Ashwani kr. Chaudhary
019
BCT- A

[Physic]

[Assignment - 1]

1) What could be the length of the simple pendulum that would have a same time period.

Sol^n:

Consider a meter scale of mass 'm' is suspended at point 'O'. The center of gravity of the body (CG) is at a distance 'l' from 'O'. If the meter scale is displaced by small angle θ & released then we have,

$$\tau = -mgl\theta \quad \text{---(i)}$$

Also, from Newton's 2nd law of motion

$$\tau = I\alpha \quad \text{---(ii)}$$

$I \rightarrow$ moment of Inertia of scale about axis
$\alpha \rightarrow$ angular acceleration.

So,

$$-mgl\theta = I\alpha \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{mgl\theta}{I}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{mgl\theta}{I} = 0$$

$$\Rightarrow \omega^2 = \frac{mgl}{I}$$

$$\Rightarrow \omega = \sqrt{\frac{mgl}{I}}$$

We have,

$$T = 2\pi\sqrt{\frac{I}{mgl}}.$$

Now, from parallel axis theorem

$$I = I_{CG} + ml^2$$

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$$I = m(k^2+l^2)$$

$$T = 2\pi\sqrt{\frac{l^2+k^2}{lg}} \quad \text{— (iii)}$$

Also,

$$I_{CG} = mk^2 = \frac{ml^2}{12} \quad , \quad L = 1m$$

$$k = \frac{1}{\sqrt{12}}$$

So, $$T = 2\pi\sqrt{\frac{12 \cdot l^2+1}{12\, l\, g}}$$

For the above case $l = 0.5m$

$$T = 2\pi\sqrt{\frac{12\times 0.5^2+1}{12\times 0.5\times 9.8}}$$

$$= 1.638 \text{ sec}$$

To be single pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$L = l + \frac{1}{12l}$$

$$= 0.5 + \frac{1}{12\times 0.5}$$

$$= 0.66 \text{ m} \#$$

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2) A uniform circular disc of radius R oscillates in a vertical plane about a horizontal axis show the disc will oscillate with min^m time period when the distance of the axis of rotation from the centre is $R/\sqrt{2}$.

⇒ Here,

the circular disc oscillates as a compound pendulum of length 'l' whose time period is

$$T = 2\pi\sqrt{\frac{k^2+l^2}{gl}}$$

where, k is radius of gyration about an axis through it C.G. parallel to the axis of dispension. We know that the time period of compound pendulum is minimum if its length is equal to its radius of gyration about its C.G. ie ($l = k$).

$$T = 2\pi\sqrt{\cancel{\frac{k^2+k}{k^2}}\ \frac{k^2+k^2}{gk}}$$

$$= 2\pi\sqrt{\frac{2k}{g}}$$

Since, the moment of inertia of a disc about an axis ~~pa~~ perpendicular to its plane & passing through its centre,

$$I = \frac{1}{2}MR^2 = MK^2$$

$$k = \frac{R}{\sqrt{2}}$$

Therefore the disc will oscillate with min^m time period when the distance of the axis of rotation from centre is $R/\sqrt{2}$

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Q.3. A simple pendulum of length 40cm & mass 50gm is suspended in a car that is travelling with a constant speed 40m/s around a circle of radius 100m. If the pendulum goes small oscillation in a radial direction about its equilibrium position what will be its frequency of oscillation?

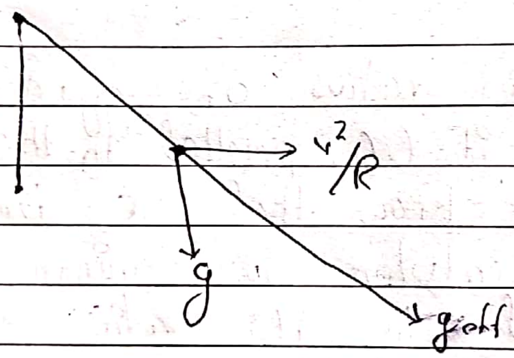
=> Soln.

we know,

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$g_{eff} = ?$

here,



v^2/R => Centripetal acceleration.

