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CW

Limit comparision ~~or~~ .

Infinite sequence

A sequence is a function whose domain is set of natural no. & the range of set of real no. so, that the ordered set of real no.

u_1, u_2, u_3, u_n is called the sequence & it is denoted by $\{u_n\}$. If the no. terms is unlimited then the sequence is said to be infinite sequence & & its general term is v_n.

eg: $1, 3, 5, 7$ ---- $(2n-1)$ + -- is infinite sequence whose general terms is $V_n = (2n-1)$

P-series or Harmonic series.

If $f(x) = \frac{1}{x^p}$ if $p > 0$ then, the series $\Sigma f(x)$ is the form

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \text{---}$$ is called p series

or Hormonic series.

Theorem :- The infinite series.

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots + \frac{1}{n^p} + \cdots$$ is

convergent if $p > 1$
& divergent if $p \leq 1$

eg: The series $1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots$ (convergent)

(2) The series $\frac{1}{1} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots$ (divergent)

(3) $\sum \frac{1}{n}$ (divergent)

(4) $\sum \frac{1}{n^2}$ (convergent)

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(1) $1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \cdots + \frac{1}{n^2+1} + \cdots$

=> given

$\frac{1}{1} + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \cdots + \frac{1}{(n^2+1)} + \cdots$

$= \sum \frac{1}{n^2+1}$

the general term of the given series
$U_n = \frac{1}{n^2+1}$

taking the serier $\sum V_n = \sum \frac{1}{n^2}$
& its general term is $V_n = \frac{1}{n^2}$

now,

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \left[\frac{1}{n^2+1} \cdot \frac{n^2}{1}\right]$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2\left[1+1/n^2\right]}$$

$$= \lim_{n\to\infty} \frac{1}{1+1/n^2}$$

$= 1$, which is finite & non-zero

Since $\Sigma v_n = \Sigma \frac{1}{n^2}$ is convergent by p-test

then by limit comparision test $\Sigma u_n = \Sigma \frac{1}{n^2+1}$ is also convergent

(2) $\frac{1}{1\cdot2\cdot3} + \frac{3}{2\cdot3\cdot4} + \frac{5}{3\cdot4\cdot5} + \ldots$

=> given

$$\frac{1}{1\cdot2\cdot3} + \frac{3}{2\cdot3\cdot4} + \frac{5}{3\cdot4\cdot5} + \ldots$$

general term of the given series

$$u_n = \frac{2n-1}{n(n+1)(n+2)} \qquad \frac{n}{n^3}$$

taking the series $\Sigma v_n = \Sigma \frac{n}{n^2}$

now,

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now

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \left[\frac{(2n-1)}{n(n+1)(n+2)} \times n^2 \right]$$

$$= \lim_{n\to\infty} \frac{n(2n-1)}{n^2+2n+n+2}$$

$$= \lim_{n\to\infty} \frac{2n^2}{n^2+3n+1}$$

$$= \lim_{n\to\infty} \frac{n^2(2-1/n)}{n^2(1+3/n+2/n^2)}$$

$$= \lim_{n\to\infty} \frac{2-1/n}{(1+3/n+2/n^2)}$$

$$= \frac{2}{1}$$

$= 2$, which is finite & non-zero

~~Then~~ $\Sigma v_n = \Sigma \frac{1}{n^2}$ is convergent by p-test

Then by limit comparision test $\Sigma u_n = \Sigma \frac{2n-1}{n(n+1)(n+2)}$ is convergent.