Electromagnetism

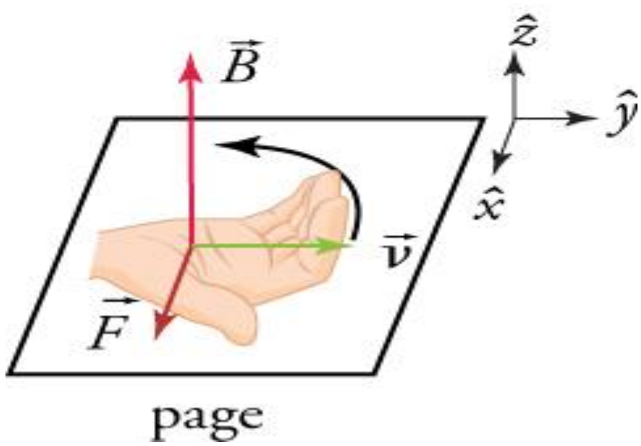
Magnetic Field(B):-

The space around a magnet around which its effect is felt is called magnetic field(B).

Magnetic Flux($\emptyset_B$):-

The total number of magnetic lines of force spreading from a magnet is called magnetic flux($\emptyset_B$).

Magnetic force:-



Magnetic force is due to the motion of electric charges. Experimentally it is found that;

1. The magnitude of magnetic force (F) exerted is directly proportional to the amount of charge (q).
2. The magnetic force depends upon the magnetic field (B).
3. The velocity of moving charge particle (v).

4. The magnetic force is proportional to sin of angle between particle velocity and magnetic field. $i.e. F \propto sin\theta$

∴ Magnetic force is given by;

$$F = Bqv \sin\theta$$

$$\overrightarrow{F} = q(\vec{v} \times \vec{B})$$

Lorentz Force:-

When a charge particle move through a region where both electric and magnetic fields are present. Then the total force is called Lorentz force.

$$i.e.\ \vec{F} = \overrightarrow{F_E} + \overrightarrow{F_B}$$

$$or, \qquad \overrightarrow{F} = q\vec{E} + q(\vec{v} \times \vec{B})$$

$$\therefore \overrightarrow{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$$

Gauss law for magnetism:-

In electrostatics Gauss law states that "the total flux through a closed surface is $1/\varepsilon_0$ times the charge enclosed by that surface.

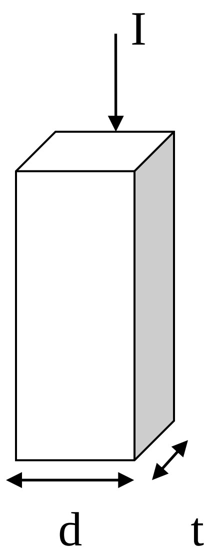
$$i.e. \oint \vec{E} \,.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$$

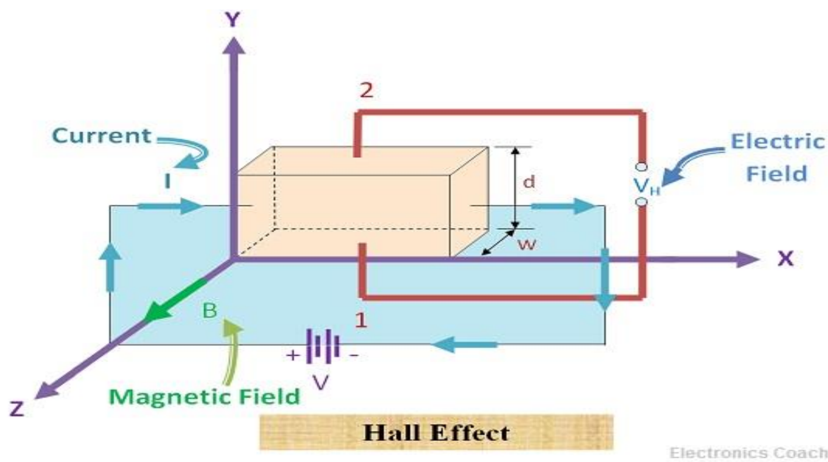
Similarly, for magnetism the total magnetic flux is proportional to the total magnetic charge but magnetic monopole does not exist as a charge in electrostatics. Thus Gauss law for magnetism can be expressed as;

$$\oint \vec{B} \,.\, \overrightarrow{dA} = 0$$

Hence, Gauss law in magnetism states that “the total magnetic flux through a closed surface is zero”.

Hall Effect:-





If a current carrying conductor is placed in a transverse magnetic field, a potential is developed in the conductor in the direction perpendicular to both current and magnetic field, this phenomena is known as Hall Effect.

Consider a rectangular strip of width ‘d’, thickness ‘t’, cross section area ‘A’ carrying current ‘I’ as shown in figure. Since, current is flowing in downward direction. So, movement of electron is in upward direction with drift velocity 'v_d'.

At equilibrium condition, the force due to magnetic field and force due to electric field are in balance. The electric field and potential difference at this condition is called hall field (E_H) and hall voltage (V_H).

$$i.e.\ eE_H = Bev_d \ldots\ldots\ldots (1)$$

Since, $J = v_d\, en$

$$or, \qquad v_d = \frac{J}{ne}$$

From equation (1),

$$eE_H = \frac{BeJ}{ne}$$

$$\frac{E_H}{JB} = \frac{1}{ne}$$

Here the term $\frac{E_H}{JB}$ is called hall coefficient (R_H).

$$\therefore\ R_H = \frac{E_H}{JB} = \frac{1}{ne} \ldots\ldots\ldots (2)$$

Again from equation (1) $eE_H = Bev_d$

Since, $E_H = \frac{V_H}{d}$

And $I = v_d enA$

$$or, \qquad v_d = \frac{I}{neA}$$

Therefore equation (1) becomes;

$$e\frac{V_H}{d} = Be\frac{I}{neA}$$

$$or, \qquad V_H = \frac{BI.d}{neA}$$

$$or, \qquad V_H = \frac{BI.d}{ne.d.t}$$

$$\therefore \text{Hall voltage } (V_H) = \frac{BI}{net} \ldots\ldots\ldots (3)$$

