Persistence and pervasiveness of corruption: New perspectives

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ABSTRACT

Among the questions which this paper addresses are, how does a culture of corruption perpetuate itself over time, what effects do different features of the economy have on the phenomenon of corruption, and why the culture of corruption might or might not alter over time?

A central feature of the present dynamic analysis is that rational individuals (citizens as well as bureaucrats) learn from their past experiences. The past economic environment thus affects current choices of individuals which, in turn, influence the future economic environment. As a result, if bureaucratic corruption has been more pervasive in the past, then different citizens are more likely to choose those behaviors (such as more extensive cheating) which induce a greater pervasiveness of corruption in the future. These inter-temporal behavioral externalities are formalized within an overlapping generations framework, and the resulting aggregate corruption and cheating is characterized and analyzed.

This positive analysis yields new perspectives as well as new results. I examine how the pervasiveness of corruption and cheating is altered by such features of the economy as the extent of government intervention, and the beliefs of the youngest generations. Among the results is that if youngest generations of citizens believe that corruption is more pervasive, then corruption actually becomes more pervasive.

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I. INTRODUCTION

The phenomenon of bureaucratic corruption is sufficiently pervasive in developing economies that it has not escaped the attention of any except the most casual observers of these economies. No matter what the nature of a transaction between the government and a member of the public might be (for example, whether the transaction involves paying taxes, receiving social benefits, or protecting one's property rights of one kind or another), it is the case more often than not that the transaction entails some degree of corruption and illegality. In fact, the problem might go deeper than that. To the extent that changes are occurring in many of these societies (for instance, new production and organization activities undertaken by private individuals and the government), the nature of change itself seems to be guided by the logic and culture of corruption.

Some development economists have also recognized the central role that corruption (and associated features such as bureaucratic ineptness, apathy and abusiveness) can play in determining the generation and distribution of real incomes in LDCs.[1] In fact, it has often been argued that the oft-observed failures of development projects and policies during the last three decades have been caused in large part by the fact that most of these projects and policies were formulated and evaluated within paradigms which abstracted from the phenomenon of corruption.

The kinds of questions which the present paper addresses are, how does a culture of corruption perpetuate itself over time, what effects do different features of the economy have on the phenomenon of corruption, and why might the culture of corruption alter or not alter over time? These questions are fundamental to an understanding of the economic environment in many LDCs, and our analysis of the questions yields a number of qualitative results as well as new perspectives.

A central element of our analysis consists of the following kinds of "dynamic behavioral externalities" which rational individuals exert on one another. An individual's choice of current behavior is in part based on his past experiences. This is because the current choice of an individual must necessarily be based on what his beliefs are concerning the nature of economic environment, and these beliefs are influenced by his past experiences. In turn, the current behavioral choices of different individuals influence the current experiences and, hence, the future behavior of other individuals in the economy. For instance, if the past experiences of the members of the public (who, for brevity, are referred as "citizens") have convinced them that bureaucratic corruption is pervasive in the economy, then these individuals are more likely to undertake those activities which are more beneficial to them in the presence of pervasive corruption (I use the short-hand expression "cheating" to refer to such activities). Such choices made by the citizens, in turn, make it more likely that the bureaucrats would find it desirable to adopt those kinds of behaviors which sustain a high level of corruption in the future. In this sense, a greater prevalence of corruption in the past induces a greater prevalence of corruption in the future.[2]

A natural framework within which to analyze the above dynamics is the overlapping generations model, because it provides an explicit basis to link the behaviors of different generations of individuals. Thus, in our analysis, each cohort lives for a finite length of time, and the behavior of the members of each generation influences (and is influenced by) the behavior of the members of other--older as well as younger--generations which are living contemporaneously. Another key advantage of this framework is that it allows significant differences in the beliefs (and hence in the behavioral choices) of similar individuals. For instance, two citizens may believe quite differently (one may believe that there is extensive corruption, while the other believes that corruption is negligible) if their past experiences are different, even though they belong to the same generation, face the same economic trade-offs, and have started their lives with the same initial beliefs. This implication of the model is consistent with the diversity in individuals' beliefs which has often been observed.[3]

The analysis developed in this paper is <u>positive</u>. It takes as given the legal and administrative structure which influences individuals' incentives. The relevant aspects of the legal and administrative structure are represented in our model through exogenous parameters, and the effects of changes in these parameters (or, alternatively, the consequences of differences in these parameters between two economies) are traced on the behaviors of different individuals as well as on the aggregate economic environment. The paper does not deal with the normative question of how a society should set its legal and administrative structure.

A related aspect which is noteworthy is that our analysis is robust to many of the details of the legal and administrative structure. For instance, what is relevant for our model of a bureaucrat's choice is a simple property that it is more attractive for an individual bureaucrat to be corrupt (or to be corrupt to a greater extent) if there is a greater prevalence of cheating among the citizens who have to deal with him. This intuitive property would be satisfied in a variety of contexts (for instance, whether the corruption is in the context of public procurement or in the context of tax collection), under a variety of bureaucratic structures (for instance, how a bureaucrat is policed and whether those responsible for policing are themselves potential participants in corruption), for a variety of mechanisms through which a bureaucrat might detect cheating among citizens (which includes the trade-off between the effort spent by the bureaucrat and the probability of his being able to detect a cheating citizen), and under a variety of reward and punishment schedules[4] (for instance, the amounts and types of bribes involved, and the punishment associated with the conviction of a corrupt bureaucrat[5]).

Since the present paper treads on ground which is relatively unexplored, I have used several simplifying assumptions to bring out what appear to me to be some of the novel aspects of the analysis. The basic model is developed in Section II, and it is analyzed in Section III. In Section IV, I show how the simplifying assumption of the basic model can be modified. Concluding remarks highlighting some of the central features of the analysis are presented at the end.

II. THE BASIC MODEL

In each period a new cohort of bureaucrats and citizens enters the economy. The life span of an individual is T periods, where $T \geq 2$. Individuals begin their lives with diverse beliefs concerning the relevant features of the economic environment (the relevant features for a bureaucrat and a citizen respectively are the extent to which cheating and corruption are prevalent in the economy),[6] and revise their beliefs based on their experiences as they progress through life. I begin the analysis with the following simple specification.

