Chapter - 02
Transformer

Introduction

Transformer is an a/c machine that i)

i) transfers electrical energy from one electric circuit to another
ii) does so without a change of frequency
iii) does so by the principle of electromagnetic induction and
iv) has electric circuits that are linked by a common magnetic circuit

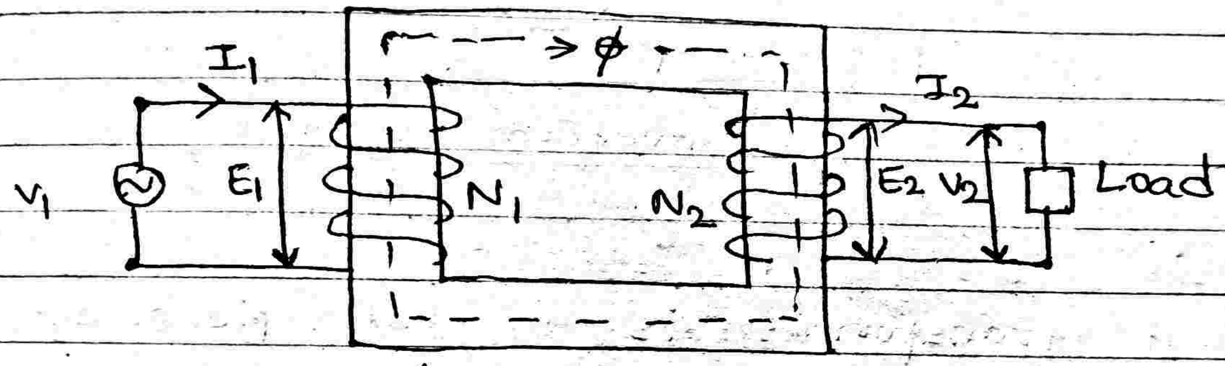


Fig. transformer

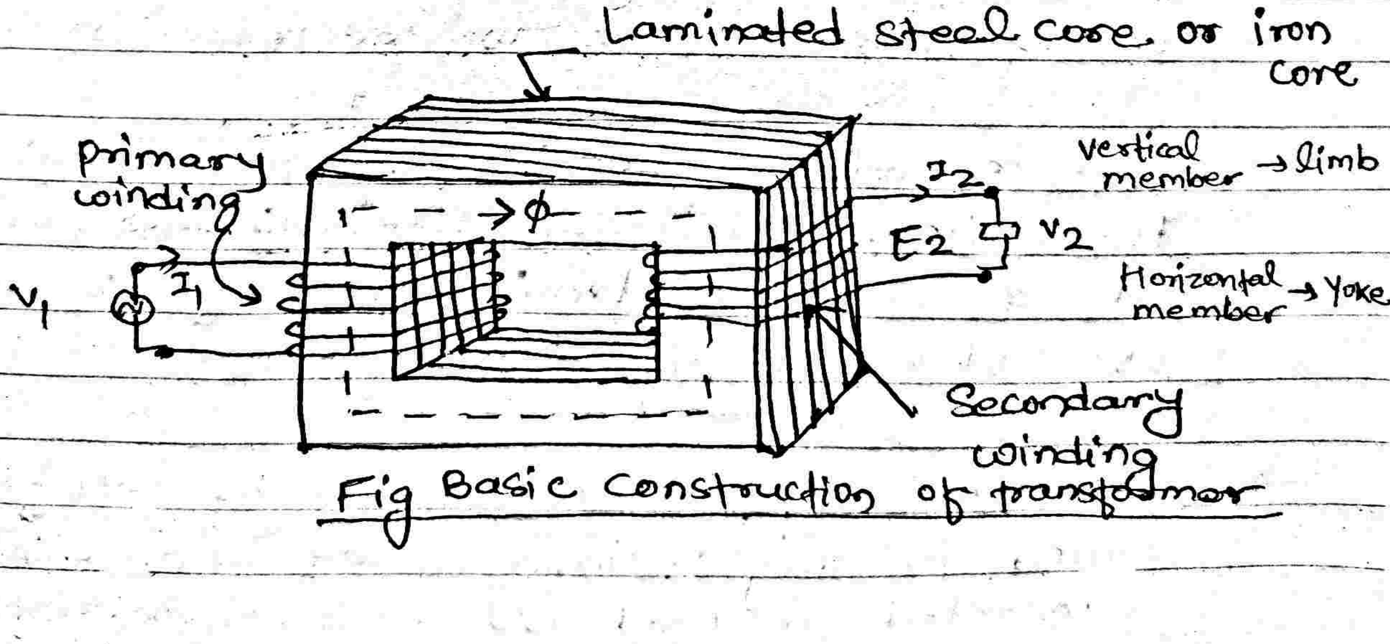


Fig Basic construction of transformer

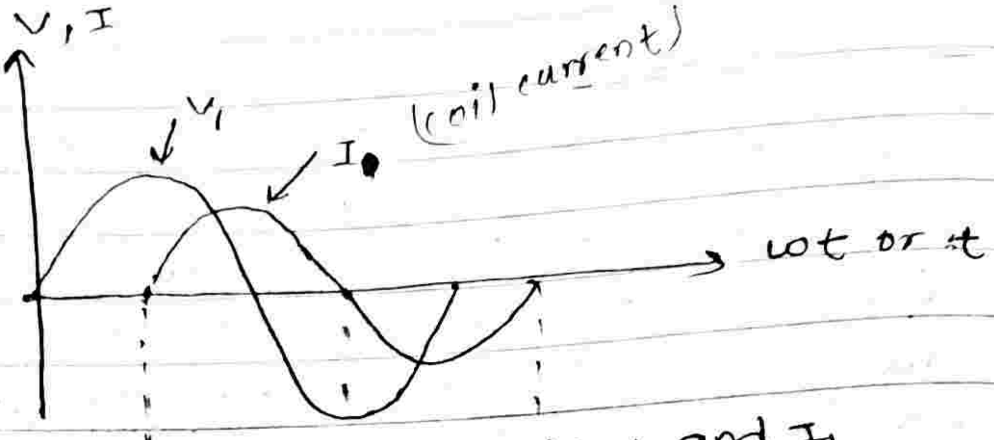


Fig. waveform of V_1 and I_1

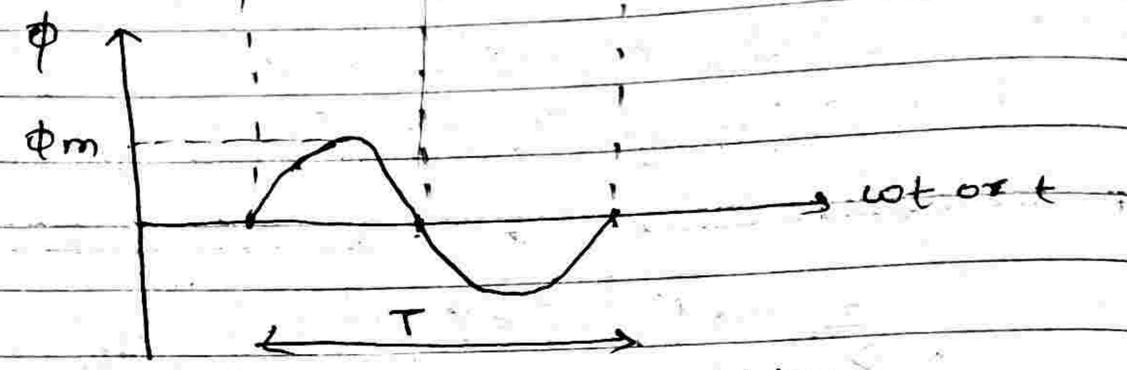


Fig waveform of flux

→ A transformer is a static piece of equipment used either for raising or lowering the voltage of ac supply with corresponding increase or decrease in current

→ It essentially consists of two windings, the primary and secondary windings, wound on a common laminated magnetic core as shown in fig above.

→ The winding connected to ac supply is called primary winding and the winding connected to load is called secondary winding

→ The alternating voltage V_1 whose magnitude is to be changed is applied on primary.

→ Depending upon number of turns of the primary (N_1) and secondary (N_2), an alternating Emf E_2 is induced in the secondary. The induced emf E_2 in secondary causes I_2 current and consequently terminal voltage V_2 will appear across load.

→ If $N_2 > N_1$ then $V_2 > V_1$ ($E_2 > E_1$), it is called step-up transformer.

→ If $N_2 < N_1$, then $V_2 < V_1$, it is called step-down transformer

Working principle of transformer

When the primary winding is connected to an ac supply mains, a current flows through it. Since this winding links with an iron core, current flows through this winding produces an alternating flux ϕ in the core. Since this flux is alternating and links with secondary winding also, induces an emf in the secondary winding. The frequency of induced emf in the secondary winding is same as that of the flux or that of supply voltage. The induced emf in the secondary winding enables it to deliver current to an external load connected across it.

Thus the energy is transfered from primary winding to the secondary winding by means of electromagnetic induction without any change in frequency. The flux ϕ of the iron core links not only secondary winding but also the primary winding so produces self induced emf in the primary winding. This induced emf in the primary winding opposes the applied voltage and therefore it is known as back emf of the primary.

Emf equation of a transformer

when an alternating voltage (sinusoidal) is applied to the primary winding of a transformer, an ac flux (sinusoidal), as shown in the fig below is set up in the iron core which links both windings (primary and secondary).

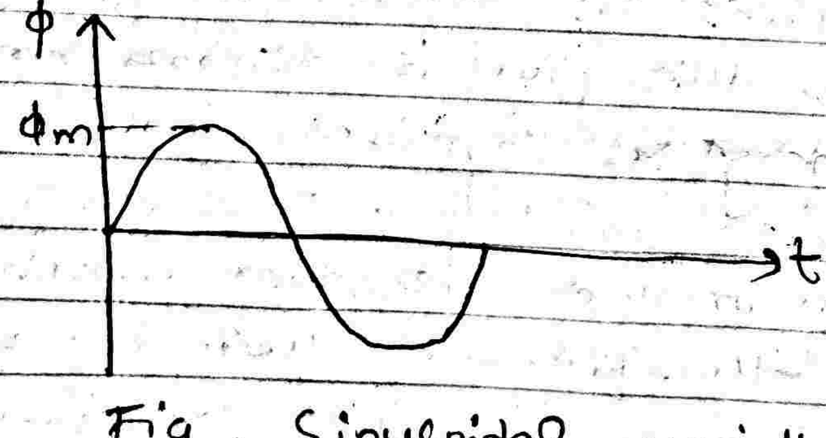


Fig. Sinusoidal variation of flux with time

The instantaneous value of sinusoidally varying flux may be given as,

$$\phi = \phi_m \sin\omega t \quad — ①$$

The instantaneous value of emf induced in the primary is,

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin\omega t)$$

$$= -\phi_m N_1 \omega \cos\omega t$$

$$e_1 = \underbrace{2\pi f N_1 \phi_m}_{e_{1m}} \sin(\omega t - 90) \quad — ②$$

From eqn ① and eqn ②, it is clear that e_1 lags behind ϕ by 90.

$$\underset{V_m}{e_{1m}} = 2\pi f N_1 \phi_m \quad — ③ \quad [\text{From eqn ②}]$$

The rms value of primary induced emf

$$E_1 = \frac{e_{1m}}{\sqrt{2}} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}} = 4.44 f N_1 \phi_m$$

$$E_1 = 4.44 f N_1 \phi_m$$

Similarly,

$$E_2 = 4.44 f N_2 \phi_m$$

Voltage transformation Ratio (Turn Ratio)

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

where, K is called transformation ratio.

Some points related to transformer

i) The transformer action is based on the laws of electromagnetic induction.

ii) There is no electrical connection between primary and secondary. The ac power is transferred from primary to secondary through magnetic flux.

iii) There is no change in frequency i.e output power has the same frequency as input power.

iv) The losses that occur in a transformer are -

a) core losses - eddy current and hysteresis losses

b) Copper losses - in the resistance of the windings.

→ These losses are very small so that output power is nearly equal to input power.

→ Transformer has high efficiency.

Ideal transformer

(Imaginary transformer)

An ideal transformer is one that has

i) no winding resistance. (pure inductive)

ii) no leakage flux i.e the same flux links both windings.

(100% flux passes through core and links both windings)

iii) no iron losses (i.e eddy current and hysteresis losses) in the core.

iv) zero magnetising current i.e core has infinite permeability and zero reluctance.

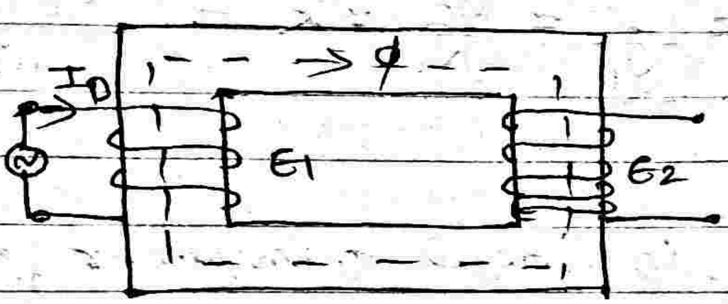
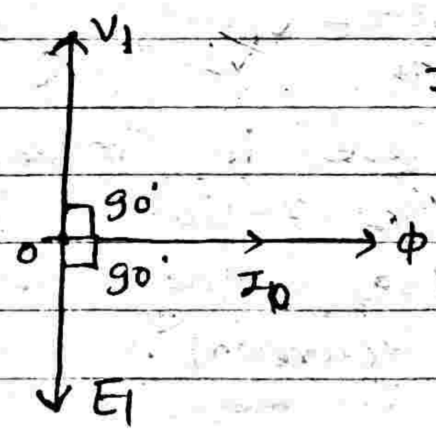


Fig Ideal transformer on no load



I_0 → used for magnetising
↳ very very less current

→ E_1 and V_1 are 180° out of phase

→ current lags 90° behind [redacted] supply voltage V_1 as it is purely inductive.

Voltage transformation ratio (K)

we have,

$E_1 = 4.44 f N_1 \phi m$

$E_2 = 4.44 f N_2 \phi m$

$E_2/E_1 = N_2/N_1 = K$

K = transformation ratio

For ideal transformer,

i) $E_1 = V_1$, and $E_2 = V_2$

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K = \frac{\text{Secondary voltage}}{\text{primary voltage}}$$

ii) no losses in ideal transformer,

$V_1 I_1 = V_2 I_2$

$I_2/I_1 = V_1/V_2 = 1/K$

→ currents are in inverse ratio of voltage transformation ratio.

→ So if raise voltage then there will be decrease in current.

Ideal transformer on Load

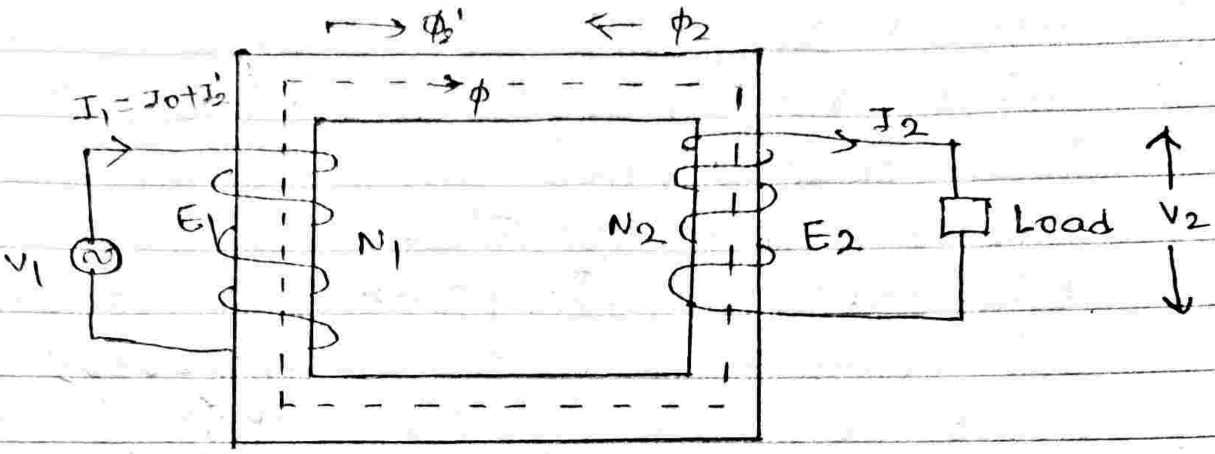


Fig. An ideal transformer on load

The no load current I_0 sets up mmf $N_1 I_0$ producing flux ϕ in the core. when load is connected, current I_2 flows in the Secondary winding which also set up a mmf $N_2 I_2$ producing secondary flux ϕ_2 which opposes main flux ϕ according to lenz law. This weakens the main flux momentarily i.e the back emf E_1 also tends to reduce the difference between V_1 and E_1 will cause more current to be drawn from the source, until the back emf E_1 balances the voltage V_1. Let I_2' be the additional primary current. The current I_2' is called counter - balancing current because it is in phase opposition with the Secondary current I_2. Such that the I_2' set some extra mmf $N_1 I_2'$ producing flux ϕ_2' which will be in the same direction of main flux ϕ and can cancel the flux ϕ_2 produced by mmf $N_2 I_2$ and tries to maintain the original value of main flux.

