Electrostatics

Electric Charge:-

When two bodies are rubbed, there is transference of electrons in the outermost orbit from atoms of one body to another. The body which gains the electron will be negatively charged and which losses electron will be positively charged.

According to modern electron theory, the state of an atom after loss or gain of electron is called the charged state. And the new form of atom is called charge.

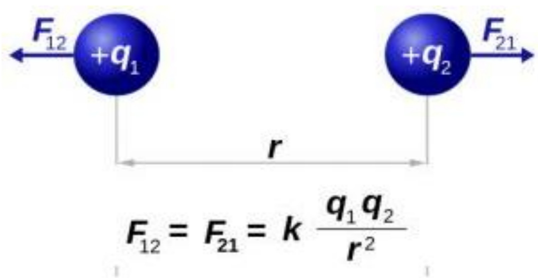
Its S.I unit is Coulomb.

Electric force (Coulomb's law):-

The force of attraction or repulsion between two charges is directly proportional to the magnitude of charges and inversely proportional to the square of the distance between them. If distance between two charges q_1 and q_2 is r. then Coulomb's law states that;

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\therefore F = k\frac{q_1 q_2}{r^2}$$



Where k is proportionality constant known as Coulomb's constant. K depends upon the nature of the medium.

For air medium or vacuum; $k = \frac{1}{4\pi\varepsilon_0}$

$$\therefore F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Where, $\varepsilon_0 = 8.85 \times 10^{-12}\ Farad/m$, is known as permittivity of free space.

In CGS system (for air medium) k = 1

$$\therefore F = \frac{q_1 q_2}{r^2}$$

But if there is a medium of permittivity, then;

$$\therefore F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$

Electric field intensity:-

The electric field intensity at a point in an electric field is defined as the force experienced by unit test charge at that point.

$$i.e. \vec{E} = \frac{\vec{F}}{q_0}$$

We have, $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$

$$\therefore \ \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}$$

Electric flux($\emptyset_E$):-

The electric flux at a point in electric field is defined as the number of electric lines of force passing per unit area perpendicular to the direction of line of force.

Generally, the number of lines of Electric field is electric flux. This means the Electric field at a point is given by electric flux passing per unit area perpendicular to direction of lines of force at that point.

$$i.e. \vec{E} = \frac{d\emptyset}{dA}$$

$$or, \qquad d\emptyset = \vec{E}.\overrightarrow{dA}$$

$$\therefore \emptyset = \oint \vec{E}.\overrightarrow{dA}$$

Gauss law for electrostatics:-

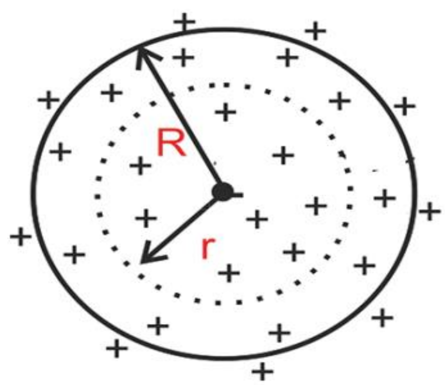
It states that the total electric flux through a close surface enclosing charge is equal to $1/\varepsilon_0$ times the charge enclosed by that surface.

$$i.e. \emptyset = \frac{q}{\varepsilon_0}$$

$$\oint \vec{E}.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$$

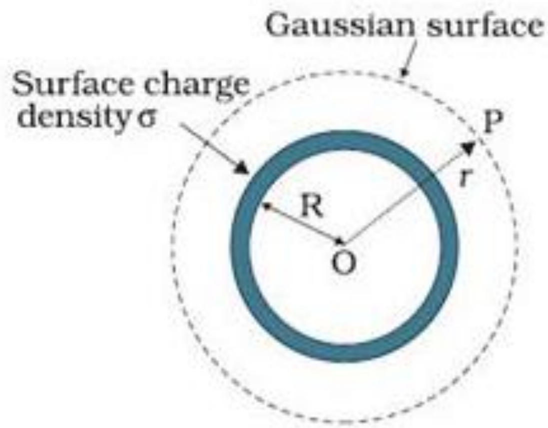
Application of Gauss law:-

1) Electric field due to non-conducting spherical symmetric distribution (Electric field due to charged sphere):-



Consider a spherical charge distribution of total charge ‘q’ and radius ‘R’. For spherical charge distribution the charged density ‘ρ’ remains same for all the points lying at equal distance from centre. To find an expression for electric field for points outside, inside and on the surface of charge distribution we use Gauss law.

Electric field outside the sphere:-



For this we construct a Gaussian surface ‘r’ such that, $r > R$, since, all the charge is enclosed by Gaussian surface. So electric flux;

$$\oint \vec{E}.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$$

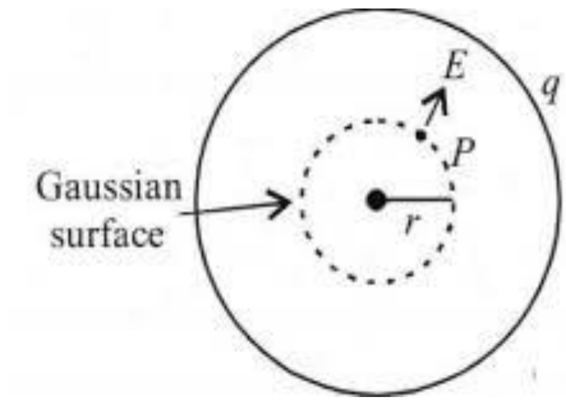
$$or, \;\; E.A = \frac{q}{\varepsilon_0}$$

$$or, \;\; E.4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\therefore \;\; E = \frac{q}{4\pi\varepsilon_0 r^2}$$

This is the electric field intensity due to spherical charge distribution of the total charge at any point on Gaussian surface.