In each period a citizen encounters one bureaucrat, and a bureaucrat encounters a certain number of citizens (this number is denoted at present by the parameter M). Individuals face binary choices in each period: a bureaucrat's choices are to be corrupt or not corrupt, and a citizen's choices are to cheat or not cheat. The actual participants of different encounters are determined randomly; that is, there is an equal probability that a citizen might encounter any one of the large number of bureaucrats in the economy, and there is an equal probability that one of the even larger number of citizens in the economy may belong to the subset of individuals whom a bureaucrat encounters within a particular period. An individual must make his behavioral choice before the actual encounter; the choice is therefore determined in part by the individual's beliefs and past experiences.

The description and analysis of the choices of a citizen and a bureaucrat is presented in the next three subsections. This is followed by a characterization of a societal equilibrium in corruption and cheating.

A. A Citizen's Choices

For a citizen who chooses to cheat in a particular period, the (expected) utility is u^{00} if he encounters a corrupt bureaucrat, and the utility is u^{01} if he encounters a bureaucrat who is not corrupt. The corresponding utilities for a citizen who chooses not to cheat are u^{10} and u^{11}. I assume that $u^{00} > u^{10}$, and $u^{11} > u^{01}$. That is, cheating is more profitable for a citizen if he were to encounter a corrupt bureaucrat, and not cheating is more profitable otherwise.[7] By making this assumption, I also abstract from those cases where a citizen's choice is trivial in the sense that it is entirely unaffected by what his beliefs are.

The choice of a citizen in each period depends therefore on what his mean estimate of the "level of corruption" (that is, the proportion of corrupt bureaucrats out of the total population of bureaucrats) is, because this estimate represents the probability which this citizen associates with the event of encountering a corrupt bureaucrat. Let $\hat{C}(t, s(t), a)$ denote this estimate for a citizen who has lived for t periods, who has encountered s(t) corrupt bureaucrats during the past t periods (obviously, $t \geq s(t) \geq 0$), and whose characteristics are denoted by the vector a. It follows then that this citizen will choose to cheat in the (t + 1)st period of his life if and only if[8]

(1) $$\hat{C}(t, s(t), a) \geq u,$$

where $u = (u^{11} - u^{01})/[(u^{00} - u^{10}) + (u^{11} - u^{01})]$. The summary parameter u can be viewed as the "relative cost of cheating" because it is increasing in u^{11} and u^{10}, and it is decreasing in u^{01} and u^{00}. Also, our assumptions concerning the pay-offs imply that $1 > u > 0$.

The estimate $\hat{C}$ is in general increasing in $s(t)$. This is because if a citizen has encountered a larger number of corrupt bureaucrats in the past, then his current estimate of the level of corruption must be larger. The criterion (1) thus can be reexpressed as:

$$(2) \qquad s(t) \geq \bar{r}(t, u, a),$$

where $\bar{r}$ is defined implicitly by the equation $\hat{C}(t, \bar{r}, a) = u$. The economic meaning of (2) is transparent. A citizen chooses to cheat in a particular period of his life if the number of his past encounters with corrupt bureaucrats exceeds his "reservation level," $\bar{r}$, for that period.

The above description of a citizen's choices does not depend in any way on what his initial beliefs are. To make the analysis more tractable, however, I shall assume that the initial beliefs of a citizen can be represented by a beta distribution with parameters $a \equiv (a_1 > 0, a_2 > 0)$. This assumption is not specially restrictive because other types of initial beliefs can be approximated to a reasonable degree by a beta distribution with appropriately chosen parameters. The assumption of beta initial beliefs yields the following simple expression for $\hat{C}$.[9]

$$(3) \qquad \hat{C}(t, s(t), a_1, a_2) = \frac{a_1 + s(t)}{a_1 + a_2 + t} .$$

The initial estimate of the level of corruption (that is, the estimate with which a citizen begins his life) is obtained by substituting $s(t) = t = 0$ into (3). This yields

$$(4) \qquad \hat{C}(0,\ 0,\ a_1,\ a_2) = \frac{a_1}{a_1 + a_2}\ .$$

The effect of parameters (a_1, a_2), which play an important role in later analysis, can be seen from (4). A citizen with a larger a_1 or a smaller a_2 believes in the beginning of his life that the level of corruption in the economy is higher.

The main advantage of the above assumption concerning the functional form of initial beliefs is that it yields a closed-form solution for the reservation level $\bar{r}$. In fact, using (3) in combination with (1) and (2), one obtains

$$(5a) \qquad \bar{r}(t,\ u,\ a_1,\ a_2) = (a_1 + a_2 + t)u - a_1\ .$$

To complete the model of a citizen's behavior, I make a minor modification which turns out to be useful later on. Note that the reservation level, $\bar{r}$, is in general not an integer, whereas the number of corrupt bureaucrats a citizen has encountered in the past, $s(t)$, is an integer. To make the two compatible, I define an integer version of $\bar{r}$; that is,

$$(5b) \qquad r = [\bar{r}]_+$$

where the symbol $[\ ]_+$ denotes the standard "rounding off" function; that is, the function is same as its argument if the argument is an integer, otherwise the function equals the next integer higher than the argument. For brevity, r is referred to as the "reservation score."

To summarize, then, the criterion which a citizen employs for his choice in the $(t + 1)$st period of his life is:

(6) Cheat if and only if $s(t) \geq r(t, u, a_1, a_2)$,

where r is given by (5a) and (5b).

B. A Bureaucrat's Choices

For simplicity I assume at present that if a bureaucrat is corrupt during a particular period then his utility is linearly increasing in b which denotes the number of citizens who have chosen to cheat among those whom the bureaucrat encounters during that period. The utility of a corrupt bureaucrat is thus denoted by $U^0(b) = U^{01} + bU^{00}$, where U^{00} is positive. Obviously, $M \geq b \geq 0$. The utility of a bureaucrat who is not corrupt is denoted by U^1. I assume that $U^0(M) > U^1 > U^0(0)$. That is, the utility of a bureaucrat who is not corrupt lies between the maximum possible utility of a corrupt bureaucrat (which occurs when all citizens whom he encounters have chosen to cheat) and the minimum possible utility of a corrupt bureaucrat (which occurs when none of the citizens whom he encounters have chosen to cheat).[10]

Since the description of a bureaucrat's choice is analogous to that of a citizen's choice discussed earlier, I leave out the details and only note those expressions which are relevant for later analysis. Let $\hat{c}(t, S(t), M, A)$ denote the "level of cheating" in the economy (that is, the proportion of citizens who are cheating) as estimated by a bureaucrat who has lived for t periods, who has found S(t) citizens to be cheating among those whom he has encountered during the past t periods, and whose characteristics are denoted by the vector A. Then, this bureaucrat would choose to be corrupt in the $(t + 1)$st period of his life if and only if

$$(7)\qquad \hat{c}(t,\ S(t),\ M,\ A) \geq U,$$

where the parameter $U = (U^1 - U^{01})/MU^{00}$ represents the "relative cost of corruption" because it is increasing in U^1 and it is decreasing in U^{01} and U^{00}. Also, it is apparent that $1 > U > 0$.

