Chapter - 2

Interest and Time value of money

- 6 marks

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Time value of money:

It is defined as time dependent value of money stemming both from change in purchasing power of money (inflation / defletion) & from the real earning potential of alternatives investment over time.

Simple Interest:

$$I = P * r * N$$

Compound Interest:

$$I_1 = i * P$$
$$F_1 = P + i * P$$
$$= P(1+i)$$
$$I_2 = i * F_1$$
$$F_2 = F_1 + F_2$$
$$= F_1 + iF_1$$
$$= F_1(1+i)$$
$$= P(i+1)(1+i)$$
$$= P(1+i)^2$$
$$F_2 = P(1+i)^2$$

$$\vdots$$

$$\boxed{F_n = P(1+i)^n}$$

Nominal Interest Rate:

It is periodic interest rate times the number of period per year.

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Effective Interest Rate:

The actual rate of interest earned during a year is known as effective interest rate. It can be calculated as:

$$I_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

m = Compounding period per year.

~~Conf~~ Continuous compounding.

$$i = \lim_{m \to \infty} \left(1 + \frac{r}{m}\right)^m - 1$$

$$i = e^r - 1$$

$$\boxed{e^r = 1 + i}$$

Q. What is the effective interest rate of nominal interest rate 10% p.a. If the compounding is:

a.) yearly
b.) quaterly
c.) monthly
d.) daily
e.) Continuously.

Here,

r = 10% p.a.

a.) Yearly

For yearly,

m = 1.

$$i = \left(1 + \frac{r}{m}\right)^m - 1$$

$= \left(1 + \frac{0.1}{1}\right)^1 - 1$

$= 0.1$

$= 10\%$

b.) For quarterly,

$m = 4$

$i = \left(1 + \frac{0.1}{4}\right)^4 - 1$

$= 0.1038$

$= 10.38\%$

c.) For monthly

$m = 12$

$i = \left(1 + \frac{0.1}{12}\right)^{12} - 1$

$= 0.1047$

$= 10.47\%$

d.) For daily

$m = 365$

$i = \left(1 + \frac{0.1}{365}\right)^{365} - 1$

$= 0.1051$

$= 10.51\%$

e.) Continuously,

$e^r = 1 + i$

$\therefore i = e^r - 1$

$= e^{0.1} - 1$

$= 0.1051$

$= 10.51\%$

Q. If you deposit Rs. 10,000 in a saving account now which gives 10% nominal interest rate. What'll be the amount after five years if the interest is compounded

a) semi-annually

b) monthly.

Soln:

$r = 10\%$

$P = \text{Rs. } 10,000$

$N = 5$ years.

a) Semi-annualy:

$$i = \left(1 + \frac{r}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.1}{2}\right)^2 - 1$$

$$= 0.1025$$

$$= 10.25\%$$

$$\therefore F_5 = P(1+i)^5$$

$$= 10,000(1 + 0.1025)^5$$

$$= \text{Rs. } 16288.95$$

b) Monthly:

$$i = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 0.1047$$

$$= 10.47\%$$

$$\therefore F_5 = P(1+i)^5$$

$$= 10,000(1 + 0.1047)^5$$

$$= \text{Rs. } 16452.12$$

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Q. A person deposit a sum of Rs. 5,000 in a bank at a nominal ~~ra~~ interest rate of 12% for 10 years. The compounding is quaterly. Find the maturity of deposit after 10 years.

Here,

$r = 12\%$

$N = 10$ years

$P = $ Rs. 5,000

Quaterly.

$$i = \left(1 + \frac{r}{m}\right)^{m} - 1$$

$$= \left(1 + \frac{0.12}{4}\right)^{4} - 1$$

$$= 0.1255$$

$$= 12.55\%$$

$$\therefore F_{10} = 5000\,(1 + 0.1255)^{10}$$

$$= \text{Rs. } 16,390.189 \,//$$

Conversion Table:

1.) To find P and given A

functionality $\left(\frac{P}{A}, i\%, n\right)$

$$P = A\left[\frac{(1+i)^{n} - 1}{(1+i)^{n} \times i}\right]$$

2.) To find A and given P.

functionality $\left(\frac{A}{P}, i\%, n\right)$

$$A = P\left[\frac{(1+i)^{n} \times i}{(1+i)^{n} - 1}\right]$$

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3) To find P and given F
functionality $\left(\frac{P}{F}, i\%n\right)$

$$P = F(1+i)^{-n}$$

4.) To find F and given P
functionality $\left(\frac{F}{P}, i\%n\right)$

$$F = P(1+i)^{n}$$

5.) To find F and given A
functionally $\left(F/A, i\%n\right)$

$$F = A\left(\frac{(1+i)^{n}-1}{i}\right)$$

6. To find A and given F
functionally $\left(A/F, i\%n\right)$

$$A = F\left(\frac{i}{(1+i)^{n}-1}\right)$$

Q) Suppose you have invested Rs. 1,000 at present. How long it takes for your investment to double if the interest rate is 8% compounded anually?

