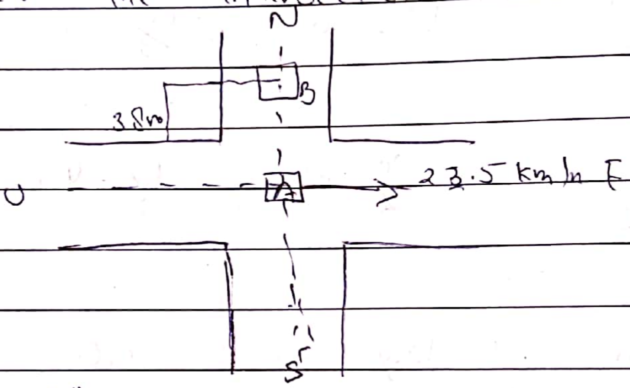
Kinematics

Assignment 1

1) Automobile (A) is travelling with the constant speed of 23.5 km/hr. As automobile A crosses the intersection shown automobile B starts from rest 38 m north of the intersection & move across south with the constant acceleration of $2.6\ m/s^2$. Determine the position, velocity & acceleration of B relative to A, 10 sec after A crosses the intersection.



In automobile A,

$$v_A = 23.5\ km/hr$$

$$= \frac{23.5 \times 1000}{60 \times 60} = 6.52\ m/s.$$

$$\vec{v_A} = 6.52\vec{i}$$

$$a_A = \frac{dv_A}{dt} = 0$$

$$\vec{r_A} = ?$$

now,

$$\frac{d\vec{r_A}}{dt} = \vec{v_A}$$

$$d\vec{r_A} = v_A\, dt$$

$$r_A = \int v_A\, dt$$

$$= \int 23.5\, dt$$

$$= \int_B (23.5t) + c$$

At, $t = 0$; $v_A = 0$ $\Rightarrow c = 0$

$$v_A = 23.5t$$

$$\Rightarrow \vec{v_A} = (23.5t)\vec{i}$$

$$\Rightarrow \vec{v_A} = (6.52t)\vec{i}$$

For automobile B,

$$a_B = -2.6 m/s^2$$

$$= -2.6\vec{j}$$

$$\vec{a_B} = -2.6\vec{j}$$

$$v_B = ?$$

Now, $a_B = \frac{dv}{dt}$

$$\Rightarrow v_B = \int a_B dt$$

$$= \int -2.6 \, dt$$

$$= (-2.6t) + c_1$$

$$= -2.6t\vec{j}$$

At $t = 0$

$$v_B = 0$$

$$v_B = -2.6t + c_1$$

$$0 = 0 + c_1$$

$$\Rightarrow c_1 = 0.$$

now

$$\left(\frac{dr_B}{dt}\right) = v_B$$

$$r_B = \int v_B dt$$

$$= \int -2.6t \, dt$$

$$= \frac{-2.6\,t^2}{2} + C_2$$

$$= (-1.3t^2 + C_2)\vec{j}$$

When $t = 0$

$r_B = 38m$

$38 = 0 + C_2$

$\Rightarrow C_2 = 38.$

Now, position of B. w.r.t. A at 10 sec

$\vec{r_A} = 65.2\vec{i}$ $\quad$ $\vec{r_B} = -92\vec{j}$

$\vec{V_A} = 6.52\vec{i}$ $\quad$ $\vec{V_B} = -26\vec{j}$

$\vec{a_A} = 0$ $\quad$ $\vec{a_B} = -2.6\vec{j}$

Now,

relative position of B w.r.t A $(\vec{r_{B/A}}) = \vec{r_B} - \vec{r_A}$

$= -92\vec{j} + 65.2\vec{i}$

$$|\vec{r_{B/A}}| = \sqrt{(-92)^2 + (65.2)^2}$$

$$= 112.76$$

$$\tan\theta = \frac{92}{65.2} = 54.67^\circ$$

relative velocity $(\vec{V_{B/A}}) = \vec{V_B} - \vec{V_A}$

$= -6.52\vec{i} - 26\vec{j}$

$= \sqrt{(6.52)^2 + (26)^2}$

$= 26.80$ m/s.

$$\tan\theta = \left(\frac{26}{6.52}\right) = 75.92^\circ$$

And

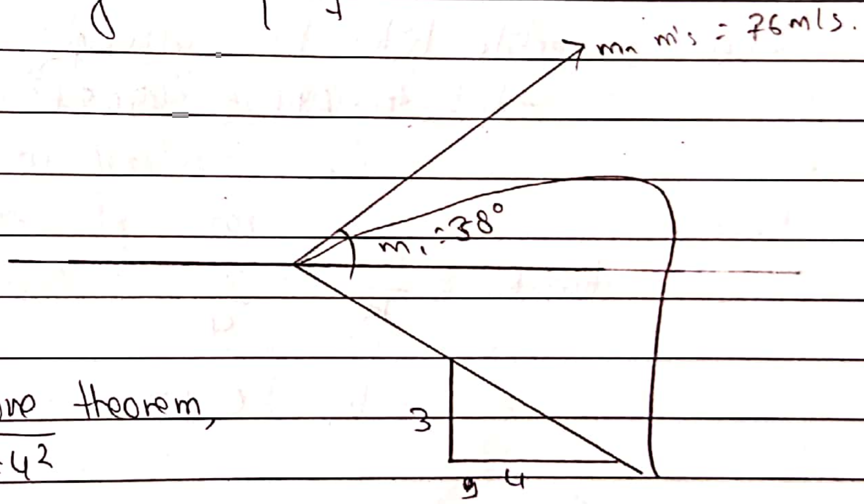
Acceleration $(\vec{a}_{B/A}) = \vec{a}_B - \vec{a}_A$

$= -2.6\vec{j} + 0$

$= -2.6\vec{j}$

$= -2.6 m/s^2 (\downarrow)$

2) A projectile is projected with an angle of 38 degree with an initial velocity of 76m/s as shown in fig. below. Find the total time of flight & sloping distance covered by the projectile.



