[Test - 1] Maths

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[BCT-A . 019]

Q) $\lim_{t \to 0} \left(\frac{\sin t}{t}\right)^{1/t^2}$

Sol^n.

let $\lim_{t \to 0} A = \lim_{t \to 0} \left(\frac{\sin t}{t}\right)^{1/t^2}$

$$\log \left(\lim_{t \to 0} A\right) = \lim_{t \to 0} \frac{1}{t^2} \log \frac{\sin t}{t}$$

$$= \lim_{t \to 0} \frac{\log \frac{\sin t}{t}}{t^2}$$

$$= \lim_{t \to 0} \frac{\frac{\cos t}{\sin t} - \frac{1}{t}}{2t}$$

$$= \lim_{t \to 0} \frac{t\cos t - \sin t}{2t^2 \sin t}$$

$$= \lim_{t \to 0} \frac{t\cos t - \sin t}{2t^3 \left(\frac{\sin t}{t}\right)}$$

$$= \lim_{t \to 0} \frac{t\cos t - \sin t}{2t^3}$$

$$= \lim_{t \to 0} \frac{-t\sin t + \cancel{\cos t} - \cancel{\cos t}}{6t^2}$$

$$= \lim_{t \to 0} \frac{-t\sin t}{6t^2}$$

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$$= \lim_{t \to 0} -\frac{1}{6} \left(\frac{\sin t}{t} \right)$$

$$= -\frac{1}{6}$$

or. $\lim_{t \to 0} A = e^{-1/6}$

$$\lim_{t \to 0} \left(\frac{\sin t}{t} \right)^{1/t^2} = e^{-1/6} \#$$

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Q 1)

Sol^n,

$$y = e^{\sin^{-1} x}$$

diff. w.r.t. x, we get

$$\Rightarrow y_1 = e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1 = \frac{y}{\sqrt{1-x^2}}$$

$$\Rightarrow y_1^2 = \frac{y^2}{1-x^2}$$

$$\Rightarrow y_1^2 (1-x^2) = y^2$$

diff. w.r.t. x we get

$$\Rightarrow 2y_1 y_2 (1-x^2) + y_1^2(-2x) - 2yy_1 = 0$$

$$\Rightarrow 2y_1 [y_2 (1-x^2) - xy_1 - y] = 0$$

$$\Rightarrow y_2 (1-x^2) - xy_1 - y = 0$$

diff. ~~w.r.t x, we get~~ it up to n^{th}. term using leibinitz theorem

$$\Rightarrow y_{n+2}(1-x^2) + ny_{n+1}(-2x) + n(n-1)y_n(-2)$$
$$- [y_{n+1} x + ny_{n-1}] - y_n = 0$$

$$\Rightarrow (1-x^2) y_{n+2} - 2nxy_{n+1} - n(n-1) \cdot y_n - xy_{n+1}$$
$$- ny_n - y_n = 0$$

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or, $(1-x^2)y_{n+2} - xy_{n+1}(2n+1) - n^2y_n + \cancel{ny_n} - \cancel{ny_n}$

$- y_n = 0$

$\therefore (1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+1)y_n = 0$ #

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Q3) $x(x-y)^2 - 3(x^2-y^2) + 8y = 0$

sol^n,

given curve is

$$x(x-y)^2 - 3(x^2-y^2) + 8y = 0$$
$$x^3 - 2x^2y + xy^2 - 3x^2 + 3y^2 + 8y = 0$$

This is the eq^n of 3rd degree, it has at most three asymptotes & x^3 is present. so there is no horizontal asymptote. but y^3 is absent, there is vertical asymptote

ie $x + 3 = 0$

For oblique asymptote, putting 1 for x & m for y in 3rd, 2nd & 1st degree term to obtain $\phi_3(m)$, $\phi_2(m)$ & $\phi_1(m)$ respectively.

$\phi_3(m) = 1 - 2m + m^2$, $\phi_2(m) = -3 + 3m^2$, $\phi_1(m) = 8m$

$\phi_3'(m) = 2m - 2$, $\phi_2'(m) = 6m$

$\phi_3''(m) = 2$

now,

$\phi_3(m) = 0$ gives

$m^2 - 2m + 1 = 0$, $m = 1, 1$

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since, $\phi_3'(1) = 0$, m has respected root,

$\phi_3''(1) = 2 \neq 0$

we have,

$$\frac{c^2}{2!} \phi_3''(1) + \frac{c}{1!} \phi_2'(1) + \phi_1(1) = 0$$

$$\frac{c^2}{2!} \cdot 2 + \frac{c}{1} \cdot 6 + 8 = 0$$

$$c^2 + 6c + 8 = 0$$

$$(c+4)(c+2) = 0,$$

$$c = -2, 4$$

Thus, the oblique asymptote is

$$y = mx + c$$

putting the value of m & c in this eqn, we get

$$y = 1 \cdot x - 2 \text{ \&}$$

$$y = 1 \cdot x - 4$$

Hence the asymptotes are,

$$x + 3 = 0$$

$$x - y = 2$$

$$x - y = 4$$

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Q.N.6)

$$xy\frac{dy}{dx} = 2y^2 - x^2$$

Sol^n,

$$\frac{dy}{dx} = \frac{2y^2 - x^2}{xy} \quad \text{---(i)}$$

eq^n is a homogenous eq^n

so, put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{2v^2x^2 - x^2}{vx^2}$$

$$v + x\frac{dv}{dx} = \frac{2v^2-1}{v}$$

$$x\frac{dv}{dx} = \frac{2v^2-1}{v} - v$$

$$x\frac{dv}{dx} = \frac{2v^2-1-v^2}{v}$$

$$x\frac{dv}{dx} = \frac{v^2-1}{v}$$

$$\frac{v}{v^2-1}\,dv = \frac{1}{x}\,dx$$

Integrating on both side

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$\Rightarrow \int \frac{v}{v^2-1}\, dv = \int \frac{1}{x}\, dx$

