ENERGY AND POTENTIAL

4.1 INTRODUCTION

In chapters 2 and 3, we became acquainted with Coulomb's law and its use in finding the electric field about several simple distributions of charge, and also with Gauss' law and its application in determining the field about some symmetrical charge arrangements. The use of Gauss' law was invariably easier for these highly symmetrical distributions because the problem of integration always disappeared when the proper closed surface was chosen.

However, if we had attempted to find a slightly more complicated field, such as that of two unlike point charges separated by a small distance, we would have found it impossible to choose a suitable gaussian surface and obtain an answer. Coulomb's law, however, is more powerful and enables us to solve problems for which Gauss' law is not applicable. The application of Coulomb's law is laborious, detailed, and often quite complex, the reason for this being precisely the fact that the electric field intensity, a vector field, must be found directly from the charge distribution. Three different integrations are needed in general, one for each component, and the resolution of the vector into components usually adds to the complexity of the integrals.

Finding potential first and using it to find the electric field intensity by some simple straightforward procedure, such as differentiation can relieve us from the burden of complexity we have discussed just now.

4.2 ENERGY EXPENDED IN MOVING A POINT CHARGE IN AN ELECTRIC FIELD

Let's consider a charge, Q in an electric field having intensity, $\vec{E}$. There will be a force on Q due to this electric field which is given as

$$\vec{F}_E = Q\vec{E}$$

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If we wish to calculate the value of the force in the direction $d\vec{l}$, we should write as

$$F_{EL} = \vec{F}_E \cdot \hat{a}_L = Q\vec{E} \cdot \hat{a}_L$$

Now, to displace the charge Q by a distance $d\vec{l}$ against the electric field, we have to exert a force equal and opposite to that exerted by the electric field. That is,

$$F_{appl} = -Q\vec{E} \cdot \hat{a}_L$$

This small work done by external source is

$$dW = F_{appl}\, dl = -Q\vec{E} \cdot \hat{a}_L\, dl = -Q\vec{E} \cdot d\vec{l}$$

Total work done in moving the charge through a finite distance can be obtained by integrating

$$W = \int_{init}^{final} dW = \int_{init}^{final} -Q\vec{E} \cdot d\vec{l} = -Q\int_{init}^{final} \vec{E} \cdot d\vec{l}$$

4.3 THE LINE INTEGRAL

The equation,

$$W = -Q\int_{init}^{final} \vec{E} \cdot d\vec{l}$$

$$\text{or,}\quad W = -Q\int_{init}^{final} E_L\, dl$$

where E_L = component of $\vec{E}$ along $d\vec{l}$ is an example of a line integral. Choosing a path, breaking it up into a large number of very small segments, multiplying the component of the field along each segment by the length of the segment and then adding the results for all the segments is what the above line integral signifies. The value of the integral is obtained exactly if we break the path into infinite no. of segments.

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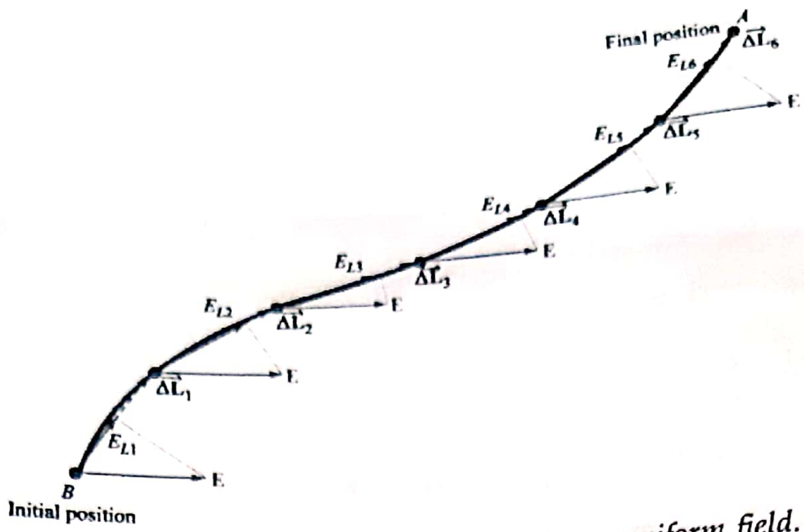

Figure 4.1 *A graphical interpretation of a line integral in a uniform field. The line integral of E between points B and A is independent of the path selected, even in a non uniform field; this result is not, in general, true for time-varying fields.*

Consider a path as shown in the figure above with an initial position B and a final position A in a uniform electric field. The work done in moving a charge Q from B to A is approximately given as

$$W = -Q\,(E_{l1}\Delta l_1 + E_{l2}\Delta l_2 + \ldots\ldots\ldots\ldots + E_{l6}\,\Delta l_6)$$

In vector form,

$$W = -Q\,(\vec{E}_1.\Delta\vec{l}_1 + \vec{E}_2.\,\Delta\vec{l}_2 + \ldots\ldots\ldots\ldots + \vec{E}_6.\Delta\vec{l}_6)$$

As $\quad \vec{E}_1 = \vec{E}_2 = \ldots\ldots\ldots\ldots = \vec{E}_6$

$$W = -Q\vec{E}.\,(\Delta\vec{l}_1 + \Delta\vec{l}_2 + \ldots\ldots\ldots\ldots + \Delta\vec{l}_6)$$

$$\therefore\ W = -Q\vec{E}.\vec{L}_{BA}$$

where $\vec{L}_{BA}$ = vector drawn from the initial to the final point.

The above equation suggests that the work done depends only on Q, $\vec{E}$ and $\vec{L}_{BA}$. It doesn't depend on the particular path we have selected along which to carry the charge.

4.4 DEFINITION OF POTENTIAL DIFFERENCE AND POTENTIAL

Potential difference is defined as the work done (by an external source) in moving a unit positive charge from one point to another in an electric field.

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$$\text{Potential difference } (V) = \frac{W}{Q} = \frac{-Q\int_{init}^{final} \vec{E}\,.\,d\vec{l}}{Q}$$

$$\therefore V = -\int_{init}^{final} \vec{E}\,.\,d\vec{l}$$

The unit of potential difference is volt. V_{AB} will denote the difference between potential at point A and that at B.

$V_{AB} = V_A - V_B = -\int_B^A \vec{E}\,.\,d\vec{l}$ and is the work done in moving the unit positive charge from B to A. It is noted that V_A and V_B shall have the same zero reference point.

The potential difference between points A and B at radial distances r_A and r_B from a point charge Q located at origin is calculated as

$$V_{AB} = -\int_B^A \vec{E}\,.\,d\vec{l}$$

where $\vec{E} = E_r\,\hat{a}_r = \frac{Q}{4\pi\varepsilon_o r^2}\hat{a}_r, \quad d\vec{l} = dr\,\hat{a}_r$

$$= -\int_{r_B}^{r_A} \frac{Q}{4\pi\varepsilon_o r^2}\hat{a}_r .\, dr\,\hat{a}_r$$

$$= -\int_{r_B}^{r_A} \frac{Q}{4\pi\varepsilon_o r^2}dr = \frac{-Q}{4\pi\varepsilon_o}\int_{r_B}^{r_A}\frac{1}{r^2}dr = \frac{-Q}{4\pi\varepsilon_o}\left(\frac{r^{-2+1}}{-2+1}\right)_{r_B}^{r_A} = \frac{Q}{4\pi\varepsilon_o}\left(r^{-1}\right)_{r_B}^{r_A}$$

$$\boxed{\therefore V_{AB} = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)}$$

For $r_B > r_A$, V_{AB} is +ve which signifies that energy is expended by the external source to bring the unit positive charge from r_B to r_A.

If we let $r_B \to \infty$, then, the potential at r_A becomes

$$V_A = \frac{Q}{4\pi\varepsilon_o r_A}$$

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or simply, we may write

$$\boxed{V = \frac{Q}{4\pi\varepsilon_o r}}$$

This expression defines the potential at any point distant r from a point charge Q at the origin, the potential at infinite radius being taken as the zero reference.

Interpreted physically, the above expression says $\frac{Q}{4\pi\varepsilon_o r}$ joules of work must be done in carrying a 1-C charge from infinity to any point r meters from the charge Q.

Thus, potenital at a point can be defined as the work done in bringing a unit positive charge from the zero reference to the point.

A convenient method to express the potential without selecting a specific zero reference entails identifying r_A as r and letting $\frac{Q}{4\pi\varepsilon_o r_B}$ be a constant. Then,

$$\boxed{V = \frac{Q}{4\pi\varepsilon_o r} + C}$$

and C may be selected so that $V = 0$ at any desired value of r.

Note that the potential is a scalar field and does not involve any unit vectors.

SOME IMPORTANT FORMULAE

$$V_P = \frac{Q}{4\pi\varepsilon_o R} + C \quad \textbf{(point charge)}$$

$$V_L = \frac{-\rho_L}{2\pi\varepsilon_o} \ln(r) + C \quad \textbf{(line charge)}$$

$$V_S = \frac{-\rho_S x}{2\varepsilon_o} + C \text{ or } \frac{-\rho_S y}{2\varepsilon_o} + C \text{ or } \frac{-\rho_S z}{2\varepsilon_o} + C \quad \textbf{(surface charge).}$$

4.5 THE POTENTIAL FIELD OF A POINT CHARGE

In the field of a point charge Q placed at the origin (see **Fig.** 4.2), potential difference between two points located at $r = r_A$ and $r = r_B$ is

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$$V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) \quad \text{.............. (i)}$$

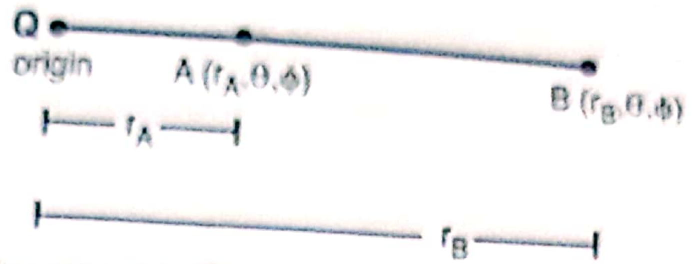

Figure 4.2 For calculating potential difference due to point charge between the two points (lying on the same radial line).

The equation (i) is the expression for the two points lying on the same radial line (having same θ and ϕ).

Now, let's assume that the two points do not lie on the same radial line but their position be A (r_A, θ_A, ϕ_A) and B (r_B, θ_B, ϕ_B) as shown (see Figure 4.3).

