Selective abortion of females: An analysis of some emerging parental choices

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ABSTRACT

During the last two decades, the technology for the *in vivo* detection of the gender of a fetus has become available, to various degrees, in many less developed countries and newly industrialized countries. Subject to the limited availability of data, it appears that in several of these countries an emerging use of this technology is in selectively aborting female fetuses. Regardless of what the present extent of such abortions might be, a likely effect of some ongoing technological trends is that increasingly larger proportions of parents will in the future be able to selectively abort females if they wish to do so.

This paper considers parents whose propensity for sons is sufficiently high that they use, or will use in the future, gender-detection technology with the primary motivation of potentially exercising the choice of selectively aborting females. The paper focuses on parental choices concerning conceptions and selective abortions. These two choices are dynamically linked (e.g., whether a set of parents choose to undertake a conception depends partly on what their future choice concerning abortion will be when they learn the gender of the fetus). The paper analyzes some qualitative properties of these choices and attempts to assess some associated patterns of behavior concerning conceptions and selective abortions that the parents undertake and daughters that they choose to have. This includes assessing how the behavior of a set of parents might depend on the number of sons they have, the propensity for sons they have, and the costs and inconveniences of abortions they face.

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AN ANALYSIS OF
SOME EMERGING PARENTAL CHOICES

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I. A BRIEF BACKGROUND

Substantial progress has been made during the post-War period in the technology for the *in vivo* testing of fetuses for a number of diseases and abnormalities. This technology, which includes amniocentesis, is now widely available in most industrialized countries. As often happens with many innovations, this technology has a byproduct with its own potential consequences: for the first time in history, parents can know with near certainty the gender of a fetus. Consequently, they can use this technology to attempt to choose the gender composition of their children by undertaking gender-specific abortions.

During the last two decades, the technology for gender detection has become available, to various degrees, in many less developed countries and newly industrialized countries (for brevity, I use the phrase "LDCs" for these countries). Subject to the limited availability of data, it appears that in several of these countries an emerging use of this technology is in selectively aborting females. Evidently as a response to this perceived trend, some governments (e.g., in China and in certain provinces in India) have in the last five years enacted laws to prevent gender-specific abortions, for example, laws that prohibit medical professionals from revealing the gender of a fetus. It is reasonable to conjecture that such laws may raise the cost of gender-specific abortions somewhat, but will have little effect on the availability of such abortions.

In order to gauge the extent to which females are being selectively aborted, key data that one would look at would include the gender ratio at birth; that is, the number of boys born per 100 girls born. One would compare recent gender ratios at birth in various LDCs with the "conventional" gender ratio at birth; that is, the ratio without any gender-specific abortions. The conventional ratio is believed to be between 103 and 106 (Coale (1984), Johansson and Nygren (1991) and references therein). However, the information that is presently available from such comparisons of gender ratios at birth is very limited.

Table 1 presents some reported gender ratios at birth in South Korea and China for several years during the 1980s. In both countries, these ratios were closer to the conventional levels in the earlier years and are higher in recent years. This is consistent with a gradual emergence of the practice of selective abortions of females. The interpretation of such data, especially of the Chinese data, requires caution. The Chinese "one child per couple" policy, officially announced in 1979, may have contributed to an increase in the reported gender ratio at birth. In particular, parents may have increased the use of such practices as female infanticide (i.e., killing a girl soon after her birth), underreporting live female births, giving away one's daughters for adoption, and sending them away to be raised by relatives in remote areas (Hull (1990)). Working in the opposite direction, some pressure perhaps exists on government officials to attempt to lower the reported gender ratio at birth if a high ratio actually exists.

TABLE 1

Gender ratio at birth[1]
(Number of boys born per 100 girls born)

Year	South Korea	China
1980	103.9	107.4
1981	107.2	107.1
1982	106.8	107.2
1983	107.5	107.9
1984	108.6	108.5
1985	109.8	111.4
1986	112.1	112.3
1987	109.2	111.0
1988	113.8	
1989	113.1	

Regardless of what the present extent of the selective abortion of females might be, there are some ongoing technological trends that are likely to influence its future extent. In most LDCs, gender-detection services are currently available primarily in the cities and they are expensive by local standards. Consequently, only a fraction of the parental population can presently afford this technology. However, as is common for medical technologies with a potentially large number of users, the cost of using gender-detection tests is likely to decrease in the future. Also, such tests may become

[1]The ratios for South Korea are from National Bureau of Statistics (1990, pp. 61-79). The ratios for China are from Johansson and Nygren (1991, p. 39), citing Chinese two-per-thousand fertility surveys. The *New York Times* (June 17, 1991) notes the ratios 112.5 and 113.3 for China for 1989 and 1990 respectively, citing Chinese censuses and various state agencies as sources.

administrable at earlier stages in a pregnancy than are currently feasible. Likewise, the cost and inconvenience of abortion (including the high health and mortality risks for women that it currently entails in many LDCs) are likely to continue to decrease, and may decrease significantly due to such innovations as abortion-inducing pills. A likely effect of these trends is that increasingly larger proportions of parents in LDCs will be able to selectively abort females if they wish to do so.

The situation concerning gender-specific abortions is believed to be different in industrialized countries. When such abortions are undertaken in these countries, they are believed to be primarily intended to balance the gender composition of children; for example, parents may want one daughter and one son rather than two daughters or two sons. On the other hand, it has been noted that a male fetus is rarely aborted in countries such as China and those in South Asia.[2] This is consistent with the prevalence of several practices in these countries that raise the mortality of women and girls (see Miller (1981) for a survey of a number of studies of such practices in South Asia).

II. AN OUTLINE OF THE PAPER

This paper considers only those parents in LDCs whose propensity for sons is sufficiently high that they use, or will use in the future, gender-detection technology with the primary motivation of potentially exercising the choice of selectively aborting female fetuses. I do not consider those parents whose motivation in using this technology is to potentially exercise the choice of balancing the gender composition of their children or of selectively aborting male fetuses. A set of parents may have a propensity for sons because of their pure preferences (e.g., even if all of the pecuniary costs and benefits are identical from having one more son or one more daughter, the parents may prefer the former) as well as the pecuniary considerations that may induce them to want to have a son rather than a daughter. For brevity, I use the following shorthand phrases throughout the paper: "selective

[2]According to a survey done in Bombay, only one of the 8,000 abortions performed after gender detection was of a male (see Freed and Freed (1989, p. 150)). An analogous disproportion has been noted in China (see, e.g., Hull (1991, p. 74)).

abortions" mean selective abortions of female fetuses, "females" and "males" respectively mean female and male fetuses, and "daughters" and "sons" mean the offspring that the parents choose to have after they have exercised their choices concerning abortions.

The focus of this paper is on examining parental choices concerning conceptions and selective abortions. These two choices are linked. For example, whether a set of parents choose to undertake a particular conception will, in general, depend on what they anticipate to be their subsequent choice concerning abortion when they learn the gender of the fetus. I focus on these parental choices because they have been examined far less in the literature than many other fertility-related choices.

Within the above focus, a primary objective of this paper is to analyze some qualitative properties of these parental choices, and to attempt to assess some associated patterns of behavior concerning conceptions and selective abortions that the parents undertake and the number of daughters that they choose to have. This includes assessing how the behavior of a set of parents might depend on the number of sons they have, the propensity for sons they have, and the costs of abortions they face. As in most qualitative economic analyses, the results presented in this paper are based on specific assumptions, here those associated with the model of the parents' choices. I examine some variations of these assumptions. An analysis such as the present one can also serve as a stepping stone for exploring parental behavior under other plausible assumptions.