The mobility (μ) of charge carrier is defined as the drift velocity per unit applied electric field.

$$i.e.\ \mu = \frac{v_d}{E}$$

Since, $J = v_d\, en$

$$v_d = \frac{J}{ne}$$

$$\therefore\ \mu = \frac{J}{neE}$$

Also, $J = \sigma E$

$$\therefore\ \mu = \frac{\sigma}{ne} = \sigma R_H = \frac{R_H}{\rho} \ldots\ldots\ldots (4)$$

Now from equation (3);

$$V_H = \frac{BI}{net}$$

$$or, \qquad \frac{V_H}{I} = \frac{B}{net}$$

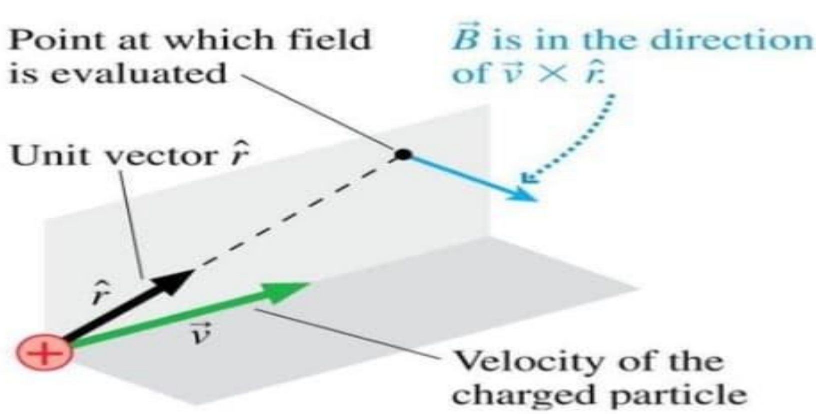
$$\therefore \text{Hall resistance } (R) = \frac{B}{net} \ldots\ldots\ldots (5)$$

From this relation it is expected to increase Hall resistance linearly with magnetic field 'B'. However Von Klitzing showed that the Hall resistance increases with 'B'. Such effect is known as quantum Hall effect.

Biot and Savart's Law:-

They derived the equation that gives the magnetic field produced due to a current carrying segment.

Magnetic field of moving point charge:-



Experimentally it is found that the magnetic field 'B' at a point at distance 'r' from the moving point charge 'q' with velocity 'v' is

1. Directly proportional to the amount of charge 'q'.
2. Directly proportional to the velocity of charge 'v'.
3. Directly proportional to the sine of angle between 'v' and 'r'.
4. Inversely proportional to the square of distance of point from moving point charge.

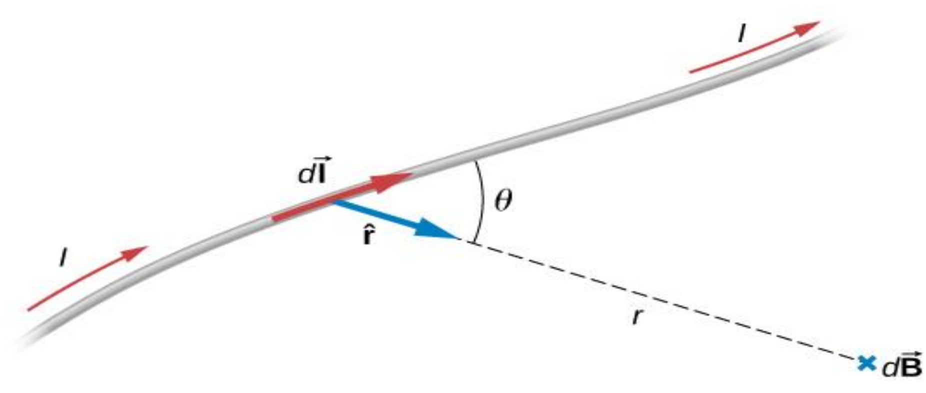
$$\therefore B \propto \frac{qvsin\theta}{r^2}$$

$$\therefore B = \frac{\mu_0}{4\pi}\frac{qvsin\theta}{r^2}$$

Where $\frac{\mu_0}{4\pi}$ is constant. μ_0 is permeability of free space.

$$i.e.\ \mu_0 = 4\pi \times 10^{-7}\ H/m$$

Magnetic field due to small current carrying element:-



Consider a short segment ‘dl’ of a current carrying conductor. The volume of the segment is $A.dl$, where A is the cross sectional area of conductor. Suppose there are ‘n’ charge particle per unit volume. Each of charge ‘q’ then the total charge in the segment is;

$$dq = nqAdl$$

Assuming all the moving charge in this segment as an equivalent single charge of velocity ‘v’, we have;

$$B = \frac{\mu_0}{4\pi} \frac{qvsin\theta}{r^2}$$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{dqvsin\theta}{r^2}$$

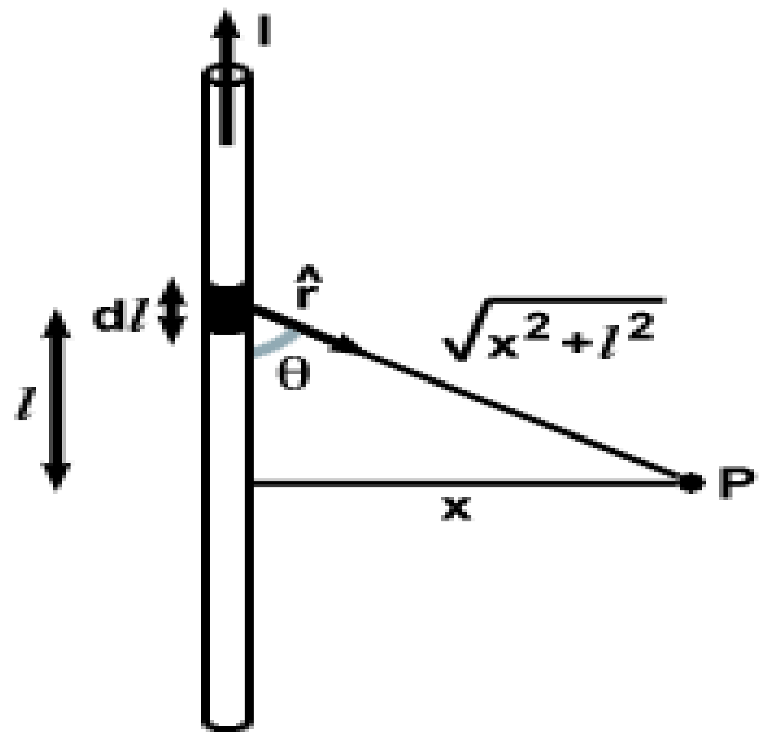
$$or, \qquad dB = \frac{\mu_0}{4\pi} \frac{nqAdlvsin\theta}{r^2}$$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{Idlsin\theta}{r^2}$$

Which is required magnetic field due to small current element.

