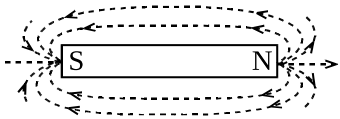
Chapter – 1

MAGNETIC CIRCUITS AND INTRODUCTION

Magnetic Field Theory Review

Magnetic field

Magnetic field is the space around a magnet within which the magnet has affect on the magnetic materials.



$$F = \frac{1}{4\pi\mu} \frac{m_1 m_2}{d^2}$$

Where, $\mu = \mu_o * \mu_r$

Where,

m_1 = Magnetic pole strength of the first pole (wh)

m_2 = Magnetic pole strength of the second pole

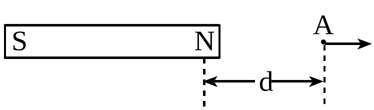
r = Distance between two poles (m)

μ_o = Permeability of free space = $4\pi \times 10^{-7}$ H/M

μ_r = Relative permeability of the medium on which the two poles are lying

Magnetic field intensity (H) :

Magnetic field intensity at any point of magnetic field is defined as the force experienced by a unit north pole at that point.



$$H_A = \frac{m}{4\pi\mu d^2} \text{ (Amp/m)}$$

Magnetic Flux density (B):

It is defined as the magnetic flux per unit area.

$B = \frac{\phi}{A}$ wb/m^2 or Tesla

Work law and it's Applications

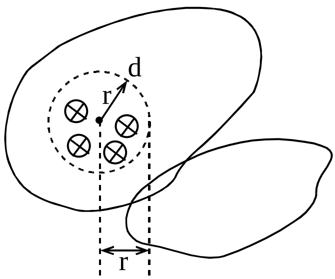
The work law states that the net work done by a unit N - Pole in moving around any closed path in a magnetic field is equal to the "amp-turns " (NI) linked with the closed path.

Mathematically,

$\oint H.dr$ (work done in a closed path. per unit pole) = NI

Application

1.



For Path-d, linking N-conductor, then the work done in moving a unit N-Pole around the circular path is given by,

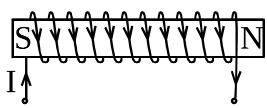
$$\oint H.dr = NI$$

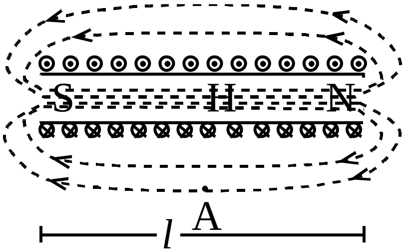
or, $\oint H.dr = NI$

or, $H*2\pi r = NI$

$$H = \frac{NI}{2\pi r}$$

2. Magnetic field due to a solenoid (electromagnet):





Assume that 'H' remains constant throughout the length 'l' of the solenoid and is negligible outside the solenoid. If a unit N-pole is moved around a closed path in a direction opposite to H the work done is given by work law as ,

Work done against the magnetizing force.

$H \times l = NI$

$H = \frac{Ni}{l}$ = Magnetizing force inside the solenoid

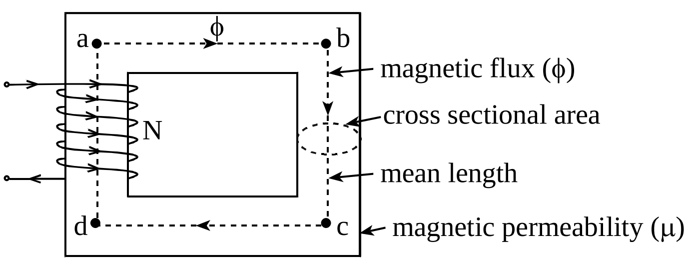
Hence,

$F = \frac{m_1 m_2}{4\pi\mu d^2}$ $\quad\quad$ $H = \frac{F}{M} = \frac{m}{4\pi\mu d^2}$ (A/M)

$B = \frac{\phi}{A}$ (wb/m^2) $\quad\quad$ $B = \mu H = \mu \frac{m}{4\pi\mu d^2} = \frac{m}{4\pi d^2}$ $\quad\quad$ (wb/m^2)

1.1 Magnetic Circuit

A magnetic circuit is the path followed by a magnetic flux (Analogous to the electric circuit as the path followed by electric circuit). It is analogous to electrical current (I).



