Chapter - 7

Assignment 7

1) Explain the scope of heat transfer in engineering

⇒ Scope of heat transfer of engineering

- Internal combustion engine
- Refrigeration & air conditioning units.
- Design of cooling system for electric motor, generators & transfer.
- Thermal control of space vehicle.
- Dispersion of atomspheric pollutant.

2) Difference betn thermodynamics of heat transfer with eg:

Thermodynamics	Heat transfer.
It deals with the equilibrium states of matter.	It is inherently a non-equilibrium process
There should not be a temp^r gradient	Temp. gradient must exist for ~~example~~ exchange of heat to take place.
It helps to determine the quantity of works heat interaction when a system change from one equilibrium state to another.	It helps to predic the distribution of temp. of temp. & to determine.

Scanned with CamScanner

3) Diff. betn steady state & unsteady state heat transfer.

Steady state heat transfer	Unsteady state heat transfer.
• There is no change in heat transfer with time at any point in direction within the medium is called steady state heat transfer.	• Heat transfer change with time.

mathematically,

$$\frac{\partial z}{\partial T} = 0$$

Q4) Explain the different modes of heat transfer.

i) Conduction

Conduction is the transfer of heat from one part of a substance to another part of the same substance or from one substance to another in physical contact with it. Without app. displacement of molecules forming the substance.

Fourier law of heat conduction states that "The rate of flow of heat through a simple homogenous solid is directly proportional to the area of the section at ~~origh~~ right angles to the direction of heat flow, & to temp'r gradient

Convection

Convention is the transfer of heat within a fluid by mixing of one portion of the fluid with another

The rate eqn for the convection heat transfer betn a surface is an adjacent fluid is pre.described

by Newton law of cooling.

$$Q = h \times A (t_s - t_f)$$

where, h = coeff. of condective heat transfer

Radiation

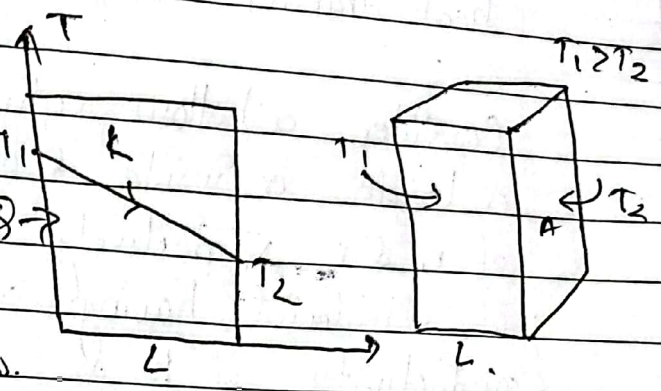
Radiation is mode of heat transfer which doesnot required any medium ie. heat transfer takes place in vacuum. in the form of electromagnetic wave. In radiation, it is the temp. of emitting surface that control the quality of energy transmitted

Q.5. Diff. betn forced & Free convection.

Free convection	Forced convection
• Heat transfer process in which flow of fluid is cause by density gradient	• Heat transfer process in which flow of fluid is caused by some external device such as pump Fan, blower etc.
• For examples: cooling of a room without a fan by natural circulation of air	• For exam cooling of room of a fan.
• During the free convection flow rate of fluid in usually low as heat transfer take place at very low rate.	• Heat transfer rate is relatively higher in forced convection due to increased mass flow rate of fluid

Scanned with CamScanner

Q.6) Derive an expression for stead istate conduction heat transfer through plane wall



Consider a plane wall made up meterial having a thermal conductivity of k, thickness L, cross sectional area of A as shown in figure. It is oppo. face are exposed to uniform temp. T_1 & T_2 respectively. Here, the temp^r gradient exist only in one ie x-direction it is one dimensional steady state heat conduction.