Here,

$i = 8\%$

P = Rs. 1,000

F = 2P

Rs. 2000

$n = ?$

We have

$$F = P(1+i)^n$$

$$\frac{2000}{1000} = (1+0.08)^n$$

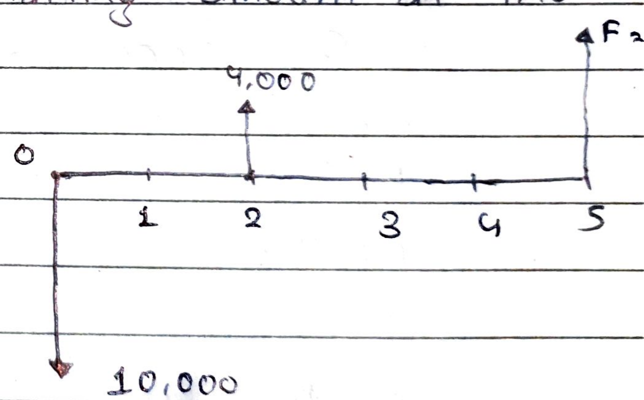
$$2 = (1.08)^n \Rightarrow \log 2 = n \log_{10} 1.08$$

$$n = \frac{\log 2}{\log(1.08)} = 9.006$$

Types of cash Flow

a) Single cash flow

Mr. X deposits Rs. 10,000 now in bank which gives ~~10~~ 8 % per year. He draws Rs. 4,000 at the end of second year. What'll be the remaining amount at the end of 5th year?



Here, First method:

$i = 8\%$

$P = \text{Rs. } 10,000$

For $n = 2$.

$$F_1 = P(1+i)^n$$
$$= \text{Rs. } 11664$$

Now, Second case.

$$P = F_1 - \text{Rs. } 4,000$$
$$= \text{Rs. } 11664 - \text{Rs. } 4,000$$
$$= \text{Rs. } 7664$$

$$\therefore F = P(1+0.08)^3$$
$$= 7664(1.08)^3$$
$$= \text{Rs. } 9654.433 \text{ //}$$

Second method :

$P = F(1+i)^{-n}$

$= 4000(1+0.08)^{-2}$

$= 3429.35$

$\therefore P' = 10000 - 3429.35$

$= 6570.64$

$F = P'(1+0.08)^5$

$= 6570.64(1.08)^5$

$= 9654.33$ //

b) Equal payment Series,

Q. How much money should you deposit in a saving account earning 10% interest rate compounded anually? so that you may make 8 end of year withdraw of Rs. 2000 each

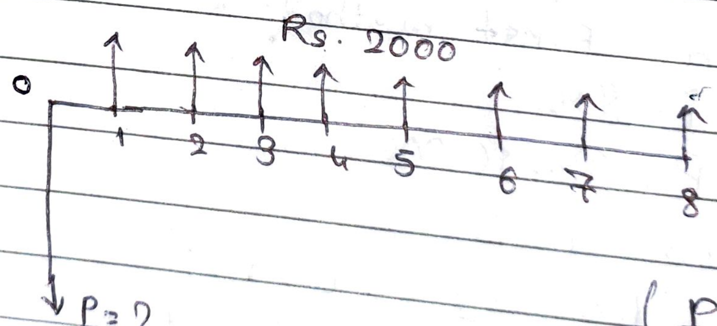
Here

$P = ?$

$i = 10\%$

$n = 8$

$A = Rs. 2000$



$\left(\frac{P}{A}, i\%, n\right)$

$$P = A\left[\frac{(1+i)^n - 1}{(1+i)^n \times i}\right]$$

$$= 2000\left[\frac{(1+0.1)^8 - 1}{(1+0.1)^8 \times 0.1}\right]$$

$= 2000 \times 5.334$

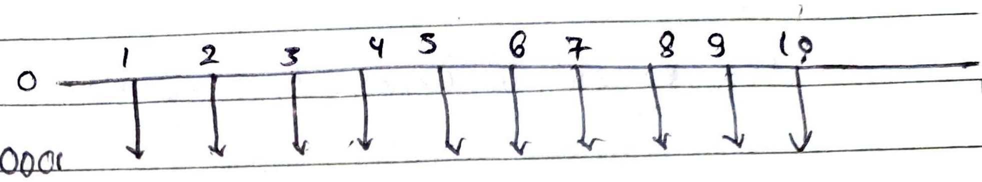
$= 10669.8$

Q. Mr. Jha wants to have Rs. 10,00,000 for the studies of his daughter after the period of 15 years how much money does he has to deposit each year for 10 continuous year in a saving account that earns ~~earns~~ 8% interest annually? 10,00,00

Here,

$i = 8\%$

~~A =~~



Transferring Rs. 10,00,000 from 15 to 10,

A = ?