In an electric circuit, the current flows due to an emf source. Similarly in magnetic circuits the magnetic flux is produced by a quantity known as MMF (Magneto Motive Force)

MMF = NI

Where, N = number of turns in the winding

I = current flow through the winding

The current flow in any electric circuit is opposed by the resistance of the path. Similarly, the magnetic flux flow in a magnetic circuit is opposed by the Reluctance (R) nature of the path.

$$R = \frac{l}{\mu A} = \frac{l}{\mu_o \mu_r A}$$

where, l = mean length of the magnetic path

μ_o = permeability of the core

A = cross sectional area of core

μ_r = relativity of the core

$$\text{Permeance (P)} = \frac{1}{R}$$

1.2 Ohm's Law of Magnetic Flux

Electric Ciruit *Magnetic Ciruit*

Figure 1-1: Analogy between Electric and Magnetic Circuit

Ohm's law: $\frac{V}{I} = R$

$V = IR$

$$\frac{F}{\varnothing} = R$$

$$F = \varnothing R$$

$$\varnothing = B \times A$$

$$\Rightarrow \varnothing = \mu H \times A$$

$$\Rightarrow \varnothing = \frac{\mu NI}{l} \times A$$

$$\varnothing = \frac{NI}{(l/\mu A)}$$

Hence,

Ohm's law in magnetic circuit is,

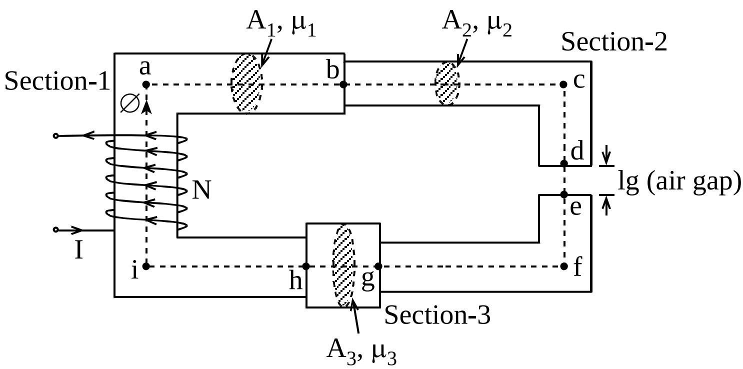
MMF (F) = ϕR

Then, the analogy is as:

	Electrical		Magnetic circuit
1	Current (I)	1	Magnetic flux (ϕ)
2	EMF (E)	2	MMF (F)
3	Resistance (R)	3	Reluctance (R)
4	$I = \frac{E}{R}$	4	$\phi = \frac{mmf}{R}$

1.3 Series and Parallel Magnetic Circuits

1.3.1 Series Magnetic Circuit



If the magnetic flux does not divide and passes through the different section of the core, then those sections are said to be in series forming a series magnetic circuit as shown in fig.

Here, mmf = NI and the resultant flux 'ϕ' flow through each section of the core. Now, reluctance of each part/ section can be calculated as:

Section-1:

L_1= ba + ai + ih

Area = A_1 and Permeability = μ_1

□ $R_1 = \dfrac{L_1}{\mu_1 A_1}$

Section - 2:

$L_2 = bc + cd + ef + fg$

Area = A_2 and permeability = μ_2

□ $R_2 = \dfrac{l_2}{\mu_2 A_2}$

Section - 3:

$L_3 = hg$; Area = A_3 and permeability = μ_3

□ $R_3 = \dfrac{L_3}{\mu_3 A_3}$

Air gap:

Length = lg; Area = Ag and permeability = μ_o

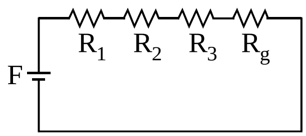
□ $R_g = \dfrac{l_g}{\mu_o A_g}$

Then, total reluctance in series is given by,

$R = R_1 + R_2 + R_3 + R_g$

□ $\phi = \dfrac{mmf}{R}$

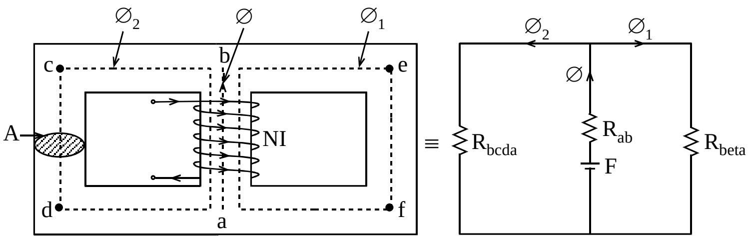
□ $\phi = \dfrac{NI}{R = R_1 + R_2 + R_3 + R_g}$



