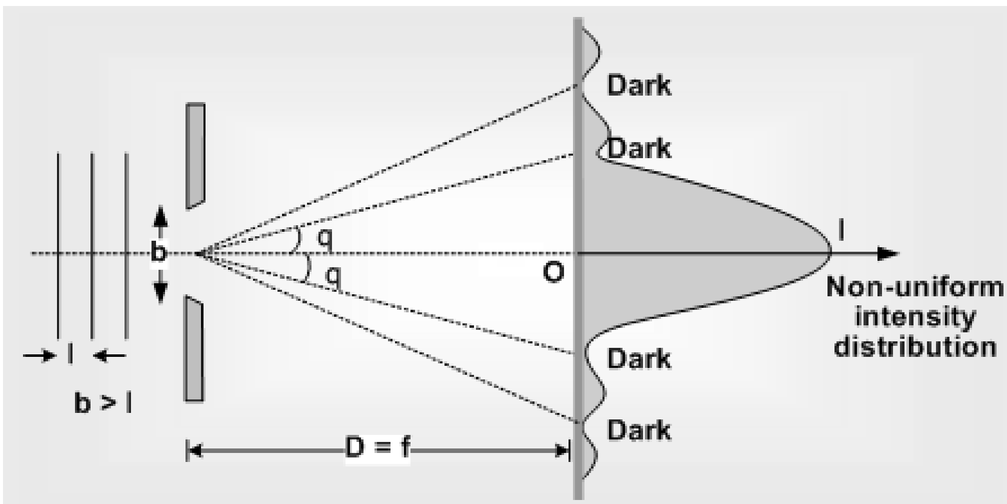
Diffraction

The phenomenon of spreading or bending of light wave when it passes through a narrow opening is known as diffraction. The intensity distribution of light on the screen is called the diffraction pattern.



Difference between Interference and diffraction:-

	Interference	Diffraction
1.	It is due to the superposition of secondary wavelets from two different wavefronts produced by two coherent sources.	It is due to the superposition of secondary wavelets emitted from various points of the same wave front.
2.	Fringes are equally spaced.	Fringes are unequally spaced.
3.	Bright fringes are of same intensity	Intensity falls rapidly
4.	Comparing with diffraction, it has large number of fringes	It has less number of fringes.

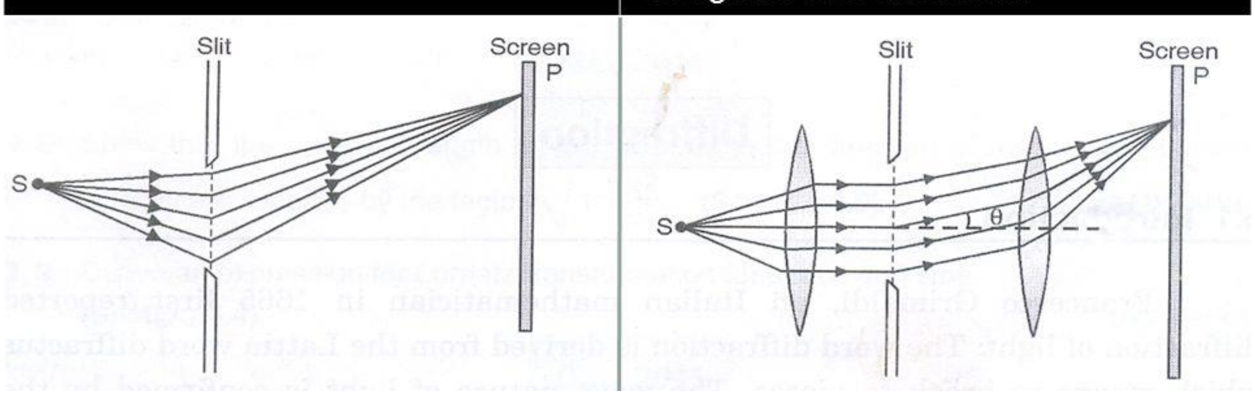
Fresnel and Fraunhoffer diffraction:-

Diffraction pattern are usually classified into two categories depending on where the source and screen are placed. When the source and screen is near the obstacle the wave fronts are spherical and the pattern is complex, this is called near field diffraction or Fresnel diffraction.