$F = Rs.\ 10,00,000$

$n = 5$ years

$P = ?$

We have

$P = F(1+i)^{-n}$

$= 10,00,000\,(1+0.08)^{-5}$

$= Rs.\ 6,80,583.197.$

Now,

To find A,

$$\cancel{A = P\left[\frac{(1+i)^n \times i}{(1+i)^n - 1}\right]}$$

$$\cancel{= 680583\left[\frac{1.08^{10} \times 0.08}{1.08^{10} - 1}\right]}$$

$$A = F\left(\frac{i}{(1+i)^n - 1}\right) = 680583.19\left(\frac{0.08}{1.08^{10} - 1}\right)$$

$= Rs.\ 46980.3$

Imp. Q. What'll be the amount at the end of 10 years if you deposit Rs. 5,000 per month for 5 years continuously if the nominal interest rate is 10% compounded qustesly?

[For 1st 5 years, interest'll be calculated monthly so, monthly interest rate is required and after 5 years, annual interest rate is used.]

Here,

For yearly, $i_{eff} = \left(1+\frac{r}{m}\right)^m - 1$

$$= \left(1+\frac{0.1}{4}\right)^4 - 1$$

$$= 10.38\%$$

$$i_{month} = (1+i_{eff})^{1/m} - 1$$

$$= (1+0.1038)^{1/12} - 1$$

$$= 1.0082 - 1$$

$$= 0.826\%$$

For 1st 5 year: $A = 5000$

$n = (5 \times 12)$ months $= 60$ $\qquad \left(\frac{F}{A}, i\%, n\right)$

$i = 0.826\%$

$F = ?$

We have,

$$F = A\left(\frac{(1+i)^n - 1}{i}\right)$$

$$= 5000\left(\frac{(1.00826)^{60} - 1}{0.00826}\right)$$

$$= \text{Rs. } 3{,}86{,}286.19$$

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After 5 years.

$P =$ Rs. 386286.19 $\quad \left(\frac{F}{P}, i\%, n\right)$

$i = i_{eff} = 10.38\%$

$n = 5$ years

$F = ?$

$F = P(1+i)^n$

$= 386286.19(1.1038)^5$

$=$ Rs. 6,32,937.94 //

Q. Mr. X receives a loan of Rs. 1,20,000 from a bank at an interest rate of 12% per year. He wishes to repay the loan in monthly installment with Rs. 3000 per month. How many installment are necessary to complete his payment?

Here $i_{eff} = 12\%$ p.a.

$i_{month} = (1 + i_{eff})^{1/m} - 1$

$= (1+0.12)^{1/12} - 1$

$= 0.948\%$

$P = A\left(\frac{(1+i)^n - 1}{(1+i)^n \times i}\right)$

or $120000 = 3000\left(\frac{(1+0.00948)^n - 1}{(1+0.00948)^n \times 0.00948}\right)$

or $0.3792\,(1.00948)^n = (1.00948)^n - 1$

or $1 = (1 - 0.3792) \times (1.00948)^n$

or $1.610 = (1.00948)^n$

or $n = \log_{1.00948}(1.610)$

$= 50.5$ months.

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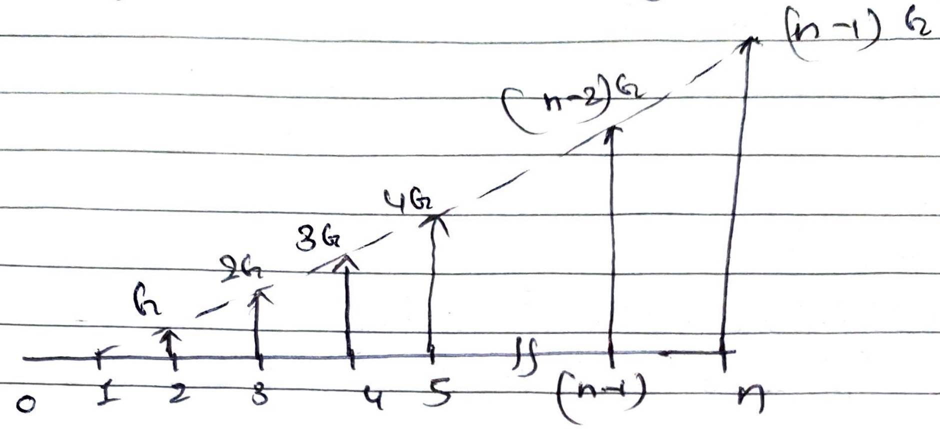
3. Linear Gradient Series

