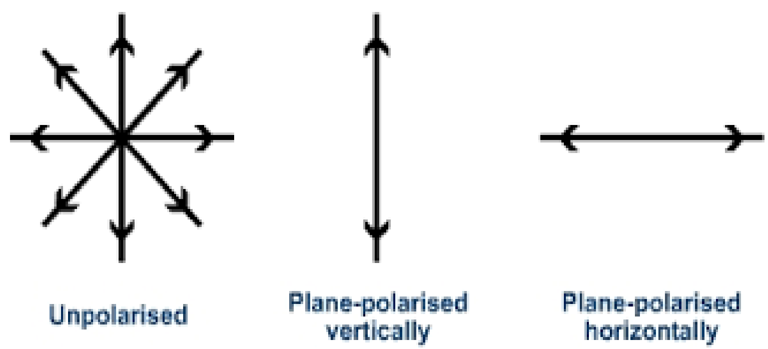
Polarization

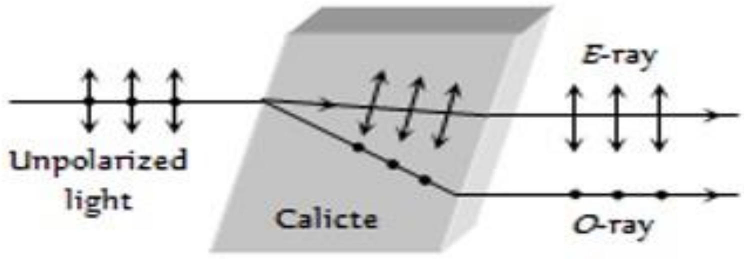
Polarization of light is a phenomena due to which the vibration of light are take place only in the particular plane, such a light is called plane polarize light. The plane in which the vibration of polarize light takes place is called the plane of vibration.



The material which polarized transverse wave is called "Polaroid". Calcite crystal, tourmaline crystal and Nichol prism are some example of Polaroid.

Polarization by double refraction:-

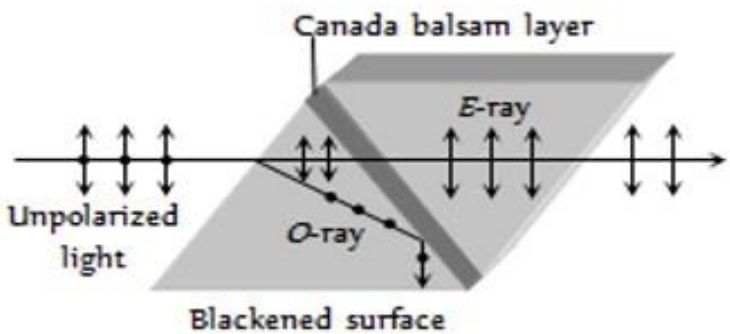
The speed of light is same in all direction in isotropic substances. However in some crystal the speed of light is different in different direction. Such materials are known as anisotropic substance. These substances are said to be double refracting substances as they give raise to an unusual phenomena.



When a small object is viewed through a crystal of calcite ($CaCo_3$) or quartz (SiO_2), there will be two refracted images. This is because of an unusual two types of refraction or there are two refracted rays. One refracted ray called a ordinary ray (0-ray) passes normal way but the other ray extra ordinary ray (e-ray) refracted with different angle. The o- ray obeys Snell's law, where as the e-ray does not. The e-ray and o-ray are found to be plane polarized in mutual perpendicular direction. Sugar solution, ice are other example of double refracting objects. In some crystal the velocity of o-ray is greater than e-ray ($\mu_o < \mu_e$), such crystals are known as positive crystal, where as the crystal in which velocity of e-ray is greater than o-ray ($\mu_e < \mu_o$), are known as negative crystal.

Nicol prism:-

Nicol prism is a simple optical device for producing and analyzing the plane polarized light. It was invented by William Nicol in 1826. It is based on the principle of double refraction. The ordinary and extra ordinary rays are polarized and are perpendicular to each other. A Nicol prism can separate the o-ray and e-ray from an un-polarized light. It is an optical device made from a natural crystal of calcite. It is used in different optical instrument to produce and analyze polarized light.