If $\bar{R}(t, U, M, A)$ represents the bureaucrat's reservation level, then (7) is equivalently represented as

$$(8)\qquad S(t) \geq \bar{R}(t,\ U,\ M,\ A).$$

When a bureaucrat's initial beliefs are represented by a beta distribution with parameters $A \equiv (A_1 > 0,\ A_2 > 0)$, then we obtain the following expressions which have familiar interpretations

$$(9)\qquad \hat{c}(t,\ S(t),\ M,\ A_1,\ A_2) = \frac{A_1 + S(t)}{A_1 + A_2 + tM},$$

$$(10)\qquad \hat{c}(0,\ 0,\ M,\ A_1,\ A_2) = A_1/(A_1 + A_2)\ , \quad \text{and}$$

$$(11a)\qquad \bar{R}(t,\ U,\ M,\ A_1,\ A_2) = (A_1 + A_2 + tM)U - A_1\ .$$

The role of parameters (A_1, A_2) can be seen from (10). A bureaucrat with a larger A_1 or a smaller A_2 believes in the beginning of his life that the level of cheating is higher in the economy. Finally, if

$$(11b)\qquad R = [\bar{R}]_+$$

represents the reservation score, then the behavioral choice of a bureaucrat, in the $(t + 1)$st period of his life, is determined by the criterion:

(12) Be corrupt if and only if $S(t) \geq R(t, U, M, A_1, A_2)$

where R is given by (11a) and (11b).

C. Properties of Individuals' Behavior

The patterns of an individual's behavior predicted by the preceding models are appealing. Consider for instance a citizen who had chosen to cheat in the last period. Then the decision rule (6) predicts that this citizen will also choose to cheat in the current period if he has encountered a corrupt bureaucrat in the last period.[11] Likewise, if a citizen had chosen not to cheat in the last period and had encountered a bureaucrat who is not corrupt, then he will choose not to cheat in the current period as well. The reason is simple. Since an individual chooses the most profitable behavior in every period, given his current beliefs, he has no reason to alter his behavior in the next period if his recent experiences have not contradicted his previously held beliefs.

Changes in the behavior of a bureaucrat between any two successive periods can be analyzed similarly. For instance, if a bureaucrat was corrupt in the last period, and if most of the citizens whom he encountered in the last period had chosen to cheat, then this bureaucrat will be corrupt in the current period as well. These results are summarized as follows.

PROPOSITION 1. A bureaucrat or a citizen does not alter his behavior between two successive periods unless he has been "disappointed" by his actual experience; that is, unless his recent experience was different from what would have been most profitable to him, given his choice of behavior in the last period.

We next note some intuitive properties of an individual's reservation score which are useful in later analysis. The dependence of an individual's reservation score on his age and on the relative cost of alternative behaviors follows directly from (5a), (5b), (11a) and (11b):

(13) $\partial r/\partial t \geq 0$, $\partial r/\partial u \geq 0$, and

(14) $\partial R/\partial t \geq 0$, $\partial R/\partial U \geq 0$.

That is, an older individual can not have a smaller reservation score; and if the relative cost of cheating (corruption) is higher, then the reservation score of any citizen (bureaucrat) can not be smaller.

Finally, consider the effect of an individual's initial beliefs on his reservation scores. One would expect the initial beliefs to play a more significant role in influencing an individual's choice in the earlier phase of his life because he has that much less experience during this phase. But at the same time, given our assumption that the individual's life span is finite, the initial beliefs of an individual continue to exert a systematic effect on his reservation scores throughout his life. To see one such effect, consider a bureaucrat who was initially corrupt (that is, he was corrupt in the first period of his life) and compare him to another bureaucrat who was initially not corrupt. Then it is straightforward to ascertain that:[12]

PROPOSITION 2. <u>An initially corrupt bureaucrat can not have a smaller reservation score during any period of his life than the corresponding reservation score of a bureaucrat who was not initially corrupt</u>.

D. Equilibrium in Aggregate Behavior

Based on the preceding models of behavior for a bureaucrat and a citizen, or based on other similar models, it is straightforward to understand the evolution of the economy. The subset of citizens who choose to cheat in the current period are those for whom the condition (6) is satisfied. In turn, how large this subset is depends stochastically on how prevalent bureaucratic corruption has been in the past. Likewise, the subset of bureaucrats who choose to be corrupt in the current period are those for whom the condition (12) is satisfied, and the size of this subset is determined by the extent to which citizens have chosen to cheat in the past. These two groups thus exert explicit behavioral externalities on one another. Therefore, given a set of initial conditions and a set of parameters representing the economy, it is possible to characterize the stochastic time-pattern of individuals' aggregate behavior in the economy.

I focus in this paper on analyzing steady-state equilibria in which the level of corruption or the level of cheating does not alter from period to period.[13] Let C and c denote respectively the steady-state levels of corruption and cheating. The set of equations which determine the steady-states are derived below.

Steady-state level of cheating: Let c^{t+1} denote the level of cheating in the (t + 1)st generation; that is, in the generation which has already lived for t periods. Then $c = (\sum_{t=0}^{T-1} c^{t+1})/T$. The level of cheating in the youngest generation is obtained directly from the parameters of the economy because the citizens in this generation have had no exposure to the economic environment. In fact, for t = 0 and s(t) = 0, expressions (2) and (5a) yield

$$(15) \qquad c^1 = \int_{a_1 \geq a_2 u/(1-u)} dg_1(a_1)dg_2(a_2)$$

where $g_1(a_1)$ and $g_2(a_2)$ are the distribution functions of a_1 and a_2 respectively. It is also being assumed here that the distribution of initial beliefs (across members of a cohort) is the same for all cohorts.