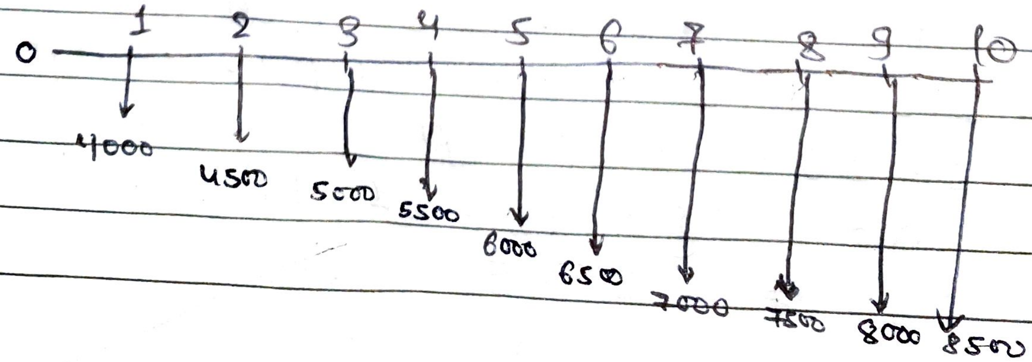
a) To find F when G is given,

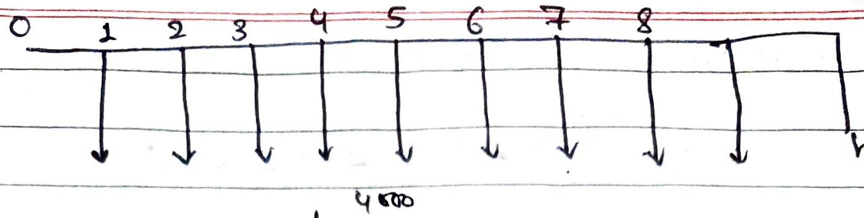


$\left(\frac{F}{G}, i\%, n\right)$

$$F = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i}\right] - \frac{nG}{i}$$

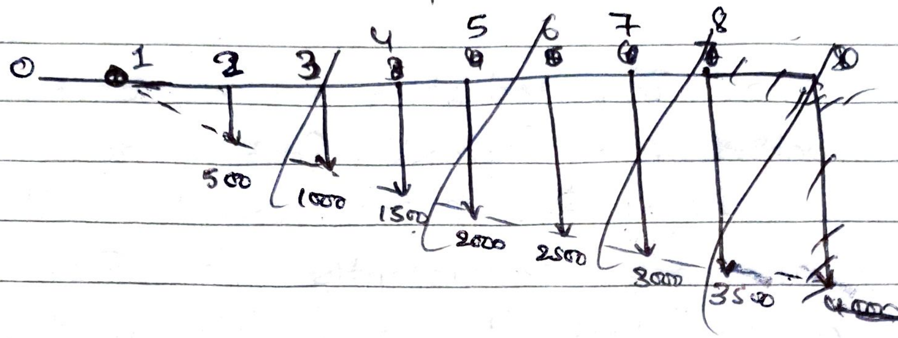
Q. A person is planning for his retired life. He has 10 more year of service. He would like to deposit 20% of his salary which is Rs. 4,000 at the end of 1st year & thereafter he wishes to deposit the amount with an annual increase of Rs. 500 for the next 9 year with an interest rate of 15%. Find the total amount at the end of 10 years of the above service.

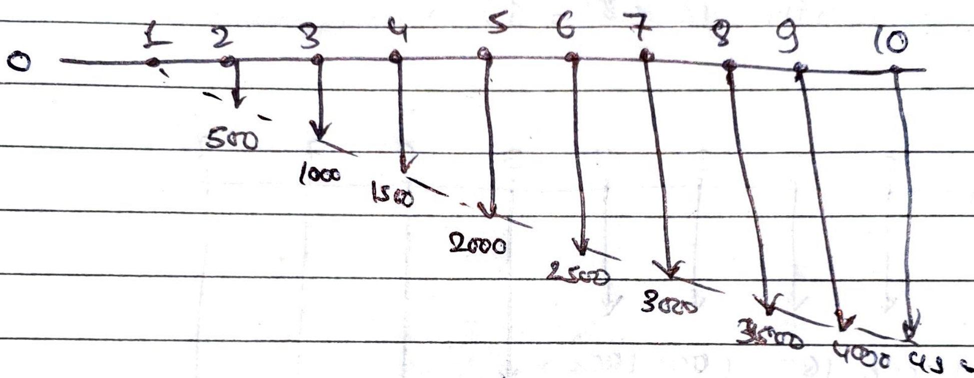


$=$ 

$\left(\frac{F}{A}, i\%n\right)$

$+$





$\left(\frac{G}{A}, i\%N\right)$

Now

$$F = \left\{ \frac{G}{i}\left[\frac{(1+i)^n - 1}{i}\right] - \frac{nG}{i} \right\} + \left\{ A\left(\frac{(1+i)^n - 1}{(1+i)^n i}\right) \right.$$

$$= \frac{500}{0.15}\left[\frac{(1.15)^{10} - 1}{0.15}\right] - \frac{10 \times 500}{0.15} + 4000\left(\frac{1.15^{10} - 1}{1.15^{10} \times 0.15}\right)$$

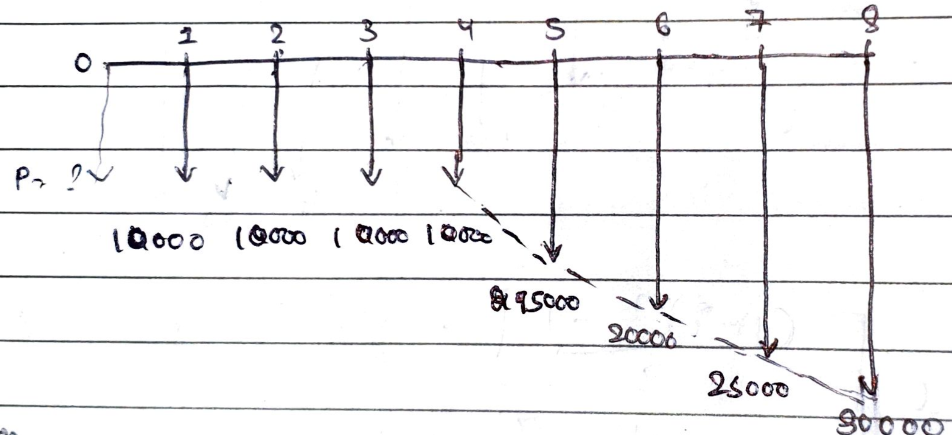
$=$ Rs. 115560.6 //

a.) To find P when G is given.