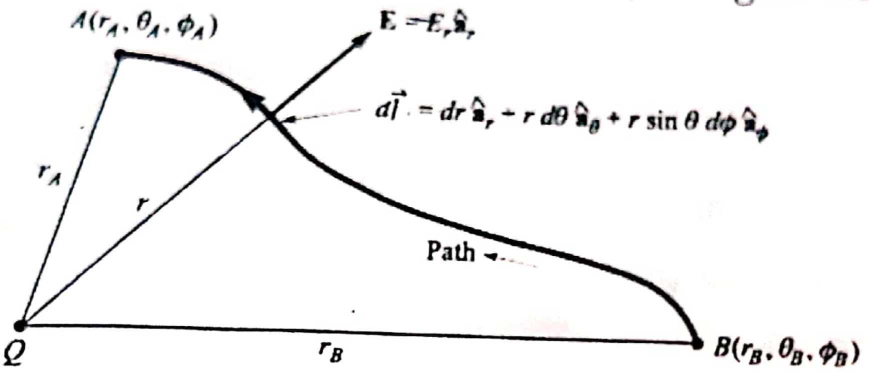

Figure 4.3 A general path between general points B and A in the field of a point charge Q at the origin. The potential difference V_{AB} is independent of the path selected.

$$V_{AB} = -\int_B^A \vec{E}\cdot d\vec{l}$$

$$= -\int_B^A (E_r\, \hat{a}_r)\cdot(dr\, \hat{a}_r + r\, d\theta\, \hat{a}_\theta + r \sin\theta\, d\phi\, \hat{a}_\phi)$$

$$= -\int_B^A E_r\, dr = -\int_{r_B}^{r_A} \frac{Q}{4\pi\varepsilon_0 r^2}\, dr = \frac{-Q}{4\pi\varepsilon_0}\int_{r_B}^{r_A} \frac{1}{r^2}\, dr$$

$$\boxed{\therefore V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)}$$

gain, this expression clearly shows that the potential difference between two oints in the field of a point charge depends only on the distance of each point from the charge and does not depend on the particular path used to carry our unit charge from one point to another.

This value is obtained from equation (ii), if we substitute $r_A = r$ and $r_B \to \infty$.

4.6 THE POTENTIAL FIELD OF A SYSTEM OF CHARGES: CONSERVATIVE CHARGES

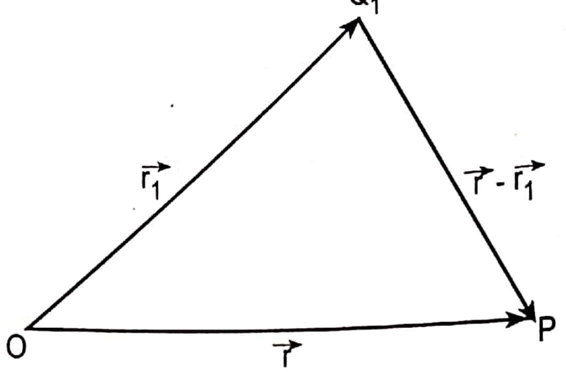

Figure 4.4 Calculation of potential at P due to point charge Q_1.

Let the position vector of a point charge be $\vec{r}_1$ and that of P where we want to find V be $\vec{r}$. The value of V due to Q_1 at point P is

$$V(\vec{r}) = \frac{Q_1}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_1|}$$

Similarly, if we add another charge Q_2 whose position vector is $\vec{r}_2$, then the value of the potential due to these two charges at point P is

$$V(\vec{r}) = \frac{Q_1}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_2|}$$

For a system of n point charges,

$$V(\vec{r}) = \frac{Q_1}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_1|} + \frac{Q_2}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_2|} + \dots\dots\dots + \frac{Q_n}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_n|}$$

$$\therefore V(\vec{r}) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_m|}$$

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Now, suppose that each point charge be a small element of a continuous volume charge distribution $\rho_v \Delta v$, then

$$V(\vec{r}) = \frac{\rho_v(\vec{r}_1)\,\Delta v_1}{4\pi\varepsilon_o |\vec{r} - \vec{r}_1|} + \frac{\rho_v(\vec{r}_2)\,\Delta v_2}{4\pi\varepsilon_o |\vec{r} - \vec{r}_2|} + \dots\dots\dots\dots + \frac{\rho_v(\vec{r}_n)\,\Delta v_n}{4\pi\varepsilon_o |\vec{r} - \vec{r}_n|}$$

And if we assume these small elements to be infinite in number, we will obtain

$$V(\vec{r}) = \int_{vol} \frac{\rho_v(\vec{r}')\,dv'}{4\pi\varepsilon_o |\vec{r} - \vec{r}'|}$$

where $\rho_v(\vec{r}')\,dv'$ denotes amount of charge located at $\vec{r}'$.

The potential $V(\vec{r})$ is determined with respect to a zero reference potential at infinity and is an exact measure of the work done in bringing a unit charge from infinity to the field point at $\vec{r}$ where we are finding the potential.

Similarly, for line charge,

$$V(\vec{r}) = \int \frac{\rho_L(\vec{r}')\,dl'}{4\pi\varepsilon_o |\vec{r} - \vec{r}'|}$$

and for surface charge,

$$V(\vec{r}) = \int_S \frac{\rho_s(\vec{r}')\,dS'}{4\pi\varepsilon_o |r - r'|}$$

Any field that satisfies the equation, $\oint \vec{E}\,.\,d\vec{l} = 0$ is said to be a **conservative field.** The name arises from the fact that no work is done (or that energy is conserved) around a closed path. The equation $\oint \vec{E}\,.\,d\vec{l} = 0$ is true for static fields and is not correct when either electric field or magnetic field varies with time.

4.7 EQUIPOTENTIAL SURFACE

Equipotential surface is a surface composed of points having the same value of potential. Moreover, no work is done in moving a unit charge around equipotential surface.

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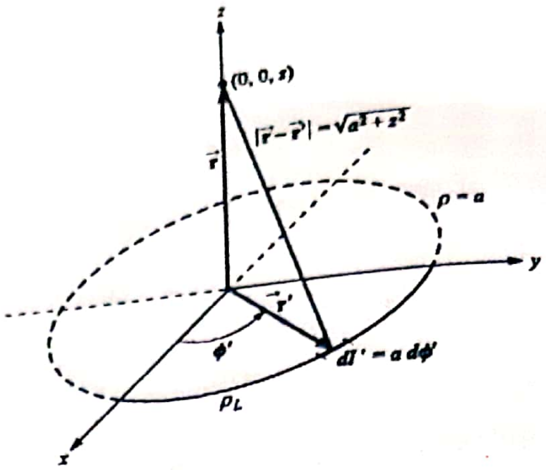
The equipotential surfaces in the potential field of a point charge are spheres centered at the point charge.

1. **Find the potential at a point on the z-axis due to the uniform line charge ρ_L in the form of a ring $\rho = a$ in the $z = 0$ plane as shown in the Figure.**



Solution:

We have, $V = \int \dfrac{\rho_L\, dl'}{4\pi\varepsilon_0 \,|\vec{r} - \vec{r}'|}$

Here, $dl' = a\, d\phi'$, $\vec{r} = z\,\hat{a}_z$, $\vec{r}' = a\,\hat{a}_\rho$

$$\vec{r} - \vec{r}' = z\hat{a}_z - a\,\hat{a}_\rho$$

$$|\vec{r} - \vec{r}'| = \sqrt{z^2 + a^2}$$

So, $V = \int \dfrac{\rho_L\, a\, d\phi'}{4\pi\varepsilon_0 \sqrt{z^2 + a^2}}$

$$= \frac{\rho_L\, a}{4\pi\varepsilon_0 \sqrt{z^2 + a^2}} \int_{\phi=0}^{2\pi} d\phi = \frac{\rho_L\, a}{4\pi\varepsilon_0 \sqrt{z^2 + a^2}} (2\pi - 0)$$

$$\therefore \quad V = \frac{\rho_L\, a}{2\varepsilon_0 \sqrt{z^2 + a^2}}$$

4.8 POTENTIAL GRADIENT

Consider a region as shown in figure where, both $\vec{E}$ and V varies at every point.

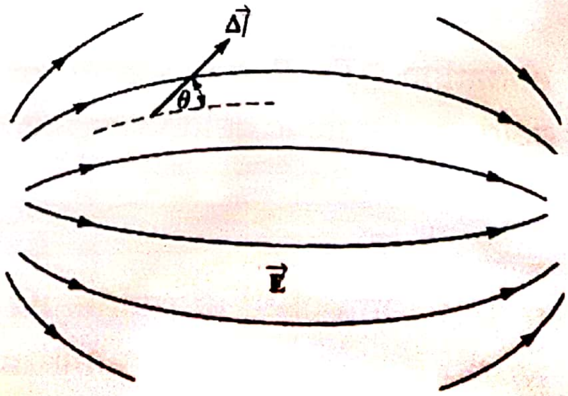


Figure 4.5 Streamlines of electric field. The source of the field are not shown.

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Small work done in moving a unit charge through a small incremental distance $\Delta\vec{l}$ is given by

$$\Delta V = -\vec{E} \cdot \Delta\vec{l}$$

Let θ be the angle between $\Delta\vec{l}$ and $\vec{E}$, then

$$\Delta V = -E\,\Delta l \cos\theta$$

or, $$\frac{\Delta V}{\Delta l} = -E\cos\theta$$

Taking limit as $\Delta l \rightarrow 0$, we have

$$\lim_{\Delta l \to 0} \frac{\Delta V}{\Delta l} = -E\cos\theta$$

or, $$\frac{dV}{dl} = -E\cos\theta$$

For $\theta = 180°$, $\frac{dV}{dl} \rightarrow \left.\frac{dV}{dl}\right|_{max}$

and, $$\left.\frac{dV}{dl}\right|_{max} = E \quad \text{.............} \quad (i)$$

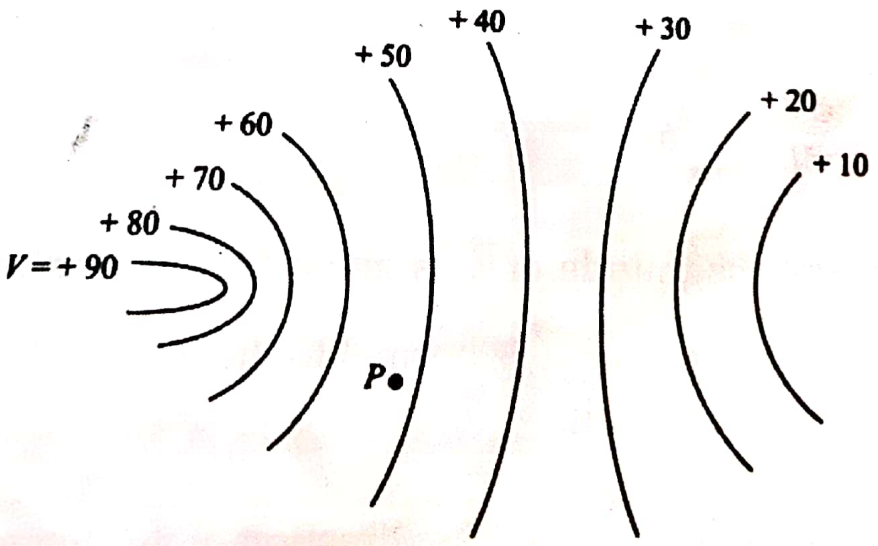

Figure 4.6 Illustration of equipotential surfaces where at any point, the $\vec{E}$ field is normal to the equipotential surface (shown as lines in the two-dimensional sketch) passing through that point and is directed towards the more negative surfaces.

