classmate
Date
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Assignment - 1

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2074 Ashwin

(b) Two AC represented by the eqⁿ $i_1 = 7\sin\omega t$ & $i_2 = 10\sin(\omega t + \pi/2)$ are fed into a common conductor. Find the eqⁿ for the resultant current & it's RMS. value.

Solⁿ

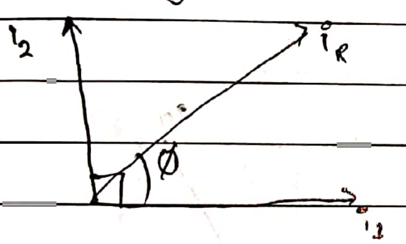
$i_1 = 7\sin\omega t$ - (1)

$i_2 = 10\sin(\omega t + \frac{\pi}{2})$ - (2)

representing above eqⁿ is phasor diagram

$\Delta\phi = \pi/2 \rightarrow (\omega t + \pi/2 - \omega t)$

$= \pi/2$



Here i_2 is leading i_1 by $\pi/2$ so,

$i_1 = 7\angle 0$

$i_2 = 10\angle \pi/2$

So,

$i_1 = 7[\cos 0 + i\sin 0]$

$i_2 = 10[\cos\pi/2 + i\sin\pi/2]$

Adding we get

$i_1 = 7 + 0i$

$i_2 = 0 + 10i$

$i_{12} = 7 + 10i$

Now,

$\phi = \tan^{-1}(\frac{10}{7}) = 55.007$

$$I_m = \sqrt{7^2+10^2} = 12.2\ A$$

so,

$$I_R = I_m \sin(\omega t + \phi)$$
$$= 12.2 \times \sin(\omega t + 55.007)$$

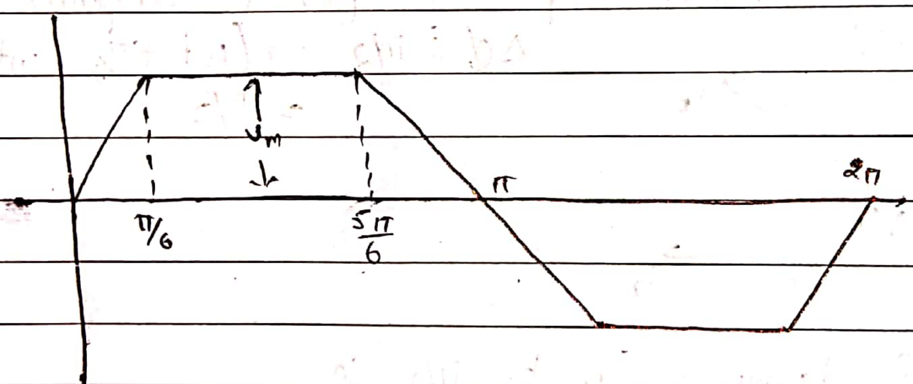
so,

$$I_{resultant} = 12.2 \sin(\omega t + 55.007)$$

now,

$$I_{RMS} = \frac{I_0}{\sqrt{2}} = \frac{12.2}{\sqrt{2}}$$
$$= 8.6\ A \quad \#$$

2074 chaitra



Calculate the average (half period)) value of rms value of the wave form shows above.

$$\text{Average value} = \frac{\text{Area under curve}}{\text{Time period}}$$

Here, for $0 < t \leq \pi/6$

$y = mx$

or $v = \frac{V_m}{\pi/6}\theta$ so, $v = \frac{V_m}{\pi/6}\theta$

for $\frac{\pi}{6} < t < \frac{5\pi}{6}$

$y = V_m$

or $v_{e2} = V_m$

for, $\frac{5\pi}{6} < t < \pi$

$$y - V_m = \frac{0 - V_m}{\pi - \frac{5\pi}{6}}\left(\theta - \frac{5\pi}{6}\right)$$