Though almost all fertility-related decisions are dynamic, a long analytical tradition in economics, beginning with Becker (1960), has fruitfully employed single-stage models to analyze these choices. In these models, a set of parents make all of their choices, including fertility-related choices such as the number of conceptions they wish to undertake, at one time. While such an abstraction has proven itself to be a greatly useful starting point for economic analysis, it does not accommodate choices that include selective abortions. This is because here the parents must first make the choice to conceive. Then, after undertaking gender detection, they must make another choice, this time concerning abortion. Therefore, at the minimum, a two-stage dynamic stochastic model is needed to explore the problem at hand. I work with such a model. I also reinterpret some of the resulting analysis within the context of a more general dynamic model.

The paper uses discrete variables to depict the parents' choices of conceptions, abortions, and the numbers of sons and daughters. This adds realism and it also avoids some special difficulties that arise in depicting abortions if one were to use continuous variables. On the other hand, the paper abstracts from many aspects of fertility. This is to keep the paper within a manageable length, and to follow the objective of the paper, outlined earlier. For example, the paper abstracts from "gender-preselection," that is, parents' attempts to increase, through behavior or technology, the probability that a conception results in the fetus of the desired gender. It also abstracts from child mortality and from errors in gender detection; the latter error is estimated to be less than 0.5 percent (Golbus *et al.* (1979)). Brief remarks on gender-preselection, including on its current feasibility, as well as on some other abstractions are presented later.

This paper's focus on some micro aspects of parental behavior should not be taken to suggest that many other questions raised by the possible emergence of the practice of selective abortions are any less compelling. Among the aggregate, society-level questions are: How might this practice affect future roles and treatment of women and men, family formation, population growth, and other aspects of the economy? How might parental behavior change in the future, in turn, when some of the aggregate consequences of this practice begin to become apparent? What kinds of interventions, including the enactment of laws, might societies undertake? Selective abortions also raise many ethical and moral issues.[3] Further, to many, selective abortions are deeply reprehensible. Though the present paper does not deal with aggregate issues, it has some complementarity with analyses of these issues. For instance, a parent-level analysis often provides a useful backdrop for the study of some aggregate issues. Likewise, no matter how deeply reprehensible a phenomenon might be, it is often useful to attempt to understand the underlying micro behavior.

An important literature on fertility has studied the role of parental preferences for the genders of their children.[4] Since gender-specific abortions have become a matter of potential concern only recently, it is natural that this literature has not dealt with this parental choice (Bloom and Grenier

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[3]Harris (1992) presents an analysis that, while not dealing specifically with gender-specific abortions, examines some ethical dilemmas that accompany advances in medical technology.

[4]See Ben-Porath and Welch (1976, 1980) for early economic work, and Bloom and Grenier (1983) and Leung (1988) for more recent contributions. See Cleland *et al.* (1983) and Goody (1981) for cross-country demographic analyses, and the former for an analysis of self-reported parental attitudes.

(1988) recognize that gender selection will affect parental choices, but abstract from it in their analysis of parental behavior). Even in the presence of gender-detection technology, this literature provides models to study the choices of those parents in LDCs who, for whatever reasons, do not consider gender-specific abortions. On the other hand, the present paper focuses, as mentioned earlier, on parents with high propensities for sons who do consider selective abortions. The behavior of these parents is affected by their propensities as well as by the available technology and the cost of using it. In the absence of gender-detection technology, these parents can implement their propensities, if they wish, through such methods as female infanticide and choices that affect gender-specific child mortality. Many of these parents who may be inhibited in implementing their propensities (e.g., due to the non-pecuniary costs of the methods just noted) may be less inhibited given the availability of selective abortions.[5]

Section III describes a model of the parents' choices. It also presents some comparative statics results that are employed repeatedly in the paper. Section IV examines the parents' choices concerning selective abortions. Section V examines their choices concerning conceptions. Section VI notes some extensions of the preceding analysis. Section VII presents some remarks. Proofs of the propositions presented later are outlined in an Appendix, except when the proof of a proposition follows readily from the material preceding it.

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[5]Pickens and Jacobson (1986), though dealing with gender-preselection rather than gender-specific abortions, present a model that is typical of a line of formal demographic work in this general area. The paper calculates the probability distribution for the total number of children that a set of parents will have. The parents wish to have particular ideal numbers of sons and daughters. They begin births without gender-preselection. If they have fewer than the ideal number of sons (respectively, daughters), but one more than the ideal number of daughters (respectively, sons), then they switch to a preselection technology that increases the probability of producing sons (respectively, daughters), and continue to use this technology until they have the ideal number of sons (respectively, daughters). The parents' choices do not reflect any consideration of the cost and benefits of the choice of technology or of various outcomes. Ahn (1991) estimates values of sons and daughters in South Korea, based on a model that abstracts from gender-detection and abortions, and uses these estimates to simulate some aspects of selective abortions.

III. A MODEL

Consider a particular set of parents. Let n denote the number of conceptions that they undertake. n is a nonnegative integer, not larger than some biologically-determined upper bound. After choosing n, the parents ascertain, without error, the genders of their conceptions. That is, they observe the number of males, denoted by the random variable m (where $n \geq m \geq 0$), and the number of females, $n-m$. The gender outcome of a conception is assumed to be stochastically independent. Hence, the probability that there are m males out of n conceptions is the binomial density $b(m, n, q) \equiv \binom{n}{m}q^m(1-q)^{n-m}$. Here q is the probability that a conception results in a male. At present, the value of q is taken to be one-half. Other values of q, as well as a more general treatment in which the parents' choices are based on a perceived probability distribution of q, rather than on any single value of q, are considered later.

Having observed m, the parents choose the number of sons and daughters they wish to have. Let M and F respectively denote these. Recall that the parents under consideration here do not abort any males. Hence, $M = m$. The parents choose F under the constraint that $n-m \geq F \geq 0$. That is, the parents cannot have more daughters than the available number of females. The number of females aborted is $n-m-F$.

Some Comparative Statics Results to be Used Later

This subsection states some comparative statics results that are later used repeatedly. These results are independent of any particular economic meaning attributed to the variables and functions.

Let $G(s, \alpha)$ denote a function defined over an integer variable s and a parameter α. The parameter α can be either continuous or restricted to only integer values. The variable s belongs to a nonempty set containing all integers from a finite lower bound to a finite upper bound. Hence, for any given value of α, one or more value of s exist that maximize $G(s, \alpha)$. Let $s(\alpha)$ denote the largest of these maximizing values of s. The necessary and sufficient conditions that $s(\alpha)$ satisfies are

(1) $G(s(\alpha), \alpha) \geq G(s(\alpha)-i, \alpha)$ for $i \geq 1$; and $G(s(\alpha), \alpha) > G(s(\alpha)+i, \alpha)$ for $i \geq 1$.

Define $G_s(s, \alpha) \equiv G(s+1, \alpha) - G(s, \alpha)$ and $G_{ss}(s, \alpha) \equiv G_s(s+1, \alpha) - G_s(s, \alpha)$. These are the discrete counterparts of the first and second derivatives, respectively. In the rest of the paper, letter subscripts of functions denote such derivatives. Let α and β denote two different values of the parameter. Then, as shown in the Appendix, the following results hold for any integer value of k.

Lemma 1.

(2) If $G_s(s-k, \beta) \geq G_s(s, \alpha)$, then $s(\beta) \geq s(\alpha)-k$.