Application of Biot and Savert law:-

1) Magnetic field due to a long straight current carrying conductor:-



Consider a long straight conductor carrying current ‘I’. ‘*dl’* is the small element on this at a distance ‘*r*’ from point ‘P’, Where the magnetic field due to conductor is to be determined. The point ‘P’ is at a distance ‘*x’* from the mid-point of conductor. Then from Biot-Savart law;

$$dB = \frac{\mu_0}{4\pi}\frac{Idlsin\theta}{r^2}$$

The total magnetic field ‘B’ at point ‘P’ due to both lower and upper half part of the conductor is;

$$B = 2\int_0^\infty dB = \frac{\mu_0 I}{2\pi}\int_0^\infty \frac{sin\theta}{r^2}\,.dl$$

From figure; $r^2 = x^2 + l^2$

And $sin\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+l^2}}$

$$\therefore B = \frac{\mu_0 I}{2\pi}\int_0^{\infty} \frac{x}{\sqrt{x^2+l^2}}\frac{1}{(x^2+l^2)}\,.dl$$

$$or, \qquad B = \frac{\mu_0 I}{2\pi}\int_0^{\infty} \frac{x}{(x^2+l^2)^{3/2}}.dl$$

Let $l = xtan\theta \ \Rightarrow dl = xsec^2\theta\ d\theta$

When, $l = 0, \theta = 0\ and\ when\ l = \infty, \theta = \pi/2$

$$\therefore\ B = \frac{\mu_0 I}{2\pi}\int_0^{\pi/2} \frac{x}{(x^2+x^2tan^2\theta)^{3/2}}.xsec^2\theta d\theta$$

$$or, \qquad B = \frac{\mu_0 I}{2\pi}\int_0^{\pi/2} \frac{x^2}{\mathrm{x^3sec^3\theta}} sec^2\theta d\theta$$

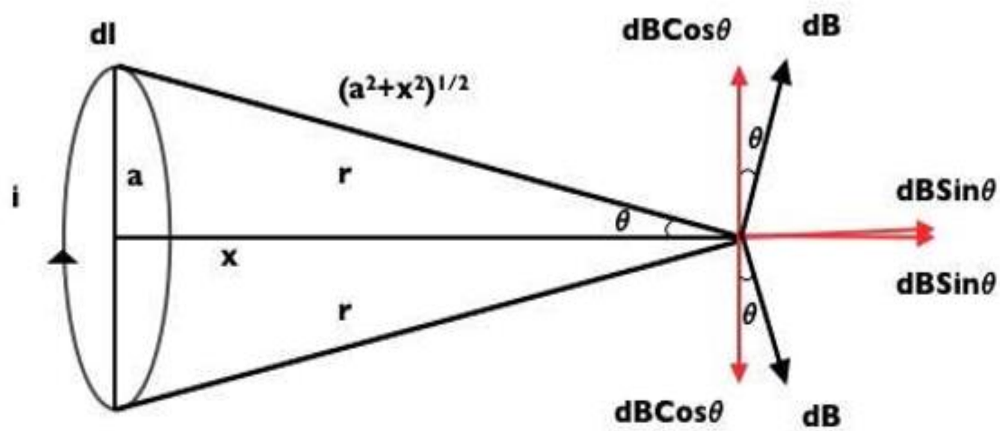
$$or, \qquad B = \frac{\mu_0 I}{2\pi x}\int_0^{\pi/2} \frac{1}{\mathrm{sec\theta}} d\theta$$

$$or, \qquad B = \frac{\mu_0 I}{2\pi x}\int_0^{\pi/2} cos\theta\, d\theta$$

$$or, \qquad B = \frac{\mu_0 I}{2\pi x}\left[[sin\theta]_0^{\frac{\pi}{2}}\right]$$

$$\therefore\ B = \frac{\mu_0 I}{2\pi x}$$

2) Magnetic field due to a current in a circular loop:-



Consider a circular conductor with radius ‘a’ carrying current ‘I’ let us take an element ‘dl’ of the conductor. From figure the angle between $\overrightarrow{dl}$ and r is 90°. therefore, magnitude of magnetic field due to element *‘dl’* at point ‘P’ at distance ‘r’ from the element is given by;

$$dB = \frac{\mu_0}{4\pi}\frac{Idlsin\theta}{r^2}$$

$$or, \qquad dB = \frac{\mu_0}{4\pi}\frac{Idlsin90}{r^2}$$

$$or, \qquad dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

$$\therefore\ dB = \frac{\mu_0}{4\pi}\frac{Idl}{x^2 + a^2}$$

If we consider another element 'dl^I' at opposite to the 'dl' the cosine components of magnetic fields due to them cancel each other. By considering such elements all round the circumference we see that the cosine component of magnetic field cancel out and only the sine components of magnetic field due to all elements constitute the total magnetic field.