Equation (i) reveals that

(i) The magnitude of the electric field intensity is given by the maximum value of the rate of change of potential with distance.

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(ii) This maximum value is obtained when the direction of $\vec{E}$ is opposite to the direction of $\Delta\vec{l}$, or in other words, the direction of $\vec{E}$ is opposite to the direction in which the potential is increasing the most rapidly.

From the sketch, the direction in which the potential is changing (increasing) the most rapidly appears to be left and slightly upward. The electric field intensity is therefore oppositely directed, or to the right and slightly downward at P.

It seems likely that the direction in which the potential is increasing the most rapidly is perpendicular to the equipotentials (in the direction of increasing potential). This logic is correct because if $\Delta\vec{l}$ is directed along an equipotential, $\Delta V = 0$.

This makes $\Delta V = -\vec{E}\cdot\Delta\vec{l} = 0$

Since neither $\vec{E}$ nor $\Delta\vec{l}$ is zero; $\vec{E}$ must be perpendicular to this $\Delta\vec{l}$ or perpendicular to the equipotentials.

Let $\hat{a}_N$ be a unit vector normal to the equpotential surface and directed toward the higher potentials. Then,

$$\vec{E} = -\frac{dV}{dl}\bigg|_{max}\hat{a}_N$$

which shows that the magnitude of $\vec{E}$ is given by the maximum space rate of change of V and the direction of $\vec{E}$ is normal to the equipotential surface (in the direction of decreasing potential).

Since $\frac{dV}{dl}\Big|_{max}$ occurs when $\Delta\vec{l}$ is in the direction of $\hat{a}_N$, we may write

$$\frac{dV}{dl}\bigg|_{max} = \frac{dV}{dN}$$

Thus,

$$\vec{E} = -\frac{dV}{dN}\hat{a}_N \qquad \text{..................(ii)}$$

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The gradient of a scalar field T is defined as

$$\text{Gradient of } T = \text{grad } T = \frac{dT}{dN}\hat{a}_N \quad \text{............ (iii)}$$

where $\hat{a}_N$ is a unit vector normal to the equipotential surfaces, and that normal is chosen which points in the direction of increasing values of T.

From equation (ii) & (iii), we have

$$\boxed{\vec{E} = -\text{ grad } V} \quad \text{............. (iv)}$$

Since V is a unique function of x, y and z,

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \quad \text{............... (v)}$$

Again, $dV = -\vec{E}\,.\,\vec{dl} = -(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z).(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$

$$\therefore \quad dV = -E_x\,dx - E_y\,dy - E_z\,dz \quad \text{.................. (vi)}$$

Comparing equation (v) & (vi), we get

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

$$\therefore \quad \vec{E} = E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z$$

$$= -\left(\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right) \quad \text{............ (vii)}$$

Comparing equation (iv) and (vii),

$$\text{grad } V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z \quad \text{............. (viii)}$$

Also we have,

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

and $$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z \quad \text{........... (ix)}$$

The equation (viii) and (ix) shows

$$\text{grad } V = \nabla V$$

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In general, for any scalar T

$$\text{grad } T = \nabla T$$

Now, we may rewrite equation (iv) as

$$\boxed{\vec{E} = -\nabla V}$$

IMPORTANT EXPRESSIONS

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z \qquad \textbf{(rectangular)}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z \qquad \textbf{(cylindrical)}$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi \qquad \textbf{(spherical)}$$

4.9 THE DIPOLE

An electric dipole, or simply a dipole, is the name given to two point charges of equal magnitude and opposite sign, separated by a distance which is small compared to the distance to the point P at which the value of $\vec{E}$ and V are to be evaluated.

Consider a dipole formed by two charges +Q and – Q separated by a distance d as shown in the figure below, the +Q & – Q charges being positioned in rectangular coordinate system at $(0, 0, \frac{1}{2}d)$ and $(0, 0, -\frac{1}{2}d)$, respectively.

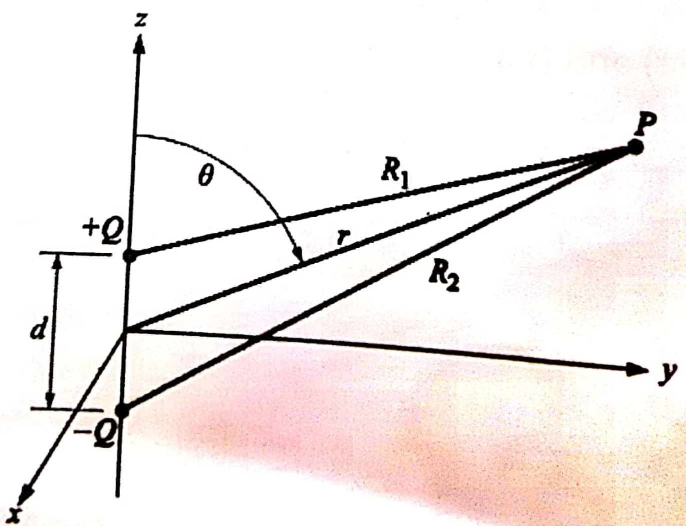

Figure 4.7 Illustration of dipole. P is the point at which field is to be determined.

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Let point P be described in spherical coordinate system as P (r, θ, φ = 90°) and the distances from Q and – Q to P be R_1 and R_2 respectively.

Now, the total potential at point P is

$$V = \frac{+Q}{4\pi\varepsilon_o R_1} + \frac{(-Q)}{4\pi\varepsilon_o R_2}$$

$$= \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $$V = \frac{Q}{4\pi\varepsilon_o}\frac{R_2 - R_1}{R_1 R_2} \qquad \text{.................. (i)}$$

Case - I: When point P is on z = 0 plane (θ = 90°), then $R_1 = R_2$

$\therefore V = 0$

Case - II: When point P is at large distance, R_1 is parallel to R_2, and we may redraw the diagram as

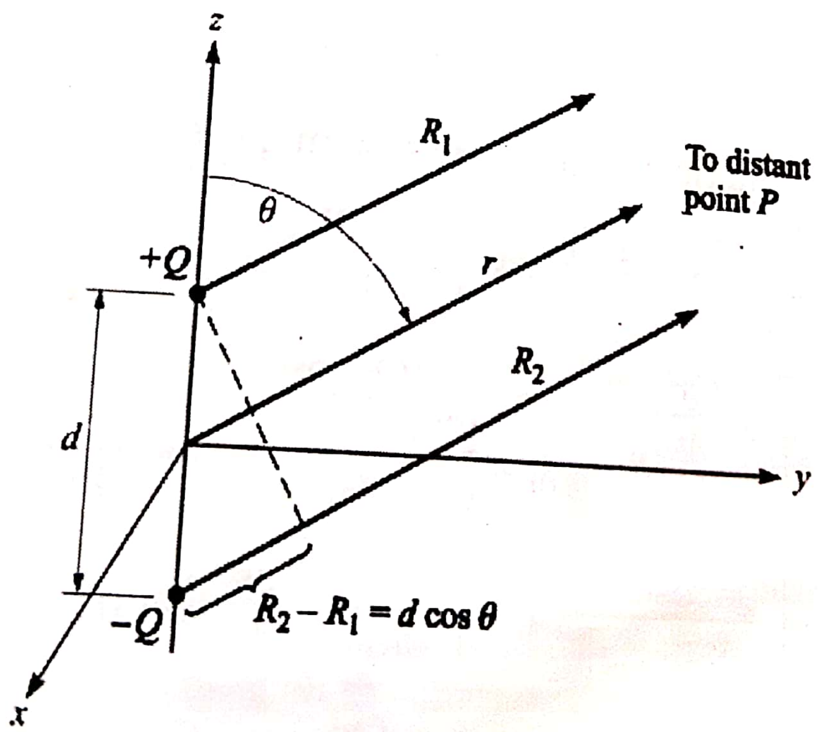

Figure 4.8 The point P is at large distance such that R_1 is essentially parallel to R_2.

$$\cos\theta = \frac{b}{h} = \frac{R_2 - R_1}{d} \Rightarrow R_2 - R_1 = d\cos\theta$$

Also, $R_1 \approx R_2$, $R_1R_2 = r^2$

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$\therefore$ From equation (i),

$$V = \frac{Q}{4\pi\varepsilon_o}\frac{d\cos\theta}{r^2} \quad \text{..........(ii)}$$

Since $d\cos\theta = \vec{d}.\hat{a}_r = d\hat{a}_z.\,\hat{a}_r$, equation (ii) may also be written as

$$V = \frac{\vec{p}.\hat{a}_r}{4\pi\varepsilon_o r^2} = \frac{\vec{p}.\,\vec{r}}{4\pi\varepsilon_o r^3}$$

where $\vec{p} = Q\,\vec{d}$ is the dipole moment directed from -Q to +Q, and p = Qd is the magnitude of dipole moment.

If the dipole center is not at origin but at $\vec{r}'$, then

$$V = \frac{\vec{p}.(\vec{r} - \vec{r}')}{4\pi\varepsilon_o|\,\vec{r} - \vec{r}'\,|^3}$$

Then, $\vec{E} = -\nabla V$

Writing ∇V in spherical co-ordinate system gives

$$\vec{E} = -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right)$$

$$= -\left[\frac{\partial\left(\frac{Qd\cos\theta}{4\pi\varepsilon_o r^2}\right)}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial\left(\frac{Qd\cos\theta}{4\pi\varepsilon_o r^2}\right)}{\partial\theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial\left(\frac{Qd\cos\theta}{4\pi\varepsilon_o r^2}\right)}{\partial\phi}\hat{a}_\phi\right]$$

$$= -\left[\frac{Qd\cos\theta}{4\pi\varepsilon_o}\frac{\partial r^{-2}}{\partial r}\hat{a}_r + \frac{1}{r}\frac{Qd}{4\pi\varepsilon_o r^2}\frac{\partial\cos\theta}{\partial\theta}\hat{a}_\theta + 0\right]$$

$$= -\left[\frac{Qd\cos\theta}{4\pi\varepsilon_o}(-2r^{-3})\,\hat{a}_r + \frac{Qd}{4\pi\varepsilon_o r^3}(-\sin\theta)\,\hat{a}_\theta\right]$$

$$= -\left(-\frac{Qd\cos\theta}{2\pi\varepsilon_o r^3}\hat{a}_r - \frac{Qd\sin\theta}{4\pi\varepsilon_o r^3}\hat{a}_\theta\right)$$

$\therefore\ \vec{E} = \dfrac{Qd}{4\pi\varepsilon_o r^3}(2\cos\theta\,\hat{a}_r + \sin\theta\,\hat{a}_\theta)$ which is the required expression.