We can show that the flux ϕ_2 and ϕ_2' are equal in magnitude as follows -
we have,

Output VA $= V_2 I_2$

and additional power input $= V_1 I_2'$

$$\left[\begin{array}{c} I_1 = I_0 + I_2' \\ I_0 \simeq 0 \\ I_1 \simeq I_2' \end{array} \right]$$

For power balance,

$$V_2 I_2 = V_1 I_2'$$

$$\frac{V_2}{V_1} = \frac{I_2'}{I_2} = \frac{N_2}{N_1}$$

$$N_1 I_2' = N_2 I_2 \quad - (1)$$

$$\phi_2 = \frac{N_2 I_2}{R} - (ii) \qquad \phi_2' = \frac{N_1 I_2'}{R} \quad (iii)$$

From eqn (1), eqn (ii) and eqn (iii)

$$\boxed{\phi_2 = \phi_2'}$$

It is evident from this discussion that ϕ_2 and ϕ_2' are of equal magnitude and always oppose each other (lenz law) and cancel out completely. Hence the net magnetic flux in the core of a transformer is always irresponsible of the load. (constant)

From above,

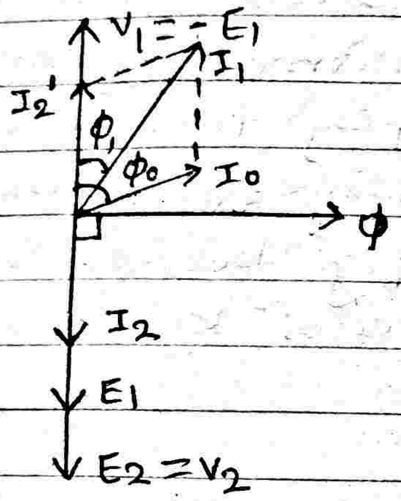
$$\frac{I_2'}{I_2} = \frac{N_2}{N_1} = \frac{I_1}{I_2} \qquad \left[\begin{array}{c} I_1 = I_0 + I_2' \\ I_0 \simeq 0 \end{array} \right]$$

$$\boxed{I_2' = K I_2} \qquad K = N_2/N_1$$

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1} = K}$$

The phasor diagrams of ideal transformer on load a) resistive load
b) Inductive load
c) capacitive load

$\Rightarrow$ $V_1 = -E_1$ and $V_2 = E_2$ as voltage drops in both windings are negligible.
$I_1 = I_0 + I_2'$



$\boxed{I_2' = K I_2}$

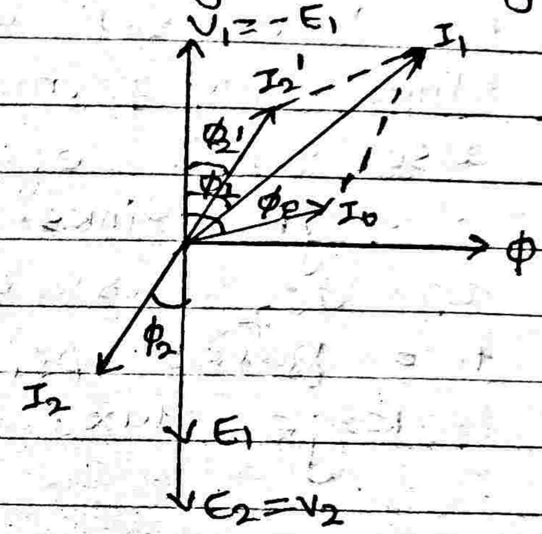


Fig ⓐ resistive load

Fig ⓑ Inductive load

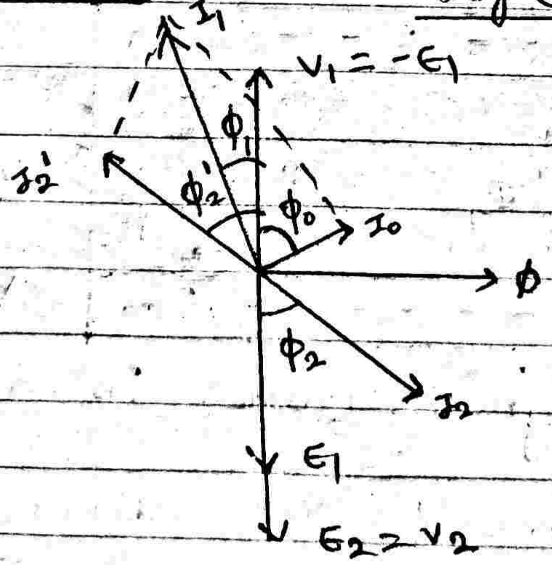


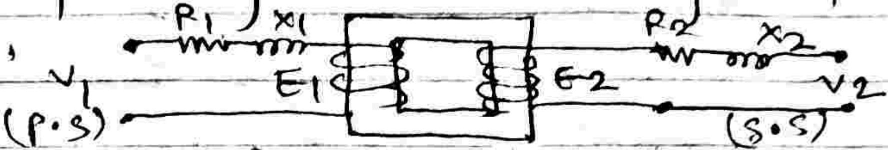
Fig ⓒ capacitive load

practical transformer

The practical transformer has

i) winding resistance.
ii) iron losses and copper losses
iii) leakage reactance.

i) <u>winding resistance</u> — Since the windings consists of copper conductors, it immediately follows that both primary and secondary will winding resistance,

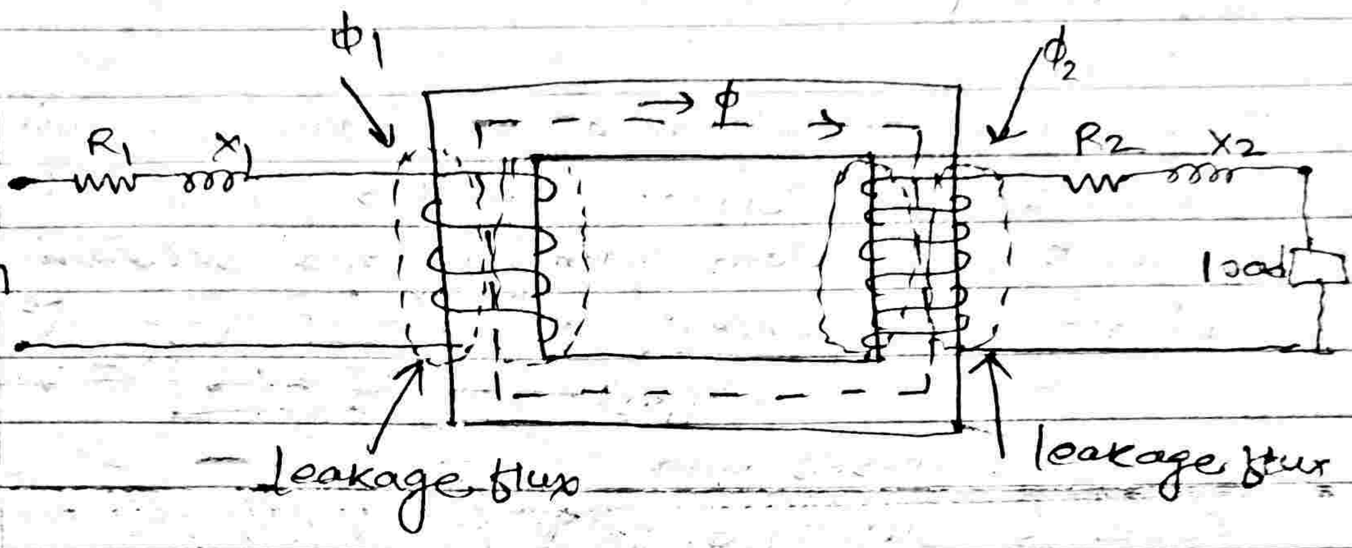


ii) <u>iron losses and copper losses</u> — Since iron core is subjected to alternating flux, there occurs eddy current and hysteresis losses in it. These two losses together are known as iron losses or corelosses. These two losses depend upon supply frequency, maximum flux density in core.

I^2R loss in the winding called copper loss.

iii) <u>leakage reactance</u> — Both primary and secondary currents produce flux. The flux ϕ which links both the windings is useful flux and is called the mutual flux. However primary winding will produce some flux ϕ_1, will not link with secondary winding similarly secondary will produce some flux ϕ_2, which will not link with primary winding. Such flux ϕ_1 or ϕ_2 which link only one winding is called leakage flux.

The effect of leakage flux is that it will introduce inductive reactance x_1 in primary x_2 in Secondary.



practical transformer on Noload

Consider a practical transformer on no load i.e Secondary on open-circuited as shown below in fig -

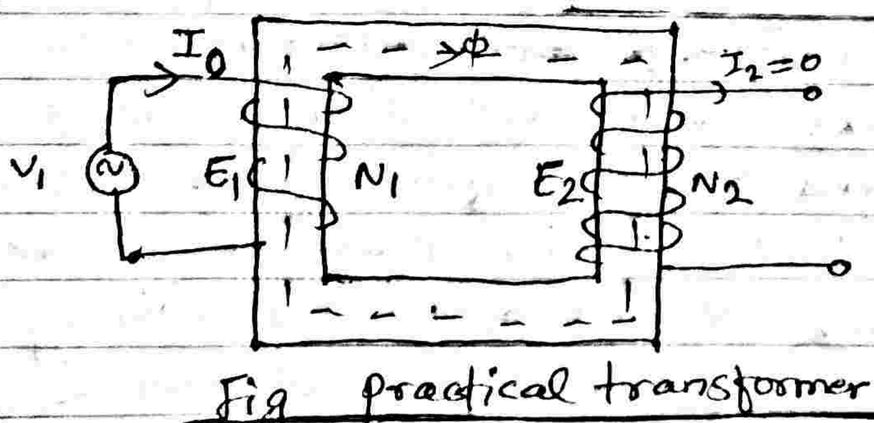


Fig practical transformer on No load

→ The primary will draw a small current I_0 to supply i) iron loss ii) a very small copper loss in the primary.

($I_0^2 R_1$ is neglected due to very small value of I_0)

→ Hence the primary no load current is not 90° behind the applied voltage V_1 (lags by an angle ($\phi_0 < 90°$)

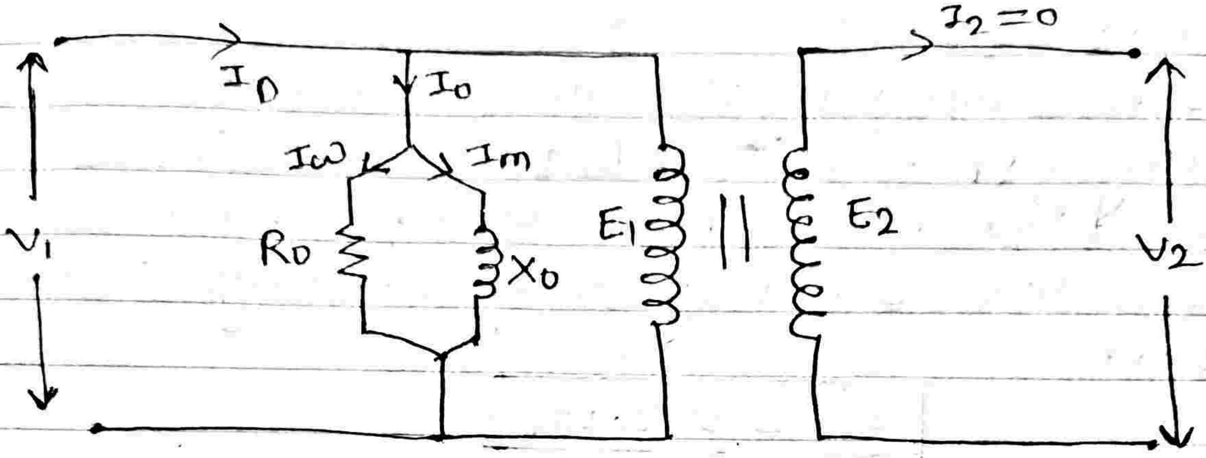
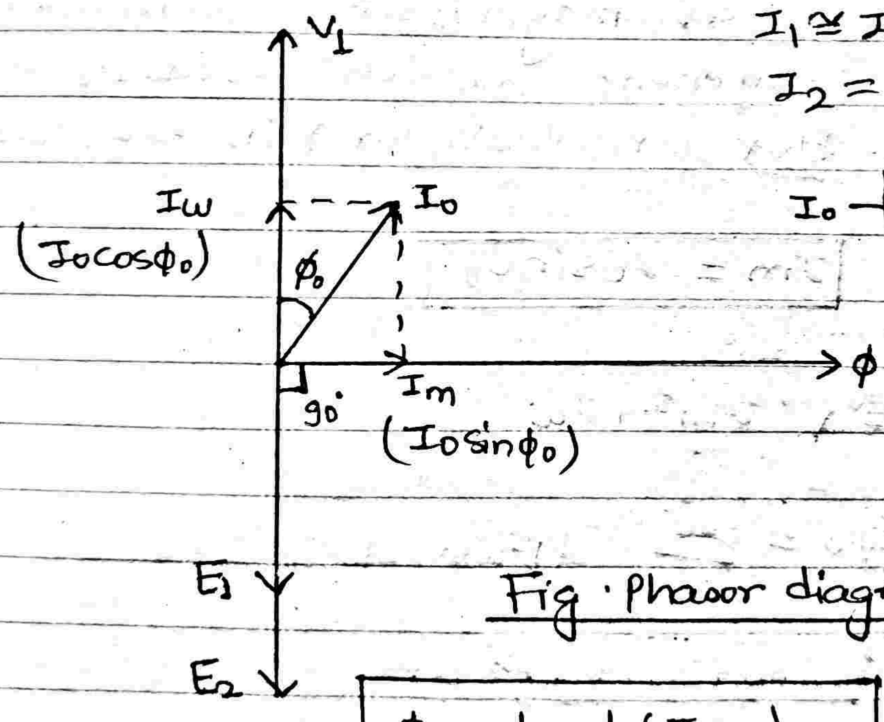


Fig. No load equivalent circuit of practical transformer



$I_1 \cong I_0$

$I_2 = 0$

I_0 → magnetising
→ loss

Fig · Phasor diagram

$$\boxed{\phi_0 = \tan^{-1}\left(\frac{I_m}{I_w}\right)}$$

I_m = magnetising component

I_w = active or working or iron loss component

$$\boxed{I_m = \frac{V_1}{X_0}, \quad I_w = \frac{V_1}{R_0}}$$

input power = core or iron loss $= \frac{V_1^2}{R_0} = V_1 I_0 \cos\phi_0$

No load current is resolved into two components –

a) The component I_w in phase with applied voltage V_1. This is known as active or working or iron loss component and supplies iron loss and very small primary copper loss.

$$\boxed{I_w = I_0 \cos\phi_0}$$

b) The component I_m lagging behind V_1 by $90°$ and is known as magnetising component. It is this component which produces the alternating flux (mutual flux) in the core.

$$\boxed{I_m = I_0 \sin\phi_0}$$

$$I_0 = \sqrt{I_m^2 + I_w^2}$$

$$\cos\phi_0 = \frac{I_w}{I_0} \quad \text{(From above)}$$

No load powerfactor

No load input power (W_0) = Iron losses

$= I_w^2 R_0$

$= \frac{V_1^2}{R_0}$

Q.1) A 11KV/400V distribution transformer takes a no load primary current of 1amp at a power factor of 0.24 lagging.
Find i) The coreloss current (ii) magnetising current
iii) The iron loss

=> soln

we have given,
No load current (I_0) = 1A
power factor ($\cos\phi_0$) = 0.24 lagging
primary voltage (V_1) = 11kV

i) coreloss current

→ coreloss current (I_w) = $I_0 \cos\phi_0 = 1 * 0.24$
$= 0.24$ A

ii) magnetising current (I_m) = $I_0 \sin\phi_0 = I_0\sqrt{1-\cos^2\phi_0}$
$= 1 * \sqrt{1-0.24^2}$
$= 0.971$ A

iii) Iron loss

→ Iron loss $= V_1 I_0 \cos\phi_0 = 11000 * 1 * 0.24$
$= 2640$ W
$= 2.64$ kW

Iron loss = 2.64 kW.

Q.2) A single phase transformer of 3300/220V, 50Hz takes a no load current of 0.8A and 500W. Calculate i) active and magnetising current ii) its pf

→ soln.

we have given,

No load current (I_0) = 0.8A
No load power = 5000
Primary voltage = 3300V

i) active and magnetising current
→ active (I_w) = $I_0 \cos\phi_0$ = $\frac{\text{No load power}}{\text{primary voltage}}$

[No load power (p_0) = $I_w * V_1$] $= \frac{500}{3300} = 0.152$ A

magnetising current (I_m) $= \sqrt{I_0^2 - I_w^2}$
$= \sqrt{0.8^2 - 0.15^2}$
$= 0.785$ A

ii) $\cos\phi_0 = \frac{I_w}{I_0} = \frac{0.152}{0.8} = 0.19$ lagging

Q.3) A voltage $V = 200\sin 314t$ is applied to the transformer winding in a no load test. The resulting current is found to be
$i = 3\sin(314t - 60°)$.
Determine the coreloss and the parameters of no-load approximate equivalent circuit.