If the source, obstacle and screen are far enough away, that all lines from the source to the obstacle can be considered parallel. The phenomenon is called far field diffraction or Frounhoffer diffraction. The diffraction pattern in this type of diffraction is simple to analyze.

Distinguish between Fresnel & Fraunhoffer diffraction

Fresnel Diffraction	Fraunhoffer Diffraction
• The source and the screen are at finite distance from the obstacle.	• The source and the screen or both are effectively at infinite distance from the obstacle.
• Observation of Fresnel diffraction does not require any lenses.	• The conditions required for the Fraunhoffer diffraction are achieved using two convex lenses.



Frounhofer’s single slit diffraction:-

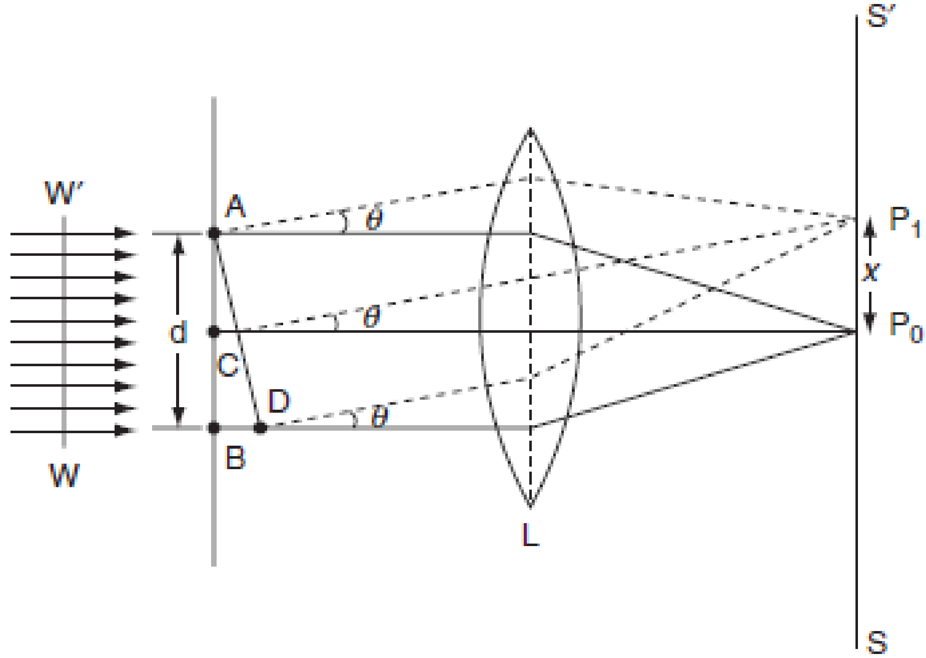


Figure shows plane parallel rays incident on a slit of width ‘d’. Since, screen is assumed to be far away, the rays heading for any point are essentially parallel. Consider rays that pass straight through in which $\theta = 0$.These rays are in phase, so there will be central bright spot on the screen. Now, consider rays moving at an angle of θ. In such a way that $BD = \lambda$. In this case the ray passing through the centre of the slit will travel one half wavelength of bottom of the slit. Therefore, these two rays will be exactly out of phase with each other and will be destructively interfere. So all the ray interferes in pairs and the screen will be dark at this particular angle. So from figure path difference between the rays from the top and centre of the slit is;

$$\frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

And condition for first dark pattern is;

$$d\ sin\ \theta = \lambda$$

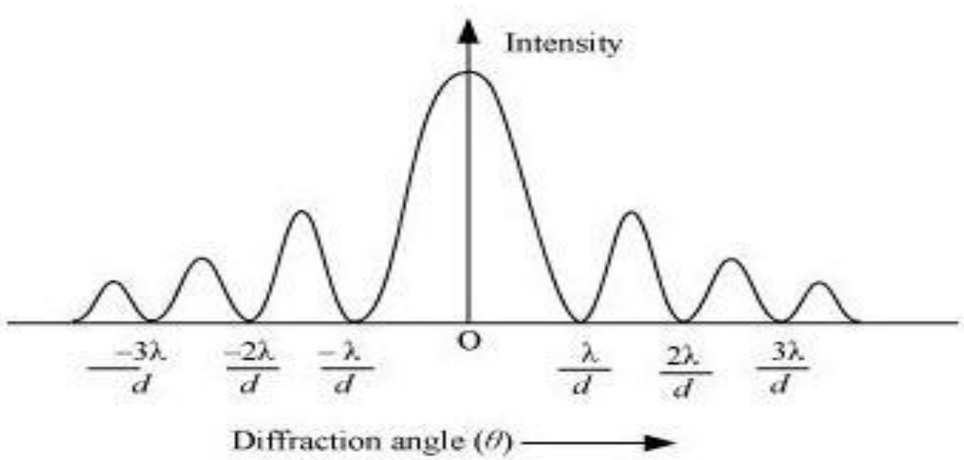
By continuing this process of dividing the slit, we find that;

