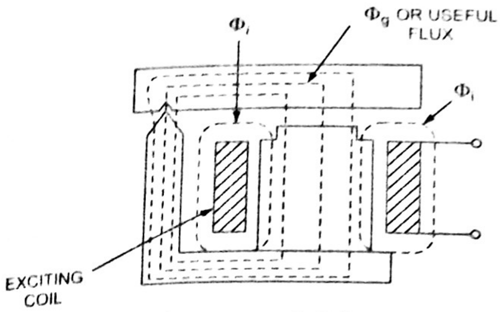
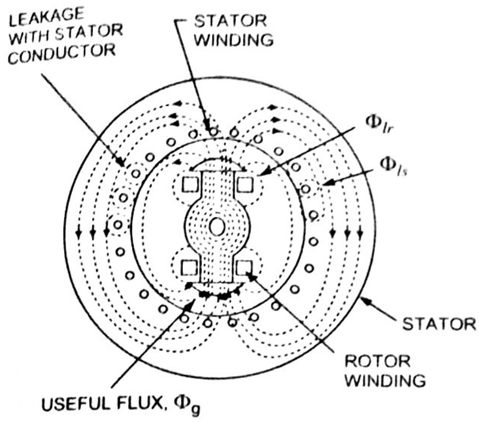


(*a*) *Electromagnetic Relay*



(*b*) *Two-Pole Electrical Machine*

Fig. 1.31 *Examples of Leakage*

Magnetic cores are made up of thin, lightly insulated laminations so as to reduce the power loss in cores owing to eddy current phenomenon.* The result is that the net cross-sectional area of the core occupied by the magnetic material is less than its gross cross-sectional area. The ratio of net cross-sectional of the core to its gross cross-sectional area is known as *stacking factor*. It is always less than unity and its value depends upon the thickness of laminations, may vary from 0.5 to 0.95 approaching unity with the increase in thickness of laminations.

1.20.8. Flux Linkage. Consider a N turn coil of any shape. On one side of the coil, all turns would carry current in one direction and those on the other side in opposite direction. As the number of turns are more, each turn of the coil will carry the same current, the mmf produced would be more and therefore more flux. If the total flux is Φ webers, the flux linkage with each turn is Φ weber turn. Hence, the flux linkage with the coil having N turns is

$$\psi = \Phi N \text{ weber-turns} \qquad \ldots(1.24)$$

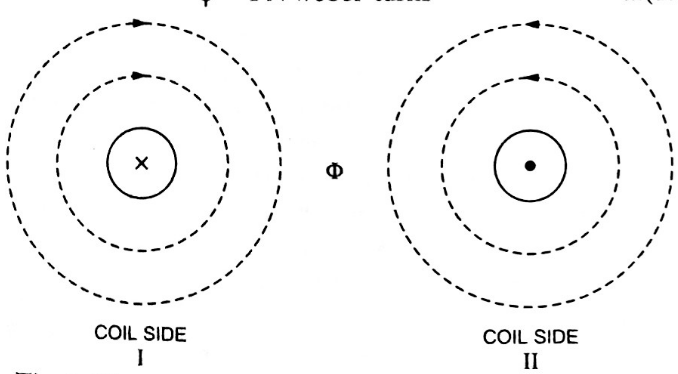


Fig. 1.32 *Flux Linkage With a Closed One-Turn Circuit*

Example 1.3. An electromagnet has an air gap of 4 mm and flux density in the gap is 1.3 Wb/m². Determine the ampere-turns for the gap. [*U.P. Technical Univ. Even Semester 2006-07; Mahamaya Technical Univ. Odd Semester 2011-12*]

Solution: Magnetising force, $H_g = \dfrac{B}{\mu_0 \mu_r} = \dfrac{1.3}{4\pi \times 10^{-7} \times 1}$

$= 1.035 \times 10^6$ AT/m $\quad \because \mu_r$ for air = 1

Ampere-turns required for the gap $= H_g l_g = 1.035 \times 10^6 \times 4 \times 10^{-3}$

$= 4.140$ **Ans.**

Example 1.4. A wrought iron bar 30 cm long and 2 cm in diameter is bent into a circular shape as given in Fig. 1.33. It is then wound with 500 turns of wire. Calculate the current required to produce a flux of 0.5 mWb in magnetic circuit with an air gap of 1 mm; μ_r (iron) = 4,000 (assume constant).

[*U.P. Technical Univ. Even Semester 2004-05*]

Solution: Flux to be produced, $\Phi = 0.5$ mWb $= 0.5 \times 10^{-3}$ Wb

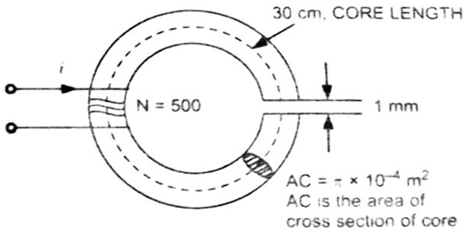


Fig. 1.33

Area of cross section of core, $a = \pi \times 10^{-4}$ m²

$$\text{Flux density required, } B = \frac{\Phi}{a} = \frac{0.5 \times 10^{-3}}{\pi \times 10^{-4}} = \frac{5}{\pi} = 1.5915 \text{ T}$$

Air-gap length, $l_g = 1$ mm $= 0.001$ m

Core length, $l_i = 30$ cm $= 0.3$ m

Total ampere-turns required,

$$AT = \frac{B}{\mu_0} l_g + \frac{B}{\mu_0 \mu_r} l_i$$

$$= \frac{1.5915}{4\pi \times 10^{-7}} \times 0.001 + \frac{1.5915}{4\pi \times 10^{-7} \times 4{,}000} \times 0.3$$

$= 1{,}266 + 95 = 1{,}361$

$$\text{Current required, } I = \frac{AT}{N} = \frac{1{,}361}{500} = 2.72 \text{ A } \textbf{Ans.}$$

Example 1.5. A flux density of 2.25 Wb/m² is required in 3 mm air gap of an electromagnetic having an iron path 1 m long. Calculate the magnetizing force and current required if the electromagnet has 1,275 turns. Assume relative permeability of iron to be 1,200.

[*M.D. Univ. Electromechanical Energy Conversion December-2013*]

Solution: Flux density to be produced in the air gap,

$B_g = 2.25$ Wb/m²

$\mu_0 = 4\pi \times 10^{-7}$ H/m

Magnetising force for air gap,

$$H_g = \frac{B_g}{\mu_0} = \frac{2.25}{4\pi \times 10^{-7}} = 1{,}790 \times 10^3 \text{ AT/m} \quad \textbf{Ans.}$$

Flux density required in iron path,

$B_i = B_g = 2.25$ Wb/m² neglecting magnetic leakage

---

*Refer to Art. 1.29.2.

Relative permeability of iron,

$\mu_r = 1{,}200$

Magnetising force for iron path,

$$H_i = \frac{B_i}{\mu_0 \mu_r} = \frac{2.25}{4\pi \times 10^{-7} \times 1{,}200} = 1{,}492 \text{ AT/m} \quad \textbf{Ans.}$$

Length of air gap, $l_g = 3 \text{ mm} = 0.003 \text{ m}$

Length of iron path, $l_i = 1 \text{ m}$

Total ampere-turns required,

$$AT = H_g l_g + H_i l_i$$
$$= 1{,}790 \times 10^3 \times 0.003 + 1{,}492 \times 1 = 6{,}862$$

$$\text{Current required, } I = \frac{AT}{N} = \frac{6{,}862}{1{,}275} = 5.38 \text{ A} \quad \textbf{Ans.}$$

Example 1.6. A ring of ferromagnetic material has a rectangular cross section. The inner diameter is 7.4 in., the outer diameter is 9 in., and the thickness is 0.8 in. There is a coil of 600 turns wound on the ring. When the coil carries a current of 2.5 A, the flux produced in the ring is 1.2 × 10⁻³ Wb. Find: (i) Magnetic field intensity (ii) Reluctance (iii) Permeability.

[U.P. Technical Univ. Even Semester 2007-08; Odd Semester 2013-14]

Solution: Number of turns in the coil, $N = 600$

Current carried by coil, $I = 2.5 \text{ A}$

Flux produced, $\Phi = 1.2 \times 10^{-3}$ Wb

Mean length of magnetic flux path is that of a circle midway between the inside and outside diameter i.e.

$$l = \frac{\pi(7.4 + 9)}{2} = 25.76 \text{ in} = 25.76 \times 0.0254 = 0.6543 \text{ m}$$

Cross-sectional area of the toroidal core,

$$a = \frac{9 - 7.4}{2} \times 0.8 = 0.64 \text{ in}^2 = 4.13 \times 10^{-4} \text{ m}^2$$ 1 inch = 0.0254 m

(i) Magnetic flux density, $B = \frac{\Phi}{a} = \frac{1.2 \times 10^{-3}}{4.13 \times 10^{-4}} = 2.9 \text{ Wb/m}^2$ **Ans.**

(ii) Reluctance, $S = \frac{MMF}{\Phi} = \frac{NI}{\Phi} = \frac{600 \times 2.5}{1.2 \times 10^{-3}}$

$= 1{,}250 \times 10^3$ AT/Wb **Ans.**

(iii) Permeability, $\mu = \frac{l}{Sa} = \frac{0.6543}{1{,}250 \times 10^3 \times 4.13 \times 10^{-4}}$

$= 1.267 \times 10^{-3}$ H/m **Ans.**

Relative permeability, $\mu_r = \frac{\mu}{\mu_0} = \frac{1.267 \times 10^{-3}}{4\pi \times 10^{-7}} = 1{,}008$ **Ans.**

Example 1.7. A ring of iron has a mean diameter of 15 cm, a cross section of 1.5 cm² and has a radial air gap of 0.5 mm cut in it. It is uniformly wound with 1, 500 turns of insulated wire and a current of 1.2 A produces a flux of 0.1 mWb across the air gap. Calculate the relative permeability of iron on the assumption that there is no magnetic leakage.

