$y^{\frac{1}{2}}$

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Assignment - 4.

Beta Gamma - Function

1) $\int_0^\infty \sqrt{y}\, e^{-y^2} dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}} dy = \frac{\pi}{2\sqrt{2}}$

=> Here,

$$\int_0^\infty \sqrt{y}\, e^{-y^2} dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}} dy$$

put $y^2 = t$

$2y\, dy = dt$

$dy = \frac{dt}{2t^{1/2}}$

When $y=0$, $t=0$, when $y=\infty$, $t=\infty$

So, $\int_0^\infty \sqrt{y}\, e^{-y^2} dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}} dy$

$$= \frac{1}{2}\int_0^\infty \frac{t^{1/4} e^{-t}}{t^{1/2}} dt \times \frac{1}{2}\int_0^\infty \frac{e^{-t}\, dt}{t^{1/4}\, t^{1/2}}$$

$$= \frac{1}{2}\int_0^\infty t^{1/4-\frac{1}{2}} e^{-t} dt \times \frac{1}{2}\int_0^\infty e^{-t} t^{-\frac{1}{4}-\frac{1}{2}} dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t} t^{-1/4} dt \times \frac{1}{2}\int_0^\infty e^{-t} t^{-3/4} dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t} t^{3/4-1} dt \times \frac{1}{2}\int_0^\infty e^{-t} t^{1/4-1} dt$$

$$= \frac{1}{2}\Gamma\left(\frac{3}{4}\right) \times \frac{1}{2}\Gamma\left(\frac{1}{4}\right)$$

$$= \frac{1}{4}\Gamma\left(\frac{1}{4}\right)\Gamma\left(1-\frac{1}{4}\right)$$

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$$= \frac{1}{4} \frac{\pi}{\sin \pi/4}$$

$$= \frac{\pi}{4 \cdot \frac{1}{\sqrt{2}}} = \frac{\pi}{2\sqrt{2}}$$

Q.2)(a) $\int_0^1 x^6 \sqrt{1-x^2}\, dx.$

$\Rightarrow$ here,

$$\int_0^1 x^6 \sqrt{1-x^2}\, dx$$

put $x^2 = t$, $2x\,dx = dt$

$$dx = \frac{dt}{2\sqrt{t}}$$

When $x=0$, $t=0$, when $x=1$, $t=1$

$$\int_0^1 x^6 \sqrt{1-x^2}\, dx$$

$$= \int_0^1 \frac{t^3 (1-t)^{1/2}}{2t^{1/2}}\, dt$$

$$= \frac{1}{2} \int_0^1 t^{5/2} (1-t)^{1/2}\, dt$$

$$= \frac{1}{2} \int_0^1 t^{\frac{7}{2}-1} (1-t)^{\frac{3}{2}-1}\, dt$$

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$$= \frac{1}{2} \beta\left(\frac{7}{2}, \frac{3}{2}\right)$$

$$= \frac{1}{2} \frac{\Gamma(\frac{7}{2})\Gamma(\frac{3}{2})}{\Gamma(\frac{7}{2}+\frac{3}{2})}$$

$$= \frac{1}{2} \frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma(\frac{1}{2})\ \frac{1}{2}\Gamma(\frac{1}{2})}{\Gamma(\frac{10}{2})}$$

$$= \frac{\frac{5}{\cancel{2}}\cdot\frac{\cancel{3}}{2}\cdot\frac{1}{2}\sqrt{\pi}\ \frac{1}{2}\sqrt{\pi}}{2\times4\times\cancel{3}\times2\times1}$$

$$= \frac{5}{256}\pi$$ #

b) $\int_0^{2a} x^5 \sqrt{2ax-x^2}\, dx$

=> Here,

$$\int_0^{2a} x^5 \sqrt{2ax-x^2}\, dx.$$

put $x = 2a\sin^2\theta$, $dx = 4a\sin\theta.\cos\theta.d\theta$

when $x=0$, $\theta=0$ when $x=2a$, $\theta=\pi/2$

$$\int_0^{2a} x^5 \sqrt{2ax-x^2}\, dx$$

$$= \int_0^{\pi/2} (2a)^5 \sin^{10}\theta \sqrt{4a^2\sin^2\theta - 4a^2\sin^4\theta}\ 4a\sin\theta.\cos\theta\, d\theta$$