Now, by fourier law.

$$\dot{Q} = -kA\frac{dT}{dx}$$

$$Q\,dx = -kA\,dT$$

Integrating with associated boundary conditions

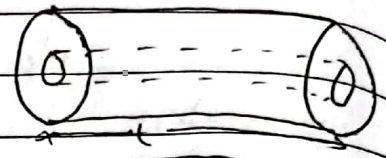
$$Q\int_0^L dx = -\int_{T_1}^{T_2} kA\,dT$$

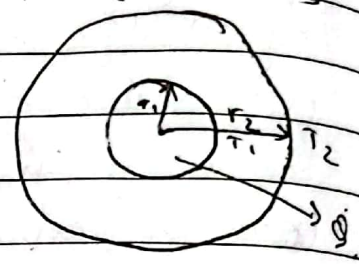
$$\Rightarrow QL = -KA(T_2 - T_1)$$

$$\boxed{\Rightarrow \dot{Q} = \frac{KA(T_1 - T_2)}{L}}$$

Scanned with CamScanner

7) Derive an expression for steady state radial conduction heat transfer through a hollow cylinder.





Consider a hollow cylinder having a length or inside & outside radii of r_1 & r_2 respectively made up of a material having a thermal conductivity of k as shown in figure. Inside & outside curved surface are exposed ~~curved~~ uniform temp^r. T_1 & T_2 respectively.

by Fourier's law

$$\dot{Q} = -kA\frac{dT}{dr}$$

$$\dot{Q} = -k(2\pi r l)\frac{dT}{dr}$$

$$Q\frac{dr}{r} = (-2\pi k)dT$$

Integrating with associated boundary condition

$$Q\int_{r_1}^{r_2}\frac{dr}{r} = -\int_{T_1}^{T_2} k(2\pi r l)\,dT$$

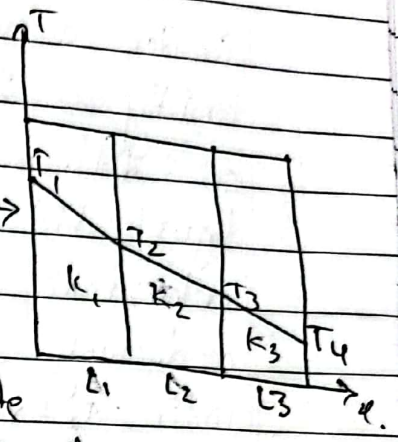
$$\Rightarrow Q\ln\left(\frac{r_2}{r_1}\right) = -2\pi k l(T_2 - T_1)$$

$$\boxed{\Rightarrow \dot{Q} = \frac{2\pi k l(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}}$$ #

Scanned with CamScanner

Q8 Derive an expression for steady state heat transfer through a composite wall consisting of three different material.

Consider a composite of wall consisting of three layers of three diff. material shown in fig 7.3 Thickness of each layer are L_1, L_2 & L_3 respestively. & Thermal conductivitive of each layer are k_1, k_2, k_3 respectively. The oppo. face of the composite wall subjected to temp. of T_1 & T_4 & interface temp. are T_2 & T_3 respectively.



for steady heat transfer heat flowing through each layer should be same

$$Q_1 = \frac{k_1 A (T_1 - T_2)}{L_1} \Rightarrow T_1 - T_2 = \frac{Q_1 L_1}{k_1 A} \quad -(1)$$

$$Q_2 = \frac{k_2 A (T_2 - T_3)}{L_2} \Rightarrow T_2 - T_3 = \frac{Q_2 L_2}{k_2 A} \quad -(2)$$

$$Q_3 = \frac{k_3 A (T_3 - T_4)}{L_3} \Rightarrow T_3 - T_4 = \frac{Q_3 L_3}{k_3 A} \quad -(3)$$

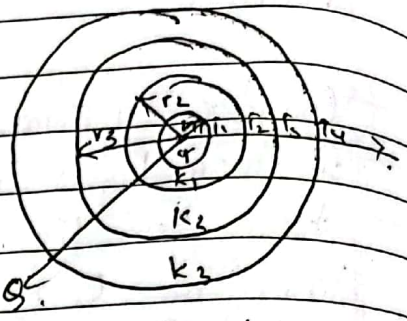
Adding eqn (1) (2) (3) we get

$$T_1 - \cancel{T_2} + \cancel{T_2} - \cancel{T_3} + \cancel{T_3} - T_4 = \frac{Q_1 L_1}{k_1 A} + \frac{Q_2 L_2}{k_2 A} + \frac{Q_3 L_3}{k_3 A}$$

$$\boxed{Q = \frac{A (T_1 - T_4)}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)}}$$