$$P = \frac{G}{i^2}\left[\frac{(1+i)^n - 1 - ni}{(1+i)^n}\right]$$

Q. An engineer has inspected the average cost on a cement production for 8 years. Cost average were ~~estim~~ steady ~~for~~ at Rs. 10,000 for the 1st ~~to~~ 4 years, but increase consistently by Rs. 5,000 per year for the last 4 year. Calculate the total present worth if i = 10%.

Here,



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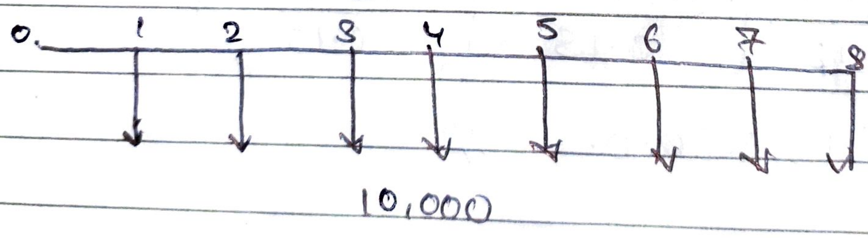
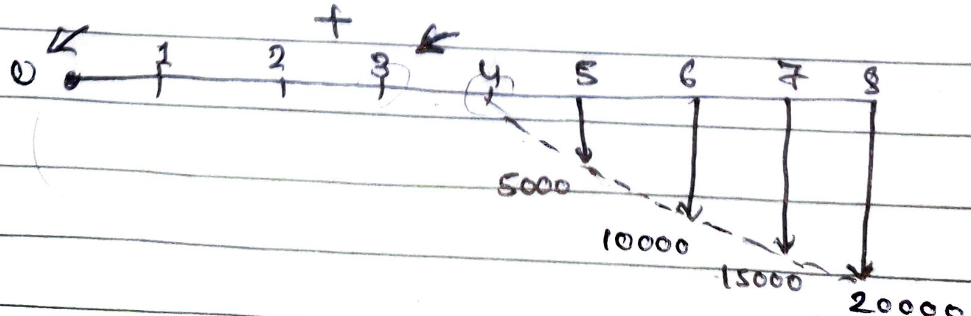


$\left(\frac{P}{A}, i\%, n\right)$

+



$\left(\frac{P}{G}, i\%, n\right)$

$\Downarrow$

$\left(\frac{P}{F}, i\%, n\right)$

For gradient

$$P = \frac{G}{i^2}\left[\frac{(1+i)^n - 1 - ni}{(1+i)^n}\right]$$

$$= \frac{5000}{0.1^2}\left[\frac{(1.1)^5 - 1 - 5\times 0.1}{(1.1)^5}\right]$$

$$\simeq 34309.007$$

$$= 34309.$$

Now,

$$P = F(1+i)^{-n}$$

$$= 34309(1+0.1)^{-3}$$

$$= 25776.86$$

Also, For 10,000

$$P = A\left(\frac{(1+i)^n - 1}{i\times(1+i)^n}\right)$$

$$= 10000\left(\frac{1.1^8 - 1}{0.1\times 1.1^8}\right)$$

$$= \cancel{44}\ 53349.26$$

$$\therefore P = 25776.86 + 53349.26$$

$$= \text{Rs } 79126.12 \quad //$$

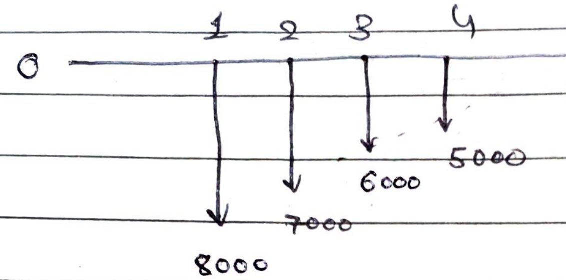
Imp.

