TRANSMISSION LINE

11.1 INTRODUCTION

Transmission lines are the inter connections that convey electromagnetic energy from one point to another. The energy may be for light, heat, mechanical work, or information - speech, music, pictures, data.

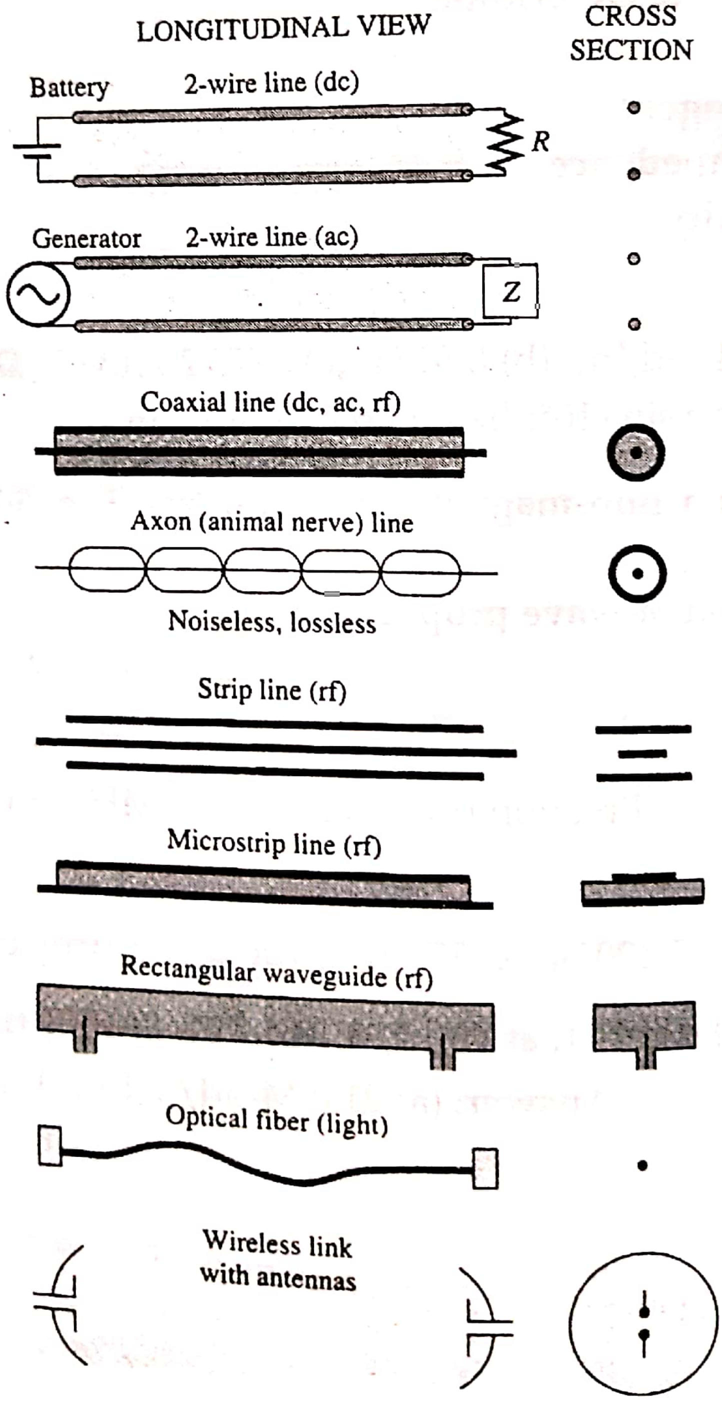

Figure 11.1 A few examples of transmission lines shown in longitudinal and cross-section views.

Examples of transmission lines include the connection between a transmitter and an antenna, connections between computers in a network, or connections between a hydroelectric generating plant and a substation several hundred miles away. Other familiar examples include the interconnects between components of a stereo system and the connection between a cable service provider and our television set. Examples that are less familiar include the connections between devices on a circuit board that are designed to operate at high frequencies.

We shall note here that transmission lines are operated at low frequencies in power distribution unlike communications where transmission lines are operated at high frequencies.

11.2 TRANSMISSION LINE PARAMETERS

The key difference between circuit theory and transmission line theory is electrical size. Circuit analysis assumes that the physical dimensions of the network are much smaller than the electrical wavelength, while transmission lines may be a considerable fraction of a wavelength, or many wavelengths, in size. Thus, a transmission line is a distributed-parameter network, where voltages and currents can vary in magnitude and phase over its length, while ordinary circuit analysis deals with lumped elements, where voltage and current do not vary appreciably over the physical dimension of the elements. It is customary and convenient to describe a transmission line in terms of its line parameters, which are its resistance per unit length R, inductance per unit length L, conductance per unit length G, and capacitance per unit length C.

- Resistance represents the imperfection of the conductor.
- Inductance is due to the magnetic field surrounding the conductors of a transmission line when a current flows through them.
- Capacitance exists as a result of the electric field between conductors of a transmission line.
- Conductance is due to the insulation of the line allowing some current to leak from one conductor to the other and hence, represents the imperfection of the insulator.

It should be noted that

1. The line parameters R, L, G, and C are not discrete or lumped but distributed as shown in Figure 11.2. By this we mean that the parameters are uniformly distributed along entire length of the line.

   We again note here that the basic elements in circuits, such as resistors, capacitors, inductors, and the connections between them, are considered "lumped" elements if the time delay in traversing the elements is negligible. On the other hand, if the elements or interconnections are large enough, it may be necessary to consider them as "distributed" elements.

2. For each line, the conductors are characterized by σ_c, μ_c, $\varepsilon_c = \varepsilon_0$, and the homogeneous dielectric separating the conductors is characterized by σ, μ, ε.

3. $G \neq \frac{1}{R}$; R is the ac resistance per unit length of the conductors comprising the line and G is the conductance per unit length due to dielectric medium separating the conductors.

4. Internal inductance $L_{in} \left(= \frac{R}{\omega}\right)$ is considered negligible owing to high frequencies at which most communication systems operate. External inductance per unit length; that is $L = L_{ext}$ is what that is accountable.

5. For each line,

   $$LC = \mu\varepsilon \text{ and } \frac{G}{C} = \frac{\sigma}{\varepsilon}$$

A transmission line is often schematically represented as a two-wire line since transmission lines (for transverse electromagnetic [TEM] wave propagation) always have at least two conductors.

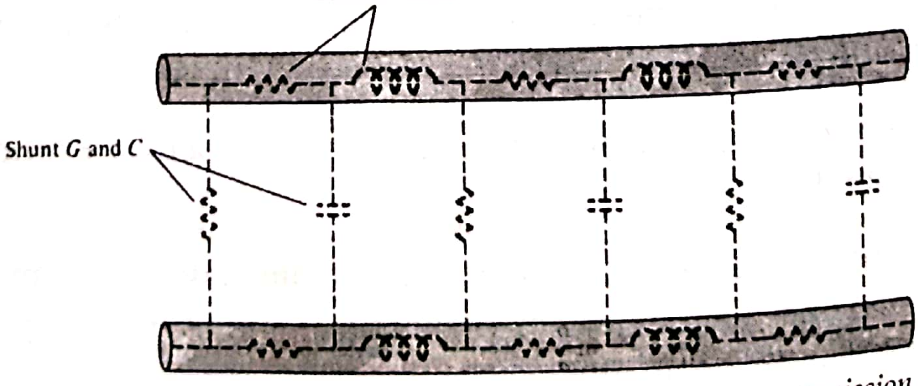

Figure 11.2 Distributed parameters of a two conductor transmission line.