Next consider any one of the older generations of citizens. The probability that a citizen chooses to cheat in the (t + 1)st period of his life is the same as the probability of (6) being satisfied. Now whether a citizen encounters a corrupt bureaucrat during any single period is a Bernoulli trial with success probability C. Thus the probability of (6) being satisfied is same as the probability of s(t) or more successes out of t Bernoulli trials where the probability of success in each trial is C. Accordingly, the probability of (6) being satisfied is given by the Binomial distribution function[14]

$$(16) \qquad B(r(t, u, a_1, a_2), t, C) = \sum_{j=r(t,u,a_1,a_2)}^{t} \binom{t}{j} C^j (1-C)^{t-j}.$$

By aggregating (16) over all members of the (t + 1)st generation, we obtain the level of cheating in this generation:

$$(17) \qquad c^{t+1} = \int B(r(t, u, a_1, a_2), t, C)dg_1(a_1)dg_2(a_2).$$

Further, by aggregating (15) and (17) across generations, we obtain the economy-wide level of cheating. That is

$$(18)\qquad c = \frac{1}{T}\sum_{t=0}^{T-1} c^{t+1} = f(C)$$

$$\equiv \frac{1}{T}c^{1} + \frac{1}{T}\sum_{t=1}^{T-1} \int B(r(t,\ u,\ a_1,\ a_2),\ t,\ C)dg_1(a_1)dg_2(a_2),$$

where f(C) is a short-hand expression for the function at the right hand side of (18).

<u>Steady-state Level of Corruption</u>: The derivation of the steady-state level of corruption is quite similar to that above. The details are therefore omitted and only the relevant expressions are presented. The level of corruption in the youngest generation of bureaucrats is

$$(19)\qquad C^{1} = \int_{A_1 \geq A_2 U/(1-U)} dG_1(A_1)dG_2(A_2),$$

where $G_1(A_1)$ and $G_2(A_2)$ are the distribution functions of A_1 and A_2. The probability that a bureaucrat, whose initial beliefs are represented by $(A_1,\ A_2)$, would choose to be corrupt during the (t + 1)st period of his life is

$$(20)\quad B(R(t,\ U,\ M,\ A_1,\ A_2),\ tM,\ c) = \sum_{j=R(t,U,M,A_1,A_2)}^{tM} \binom{tM}{j} c^{j}(1-c)^{tM-j}.$$

Therefore, the level of corruption in the (t + 1)st generation is

$$(21)\qquad C^{t+1} = \int B(R(t,\ U,\ M,\ A_1,\ A_2),\ tM,\ c)dG_1(A_1)dG_2(A_2).$$

Finally, the economy-wide level of corruption is obtained by aggregating over (19) and (21):

$$(22) \qquad C \equiv \frac{1}{T}\sum_{t=0}^{T-1} C^{t+1} = F(c)$$

$$\equiv \frac{1}{T}C^1 + \frac{1}{T}\sum_{t=1}^{T-1} \int B(R(t,\ U,\ M,\ A_1,\ A_2),\ tM,\ c)dG_1(A_1)dG_2(A_2),$$

where F(c) represents the function on the right hand side of (22). The above equation, in combination with (18), determines the steady-state levels of cheating and corruption.

III. QUALITATIVE ANALYSIS OF AGGREGATE CORRUPTION AND CHEATING

The model developed above yields a number of insights; these are summarized in this section. I begin with some important issues concerning the multiplicity and stability of equilibria. It is then argued that under plausible assumptions we should expect a diversity in individuals' behavior rather than uniformity. The final subsection is devoted to ascertaining the effects of changes in the parameters of the economy on societal corruption and cheating.

A. Multiplicity and Stability of Equilibria

Note from (18) and (22) that c is a polynomial in C, and C is a polynomial in c. Also, the highest possible order of each of these polynomials is T − 1. Thus, unless there are special circumstances (for instance, the life span of individuals is too short; say, T = 2), it would be the case that there are multiple equilibria. Therefore, two economies with identical set of parameters can have significantly different levels of corruption and cheating. The reason for the multiplicity of equilibria is that the current behavior of individuals is explicitly influenced by their past behaviors. Therefore, the particular steady-state to which an economy settles (when it does) is influenced by the history of the economy preceding the steady-state.[15]

In the analysis below, we restrict our attention to only those equilibria which are locally stable; this is because the economy does not return to a locally unstable equilibrium after a small perturbation. A necessary condition for the local stability of an equilibrium defined by (18) and (22) is

$$(23) \qquad f_C F_c < 1,$$

where a subscript of f or F denotes the variable with respect to which a partial derivative is being taken. The stability condition (23) plays a central role in the comparative dynamics analysis presented below. Therefore, a derivation of this stability condition is provided in Appendix I.

Also, it is obvious that our interest is limited to only those solutions of (18) and (22) for which both c and C are not smaller than zero or larger than one. This is because other values of c and C do not have an economic meaning in the present context.

B. Diversity versus Uniformity in Individuals' Behavior

Is a complete uniformity in individuals' behavior (for instance, an economy in which all bureaucrats are corrupt, or no bureaucrat is corrupt) a likely possibility? The analysis below shows that the answer is no. That is, under plausible assumptions we should expect a diversity in individuals' behavior.

Consider the possibility of an equilibrium in which no bureaucrat is corrupt (that is, the "corner" equilibrium where $C = 0$). Such an equilibrium obviously requires the youngest generations of bureaucrats to be free of corruption. Furthermore, this corner equilibrium is infeasible if even a few members of the youngest generations of citizens are choosing to cheat. This is because the arrival of these few citizens into the economy in each period induces, given sufficiently long life span, at least some bureaucrats (those who happen to encounter these citizens) to believe that it is in their interest to be corrupt. The corresponding choice of these bureaucrats to be corrupt, in turn, leads to more citizens deciding to cheat in the future. The resulting equilibrium must therefore entail at least some cheating as well as some corruption.

A noteworthy aspect of the above argument is that it is effective only if the life span of bureaucrats is not too brief. If the life span is too brief then it is possible that the behavioral externalities generated by a small subset of individuals may die out before they have had an opportunity to affect the economy's equilibrium.