⇒ Solⁿ,

by using pythagore theorem,

$a = \sqrt{3^2+4^2}$

$= 5$

also,

$\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$, $\tan\theta = \frac{3}{4}$.

For (horizontal motion) · $(\xrightarrow{+})$

V_x = constant

$(V_x)_0 = 76 \cdot \cos 38$

$= 59.88$ m/s.

we know,

$x = V_x t$

When projectile hits the ground.

$R = 59.88 t$ —— (i)

vertical motion (↑+)

$a = -9.8\ m/s^2$

we know,

$S = ut + \frac{1}{2}at^2$

$y = (V_y)t + \frac{1}{2}(-g)t^2$

$= (76 \sin 38)t - \frac{1}{2} \times 9.8 \times t^2$

$= 46.79t - 4.905t^2$

When projectile hits the ground, (ie $y = -h$)

$-h = 46.79t - 4.905t^2$ —(ii)

Since,

$\tan\theta = \frac{h}{R} = \frac{3}{4}$

$h = 0.75R$ —(iii)

From eq^n (ii)

$-0.75 \times (59.88t) = 46.79t - 4.905t^2$

$-44.9t = 46.79t - 4.905t^2$

$4.205t^2 = 91.7t$

$t = 18.69$ sec

Then

$R = 52.88t$

$= 52.88 \times 18.69$

$= 1119.46$ m.

Now,

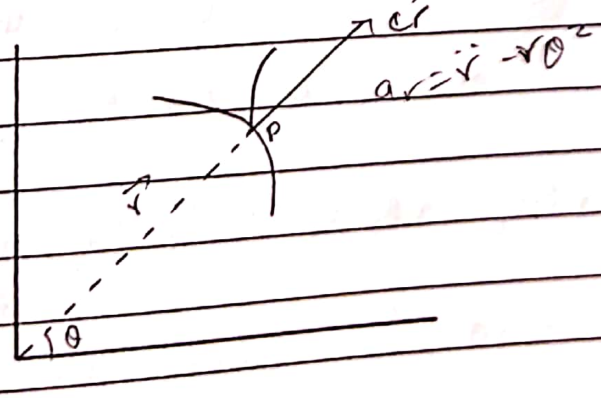
$$\cos\theta = \frac{B}{\text{slope}}$$

$$\text{Slope} = \frac{B}{\cos\theta}$$

$$= \frac{1119.46}{4/5}$$

$$\text{Slope} = 1399.325 \text{ m}$$ #

3) Deduce the relation for radical & transverse components of velocity & acceleration when a particle is moving curvilinearly. For the pully system as shown in fig below, calculate the velocity & acceleration of block C. If the velocity & acceleration of block A & B are 2 m/s (↓), 1 m/s² (↑) 3 m/s (↑) & 2 m/sec² (↓).



$$\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta$$

$$\frac{d\hat{e}_\theta}{d\theta} = -\hat{e}_r$$

r = Position vector (along radial direction)

$= r\hat{e}_r$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$= \frac{d}{dt}(r\hat{e}_r)$$

$$\Rightarrow \frac{r\,d}{dt}(\hat{e}_r) + \left(\frac{dr}{dt}\hat{e}_r\right)$$

$$\Rightarrow r\frac{d}{d\theta}\hat{e}_r \cdot \frac{d\theta}{dt} + \frac{dr}{dt}\hat{e}_r$$

$$\boxed{\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$$

now,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta)$$

$$= \left(\frac{d\dot{r}}{dt}\right)\hat{e}_r + \dot{r}\frac{d\hat{e}_r}{dt} + (r\dot{\theta})\frac{d\hat{e}_\theta}{dt} + \frac{d}{dt}(r\dot{\theta})\hat{e}_\theta$$

$$= \ddot{r}\hat{e}_r + \dot{r}\left(\frac{d\hat{e}_r}{d\theta}\cdot\frac{d\theta}{dt}\right) + r\dot{\theta}\left(\frac{d\hat{e}_\theta}{d\theta}\cdot\frac{d\theta}{dt}\right) + \frac{d(r\dot{\theta})}{dt}\hat{e}_\theta$$

$$= \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\dot{\theta}(-\dot{\theta}\hat{e}_r) + \hat{e}_\theta\left[\frac{dr}{dt}(\dot{\theta}) + \frac{d\dot{\theta}}{dt}r\right]$$

$$= \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r + \hat{e}_\theta(\dot{\theta}\dot{r} + r\ddot{\theta})$$

