Chapter 3
Continuous System Simulation

Continuous System

- A continuous system is one in which the state variable(s) change continuously over time.
- A continuous system is one in which the predominant activities of the system causes smooth changes in the attributes of the system entities.
- Changes in the state variable(s) are predominantly continuous and smooth without any delay.
- When such systems are modeled mathematically, the attributes of the system are controlled by a continuous functions.
- The continuous system is modeled using the differential equations.

Differential and Partial Differential Equations

Differential Equation

- The equation that consists of the higher order derivatives of the dependent variable is known as differential equations.
- A differential equation is a mathematical equation that relates some function with its derivatives.

Non-Linear Differential Equation

Let us assume an equation $M\ddot{x} + D\dot{x} + Kx = KF(t)$. Here M,D,K are constants, x is dependent variable and t is independent variable.

- The differential equation is said to be non-linear if any product exists between the dependent variable and its derivative or between the derivatives themselves.
- The differential equation is said to be non-linear if the dependent variable or any of its derivatives are raised to a power or are combined in other way like multiplication.

- $$\frac{d^3y}{dx^3} - \boxed{\left(\frac{dy}{dx}\right)^2} + 4y = 4e^x \cos x$$

Product between two derivatives ---- non-linear

$$\frac{dy}{dx} + 4\boxed{y^2} = \cos x$$

Product between the dependent variable themselves ---- non-linear

Linear Differential Equation

- The differential equation is said to be linear if no any product exists between the dependent variable and its derivative or between the derivatives themselves.
- The differential equation is said to be linear if any of the dependent variables and its derivatives have power of one(i.e. no higher powers) and are multiplied by the constant.
- Example: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 9y = 0$, $\frac{dy}{dx} + y = 0$, etc.

Partial Differential Equation

- When more than one independent variable occurs in a differential equation, the equation is said to be partial differential equation.
- A partial differential equation (PDE) is an equation involving functions and their partial derivatives.

Necessity of Differential Equation

1. Most physical and chemical process occurring in the nature involves rate of change, which requires differential equations to provide mathematical model.
2. It can be used to understand **general effects of growth trends** as differential equations can represent a growth rate.

Continuous System Models

- Models developed from continuous systems.
- The continuous system is modeled using the differential equations.
- When such systems are modeled mathematically, the attributes of the system are controlled by a continuous functions.
- In a continuous system, the relationships describe the rate at which system attributes change. So the model consists of differential equation.

Analog Computers

- An analog computer is a type of computer that uses the **continuously changeable aspects of physical phenomena** such as electrical, mechanical, or hydraulic quantities to model the problem being solved.
- Analog computers are those computers that are unified with devices like **adder and integral so as to simulate the continuous mathematical model of the system, which generates continuous outputs.**
- The most widely used form of analog computer is the electronic analog computer, based on the **use of high gain dc(direct current) amplifiers, called operational amplifiers.**
- In such analog computer, **voltages are equated to mathematical variables** and **the op amps can add and integrate the voltages.**
- With appropriate circuits, an amplifier can be <u>made to add several input voltages, each representing a variable of the model</u>, to produce a voltage representing the sum of the input variables.
- Different scale factors can be used on the inputs or input values to represent coefficients of the model equations.