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$$= 256a^7 \int_0^{\pi/2} \sin^{12}\theta \cdot \cos^{2}\theta$$

$$= 256a^7 \frac{\Gamma\left(\frac{12+1}{2}\right)\Gamma\left(\frac{2+1}{2}\right)}{2\Gamma\left(\frac{12+2+2}{2}\right)}$$

$$= 256a^7 \frac{\frac{11}{2}\cdot\frac{9}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}\cdot\frac{1}{2}\sqrt{\pi}}{2\times 7\times 6\times 5\times 4\times 3\times 2\times 1}$$

$$= \frac{33\pi a^7}{16} \quad \#$$

$\cos^2 a - \sin^2 a$

$2\cos^2 a - 1$

$1 - 2\sin^2 a$

3)a) $\int_0^{\pi/6} \cos^2 6\theta \sin^4 3\theta \, d\theta = \frac{7\pi}{192}$

let $I = \int_0^{\pi/6} \cos^2 6\theta \sin^4 3\theta \, d\theta$

$$= \int_0^{\pi/6} (\cos 2\cdot 3\theta)^2 \sin^4 3\theta \, d\theta$$

$$= \int_0^{\pi/6} (1 - 2\sin^2 3\theta)^2 \sin^4 3\theta \, d\theta$$

$$= \int_0^{\pi/6} (1 - 4\sin^2 3\theta + 4\sin^4 3\theta) \sin^4 3\theta \, d\theta$$

$$= \int_0^{\pi/6} (\sin^4 3\theta - 4\sin^6 3\theta + 4\sin^8 3\theta) \, d\theta$$

put,

$$3\theta = t$$

$$3\,d\theta = dt$$

when $\theta = 0,\ t = 0$

when $\theta = \pi/6,\ t = \pi/2$

$$= \frac{1}{3}\int_0^{\pi/2} (\sin^4 t - 4\sin^6 t + 4\sin^8 t)\, dt$$

using gamma function, we get,

$$= \frac{1}{3}\,\frac{\sqrt{\pi}\,\Gamma\left(\frac{4+1}{2}\right)}{2\,\Gamma\left(\frac{4+2}{2}\right)} - \frac{4}{3}\,\frac{\sqrt{\pi}\,\Gamma\left(\frac{6+1}{2}\right)}{\cancel{2}\,\Gamma\left(\frac{6+2}{2}\right)} + \frac{\cancel{4}^{2}\sqrt{\pi}\,\Gamma\left(\frac{8+1}{2}\right)}{3\cdot\cancel{2}\,\Gamma\left(\frac{8+2}{2}\right)}$$

$$= \frac{\sqrt{\pi}\,\frac{\cancel{3}}{2}\cdot\frac{1}{2}\sqrt{\pi}}{\cancel{3}\times\cancel{2}\times 2\times 1} + \frac{\cancel{4}^{2}\sqrt{\pi}\,\frac{7}{2}\cdot\frac{\cancel{5}}{2}\cdot\frac{\cancel{3}}{2}\cdot\frac{1}{2}\sqrt{\pi}}{3\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} - \frac{\cancel{2}\sqrt{\pi}\,\frac{5}{2}\cdot\frac{\cancel{3}}{2}\cdot\frac{1}{2}\sqrt{\pi}}{\cancel{3}\times 3\times\cancel{2}\times 1}$$

$\cancel{3\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}$

$$= \frac{\pi}{16} - \frac{5\pi}{24} + \frac{35\pi}{192}$$

$$= \frac{12\pi - 40\pi + 35\pi}{192}$$

$$= \frac{7\pi}{192}$$

\#\#

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Assignment - 4.

Q.3) (b)

$$\int_0^{\pi/4} \sin^4 x \cos^2 x \, dx = \frac{3\pi - 4}{192}$$

=> Here,

$$\int_0^{\pi/4} \sin^4 x \cos^2 x \, dx = \int_0^{\pi/4} (\sin^2 x)^2 \cos^2 x \, dx$$

$$= \int_0^{\pi/4} \left(\frac{1-\cos 2x}{2}\right)^2 \cdot \left(\frac{1+\cos 2x}{2}\right) dx$$

$$= \frac{1}{8} \int_0^{\pi/4} (1 - 2\cos 2x + \cos^2 2x)(1 + \cos 2x) \, dx$$

$$= \frac{1}{8} \int_0^{\pi/4} (1 - 2\cos 2x + \cos^2 2x + \cos 2x - 2\cos^2 2x + \cos^3 2x) \, dx$$