Air gap has very high reluctance with compare to iron core, it reduces the magnetic flux in the circuit. It is quite similar to addition of very high resistance in series with low resistance in case of series electric circuit.

1.3.2 Parallel Magnetic Circuit

If the magnetic flux produced by mmf, divides into two or more parallel paths in some sections of the magnetic circuit in a core, then those section are said to be in parallel as shown in figure.

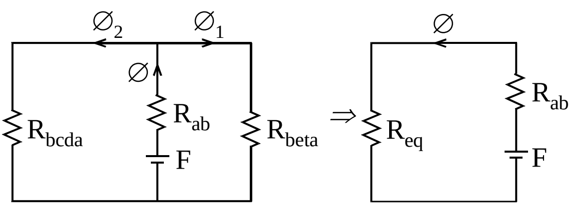


Thus, in this case

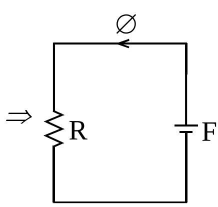
$$R_{ab} = \frac{l_{ab}}{\mu A} \quad ; \quad R_{bcda} = \frac{l_{bcda}}{\mu A}$$

$$R_{beta} = \frac{l_{beta}}{\mu A}$$

And, thus as in electrical circuit, the magnetic equivalent reluctance can be calculated.



$$R_{eq} = \frac{R_{bcda} \times R_{befa}}{R_{bcda} + R_{befa}}$$



$$R = R_{eq} + R_{ab}$$

$$\Rightarrow \varnothing_{net} = \varnothing = \frac{F}{R}$$

$$\therefore \quad \varnothing = \frac{\text{mmf}}{R_{ab} + \dfrac{R_{bcda} \times R_{befa}}{R_{bcda} + R_{befa}}}$$

$$\Rightarrow \quad \varnothing = \frac{NI}{R_{ab} + \dfrac{R_{bcda} \times R_{befa}}{R_{bcda} + R_{befa}}}$$

1.4 B–H Relationship (Magnetization Characteristics)

$$B = \mu H;\ H = \frac{NI}{l} \Rightarrow H \propto I$$

$$B = \frac{\varnothing}{A} \Rightarrow B \propto \varnothing$$

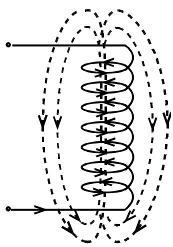


Figure 1-2: Coil without Core

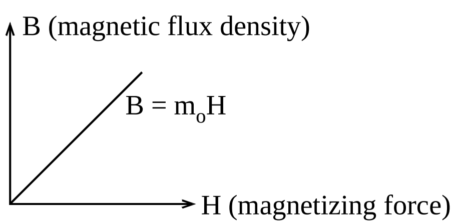
In free space, the magnetic flux density (B) is directly proportion to the magnetizing force (H).

i.e. $B \alpha\ H$

or, $B = \mu_o H$, where μ_o = permeability of free space

$= 4\pi \times 10^{-7}$ H/M

Here, the relationship between B and H is linear one.



However, this is not the case in magnetic material used as core, as in electrical machines and transformers. The relationship is strictly non-linear. A typical B-H current for a magnetic material is shown below.

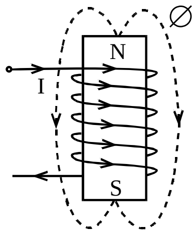
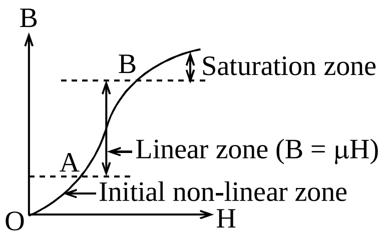


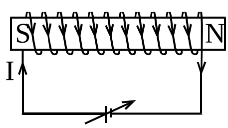
Figure 1-3: Coil with Iron Core (Ferromagnetic Material)



1.5 Hysteresis Curve/ B.H Curve / Magnetizing Curve

1.5.1 Hysteresis Loss in DC Excitation

Consider an electromagnetic supplied by a variable DC supply. The magnetizing force inside the core is given by, $H = \frac{NI}{l}$, Hence varying I , H in the material can be varied and accordingly "B" will also vary.