Q.) Suppose that one cash flow as follows:

EOY	Net Cash Flow
1	− 8000
2	− 7000
3	− 6000
4	− 5000

Calculate the present equivalent at $i = 15\%$

Here,



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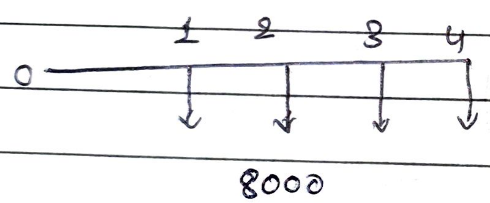
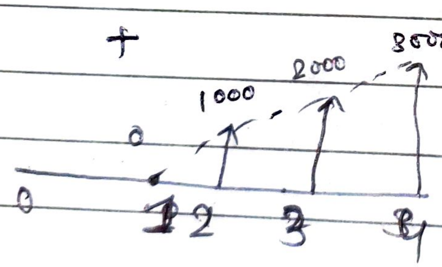


$\left(\frac{P}{A}, i\%\, n\right)$

+



$\left(\frac{P}{G}, i\%\, n\right)$

For 8000,

$$P = A\left[\frac{(1+i)^n - 1}{(1+i)^n \times i}\right]$$

$$= 8000\left[\frac{1.15^4 - 1}{1.15^4 \times 0.15}\right]$$

$$= 22{,}839.82$$

Then,

$$\frac{P}{G} = \frac{G}{i^2}\left[\frac{(1+i)^n - 1 - ni}{(1+i)^n}\right]$$

$$= \frac{1000}{0.15^2}\left[\frac{1.15^4 - 1 - 4 \times 0.15}{1.15^4}\right]$$

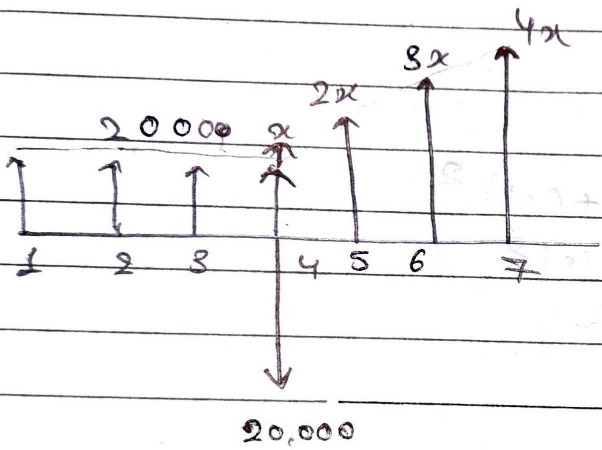
$$= 3786.43$$

Now,

$\therefore$ Net Cash Flow $= -22839.82 + 3786.43$

$= -19053.38$ //

Q. Find the value of x : if $i = 10\%$:



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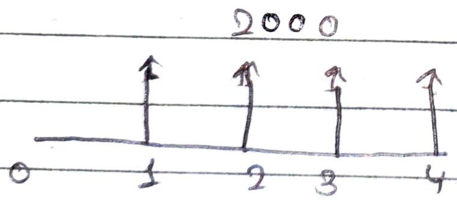
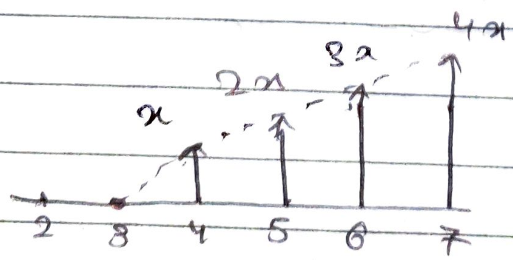


$(F/A, i\%, n)$

+



$\left(\frac{P}{G}, i\%, n\right) \times \left(\frac{F}{P}, i\%, n\right)$

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$$F_1 = A\left(\frac{(1+i)^n - 1}{i}\right)$$

$$= 2000\left(\frac{1.1^4 - 1}{0.1}\right)$$

$$= 9282.$$

Then,
For Gradient

$$P = \frac{G}{i^2}\left[\frac{(1+i)^n - 1 - ni}{(1+i)^n}\right]$$

$$= \frac{x}{0.1^2}\left[\frac{1.1^5 - 1 - 5\times0.1}{1.1^5}\right]$$

$$= 6.86x.$$

Then,

$$F_2 = P(1+i)^n$$

$$= 6.86x(1+0.1)^2$$

$$= 6.8x \times 1.1^2$$

$$= 8.3006x.$$

Now
At Year = 4,

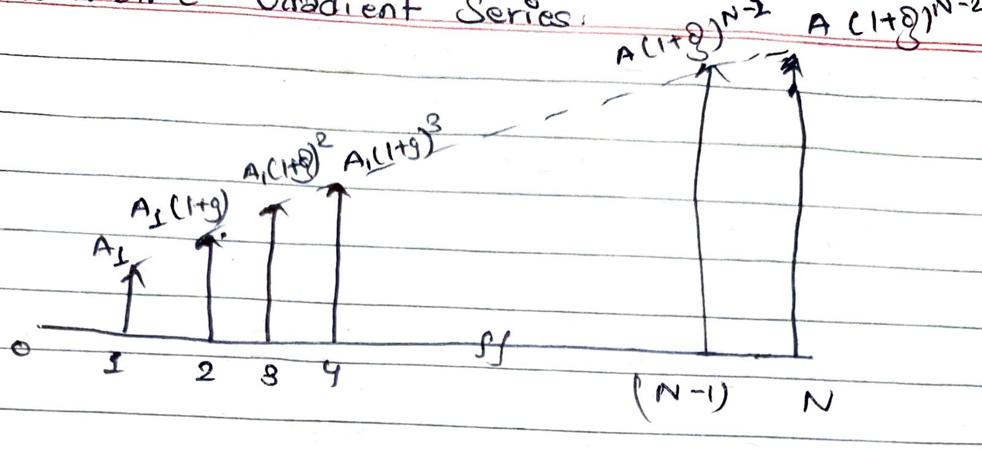
$$F_1 + F_2 - 20000 = 0$$

or $9282 + 8.3006x = 20000$

or $8.3x = 10718$

$\therefore x = 1291.$ //

Geometric Gradient Series:



To find P when g, i, A is given

$\left(\frac{P}{A}, g\%, i\%, n\right)$

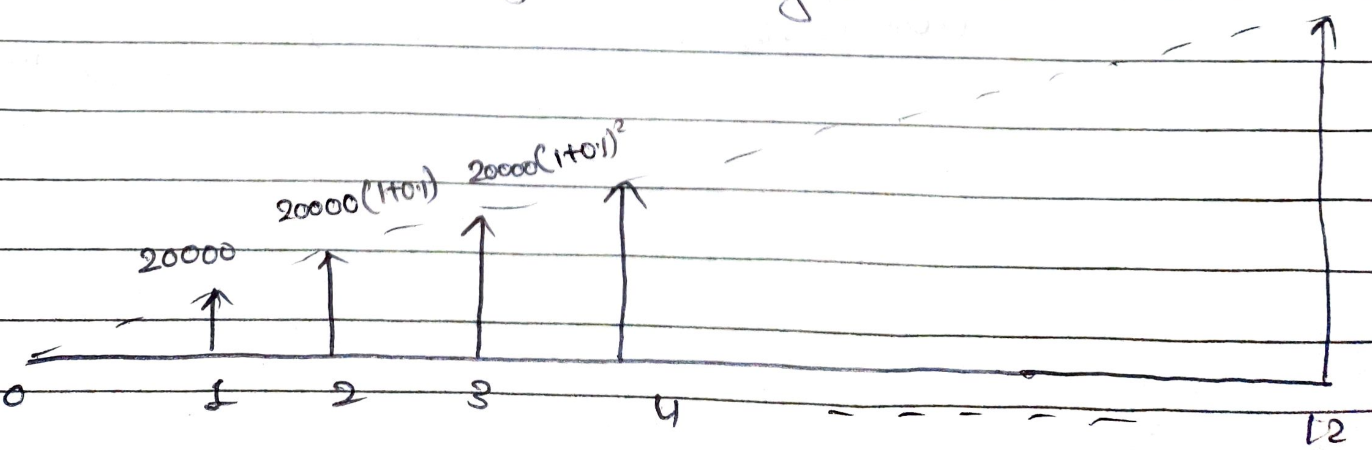
If $i \neq g$

$$P = \frac{A\left[1 - \left(\frac{1+g}{1+i}\right)^n\right]}{i-g}$$

If $i = g$,

$$P = \frac{NA_1}{1+i}$$

Q. The revenue produced by a company at the end of 1st year is Rs. 20,000 and increases by 10% per year, interest rate is 8% per year. What is the present equivalent worth? for 12 years



$$P = 20000 \frac{\left[1 - \left(\frac{1+0.1}{1+0.08} \right)^{12} \right]}{0.08 - 0.1}$$

$$= 246313.08$$

To find F when g, i, A is given $\left(\frac{F}{A}, g\%, i\%, n \right)$

if $i \neq g$

$$F = A \left[\frac{[(1+i)^n - (1+g)^n]}{i-g} \right]$$

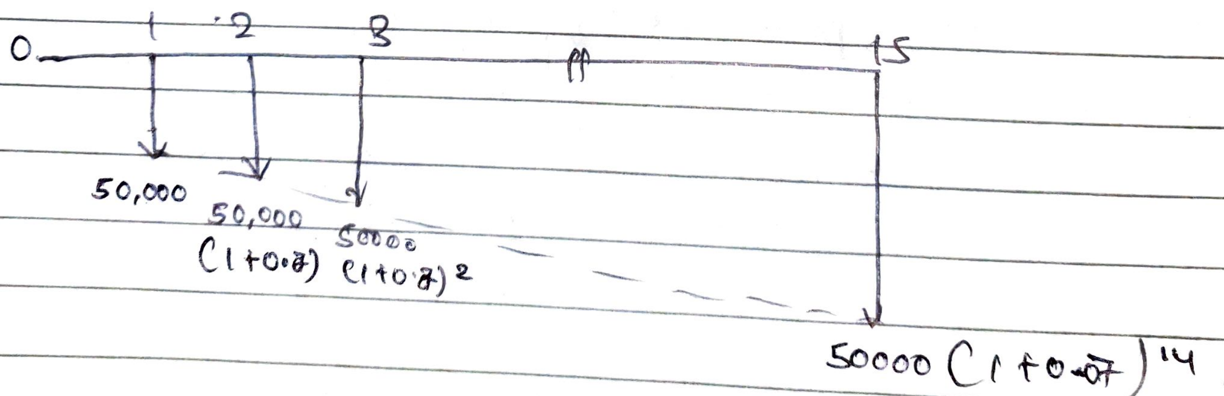
if $i = g$

$$F = NA(1+i)^{n-1}$$

Q. A computer engineer is planning to plane a total of 20 % of his salary which is Rs. 2,50,000 per year. He expects 7 % increase in salary for next 15 years if the mutual fund results in 10 % annual return. What'll be the amount at the end of 15 years?