$$\therefore B = \int dB sin\theta \ = \ \frac{\mu_0 I}{4\pi}\int_0^{2\pi a} \frac{dl}{\mathrm{x}^2 + \mathrm{a}^2}.sin\theta$$

$$= \ \frac{\mu_0 I}{4\pi}\int_0^{2\pi a} \frac{dl}{(\mathrm{x}^2 + \mathrm{a}^2)}.\frac{a}{(x^2 + a^2)^{1/2}}$$

$$= \frac{\mu_0 I}{4\pi}\int_0^{2\pi a} \frac{adl}{(x^2 + a^2)^{3/2}}$$

$$= \frac{\mu_0 Ia}{4\pi(x^2 + a^2)^{3/2}}.[l]_0^{2\pi a}$$

$$= \frac{\mu_0 Ia}{4\pi(x^2 + a^2)^{3/2}}.2\pi a$$

$$\therefore B = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

If the coil consists 'N' loop having same radius then;

$$\therefore B = \frac{\mu_0 N I a^2}{2(x^2 + a^2)^{3/2}}$$

The magnetic field at *x = 0*, i.e. at the centre of the coil is maximum and is given by;

$$B = \frac{\mu_0 N I}{2a}$$

3) Magnetic field due to a current in a circular arc of wire (curved wire segment):-

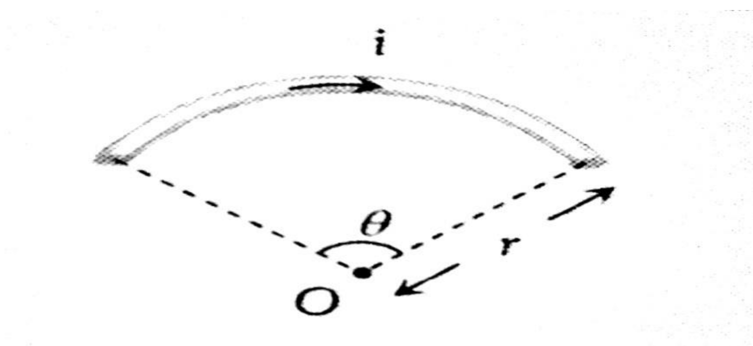


Figure shows a circular arc of wire with central angle $'\theta'$, radius 'r' and centre 'O' carrying current 'I' as shown in figure. At 'O' each current element of the wire produces a magnetic field given by;

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$

The angle between vector $\overrightarrow{dl}$ and 'r' is 90^0 as shown in figure.

$$\therefore dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

Here, $dl = rd\theta$

$$or, \qquad dB = \frac{\mu_0}{4\pi}\frac{Ird\theta}{r^2}$$

$$\therefore dB = \frac{\mu_0 I}{4\pi r}d\theta$$

The total magnetic field due to arc with central angle θ is;

$$B = \int_0^\theta dB = \int_0^\theta \frac{\mu_0}{4\pi}\frac{Id\theta}{r}$$

$$\therefore B = \frac{\mu_0 I}{4\pi r}.\theta$$

This is the expression for magnetic field due to the circular arc at its centre. The magnetic field due to full circle of current is;

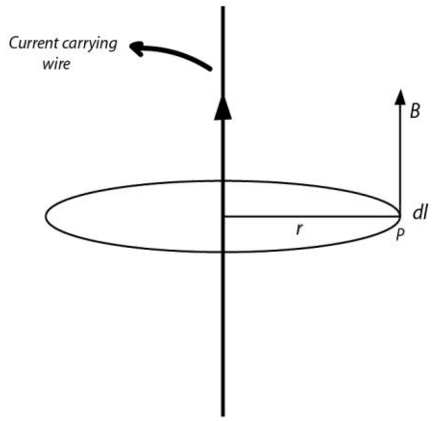
$$B = \frac{\mu_0 I}{4\pi r}.2\pi$$

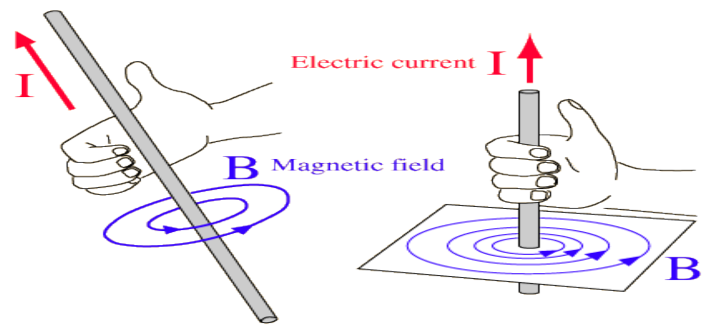
$$\therefore B = \frac{\mu_0 I}{2r}$$

If there are N number of circle,

$$B = \frac{\mu_0 NI}{2r}$$

Ampere's theorem:-





Ampere's law is used to find the magnetic field due to symmetric current distribution for which line integral of B can be determined. The magnetic field due to straight current carrying conductor at point 'P' at distance 'R' from the conductor is;

$$B = \frac{\mu_0 I}{2\pi R}$$

This value of magnetic field is same for all the points at distance 'R' in the circle of radius 'R'. The direction of magnetic field at any point on the circle is given by the tangent drawn at that point. This means the magnetic field 'B' and the line element 'dl' are along the same direction at that point. Therefore angle between 'B" and 'dl' is zero.

$$\oint \vec{B}.\overrightarrow{dl} = \oint Bdlcos\theta = \oint Bdl = \oint_{0}^{2\pi R} \frac{\mu_0 I}{2\pi R} dl = \frac{\mu_0 I}{2\pi R} . 2\pi R$$

$$\therefore \oint \vec{B}.\overrightarrow{dl} = \mu_0 I$$

Application of Ampere's theorem:-

Magnetic field due to long straight current carrying conductor:-

1. Outside the straight conductor:-

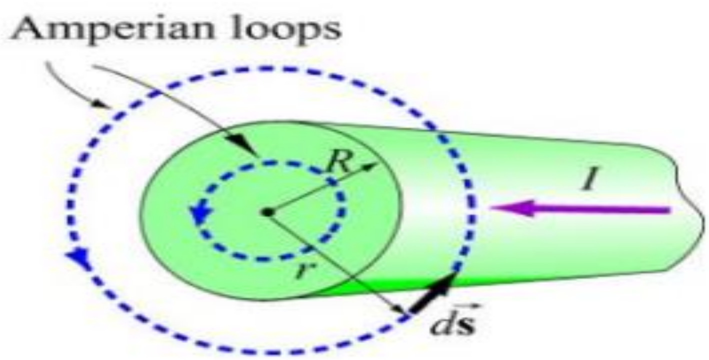


Figure shows a cross-section of long straight wire carrying current 'I'. The magnitude of 'B' has same value on the Amperion loop of radius 'r'. The Amperion loop can be drawn such that it contains the point where magnetic field is to be determined.