[M.D. Univ. Electromechanical Energy Conversion December 2005]

Solution: Flux to be produced in the air gap, $\Phi_g = 0.1$ mWb

$= 0.1 \times 10^{-3}$ Wb

Flux to be produced in the iron path, $\Phi_i = 0.1$ mWb

∵ there is no magnetic leakage

Area of cross section, $a = 1.5 \text{ cm}^2 = 1.5 \times 10^{-4} \text{ m}^2$

Length of air gap, $l_g = 0.5 \text{ mm} = 0.5 \times 10^{-3}$ m

Length of iron path, $l_i = \pi \times 0.15 - 0.5 \times 10^{-3} = 0.4707$ m

Number of turns provided, $N = 1{,}500$

Current flowing through the coil, $I = 1.2$ A

Total ampere-turns provided, $AT = N \times I = 1{,}500 \times 1.2 = 1{,}800$

Ampere-turns required for air gap,

$$AT_g = \frac{\Phi_g / a}{\mu_0} l_g$$
$$= \frac{0.1 \times 10^{-3}}{4\pi \times 10^{-7} \times 1.5 \times 10^{-4}} \times 0.5 \times 10^{-3} = 265$$

Ampere-turns provided for iron path,

$$AT_i = AT - AT_g = 1{,}800 - 265 = 1{,}535$$

Thus ampere-turns provided for iron path,

$$AT_i = \frac{\Phi_i / a_i}{\mu_0 \mu_r} l_i = AT - AT_g$$

or $\frac{0.1 \times 10^{-3}}{4\pi \times 10^{-7} \times \mu_r \times 1.5 \times 10^{-4}} \times 0.4707 = 1{,}535$

or Relative permeability of iron, $\mu_r = \frac{0.1 \times 10^{-3} \times 0.4707}{4\pi \times 10^{-7} \times 1.5 \times 10^{-4} \times 1{,}535}$

$= 163$ **Ans.**

Example 1.8. A steel ring 12 cm mean radius and of circular cross section 1 cm in radius has an air gap 2 mm length. It is wound uniformly with 550 turns of wire core carrying 3 A. Neglecting magnetic leakage. The air gap takes 60% of total mmf. Find total reluctance.

[M.D. Univ. Electromechanical Energy Conversion December-2013]

Solution: Area of cross section of steel ring,

$$a = \pi r^2 = \pi \times (0.01)^2 = \pi \times 10^{-4} \text{ m}^2$$

Total ampere-turns provided,

$$AT = NI = 550 \times 3 = 1{,}650$$

Ampere-turns provided for air gap,

$$AT_g = 60\% \text{ of total mmf} = 0.6 \times 1{,}650 = 990$$

Ampere-turns provided for iron path,

$$AT_i = AT - AT_g = 1{,}650 - 990 = 660$$

Flux density in air gap,

$$\boxed{B_g = \frac{AT_g \times \mu_0}{l_g}} = \frac{990 \times 4\pi \times 10^{-7}}{2 \times 10^{-3}} = 0.622 \text{ T}$$

Magnetic flux in air gap,

$$\Phi_g = B_g \times a = 0.622 \times \pi \times 10^{-4} = 0.195 \text{ mWb}$$

Magnetic flux in iron path,

$\Phi_i = \Phi_g = 0.195$ mWb neglecting magnetic leakage

Total reluctance,

S = Air-gap reluctance + reluctance for iron path

$$= \frac{AT_g}{\Phi_g} + \frac{AT_i}{\Phi_i} = \frac{AT}{\Phi} = \frac{1{,}650}{0.195 \times 10^{-3}} = 0.846 \times 10^7 \text{ AT/Wb} \quad \textbf{Ans.}$$

Example 1.9. A circular iron ring having a cross-sectional area of 10 cm² and a length of 4π cm in iron has an air gap of 0.4π mm made a saw cut. The relative permeability of iron is 10³ and permeability of free space is 4π × 10⁻⁷ H/m. The ring is wound with a coil of 2,000 turns and carries 2 mA. Determine the air-gap flux neglect leakage and fringing. *[Pb. Technical Univ. December-2013]*

Solution: Let the flux density in iron as well as air gap be B Wb/m²

Total ampere-turns provided,

$$AT = N \times I = 2{,}000 \times 2 \times 10^{-3} = 4$$

Total ampere-turns required

= AT required for air gap + AT required for iron path

or Total ampere-turns provided

$$= \frac{B}{\mu_0} l_g + \frac{B}{\mu_0 \mu_r} l_i$$

$$\text{or } 4 = \frac{B}{\mu_0}\left(l_g + \frac{l_i}{\mu_r}\right)$$

$$\text{or Flux density, } B = \frac{4 \times \mu_0}{l_g + \frac{l_i}{\mu_r}}$$

$$= \frac{4 \times 4\pi \times 10^{-7}}{0.4\pi \times 10^{-3} + \frac{4\pi \times 10^{-2} - 0.4\pi \times 10^{-3}}{1,000}}$$

$$= \frac{16\pi \times 10^{-7}}{0.4\pi \times 10^{-3} + 0.0396\pi \times 10^{-3}}$$

$$= \frac{16 \times 10^{-7}}{0.4396 \times 10^{-3}} = 3.64 \times 10^{-3} \text{ T} \quad \textbf{Ans.}$$

Example 1.10. An iron ring is made up of three parts: $l_1 = 10$ cm, $A_1 = 8$ cm^2, $l_2 = 8$ cm, $A_2 = 3$ cm^2, $l_3 = 6$ cm, $A_3 = 2.5$ cm^2. It is wound with a coil of 250 turns. Calculate current required to produce flux of 0.4 mWb.

$\mu_1 = 2,670$, $\mu_2 = 1,050$, $\mu_3 = 600$.

[U.P. Technical Univ. Even Semester 2007-08]

Solution: Total reluctance,

$$S = \frac{l_1}{\mu_0 \mu_{r_1} a_1} + \frac{l_2}{\mu_0 \mu_{r_2} a_2} + \frac{l_3}{\mu_0 \mu_{r_3} a_3}$$

$$= \frac{0.1}{4\pi \times 10^{-7} \times 2,670 \times 8 \times 10^{-4}} + \frac{0.08}{4\pi \times 10^{-7} \times 1,050 \times 3 \times 10^{-4}} + \frac{0.06}{4\pi \times 10^{-7} \times 600 \times 2.5 \times 10^{-4}}$$

$= 0.37255 \times 10^5 + 2.021 \times 10^5 + 3.183 \times 10^5$

$= 5.57655 \times 10^5$ AT/Wb

MMF = Flux Φ × Reluctance S $= 0.4 \times 10^{-3} \times 5.57655 \times 10^5$

= 223 ampere-turns

Current required,

$$I = \frac{MMF}{N} = \frac{223}{250} = 0.892 \text{ A} \quad \textbf{Ans.}$$

Example 1.11. A cast steel ring has a circular cross section of 3 cm in diameter and a mean circumference of 80 cm. A 1 mm air gap is cut in the ring which is wound with a coil of 600 turns. Estimate the current required to establish a flux of 0.75 mWb in the air gap. Neglect leakage.

Magnetization data:

H (AT/M)	200	400	600	800	1,000	1,200	1,400	1,600	1,800
B (T)	0.1	0.32	0.6	0.9	1.08	1.18	1.27	1.32	1.36

[U.P. Technical Univ. Electromechanical Energy Conversion-I, 2004-2005]

Solution: Area of x-section, $a = \frac{\pi}{4}(d)^2 = \frac{\pi}{4}(0.03)^2 = 0.0007 \text{ m}^2$

Flux density in the air gap, $B_g = \frac{\Phi_g}{a} = \frac{0.75 \times 10^{-3}}{0.0007} = 1.07 \text{ T}$

Ampere-turns required for air gap,

$$AT_g = \frac{B_g}{\mu_0} \times l_g = \frac{1.07}{4\pi \times 10^{-7}} \times 1 \times 10^{-3} = 850$$

Flux density in iron path,

$B_i = B_g = 1.07$ T $\quad \because$ leakage in negligible.

From Fig. 1.34, magnetising force, H corresponding to flux density B of 1.07 T is 980 AT/m

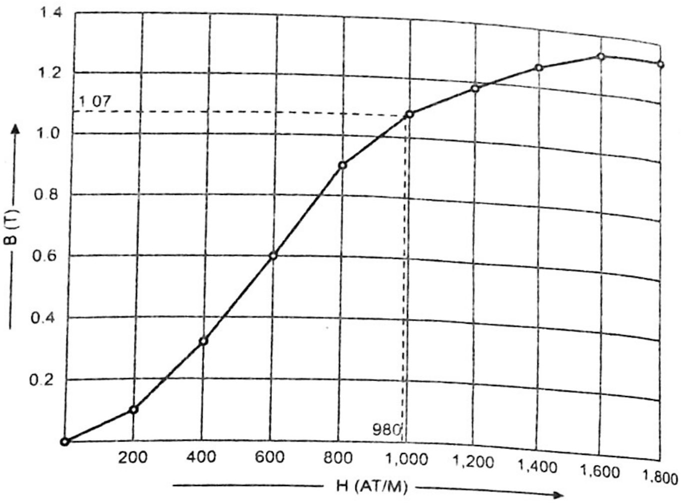


Fig. 1.34 *Magnetization Curve (Example 1.11)*

AT required for iron path, $AT_i = H \times l_i = 980 \times (80 - 0.1) \times 10^{-2}$

$= 783$

Total ampere-turns required, $AT = AT_g + AT_i = 850 + 783 = 1,633$

$$\text{Current required, } I = \frac{AT}{N} = \frac{1,633}{600} = 2.72 \text{ A} \quad \textbf{Ans.}$$

Example 1.12. A circular iron ring has a mean circumference of 1.5 m and a cross-sectional area of 100 cm^2. A saw cut of 0.4 cm wide is made in the ring. Calculate the magnetizing current required to produce a flux of 0.8 mWb in the air gap if the ring is wound with a coil of 350 turns. Assume relative permeability of iron as 400 and leakage factor 1.25.