(3) If $G_s(s-k, \beta) \leq G_s(s, \alpha)$, then $s(\beta) \leq s(\alpha)-k$. □

A noteworthy aspect of these results is as follows. When dealing with a decision variable that is continuous, rather than discrete as here, comparative statics results typically require that the maximand G satisfy some concavity properties. No such properties are needed here.

A Specification of the Parents' Choice Problem

Consider the following description of the parents' choices:

(4) $\text{Max}_n : -C(n) + \Sigma_m b(m, n, q) \ [\text{Max}_F : \{u(m, F) - \ell(n-m-F) | n-m \geq F \geq 0\}]$.

Here, $C(n)$ is the cost of n conceptions, including the cost of gender detection. The parameter ℓ represents the cost and inconvenience of an abortion. The expected utility from choosing F daughters, given that there are m sons, is $u(m, F)$. In (4) and in the rest of the paper, the range of a summation is suppressed if the summation is over the entire range; for example, the range is $m = 0$ to n in (4).

The specification in (4) is based on an indirect utility function. The function $u(m, F)$ represents the parents' utility after netting out the other choices that they make (i.e., choices other than those of n and F), having taken into account the constraints that they face other than that, noted in (4), on the choice of F. The use of indirect utility functions has been less common in the economic analyses of fertility than in many other areas. Hence, the last section presents some remarks on the approach adopted here, and on that of working with a direct utility function. As discussed there, both approaches entail their own sets of simplifications and assumptions, especially if one's objective is to analyze the qualitative properties of the optimal choices, rather than to only describe an optimization problem. As was noted earlier, my objective here is not only to explore some qualitative properties of parental choices, but also to focus on their choices concerning conceptions and selective abortions, as (4) does, rather than on a potentially large number of their other choices. Some extensions of the specification in (4) are discussed in the last section.

Recalling our notation, $u_F(M, F) \equiv u(M, F+1) - u(M, F)$ is the marginal utility of a daughter if the parents have M sons and F daughters. The corresponding marginal utility of a son is $u_M(M, F)$. The properties of u that are employed in the later analysis are

$$u_{FF}(M, F) \leq u_{MF}(M, F) \leq 0 \ . \tag{5}$$

This expression has three parts. The inequality $u_{FF}(M, F) \leq 0$ says that the marginal utility of a daughter does not increase if the parents have one more daughter. The inequality $u_{MF}(M, F) \leq 0$ says that daughters and sons are weak substitutes. The inequality $u_{FF}(M, F) \leq u_{MF}(M, F)$ says that if the parents have one more daughter, then the marginal utility of a daughter is not larger than that when they have one more son.

Let the parameter λ represent the parents' propensity for sons. This parameter reflects the effects of the parents' pure preferences for sons as well as of their pecuniary considerations regarding sons and daughters. A larger value of λ represents a greater propensity for sons. It is assumed that

$$\frac{\partial}{\partial\lambda}u_M(M, F, \lambda) \geq -\frac{\partial}{\partial\lambda}u_F(M, F, \lambda) \geq 0 \ . \tag{6}$$

To interpret (6), suppose, for a moment, that both of the inequalities in (6) are strict. This implies that a greater propensity for sons raises the marginal utility of a son, lowers the marginal utility of a daughter, and that the magnitude of the former increase is larger than that of the latter decrease. Property (6) is weaker than the preceding statement because both of the inequalities in (6) are weak. In particular, (6) allows for the plausible case in which a greater propensity for sons does not alter the marginal utility of a daughter, but raises the marginal utility of a son. Of course, the propensity for sons can have a nontrivial effect on the parents' behavior only if at least one of two inequalities in (6) is strict for at least one value of (M, F).

IV. THE PARENTS' CHOICES CONCERNING DAUGHTERS

The decision problem in (4) can be restated as

$$\text{(7)} \qquad \text{Max}_n : w(n) \ , \quad \text{where} \quad w(n) \equiv -C(n) + \Sigma_m b(m,\ n,\ q)V(m,\ n) \ , \quad \text{and}$$

$$\text{(8)} \qquad V(m,\ n) \equiv \text{Max}_F : \{u(m,\ F) - \ell(n-m-F) \,|\, n-m \geq F \geq 0\} \ .$$

If more than one value of F is optimal in (8), then the parents are assumed to choose the largest of these values. Likewise, if more than one value of n is optimal in (7), then the parents are assumed to choose the largest of these values.

I begin with the second stage of the above decision problem, which is (8), and then proceed to the first stage, which is (7). Let $F(m,\ n)$ denote the parents' optimal choice of F. Naturally, $F(m,\ n)$ is defined only for $m = 0$ to n. The rest of this section analyzes $F(m,\ n)$. For this analysis to be nontrivial, it is assumed that $n \geq 1$; that is, the parents undertake at least one conception.

A convenient way to deal with (8) is to define $\phi(m)$ to be the largest value of ϕ that solves the problem

$$\text{(9)} \qquad \text{Max}_\phi : \{u(m,\ \phi) - \ell(n-m-\phi) \,|\, \phi \geq 0\} \ .$$

Then, it is straightforward to show that

$$\text{(10)} \qquad F(m,\ n) = \text{Min}\{\phi(m),\ n-m\} \ .$$

An interpretation of $\phi(m)$ is the "desired" number of daughters if the parents have m sons. This is because, given m sons, the parents would like to have $\phi(m)$ daughters if there were no constraints on the availability of females. At this stage of the decision, however, only $n-m$ females are available, since the parents undertook n conceptions and these resulted in m males. Hence, as (10) shows, the number of daughters that the parents choose is the minimum of the desired number of daughters and the available number of females. Proposition 1 describes some properties of $\phi(m)$. These properties are also useful later.

Proposition 1. If the parents have one more son, then the desired number of daughters decreases by one or remains unchanged. That is,

$$\text{(11)} \qquad 0 \geq \phi(m+1) - \phi(m) \geq -1 \ . \ \square$$

Since daughters and sons are weak substitutes, the marginal utility of a daughter does not increase if there is one more son. Hence, as the first part of (11) states, the desired number of daughters does not increase if the parents have one more son. Next, suppose that the parents have one fewer daughter and one more son. Then, recalling the interpretation of the last part of (5), the marginal utility of a daughter does not decrease. Hence, as the second part of (11) states, the desired number of daughters does not decrease by more than one if the parents have one more son.

Proposition 2 describes an intuitive pattern of the number of daughters that the parents have depending on how many sons they have.

Proposition 2. For any given number of conceptions, n, that the parents undertake, there exists a unique "threshold" number of sons, $\mu(n)$, such that $n \geq \mu(n) \geq 0$. If the parents have fewer sons than this threshold number, then they abort at least one female and choose to have the desired number of daughters, $\phi(m)$; otherwise, they do not abort any female and choose to have as many daughters as the available number of females, n-m. That is,

$$F(m, n) = \phi(m) \leq n-m-1 \quad \text{if} \quad m \leq \mu(n) - 1 \text{ , and} \tag{12}$$

$$F(m, n) = n-m \quad \text{if} \quad m \geq \mu(n) \text{ . } \square \tag{13}$$

Obviously, it is possible for the parents to have $\mu(n) = 0$, in which case (12) is not relevant for them because, by definition, $m \geq 0$. Also, note that $\phi(m)$, $F(m, n)$, and the other choices of the parents discussed later depend on the parameters that they face. For brevity, this dependence is suppressed throughout the paper except when the context requires it to be made explicit.

Proposition 3 presents an alternative way of describing some patterns of selective abortions. It relates these abortions to the number of females the parents have.