⇒ Soln -
we have given,
No load current (I_0) $= 3/\sqrt{2} = 2.12$ A

coreloss = input at no load
$= V I_0 \cos\phi_0$
$= \frac{200}{\sqrt{2}} * 3/\sqrt{2} * \cos(60)$

$I_0 = 150W$

$I_w = I_0 \cos\phi_0 = 2.12 * \cos 60° = 1.836A$

$I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(2.12)^2 - (1.06)^2}$

$=$

No load resistance $(R_0) = V/I_w = \frac{200/\sqrt{2}}{1.06}$

$= 133.42 \Omega$

No load reactance $(X_0) = V/I_m = \frac{200/\sqrt{2}}{1.836}$

$= 77 \Omega$

Q.4) In no load test of single phase transformer the following data were obtained : primary voltage 230 V, secondary voltage 115 V primary current 0.5A, power input 40W. Find i) turn ratio ii) magnetising component of no load iii) working of primary winding is 0.5 Ω.

=> we have given,

No load power input to transformer,

$P_0 = 40W$

No load primary current $(I_0) = 0.5A$

primary voltage $(V_1) = 230V$

Secondary voltage $(V_2) = 115V$

$R_1 = 0.5 \Omega$

i) turn ratio

→ $k = V_2/V_1 = \frac{115}{230} = 0.5$

ii) magnetising component

→ $I_m = \frac{P_i}{V_1} = \frac{39.875}{230} = 0.173$

$I_m = 0.173\ A$

iii) working component of no load current

→ $I_w = \sqrt{I_0^2 - I_m^2} = \sqrt{0.5^2 - 0.173^2}$

$= 0.469\ A$

iv) Iron loss

→

Copper loss in primary winding

$P_c = I_0^2 R_1 = 0.5^2 * 0.5 = 0.125$

Iron loss $P_i = P_0 - P_c = 40 - 0.125$

$= 39.875\ W$

Resistance and leakage reactance

In actual transformer both the windings, primary and secondary windings have finite resistance R_1 and R_2 which cause copper losses and voltage drop in them.

It was assumed that entire flux ϕ, developed by the primary winding links secondary winding but it is impossible to realize this condition. However, part of the flux set up by the secondary winding links only primary turns named ϕ_{l_1} and also some of flux set up by the secondary winding links only secondary turns named as ϕ_{l_2} as shown in fig below: These two fluxes ϕ_{l_1} and ϕ_{l_2} are known as leakage flux.

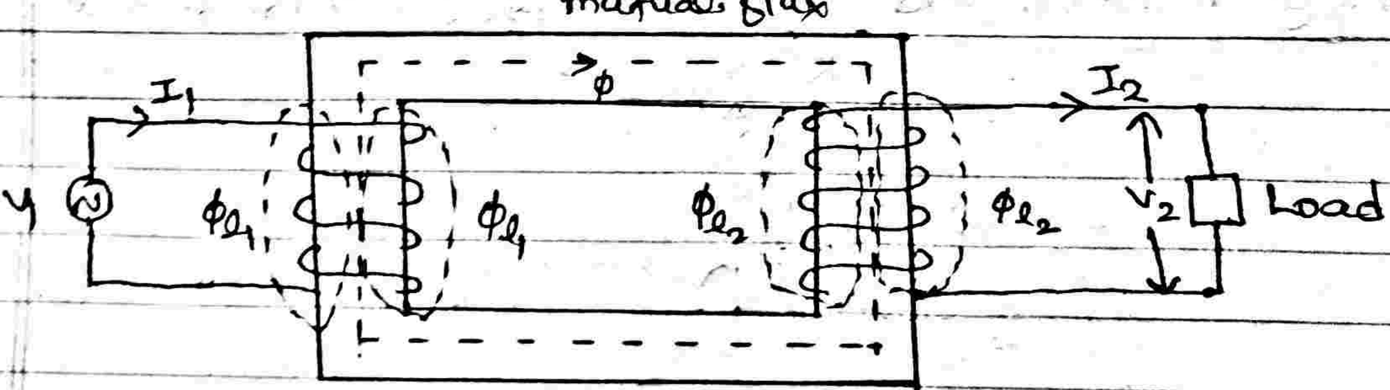
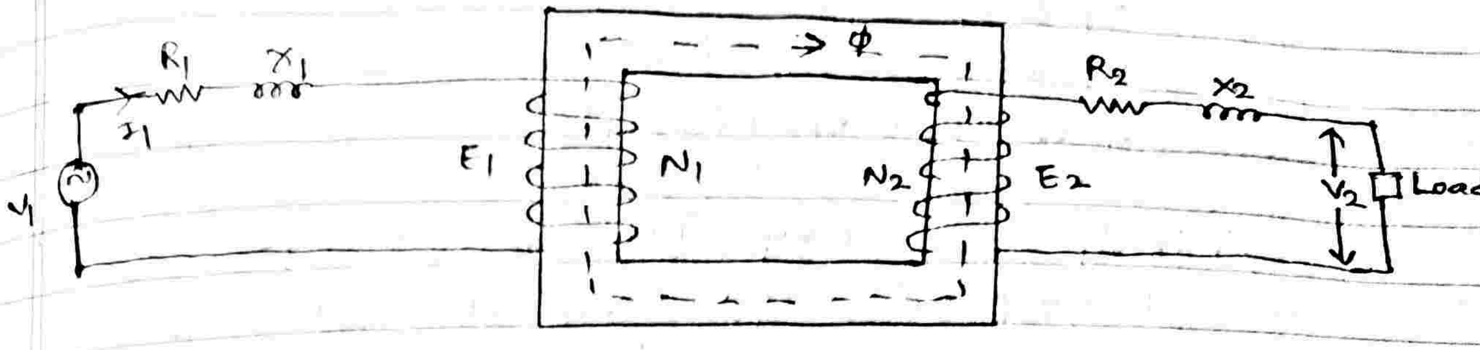


Fig: magnetic fluxes in a transformer

To represent the effect of leakage flux, we add a reactance in the equivalent circuit as follow:



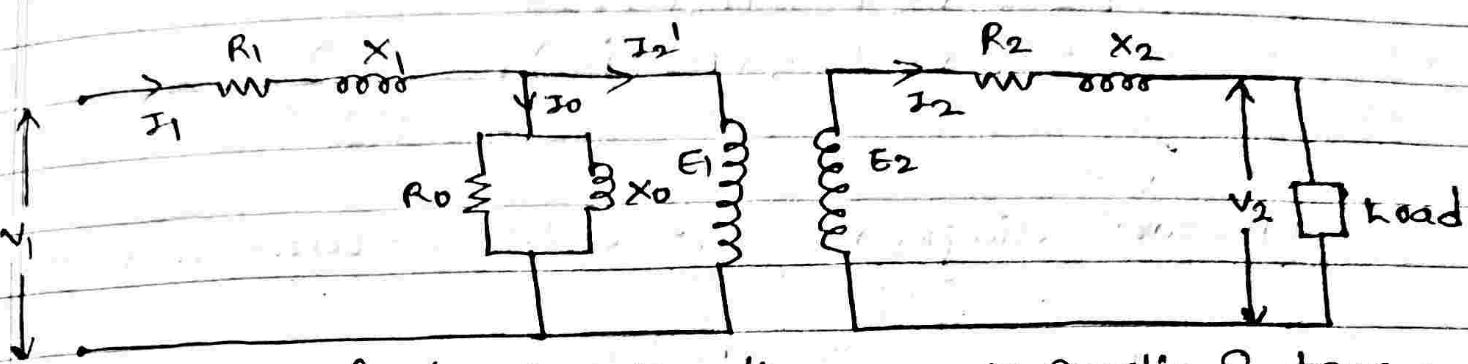


Fig. equivalent circuit diagram of practical tr-er

R_1 = internal resistance of pw
X_1 = leakage reactance of pw
R_2 = Internal resistance of sw
X_2 = leakage reactance of sw
Z_1 = $R_1 + jX_1$ = impedance of pw
Z_2 = $R_2 + jX_2$ = impedance of sw

Since E_1 is the ~~applied~~ voltage induced in the primary winding, it is equal and opposite to the component of the applied voltage at the ideal transformer winding. Let V_1' be the voltage applied to the primary of the ideal transformer to nuetr-alize the effect of induced voltage E_1. Thus V_1' is equal and opposite to E_1. The phasor sum I_1R_1, I_1X_1 and V_1' is equal to the supply voltage V_1.

$$V_1 = V_1' + I_1R_1 + jI_1X_1$$
$$V_1 = -E_1 + I_1R_1 + jI_1X_1$$
$$V_1 = -\dot{E}_1 + I_1(R_1 + jX_1) \quad — (1)$$

Similarly secondary induced voltage E_2 is the phasor sum of V_2, I_2R_2 and I_2X_2.

$$E_2 = V_2 + I_2R_2 + jI_2X_2$$
$$E_2 = V_2 + I_2(R_2 + jX_2) \quad — (2)$$

Phasor diagram of actual transformer on Load

The phasor diagrams of actual (practical) transformer on load (i) pure resistive (ii) resistive - Inductive and (iii) resistive - capacitive loads are shown below in fig (a), fig (b) and fig (c) respectively.

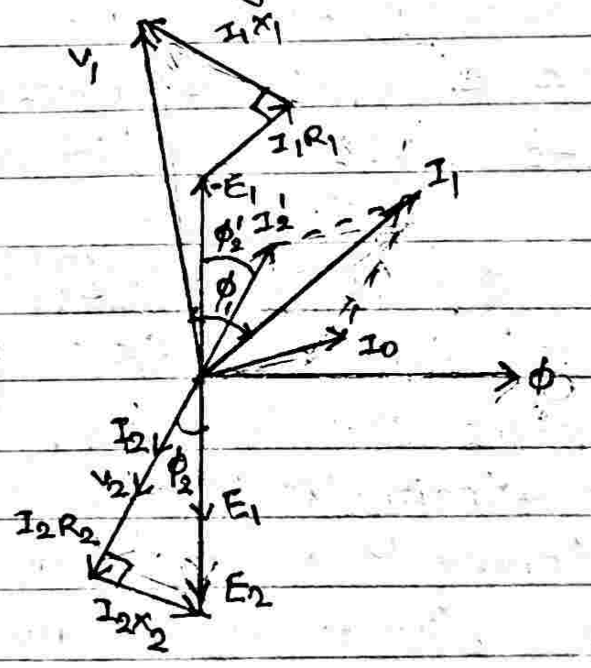


Fig (a) Phasor diagram of actual transformer on pure resistive load

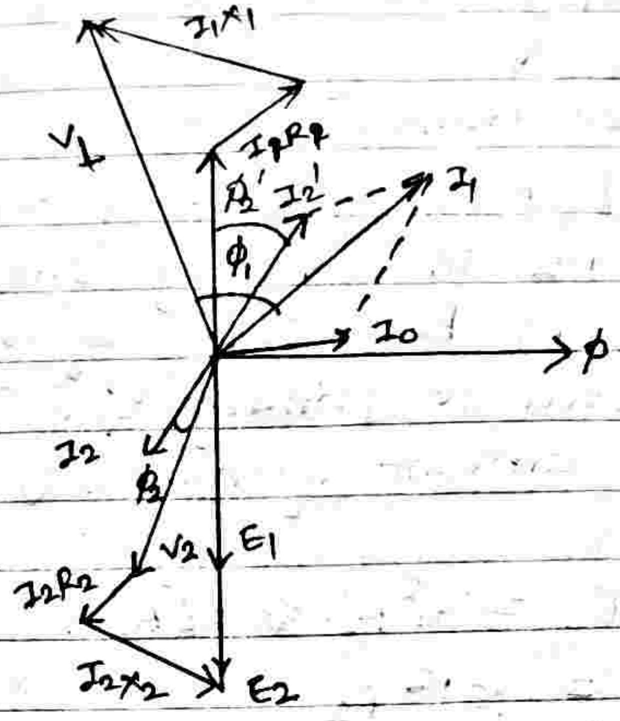


Fig (b) resistive - inductive load

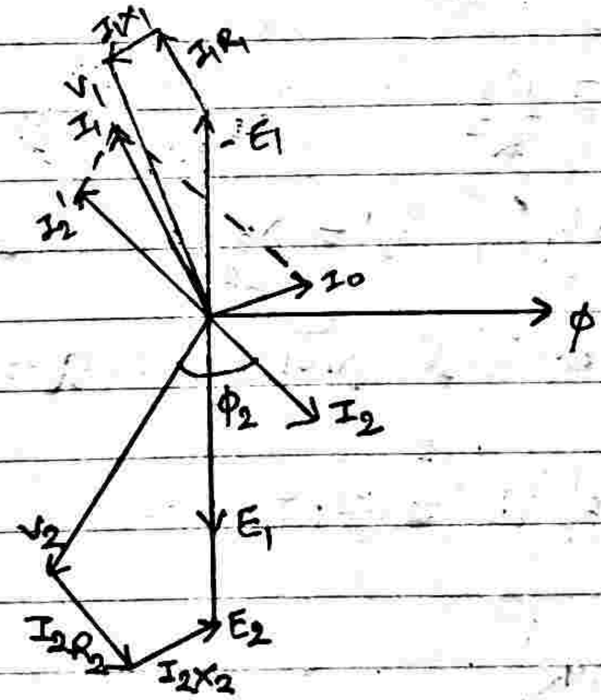


Fig (c) resistive capacitive load

Transformation of Impedance

The transformation of Impedance is done for the simplicity of mathematical analysis. The resistance and leakage reactance in secondary winding can be transferred to primary side so that effect of (R_2') in primary side is equivalent to effect of R_2 in secondary side.

The R_2 produces heat loss of $I_2^2 R_2$ when R_2 is transferred to primary side, it should produce same amount of heat loss. i.e

$$I_2^2 R_2 = I_1^2 R_2' \approx I_2'^2 R_2'$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

we know,

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = k$$

$$R_2' = \left(\frac{1}{k}\right)^2 R_2$$

$$R_2' = \frac{R_2}{k^2}$$

Similarly, $X_2' = \frac{X_2}{k^2}$ and $Z_L' = \frac{Z_L}{k^2}$

Secondary terminal voltage referred to primary side,

$$V_2' = I_2' Z_L' = I_1 Z_L' = (k I_2)\left(\frac{Z_L}{k^2}\right) = \frac{I_2 Z_L}{k}$$

$$V_2' = (V_2/k)$$

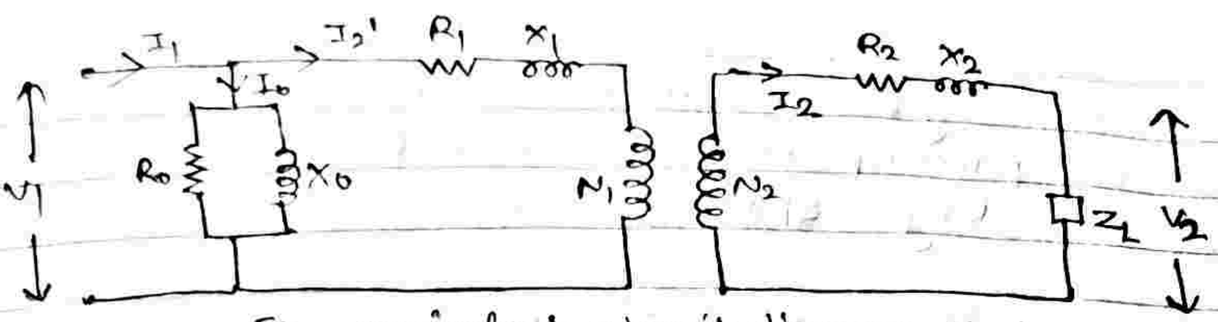


Fig. equivalent circuit diagram of tr-er

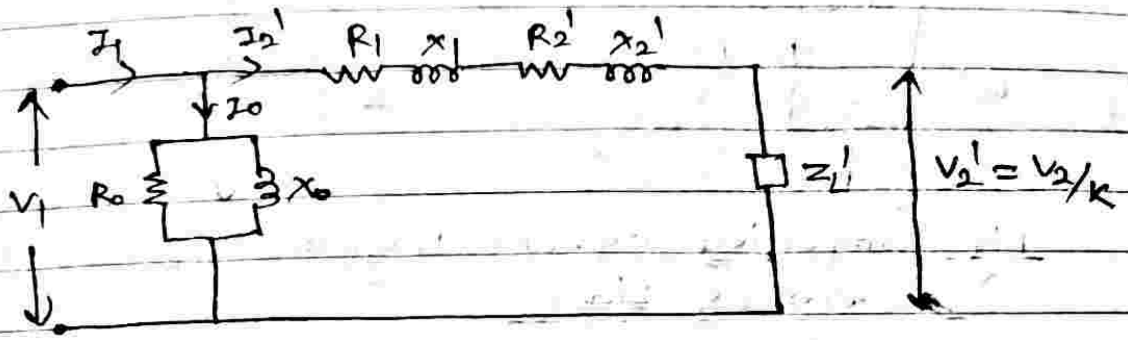


Fig. equivalent circuit diagram referred to primary side

Also,

$R_{01} = (R_1 + R_2') = R_1 + \frac{R_2}{K^2}$ = equivalent resistance referred to primary side.

$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$ = equivalent leakage reactance referred to primary side.