Scanned with CamScanner

9) Expression for steady state heat transfer through a composite cylinder consisting of three different



Consider a composite hollow cylinder consisting of 3 cylinderical layer of 3 diff. material as shown in figure. First layer with inner & outer radii of r_2 & has a thermal conductivity of k_1. Second layer with inner & outer radii r_2 & r_3 respectively with thermal conductivity of k_2 & third layer with ~~thermal~~ inner & outer radii r_3 & r_4 has a thermal conductivity of k_3. Inner & outer surface ~~of~~ the composite layers are subjected to temperature of t_1 & T_4 & interface temperature are T_2 & T_3 respectively.

For steady state heat transfer, heat flowing through each layer should be same ie

$$Q = \frac{2\pi k_1 L (T_1 - T_2)}{\ln(r_2/r_1)} \Rightarrow T_1 - T_2 = \frac{Q \ln(r_2/r_1)}{2\pi k_1 L} \quad (1)$$

$$Q = \frac{2\pi k_2 L (T_2 - T_3)}{\ln(r_3/r_2)} \Rightarrow T_2 - T_3 = \frac{Q \ln(r_3/r_2)}{2\pi k_2 L} \quad (2)$$

$$Q = \frac{2\pi k_3 L (T_3 - T_4)}{\ln(r_4/r_3)} \Rightarrow T_3 - T_4 = \frac{Q \ln(r_4/r_3)}{2\pi k_3 L} \quad (3)$$

adding (1) (2) & (3) we get

$$T_1 - T_2 + T_2 - T_3 + T_3 - T_4 = \frac{Q}{2\pi L}\left[\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{\ln(r_4/r_3)}{k_2}\right]$$

$$\boxed{\Rightarrow Q = \frac{(T_1 - T_4)\, 2\pi L}{\left[\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{\ln(r_4/r_3)}{k_3}\right]}}$$

Scanned with CamScanner

10) Thermal resistance. define. Derive expressions for thermal resistance for.

a) Conduction heat transfer trough a planer wall

Heat transfer through a plane wall is given by

$$Q = \frac{T_1 - T_2}{L/Ak} \quad - eq.$$

comparining eq^n with $Q = \frac{\Delta T}{R_{th}}$

$$\Rightarrow R_{th} = \frac{L}{Ak}$$

=> Thermal resistance is defined as the properties of the substance by the virtue of which it opposes the flow of heat.

~~Heat transfer through plane wall, is given by~~

Q

b) Conduction heat transfer though a hollow cylinder

Heat transfer through a hollow cylinder is given by

$$Q = \frac{T_1 - T_2}{\frac{\ln(r_2/r_1)}{2\pi kL}}$$

comparising above eq^n with $Q = \frac{\Delta T}{R_{th}}$.

$$\Rightarrow R_{th} = \frac{\ln(r_2/r_1)}{2\pi kL}$$

Scanned with CamScanner

c) Convection heat transfer through a fluid

Convection heat transfer through fluid layer given by eqⁿ can also be expressed as

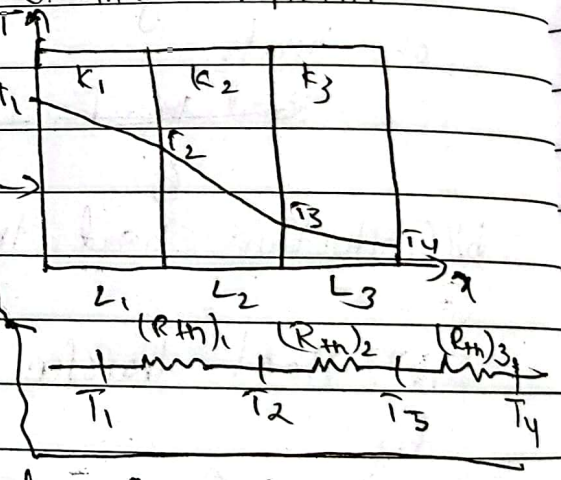
$$Q = \frac{T_s - T_\infty}{1/hA}$$

comparing above eqⁿ with $Q = \frac{\Delta T}{R_{th}}$

$$\boxed{\Rightarrow R_{th} = \frac{1}{hA}}$$

Q.11 Use electrical analogy approach to derive following.
a) a composite wall consisting of three different material
b) a composite cylinder consisting of three different

a) ⇒ :