Parallel reasoning suggests the infeasibility of other corner equilibria; that is, of the equilibrium where all bureaucrats are corrupt, and of the equilibria where all or none of the citizens choose to cheat. These conclusions are summarized below.[16]

PROPOSITION 3. <u>If the life span of individuals is not too brief then</u>

(a) $C > 0$ <u>even if</u> $C^1 = 0$, <u>provided</u> $c^1 > 0$.

(b) $C < 1$ <u>even if</u> $C^1 = 1$, <u>provided</u> $c^1 < 1$.

(c) $c > 0$ <u>even if</u> $c^1 = 0$, <u>provided</u> $C^1 > 0$.

(d) $c < 1$ <u>even if</u> $c^1 = 1$, <u>provided</u> $C^1 > 0$.

Next, consider the polar case where there is complete absence of corruption and cheating in the youngest generations (that is, $C^1 = 0$ and $c^1 = 0$). Then it is straightforward to verify that an equilibrium in which there is no corruption or cheating in the economy is feasible; that is, $C = 0$ and $c = 0$ is a feasible solution of (18) and (22). Yet there is no economic reason to expect this particular corner equilibrium to be necessarily stable. On the contrary, examples can be constructed in which such an equilibrium is not stable (that is, it does not satisfy the condition (23)), particularly if the life span of individuals is not too brief. To see the reason, consider a slight perturbation of this corner equilibrium which introduces a small number of corrupt bureaucrats in the economy. Then, given sufficient life span, these handful of bureaucrats can initiate a chain reaction of behavioral externalities (inducing citizens to cheat and, in turn, inducing other bureaucrats to be corrupt) so that the economy can not return to the same equilibrium. Thus

PROPOSITION 4. <u>Some bureaucrats might be corrupt and some citizen might choose to cheat even if the youngest generations of bureaucrats is free of corruption and the youngest generations of citizens is free of cheating</u>.

Analogously, though the corner equilibrium in which all bureaucrats are corrupt and all citizens cheat (that is, $C = 1$ and $c = 1$) is feasible for certain sets of parameters (for which, $C^1 = 1$ and $c^1 = 1$), this corner equilibrium also may not be stable.

C. Effects of Economy's Parameters on Aggregate Corruption and Cheating

The method for comparative dynamics analysis is as follows. Let Θ denote a parameter which affects the function f in (18), and let $\emptyset$ denote a parameter which affects the function F in (22). That is, the equation system

(24) $\quad c = f(C, \Theta)$, and $C = F(c, \emptyset)$

defines the equilibrium values of c and C. Then perturbing the above equation system in the neighborhood of a locally stable equilibrium, and using (23), it is easily ascertained that[17]

(25) $\quad \mathrm{sgn}(dc/d\Theta) = \mathrm{sgn}(dC/d\Theta) = \mathrm{sgn}(f_\Theta)$, and

(26) $\quad \mathrm{sgn}(dc/d\emptyset) = \mathrm{sgn}(dC/d\emptyset) = \mathrm{sgn}(F_\emptyset)$.

I now employ the above expressions to determine the effects of different parameters on a societal equilibrium.

Effect of Changes in the Cost of Cheating and the Cost of Corruption: In many cases, a reduction in government intervention reduces the opportunities of corruption available to bureaucrats, which in turn would increase the relative cost of corruption for an individual bureaucrat.[18] The consequences of such a change in the relative cost on the societal equilibrium are easily ascertained. If this change in incentives prevents even a few of the bureaucrats from being corrupt, as one would expect to be the case, then the incidence of cheating would decline in the future because the citizens would encounter fewer corrupt bureaucrats. In turn, this would reduce the future incidence of corruption. As a result of this chain reaction, the economy would settle to an equilibrium in which the level of corruption as well as the level of cheating is lower.

To confirm the above intuition, recall from (14) that the reservation score of any bureaucrat can not decrease if the relative cost of corruption, U, is larger. Also, from (20), B must decrease if R increases. Now to avoid unnecessary details, I assume that if U is larger, then the reservation score for at least one bureaucrat increases during at least one period of his life. Correspondingly, in the right hand side of (22), B is smaller for at least one set of (t, A_1, A_2). Thus $F_U < 0$ and, from (26), $dc/dU < 0$ and $dC/dU < 0$. A similar analysis of the effect of a change in the relative cost of cheating, u, shows that $dc/du < 0$ and $dC/du < 0$. We therefore obtain

PROPOSITION 5. A larger relative cost of corruption or a larger relative cost of cheating lowers the level of corruption as well as the level of cheating.

Effect of Changes in Initial Beliefs: The initial beliefs of a bureaucrat are represented in the present model by (A_1, A_2). As (10) indicates, a bureaucrat with a larger A_1 or a smaller A_2 begins his life with a larger estimate of the level of cheating in the economy. A first-order stochastic improvement in the distribution of A_1, or a first-order stochastic worsening in the distribution of A_2, therefore represents that the youngest generations of bureaucrats believe that there is a greater prevalence of cheating in the economy. Likewise, a first-order stochastic improvement in the distribution of a_1, or a first-order stochastic worsening in the distribution of a_2, represents that the youngest generations of citizens believe that there is a greater prevalence of corruption in the economy.

Each of the preceding four stochastic changes have similar implications. Consider a first-order stochastic improvement in the distribution of A_1. From (11a), this means that the distribution of reservation levels across bureaucrats shifts to the left. Consequently, at least a few more bureaucrats would be corrupt than what would have been the case in the absence of the stochastic change. This, in turn, sets up a chain reaction encouraging more citizens to cheat and more bureaucrats to be corrupt. The new equilibrium to which the economy would settle would thus entail higher levels of corruption and cheating. The effects of other stochastic changes are traced similarly. These results are summarized below, and a formal derivation is presented in Appendix 2.

PROPOSITION 6. <u>If the youngest generations of bureaucrats believe that the level of cheating is higher in the economy, or if the youngest generations of citizens believe that the level of corruption is higher in the economy, then the actual level of corruption as well as the actual level of cheating in the economy is higher</u>.

<u>Effect of Age on Different Generations' Behavior</u>: A difference between an older and a younger generation is that the former has had more opportunities to observe the economic environment. Our objective then is to ascertain whether the levels of corruption and cheating are higher or lower in older generations.

