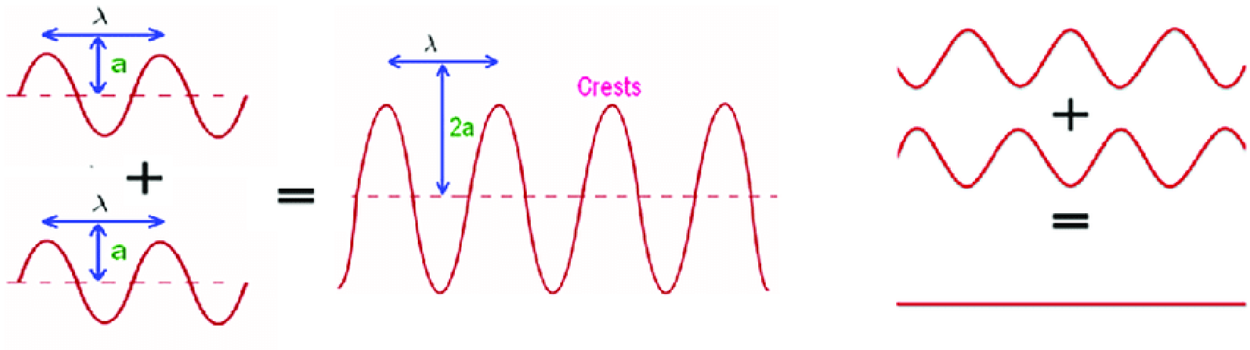
Interference

The phenomenon of non uniform distribution of light energy in the medium due to the superposition of two light waves is called interference. There are two types of interference;

1. **Constructive interference (Bright band):-** When two coherent waves superimpose which are in phase, the resultant wave is formed with amplitude sum of the amplitude of the individual wave. If two coherent waves each with amplitude A superimpose, the amplitude of resultant wave is;
   $\mathrm{A_r = A + A = 2A.}$
   $\therefore$ Intensity of resultant wave $\mathrm{I_r \propto A_r^2 = 4A^2 i.e.}$ maximum, this type of interference is called constructive interference.

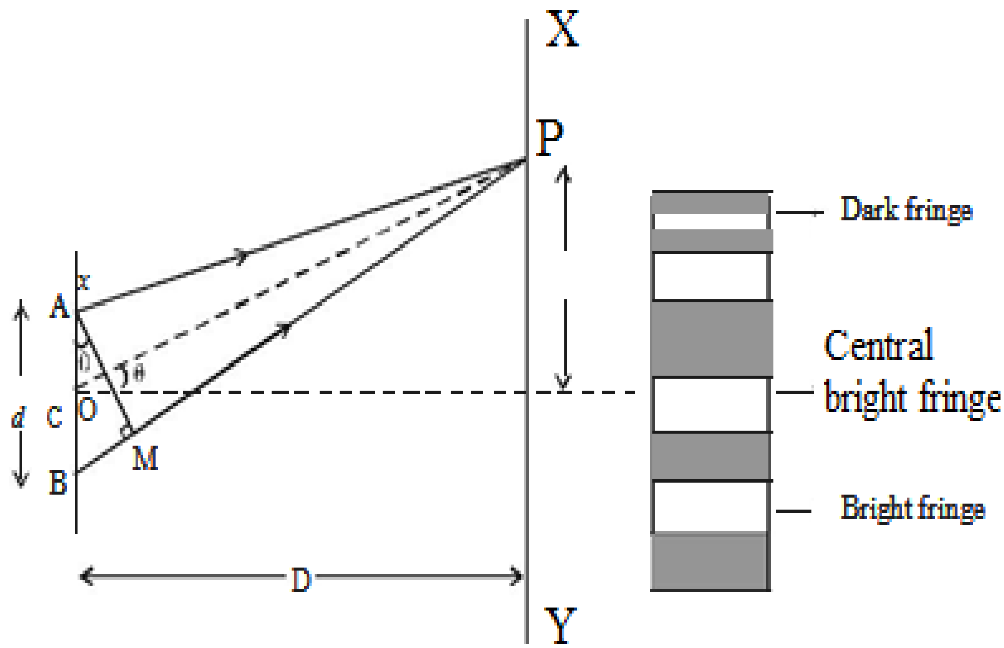
2. **Destructive interference (Dark band):-** When two coherent waves superimpose, which are out of phase. The amplitude of resultant wave is;
   $\mathrm{A_r = A + (-A) = 0.}$
   $\therefore$ Intensity of resultant wave $\mathrm{I_r \propto A_r^2 = 0}$ i.e. minimum, this type of interference is called destructive interference.