In this case, the same amount of heat flows through each layer hence all the thermal resistance are arranged in series. Whenever heat flows through a thermal resistance, there will be a temperature drop ~~long low~~ drop in case of electric circuit. Thermal resistance for each layer of the plane wall is given as,

$$(R_{th})_1 = \frac{L_1}{Ak_1} \quad \& \quad (R_{th})_2 = \frac{L_2}{Ak_2} \quad \& \quad (R_{th})_3 = \frac{L_3}{Ak_2}$$

eqⁿ. valentent resistance is given as:

$$req = (R_{th})_1 + (R_{th})_2 + (R_{th})_3$$

$$= \frac{1}{A}\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right) \quad \text{--- (1)}$$

Scanned with CamScanner

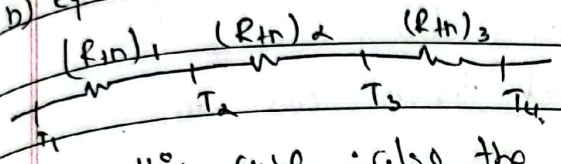
~~eq^n Valentent resistance is given as~~

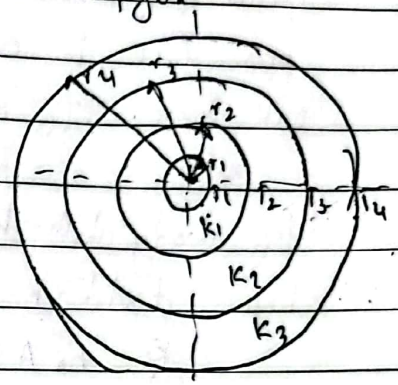
~~$R_{eq.} = (R_{th})_1 + (R_{th})_2 + (R_{th})_3$~~

then overall heat ~~therm~~ transfer for composite plane wall is given by.

$$Q = \frac{\Delta T}{R_{eq.}} = \frac{T_1 - T_4}{\frac{1}{A}\left(\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}\right)} = \frac{A(T_1 - T_4)}{\left(\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}\right)}$$

b) eq^n circit for composite cylinder shown in figure





In this case also the same amount of heat flow through each layer hence all the thermal resistance are arranged in series. thermal resistance for each layer of the composite cylinder are given as

$$(R_{th})_1 = \frac{\ln(r_2/r_1)}{2\pi K_1 L} \qquad (R_{th})_2 = \frac{\ln(r_3/r_2)}{2\pi K_2 L} \qquad (R_{th})_3 = \frac{\ln(r_4/r_3)}{2\pi K_3 L}$$

eq. of resistance.

$$R_{eq} = (R_{th})_1 + (R_{th})_2 + (R_{th})_3$$

$$= \frac{1}{2\pi L}\left[\frac{\ln(r_2/r_1)}{K_1} + \frac{\ln(r_3/r_2)}{K_2} + \frac{\ln(r_4/r_3)}{K_3}\right]$$

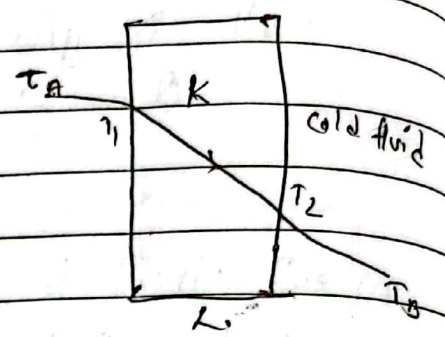
then, overall heat transfer for composite cylinder is given by.