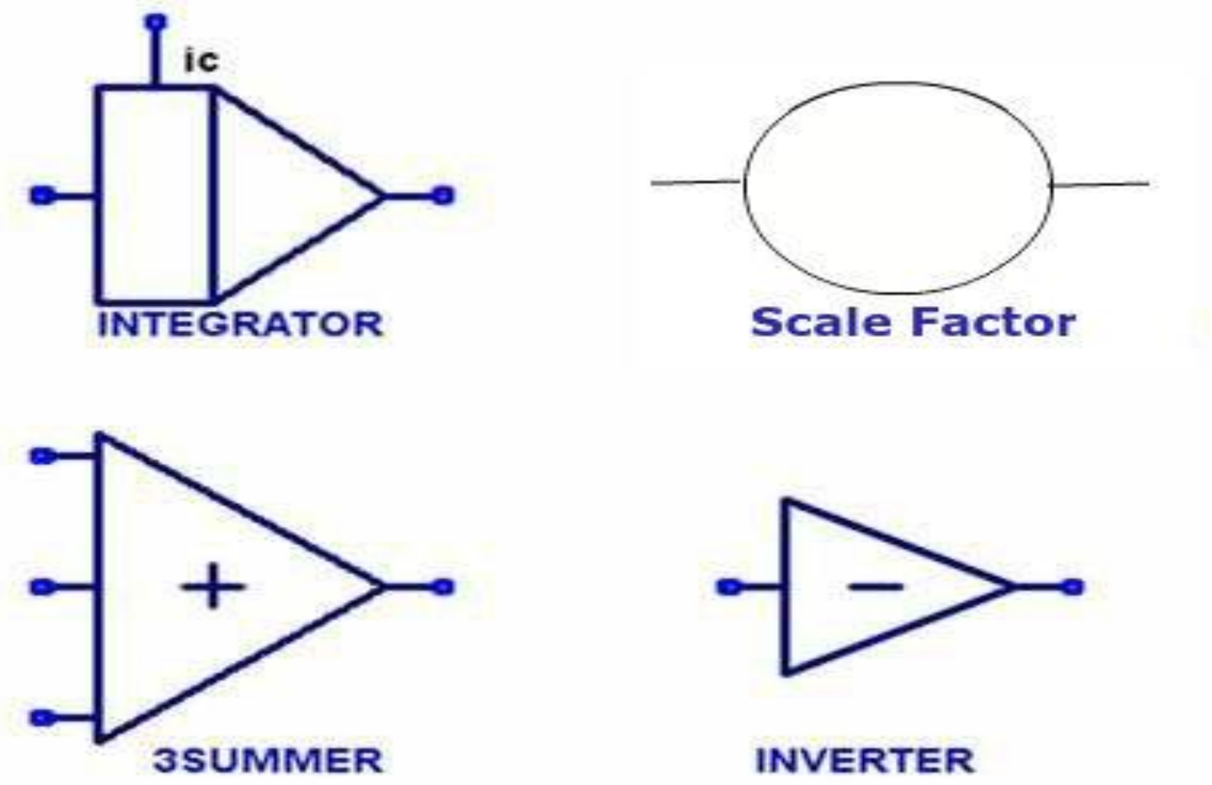
- Another circuit arrangement produces an integrator for which the output is the integral with respect to time of a single input voltage or the sum of several input voltages.
- All voltages can be positive or negative to correspond to the sign of the variable represented.
- Sign inverter can be used to reverse the sign of the input as per the requirement of the model equation.
- Electronic analog computers are limited in accuracy for several reasons. It is difficult to carry the accuracy of measuring a voltage beyond a certain point.
- A number of assumptions are made in deriving the relationships for operational amplifiers, none of which is strictly true. So, amplifiers do not solve the mathematical model with complete accuracy.
- Another type of difficulty is presented by the fact that the operational amplifiers have a limited dynamic range of output, so that scale factors must be introduced to keep within the range.
- As a consequence, it is difficult to maintain an accuracy better than 0.1% in an electronic analog computer.

- A digital computer is not subject to the same type of inaccuracies.
- Virtually any degree of accuracy can be programmed and, with the use of floating-point representation of numbers, an extremely wide range of variations can be tolerated.
- A digital computer also has the advantage of being easily used for many different problems.
- An analog computer must usually be dedicated to one application at a time, although time-sharing sections of an analog computer has become possible.
- In spite of the widespread availability of digital computers, many users prefer to use analog computers. There are several considerations involved.
- The analog representation of a system is often more natural in the sense that it directly reflects the structure of the system; thus simplifying both the setting up of a simulation and the interpretation of the results.
- Under certain circumstances, an analog computer is faster than a digital computer, principally because it can solve many equations in a truly simultaneous manner.

- Whereas a digital computer can be working only on one equation at a time, giving the appearance of simultaneity by interfacing the equations.
- On the other hand, the possible disadvantages of analog computers, such as limited accuracy and the need to dedicate the computer to one problem, may not be significant.