Proposition 3.

(a) Suppose the parents do not abort any female when they have a given number of females. They will not abort any female when they have fewer females.

(b) Suppose the parents abort at least one female when they have a given number of females. They will abort at least one female when they have more females.

(c) Suppose the parents undertake n conceptions, abort at least one female when they have n females, but do not abort the female when they have only one female. They will abort at least one female if they have more than $n - \mu(n)$ females (here $n-1 \geq n - \mu(n) \geq 1$), but will not abort any female otherwise. □

Figure 1 illustrates an example of $F(m, n)$. Here $n = 6$. The line with large dashes has a 45° slope. The circles on this line indicate the available number of females, $n-m$, for different values of m. The squares on the other line indicate the desired number of daughters, $\phi(m)$. The minimum of these two lines, the solid line, indicates the parents' choices of the number of daughters. The difference between the two lines, if it is positive, is the number of selective abortions. In this figure, the origin represents the outcome in which the parents have no sons, and $m = 5$ represents the outcome in which they have only one female. In the former case, the parents abort two females but none in the latter. Correspondingly, this figure is an example of part (c) of Proposition 3. The threshold number of sons, $\mu(6)$, here is 3.

The Effects of the Propensity for Sons and of the Cost of Abortion on the Number of Daughters

These effects are what we would intuitively expect them to be.

Proposition 4.

(a) Consider two sets of parents who have different propensities for sons, but are otherwise identical. The parents with a greater propensity will have fewer or the same number of daughters.

(b) If the cost of abortion decreases, then the parents will have fewer or the same number of daughters. □

The Effects of the Number of Conceptions on the Number of Daughters

Suppose the number of conceptions increases from n to $n+1$. Its effect on the parents' choice of the number of daughters depends partly on whether the $(n+1)$-st conception results in a female or a

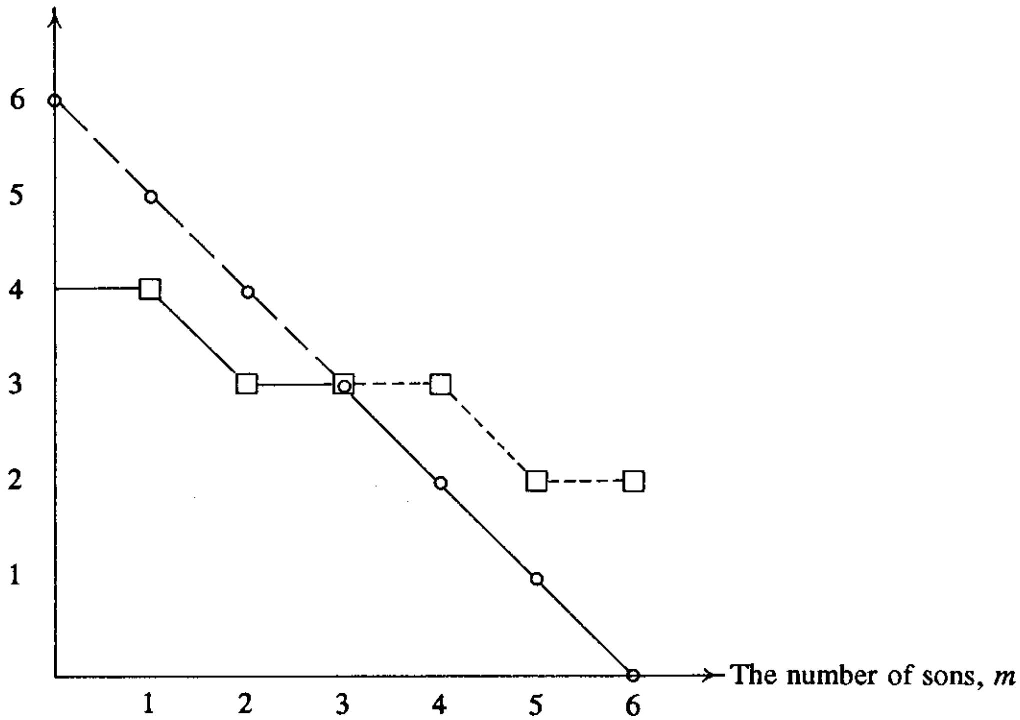

FIGURE 1

The number of conceptions, n, is 6. The circles represent the available number of females, $n-m$. The squares represent the desired number of daughters, $\phi(m)$. The integer values on the solid line represent the parents' choice of the number of daughters, $F(m, n)$.

male. These two outcomes are therefore evaluated separately. This evaluation plays an important role later in the paper.

The additional conception results in a female. Here we compare the parents' choice of the number of daughters in the case in which there are m sons out of $n+1$ conceptions to that in the case in which there are m sons out of n conceptions. The parents' choice is $F(m, n+1)$ in the former case and $F(m, n)$ in the latter. The desired number of daughters is the same in both cases, namely, $\phi(m)$. The available number of females is one larger in the former case; that is, it is $n-m+1$ in the former case, and $n-m$ in the latter.

To make this comparison, assign each of the values of m, from $m = 0$ to n, to one of the following two regimes: with m sons out of $n+1$ conceptions, the parents (i) abort at least one female, and (ii) do not abort any female. From Proposition 2, these regimes are respectively described by $m \leq \mu(n+1) - 1$ and $m \geq \mu(n+1)$. Recalling (10), an alternative description of these regimes, to be used below, is

$$\phi(m) \leq n-m \text{ , and} \tag{14}$$

$$\phi(m) \geq n-m+1 \text{ ,} \tag{15}$$

respectively. In the first regime, the desired number of daughters, $\phi(m)$, is strictly smaller than the available number of females, $n-m+1$. Hence, the parents choose to have $\phi(m)$ daughters. That is, $F(m, n+1) = \phi(m)$. In the second regime, the desired number of daughters is not smaller than the available number of females. Hence, the parents choose to have $n-m+1$ daughters. That is, $F(m, n+1) = n-m+1$.

Now consider the parents' choices, corresponding to each of the above two regimes, (14) and (15), when there are m sons out of n conceptions. It follows from (14) that, under the first regime, the desired number of daughters, $\phi(m)$, is not larger than the available number of females, $n-m$. Hence, the parents choose to have $\phi(m)$ daughters. That is, $F(m, n) = \phi(m)$. Next, consider the second regime. From (15), the desired number of daughters, $\phi(m)$, exceeds the available number of females, $n-m$. Hence, the parents choose to have $n-m$ daughters. That is, $F(m, n) = n-m$. Accordingly, $F(m, n+1)$ is the same as $F(m, n)$ in the first regime, and it is one larger than $F(m, n)$ in the second.

Further, it can be shown that the threshold number of sons, $\mu(n)$, is nondecreasing in the number of conceptions, n. These results are summarized below.

Proposition 5.

$$F(m,\ n+1) = F(m,\ n) = \phi(m) \quad \text{if} \quad m \le \mu(n+1) - 1 \ ; \quad \text{and} \tag{16}$$
$$F(m,\ n+1) = n-m+1 \quad \text{and} \quad F(m,\ n) = n-m \quad \text{if} \quad m \ge \mu(n+1) \ .$$

$$\mu(n+1) \ge \mu(n) \ . \quad \square \tag{17}$$

The additional conception results in a male. Here we compare the parents' choice of the number of daughters in the case in which there are $m+1$ sons out of $n+1$ conceptions to that in the case in which there are m sons out of n conceptions. The parents' choice is $F(m+1,\ n+1)$ in the former case and $F(m,\ n)$ in the latter. The available number of females is the same in both cases, namely, $n-m$. The desired number of daughters is $\phi(m+1)$ in the former case and $\phi(m)$ in the latter. From (11), $\phi(m+1)$ is the same as $\phi(m)$ or one smaller. To distinguish between these two possibilities, define the set

$$\omega \equiv \{m \,|\, \phi(m+1) = \phi(m)\} \ . \tag{18}$$

That is, a value of m belongs to ω if a unit increase in the number of sons does not alter the parents' desired number of daughters.