$$V - V_m = \frac{-V_m}{\pi/6}\left(\theta - \frac{5\pi}{6}\right)$$

$$V_3 = V_m - V_m\left(\frac{\theta - \frac{5\pi}{6}}{\pi/6}\right)$$

now,

$$V_{avg} = \frac{\text{Area under curve}}{\text{Time period}}$$

$$= \underbrace{\int_0^{\pi/6} \frac{V_m\,\theta\,d\theta}{\pi/6}}_{(I_1)} + \underbrace{\int_{\pi/6}^{5\pi/6} V_m\,d\theta}_{(I_2)} + \underbrace{\int_{5\pi/6}^{\pi} V_3\,d\theta}_{(I_3)}$$

$$= \frac{\int_0^{\pi/6} \frac{6}{\pi} V_m\,\theta\,d\theta + \int_{\pi/6}^{5\pi/6} V_m\,d\theta + \int_{5\pi/6}^{\pi} V_3\,d\theta}{\pi}$$

Now, let

$$I_1 = \int_0^{\pi/6} \frac{6}{\pi} V_m\,\theta\,d\theta = \frac{6}{\pi} V_m \left[\frac{\theta^2}{2}\right]_0^{\pi/6} = \frac{\cancel{6}}{\cancel{\pi}} V_m \frac{\pi^{\cancel{2}}}{\cancel{36}\times 2}$$

$$= \frac{V_m\,\pi}{12}$$

$$I_2 = \int_{\pi/6}^{5\pi/6} V_m\,d\theta = V_m\left[\frac{5\pi}{6} - \frac{\pi}{6}\right] = \frac{2}{3}\pi V_m$$

$$I_3 = \int_{5\pi/6}^{\pi} V_m\,d\theta = \int_{5\pi/6}^{\pi} \frac{V_m}{\pi}(6\theta - 5\pi)\,d\theta$$

$$= V_m\left[\pi - \frac{5\pi}{6}\right] - \left[\frac{V_m}{\pi}\left(\int_{5\pi/6}^{\pi} 6\theta\, d\theta - \int_{5\pi/6}^{\pi} 5\pi\, d\theta\right)\right]$$

$$= V_m \frac{\pi}{6} - \left[\frac{V_m}{\pi}\left(6\left[\frac{\theta^2}{2}\right]_{5\pi/6}^{\pi} - 5\pi\left[\theta\right]_{5\pi/6}^{\pi}\right)\right]$$

$$= V_m \frac{\pi}{6} - \left[\frac{V_m}{\pi}\left(6\left(\frac{\pi^2}{2} - \frac{25\pi^2}{36\times 2}\right) - 5\pi\left(\pi - \frac{5\pi}{6}\right)\right)\right]$$

$$= V_m \frac{\pi}{6} - \left[\frac{V_m}{\pi}\left(6\pi^2\left(\frac{1}{2} - \frac{25}{72}\right) - 5\pi^2\left(1 - \frac{5}{6}\right)\right)\right]$$

$$= V_m \frac{\pi}{6} - \left[\frac{V_m}{\pi}\left(6\,\pi^2 \times \frac{11}{72} - 5\pi^2 \times \frac{1}{6}\right)\right]$$

$$= V_m \frac{\pi}{6} - \left[\frac{V_m}{\pi}\left(\frac{11\pi^2}{12} - \frac{5\pi^2}{6}\right)\right]$$

$$= V_m \frac{\pi}{6} - \left[\frac{V_m}{\pi} \times \frac{1}{12}\pi^2\right]$$

$$= V_m \cdot \pi/6 - \frac{V_m \pi}{12}$$

$$= \frac{V_m}{12}.$$

Now,

$$V_{avg} = \frac{I_1 + I_2 + I_3}{\pi}$$

$$= \frac{\frac{V_m \pi}{12} + \frac{2}{3}\pi V_m + V_m \frac{\pi}{12}}{\pi}$$

$$= \frac{5/6\ V_m \pi}{\pi}$$

$$= 5/6\ V_m \ \#$$

Now For RMS,

$$V_{RMS} = \sqrt{\frac{1}{T_0}\int_0^{T} V_m^2\, d\theta}$$

$$V^2_{rms} = \frac{1}{\pi}\left[\int_0^{\pi/6}\left(\frac{6}{\pi}V_m\theta\right)^2 d\theta + \int_{\pi/6}^{5\pi/6} V_m^2\, d\theta + \int_{5\pi/6}^{\pi} V_3^2\, d\theta\right]$$