To explore an intuitive answer, consider a bureaucrat's behavior in two successive periods. If most of the citizens he has encountered in the last period had chosen to cheat (an event which is more likely to occur if the level of cheating in the economy is higher), then this bureaucrat will revise upwards his estimate of the level of cheating in the economy. Correspondingly, the possibility of his being corrupt (rather than not corrupt) during the current period would increase. The preceding observation then suggests that the level of corruption in an older generation of bureaucrats is likely to be higher, rather than lower, if the level of cheating is higher. This intuition is not entirely complete however because bureaucrats' choices are also affected by the relative cost of corruption (that is, a larger cost makes it less attractive to be corrupt) and by their initial beliefs (for instance, a person with very strongly held initial beliefs is less likely to alter his behavior with time and experience). To take into account all of these aspects, we obtain the following result from an approximated version of (21):[19]

$$(27) \qquad \operatorname{sgn}(\partial C^{t+1}/\partial t) = \operatorname{sgn}(c - U),$$

if the youngest generations of bureaucrats are indifferent between being corrupt or not corrupt.

The corresponding expression to ascertain the effect of age on the level of cheating in different generations of citizens, obtained from an approximated version of (17), is:

$$(28) \qquad \operatorname{sgn}(\partial c^{t+1}/\partial t) = \operatorname{sgn}(C - u),$$

if the youngest generations of citizens are indifferent between cheating and not cheating. These results are summarized below.

PROPOSITION 7. An older generation of bureaucrats is more corrupt than younger generations if the level of cheating in the economy is larger than the relative cost of corruption, and if the youngest generations of bureaucrats are indifferent between being corrupt or not corrupt. Analogously, an older generation of citizens cheats to a greater extent than younger generations if the level of corruption in the economy is larger than the relative cost of cheating, and if the youngest generations of citizens are indifferent between cheating or not cheating.

Effects of Changes in the Life Span: To look into the effects of a change in the life span (from T to T + 1) on the societal equilibrium, we obtain the following two results from (18), (22), (25), and (26).[20]

(29) A longer life span of bureaucrats increases (decreases) c as well as C, if the current C^T is larger (smaller) than C.

(30) A longer life span of citizens increases (decreases) c as well as C, if the current c^T is larger (smaller) than c.

I focus on the interpretation of (29); the interpretation of (30) is similar. The result (29) is intuitive because if the level of corruption in the oldest generation is higher than the economy-wide level of corruption, then the direct effect of a longer life span of bureaucrats is to increase the economy-wide level of corruption (because the oldest generation gets to live for one more period, and it therefore increases the average) and the indirect effect is to induce a greater extent of cheating and corruption. The new equilibrium therefore entails a higher level of corruption as well as a higher level of cheating in the economy.

Next, recall that Proposition 7 provides a set of sufficient conditions for the monotonicity of c^{t+1} and C^{t+1} with respect to t. We use this proposition to obtain the following, easily understandable, result for the case where the youngest generations of bureaucrats are indifferent between being corrupt or not corrupt, and the youngest generations of citizens are indifferent between cheating or not cheating.

PROPOSITION 8. A longer life span of bureaucrats increases the level of corruption as well as the level of cheating if the current level of cheating is larger than the relative cost of corruption. Similarly, a longer life span of citizens increases the level of corruption as well as the level of cheating if the current level of corruption is larger than relative cost of cheating.

Effect of Changes in M: We finally ascertain the effect of a change in the number of citizens encountered by a bureaucrat during a single period. There are two effects of an increase in M, say from M to (M + 1), on the behavior of a bureaucrat in the (t + 1)st generation: (i) his sample size increases by t because he now obtains one more observation in each of the past M periods, and (ii) his reservation level increases because he now needs a greater evidence of cheating (that is, a larger number of past encounters with citizens who had chosen to cheat) to be convinced that it is in his interest to be corrupt.

The overall effect of an increase in M on the probabilities of different bureaucrats being corrupt or not corrupt is therefore ambiguous, as can be seen from (20). It is intuitive however to expect that if the current level of cheating is high (say, the current c is close to one) then a larger M would increase the probabilities of different bureaucrats being corrupt. This is because a larger M would make it more likely that the increased reservation levels are satisfied. If this is the case, then the level of corruption and cheating in the economy would increase. On the other hand, if the level of cheating is low (say, the current c is close to zero) then one might expect that a larger M would reduce the level of corruption and cheating. The above intuition is supported, with some qualifications, by the following result (for a proof, see Appendix 2).

PROPOSITION 9. A larger M implies an increased (decreased) level of corruption as well as an increased (decreased) level of cheating in the economy if the current level of cheating is larger (smaller) than the relative cost of corruption and if the youngest generations of bureaucrats are indifferent between being corrupt or not corrupt.

IV. MORE GENERAL MODELS

The simple model analyzed in preceding sections can be generalized in a variety of ways. In each case, there would be a concomitant change in some of the details of the analysis but the qualitative issues which we have emphasized would not be significantly affected. Some of these generalizations are briefly discussed in this section.

Heterogeneity in Individuals' Characteristics: The only relevant source of heterogeneity in individuals' characteristics in the preceding model is that due to their initial beliefs. To see how other kinds of heterogeneity can be introduced, consider the case where the benefits to a citizen from alternative actions (that is, from cheating versus not cheating) depend in part on his income. Let the parameter a_3 denote the income of a citizen, and let his relative cost of cheating be represented as $u(a_3)$. It is then straightforward to characterize the societal equilibrium for any given distribution of incomes in the economy. Furthermore, intuitive results of the following kind are easily established: <u>If a citizen's relative cost of cheating is decreasing in his income, then a first-order stochastic improvement in the distribution of incomes results in a higher level of corruption as well as a higher level of cheating</u>. The effects of other types of heterogeneity (such as the heterogeneity in individuals' tastes for cheating or corruption, and the heterogeneity in their risk-aversion) can be assessed similarly.

<u>Sources of an Individual's Learning</u>: The exogenous parameters representing the nature of an individual's learning can be given more general meaning than that in the preceding model. For instance, the number of citizens that a bureaucrat encounters during a period or the number of bureaucrats a citizen encounters during a period may be stochastic rather than fixed parameters. Other sources of learning can also be included in the analysis (for instance, interaction with one's peers) but in doing so one needs to take into account the direct costs of such learning as well as the indirect costs of potential miscommunication. At yet another level of generalization, the nature of an individual's learning may in part be influenced by his own choices; for instance, whether a citizen encounters a bureaucrat in a particular period (or how many bureaucrats he encounters) is partly a matter of the citizen's own choice which is, in turn, determined by his beliefs and by the pecuniary trade-offs he

faces. In each case, the basic feature of the analysis would remain similar to that in preceding sections.