Nicol prism is made by cutting a calcite crystal along a diagonal as shown in figure. They are cemented back together with a layer of Canada balsam. It is a transparent medium such that optically denser than calcite for e-ray and less denser for o-ray. In other words calcite is negative crystal with $\mu_o > \mu_e$ and Canada balsam has refractive index between μ_o and μ_e. When an ordinary un-polarized light incident on the first calcite crystal surface. It breaks in to o-ray and e-ray. The o-ray is totally internally reflected whereas the e-ray passes into emerged parallel to the incident ray as shown in figure. In this way Nicol prism can transmit only plane polarized light from the un-polarized light.

Optical Activity:-

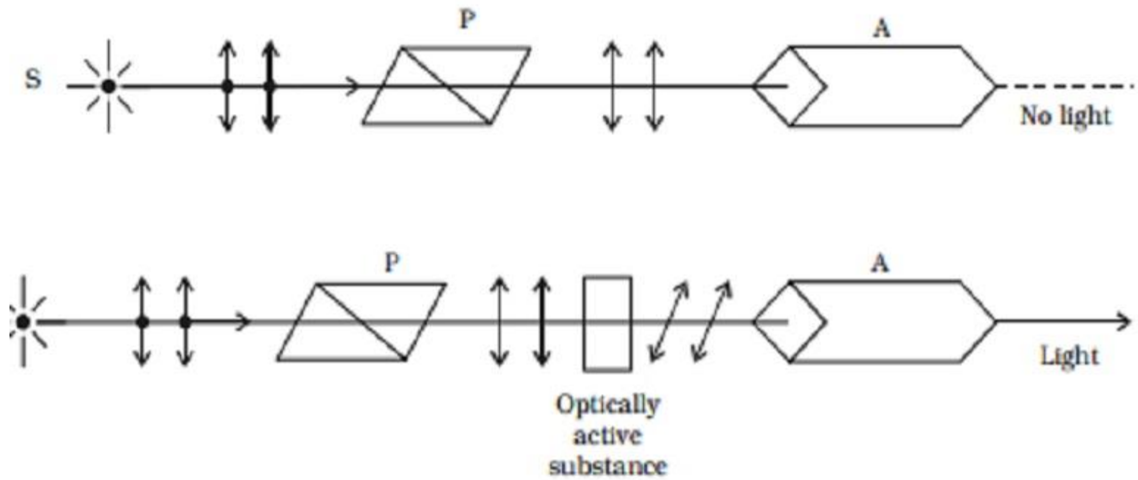


Fig Optical activity

Consider a set of Nicol prism placed in such a way that there is no emergent light coming out from the analyzer as shown in figure. In this particular condition the e-ray in analyzer is totally internally reflected. But if quartz plate is kept between prism in such a way that the emergent ray from polarizer falls at an angle 90^o, with the plate there is light emerging out from analyzer as well which is as shown in figure below.

The plane of vibration of plane polarized light from polarizer entering the quartz plate is gradually rotated, the amount of rotation of the plane of vibration is depends on the thickness of the quartz plate and wavelength of light. This property of rotating the plane of vibration by crystal is known as optical activity. E.g. sugar crystal, sugar solution in water, sodium chloride, quartz crystal and turpentine etc.

There are two types of optically active substances. In the first type of substances the rotate the plane of vibration to the right or clockwise direction such substances are known as dextrorotatory substances. In the second type of substances they rotate the plane of vibration to the left or anti-clockwise direction such substances are known as laevorotatory substances.

Specific rotation:-

The specific rotation is the rotation produced by one decimeter long solution containing one gram of optically active substance in one cubic centimeter of the solution.