Electric field inside the sphere:-



In this case, Gaussian surface of radius ‘r,’ lies inside the charge sphere. Such that, $r < R$. In this case the part of charge lying outside the Gaussian surface does not contribute to setup electric field.

Let q^1 represent a part of charge enclosed by Gaussian surface. Then, using Gauss law;

$$\oint \vec{E}.\overrightarrow{dA} = \frac{q^1}{\varepsilon_0}$$

$$or, \;\; E.A = \frac{q^1}{\varepsilon_0}$$

$$or, \;\; E.4\pi r^2 = \frac{q^1}{\varepsilon_0}$$

$$\therefore \;\; E = \frac{q^1}{4\pi\varepsilon_0 r^2} \dots \dots \dots (1)$$

Since charge density $\rho = q/V$ is uniform;

$$i.e.\ \rho = \frac{q}{\frac{4}{3}\pi R^3} = \frac{q^1}{\frac{4}{3}\pi r^3}$$

$$\therefore \quad q^1 = \frac{qr^3}{R^3} \ldots\ldots\ldots (2)$$

Now from equation (1) and (2)

$$E = \frac{q}{4\pi\varepsilon_0 r^2}\frac{r^3}{R^3}$$

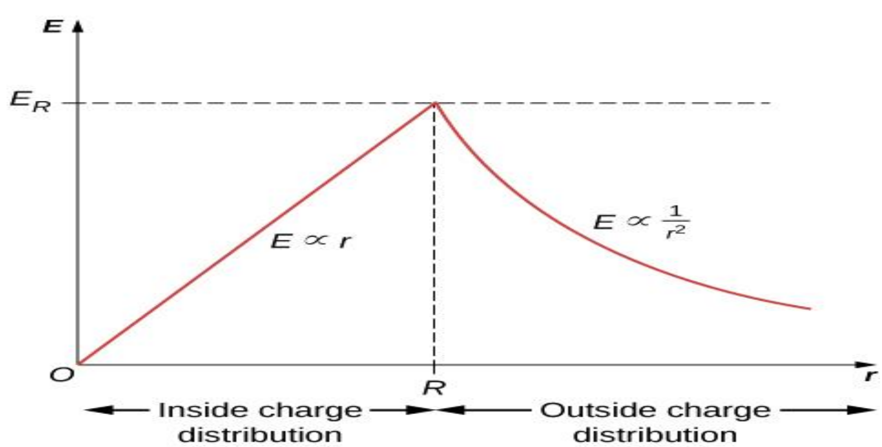
$$\therefore\ E = \frac{q}{4\pi\varepsilon_0}\frac{r}{R^3}$$

Which is required electric field inside the sphere in terms of, total charge 'q'.

Electric field on the surface of sphere:-

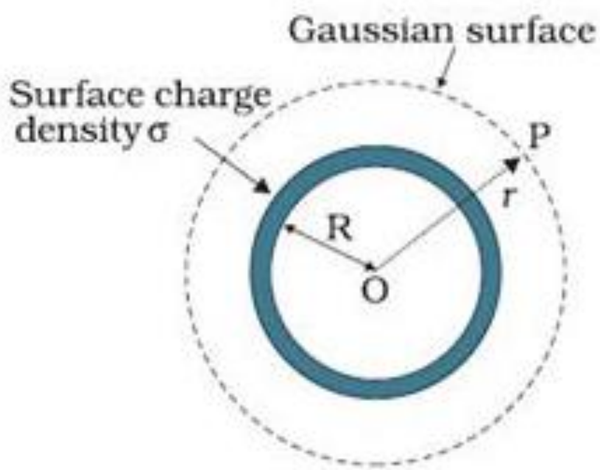
At surface of the sphere, r = R;

$$\therefore\ E = \frac{q}{4\pi\varepsilon_0 R^2}$$



2) Electric field due to conducting spherical symmetric distribution:-

Electric field outside the sphere:-



For this we construct a Gaussian surface 'r' such that $r > R$. since all the charge is enclosed by Gaussian surface. So electric flux;

$$\oint \vec{E}.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$$

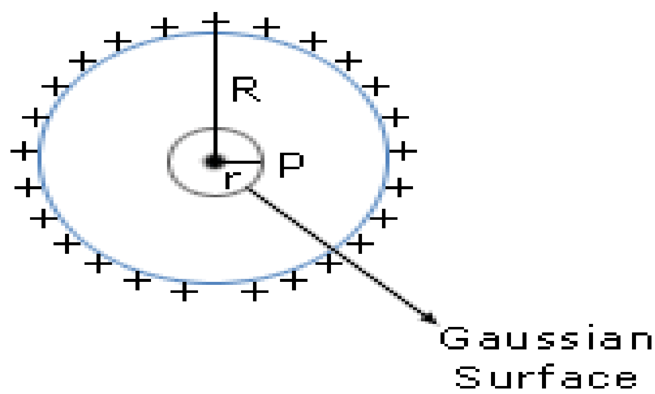
$$or, \; E.A = \frac{q}{\varepsilon_0}$$

$$or, \; E.4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\therefore \; E = \frac{q}{4\pi\varepsilon_0 r^2}$$

This is the electric field intensity due to conducting spherical charge distribution of the total charge at outside the sphere.

Electric field inside the sphere:-



In this case Gaussian surface does not enclosed any charges then electric field becomes zero inside the conducting sphere.

