CW

68. $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$

=> Soln.

$$J_n(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+n+1)} \left(\frac{x}{2}\right)^{n+2m}$$

put $n = \frac{1}{2}$

$$J_{\frac{1}{2}}(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+\frac{1}{2}+1)} \left(\frac{x}{2}\right)^{\frac{1}{2}+2m}$$

$$= \left(\frac{x}{2}\right)^{1/2} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+3/2)} \left(\frac{x}{2}\right)^{2m}$$

$$= \sqrt{\frac{x}{2}} \left[\frac{(-1)^0}{0!\,\Gamma(0+3/2)} \left(\frac{x}{2}\right)^{2\cdot 0} + \frac{(-1)^1}{1!\,\Gamma(1+\frac{3}{2})} \left(\frac{x}{2}\right)^{2\cdot 1} + \frac{(-1)^2}{2!\,\Gamma(2+3/2)} \left(\frac{x}{2}\right)^{3\cdot 1}\right.$$

$$= \sqrt{\frac{x}{2}} \left[\frac{1}{\frac{1}{2}\sqrt{\pi}} - \frac{1}{\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}} \frac{x^2}{4} + \frac{1}{2\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}} \frac{x^4}{16} \right]$$

$$= \sqrt{\frac{x}{2}} \frac{1}{\frac{\sqrt{\pi}}{2}} \left[1 - \frac{x^2}{3\cdot 2} + \frac{x^4}{5\cdot 4\cdot 3\cdot 2} - \cdots \right]$$

$$= \sqrt{\frac{x}{2}} \frac{2}{\sqrt{\pi}} \frac{1}{x} \left[x - \frac{x^3}{3\cdot 2\cdot 1} + \frac{x^5}{5\cdot 4\cdot 3\cdot 2\cdot 1} - \cdots \right]$$

$$= \sqrt{\frac{2}{\pi x}} \left[\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \right]$$

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x.$$

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Q.1 $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

we have

$$J_n(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+n+1)} \left(\frac{x}{2}\right)^{n+2m}$$

put $n = \frac{-1}{2}$

$$J_{-\frac{1}{2}}(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m-\frac{1}{2}+1)} \left(\frac{x}{2}\right)^{-\frac{1}{2}+2m}$$

$$= \left(\frac{x}{2}\right)^{-\frac{1}{2}} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+\frac{1}{2})} \left(\frac{x}{2}\right)^{2m}$$

$$= \sqrt{\frac{2}{x}} \left[\frac{(-1)^0}{0!\,\Gamma(0+\frac{1}{2})} \left(\frac{x}{2}\right)^{2.0} + \frac{(-1)^1}{1!\,\Gamma(1+\frac{1}{2})} \left(\frac{x}{2}\right)^{2.1} + \frac{(-1)^2}{2!\,\Gamma(2+\frac{1}{2})} \left(\frac{x}{2}\right)^{2.2} + \cdots \right]$$

$$= \sqrt{\frac{2}{x}} \left[\frac{1}{\sqrt{\pi}} - \frac{1}{\frac{1}{2}\sqrt{\pi}} \left(\frac{x}{2}\right)^{2.1} + \frac{1}{2!\,\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}} \left(\frac{x}{2}\right)^{2.2} + \cdots \right]$$

$$= \sqrt{\frac{2}{x}} \cdot \frac{1}{\sqrt{\pi}} \left[1 - \frac{x^2}{2.1} + \frac{x^4}{4.3.2.1} - \cdots \right]$$

$$= \sqrt{\frac{2}{\pi x}} \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \right]$$

$$\therefore J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x.$$

proved

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88. $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$.

=> we known, the recurrence relation of the Bessel's function

$$2n\, J_n(x) = x\left[J_{(n-1)}(x) + J_{n+1}(x)\right] \quad - (1)$$

put $n = \frac{1}{2}$

$$2.\frac{1}{2} J_{\frac{1}{2}}(x) = x\left[J_{(\frac{1}{2}-1)}(x) + x J_{\frac{3}{2}}(x)\right]$$

$$J_{\frac{1}{2}}(x) = x J_{-\frac{1}{2}}(x) + x J_{\frac{3}{2}}(x)$$

$$x J_{\frac{3}{2}}(x) = \left[J_{\frac{1}{2}}(x) - x J_{-\frac{1}{2}}(x)\right]$$

since $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$$

$$x J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x - x \sqrt{} \cos x.$$

$$= \sqrt{\frac{2}{\pi x}} \left[\sin x - x \cos x\right]$$

H.P.

$$\Rightarrow J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x} - \cos x\right]$$

i) $J_{-\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(-\frac{\cos x}{x} - \sin x\right)$

Sol^n.

$$2n J_n(x) = x\left[J_{n-1}(x) + J_{n+1}(x)\right] \quad \text{—} \ (1)$$

put $n = -\frac{1}{2}$ in (1)

$$2 \cdot -\frac{1}{2} J_{-\frac{1}{2}}(x) = x\left[J_{-\frac{1}{2}-1}(x) + J_{-\frac{1}{2}+1}(x)\right]$$

$$-J_{-\frac{1}{2}}(x) = x\left[J_{-3/2}(x) + J_{\frac{1}{2}}(x)\right]$$

$$-J_{-\frac{1}{2}}(x) = x J_{-\frac{3}{2}}(x) + x J_{\frac{1}{2}}(x)$$

$$x J_{-\frac{3}{2}}(x) = -J_{-\frac{1}{2}}(x) - x J_{\frac{1}{2}}(x)$$

$$x J_{-\frac{3}{2}}(x) = -\left(\sqrt{\frac{2}{\pi x}}\cos x + x\sqrt{\frac{2}{\pi x}}\sin x\right)$$

$$x J_{-\frac{3}{2}}(x) = \left(-\sqrt{\frac{2}{\pi x}}\cos x - x\sqrt{\frac{2}{\pi x}}\sin x\right)$$

$$x J_{-\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(-\cos x - x\sin x\right)$$

$$J_{-\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{-\cos x}{x} - \sin x\right)$$

Prove