$$\Rightarrow \vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$$

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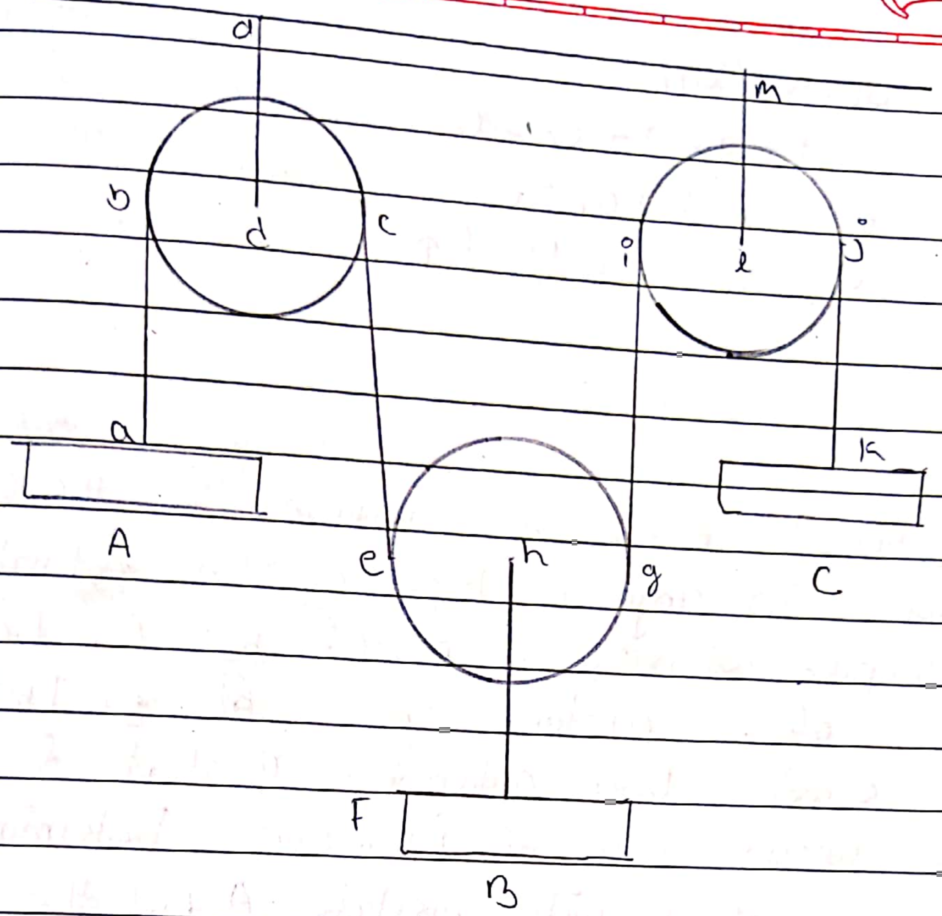

$V_A = 2\,m/s\ (\downarrow)$ $\qquad$ $V_B = 3\,m/s\ (\uparrow)$

$a_A = 1\,m/s^2\ (\uparrow)$ $\qquad$ $a_B = 2\,m/s^2\ (\uparrow)$

length of rope constant

$ab + bc + ce + eg + gi + ij + jk = \text{constant}$

or, $ab + ce + gi + jk = \text{constant}$

or $(ab + dl) + 2(ce + hf + dl) + (gi + lm + hf) + (jk + lm)$
$= \text{constant}$

or $x_A + x_B + x_B + x_C = \text{constant}$

$x_A + 2x_B + x_C = \text{constant}$

$\therefore V_A + 2V_B + V_C = 0$

$a_A + 2a_B + a_C = 0$

now

or $-2 + 2\times 3 + V_C = 0$

or $-2 + 6 + V_C = 0$

or $V_C = -4\,m/s$

$\therefore V_C = 4\,m/s\ (\downarrow)$

for acceleration

$\Rightarrow 1+2\times-2+a_c=0$

$\Rightarrow 1-4+a_c=0$

$\Rightarrow a_c = 3 m/s^2 \ (\uparrow)$

4) Two automobile A & B are travelling in the same direction in adjacent highway lane Automobile B is stopped when it is passed by A, which travel at a constant speed of 36 km/hr. Two seconds later automobile B start & accelerate at a constant rate of 1.5 m/sec². Determine (a) when & where B will overtake A (b) the speed of B at that time

$\Rightarrow$ Solⁿ,

$v_A = 36$ km/hr

$= \frac{36\times1000}{60\times60}$

$= 10$ m/s.

$a_A = 0$ $\qquad \frac{dv_B}{dt} = 0$

$\vec{r}_A = \int v_a \, dt$

$\vec{r}_A = 10t + c$ —— (i)

At $t=0$ & $r_A = 0$,

From (i)

$r_A = 10t$

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For automobile A,
$v_A = 10 m/s$
$a_A = 0 m/s^2$
$r_A = 10t$

For automobile B.
$a_B = 1.5 m/s^2$
$v_B = ?$

$a_B = \frac{dv_B}{dt}$

$\Rightarrow v_B = \int 1.5\, dt$
$v_B = 1.5t + C_1$ — (ii)

At $t = 2$, $v_B = 0$
$C_1 = -3$

From (ii)
$v_B = 1.5t - 3$

$\frac{dr_B}{dt} = v_B$

$\Rightarrow r_B = \int (1.5t - 3)\, dt$

$= \frac{1.5t^2}{2} - 3t + C_2$ — (iii)

At $t = 2$, $r_B = 0$
$C_2 = 3$

from (iii)
$r_B = 0.75t^2 - 3t + 3$ — (iv)

when B overtake.
$r_B - r_a = 0$
or $0.75t^2 - 3t + 3 - 10t = 0$

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or, $0.75t^2 - 13t + 3 = 0$

$t = 17.1$ sec

From (iv)

$r_B = 0.75 \times (17.10)^2 - 3 \times 17.10 + 3.$

$r_B = 171.0m$

Also,

$v_B = 1.5t - 3$

$= 1.5 \times 17.1 - 3$

$= 22.65$ m/s.

5) A ball is throw vertically upword from the 12m level in an elevator shaft, with an initial velocity of 18m/sec. At the same time an open-pletform passes the 5m level, moving upword with a constant velocity of 2m/sec. Determine ()

a) When & where the ball will hit the elevator.

b) The relative velocity of the ball with respect to the elevator when the ball hits the elevator.

⇒ Sol^n,

vertical motion (r-t)

here,

$a_B = -9.81$

$a_B = \frac{dv_B}{dt}$

$v_B = \int a \, dt$

$v_B = -9.81t + c$ — (i)

when $v_B = 18 m/s$ & $t=0$

$$18 = -9.81 \times 0 + C$$

$$C = 18$$

so,

$$v_B = 18 - 9.81t \quad \text{--- (ii)}$$

also,

$$v_B = \frac{dy_B}{dt}$$

$$y_B = \int v\,dt$$

$$= \int (18 - 9.81t)\,dt$$

$$y_B = 18t - \frac{9.81t^2}{2} + C_1 \quad \text{--- (iii)}$$

when, $y = 12$ & $t=0$

$$12 = 18 \times 0 - \frac{9.81 \times 0}{2} + C_1$$

$$C_1 = 12$$

now,

$$y_B = 18t - 4.905t^2 + 12 \quad \text{--- (iii)}$$

At top, velocity of Ball $(v) = 0$.