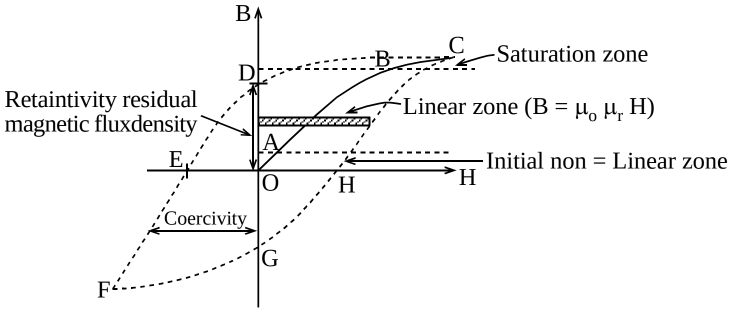


Figure 1-4: Hysteresis Loop

There is a loss in the process of magnetization and demagnetization in the form of heat and is called hysteresis loss, due to the property of magnetic material known as retentivity.

The magnetic flux at any instant is given by,

$\phi(t) = B(t).A$

Then emf induced in the coil according to Faraday's law of electromagnetic induction,

$$e = N\frac{d\phi}{dt} = N.\frac{d(B.A)}{dt} = NA\frac{dB}{dt} \quad .... \text{(i)}$$

and also, magnetizing force is ,

$$H = \frac{NI}{l}$$

$$\text{Thus, } P = e.I = N.A\frac{dB}{dt}.\frac{H.l}{N} \qquad » P = A.L.H\frac{dB}{dt}$$

Energy spent in small time interval, (dw) = P.dt.

or, dw = A.L.H dB

Energy spent in one cycle of magnetization (i.e the complete hysteresis loop) = $\oint dw$

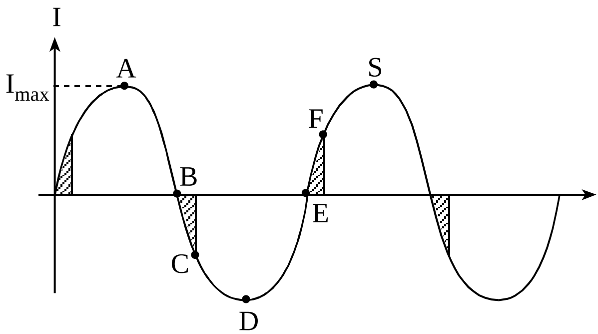
$$W = Al\oint H.dB \qquad » H.dB = \text{Shaded area}$$

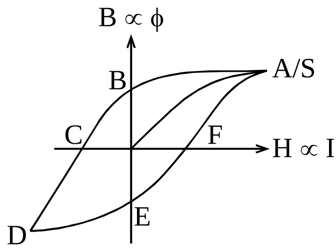
$$» \oint H.dB = \text{Complete area of the loop}$$

$$\frac{W}{Al} = \oint H.dB$$

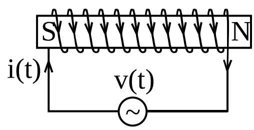
∴ Energy loss per unit volume = Area of the loop.

1.5.2 Hysteresis Loss in AC Excitation





With the varying voltage source, the core inside the coil gets magnetized and demagnetized in each cycle causing hysteresis loss. The power loss due to hysteresis.



$P_n = \eta B_m^{1.6} fv$ (watts)

where, η = steinmetz constant

= 502 J/m^3 (sheet steel)

= 181J/m^3 (silicon steel)

B_m = max. flux density in the core

f = freq. of exciting current

v = volume of iron core

Addition of silicon reduces 'n', hence reduces hysteresis loss.

1.6 Eddy Current Loss

The time varying flux in the core induces emf in the coils according to Faraday's law of electromagnetic induction. But since the core itself is a conductor (all magnetic material are) emf will also be induced in the core resulting circulating currents in the core. These currents are known as eddy current and have a Power loss (I^2R) associated with it. This loss being known as eddy current loss.