$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$ = equivalent impedance referred to primary side

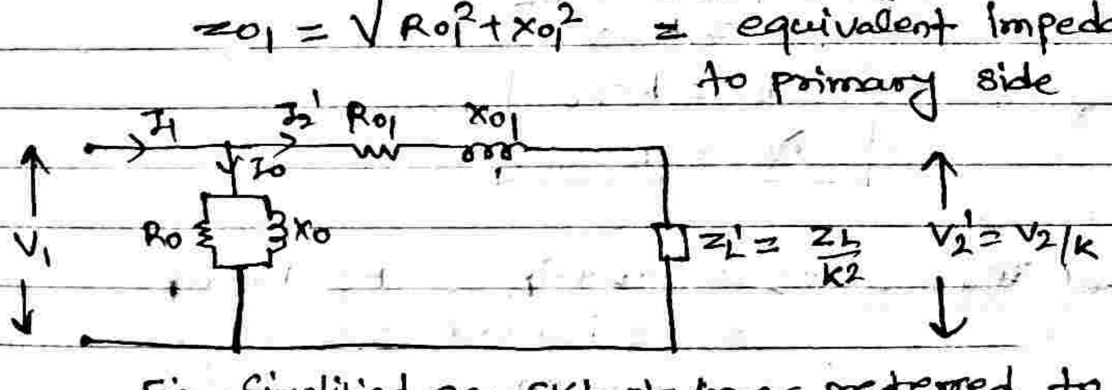


Fig. Simplified eq. ckt of tr-er referred to p.s

Similarly, resistance and leackage reactance of primary winding can be transferred to Secondary side.

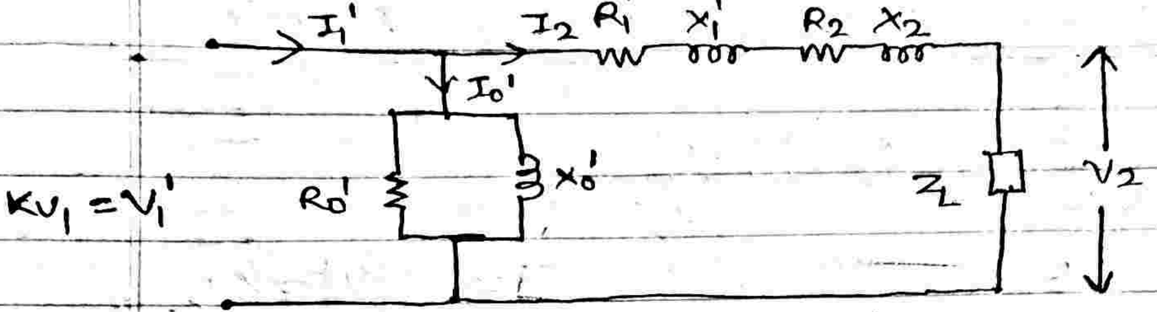


Fig Simplified eq. ckt diagram referred to Secondary side.

For heat loss to be same,

$$(I_2')^2 R_1 = I_2^2 R_2'$$

$$\left(\frac{I_2'}{I_2}\right)^2 = \left(\frac{R_1'}{R_1}\right)$$

$$\left(\frac{I_1}{I_2}\right)^2 = \left(\frac{R_1'}{R_1}\right) = k^2$$

$$\boxed{R_1' = k^2 R_1}$$

Similary,

$$\boxed{\begin{array}{c} x_1' = k^2 x_1 \\ R_0' = k^2 R_0 \\ x_0' = k^2 x_0 \end{array}}$$

Also, $R_{02} = R_2 + R' = R_2 + k^2 R$

= equivalent resistance of tr-er referred to Secondary side

$X_{02} = X_2 + X_1' = X_2 + k^2 X_1$
= equivalent leakage reactance of tr-er referred to secondary side.

$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}$ = eq. impedance

$$\frac{(V_1')^2}{R_0'} = \frac{(V_1)^2}{R_0}$$

$$\frac{V_1^2}{k^2 R_0} = \frac{V_1^2}{R_0} \Rightarrow V_1'^2 = k^2 V_1^2$$

$\boxed{V_1' = k V_1}$ primary voltage referred to secondary side.

$$I_0' = \frac{V_1'}{Z_0'} = \frac{V_1'}{R_0' + jX_0'} = \frac{kV_1}{k^2(R_0 + jX_0)} = I_0/k$$

$I_0' = I_0/k$ primary no load current refr. to secondary side.

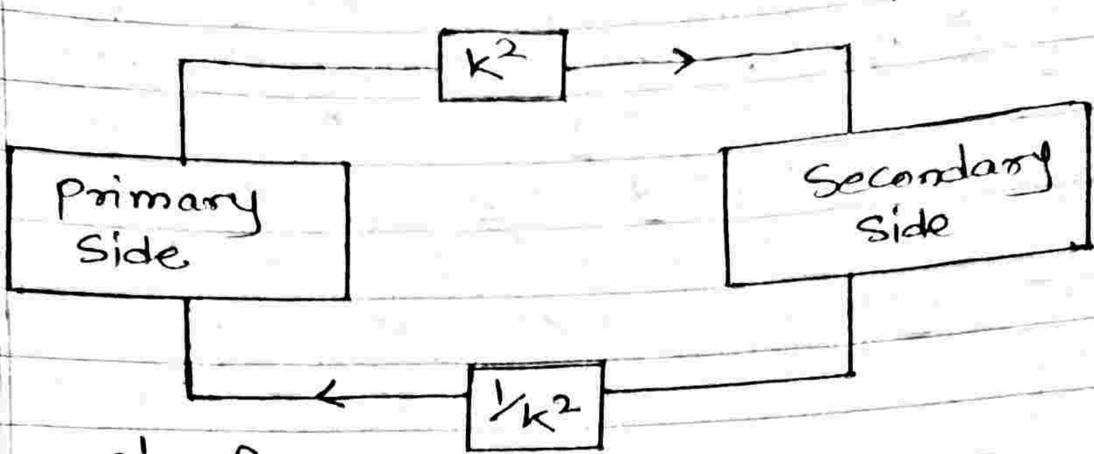
$$I_1' = I_0' + I_2 = \left(\frac{I_0}{k} + \frac{I_2'}{k}\right) = \frac{I_0 + I_2'}{k} = I_1/k$$

$I_1' = I_1/k$ primary current referred to secondary side.

Summary

1) For impedance R and X

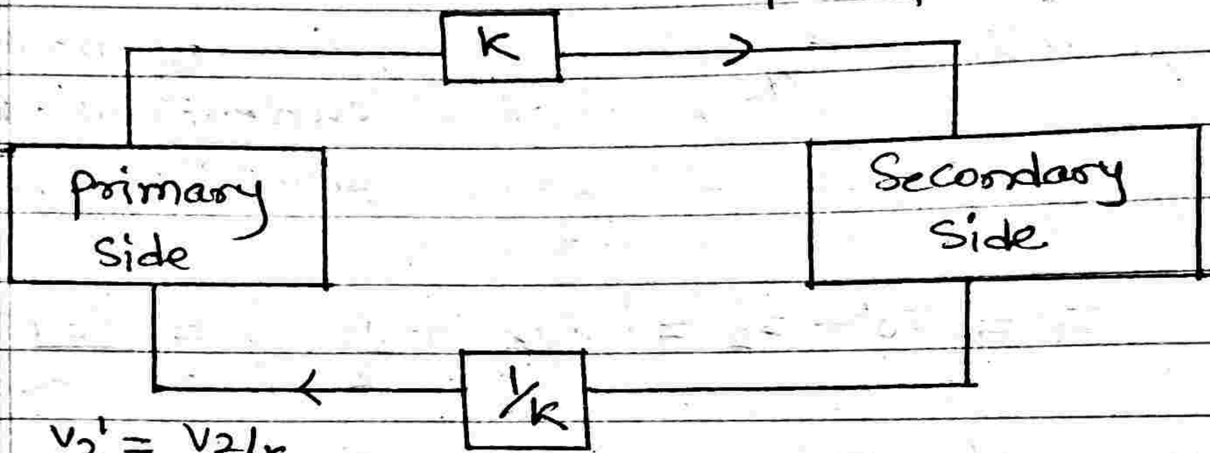
$R_1' = k^2 R_1$



$R_2' = \frac{R_2}{k^2}$

2) For voltage

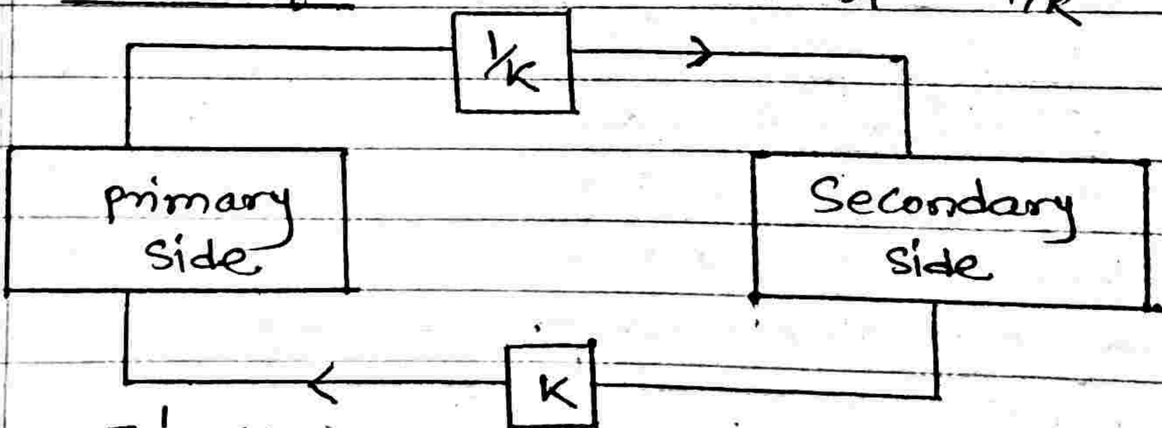
$V_1' = kV_1$



$V_2' = V_2/k$

3) Current

$I_1' = I_1/k$



$I_2' = kI_2$

Q1) A 25KVA, 2200/220V, 50Hz, 1-phase transformer has following parameters :

$R_1 = 1.75\ \Omega$, $R_2 = 0.0045\ \Omega$, $X_1 = 2.6\ \Omega$, $X_2 = 0.0075\ \Omega$

calculate : i) Equivalent resistance referred to primary and Secondary ii) Equivalent reactance referred to primary and secondary

⇒ Soln -

Transformation ratio $(K) = \frac{V_2}{V_1} = \frac{220}{2200} = 0.1$

i) Equivalent resistance referred to primary

$$R_{01} = R_1 + \frac{R_2}{K^2} = 1.75 + \frac{0.0045}{0.1} = 2.2\ \Omega$$

Equivalent resistance referred to secondary

$$R_{02} = K^2 R_1 + R_2 = (0.1)^2 * 1.75 + 0.0045$$
$$= 0.022\ \Omega$$

ii) Equivalent reactance referred to primary

$$X_{01} = X_1 + \frac{X_2}{K^2} = 2.6 + \frac{0.0075}{(0.1)^2} = 3.35\ \Omega$$

Equivalent reactance referred to secondary

$$X_{02} = K^2 X_1 + X_2 = (0.1)^2 * 2.6 + 0.0075$$
$$= 0.0335\ \Omega$$

Q.2) A 30KVA, 2000/200 V, Single - phase, 50Hz transformer has primary resistance of 3.5Ω and reactance of 4.5Ω. The secondary resistance and reactance are 0.015Ω and 0.02Ω respectively. Find i) equivalent resistance, reactance and impedance referred to the primary side ii) total copper loss in the transformer.

⇒ Soln -

Transformation ratio $(K) = V_2/V_1 = \frac{200}{2000} = 0.1$

Full load primary current $(I_1) = \frac{30 \times 1000}{2000}$

$= 15 A$

Equivalent resistance referred to primary

$$R_{01} = R_1 + \frac{R_2}{K^2} = 3.5 + \frac{0.015}{(0.1)^2} = 5 \Omega$$

Equivalent reactance referred to primary

$$x_{01} = x_1 + \frac{x_2}{K^2} = 4.5 + \frac{0.02}{(0.1)^2} = 6.5 \Omega$$

Equivalent Impedance referred to primary

$$Z_{01} = \sqrt{(R_{01})^2 + (X_{01})^2}$$

$$= \sqrt{5^2 + 6.5^2}$$

$$= 8.2 \Omega$$

Total copper loss in tr-er $= I_1^2 R_{01}$

$= 15^2 * 5 = 1125$ W

Testing of transformer

1) Polarity testing -

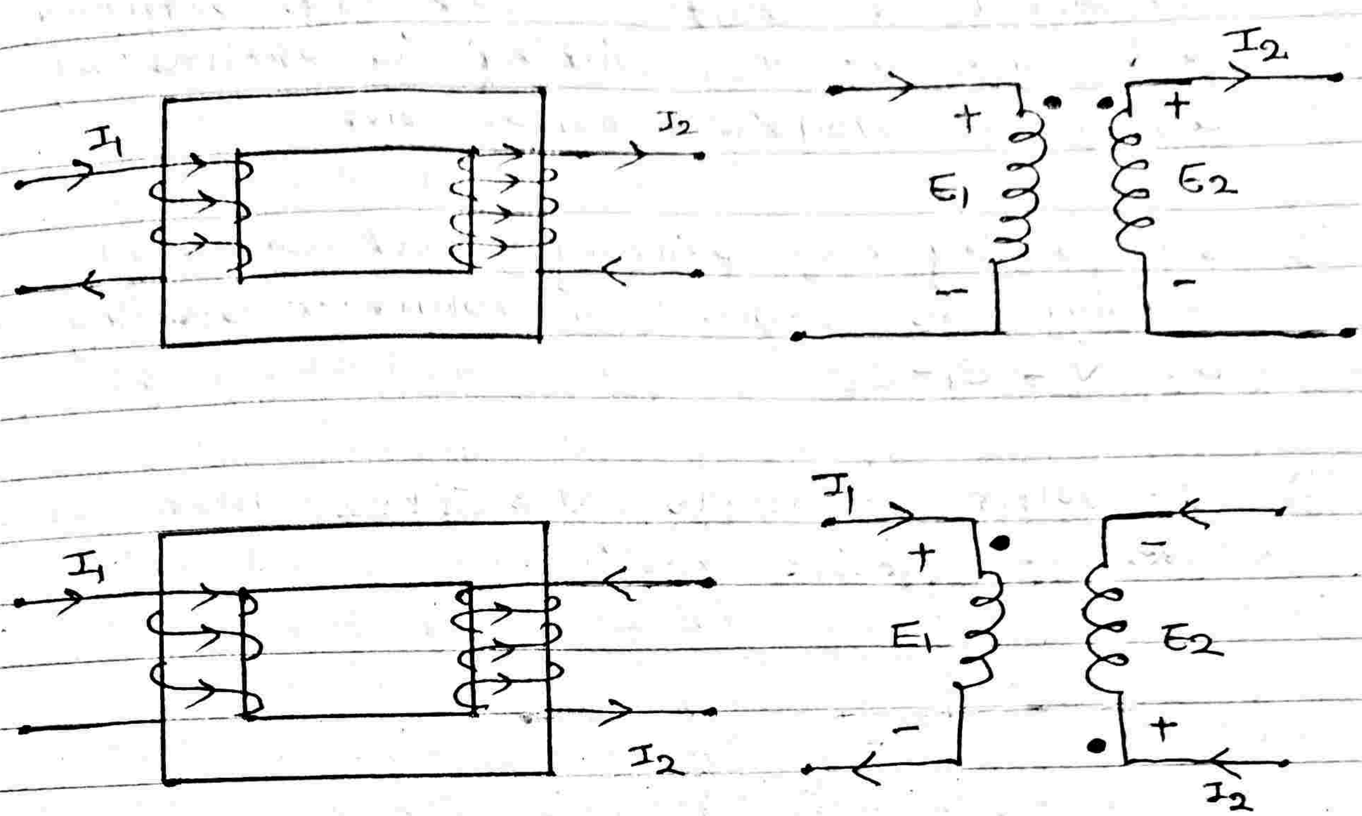


Fig polarity of transformer windings

Dot convention

positive terminal → dot (•) -

p.s → current enters at dot (•)

s.s → current leaves from dot (•)

⇒ polarity test are performed to determine the terminals having same instantaneous polarity.

usually polarity is indicated by the dot convention as shown in fig above. Similar polarity ends of a two winding transformer are those ends that acquire simultaneously positive or negative polarity because of emf induced in them.

In polarity test, the two windings are Connected in series across the voltmeter and one of the winding is excited by a Suitable voltage source and

i) If polarity of primary and Secondary winding is Same then voltmeter reading will be $V = E_1 - E_2$

ii) If voltmeter reads $V = E_1 + E_2$ then windings are of opposite polarity.

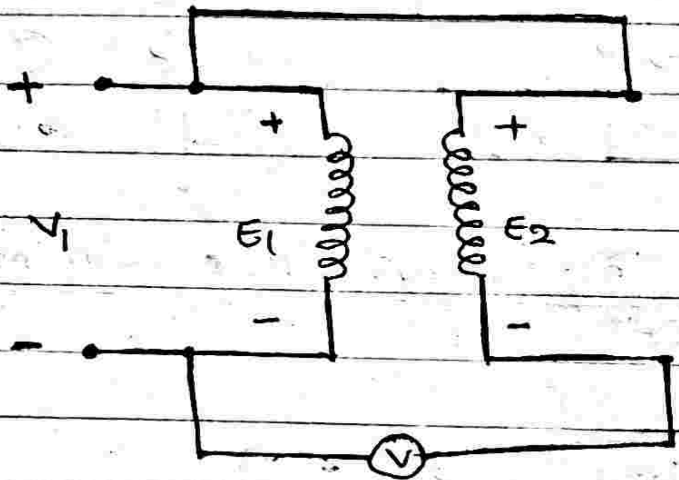


Fig. polarity test

2) Open circuit and short-circuit tests -

open circuit and short circuit tests are performed to determine the circuit constants, efficiency and regulation without actually loading transformer. These tests give more accurate results than those obtained by taking measurements on fully loaded transformer. Also, the power consumption in these tests is very small compared with the full - load output of the transformer.

a) open - circuit test (No load test) -

→ This test is performed on L.V side

→ It is done at rated voltage and frequency.