Electric field due to cylindrical charge distribution or electric field due to a line of charge:-

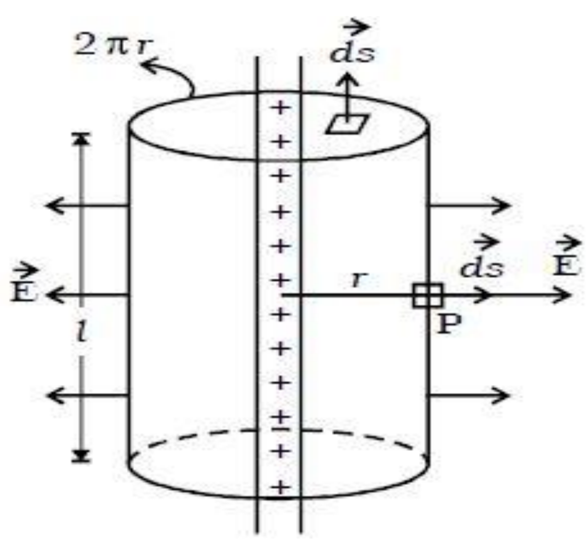


Figure shows a cross section of an infinitely long cylindrical rod made up of non-conducting material with linear charge density λ. To find the electric field intensity at a point 'p' at distance 'r', from the line charge. We consider a circular cylinder of radius 'r', and height 'l', as a Gaussian surface.

Now, Applying Gauss law;

$$\oint \vec{E}.\overrightarrow{dA} = \frac{q}{\varepsilon_0}$$

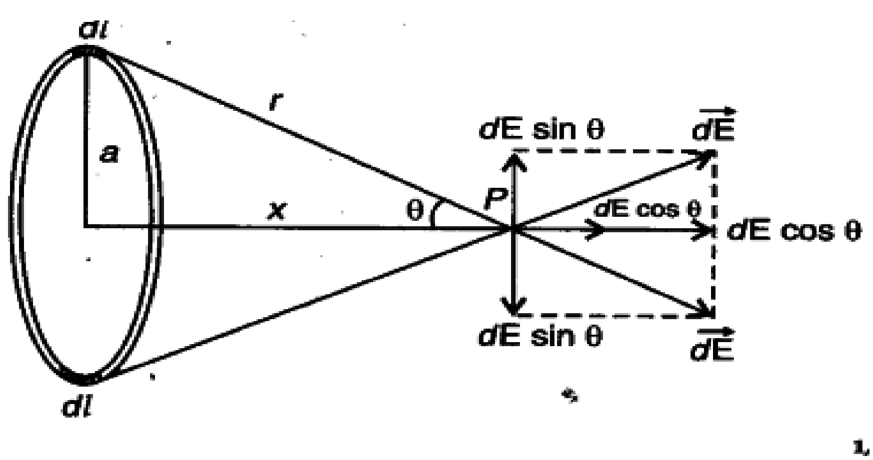
$$or, \quad E.A = \frac{q}{\varepsilon_0}$$

$$or, \qquad E.2\pi rl = \frac{q}{\varepsilon_0}$$

$$or, \qquad E.2\pi rl = \frac{\lambda l}{\varepsilon_0}$$

$$\therefore E = \frac{\lambda}{2\pi r\varepsilon_0}$$

Electric field due to a ring of charge:-



Consider a ring of radius a carrying uniformly distributed positive charge q with linear charge density λ. The ring is divided into elementary segments of each of length dl. Let the electric field intensity dE due to this segment makes an angle θ with vertical. So, it can be resolved in to two components. If we consider the effect of whole ring, $dE \sin\theta$ components get canceled out and resultant field is;

$$E = \int dE \cos\theta, \qquad Where\ dE = \frac{dq}{4\pi\varepsilon_0 r^2} = \frac{\lambda dl}{4\pi\varepsilon_0 (a^2 + x^2)}$$

$$And, \qquad \cos\theta = \frac{x}{\sqrt{a^2 + x^2}}$$

$$Now, \qquad E = \frac{\lambda x}{4\pi\varepsilon_0 (a^2 + x^2)^{\frac{3}{2}}} \int_0^{2\pi a} dl$$

$$or, \qquad E = \frac{2\pi\lambda a x}{4\pi\varepsilon_0 (a^2 + x^2)^{\frac{3}{2}}}$$

$$\therefore E = \frac{qx}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}$$

where, $q = 2\pi\lambda a \quad \because$ liner charge density $(\lambda) = \dfrac{q}{2\pi a}$