[M.D. Univ. Electromechanical Energy Conversion, May-2007]

Solution: Area of cross section, $a = 100 \text{ cm}^2 = 0.01 \text{ m}^2$

Flux to be produced in the air gap, $\Phi_g = 0.8 \text{ mWb} = 0.8 \times 10^{-3}$ Wb

$$\text{Flux density in the air gap, } B_g = \frac{\Phi_g}{a} = \frac{0.8 \times 10^{-3}}{0.01} = 0.08 \text{ T}$$

Ampere-turns required for air gap,

$$AT_g = \frac{B_g}{\mu_0} \times l_g = \frac{0.08}{4\pi \times 10^{-7}} \times 0.4 \times 10^{-2} = 255$$

Flux through iron path, $\Phi_i = \Phi_g \times$ leakage factor

$= 0.8 \times 10^{-3} \times 1.25 = 1 \times 10^{-3}$ Wb

Flux density in the iron path,

$$B_i = \frac{\Phi_i}{a} = \frac{1 \times 10^{-3}}{0.01} = 0.1 \text{ T}$$

Ampere-turns required for iron path,

$$AT_i = \frac{B_i}{\mu_0 \mu_r} l_i = \frac{0.1 \times (1.5 - 0.004)}{4\pi \times 10^{-7} \times 400} = 298$$

Total ampere-turns required,

$$AT = AT_g + AT_i = 255 + 298 = 553$$

$$\text{Current required, } I = \frac{AT}{N} = \frac{553}{350} = 1.58 \text{ A } \textbf{Ans.}$$

Example 1.13. A rectangular magnetic core shown in Fig. 1.35 (a) has square cross section of area 16 cm². An air gap of 2 mm is cut across one of its limbs. Find the exciting current needed in the coil having 1,000 turns wound on the core to create an air-gap flux of 4 m Wb. The relative permeability of the core is 2,000.

[U.P. Technical Univ. February-2002]

Solution: Flux to be created, $\Phi = 4 \text{ mWb} = 0.004 \text{ Wb}$

Area of x-section, $a = 16 \text{ cm}^2 = 16 \times 10^{-4} \text{ m}^2$

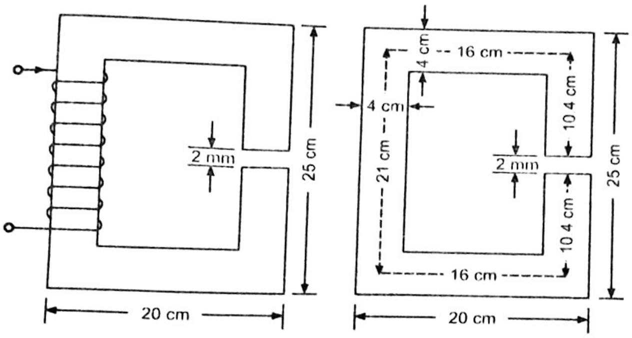


Fig. 1.35

$$\text{Fluse density required, } B = \frac{\Phi}{a} = \frac{4\times10^{-3}}{16\times10^{-4}} = 2.5 \text{ T}$$

$$\text{Length of air gap } l_g = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$\text{Each side of cross section} = \sqrt{16} = 4 \text{ cm}$$

$$\text{Length of iron path, } l_i = \left(25 - 2\times\frac{4}{2} + 20 - 2\times\frac{4}{2}\right)\times 2 - 0.2$$

$$= 73.8 \text{ cm} = 0.738 \text{ m}$$

Total ampere-turns required,

$$AT = \frac{B}{\mu_0} l_g + \frac{B}{\mu_0 \mu_r} l_i$$

$$= \frac{2.5\times2\times10^{-3}}{4\pi\times10^{-7}} + \frac{2.5\times0.738}{2{,}000\times4\pi\times10^{-7}}$$

$$= 3{,}979 + 734 = 4{,}713$$

Number of turns on the coil = 1,000

$$\text{Exciting current required, } I = \frac{AT}{N} = \frac{4{,}713}{1{,}000} = 4.713 \text{ A } \textbf{Ans.}$$

Example 1.14. The magnetic circuit frame shown in Fig. 1.36 is built of iron of square cross section of 3 cm. Each air gap is 2 mm wide and each of the coil is wound with 1,000 turns. The relative permeability of part A and B may be taken as 1,000 and 1,200 respectively. Calculate (a) reluctance of part A (b) reluctance of two air gaps (c) total reluctance of complete magnetic path (d) total mmf (e) total flux and (f) flux density.

Solution: Cross-sectional area,

$$a = (0.03)^2 = 0.0009 \text{ m}^2$$

Mean length flux path in part A,

$$l_A = 20 - (1.5 + 1.5) = 17 \text{ cm or } 0.17 \text{ m}$$

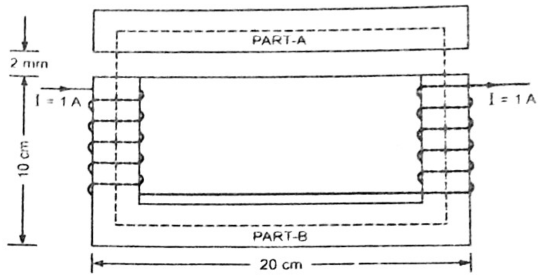


Fig. 1.36

Mean length flux path in part B,

$$l_B = (20 - 1.5 - 1.5) + (10 - 1.5) + (10 - 1.5)$$

$$= 34 \text{ cm or } 0.34 \text{ m}$$

Relative permeability of part A, $\mu_{r\,A} = 1{,}000$

Relative permeability of part B, $\mu_{rB} = 1{,}200$

Length of two air gaps, $l_g = 2 \times 2 = 4 \text{ mm} = 0.004 \text{ m}$

$$(a)\ (i) \text{ Reluctance of part A, } S_A = \frac{0.17}{4\pi\times10^{-7}\times1{,}000\times0.0009}$$

$$= 1{,}50{,}313 \text{ AT/Wb } \textbf{Ans.}$$

$$(ii) \text{ Reluctance of part B, } S_B = \frac{l_B}{\mu_0\,\mu_{rB}\times a}$$

$$= \frac{0.34}{4\pi\times10^{-7}\times1{,}200\times0.0009}$$

$$= 2{,}50{,}521 \text{ AT/Wb}$$

(*b*) Reluctance of two air gaps,

$$S_g = \frac{l_g}{\mu_0 a} = \frac{0.004}{4\pi\times10^{-7}\times0.0009}$$

$$= 35{,}36{,}776 \text{ AT/Wb } \textbf{Ans.}$$

$$(c) \text{ Total reluctance, } S = S_A + S_B + S_g$$

$$= 1{,}50{,}313 + 2{,}50{,}521 + 35{,}36{,}776$$

$$= 39{,}37{,}610 \text{ AT/Wb } \textbf{Ans.}$$

(*d*) Total mmf = 2 × 1,000 × 1 = 2,000 AT **Ans.**

because mmf is produced by two coils on part B each having 1,000 turns and carrying a current of 1 A

$$(e) \text{ Total flux} = \frac{\text{MMF}}{\text{Reluctance}} = \frac{2{,}000}{39{,}37{,}610}$$

$$= 5.08 \times 10^{-4} \text{ Wb.} \quad \textbf{Ans.}$$

$$(f) \text{ Flux density, } B = \frac{\Phi}{a} = \frac{5.08\times10^{-4}}{0.0009}$$

$$= 0.564 \text{ Wb/m}^2 \text{ (or Tesla) } \textbf{Ans.}$$

Example 1.15. A rectangular iron core shown in the figure (Fig. 1.37) has mean length of a magnetic circuit of 100 cm, cross section of 2 cm × 2 cm and relative permeability of 1,450. A cut of size 5 mm in the core has been made. The three coils A, B and C on the core have number of turns $N_A = 335$, $N_B = 600$ and $N_C = 600$, and the respective currents flowing are 1.5 A, 4 A and 3 A. The direction of currents are as shown. Find the air-gap flux, neglecting fringing of flux.

Solution: It is observed that flux produced by coils B and C are in the same direction but that by coil A is in opposite direction.

Hence net available ampere-turns,

$$= AT_B + AT_C - AT_A$$

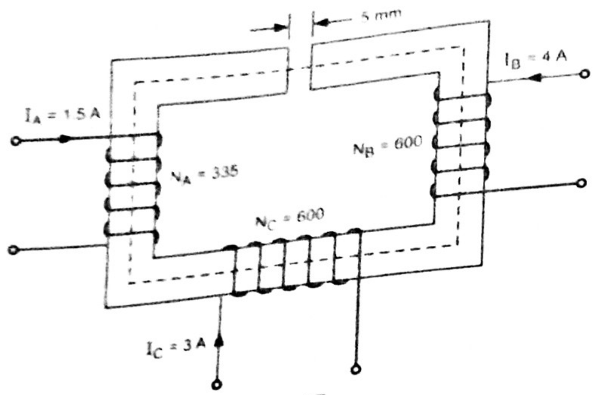


Fig. 1.37

$= 600 \times 4 + 600 \times 3 - 335 \times 1.5$

$= 3{,}697.5$

Mean length of iron path, $l_i = 100$ cm $= 1$ m

Area of cross section, $a = 2 \times 10^{-2} \times 2 \times 10^{-2} = 4 \times 10^{-4}$ m^2

Relative permeability of iron path,

$\mu_r = 1{,}450$

$$\text{Reluctance of iron path, } S_i = \frac{l_i}{\mu_0 \mu_r a}$$

$$= \frac{1}{4\pi \times 10^{-7} \times 1{,}450 \times 4 \times 10^{-4}}$$

$= 1.372025 \times 10^6$ AT/Wb

Length of air gap $l_g = 5$ mm $= 5 \times 10^{-3}$ m

$$\text{Reluctance of air gap, } S_g = \frac{l_g}{\mu_0 \times a} = \frac{5 \times 10^{-3}}{4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$= 9.947184 \times 10^6$ AT/Wb

Total reluctance of the circuit,

$S = S_i + S_g$

$= (1.372025 \times 10^6 + 9.947184 \times 10^6)$ AT/Wb

$= 11.31921 \times 10^6$ AT/Wb

$$\text{Flux in the air-gap, } \Phi = \frac{\text{Total available AT}}{\text{Total reluctance}} = \frac{3{,}697.5}{11.31921 \times 10^6}$$

$= 0.3267 \times 10^{-3}$ Wb or 0.3267 mWb **Ans.**

Example 1.16. The magnetic circuit of the figure given below has cast steel core with dimensions as shown:

Mean length from A to B through either outer limb = 0.5 m.