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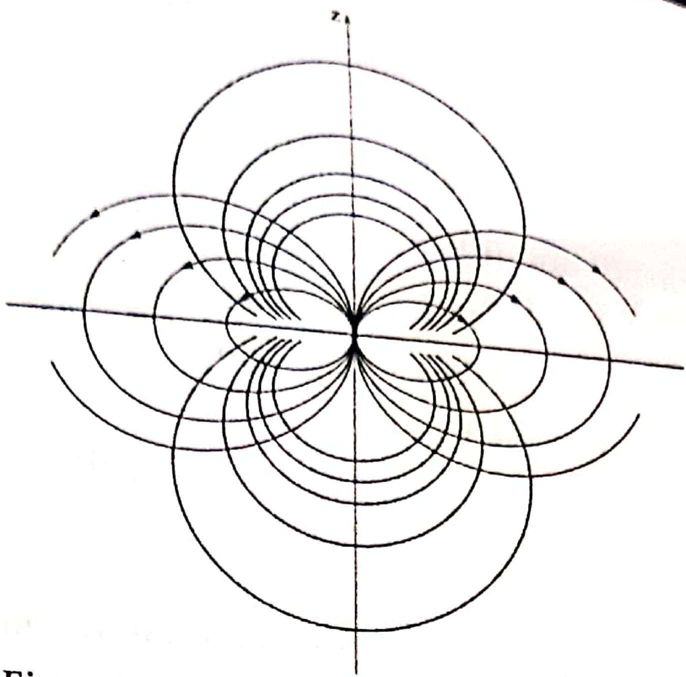

Figure 4.9 The electrostatic field of a point dipole.

4.10 ENERGY DENSITY IN THE ELECTROSTATIC FIELD

To find the potential energy present in a system of charges, we must find the work done by an external source in positioning the charges.

Consider an empty universe at first. We don't have to do work to bring a charge Q_1 from infinity to any position as there exists no field.

Work done in positioning Q_2 in the field of $Q_1 = Q_2\, V_{2,1}$

$V_{2,1}$ = the potential at the location of Q_2 due to Q_1

Work done in positioning Q_3 in the field of Q_1 and Q_2

$$= Q_3\, V_{3,1} + Q_3\, V_{3,2}$$

Similarly, work done in positioning Q_4

$$= Q_4\, V_{4,1} + Q_4\, V_{4,2} + Q_4\, V_{4,3}$$

and so forth.

The total work done in positioning all the charges

= potential energy of field

$$= W_E = Q_2 V_{2,1} + Q_3 V_{3,1} + Q_3\, V_{3,2} + Q_4\, V_{4,1} + Q_4\, V_{4,2} + Q_4\, V_{4,3} + \text{-----} \quad \text{......} \quad \text{(i)}$$

We have,

$$Q_3 V_{3,1} = Q_3 \frac{Q_1}{4\pi\varepsilon_o R_{13}} = Q_1 \frac{Q_3}{4\pi\varepsilon_o R_{31}} = Q_1\, V_{1,3}$$

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where R_{13} and R_{31} each represent the scalar distance between Q_1 and Q_3

Now we can rewrite W_E as

$$W_E = Q_1 V_{1,2} + Q_1 V_{1,3} + Q_2 V_{2,3} + Q_1 V_{1,4} + Q_2 V_{2,4} + Q_3 V_{3,4} + \text{-----} \quad \ldots\ldots\ldots \text{(ii)}$$

Adding expressions (i) and (ii),

$$\begin{aligned} 2W_E = {} & Q_1(V_{1,2} + V_{1,3} + V_{1,4} + \ldots\ldots\ldots\ldots) \\ & + Q_2(V_{2,1} + V_{2,3} + V_{2,4} + \ldots\ldots\ldots\ldots) \\ & + Q_3(V_{3,1} + V_{3,2} + V_{3,4} + \ldots\ldots\ldots\ldots) \\ & + \text{-------} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)} \end{aligned}$$

Here,

$V_{1,2} + V_{1,3} + V_{1,4} + \ldots\ldots\ldots\ldots = V_1$ which is the potential at the location of Q_1 due to the presence of Q_2, Q_3, Q_4, ...

$V_{2,1} + V_{2,3} + V_{2,4} + \ldots\ldots\ldots\ldots = V_2$ which is the potential at the location of Q_2 due to the presence of Q_1, Q_3, Q_4, ... and so on.

∴ Equation (iii) gets reduced to

$$2W_E = Q_1 V_1 + Q_2 V_2 + Q_3 V_3 + \ldots\ldots\ldots\ldots)$$

$$\text{or, } W_E = \frac{1}{2}(Q_1 V_1 + Q_2 V_2 + Q_3 V_3 + \ldots\ldots\ldots\ldots)$$

$$\therefore \quad W_E = \frac{1}{2}\sum_{m=1}^{m=N} Q_m V_m$$

For continuous charge distribution,

$$W_E = \frac{1}{2}\int_{vol} \rho_v V \, dv$$

From Maxwell's first equation,

$$\nabla.\vec{D} = \rho_v$$

$$\therefore \quad W_E = \frac{1}{2}\int_{vol} (\nabla.\vec{D}) V \, dv \qquad \ldots\ldots\ldots\ldots \text{(iv)}$$

Consider a vector identity which is true for any scalar V and any vector $\vec{D}$.

$$\nabla.(V\vec{D}) = V(\nabla.\vec{D}) + \vec{D}.(\nabla V)$$

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or, $V(\nabla.\vec{D}) = \nabla.(V\vec{D}) - \vec{D}.(\nabla V)$

Equation (iv) therefore becomes

$$W_E = \frac{1}{2}\int_{vol} [\nabla.(V\vec{D}) - \vec{D}.(\nabla V)]\, dv$$

or, $$W_E = \frac{1}{2}\int_{vol} \nabla.(V\vec{D})\, dv - \frac{1}{2}\int_{vol} \vec{D}.(\nabla V)\, dv$$

Using the divergence theorem,

$$\int_{vol} \nabla.(V\vec{D})\, dv = \oint_S (V\vec{D}).d\vec{S}$$

$$W_E = \frac{1}{2}\oint_S (V\vec{D}).d\vec{S} - \frac{1}{2}\int_{vol} \vec{D}.(\nabla V)\, dv$$

The first integral encloses the surface surrounding the universe. V is decreasing as rapidly as $\frac{1}{r}$, $\vec{D}$ is decreasing as rapidly as $\frac{1}{r^2}$ but $d\vec{S}$ is increasing only as r^2. This means the first integral is decreasing as rapidly as $\frac{1}{r}$ in total. And if we take limit as $r \rightarrow \infty$, the first integral becomes zero.

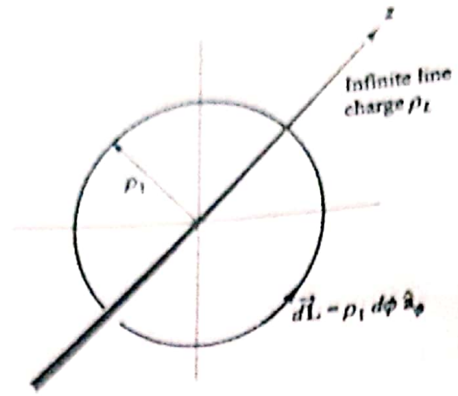
So, $$W_E = -\frac{1}{2}\int_{vol} \vec{D}.(\nabla V)\, dv$$

Substituting $\vec{E} = -\nabla V$ and $\vec{D} = \varepsilon_o \vec{E}$, we have

$$W_E = \frac{1}{2}\int_{vol} \vec{D}.\vec{E}\, dv = \frac{1}{2}\int_{vol} \varepsilon_o E^2\, dv$$

$$\boxed{\therefore w_E = \frac{dW_E}{dv} = \frac{1}{2}\vec{D}.\vec{E} = \frac{1}{2}\varepsilon_o E^2}$$

2. **Find the work done in carrying the positive charge Q about a circular path centered at the line charge. The line charge is located on z-axis extending from $-\infty$ to $+\infty$ and the radius of the circular path is ρ_1.**



Solution:

Electric field due to line charge is $\vec{E} = \dfrac{\rho_L}{2\pi\varepsilon_o\rho_1}\hat{a}_\rho$

In cylindrical coordinate system,

$$\vec{dl} = d\rho\, \hat{a}_\rho + \rho d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$$

For the given circular path, $d\rho = 0$, $dz = 0$

$$\therefore \quad \vec{dl} = \rho d\phi\, \hat{a}_\phi = \rho_1\, d\phi\, \hat{a}_\phi$$

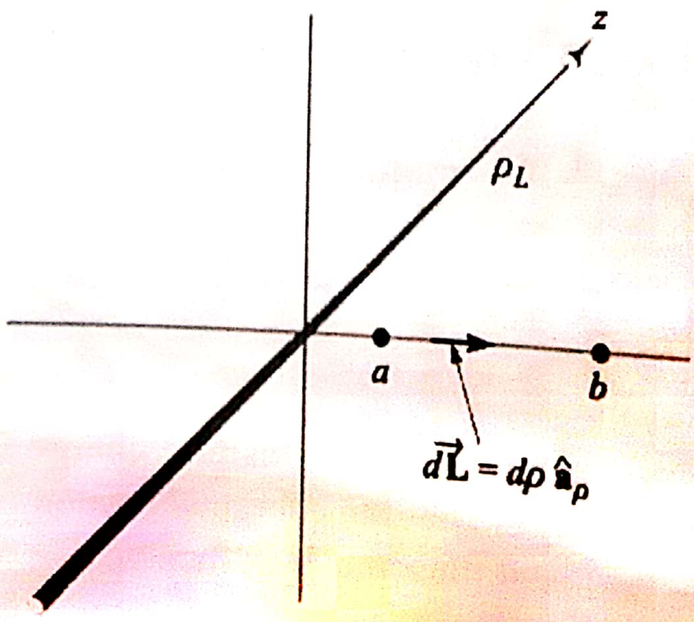
Now, work done is given as

$$W = -Q\int_{\text{init}}^{\text{final}} \vec{E}.\, \vec{dl}$$

$$= -Q\int_{\text{init}}^{\text{final}} \frac{\rho_L}{2\pi\varepsilon_o\rho_1}\hat{a}_\rho \,.\, \rho_1 d\phi\, \hat{a}_\phi = -Q\int_0^{2\pi} \frac{\rho_L}{2\pi\varepsilon_o} d\phi\, \hat{a}_\rho .\, \hat{a}_\phi$$

$$\therefore\; W = 0 \quad (\because \hat{a}_\rho .\, \hat{a}_\phi = 0)$$

3. **Calculate the work done in carrying the positive charge Q from $\rho = a$ to $\rho = b$ along a radial path as shown in the figure.**