<u>A Bureaucrat's Trade-offs</u>: The simple reduced-form representation of a bureaucrat's trade-offs which was employed in preceding sections is: a bureaucrat's choice is binary (to be corrupt or not corrupt) and his gains from being corrupt are linearly increasing in the number of citizens who are cheating. To generalize this specification, consider the following two steps.

First, let a bureaucrat's choice remain binary, which would be the case if the probability of convicting a corrupt bureaucrat and the punishment imposed upon a corrupt bureaucrat are not significantly different whether he is corrupt with respect to one or more than one citizen. Also, recalling the notations defined earlier, let $V(b) = U^0(b) - U^1$ denote the difference between a bureaucrat's ex-post utilities from being corrupt versus being not corrupt. Then, for a wide range of transactions between bureaucrats and citizens, we would expect $V(b)$ to be increasing in b, but not always linearly as was assumed in preceding sections. Further, if the ex-ante (expected) value of $V(b)$ is represented as $\overline{V}(S(t), t, M, A)$, then we would expect $\overline{V}$ to be increasing in $S(t)$. That is, the ex-ante utility from being corrupt (rather than not corrupt) would be higher for a bureaucrat if he has encountered a larger number of citizens who are cheating. Now, since a bureaucrat's rational choice is to be corrupt if and only if $\overline{V} \geq 0$, it is easily verified that this choice can be represented by a criterion such as (12). The corresponding societal equilibrium is accordingly characterized.

Next, consider an illustration of those situations where a bureaucrat can be partially corrupt. Let α denote the fraction of citizens to whom a bureaucrat offers corruption as a basis for transaction, where these offers are accepted by the citizens who have chosen to cheat. The ex-ante utility in this case can be represented in a reduced form as $\overline{V}(S(t), t, M, A, \alpha)$, where the bureaucrat chooses α to maximize this ex-ante utility. It would then be the case for a wide variety of circumstances that the optimal α, denoted by $\alpha^*(S(t), t, M, A)$, is increasing in $S(t)$. That is, the extent of partial corruption currently chosen by a bureaucrat is higher if he has encountered in the past a larger number of citizens who are cheating. Once again, then, it can be shown that the behavioral externalities exerted by individuals on one another and the resulting societal equilibrium has qualitative properties similar to those analyzed earlier.

Finally, for simplicity, I have assumed thus far that an individual's relative pay-offs (specifically, the relative cost of cheating to a citizen and the relative cost of corruption to a bureaucrat) are not affected by the levels of cheating and corruption in the economy. On the other hand, we would in general expect changes in the societal environment to influence the kinds of economic activities which individuals do or do not undertake. A simple example is the possibility that a higher level of corruption lowers the amounts of bribes which a corrupt bureaucrat can potentially extract, and this in turn reduces the attractiveness of corruption to a bureaucrat. Such modifications alter some of the details of the analysis (for instance the extreme right hand side of (22) will now be a function of c as well as C) but not its nature.

V. CONCLUDING REMARKS

This paper has developed an economic model to analyze the evolution and the perpetuation of bureaucratic corruption, and to identify how such processes might be related to the characteristics of the economy. I conclude the paper with remarks highlighting some of the central features of the analysis presented earlier. These remarks also underscore some of the limitations of the analysis which have not been previously noted.

The basic source of dynamics in our analysis is that an individual's past experiences of dealing with the economic environment provide him with inference and partial knowledge concerning the environment which he is currently facing.[21] The economic environments of previous periods (which are themselves aggregations of how individuals had chosen to behave in those respective periods), therefore, affect individuals' rational choices in the current period. The resulting economic environment of the current period, in turn, influences the economic environments of future periods. It is apparent that such a dynamics is relevant not only to the phenomenon of bureaucratic corruption but also to those other aspects of human behavior where past experiences influence current economic choices.[22]

An advantage of an explicit historical process of the kind described above is that one can, in principle, study the pattern of evolution of the economy. For instance, given a set of initial conditions, and a set of random shocks to which the economy might be subject, it is possible to calculate the probabilities of different patterns of bureaucratic corruption through which the economy might evolve over time. Depending on the underlying parameters, such a model can predict patterns consistent with rapid changes in societal behavior (for instance, cases where a set of shocks lead the economy rapidly into hyper-corruption) as well as patterns which are consistent with a relative lack of change in societal behavior (for instance, cases in which the effects of temporary reforms in the salary or the policing structure of bureaucrats die out, given the forces of persistence, before they have had an opportunity to affect the level of corruption which will be sustained in the economy).

For brevity, I have not analyzed this kind of dynamics in the present paper. Instead, I have focused on those steady-states of the economy in which, in the absence of external shocks, certain features of the economy (specifically, the level of cheating and the level of corruption in the economy) do not change from period to period. I have then analyzed these steady-states (which, in general, are multiple) to derive a number of qualitative results; in particular, a result showing that the economy-wide extremes of corruption and cheating might be unlikely, and several results identifying the effects of the economy's parameters on the steady-state levels of corruption and cheating.

Moreover, the historical process described earlier allows us, once again, to be explicit about how different steady-states might be reached by the economy.[23] For instance, two economies with identical underlying parameters may reach two different steady-states if their initial conditions were different, or if they had faced different kinds of shocks in the past, or if they had faced similar shocks but at different times in the past. In any event, once the patterns of economic environment in these two otherwise identical economies begin to diverge, there is no natural force in these economies to reduce and eliminate these differences.

Finally, an important aspect of our analysis is that it applies to economies with a variety of legal, political and administrative structures. A primary role of these structures is to influence the nature of pecuniary trade-offs faced by individuals (for instance, the relative cost of corruption faced by a bureaucrat, and the relative cost of cheating faced by a citizen). Obviously, therefore, these structures affect the precise dynamic path of corruption and cheating in the economy as well as the precise steady-state to which the economy might settle. Yet, no matter what the nature of legal and bureaucratic structure might be, the positive analysis on which we have focused remains unaffected so long as an individual's choice is partly a consequence of what his beliefs are concerning the nature of the environment he faces, and so long as these beliefs are influenced by the individual's past experiences of dealing with the environment.