From Ampere's theorem;

$$\oint \vec{B}.\vec{dl} = \mu_0 I$$

$$or, \qquad \oint \vec{B}\vec{dl}\, cos\theta = \mu_0 I$$

The tangent drawn at ‘dl’ gives the direction of ‘B’ and has same direction as ‘dl’.

$$\therefore \oint \vec{B}\vec{dl}\, cos\theta = B\oint dl\,.cos0 = B\oint_0^{2\pi r} dl = B.2\pi r$$

$$\therefore B.2\pi r = \mu_0 I$$

$$\therefore B = \frac{\mu_0 I}{2\pi r}$$

2. Inside the straight conductor:-

In this case the radius of Amperion loop ‘r’ is smaller than that of wire. Let the current enclosed by Amperion loop be ‘I^1’.

From Ampere’s theorem;

$$\oint \vec{B}.\vec{dl} = \mu_0 I^I$$

$$or, \qquad \oint \vec{B}\vec{dl}\, cos\theta = \mu_0 I^I$$

The tangent drawn at ‘dl’ gives the direction of ‘B’ and has same direction as ‘dl’.

$$\therefore \oint \vec{B}\vec{dl}\, cos\theta = B\oint dl\,.cos0 = B\oint_{0}^{2\pi r} dl = B.2\pi r$$

$$\therefore B.2\pi r = \mu_0 I^I$$

$$\therefore B = \frac{\mu_0 I^I}{2\pi r}$$

Since, the current density is same inside Amperion loop and outside of Amperion loop.

$$i.e.\ \frac{I}{\pi R^2} = \frac{I^I}{\pi r^2}$$

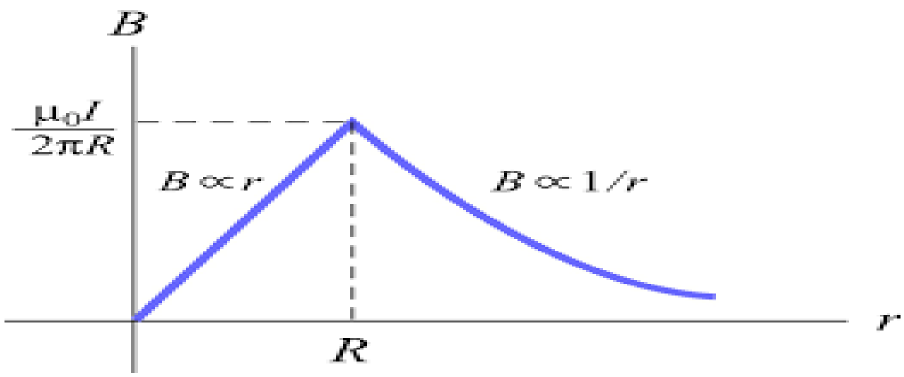
$$I^I = \frac{r^2}{R^2}.I$$

$$\therefore B = \frac{\mu_0}{2\pi r}\frac{r^2}{R^2}.I$$

$$\therefore B = \frac{\mu_0}{2\pi}\frac{r}{R^2}.I$$

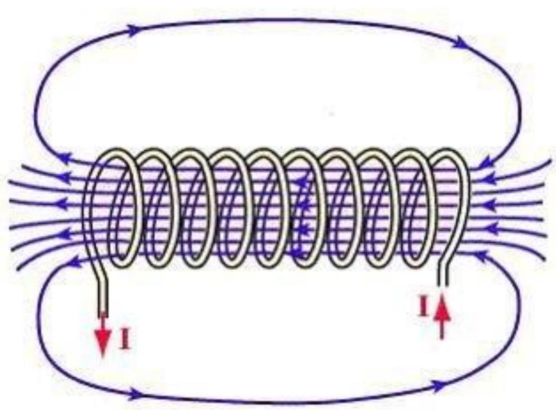
This shows that magnetic field is zero at the centre and maximum at the surface where r = R.

$$i.e.\ B_{max} = \frac{\mu_0 I}{2\pi R}$$



Magnetic field due to a solenoid:-

A solenoid is a long coil of wire made of many loops, each producing a magnetic field. Inside the solenoid, the magnetic field is parallel to the axis. Outside the solenoid, the magnetic field is zero.



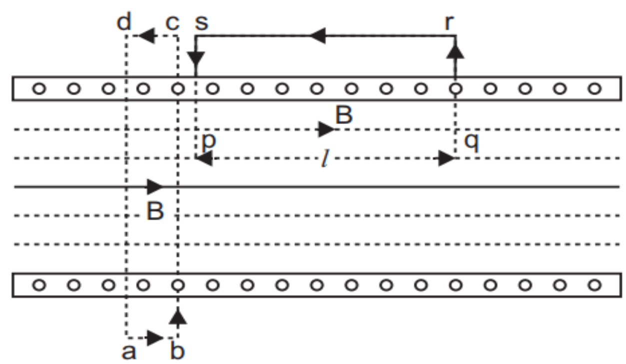


Figure:- (i) Magnetic field lines for a solenoid of finite length and (ii) application of Ampere's law in a section of a solenoid.

At a point outside the solenoid, the magnetic fields due to neighboring loops oppose each other and at a point inside the solenoid, the magnetic fields are in the same direction. As a result of it the effective magnetic field outside the solenoid becomes zero, where as the magnetic field inside the solenoid becomes strong and uniform and acting along the axis of solenoid.