Constructive interference

Destructive interference

Analytical (mathematical) treatment of interference:-



Le the amplitude of the waves be A and the phase difference between two waves reaching the point P be δ. The displacement from a and b are denoted by y_1 and y_2. The displacement equation can be written as;

$$y_1 = A\sin wt$$

$$y_2 = A\sin(wt + \delta)$$

From the principle of superposition the resultant displacement is;

$$y = y_1 + y_2$$

$$or, y = A\sin wt + A\sin\,(wt + \delta)$$

$$or, y = A\sin wt + A\sin wt \cos\delta + A\cos wt \sin\delta$$

$$or, y = A\sin wt\,(1 + \cos\delta) + \ A\cos wt \sin\delta$$

$$Let, \qquad A(1 + \cos\delta) = R\cos\theta \ldots\ldots\ldots (1)$$

$$A\sin\delta = R\sin\theta \ldots\ldots\ldots.(2)$$

$$\therefore y = R\cos\theta \sin wt + R\sin\theta \cos wt$$

$$\therefore y = R\sin(wt + \theta)$$

This implies the resultant wave is simple harmonic with an amplitude R.

Squaring and adding equation (1) and (2) we get;

$$R^2\sin^2\theta + R^2\cos^2\theta = \ A^2\sin^2\delta + A^2(1 + \cos\delta)^2$$

$$or,\ A^2\sin^2\delta + A^2 + 2A^2\cos\delta + \ \ A^2\cos^2\delta = R^2$$

$$or, R^2 = 2\,A^2 + \ 2A^2\cos\delta$$

$$or, R^2 = 2\,A^2(1 + \cos\delta)$$

$$or, R^2 = 2\ A^2 \times 2cos^2\frac{\delta}{2}$$

$$or, R^2 = 4\ A^2 cos^2\frac{\delta}{2}$$

Since, the intensity at a point is given by the square of the amplitude.

$$\therefore I = R^2 = 4\ A^2 cos^2\frac{\delta}{2}$$

When the phase difference $\delta = 0, 2\pi, 4\pi, 6\pi \dots \dots n(2\pi),$

Or path difference = $0, \lambda, 2\lambda, 3\lambda \dots \dots \dots .. n(\lambda),$

Then, $I = 4A^2 = maximum\ intensity$

When the phase difference $\delta = \pi, 3\pi, 5\pi, 7\pi \dots \dots (2n+1)\pi,$

Or path difference = $\lambda/2, 3\lambda/2, 5\lambda/2 \dots \dots \dots .. (2n+1)\lambda/2,$

Then, $I = 0 = minimum\ intensity$

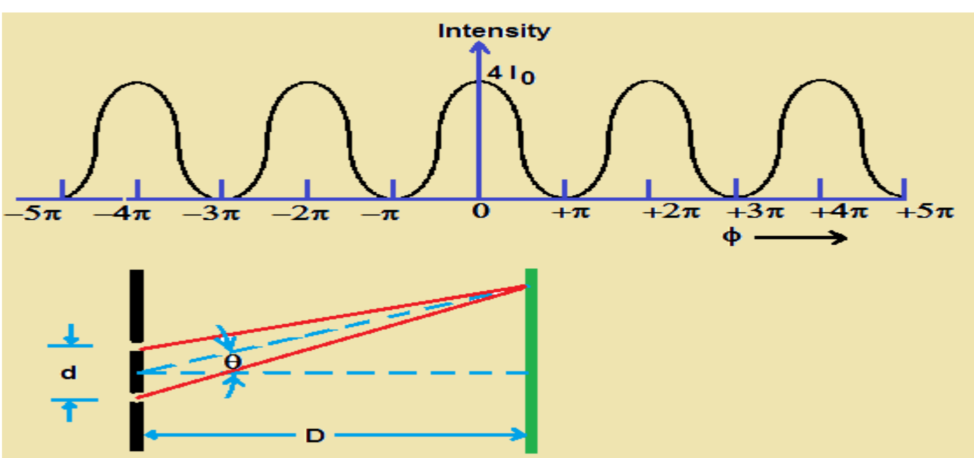
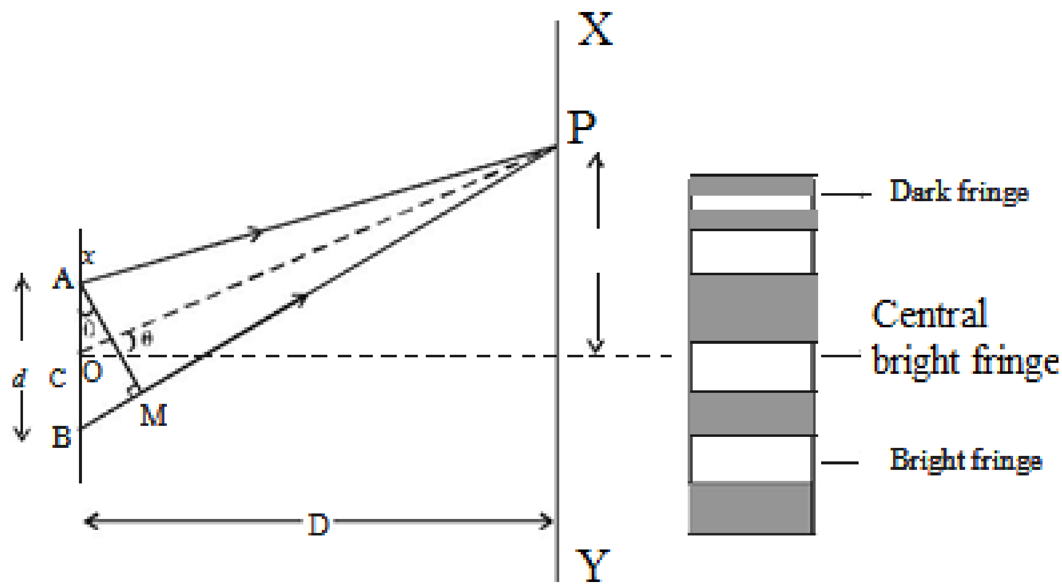