I_1 $\qquad$ I_2 $\qquad$ I_3.

$$I_1 = \int^{\pi/6} \frac{36}{\pi^2} V_m^2 \cdot \theta^2 d\theta \Rightarrow \frac{36}{\pi^2} V_m^2 \left[\frac{\theta^3}{3}\right]_0^{\pi/6}$$

$$= \frac{36}{\pi^2} V_m^2 \left[\frac{\pi^2}{6\times 3}\right]$$

$$= \frac{V_m^2 \pi}{18} \quad \text{---(i)}$$

$$I_2 = \int_{\pi/6}^{5\pi/6} V_m^2 d\theta \Rightarrow V_m^2 \int_{\pi/6}^{5\pi/6} d\theta \quad = V_m^2\left(\frac{5\pi}{6} - \frac{\pi}{6}\right)$$

$$= \frac{4}{6}\pi V_m^2$$

$$= \frac{2}{3}\pi V_m^2$$

$$I_3 = \int_{5\pi/6}^{\pi} \left[V_m - \frac{V_m}{\pi}(6\theta - 5\theta)\right]^2 d\theta$$

$$= \int_{5\pi/6}^{\pi} V_m^2 \left(1 - \frac{1}{\pi}(6\theta - 5\pi)\right)^2 d\theta.$$

$$= V_m^2 \left[\int_{5\pi/6}^{\pi} d\theta - \left(\frac{2}{\pi}\int_{5\pi/6}^{\pi} 6\theta\, d\theta + \int_{5\pi/6}^{\pi} 5\pi\, d\theta\right) + \right.$$

$$\frac{1}{\pi^2}\left(\int_{5\pi/6}^{\pi} 36\theta^2 d\theta - \int_{5\pi/6}^{\pi} 60\pi\theta\, d\theta\right.$$

$$+ \int_{5\pi/6}^{\pi} 25\pi^2 d\theta$$

$$= V_m^2\left[\left(\pi - \frac{5\pi}{6}\right) - \left(\frac{2}{\pi}\left(6\times\left[\frac{\theta^2}{2}\right]_{5\pi/6}^{\pi} + 5\pi\left(\pi - \frac{5\pi}{6}\right)\right)\right] + \frac{1}{\pi^2}\left[36\left[\frac{\theta^3}{3}\right]_{5\pi/6}^{\pi} - \right.$$

$$60\pi\left[\frac{\theta^2}{2}\right]_{5\pi/6}^{\pi} + 25\pi^2\left[\pi - \frac{5\pi}{6}\right].$$

Now,

$$V_{rms}^2 = \left(\frac{V_m^2\pi}{18} + \frac{2}{3}\pi V_m^2 + \frac{38}{9}\pi V_m^2\right) \times \frac{1}{\pi}$$