Here,

A = 20% of Rs. 2,50000 = Rs. 50,000



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Now

$$F = A\left[\frac{(1+i)^n - (1+g)^n}{i-g}\right]$$

$$= 50000\left[\frac{(1.1)^{15} - (1.07)^{5}}{0.1 - 0.07}\right]$$

$= Rs\ 23,63,694.38$.

Q. A person invest a sum of 50,000 in bank at a nominal interest rate of 18% for 15 years? The compounding is monthly. Find the total amount deposited after 15 years?

Solution

$i = 18\%$

$$i_{eff} = \left(1+\frac{r}{m}\right)^m - 1$$

$$= \left(1+\frac{0.18}{12}\right)^{12} - 1$$

$= 0.195$

$= 19.5\%$

$$\cancel{F = A\left[\frac{(1+i)^n - 1}{i}\right]}$$

$$\cancel{= 50000 \times \left[\frac{(1+0.195)^{15} - 1}{0.195}\right]}$$

$=$

$F = P(1+i)^n$

$= 50000(1+0.195)^{15}$

$= 723583.381$ //

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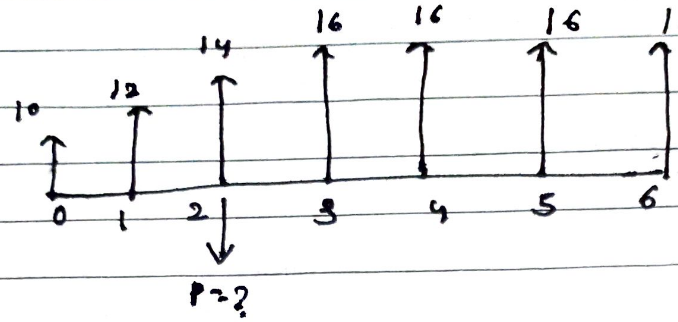
Q. Find the value of P if $i = 10\%$. Use gradient formula:



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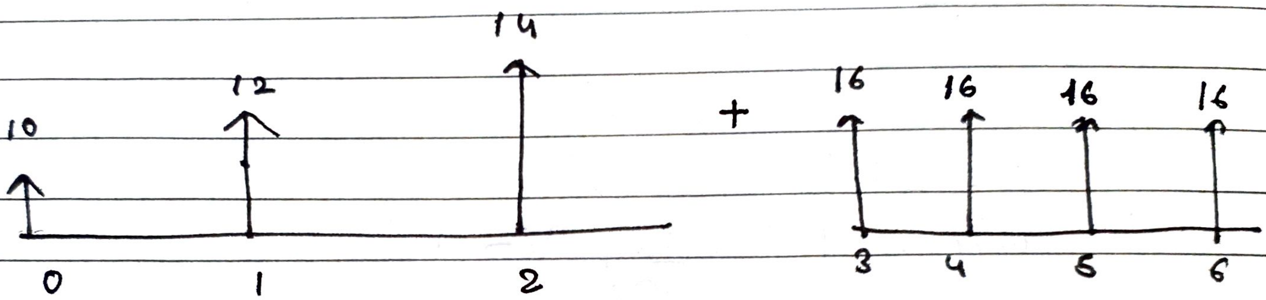
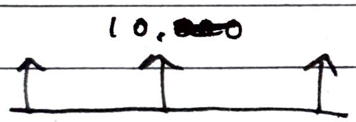


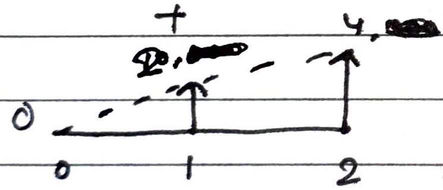
$\left(\frac{P}{A}, i\%, n\right)$

$=$



$\left(\frac{F}{A}, i\%, n\right)$

$+$



$\left(\frac{F}{G}, i\%, n\right)$

For 16,

$$P = A\left[\frac{(1+i)^n - 1}{(1+i)^n \times i}\right]$$

$$= 16\left[\frac{(1+0.1)^4 - 1}{(1+0.1)^4 \times 0.1}\right]$$

$$= 50.71.$$

$$F = \frac{G}{i}\left[\frac{(1+i)^n - 1}{i}\right] - \frac{nG}{i}$$

$$= \frac{2}{0.1}\left[\frac{(1.1)^3 - 1}{0.1}\right] - \frac{3 \times 2}{0.1}$$

$$= 6.2$$

Now,

$$F = A\left(\frac{(1+i)^n - 1}{i}\right)$$

$$= 33.1$$

$P = 50.71 + 33.1 + 6.2 = 80.01.$