Mean length from A to B through the central limb = 0.2 m.

In the magnetic circuit shown it is required to establish a flux of 0.75 mWb in the air gap of the central limb. Determine the mmf of the exciting coil if for the core material relative permeability μ_r = 5,000. Neglect fringing. *[RGPV December-2003]*

Solution: Flux in the air gap of central limb,

$\Phi_{g_1} = 0.75$ m Wb $= 7.5 \times 10^{-4}$ Wb

Area of cross section,

$a_{g_1} = 2 \times 1 \times 10^{-4} = 2 \times 10^{-4}$ m^2

Flux density,

$$B_{g_1} = \frac{\Phi_{g_1}}{a_{g_1}} = \frac{7.5 \times 10^{-4}}{2 \times 10^{-4}} = 3.75 \text{ T}$$

Ampere-turns required,

$$AT_{g_1} = H_g l_g = \frac{B_{g_1}}{\mu_0} l_{g_1} = \frac{3.75}{4\pi \times 10^{-7}} \times 0.02 \times 10^{-2} = 596$$

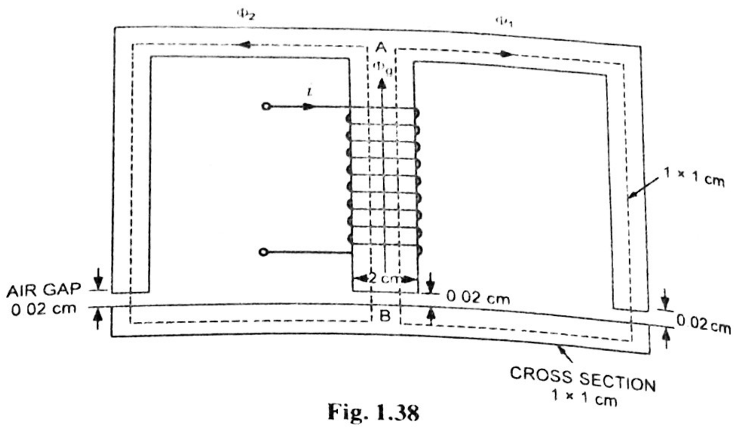


Fig. 1.38

Flux in central limb path AB,

$\Phi_{AB_1} = 7.5 \times 10^{-4}$ Wb

Flux density in limb path AB,

$$B_{AB_1} = \frac{\Phi_{AB}}{a_{AB}} = \frac{7.5 \times 10^{-4}}{2 \times 10^{-4}} = 3.75 \text{ T}$$

Length of limb path AB,

$l_{AB_1} = 0.2$ m $- 0.0002$ m $= 0.1998$ m

Relative permeability of core material,

$\mu_r = 5{,}000$

Ampere-turns required,

$$AT_{AB_1} = \frac{B_{AB}}{\mu_0 \mu_r} \times l_{AB_1} = \frac{3.75}{4\pi \times 10^{-7} \times 5{,}000} \times 0.1998 = 120$$

Flux in the outer limb path,

$$\Phi_{AB_2} = \frac{\Phi_{g_1}}{2} = \frac{7.5 \times 10^{-4}}{2} = 3.75 \times 10^{-4} \text{ Wb}$$

Area of cross section,

$a_2 = 1 \times 1 \times 10^{-4} = 1 \times 10^{-4}$ m^2

Flux density in the outer limb path

$$= \frac{3.75 \times 10^{-4}}{1 \times 10^{-4}} = 3.75 \text{ T}$$

Ampere-turns required for gap in the outer limb,

$$AT_{g_2} = \frac{B_{g_2}}{\mu_0} l_{g_2} = \frac{3.75}{4\pi \times 10^{-7}} \times 0.02 \times 10^{-2} = 596$$

Ampere-turns required for the outer limb,

$$AT_{AB_2} = \frac{B_{AB_2}}{\mu_0 \mu_r} \times l_{AB_2} = \frac{3.75 \times (0.5 - 0.0002)}{4\pi \times 10^{-7} \times 5{,}000} = 300$$

Total ampere-turns (mmf) required

$= AT_{g_1} + AT_{AB_1} + AT_{g_2} + AT_{AB_2}$

$= 596 + 120 + 596 + 300 = 1{,}612$ **Ans.**

Example 1.17. Fig. 1.39(*a*) shows a magnetic circuit formed by an ideal core material. Determine the magnetic flux density in the air gap. *[GATE 1997]*

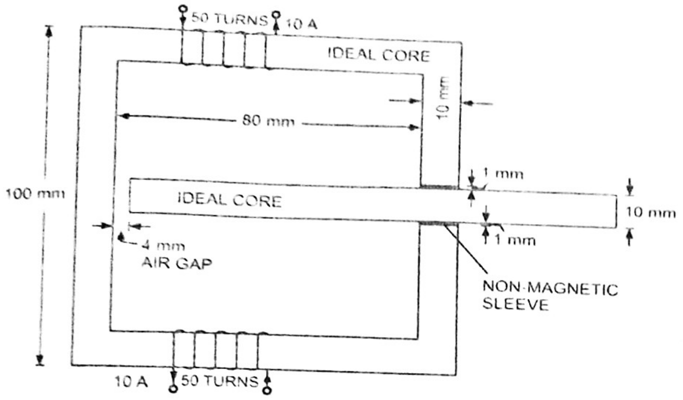
Solution: The equivalent circuit is given in Fig. 1.39(*b*) where S_s is reluctance of nonmagnetic sleeve and S_g is the reluctance of air gap.

Magnetising force, $F_1 = F_2 = 50 \times 10 = 500$ AT

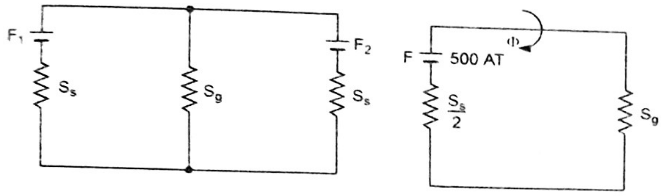
Length of nonmagnetic sleeve, $l_s = 1$ mm

Length of air gap, $l_g = 4$ mm

Since core is ideal, its reluctance is zero.



(a)



(b)

Fig. 1.39

Reluctance of nonmagnetic sleeve, $S_s = \dfrac{l_s}{\mu_0 \mu_r a}$

where $l_s = 1$ mm, $\mu_0 = 4\pi \times 10^{-7}$, and $\mu_r = 1$ being nonmagnetic

So $S_s = \dfrac{1 \times 10^{-3}}{4\pi \times 10^{-7} a}$

and $S_g = \dfrac{l_g}{\mu_0 a_g} = \dfrac{4 \times 10^{-3}}{4\pi \times 10^{-7} \times a}$

Total reluctance of magnetic circuit,

$$S = \frac{S_s}{2} + S_g = \frac{1 \times 10^{-3}}{2 \times 4\pi \times 10^{-7} a} + \frac{4 \times 10^{-3}}{4\pi \times 10^{-7} a} = \frac{4.5}{4\pi \times 10^{-4} a}$$

Magnetic flux density,

$$B = \frac{\Phi}{a} = \frac{MMF}{S \times a} = \frac{500}{\dfrac{4.5}{4\pi \times 10^{-4} a} \times a} = \frac{500 \times 4\pi \times 10^{-4}}{4.5}$$

$= 0.14$ Wb/m^2 **Ans.**

Example 1.18. A cast steel magnetic structure made of a bar of section 2 cm × 2 cm is shown in Fig. 1.40. Determine the current that the 500 turn magnetizing coil on the left limb should carry so that a flux of 2 mWb is produced in the right limb. Take $\mu_r = 600$ and neglect leakage. *[Chhatisgarh Vivekanand Technical Univ. 2006-07]*

Solution: Flux created by magnetizing coil, say Φ, is divided at junction point into two paths depending upon the reluctances of portions B and C respectively.