$$d\ sin\ \theta = n\ \lambda \quad where, n = 1,2,3 \ldots \ldots$$

Note that $n \neq 0,\ since\ \theta = 0,$ corresponds to the central maxima not minima.

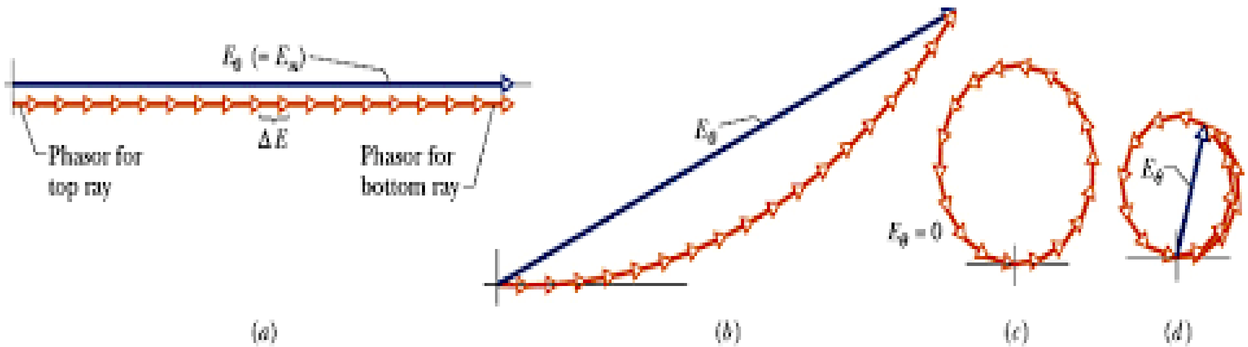
In addition to the central maxima at $\theta = 0$, there are secondary maxima on either side of centre maxima. These are situated in a direction in which the path difference is an odd multiple of $\lambda/2$. Hence for secondary maxima;

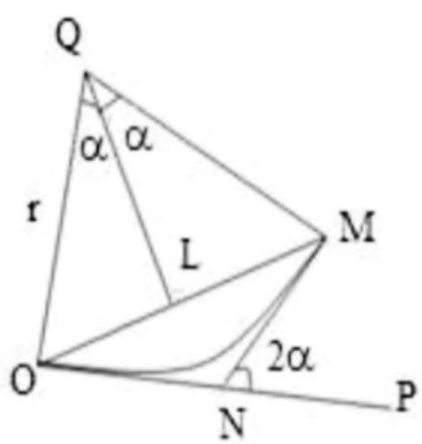
$$d\ sin\ \theta = (2n+1)\frac{\lambda}{2}$$

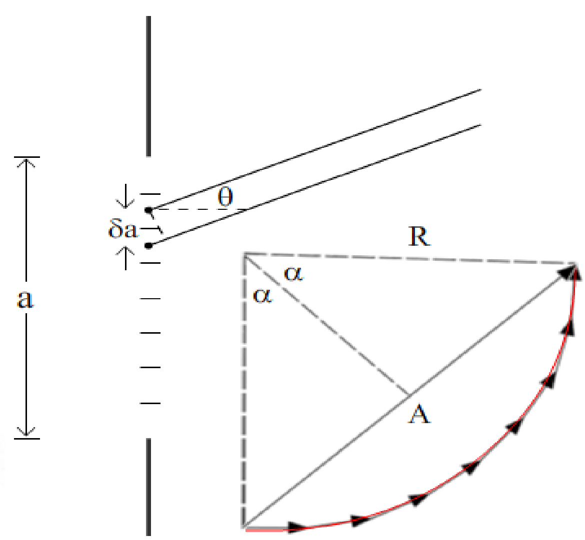


Intensity Distribution In Diffraction Pattern due to a single slit:-

Bending of light from the edge of the obstacle and spreading around the geometrical shadow is known as diffraction of light. The total phase difference between the wavelets from the top and bottom edge of the slit of width 'a' is α. As the wave front is divided in to a large number of strips the resultant amplitude due to all the individual strips can be obtained by the vector polygon method.