From (ii)

$$0 = 18 - 9.81t$$

$$t = \frac{18}{9.81}$$

$$= 1.83 \text{ sec}$$

from (ii)

$$y_B = 18 \times 1.83 - 4.205 \times (1.83)^2 + 12$$

$$= 28.51 m$$

now,

$$v_e = 2 m/s$$

$$v_c = \frac{dy_c}{dt}$$

$$\Rightarrow y_c = \int v_c \, dt$$
$$= \int 2 \, dt$$

$$y_c = 2t + C_2 \quad \text{---(v)}$$

When $t = 0$ & $y_c = 5$
$$5 = 2 \times 0 + C_2$$
$$C_2 = 5$$

So,
$$y_c = 2t + 5 \quad \text{---(vi)}$$

When the ball hits the elevator.
$$y_B - y_c = 0$$
$$\text{or } 18t - 4.905t^2 + 12 - 2t - 5 = 0$$
$$\text{or } -4.905t^2 - 16t + 8 = 0$$
$$\therefore t = 3.7 \text{ sec.}$$

From (iv)
$$y_B = 18 \times 3.7 - 4.905 \times (3.7)^2 + 12$$
$$= 11.45 \text{ m}$$

Now,
$$v_B - v_c = 18 - 9.81t - 2$$
$$= 16 - 9.81t$$
$$= -20.3 \text{ m/s}$$
$$= 20.3 \text{ m/s} (\downarrow).$$
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6) Define statics, Dynamics, kinetics, kinematic uniform rectilinear motion & uniformly accelerated rectilinear motion. Also derive the equation of motion for uniform rectilinear motion & uniform rectilinear motion & uniformly rectilinear motion.

⇒ Statics is defined as the branch of mechanics concerned with bodies at rest & force in equilibrium.

⇒ Dynamics is defined as the branch of mechanics concerned with motion of bodies under action of force.

⇒ kinetics is defined as branch of mechanics concerned with motion of object with out reference to forces which cause motion.

⇒ When an object travel at a constant speed with zero acceleration is known as uniform rectiliner motion.

⇒ A body moves with constant accelerating motion or uniformly accelerated rectilinear motion, when its trajectory is a st. line & accⁿ is constant.

⇒ Uniform motion means covering equal distance over equal interval of time. In this case velocity is constant throughout the motion & hence acceleration is zero for every value of t

$$v = \frac{dx}{dt} = \text{constant} = u \text{ (say)}$$

$$\Rightarrow \frac{dx}{dt} = u$$

$$\Rightarrow dx = u\,dt$$

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$x = 4t + C_1$

At $t \to 0$, $x \to \cancel{x_0}\, 0$

$\therefore C_1 = \cancel{x_0}\, 0$.

$x = x_0 + 4t$

For uniformly rectilinear accelerated motion. In this motion, acceleration is constant.

$$\frac{dv}{dt} = \text{constant} = a$$

$$dv = a\,dt$$

$$v = at + C_1$$

At $t = 0$, $v = v_0$

so, $C_1 = v_0$

$$v = v_0 + at$$

Also, $v = \frac{dx}{dt}$

$$\Rightarrow dx = (v_0 + at)\,dt$$

$$x = v_0 t + \frac{at^2}{2} + C_1$$

At $t \to 0$; $x \to x_0$

$\Rightarrow C_1 = x_0$

$x = x_0 + v_0 t + \frac{at^2}{2}$ is required eqn

Also, $a = \frac{v\,dv}{dx}$

$$\Rightarrow v\,dv = a\,dx$$

$$\Rightarrow \frac{v^2}{2} = ax + C_1$$

At $t=0$, $v = v_0$, $x = x_0$

$$\Rightarrow \frac{v_0^2}{2} = ax_0 + C_1$$

$$\Rightarrow C_1 = \frac{v_0^2}{2} - ax_0$$

$$\Rightarrow \frac{v^2}{2} = ax + \frac{v_0^2}{2} - ax_0$$

$$\text{or } \frac{v^2}{2} - \frac{v_0^2}{2} = ax - ax_0$$

$$\therefore v^2 = v_0^2 + 2a(x - x_0) \quad \#$$

I) Deduce the relation for tangential & normal components of acceleration when a particle is moving curvilinearly. The acceleration of a particle is defined by the relation $a = kt^2$. knowing that velocity is -32 m/s when time is zero second & again velocity is $+32$ m/s when time is 4 sec. Determine the value of the constant k & write the motion of eqn knowing that the position of the particle is zero at the instant of 4 sec

$\Rightarrow$ Sol^n.

$$a = kt^2$$

$$\frac{dv}{dt} = kt^2$$

$$v^2 = \frac{kt^3}{3} + c$$

Now,

At $t = 0$, $v = -32$

$\Rightarrow c = -32$

$$v^2 = \frac{kt^3}{3} - 32$$

Now, when $t = 4$

$v = 32$ m/s

or $32 = k \times \frac{(4)^3}{3} - 32$

or $k = \frac{\cancel{64} \times 3}{\cancel{4 \times 4 \times 4}}$

$\therefore k = 3$

Now

$$v = \frac{3t^3}{3} - 32$$

$$x = \frac{\cancel{3}t^4}{\cancel{3} \times 4} - 32t + c$$

At $t = 4$, $c = 64$

$$x = \frac{t^4}{4} - 32t + 64 \#$$

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9) The motion of a vibrating particle is defined by the eq^n $x = 100 \sin \pi t$ & $y = 25 \cos 2\pi t$ where x & y are expressed in millimeter & t is in seconds.

a) Determine the velocity & acceleration when $t = 1$ sec

b) Show that the path of particle is parabolic.