To make the comparison at hand, consider again the two regimes, (14) and (15). From (16), we already know some properties of the choice $F(m,\ n)$ corresponding to these regimes. We therefore examine, corresponding to each of these two regimes, the choice $F(m+1,\ n+1)$. Under the first regime, it follows from (14) and the first part of (11) that $\phi(m+1) \le n-m$. That is, the desired number of daughters, $\phi(m+1)$, is not larger than the available number of females, $n-m$. Hence, the parents choose to have $\phi(m+1)$ daughters. That is, $F(m+1,\ n+1) = \phi(m+1)$. Recalling (18), $\phi(m+1)$ equals $\phi(m)$ if m belongs to ω; otherwise $\phi(m+1) = \phi(m) - 1$.

Next, consider the second regime. From (15) and the second part of (11), it follows that $\phi(m+1) \ge n-m$. That is, the desired number of daughters, $\phi(m+1)$, is not smaller than the available numbers of females, $n-m$. Hence, the parents choose to have $n-m$ daughters. That is, $F(m+1,\ n+1) = n-m$. Now, the comparison at hand can be made by recalling the values of

$F(m, n)$ from (16). $F(m+1, n+1)$ equals $F(m, n)$ in the first regime if m belongs to ω. The same holds in the second regime. On the other hand, $F(m+1, n+1)$ is one smaller than $F(m, n)$ in the first regime if m does not belong to ω. The proposition below states these conclusions.

Proposition 6.

$$\begin{aligned} &F(m+1,\ n+1) = F(m,\ n) = \phi(m) \text{ if } m \leq \mu(n+1) - 1 \text{ and } m \in \omega\ ; \\ (19)\quad &F(m+1,\ n+1) = \phi(m)-1 \text{ and } F(m,\ n) = \phi(m) \text{ if } m \leq \mu(n+1) - 1 \text{ and } m \notin \omega\ ; \text{ and} \\ &F(m+1,\ n+1) = F(m,\ n) = n-m \text{ if } m \geq \mu(n+1)\ .\ \square \end{aligned}$$

V. THE PARENTS' CHOICES CONCERNING CONCEPTIONS

While choosing the number of conceptions, n, the parents take into account that they will choose the optimal number of daughters in each state-of-the-world they will face. Therefore, substituting $F = F(m, n)$ into (7) and (8), the parents' choice problem becomes

$$(20) \qquad \text{Max}_n : w(n)\ , \quad \text{where}$$

$$(21) \qquad w(n) = -C(n) + \Sigma_m\, b(m,\ n,\ q)[u(m,\ F(m,\ n)) - \ell(n - m - F(m,\ n))]\ .$$

A first step in the analysis of the choice of the number of conceptions is to examine the effect that a change in n has on the parents. From (21),

$$(22) \qquad \begin{aligned} w_n(n) = &-C_n(n) + q\Sigma_m b(m,\ n,\ q)[X(m,\ n) - \ell x(m,\ n)] \\ &+ (1-q)\Sigma_m b(m,\ n,\ q)[Y(m,\ n) - \ell y(m,\ n)]\ , \quad \text{where} \end{aligned}$$

$$(23) \qquad x(m,\ n) \equiv -F(m+1, n+1) + F(m, n)\ ,\ X(m,\ n) \equiv u(m+1, F(m+1, n+1)) - u(m, F(m, n))\ ,$$

$$(24) \qquad y(m,\ n) \equiv 1 - F(m, n+1) + F(m, n),\ \text{and}\ Y(m,\ n) \equiv u(m, F(m, n+1)) - u(m, F(m, n))\ .$$

Expression (22), and (25) to be presented below, are derived in the Appendix.

An explanation of (22) to (24) is as follows. $w_n(n)$ is the marginal valuation that the parents place on undertaking $n+1$ instead of n conceptions. Consider the state-of-the-world in which there are m males out of n conceptions. The probability of this state is $b(m, n, q)$. Further, consider the outcome in which the $(n+1)$-st conception results in a male. The probability of this outcome is q. The parents' choice of the number of daughters changes from $F(m, n)$ to $F(m+1, n+1)$, and the number of available females remains unchanged. Hence, $x(m, n)$, defined in (23), is the extra number of abortions that the parents undertake. Likewise, $X(m, n)$, also defined in (23), is the change in utility. Putting X and x together and recalling that ℓ is the cost per abortion, $X(m, n) - \ell x(m, n)$ is the parents' net gain. A summation of $X(m, n) - \ell x(m, n)$ over the relevant states-of-the-world (i.e., from $m = 0$ to n) leads to the middle part of the expression on the right-hand side of (22).

The last part of the right-hand side of (22) has an analogous explanation. Consider the outcome in which the $(n+1)$-st conception results in a female. The probability of this outcome is $1-q$. For a given m, the parents' choice of the number of daughters changes from $F(m, n)$ to $F(m, n+1)$, and the number of available females increases by one. Hence, $y(m, n)$, defined in (24), is the extra number of abortions that the parents undertake. Likewise, $Y(m, n)$, also defined in (24), is the change in utility. This leads to the last part of the expression on the right-hand side of (22). The first part of this expression represents the increase in the cost of conceptions.

The expression below is helpful in assessing some of the properties of w_n and of the optimal number of conceptions.

$$
\begin{aligned}
w_n(n) = & -C_n(n) \\
& + \Sigma_{\substack{m \leq \mu(n+1)-1 \\ m \in \omega}} b(m, n, q)[qu_M(m, \phi(m)) - (1-q)\ell] \\
& + \Sigma_{\substack{m \leq \mu(n+1)-1 \\ m \notin \omega}} b(m, n, q)\{q[u_M(m, \phi(m)-1) - u_F(m, \phi(m)-1)] - \ell\} \\
& + \Sigma_{m \geq \mu(n+1)} b(m, n, q)[qu_M(m, n-m) + (1-q)u_F(m, n-m)] .
\end{aligned}
\tag{25}
$$

An interpretation of (25) is as follows. Consider the states-of-the-world in which there are m sons out of n conceptions; that is, $m = 0$ to n. Partition these into the following three exhaustive subsets: with m sons out of $n+1$ conceptions, the parents: (i) abort at least one female, and their desired number of daughters does not change if they have $m+1$ sons instead of m, (ii) abort at least one female, and their desired number of daughters changes if they have $m+1$ sons instead of m, and (iii) do not abort any female. These subsets respectively correspond to the three subsets of the values of m in (19). From Propositions 5 and 6, we have some information on how the parents' choice of the number of daughters changes when they undertake $n+1$ instead of n conceptions, and the additional conception results, respectively, in a female and a male. This information, used in conjunction with (22), yields (25).

For example, in the first subset, we know from (18) and (19) that $F(m, n) = \phi(m)$ and that the parents' choice of the number of daughters does not change whether the additional conception results in a female or a male. Hence, the parents gain extra utility $u_M(m, \phi(m))$ if the additional conception results in a male, whereas they undertake one more abortion (incurring the extra cost ℓ) if the additional conception results in a female. The net effect, weighted by the appropriate probabilities and summed over the relevant states, is described in the second part of the expression on the right-hand side of (25). The remaining parts of this expression have analogous interpretations.