Figure:- Intensity distribution in interference

Young's double slit experiment:-



Let A and B be two narrow slits illuminated by monochromatic light from the source. Let the distance between A and B be 'd' which is very small as compared to the slit screen distance 'D' . The Point O is equidistance from A and B will meet each other on screen. Thus there will be central bright fringe at the center.

The intensity at any point P at a distance 'y' from the center on the screen can be calculated by finding the path difference (BP-AP). Joining AP and BP and draw AM perpendicular to BP. Then,

$$BP - AP = BM \ldots\ldots\ldots (i)$$

From right angle triangle ABM,

$$BM = d \sin\theta \ldots\ldots\ldots (ii)$$

If this path difference contained a whole number of wavelength, then constructive interference will be observed.

$$i.e.\ \ d\sin\theta = n\lambda \ldots\ldots\ldots (iii)$$

If path difference is odd number of half wavelength then the destructive interference will be occur.

$$i.e.\ \ d\sin\theta = (n+\frac{1}{2})\lambda \ldots\ldots\ldots (iv)$$

Where ‘n’ is called the order of interference fringes now consider a case for constructive interference at P. Then,

$$d\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{d}. \ldots\ldots\ldots (v)$$

Let ‘y’ be distance at point P from center. We have from figure,

$$y = D\tan\theta \ldots\ldots\ldots (vi)$$

For small angles we can write,

$$y = D\tan\theta = D\sin\theta$$

$$\therefore y = \frac{Dn\lambda}{d} \quad and \ \ \lambda = \frac{dy}{Dn}$$

We define fringe width as the distance between constructive - constructive (or destructive - destructive) Interference and is denoted by β. Then

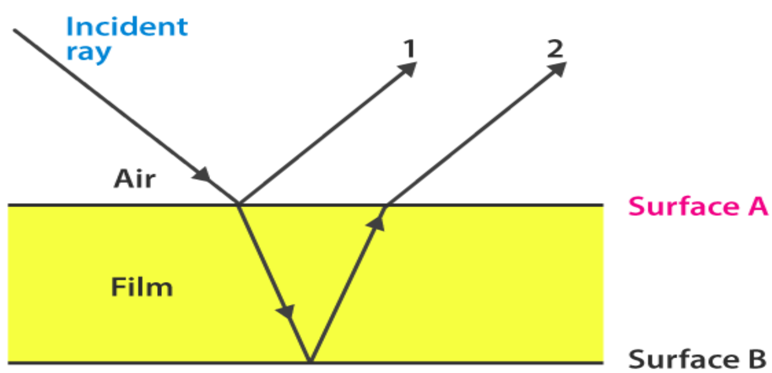
$$\beta = y_{n+1} - y_n$$

$$or, \qquad \beta = \frac{D(n+1)\lambda}{d} - \frac{Dn\lambda}{d}$$

$$\therefore \beta = \frac{D\lambda}{d}$$

Using this equation Young was able to determine the wavelength of light used in experiment.