$$= \frac{1}{8} \int_0^{\pi/4} (1 - \cos 2x) \, dx - \frac{1}{8} \int_0^{\pi/4} (\cos^2 2x - \cos^3 2x) \, dx$$

$$= \frac{1}{8} \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/4} - \frac{1}{8} \int_0^{\pi/4} (\cos^2 2x - \cos^3 2x) \, dx$$

put, $2x = \theta$, $2dx = d\theta$

when, $x = 0$, $\theta = 0$

when, $x = \pi/4$, $\theta = \pi/2$

$$= \frac{1}{8}\left(\frac{\pi}{4} - \frac{1}{2}\right) - \frac{1}{8} \int_0^{\pi/4} (\cos^2\theta - \cos^3\theta) \frac{d\theta}{2}$$

$$= \frac{\pi}{32} - \frac{1}{16} - \frac{1}{16} \int_0^{\pi/2} (\cos^2\theta - \cos^3\theta) \, d\theta$$

$$= \frac{\pi}{32} - \frac{1}{16} - \frac{1}{16} \frac{\sqrt{\pi}\, \Gamma\left(\frac{2+1}{2}\right)}{2\, \Gamma\left(\frac{2+2}{2}\right)} + \frac{1}{16} \frac{\sqrt{\pi}\, \Gamma\left(\frac{3+1}{2}\right)}{2\, \Gamma\left(\frac{3+2}{2}\right)}.$$

$$= \frac{\pi}{32} - \frac{1}{16} - \frac{1}{32} \frac{\sqrt{\pi} \cdot \frac{1}{2} \cdot \sqrt{\pi}}{1} + \frac{1}{16} \frac{\sqrt{\pi}}{2 \cdot \frac{3}{2} \cdot \frac{1}{2} \sqrt{\pi}}.$$

$$= \frac{\pi}{32} - \frac{1}{16} - \frac{\pi}{64} + \frac{1}{24}$$

$$= \frac{\pi}{64} - \frac{1}{48}$$

$$= \frac{3\pi - 4}{192}$$

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ii) $\int_0^a x^3(a^2-x^2)^{5/2}\,dx$

let $I = \int_0^a x^3(a^2-x^2)^{5/2}\,dx$

put $x = a\sin\theta$ $\Rightarrow dx = a\cos\theta\,d\theta$

when, $x=0$ $\theta=0$ & when $x=a$ $\theta=\pi/2$

$$I = \int_0^a x^3(a^2-x^2)^{5/2}\,dx$$

$$= \int_0^{\pi/2} a^3\sin^3\theta\,(a^2-a^2\sin^2\theta)^{5/2}.\,a\cos\theta\,d\theta$$

$$= a^4\int_0^{\pi/2} \sin^3\theta.\cos\theta.\,a^5\{\cos^5\theta\}\,d\theta$$

$$= a^9\int_0^{\pi/2} \sin^3\theta.\cos^6\theta\,d\theta$$

$$= \frac{a^9\,\Gamma\left(\frac{3+1}{2}\right)\Gamma\left(\frac{6+1}{2}\right)}{2\,\Gamma\left(\frac{3+6+2}{2}\right)}$$

$$= \frac{a^9\,\Gamma 2.\,\Gamma\left(\frac{7}{2}\right)}{2\Gamma\frac{11}{2}}$$

$$= \frac{a^9\times 1\times \cancel{\frac{5}{2}\frac{3}{2}\frac{1}{2}\sqrt{\pi}}}{\cancel{2}.\frac{9}{\cancel{2}}.\frac{7}{2}.\cancel{\frac{5}{2}.\frac{3}{2}.\frac{1}{2}\sqrt{2}}}$$

$$= \frac{2a^9}{63}$$ #

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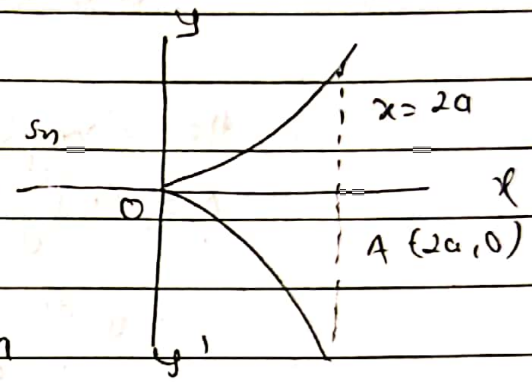
Assignment -5
[Quadrate]

Q1. Find the area of the curve $y^2(2a-x) = x^3$ & its asymptotes.