The line integral of magnetic field $\vec{B}$ over the closed path $pqrs$ is;

$$\oint \vec{B}.\vec{dl} = \oint_p^q \vec{B}.\vec{dl} + \oint_q^r \vec{B}.\vec{dl} + \oint_r^s \vec{B}.\vec{dl} + \oint_s^p \vec{B}.\vec{dl}$$

$$\oint \vec{B}.\vec{dl} = \oint_p^q Bdl\cos 0 + \oint_q^r Bdl\cos 90 + \oint_r^s 0.dl + \oint_s^p Bdl\cos 90$$

The first integral on the right of above equation is Bl, where B is the magnitude of the uniform magnetic field inside the solenoid and l is the length of segment from $p\ to\ q$. The second and fourth integrals are zero because for every element dl of these segments, $\vec{B}$ either is perpendicular to dl or is zero. And thus the product $\vec{B}.\vec{dl}$ is zero. The third integral, which is taken along a segment that lies outside the solenoid, is zero because $B = 0$ at all external points.

$$\therefore \oint \vec{B}.\vec{dl} = Bl + 0 + 0 + 0 = Bl$$

Total current through the rectangle $pqrs$ is;

$$\Sigma I = \text{number of turns in rectangle } \times \text{current}$$

$$\Sigma I = nlI$$

From Ampere's circuital law;

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \Sigma I$$

$$Bl = \mu_0 nlI$$

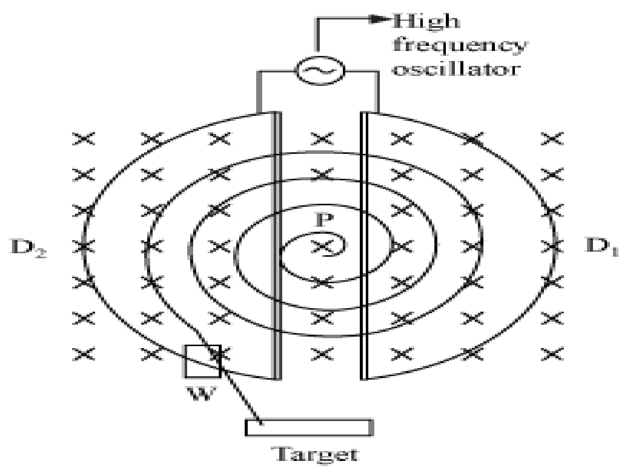
$$\therefore B = \mu_0 nI$$

$$or, \quad B = \frac{\mu_0 NI}{l}$$

Which is required magnetic field due to solenoid.

Cyclotron:-

The cyclotron is a particle accelerator which is consists of two large dipole magnets designed to produce a semicircular region of uniform magnetic field, directed uniformly downward.



Two Dee's are placed back- to back with their straight sides parallel but slightly separated. Now, in order to produce an electric field across this gap we apply an oscillating voltage. Particles, which are injected in to the magnetic field region of D, trace out the semicircular path until they reach the gap. However, as the particles pass across the gap they are accelerated by the applied electric field. After gaining energy, these particles follow a semicircular path in the next D with larger radius.

The cyclotron uses electric and magnetic fields and the whole accelerator remains in a uniform magnetic field. Here the Lorentz force due to the magnetic field provides the centripetal force for the circular motion with radius R. It means for a charged particle of charge q and mass m circulating with velocity v is, $Bqv = \frac{mv^2}{R}$.

$$\therefore R = \frac{mv}{Bq}, \quad or, v = \frac{qBR}{m}$$

The time period of motion is;

$$T = \frac{2\pi R}{v} = \frac{2\pi}{v} . \frac{mv}{qB} = \frac{2\pi m}{qB}$$

Now, the frequency of the cyclotron is given by;

$$f = \frac{qB}{2\pi m}$$

The kinetic energy of the charged particle is;

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m(\frac{qBR}{m})^2$$

$$\therefore E = \frac{B^2q^2R^2}{2m}$$

Also,

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m(R\omega)^2$$

$$= \frac{1}{2}mR^2 4\pi^2 f^2 \quad (\because \ \omega = 2\pi f)$$

$$\therefore E = 2mR^2\pi^2 f^2$$

$$\text{So,} \qquad E \propto R^2, \quad E \propto B^2 \text{ and } E \propto f^2$$

It is clear from this expression that, for obtaining high energy, the strength of the magnetic field should be large and radius of machine also be as large as possible.

Limitation:-

We have,

$$T = \frac{2\pi m}{qB} \text{ and } f = \frac{qB}{2\pi m}$$

The cyclotron fails to operate at high energies because one of its assumptions that the frequency of rotation of an ion circulating in a magnetic field is independent of its speed is true only for speeds much less than that of light. As the particle speed increases, we must use the relativistic mass ‘m’ in above equation.

$$f = \frac{qB\sqrt{1-\frac{v^2}{c^2}}}{2\pi m_0}$$

a) If frequency varies with field variation then the cyclotron is called **synchrocyclotron.**

b) If frequency remains constant with field variation then the cyclotron is called **Synchrotron**.

Because product of B and $\sqrt{1-\frac{v^2}{c^2}}$ always remains constant.

Numerical Examples:-

1.In a hall experiment a current of $3A$ is sent length wise through a conductor $1\,cm$ width $4\,cm$ long and $10\,\mu m$ thick produces a transverse hall voltage of $10\,\mu V$. When a magnetic field of $1.5\,Tesla$ is passed perpendicularly through the thickness of conductor. From these data find (i) The drift velocity of charge carrier and (ii) number density of charge carrier.

Solution:-

Current $(I) = 3A$

Length $(L) = 4\ cm = 0.04m$

Width $(d) = 1\ cm = 0.01\ m$

Thickness $(t) = 10\ \mu m = 10 \times 10^{-6} m$

Hall voltage $(V_H) = 10\ \mu V = 10 \times 10^{-6} V$

Magnetic field $(B) = 1.5\ T$

(i) We have, $eE_H = Bev_d$

$$v_d = \frac{E_H}{B} = \frac{V_H}{d.B} = \frac{10 \times 10^{-6}}{0.01 \times 1.5}$$

$\therefore$ drift velocity $(v_d) = 6.67 \times 10^{-4}\ m/sec$

(ii) hall voltage $(V_H) = \frac{B.I}{net}$

$$or, n = \frac{B.I}{V_H et} = \frac{1.5 \times 3}{10 \times 10^{-6} \times 1.6 \times 10^{-19} \times 10 \times 10^{-6}}$$

$$\therefore n = 2.8 \times 10^{29}\ electrons/m^3$$

2. A copper strip 2 cm wide and 1 mm thick is placed in a magnetic field of $1.5\ T$. If a current of $200\ A$ is set up in the strip, Calculate (i) hall voltage and (ii) hall mobility, if the number of electron per unit volume is $8.4\times10^{28}/m^3$ and resistivity $1.72 \times 10^{-8} \Omega m$.