Reluctance of portion B, $S_B = \dfrac{l_B}{\mu_0 \mu_r \times a} = \dfrac{0.15}{\mu_0 \mu_r a}$

Reluctance of portion C, $S_C = \dfrac{l_C}{\mu_0 \mu_r a} = \dfrac{0.25}{\mu_0 \mu_r a}$

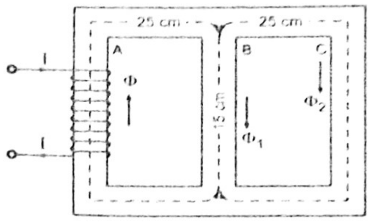


Fig. 1.40

Flux in portion C,

$\Phi_C = 2$ m Wb $= 0.002$ Wb (given)

Flux in portion B,

$$\Phi_B = \Phi_C \times \frac{S_C}{S_B} = 2 \times 10^{-3} \times \frac{0.25/\mu_0 \mu_r a}{0.15/\mu_0 \mu_r a} = 3.33 \times 10^{-3} \text{ Wb}$$

Total flux in portion A,

$\Phi = \Phi_B + \Phi_C = 3.33 \times 10^{-3} + 2 \times 10^{-3} = 5.33 \times 10^{-3}$ Wb

Flux density in portion A,

$$B_A = \frac{\Phi}{a} = \frac{5.33 \times 10^{-3}}{4 \times 10^{-4}} = 13.33 \text{ Wb/m}^2$$

AT required for portion A,

$$AT_A = \frac{B_A \times l_A}{\mu_0 \mu_r} = \frac{13.33 \times 0.25}{4\pi \times 10^{-7} \times 600} = 4{,}416$$

Flux density in path B,

$$B_B = \frac{3.33 \times 10^{-3}}{4 \times 10^{-4}} = 8.33 \text{ Wb/m}^2$$

AT required for portion B,

$$AT_B = \frac{B_B\, l_B}{\mu_0 \mu_r} = \frac{8.33 \times 0.15}{4\pi \times 10^{-7} \times 600} = 1{,}658$$

Total ampere-turns required,

$AT = AT_A + AT_B = 4{,}416 + 1{,}658 = 6{,}074$ **Ans.**

Required current,

$$I = \frac{AT}{N} = \frac{6{,}074}{500} = 12.15 \text{ A}$$ **Ans.**

Example 1.19. A ring of cast steel has an external diameter of 24 cm and a square x-section of 3 cm side. Inside and across the ring an ordinary steel bar 18 × 3 × 0. 4 cm is fitted with negligible gap. Calculate the number of ampere-turns required to be applied to one half of the ring to produce a flux density of 1 Wb/m^2 in the other half. Neglect leakage.

The B-H characteristics area follows:

	For cast steel			For ordinary steel		
B(Wb/m^2)	**1.0**	**1.1**	**1.2**	**1.2**	**1.4**	**1.45**
H(AT/m)	**900**	**1,020**	**1,230**	**590**	**1,200**	**1,650**

Solution: External diameter of the ring $= 24$ cm $= 0.24$ m

Internal diameter of the ring $= 24 - 6 = 18$ cm $= 0.18$ m

Mean diameter of the ring, $D = \dfrac{0.24 + 0.18}{2} = 0.21$ m

Mean length of the ring $= \pi D = \pi \times 0.21 = 0.66$ m

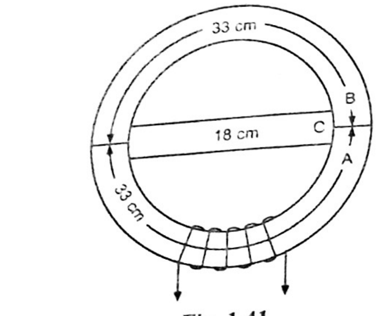


Fig. 1.41

Length of upper portion of ring, l_a = length of lower portion of ring l_a

$$= \frac{0.66}{2} = 0.33 \text{ m}$$

Area of x-section of ring $a_A = a_B = 0.03 \times 0.03 = 9 \times 10^{-4}$ m^2

Length of cross-bar, l_c = 18 cm = 0.18 m

Area of cross section of cross-bar, $a_c = 0.03 \times 0.004 = 12 \times 10^{-5}$ m^2

This is a case of parallel magnetic circuit.

Flux created in A divides itself when it comes to the junction point depending upon the reluctance of portions B and C respectively.

$$i.e.\ \Phi_A = \Phi_B + \Phi_C$$

Flux density in the upper half, B_B = 1 T

For cast steel, against B = 1 T ; H = 900 AT/m

∴ Total ampere-turns required for portion B = 900 × 0.33 = 297

Since portions B and C are in parallel

∴ AT required for portion C

= AT required of portion B = 297

Magnetising force for the portion C,

$$H = \frac{297}{0.18} = 1{,}650 \text{ AT/m}$$

For ordinary steel density,

B_c = 1.45 T when H = 1,650 AT/m

Flux in portion B,

$$\Phi_B = B_B \times a_B = 1 \times 9 \times 10^{-4} = 9 \times 10^{-4} \text{ Wb}$$

Flux in portion C,

$$\Phi_C = B_C \times a_C = 1.45 \times 12 \times 10^{-5} = 1.74 \times 10^{-4} \text{ Wb}$$

Flux in portion A,

$$\Phi_A = \Phi_B + \Phi_C = 9 \times 10^{-4} + 1.74 \times 10^{-4} = 10.74 \times 10^{-4} \text{ Wb}$$

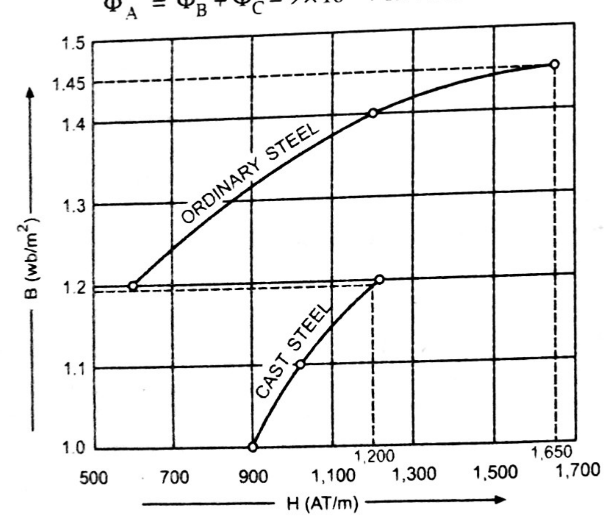


Fig. 1.42

Flux density in portion A,

$$B_A = \frac{\Phi_A}{a_A} = \frac{10.74 \times 10^{-4}}{9 \times 10^{-4}} = 1.1933 \text{ T}$$

From the B-H curve drawn between H and B for cast steel magnetising force corresponding to flux density of 1.1933 T is 1,200 AT/m.

AT required for portion A = 1200 × 0.33 = 396

Total ampere-turns required = AT for portion A + AT for portion B (or AT for portion C)

= 396 + 297 = 693 **Ans.**

★ 1.21 MAGNETICALLY INDUCED EMFS (OR VOLTAGES)

A very important effect of a magnetic field on an electric circuit is that when the flux linking the circuit changes, an emf is induced. Electromagnetic induction of emf (or voltage) is basic to the operation of transformers, generators (ac or dc) and motors (ac or dc). The effect is described by Faraday's law, which states that *the magnitude of emf (or voltage) is directly proportional to the rate of change of flux linkage or to the product of number of turns and rate of change of flux linking the coil.*

$$i.e.\ \text{induced emf, } e = N\frac{d\Phi}{dt} \qquad \ldots(1.25)$$

where $N\frac{d\Phi}{dt}$ is the product of number of turns and rate of change of linking flux and is termed as *rate of change of flux linkage.*

The direction of induced emf is governed by Lenz's law which states that *the direction of induced emf or voltage is such that the current produced by it sets up a magnetic field opposing the cause that produces it.*

A minus (–) sign is required to be placed before the right hand side quantity of Eq. (1.25) just to indicate the phenomenon explained by Lenz's law. Thus the expression for induced emf becomes as

$$e = -N\frac{d\Phi}{dt} \text{ volts} \qquad \ldots(1.26)$$

where Φ is in webers and time t is in seconds.

The induced emf or voltage appears in any circuit which may be linked by the changing flux, including the circuit giving rise to flux. Thus, current in a circuit produces a magnetic field linking the circuit and permeating the medium around it. Growth, decay or any other change of current with time causes corresponding changes of the magnetic flux and induces an emf in the circuit.

1.21.1. Dynamically Induced EMF. We have learnt that when the flux linking with the coil or circuit changes, an emf is induced in the coil or circuit.

EMF can be induced by changing the flux linking in two ways :

(*i*) By increasing or decreasing the magnitude of the current producing the linking flux. In this case there is no motion of the conductor or of coil relative to the field and, therefore, emf induced in this way is known as *statically induced emf.*

Coefficient of Mutual Induction. Mutual inductance may be defined as the ability of one coil or circuit to induce an emf in a nearby coil by induction when the current flowing in the first coil is changed. The action is also reciprocal *i.e.* the change in current flowing through second coil will also induce an emf in the first coil. The ability of reciprocal induction is measured in terms of the coefficient of mutual induction M.

The coefficient of mutual induction (M) can be determined from any one of the following three relations.

First Method. In case the dimensions of the coils are given, he coefficient of mutual induction may be determined from the elation

$$M = \frac{N_1 N_2 a \mu_0 \mu_r}{l} \text{ henrys} \quad ...(1.33) \text{ [Refer to Art. 1.21.2 (b)]}$$

Second Method. In case the magnitude of induced emf in e second coil for a given rate of change of current in the first il is known, mutual inductance between the coil may be termined from the following relation

$$e_m = M\frac{di_1}{dt} \quad \text{[Refer to Art. 1.21.2 (b)]}$$

$$\text{or } M = \frac{e_m}{di_1/dt} \quad ...(1.34)$$

Third Method. In case the number of turns of the coil and linking with this coil per ampere of current in another coil is wn, the mutual inductance of the coil may be determined the following relation

$$M = N_2 \frac{\Phi_2}{i_1} \text{ Henry} \quad ...(1.35)$$

3. Leakage Reactance. A coil current varying sinusoidally be expressed as $i = I \sin \omega t$ and therefore, change of current time is given as

$$\frac{di}{dt} = \omega I \cos \omega t$$

duced emf is given as

$$e = L\frac{di}{dt} = \omega L I \cos \omega t$$

om the above relationship it is observed that ωL must be ed in ohms ωL is known as *self-inductive reactance* and olized as X. If there is mutual inductance, the reactance used will X_m, which is proportional to ωM.

en leakage flux is associated with a winding carrying state" alternating current, a reactive voltage is induced. netic reluctance offered to leakage flux is predominantly e air path under normal conditions and therefore, this ies directly as the current, providing a constant ce. Consequently the reactive voltage may be considered e of constant leakage reactance and will be symbolized determining factor in the machine performance.