=> Sol^n

The curve $y^2(2a-x) = x^3$

The eq^n of its asymptote $x = 2a$. so required area



$$A = 2\int_0^{2a} y\,dx = 2\int_0^{2a} x\sqrt{\frac{x}{2a-x}}\,dx$$

Put $x = 2a\sin^2\theta$

$dx = 4a\sin\theta\cos\theta\,d\theta$

when $x = 0$, $\theta = 0$, when $x = 2a$, $\theta = \pi/2$.

So,

$$A = 2\int_0^{\pi/2} 2a\sin^2\theta.\sqrt{\frac{2a\sin^2\theta}{2a(1-\sin^2\theta)}}\;4a\sin\theta\cos\theta\,d\theta$$

$$= 16a^2\int_0^{\pi/2}\sin^4\theta\,d\theta$$

$$= \frac{16a^2\sqrt{\pi}\;\Gamma\left(\frac{4+1}{2}\right)}{2\Gamma\left(\frac{4+2}{2}\right)}$$

$$= \frac{8a^2\sqrt{\pi}\,\frac{3}{2}.\frac{1}{2}\sqrt{\pi}}{2.1}$$

$$= 3\pi a^2 \text{ sq. unit}$$

Ans

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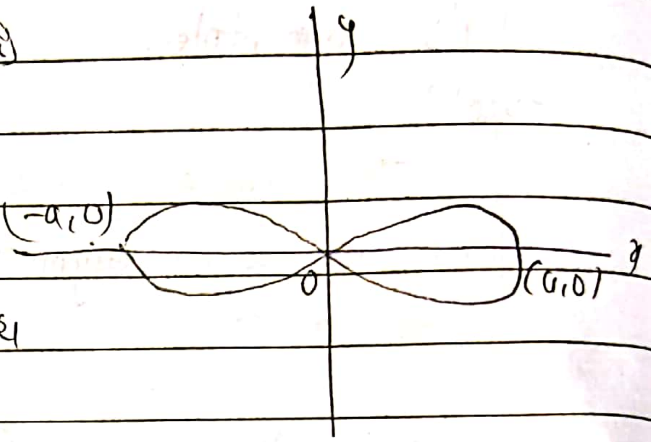
Q) Find the area of two loop of the curve
$$a^2y^2 = a^2x^2 - x^4.$$

→ sol^n,

the eq^n of the curve is
$$a^2y^2 = a^2x^2 - x^4$$



So the reqd. area
$$A = 4\int_0^a y\,dx$$

$$= 4\int_0^a \frac{x}{a}\sqrt{a^2-x^2}\,dx$$

put $a^2 - x^2 = t^2$

$$-x\,dx = t\,dt$$

when $x=0$, $t=a$, when $x=a$, $t=0$

$$= \frac{4}{a}\int_a^0 t(-t\,dt)$$

$$= -\frac{4}{a}\left[\frac{t^3}{3}\right]_a^0$$

$$= -\left[0 - \frac{a^3}{3}\right] = \frac{4}{a}\cdot\frac{a^3}{3}$$

$$= \frac{4a^2}{3} \text{ sq. unit}$$

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8. Find the area bounded by curve
$$x^2 y = a^2(a-y) \text{ \& the } x\text{-axis.}$$