For maximum Electric field, $\dfrac{dE}{dx} = 0$

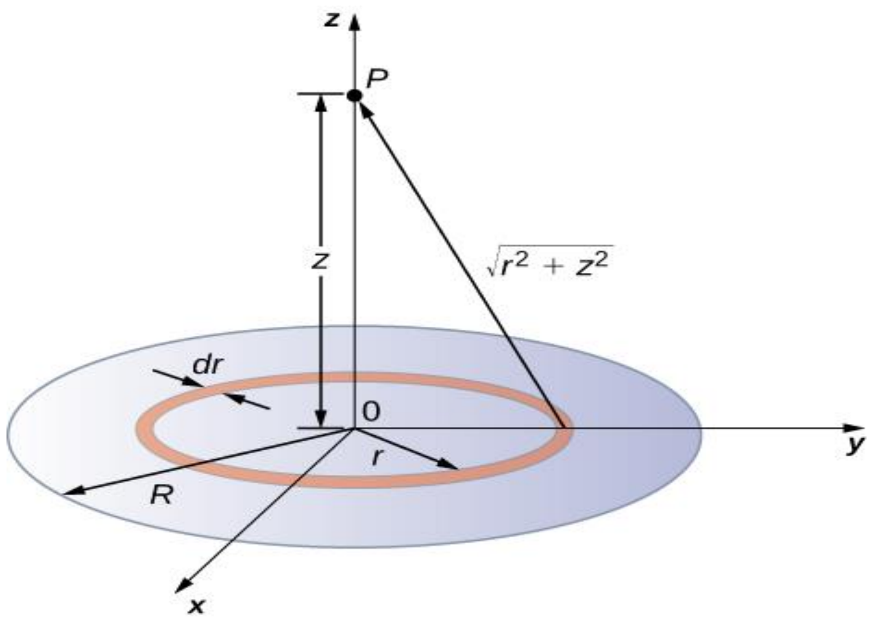
$$\frac{dE}{dx} = \frac{q}{4\pi\varepsilon_0}\frac{d}{dx}\left(\frac{x}{(a^2+x^2)^{\frac{3}{2}}}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{a^2+x^2-3x^2}{(a^2+x^2)^{\frac{3}{2}}}\right)$$

$$Now, \qquad \frac{dE}{dx} = 0 \quad or, a^2+x^2-3x^2 = 0$$

$$\therefore x = \pm\frac{a}{\sqrt{2}}$$

Hence, the electric field is maximum at $\pm\frac{a}{\sqrt{2}}$.

Electric field due to a circular disc:-



Consider a disc of radius R has uniform surface charge density σ. To find the electric field at a point P along its central axis at z distance from the center, we consider the disc as a set of concentric rings. We calculate the electric field at P due to one such ring. Let the radius of ring be r and thickness of the ring be dr. By symmetry, the field at an axial point must be along the central axis.

The ring of radius r and width dr has surface area $2\pi rdr$.

So the charge on the ring of width dr is $dq = 2\pi\sigma rdr$.

Now, the electric field at P due to this ring is

$$dE = \frac{dqz}{4\pi\varepsilon_0(z^2 + r^2)^{\frac{3}{2}}}$$

$$= \frac{2\pi\sigma zrdr}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\frac{rdr}{(z^2 + r^2)^{3/2}}$$

Total electric field at point P is given by;

$$E = \int_0^R dE = \frac{\sigma z}{2\varepsilon_0}\int_0^R \frac{rdr}{(z^2 + r^2)^{3/2}}$$

Put $r = z\tan\theta$ Then, $dr = z\sec^2\theta\, d\theta$

When $r = 0, \qquad \theta = 0^0,$

and when $r = R, \qquad \theta = \tan^{-1}(\frac{R}{Z})$

$$E = \frac{\sigma z}{2\varepsilon_0}\int_0^\theta \frac{z\tan\theta\ .z\sec^2\theta\, d\theta}{(z^2 + z^2\tan^2\theta)^{3/2}} = \frac{\sigma}{2\varepsilon_0}\int_0^\theta \frac{\tan\theta\ .\sec^2\theta\, d\theta}{(\sec^2\theta)^{3/2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\int_0^\theta \tan\theta .\cos\theta\, d\theta = \frac{\sigma}{2\varepsilon_0}\int_0^\theta \sin\theta\, d\theta = \frac{\sigma}{2\varepsilon_0}[1 - \cos\theta]$$

$$\therefore \; E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

Electric potential energy (U):-

The amount of work done required to bringing a unit positive test charge (q_0) from infinity to any point in the electric field of another charge q is called electric potential energy.

$$i.e.\,\text{Electric potential energy }(U) = -W$$

$$= -\oint_{\infty}^{r} \vec{F}.\overrightarrow{dr}$$

$$= \frac{-qq_0}{4\pi\varepsilon_0}\oint_{\infty}^{r} r^{-2}\,dr$$

$$\therefore U = \frac{qq_0}{4\pi\varepsilon_0 r}$$

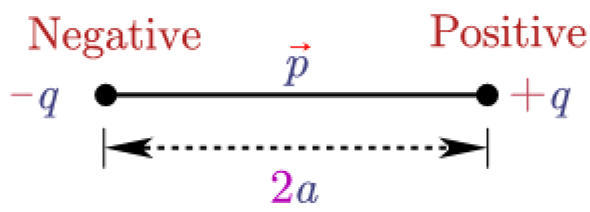
Electric potential (V):-

The electric potential energy per unit test charge (q_0) at a point inside an electric field is called electric potential.

$$i.e. V = \frac{U}{q_0}$$

$$\therefore V = \frac{q}{4\pi\varepsilon_0 r}$$

Electric dipole:-

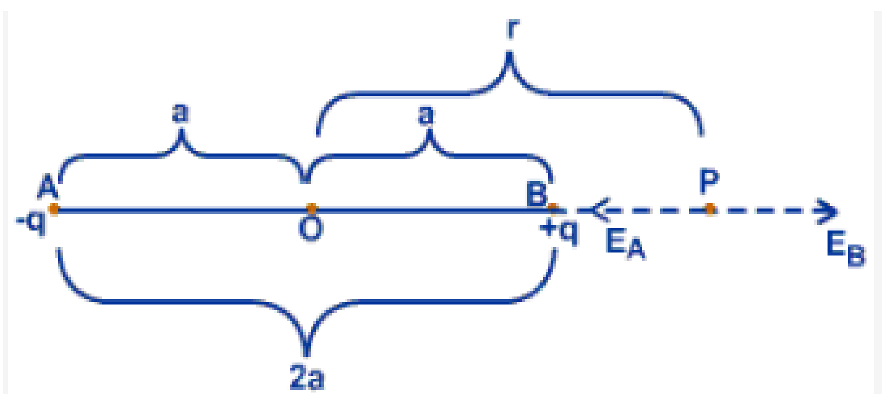


Electric Dipole

A set of two equal and opposite charges separated by a distance is called an electric dipole. The separation between the two charges is known as length of dipole and is denoted by '2a'. The product of magnitude of one of the charge and separation of charges in dipole is called dipole moment.

$$i.e. P = 2qa$$

Electric field along the axial line of dipole:-



Let E_A and E_B be the electric fields at axial line of dipole with separation '2a'.