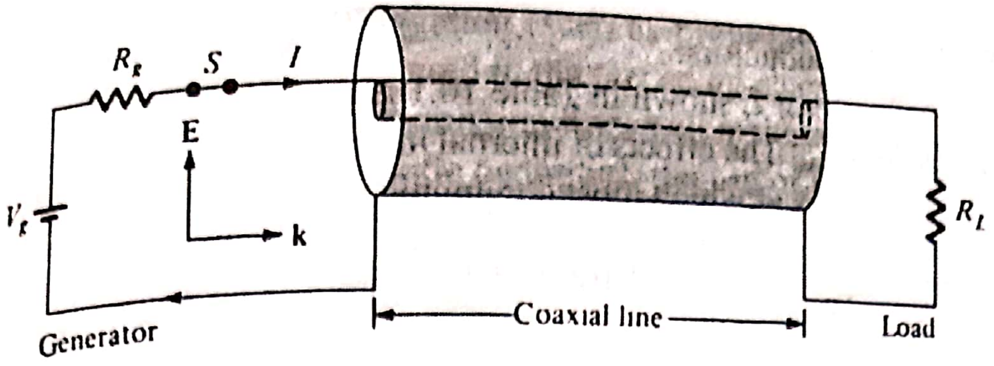

Figure 11.3 Coaxial line connecting the generator to the load.

11.3 TRANSMISSION LINE EQUATIONS

A two conductor transmission line supports a TEM wave; that is, the electric and magnetic fields on the line are transverse to the direction of wave propagation. In TEM waves, $\vec{E}$ and $\vec{H}$ are related to voltage V and current I, respectively as,

$$V = -\int \vec{E} \cdot d\vec{l}\ , \quad I = \oint \vec{H} \cdot d\vec{l}$$

So, instead of solving $\vec{E}$ and $\vec{H}$ (i.e. solving Maxwell's equation and boundary conditions), we will use circuit quantities V and I in solving the transmission line problem; this way proves simpler and more convenient.

Consider an incremental portion of length Δz of a two-conductor transmission line as shown in Figure 11.4.

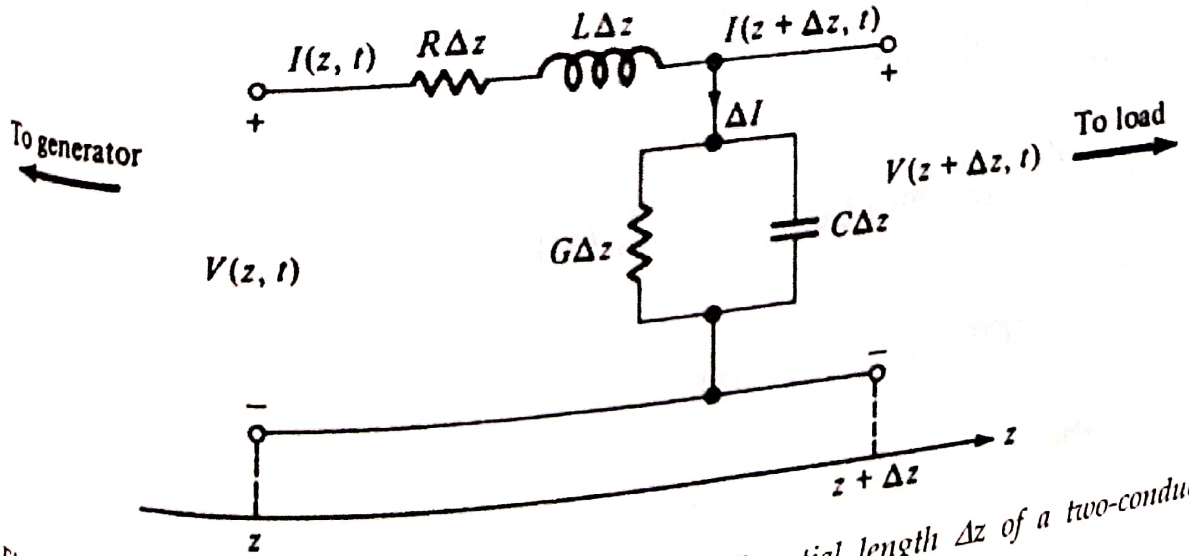

Figure 11.4 *L-type equivalent circuit model of a differential length Δz of a two-conductor transmission line.*

Applying KVL to the outer of the circuit, we have

$$V(z,t) = R\,\Delta z\, I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z, t)$$

$$\text{or,}\quad -\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

Taking limit as $\Delta z \rightarrow 0$ will result

$$-\frac{\partial V(z,t)}{\partial z} = RI\,(z,t) + L\frac{\partial I(z,t)}{\partial t} \quad \text{......................(i)}$$

Applying KCL to the main node of the circuit, we get

$$I(z,t) = I(z+\Delta z, t) + \Delta I$$

$$\text{or,}\quad I(z,t) = I(z+\Delta z,t) + G\Delta z\, V(z+\Delta z,t) + C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$\text{or,}\quad -\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = GV(z+\Delta z,t) + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

Taking limit as $\Delta z \rightarrow 0$ results

$$-\frac{\partial I(z,t)}{\partial z} = GV(z,t) + C\frac{\partial V(z,t)}{\partial t} \quad \text{......................(ii)}$$

Equations (i) & (ii) are now written in phasor notation as

$$\frac{-\partial V_S(z)}{\partial z} = RI_S(z) + j\omega LI_S(z)$$

$$\text{or,}\quad \frac{-dV_S}{dz} = (R+j\omega L)\, I_S \quad \text{.................... (iii)}$$

$$\frac{-\partial I_S(z)}{\partial z} = GV_S(z) + j\omega CV_S(z)$$

$$\text{or,}\quad \frac{-dI_S}{dz} = (G+j\omega C)V_S \quad \text{..........(iv)}$$

Taking second derivative of V_S in equation (iii) and then substituting equation (iv), we get

$$\frac{d^2V_S}{dz^2} = (R + j\omega L)(G + j\omega C) V_S$$

$$\text{or,} \quad \frac{d^2V_S}{dz^2} - \gamma^2 V_S = 0 \text{ ....(v)}$$

where $\boxed{\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}}$

Similarly, taking second derivative of I_s in equation (iv) and then substituting equation (iii), we get

$$\frac{d^2I_S}{dz^2} - \gamma^2 I_S = 0 \text{ ....(vi)}$$

The solutions of the linear homogeneous differential equations (v) and (vi) are

$$V_S(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \text{ and } I_S(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

where V_0^+, V_0^-, I_0^+, and I_0^- are wave amplitudes ; the "+" and "–" signs, respectively, denote wave travelling along +z and –z directions.

The instantaneous expression for voltage is

$$V(z,t) = \text{Re}\,[V_S(z)e^{j\omega t}] = \text{Re}\,[(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z})\, e^{j\omega t}]$$

$$= \text{Re}\,[V_0^+ e^{-\gamma z} e^{j\omega t}] + \text{Re}\,[V_0^- e^{\gamma z})\, e^{j\omega t}]$$

$$= \text{Re}\,[V_0^+ e^{-\alpha z} e^{-j\beta z} e^{j\omega t}] + \text{Re}\,[V_0^- e^{\alpha z} e^{j\beta z} e^{j\omega t}]$$

$$= e^{-\alpha z} V_0^+ \text{Re}\,[e^{j(\omega t-\beta z)}] + V_0^- e^{\alpha z} \text{Re}\,[e^{j(\omega t+\beta z)}]$$

$$\therefore V(z,t) = V_0^+ e^{-\alpha z} \cos(\omega t-\beta z) + V_0^- e^{\alpha z} \cos(\omega t+\beta z)$$

11.4 CHARACTERISTIC IMPEDANCE

The characteristic impedance Z_o of the line is the ratio of positively traveling voltage wave to current wave at any point on the line. The input impedance of a transmission line becomes constant when the length of the transmission line approaches infinity. The characteristic impedance Z_0 may therefore be defined as the impedance measured at the input of line when its length is infinite.