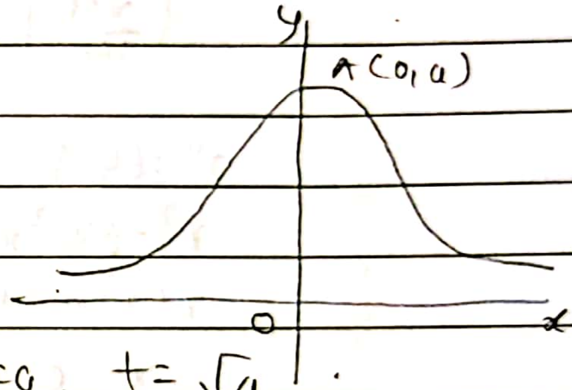
$\Rightarrow$ The eqⁿ of the curve is

$$x^2 y = a^2(a-y)$$

$$x = a\sqrt{\frac{a-y}{y}} \quad \text{for a positive \& } a > y.$$

The area bounded by the curve & x-axis,

$$A = \int_0^a x\,dx$$



$$= \int_0^a a\sqrt{\frac{a-y}{y}}\,dy$$

put $y = t^2$, $dy = 2t\,dt$

when $y = 0$, $t = 0$, when $y = a$, $t = \sqrt{a}$

$$A = a\int_0^{\sqrt{a}} \frac{\sqrt{a-t^2}}{t}(2t\,dt)$$

$$= 2a\int_0^{\sqrt{a}} \sqrt{a-t^2}\,dt$$

$$= 2a\left[\frac{t\sqrt{a-t^2}}{2} + \frac{a}{2}\sin^{-1}\left(\frac{t}{\sqrt{a}}\right)\right]_0^{\sqrt{a}}$$

$$= 2a\left[\frac{t\sqrt{a-t^2}}{2} + \frac{a}{2}\sin^{-1}\left(\frac{t}{\sqrt{a}}\right)\right]_0^{\sqrt{a}}$$

$$= 2a\left[0 + \frac{a}{2}\cdot\frac{\pi}{2} - 0\right]$$

$$= \frac{\pi a^2}{2} \text{ sq. unit}$$

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(1) Show that the area bounded by curve $y^2 = 4ax$ & $x^2 = 4ay$ is $\frac{16}{3}a^2$.

$\Rightarrow$ Sol^n.

given, parabolas $y^2 = 4ax$ & $x^2 = 4ay$ ~~is $\frac{16a^2}{3}$~~

point of intersection of two curves are

$$\left(\frac{x^2}{4a}\right)^2 = 4ax$$

$$\left(\frac{x^4}{16a^2}\right) = 4ax$$

$$x^4 = 64a^3x$$

$$x^4 - 64a^3x = 0$$

$$x(x^3 - 64a^3) = 0$$

$$x = 0 \; ; \; x = 4a$$

Also $y = 0$, $y = 4a$

the point of intersection of 2 curve are $(0,0)$ & $(4a, 4a)$

area of the two region bet^n the 2 curve

= area of the shaded region

$$= \int_0^{4a} [y_2 - y_1] \, dx$$

$$= \int_0^{4a} \left[\sqrt{4ax} - \frac{x^2}{4a} \right] dx$$

On integrating this we get

$$= \left[4a^{1/2} \frac{x^{3/2}}{3} - \frac{x^3}{12a} \right]_0^{4a}$$

$$= \left[\frac{32a^2}{3} - \frac{16a^2}{3} \right]$$

$$= \frac{16a^2}{3} \; \#$$

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5) Find the area betn the cardiod $r = a(1+\cos\theta)$ & circle $r = \frac{3}{2}a$.

⇒ Soln,

the cardiod is

$$r = a(1+\cos\theta)$$

& circle is

$$r = \frac{3a}{2}$$

Solving (1) & (2)

The common point is $\left(\frac{3a}{2}, \frac{\pi}{3}\right)$.

Area,

$$= 2\{\text{Area OCP} + \text{Area PQO}\}$$

$$= 2\left\{\frac{1}{2}\int_0^{\pi/3}\left(\frac{3a}{2}\right)^2 d\theta + \frac{1}{2}\int_{\pi/3}^{\pi} a^2(1+\cos\theta)^2 d\theta\right\}$$

$$= \int_0^{\pi/3} \frac{9a^2}{4} d\theta + a^2\int_{\pi/3}^{\pi}(1+2\cos\theta+\cos^2).d\theta$$

$$= \frac{9a^2}{4}[\theta]_{\pi/3}^{\pi} + a^2\int_{\pi/3}^{\pi}\left[1+2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right]d\theta$$

$$= \frac{9a^2}{4}\left(\frac{\pi}{3}-0\right) + a^2\left[\theta+2\sin\theta+\frac{1}{2}\theta+\frac{\sin 2\theta}{4}\right]_{\pi/3}^{\pi}$$

$$= \frac{3\pi a^2}{4} + a^2\left[\pi+0+\frac{\pi}{2}+0-\frac{\pi}{3}-2\frac{\sqrt{3}}{2}-\frac{1}{2}\cdot\frac{\pi}{3}-\frac{1}{4}\frac{\sqrt{3}}{2}\right]$$

$$= \frac{3\pi a^2}{4} + a^2\left(\pi - \frac{9\sqrt{3}}{8}\right) = \frac{7\pi a^2}{4} - \frac{9\sqrt{3}\,a^2}{8}$$

$$= a^2\left(\frac{7\pi}{4}, - \frac{9\sqrt{3}}{8}\right) \text{ sq. unit.}$$

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Assignment - 6
[Volume & Surface]

1) Find the vol. & surface area of the solid generated by the revolutate of the cardiod $r = a(1-\cos\theta)$ about initial line.