Using the principle of superposition, the resultant electric field at point 'P' at a distance 'r' from centre of the dipole is

$$\vec{E} = \overrightarrow{E_B} + \overrightarrow{E_A}$$

$$or, \qquad E = \frac{q}{4\pi\varepsilon_0(r-a)^2} + \frac{-q}{4\pi\varepsilon_0(r+a)^2}$$

$$or, \qquad E = \frac{q}{4\pi\varepsilon_0}[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}]$$

$$or, \qquad E = \frac{q}{4\pi\varepsilon_0}[\frac{(r+a)^2}{(r-a)^2}\frac{-(r-a)^2}{(r+a)^2}]$$

$$or, \qquad E = \frac{q}{4\pi\varepsilon_0}[\frac{r^2 + 2ra + a^2 - r^2 + 2ra - a^2}{(r^2-a^2)^2}]$$

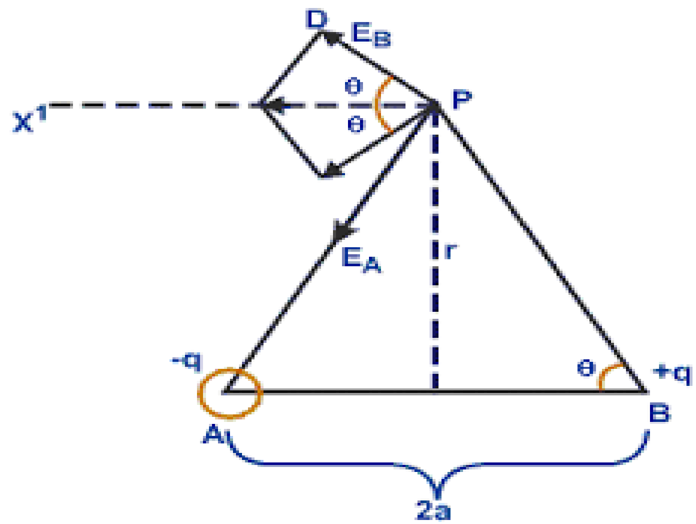
$$or, \qquad E = \frac{q}{4\pi\varepsilon_0}[\frac{4ra}{(r^2-a^2)^2}]$$

$$\therefore E = \frac{P}{2\pi\varepsilon_0}.\frac{r}{(r^2-a^2)^2} \qquad where\ P = 2qa$$

For short dipole $r \gg a$ then we get;

$$\therefore E = \frac{P}{2\pi\varepsilon_0 r^3}$$

Electric field along the equatorial line of dipole (Across the dipole):-



Let E_A and E_B be the electric fields due to -q and +q at a point 'P' on the equatorial line of dipole with separation '2a'. The distance of point 'P' from the centre of dipole is 'r'. The distance of point 'P' from each charge is;

$$AP = BP = \sqrt{r^2 + a^2}$$

The vertical component of E_A and E_B cancel each other and the resultant electric field at point 'P' is due to the horizontal components of E_A and E_B.

$$\therefore E = |E_A|cos\theta + |E_B|cos\theta$$

Since, $|E_A| = |E_B| = \frac{q}{4\pi\varepsilon_0(r^2+a^2)}$

$$\therefore E = 2|E_+|cos\theta$$

$$= \frac{2q}{4\pi\varepsilon_0(r^2+a^2)}.cos\theta$$

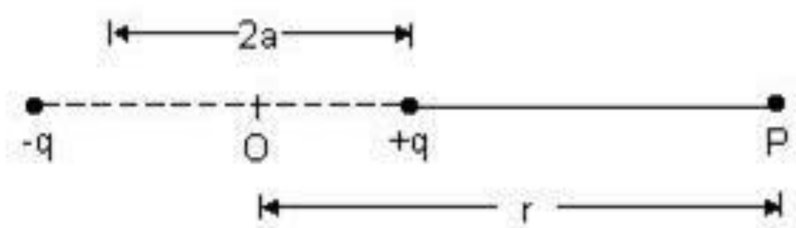
$$= \frac{2q}{4\pi\varepsilon_0(r^2+a^2)}.\frac{a}{\sqrt{r^2+a^2}}$$

$$\therefore E = \frac{P}{4\pi\varepsilon_0(r^2+a^2)^{3/2}} \quad where\ P = 2qa$$

For short dipole, $r \gg a$;

$$\therefore E = \frac{P}{4\pi\varepsilon_0 \mathrm{r}^3}$$

Electric potential along the axial line of dipole (Electric potential due to the dipole):-



