COMP-4540
Design and Analysis of Algorithms
Fall 2024

## Assignment 4 (Optional)

Due Date: Dec 5 (11:59p.m.)

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1. The decision problem Π:

   Given a deck of $n + 1$ cards each of which is imprinted with two columns of circles along the two sides. Each column contains m circles and some of the circles are punched out to make holes so that one can see though. One of the cards, called the TRUE card, has all the circles in one column punched out and all the circles in the other column left intact.

   Determine if there is a way of stacking up the cards with the TRUE card on top so that all the holes are covered (i.e. no one can see through). Note that the two faces of each card are indistinguishable (i.e. you may flip the card over).

   Prove that 3-SAT $\propto \Pi$.

The TRUE card   The card for variable x_i
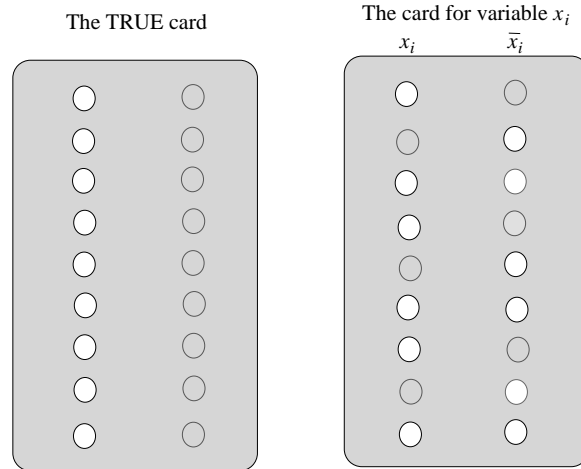
[**Hints:**

   - For each variable u in the given problem instance of 3-SAT, create one card for u with one column corresponds to u and the other column corresponds to $\overline{u}$.

   - The number of circles in each column is the number of clauses in the given problem instance of 3-SAT.

   - Circles on the card for variable u are punched out based on the locations of the occurrences of u and $\overline{u}$ in the given problem instance of 3-SAT.]