L.V	H.V
V ↓ I ↑	V ↑ I ↓

No load current is (2-5)% of full load current

→ The main purpose of this test is to obtain:

i) Shunt branch parameters of the eq. ckt

ii) No load power loss.

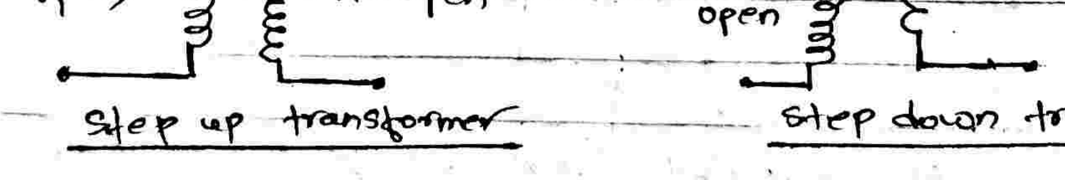
iii) No load current

iv) No load power factor

Procedure :

i) The high voltage winding is kept open and low voltage winding is supplied by rated voltage.

ii) Ammeter, Voltmeter and wattmeter are connected as shown in figure.



Step up transformer

Step down transformer

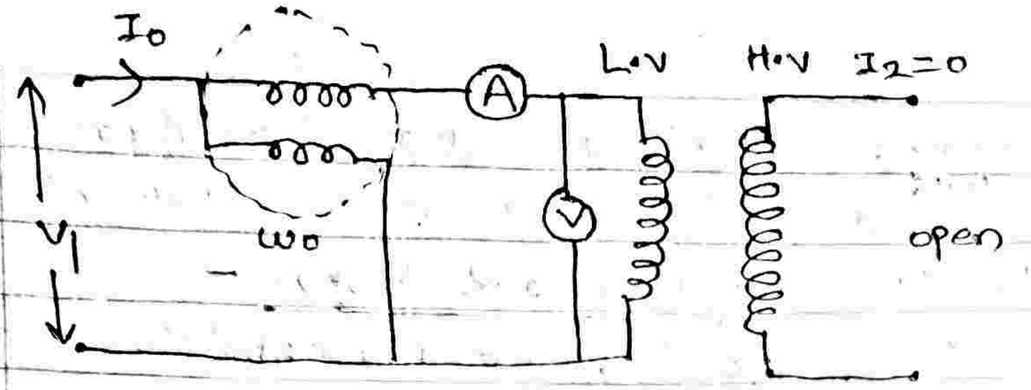


Fig open circuit test on transformer

Let V_1 = voltmeter reading (rated voltage)
I_0 = Ammeter reading (no load current)
W_0 = wattmeter reading (no load power loss)
↓
iron loss

As the no load current is very small with compare to full load current and the series resistance and reactance R_1 and x_1 are also small. copper loss at no load can be neglected and the wattmeter reading can be considered as no load power loss or iron loss of the transformer. The equivalent ckt at no load condition can be realised as shown in fig below:

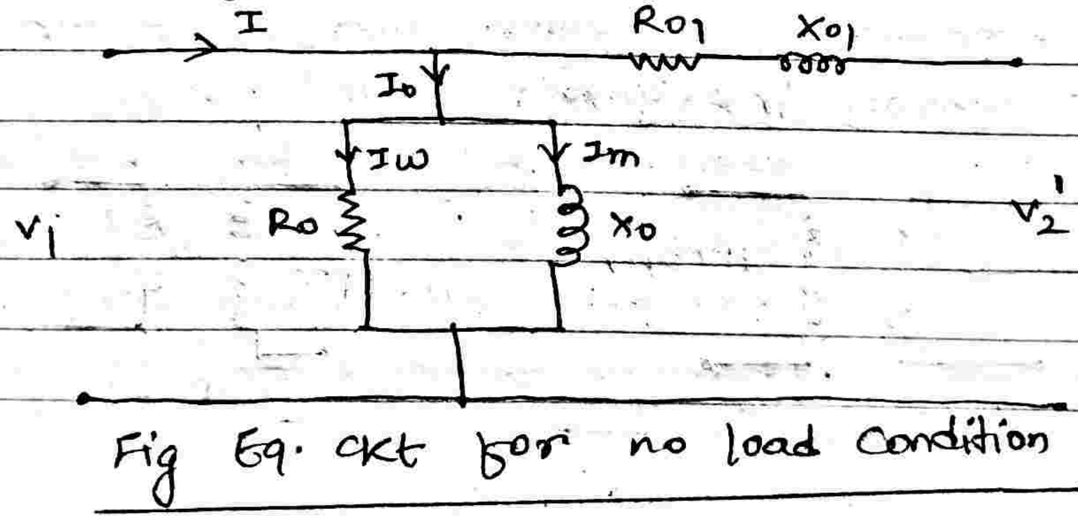


Fig Eq. ckt for no load condition

$$W_0 = V_1 I_0 \cos\phi_0$$

$\cos\phi_0$ = No load power factor

$$I_w = I_0 \cos\phi_0$$

$$I_m = I_0 \sin\phi_0$$

$$R_0 = \frac{V_1}{I_w} = \frac{V_1}{I_0 \cos\phi_0}$$

$$X_0 = \frac{V_1}{I_m} = \frac{V_1}{I_0 \sin\phi_0}$$

Hence R_0 and X_0 are obtained in above analysis

b) **Short circuit test (Impedance test)** –

→ This test is performed on H.V side.

→ The main purpose of this test is to obtain the series resistance and leakage reactance and copper loss of transformer at full load.

Procedure:

i) In this test (LV) winding is shorted (Short circuited) by a thick wire and variable voltage is applied to H.V winding. The low input voltage is gradually increased till at voltage (V_{sc}), full load current I_1, flows in the primary. Then I_2 in the secondary also has full load value since $\left(I_1/I_2 = \frac{N_2}{N_1}\right)$

There is no output from transformer under short circuit conditions, and therefore input power is all loss and this loss is entirely copper loss and iron loss is negligible in this case because vsc is very small.

ii) Ammeter, voltmeter and wattmeter are connected as shown.

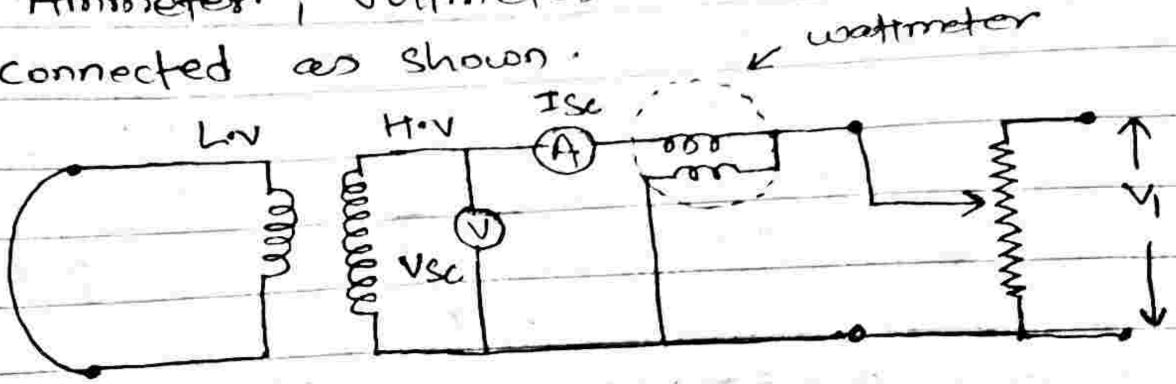


Fig Short circuit test

The equivalent circuit of the transformer for short circuit test is shown below:

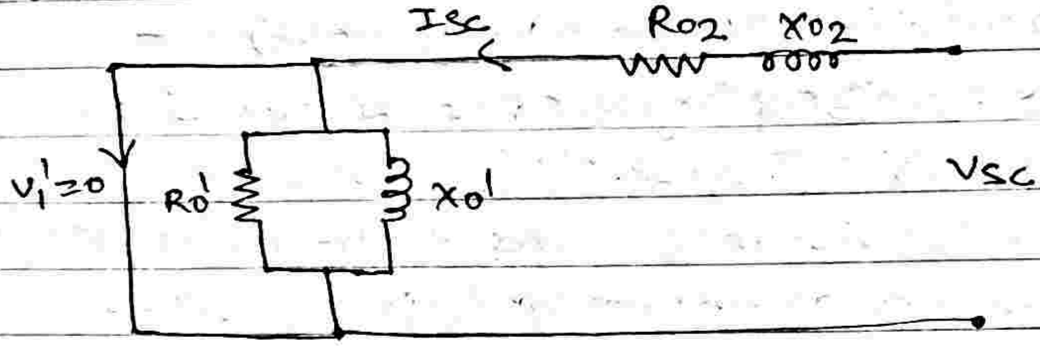


Fig. Eq. ckt for short circuit test

Now we take readings of ammeter, voltmeter and wattmeter as,

I_{sc} = ammeter reading
V_{sc} = voltmeter reading
W_{sc} = wattmeter reading

Here, shunt branch is shorted and no current flows through shunt branch

$$W_{sc} = I_{sc}^2 R_{02}$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2}$$

also,

$$Z_{02} = \frac{V_{sc}}{I_{sc}}$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2}$$

Hence parameters of equivalent circuit can be calculated.

Q) Why short circuit test is performed on H.V side?

⇒ The reasons for shorting L.V side and taking measurements on the hv side are as follows:

i) The rated current on hv side is lower than that on L.V side. This current can be safely measured with the available laboratory ammeters.

ii) Since the applied voltage is less than 5% of the rated voltage of the winding, greater accuracy in the reading of the voltmeter is possible when the hv side is used as the primary.

Q.1) A 50KVA, 2200/110 transformer when tested gave the following results :

OC test : 400W 10A 110V

SC test : 808W 20.5A 90V

Compute all the parameters of the equivalent ckt referred to H.V side and L.V side of the transformer.

⇒ Soln -

Here,

$S = 50KVA$

$V_1 = 2200V$ $V_2 = 110V$

H.V → primary L.V → Secondary

Now, <u>OC test</u>

$W_0 = 400W$ $I_0 = 10A$ $V_1 = 110V$

<u>SC test</u>

$W_{SC} = 808W$ $I_{SC} = 20.5A$ $V_{SC} = 90V$

From O.C test :-

Referred to L.V side

$$\cos\phi_0 = \frac{W_0}{V_1 I_0} = \frac{400}{110*10} = 0.36$$

$$R_0 = \frac{V_1}{I_0\cos\phi_0} = \frac{110}{10*0.36} = 30.55\,\Omega$$

$$X_0 = \frac{V_1}{I_0\sin\phi_0} = \frac{110}{10*\sqrt{1-0.36^2}} = 11.79$$

Referred to H.V side

$$R_0^1 = \frac{R_0}{K^2} = \frac{30.55}{0.05^2} = 12220\,\Omega$$

$$X_0^1 = \frac{X_0}{K^2} = \frac{11.79}{0.05^2} = 4720\,\Omega$$

From S.C test,

Referred to H.V side,

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{808}{(20.5)^2} = 1.922\,\Omega$$

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{90}{20.5} = 4.39\,\Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{4.39^2 - 1.922^2}$$

$$X_{01} = 3.946\,\Omega$$

Referred to L.V side,

$$R_{02} = 1.922 * (0.05)^2 = 4.805 * 10^{-3}\,\Omega$$

$$X_{02} = 3.946 * (0.05)^2 = 9.865 * 10^{-3}\,\Omega$$

Q.3) A 20kVA, 250/2500 V, 50Hz, 1-ϕ transformer gave the following test results:

OC (LV side)	250V	1.4A	105W
SC (HV side)	120V	8A	320W

Find the parameters and draw the equivalent circuit of 1-ϕ transformer as referred to:
i) Low voltage side ii) High voltage side

$\Rightarrow$ Soln -

we have given,

$S = 20kVA$

$V_1 = 250V \quad V_2 = 2500V$

OC (LV side) test

$V_1 = 250V \quad I_0 = 1.4A \quad W_0 = 105W$

SC (HV side) test

$V_{SC} = 120V \quad I_{SC} = 8A \quad W_{SC} = 320W.$

From OC test

referred to L.V side,

$$\cos\phi_0 = \frac{W_0}{V_1 I_0} = \frac{105}{250 * 1.4} = 0.3$$

$$R_0 = \frac{V_1}{I_0 \cos\phi_0} = \frac{250}{1.4 * 0.3} = 595.23\Omega$$

$$X_0 = \frac{V_1}{I_0 \sin\phi_0} = \frac{250}{1.4 * \sqrt{1-0.3^2}} = 187.19\Omega$$

No load resistance and reactance referred to hv side

$$R_0' = K^2 R_0 = \left(\frac{2500}{250}\right)^2 * 595.23 = 59523\Omega$$

$$X_0' = K^2 X_0 = (10)^2 * 187.19 = 18719\Omega$$

From s.c test,

Equivalent impedance referred to hv side,

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{120}{8} = 15\,\Omega$$

Equivalent resistance referred to hv side,

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{320}{8^2} = 5\,\Omega$$

Equivalent reactance referred to hv side,

$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2} = \sqrt{15^2 - 5^2} = 14.14\,\Omega$$

Equivalent resistance and reactance referred to L.V side,

$$R_{01} = \frac{R_{02}}{K^2} = \frac{5}{10^2} = 0.05\,\Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{14.14}{10^2} = 0.1414\,\Omega$$

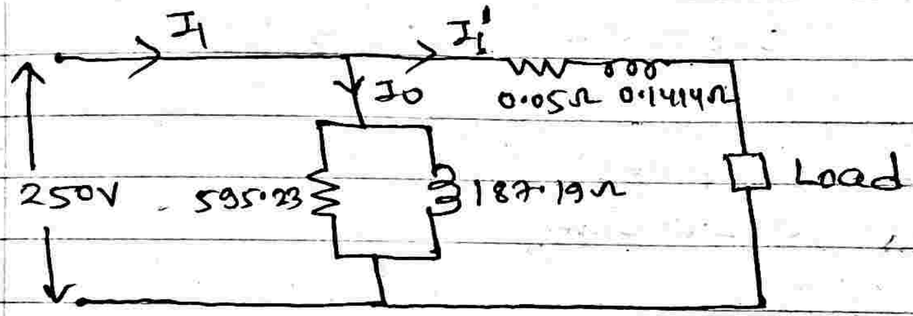


Fig. Eq. ckt referred to LV side (primary side)

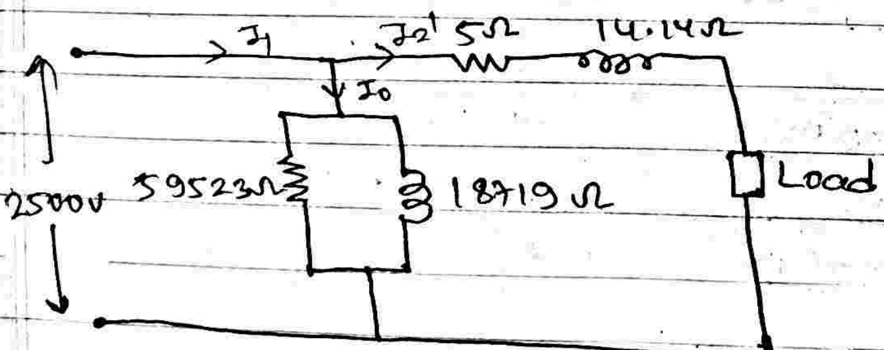


Fig. Eq. ckt referred to H.V side (Secondary side)

Q.3) A 20KVA, 2500/250V, 50Hz, single-phase transformer gave the following test results:
Open-circuit test (LV side) - 250V, 1.4A, 105W
Short-circuit test (HV side) - 104V, 8A, 320W.
Compute the parameters of the equivalent circuit referred to high-voltage and low-voltage sides. Also draw the exact equivalent circuit referred to low-voltage side.