Consider an electric dipole with separation ‘2a’. P is any point at a distance ‘r’ from centre of dipole at which electric potential is to be determined. Using the principle of superposition, the resultant electric field at point ‘P’at a distance ‘r’ from centre of the dipole is

$$\vec{V} = \overrightarrow{V_-} + \overrightarrow{V_+}$$

$$or, \qquad V = \frac{-q}{4\pi\varepsilon_0(r+a)} + \frac{q}{4\pi\varepsilon_0(r-a)}$$

$$or, \qquad V = \frac{q}{4\pi\varepsilon_0}[\frac{-1}{r+a} + \frac{1}{r-a}]$$

$$or, \qquad V = \frac{q}{4\pi\varepsilon_0}[\frac{-r+a+r+a}{r^2-a^2}]$$

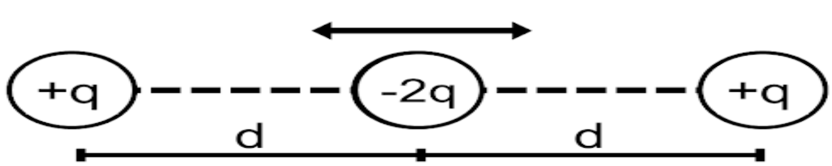
$$or, \qquad V = \frac{q}{4\pi\varepsilon_0}[\frac{2a}{r^2-a^2}]$$

$$\therefore \; V = \frac{1}{4\pi\varepsilon_0}\frac{P}{(r^2-a^2)}$$

In case of very short dipole, $r \gg a$;

$$\therefore \; V = \frac{P}{4\pi\varepsilon_0 r^2}$$

Electric Quadrupole:-

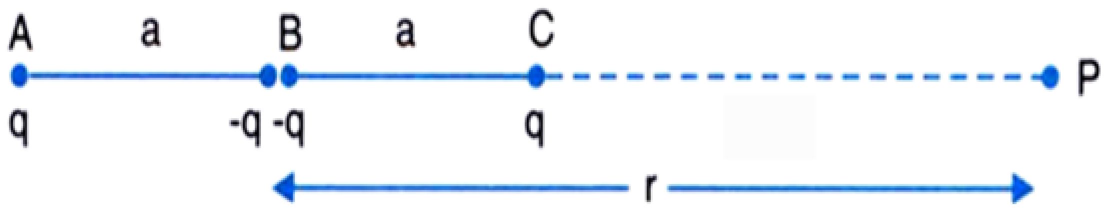


The arrangement of four equal and opposite charges or arrangement of two dipoles is called quadrupole. The quadrupole momentum is defined by the relation;

$$Q = 2qd^2$$

Where, q is magnitude of each charge in quadrupole and 2d is quadrupole separation.

Electric field due to Quadrupole:-



Consider a linear quadrupole of separation 2a with magnitude of each charge q as shown in figure. P is the point on the axial line of quadrupole at distance ‘r’ from its centre.

According to principle of superposition, the net electric field at point ‘P’ is the resultant of electric field due to all charges at A, B and C.

$$i.e \;\; E = E_A + E_B + E_C$$

$$or, \qquad E = \frac{q}{4\pi\varepsilon_0(r+a)^2} + \frac{-2q}{4\pi\varepsilon_0 r^2} + \frac{q}{4\pi\varepsilon_0(r-a)^2}$$

$$or, \qquad E = \frac{q}{4\pi\varepsilon_0}[\frac{1}{(r+a)^2} + \frac{1}{(r-a)^2} - \frac{2}{r^2}]$$

$or,$

$$E = \frac{q}{4\pi\varepsilon_0}[\frac{r^2(r-a)^2 + r^2}{(r+a)^2} \frac{(r+a)^2 - 2}{(r-a)^2} \frac{(r+a)^2(r-a)^2}{r^2}]$$

$$or, \qquad E = \frac{q}{4\pi\varepsilon_0}[\frac{6r^2a^2 - 2a^4}{r^2(r^2-a^2)^2}]$$

$$or, \qquad E = \frac{q2a^2}{4\pi\varepsilon_0}[\frac{3r^2 - a^2}{r^2(r^2-a^2)^2}]$$

$$E = \frac{Q}{4\pi\varepsilon_0}\left[\frac{3r^2 - a^2}{r^2(r^2-a^2)^2}\right] \; where \; Q = 2qa^2$$

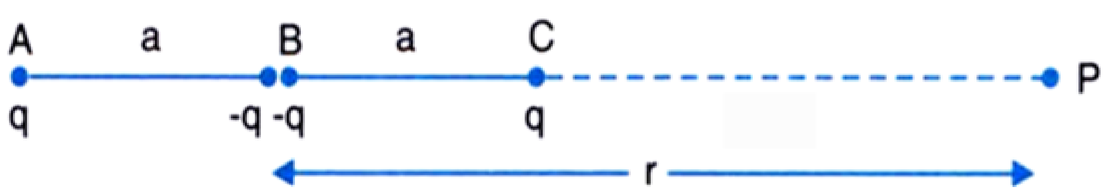
For a short Quadrupole; $r \gg a$

$$E = \frac{Q}{4\pi\varepsilon_0}\left[\frac{3r^2}{r^2r^4}\right]$$

$$\therefore \; E = \frac{3Q}{4\pi\varepsilon_0 r^4}$$

Which is required electric field, due to Electric Quadrupole.

Electric potential due to linear Quadrupole:-



Consider a linear quadrupole of separation 2a with magnitude of each charge q as shown in figure. P is the point on the axial line of quadrupole at distance ‘r’ from its centre.