Solution:-

Width $(d) = 2\ cm = 0.02\ m$

Thickness $(t) = 1mm = 1 \times 10^{-3} m$

Magnetic field $(B) = 1.5\ T$

Current $(I) = 200\ A$

Number of electron per unit volume(n) = $8.4 \times 10^{28}/m^3$

Resistivity $(\rho) = 1.72 \times 10^{-8} \Omega m$

We have, hall voltage $(V_H) = \dfrac{BI}{net}$

$$= \frac{1.5 \times 200}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 1 \times 10^{-3}}$$

$$= 2.23 \times 10^{-5}\ V$$

Now, hall mobility $(\mu) = \dfrac{\sigma}{ne} = \dfrac{1}{\rho ne}$

$$= \frac{1}{1.72 \times 10^{-8} \times 8.4 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= 4.32 \times 10^{-3}\ \Omega^{-1} m^2 C$$

3.The magnetic field on the axis of a current carrying circular loop of radius R, at distance $x \gg R$ from centre of the loop is B. If the radius of loop is double, keeping the current unchanged. Find the magnetic field at a point at $x \gg R$.

Solution:-

We have magnetic field at distance x from centre of the coil is;

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

According to the question $x \gg R$,

$$\therefore B = \frac{\mu_0 I R^2}{2x^3}$$

When $R^I = 2R$

$$\therefore\ B^I = \frac{\mu_0 I (2R)^2}{2x^3} = 4\frac{\mu_0 I R^2}{2x^3}$$

$$\therefore\ B^I = 4B$$

4. In the hydrogen atom the electron moves around the proton with a speed of $2 \times 10^6\ m/sec$. In a circular orbit of radius $5 \times 10^{-11}\ m$. What is the strength of magnetic field at the centre of the orbit?

Solution:-

Velocity $(v) =\ 2 \times 10^6\ m/sec$

Radius $(r) = 5 \times 10^{-11}\ m$

The time period of electron in the circular orbit is;

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 5 \times 10^{-11}}{2 \times 10^{6}} = 1.57 \times 10^{-16}\ sec$$

$$\therefore \text{current } (I) = \frac{e}{T} = \frac{1.6 \times 10^{-19}}{1.57 \times 10^{-16}}$$

∴ Magnetic field at the centre of the orbit is;

$$B = \frac{\mu_0 I}{2r} = \frac{4\pi \times 10^{-7} \times 1.6 \times 10^{-19}}{2 \times 5 \times 10^{-11} \times 1.57 \times 10^{-16}}$$

$$\therefore B = 12.8\ T$$

5. A coil of $50\ turns$ and $10\ cm$ diameter is made out of wire of resistivity $2 \times 10^{-6}\ \Omega cm$. The coil is connected to a source of emf $10\ V$ and negligible internal resistance. (i) Find the current through the coil. (ii) What must be the current and potential difference in the coil, when $B = 0.314 \times 10^{-4}\ T$ at the centre of the coil?

Solution:-

Number of turns $(N) = 50\ turns$

Diameter $(d) = 10\ cm = 0.1\ m$

∴ Radius $(r) = 0.05\ m$

Resistivity $(\rho) = 2 \times 10^{-6}\ \Omega cm = 2 \times 10^{-8}\ \Omega m$

∴ Resistance of coil $(R) = \frac{\rho l}{A}$

$$or, \qquad R = \frac{\rho \times 2\pi r . N}{\pi r^2} = \frac{2 \times 2 \times 10^{-6} \times 50}{(0.05)^2}$$

$$= 4 \times 10^{-5}\ \Omega$$

(i) current $(I) = \dfrac{V}{R} = \dfrac{10}{4 \times 10^{-5}} = 2.5 \times 10^5\ A$

(ii) Magnetic field $(B) = \dfrac{\mu_0 N I}{2r}$

$$\therefore I = \frac{2Br}{\mu_0 N} = \frac{2 \times 0.314 \times 10^{-4} \times 0.05}{4\pi \times 10^{-7} \times 50} = 5 \times 10^{-2}\ A$$

The potential difference across the coil is;

$$V = IR$$

$$= 5 \times 10^{-2} \times 4 \times 10^{-5}$$

$$\therefore V = 2 \times 10^{-6}\ V$$

6. A long coil consisting of $50\ turns$ with diameter $1.2\ m$ carries a current of $10\ A$. (i) Find a magnetic field at a point along the axis $90\ cm$ from the centre. (ii) At what distance from the centre along the axis the field is $(\frac{1}{8})^{th}$ as the centre.

Solution:-

Number of turns $(N) = 50$

Radius $(R) = 0.6\ m$

Current $(I) = 10\ A$

Distance $(x) = 90\ cm = 0.9\ m$

(i) We know that;

$$B = \frac{\mu_0 NIR^2}{2(x^2 + R^2)^{3/2}}$$

$$= \frac{4\pi \times 10^{-7} \times 50 \times 10 \times (0.6^2)}{2(0.9^2 + 0.6^2)^{3/2}}$$

$$\therefore B = 8.93 \times 10^{-5}\ T$$

(ii) Magnetic field at centre;

$$B^I = \frac{\mu_0 NI}{2R} = \frac{\mu_0 NIR^2}{2R^3}$$

Let at a distance x from the centre of coil, the field is one eighth as the centre.

$$\frac{\mu_0 NIR^2}{2(x^2 + R^2)^{3/2}} = \frac{1}{8}\frac{\mu_0 NIR^2}{2R^3}$$

$$or, (x^2 + R^2)^{\frac{3}{2}} = 8R^3$$

$$or, x^2 + R^2 = (8R^3)^{2/3}$$

$$or, \quad x^2 + R^2 = 2R^2$$

$$or, x^2 = 3R^2$$

$$\therefore x = 3 \times (0.6)^2 = 1.039\ m$$

7. A solenoid is $1\ m$ long and $3\ cm$ in diameter. It has 5 layer of winding of $850\ turns$, carries a current of $5\ A$. (i) Find magnetic field at its centre. (ii) Calculate the magnetic flux for a cross section of solenoid at its centre.