⇒ Soln-

we have given,

$S = 20KVA$

$V_1 = 2500V \quad V_2 = 250V$

open-circuit test (LV side)

$V_1 = 250V \quad I_o = 1.4A \quad W_o = 105W.$

Short circuit test (HV side)

$V_{sc} = 104V \quad I_{sc} = 8A \quad W_{sc} = 320W.$

From open circuit test, referred to LV side,

$$\cos\phi_o = \frac{W_o}{V_1 I_o} = \frac{105}{250 * 1.4} = 0.3$$

$$R_o = \frac{V_1}{I_o \cos\phi_o} = \frac{250}{1.4 * 0.3} = 595.23\,\Omega$$

$$X_o = \frac{V_1}{I_o \sin\phi_o} = \frac{250}{1.4 * \sqrt{1-0.3^2}} = 187.19\,\Omega$$

No load resistance and reactance referred to hv side.

$$R_0' = \frac{R_0}{K^2} = \frac{595.23}{\left(\frac{250}{2500}\right)^2} = 59523\,\Omega$$

$$X_0' = \frac{X_0}{K^2} = \frac{187.19}{(0.1)^2} = 18719\,\Omega$$

From short circuit test (HV),

Equivalent impedance referred to hv side,

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{104}{8} = 13\,\Omega$$

Equivalent resistance referred to hv side,

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{320}{8^2} = 5\,\Omega$$

Equivalent resistance and reactance referred to Lv side,

$$R_{01} = K^2 R_{02} = (0.1)^2 * 5 = 0.05\,\Omega$$

$$X_{01} = (0.1)^2 X_{02} = (0.1)^2 * 13 = 0.13\,\Omega$$

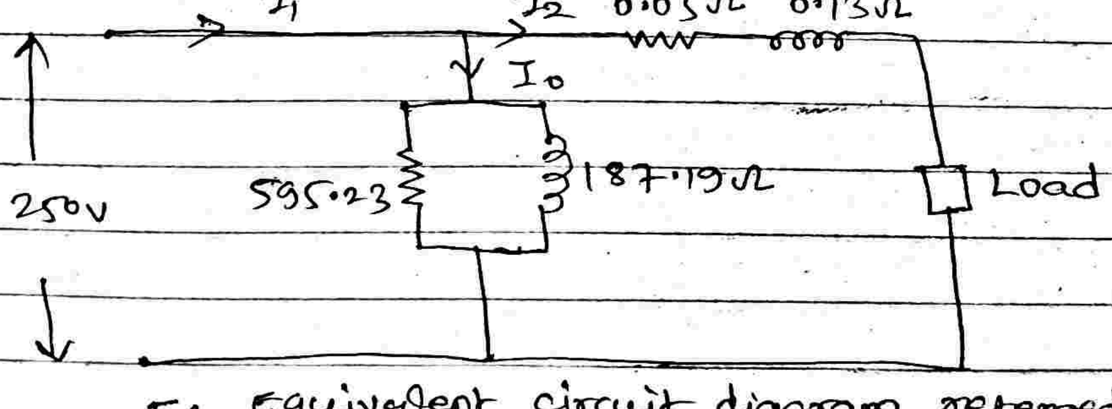


Fig. Equivalent circuit diagram referred to L.V side

Losses and Efficiency of transformer

The transformer has no moving parts so that its efficiency is much higher than rotating machines.

Usually there are no such devices, which are 100% efficient. Similarly power loss takes place within the transformer, while it transfers power from one circuit to another circuit.

The various power losses occuring in a transformer are –

1) **Iron or Core loss** – Iron loss is caused by the alternating flux in the core and consists of hysteresis and eddy current losses.

a) Hysteresis loss – The core of a transformer is subjected to an alternating magnetising force and for each cycle of emf, a hysteresis loop is traced out. The hysteresis loss per second is given by –

$$\boxed{\text{Hysteresis loss}(P_h) = \eta (B_{max})^x f v \text{ joules per sec or watt}}$$

where, f is supply frequency

v is the volume of core in cubic metres.

η is the hysteresis coefficient

B_{max} is peak value of flux density in the core and x lies between 1.5 and 2.5 depending upon material and is often taken as 1.6.

b) **Eddy current loss** - If the magnetic circuit is made up of iron and if the flux in the circuit is variable, current will be induced by induction in the iron circuit itself. All such currents are known as eddy currents.

$$\boxed{P_e = K_e (B_{max})^2 f^2 t^2 V \text{ watts or joules/sec}}$$

Hysteresis and eddy current loss depend upon the maximum flux density in the core and supply frequency. Since it has determined that the mutual flux varies somewhat with load i.e (1 to 3% from no load to full load), the core losses will vary somewhat with load and its power factor. This loss is assumed to be constant from no load to full load being very small and negligible.

$$\boxed{\begin{aligned} \text{Iron loss (Pi) or core loss} &= \text{Hysteresis loss} + \text{Eddy current loss} \\ &= \text{Constant losses} \end{aligned}}$$

Note -

i) Hysteresis loss can be minimised by using steel of silicon content for core.

whereas eddy current loss are minimised by using thin lamination (0.3 to 0.5 mm) (iron laminated).

2) copper loss - These losses occur in both the primary and secondary windings due to their ohmic resistance. These can be determined by short-circuit test.

$$\boxed{\begin{aligned} \text{Total cu losses } (P_c) &= I_1^2 R_1 + I_2^2 R_2 \\ &= I_1^2 R_{01} \text{ or } I_2^2 R_{02} \end{aligned}}$$

$$\boxed{\begin{aligned} \text{Total losses in a transformer } &= P_i + P_c \\ &= \text{constant losses} + \\ &\quad \text{variable losses} \end{aligned}}$$

3) Stray load loss - leakage fields presents in the transformers include eddy currents in the conductors, tank wall, bolts etc. These eddy current loss gives rise to stray load loss

4) Dielectric loss - This loss occur in the insulating materials particularly in oil and solid materials

⇒ These stray load loss and dielectric loss are very small and are neglected. Thus major losses are by far the first two losses i.e. iron loss or coreloss (constant loss) and copper loss (variable loss).

Efficiency of transformer

The efficiency (η) of a transformer is defined as the ratio of output power to the input power, the two measured in same units.

$$\eta = \frac{\text{output power } (P_{out})}{\text{Input power } (P_{in})} * 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + \text{losses}} * 100\%$$

output power $(P_{out}) = V_2 I_2 \cos\phi_2$ watts

$\phi_2 \rightarrow$ phase angle between V_2 and I_2

Input power $(P_{in}) = V_1 I_1 \cos\phi_1$ watts

$\phi_1 \rightarrow$ Phase angle between V_1 and I_1.

$$P_{out} = P_{in} - \text{losses}$$

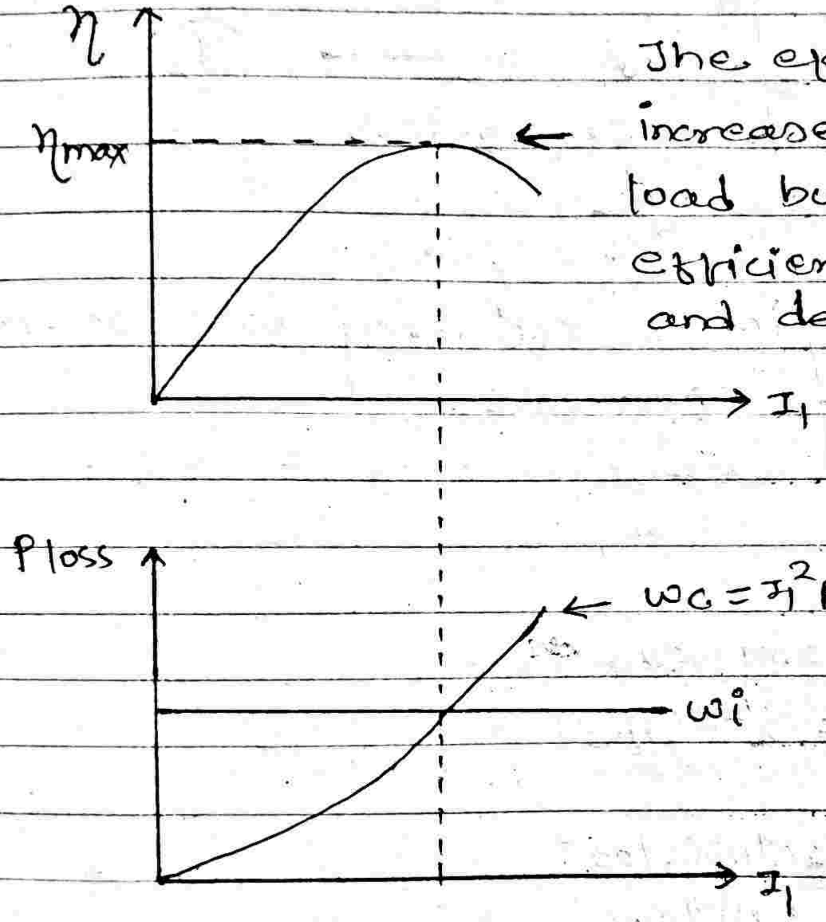
$$= P_{in} - \text{Iron loss} - \text{copper loss}$$

$$P_{out} = P_{in} - W_i - I_1^2 R_{01}$$

$$\eta = \frac{V_1 I_1 \cos\phi_1 - W_i - I_1^2 R_{01}}{V_1 I_1 \cos\phi_1} * 100\%$$

$$\eta = \frac{V_1 I_1 \cos\phi_1 - w_i - I_1 R_{01}}{V_1 I_1 \cos\phi_1} \quad \text{--- (i)}$$

From eqn (i), it is clear that the efficiency depends upon the load current. If we plot a curve showing the efficiency of transformer at different values of load current, the curve will be as shown below :-



The efficiency of a tr-er increases with increase in load but after certain load, efficiency becomes maximum and decreases further increase in load.

Fig. Efficiency and power loss curve of transformer

Eqn (i) can be written as,

$$\eta = 1 - \frac{w_i}{V_1 I_1 \cos\phi_1} - \frac{I_1 R_{01}}{V_1 \cos\phi_1}$$

Differentiating w.r.t. to I_1

$$\frac{d\eta}{dI_1} = 0 + \frac{w_i}{V_1 I_1^2 \cos\phi_1} - \frac{R_{01}}{V_1 \cos\phi_1}$$

The efficiency (η) will be maximum when

$$\frac{d\eta}{dI_1} = 0$$

i.e. $\frac{w_i}{V_1 I_1^2 \cos\phi_1} - \frac{R_{01}}{V_1 \cos\phi_1} = 0$

or, $w_i = I_1^2 R_{01}$ —— (ii)

$\therefore$ Iron loss = Copper loss

$\therefore$ $I_1 = \sqrt{\frac{w_i}{R_{01}}}$

Hence, maximum efficiency will occur when iron loss = copper loss.

Note :-

1) Copper at any load

$w_{cu}(x) = x^2 w_{cu}(f)$

where,

$$x = \frac{\text{actual load}}{\text{full load}}$$

2) Above eqn (ii) can be written as

$$w_i = x^2 w_c$$

$$x = \sqrt{\frac{w_i}{w_c}} = \sqrt{\frac{P_i}{P_c}}$$

x is fraction of full load KVA at which efficiency is maximum.

Q.1) calculate the efficiency at half - full load of a 100 KVA transformer for pf of unity and 0.8. The copper loss is 1000w at full load and iron loss is 1000w.

=> soln -

we have given,

Iron loss at full load (P_i) = 1000w

copper loss at full load = 1000w

copper loss at half load = $x^2 P_{cu(f)}$

$= \left(\frac{1}{2}\right)^2 * 1000$

$P_{cu}(x) = \frac{1}{4} * 1000 = 250\ w$

Transformer output on half-full load and unity pf

$P_{out_1} = \frac{1}{2} * 100 * 1 = 50\ kw$

Transformer output on half-full load and 0.8 pf.

$P_{out_2} = \frac{1}{2} * 100 * 0.8 = 40 kw.$

Transformer efficiency (η_1) $= \dfrac{P_{out_1}}{P_{out_1} + P_C + P_i} * 100\ \%$

$= \dfrac{50}{50 + \frac{250}{1000} + \frac{1000}{1000}} * 100\ \%$

$= \dfrac{50}{50 + 0.25 + 1} * 100\ \%$

$\eta_1 = 0.9756 * 100\ \%$

$\eta_1 = 97.56\ \%$

$$\eta_2 = \frac{Pout_2}{Pout_2 + Pc + Pi} * 100\%$$

$$= \frac{40}{40 + 0.25 + 1} * 100\%$$

$$\eta_2 = 0.969\ \%$$

Q.2) A 30 kVA, single phase transformer has an iron loss of 457 and copper loss of 125 W when delivering half - full load. At what percent of the full load will the transformer have maximum efficiency?

⇒ Soln - we have given,

Iron loss = 457 W

copper loss at half-full load = 125 W

copper loss at full load,

we have,

$$W_{cu}(x) = x^2\, W_{cu}(f)$$

$$W_{cu}(f) = \frac{W_{cu}(x)}{x^2}$$

$$= \frac{W_{cu}(\frac{1}{2})}{(\frac{1}{2})^2} = \frac{125}{(\frac{1}{2})^2}$$

$$W_{cu}(f) = 500\,W$$

Let the transformer maximum efficiency occur at x % of full load,

$$\frac{x}{100} = \sqrt{\frac{Pi}{Pc}}$$

$$x = \sqrt{\frac{Pi}{Pc}} * 100\%$$

$$\therefore x = \sqrt{\frac{457}{500}} * 100\%$$

$$\therefore x = 95.6\%$$

Q.3) A single-phase transformer working at unity pf has an efficiency of 90% on both half load and at full load of 500 kW. Determine i) iron loss ii) full-load copper loss iii) efficiency at 75% of full load iv) maximum efficiency.

⇒ Soln -

we have given,

At full load and unity pf.

power output (P_{out}) = 500 kW

efficiency (η) = 90% = 0.9

power input (P_{in}) = $\frac{P_{out}}{\eta} = \frac{500}{0.9} = 555.5$ kW

Total loss = $P_i + P_c = P_{in} - P_{out} = 555.5 - 500$

$P_i + P_c = 55.5$ kW — 1

At half full load and unity pf

power output (P'_{out}) = $\frac{1}{2} * 500 = 250$ kW

Power input (P'_{in}) = $\frac{P'_{out}}{\eta} = \frac{250}{0.9} = 277.8$ kW

Total loss = $P_i + (\frac{1}{2})^2 P_c = P'_{in} - P'_{out} = 277.8 - 250$

$P_i + \frac{1}{4} P_c = 27.8$ kW — 2

Solving eqn(1) and eqn(2) we get

$P_i = 18.6$ kw

$P_c = 36.9$ kw

i) iron loss = 18.6 kw

ii) full load copper loss = 36.9 kw.

iii) Efficiency at 75% of full load.

$\Rightarrow$

$$P_{out} = \frac{3}{4} * 500 = 375 \text{ kw.}$$

$$\text{total losses} = P_i + \left(\frac{3}{4}\right)^2 P_c$$

$$= 18.6 + \left(\frac{3}{4}\right)^2 * 36.9$$

$$= 39.36 \text{ kw.}$$

$$\eta = \frac{P_{out}}{P_{out} + P_i + P_c} = \frac{375}{375 + 39.36}$$

$$\eta = 90.5\%$$

iv) maximum efficiency.

$\Rightarrow$

$$x = \sqrt{\frac{P_i}{P_c}} = \sqrt{\frac{18.6}{36.9}} = 0.7$$

output power at maximum efficiency

$$(P_{out})_{\eta max} = 0.7 * 500 = 354.98$$

$$= 355 \text{ kw}$$

$$\text{Efficiency } (\eta)_{max} = \frac{(P_{out})_{\eta max}}{(P_{out})_{\eta max} + 2 * \text{iron loss}}$$

[Iron loss = copper loss]

$$\eta_{max} = \frac{355}{355 + 2 * 18.6} * 100\%$$

$$\eta_{max} = 90.515\%$$

All day efficiency

→ we call it energy efficiency also.

→ There are certain types of transformers whose performance cannot be judge by ordinary or commerical efficiency. For instance, distribution transformers are energised for 24 hours, but they deliver very light loads for major portion of the day. Thus iron loss or core loss occur for the whole day but copper loss occurs only when the transformer is loaded.

→ The performance of such a transformer must be judge by its all-day efficiency, also called energy efficiency or operational efficiency which is computed on the basis of energy consumed during the whole day (24hours)

→ The all-day efficiency is defined as ratio of energy (kwh) output over 24 hrs to the energy input over the same period. i.e.

$$\boxed{\text{All-day efficiency} = \frac{\text{output in kwh}}{\text{Input in kwh}}}$$

→ Since distribution transformer does not supply the rated load for whole day so all-day efficiency of such transformer will be lesser than ordinary or commercial efficiency.

→ For determination of all-day efficiency of a transformer, it is necessary, ofcourse, to know how the load varies from hour to hour during the day.