Here, the amplitudes are small and the phase difference increases by infinitesimally small amounts from strip to strip.

Thus the variation polygon coincides with the circular arc OM. OP gives the direction of the initial vector and NM the direction of the final vector due to the secondary waves from A. Q is the center of the circular arc.

$$< MNP = 2\alpha \text{ and } < OQM = 2\alpha$$

In the $\Delta\, OQL, \sin\alpha = \frac{OL}{r}; OL = r\ sin\alpha$

Where r is the radius of the circular arc.

$$\therefore \text{Chord OM} = 2\text{OL} = 2\text{r} \sin\alpha \ldots\ldots\ldots (1)$$

The length of the arc OM is proportional to the width of the slit.

$\therefore$ Length of the arc OM $=$ Ka, where K is constant and a is the width of the slit.

Also, $2\alpha = \frac{\text{arc OM}}{\text{radius}} = \frac{\text{Ka}}{\text{r}} \quad \Rightarrow 2r = \frac{Ka}{\alpha}$

Substituting this value of 2r in equation (1)

$$\text{Chord OM} = \frac{\text{Ka}}{\alpha} \sin\alpha$$

But, Chord OM $=$ A is the amplitude of resultant.

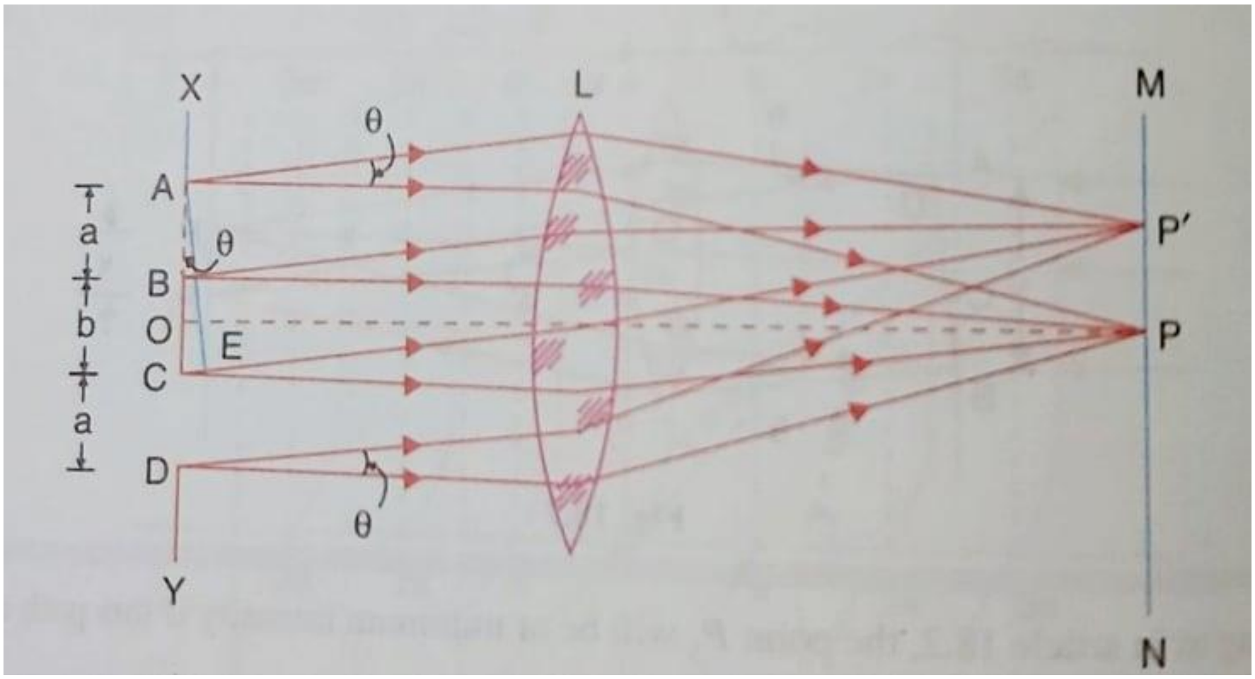
$$\text{A} = (\text{Ka})\frac{\sin\alpha}{\alpha} \quad \Rightarrow A = A_o \frac{\sin\alpha}{\alpha}$$