=> Sol^n,

$$x = 100 \sin \pi t \text{ mm} = 0.1 \sin \pi t \text{ m}$$

$$y = 25 \cos 2\pi t \text{ mm} = 0.025 \cos 2\pi t.$$

now,

$$v_x = 0.1 \pi \cos \pi t$$

$$v_y = -0.025 \times 2\pi \sin \pi t$$

again

$$a_x = -0.1\pi \times \pi \sin \pi t$$

$$a_y = -0.025 \times 4\pi^2 \cos 2\pi t.$$

Now At $t = 1$,

$$v_x = 0.31 \text{ m/s}$$

$$v_y = -0.017 \text{ m/s}$$

$$a_x = -0.05 \text{ m/s}^2$$

$$a_y = -0.98 \text{ m/s}^2$$

now,

$$y = 0.025 (1 - 2\sin^2 \pi t)$$

$$= 0.025 - 0.05 \sin^2 \pi t$$

$$= 0.025 - \frac{x^2}{0.01} \times 0.05$$

$$= 0.025 - 5x^2$$ which is parabolic

~~which~~

//

10) Two cars A & C travel along the same straight route. At any time t their distance x_a & x_c from the starting points are given by.

$$x_a = 2.5t + 1.2t^2$$
$$x_c = 7t^2 - 0.25t^3$$

where t in seconds & x_a & x_c are in meters

=> Solⁿ

$$x_a = 2.5t + 1.2t^2$$
$$x_c = 7t^2 - 0.025t^3$$

At, $t=0$, $x_a = x_c = 0$, so they start at same point.

$$v_A = 2.5 + 2.4t$$
$$v_B = 14t - 0.75t^2$$

$$a_A = 2.4$$
$$a_B = 1.4 - 1.5t$$

a) Which car is ahead just after they leave the starting point?

=> let's say, after 1 sec

$$x_B = 7 - 0.25 = 6.75m$$
$$x_A = 2.5 + 1.2 = 3.7$$

B is ahead at the start.

b) At what time are the cars at the same point:

=> $x_B = x_A$

or $7t^2 - 0.25t^3 = 2.5t + 1.2t^2$

or $5.8t^2 - 2.5t - 0.25t^3 = 0$

or $0.25t^3 + 2.5t - 5.8t^2 = 0$

or $t(0.25t^2 - 5.8t + 2.5) = 0$

$t = 22.7$ sec, 0 sec.

c) At what time is the distance betn A & E neither increasing or decreasing?

=>

$$\frac{d(x_B - x_A)}{dt} = 0$$

$$x_B - x_A = 7t^2 - 0.25t^3 - 2.5t - 1.2t^2$$
$$= 5.8t^2 - 0.25t^3 - 2.5t$$

Now,

$$\frac{d(x_B - x_A)}{dt} = 11.6t - 0.7t^2 - 2.5 = 0$$

or $0.75t^2 - 11.6t + 2.5 = 0$

=> $t = 15.24$ sec.//

d) At what time do A & E have the same acceleration

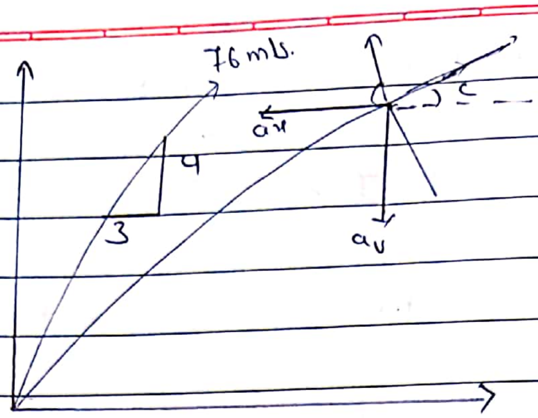
=>

$2.4 = 14 - 1.5t$

or $1.5t = 11.6$

$\therefore$ $t = 7.73$ sec.

Q.11) A particle starting from origin is subjected to acceleration, such that $a_x = -3.0$ m/sec^2 & $a_y = -11$ m/s^2. If the inital velocity is 70 m/sec. directed at a slope of 4V: 3H. Compute the radius of curvature of the path after 7 sec. Also find the position at end of ~~4~~ sec.



=> Sol^n

$$\tan\theta = \frac{4}{3} = 1.33$$

$$\sin\theta = \frac{4}{5} = 0.80$$

$$\cos\theta = \frac{3}{5} = 0.60$$

components of initial velocity.

$$(V_0)_x = 76\cos\theta = 76 \times 0.6 = 45.6 \text{ m/s.}$$

$$(V_0)_y = 76\sin\theta = 76 \times 0.8 = 60.8 \text{ m/s.}$$

componet of inital velocity at the end of 7sec

$$V_{7x} = (V_0)_x + a_x t$$
$$= 45.6 + (-3) \times 7$$
$$= 24.6 \text{ m/s.}$$

$$V_{7y} = (V_0)_y + a_y t$$
$$= 60.8 + (-11) \times 7$$
$$= -16.7 \text{ m/s.}$$

Inclination of V_7 with x-axis.

$$= \tan^{-1}\frac{V_{7y}}{V_{7x}}$$
$$= \tan^{-1}\frac{16.7}{24.6}$$
$$= 34.17^\circ$$

Normal component of acceleration

$a_n = a_y \cos\alpha + a_x \sin\alpha$

$= 11 \cos 34.17 - 3 \sin 34.17$

$= 9.10 - 1.68$

$= 7.41 \; m/s^2$

since, $a_n = \dfrac{v_7^2}{\rho_7}$

$\Rightarrow \rho_7 = \dfrac{v_7^2}{a_n}$

$= \dfrac{v_{7x}^2 + v_{7y}^2}{a_x}$

$= \dfrac{(24.6)^2 + (-16.7)^2}{7.41}$

$= 119.30 m$

Hence, the radius of curvature at the end of 7sec $(\rho_7) = 119.3$ m.