$$Q = \frac{\Delta T}{R_{eq.}} = \frac{(T_1 - T_4)\,2\pi L}{\frac{\ln(r_2/r_1)}{K_1} + \frac{\ln(r_3/r_2)}{K_2} + \frac{\ln(r_4/r_3)}{K_3}}$$

Scanned with CamScanner

12 For overall heat transfer coeff. for a plane well subjected to convection medium on both sides

⇒ Consider a plane well having a thickness of L & a thermal conductivity of k as shown in fig 1. one of its face is exposed to a hot fluid with temp T_A & convective heat transfer coeff. h_A while the oppo. face is exposed to a cold fluid with temp. T_B & convective heat transfer coeff h_B



for steady state heat transfer, heat flow through each layer should be same

$$Q = h_A A (T_A - T_1) \quad \& \quad Q = h_B A (T_2 - T_B)$$

$$\Rightarrow T_A - T_1 = \frac{Q}{h_A A} \quad \text{—(i)} \qquad \Rightarrow T_2 - T_B = \frac{Q}{h_B A} \quad \text{—(iii)}$$

lly,

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2)$$

$$\Rightarrow T_1 - T_2 = \frac{\dot{Q} L}{kA} \quad \text{—(ii)}$$

adding eqn (i) (ii) & (iii)

$$T_A - T_1 + T_1 - T_2 + T_2 - T_B = \frac{\dot{Q}}{h_A A} + \frac{\dot{Q} L}{kA} + \frac{\dot{Q}}{h_B A}$$

$$\Rightarrow Q = \frac{(T_A - T_B) A}{\left(\frac{1}{h_A} + \frac{L}{k} + h_B\right)}$$

comparing the above eqn with

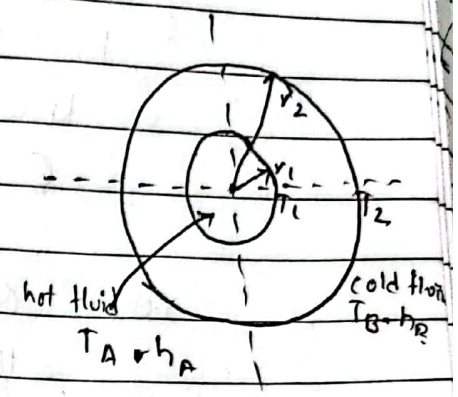
$$Q = UA(T_A - T_B)$$

$$\Rightarrow U = \frac{1}{\left(\frac{1}{h_A} + \frac{1}{k} + h_B\right)}$$ is called over heat transfer coeff.

Scanned with CamScanner

13) For inside & outside overall heat transfer coeff. for a hollow cylinder subjected to convection medium on both sides.

⇒ Consider a hollow cylinder with inner & outer radii r_1 & r_2, length L & thermal conductivity k as shown in fig. The inner surface of the cylinder exposed to hot fluid with temp. T_A & convective heat transfer coeff. of h_A where the outer surface of the cylinder is exposed to a cold fluid with temp. T_B & convective heat transfer coeff. of T_B.



$$Q = h_A A_1 (T_A - T_1)$$

$$\Rightarrow T_A - T_1 = \frac{Q}{h_A A_1} \quad \text{—(i)}$$

$$Q = \frac{2\pi k L}{\ln(r_2/r_1)} (T_1 - T_2)$$

$$\Rightarrow T_1 - T_2 = \frac{\ln(r_2/r_1)\, Q}{2\pi k L} \quad \text{—(ii)}$$

$$Q = h_B \cdot A_2 (T_2 - T_B)$$

$$\Rightarrow T_2 - T_B = \frac{Q}{h_B \cdot A_2} \quad \text{—(iii)}$$

adding eqn (i) (ii) & (iii)

$$T_A - T_B = Q\left(\frac{1}{h_A A_1} + \frac{\ln(r_2/r_1)}{2\pi k L} + \frac{1}{h_B A_2}\right)$$

$$\Rightarrow Q = \frac{T_A - T_B}{\left(\frac{1}{h_A A_1} + \frac{\ln(r_2/r_1)}{2\pi k L} + \frac{1}{h_B A_2}\right)}$$