The intensity I at the point is given by;

$$I = A^2 = A_o^2 (\frac{\sin\alpha}{\alpha})^2$$

$$\therefore I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$

Frounhofer's Double slit diffraction:-



Suppose there are two parallel rectangular slits of equal width. The distance between the corresponding points of the two slits will be $(a + b) = d$, where a is transparent portion and b is opaque. Let a monochromatic light of wavelength λ is incident on a double slits. Suppose each slit diffracts the beam in a direction making an angle θ with the direction of incident beam. The path difference between the rays from the corresponding points A and C of the two slits.

$$CE = (a + b)\sin\theta \ldots\ldots\ldots (i)$$

If this path difference is equal to odd multiple of λ/2, θ gives the direction of minima due to interference of the secondary waves from the two slits.

$$\therefore CE = (a+b)\sin\theta_n = (2n+1)\frac{\lambda}{2} \dots\dots (ii)$$

Putting $n = 1,2,3, etc,$ the values of $\theta_1, \theta_2, \theta_3, etc,$ corresponding to the direction of minima can be obtained.

From equation (ii),

$$\sin\theta_n = \frac{(2n+1)\lambda}{2(a+b)}$$

On the other hand, If the path difference is equal to even multiple of λ/2, θ gives the direction of maxima due to interference of the secondary waves from the two slits.

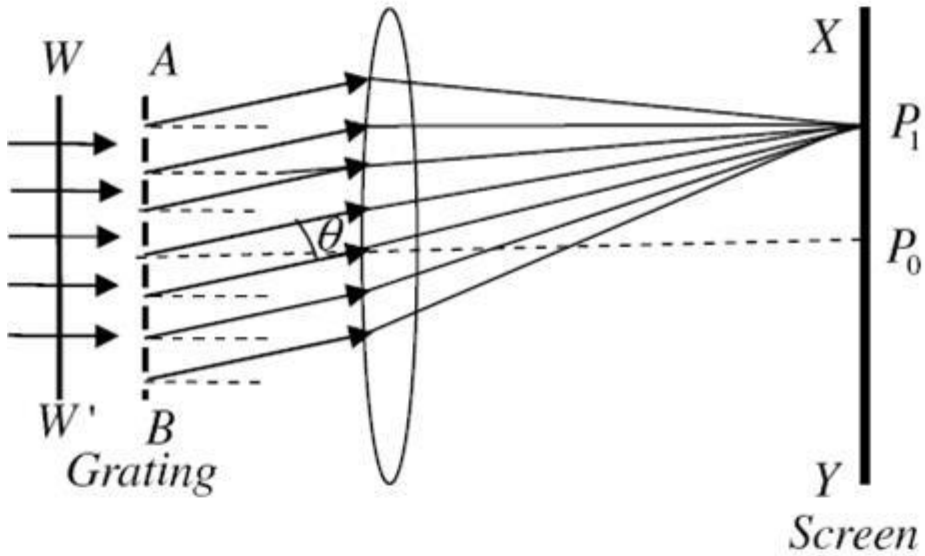
$$\therefore CE = (a+b)\sin\theta_n = 2n\frac{\lambda}{2} \dots\dots (iii)$$

Putting $n = 1,2,3, etc,$ the values of $\theta_1, \theta_2, \theta_3, etc,$ corresponding to the direction of maxima can be obtained.