Analog Methods

- The general method by which analog computers are applied can be demonstrated using the second-order differential equation.
- The general method to apply analog computers for the simulation of continuous system models involves following components:



- The equation representing the car wheel system(automobile suspension problem) is

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

$$\text{or, } M\ddot{x} = KF(t) - D\dot{x} - Kx$$

Suppose a variable representing the input F(t) is supplied, and assume for the time being that there exist variables representing x and $\frac{dx}{dy}$ i.e. $\dot{x}$.

- These three variables can be scaled and added with a summer to produce a voltage representing $M\ddot{x}$.
- This variable($M\ddot{x}$) is first scaled by a scaling factor $\frac{1}{M}$ and the result is supplied to an integrator which produces $\frac{dx}{dy}$ i.e. $\dot{x}$.
- Also an inverter is used which changes the sign of the variable and hence produces $-\dot{x}$. Again this variable is fed to an integrator which produces $-x$.

 For convenience, a further sign inverter is included to produce $+x$ as an output.
 Block Diagram to solve the automobile suspension problem is shown below:

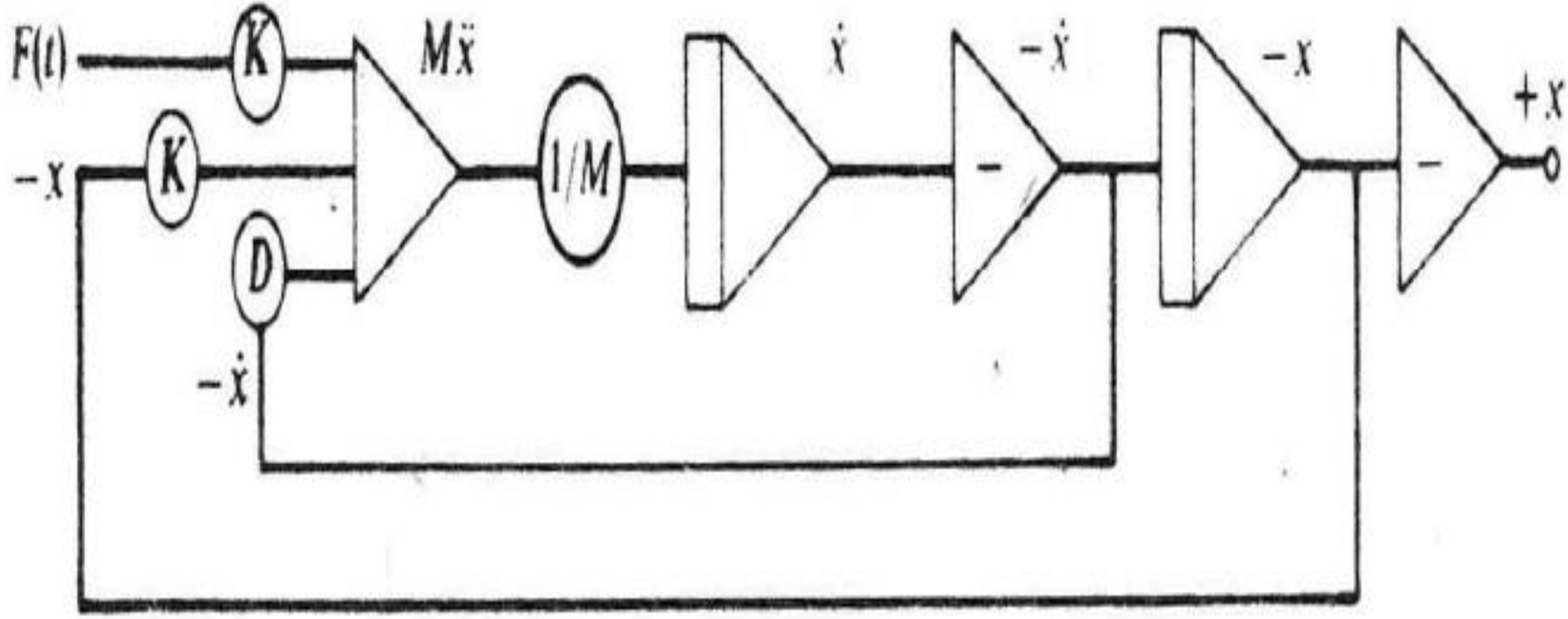


Figure: Diagram for automobile suspension problem

- The addition on the left, with its associated scaling factors, corresponds to the addition of the variables representing the three forces on the wheel, producing a variable representing $M\ddot{x}$.