Substituting the value of V_s and I_s in (iii), we have

$$\frac{-d}{dz}(V_0^+ e^{-\gamma z} + V_0^- e^{-\gamma z}) = (R + j\omega L)(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z})$$

or, $\gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{\gamma z} = (R + j\omega L) I_0^+ e^{-\gamma z} + (R + j\omega L) I_0^- e^{\gamma z}$

Equating coefficient of terms $e^{\gamma z}$ and $e^{-\gamma z}$, we have

$$\frac{V_0^+}{I_0^+} = \frac{R + j\omega L}{\gamma}\ ; -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma}$$

$$\therefore Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R + j\omega L}{\gamma}$$

Similarly, if we substitute the value of V_s and I_s into (iv), the result is

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{\gamma}{G + j\omega C}$$

But, $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$\boxed{\therefore Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0}$$

So far we have considered the lossy types of the transmission line. This is the general case in which the conductors comprising the line are imperfect ($\sigma_c \neq \infty$) and the dielectric in which the conductors are embedded is lossy ($\sigma \neq 0$). Now we deal with two special cases.

i. **Lossless Line (R = 0 = G)**

A lossless line is one in which the conductors of the line are perfect ($\sigma_c \approx \infty$) and the dielectric medium separating them is lossless ($\sigma \approx 0$)

For such a line,

$$\boxed{R = 0 = G} \quad \text{.................... (i)}$$

We deduced that

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \quad \text{............ (ii)}$$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad \text{............ (iii)}$$

Putting equation (i) in (ii), we get

$$\gamma = \alpha + j\beta = \sqrt{(0 + j\omega L)(0 + j\omega C)} = j\omega\sqrt{LC} = 0 + j\omega\sqrt{LC}$$

$$\boxed{\begin{aligned} \therefore \alpha &= 0 \\ \beta &= \omega\sqrt{LC} \end{aligned}}$$

Similarly, substituting equation (i) in (iii), we get

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{0 + j\omega L}{0 + j\omega C}} = \sqrt{\frac{L}{C}} = \sqrt{\frac{L}{C}} + j0$$

$$\boxed{\begin{aligned} \therefore R_0 &= \sqrt{\frac{L}{C}} \\ X_0 &= 0 \end{aligned}}$$

ii. **Distortionless Line $\left(\frac{R}{L} = \frac{G}{C}\right)$**

A signal normally consists of a band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line as α is frequency dependent. This results in distortion.

A distortionless line is one in which the attenuation constant α is frequency independent while the phase constant β in linearly dependent on frequency.

For such a line,

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

The expression for γ and Z_0 will therefore be

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + \frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG\left(1 + \frac{j\omega C}{G}\right)\left(1 + \frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG}\left(1 + \frac{j\omega C}{G}\right)$$

$$= \sqrt{RG} + j\sqrt{RG}\,\frac{\omega C}{G}$$

$$= \sqrt{RG} + j\sqrt{\left(\frac{LG}{C}\right) G \left(\frac{\omega C}{G}\right)}$$

$$= \sqrt{RG} + j\omega\sqrt{LC}$$

$$\boxed{\begin{aligned} \therefore \alpha &= \sqrt{RG} \\ \beta &= \omega\sqrt{LC} \end{aligned}}$$

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{R\left(1 + \frac{j\omega L}{R}\right)}{G\left(1 + \frac{j\omega C}{G}\right)}}$$

$$= \sqrt{\frac{R\left(1 + \frac{j\omega C}{G}\right)}{G\left(1 + \frac{j\omega C}{G}\right)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$$\boxed{\begin{aligned} \therefore R_0 &= \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \\ X_0 &= 0 \end{aligned}}$$

A lossless line is also a distortionless line, but a distortionless line is not necessarily lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortionless.

Understanding more:

Why is the condition $\frac{R}{L} = \frac{G}{C}$ required to be satisfied for a distortionless transmission line?

The distortion happens in two ways. Firstly, the attenuation of the line can vary with frequency which results in a change to the shape of a pulse transmitted down the line. Secondly, and usually more problematically, distortion is caused by a frequency dependance on phase velocity of the transmitted signal frequency components. If different frequency components of the signal are transmitted at different velocities, the signal becomes "smeared out" in space and time, a form of distortion called dispersion.

The transmission function of a transmission line is defined in terms of its input and output voltages when correctly terminated (that is, with no reflections) as

$$\frac{V_{in}}{V_{out}} = e^{\gamma x}$$

where x represent distance from the transmitter in meters, $\gamma = \alpha + j\beta$ are the secondary line constants, α being the attenuation in nepers per metre and β being the phase change constant in radians per metre.

Note: The primary constants are L, C, R, G (all expressed in per m).

For no distortion, α is required to be constant with angular frequency ω, while β must be proportional to ω. This requirement for proportionality to frequency is due to the relationship between the velocity v and phase constant β being given by $v = \frac{\omega}{\beta}$ and the requirement that the phase velocity v be constant at all frequencies.

The relationship between the primary and secondary line constants is given by

$$\gamma^2 = (\alpha + j\beta)^2 = (R + j\omega L)(G + j\omega C)$$

which has to be of the form $(A + j\omega B)^2$ in order to meet the distortionless condition. The only way this can be so is if $(R + j\omega L)$ and $(G + j\omega C)$ differ by no more than a constant factor. Since both have a real and imaginary part, the real and imaginary parts must independently be related by the same factor, so that

$$\frac{R}{G} = \frac{j\omega L}{j\omega C}$$

Hence, we get $\frac{R}{L} = \frac{G}{C}$ and is the required condition for the distortionless transmission line also known as Heaviside condition.

Explain and prove that every lossless line is also distortionless but every distortionless lines may not be lossless.

For a transmission line to be distortionless, it must satisfy the following condition

$$\frac{R}{L} = \frac{G}{C} \quad \text{........................} \quad (1)$$

The losses in the transmission line are due to the presence of series resistance and shunt conductance. For an ideal lossless transmission line, the values of series resistance and shunt conductance must be zero.

i.e., $R = G = 0$ .... (2)

If equation (2) is satisfied, it makes the transmission line lossless and using the same condition, equation (1) is also satisfied. Hence, we can say that, a lossless transmission line is also distortionless.

However, if a transmission line is distortionless, it will satisfy the equation (1), but the same condition may not satisfy equation (2). Hence, we can say that a distortionless transmission line may not be lossless.

Distortionless Condition for Telephone Cables

For telephone cables, the ratio $\frac{R}{L} >> \frac{G}{C}$.

G could be increased but it would greatly increased the attenuation. R could be reduced but it results in larger diameter of conductor. C could be reduced by larger spacing between conductor but this has practical limit. So, common method is to increase L by inductive loading.

Summary

Case	Propagation constant ($\gamma = \alpha + j\beta$)	Characteristics impedance ($Z_0 = R_0 + jX_0$)
General	$\sqrt{(R + j\omega L)\,(G + j\omega C)}$	$\sqrt{\frac{R + j\omega L}{G + j\omega C}}$
Lossless	$0 + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j0$
Distortionless	$\sqrt{RG} + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j0$

11.5 REFLECTION COEFFICIENT, VSWR, INPUT IMPEDANCE

We know,

$$V_s = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z} \quad \text{..........} \quad (i)$$

$$I_s = I_o^+ e^{-\gamma z} + I_o^- e^{-\gamma z}$$

$$I_s = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z} \qquad \text{.......... (ii)}$$

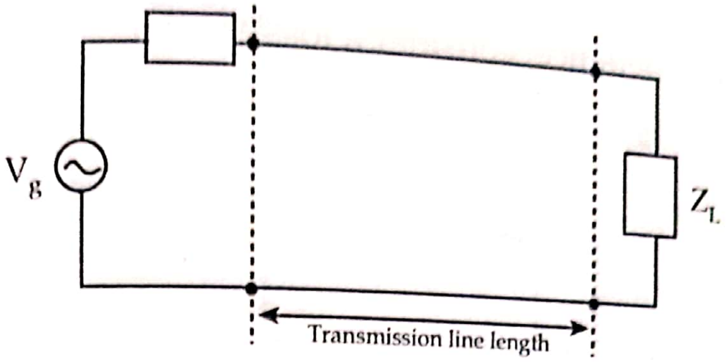

Figure 11.5 For calculating reflection coefficient

Applying boundary conditions to equation (i) and (ii) at z = 0 (the load end)

$$V_s(z = 0) = V_o^+ + V_o^-$$

$$I_s(z = 0) = \frac{V_o^+}{Z_o} - \frac{V_o^-}{Z_o}$$

The ratio of $\left.\frac{V_s}{I_s}\right|_{z=0}$ is the load impedance Z_L.

$$Z_L = \left.\frac{V_s}{I_s}\right|_{z=0} = Z_o\left[\frac{V_o^+ + V_o^-}{V_o^+ - V_o^-}\right]$$

$$\text{or, } Z_L = Z_o\left[\frac{1 + \frac{V_o^-}{V_o^+}}{1 - \frac{V_o^-}{V_o^+}}\right]$$

$$\text{or, } Z_L = Z_o\left[\frac{1+\Gamma}{1-\Gamma}\right]$$

where Γ is the reflection coefficient. The above expression can be simplified to yield

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o}$$

The current reflection coefficient at any point is the negative of voltage reflection coefficient.