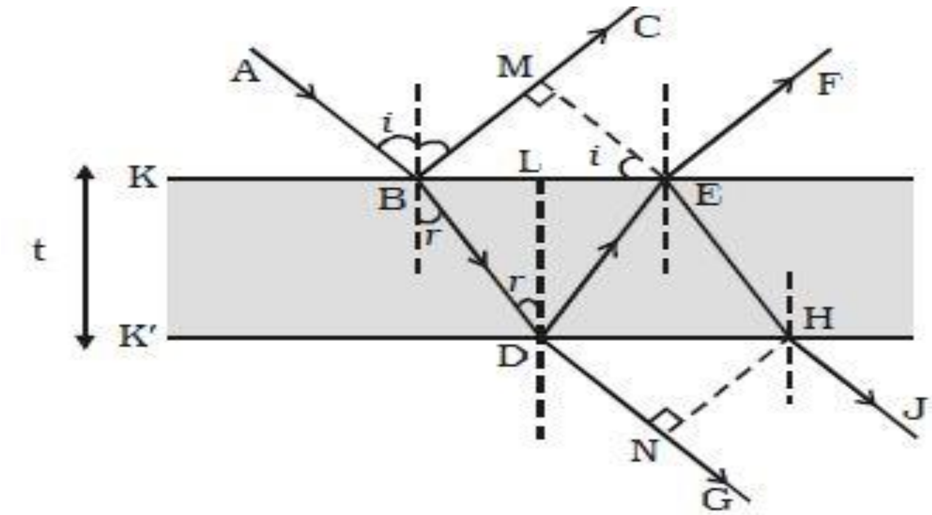
Interference in thin film:-



An optical medium is called a thin film, when its thickness is about the order of one wavelength of light in visible region. Thus a film of thickness in the range of 0.5μm to 10μm. A thin film may be a thin sheet of transparent material such as glass, mica, an air film enclosed between two transparent plates.

When light is incident on such a film, a small part of it gets reflected from the top surface and major part is transmitted into the film. Again a small part of the transmitted component is reflected back in to the film by the bottom surface and rest of its emerge out of the film. When light encounter a medium that has a higher refractive index, the reflected light or wave suffer a phase change π.

Interference in plane parallel film due to reflected rays:-



Consider a transparent film of uniform thickness t bounded by two parallel surfaces as shown in figure. Let the refractive index of the material be μ. The film is surrounded by air on the both sides. Let us consider plane wave from a monochromatic source falling on the film at an angle of incidence i. Part of a ray such as AB is reflected along BC and part of it is transmitted in the film along BD. The transmitted ray BD makes an angle r with normal to the surface at the point D. The ray partially reflected back in to the film along DE, while a major part refracts into the surrounding medium along DG. Part of the reflected ray DE is transmitted at the upper surface and travel along EF.

Let EM be normal to BC, from point M and E onwards the rays MC and EF travel equal path. The ray BM travel in air while the ray BE travels in the film of refractive index μ along the path BD and DE.

The geometrical path difference between the two rays is;

$$(BD + DE) - BM$$

Therefore, the optical path difference $(\Delta) = \mu(BD + DE) - 1(BM) \dots \dots \dots .(i)$

$$In\ \Delta\ BDL;\quad BD = \frac{DL}{\cos r} = \frac{t}{\cos r} \quad (where, DL = t)$$

$$\therefore\ BD + DE = \frac{2t}{\cos r} \dots \dots \dots (ii)$$

$$Also, BL = DL \tan r = t \tan r$$

$$\therefore BE = 2t \tan r$$

$$\therefore BM = BEsin\ i = 2t \tan r \sin i \dots \dots \dots (iii)$$

From Snell's law;

$$\sin i = \mu \sin r$$

$$\therefore BM = 2t \tan r. \mu \sin r = 2\mu t \frac{sin^2 r}{\cos r} \dots \dots \dots (iv)$$

Now, from equation (i), (ii) and (iv);

$$\Delta = \left(\mu \times \frac{2t}{\cos r}\right) - (2\mu t \frac{sin^2 r}{\cos r})$$

$$or, \qquad \Delta = \frac{2\mu t}{\cos r}\ (1 - sin^2 r)$$

$$or, \qquad \Delta = \frac{2\mu t}{\cos r}\ (cos^2 r)$$

$$\therefore\ \Delta = 2\mu t \cos r$$

This is the optical path difference and not the total true optical path difference. Ray 1 and 2 travels in two different medium, when light encounter a medium of higher refractive index, the reflected waves suffers a phase change of π. $i.e. path\ change\ of\ \lambda/2$.

Therefore, the net optical path difference becomes;

$$\Delta = \ 2\mu t \cos r + \frac{\lambda}{2}$$

Condition for maxima:-

The condition for constructive interference in the air film to appear bright is;

$$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$or, 2\mu t \cos r = n\lambda - \frac{\lambda}{2}$$

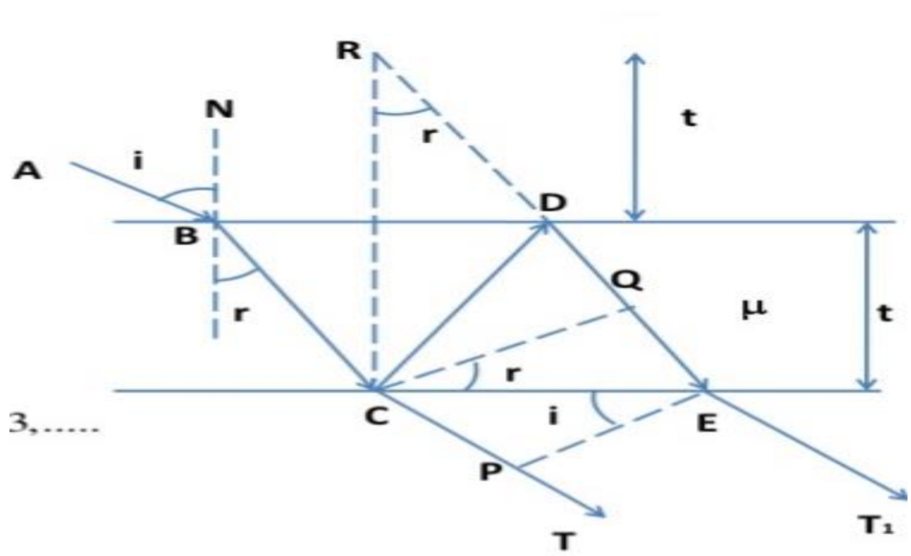
$$\therefore 2\mu t \cos r = (2n-1)\frac{\lambda}{2}\ \ where, n = 1,2,3, \dots \dots$$