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Solution:

In cylindrical co-ordinate system, $\vec{dl} = d\rho\, \hat{a}_\rho + \rho\, d\phi\, \hat{a}_\phi + dz\, \hat{a}_z$

Since $d\phi = 0$, $dz = 0$, $\vec{dl} = d\rho\, \hat{a}_\rho$

Now, work done is

$$W = -Q \int_{init}^{final} \vec{E} \cdot \vec{dl}$$

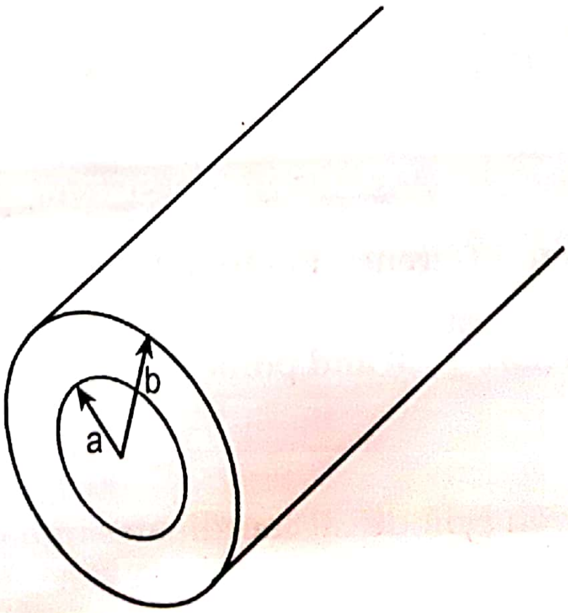
$$= -Q \int_{init}^{final} \frac{\rho_L}{2\pi\varepsilon_o \rho} \hat{a}_\rho . d\rho\ \hat{a}_\rho$$

$$= \frac{-Q\rho_L}{2\pi\varepsilon_o} \int_{\rho = a}^{b} \frac{d\rho}{\rho}$$

$$= \frac{-Q\rho_L}{2\pi\varepsilon_o} [ln\ \rho]_a^b = \frac{-Q\rho_L}{2\pi\varepsilon_o} [ln\ b - ln\ a]$$

$$\therefore \quad W = \frac{-Q\rho_L}{2\pi\varepsilon_o} ln \frac{b}{a}$$

4. **Find the energy stored in a coaxial cable of length L.**



Solution:

$$W_E = \frac{1}{2} \int_{vol} \vec{D} . \vec{E}\, dv$$

$$= \frac{1}{2} \int_{vol} \varepsilon_o \vec{E} . \vec{E}\, dv = \frac{1}{2} \int_{vol} \varepsilon_o E^2\, dv$$

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For a coaxial cable,

$\vec{E} = \frac{a\rho_s}{\varepsilon_o \rho}\hat{a}_\rho$, ρ_s = surface charge density on the inner conductor

dv (in cylindrical coordinate system) = $\rho d\rho\, d\phi\, dz$

or, $W_E = \frac{1}{2}\int_{vol} \varepsilon_o \left(\frac{a\rho_s}{\varepsilon_o \rho}\right)^2 (\rho d\rho\, d\phi\, dz)$

$$= \frac{1}{2}\int_{vol} \varepsilon_o \frac{a^2\rho_s^2}{\varepsilon_o^2 \rho^2}(\rho d\rho\, d\phi\, dz)$$

$$= \frac{1}{2}\frac{a^2\rho_s^2}{\varepsilon_o}\int_{vol}\frac{d\rho}{\rho}\, d\phi\, dz$$

$$= \frac{1}{2}\frac{a^2\rho_s^2}{\varepsilon_o}\int_{z=0}^{L}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\frac{1}{\rho}\, d\rho\, d\phi\, dz$$

$$= \frac{1}{2}\frac{a^2\rho_s^2}{\varepsilon_o}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\frac{1}{\rho}\, d\rho\, d\phi\ (L-0)$$

$$= \frac{L}{2}\frac{a^2\rho_s^2}{\varepsilon_o}\int_{a}^{b}\frac{1}{\rho}\, d\rho\ (2\pi - 0) \quad = \frac{L\pi a^2\rho_s^2}{\varepsilon_o}\int_{a}^{b}\frac{d\rho}{\rho}$$

$$\therefore\ W_E = \frac{L\pi a^2\rho_s^2}{\varepsilon_o}\, ln\frac{b}{a}$$

PROBLEMS SOLVED AND SCRAMBLED

1. Calculate the potential difference V_{AB} for a line charge $\rho_L = 0.25\times10^{-9}$ C on the z-axis where point A is (2m, $\frac{\pi}{2}$, 0) and point B is (4m, $\frac{\pi}{2}$, 5 m).

Solution:

First converting the given cylindrical coordinates into rectangular coordinates.

For $(2, \frac{\pi}{2}, 0) \leftrightarrow (\rho, \phi, z)$, we have

$x = \rho\cos\phi = 2\cos\frac{\pi^c}{2} = 0$, $y = \rho\cos\phi = 2\sin\frac{\pi^c}{2} = 2$, z=z = 0

$\therefore (x, y, z) = (0, 2, 0)$

For $(4, \frac{\pi}{2}, 5) \leftrightarrow (\rho, \phi, z)$, we have

$x = \rho \cos\phi = 4\cos\frac{\pi^c}{2} = 0$, $y = \rho \sin\phi = 4\sin\frac{\pi^c}{2} = 4$, $z = z = 5$

$\therefore (x, y, z) = (0, 4, 5)$

We have,

$$V = -\frac{\rho_L}{2\pi\varepsilon_o} \ln(r) + C$$

$$V_A = -\frac{\rho_L}{2\pi\varepsilon_o} \ln(r_A) + C$$

$$V_B = -\frac{\rho_L}{2\pi\varepsilon_o} \ln(r_B) + C$$

$$V_{AB} = V_A - V_B = [-\frac{\rho_L}{2\pi\varepsilon_o} \ln(r_A) + C] - [-\frac{\rho_L}{2\pi\varepsilon_o} \ln(r_B) + C]$$

$$= \frac{\rho_L}{2\pi\varepsilon_o} [\ln(r_B) - \ln(r_A)] = \frac{\rho_L}{2\pi\varepsilon_o} \ln\frac{r_B}{r_A}$$

$\vec{r_A}$ = F.P. - S.P. = (0, 2, 0) – (0, 0, 0) = 2 $\hat{a}_y$; $r_A = 2$

$\vec{r_B}$ = F.P. – S.P. = (0, 4, 5) – (0, 0, 5) = 4 $\hat{a}_y$; $r_B = 4$

$$\therefore V_{AB} = \frac{0.25\times10^{-9}}{2\pi\times8.854\times10^{-12}} \ln\left(\frac{4}{2}\right) = 3.119 \text{ V}$$

2. **If $V = (60 \sin\theta)/r^2$ V in free space and point P is located at r = 3 m, θ = 60°, ϕ = 25°, find (a) V_P (b) $\vec{E_p}$ (c) dV/dN at P (d) $\hat{a}_N$ at P (e) ρ_v at P**

Solution:

a. $V_P = 60 \sin60°/3^2 = 5.774$ V

b. $$\vec{E} = -\nabla V = \left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right]$$

$$= -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + 0\right]$$

$$= -\left[\frac{-120\sin\theta}{r^3}\hat{a}_r + \frac{60\cos\theta}{r^3}\hat{a}_\theta\right]$$

$$\vec{E_p} = \frac{120\sin60°}{3^3}\hat{a}_r - \frac{60\cos60°}{3^3}\hat{a}_\theta = (3.849\,\hat{a}_r - 1.111\,\hat{a}_\theta) \text{ V/m}$$

c. $$\frac{dV}{dN}\Big|_P = |\vec{E_p}| = \sqrt{3.849^2 + 1.111^2} = 4.006 \text{ V/m}$$

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d. $\hat{a}_N|_P = -\dfrac{\vec{E}_P}{|\vec{E}_P|} = \dfrac{-(3.849\,\hat{a}_r - 1.111\,\hat{a}_\theta)}{4.006} = -0.961\,\hat{a}_r + 0.277\,\hat{a}_\theta$

e. $\vec{D} = \varepsilon_o \vec{E} = \varepsilon_o \left[\dfrac{120\sin\theta}{r^3}\hat{a}_r - \dfrac{60\cos\theta}{r^3}\hat{a}_\theta\right]$

$$\rho_v = \nabla . \vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{120\varepsilon_o \sin\theta . r^2}{r^3}\right] + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{-60\varepsilon_0\cos\theta\sin\theta}{r^3}\right]$$

$$= \frac{-120\varepsilon_0\sin\theta}{r^4} \quad \frac{-60\varepsilon_0\cos 2\theta}{r^4\sin\theta}$$

$$\rho_v|_P = \frac{-120\varepsilon_0\sin 60^\circ}{3^4} \quad \frac{-60\varepsilon_0\cos 120^\circ}{3^4\sin 60} = -7.573\ \text{pC/m}^3$$

3. **A uniform sheet of charge $\rho_S = 40\,\varepsilon_o\ C/m^2$ is located in the plane $x = 0$ in free space. A uniform line charge $\rho_L = 0.6\ nC/m$ lies along the line $x = 9$, $y = 4$ in free space. Find the potential at point P(6, 8, -3) if $V = 10V$ at A(2, 9, 3).**

Solution:

Potential due to line charge is

$$V_L = -\int \frac{\rho_L}{2\pi\varepsilon_o r}\,dr = \frac{-\rho_L}{2\pi\varepsilon_o}\ln(r) + C_1$$

Potential due to sheet of charge is

$$V_S = -\int \frac{\rho_S}{2\varepsilon_o}\,dx = \frac{-\rho_S}{2\varepsilon_o}x + C_2$$

Total potential is expressed as

$$V = V_L + V_S = \frac{-\rho_L}{2\pi\varepsilon_o}\ln(r) - \frac{\rho_S}{2\varepsilon_o}x + C_1 + C_2$$

$$= \frac{-\rho_L}{2\pi\varepsilon_o}\ln(r) - \frac{\rho_S}{2\varepsilon_o}x + C \quad \text{.....................(i)}$$

To find C, we use the condition that, at (2, 9, 3), $V = 10$.