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

To find input impedance, consider a lossless line.

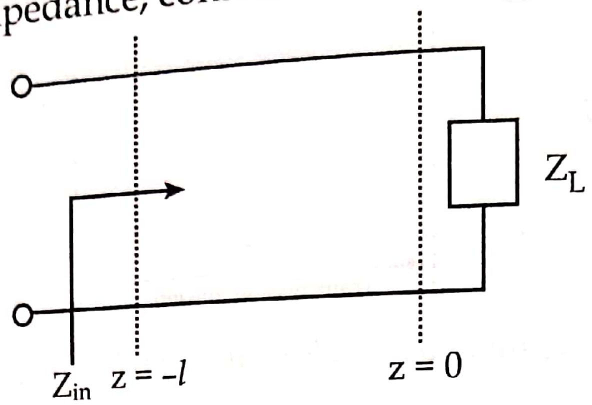

Figure 11.6 For calculating input impedance of a transmission line.

$$Z_{in}(z = -l) = \frac{V_s(-l)}{I_s(-l)} \quad \text{.............(1)}$$

We know,

$$V_s = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

$$V_s(z=-l) = V_o^+ e^{j\beta l} + V_o^- e^{-j\beta l}$$

$$= V_o^+ (\cos\beta l + j\sin\beta l) + V_o^- (\cos\beta l - j\sin\beta l)$$

$$= (V_o^+ + V_o^-)\cos\beta l + j\sin\beta l\,(V_o^+ - V_o^-)$$

$$= V_L \cos\beta l + j Z_o I_L \sin\beta l$$

$$= V_L\left[\cos\beta l + j\left(\frac{Z_o}{Z_L}\right)\sin\beta l\right] \quad \text{....... (2)}$$

$$I_s = I_o^+ e^{-j\beta z} + I_o^- e^{j\beta z}$$

$$I_s(z=-l) = I_o^+ e^{j\beta l} + I_o^- e^{-j\beta l}$$

$$= I_o^+ (\cos\beta l + j\sin\beta l) + I_o^- (\cos\beta l - j\sin\beta l)$$

$$= (I_o^+ + I_o^-)\cos\beta l + j\sin\beta l\,(I_o^+ - I_o^-)$$

$$= I_L \cos\beta l + j\frac{V_L}{Z_o}\sin\beta l \quad \left[\because \frac{V_L}{Z_o} = \frac{V_o^+ + V_o^-}{Z_o} = I_o^+ - I_o^-\right]$$

Using $V_L = \frac{I_L Z_L}{Z_o}$, we have,

$$I_s(z=-l) = I_L\left[\cos\beta l + \left(\frac{Z_L}{Z_o}\right)\sin\beta l\right] \quad \text{..............(3)}$$

Substituting values of (2) and (3) in (1),

$$Z_{in} = \frac{V_L\left[\cos\beta l + j\left(\frac{Z_o}{Z_L}\right)\sin\beta l\right]}{I_L\left[\cos\beta l + j\left(\frac{Z_L}{Z_o}\sin\beta l\right)\right]}$$

$$= Z_L\frac{\left[\cos\beta l + j\left(\frac{Z_o}{Z_L}\right)\sin\beta l\right]}{\left[\cos\beta l + j\left(\frac{Z_L}{Z_o}\right)\sin\beta l\right]}$$

$$= Z_o\left[\frac{Z_L\cos\beta l + jZ_o\sin\beta l}{Z_o\cos\beta l + jZ_L\sin\beta l}\right]$$

$$\boxed{\therefore Z_{in} = Z_o\left[\frac{Z_L + jZ_o\tan\beta l}{Z_o + jZ_L\tan\beta l}\right]}$$

Now, generalizing for lossy medium, we have

$$\boxed{Z_{in} = Z_o\left[\frac{Z_L + jZ_o\tanh\gamma l}{Z_o + jZ_L\tanh\gamma l}\right]}$$

11.6 IMPEDANCE MATCHING

When $Z_o \neq Z_L$, we say that the load is mismatched and a reflected wave exists on the line. However, for maximum power transfer, it is desired that the load be matched to the transmission line ($Z_o=Z_L$) so that there is no reflection ($|\Gamma| = 0$ or $s = 1$).

Impedance matching is important for the following reasons:

i. Maximum power is delivered when the load is matched to the line (assuming the generator is matched), and power loss in the feed line is minimized.

ii. Impedance matching sensitive receiver components (antenna, low-noise amplifier, etc.) may improve signal-to-noise ratio of the system.

iii. Impedance matching in a power distribution network (such as an antenna array feed network) may reduce amplitude and phase errors.

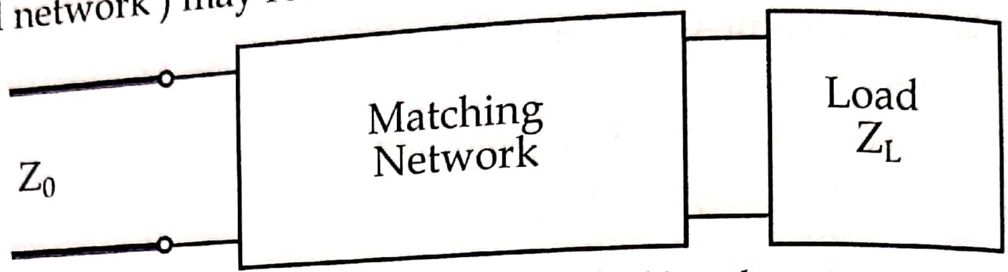

Figure 11.7 A lossless network matching an arbitrary load impedance to a transmission line.

Several methods are used to match a load Z_L to the line, or to match cascaded lines with different characteristic impedances. Matching networks can be placed at the load or at some position along the line.

(i) Quarter-Wave Transformer

The quarter-wave transformer is a useful and practical circuit for impedance matching and also provides a simple transmission line circuit that further illustrates the properties of standing waves on a mismatched line.

This method inserts a transmission line $\lambda/4$ (with characteristic impedance Z_o') prior to the load as shown in Figure 11.8. The $\lambda/4$ section of the transmission line is called a quarter-wave transformer because it is used for impedance matching like an ordinary transformer.

If Z_o' is the characteristic impedance of quarter-wave transformer, then we have

$$Z_{in} = Z_o' \left[\frac{Z_L + jZ_o' \tan \beta l}{Z_o' + jZ_L \tan \beta l} \right]$$

For $l = \lambda/4$, $\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$. To evaluate the above expression, we divide the numerator and denominator by $\tan \beta l$ and take the limit as $\beta l \to \frac{\pi}{2}$ to get

$$Z_{in} = \frac{Z_o'^2}{Z_L} \Rightarrow Z_o' = \sqrt{Z_{in} Z_L}$$

In order for $\Gamma = 0$, we must have $Z_{in} = Z_o$.

$$\therefore Z_o' = \sqrt{Z_o Z_L}$$

For example, if a 120 Ω load is to be matched to a 75 Ω line, the quarter-wave transformer must have a characteristic impedance of $\sqrt{(75)(120)} \simeq 95\ \Omega$. This 95 Ω quarter-wave transformer will also match a 75 Ω load to a 120 Ω line.