This loss depends upon the

i. resistivity of the material
ii. mean length of the path of the circulating current for a given cross-sectional area.

The eddy current loss in the core is given by,

$$P_e = KB_m^2 f^2 t^2 V \text{ (watts)}$$

where,

B_m = maximum value of the flux density in the core

f = frequency of exciting current

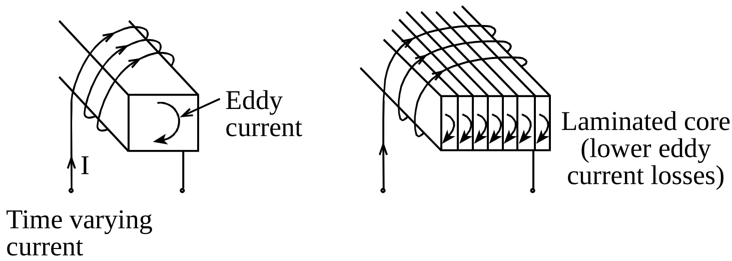
V = volume of iron core

t = thickness of each lamination

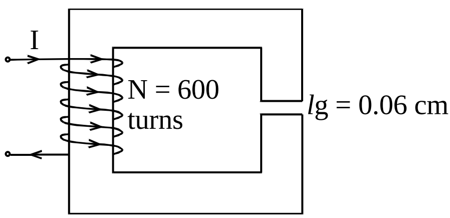
K = constant, depending upon the nature of the core

In practical applications, the eddy current losses can be reduced by

i. Adding silicon to steel which will give a high resistivity of the material
ii. By dividing up the solid core into thin laminations while making sure that each lamination is insulated from each other.



Q. **For the magnetic circuit shown below calculate the value of current 'I' required to produce a magnetic flux density of 1.2 Tesla. Given that cross-sectional area of the core is 16 sq mm, air gap length (l_g)=0.06cm and length of core (l_c) = 40cm take μ_r = 6000.**



Solution:

Total flux required (ϕ) = B × A

$$= 1.2 \times 16\times10^{-4} = 19.2\times10^{-4} \text{ wb}$$

Reluctance of core = $R_e = \dfrac{l}{\mu A}$

$$= \frac{40\times10^{-2}}{4\pi\times10^{-7}\times6000\times16\times10^{-4}}$$

$$= 33157.28$$

Reluctance of air gap = $R_g = \dfrac{l_g}{\mu A}$

$$= \frac{0.06\times10^{-2}}{4\pi\times10^{-7}\times16\times10^{-4}}$$

$$= 298415.52$$

Total reluctance of the circuit (R) = $R_c + R_g$

$$= 331572.8$$

Now,

$$\phi = \frac{NI}{R}$$

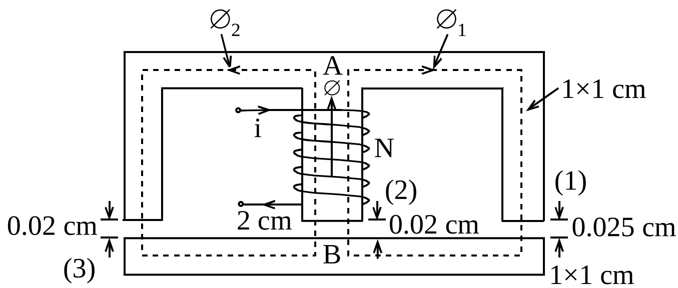
$$\text{or, } I = \frac{\phi R}{N} = \frac{19.2\times10^{-4}\times331572.8}{600} = 1.06 \text{ Amp}$$

Q. For the given magnetic circuit cost steel core with dimensions as shown:

Mean length from A to B through either outer limb = 0.5m

Mean length from A to B through central limb = 0.2m

In the magnetic circuit shown it ios required to establish a flux of 0.75 mwb in the air-gap of the central limb. Determine the mmf of the exciting coil it for the core material (a) $\mu_r = \infty$ (b) $\mu_r = 5000$



