

Direct Current

- 1) What will be the conductivity of sodium metal having electron density $2.5 \times 10^{28} \text{ m}^{-3}$ & relaxation time $3 \times 10^{-14} \text{ sec}$?

\Rightarrow Solⁿ,

$$n = 2.5 \times 10^{28} \text{ m}^{-3}$$

$$\tau = 3 \times 10^{-14} \text{ sec}$$

$$\sigma = ?$$

we know,

$$\sigma = \frac{ne^2\tau}{m_e}$$

$$\frac{1}{\sigma} = \frac{m_e}{ne^2\tau}$$

$$\sigma = \frac{ne^2\tau}{m_e}$$

$$\sigma = \frac{2.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 3 \times 10^{-14}}{9.1 \times 10^{-31}}$$

$$\therefore \sigma = 2.1 \times 10^7 \text{ mho/m}$$

- 2) The current density in a cylindrical wire of radius $R = 2 \text{ mm}$ & uniform CSA is given by $J = 2 \times 10^5 \text{ Am}^2$. What is the current through the outer portion of the wire both radial distance $R/2$ & R ?

\Rightarrow Solⁿ

$$R = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$J = 2 \times 10^5 \text{ Am}^2$$

$$r = \frac{R}{2} = 10^{-3} \text{ m}$$

we know,

$$J = \frac{I}{A}$$

$$I = J \cdot A$$

$$I = 2 \times 10^5 \times \pi (R^2 - r^2)$$

$$I = 2 \times 10^5 \times \pi \times ((2 \times 10^{-3})^2 - (10^{-3})^2)$$

$$I = 1.884 \text{ A}$$

3) Calculate the (a) mean free time and (b) mean free path betⁿ collision for the conduction electron in copper having electron density $8.4 \times 10^{22} \text{ cm}^{-3}$ & resistivity $1.7 \times 10^{-8} \Omega \text{ m}$. [charge of e^- is $1.6 \times 10^{-19} \text{ C}$, mass of electron is $9.1 \times 10^{-31} \text{ kg}$, $1.6 \times 10^{-19} \text{ C}$, mass of electron is $1.6 \times 10^{-26} \text{ mls}$]. effective speed of

$\Rightarrow \text{sol}^n$

$$n = 8.4 \times 10^{22} / \text{cm}^3 = 8.4 \times 10^{28} / \text{m}^3$$

$$\rho = 1.7 \times 10^{-8} \Omega \text{ m}.$$

(a) mean free time

$$\tau = \frac{m_e}{n e^2 \rho}$$

$$= \frac{9.1 \times 10^{-31}}{8.4 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.7 \times 10^{-8}}$$

$$= 2.48 \times 10^{-14} \text{ sec}$$

(b) mean free path

$$\lambda = \tau \times \text{effective speed of } e^-$$

$$= 2.48 \times 10^{-14} \times 1.6 \times 10^6$$

$$= 3.98 \times 10^{-8} \text{ m}$$

- 4) A copper wire of CSA $3 \times 10^{-6} \text{ m}^2$ carries a steady current of 60 A, assuming one e^- per atom. Calculate (i) free e^- density & (ii) average drift velocity.

$$\text{density of Cu} = 8.9 \times 10^3 \text{ kg/m}^3$$

$$\text{mass of Cu} = 64$$

$$\text{Avogadro's no.} = 6.02 \times 10^{23} / \text{mole}$$

\Rightarrow Solⁿ,

$$A = 3 \times 10^{-6} \text{ m}^2$$

$$I = 60 \text{ A}$$

$$d = 8.9 \times 10^3 \text{ kg/m}^3$$

$$M = 64 \text{ gm} = 64 \times 10^{-3} \text{ kg}$$

$$N_A = 6.02 \times 10^{23} / \text{mole}$$

$$n = ?$$

$$v_d = ?$$

We know

$$n = \frac{N_A d}{M}$$

$$= \frac{6.02 \times 10^{23} \times 8.9 \times 10^3}{64 \times 10^{-3}}$$

$$= 8.37 \times 10^{28} / \text{m}^3$$

&

$$I = v_d e n A$$

$$v_d = I / e n A$$

$$v_d = \frac{60}{1.6 \times 10^{-19} \times 8.37 \times 10^{28} \times 3 \times 10^{-6}}$$

$$v_d = 1.49 \times 10^{-3} \text{ m/s}$$

5. Two conductors are made of the same material & have the same length. Conductor A is a solid wire of diameter 1mm & B is a hollow tube of outer diameter 2mm & inner diameter 1mm. What is the resistance ratio R_A/R_B measured betⁿ their ends?