$$i.e.\ S = \frac{10\theta}{lC}$$

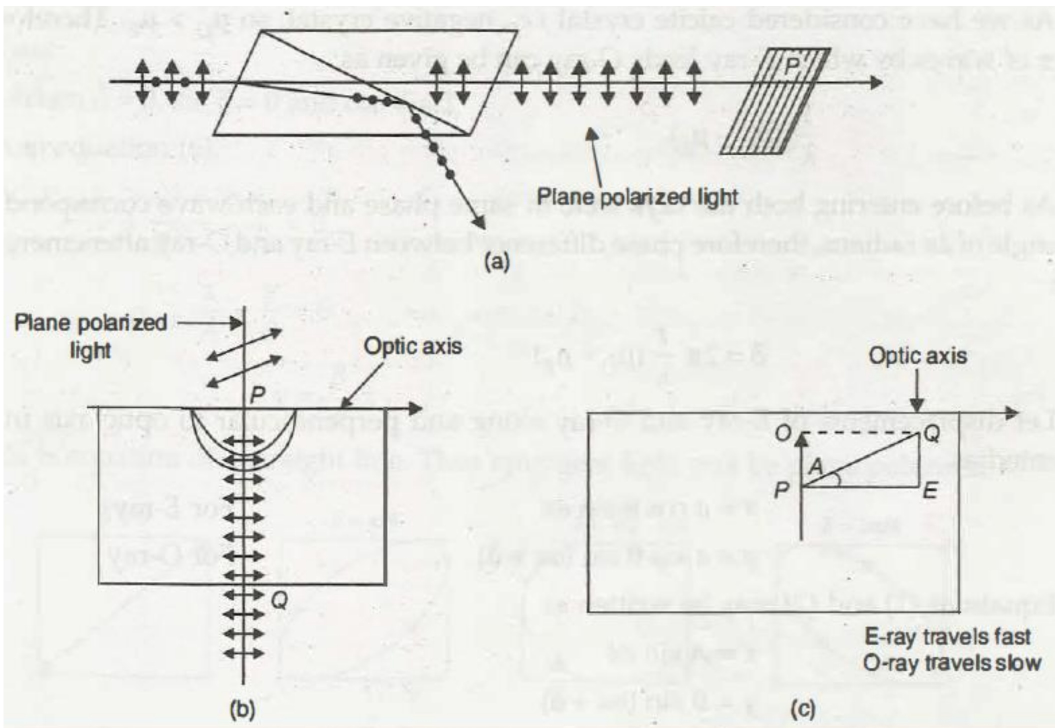
$$\therefore\ SlC = 10\theta$$

Where θ is angle of rotation, l is length of solution in cm, C is concentration of active substances in gm/cm^3 and S is specific rotation at given temperature for given wavelength of light.

Mathematical treatment of linearly, circularly and elliptically polarized light:-

Since, the light is made to incident normally on a crystal so the e-ray and o-ray travel along the same direction with the different velocity. Though e-ray and o-ray travel same direction but their plane of vibration is perpendicular to each other. So, if we resolve the amplitude ‘A’ of the incident wave. $A\sin\theta$ and

$A\cos\theta$ will represent the amplitude of o-ray and e-ray as shown in figure.



Therefore amplitude of $e - ray = A\cos\theta$

And, amplitude of $O - ray = A\sin\theta$.

Since, both the vibration moves with different velocity in the same direction. So, phase difference δ is introduced between e-ray and o-ray. Therefore, the displacement of e-ray is;

$$x = A\cos\theta\sin(wt + \delta) \ldots\ldots\ldots (i)$$

And the displacement of o-ray along the direction perpendicular to e-ray is;

$$y = A\sin\theta\sin wt \ldots\ldots\ldots (ii)$$

Let, $A\cos\theta = a \quad and \quad A\,sin\theta = b$

From equation (i);

$$x = \mathrm{a}\sin(wt + \delta)$$

$$or, \qquad \frac{x}{a} = \sin wt.\cos\delta + \cos wt.\,sin\delta \dots\dots\dots (iii)$$

And from (ii);

$$y = \mathrm{b}\sin wt$$

$$or, \quad \frac{y}{b} = \sin wt \dots\dots\dots (iv)$$

And $\cos wt = \sqrt{1 - sin^2wt} = \sqrt{1 - \frac{y^2}{b^2}} \dots\dots\dots. (v)$

Putting value of $\sin wt \;\; and \cos \; wt \; in \; (iii)$;

$$\frac{x}{a} = \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}}\sin\delta$$

$$or, \qquad \frac{x}{a} - \frac{y}{b}\cos\delta = \sqrt{1 - \frac{y^2}{b^2}}\sin\delta$$