From equation (iii),

$$\sin\theta_n = \frac{n\lambda}{(a+b)}$$

Diffraction grating:-



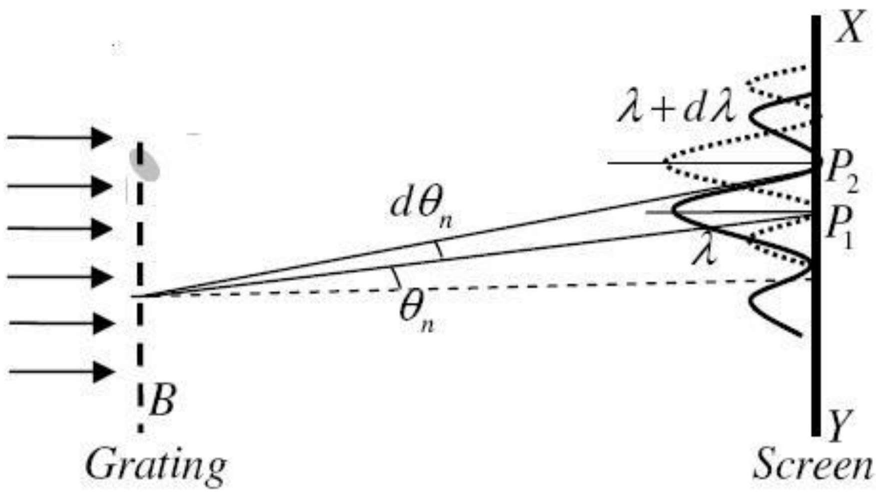
A diffraction grating consists of thousands of very fine equally spaced parallel slit. Gratings are made by ruling lines on a glass plate with a diamond tip. A diffraction containing slit is called a transmission grating. There is another type of grating called as reflection grating. Reflection grating can be made by ruling the lines on a metallic or glass surface from which light is reflected.

Gratings are extremely important in the analysis of light emitted by atoms and molecules. The advantage of grating is that the wavelength can be determined perfectly.

Let assume parallel ray of light is incident on the grating as shown in figure. We should note that the light ray that passes through each slit with deviation $\theta = 0$, interfere constructively and will produce a bright spot at the centre. When the path difference between 1 and 2 in figure is λ, they interfere constructively, same holds for 2 and 3 as shown in figure. Any path difference is equal to an integral multiple of wavelength would lead to constructive interference. Hence the path difference between the rays from adjacent slit is $d \sin\theta$ where d is the slit separation. Therefore, the position of the principle maxima is given by;

$$d \sin\theta = n\lambda \quad where, n = 0,1,2,3 \ldots\ldots$$

Dispersive power of a grating:-



Spreading the diffraction lines associated with various wavelengths by the grating is called dispersion. Ratio of the difference in the angle of diffraction of any two neighboring lines to the difference in wavelength between the two spectral lines is called dispersive power. $i.e. \frac{d\theta}{d\lambda}$

We have, $d \sin\theta = n\lambda$

Now, differentiating;

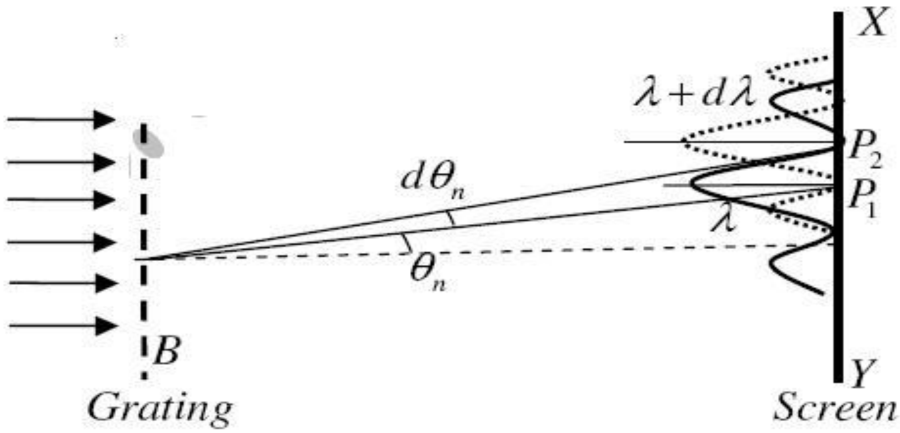
$$\cos\theta d\theta = \frac{n}{d} d\lambda$$

$$or, \qquad \frac{d\theta}{d\lambda} = \frac{n}{d\cos\theta} = \frac{n}{d\sqrt{1-sin^2\theta}}$$

$$\therefore \frac{d\theta}{d\lambda} = \frac{n}{d\sqrt{d^2 - n^2\lambda^2}}$$

Hence, the dispersive power of grating depends on the order, nature of grating and nature of light.