To find position,

$x_4 = (V_0)_x t + \frac{1}{2} a_x t^2$

$= (45.6 \times 4) + \frac{1}{2} \times (-3) \times 4^2$

$= 182.4 - 24$

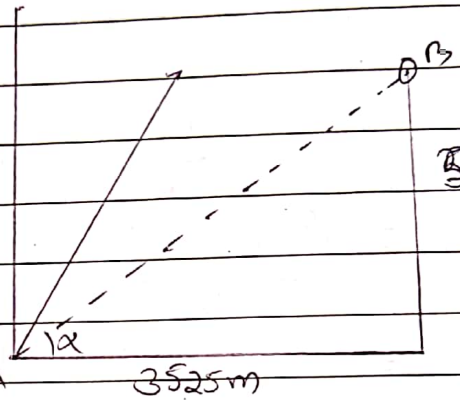
$= 158.4 m$

$y_4 = (V_0)_y t + \frac{1}{2} a_y t^2$

$= (60.8 \times 4) + \frac{1}{2} \times (-11) \times 4 \times 4$

$= 243.2 - 88 \quad = 155.2$ m

12) Define average & instantaneous velocity. A projectile is fired from position A with an initial velocity of 235 m/s at a target B on right located 587.5 m above position A. The horizontal distance betn A & B is 3525 m. Determine the firing angle neglecting air resistance.



=> Soln,

horizontal: $(\xrightarrow{+})$

$V_x = (V_x)_0 = 235 \cos\alpha$

$x = (235\cos\alpha)t$

$t = \frac{3525}{255\cos\alpha}$

Vertical : $(\uparrow +)$

$(V_y)_0 = 235 \sin\alpha$

$a = -9.81$

$\Rightarrow y = (V_y)_0 t + \frac{1}{2}at^2$

or $587.5 = (\cancel{235}\sin\alpha)\cdot\frac{3525}{\cancel{235}\cos\alpha} + \frac{1}{2}\times(-9.81)\times\left(\frac{3525}{235\cos\alpha}\right)^2$

or $587.5 = \frac{3525\sin\alpha}{\cos\alpha} - 4.905\times\left(\frac{3525}{235\cos\alpha}\right)^2$

or $587.5 = 3525\left(\frac{\sin\alpha}{\cos\alpha} - \frac{4.905\times 3525}{(235\cos\alpha)^2}\right)$

or $\frac{1}{6} = \tan\alpha - 0.313\sec^2\alpha$

or $\frac{1}{6} = \tan\alpha - 0.313(1+\tan^2\alpha)$

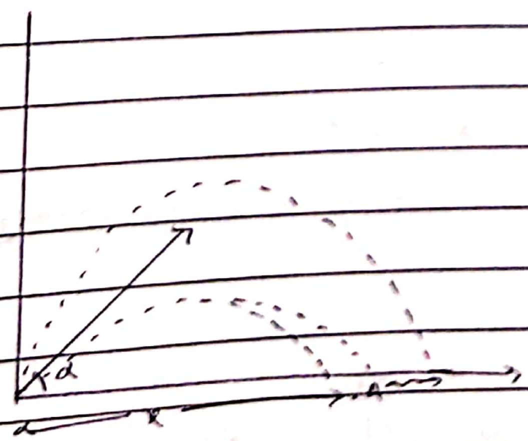
or $\frac{1}{6} = \tan\alpha - 0.313 + 0.313\tan^2\alpha$

or $0.313\tan^2\alpha + \tan\alpha - 0.479 = 0$

$\Rightarrow \tan\alpha = 0.422$ & -3.617

$\Rightarrow \alpha = 22.87°$ or $74.54°$ #

Q(13) A projectile is aimed at a mark A on the horizontal plane through the point of projection & falls 235m short when the angle of projection is 15° while overshoots the mark by 25m when the inclination is 40°. Calculate the distance of the target & required of projection, if the velocity remain constant.



=> Sol^n.

Horizontal ($\xrightarrow{+}$)

$x = (V_x)_0 t$

$x = (v\cos\alpha)\, t$ —— (i)

case (i)

For $\alpha = 15^\circ$

$(x, -235) = (v\cos 15^\circ)\, t_1$ —— (ii)

case (ii)

For $\alpha = 40^\circ$

$(x, +25) = (v\cos 40^\circ)\, t_2$ —— (iii)

For vertical direction (↑ +ve)

$a = -9.81$

$y = (V_y)_0 + \frac{1}{2} at^2$

$= (v\sin\alpha)\, t + \frac{1}{2}(-9.81)t^2$

$= v\sin\alpha t - 4.92\, t^2$

$0 = v\sin\alpha t - 4.90\, t^2$

or $4.905t = v\sin\alpha$

case (i)

$\alpha = 15^\circ$

$4.905t_1 = v\sin 15$ —— (4)

case (ii)

$\alpha = 40^\circ$

$4.905t_2 = v\sin 40$ —— (5)