For (2, 9, 3) as field point,

$\vec{r}$ = F.P. – S.P.

$= (2, 9, 3) - (9, 4, 3) = -7\,\hat{a}_x + 5\,\hat{a}_y\ ;\ r = \sqrt{(-7)^2 + (5)^2} = \sqrt{74}$

$x = 2 - 0 = 2$

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From equation (i),

$$10 = \frac{-0.6\times10^{-9}}{2\pi\times8.854\times10^{-12}} \ln\left(\sqrt{74}\right) - \frac{40\varepsilon_o}{2\varepsilon_o}(2) + C$$

$\therefore C = 73.210$ V

Equation (i) therefore becomes

$$V = \frac{-\rho_L}{2\pi\varepsilon_o}\ln(r) - \frac{\rho_s}{2\varepsilon_o}x + 73.21$$

For (6, 8, –3) as field point,

$\vec{r}$ = F.P. – S.P. =(6, 8, –3) – (9, 4, –3) = $-3\hat{a}_x + 4\hat{a}_y$; $r = \sqrt{(-3)^2 + (4)^2} = 5$

$x = 6 - 0 = 6$

$$V = \frac{-0.6\times10^{-9}}{2\pi\times8.854\times10^{-12}}\ln(5) - \frac{40\varepsilon_o}{2\varepsilon_o}(6) + 73.21 = -64.1482 \text{ V}$$

4. **Given the potential field $V = 100\,xz/(x^2+4)$ volts in free space: (a) Find $\vec{D}$ at the surface z = 0. (b) Show that the z =0 surface is an equipotential surface. (c) Assume that the z = 0 surface is a conductor and find the total charge on that portion of the conductor defined by $0<x<2$, $-3<y<0$.** *[2069 Chaitra]*

Solution:

a. $$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$= -\left[100z\frac{\partial}{\partial x}\left(\frac{x}{x^2+4}\right)\hat{a}_x + 0 + \frac{100x}{x^2+4}\frac{\partial z}{\partial z}\hat{a}_z\right]$$

$$= -\left[100z\left\{\frac{4-x^2}{(x^2+4)^2}\right\}\hat{a}_x + \left\{\frac{100x}{x^2+4}\right\}\hat{a}_z\right] \text{V/m}$$

$$\vec{D} = \varepsilon_o\vec{E} = -\varepsilon_o\left[100z\left\{\frac{4-x^2}{(x^2+4)^2}\right\}\hat{a}_x + \left\{\frac{100x}{x^2+4}\right\}\hat{a}_z\right] \text{C/m}^2$$

$$\vec{D}\Big|_{z=0} = -\varepsilon_o\left[0 + \frac{100x}{x^2+4}\hat{a}_z\right] = \frac{-100\varepsilon_0 x}{x^2+4}\hat{a}_z \text{ C/m}^2$$

b. $$V = \frac{100xz}{x^2+4} \text{ V}$$

At z = 0, V =0 for all values of x and y. Therefore, z =0 surface is a[n] equipotential surface.

c. The total charge on the conductor is

$$Q = \oint_S \vec{D} \cdot d\vec{S}$$

For $z = 0$, $\vec{D} = \frac{-100\varepsilon_o x}{x^2+4} \hat{a}_z$ C/m^2

$$Q = \oint_S \left(\frac{-100\varepsilon_o x}{x^2+4} \hat{a}_z\right) \cdot (dx\, dy\, \hat{a}_z) = \int_{x=0}^{2} \int_{y=-3}^{0} \left(\frac{-100\varepsilon_o x}{x^2+4}\right) dx\, dy$$

$$= \int_{x=0}^{2} \frac{-300\varepsilon_o x}{x^2+4}\, dx$$

$$= -150\varepsilon_o \int_{x=0}^{2} \frac{2x}{x^2+4}\, dx$$

$$= -150\, \varepsilon_o \left[ln\,(x^2+4)\right]_0^2 = -150\, \varepsilon_o \left[ln\, 8 - ln\, 4\right]$$

$$\therefore Q = -150\, \varepsilon_o\, ln\, \frac{8}{4} = -150\, \varepsilon_o\, ln\, 2\, C$$

5. **Given the potential $V = \frac{10}{r^2} \sin\theta \cos\phi$,**

(a) **Find the electric flux density $\vec{D}$ at $\left(2, \frac{\pi}{2}, 0\right)$.**

(b) **Calculate the work done in moving a 10 μC from point A(1, 30°, 120°) to B(4, 90°, 60°).**

Solution:

a. $\vec{D} = \varepsilon_o \vec{E}$

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi\right]$$

$$= -\left[\frac{\partial\left(\frac{10}{r^2}\sin\theta\cos\phi\right)}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial\left(\frac{10}{r^2}\sin\theta\cos\phi\right)}{\partial\theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial\left(\frac{10}{r^2}\sin\theta\cos\phi\right)}{\partial\phi}\hat{a}_\phi\right]$$

$$= -\left[(-20r^{-3}\sin\theta\cos\phi)\,\hat{a}_r + \left(\frac{10}{r^3}\cos\theta\cos\phi\right)\hat{a}_\theta - \left(\frac{10}{r^3}\sin\phi\right)\hat{a}_\phi\right]$$

$$= \left[\left(\frac{20}{r^3}\sin\theta\cos\phi\right)\hat{a}_r - \left(\frac{10}{r^3}\cos\theta\cos\phi\right)\hat{a}_\theta + \left(\frac{10}{r^3}\sin\phi\right)\hat{a}_\phi\right]$$

At $\left(2, \frac{\pi}{2}, 0\right)$,

$$\vec{E} = \left[\left(\frac{20}{2^3}\sin 90^\circ \cos 0^\circ\right)\hat{a}_r - \left(\frac{10}{2^3}\cos 90^\circ \cos 0^\circ\right)\hat{a}_\theta + \left(\frac{10}{2^3}\sin 0^\circ\right)\hat{a}_\phi\right]$$

$$= 2.5\ \hat{a}_r \frac{V}{m}$$

$$\therefore \vec{D} = \varepsilon_o \vec{E} = 8.854 \times 10^{-12} \times 2.5\ \hat{a}_r = 2.2135 \times 10^{-11}\ \hat{a}_r = 22.135\ \hat{a}_r\ pC/m^2$$

b. $$W = -Q\int_A^B \vec{E} \cdot d\vec{l} = QV_{BA} = Q\,(V_B - V_A)$$

$$= 10\times10^{-6}\left(\frac{10}{4^2}\sin 90^\circ \cos 60^\circ - \frac{10}{1^2}\sin 30^\circ \cos 120^\circ\right)$$

$$= 28.125\times10^{-6}\,J$$

6. **Two point charges -4 μC and 5 μC are located at (2, -1, 3) and (0, 4, -2) respectively. Find the potential at (1, 0, 1) assuming zero potential at infinity.**

Solution:

$$V = V_{P1} + V_{P2} = \frac{Q_1}{4\pi\varepsilon_o R_1} + \frac{Q_2}{4\pi\varepsilon_o R_2} + C \qquad \text{......(i)}$$

To determine the value of C, we use the condition that V= 0 at (∞,∞,∞).

For (∞, ∞, ∞) as field point,

$$\vec{R_1} = (\infty, \infty, \infty) - (2, -1, 3);\ R_1 = \infty$$

$$\vec{R_2} = (\infty, \infty, \infty) - (0, 4, -2);\ R_2 = \infty$$

$$\therefore 0 = \frac{-4\times10^{-6}\times9\times10^9}{\infty} + \frac{5\times10^{-6}\times9\times10^9}{\infty} + C \Rightarrow C = 0$$

Equation (i) is then reduced to

$$V = \frac{Q_1}{4\pi\varepsilon_o R_1} + \frac{Q_2}{4\pi\varepsilon_o R_2} + 0 = \frac{Q_1}{4\pi\varepsilon_o R_1} + \frac{Q_2}{4\pi\varepsilon_o R_2}$$

For (1, 0, 1) as field point,

$$\vec{R_1} = (1, 0, 1) - (2,-1,3) = -\hat{a}_x + \hat{a}_y - 2\hat{a}_z;\ R_1 = \sqrt{(-1)^2+(1)^2+(-2)^2} = \sqrt{6}$$

$$\vec{R_2} = (1, 0, 1) - (0, 4, -2) = \hat{a}_x - 4\,\hat{a}_y + 3\hat{a}_z;\ R_2 = \sqrt{(1)^2+(-4)^2+(3)^2} = \sqrt{26}$$

$$\therefore V = \frac{-4\times10^{-6}\times9\times10^9}{\sqrt{6}} + \frac{5\times10^{-6}\times9\times10^9}{\sqrt{26}} = -5871.712\ V$$

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7. **Let a uniform surface charge density of 5 nC/m² be present at z = 0 plane, a uniform line charge density of 8 nC/m be located at x = 0, z = 4, and a point charge of 2 μC be present at P(2, 0, 0). If $V = 0$ at M (0, 0, 5), find the potential at N (1, 2, 3).** [2063 Kartik]

Solution:

$$V = \frac{Q}{4\pi\varepsilon_o R_1} - \frac{\rho_L}{2\pi\varepsilon_o} ln(R_2) - \frac{\rho_S z}{2\varepsilon_o} + C$$

At M (0, 0, 5), $V = 0$ (given)

$\vec{R_1}$ = F.P. - S.P. = (0, 0, 5) – (2, 0, 0) = – $2\hat{a}_x + 5\hat{a}_z$; $R_1 = \sqrt{(-2)^2 + (5)^2} = \sqrt{29}$

$\vec{R_2}$ = F.P. – S.P. = (0, 0, 5) – (0, 0, 4) = $+\hat{a}_z$; $R_2 = \sqrt{(1)^2} = 1$

z = 5 – 0 = 5

$$\therefore \quad 0 = \frac{2 \times 10^{-6} \times 9 \times 10^9}{\sqrt{29}} - 8 \times 10^{-9} \times 2 \times 9 \times 10^9\, ln\,(1) - \frac{5 \times 10^{-9} \times 5}{2 \times 8.85 \times 10^{-12}} + C$$

$\Rightarrow \quad C = -1.928 \times 10^3$ V

Equation (i) is then reduced to

$$V = \frac{Q}{4\pi\varepsilon_o R_1} - \frac{\rho_L}{2\pi\varepsilon_o} ln\,(R_2) - \frac{\rho_S z}{2\varepsilon_o} - 1.928 \times 10^3$$