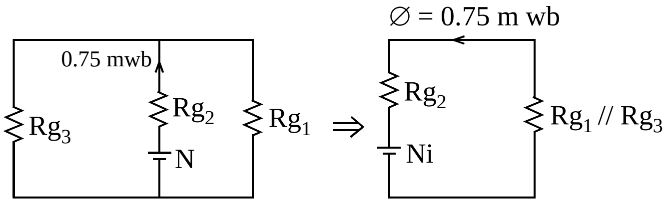
a. $\mu_r = \infty$, i.e there are no mmf drops in the magnetic core in this figure two outer limbs are parallel magnetic ckt. Now, air gap reluctances are :

$$R_{g1} = \frac{0.025\times10^{-2}}{4\pi\times10^{-7}\times1\times10^{-4}} = 1.99\times10^{6} \text{ w/m}$$

$$R_{g2} = \frac{0.02\times10^{-2}}{4\pi\times10^{-7}\times2\times10^{-4}} = 0.796\times10^{6}$$

$$R_{g3} = \frac{0.02\times10^{-2}}{4\pi\times10^{-7}\times1\times10^{-4}} = 1.592\times10^{6}$$

The electrical analog of the magnetic ckt is



Or, mmf = $ØR = 0.75\times10^{-3} (R_{g1} // R_{g2} + R_{g3})$

$= 1230$ AT

b. $\mu_r = 5000$, mmf = 1466 AT

1.7 Faraday's Law of Electromagnetic Induction, Statically and Dynamically Induced emf

A Gentlemen named Michael Faraday Paved a history in 1891 by discovering the relationship between electricity and magnetism. He observed the momentary with the circuit, when magnetic flux linking with the circuit changes momentary with respect to time. After his detail, study of the phenomenon, he formulated some laws, which are well known as "Faraday's law of electromagnetic induction".

i. First law:

"Whenever the magnetic flux linked with a conductor changes with respect to time, an emf will be induced in it." (magnetic flux linkage change = Cuts magnetic flux)

ii. Second law:

"The magnitude of emf induced is equal to the time rate of change of magnetic flux linkage"

The magnetic flux-linkage could be changed in two different ways. Therefore, emf could be produced in two different ways.

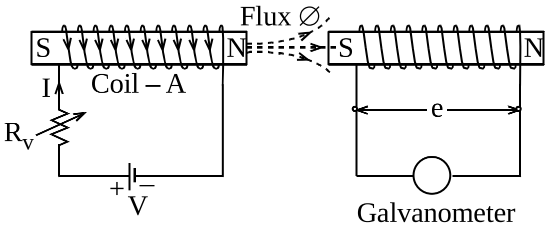
a. Statically induced emf

b. Dynamically induced emf

$$\left(e = N\frac{dØ}{dt} = N\frac{d(BA)}{dt} = BN\frac{da}{dt} + NA\frac{dB}{dt} \right)$$

1.7.1 Statically Induced emf: (Coil Stationary, Field Changes)

In this method, there is no physical movement of conductor or coil, only the magnitude of magnetic flux is changed. Hence changing the flux-linkage.



When the value of DC current is varied by varying the resistance R_v in coil-A, the produced magnetic flux also varies. When current is not varied, the magnetic field remains constant. The produced magnetic flux links the coil-B. When I is increased, change in flux-linkage i.e increase in it occurs, and the galvanometer shows a deflection in one direction indicating induced emf causing current flow. When I is decreased, the observation is just opposite.

If the magnetic flux in the coil-B changes from $\emptyset_1$ to $\emptyset_2$ in a small time interval from t_1 to t_2, then according to second statement given by Faraday's law of electromagnetic induction, emf induced in a single turn of coil - B is given by,

$$E \text{ (per turn)} = \frac{\emptyset_2 - \emptyset_1}{t_2 - t_1} = \frac{d\emptyset}{dt} = \text{Rate of change of flux}$$

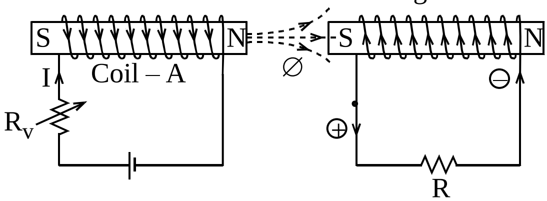
For 'N' number of turns in the coil -B, then total emf induced across the coil is given by:

$$E = N\frac{d\emptyset}{dt} \text{ volts}$$

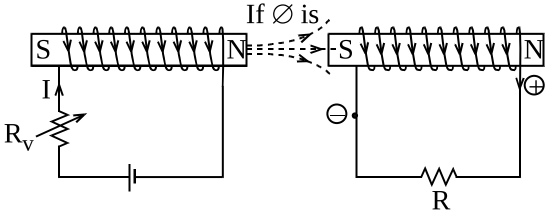
Taking the Lenz's law statements "Direction of induced current\emf in the conductor will be such that the magnetic field set up by the induced current opposes the cause by which the current/emf was induced."