Divi (4) by (5)

$$\frac{t_1}{t_2} = \frac{\sin 15}{\sin 40} \quad — (6)$$

again div eq^n (ii) by (1)

$$\frac{x_1 - 235}{x + 25} = \frac{v\cos 15}{v\cos 40} \cdot \frac{t_1}{t_2}$$

$$\Rightarrow \frac{x_1 - 235}{x_1 + 25} = \frac{\cos 15}{\cos 40} \cdot \frac{\sin 15}{\sin 40}$$

$$\Rightarrow \frac{x_1 - 235}{x_1 + 25} = 0.51$$

$$\Rightarrow x_1 - 235 = 0.51 x_1 + 12.75$$

$$\Rightarrow 0.49 x_1 = 247.75$$

$$x_1 = 505.61 \text{ m}$$

Now,

$$x_1 = v\cos\alpha t$$

$$y = (v\sin\alpha)t - 4.905t^2$$

$$0 = (v\sin\alpha)t - 4.905t^2$$

$$4.905\, t = v\sin\alpha$$

now

$$\frac{t}{t_1} = \frac{v\sin\alpha}{v\sin 15}$$

$$\frac{t_0}{t_1} = \frac{\sin\alpha}{\sin 15}$$

And,

$$\Rightarrow \frac{x_1}{x_1 - 235} = \frac{v\cos\alpha t}{(v\cos 15)t_1}$$

$$\Rightarrow \frac{505.6}{505.6 - 235} = \frac{\cos\alpha}{\cos 15} \cdot \frac{t}{t_1}$$

or $1.86 = \frac{2 \cdot \cos\alpha}{2\cos 15} \cdot \frac{\sin\alpha}{\sin 15}$

or $1.86 = \frac{\sin(2\alpha)}{\sin 30}$

or $0.93 = \sin 2\alpha$

$\alpha = 34.21^\circ$ #

(14) For partile moving retilinearly $a = -8x^2$, where a is the acceleration in m/s^2 & x is in meter unit. It is known that when $t = 1$ sec $x = 4m$ & $v = 2m/sec^2$. Determine its acceleration when $t = 2$ se

=> Soln

$a = v\frac{dv}{dx} = -8x^2$

$v\,dv = -8x^2\,dx$

Integrating

$\frac{v^2}{2} = 8x^{-1} + C_1$

when, $v = 2m/s$ $x = 4m$. Then $C_1 = 0$

$v^2 = 16x^{-1}$

again,

$v = \frac{dx}{dt} \Rightarrow \sqrt{16x^{-1}} = \frac{dx}{dt}$

$\frac{dx}{dt} = \pm 4x^{-1/2}$ — (i)

Case I,

taking +ve sign only, $\frac{dx}{dt} = 4x^{-1/2}$

$x^{1/2}\,dx = 4\,dt$

Integrating

$$\frac{2x^{3/2}}{3} = 4t + C_2$$

when $t = 1$ sec, $x = 4$m Then $C_2 = 4/3$

or $x^{3/2} = \frac{3}{2}\left(4t + \frac{4}{3}\right)$

$$x^{3/2} = 6t + 2$$

$$x = (6t+2)^{2/3} \quad \text{—— } (2)$$

Eq^n (2) is eq^n of motion

Differentiating eq^n (2) w.r.t. time, we get

$$v = \frac{dx}{dt} = \frac{d}{dt}(6t+2)^{2/3}$$

$$= \frac{2}{3}(6t+2)^{-1/3} \times 6$$

$$= 4(6t+2)^{-1/3}$$

again, differentiating wrt. time

$$a = \frac{dv}{dt} = \frac{d}{dt}\{4(6t+2)^{-1/3}\}$$

$$= -\frac{4}{3}(6t+2)^{-4/3} \times 6)$$

$$= -8(6t+2)^{-4/3}$$

putting $t = 2$sec.

acceleration $= -0.24$ m/s^2 ←

(a_2)

Case II

Taking $-$ve sign, we get, $x^{1/2}\,dx = -4\,dt$

Integrating, $\frac{2}{3} x^{3/2} = -4t + C_3$

putting $x = 4m$ & $t = 1 sec$

$C_3 = \frac{28}{3}$

Hence, $x^{3/2} = -6t + 4$

$x = (-6t + 4)^{2/3}$ — (3)

Differentiating above eq^n w.r.t time.

$v \frac{dx}{dt} = \frac{2}{3} (-6t + 14)^{-1/3} \times (-6)$

$v = -4(-6t + 14)^{-1/3}$

Again, differentiating w.r.t time

$a = \frac{dv}{dt} = \frac{d}{dt} \left\{ -4(-6t + 14)^{-1/3} \right\}$

$= -4 \times \left(-\frac{1}{3}\right) (-6t + 14)^{-4/3} \times$

$= -8 (-6t + 14)^{-4/3}$

At $t = 2 sec$,

Acceleration $(a_2) = -3.18 \, m/sec^2$

15) The position of a particle which move along a st. line is defined by the relation $x = \frac{t^3}{3} - 6t^2 - 15t$. where x is in meter. & t is in seconds. Determine

i) The time at which velocity will be zero.

i) soln

$$x = \frac{t^3}{3} - 6t^2 - 15t$$

$$v = \frac{dx}{dt} = t^2 - 12t - 15$$

At $v = 0$,

$$t^2 - 12t - 15 = 0$$

$$t = 13.14 \text{ sec}$$

Time at which velocity will be zero = 13.14s

ii) The position & distance travelled by particle at that time.

=> soln,

$t = 0$, when it starts

when $t = 0$, $x = 0$

$t = 13.14$ sec

$x = -476.8$

Total distance travelled = 476.8m

& position at 13.14sec = 476.8m

ii) For acceleration

$$a = \frac{dv}{dt} = 2t - 12$$

At $t = 13.14$

$$a = 2 \times 13.14 - 12$$
$$= 14.28 \, m/s^2$$

16) The acceleration of the particle is directly proportional to time (t) At $t = 0$, velocity of particle is (v) = 16 m/sec. knowing that velocity (v) = 15m/sec & that $x = 20m$. when $t = 1$. Determine the velocity, the position & total distance travelled when $t = 7$ sec.