.20. Find the induced emf in a conductor of length 150 cm an angle of 30° to the direction of uniform magnetic field sity 1.2 Wb/m² with a velocity of 60 m/s.

Flux density B $= 1.2$ T

ngth of conductor, l = 150 cm = 1.5 m

ity of conductor, v = 60 m/s

Angle of movement of conductor from the direction of magnetic field.

$$\theta = 30^\circ$$

$$\text{EMF induced, } e = Blv \sin\theta = 1.2 \times 1.5 \times 60 \times \sin 30^\circ = 108 \times 0.5 = 54 \text{ V } \textbf{Ans.}$$

Example 1.21. A conductor of active length 30 cm carries a current of 100 A and lies at right angles to a magnetic field of strength 0.4 Wb/m². Calculate the force in newtons exerted on it.

If the force causes the conductor to move at a velocity of 10 m/s, calculate (*i*) the emf induced in it and (*ii*) the power in watts developed by it.

Solution: Length of conductor, l = 30 cm = 0.3 m

Current flowing through conductor,

$$I = 100 \text{ A}$$

$$\text{Field strength, } B = 0.4 \text{ Wb/m}^2$$

$$\text{Force exerted on the conductor, } F = BIl = 0.4 \times 100 \times 0.3 = 12 \text{ N } \textbf{Ans.}$$

$$\text{Conductor velocity, } v = 10 \text{ m/s}$$

$$\text{Induced emf, } e = Blv = 0.4 \times 0.3 \times 10 = 1.2 \text{ V } \textbf{Ans.}$$

$$\text{Power developed, } P = F \times v = 12 \times 10 = 120 \text{ watts } \textbf{Ans.}$$

Example 1.22. A square coil of 10 cm side and with 100 turns is rotated at a uniform speed of 500 rpm about an axis at right angle to a uniform field of 0.5 T. Calculate the instantaneous value of induced emf when the plane of coil is (*i*) at right angle to the plane of the field (*ii*) at 30° to the plane of the field and (*iii*) in the plane of the field.

Solution: As there are two active sides in the coil, emf induced in the coil is given by $e = 2Blv \sin\theta$ volts

where θ is the angle of the coil movement with the direction of the field.

(*a*) *When plane of the coil is at right angle to the plane of the field.*

The coil moves at an angle of 0° with the direction of the field *i.e.* moves just parallel to the lines of force or $\theta = 0$ or $\sin\theta = 0°$

$\therefore$ emf induced, $e = 0$ **Ans.**

(*b*) *When the plane of the coil makes an angle of 30° with the plane of the field*

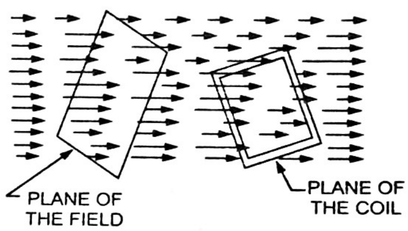


Fig. 1.48

The coil moves at an angle of (90° – 30°) with the direction of the field *i.e.* $\theta = 60°$ and $\sin 60° = \frac{\sqrt{3}}{2}$

$$\text{Flux density, } B = 0.5 \text{ T}$$

$$\text{Length of coil, } l = 10 \text{ cm} = 0.1 \text{ m}$$

$$\text{Number of turns } N = 100$$

$$\text{Velocity, } v = \omega r \text{ m/s} = \frac{2\pi rpm}{60} r \text{ m/s}$$

$$= \frac{2\pi \times 500}{60} \times 0.05 = 2.62 \text{ m/s}$$

$$\text{EMF induced, } e = 2BlvN\sin\theta$$

$$= 2 \times 0.5 \times 0.1 \times 2.62 \times 100 \times \frac{\sqrt{3}}{2}$$

$$= 22.65 \text{ volts } \textbf{Ans.}$$

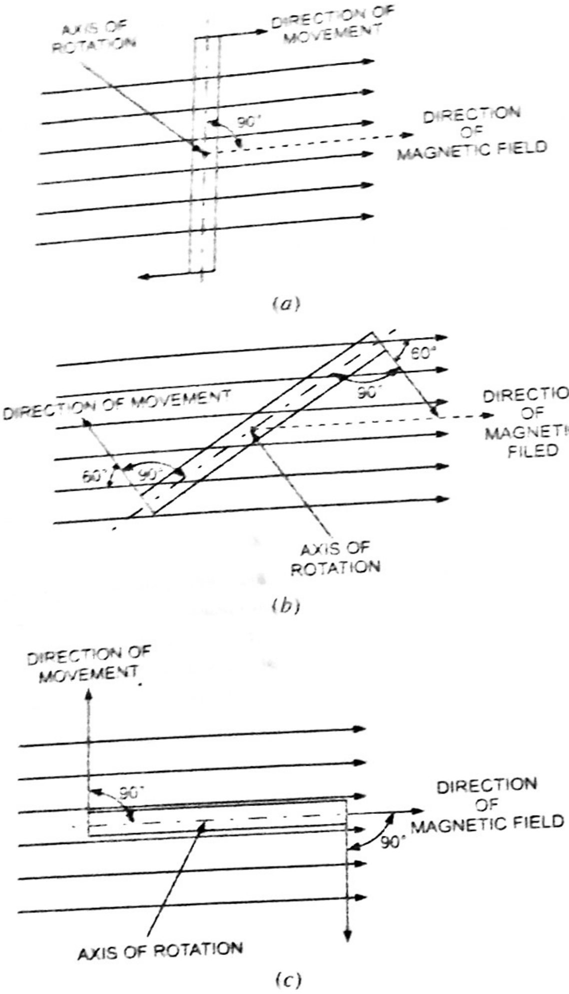


Fig. 1.49

(c) When the plane of the coil is in the plane of the field

In this position, the coil moves at right angle to the field direction

$$i.e.\ \theta = 90° \text{ and } \sin 90° = 1$$

$$\therefore \text{ EMF induced, } e = 2\,B\,l\,v\,N\sin\theta$$

$$= 2 \times 0.5 \times 0.1 \times 2.62 \times 100 \times 1 = 26.2 \text{ V} \textbf{ Ans.}$$

Example 1.23. A square coil of 10 turns and 10 cm side is moved through a steady magnetic field of 1 Wb/m^2 at a constant velocity of 2 m/s with its plane perpendicular to the field as shown in Fig. 1.50. Plot the variation of induced emf as the coil moves along the field. *[GATE 1997]*

Solution: Number of turns of coil, N = 10

Magnetic field strength B = 1 Wb/m^2

Movement velocity, v = 2 m/s

Coil side, a = 10 cm = 0.1 m

When the coil enters, let x length of the loop lie inside the field, therefore,

$$\text{Flux } \Phi = B \times A = B \times (x \times a) = B\,x\,a$$

$$\text{Rate of change of flux, } \frac{d\Phi}{dt} = Ba\frac{dx}{dt} = Bav$$

$$\text{Induced emf, } e = N\frac{d\Phi}{dt} = N\,Bav$$

$$= 10 \times 1 \times 0.1 \times 2 = 2 \text{ V}$$

Plot of variation of induced emf is given in Fig. 1.51.

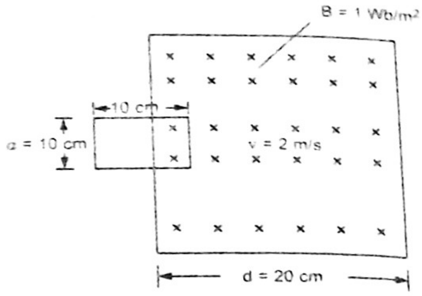


Fig. 1.50

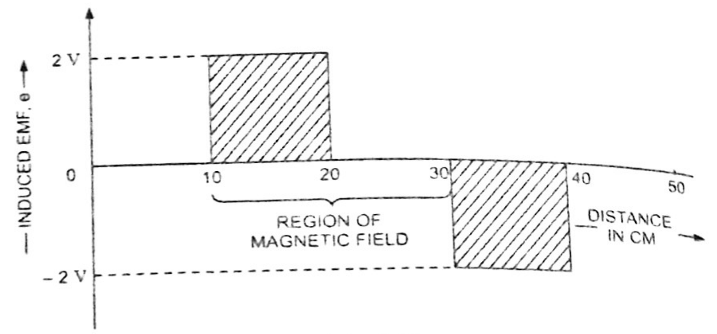


Fig. 1.51

Example 1.24. A rectangular loop of sides a and b has its plane normal to a magnetic flux density of strength $B_0 \sin \omega t$. What is the voltage induced in the above loop ? *[GATE 1997]*

Solution: Area of rectangular loop,

$$A = a \times b$$

$$\text{Magnetic flux density, } B = B_0 \sin \omega t$$

$$\text{Voltage induced, } v = -\int \frac{\delta B}{\delta t}\, dS$$

$$= -\int_0^a\int_0^b B_0\,\omega \cos \omega t\, dx\, dy$$

$$= B_0\,\omega \cos \omega t \times a \times b = B_0\,\omega\,A \cos \omega t \quad \textbf{Ans.}$$

Example 1.25. A solenoid has 1,200 turns and carries a current of 2 A. The iron core has a length of 0.4 m and cross section of 80 cm^2, the relative permeability is 1,000. Calculate the self-induced emf in the solenoid, if the current is switched off in 0.01 second.