=> Sol^n

OA = 2a so that x-coordinate of A is -2a

the reqd. vol. is

$$V = \pi\int_{-2a}^{0} y^2\,dx$$

the eq^n of cardioid

$$r = a(1-\cos\theta)$$

put $x = r\cos\theta = a(1-\cos\theta)\cos\theta$

$y = r\sin\theta = a(1-\cos\theta)\sin\theta$

differentiating

$$dx = (-a\sin\theta + 2a\sin\theta\cos\theta)\,d\theta$$

when $x=0$, $\theta = 0$ & when $x=-2a$, $\theta=\pi$

$$V = \pi\int_{-2a}^{0} a^2(1-\cos\theta)^2\sin^2\theta\,(-a\sin\theta + 2a\sin\theta\cos\theta)\,d\theta$$

$$= \pi a^3\int_{\pi}^{0}(1-\cos\theta)^2\sin^2\theta\,(1-2\cos\theta)\,d\theta$$

$$= \pi a^3\int_{0}^{\pi} 4\sin^4\frac{\theta}{2}\; 8\sin^3\frac{\theta}{2}\cos^3\frac{\theta}{2}\left(3-4\cos^2\frac{\theta}{2}\right)d\theta$$

$$= 96\pi a^3\int_0^{\pi}\sin^7\frac{\theta}{2}\cos^3\frac{\theta}{2}\,d\theta - 128\pi a^3\int_0^{\pi}\sin^7\frac{\theta}{2}\cos^5\frac{\theta}{2}\,d\theta$$

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put $\theta/2 = t$, then, $\frac{1}{2}\, d\theta = dt$

when $\theta = 0,\ t = 0,\ \theta = \pi,\ t = \pi/2$

$$v = 96\,\pi a^3 \int_0^{\pi/2} 2\sin^7 t \cos^3 t \, dt - 128\,\pi a^3 \int_0^{\pi/2} 2\sin^7 t \cos^5 t \, dt$$

$$= 96\,\pi a^3 \frac{2\,\Gamma(4)\,\Gamma(2)}{2\,\Gamma(6)} - 128\,\pi a^3 \frac{2\,\Gamma(4)\,\Gamma(3)}{2\,\Gamma(7)}$$

$$= \pi a^3 \quad \frac{96 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} - 128 \cdot \frac{3 \cdot 2 \cdot 2}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}.$$

$$= \pi a^3 \left(\frac{24}{5} - \frac{32}{15} \right)$$

$$= \pi a^3 \frac{40}{15} \quad = \frac{8}{3}\,\pi a^3$$

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2) Find the vol. & surface area of the solid formed by the revolution of cycloid
$x = a(\theta + \sin\theta)$; $y = a(1-\cos\theta)$ about the target at the vertex / base / x-axis

=) the eq^n of cycloid is,

$$x = a(\theta + \sin\theta), \quad y = a(1-\cos\theta)$$

differentiating,

$$\frac{dx}{d\theta} = a(1+\cos\theta), \quad \frac{dy}{d\theta} = a\sin\theta$$

$$\frac{dy}{dx} = \frac{\sin\theta}{1+\cos\theta}$$

The surface area generated by revolving about x-axis, defined a,

$$S = \int_{-a\pi}^{a\pi} 2\pi y \frac{ds}{dx}\, dx$$

$$= \int_{-a\pi}^{a\pi} 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-\pi}^{\pi} 2\pi\, a(1-\cos\theta) \sqrt{1+\left(\frac{\sin\theta}{1+\cos\theta}\right)^2}\, a(1+\cos\theta)\, d\theta$$

$$= 2\pi a^2 \cdot 2 \int_0^{\pi} (1-\cos\theta) \frac{\sqrt{1+2\cos\theta+\cos^2\theta+\sin^2\theta}}{1+\cos\theta} (1+\cos\theta)\, d\theta$$