The Effects of an Additional Conception on the Number of Abortions

Recall expression (25) for the marginal valuation of an additional conception. The cost of abortion, ℓ, appears in this expression only in those states in which the additional conception alters the number of abortions that the parents undertake. Looking at (25), it is apparent that, for the values of m belonging to the first subset, the parents undertake one extra abortion if the additional conception results in a female, but the same number of abortions otherwise. For the values of m belonging to the second subset, an additional conception leads, regardless of its gender outcome, to an extra abortion. For the values of m belonging to the third subset, an additional conception, regardless of its gender outcome, does not alter the number of abortions. The following proposition is a direct consequence of these observations.

Proposition 7. An additional conception increases the number of abortions of females by one or leaves it unchanged. □

The Effects of the Propensity for Sons and of the Cost of Abortion on the Choice Concerning Conceptions

It can be seen from expression (25) that a change in the parameters λ or ℓ has two effects on the marginal valuation of an extra conception, w_n. First, it has the indirect effect that the desired number of daughters, $\phi(m)$, may change for some or all values of m. Second, even if the $\phi(m)$ do not change for any relevant value of m, a change in λ or ℓ has a direct effect on w_n. Consider those small increases in λ, from λ to λ', and those small decreases in ℓ, from ℓ to ℓ', for which the indirect effect is absent. That is,

$$\phi(m, \lambda') = \phi(m, \lambda) \quad \text{and} \quad \phi(m, \ell') = \phi(m, \ell) \;, \tag{26}$$

for the relevant values of m. Then the following results hold.

Proposition 8.

(a) Consider two sets of parents who have different propensities for sons, but are otherwise identical. The parents with a greater propensity will undertake more or the same number of conceptions.

(b) If the cost of abortion decreases, the parents will undertake a larger or the same number of conceptions. □

To interpret these results, note that, from (25), one can identify the effects of an extra conception on the number of sons and daughters that the parents have in each of the three subsets of the states-of-the-world described there. In the first subset, the parents have one more son and the same number of daughters as before if the additional conception results in a male, and the same numbers of sons and daughters as before if the additional conception results in a female. Hence, a greater propensity for sons implies that the parents' marginal valuation of an extra conception does not decrease in the states in the first subset. It can be ascertained that the same conclusion holds for the states in the other two subsets. Accordingly, a greater propensity for sons does not decrease the overall marginal valuation of an extra conception. Hence, as part (a) of Proposition 8 states, a greater propensity for sons does not decrease the optimal number of conceptions. Next, Proposition 7 shows that the number of abortions does not decrease if the parents undertake one more conception. Accordingly, the marginal valuation of an extra conception does not decrease if the cost of abortion decreases. Hence, as part (b) of Proposition 8 states, a decrease in the cost of abortion does not decrease the optimal number of conceptions.

Note that if the changes in the parameters λ and ℓ are large, then (26) may not hold. From Proposition 4, a larger λ or a smaller ℓ can reduce the desired number of daughters, $\phi(m)$, for some relevant values of m. This indirect effect can alter the marginal valuation of an extra conception. The combination of the direct and the indirect effects of a parameter change will then determine how the parents' optimal choice of conceptions is affected.

VI. SOME EXTENSIONS

This section briefly discusses some of the directions in which the earlier analysis can be extended.

(i) The earlier analysis abstracted from the sequential nature of the decisions to conceive. To a limited extent, the analysis can be reinterpreted in this direction. Consider the parents' last choice of conception, given that they already have m' sons and F' daughters from their previous choices. Suppose the marginal utility of a son does not increase if the parents have one more son; that is, $u_{MM} \leq 0$. Replace u in (4) by $u(m' + m, F' + F)$. Restrict n to be one or zero, representing respectively whether or not the parents choose to undertake the last conception. Let $n(m', F')$ denote the parents' optimal choice of n. If the parents undertake this conception and it results in a female, then let $\bar{F}(m', F')$ denote the parents' optimal choice of F. That is, $\bar{F}(m', F')$ is one if the parents choose not to abort the female; otherwise it is zero. Using these, one can examine some properties of the last choice of conception, and of the subsequent choice of selective abortion in the case that the parents undertake this conception. As an illustration, it can be shown that the following results hold.

Proposition 9.

(a) Suppose that from their previous choices the parents have either a larger number of sons or a larger number of daughters. Then, the last conception will not be more attractive for them. If they undertake this conception and it results in a female, then aborting it will not be less attractive for them. That is, $n(m', F')$ and $\bar{F}(m', F')$ are nonincreasing in m' and F'.

(b) Suppose that from their previous choices the parents have one fewer son and one more daughter. If they undertake the last conception and it results in a female, then aborting it will not be less attractive for them. That is, $\bar{F}(m', F') \geq \bar{F}(m'-1, F'+1)$. □

A further implication of the above proposition is as follows. From parts (a) and (b), respectively, $\bar{F}(m', F') \geq \bar{F}(m', F'+1)$ and $\bar{F}(m', F') \geq \bar{F}(m'-1, F'+1)$. These yield

$$(27) \qquad \bar{F}(m', F') \geq \bar{F}(m'', F'') \text{ if } m'' + F'' \geq m' + F' \text{ and } F'' \geq F' .$$

That is, suppose that from their previous choices the parents have the same or a larger number of children (daughters plus sons) but no fewer daughters. If they undertake the last conception and it results in a female, then aborting it will not be less attractive for them.

Some recent gender ratios at birth have been observed to be larger for higher-order births. For South Korea for 1989, a reported gender ratio at birth is 217.2 for the women giving their fourth birth, compared to 113.6 for those giving their second birth (National Bureau of Statistics (1990, p. 61)). A corresponding set of numbers for China for 1986 are 129.1 and 117.3 (Hull (1991, p. 68)).

Now consider two large groups of parents who have undertaken their last conceptions. The distribution of the function u and the parameter ℓ across the parents within each group is the same for both groups. Suppose that from their previous choices the parents in the first group have a larger number of sons or daughters, or both, as compared to what the parents in the second group have. Hence, the birth from the last conception, if any, will be a higher-order birth for the parents in the first group. It follows from part (a) of Proposition 9, then, that the resulting gender ratio at birth will not be smaller for the first group. Though this comparison is restricted to the births undertaken by parents with particular profiles, it suggests a pattern of gender ratios at birth that is consistent with the observations in the preceding paragraph.

Next, suppose that the parents in the first group described above have from their previous choices no fewer children and no fewer daughters, as compared to what the parents in the second group have. Then, expression (27) implies that, for the births under consideration, the gender ratio at birth will, once again, not be smaller for the first group.

(ii) In the specification in (4), the parents' valuation of the choice (n, F) is $u(m, F) - \ell(n-m-F) - C(n)$ if n conceptions result in m sons. Instead, suppose this valuation is

$$(28) \qquad u(m, F) - K(n, n-m-F) .$$

That is, instead of having additive costs of conceptions and abortions, (28) specifies a function that aggregates these costs and that depends jointly on the number of conceptions and on the number of abortions. Let $a \equiv n-m-F$ denote the number of abortions. Suppose $K_{aa} \geq 0$ and $K_{na} \geq 0$. That is, K is convex in the number of abortions, and the marginal change in K due to an extra abortion is not smaller if the number of conceptions is larger. Suppose that an increase in the cost of abortion is now represented by a change in K_a such that it is larger for at least one value of (n, a) and not smaller for any value of (n, a).