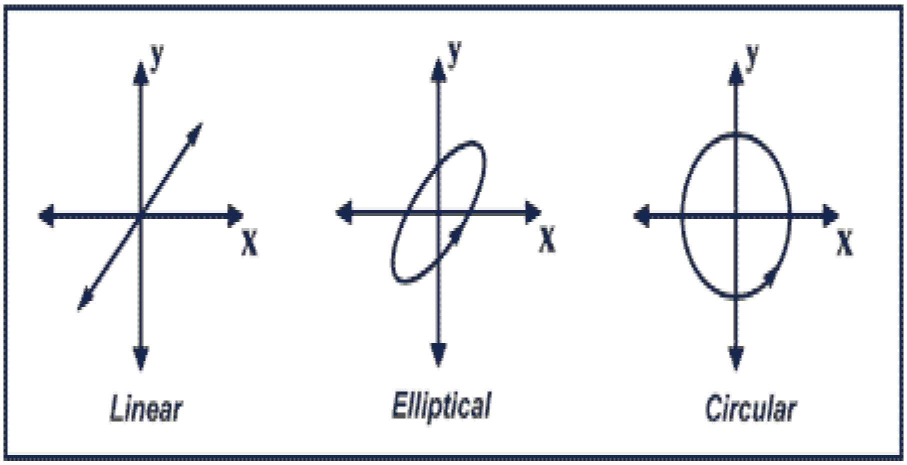
Squaring both sides;

$$(\frac{x}{a} - \frac{y}{b}\cos\delta)^2 = \left(1 - \frac{y^2}{b^2}\right) sin^2\delta$$

$$or, \qquad \frac{x^2}{a^2} - 2.\frac{xy}{ab}\cos\delta + \frac{y^2}{b^2}cos^2\delta = sin^2\delta - \frac{y^2}{b^2}sin^2\delta$$

$$or, \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} - 2.\frac{xy}{ab}\cos\delta = sin^2\delta \ldots\ldots\ldots (vi)$$

Which is the equation of ellipse (oblique), so in general the emergent light is elliptically polarized. The exact nature of emergent light depends upon the value of δ.



Case I:-

$$If\ \delta = 0, 2\pi, 4\pi \ldots\ldots .2n\pi\ \ then \cos\delta = 1\ and\ \ sin\delta = 0$$

From equation (vi); $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2.\frac{xy}{ab} = 0$

$$or, \qquad (\frac{x}{a} - \frac{y}{b})^2 = 0$$

$$or, \qquad \frac{x}{a} - \frac{y}{b} = 0$$

$$\therefore y = \frac{b}{a}x \ldots\ldots\ldots (vii)$$

Which is the equation of straight line. So the light is plane polarized.

Case II:-

$$If\ \delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots\dots(2n+1)\frac{\pi}{2}$$

$$then \cos\delta = 0\ and\ \ sin\delta = 1$$

From equation (vi); $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Which is the equation of ellipse. So, the emergent light is elliptically polarized.

Case III:-

$$If\ \delta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots\dots(2n+1)\frac{\pi}{2}\ \ and\ \ a = b,$$

$$then \cos\delta = 0\ and\ \ sin\delta = 1$$

From equation (vi); $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$

$$x^2 + y^2 = a^2$$

Which is the equation of circle. So, the emergent light is circularly polarized.

Phase difference between e-ray and o-ray:-

The two waves travel along the same direction in the crystal but with different velocity. As a result the wave emerges from the crystal. An optical path difference would have developed between them. The optical path difference can be calculated as;

Let ‘t’ be the thickness of the crystal. The optical path for e-ray within the crystal $= \mu_e t$.

And optical path for o-ray within the crystal $= \mu_o t$.

Therefore the optical path difference between e-ray and o-ray is;

$$\Delta = (\mu_e - \mu_o)t$$

$$\therefore \text{ phase difference } (\delta) = \frac{2\pi}{\lambda}(\mu_e - \mu_o)t$$

Quarter wave plate:-

If the path difference between e-ray and o-ray is equal to $\frac{\lambda}{4}$. Then the plate is called quarter wave plate.

$$i.e. (\mu_e - \mu_o)t = \frac{\lambda}{4} \quad (\text{for} + \text{ve crystal})$$

$$\therefore t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

$$\text{and for} - \text{ve crystal;}$$

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

Half wave plate:-

If the path difference between e-ray and o-ray is equal to $\frac{\lambda}{2}$. Then the plate is called half wave plate.

