1. Describe about the symmetry poperty of circle. Determine the pixel positions of following curve in first quadrants using midpoint algorithm.

$$\frac{x^2}{64} + \frac{y^2}{36} = 1$$

ans:- Symmetry property of circle describes about the ability of circle to be divided into eight octants which are identical to each other. By using this property, we trace the pixels of one octant and then use quadrant symmetry and rotational symmetry to easily trace other octant pixels to form a complete circle. It reduces the computational complexity.

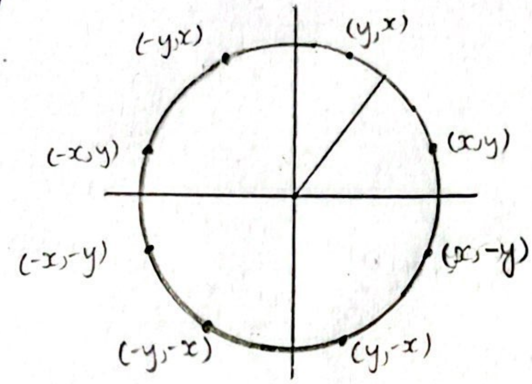


fig: Symmetry property of circle

The given curve is $\frac{x^2}{64} + \frac{y^2}{36} = 1$. Comparing with $\frac{(x-x_c)^2}{r_x^2} + \frac{(y-y_c)^2}{r_y^2} = 1$,

$(x_c, y_c) = (0,0)$

$r_x = 8$

$r_y = 6$

Now calculating the initial decision parameter,

$$P_{10} = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$

$$= 6^2 - 8^2 . 6 + \frac{1}{4} . 8^2$$

$$= -332$$

Now starting from (0,6) and using the mid point algorithm, we plot the points from the table:

K	P_{1K}	x_{K+1}	y_{K+1}	(x_{K+1}, y_{K+1})	$2r_y^2x$	$2r_x^2y$
0	-332	1	6	(1, 6)	72	768
1	-224	2	6	(2,6)	144	768
2	-44	3	6	(3,6)	216	768
3	208	4	5	(4,5)	288	640
4	-108	5	5	(5,5)	360	640
5	288	6	4	(6,4)	432	512
6	244	7	3	(7,3)	504	384

Here, as $2r_y^2x$ became greater than $2r_x^2y$, we stop computing for region 1 and now we plot for region 2. So, calculating initial decision parameter:

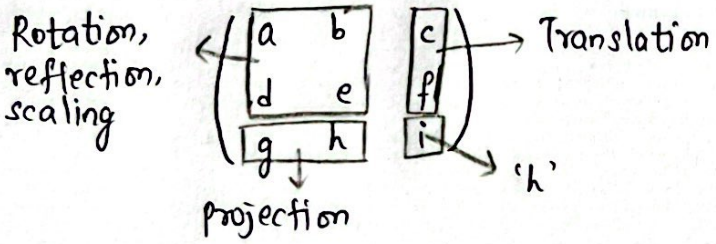
$$P_{20} = r_y^2(x_K + 1/2)^2 + r_x^2(y_K - 1)^2 + r_x^2 - r_y^2$$
$$= 6^2(7 + 1/2)^2 + 8^2(3-1)^2 + 6^2 - 8^2$$
$$= -23$$

K	P_{2K}	x_{K+1}	y_{K+1}	(x_{K+1}, y_{K+1})	$2r_y^2x$	$2r_x^2y$
0	-23	8	2	(8,2)	576	256
1	361	8	1	(8, 1)	576	128
2	297	8	0	(8,0)	576	0

Thus, the points of pixel positions are (0,6), (1,6), (2,6), (3,6), (4,5), (5,5) (6,4), (7,3), (8,2), (8,1) and (8,0).

Q. What do you mean by homogeneous coordinates? Rotate a triangle A(7,5), B(4,2), C(11,3) by 45 degree clockwise about an arbitary pivot point (1,1).

ans:- Homogeneous Coordinates are mathematical concept used in computer graphics to represent points, vector and transformation in higher - dimentional spaces. They provide convenient and effective way to perform operation like translation, scaling, rotation and perspective transformation. In 2D graphics coordinate is represented as (x,y). If points are expressed in homogeneous coordinates, we add a third coordinate h to point (x,y) as (hx, hy, h). Here h is normally set to 1. If value of h is more than 1, then all coordinates are scaled by this value.