Then, it can be ascertained that the above reformulation yields Propositions 1 to 4. The desired number of daughters now may depend on the number of conceptions; accordingly, this number is now expressed as $\phi(m, n)$. Further, it can be shown that if the parents undertake a larger number of conceptions, then their desired number of daughters is not smaller. That is, $\phi(m, n+1) \geq \phi(m, n)$. The reason for this conclusion is as follows. For any given number of daughters that the parents may wish to have, a larger number of conceptions implies a larger number of abortions. Since both the number of conceptions and the number of abortions increase, the marginal cost of an abortion does not decrease. Hence, the marginal net gain from having one more daughter does not decrease.

(iii) Some of the aspects of fertility from which the model of the parents' choice in (4) abstracts were noted in Section II. The model also abstracts from, but can easily incorporate, such aspects as the stochastic nature of conceptions, unintended conceptions, miscarriages, and abortions that are not gender-specific (e.g., an abortion undertaken without ascertaining the gender of the fetus, or that undertaken for medical considerations).

The earlier analysis was based on the assumption that the probability that a given conception results in a male, q, is one-half. The relevant probability here is that perceived by the parents. It can be verified that all of the propositions in the paper remain unchanged if q is larger than one-half. If q is smaller than one-half, then all of the propositions except part (a) of Proposition 8 remain unchanged. With the same exception, the propositions also hold if the parents' choices are based on a probability distribution of q, rather than on any single value of q.

V. SOME REMARKS

Remarks on the Use of Indirect Utility Functions

Consider the following decision problem based on a direct utility function. The parents first choose the number of conceptions, n. Then, after learning the genders of their conceptions, they choose the number of daughters, F, as well as make a number of other choices that are denoted by the vector z. Let $h(m, F, n, z)$ denote the parents' direct utility if their choice is (n, F, z) and if n conceptions result in m sons. Let the vector inequality $H(m, F, n, z) \leq 0$ denote the vector of constraints that the parents face in addition to the constraint $n-m \geq F \geq 0$. The preceding constraints and the direct utility incorporate the relevant considerations of the parents, including the costs of conceptions and abortions. Then the parents' choice problem can be stated as

$$\text{(29)} \qquad \text{Max}_n : \Sigma_m\, b(m, n, q))[\text{Max}_F : \{J(m, F, n) | n\text{-}m \geq F \geq 0\} \text{ , where}$$

$$\text{(30)} \qquad J(m, F, n) = \text{Max}_z : \{h(m, F, n, z) | H(m, F, n, z) \leq 0\} \text{ .}$$

That is, $J(m, F, n)$ is the parents' indirect utility after netting out the vector of choices z. It is apparent from (29) that the specifications in (4) and (28) are respectively based on the simplifications that

$$\text{(31)} \quad J(m, F, n) = u(m, F) - C(n) - \ell(n-m-F) \text{ , and } J(m, F, n) = u(m, F) - K(n, n-m-F) \text{ .}$$

A primary motivation for simplifications such as those in (31) is their tractability, which is important for qualitative analyses such as the one presented in this paper. For similar motivations, a variety of simplifications as well as a number of functional forms of indirect utility functions have been employed for studying many economic problems (see, for example, Blackorby, Primont and Russell (1978) and Diewert (1982)).

Note that a different kind of assumption is implicit in the use of direct utility functions. Suppose that the researcher's objective is not limited to describing a choice problem, such as (29), but that it includes analyzing properties of the optimal choices. Then, the researcher is typically restricted to dealing explicitly with only some of the choice variables, leaving out a potentially large number of other choices. For example, an analysis of fertility-related choices may typically incorporate the choice between the parents' own consumption and the expenditures on children, including on children's quality, but may leave out an explicit treatment of labor-leisure choices, household production choices, human capital formation choices, choices of the composition of the consumption bundle, and a myriad other choices. In such cases, the researcher typically makes some tacit assumptions, for example, that the choices that are not explicitly considered are being netted out, in a manner analogous to that in the definition of J above, or that these choices have been made outside the model and are fixed at those levels.

Remarks on Technological Innovations

Innovations that potentially affect fertility-related choices continue to take place. One aspect of these innovations is that parental actions that were technologically infeasible earlier become feasible (e.g., gender-specific abortions). Another aspect is an almost continual reduction in the overall costs and inconveniences of undertaking such actions. Yet another aspect is that the rate at which the knowledge of some of these innovations diffuses across populations is rapid. For example, amniocentesis was introduced in South Asia for research-related purposes around mid-1970s. It appears a reasonable guess that, by now, a vast majority of the parents in this region already know that this technology can be used for gender-specific abortions, even though many of them may be unaware of the medical uses of this technology, and even though it may presently be too expensive or inaccessible for many of them.

A natural next stage of innovation would be the technological and economic feasibility of gender-preselection. It is believed that even though most of the existing approaches to gender-preselection are currently impractical and clinically unproven, several of them show partial effectiveness under experimental conditions (see Zarutskie *et al.* (1989) for a survey of the medical literature). Given the rate of scientific advance in this area, it is likely that inexpensive technologies that make gender-preselection possible to a significant degree will become available in the foreseeable future. If so, the fertility-related choices of many parents in several less developed countries and newly industrialized countries may undergo changes analogous to, if not more significant than, what may be induced by gender-specific abortions, as they continue to become less expensive and more convenient and widely available.

APPENDIX

Proof of Lemma 1. For $i \geq 1$, define $\xi(s, i, \alpha) \equiv G(s, \alpha) - G(s-i, \alpha)$. The definition of G_s implies

(A1) $$\xi(s, i, \alpha) = \Sigma_{j=0}^{i-1} G_s(s-j-1, \alpha) .$$

Since $s(\alpha)$ is the largest optimal value of s at α, it follows from the definition of a that

(A2) For any given s^*, if $\xi(s^*, i, \alpha) \geq 0$ for $i \geq 1$ then $s(\alpha) \geq s^*$.

By the hypothesis of (2), $G_s(s-k-j-1, \beta) \geq G_s(s-j-1, \alpha)$. Using (A1), summation of the preceding inequality from $j = 0$ to $i-1$ yields $\xi(s-k, i, \beta) \geq \xi(s, i, \alpha)$. Substitution of $s = s(\alpha)$ in the preceding yields $\xi(s(\alpha) - k, i, \beta) \geq \xi(s(\alpha), i, \alpha)$. The first part of (1) yields $\xi(s(\alpha), i, \alpha) \geq 0$. Hence, $\xi(s(\alpha) - k, i, \beta) \geq 0$ for $i \geq 1$. In turn, (A2) yields (2). The proof of (3) is analogous.

Proof of Proposition 1. Define $v(m, \phi) \equiv u(m, \phi) - \ell(n-m-\phi)$. The argument n of the function v is suppressed because it is not relevant here. Hence,

(A3) $$v_\phi(m, \phi) = u_F(m, \phi) + \ell .$$

From (5) and (A3), $v_\phi(m+1, \phi) - v_\phi(m, \phi) = u_{MF}(m, \phi) \leq 0$. In turn, the use of (3) yields $\phi(m+1) \leq \phi(m)$, which is the first part of (11). Next, (5) and (A3) yield $v_\phi(m+1, \phi-1) - v_\phi(m, \phi) = u_F(m+1, \phi-1) - u_F(m, \phi) = u_{MF}(m, \phi-1) - u_{FF}(m, \phi-1) \geq 0$. In turn, the use of (2) yields $\phi(m+1) \geq \phi(m) - 1$, which is the second part of (11).