- The scale is changed to produce $\ddot{x}$ from $M\ddot{x}$ and the result is integrated twice to produce $\dot{x}$ and x.
- Sign changers are introduced so that variables of the correct sign can be fed back to the adder, and the output can be given in convenient form.
- With an electronic analog computer, the variables that have been described would be voltages, and the symbols would represent operational amplifiers arranged as adders, integrators, and sign changers.
- The above figure would then represent how the amplifiers are interconnected to solve the equation.
- There can be several ways of drawing a diagram for a particular problem, depending upon which variables are of interest, and on the size of the scale factors.

- When a model has more than one independent variable, a separate block diagram is drawn for each independent variable and where necessary, interconnections are made between the diagrams.

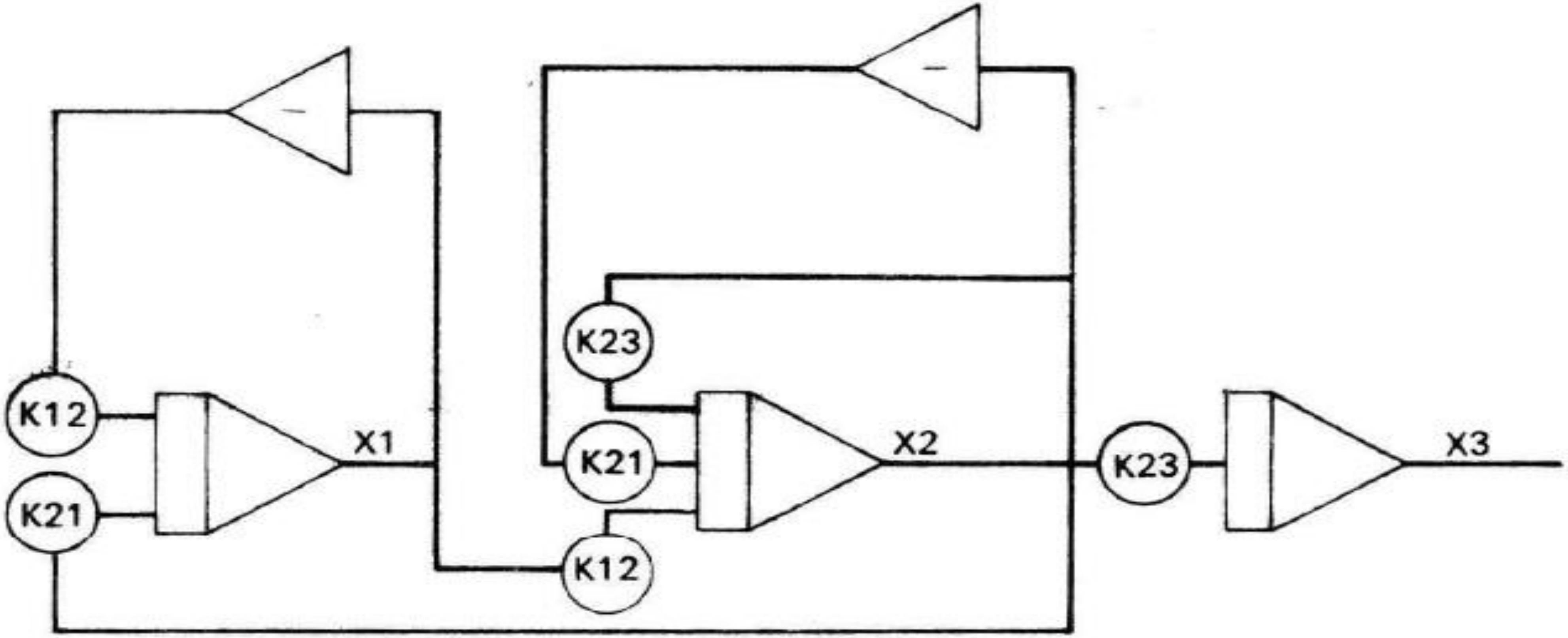


Figure: Analog Computer Model of the Liver

- There are three integrators, shown at the bottom of the figure. Reading from left to right, they solve the equations for x_1, x_2 and x_3.
- Interconnections between the three integrators, with sign changers where necessary, provide inputs that define the differential coefficients of the three variables.
- The first integrator, for example, is solving the equation