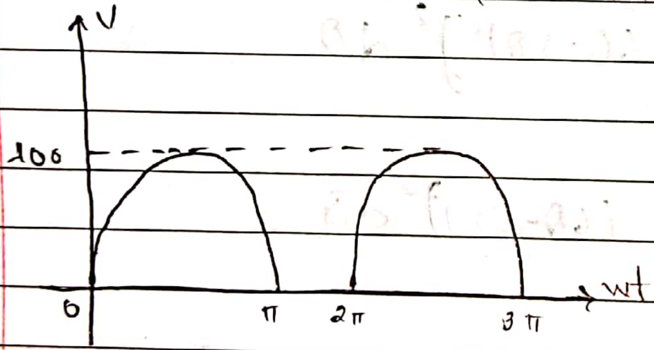
$$= V_m^2 \cancel{\pi} \left(\frac{1}{18} + \frac{38}{9} + \frac{2}{3}\right) \times \frac{1}{\pi}$$

$$V_{rms}^2 = V_m^2 \times \frac{89}{18}$$

$$V_{rms} = 2.22\ V_m$$

2073 Chrawon

① Calculate (i) average rate
(ii) rms value of voltage



Here,

$V = V_m \sin wt = 10 \sin wt$

now

$$V_{avg} = \frac{1}{T}\left[\int_0^{\pi} 10 \sin wt\, dwt + \int_{\pi}^{2\pi} 0\, dt\, wt\right]$$

$$= \frac{1}{2\pi}\left[10 \int_0^{\pi} \sin wt\, dwt\right]$$

$$= \frac{1}{2\pi}\left[-10\left[\cos wt\right]_0^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[-10(\cos\pi - \cos 0)\right]$$

$$= \frac{10}{\pi} = 3.18 \text{ volt}$$

Now,

$$V_{rms} = \sqrt{\frac{1}{T_0}\int_0^{T_2} V_m \, d\omega t}$$

$$V_{rms}^2 = \frac{1}{2\pi}\int_0^{\pi} \cancel{100 \sin^2 \omega t \, d\omega t}$$

$$V_{rms}^2 = \frac{1}{2\pi} \times 100 \, V_m^2 \left[\frac{1}{2} + \frac{\cos 2\omega t}{2}\right] d\omega t$$

$$= \frac{1}{2\pi} \times 100 \left[\frac{1}{2}\int_0^{\pi} d\omega t + \frac{1}{2}\int_0^{\pi} \cos 2\omega t \, d\omega t\right]$$

$$= \frac{50}{\pi}\left[\frac{1}{2}[\pi] + \frac{1}{2}\left[\frac{\sin 2\omega t}{2}\right]_0^{\pi}\right]$$

$$= \frac{50}{\pi}\left[\frac{\pi}{2} + \frac{1}{4} \times 0\right]$$

$$= \frac{50}{\pi} \times \frac{\pi}{2}$$

$$= 25$$

$V_{rms} = 5$ volt //

2068 shrawan

a) The impressed voltage & current flowing through a circuit are:

$v = 500 \sin(400t + \pi/3)$ & $i = 5 \sin(400t - \pi/6)$.

Find:

(i) Rms & avg value of v & i

=> Sol^n,

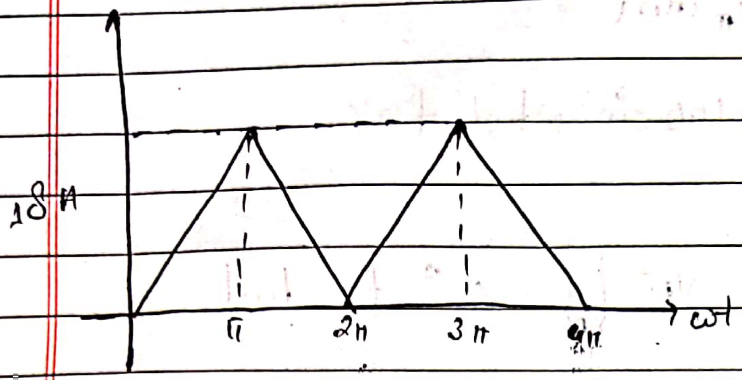
$V_0 = 500$ & $I_0 = 5$

$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{500}{\sqrt{2}} V$ $\qquad$ $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} A$

$V_{avg} = \frac{2}{\pi} \times 500 = \frac{1000}{\pi} V$ $\qquad$ $I_{avg} = \frac{2}{\pi} \times 5 = \frac{10}{\pi} A$ //