Scanned with CamScanner

Numerical

1. Sol^n,

length $(l) = 2m$

radius $(r_1) = 1.75 \times 10^{-3} m$

surface tempr. $(T_1) = 150°C$

coeff. of heat transfer,

$h_B = ?$

we know, that,

$Q = VI$

$= 50 \times 2 = 100 W$

now, $Q = h_B (T_1 - T_B) \cdot r \times 2\pi l$

$$\frac{100}{(150-20) 2 \times 2\pi l} = h_B$$

$\Rightarrow h_B = 34.979 \cdot W/m^2 k$

Q.2) Sol^n,

$T_3 = 25°C = 25 + 273 = 298 K$

$L = 0.15 m$

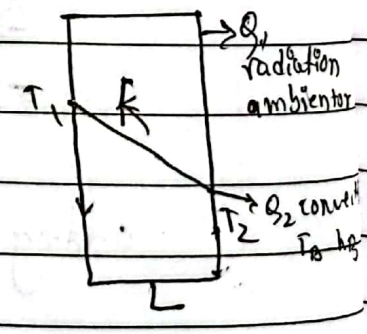
thermal conductivity of brick wall $= 1.2 W/mk$

$T_2 = 100°C = 373 K$

Surface emissivity $= (\epsilon) = 0.8$

convection coefficient $(h_B) = 20 W/m^2 k$

$\sigma = 5.67 \times 10^{-8} W/m^2 k^4$



$Q_{conduction} = Q_{convection} + Q_{radiation}$

$$\frac{kA(T_1 - T_2)}{L} = h_B A (T_2 - T_B) + \epsilon \sigma A (T_2^4 - T_B^4)$$

$$T_1 = T_2 + \frac{L}{k}\left[h_B (T_2 - T_B) + \epsilon \sigma (T_2^4 - T_B^4)\right]$$

$$= 373 + \frac{0.15}{1.2}\left[20(100-25) + 0.8 + 5.67 \times 10^{-8} \times (373^4 - 298^4)\right]$$

$= 352.533°C$.

Scanned with CamScanner

3) Sol^n,

length $L = 300\,mm$

$= 0.3\,m$

$\Rightarrow L_1 = 300 - L_2$

now,

$$\frac{Q_2}{A} = \frac{1300 - 1100}{\left(\frac{300 - L_2}{k_1}\right)}$$

$$= \frac{200}{300 - L_2} \times 1 \qquad = \frac{200}{300 - L_2} \quad \text{—(i)}$$

also,

$$\frac{Q_2}{A} = \frac{1100 - 300}{\frac{L_2}{k_2}}$$

$$= \frac{800 \times 0.2}{L_2} \quad \text{—(ii)}$$

eq^n, (i) & (ii)

$$\Rightarrow \frac{800 \times 0.2}{L_2} = \frac{200}{300 - L_2}$$

$$\Rightarrow \frac{160}{L_2} = \frac{200}{300 - L_2}$$

$$\Rightarrow 48000 - 160 L_2 = 200 L_2$$

$$\Rightarrow L_2 = 133.33\,mm \quad \text{—(i)}$$

$$\Rightarrow L_1 = 166.67\,mm \quad \text{—(ii)}$$

also,

$$\frac{Q}{A} = \frac{1100 - 300}{\left(\frac{0.1333}{0.2}\right)} = 1200.03\ W/m^2$$

$$= 1.20003\ kW/m^2$$

hence,

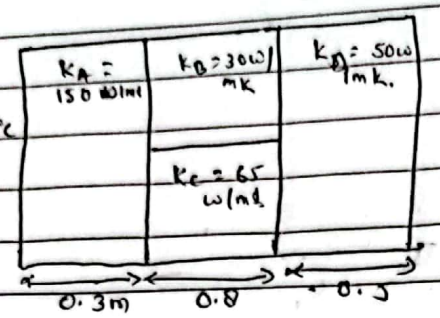
reqd. $L_1 = 166.67\,mm$

$L_2 = 133.33\,mm$

& $\frac{Q}{A} = 1.20003\ kW/m^2$.