Proofs of Propositions 2, 3 and 4. Define

(A4) $$\eta(m) \equiv \phi(m) + m .$$

From (10) and the second part of (11), respectively,

(A5) $$F(m, n) = -m + \text{Min}\{\eta(m), n\} , \quad \text{and}$$

(A6) $$\eta(m+1) \geq \eta(m) .$$

Define $\mu(n)$ to be the smallest nonnegative value of m for which $\eta(m) \geq n$. That is,

(A7) $$\mu(n) \equiv \text{Min}_m : \{m \mid \eta(m) \geq n, m \geq 0\} .$$

From (9), $\phi(m)$ is nonnegative. Hence, (A4) implies $\eta(n) \geq n$. From (A7), in turn, $\mu(n)$ exists, it is unique, and $n \geq \mu(n) \geq 0$. Further, (A6) and (A7) imply

(A8) $$\eta(m) \leq n-1 \text{ iff } m \leq \mu(n) - 1 , \text{ and } \eta(m) \geq n \text{ iff } m \geq \mu(n) .$$

Expressions (12) and (13) follow from (A4), (A5) and (A8).

To prove Proposition 3, let $f \equiv n-m$ denote the number of females and let $a(m, n) \equiv f-F(m, n)$ denote the number of abortions. From (12) and (13) respectively,

(A9) $$a(m, n) \geq 1 \text{ if } f \geq n - \mu(n) + 1 , \text{ and}$$

(A10) $$a(m, n) = 0 \text{ if } f \leq n - \mu(n) .$$

From (A10), if $a(m, n) = 0$ for $f = f^*$, then, as part (a) of the proposition states, $a(m, n) = 0$ for $f < f^*$. Likewise, from (A9), if $a(m, n) \geq 1$ for $f = f^{**}$, then, as part (b) of the proposition states, $a(m, n) \geq 1$ for $f > f^{**}$. Further, if $a(m, n) \geq 1$ for $f = n$, and $a(m, n) = 0$ for $f = 1$, then, from (A9) and (A10), $n-1 \geq n - \mu(n) \geq 1$. This and (A9) and (A10) yield part (c) of the proposition.

To prove Proposition 4, consider $\lambda' > \lambda$, and $\ell' < \ell$. From (6) and (A3), $v_\phi(m, \phi, \lambda') \leq v_\phi(m, \phi, \lambda)$. From (A3), $v_\phi(m, \phi, \ell') < v_\phi(m, \phi, \ell)$. These inequalities and (3) yield $\phi(m, \lambda') \leq \phi(m, \lambda)$ and $\phi(m, \ell') \leq \phi(m, \ell)$. In turn, (10) yields $F(m, n, \lambda') \leq F(m, n, \lambda)$ and $F(m, n, \ell') \leq F(m, n, \ell)$, as stated in the proposition.

Proofs of Propositions 5 and 6. Expressions (A6) and (A7) yield (17). From (A4) and (A5),

(A11) $$F(m, n) = \phi(m) \text{ if } \eta(m) \leq n , \text{ and } F(m, n) = n-m \text{ if } \eta(m) \geq n+1 .$$

From (A5), $F(m, n+1) = -m + \text{Min}\{\eta(m), n+1\}$. This and (A4) yield

(A12) $$F(m, n+1) = \phi(m) \text{ if } \eta(m) \leq n , \text{ and } F(m, n+1) = n-m+1 \text{ if } \eta(m) \geq n+1 .$$

From (A8),

(A13) $$\eta(m) \leq n \text{ iff } m \leq \mu(n+1)-1 , \text{ and } \eta(m) \geq n+1 \text{ iff } m \geq \mu(n+1) .$$

Expressions (A11), (A12) and (A13) yield (16).

To prove Proposition 6, note from (11) and (A4) that $\eta(m+1) = \eta(m)$ or $\eta(m) + 1$. Hence, if $\eta(m) \leq n$ then $\eta(m+1) \leq n+1$, and if $\eta(m) \geq n+1$ then $\eta(m+1) \geq n+1$. Next, from (A5),

$F(m+1, n+1) = -(m+1) + \text{Min}\{\eta(m+1), n+1\}$. The preceding relationships and (A4) yield

(A14) $F(m+1, n+1) = \phi(m+1)$ if $\eta(m) \leq n$, and $F(m+1, n+1) = n-m$ if $\eta(m) \geq n+1$.

Expressions (18), (A11), (A13) and (A14) yield (19).

Derivation of (22) and (25). The following result holds for any function g. If $G(n) \equiv \Sigma_m b(m, n, q)g(m, n)$, then

(A15) $$G_n(n) = \Sigma_m b(m, n, q)[qg_m(m, n+1) + g_n(m, n)] .$$

This result can be proved using: (i) the partial summation formula $\Sigma_m b(m, n, q)g(m, n) = -\Sigma_{m=0}^{n-1} B(m, n, q)g_m(m, n) + B(n, n, q)g(n, n)$, where $B(m, n, q) \equiv \Sigma_{j=0}^{m} b(j, n, q)$ is the cumulative binomial probability, and (ii) the identity $B(m, n+1, q) - B(m, n, q) = -qb(m, n, q)$, which follows from expression (10.7) in Feller (1968, p. 173). Now, recall (21) and substitute $g(m, n) = u(m, F(m, n)) - \ell(n - m - F(m, n))$ into (A15). This and a series of rearrangements yield (22).

The derivation of (25) is as follows. If $m \in \omega$ and $m \leq \mu(n+1) - 1$, then, from (16) and (19), $F(m, n) = F(m, n+1) = F(m+1, n+1) = \phi(m)$. Hence, (23) and (24) yield $x(m, n) = 0$, $X(m, n) = u_M(m, \phi(m))$, $y(m, n) = 1$ and $Y(m, n) = 0$. Next, if $m \notin \omega$ and $m \leq \mu(n+1) - 1$, then, from (16) and (19), $F(m, n) = F(m, n+1) = \phi(m)$ and $F(m+1, n+1) = \phi(m) - 1$. Hence, (23) and (24) yield the same values of $Y(m, n)$ and $y(m, n)$ as before, but now $X(m, n) = u(m+1, \phi(m)-1) - u(m, \phi(m)) = u_M(m, \phi(m)-1) - u_F(m, \phi(m)-1)$ and $x(m, n) = 1$. Finally, if $m \geq \mu(n+1)$, then, from (16) and (19), $F(m, n) = F(m+1, n+1) = n-m$, and $F(m, n+1) = n-m+1$. Hence, (23) and (24) yield $x(m, n) = 0$, $X(m, n) = u_M(m, n-m)$, $y(m, n) = 0$ and $Y(m, n) = u_F(m, n-m)$. Substitution of the preceding three sets of expressions for x, X, y and Y into (22) yields (25).

Proof of Proposition 8. From (26), (A4) and (A7), the changes in the parameters λ and ℓ do not alter $\mu(n+1)$. Hence, (6), (25), and the assumption that $q = 1/2$, yield $w_n(n, \lambda' | \phi(m, \lambda') = \phi(m, \lambda)) \geq w_n(n, \lambda)$ and $w_n(n, \ell' | \phi(m, \ell') = \phi(m, \ell)) \geq w_n(n, \ell)$. In turn, the use of (2) yields the desired results. That is, if $n(\lambda)$ and $n(\ell)$ denote respectively the optimal choices of n at λ and ℓ, then $n(\lambda') \geq n(\lambda)$ and $n(\ell') \geq n(\ell)$.

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