According to principle of superposition, the net electric potential at point ‘P’ is the resultant of electric potential due to all charges at A, B and C.

$$i.e\ \ V = V_A + V_C + V_B$$

$$V = \frac{q}{4\pi\varepsilon_0(r+a)} + \frac{q}{4\pi\varepsilon_0(r-a)} - \frac{2q}{4\pi\varepsilon_0 r}$$

$$or, \qquad V = \frac{q}{4\pi\varepsilon_0}[\frac{1}{r+a} + \frac{1}{r-a} - \frac{2}{r}]$$

$$or, \qquad V = \frac{q}{4\pi\varepsilon_0}[\frac{r(r-a) + r(r+a) - 2(r^2-a^2)}{r(r^2-a^2)}]$$

$$or, \qquad V = \frac{q}{4\pi\varepsilon_0}[\frac{2a^2}{r(r^2-a^2)}]$$

$$or, \qquad V = \frac{Q}{4\pi\varepsilon_0}\frac{1}{r(r^2-a^2)} \quad where, Q = 2qa^2$$

In case of very short quadrupole we have

$$\therefore\ V = \frac{Q}{4\pi\varepsilon_0 r^3}$$

Numerical Examples:-

1. **Three charges $\mathbf{1 \times 10^{-7}}$ Columb, $\mathbf{-4 \times 10^{-7}}$ Columb and $\mathbf{2 \times 10^{-7}}$ Columb are placed at the three verticies of an equilateral triangle of side $\mathbf{0.1}\ \boldsymbol{m}$. Find the minimum amount of work required to dismantle this structure. Or what is the mutual potential energy of system of the charges?**

Solution:-

$$\text{Mutual potential energy (U)} = \mathrm{U_{12} + U_{23} + U_{31}}$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0 r} + \frac{q_2 q_3}{4\pi\varepsilon_0 r} + \frac{q_3 q_1}{4\pi\varepsilon_0 r}$$

$$= \frac{1}{4\pi\varepsilon_0 r}\left(-4 \times 10^{-14} - 8 \times 10^{-14} + 2 \times 10^{-14}\right)$$

$$= \frac{-1 \times 10^{-13}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times 0.1}$$

$$\therefore \mathrm{U} = -8.99 \times 10^{-3}\ \mathrm{J}$$

2. **27 identical drops of mercury are charged simultaneously to the same potential of $\mathbf{10}$ V. What will be the potential if all the drops are made to combine to form one large drop? Assume the drop to be spherical.**

Solution:-

Let r be the radius of each small drops, R be the radius of large drop and q be the charge on each drops. Then;

$$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

$$i.e. R = 3r$$

The electric potential of each small drop is;

$$V_s = \frac{q}{4\pi\varepsilon_0 r}$$

And for large drop;

$$V_L = 27\frac{q}{4\pi\varepsilon_0 R} = 27\frac{q}{4\pi\varepsilon_0 3r}$$

$$= \frac{9q}{4\pi\varepsilon_0 r} = 9V_s = 9 \times 10$$

$$\therefore\ V_L = 90\,Volts$$

3. A spherical drop of water carrying a charge of 30 PC has potential of 500 volts at its surface. (i) What is the radius of the drop? (ii) If two drops of same charge and radius combined to form a single spherical drop, what is the potential at the surface of new drop?

Solution:-

$$\text{q} = 30\ \text{PC} = 30 \times 10^{-12}\ \text{C}$$

$\mathrm{V} = 500$ volts

(i)We have; $\mathrm{V} = \dfrac{\mathrm{q}}{4\pi\varepsilon_0 \mathrm{r}}$

or, $\quad \mathrm{r} = \dfrac{30 \times 10^{-12}}{500} \times 9 \times 10^{9}$

$\therefore \mathrm{r} = 5.4 \times 10^{-4}$ m

(ii) Also from question, $\quad \dfrac{4}{3}\pi\mathrm{R}^3 = 2 \times \dfrac{4}{3}\pi\mathrm{r}^3$

$or, \quad R^3 = 2r^3$

$\therefore R = 2^{1/3}r = 6.8 \times 10^{-4}$ m

$$\therefore \mathrm{V} = \frac{2\mathrm{q}}{4\pi\varepsilon_0 \mathrm{R}} = \frac{2 \times 9 \times 10^9 \times 30 \times 10^{-12}}{6.8 \times 10^{-4}}$$
$$= 794.12 \text{ Volts}$$

4. Two small spheres of charge $10\,\mu C$ and $40\,\mu C$ are placed 5 cm apart. Find the location of a point between them where field strength is zero.

Solution:-

From question; $\mathrm{E}_1 = \mathrm{E}_2$

Consider the field strength is zero at x distance from first charge then;

$$\frac{q_1}{4\pi\varepsilon_0 x^2} = \frac{q_2}{4\pi\varepsilon_0 (5-x)^2}$$

$$or, \frac{10\times10^{-6}}{x^2} = \frac{40\times10^{-6}}{(5-x)^2}$$

$$or, \frac{1}{x^2} = \frac{4}{(5-x)^2}$$

$$or, 4x^2 = (5-x)^2$$

$$or, 2x = 5 - x$$

$$\therefore x = 1.67\ cm$$

$i.e.$ The location of the point is 1.67 cm from first charge.

5. Assume that earth have surface charge density of $1.6 \times 10^{-10}\ electron/m^2$. Calculate earth's electric field and potential on the earth surface. Given that radius of earth is $6400\ km$.