$$\dot{x}_1 = -k_{12}x_1 + k_{21}x_2$$

- The second integrator is solving the equation

$$\dot{x}_2 = -k_{21}x_2 + k_{12}x_1 + k_{23}x_2$$

$$\text{or, } \dot{x}_2 = k_{12}x_1 - (k_{21} - k_{23})x_2$$

- Similarly the third integrator is solving the equation

$$\dot{x}_3 = k_{23}x_2$$

Hybrid Simulation

- In case of hybrid simulation, the system is of neither a pure continuous nor a pure discrete in nature.
- For simulating such system, the combination of analog and digital computers are used. Such setup is known as hybrid computers.
- Hybrid computers are computers that exhibit features of analog computers and digital computers.
- The simulation provided by the hybrid computers is known as hybrid simulation.
- The term hybrid is reserved for the case in which functionally distinct analog and digital computers are linked together for the purpose of simulation.
- Hybrid computers can be used to simulate systems that are mainly continuous, but have some digital elements.
- One computer may be simulating the system being studied, while the other is providing the simulation of the environment where the system is to operate.

- The major difficulty in use of hybrid simulation is that it requires **high speed converters** to transform signals from analog to digital form and vice versa.
- High speed converters are required to transform signals from one form of representation to the other form.
- The availability of mini-computers has made hybrid simulation easier by lowering costs and allowing computers to be dedicated to an application.
- For Example: An artificial satellite for which both the continuous equations of motion and the digital signals need to be simulated.

Digital-Analog Simulators

- Digital Analog simulators indicates the use of programming languages in digital computer to simulate the continuous system.
- They allow a continuous model to be programmed on a digital computer.
- The language is composed of macro-instructions which are able to act as **adder**, **integrator** and **sign-changer**.
- A program is written to link these macro-instruction essentially in the same manner as operational amplifiers are connected in analog computers.
- More powerful techniques of applying digital computers to the simulation of continuous system have been developed.

Continuous System Simulation Languages(CSSLs)

- Continuous system simulation languages are high level programming languages which facilitate modelling and simulation of systems characterized by ordinary and partial differential equations.
- CSSLs help to model and study basically continuous systems formulated as block diagrams or in Ordinary Differential Equations(ODE).
- They allow a problem to be programmed directly from the equations of mathematical model rather than breaking those equations into functional elements.
- CSSLs can easily include macros and sub-routines that perform the function of specific analog elements.
- CSSLs include a variety of algebraic and logical expressions to describe the relations between variables.
- They, therefore, remove the orientation towards linear differential equations which characterizes analog methods.

Application Areas of CSSLs

1. **Vehicle Development:** Used in the fields of hydraulic systems(injection pumps, breaks), vehicle-ground interactions, dynamics of accident, etc.
2. **Missiles:** Can be used in autopilot mechanisms, flight systems, all kinds of control loops, etc.
3. **Peripheral System for Computers:** Can be used in electrical-mechanical interaction, diskdrives, pendrives, printers, etc.
4. **Environmental Analysis:** Can be used in the field of environmental analysis growth of plants, spread of harmful substances, etc.
5. **Chemical Processes:** Can be used in the study of diffusion process, in heat-exchangers, chemical-plants, etc.
6. **Electrical Supply:** Can be used in power plants, pumps, power distribution plants, control loops, etc.