Condition for minima (darkness):-

$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\therefore 2\mu t \cos r = n\lambda \qquad where, n = 0,1,2,3 \ldots\ldots$$

Interference in plane parallel films due to transmitted rays:-



Consider a thin transparent film of thickness t and refractive index μ. A ray AB after refraction goes along BC. At C it is partially reflected along CD and partially refracted through CT. The ray CD after reflection at D finally emerges along ET_1. Here at C and D reflection takes place at rare medium. Therefore, no phase change occurs. Draw CQ normal to DE and EP normal to CT. The optical path difference between ET_1 and CT is;

$$\Delta = \mu(CD + DE) - 1(CP)$$

$$\text{Now, } \mu = \frac{\sin i}{\sin r} = \frac{CP/CE}{QE/CE} = \frac{CP}{QE}$$

$$or, \qquad CP = \mu\, QE$$

In figure, $< CRD = r\; where,\; DR = CD = DE$

$$and\;\; CD + DE = RE$$

$$\therefore\; \Delta = \mu(RE) - \mu(QE)$$

$$= \mu(RE - QE)$$

$$\therefore\; \Delta = \mu(RQ)$$

$$\text{In } \Delta\, CRQ, \cos r = \frac{RQ}{CR}$$

$$RQ = CR \cos r$$

$$But, \qquad CR = 2t$$

$$\therefore\; RQ = 2t \cos r$$

$$\therefore\; \Delta = \mu\, RQ \;\; becomes;$$

$$\therefore\; \Delta = 2\mu t \cos r$$

Condition for maxima (Brightness):-

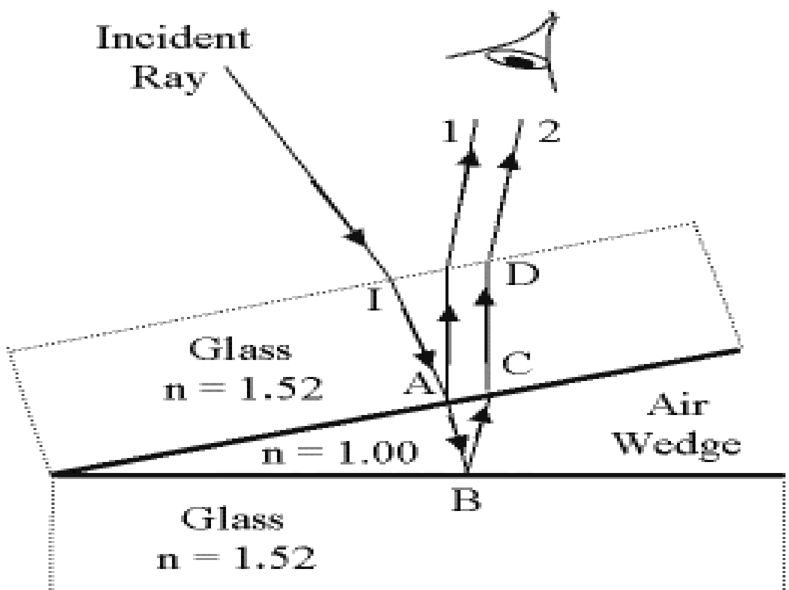
When the optical path difference $2\mu t \cos r = n\lambda$, then bright fringe occurs.

Condition for minima(Darkness):-

When the optical path difference $2\mu t\cos r = (2n+1)\lambda/2$, then dark fringe occurs.

Interference in wedge shape thin film:-

A thin film having zero thickness at one end and progressively increasing to a particular thickness at the other end is called a wedge. A thin wedge of air film can be formed by two glasses slides on each other at one edge and separated by a thin spacer at the opposite edge.