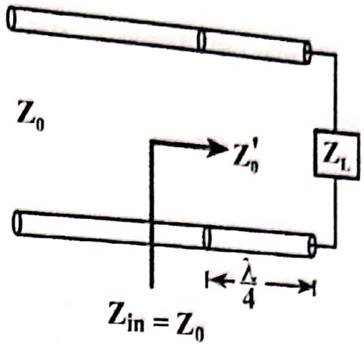

Figure 11.8 Impedance matching using a quarter-wave transformer.

Disadvantage:

The reflected wave (or standing wave) is eliminated only at the desired wavelength (or frequency f); there will be reflection at a slightly different wavelength. Thus, the main disadvantage of the quarter-wave transformer is that it is a narrow-band or frequency-sensitive device.

(ii) Single-Stub Matching

The drawback of a quarter-wave transformer can be eliminated if we employ single-stub matching technique.

This method uses one shorted stub, of length ℓ, placed at a distance d from the load as shown in Figure 11.9. The match is achieved by varying the location of the stub and the length of the stub. Notice that the stub has the same characteristic impedance as the main line.

We select d so that the admittance Y seen looking into the line at distance d from the load is of the form Y_0+jB. Then, the stub susceptance is chosen as $-jB$, resulting in a matched condition.

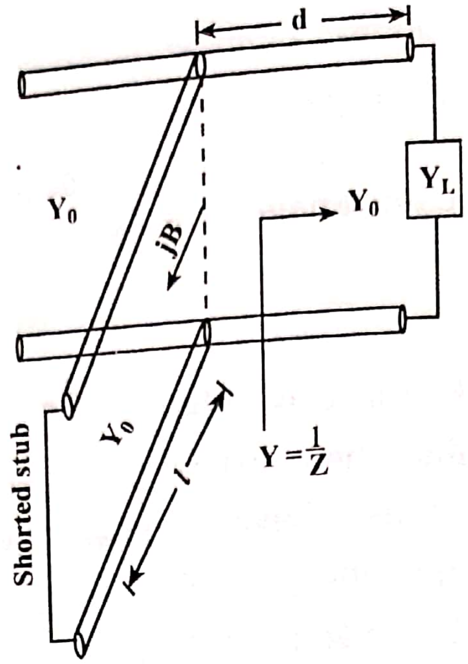

Figure 11.9 Single-stub matching technique.

(iii) Double-Stub Matching

In the single-stub matching technique, it is necessary to vary the distance between the stub and the load, as well as the length of the stub, in order to achieve a match for different loads or for different frequencies. This can be inconvenient for some arrangements of lines. When two stubs are used, it is possible to fix their locations and achieve a match for a wide range of loads by adjusting the length of the stubs. The technique of employing two stubs across the transmission line for matching load is called double-stub matching.

Stub 1 is nearest to the load and is frequently connected at the load. Common separations for the two stubs are $\lambda/4$ and $3\lambda/8$.

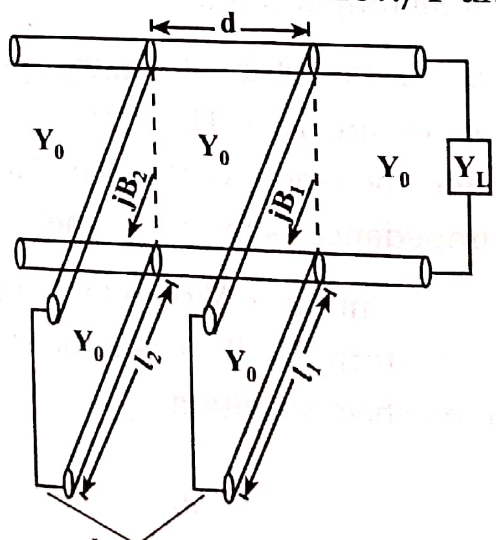

Figure 11.10 Double-stub matching technique.

Advantages of the Smith chart

- It is a direct graphical representation, in the complex plane, of the complex reflection coefficient.
- It is Reimann surface, in that it is cyclical in numbers of half-wavelength along the line. As the standing wave pattern repeats every half wavelength, this is entirely appropriate. The number of half wavelengths may be represented by the winding number.
- It may be used either as an impedance or admittance calculator, merely by turning it through 180 degrees.
- The inside of the unity gamma circular region represents the passive reflection case, which is most often the region of interest.
- Transformation along the line (if lossless) results in a change of the angle, and not the modulus or radius of gamma. Thus, plots may be made quickly and simply.
- Many of the more advanced properties of microwave circuits, such as noise figure and stability regions, map onto the Smith chart as circles.
- The "point at infinity" represents the limit of very large reflection gain, and so therefore need never be considered for practical circuits.
- The real axis maps to the standing wave ratio (SWR) variable. A simple transfer of the plot locus to the real axis at constant radius gives a direct reading of the SWR.

11.8 WAVEGUIDES

Like a transmission line, waveguide is a structure which can be used to guide EM energy from one point (generator) to another (load). However, a waveguide differs from a transmission line in some respects. A transmission line can support only a transverse electromagnetic (TEM) wave, whereas a waveguide can support many possible field configurations. A transmission line may operate from dc (f=0) to a very high frequency, a waveguide can operate only above a certain frequency called the cutoff frequency and therefore acts as a **high-pass filter**. Thus, waveguides cannot transmit dc, and they become excessively large at frequencies below microwave frequencies. Specially constructed hollow metallic pipes are generally used as waveguides.

Although a waveguide may assume any arbitrary shape but uniform cross section, common waveguides are either rectangular or circular. Typical waveguides are shown in figure below.

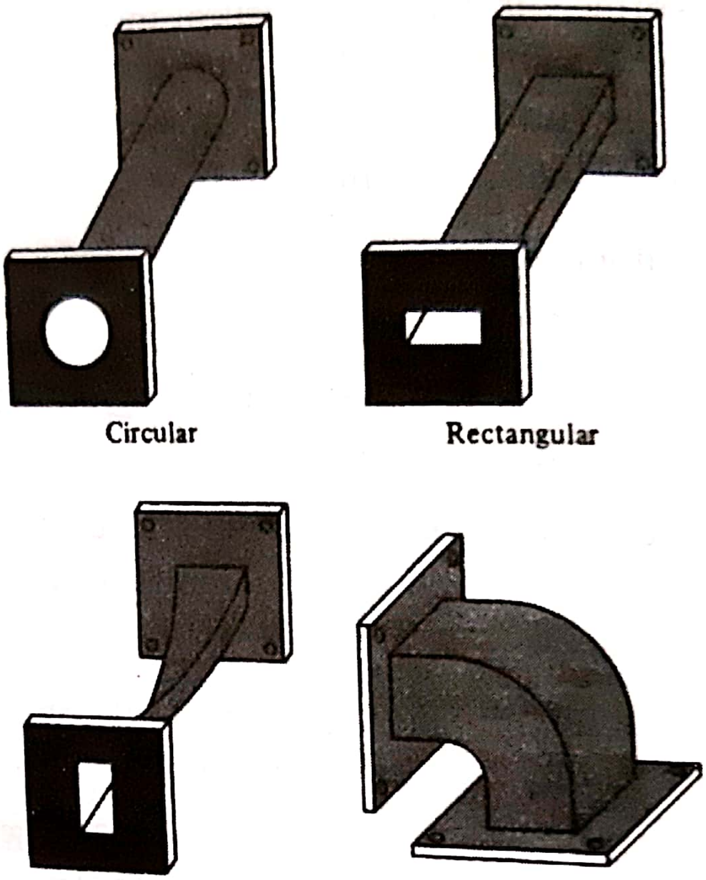

Figure 11.17 Typical waveguides.