Voltage Regulation

When a transformer is loaded with a constant primary voltage, secondary voltage varies because of voltage drop in its internal resistance and leakage reactance. The quality of a transformer from the point of view of voltage drop is expressed in terms of a quantity known as the voltage regulation. The voltage regulation of a transformer is thus defined as the change in the magnitude of the output voltage from full load to no load expressed usually as the % of full load voltage or No-load voltage.

$$\boxed{\begin{aligned} V_{reg} &= \frac{OV_2 - fV_2}{fV_2} * 100\% \Rightarrow \text{reg. up} \\[2ex] V_{reg} &= \frac{OV_2 - fV_2}{OV_2} * 100\% \Rightarrow \text{reg. down} \end{aligned}}$$

OV_2 = No-load terminal voltage (Secondary)
fV_2 = full load

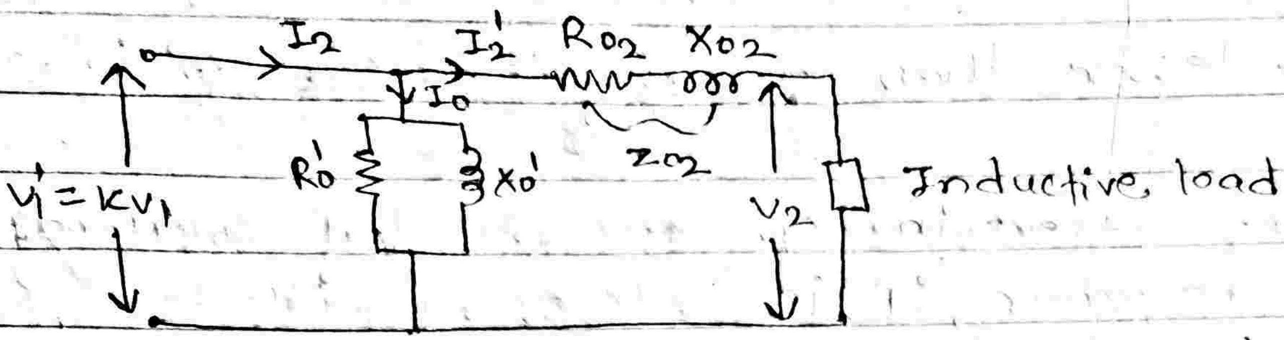


Fig (i) Equivalent ckt referred to secondary side

$\boxed{I_2' \simeq I_2}$

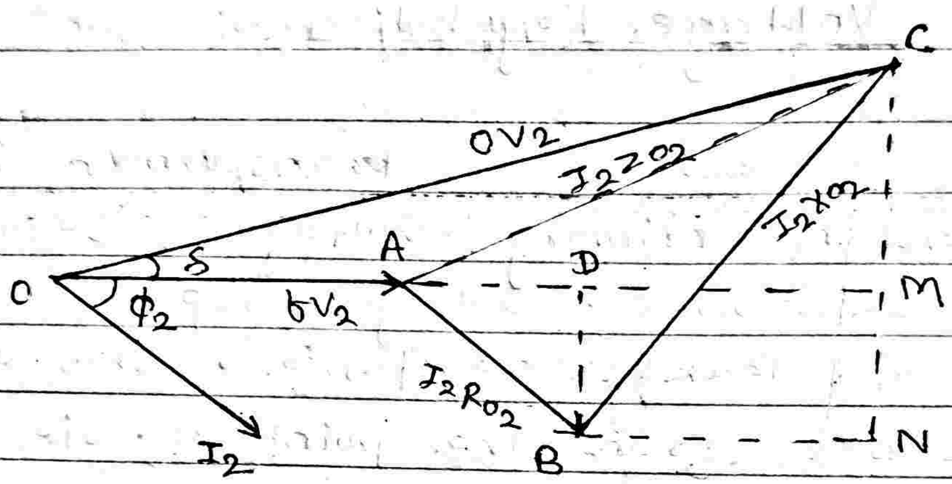


Fig (ii) phasor diagram of equivalent ckt referred to secondary side.

Here, the total voltage drop $= I_2Z_{02} = AC$

Assume that the angle δ is very small then

$AC \approx AM$

or $AC = AD + DM$

or $AC = AD + BN = I_2R_{02}\cos\phi_2 + I_2X_{02}\sin\phi_2$

From the phasor diagram it is clear that,

$OV_2 - \not{b}V_2 = AC$

Voltage regulation $= \dfrac{OV_2 - \not{b}V_2}{\not{b}V_2}$

$$V_{reg} = \frac{I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2}{{}_{f}V_2}$$

$$= \frac{I_2 R_{02} \cos\phi_2}{{}_{f}V_2} + \frac{I_2 X_{02} \sin\phi_2}{{}_{f}V_2}$$

$$V_{reg} = R(pu)\cos\phi_2 + X(pu)\sin\phi_2$$

where,

$$R(pu) = \frac{I_2 R_{02}}{{}_{f}V_2}$$ = percentage resistance of the transformer

$$X(pu) = \frac{I_2 X_{02}}{{}_{f}V_2}$$ = percentage reactance of the transformer

$$Z_{pu} = \sqrt{R(pu)^2 + X_{pu}^2}$$ is known as percentage impedance.

when the load is capacitive (pf is leading),

$$V_{reg} = R(pu)\cos\phi_2 - X(pu)\sin\phi_2$$

Condition for zero voltage regulation

$$V_{reg} = \frac{{}_{0}V_2 - {}_{f}V_2}{{}_{f}V_2}$$

→ V_{reg} can be zero when ${}_{0}V_2 - {}_{f}V_2 = 0$ and this can be possible only when load is leading

$$V_{reg} = 0$$

$$R(pu)\cos\phi_2 - X_{pu}\sin\phi_2 = 0$$

$$\boxed{\tan\phi_2 = \frac{R_{pu}}{X_{pu}} = \frac{R_{02}}{X_{02}}}$$

Condition for maximum voltage regulation

$$V_{reg} = \frac{{}_0V_2 - {}_fV_2}{{}_fV_2}$$

⟹ Vreg. can be max^m when ${}_0V_2 - {}_fV_2 = $ max^m and this can be possible only when load is lagging.

$$\frac{d}{d\phi_2}\left(R_{pu}\cos\phi_2 + X_{pu}\sin\phi_2\right) = 0$$

$$-R_{pu}\sin\phi_2 + X_{pu}\cos\phi_2 = 0$$

$$\boxed{\tan\phi_2 = \frac{X_{pu}}{R_{pu}} = \frac{X_{02}}{R_{02}}}$$

Q) In a 500KVA, 3300/500V, 50Hz single-phase transformer, the maximum efficiency of 98% occurs at 75% of load at a pf of unity. Calculate the voltage Regulation at full load and 0.8 pf lagging if the impedance is 0.1.

⟹ Soln –

At 3/4 full load and pf of unity.

$$\text{power output } (P_{out}) = 3/4 * 500 * 1 = 375\,kW$$

$$\text{maximum efficiency } (\eta_{max}) = 98\% = 0.98$$

$$\text{power input } (P_{in}) = \frac{P_{out}}{\eta} = 382.65\,kW$$

$$\text{Total loss} = P.\text{Input} - P\text{output}$$

$$\text{Total loss} = 382.65 - 375 = 7.65\,kW$$

This loss is equally divided between iron and copper loss so

$$\left(\frac{3}{4}\right)^2 P_c = P_i = \frac{7.65}{2} = 3.82 \text{ kW}$$

$$P_c = \left(\frac{4}{3}\right)^2 * 3.825 = 6.8 \text{ kW}$$

$$\text{Percentage resistance} = \frac{I_2 R_{02}}{V_2} * 100$$

$$= \frac{I_2^2 R_{02}}{I_2 V_2} * 100$$

$$= \frac{6.8}{500} * 100 = 1.36\ \%$$

$$\text{percentage impedance} = 100 * 0.1 = 10\ \%$$

$$\text{percentage reactance} = \sqrt{10^2 - 1.36^2} = 9.907\ \%$$

$$\text{Percentage Regulation} = \frac{I_2 R_{02} \cos\phi_2 + I_2 X_{02} \sin\phi_2}{V_2} * 100$$

$$= \left[\frac{I_2 R_{02}}{V_2} \cos\phi_2 + \frac{I_2 X_{02}}{V_2} \sin\phi_2\right] * 100$$

$$= [1.36 * 0.8 + 9.907 * 0.6] * 100$$

$$= 7.0322\ \%$$

Instrument transformer

Instrument transformer are high accuracy class electrical devices used to isolate or transform voltage or current levels. The most common uses of instrument transformer is to operate instruments or metering from high voltage or high current circuits and safely isolating secondary control circcuitary from the high voltage or high currents. The primary winding of the transformer is connected to high voltage or high current circuit and meter or relay is connected to the secondary circuits.

Instrument transformers are of two types and they are :-

1) current transformer (CT)
2) potential transformer (PT)

1) current transformer (CT) – A current transformer is a device which is used for transformation of current at a higher value to a lower value with respect to the earth potential, It is used with AC instruments for measuring the high value of currents.

→ The current is too high, and it is very difficult to measure them directly thus the current transformer is used which decreases the high value of current into fractional value which is easy to measure by the instruements.

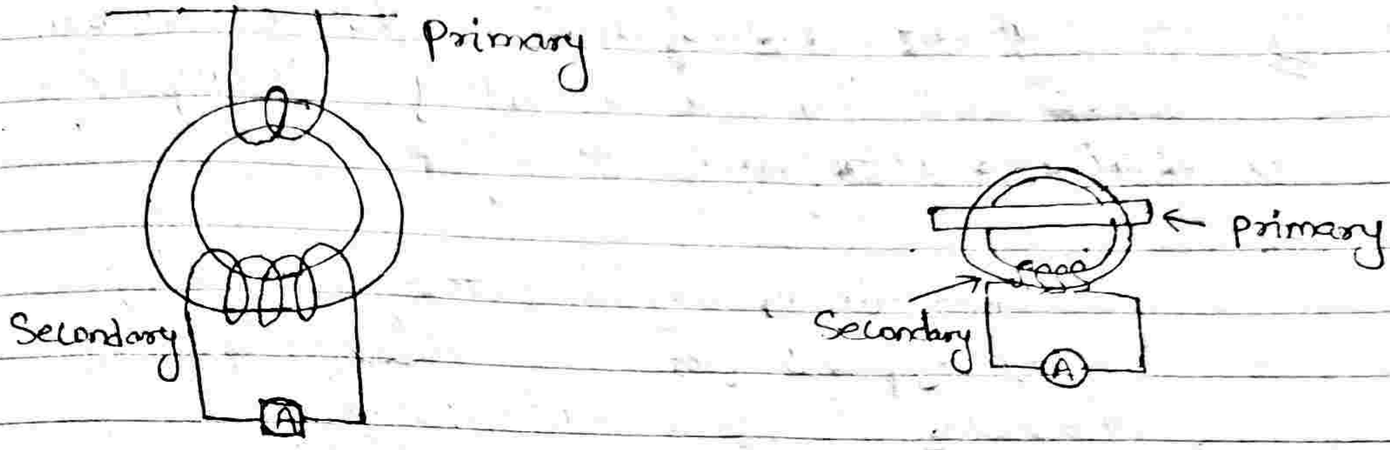


Fig current transformer

→ The conductor carrying large current passes through a circular laminated iron core.

→ The conductor constitutes a one turn primary winding and large numbers of turns in secondary winding.

→ Due to transformer action, the secondary current is transformed to a low value which can be measured by ordinary meters.

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}$$

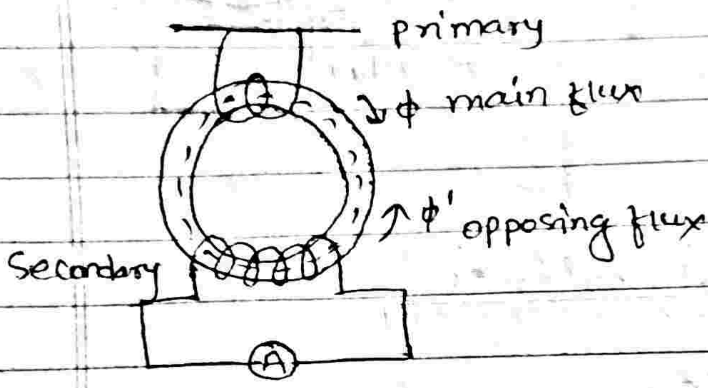
$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$

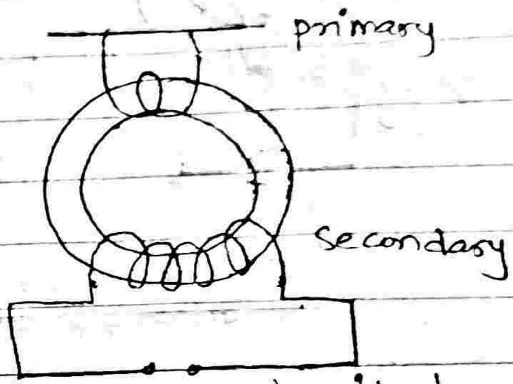
→ CT ratio 1000A/1A means CT will reduce 1000A current into 1A current.

→ CT is always connected in series with line.

Q) A current transformer must never be operated with its secondary winding open circuited. Explain.

=>





High current is applied to primary winding and it will generate high amount of flux. If the secondary side is open circuited then secondary current will be zero and no flux is generated means no opposing flux is generated in secondary side which will cause serious overheating of the core (may burn) Second thing is, if secondary is open circuited then voltage may increase dangerously very high.

2) <u>Potential transformer</u> – PT is a device which is used to measure high alternating voltage. It is essentially a step down transformer having small number of secondary turns as shown in fig below :

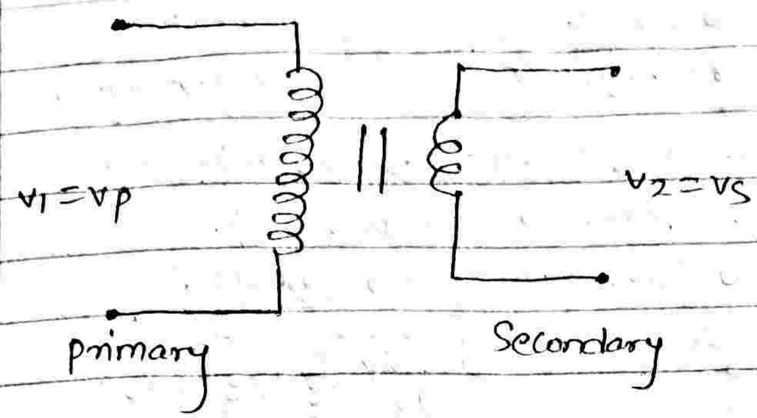


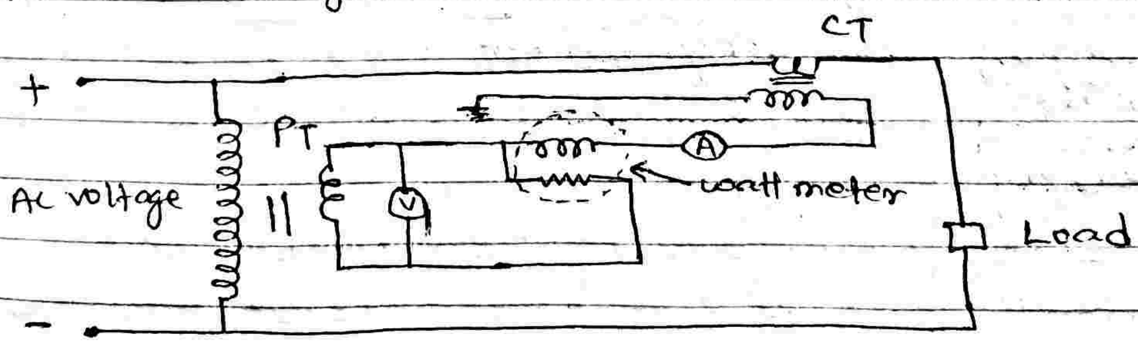
Fig potential transformer

→ The high alternating voltage to be measured is connected to primary directly and the low voltage winding is connected to voltmeter.

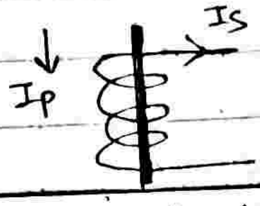
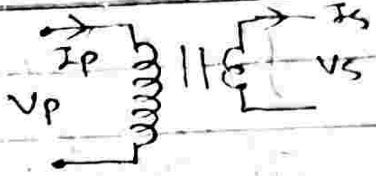
$$\boxed{\begin{aligned} \frac{V_p}{V_S} &= \frac{N_p}{N_S} \\ \frac{I_1}{I_2} &= \frac{N_1}{N_2} \end{aligned}}$$

→ PT ratio 11KV / 110V means PT reduces 11KV into 110V

→ PT is always connected in parallel with line.



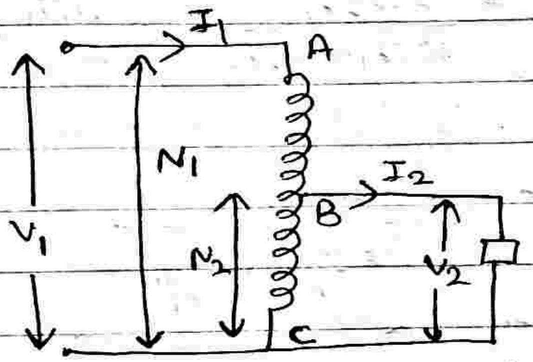
Comparision of CT and PT

	Basis for comparision	current transformer	potential transformer
1)	Definition	Transform the current from high value to the low value.	Transform the voltage from high value to the low value.
2)	circuit symbol		
3)	core	usually buit up with lamination of silicon steel	It is made up of with high quality steel operating at low flux densities
4)	primary winding	It carries the current to be measured.	It carries the voltage to be measured
5)	Secondary winding	It is connected to instruments (ammeter)	It is connected to meter (voltmeter)
6)	Connection	connected in series with the instruments	Connected in parallel with the instruments.
7)	primary circuit	has small number of turns	has large number of turns
8)	selondary circuit	has large number of turns	has small number of turns.
9)	Transformation ratio.	High	low
10)	Impedance	low	high
11)	Types	two types (wound and closed core)	two types (electromagnetic and capacitor voltage.

Auto Transformer

An auto transformer is a special type of transformer with only one winding. A part of the winding is common to both primary and secondary side. In Auto transformer, primary and secondary windings are connected electrically as well as magnetically.