Let us suppose we are having two planes inclined at a small angle θ. The space between the planes is filled up by air so it forms a wedge shape thin film of air. Now, if light is incident on this film then the interference will take place between reflected rays. The condition for bright fringe will be;

$$2\mu t \cos r = (2n - 1)\frac{\lambda}{2}$$

$$for\ air, \mu = 1$$

$$for\ normal\ incidence, \qquad r = 0$$

$$\therefore \cos r = 1$$

$$so, \qquad 2t = (2n - 1)\frac{\lambda}{2}$$

$$\therefore t = (2n - 1)\frac{\lambda}{4}$$

And condition for dark fringe will be;

$$2\mu t \cos r = n\lambda$$

For air medium and normal incidence;

$$t = \frac{n\lambda}{2}$$

If the wedge shaped film has refractive index μ then the thickness of the film increases by $\frac{\lambda}{2\mu}$ in consecutive bright or dark fringe.

Newton’s ring:-

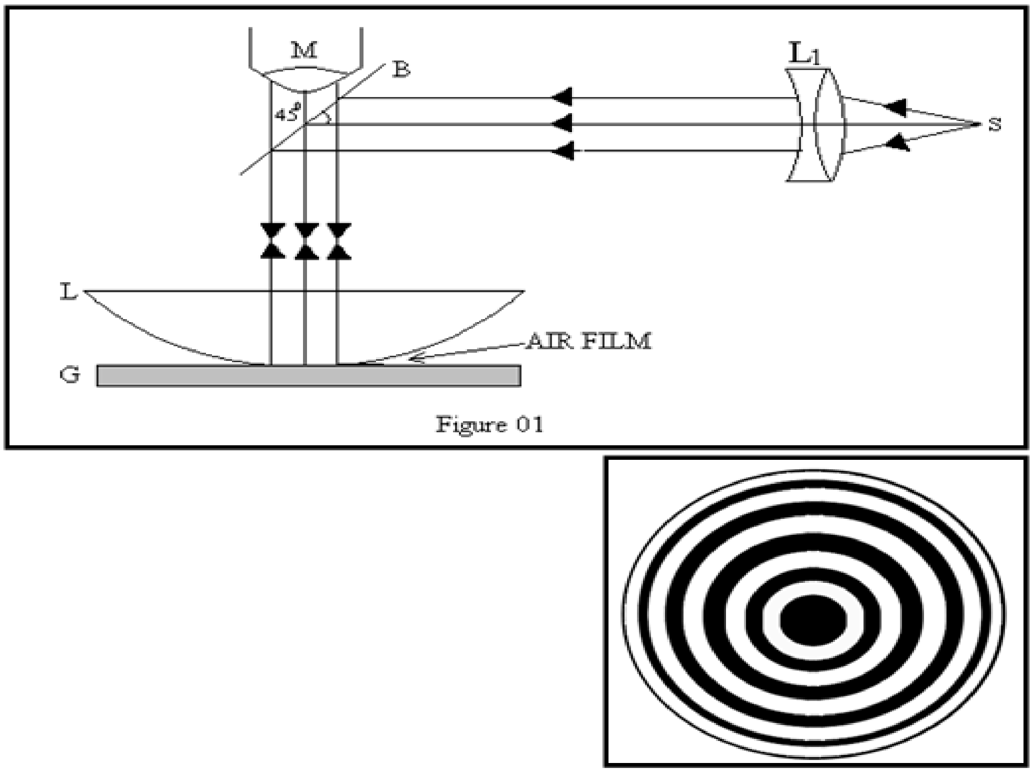


Figure 01

When a lens or curve glass surface is placed in contact with a plane glass surface as in figure, a thin air film is formed. When such film is exposed by mono-chromatic light, a series of concentric ring appear called Newton’s ring. Therefore, Newton’s rings are concentric ring formed when light falls on a uniform wedges shaped film. These fringes are due to the interference between rays reflected from top and bottom surface of the air gap between the two pieces of glass.

Interference in Newton’s ring:-

When monochromatic light is allowed in the experimental setup as shown in figure, Newton’s rings are observed. The path difference between the rays reflected on the upper and lower surface of the film is;

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2} \dots \dots \dots (i)$$

For almost normal incidence of light in air film, $r \cong 0\ and\ \mu = 1$

So path difference will be;

$$\Delta = 2t + \frac{\lambda}{2} \dots \dots \dots (ii)$$

At a point of contact; $t = 0,$

So, path difference $\Delta = \frac{\lambda}{2},$

Hence, central spot is dark.

The condition for bright ring is;

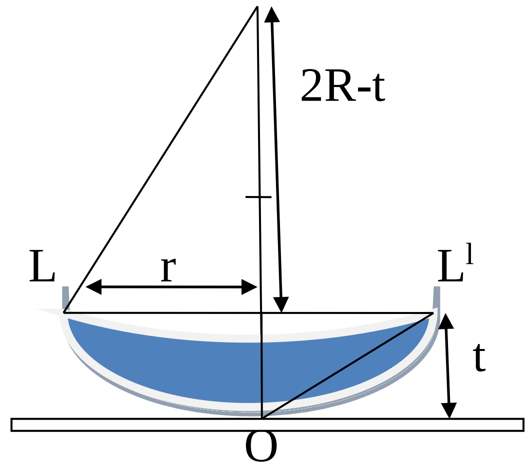
$$2t + \frac{\lambda}{2} = n\lambda$$

$$or, \qquad 2t = (2n-1)\frac{\lambda}{2} \dots \dots \dots (iii) \quad where, n = 1,2,3,4 \dots \dots$$