Given,
The composite matrix to rotate a point by 45° clockwise about (1,1) is calculated as

$$\text{composite matrix (C.M.)} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & 0.7071 & -0.414 \\ -0.7071 & 0.7071 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, the co-ordinates of image will be,

$$P' = C\cdot M \times P = \begin{bmatrix} 0.7071 & 0.7071 & -0.414 \\ -0.7071 & 0.7071 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 4 & 11 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 8.071 & 3.8284 & 9.485 \\ -0.414 & -0.414 & -4.656 \\ 1 & 1 & 1 \end{bmatrix}$$

Hence, the co-ordinates of image points will be A'(8,0), B'(3,0) and C'(9,-4)

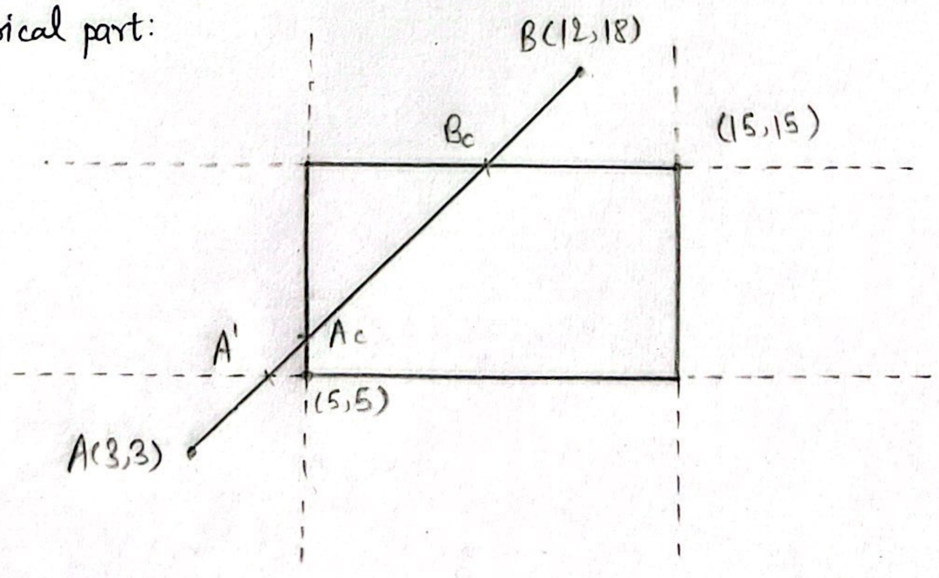
3. Compare Cohen Sutherland and Liang Barsky Line clipping algorithm. Given a clipping window with diagonal coordinates (5,5) and (15,15). Use Cohen Sutherland line clipping algorithm to clip the with endpoints A(3,3) and B (12,18).

= The Comparision between Cohen Sutherland and Liang Barsky Line clipping algorithm is given below:

Cohen Sutherland Algorithm	Liang Barsky Algorithm
• It repeatedly calculates intersection along a line path even though the line may be completely outside the clip window.	• In this algorithm, window intersections are calculated only once when final values have been computed.
• It follows the encoding approach where a point is encoded based on the position of the point.	• It follows the parametric approach by using the parametric equation of line.
• It is a less efficient method and is suitable only on a rectangular clip window.	• It is a more efficient method and is suitable for 1-D, 2-D, 3-D lines.

Numerical part:

Given



From the question

$x_{min} = 5$, $y_{min} = 5$

$x_{max} = 15$, $y_{max} = 15$

Now, using the Cohen Sutherland Algorithm, assigning code to the line points $A(3,3)$ and $B(12,18)$

$A(3,3) = 0110$
$B(12,18) = 1000$

Performing logical AND we get = 0000 so, clipping is needed. Then we get the point,

$A'(x, 5)$

Now,

$5-3 = m(x-3)$

$2 = \frac{18-3}{12-3}(x-3)$

or, $2 = \frac{15}{9}(x-3)$

$\therefore x = 4.2 \approx 4$

So, $A'(4,5)$ is the new point.