Solution: Area of x-section of iron core,

$$a = 80 \text{ cm}^2 = 0.008 \text{ m}^2$$

Self-inductance of coil,

$$L = \frac{N^2 a \mu_r \mu_0}{l} = \frac{(1{,}200)^2 \times 0.008 \times 1{,}000 \times 4\pi \times 10^{-7}}{0.4}$$

$$= 36.191 \text{ H}$$

Rate of change of current,

$$\frac{di}{dt} = \frac{2-0}{0.01} = 200 \text{ A/s}$$

Self-induced emf,

$$e = L\frac{di}{dt} = 36.191 \times 200 = 7{,}238 \text{ V} \textbf{ Ans.}$$

Example 1.26. A coil of 300 turns, wound on a core of nonmagne[tic] material, has an inductance of 10 mH. Calculate (*i*) the flux produ[ced] by a current of 5 A and (*ii*) the average value of emf induced w[hen] a current of 5 A is reversed in 8 milliseconds.

Solution: (*i*) Flux produced, $\Phi = \frac{iL}{N} = \frac{5 \times 0.01}{300} = 0.1667$ mWb **Ans.**

$$\because L = \frac{N\Phi}{i}$$

Rate of change of current, $\frac{di}{dt} = \frac{5-(-5)}{0.008} = 1{,}250$ A/s

Induced emf, $e = L\frac{di}{dt} = 0.01 \times 1{,}250 = 12.5$ V **Ans.**

Example 1.27. A coil of resistance 90 Ω is placed in a magnetic field of 1 mWb. The coil has 100 turns and a galvanometer of 405 Ω resistance is connected in series with it. Find the average emf and current if the coil is moved in 1/10th second from the given field to a field of 0.2 mWb.

[M.D. Univ. Electromechanical Energy Conversion, December-2010]

Solution: Number of turns on the coil,

$$N = 100$$

Change of flux in 1/10 second,

$$d\Phi = 0.001 - 0.0002 = 0.0008 \text{ Wb}$$

Rate of change of flux,

$$\frac{d\Phi}{dt} = \frac{0.0008}{0.1} = 0.008 \text{ Wb/second}$$

Induced emf,

$$e = N\frac{d\phi}{dt} = 100 \times 0.008 = 0.8 \text{ V} \quad \textbf{Ans.}$$

Current through coil,

$$I = \frac{e}{\text{Coil resistance + galvanometer resistance}}$$

$$= \frac{0.8}{90+405} = \frac{0.8}{495} = 0.00162 \text{ A or } 1.62 \text{ mA} \quad \textbf{Ans.}$$

Example 1.28. A coil of 1,000 turns is wound on a laminated core of steel having a cross section of 5 cm². The core has an air gap of 2 mm cut at right angle. What value of current is required to have an air gap flux density of 0.5 T ? Permeability of steel may be taken as infinity. Determine the coil inductance.

Solution: Total ampere-turns required,

$$AT = \frac{B}{\mu_0\mu_r}l_i + \frac{B}{\mu_0}l_g = 0 + \frac{0.5}{4\pi\times10^{-7}}\times2\times10^{-3} = 796 \quad \because \mu_r = \infty$$

Current required,

$$i = \frac{AT}{N} = \frac{796}{1{,}000} = 0.796 \text{ A } \textbf{Ans.}$$

Inductance of coil,

$$L = \frac{N\Phi}{i} = \frac{1{,}000\times0.5\times5\times10^{-4}}{0.796} = 0.314 \text{ H } \textbf{Ans.}$$

Example 1.29. The resistance and inductance of a coil are 3 Ω and 0.1 m H respectively. What potential difference exists at the terminals of the solenoid at the instant when the current is 1 A, but increasing at the rate of 10,000 A per second?

Solution: Resistance of the coil, R = 3 Ω

Inductance of the coil, $L = 0.1$ m H $= 1 \times 10^{-4}$ H

Current flowing through the coil, I = 1.0 A

Rate of increase of current, $\frac{di}{dt} = 10{,}000$ A/s

Potential difference across the solenoid,

V = Voltage drop in resistance + induced voltage in the coil

$$= iR + L\frac{di}{dt} = 1 \times 3 + 1 \times 10^{-4} \times 10{,}000 = 4 \text{ V } \textbf{Ans.}$$

Example 1.30. Find the inductance of a coil in which a current of 0.2 A increasing at the rate of 0.4 A per second represents a power flow of 0.4 watt.

Solution: Power flow, $p = 0.4$ watt

Current, $i = 0.2$ A

Induced emf, $e = \frac{p}{i} = \frac{0.4}{0.2} = 2$ V $\quad \because p = ei$

Rate of increase of current, $\frac{di}{dt} = 0.4$ A/s

Inductance of coil, $L = \frac{e}{di/dt} = \frac{2}{0.4} = 5$ H **Ans.**

Example 1.31. Two coils having 100 and 150 turns respectively are wound side by side on a closed iron circuit of section 125 cm² and mean length 200 cm. If the permeability of iron is 2,000, calculate (*a*) self-inductance of each coil (*b*) mutual inductance between them (*c*) the emf induced in the second coil if current in first coil changes from 0 to 5 A in 0.02 *s*.

Solution: Area of x-section of iron circuit, $a = 125 \text{ cm}^2 = 0.0125 \text{ m}^2$

Length of iron circuit, $l = 200$ cm $= 2.0$ m

(*a*) Self-inductance of first coil,

$$L_1 = \frac{N_1^2\mu_0\mu_r a}{l} = \frac{(100)^2\times4\pi\times10^{-7}\times2{,}000\times0.0125}{2.0}$$

$= 157.1$ m H **Ans.**

Self-inductance of second coil,

$$L_2 = \frac{N_2^2\mu_0\mu_r a}{l} = \frac{(150)^2\times4\pi\times10^{-7}\times2{,}000\times0.0125}{2.0}$$

$= 353.4$ mH **Ans.**

(*b*) Mutual inductance between coil,

$$M = \frac{N_1N_2\mu_0\mu_r a}{l}$$

$$= \frac{100\times150\times4\pi\times10^{-7}\times2{,}000\times0.0125}{2.0} = 235.6 \text{ mH } \textbf{Ans.}$$

Rate of change of current in first coil,

$$\frac{di_1}{dt} = \frac{5-0}{0.02} = 250 \text{ A/s}$$

(*c*) EMF induced in second coil,

$$e_2 = M\frac{di_1}{dt} = 235.6 \times 10^{-3} \times 250 = 58.9 \text{ V } \textbf{Ans.}$$

Example 1.32. Two coils have a mutual inductance of 0.3 H. If the current in one coil is varied from 5 A to 2 A in 0.4 s, calculate (*i*) the average emf induced in the second coil (*ii*) the change of flux linked with the second coil assuming that it is wound with 200 turns.

Solution: Mutual inductance,

$$M = 0.3 \text{ H}$$

Rate of change of current in one coil,

$$\frac{di_1}{dt} = \frac{5-2}{0.4} = 7.5 \text{ A/s}$$

(*i*) EMF induced in second coil,

$$e_2 = M\frac{di_1}{dt} = 0.3 \times 7.5 = 2.25 \text{ V } \textbf{Ans.}$$

(*ii*) Since mutually induced emf,

$$e_2 = N_2 \frac{d\Phi_2}{dt}$$

∴ Change of flux linked with the second coil,

$$\frac{d\Phi_2}{dt} = \frac{e_2}{N_2} = \frac{2.25}{200} = 0.01125 \text{ Wb/s } \textbf{Ans.}$$

Example 1.33. A long single layer solenoid has an effective diameter of 10 cm and is wound with 2,500 T/m. There is a small concentrated coil having its plane lying in the centre cross-sectional plane of the solenoid. Calculate the mutual inductance between the two coils if the concentrated coil has 120 turns on an effective diameter of (*a*) 8 cm and (*b*) 12 cm.

Solution: (*a*) When the effective diameter of concentrated coil is 8 cm (less than that of solenoid)

Flux density at the centre of the solenoid,

$$B = \mu_0 H = \frac{\mu_0 N I_1}{l} = 2{,}500 \times 4\pi \times 10^{-7} \times I_1$$

$$= 0.00314\, I_1 \text{ Wb/m}^2$$

where I_1 is the current flowing through the solenoid and N/l is equal to number of turns per metre length of solenoid *i.e.* 2,500

Area of search coil,

$$a_S = \frac{\pi}{4}(0.08)^2 = 0.005 \text{ m}^2$$

Flux linking with search coil,

$$\Phi_2 = B\, a_S = 0.00314\, I_1 \times 0.005 = 15.79 \times 10^{-6}\, I_1 \text{ Wb}$$

Mutual inductance between the coil,

$$M = \frac{N_2 \Phi_2}{I_1} = \frac{120 \times 15.79 \times 10^{-6}\, I_1}{I_1} = 1.895 \text{ mH } \textbf{Ans.}$$

(*b*) When the effective diameter of the concentrated coil is 12 cm (more than that of the solenoid)

In this case the effective area of the search coil will be equal to that of long solenoid as the field strength outside the solenoid is negligible.

$$\text{So, } \Phi_2 = 0.00314\, I_1 \times \frac{\pi}{4}(0.10)^2 = 24.67\, I_1 \times 10^{-6} \text{ Wb}$$

$$\text{and } M = \frac{N_2 \Phi_2}{I_1} = \frac{120 \times 24.67 \times I_1 \times 10^{-6}}{I_1} = 2.96 \text{ mH } \textbf{Ans.}$$