⇒ solⁿ,

For conductor A;

$$\text{Area } (A_A) = \frac{\pi d^2}{4} = \frac{\pi (1)^2}{4} = 0.785 \text{ mm}^2$$

For conductor B;

$$\begin{aligned} \text{Area } (A_B) &= \pi \left(\frac{d_o^2}{4} - \frac{d_i^2}{4} \right) = \pi \left(\frac{2^2}{4} - \frac{1^2}{4} \right) \\ &= 2.356 \text{ mm}^2 \end{aligned}$$

we know,

$$\text{Resistance } (R) = \frac{\rho l}{A}$$

Since the two conductors are made of same material & have the same length; $R \propto 1/A$
 i.e.,

$$R_A \propto 1/0.785 \quad (i) \quad \& \quad R_B \propto 1/2.356 \quad (ii)$$

Dividing (i) by (ii)

$$\frac{R_A}{R_B} = \frac{1}{0.785} \times \frac{2.356}{1}$$

$$\therefore \frac{R_A}{R_B} \approx \frac{3.001}{1} \approx 3:1$$

6) What is the average time betⁿ collision of free electrons in a copper wire?

$$At. wt = 63 \text{ g/mol.}$$

$$\text{density} = 9 \text{ gm/cc}$$

$$g = 1.7 \times 10^{-4} \Omega m$$

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

\Rightarrow Solⁿ,

$$At. wt. (m) = 63 \text{ g/mol} = 63 \times 10^{-3} \text{ kg/mol}$$

$$\text{density } (d) = 9 \text{ gm/cc} = 9 \times 10^3 \text{ kg/m}^3$$

$$g = 1.7 \times 10^{-4} \Omega m$$

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

$$T = ?$$

We have,

$$g = \frac{m e}{n e^2 T}$$

$$T = \frac{m}{n e^2 g}$$

now,

$$n = \frac{d \cdot N_A}{m}$$

$$= \frac{9 \times 10^3 \times 6.02 \times 10^{23}}{63 \times 10^{-3}}$$

$$= 8.6 \times 10^{23} / m^2$$

Then,

$$T = \frac{9.1 \times 10^{-31}}{8.6 \times 10^{23} \times (1.6 \times 10^{-19})^2 \times 1.7 \times 10^{-4}}$$

$$= 2.43 \times 10^{-8} \text{ sec.}$$

Q.7) A current of $1.2 \times 10^{-10} \text{ A}$ exist in a copper wire. (At. wt = 63 g/mol., density = 9 g/mlcc) where diameter is 2.5 mm & resistivity is $1.7 \times 10^{-8} \Omega \text{ m}$. Assuming current to be uniform, calculate

(a) Current density (J)

(b) Electrical conductivity (σ)

(c) mobility of electron.

→ Soln,

$$\text{Current (I)} = 1.2 \times 10^{-10} \text{ A}$$

$$\text{At. wt} = 63 \text{ g/mol}$$

$$\text{density (d)} = 9 \text{ g/mlcc}$$

$$\text{radius (r)} = \frac{d}{2} = \frac{2.5}{2} = 1.25 \times 10^{-3} \text{ m}$$

$$\rho = 1.7 \times 10^{-8} \Omega \text{ m}$$

We know,

$$n = \frac{N_A}{m/d}$$

$$= \frac{d N_A}{m}$$

$$= \frac{9 \times 6.023 \times 10^{23}}{63}$$

$$= 8.60 \times 10^{22} / \text{cc}$$

$$= 8.6 \times 10^{28} / \text{m}^3$$

Nxv

$$(A) \quad J = \frac{I}{A} = \frac{1.2 \times 10^{-10}}{\pi \times r^2} = \frac{1.2 \times 10^{-10}}{\pi \times (1.25 \times 10^{-3})^2}$$

$$J = 2.44 \times 10^{-5} \text{ A/m}^2$$

$$(b) \quad \text{Electrical conductivity } (\sigma) = \frac{1}{\rho}$$

$$= \frac{1}{1.7 \times 10^{-8} \text{ } \Omega \cdot \text{m}}$$

$$= 5.8 \times 10^7 / \Omega \cdot \text{m}$$

$$(c) \quad \text{mobility of electron } (\mu) = \frac{\sigma}{ne}$$

$$= \frac{5.8 \times 10^7}{8.6 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= 4.27 \times 10^{-3} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$