The figure shows a single phase auto transformer having N_1 turns in the primary and N_2 turns tapped for lower secondary voltage. The winding section BC with N_2 turns is common to both primary and secondary side.



$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

Fig Step down auto transformer

If I_2 = current drawn by the load

I_1 = current drawn from supply

current in section BC $= I_2 - I_1$

Cu saving $\Rightarrow$ weight of Cu $\propto N \cdot I$

weight of copper in section AB $\propto (N_1 - N_2) I_1$

" " " " BC $\propto (N_2)(I_2 - I_1)$

Total weight of copper used in auto transformer

$(W_{auto}) \propto (N_1 - N_2) I_1 + N_2 (I_2 - I_1)$

If a two winding transformer were to perform the same duty, then weight of copper in two winding transformer $(w_{tw}) = N_1I_1 + N_2I_2$

Now,

$$\frac{W_{auto}}{w_{tw}} = \frac{N_1I_1 - N_2I_1 + N_2I_2 - N_2I_1}{N_1I_1 + N_2I_2}$$

$$\frac{W_{auto}}{w_{tw}} = 1 - \frac{2N_2I_1}{N_1I_1 + N_2I_2}$$

$$\frac{W_{auto}}{w_{tw}} = 1 - \frac{2N_2I_1}{N_1I_1 + N_2I_2} \times \frac{N_1I_1}{N_1I_1}$$

$$\frac{W_{auto}}{w_{tw}} = 1 - \frac{2N_2/N_1}{1 + \frac{N_2}{N_1} \times \frac{I_2}{I_1}}$$

$$\frac{W_{auto}}{w_{tw}} = 1 - \frac{2K}{1 + \cancel{K}/\cancel{K}} = 1 - K$$

$$\boxed{W_{auto} = (1-K)\, w_{tw}}$$

Case I - if $V_1 = 220V$ and $V_2 = 200V$ then,

$$K = \frac{200}{220} = 0.909 \approx 1$$

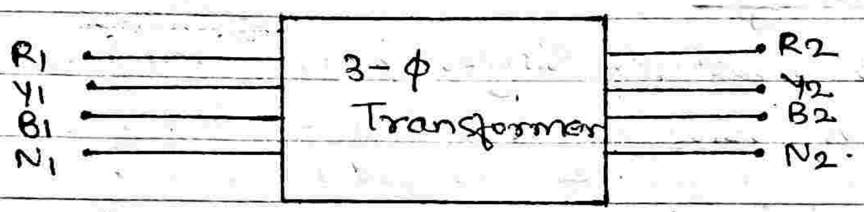
$$w_{auto} = (1 - 0.909)\, w_{tw} = 0.091\, w_{tw}$$

$\Rightarrow$ So the weight of Cu required in auto transformer is 9.1 % of the normal two winding transformer.

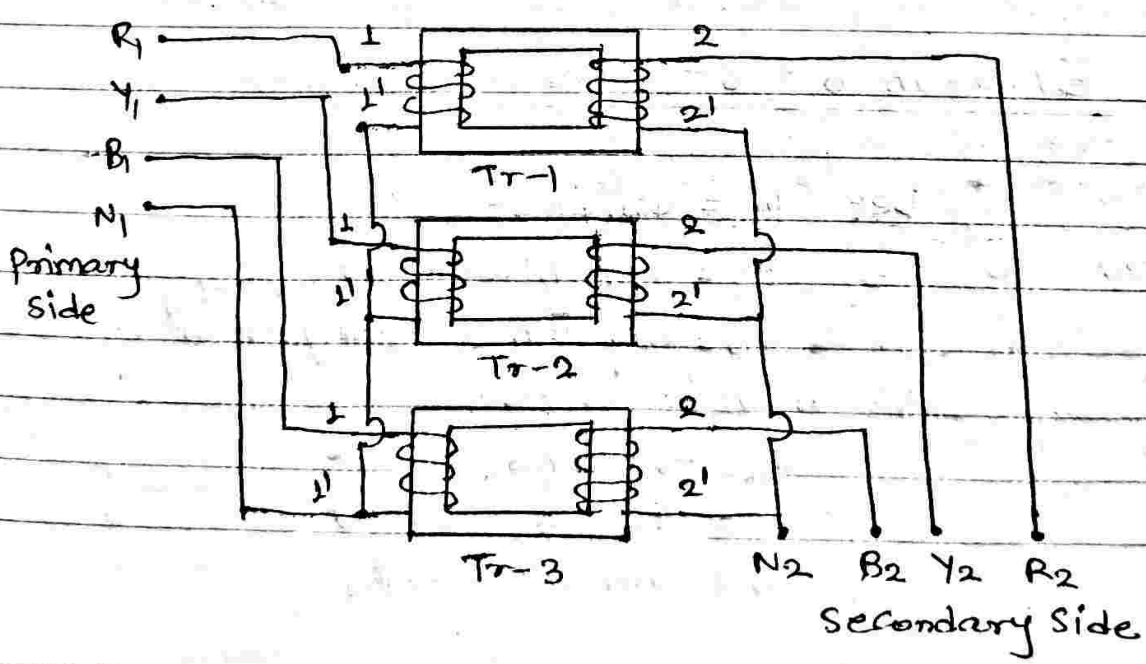
Case–II – If $V_1 = 220V$ and $V_2 = 6V$, than $K = \frac{6}{220} = 0.0272$
(very less than 1)
$W_{auto} = 0.9728\, W_{two}$

⇒ The weight of the copper required is 97.2% of the normal two winding transformer. Hence, the saving in copper in an auto transformer is only significant when K approaches unity.

Three phase transformer



3 Nos of 1-ϕ transformer can be used to step up or step down the 3-ϕ ac voltage.



Advantage and disadvantage of using the Scheme shown above

Advantages

1) Usually a single unit of 3-ϕ transformer is quite large compared to a single phase unit. The transportation becomes easier.

2) During maintenance only one of the units becomes unavailable so the system becomes more reliable.

Disadvantage

1) Using 3 separate single phase transformer is more expensive than using a single 3 phase unit.

2) This kind of scheme is less efficient.

3) It occupies more space.

Evolution of 3-phase transformer

Let $\phi_R = \phi_m \sin\omega t$

$\phi_Y = \phi_m \sin(\omega t - 120)$

$\phi_B = \phi_m \sin(\omega t - 240) = \phi_m \sin(\omega t + 120^\circ)$

As current is also,

$I_R = I_m \sin\omega t$

$I_Y = I_m \sin(\omega t - 120^\circ)$

$I_B = I_m \sin(\omega t - 240)$

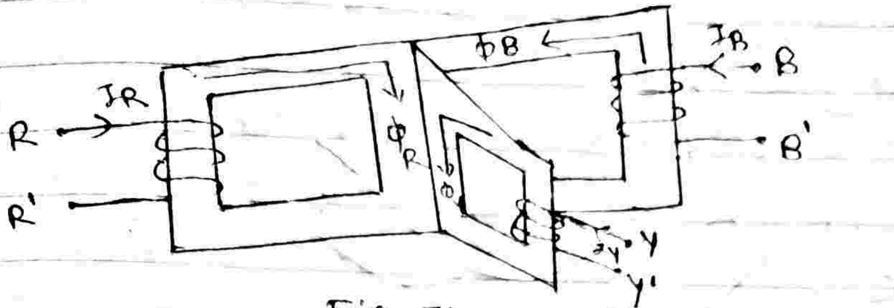


Fig Three units of single phase transformer

$\phi \propto I$

The total flux to the common limb is,

$$\phi_t = \phi_R + \phi_Y + \phi_B \quad [\because \text{flux vector are in same direction}]$$

$$\phi_t = \phi_m \sin\omega t + \phi_m \sin(\omega t - 120) + \phi_m \sin(\omega t + 120)$$

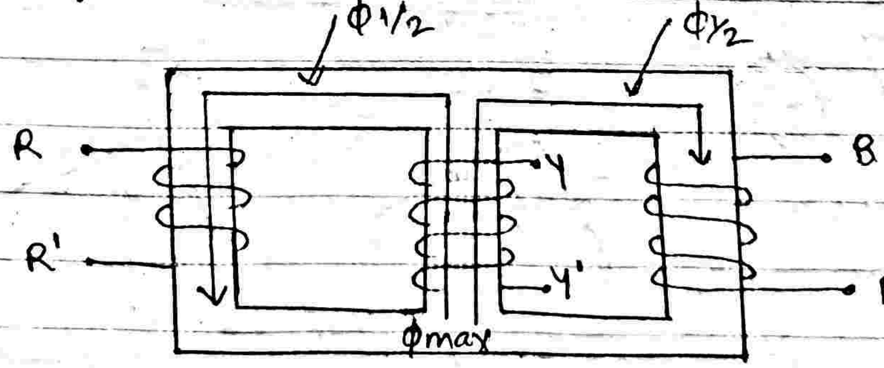
$$= \phi_m \sin\omega t + \phi_m \sin\omega t * \cos 120 - \phi_m \cos\omega t * \sin 120° + \phi_m \sin\omega t \cos 120° + \phi_m \cos\omega t \sin 120°$$

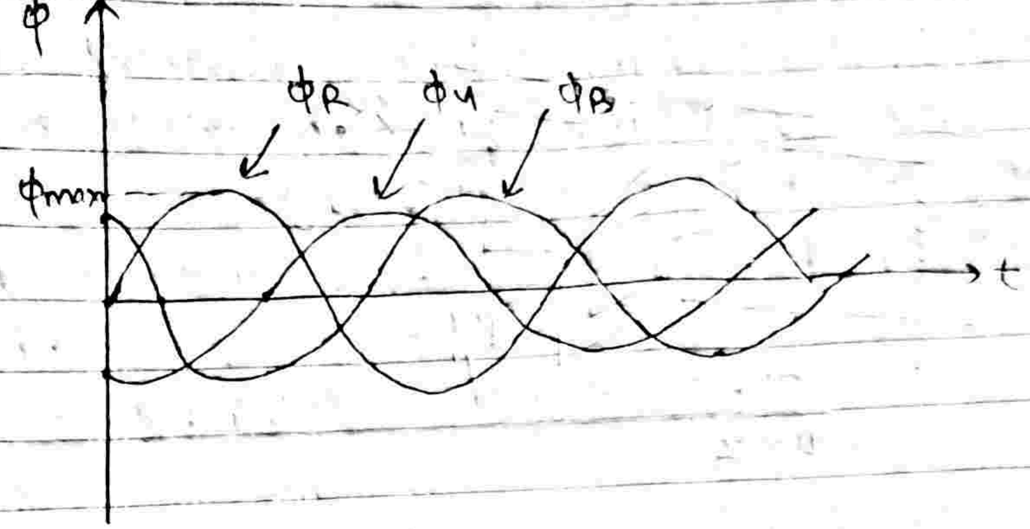
$$\phi_t = \phi_m \sin\omega t - \phi_m \sin\omega t$$

$$\phi_t = 0$$

$$\boxed{\therefore \phi_t = 0}$$

Hence no flux flows through the central core therefore central core can be removed and modified design of core can be made as follow:-

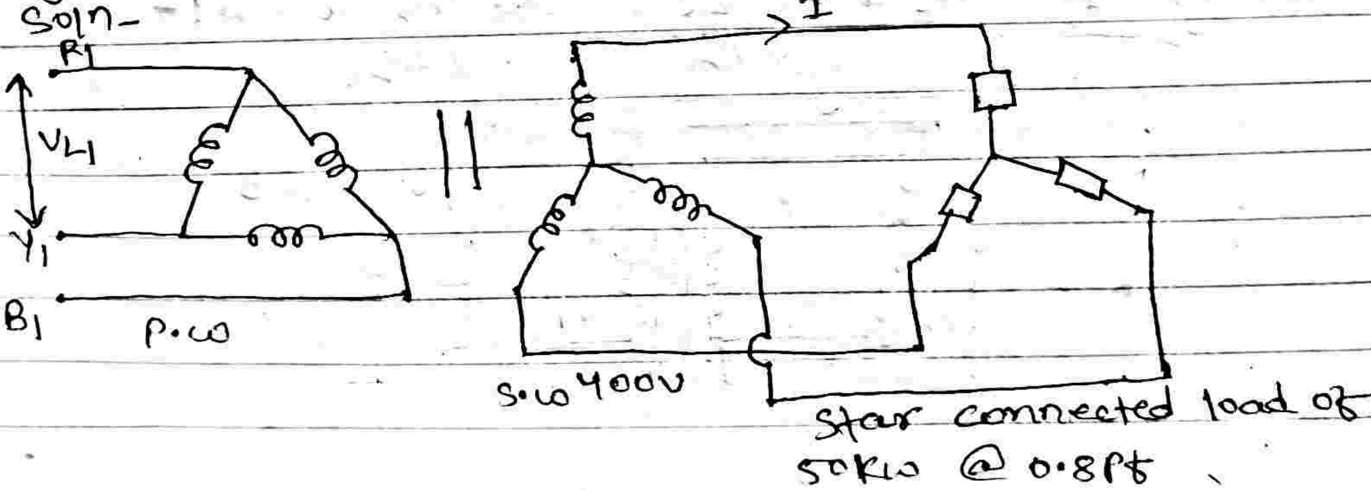




$$\phi_Y = \phi_{max}$$
$$\phi_R = -\tfrac{1}{2}\,\phi_{max}$$
$$\phi_B = -\tfrac{1}{2}\,\phi_{max}$$

Q) A 3-phase, 50Hz 111/400 V Delta/star (Δ-Y) transformer has a balanced star connected load of 90 kW at 0.8 lagging pf. calculate the secondary lagging current, primary phase current and the primary line current assuming that the transformer is an ideal core.

⇒ soln-



In delta connection

$V_L = V_{ph}$

$I_L = \sqrt{3}\ I_P$

$P = \sqrt{3}\ I_L V_L \cos\phi$

$P = 3 V_P I_P \cos\phi$

In Star connection

$I_L = I_P$

$V_L = \sqrt{3}\ V_P$

$P = \sqrt{3}\ V_L I_L \cos\phi$

$P = 3 V_P I_P \cos\phi$

Load output power $= 90KW$

$P = \sqrt{3}\ V_{L2} I_{L2} \cos\phi$

$90 * 10^3 = \sqrt{3} * 400 * I_{L2} * 0.8$

$I_L = 162.38A = I_{P2}$

Ideal transformer mean i/p power = o/p power

i/p power = o/p power

Delta $V_{P1} * I_{P1} = V_{P2} * I_{P2}$

V_L $11 * 10^3 * I_{P1} = \frac{400}{\sqrt{3}} * 162.38$

$I_{P1} = 3.409A$

$I_{P1} = 3.409\ A$

$I_{L1} = \sqrt{3}\ I_{P1} = \sqrt{3} * 4.09 = 5.905A.$

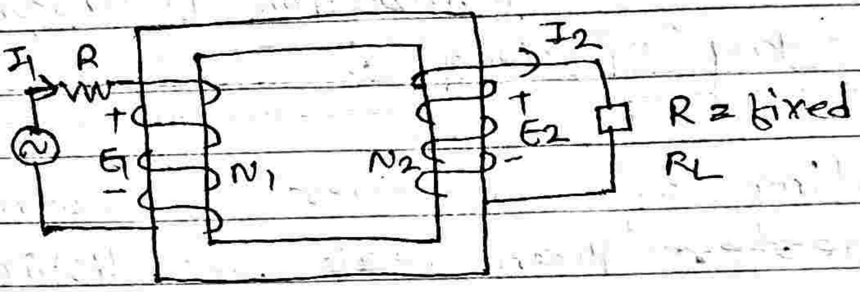
Application of transformer

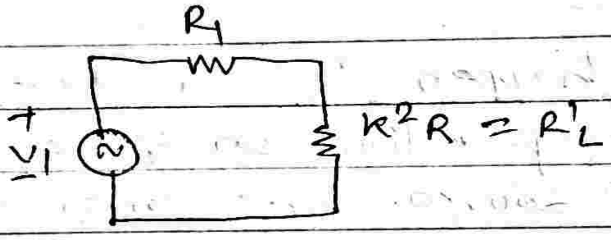
1) Step up and down the voltage.

⇒ $E_1/E_2 = N_1/N_2$

⇒ depends on N_1 and N_2, we can change voltage level without changing its frequency and its power.

2) Impedance matching for maximum power transfer.





⇒ we can adjust value K and make $R_1 = R'_L$ impedance matching

⇒ audio amplifier (use)

3) Filtration of DC

⇒ filter dc signals

If we give DC Source (supply) to transformer then ϕ will be constant (rate of change is zero) then induced emf in secondary will be zero.

$f = 0 \quad , X_1 = 0 \quad X_1 = 2\pi f L_1 = 0$

$|z_1| = \sqrt{R_1^2 + X_1^2} = R_1$

$z = R_1$

z is reduced $\uparrow I_1 = V/z_1 \downarrow$

⇒ It means it will draw heavy current (primary current) which may burn primary winding thatswhy transformer will not work on dc supply but at steady state condition.

⇒ It can induce emf at the time of switching on.

⇒ If DC supply to ideal transformer (e_1 = finite) then $e_2 = 0$, but no burning of winding as there is no winding resistance in primary and secondary.

⇒ If DC supply to practical transformer then $e_2 = 0$, and the primary will burn out due to heavy current

⇒ Prevents dc signal to pass.

4) Isolation of circuit

⇒ In transformer primary and secondary are electrically isolated.