = 0$

$2x + 2 - y + 2 = 0$

$$\boxed{2x - y + 4 = 0}$$

which is reqd eqn of conjugate axis

Date:

Page:

7 i)

eq^n of conic

$$x^2+xy+y^2+x+y-1=0$$

comparing with $ax^2+2hxy+by^2+2gx+2fy+c=0$

$a=1$ $\quad 2h=1$ $\quad 2g=1$ $\quad 2f=1$

$b=1$ $\quad h=1/2$ $\quad g=1/2$ $\quad f=1/2$

$c=-1.$

now

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$$

$$= 1\times1\times(-1) + \cancel{2}\times\frac{1}{\cancel{2}}\times\frac{1}{2}\times\frac{1}{2} - 1\times\left(\frac{1}{2}\right)^2 - 1\times\left(\frac{1}{2}\right)^2 - (-1)\left(\frac{1}{2}\right)^2$$

$$= -1 + \frac{\cancel{2}\,1}{\cancel{4}} - \frac{1}{4} - \frac{1}{4} + \frac{1}{4}$$

$$= -1 \neq 0$$

$$ab-h^2 = 1\times1-\left(\frac{1}{2}\right)^2.$$

$$= 1-\frac{1}{4} = \frac{3}{4} > 0$$

The give conic is ellips

Date: ....

Page: ....

1. Centre

let $\phi = x^2 + xy + y^2 + x + y - 1$

$\frac{\partial \phi}{\partial x} = 2x + y + 1$ & $\frac{\partial \phi}{\partial y} = x + 2y + 1$

For the centre $\frac{\partial \phi}{\partial x} = 0 \Rightarrow 2x + y + 1 = 0$ — (i)

& $\frac{\partial \phi}{\partial y} = 0 \Rightarrow x + 2y + 1 = 0$ — (ii)

solving (i) & (ii) we get

$x = -1/3$ $y = -1/3$.

centre $(x, y) = \left(\frac{-1}{3}, -\frac{1}{3}\right)$

2) Eqn of conic refered to the centre (α, β) shifting the origin to origine to centre $(\alpha, \beta) = (-1/3, -1/3)$

so, put $x = X - 1/3$

$y = Y - 1/3$.

Narayani

Date:
Page:

$$\Rightarrow a\frac{(h^2-ab)}{\Delta}x^2 + 2h\frac{(h^2-ab)}{\Delta}xy + b\frac{(h^2-ab)}{\Delta}y^2 = 1$$

$$1\times\left(\frac{-3/4}{-1}\right)x^2 + 2\times\frac{1}{2}\left(\frac{-3/4}{-1}\right)xy + 1\times\left(\frac{-3/4}{-1}\right)y^2$$

$$\frac{3}{4}x^2 + \frac{3}{4}xy + \frac{3}{4}y^2 = 1$$

Comparing with $Ax^2 + 2Hxy + By^2 = 1$

$A = 3/4$ $\qquad 2H = 3/4$ $\qquad B = 3/4$

$H = \frac{3}{8}$

3) Length of Axis

By formula

$$\frac{1}{r^4} - \frac{1}{r^2}(A+B) + (AB - H^2) = 0$$

$$\frac{1}{r^4} - \frac{1}{r^2}\left(\frac{3}{4}+\frac{3}{4}\right) + \left(\frac{9}{16} - \frac{9}{64}\right) = 0$$

$$\Rightarrow \frac{1}{r^4} - \frac{6}{4r^2} + 9\left(\frac{4-1}{64}\right) = 0$$

$$\Rightarrow \frac{1}{r^4} - \frac{3}{2r^2} + \frac{9\times 3}{64} = 0$$

$$\Rightarrow \frac{1}{r^4} - \frac{3}{2r^2} + \frac{27}{64} = 0.$$

$$\Rightarrow \frac{64 - 3\times 32r^2 + 27r^4}{64r^4} = 0$$

Narayani

Date: ....
Page: ....

$64 - 96r^2 + 27r^4 = 0.$

$27r^4 - 96r^2 + 64 = 0.$

$ax^2 + Bxy + C.$

$ax^2 + 2bxy + \quad \frac{b \pm \sqrt{b^2 - ac}}{2a}$

$\cancel{r^2 = \frac{16 \pm 8\sqrt{7}}{9}}$

$$r^2 = \frac{-96 \pm 32\sqrt{3}}{54}$$

$\frac{8}{3}, \frac{8}{2}$

4) Eq^n of axis

The eq^n of major axis corresponding to the value r_1^2 is.

$$\left(A - \frac{1}{r_1^2}\right)X + HY = 0$$

$$\left(\frac{3}{4} - \frac{1}{8/3}\right)X - \frac{3}{8}Y = 0$$

$$\left(\frac{3}{4} - \frac{3}{8}\right)X - \frac{3}{8}Y = 0$$

$$\left(\frac{6-3}{8}\right)X + \frac{3}{8}Y = 0$$

$$\frac{3}{8}X + \frac{3}{8}Y = 0$$

$$3X + 3Y = 0$$

$$\boxed{X + Y = 0}$$

shifting the origin becomes from centre $(-1/3, -1/3$

Date:
Page:

$$x = X - \frac{1}{3} \Rightarrow X = x + \frac{1}{3}$$

$$y = Y - \frac{1}{3} \Rightarrow Y = y + \frac{1}{3}$$

$$x + \frac{1}{3} + y + \frac{1}{3} = 0$$

$$\frac{3x+1}{3} + \frac{3y+1}{3} = 0$$

$$3x + 1 + 3y + 1 = 0$$

$$3x + 3y + 2 = 0$$ hyrm :) eq^n of major axis

To find sine & cosen

$$\text{Slope } (m) = -\frac{\text{coeff. of } x}{\text{coeff. of } y} = \frac{-3}{3} = -1$$

$$\tan\theta = -1$$

now, $$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+(-1)^2}} = \frac{1}{\sqrt{2}}$$

$$\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} = \frac{-1}{\sqrt{1+(-1)^2}} = \frac{-1}{\sqrt{2}}$$

Date: ....
Page: ....

7) Eccentricity.

$$r_2^2 = r_1^2 (e^2 - 1)$$

$$\frac{1}{3} = 1 - e^2$$

$$e^2 = 1 - \frac{1}{3} = 2/3$$

$$e = \frac{\sqrt{2}}{\sqrt{3}}.$$

6) foci $= (\alpha \pm er_1 \cos\theta, \beta \pm er_1 \sin\theta)$

$$= \left(-1/3 \pm \frac{\cancel{\sqrt{2}}}{\sqrt{3}} \cdot \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{1}{\cancel{\sqrt{2}}}, \frac{-1}{3} \pm \frac{\sqrt{2}}{\sqrt{3}} \cdot \frac{2\sqrt{2}}{\sqrt{3}} \times \left(\frac{-1}{\cancel{\sqrt{2}}}\right)\right)$$

$$= \left(-\frac{1}{3} \pm \frac{2\sqrt{2}}{3}, \frac{-1}{3} \mp \frac{2\sqrt{2}}{3}\right) \#$$

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