And the condition for minima is;

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$or, 2t = n\lambda \dots \dots \dots (iv)$$



Now, let's find the diameter of Newton's ring from the similar triangle property;

$$\frac{r}{t} = \frac{2R - t}{r}$$

$$or, r^2 = 2Rt\ (since\ t\ is\ very\ small\ so\ t^2 can\ be\ neglected)$$

$$\therefore t = \frac{r^2}{2R} \ldots \ldots .. (v)$$

Where, R is radius of curvature and r is radius of Newton's ring.

Thus for a bright ring; from equation (iii) and (v),

$$2\frac{r^2}{2R} = (2n - 1)\frac{\lambda}{2}$$

$$or, \qquad r^2 = (2n - 1)\frac{\lambda R}{2} \ldots \ldots \ldots (vi)$$

If D_n is the diameter of the n[th] bright ring then;

$$\frac{D_n^2}{4} = (2n-1)\frac{\lambda R}{2}$$

$$\therefore\ D_n = \sqrt{(2n-1)2\lambda R}$$

Similarly, for a dark ring; from equation (iv) and (v),

$$2\frac{r^2}{2R} = n\lambda$$

$$or, \qquad r^2 = n\lambda R$$

If D_n is the diameter of the nth dark ring then;

$$\frac{D_n^2}{4} = n\lambda R$$

$$\therefore\ D_n = 2\sqrt{n\lambda R}$$

Determination of wavelength of light:-

Let the diameter of nth dark ring is;

$$D_n^2 = 4n\lambda R$$

Similarly, the diameter of (n+m)th dark ring is;

$$D_{n+m}^2 = 4(n+m)\lambda R$$

$$\text{So, } D_{n+m}^2 - D_n^2 = 4m\lambda R$$

$$\therefore\ \lambda = \frac{D_{n+m}^2 - D_n^2}{4mR}$$

Determination of refractive index of liquid:-

Let $D_n^{/}$ and $D_{n+m}^{/}$ be diameter of n[th] and (n+m)[th] Newton's ring. Here we can write the equation for normal incidence and with film of refractive index μ as;

$$D_{n+m}^{/2} - D_n^{/2} = 4m\lambda R/\mu$$

Now using above relation;

$$\mu = \frac{D_{n+m}^2 - D_n^2}{D_{n+m}^{/2} - D_n^{/2}}$$

Numerical Examples:-

1. **In Newton's ring experiment diameter of 15^{th} ring was found to be $0.59\ cm$ and that of 5^{th} ring was $0.336\ cm$. If the radius of the Plano-convex lens is $100\ cm$. Calculate the wavelength of light used.**

Solution:-

Diameter of 15^{th} ring $(\mathrm{D}_{15}) = 0.59\ \mathrm{cm}$

Diameter of 5^{th} ring $(\mathrm{D}_5) = 0.336\ \mathrm{cm}$

Radius of plano $-$ convex lens $(\mathrm{R}) = 100\ cm$

Wavelength of light $(\lambda) = ?$

We have; $\lambda = \dfrac{D_{n+m}^2 - D_n^2}{4mR} = \dfrac{0.59^2 - 0.336^2}{4 \times 10 \times 100}$

$= 5.9 \times 10^{-5}$ cm

$\therefore \lambda = 5900\ A^0$

2. **Newton's ring formed by sodium light between a flat glass plate and a convex lens are viewed normally, what will be the order of the dark ring which will have double the diameter of 40th ring?**

Solutions:-

We have the relation about diameter of dark ring is;

$D_n^2 = 4n\lambda R \ldots\ldots\ldots.(i)$

$\therefore D_{40}^2 = 4 \times 40 \times \lambda R \ldots\ldots\ldots.(ii)$

Dividing equation (i)by (ii)we gae;

$$\frac{D_n^2}{D_{40}^2} = \frac{n}{40}$$

According to the question; $D_n = 2D_{40}$

$$or, \frac{4 \times D_{40}^2}{D_{40}^2} = \frac{n}{40}$$

$\therefore n = 4 \times 40 = 160$

3. In Newton's ring experiment diameter of the 10^{th} dark ring changes from $1.4\ cm$ to $1.27\ cm$. When a liquid is introduced between the lens and the plate. Calculate refractive index of liquid.

Solution:-

For air medium; $D_n^2 = 4n\lambda R \ldots\ldots\ldots$ (i)

For liquid medium; $D_n^{/2} = 4n\lambda R/\mu \ldots\ldots\ldots$ (ii)

$$\therefore\ \mu = \frac{D_n^2}{D_n^{/2}} = \frac{1.4^2}{1.27^2} = 1.215$$

4. In Newton's ring experiment radius of the 4^{th} and 12^{th} ring are $0.26\ cm$ and $0.37\ cm$ respectively. Find the diameter of 24^{th} dark ring.