$\Rightarrow \frac{1}{2} \int \frac{2v}{v^2-1}\, dv = \log x + c$

$\Rightarrow \frac{1}{2} \log (v^2-1) = \log x + \log c$

$\Rightarrow \frac{1}{2} \log (v^2-1) = \log cx$

$\Rightarrow \frac{1}{2} \log \left[\left(\frac{y}{x}\right)^2 - 1\right] = \log cx$

$\Rightarrow \frac{1}{2} \log \left[\frac{y^2}{x^2} - 1\right] = \log cx$

$\Rightarrow \frac{1}{2} \log \left[\frac{y^2-x^2}{x^2}\right] = \log cx$

$\Rightarrow \log \left[\frac{y^2-x^2}{x^2}\right]^{1/2} = \log cx$

$\Rightarrow \cancel{\log} \left[\frac{y^2-x^2}{x^2}\right]^{1/2} = cx$

$\Rightarrow \frac{\sqrt{y^2-x^2}}{(x^2)^{1/2}} = cx$

$\Rightarrow \frac{\sqrt{y^2-x^2}}{x} = cx$

$\therefore \sqrt{y^2-x^2} = cx^2$

which is reqd. solⁿ.

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Q.7)

$$\frac{dy}{dt} - \frac{\tan y}{1+t} = (1+t)e^{t}\sec y$$

Sol^n:

$$\frac{dy}{dt} - \frac{\tan y}{1+t} = (1+t)e^{t}\sec y$$

dividing by sec y.

$$\cos y\frac{dy}{dt} - \frac{\sin y}{1+t} = (1+t)e^{t}$$

put $\sin y = v$.

$$\cos y\frac{dy}{dt} = \frac{dv}{dt}$$

so, the eq^n reduce to

$$\frac{dv}{dt} - \frac{v}{1+t} = (1+t)e^{t}$$

This is linear form, $P = -\frac{1}{1+t}$, $Q = (1+t)e^{t}$

so, $IF = e^{\int P dt} = e^{-\int \frac{1}{1+t}dt}$

$$= e^{-\log(1+t)}$$

$$= \frac{1}{1+t}$$

Its general sol^n is

$$\Rightarrow v \times IF = \int Q \times (IF)\, dt$$

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$$\Rightarrow y \cdot \frac{1}{1+t} = \int e^t \, dt + c$$

$$\Rightarrow \frac{\sin y}{1+t} = e^t + c$$

$$\Rightarrow \sin y = (1+x)(e^t + c)$$

: which is reqd. solⁿ

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8) Sol^n

$$y - 2px + ayp^2 = 0$$

where $p = \frac{dy}{dx}$.

$$-2px = -y - ayp^2$$

$$x = \frac{y}{2p} + \frac{ayp^2}{2p}$$

$$2x = \frac{y}{p} + ayp \quad \text{--- (i)}$$

diff. w.r.t y, we get

$$\Rightarrow 2\frac{dx}{dy} = \frac{p - y\,\frac{dp}{dy}}{p^2} + ay\frac{dp}{dy} + ap$$

$$\Rightarrow 2 \times \frac{1}{p} = \frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy} + ay\frac{dp}{dy} + ap$$

$$\Rightarrow \frac{2}{p} - \frac{1}{p} - ap = -\frac{dp}{dy}\left(\frac{y}{p^2} - ay\right)$$

$$\Rightarrow \frac{2 - 1 - ap \times p}{p} = -\frac{dp}{dy}\left(\frac{y}{p^2} - ay\right)$$

$$\Rightarrow \frac{1 - ap^2}{p} = -\frac{dp}{dy} \cdot y\left(\frac{1 - ap^2}{p^2}\right)$$

$$\Rightarrow (1 - ap^2)\,dy = -dp \cdot y\left(\frac{1 - ap^2}{p}\right)$$

$$\text{or} \quad dy = -dp \cdot y \cdot \frac{1}{p}$$

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Integrating

$\Rightarrow \int \frac{1}{y}\,dy = -\int \frac{1}{p}\,dp$

$\Rightarrow \log y = -\log p + \log c$

$\Rightarrow \log y = \log\left(\frac{c}{p}\right)$

$\Rightarrow y = \frac{c}{p}$

$p = \frac{c}{y}$ — (2)

eliminating p from (1) & (2) we get

$2x = \frac{y}{c/y} + ay \times \frac{c}{y}$

$\Rightarrow 2x = \frac{y^2}{c} + ac$

$\therefore 2cx = y^2 + ac^2$

~~There~~ which is reqd. solⁿ.