Resolving power of grating:-



The ability of an optical instruments to produce distinctly separated images of two objects located very close to each other is called resolving power. The ratio of wavelength (λ) of any spectral line to the smallest difference in wavelength (dλ) between these lines is called resolving power (R).

$$i.e \text{ resolving power } (R) = \frac{\lambda}{d\lambda}$$

From figure above;

$$d\sin(\theta + d\theta) = n(\lambda + d\lambda) \ldots\ldots\ldots (1)$$

Also from Rayleigh's criterion;

$$d\sin(\theta + d\theta) = n\lambda + \frac{\lambda}{N} \ldots\ldots\ldots (2)$$

From equation (1) and (2);

$$n\lambda + \frac{\lambda}{N} = n(\lambda + d\lambda)$$

$$\therefore \frac{\lambda}{d\lambda} = nN$$

Therefore, resolving power $(R) = nN$

Where, n = number of order and N = number of lines.

Numerical Examples:-

1. **A diffraction grating is 3 cm wide produces the second order at 33^0 with light of wavelength 600 nm. What is the total number of lines in the grating?**

Solution:-

Given, $\theta = 33^0$, $\lambda = 600\ \text{nm} = 600 \times 10^{-9}\ \text{m}$
$= 6 \times 10^{-5}\ \text{cm}$

We kanow that; $d\sin\theta = n\lambda$

$$d = \frac{n\lambda}{\sin\theta}$$

$$or, \quad \frac{1}{N} = \frac{n\lambda}{\sin\theta}$$

$$or, N = \frac{\sin 33}{2 \times 6 \times 10^{-5}} = 4539\ lines/cm$$

$\therefore$ Total numer of lines $=$ N $\times$ width

$$= 4539 \times 3 = 13616\ lines$$

2. **Plane diffraction grating has $\boldsymbol{5000\ lines/cm}$. How many maxima can be observed, when a plane wave of wavelength $\boldsymbol{5500\ A^0}$ is made to incident on it.**

Solution:-

Given, $\mathrm{N} = 5000\ \mathrm{lines/cm}$, $\lambda = 5500\ A^0$
$= 5500 \times 10^{-8}$ cm

For maxima, $\sin\theta = 1$;

Now; $d\sin\theta = n\lambda$

$$or\ \frac{1}{N}\sin\theta = n \times 5500 \times 10^{-8}$$

$$or, \frac{1}{5000} = n \times 5500 \times 10^{-8}$$

$$\therefore n = 4$$

3. **Calculate the possible order of spectra with a plane transmission grating having $18000\ lines\ /inch$. When light of wavelength is $4500\ A^0$.**

Solution:-

Total number of lines(N) = 18000 lines/inch

$$\therefore d = \frac{2.54}{N} = \frac{2.54}{18000} = 1.41 \times 10^{-4}\ cm$$

The highest order occurs when, $\sin\theta = 1$

Wavelength $(\lambda) = 4500\ A^0 = 4500 \times 10^{-8}\ cm$

We have, $\quad d\sin\theta = n\lambda$

$$1.41 \times 10^{-4} \times 1 = n \times 4500 \times 10^{-8}$$

$\therefore$ Number of order (n) = 3

4. **Light is incident on a grating of total ruled width $5 \times 10^{-3}\ m$ with 2500 in all. Calculate the angular separation and resolving power of two sodium lines in the 1st order spectrum.(Given; $\lambda_1 = 5890\ A^o$ and $\lambda_2 = 5896\ A^o$).**