⇒ Given

$$a \propto t$$
$$a = kt$$
$$\text{or} \quad \frac{dv}{dt} = kt$$
$$\text{or} \quad v = \frac{kt^2}{2} + c$$

when $t = 0$, $v = 16$

$$c = 16$$
$$v = \frac{kt^2}{2} + 16 \quad \text{—(1)}$$

again

$$v = \frac{kt^2}{2} + 16$$
$$\text{or} \quad \frac{dx}{dt} = \frac{kt^2}{2} + 16.$$
$$x = \frac{kt^3}{6} + 16t + c_1 \quad \text{—(2)}$$

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At $t = 15$, $v = 15\ m/s$, from (1)

$$15 = \frac{k}{2} + 16$$

$\Rightarrow k = -2.$

from (2)

$$x = \frac{-2t^3}{6} + 16t + C_1$$

At $t = 1$ sec,

$$20 = -\frac{t^3}{3} + 16 + C_1$$

$$C_1 = 4.33$$

At $t = 7$ sec,

$$v = -2 \times \frac{(7)^2}{2} + 16$$

~~$\Rightarrow k = -2$~~

$$\Rightarrow v = -2 \times \frac{7^2}{2} + 16$$

$$= -33\ m/s$$

$$= 33\ m/s.$$

from (2)

$$\cancel{x = \frac{-2t^3}{6} + 16t + C_1}$$

$\cancel{\text{At } t = 1 \text{ sec.}}$

$$\cancel{20 = \frac{-t^3}{3} + 16 + C_1}$$

$$\cancel{C_1 = 4.33.}$$

Also,

$$x = \frac{-t^3}{3} + 16t + 4.33$$

$$= \frac{-7^3}{3} + 16 \times 7 + 4.33$$

$$= 2\ m$$

Here,

total distance $= 33 \times 7$

$$= 231\ m.$$

17) Rotation of the arm about O is defined by $\theta = 0.15t^2$ where θ is in radians & t is in seconds. Collar B slides along the arm such that $r = 1 - 0.12t^2$. Where r is in meter. After the arm has rotated through 38° determine.

a) total velocity of collar.
b) The total acceleration of collar.

→ sol^n

$\theta = 0.15t^2$

$\Rightarrow \frac{13 \cdot \pi}{45} = 0.15t^2$

$\Rightarrow t = 2.45$ sec

$\theta = 0.15 \times (2.45)^2$
$= 0.9$ rad/sec

$\dot{\theta} = 0.3t.$

At 2.45

$\dot{\theta} = 0.735$ rad/sec

$\ddot{\theta} = 0.3$

Now

$r = 1 - 0.12t^2$

At $t = 2.45$ sec

$r = 0.279$

$\dot{r} = -0.24t$
$= -0.588$ (At $t = 2.45$ sec)

$\ddot{r} = -0.24$

now, $\vec{v} = v_r \hat{e}_r + \hat{v}_\theta \, e_\theta$

$= r\hat{e}_r + r\theta \, \hat{e}_\theta$

$= -0.588 \, \hat{e}_r + (0.279 \times 0.735) \hat{e}_\theta$

$= -0.588 \, \hat{e}_r + 0.205 \hat{e}_\theta$

$v = \sqrt{(-0.588)^2 + (0.205)^2}$

$= 0.622$ m/s

again, $\tan \alpha = \frac{0.205}{0.588}$

$= 19.22°$

now,

$\vec{a} = (\ddot{r} - r\dot{\theta}^2) \hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \hat{e}_\theta$

$= (-0.24 - 0.279 \times (0.735)^2) \hat{e}_r + (0.279 \times 0.3 + 2 \times -0.588 \times 0.735) \hat{e}_\theta$

$= (-0.390) \hat{e}_r + (-0.781) \hat{e}_\theta.$

$a = \sqrt{(-0.390)^2 + (-0.781)^2}$

$= 0.872$ m/s^2

also

$\tan \beta = \frac{0.390}{0.761}$

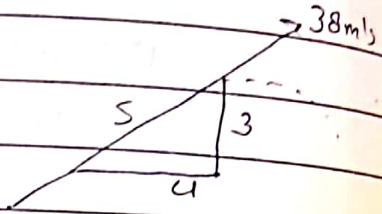
$\beta = 2653$

(c) relative acceleration of collar w.r.t arm

$\ddot{r} = (-0.24)$ m/s^2

18) A nozzle discharge ~~of~~ a stream of water in the direction 4H : 3V with an initial velocity of 38 m/s. Determine the radius of curvature of the stream

a) as it leave the nozzle.
b) at maximum height
c) when t = 3 sec



=> Soln.

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

From horizontal motion ($\rightarrow$ +ve)

$x = u_x \times t$

$= 38 \cos\theta\, t$

$x = 30.4\,t$

$\Rightarrow t = \frac{x}{30.4}$

from vertical motion, ($\uparrow$ +ve)

$Y = (V_{oy})t - \frac{1}{2} g t^2$

$= 38 \sin\theta\, t - 4.905 t^2$

$= 22.8 t - 4.905 t^2$

$= 22.8 \times \frac{x}{30.4} - 4.905 \times \frac{x^2}{(30.4)^2}$

$y = 0.75x - 0.0053x^2$

$\frac{dy}{dx} = 0.75 - 0.0106x$

$\frac{d^2y}{dx^2} = -0.0106$

a) As it leaves nozzle

$$x = 0$$

$$\frac{dy}{dx} = 0.75, \quad \frac{d^2y}{dx^2} = -0.0106$$

$$\delta = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left[1 + (0.75)^2\right]^{3/2}}{0.0106}$$

$$= 184.16 \text{ m}$$

b) At max height

$$\frac{dy}{dx} = 0$$

$$\frac{d^2y}{dx^2} = 0.0106$$

$$\delta = \frac{\left[1 + \left(dy/dx\right)^2\right]^{3/2}}{d^2y/dx^2}$$

$$= \frac{1}{0.0106}$$

$$= 94.3 \text{m}$$

c) At $t = 3$ sec

$$x = 30.4 \times 3 = 91.2$$

$$\frac{dy}{dx} = 0.75 - 0.0106 \times 91.2$$

$$\frac{d^2y}{dx^2} = -0.0106$$

now,

$$\delta = \frac{\left[1 + (-0.216)^2\right]^{3/2}}{0.0106}$$

$$= 101.01 \text{ m} \quad \#$$

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