Now, assigning the code we get = 0010

then, we have the code for $B(12,18) = 1000$

Performing logical AND we get = 0000. Thus, clipping is needed. we get the point $B_c(x, 15)$.

Now,

$15-3 = m(x-3)$

or, $12 = \frac{18-3}{12-3}(x-3)$

or, $12 = \frac{15}{9}(x-3)$

$\therefore x = 10.2 \approx 10$

Now we have the point $B_c(10,15)$ with code 0000

Now we have the point $A'(4,5)$ with code 0010

performing logical AND we get 0000.

So, clipping is needed. Now, we get the point $A_c(5,y)$.

Now,

$y-3 = \frac{18-3}{12-3}(5-3)$

or $y-3 = \frac{15}{9} \times 2$

$\therefore y = 6.33 \approx 6$

Thus, we have point $A_c(6,5)$ with code 0000

As, both points have code 0000, they are the final points. Thus, the clipped part is $A_c(6,5)$ and $B_c(10,15)$

4. Write the transformation matrix for 3D rotation about X-axis. A unit length cube is sheared with respect to y-axis with shear constant =2. Obtain the co-ordinates of all the corners of the cube after shearing.

= In 3-D, x-axis rotation is represented by:

$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z = y\sin\theta + z\cos\theta$

Thus, the matrix form will be:

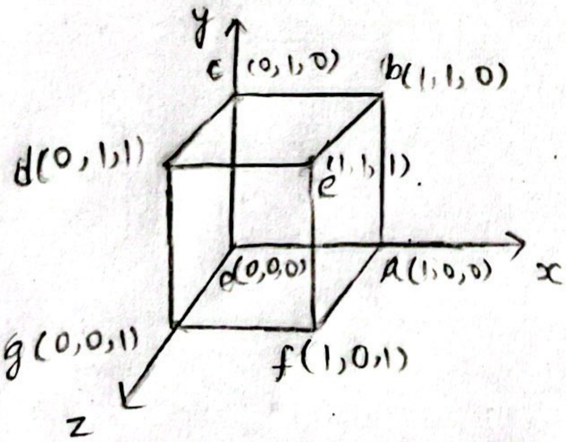
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Thus,

Transformation matrix $R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Numerical part:

Given:-



we have shear constant = 2 with respect to y-axis. It is represented by the equations below:

$x' = x + 2y$

$y' = y$

$z = z + 2y$

Thus, the matrix form is:

$(S_y) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Now, the co-ordinates of all the corners after shearing is calculated below.

$$P' = S_y \cdot P$$

$$\begin{bmatrix} x_a' & x_b' & x_c' & x_o' \\ y_a' & y_b' & y_c' & y_o' \\ z_a' & z_b' & z_c' & z_o' \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_a & x_b & x_c & x_o \\ y_a & y_b & y_c & y_o \\ z_a & z_b & z_c & z_o \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_a' & x_b' & x_c' & x_o' \\ y_a' & y_b' & y_c' & y_o' \\ z_a' & z_b' & z_c' & z_o' \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_a' & x_b' & x_c' & x_o' \\ y_a' & y_b' & y_c' & y_o' \\ z_a' & z_b' & z_c' & z_o' \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Again,

$$P' = S_y \cdot P$$

$$\begin{bmatrix} x_d' & x_e' & x_f' & x_g' \\ y_d' & y_e' & y_f' & y_g' \\ z_d' & z_e' & z_f' & z_g' \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_d' & x_e' & x_f' & x_g' \\ y_d' & y_e' & y_f' & y_g' \\ z_d' & z_e' & z_f' & z_g' \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Hence, the co-ordinates are, o'(0,0,0), a'(1,0,0), b'(3,1,2), c'(2,1,2), d'(2,1,3), e'(3,1,3), f'(1,0,1) and g'(0,0,1) *