Chaitra

Q. Calculate the rms value of current.



⇒ Here,

for $0 \le \omega t \le \pi$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{18 - 0}{\pi - 0}(x - 0)$$

$$y = \frac{18x}{\pi}$$

$$i = \frac{18}{\pi}\omega t$$

for $\pi \le \omega t \le 2\pi$

$$y - 18 = \frac{0 - 18}{2\pi - \pi}(x - \pi)$$

$$y - 18 = \frac{18}{\pi}(x - \pi)$$

$$y = 18 - \frac{18x}{\pi} + 18$$

$$y = 36 - \frac{18}{\pi}x$$

$$i = 36 - \frac{18}{\pi}\omega t$$

now,

$$I_{rms} = \sqrt{\frac{1}{T_0}\int_0^T i^2 \, d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi}\left\{\frac{18}{\pi}(\omega t)\right\}^2 d\omega t + \int_{\pi}^{2\pi}\left\{36 - \frac{18}{\pi}(\omega t)\right\}^{2} d\omega t\right]}$$

$$I_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{\pi}\frac{324}{\pi^2}\times(\omega t)^2 d\omega t + \int_{\pi}^{2\pi}\left[(36)^2 - 2\times 36\times\frac{18}{\pi}(\omega t) + \frac{324}{\pi^2}(\omega t)^2 d\omega t\right]\right].$$

$$I_{rms}^2 = \frac{1}{2\pi}\left[\frac{324}{\pi^2}\cdot\frac{\pi^3}{3} + 1296\pi - \frac{1296}{\pi}\times\frac{3\pi^2}{2} + \frac{324}{\pi^2}\times\frac{7\pi^3}{3}\right]$$

$$= \frac{1}{2\pi}\left[\frac{324\pi}{3} + 1296\pi - \frac{1296\times 3\pi}{2} + \frac{324\times 7\pi}{3}\right]$$

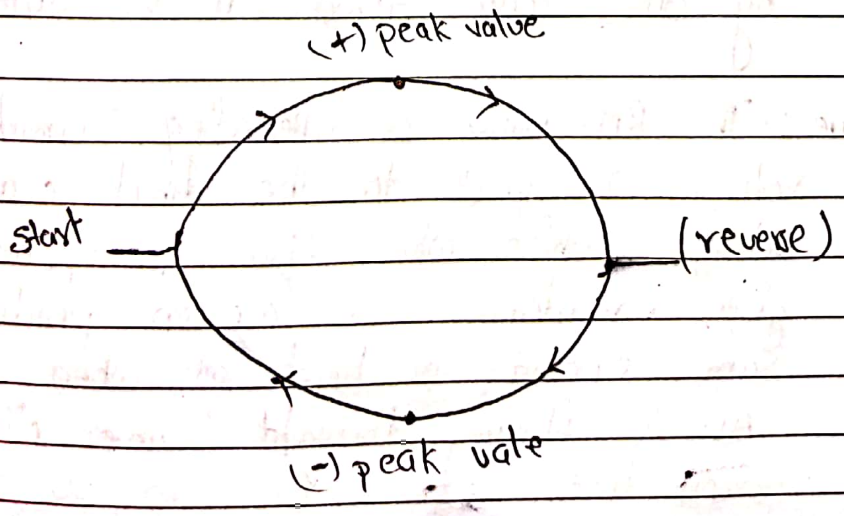
$$= \frac{1}{2\pi}\times 216\pi$$

$$I_{rms} = \sqrt{108} = 10.39\ A$$

2072

5a) Define cycle, Time, angular velocity, frequency, average & rms value of an alternating quality.

