

Linear Congruential Method

$$x_{i+1} = (a x_i + c) \bmod m \quad \text{and} \quad R_i = x_i / m$$

Where a and c are constants & x_0 is the initial seed. We select them or they are mentioned beforehand. m is taken usually as capacity power of 2 ($2^1, 2^2, \dots$) as it equals computer word.

Eg:- let $a = 13$
 $m = 26$ $x_0 = 2$ and $c = 0$.

so,
$$x_1 = (a x_0 + c) \bmod m$$
$$= (13 \times 2 + 0) \bmod 64$$
$$= 26$$

$$R_1 = \frac{26}{64}$$

$$x_2 = (a x_1 + c) \bmod m$$
$$= (13 \times 26 + 0) \bmod 64$$
$$= 18$$

$$R_2 = \frac{18}{64}$$

$$x_3 = (a x_2 + c) \bmod m$$
$$= (13 \times 18 + 0) \bmod 64$$
$$= 42$$

$$R_3 = 42/64 =$$

$$X_4 = 39$$

$$X_6 = 50$$

$$X_5 = 58$$

$$X_7 = 10$$

$$X_8 = 2$$

Here $X_9 = 26$ and values of X_i will be in loop
So, the period of Random Number Generator is 8.

Numerical 2 Generate a sequence of random number with initial seed $X_0 = 27$
 $a = 17$ $c = 43$ $m = 100$

$$X_i = (aX_{i-1} + c) \bmod m$$

$$X_1 = (17 \times X_0 + 43) \bmod 100$$

$$= (17 \times 27 + 43) \bmod 100$$

$$X_1 = 2$$

$$R_1 = \frac{X_1}{m} = \frac{2}{100} = 0.02$$

$$X_2 = (17 \times 2 + 43) \bmod 100$$

$$X_2 = 77$$

$$R_2 = \frac{77}{100} = 0.77$$

$$X_3 = (17 \times 77 + 43) \bmod 100$$

$$= 52$$

$$R_3 = 0.52$$

$$X_4 = (17 \times 52 + 43) \bmod 100$$

$$= 27$$

$$R_4 = \frac{27}{100} = 0.27$$

$$X_5 = (17 \cdot 27 + 43) \bmod 100 \\ = 2$$

Since, X_5 & X_0 is same so after this random number generator goes in a loop. So we stop & the period of length is 4.

Numerical 3 $a=5$ $c=1$ $m=8$ & $X_0=3$

Numerical 4 Max m period
 $m = 2^b = 2^3 = 8$

$$c = 7$$

$$a = 5 = 1 + 4 \times 1$$

$$X_0 = 17$$

$$X_1 = \left(\frac{5 \times X_0}{= 4} (5 \cdot 17 + 7) \bmod 8 \right)$$

$$X_2 = (5 \times 4 + 7) \bmod 8 \\ = 3$$

$$X_3 = (5 \times 3 + 7) \bmod 8 \\ = 6$$

$$X_4 = (5 \times 6 + 7) \bmod 8 \\ = 5$$

$$X_5 = (5 \times 5 + 7) \bmod 8 \\ = 0$$

$$x_6 = 7 \text{ mod } 8$$

$$= 7$$

$$x_7 = (5 \times 7 + 7) \text{ mod } 8$$

$$= 2$$

$$x_8 = (5 \times 2 + 7) \text{ mod } 8$$

$$= 1$$

$$x_9 = (5 + 7) \text{ mod } 8$$

$$= 4$$

Here the period is 8.