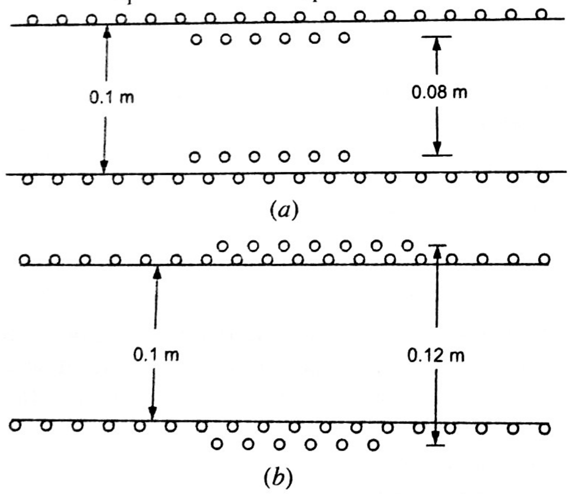


Fig. 1.52

Example 1.34. The self-inductance of a coil of 500 turns is 0.25 H. If 60% of the flux of this coil is linked with a second coil of 10,000 turns, calculate (*i*) The mutual inductance of the two coils, (*ii*) EMF induced in the second coil, when current in the first coil changes at the rate of 100 A/second. [*Rajasthan Technical Univ. 2007*]

Solution: Self-inductance of the first coil,

$$L_1 = 0.25 \text{ H}$$

Number of turns on first coil,

$$N_1 = 500$$

Flux per ampere of current in first coil,

$$\frac{\Phi_1}{i_1} = \frac{L_1}{N_1} = \frac{0.25}{500} = 0.5 \text{ mWb} \qquad \because L_1 = N_1 \frac{\Phi_1}{i_1}$$

Flux linked with second coil per ampere of current in first coil

$$\frac{\Phi_2}{i_1} = \frac{0.6\,\Phi_1}{i_1} = 0.6 \times 0.5 = 0.3 \text{ mWb}$$

(*i*) The mutual inductance of the two coils,

$$M = \frac{N_2 \Phi_2}{i_1} = 10{,}000 \times 0.3 \times 10^{-3} = 3 \text{ H } \textbf{Ans.}$$

(*ii*) EMF induced in the second coil,

$$e_2 = M \frac{di_1}{dt} = 3 \times 100 = 300 \text{ V } \textbf{Ans.}$$

Example 1.35. A coil with axial length 25 cm and diameter of 20 cm has 200 turns. It is placed in a uniform flux of 0.002 Wb/m². (*a*) If the coil is rotated at 25 revolutions per second, find the voltage induced in the coil. (*b*) What will be the force on each conductor and torque acting on the coil if it carries a current of 10 A.

[*Rajasthan Technical Univ. Electrical Machines-I, 2008*]

Solution: Length of coil, l = 25 cm = 0.25 m

Number of turns, N = 200

$$\text{Velocity, } v = \omega r \text{ m/s} = 2\pi \times rps \times r \text{ m/s} = 2\pi \times 25 \times \frac{10}{100}$$

$$= 5\pi \text{ m}$$

$$\because r = \frac{20}{2} = 10 \text{ cm} = \frac{10}{100} \text{ m}$$

Flux density, $B = 0.002 \text{ Wb/m}^2$

(*a*) Induced e.m.f., $e = NBlv = 200 \times 0.002 \times 0.25 \times 5\pi = 1.57$ V **Ans.**

(*b*) Force on each conductor,

$$F = BIl = 0.002 \times 10 \times 0.25 = 0.005 \text{ N} \quad \textbf{Ans.}$$

Torque acting on the coil,

$$T = 2 \times F \times r \times N = 2 \times 0.005 \times 0.1 \times 200$$

$$= 0.2 \text{ N-m} \quad \textbf{Ans.}$$

1.22 MUTUAL COUPLING IN A SIMPLE MAGNETIC CIRCUIT

The self-inductance of a circuit is intimately associated with the magnetic field linking the circuit. The self-inductance emf may be thought of as the emf induced in the circuit by a magnetic field produced by the circuit current.

Since a magnetic field exists in the region around the current that develops it, there is also a possibility that an emf be induced in the other circuits linked by the field. Two circuits linked by the same magnetic field are said to be *coupled* to each other. The circuit element used to represent *magnetic coupling* is shown in Fig. 1.53 (*a*) and is called *mutual inductance.* It is represented by symbol M and is measured in henrys, already stated in Art. 1.21.2 (*b*). The volt-ampere relationship is one which gives the induced emf in one circuit by a current in another and is given as

$$e_2 = M \frac{di_1}{dt}$$

With ac excitation, however, inductance enters into the steady-state performance as well; the result for most magnetic circuits, although not for all, is that, to a close approximation, the flux is determined by the impressed voltage and frequency, and the magnetizing current must adjust itself in accordance with this flux so that the relationship imposed by the magnetisation curve is satisfied.

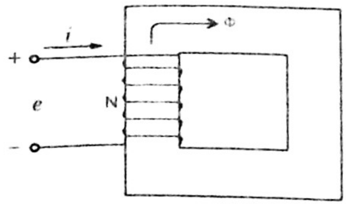


Fig. 1.59 *Magnetic Circuit With AC Excitation*

Except where preservation of linear relationship is of great importance, the normal working flux density in a magnetic circuit is kept beyond the linear portion of the magnetisation curve for the overall circuit (*i.e.* partially saturating the circuit). This is done so as to affect the economic utilisation of magnetic material. Thus accurate analysis cannot be predicted on constant self-inductance. Equivalent circuits containing parameters that do remain substantially constant are used instead. The reactive effect of the time-varying flux on the exciting circuit can readily be shown from Faraday's law,

$$e = N \frac{d\Phi}{dt}$$

Consider an iron core excited by a winding having N turns and carrying a current of i amperes (Fig. 1.59). A magnetic flux Φ is produced by the exciting current i. Let the magnetic flux Φ vary sinusoidally with time t as in

$$\Phi = \Phi_{max} \sin 2\pi f t \qquad ...(1.52)$$

Φ_{max} being the maximum value of flux in the cycle and f is the supply frequency. The induced emf in accordance with Faraday's law is

$$e = N \frac{d\Phi}{dt} = 2\pi f N \Phi_{max} \cos 2\pi f t \qquad ...(1.53)$$

and its effective or rms value is

$$E_{rms} = \frac{2\pi}{\sqrt{2}} f N \Phi_{max} = 4.44 f N \Phi_{max} \text{ volts} \qquad ...(1.54)$$

The polarity of the emf must, in accordance with Lenz's law, oppose the change in flux and, therefore, is as shown in Fig. 1.59 when the flux is increasing. Since the current produces the flux, the two may be considered in phase. From Eq. (1.53), the induced voltage leads the flux, and hence the exciting current by π/2 radians or 90°. The induced emf and the coil resistance drop oppose the applied voltage. The resistance drop does not exceed a few per cent of the applied voltage in ac machines, most transformers, and many other electromagnetic devices. To a close approximation, resistance drop may be neglected and the induced emf E and applied voltage V may be considered equal in magnitude. The flux Φ_{max} is then determined by the applied voltage V in accordance with Eq. (1.54), even if maintenance of this flux requires a magnetising current far in excess of rated current for the device.

Example 1.48. For the ac excited magnetic circuit of Fig. 1.60, calculate the excitation current and induced emf of the coil to produce a core flux of 0.6 sin 314 mWb. [*U.P. Technical Univ. September-2001*]

Solution: Maximum value of flux to be created,

$$\Phi_{max} = 0.6 \text{ mWb} = 6 \times 10^{-4} \text{ Wb}$$

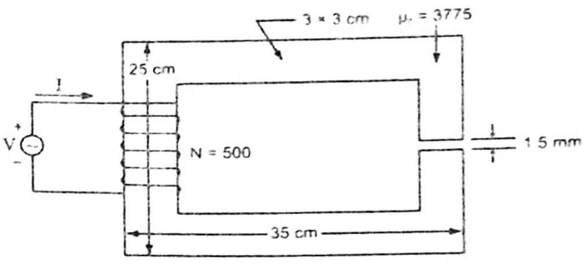


Fig. 1.60

Area of x-section, $a = 3 \times 3 = 9 \text{ cm}^2 = 9 \times 10^{-4} \text{ m}^2$

Flux density required,

$$B_{max} = \frac{\Phi_{max}}{a} = \frac{6 \times 10^{-4}}{9 \times 10^{-4}} = 0.667 \text{ T}$$

Length of air gap,

$$l_g = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

Length of iron path,

$$l_i = \left(25 - 2 \times \frac{3}{2} + 35 - 2 \times \frac{3}{2}\right) \times 2 - 0.15 = 107.85 \text{ cm}$$

Total ampere-turns required,

$$AT_{max} = \frac{B_{max}}{\mu_0} l_g + \frac{B_{max}}{\mu_0 \mu_r} l_i$$

$$= \frac{0.667 \times 1.5 \times 10^{-3}}{4\pi \times 10^{-7}} + \frac{0.667 \times 1.0785}{3{,}775 \times 4\pi \times 10^{-7}}$$

$$= 796 + 152 = 948$$

Number of turns on the coil, N = 500

Maximum value of excitation current required,

$$I_{max} = \frac{AT_{max}}{N} = \frac{948}{500} = 1.896 \text{ A}$$

RMS value of excitation current,

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1.896}{1.414} = 1.34 \text{ A} \quad \textbf{Ans.}$$

RMS value of induced emf in the coil,

$$E_{rms} = 4.44 f N \Phi_{max} = 4.44 \times \frac{314}{2\pi} \times 500 \times 0.6 \times 10^{-3}$$

$$= 66.6 \text{ V} \quad \textbf{Ans.}$$

1.29 HYSTERESIS AND EDDY CURRENT LOSSES

When magnetic circuits are subjected to time-varying flux densities, there are two causes of power loss in the form of heat in the iron core. These losses are significant in determining the heating, rating, and efficiency of rotating electrical machines, transformers, and ac operated devices.

The first loss is associated with the phenomenon of hysteresis, discussed in Art. 1.18, and is an expression of the fact that when ferromagnetic material is involved, not all the energy of the magnetic field is returned to the circuit when the mmf is removed. It is known as *hysteresis loss*. When the flux varies from + B, to − B, [Fig. 1.23] at the frequency f, the hysteresis loss per unit volume of material may be shown to be proportional to the area of the hysteresis loop and to the number of loops traversed per second.