CSMP III(Continuous System Modeling Program Version III)

- It is a program used for modeling continuous systems.
- A CSMP III program is constructed from three general types of statements:

a. Structure Statement

- Structure Statement are used to define the model.
- They consist of FORTRAN like statements, and functional blocks designed for operations that frequently occur in model definition.

b. Data Statements

- These statements are used to assign values to parameters, constants and initial conditions.

c. Control Statements

- These statements are used to specify options in the execution of the program and the choice of output.

- Structural Elements can make use of the operations of addition, subtraction, multiplication, division, exponential operations using the same notations and rules that are used in FORTRAN.
- For example, if the model includes the equation $X = \frac{6Y}{W} + (Z - 2)^2$ then following statement will le used:

$$X = 6.0*Y/W + (Z - 2)**2.0$$

- Note that real constants are specified in decimal notation. Exponent notation may also be used.
- Fixed value constants may also be declared.
- Also there are many functional blocks and functions that can be used such as exponential function, trigonometric functions and functions for taking maximum and minimum value.

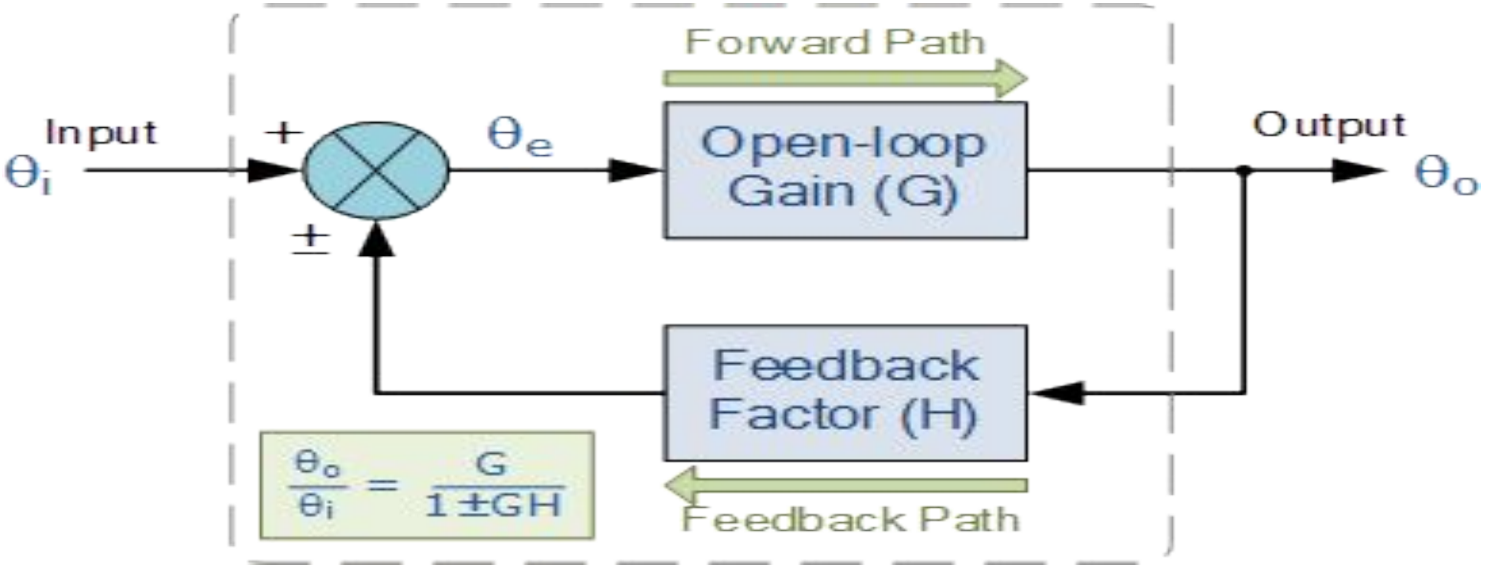
CSMP III Functional Blocks

General Form For CSMP III	Function
$Y = INTGRL(IC,X)$	
$Y = LIMIT(P_1,P_2,X)$	Used for finding limiting values. $Y = P_1$ for $X < P_1$ $Y = P_2$ for $X > P_2$ $Y = X$ for $P_1 \leq X \leq P_2$
$Y = STEP(P)$	Step Function $Y = 0$ for $t < P$ $Y = 1$ for $t \geq P$
$Y = EXP(X)$	
$Y = ALOG(X)$	For finding natural logarithm. $Y = \ln(X)$
$Y = SIN(X)$	Trigonometric Sine Function $Y = \sin(X)$