Scanned with CamScanner

Q.4)



now,

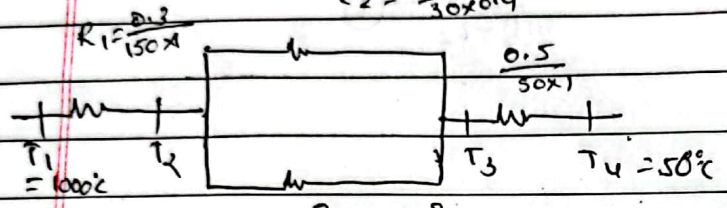
$$Q = \frac{\Delta T}{R_{th}}$$

$$Q = \frac{(1000 - 50)}{\frac{353}{12750}}$$

$$= 34313.03110 \text{ W}$$

$$= 34.3130 \text{ kW}$$

$$R_2 = \frac{0.8}{30 \times 0.4}$$



$$R_3 = \frac{0.8}{65 \times 0.4}$$

now

$$R_{eq.} = 2 \times 10^{-3} + \frac{1}{15} \,||\, \frac{4}{195} + \frac{1}{100}$$

$$= \frac{353}{12750}$$

Q.5) solⁿ,

$$\frac{Q}{L} = \frac{2\pi (600 - 100)}{\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2}}$$

$$= \frac{2\pi (500)}{\frac{\ln(0.002/0.001)}{19} + \frac{\ln(0.005/0.002)}{0.2}}$$

$$= 680.3024 \text{ w/m}$$

Again

$$\frac{Q'}{L} = \frac{2\pi (T_1 - T_2)}{\frac{\ln(r_2/r_1)}{k_1}}$$

$r_1 = 0.001$ m

$r_2 = 0.002$ m

$r_3 = 0.005$ m

$k_1 = 19$ w/mk

$k_2 = 0.2$ w/mk

$T_1 = 600°c$

$T_2 = 100°c$

Scanned with CamScanner

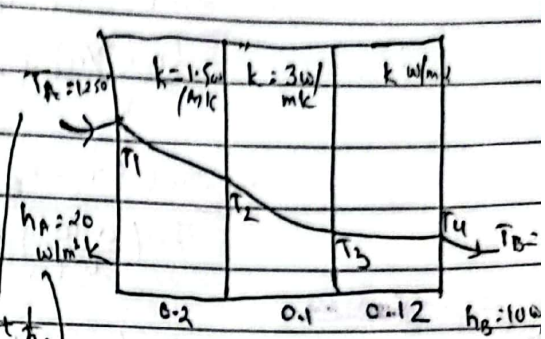
$$68.30 = \frac{2\pi (600 - T_2)}{\ln\left(\frac{0.005}{0.003}\right)}$$

$$\Rightarrow 471.54 = 2\pi (600 - T_2)$$

$$\Rightarrow T_2 = 600 - 75.049$$

$$= 524.95^\circ C.$$

Q.7) solⁿ,



$$\frac{Q}{A} = 10(300 - 25)$$

$$= 2750 \text{ W/m}^2$$

again

$$\left(\frac{T_A - T_B}{\frac{1}{h_A} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{1}{h_B}}\right)$$

$$\frac{Q}{A} = \frac{1250 - 25}{\frac{1}{20} + \frac{0.2}{1.5} + \frac{0.1}{3} + \frac{0.12}{K} + \frac{1}{10}}$$