For N(1, 2, 3) as the field point,

$\vec{R}_1$ = F.P. – S.P. = (1, 2, 3) – (2, 0, 0) = – $\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$

$R_1 = \sqrt{(-1)^2 + (2)^2 + (3)^2} = \sqrt{14}$

$\vec{R_2}$ = F.P. – S.P. = (1, 2, 3) – (0, 2, 4) = $\hat{a}_x - \hat{a}_z$

$R_2 = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

z = 3 – 0 = 3

$$\therefore V = \frac{2 \times 10^{-6} \times 9 \times 10^9}{\sqrt{14}} - 2 \times 8 \times 10^{-9} \times 9 \times 10^9\, ln\,(\sqrt{2}) - \frac{5 \times 10^{-9} \times 3}{2 \times 8.85 \times 10^{-12}} - 1.928 \times 10^3$$

$= 1.985$ kV

8. **Two uniform charges 8 nC/m are located at x = 1, z = 2 & at x = – 1, y = 2 in free space. If the potential at origin is 100 V, find V at P (4, 1, 3).** [2065 Chaitra]

Solution:

The potential is

$$V = V_{L1} + V_{L2} = -\frac{\rho_{L_1}}{2\pi\varepsilon_o} ln\,(R_1) - \frac{\rho_{L_2}}{2\pi\varepsilon_o} ln\,(R_2) + C \text{ ....(i)}$$

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At origin (0, 0, 0), $V = 100$ V. For (0, 0, 0) as field point,

$$\vec{R}_1 = (0, 0, 0) - (1, 0, 2) = -\hat{a}_x - 2\hat{a}_z\ ;\ R_1 = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$$

$$\vec{R}_2 = (0, 0, 0) - (-1, 2, 0) = \hat{a}_x - 2\hat{a}_y;\ R_2 = \sqrt{(1)^2 + (-2)^2} = \sqrt{5}$$

$$\therefore 100 = -2 \times 8 \times 10^{-9} \times 2 \times 9 \times 10^9\, ln\,(\sqrt{5}) + C \Rightarrow C = 331.443V$$

Equation (i) is then reduced to

$$V = -\frac{\rho_{L_1}}{2\pi\varepsilon_o} ln\,(R_1) - \frac{\rho_{L_2}}{2\pi\varepsilon_o} ln\,(R_2) + 331.443$$

Now, for (4, 1, 3) as field point,

$$\vec{R}_1 = (4, 1, 3) - (1, 1, 2) = 3\,\hat{a}_x + \hat{a}_z;\ R_1 = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$$

$$\vec{R}_2 = (4, 1, 3) - (-1, 2, 3) = 5\hat{a}_x - \hat{a}_y;\ R_2 = \sqrt{(5)^2 + (-1)^2} = \sqrt{26}$$

$$\therefore\quad V = -2 \times 9 \times 10^9 \times 8 \times 10^{-9}\,[ln\sqrt{26} + ln\sqrt{10}\,] + 331.443 = -68.92\ V$$

In free space, a line charge ρ_L = 80 nC/m lie along entire z-axis, while point charge of 100 nC is located at (0, 1, 0). Find the potential difference V_{PQ} given that P(2 ,1, 0) & Q(3, 2, 5). *[2068 Magh]*

Solution:

$$V_{PQ} = V_P - V_Q = \left[\frac{Q}{4\varepsilon_o R_P} - \frac{\rho_L}{2\pi\varepsilon_o} ln\,(r_P) + C\right] - \left[\frac{Q}{4\varepsilon_o R_Q} - \frac{\rho_L}{2\pi\varepsilon_o} ln\,(r_Q) + C\right]$$

$$= \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{R_P} - \frac{1}{R_Q}\right] + \frac{\rho_L}{2\pi\varepsilon_o} ln\left[\frac{r_Q}{r_P}\right]$$

$\vec{R}_P$ = field point– source point = (2, 1, 0) – (0, 1, 0) = $2\,\hat{a}_x$; $R_P = 2$

$\vec{R}_Q$ = (3, 2, 5) – (0 1, 0) = $3\,\hat{a}_x + \hat{a}_y + 5\,\hat{a}_z$; $R_Q = \sqrt{35}$

$\vec{r}_P$ = (2, 1, 0)- (0, 0, 0) = $2\,\hat{a}_x + \hat{a}_y$; $r_P = \sqrt{5}$

$\vec{r}_Q$ = (3, 2, 5)- (0, 0, 5) = $(3\hat{a}_x + 2\hat{a}_y)$; $r_Q = \sqrt{13}$

$$\therefore\ V_{PQ} = 100\times10^{-9}\times9\times10^9\left[\frac{1}{2} - \frac{1}{\sqrt{35}}\right] + 2\times9\times10^9\times80\times10^{-9}\, ln\left[\frac{\sqrt{13}}{\sqrt{5}}\right] = 984.5\ V$$

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10. For a potential field $V = r^2z^2 \sin\phi$ at P (1, 45°, 1) in cylindrical co-ordinate system, determine

(a) Potential (V)
(b) Electric field intensity ($\vec{E}$)
(c) Electric flux density ($\vec{D}$)
(d) Volume charge (ρ_v)
(e) Unit vector in the direction of $\vec{E}$

[2068 Shrawan]

Solution:

a. $V = r^2z^2 \sin\phi$

$V(1, 45°, 1) = 1 \times 1 \times \sin 45° = 0.707$ V

b. $$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$= -\left[\frac{\partial(r^2 z^2 \sin\phi)}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial(r^2z^2 \sin\phi)}{\partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}(r^2z^2 \sin\phi)\,\hat{a}_z\right]$$

$$= -\left[2r z^2 \sin\phi\,\hat{a}_r + rz^2 \cos\phi\,\hat{a}_\phi + 2zr^2 \sin\phi\,\hat{a}_z\right]$$

At point P (1, 45°, 1),

$$\vec{E} = -(2 \times 1 \times \sin 45°\,\hat{a}_r + 1 \times 1 \times \cos 45°\,\hat{a}_\phi + 2 \times 1 \times 1 \times \sin 45°\,\hat{a}_z)$$

$$= -(1.4142\,\hat{a}_r + 0.707\,\hat{a}_\phi + 1.4142\,\hat{a}_z) \text{ V/m}$$

c. $$\vec{D} = \varepsilon_o \vec{E} = 8.854 \times 10^{-12}(-1.4142\,\hat{a}_r - 0.707\,\hat{a}_\phi - 1.4142\,\hat{a}_z)$$

$$= (-12.515\,\hat{a}_r - 6.2569\,\hat{a}_\phi - 12.515\,\hat{a}_z) \text{ pC/m}^2$$

d. $$\rho_v = \nabla.\vec{D} = \frac{1}{r}\frac{\partial}{\partial r}(rD_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

The general expression for $\vec{D}$ is

$$\vec{D} = \varepsilon_o\vec{E} = -\varepsilon_o(2rz^2 \sin\phi\,\hat{a}_r + rz^2 \cos\phi\,\hat{a}_\phi + 2zr^2 \sin\phi\,\hat{a}_z)$$

$$= D_r\,\hat{a}_r + D_\phi\,\hat{a}_\phi + D_z\,\hat{a}_z$$

$\therefore D_r = -2\varepsilon_o rz^2 \sin\phi,\ D_\phi = -\varepsilon_o rz^2 \cos\phi,\ D_z = -2\varepsilon_o r^2 z \sin\phi$

$\rho_v = -\varepsilon_o[4z^2 \sin\phi - z^2\sin\phi + 2r^2 \sin\phi]$

At point P (1, 45°,1),

$\rho_v = 8.854 \times 10^{-12}[4 \times 1 \times \sin45° - 1 \times \sin 45° + 2 \sin45°] = -31.28$ pC/m³

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c. Unit vector in the direction of $\vec{E}$ is calculated as

$$\frac{dV}{dN} = |\vec{E}| = \sqrt{(1.4142)^2 + (0.707)^2 + (1.4142)^2} = 2.1212 \text{ V/m}$$

$$\hat{a}_N = -\frac{\vec{E}}{|\vec{E}|} = \frac{1.4142\,\hat{a}_r + 0.707\hat{a}_\phi + 1.4142\hat{a}_z}{2.1212}$$

$$= 0.667\,\hat{a}_r + 0.333\hat{a}_\phi + 0.667\,\hat{a}_z$$

11. **Find the potential at point P (2, 3, 3) due to a 1 nC charge located at Q (3, 4, 4), 1 nC/m uniform line charge located at x = 2, y = 1 if potential at (3, 4, 5) is 0 V.**

[2067 Mangsir]

Solution:

Potential due to point charge (V_P) = $\frac{Q}{4\pi\varepsilon_o R} + C_1$

Potential due to line charge (V_L) = $-\frac{\rho_L}{2\pi\varepsilon_o} \ln(r) + C_2$

$\therefore$ Total potential (V) = $V_P + V_L = \frac{Q}{4\pi\varepsilon_o R} - \frac{\rho_L}{2\pi\varepsilon_o}\ln(r) + C$

Now at point R (3, 4, 5),

$\vec{R}$ = F. P – S. P. = (3, 4, 5) – (3, 4, 4) = $\hat{a}_z$; $R = \sqrt{(1)^2} = 1$

$\vec{r}$ = F. P – S. P. = (3, 4, 5) – (2, 1, 5) = $\hat{a}_x + 3\,\hat{a}_y$; $r = \sqrt{(1)^2 + (3)^2} = \sqrt{10}$

$$0 = \frac{9 \times 10^9 \times 1 \times 10^{-9}}{1} - 2 \times 9 \times 10^9 \times 1 \times 10^{-9} \ln(\sqrt{10}) + C$$

$0 = 9 - 20.723 + C$

$\therefore$ $C = 11.723$

At point P (2, 3, 3),

$\vec{R}$ = (2, 3, 3) – (3, 4, 4) = $-\hat{a}_x - \hat{a}_y - \hat{a}_z$; $R = \sqrt{(-1)^2 + (-1)^2 + (-1)^2} = \sqrt{3}$

$\vec{r}$ = (2, 3, 3) – (2, 1, 3) = $2\hat{a}_y$; $r = \sqrt{(2)^2} = 2$

$V = 9 - 18\ln(2) + 11.723$

$\therefore V = 4.436$ V

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12. Two dipoles with dipole moments $-5a_z$ nC m and $9a_z$ nC m are located at points (0,0, −2) and (0, 0,3) respectively. Find the potential at the origin.

