COMP-4540/8540
Design and Analysis of Algorithms

Winter 2025

Assignment 1

Due Date: February 4 (before 11:59p.m.)

1. Prove that $n! \in o(n^n)$ using the definition of o-notation.

2. Prove or disprove: $\Theta(n) - \Theta(n-1) = \Theta(1)$.

3. Prove that $\sum_{k=1}^{n} \frac{1}{2k-1} = O(\lg \sqrt{n})$. [**Hint:** $\sum_{k=1}^{n} \frac{1}{k} = \ln n + \gamma + \frac{1}{2n} + o(\frac{1}{n})$.]

4. Rank the following functions by order of growth; i.e. find an ordering $g_1, g_2, \ldots, g_7$ of the functions such that $g_i \in o(g_{i+1})$ or $g_i \in \Theta(g_{i+1})$, for $1 \leq i < 7$. Partition your lists into equivalences classes such that functions $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. Provide proofs.

$$(\lg n)!,\ n^{\lg \lg n},\ (n+1)!,\ 2^{2^{\lg \lg n+1}}, 10^{\lg \lg n},\ n^2 \cdot 2^n,\ 6^n$$

[**Hint:** $(\lg n)! \in \Theta(n^{\lg \lg n - \lg e}\sqrt{\lg n})$]

5. To be posted

6. To be posted