$$i.e. (\mu_e - \mu_o)t = \frac{\lambda}{2} \text{ (for + ve crystal)}$$

$$\therefore t = \frac{\lambda}{2(\mu_e - \mu_0)}$$

$$\text{and for} - \text{ve crystal;}$$

$$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

Numerical Examples:-

1. **A $\mathbf{200}\,\boldsymbol{mm}$ long tube containing $\mathbf{48}\,\boldsymbol{cm^3}$of sugar solution produces an optical rotation $\mathbf{11^0}$ when placed in saccharimeter. If the specific rotation of sugar solution is $\mathbf{66^0}$. Calculate the quantity of sugar contained in the tube in the form of solution.**

Solution:-

$\text{Length}(l) = 200 \text{ mm} = 20 \text{ cm}$

$\text{Volume (V)} = 48\, cm^3$

$\text{Optical rotation } (\theta) = 11^0$

Specific rotation $(S) = 66^0$

We have, $\quad SlC = 10\theta$

$$or, C = \frac{10\theta}{Sl} = \frac{10 \times 11^0}{66^0 \times 20} = \frac{1}{12}\ gm/cc$$

$$\therefore \text{Quantity of sugar (m)} = \text{CV} = \frac{1}{12} \times 48 = 4\ gm$$

2. A sugar solution in a tube of length $20\ cm$ produces an optical rotation of 13^0. The solution is then diluted to 1/3 of its previous concentration. Find the optical rotation produced by $30\ cm$ long tube containing the diluted solution.

Solution:-

For first case;

$l_1 = 20\ cm,\ \ \theta_1 = 13^0, \qquad C_1 = C, \qquad S_1 = S$

We have, $\quad S_1 l_1 C_1 = 10\theta_1$

$$or, \quad S l_1 C = 10\theta_1 \ldots\ldots\ldots (i)$$

For second case;

$l_2 = 30\ cm,\ \ \theta_2 = ?, \qquad C_2 = \frac{1}{3}C, \qquad S_2 = S$

We have, $\quad S_2 l_2 C_2 = 10\theta_2$

$$or, \qquad Sl_2 \frac{1}{3} C = 10\theta_2 \dots \dots \dots (ii)$$

Dividing equation (ii) by (i);

$$\therefore \frac{Sl_2 \frac{1}{3} C}{Sl_1 C} = \frac{10\theta_2}{10\theta_1}$$

$$or, \qquad \frac{30 \times \frac{1}{3}}{20} = \frac{\theta_2}{13}$$

$$\therefore \theta_2 = 6.5^0$$

3. Find the specific rotation of the sample of sugar solution, if the plane of polarization is turned through 46^0. The length of the tube containing 20 % solution is $20\ cm$.

Solution:-

Optical rotation $(\theta) = 46^0$

Length$(l) = 20$ cm

Concentration $(\mathrm{C}) = 20\ \% = 0.20$

Specific rotation $(\mathrm{S}) = ?$

We know that; $Sl\mathrm{C} = 10\theta$

$$\therefore S = \frac{10\,\theta}{lC} = \frac{10 \times 46}{20 \times 0.20} = 115^0$$

4. A length of $25\,cm$ of solution containing $50\,gm$ of solute per liter causes the rotation of the plane of polarization of light by 5^0. Find the rotation of plane of polarization by length of $75\,cm$ of a solution containing $100\,gm$ of solute per liter.

Solution:-

For first case;

$l_1 = 25\,cm,\ \ \theta_1 = 5^0, \qquad C_1 = 50\,gm\,/l, \qquad S_1 = S$

We have, $\qquad S_1 l_1 C_1 = 10\theta_1$

$or, \qquad S \times 25 \times 50 = 10 \times 5$

$\therefore S = 0.04^0$

For second case;

$l_2 = 75\,cm,\ \ \theta_2 = ?, \qquad C_2 = 100\,gm/l, \qquad S_2 = S$

We have, $\qquad S_2 l_2 C_2 = 10\theta_2$

$or, 0.04 \times 75 \times 100 = 10\theta_2$

$$\therefore\ \theta_2 = \frac{0.04 \times 75 \times 100}{10} = 30^0$$

5. **A sugar solution produces an optical rotation of 9.9^0 when placed in a tube of length of $20\ cm$. If the specific rotation of the solution is 66^0. Find the concentration of the solution in gm/liter.**