Solution:

$$V = \sum_{k=1}^{2} \frac{\vec{P} \cdot \hat{a}_{R}}{4\pi\varepsilon_o r_k^2} = \frac{1}{4\pi\varepsilon_o}\left[\frac{\vec{P_1} \cdot \hat{a}_{r1}}{r_1^2} + \frac{\vec{P_2} \cdot \hat{a}_{r2}}{r_2^2}\right] = \frac{1}{4\pi\varepsilon_o}\left[\frac{\vec{P_1} \cdot \vec{r_1}}{r_1^3} + \frac{\vec{P_2} \cdot \vec{r_2}}{r_2^3}\right]$$

$\vec{P_1} = -5\times10^{-9}\hat{a}_z$, $\vec{P_2} = 9\times10^{-9}\hat{a}_z$

$\vec{r_1} = (0,0,0) - (0,0,-2) = 2\hat{a}_z$; $r_1 = |\vec{r_1}| = 2$

$\vec{r_2} = (0,0,0) - (0,0,3) = -3\hat{a}_z$; $r_2 = |\vec{r_2}| = 3$

$$V = \frac{1}{4\pi \times 8.854 \times 10^{-12}}\left[\frac{-5\times10^{-9}\hat{a}_z \cdot 2\hat{a}_z}{2^3} + \frac{9\times10^{-9}\hat{a}_z \cdot -3\hat{a}_z}{3^3}\right] = -20.25 \text{ V}$$

13. A point charge $Q_A = -1$ μC is at A(0, 0, 1) and $Q_B = 1$ μC is at B(0, 0, −1). Find $\vec{E}$ in spherical coordinate system at P(1, 2, 3).

Solution:

The system of charges mentioned in the question create a dipole.

For dipole,

$$\vec{E} = \frac{Qd}{4\pi\varepsilon_o r^3}(2\cos\theta\, \hat{a}_r + \sin\theta\, \hat{a}_\theta)$$

$Q = 1\mu C = 10^{-6}$ C(considering magnitude of either charge)

$d = 2$

$r = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{14}$

$\theta = \cos^{-1}\frac{z}{r} = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right) = 36.7°$

$\therefore \vec{E} = (550.25\, \hat{a}_r + 205.274\, \hat{a}_\theta)$ V/m

14. If $V = \rho^2 z \sin\phi$, calculate the energy within the region defined by $1<\rho<4$, $-2<z<2$, $0<\phi<\pi/3$.

Solution:

$$W_E = \frac{1}{2}\int_{vol} \varepsilon_o E^2\, dv$$

$$\vec{E} = -\nabla V$$

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$$= -\left[\frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$$= -(2\rho z \sin\phi \hat{a}_\rho + \rho z \cos\phi \hat{a}_\phi + \rho^2 \sin\phi \hat{a}_z)$$

Now, $E = \sqrt{4\rho^2 z^2 \sin^2\phi + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi}$

or, $E^2 = (4\rho^2 z^2 \sin^2\phi + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi)$

$$\therefore W_E = \frac{1}{2}\int_{\rho=1}^{4}\int_{\phi=0}^{\pi/3}\int_{z=-2}^{2}(4\rho^2 z^2 \sin^2\phi + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi)(\rho\, d\rho\, d\phi\, dz)$$

$$= 925.39 \text{ J}$$

15. **The point charges –1 nC, 4 nC, and 3 nC are located at (0, 0, 0), (0, 0, 1), and (1, 0, 0) respectively. Find the energy in the system.**

Solution:

$$W = \frac{1}{2}\sum_{m=1}^{3} Q_m V_m$$

$$= \frac{1}{2}(Q_1 V_1 + Q_2 V_2 + Q_3 V_3)$$

$$= \frac{Q_1}{2}\left[\frac{Q_2}{4\pi\varepsilon_o(1)} + \frac{Q_3}{4\pi\varepsilon_o(1)}\right] + \frac{Q_2}{2}\left[\frac{Q_1}{4\pi\varepsilon_o(1)} + \frac{Q_3}{4\pi\varepsilon_o\sqrt{2}}\right] + \frac{Q_3}{2}\left[\frac{Q_1}{4\pi\varepsilon_o(1)} + \frac{Q_2}{4\pi\varepsilon_o(\sqrt{2})}\right]$$

$$= 13.37 \text{ nJ}$$

16. **For the point P(3, 60°,2) in cylindrical co-ordinates and the potential field $V = 20(\rho + 1) z^2 \cos\phi$ V in free space, find at P:**

(i) V **(ii) E** **(iii) D**

(iv) Volume charge density ρ_v

(v) Unit normal vector $\hat{a}_N$ which is in the direction of maximum rate of increase of potential. [2074 Bhadra]

Solution:

i. $V = 20(\rho + 1)z^2\cos\phi$

$V(3, 60°, 2) = 20(3 + 1)(2)^2\cos 60° = 160\text{V}.$

ii. $\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z\right]$

$$= -\left[20z^2\cos\phi\, \hat{a}_\rho - \frac{20(\rho+1)z^2\sin\phi}{\rho}\hat{a}_\phi + 40(\rho+1)z\cos\phi\, \hat{a}_z\right]$$

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$\vec{E}(3, 60°, 2) = -[40\,\hat{a}_\rho - 92.376\,\hat{a}_\phi + 160\,\hat{a}_z]$

$E(3, 60°, 2) = \sqrt{(-40)^2 + (92.376)^2 + (-160)^2} = 189.032$ V/m.

iii. $D = \varepsilon_o E = 8.854\times10^{-12} \times 189.032 = 1.673 \times 10^{-9}$ C/m²

iv. $\rho_v = \nabla.\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$

$$\vec{D} = \varepsilon_o\vec{E} = -\left[20\varepsilon_o z^2\cos\phi\,\hat{a}_\rho - \frac{20\varepsilon_o(\rho+1)z^2\sin\phi}{\rho}\hat{a}_\phi + 40\varepsilon_o(\rho+1)z\cos\phi\,\hat{a}_z\right]$$

$$= D_\rho\hat{a}_\rho + D_\phi\hat{a}_\phi + D_z\hat{a}_z$$

$$\therefore D_\rho = -20\,\varepsilon_o\,z^2\cos\phi,\ D_\phi = \frac{20\,\varepsilon_o\,(\rho+1)z^2\sin\phi}{\rho},\ D_z = 40\,\varepsilon_o(\rho+1)z\cos\phi$$

$$\rho_v = \frac{1}{\rho}\frac{\partial}{\partial\rho}(-20\varepsilon_o\rho z^2\cos\phi) + \frac{1}{\rho}\frac{\partial}{\partial\phi}\left[\frac{20\,\varepsilon_o\,(\rho+1)z^2\sin\phi}{\rho}\right] + \frac{\partial}{\partial z}[40\,\varepsilon_o(\rho+1)z\cos\phi]$$

$$= \frac{-20\,\varepsilon_o\,z^2\cos\phi}{\rho} + \frac{20\,\varepsilon_o\,(\rho+1)\,z^2\cos\phi}{\rho^2} + 40\,\varepsilon_o(\rho+1)\cos\phi$$

$\rho_v(3, 60°, 2) = 9.8377 \times 10^{-10}$ C/m³

v. $\hat{a}_N = -\frac{\vec{E}}{|\vec{E}|} = -\frac{-(40\hat{a}_\rho - 92.376\hat{a}_\phi + 160\hat{a}_z)}{189.032} = 0.2116\hat{a}_\rho - 0.4886\hat{a}_\phi + 0.8464\hat{a}_z$

Hammered Problems

1. **Given the potential, $V = 100\,(x^2 - y^2)$ and a point P(2, –1, 3) that is stipulated to lie on a conductor-to-free-space boundary, find V, $\vec{E}$, $\vec{D}$, and ρ_s at P, and also the equation of the conductor surface.**

   Answer: $V_P = 300$ V, $\vec{E}_P = 400\hat{a}_x - 200\hat{a}_y$ V/m, $\vec{D}_P = -3.54\hat{a}_x - 1.771\hat{a}_y$ nC/m², $\rho_{s,P} = D_N = |\vec{D}_P| = 3.96$ nC/m², $xy = -2$

2. **A point charge of 5 nC is located at the origin. If $V = 2$ V at (0, 6, –8), find:**
   a. **The potential at A(–3, 2, 6)**
   b. **The potential at B (1, 5, 7)**
   c. **The potential difference V_{AB}** **Answer:** (a) 3.929 V (b) 2.696 V (c) -1.233 V

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3. Given the potential field $V = 100\sqrt{r}$ V in free space, find (a) $\vec{E}$ (b) $\vec{D}$ (c) How much charge lies within the sphere r = 0.5?

Answer: (a) $-\frac{50}{\sqrt{r}}\,\hat{a}_r$ V/m (b) $-\frac{442.5}{\sqrt{r}}\,\hat{a}_r$ pC/m² (c) -1.965 nC

4. Given the electric field $\vec{E} = \frac{1}{z^2}(8xyz\,\hat{a}_x + 4x^2z\,\hat{a}_y - 4x^2y\,\hat{a}_z)$ V/m , find the differential amount of work done in moving a 6 nC charge a distance of 2µm, starting at P(2,-2,3) and proceeding in the direction $\hat{a}_L = -\frac{6}{7}\hat{a}_x + \frac{3}{7}\hat{a}_y + \frac{2}{7}\hat{a}_z$.

Answer: –149.3 fJ

5. Given, $W = x^2y^2 + xyz$, compute ∇W and the directional derivative $\frac{dW}{dl}$ in the direction $3\,\hat{a}_x + 4\,\hat{a}_y + 12\hat{a}_z$ at (2, -1, 0).

Answer: $\nabla W = 4\,\hat{a}_x - 8\,\hat{a}_y - 2\,\hat{a}_z$, $\frac{dW}{dl} = \frac{-44}{13}$

6. A dipole of moment $\vec{p} = -4\hat{a}_x + 5\hat{a}_y + 3\hat{a}_z$ nCm is located at D(1, 2, –1) in free space. Find V at : (a) $P_A(0, 0, 0)$ (b) $P_B(1, 2, 0)$ (c) $P_C(1, 2, -2)$ (d) $P_D(2, 6, 1)$.

Answer: (a) –1.835 V (b) 26.963 V (c) –26.963 V (d) 2.055 V

7. An electric dipole of $100\,\hat{a}_z$ pC m is located at the origin. Find V and $\vec{E}$ at points (a) (0, 0, 10) (b) $(1, \frac{\pi}{3}, \frac{\pi}{2})$

Answer: (a) 9mV, $1.8\,\hat{a}_r$ mV/m (b) 0.45V, $0.9\hat{a}_r + 0.7794\,\hat{a}_\theta$ V/m

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