$$\frac{Q}{A} = \frac{1225}{\frac{19}{60} + \frac{0.12}{K}}$$

$$2750 = \frac{1225}{\frac{19}{60} + \frac{0.12}{K}}$$

$$\frac{19}{60} + \frac{0.12}{K} = \frac{49}{110}$$

$$\frac{0.12}{K} = \frac{49}{110} - \frac{19}{60}$$

$$\frac{0.12}{K} = \frac{17}{132}$$

$$K = 0.93174 \text{ W/m}^2\text{K}$$

$$\frac{Q}{A} = 20(1250 - T_1)$$

$$\frac{2750}{20} = 1250 - T_1$$

$$T_1 = 1112.5^\circ C.$$

again, $\frac{T_1 - T_2}{\frac{0.2}{1.5}} = \frac{Q}{A}$

$$\Rightarrow 1112.5 - T_2 = 1250 \times \left(\frac{0.2}{1.5}\right)$$

$$T_2 = 745.83^\circ C.$$

$T_1 = ?$

$T_2 = ?$

$T_3 = ?$

$T_4 = 300^\circ C$

$L_1 = 0.2$ m

$L_2 = 0.1$ m

$L_3 = 0.13$ m

$T_A = 1250$

$T_B = 25^\circ C$

$h_A = 20$ W/m

$h_B = 10$ W/m²

$K_1 = 1.5$

$K_2 = 3$ W

$K_3 = K$

$$T_3 = 654.167^\circ C.$$

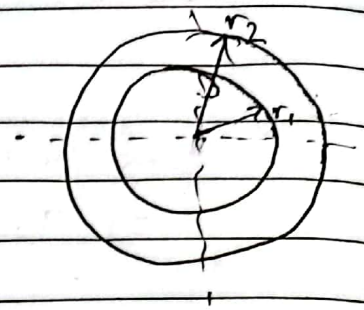
Scanned with CamScanner

9) Sol^n

given, outer radius of pipe

$(r_1) = \frac{140}{2}$ mm $= 70$ mm $= 0.7$ m

outer radius of insulation $= 0.1$ m



$r_1 = 0.07$ m

$r_2 = 0.1$ m

$k_1 = 0.5$ w/mk

$k_2 = 1$ w/mk

$h_B = 10.2$ lm^2k

$r_3 = 0.12$ m

case - I

$$Q_1 = \frac{2\pi L (T_1 - T_B)}{\frac{\ln(r_2/r_1)}{k} + \frac{1}{h_B}}$$

$$= \frac{2\pi L (T_1 - T_B)}{\ln\left(\frac{0.10}{0.7}\right) + \frac{1}{10 \times 0.1}}$$

$$= \frac{2\pi L.(T_1 - T_B)}{1.71335}$$

case II with extra layer of insulation

$h_o = 10$ w/m^{2}k

heat transfer coff. of extra layer of insulation

$$Q = \frac{2\pi L (T_1 - T_B)}{\frac{\ln(r_2/r_1)}{k} + \frac{\ln(r_3/r_2)}{k_2} + \frac{1}{h_B r_3}}$$

$$= \frac{2\pi L (T_1 - T_B)}{\frac{\ln\left(\frac{0.1}{0.07}\right)}{0.5} + \frac{\ln\left(\frac{0.17}{0.2}\right)}{1} + \frac{1}{10 \times 0.12}}$$

$$= \frac{2\pi L (T_1 - T_B)}{1.729}$$

now the percentage

Change in heat transfer rate due to extra insulation

$$\frac{Q_1 - Q_2}{Q_1} = \frac{1 - Q_2}{Q_1} = 1 - \frac{1.7135}{1.729} \times 100\%$$

$$= 0.905\%$$

(10) Sol^n.

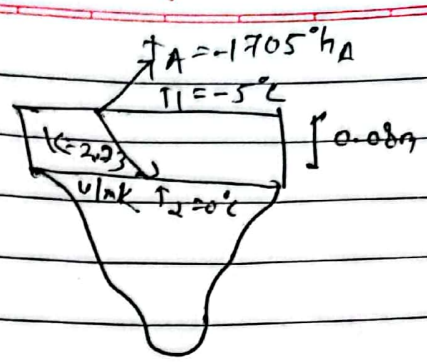
$L = 0.08\ m$

$T_A = 1205°c$

$h_A = ?$

$T_1 = -5°c$

$K_1 = 2.23\ w/mk$



now

$$h_A(-5-(12.5)) = \frac{Q}{A}$$

$$\text{or } h_A = \frac{139.375}{7.5}$$

$$= 18.583\ w/m^2k$$

also. $$\frac{Q}{A} = \frac{0-(-5)}{\frac{0.08}{2.23}}$$

$$= \frac{5 \times 2.23}{0.8}$$

$$= 139.375\ w/m^2k$$

Scanned with CamScanner