Solution:-

Surface charge density (σ) = 1.6×10^{-10} electron/m^2

Radius of the earth (R) = 6.4×10^6 m

If q is charge on earth, the electric potential on its surface is;

$$V = \frac{q}{4\pi\varepsilon_0 R} = \frac{1}{\varepsilon_0}\times\frac{q}{4\pi R} = \frac{1}{\varepsilon_0}\times\frac{R.q}{4\pi R^2} = \frac{\sigma R}{\varepsilon_0}$$

$$\text{or}, V = \frac{1.6 \times 10^{-10} \times 6.4 \times 10^{6}}{8.85 \times 10^{-12}}$$

$$\therefore V = 0.1157 \times 10^{8} \text{ Volts}$$

6. A charge of $5 \times 10^{-5}\ C$ is distributed between two sphere. It is found that they repeal each other with a force of 1 N. when their centers are 2 m apart find the charge on each sphere.

Solution:-

We have; $q_1 + q_2 = 5 \times 10^{-5}$ … … … . (i)

$$\text{Now}, F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$\text{or}, q_1 q_2 = F \times 4\pi\varepsilon_0 r^2$$

$$\therefore q_1 q_2 = 4.44 \times 10^{-10} \ldots \ldots \ldots \ldots \text{(ii)}$$

$$\text{We have}, (q_1 - q_2) = [(q_1 + q_2)^2 - 4q_1 q_2]^{1/2}$$

$$or, q_1 - q_2 = [(5 \times 10^{-5})^2 - 4 \times 4.44 \times 10^{-10}]^{1/2}$$

$$\therefore q_1 - q_2 = 2.69 \times 10^{-5} \ldots \ldots \ldots \text{(iii)}$$

Adding equation (i) and (iii) we get;

$$2q_1 = 7.69 \times 10^{-5}$$

$$\therefore q_1 = 3.84 \times 10^{-5}\ \text{C}$$

From equation (iii);

$$q_2 = q_1 - 2.69 \times 10^{-5}$$

$$\therefore q_2 = 1.15 \times 10^{-5} C$$

7. The electric potential V varies with x according to the relation $V = 5 + 4x^2$. Calculate the force experience by a negative charge of $2 \times 10^{-6}\ C$ located at $x = 0.5\ m$.

Solution:-

Potential (V) $= 5 + 4x^2$

Charge (q) $= -2 \times 10^{-6} C$

Distance (x) $= 0.5\ m$

$$E = -\frac{dV}{dx}$$

$$F = qE = -2 \times 10^{-6} \times (-8) \times 0.5$$

$$\therefore F = 8 \times 10^{-6}\ N$$

Exercise:-

1. What is electric dipole and dipole moment? Derive an expression of the electric field intensity at a point due to dipole at equatorial line?
2. What is electric quadrupole? Calculate potential for points on the axis of the quadrupole.
3. What is electric field intensity? Show that electric field for a short dipole drops inversely to cube of the distance at any point from the dipole on the axial line.
4. Derive an expression for the electric potential at a point P at an axial distance x from center of the ring of radius a and linear charge density λ. Hence develop the expression for electric field intensity at the same point.
5. Prove that the electric field due to a short dipole at a point on axial line is twice that on the equatorial line.
6. Derive the relation for the potential at any point due to an electric dipole and show that no work is done in bringing a charge from infinity to dipole along the perpendicular bisector of the dipole.
7. Determine the electric field at a distance z on the central axis from the center of a charged ring. Also find the maximum value of electric field.
8. A thin ring made of plastic of radius R is uniformly charged with linear charge density λ. Calculate the electric field intensity at any point at an axial distance y from the

center. If electron is constrained to be in axial line of the same ring, show that the motion of electron is SHM.

9. A particle of charge –q and mass m is placed midway between two equal positive charges q_o of separation d. If the negative charge –q is displaced in perpendicular direction to the line joining them and released. Show that the particle describes a SHM with a period $T = \sqrt{\frac{\varepsilon_o m\pi^3 d^3}{q q_o}}$.

10. Calculate electric field at any point in axial distance due to a dipole and a quadrupole. What conclusion you can draw from your results.

11. Charges are uniformly distributed throughout the volume of an infinitely larger cylinder of radius a. Show that the electric field at a distance r from the cylinder axis r < a is given by $E = \frac{\rho r}{2\varepsilon_o}$. Where ρ is the volume charge density.

12. Write an expression for the electric field at any point in the axial line of a charged ring. Using this equation, calculate the electric field at any point in the axial line of a charged disk.

13. For a given short electric dipole, Show that the electric potential at any point at a distance r is $V = \frac{P\cos\theta}{r^2}$, where θ is the angle made by r to the dipole moment. Using above relation fin an expression for resultant electric intensity at that point.

14. Calculate the electric field due to a uniformly charged rod of length l at a point along its long axis at a distance 'a' from its nearest end.

15. Define electric flux. Determine electric field due to an infinite line of charge.

16. To similar balls each of mass m are hung from silk threads of length l and carry similar charges q. Assume that the angle made by each thread with vertical, θ is small. Show that $x = (\frac{q^2 l}{2\pi\varepsilon_0 mg})^{1/3}$, where x is separation between the balls. Also calculate the charge q on the hung mass if $l = 1.2$ m, m $= 20$ gm and x $= 3$ cm.

17. Two equal and opposite charge of magnitude 2×10^{-7} C are 15 cm apart (i) what are the magnitude and direction of E at a point mid way between the charges? (ii) What force would act on an electron placed there?

18. The electric potential V due to a charge in the surrounding space at any point x meters from the charge is given by the relation, $\mathrm{V} = 8\mathrm{x} + 3\mathrm{x}^2$ volts. Find the electric field intensity at a point 1.5 m from the charge. Consider the medium is air.

19. Differentiate between polar and non-polar dielectrics. Using Gauss law in dielectric establish a relation of electric field with displacement vector and polarization vector and obtain the relation for free and induced charge in the dielectric.