⇒ Cycle : Cycle of an alternating quality is defined as a process during which the value of alternating quantity increase from initial reaches to peak value & reverse the direction & reach the peak value in reverse direction & reaches to inital position



ii) Time period : Time taken to complete a one cycle by alternating quantity is known as time period.

iii) Angular velocity: The rate of change in angular displacement (θ) with respect to time is known as angular velocity & is denoted by ω.

$$\omega = \frac{d\theta}{dt} \text{ (instantaneous)}.$$

iv) frequency :- The no. of complete cycle made in 1 second is known as frequency. It is denoted by 'f'.

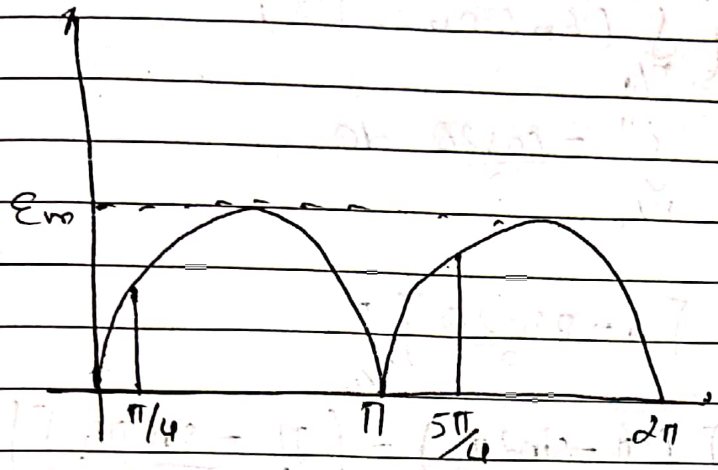
$$f = \frac{1}{T} \ Hz.$$

5) Average rate : The average value of an alternating current is equal to the value of direct current which transfers across any circuit the same amount of charge during a given time as is transfered by the same time in the same circuit.

vi) RMS value :- The RMS value of alternating current or voltage is equal to the direct current or voltage which when flow or applied to a given resistance for a given produce the same amount of heat as when AC or AV is flowing throught same circuit for same time

2072 Chaitra

3(c) Calculate the avg & rms value of full-rectified sine wave as shown below.



⇒ Solⁿ,

The wave is started from $\pi/4$, so:

$$e = 0 \quad (0 < \theta < \pi/4)$$

$$e = E_m \sin\theta \; (\pi/4 < \theta < \pi)$$

now, $$V_{avg} = \frac{1}{\pi}\left[\int_0^{\pi/4} 0\, d\theta + \int_{\pi/4}^{\pi} E_m \sin\theta\, d\theta\right]$$

$$= \frac{1}{\pi}\left[\int_{\pi/4}^{\pi} E_m \sin\theta\, d\theta\right]$$

$$= \frac{E_m}{\pi}\left[-\cos\theta\right]_{\pi/4}^{\pi}$$

$$= -\frac{E_m}{\pi}\left[\cos\pi - \cos\frac{\pi}{4}\right]$$

$$= \frac{-E_m}{\pi}\left[-1 - \frac{1}{\sqrt{2}}\right]$$

$$= 0.5433\, E_m$$

Now for RMS

$$V_{rms} = \sqrt{\frac{1}{\pi}\left[\int_0^{\pi/4} 0^2 d\theta + \int_{\pi/4}^{\pi} (E_m \sin\theta)^2 d\theta\right]}$$

$$= \frac{1}{\pi}\left[\int_{\pi/4}^{\pi} (E_m \sin\theta)^2 d\theta\right]$$

$$= \frac{E_m^2}{\pi} \int_{\pi/4}^{\pi} -\frac{\cos 2\theta}{2} d\theta$$

$$= \frac{E_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/4}^{\pi}$$

$$= \frac{E_m^2}{2\pi}\left[\left(\pi - \frac{\sin 2\pi}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin \pi/2}{2}\right)\right]$$

$$= \frac{E_m^2}{2\pi}\left[(\pi - 0) - \left(\pi/4 - \frac{1}{2}\right)\right]$$

$$V_{rms} = 0.6742\ E_m$$ #