Solution:-

$$N = 2500,\ \ width = 5 \times 10^{-3}\ m,\ \ \ n = 1$$

$\lambda_1 = 5890 \times 10^{-10} m, \;\; \lambda_2 = 5896 \times 10^{-10} m$

Total number of lines per meter$(\mathrm{N}) = \dfrac{2500}{5 \times 10^{-3}}$

$$= 5 \times 10^5$$

$$\therefore d = \frac{1}{5 \times 10^5} = 2 \times 10^{-6}\, m$$

For first order $(\mathrm{n}) = 1$

Now, $\quad \sin \theta_1 = \dfrac{\lambda_1}{\mathrm{d}} = 0.2945$

$$\therefore \; \theta_1 = \; 17^0.8^I$$

And, $\quad \sin \theta_2 = \dfrac{\lambda_2}{\mathrm{d}} = 0.2948$

$$\therefore \; \theta_2 = \; 17^0.14^I$$

$\therefore$ Angular seperation$(\theta_1 - \theta_2) = 0.66$

The resolving power of grating $(R) = \dfrac{\lambda_{avg}}{d\lambda}$

$$= \frac{5893 \times 10^{-10}}{6 \times 10^{-10}}$$

$$\therefore \; \frac{\lambda}{d\lambda} = 982.16$$

As the total number of lines on the grating is 2500 is more than the 982. Therefore the lines can be resolved.

Exercise:-

1. Explain the physical meaning of dispersive power and resolving power of a grating. Two spectral lines of wavelength λ and $\lambda + \Delta\lambda$ respectively where $\Delta\lambda \ll \lambda$. Show that there angular separation $\Delta\theta$ in a grating spectrometer is $\Delta\theta = \frac{\Delta\lambda}{\sqrt{(\frac{d}{m})^2 - \lambda^2}}$, where d and m are grating elements and no of order respectively.
2. What is diffraction of light? Explain the dispersive power and resolving power of a grating. Derive the relation and also relate between them.
3. A diffraction grating used at normal incidence gives a line (540 nm) in a certain order superposed on the violet line (405 nm) of the next higher order. How many lines per cm are there in the grating if the angle of diffraction is 30^o.
4. A grating with 250 groves/ mm is used with an incandescent light source. Assume visible spectrum to range in wavelength from 400 to 700 nm. In how many orders can one see the entire visible spectrum?
5. What is diffraction of light? Discuss the intensity distribution with special reference to diffraction of light in single slit.
6. In a Fraunhofer single slit diffraction, a convex length of focal length 20 cm is placed just after the slit of width 0.6 nm. If a plane wave of wavelength 6000 A^o falls on slits

normally, calculate the separation between the second minima on either side of central maxima.

7. Discuss the phenomenon of Fraunhofer diffraction at a single slit. Show that the relative intensities of the successive maxima are 1: $\frac{4}{9\pi^2}$: $\frac{4}{25\pi^2}$ … … …
8. Show that the intensity of second primary maxima is 1.62 % of central maxima in Fraunhofer single slit diffraction.
9. Light of wavelength 600 nm is incident normally on a slit of width 0.1 mm. Calculate the intensity at $\theta = 0^o$.
10. Light is incident normally on a grating of total ruled width $5 \times 10^{-3}\ m$ with 2500 lines in all. Find the angular separation of the sodium lines in the first order spectrum. Wavelength of lenses are $589 \times 10^{-9}\ m$ and $589.6 \times 10^{-9}\ m$. Can they be seen distinctly?
11. A screen containing two slits 0.1 mm apart is 1m from the viewing screen. Light of wavelength 500 nm falls on the slits from a distant source. Approximately how far apart will the bright interference fringes be seen on the screen?
12. What is the highest order spectrum which may be seen with monochromatic light of wavelength 600 nm by means of a diffraction grating with 4500 lines/cm.
13. A grating of width 2.8 cm has 6000 lines. What is the minimum difference in wavelength that can be resolved in second order at 550 nm?

14. In a double slit experiment, the distance between the slit is 5mm and the slits are 1 m from the screen. Two interference patterns can be seen on the screen; one due to light of wavelength 480 nm and the other due to light of wavelength 600 nm. What is the separation on the screen between the third-order bright fringes of two interference patterns?

15. What is plane diffraction grating? How it is used to find the wavelength of a monochromatic light experimentally?

16. Assume that the limits of the visible spectrum are arbitrary chosen as 430 nm and 680 nm. Calculate the no of rulings per millimeter of a grating that will spread the first-order spectrum through an angle of 20^o.

17. In a grating the sodium doublet (5890 A^o, 5896 A^o) is viewed in third order at 30^o to the normal and is resolved. Determine the grating element and the total width of the rulings.