Solution:-

Specific rotation (S) $= 66^0$

Optical rotation $(\theta) = 9.9^0$

Length $(l) = 20\ cm$

Concentration $(C) =?$

We have, $SlC = 10\theta$

$$or, C = \frac{10\theta}{Sl} = \frac{10 \times 9.9^0}{66^0 \times 20} = 0.075\ gm/cc$$

$$\therefore C = 0.075 \times 1000 = 75\ gm/l$$

6. **A beam of plane polarized light is converted into circularly polarized light by passing it through a crystal of thickness $3 \times 10^{-5}\ m$. Calculate the difference in refractive indices of two rays inside the crystal. Wavelength of light is $600\ nm$.**

Solution:-

Thickness (t) $= 3 \times 10^{-5}\ m$

Wavelength $(\lambda) = 600\, nm = 6 \times 10^{-7}\, m$

Since quarter wave plate convert plane polarized light into circularly polarized light. So we have;

$$(\mu_0 - \mu_e)t = \frac{\lambda}{4}$$

$$or, (\mu_0 - \mu_e) = \frac{6 \times 10^{-7}}{4 \times 3 \times 10^{-5}}$$

$$\therefore (\mu_0 - \mu_e) = 5 \times 10^{-3}$$

7. Plane polarized light is incident on a piece of quartz cut parallel to the axis. Find the least thickness for which the ordinary and extra ordinary rays combined to form plane polarized light. (Given:- $\mu_0 = 1.5442$, $\mu_e = 1.553$ and $\lambda = 5 \times 10^{-7} m$)

Solution:-

Here, $\mu_0 = 1.5442$, $\mu_e = 1.553$ and $\lambda = 5 \times 10^{-7} m$

Since half wave plate combine O-ray and e-ray to form plane polarized light.

$$\therefore (\mu_e - \mu_0)t = \frac{\lambda}{2}$$

$$i.e.\ t = \frac{5 \times 10^{-7}}{2(1.5533 - 1.5442)} = 2.75 \times 10^{-3}\, cm$$

Exercise:-

1. What is polarization? Describe how will you produce linearly, circularly, and elliptically polarized light. Explain with mathematical conclusion.
2. Define the polarization of light. Write its importance in different optical instruments. Derive the relation for the thickness of quarter wave plate and half wave plate.
3. What is double refraction? Explain how Nicol prism can be used as polarizer and analyzer?
4. Give rise generally to an elliptically polarized wave that can become linearly and circularly polarized wave under special condition.
5. What are retardation plates? Find out an expression to find the thickness of a retardation plate that produces elliptically polarized light.
6. What is Nicol prism? How is it constructed? Discuss some its applications.
7. Define the term "optical activity". Derive a relation for the specific rotation of any optically active substance. Also write down its applications.
8. Light of wavelength 580 nm falls on a calcite crystal of certain thickness. The emerging light is circularly polarized. What must be thickness of such crystal?
9. A 30 cm long polarimeter tube containing 50 cm^3 of sugar solution produces an optical rotation 14.5^o when placed on a polarimeter tube. If the specific rotation of the sugar

solution is 65°, calculate the quantity of sugar contained in the tube.

10. Calculate the thickness of double refracting plate capable of producing a path differences of $\lambda/4$ between extraordinary and ordinary rays of wavelength 5890 A. (Use $\mu_0 = 1.53$ and $\mu_e = 1.54$)

11. If the plane of vibration of the incident beam makes an angle of 30° with the optic axis, compare the intensities of extraordinary and ordinary light. (Hint; $\frac{I_e}{I_o} = \frac{E\ Cos^2\theta}{E\ Sin^2\theta}$)

12. A beam of polarized light is sent in to a system of two polarizing sheets, relative to a polarization direction of the incident light. The polarizing directions of the sheets are at angle θ for the first sheet and 90° for the second sheets. If the 0.1 of the incident intensity is transmitted by the two sheets, what is θ?