Solution:-

(i) Magnetic field at its centre is;

$$B = \mu_0 nI = \mu_0 \frac{N}{l} I$$
$$= 4\pi \times 10^{-7} \times 5 \times 850 \times 5$$
$$\therefore B = 0.02669\ T$$

(ii) Magnetic flux of solenoid is;

$$\emptyset_B = B \times A$$
$$= 0.02669 \times \pi r^2$$
$$= 0.02669 \times 3.14 \times (1.5)^2$$
$$\therefore \emptyset_B = 1.88 \times 10^{-5}\ weber$$

8. A solenoid $1.3\ m$ long and $2.6\ cm$ in diameter, carries a current of $18\ A$. The magnetic field inside the solenoid is $23\ Mili\ tesla$. Find the length of the wire forming the solenoid.

Solution:-

Length $(l) = 1.3\ m$

Diameter $(d) = 2.6\ cm = 0.026\ m$

Radius $(r) = 0.013\ m$

Current $(I) = 18\ A$

Magnetic field $(B) = 23\ mT = 23 \times 10^{-3}\ T$

Now magnetic field for solenoid is;

$$B = \mu_0 n I$$

$$n = \frac{B}{\mu_0 I} = \frac{23 \times 10^{-3}}{4\pi \times 10^{-7} \times 18}$$

$$\therefore n = 1016.82$$

Total number of turns is;

$$N = n \times l$$

$$= 1016.82 \times 1.3$$

$$\therefore N = 1322$$

Length of wire is;

$$L = 1322 \times 2\pi r$$

$$= 1322 \times 2 \times 3.14 \times 0.013$$

$$\therefore \text{lenth of wire} = 108 \text{ m}$$

Exercise:-

1. What is Hall Effect? Write its importance. Show that the coefficient $R_H = \frac{-1}{ne}$, Where the symbols have their own meanings.
2. What type of particles can be accelerated by a cyclotron? Explain the working of cyclotron and synchrotron with their differences.
3. Using Ampere's law, calculate the magnetic field inside, outside and on the surface of a long current carrying

conductor and hence plot a graph Between magnetic field versus distance from the center of the conductor.

4. Explain Biot-Savart law. Show that a current carrying circular coil behaves as a magnetic dipole for a large distance.
5. Derive an expression for Hall voltage. How do you differentiate the type of charge carrier from the result of Hall experiment? What is Hall resistance?
6. What is Hall Effect? Derive an expression for the Hall coefficient and established the relation between mobility of charge carrier and conductivity of material of wire.
7. What is Ampere's law? Derive the expression for magnetic flux density outside and inside a long straight conductor carrying current I.
8. Obtain an expression for magnetic field intensity due to a circular coil carrying current at its axial point.
9. Explain Hall Effect. Derive a relation for Hall resistance. From this relation explain the meaning of quantization of hall resistance.
10. Consider a circular coil of radius R carrying current I. Find the magnetic field at any point on the axis of the loop at a distance z from the center of the loop. Show that the circular current coil behaves as a magnetic dipole for large distance.
11. Compare the methods of Biot- Savart law and Ampere's law to calculate magnetic fields due to current carrying conductor. Calculate magnetic field at an axial

distance ‘x’ from the center of the circular coil carrying current.

12. Describe the principle and working of cyclotron. Show that the time taken by the ion in a Dee to travel a semicircle is exactly same whatever be its radius and velocity.

13. Deuterons in a cyclotron describe a circle of radius 0.32 m just before emerging from Dees. The frequency of the applied emf is 10 MHz. Find the flux density of the magnetic field and energy of the deuterons emerging out of the cyclotron. (mass of deuterons $= 3.32 \times 10^{-27}$ kg.)s

14. A copper strip 3 cm wide and 2 mm thick is placed in a magnetic field of 1.75 T. If a current of 150 A is set up in the strip, Calculate (i) hall voltage and (ii) hall mobility, if the number of electron per unit volume is $8.4 \times 10^{28}\ \text{electrons/m}^3$ and resistivity$1.72 \times 10^{-8}\Omega$m.

15. In a Hall Effect experiment, a current of 3.2 A lengthwise in a conductor 1.2 cm wide, 4.0 cm long and 9.5 µm thick produces a transverse hall voltage (across the width) of 40 µV when a magnetic field of 1.4 T is passed perpendicularly through the thin conductor. From these data, find (a) The drift velocity of the charge carriers and (b) the number density of charge carriers.

16. In a Hall experiment a current of 25 A is passed through a long foil of silver, which is 0.1 mm thick and 3 m long. Calculate the Hall voltage produces across the width by a flux of 1.4 Wb/m^2. If the conduction of silver is $6.8 \times 10^7\ \text{mho/m}$, estimate the mobility of electron in silver.

17. A long circuit coil consisting of 50 turns with diameter 1.2 m carries a current of 10 A. (a) Find the magnetic field at a point along the axis 90 cm from the center. (b) At what distance from the center, along the axis, the field is 1/8 greater as at the center.

18. A copper strip 150 μm thick is placed in a magnetic field of strength 0.65 T perpendicular to the plane of the strip and current of 23 A is set up in the strip. Calculate (i) The Hall voltage (ii) Hall coefficient and (iii) Hall mobility, if the number of electron per unit volume is is $8.4 \times 10^{28}\ \mathrm{electrons/m^3}$ and resistivity$1.72 \times 10^{-8}\Omega$m.