Advantages of waveguides over other transmission lines:

(a) It is easier to leave out the inner conductor than to put it in, so waveguides are simple to manufacture.

(b) A transmission line can support only transverse electromagnetic (TEM) wave whereas a waveguide can support many possible field configurations other than TEM.

(c) Since propagation is due to the reflection from the wall, the power losses in the waveguide are lower than transmission line.

(d) In a waveguide, different signals can be propagated in different mode which can be detected at receiving end such that single waveguide can transmit different signals at same time.

(e) Co-axial cable works in frequency range (0–18 GHz) whereas waveguides operating frequency can be from certain frequency (cut off) upto 325 GHz. At microwave frequency (3-300 GHz), transmission lines become inefficient due to skin effect and dielectric losses; waveguides are used at that ranges of frequencies to obtain larger bandwidth and lower signal attenuation.

(f) Power handling capacity of waveguides is higher.

Disadvantages of waveguides:

1. Physical size is the primary lower-frequency limitation of waveguides. The width of a waveguide must be approximately a half wavelength at the frequency of the wave to be transported. For example, waveguide for use at 1 megahertz would be about 500 feet wide. This makes the use of waveguides at frequencies below 100 megahertz increasingly impractical.

2. Waveguides are difficult to install because of their rigid, hollow-pipe shape. Special couplings at the joints are required to assure proper operation.

3. Also, the inside surfaces of waveguides are often plated with silver or gold to reduce skin effect losses. These requirements increase the costs and decrease the practicality of waveguide systems at any other than microwave frequencies.

Types of Waveguides

(a) Rectangular waveguides

(b) Circular waveguides

(a) Rectangular Waveguides

A rectangular waveguide is a hollow metallic tube with a rectangular cross section. The waveguide is assumed to be filled with a source- free ($\rho_v = 0$, $\vec{J} = 0$) lossless dielectric material ($\sigma \simeq 0$) and that its walls are perfectly conducting ($\sigma_c \simeq \infty$). The conducting walls of the guide confine the electromagnetic fields and there by guide the electromagnetic wave. The rectangular waveguide supports TE and TM modes (TE_{mn} and TM_{mn}), but not TEM modes, since only one conductor is present. The integer 'm' denotes the number of half waves of electric or magnetic intensity in the x direction and 'n' is the number of half waves in the y direction if the propagation of the wave is assumed in the positive z direction.

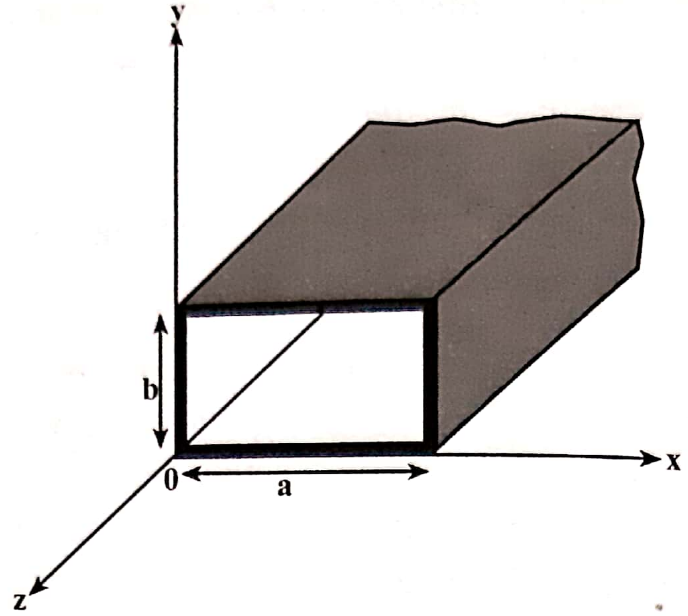

Figure 11.18 A rectangular waveguide.

If we assume a and b are the inner dimensions, for b < a, then the lowest cut off will occur for the TE_{10} mode. This is the dominant (and most important) mode in the rectangular waveguide because it can propagate alone if the operating frequency is appropriately chosen.

(b) Circular Waveguides

A circular waveguide is a tubular, circular conductor filled with a dielectric medium. Like rectangular waveguides, a circular waveguide supports TE and TM modes only.

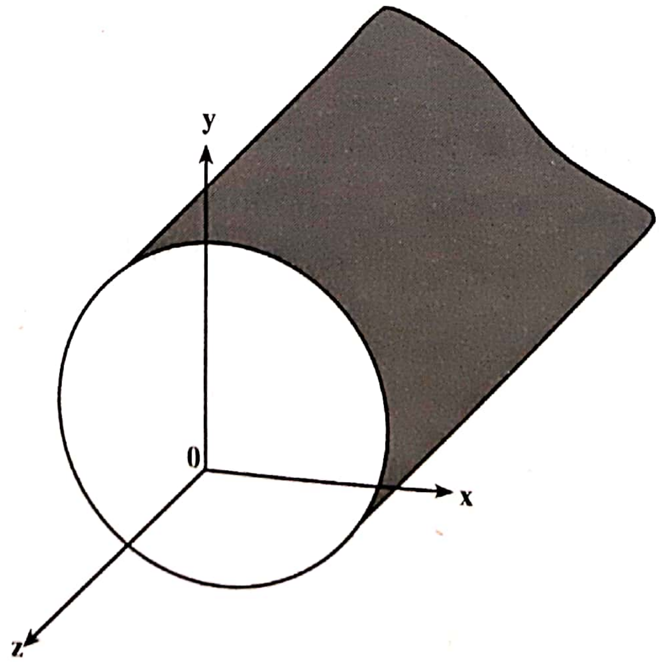

Figure 11.19 A circular waveguide.

Circular waveguides are used in specific areas of radar and communication systems, such as rotating joints used at the mechanical point where the antennas rotate.

Circular waveguides are easier to manufacture than rectangular ones. They are also easier to join, in the usual plumbing fashion. However, a disadvantage of circular waveguide is its limited dominant mode bandwidth in comparison to that of rectangular waveguide.

MODES

A configuration of the electric and the magnetic fields within a waveguide is called a mode. Waveguide supports different modes of operation depending on the presence and absence of electric field, magnetic field component. Four different mode categories can exist, namely: (a) TEM mode (b) TE mode (c) TM mode (d) HE mode.

(a) **Transverse electromagnetic (TEM) mode**

In this mode, both the electric field and magnetic fields are transverse to the direction of wave propagation. There is no field component in the direction of wave propagation. For waves propagating in the z-direction, there will be no E_z or H_z and such waves are called transverse electromagnetic (TEM) waves.

(b) **Transverse electric (TE) mode**

In TE mode, the electric field is transverse to the direction of propagation of wave. For waves propagating in the z-direction, the waves in this mode have a non zero H_z but $E_z = 0$.

(c) **Transverse magnetic (TM) mode**

In TM mode, the magnetic field is transverse to the direction of propagation of wave. For waves propagating in the z-direction, the waves in this mode have a non zero E_z but $H_z = 0$.

(d) **HE mode**

In this case, neither the electric field nor the magnetic field is transverse to the direction of wave propagation. Sometimes these modes are referred to as hybrid modes.