Solution:-

$r_4 = 0.26\ cm \qquad \therefore D_4 = 0.52\ cm$

$r_{12} = 0.37\ cm \qquad \therefore D_{12} = 0.74\ cm$

$$Now, \lambda = \frac{D_{n+m}^2 - D_n^2}{4mR} = \frac{D_{12}^2 - D_4^2}{4 \times 8 \times R}$$

$$\therefore\ \lambda R = \frac{0.5476 - 0.2704}{32} = 8.663 \times 10^{-3}$$

$Again;\ D_n^2 = 4n\lambda R$

$or,\ D_{24}^2 = 4 \times 24 \times 8.663 \times 10^{-3} = 0.8316\ cm^2$

$\therefore \; D_{24} = 0.911\ cm$

5. A parallel beam of sodium light of wavelength $\mathbf{5890 \times 10^{-8}} \boldsymbol{cm}$ is incident on a thin glass plate of refractive index $\mathbf{1.5}$ such that the angle of refraction in the plate is $\mathbf{60^0}$. Calculate the smallest thickness of the plate which will make it appear dark by reflection.

Solution:-

Wavelength $(\lambda) = 5890 \times 10^{-8} cm$

Refractive index (μ) $= 1.5$,

Angle of refraction (r) $= 60^0$

Thickness (t) = ?

We know that to appear dark fringe by reflected is;

$$2\mu t \cos r = n\lambda$$

For smallest thickness, $n = 1$

$$2 \times 1.5 \times t \times \cos 60^0 = 5890 \times 10^{-8}$$

$$\therefore t = \frac{5890 \times 10^{-8}}{1.5} = 3927 \times 10^{-8}\ cm$$

Exercise:-

1. In Newton's ring experiment, "Central spot is dark in reflected system" and "Fringes get closer as the number of order increased" explain. Is it possible to make a central spot bright in reflected system? If so how?
2. What are constructive and destructive interference? Prove that the path difference for constructive interference is integer multiple of λ and that for destructive interference is odd integer multiple of $\lambda/2$.
3. Explain how interference fringes are formed by thin wedge shaped film examined by normally reflected light. Derive a relation for the fringe width on such system of interference fringes.
4. Write down the condition for interference of light? Give the necessary theory for the interference in thin film due to reflected light.
5. Prove that interference in thin film of the reflected and transmitted light are complementary to each other.
6. What happens to the energy when waves perfectly cancel to each other in interference? Derive the relation for thin film interference by reflected light.
7. What are Newton's rings? How can you use these rings to determine refractive index of a given liquid?
8. What is interference? Explain the intensity distribution in interference with mathematical treatment.

9. What are Haidengers fringes? Describe the interference phenomena in wedge shape thin film and determine the relation of path difference.
10. Why Newton's rings are circular? Discuss and derive the necessary theory of Newton's ring experiment for transmitted light.
11. What are coherent source of light? How such sources develop in lab? Show that the square of the n^{th} dark ring by the reflected light of Newton's ring is directly proportional to the natural number.
12. Explain the circular nature of the Newton's interference fringes. Show that square of radius of n^{th} bright fringe of Newton's ring due to the reflected light is proportional to 2n-1.
13. White light is incident in soap film at an angle $\sin^{-1}(\frac{4}{5})$ and the reflected light on examination by spectrometer shows dark bands. The consecutive dark bands correspond to wavelength 6.1×10^{-5} cm and 6×10^{-5} cm. if $\mu = 1.33$ for the film, calculate the thickness.
14. In Newton's ring experiment, the radius of curvature of lens is 5 cm and the lens diameter is 20 mm. (a) How many bright fringes are produced? Assume that $\lambda = 589$ nm (b) How many bright rings are produced if the arrangement were immersed in water ($\mu = 1.33$).

15. Light of wavelength 6000 A falls normally on a thin wedge shape thin film of refractive index 1.4, forming fringes that are 2 mm apart. Find the angle of the wedge.

16. Newton's ring formed by sodium light viewed normally. What is the order of the dark ring which will have double the diameter of 50^{th} ring?

17. A soap film $5 \times 10^{-5}\ cm$ thick is viewed at an angle of 35^{o} to the normal. Find the wavelength of visible light which will be absent from the reflected light.

18. A plano-convex lens of radius 300 cm is placed on an optically flat glass plate and is illuminated by monochromatic light. The diameter of the 8^{th} dark ring in the transmitted system is 0.72 cm. Calculate the wavelength of light used.

19. Newton's rings are observed in reflected light of wavelength 5900 A. The diameter of the 10^{th} dark ring is 50 mm. Find the radius of curvature of lens and thickness of the air film.