$$E = -N\frac{d\varnothing}{dt}$$

e.q.



emf is induced to oppose the cause i.e. to decrease the flux from coil A.



emf is induced to try to increase the ∅.

Figure 1- 5: Lenz Law

1.7.2 Dynamically Induced emf

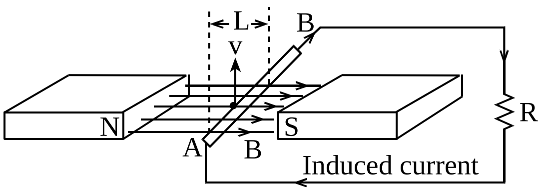


Figure 1-6: Dynamically Induced emf

In this method, Field is stationary and conductor cut across it which is responsible for the change in flux linkage.

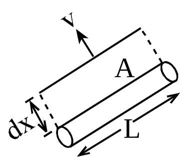


Figure 1- 7: Area swept by the conductor in dt time

As shown in the first figure, the conductor is moving upward in the magnetic field with velocity V. Here, in small time 'dt' the conductor sweeps a distance dx with velocity 'v'. Let, L be the length of conductor inside the electric field.

When the conductor moves in the magnetic field, there is a change in flux-linkage.

Now, the change in flux-linkage will be equal to the change in flux when conductor moves a distance dx, which is given by,

$d\varnothing = B \times A$; where A = area swept by conductor in dt time

or, $d\varnothing = B \times dx \times l$

or, $d\varnothing = B \times l \times v \times dt \qquad (\because v = \frac{dx}{dt})$

or, $\frac{d\varnothing}{dt} = BLv$.............. (i)

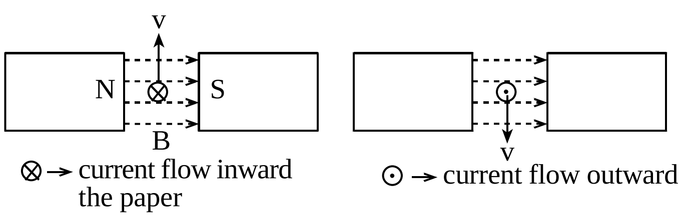
(flux change = flux cut = $l \,.\, dx \times B$)

We know that, induced emf is given by,

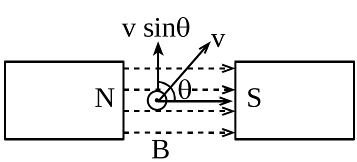
$e = \frac{d\varnothing}{dt}$ .... (ii)

Thus,

E = BLV, Here, L, B and v are vector quantity. The direction of induced emf (or current) can be finding out by using fleming's right hand rule.



If the direction of motion is inclined to the magnetic flux density as shown below,



Then, only the component of velocity perpendicular to field 'B' is taken. Since the component parallel to B has no change in flux linkage. Thus, in general, the induced emf for dynamic case is,

$E = Blv \sin\theta$

1.8 Force on Current Carrying Conductor

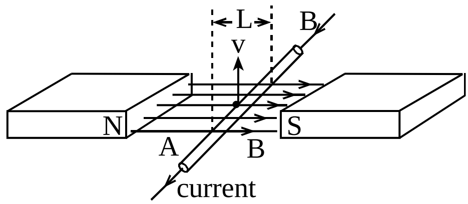


Figure 1-8: Force developed on current carrying conductor in a magnetic field

When a current carrying conductor is placed in a magnetic field, then a force will develop on the conductor, whose magnitude is given by,

$F = B.I.L$ (Newton)

Where,

B = magnetic flux density (wb/m^2)

I = current passing through the conductor (A)

L = length of conductor lying within the magnetic field (m)

And the direction of force is given by the fleming's left hand rule.