$$= 4\pi a^2 \int_0^{\pi} (1-\cos\theta)\sqrt{2(1+\cos\theta)}\, d\theta$$

$$= 4\pi a^2 \int_0^{\pi} 2\sin^2\theta/_2 \sqrt{4\cos^2\theta/_2}\, d\theta$$

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$$= 16\pi a^2 \int_0^{\pi} \sin^2 \theta/2 \cos \theta/2 \, d\theta$$

put $\theta/2 = t$, $d\theta = 2dt$,

when $\theta = 0$, $t = 0$, when $\theta = \pi$, $t = \pi/2$

$$S = 16\pi a^2 \int_0^{\pi/2} \sin^2 t \cos t \, . 2dt =$$

$$= \frac{32\pi a^2 \Gamma\left(\frac{2+1}{2}\right) \cdot \Gamma\left(\frac{1+1}{2}\right)}{2\Gamma\left(\frac{2+1+2}{2}\right)}$$

$$= \frac{32\pi a^2 \frac{1}{2}\sqrt{\pi} \, 1}{2 \cdot \frac{3}{2} \cdot \frac{1}{2} \sqrt{\pi}}$$

$$= \frac{32\pi a^2}{3} \text{ sq. unit}$$

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3) Show that the vol. of solid formed by revolving ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the line $x=2a$ is $4\pi^2a^2b$.

$\Rightarrow$ Sol^n,

eq^n of ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$V =$ Area of ellipse $\times$ Path traced by the C.G. of the ellipse when it is rotated about the line $x=2a$

$$\text{Area} = 4\int_0^a y\,dx$$

$$= 4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$= \frac{4b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{4b}{a}\left[\frac{a^2}{2}\frac{\pi}{2}\right]$$

$$= \pi ab.$$

C.G. of the ellipse is origin & the path traced by C.G.

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$= 2\pi . OC$

$= 2\pi 2a$

$= 4\pi a.$

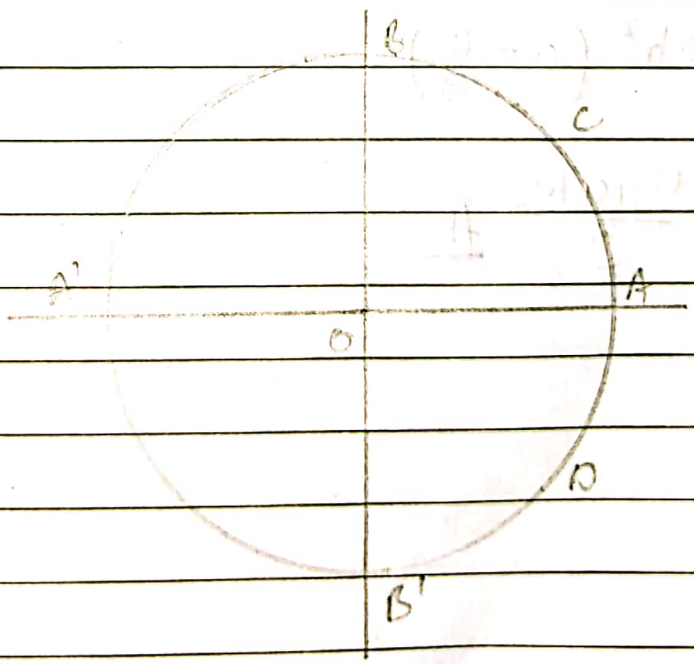
$\therefore$ req. vol. $= \pi ab . 4\pi a$

$= 4\pi^2 a^2 b$ cubic unit.

4) Find the vol. of ellipsoid solid formed by the revolution of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about x-axis.

=> given,

eq ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



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The vol. of the solid formed by the revolution of the ellipse about x-axis.

= vol. formed by the revolution of two portion ABA'

= 2 (vol. formed by the revolution of the position ACB)

$$= 2\pi \int_0^a y^2 \, dx$$

$$= 2\pi \int_0^a b^2 \left(1 - \frac{x^2}{a^2}\right) dx$$

$$= 2\pi b^2 \left[x - \frac{x^3}{3a^2} \right]_0^a$$

$$= 2\pi b^2 \left(a - \frac{a}{3}\right)$$

$$= \frac{4\pi a b^2}{3}$$ #