General Form For CSMP III	Function
$Y = COS(X)$	Trigonometric COSINE Function $Y = \cos(X)$
$Y = SQRT(X)$	
$Y = ABS(X)$	For finding the absolute value $Y = \lvert X \rvert$
$Y = AMAX1(X_1,X_2,\ldots,X_n)$	For finding the maximum value among the available values.
$Y = AMIN1(X_1,X_2,\ldots,X_n)$	For finding the minimum value among the available values.

Feedback Systems

- Feedback system is one in which the output signal is sampled and then fed back to the input to form an error signal that drives the system.
- Feedback systems have a closed loop structure that bring results from past action of the system back to control future action.
- So feedback systems are influenced by their own past behavior.
- Feedback Systems are very useful and widely used in amplifier circuits, oscillators, process control systems as well as other types of electronic systems.

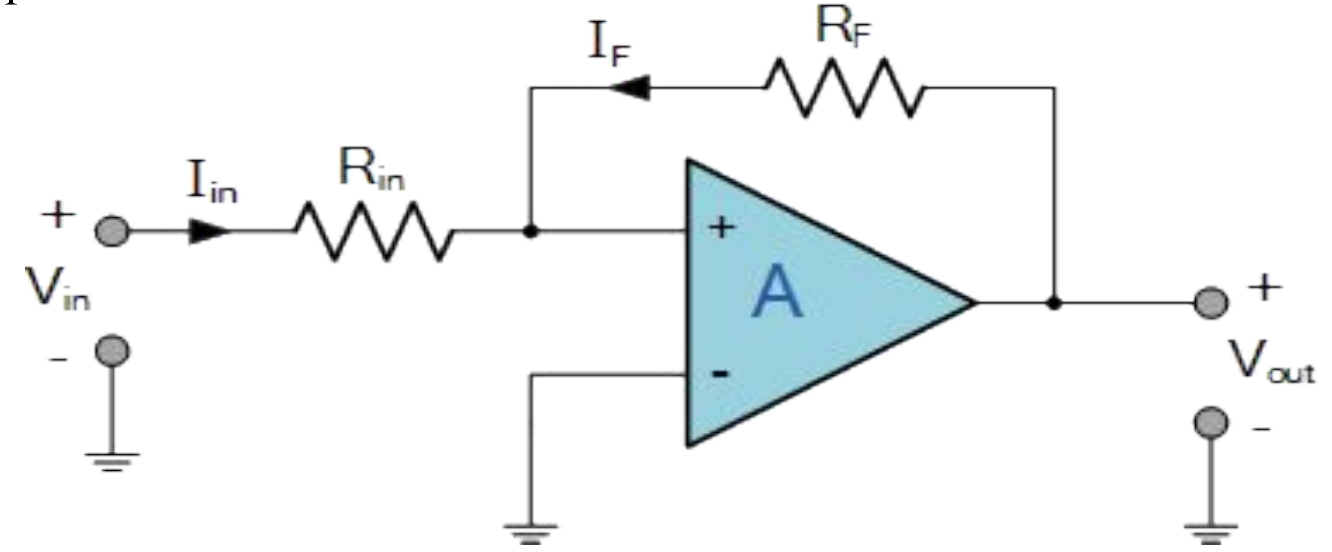


- A home heating system controlled by a thermostat is a simple example of a feedback system.
- The system has a furnace whose purpose is to heat a room and the output of the system can be measured as a room temperature.
- Depending upon whether the temperature is below or above the thermostat setting, the furnace will be turned on or off.
- There are two types of feedback systems:

1. Positive Feedback System
2. Negative Feedback System

1. Positive Feedback System

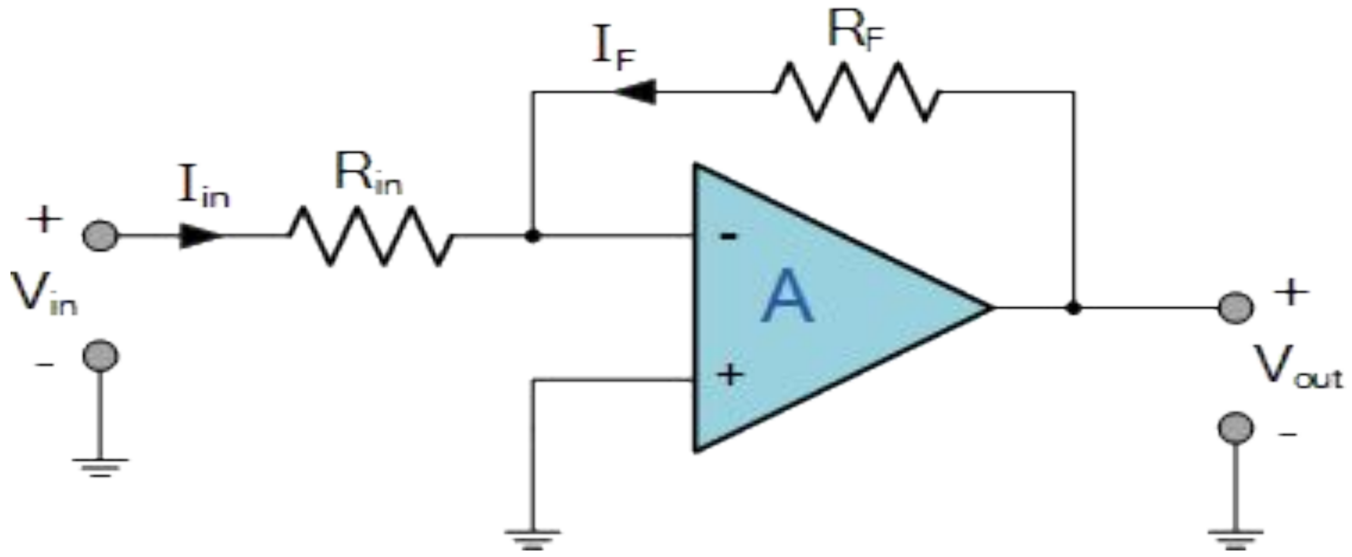
- In a positive feedback system the feedback is in-phase with the original input.
- The set point and output values are added together by the controller.
- The effect of positive (or regenerative) feedback is to "increase" the system gain, i.e, the overall gain with positive feedback applied will be greater than the gain without feedback.
- Positive feedback control of the op-amp is achieved by applying a small part of the output voltage signal at Vout back to the non-inverting (+) input terminal via the feedback resistor, R_F.



- Positive or regenerative feedback increases the gain and the possibility of instability in a system which may lead to self-oscillation and as such, positive feedback is widely used in oscillatory circuits such as Oscillators and Timing circuits.

2. Negative Feedback System

- In a negative feedback system the feedback is out-of-phase with the original input.
- The set point and output values are subtracted from each other by the controller.
- The effect of negative (or degenerative) feedback is to “reduce” the systems gain, i.e, the overall gain with negative feedback applied will be less than the gain without feedback.
- Negative feedback control of the op-amp is achieved by applying a small part of the output voltage signal at Vout back to the inverting (-) input terminal via the feedback resistor, R_F.



- The use of negative feedback in amplifier and process control systems is widespread because as a rule negative feedback systems are more stable than positive feedback systems.
- A negative feedback system is said to be stable if it does not oscillate by itself at any frequency except for a given circuit condition.
- Another advantage is that negative feedback also makes control systems more immune to random variations in component values and inputs.

Positive vs Negative Feedback System

Positive Feedback System	Negative Feedback System
The feedback is in-phase with the original input.	The feedback is out-of-phase with the original input.
The set point and output values are added together by the controller.	The set point and output values are subtracted from each other by the controller.
The overall gain with positive feedback applied will be greater than the gain without feedback.	The overall gain with negative feedback applied will be less than the gain without feedback.
Positive feedback system are more oscillatory	Negative feedback systems are more stable than positive feedback systems.