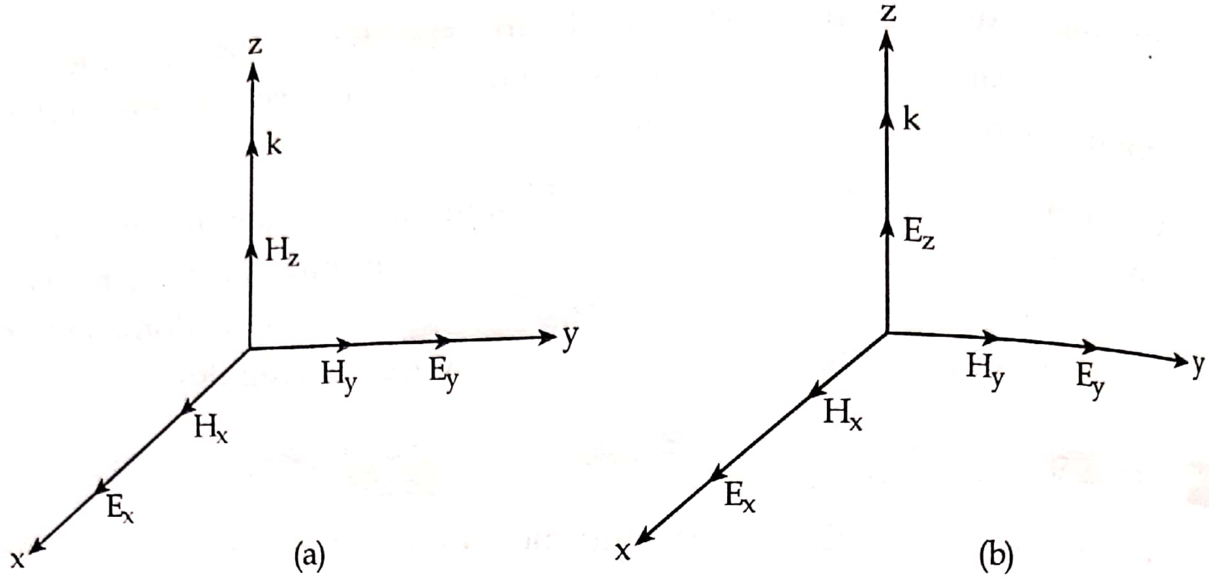

***Figure 11.20** (a)TE mode, $E_z = 0$ (b) TM mode, $H_z = 0$.*

Both TM and TE modes have characteristic cut-off frequencies. As waves of frequencies below the cut-off frequency of a particular mode cannot propagate, and power and signal transmission at that mode is possible only for frequencies higher than the cut-off frequency. Thus, waveguides operating in TM and TE modes are like high-pass filters.

Dominant mode

The mode with the lowest cut-off frequency is called dominant mode. For rectangular waveguide with b<a as shown in Figure 11.18, dominant mode is TE_{10} and TM_{11}.

Cutoff frequency

The cutoff frequency is the operating frequency below which attenuation occurs and above which propagation take place. It the operating frequency is below the cutoff frequency, the mode will not propagate. In waveguide, the lowest cut-off frequency is dominant mode.

In rectangular waveguide for $TE_{m,n}$ or $TM_{m,n}$ mode,

$$\text{Cutoff frequency } (f_c) = = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\text{Cutoff wavelength } (\lambda_c) = \frac{1}{\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}}$$

Introduction

An antenna is a structure that is generally a metallic object, often a wire or group of wires, used to convert high-frequency current into electromagnetic waves, and vice-versa. An antenna may also be defined as any device that radiates electromagnetic energy into space, where the energy originates from a source that feeds the antenna through a transmission line or waveguide. The antenna thus serves as an interface between the confining line and space when used as a transmitter or between space and the line when used as a receiver. Apart from their different functions, transmitting and receiving antennas have similar characteristics, which means that their behavior is reciprocal.

For wireless communication systems, the antenna is one of the most critical components. A good design of the antenna can relax system requirements and improve overall system performance. A typical example is TV for which the overall broadcast reception can be improved by utilizing a high-performance antenna. The antenna serves to a communication system the same purpose that eye and eyeglasses serve to a human.

Regardless of antenna type, all involve the same basic principle that radiation is produced by accelerated (or decelerated) charge. The basic equation of radiation may be expressed simply as

$$\dot{I}L = Q\dot{v} \ (\mathrm{A\,m\,s^{-1}})$$

where $\dot{I}$ = time-changing current, A s^{-1}

L = length of current element, m

Q = charge, C

$\dot{v}$ = time change of velocity which equals the acceleration of the charge, m s^{-2}

Thus, time-changing current radiates and accelerated charge radiates. For steady-state harmonic variation, we usually focus on current. For transients or pulses, we focus on charge. The radiation is perpendicular to the acceleration, and the radiated power is proportional to the square of $\dot{I}L$ or $Q\dot{v}$.

Basic Antenna Parameters (Properties)

To describe the performance of an antenna, definitions of various parameters are necessary. These include:

1. **Radiation pattern**

An antenna radiation pattern or antenna pattern is defined as a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates.

2. **Radiation power density**

Radiation power density is a quantity that is used to describe the power associated with an electromagnetic wave. The instantaneous Poynting vector $\vec{S}$ $(= \vec{E} \times \vec{H})$ gives the value of radiation power density.

3. **Radiation intensity**

Radiation intensity in a given direction is defined as the power radiated from an antenna per unit solid angle.

4. **Directivity**

Directivity of an antenna is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

5. **Gain**

Gain of an antenna in a given direction is defined as the ratio of the intensity in a given direction to the ratio of the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.

6. **Antenna efficiency**

Antenna efficiency is defined as the ratio of the aperture effective area to its actual physical area. It describes the percentage of the physical aperture area which actually captures radio frequency (RF) energy.

7. **Beamwidth**

Associated with the pattern of an antenna is a parameter designated as beamwidth. The beamwidth of a pattern is defined as the angular separation between two identical points on opposite side of the pattern maximum.

8. **Beam efficiency**

Another parameter that is frequently used to judge the quality of transmitting and receiving antennas is the beam efficiency and is defined as

Beam efficiency

$$= \frac{\text{power transmitted (received) within cone angle } \theta_1}{\text{power transmitted (received) by the antenna}}$$

where θ_1 is the half-angle of the cone within which the percentage of the total power is to be found.

9. **Bandwidth**

The bandwidth of an antenna is defined as the range of frequencies within which the performance of the antenna with respect to some characteristic conforms to a specified standard.

10. **Polarization**

Polarization of an antenna in a given direction is defined as the polarization of the wave transmitted (radiated) by the antenna.

11. **Input impedance**

Input impedance is defined as the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point.

12. **Antenna vector effective length and equivalent areas**

The vector effective length of an antenna is a vector quantity that is used to determine the voltage induced on the open-circuit terminals of the antenna when a wave impinges upon it.

Within each antenna, we can associate a number of equivalent areas. These are used to describe the power capturing characteristics of the antenna when a wave impinges on it. One of these areas is the effective area (aperture).

13. **Antenna temperature**

Antenna temperature is a parameter that describes how much noise an antenna produces in a given environment and is also referred to as antenna noise temperature.

Types of Antennas

1. **Wire antennas**

Wire antennas are seen virtually everywhere-on automobiles, buildings, ships, aircraft, spacecraft, and so on. There are various shapes of wire antennas such as a straight wire (dipole), loop, and helix which are shown in Figure 11.21. Loop antennas may take the form of a circle, rectangle, square, ellipse, or any other configuration.

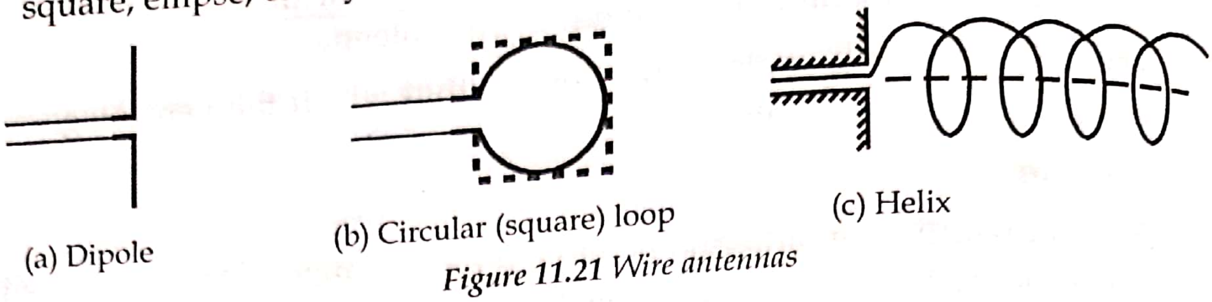

Figure 11.21 Wire antennas

2. **Aperture antennas**

Aperture antennas are very useful for aircraft and spacecraft applications, because they can be very conveniently flush-mounted on the skin of the aircraft or spacecraft. In addition, they can be covered with a dielectric material to protect them from hazardous condition of the environment.