Now,

$$g_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R^2}\right)^2} = \sqrt{9.8^2 + \left(\frac{40^2}{100}\right)^2}$$

$$= 18.76 \text{ m/s}^2$$

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{0.4}{18.76}} = 0.917 \text{ sec.}$$

$$\text{frequency} = \frac{1}{T} = \frac{1}{0.917} = 1.089 \text{ Hz.}$$

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4) The amplitude of lightly decrease by 3% during each cycle. What fraction of the energy of the oscillation is lost in each full oscillation?

=> Soln,

$E = \frac{1}{2} kA^2$ [Total Energy]

So, $E_1 = \frac{1}{2} k A_1^2$ —(i)

$E_2 = E_1 - 3\% \text{ of } E_1$

$= \frac{1}{2} kA_1^2 - \frac{3}{100} \times \frac{1}{2} k A_1^2$

$E_2 = 0.485 \, kA_1^2$

Now,

$\frac{E_2}{E_1} = \left(\frac{0.97 \cdot A_1}{A_1}\right)^2$

$\frac{E_2}{E_1} = 0.97^2$

$\frac{E_2}{E_1} = 0.9409$ #

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Q5) In damped harmonic motion, calculate the time in which (i) its amplitude & (ii) it energy falls to 1/e of its undamped value if the mass of the system is 0.25 gm & damping constant is 0.01 g/s?

⇒ Sol^n,

damping const. (b) = 0.01 g/s
$= \frac{0.01}{1000}$ kg/s.

mass (m) = 0.25 gm = $\frac{0.25}{1000}$ = 2.5×10^{-4} kg

we have,

$A = A_0 e^{(-\frac{b}{2m})t}$

$\frac{1}{e} A_0 = A_0 e^{(-\frac{b}{2m})t}$

$e^{-1} = e^{(-\frac{b}{2m})t}$

Taking log on both sides

$\ln(e^{-1}) = \ln e^{(-\frac{b}{2m})t}$

$t = \frac{2m}{b} = \frac{2\times2.5\times10^{-4}}{1\times10^{-5}} = 50$ sec,

ii) now,

$E = E_0 e^{-bt/m}$

or $\frac{1}{e} E_0 = E_0 e^{(-bt/m)}$

or $1 = \frac{bt}{m}$

$t = \frac{2.5\times10^{-4}}{10^{-5}} = 25$ sec #

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5) A particle is moving with simple harmonic motion in a st. line. If it has a speed v_1 when the displacement is x_1 & a speed v_2 when the displacement is x_2 then show that the amplitude of motion is $a = \left[\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}\right]$

⇒ Sol^n,

At x_1 velocity is v_1 & at x_2 velocity is v_2 so,

$$v_1 = \omega\sqrt{a^2 - x_1^2} \quad \text{---(i)}$$

$$v_2 = \omega\sqrt{a^2 - x_2^2} \quad \text{---(ii)}$$

squring ~~&~~ dividing both eq^n we get

$$\frac{v_1^2}{v_2^2} = \frac{a^2 - x_1^2}{a^2 - x_2^2}$$

$$a^2 v_1^2 - v_1^2 x_2^2 = a^2 v_2^2 - x_1^2 v_2^2$$

$$a^2(v_1^2 - v_2^2) = v_1^2 x_2^2 - v_2^2 x_1^2$$

$$a^2 = \frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}$$

$$a = \left(\frac{v_2^2 x_1^2 - v_1^2 x_2^2}{v_2^2 - v_1^2}\right)^{1/2}$$

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7) A 750 gm block oscillation on the end of a spring whole force constant is 56 N/m. The mass move in a fluid which offers a retistive force $F = -bv$, where $b = 0.162$ Ns/m. What is period of oscillation?

⇒ sol^n.

mass of block = 750 gm = 0.75 kg
force constant (k) = 56 N/m.
b = 0.162 Ns/m.

now, we know,

$$\text{Angular velocity } (\omega) = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$= \sqrt{\frac{56}{0.75} - \left(\frac{0.162}{2\times0.75}\right)^2}$$

$$= 8.64 \text{ rad/s.}$$

now,

$$\omega = 2\pi T$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\times3.14}{8.64}$$

$$= 0.727 \text{ sec.}$$ //

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