3. **Microstrip antennas**

Microstrip antennas are used for government and commercial applications. These antennas consist of a metallic patch on a grounded substrate. The metallic patch can take many different configurations, as shown in Figure 11.22.

The microstrip antennas are low-profile, conformable to planar and nonplanar surfaces, simple and inexpensive to fabricate using modern printed-circuit technology, mechanically robust when mounted on rigid surfaces, compatible with MMIC designs, and very versatile in terms of resonant frequency, polarization, pattern, and impedance. These antennas can be mounted on the surface of high-performance aircraft, spacecraft, satellites, missiles, cars, and even handheld mobile telephones.

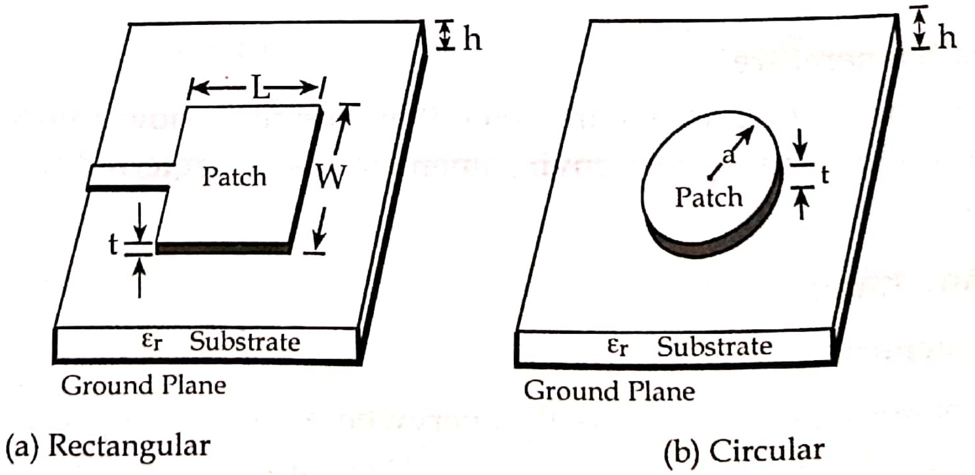

Figure 11.22 Microstrip antennas

4. Array antennas

Many applications require radiation characteristics that may not be achievable by a single element. It may, however, be possible that an aggregate of radiating elements in an electrical and geometrical arrangement (*an array*) will result in the desired radiation characteristics. The arrangement of the array may be such that the radiation from the elements adds up to give a radiation maximum in a particular direction or directions, minimum in others, or otherwise as desired.

5. Reflector antennas

Because of the need to communicate over great distances, sophisticated forms of antennas had to be used in order to transmit and receive signals that had to travel millions of miles. A very common antenna form such an application is a parabolic reflector. Another form of a reflector, although not as common as the parabolic, is the corner reflector.

6. Lens antennas

Lenses are primarily used to collimate incident divergent energy to prevent it from spreading in undesired directions. By properly shaping the geometrical configuration and choosing the appropriate material of the lenses, they can transform various forms of divergent energy into plane waves. They can be used in most of the same applications as are the parabolic reflectors, especially at higher frequencies. Their dimensions and weight become exceedingly large at lower frequencies.

11.11 RADIATION FROM A DIPOLE ANTENNA

When a piece of open transmission line is considered, there exists a standing wave as shown in Figure 11.23 (a). Because the conductors are very close to each other, the fields (the electric and the magnetic) produced by the individual conductor therefore cancel with each other. Hence, there will not be any radiation from the line. However, if a portion of the line at the open end is slowly bent outward, the cancellation of the fields gradually decrease. When the line finally takes the form as shown in Figure 11.23 (b), no more cancellation occurs and the construction radiates the EM waves out into the surrounding medium. The portion of the line, which has been bent, is the standing wave linear antenna, and is popularly known as the dipole antenna.

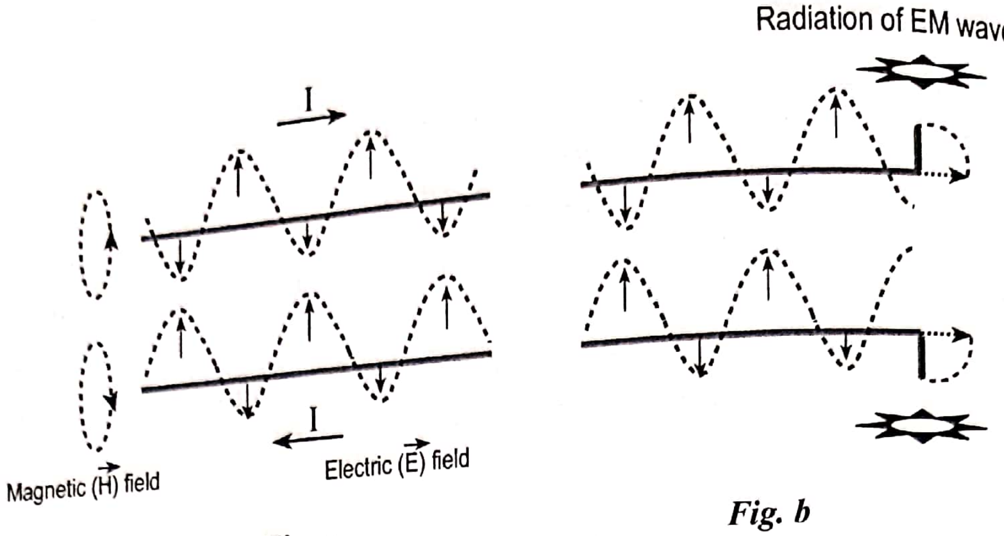

Figure 11.23 *(a) The opposite electric and the magnetic fields with no radiation, (b) a standing wave linear dipole antenna with radiation, and (c) a representation of a dipole antenna.*

PROBLEMS SOLVED AND SCRAMBLED

1. **A transmission line operating at $\omega = 10^8$ rad/s has these parameter values: R = 0.1Ω /m, L= 0.2 μH/m, G = 10 μmho/m, C= 100 pF/m Find**
   (i) α (ii) β (iii) γ (iv) v (v) Z_0

Solution:

$R = 0.1\ \Omega/m,\ L = 0.2\times10^{-6}\ H/m,\ C = 100\times10^{-12}\ F/m,\ G = 10\times10^{-6}\ mho/m$

Propagation constant $(\gamma) = \sqrt{(R+j\omega L)\ (G+j\omega C)}$

$$R+j\omega L = 0.1+j\,(10^8\times0.2\times10^{-6}) = 20\ \angle 89.71^\circ$$

$$G+j\omega C = 10\times10^{-6}+j(10^8\times100\times10^{-12})$$

$$= 0.01\ \angle 89.94^\circ$$

$$\therefore \gamma = \sqrt{(20\angle 89.71^\circ)(0.01\angle 89.94^\circ)}$$

$$= 0.447\ \angle 89.825^\circ = 1.365\times10^{-3} + j\,0.4469\ m^{-1}$$

As $\gamma = \alpha + j\beta$, $\alpha = 1.365\times10^{-3}$ Np/m, $\beta = 0.4469$ rad/m

$$\text{Velocity } (v) = f\lambda = \frac{\omega}{2\pi}\times\frac{2\pi}{\beta} = \frac{\omega}{\beta} = \frac{1\times10^8}{0.4496} = 2.22\times10^8\ m/s$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{20\angle 89.71^\circ}{0.01\angle 89.94^\circ}} = 44.72\angle -0.12^\circ = (44.71 - j0.093)\ \Omega$$