APPENDIX 1

The purpose of this Appendix is to demonstrate that (23) is a necessary condition for the local stability of the equation system (18) and (22). To do this, I begin with a dynamic model of which a special case is the equation system (18) and (22).

Let τ, $\tau - 1$, ... denote different periods. It is clear from the model described in the text (Section I.D) that the level of cheating in the current period depends on the levels of corruption in previous T − 1 periods. This dependence can be represented in general as

$$\text{(A1)} \qquad c(\tau) = z(C(\tau - 1), \ldots, C(\tau - T + 1)).$$

Likewise, the level of corruption can be represented in general as

$$\text{(A2)} \qquad C(\tau) = Z(c(\tau - 1), \ldots, c(\tau - T + 1)).$$

A special case of the above equation system, reckoned at a steady-state, is the equation system (18) and (22).

Let z_j denote the partial derivative of z with respect to its jth argument in (A1), and let Z_j denote the partial derivative of Z with respect to its jth argument in (A2). This Appendix establishes the following.

PROPOSITION 10. <u>A necessary condition for the local stability of the difference equation system (A1) and (A2) is</u>

$$\text{(A3)} \qquad \left(\sum_{j=1}^{T-1} z_j\right)\left(\sum_{j=1}^{T-1} Z_j\right) < 1.$$

The reason why this proposition is central to our purpose is that if this proposition holds then it is evident that (23) is a necessary condition for the stability of the steady-state equilibrium defined by (18) and (22).

My strategy for proving Proposition 10 is as follows. Rather than to provide a direct proof, I first transform the system (A1) and (A2) into another system in which the current c is a function of past c's, and the current C is a function of past C's. I then show that (A3) is a necessary condition for the stability of the transformed system. This proof is presented in three steps.

<u>Step 1</u>: Substituting (A2) into (A1), the current c can be expressed as a function of past values of c's. In particular

$$\text{(A4)} \qquad c(\tau) = z(Z(c(\tau - 2), \ldots, c(\tau - T)), \ldots, Z(c(\tau - T), \ldots, c(\tau - 2T + 2)),$$

which can be reexpressed, in turn, in a reduced form as

$$\text{(A5)} \qquad c(\tau) = \bar{z}(c(\tau - 2), \ldots, c(\tau - 2T + 2)).$$

In the above expression, let $\partial c(\tau)/\partial c(\tau - m)$ be denoted as e_m. Then it is easily shown that

(A6) $$\sum_{m=2}^{2T-2} e_m = (\sum_{j=1}^{T-1} z_j)(\sum_{j=1}^{T-1} Z_j).$$

This is because, from (A4) and (A5),

(A7) $$\sum_{m=2}^{2T-2} e_m = \sum_{m=2}^{2T-2} \sum_{j=1}^{T-1} z_j \frac{\partial C(\tau - j)}{\partial c(\tau - m)} .$$

Next note from (A2) that

(A8) $$\frac{\partial C(\tau - j)}{\partial c(\tau - m)} = Z_{m-j} \quad \text{if} \quad j + T - 1 \geq m \geq j + 1, \text{ and}$$

$$= 0 \qquad \text{otherwise.}$$

Using (A8), the right hand side of (A7) becomes $\sum_{j=1}^{T-1} z_j \sum_{m=j+1}^{j+T-1} Z_{m-j}$. A rearrangement of the subscripts in the preceding expression allows (A7) to be rewritten as (A6).

Using an analogous reasoning, the current value of C can be expressed as a function of the past C's. That is

(A9) $$C(\tau) = \bar{Z}(C(\tau - 2), \ldots, C(\tau - 2T + 2));$$

and if $\partial C(\tau)/\partial C(\tau - m)$ is denoted as E_m, then it is easily established that

(A10) $$\sum_{m=2}^{2T-2} E_m = (\sum_{j=1}^{T-1} z_j)(\sum_{j=1}^{T-1} Z_j).$$

<u>Step 2</u>: I now show that a necessary condition for the local stability of (A5) is

(A11) $$\sum_{m=2}^{2T-2} e_m < 1.$$

To show this, I use a standard technique of converting the difference equation (A5), which is of order 2T − 2, into a system of first-order difference equations. Define $x^1(\tau) \equiv c(\tau)$, and $x^j(\tau) \equiv x^{j-1}(\tau - 1)$ for $j = 2$ to $2T - 2$. Define the column vector $x(\tau) \equiv [x^1(\tau), \ldots, x^{2T-2}(\tau)]$. Then equation (A5) can be equivalently expressed as the vector equation

(A12) $$x(\tau) = X(x(\tau - 1)),$$

where the derivatives of X with respect to $x(\tau - 1)$ are represented by the companion matrix

(A13) $$\begin{bmatrix} 0 & e_2 & e_3 & \cdot\ \cdot & e_{2T-3} & e_{2T-2} \\ 1 & 0 & 0 & \cdot\ \cdot & 0 & 0 \\ 0 & 1 & 0 & \cdot\ \cdot & 0 & 0 \\ \vdots & & & & & \\ 0 & 0 & 0 & \cdot\ \cdot & 1 & 0 \end{bmatrix}$$

A necessary and sufficient condition for (A5) to be stable is that the magnitude of every eigenvalue of (A13) should be smaller than one (for a proof of this result, see Hirsch and Smale, 1974, pp. 280–81).

To see that (A11) is necessary for the stability of (A5), note that the characteristic polynomial of matrix (A13) is

(A14) $$P(\lambda) = \lambda^{2T-2} - e_2\lambda^{2T-4} - e_3\lambda^{2T-5} - \dots - e_{2T-2} ,$$

where λ denotes an eigenvalue of matrix (A13). Now suppose (A11) does not hold. Then (A14) shows that $P(1) \leq 1$. This violates one of the Schur-Cohn criteria for the roots of the polynomial (A14) to have magnitudes smaller than one (see LaSalle, 1986, p. 27, for a description of these standard criteria). Therefore, (A11) is necessary for the stability of (A5).

Step 3: The proof that $\sum_{m=2}^{2T-2} E_m < 1$ is a necessary condition for the stability of (A9) is identical to that in Step 2. From (A6) and (A10), therefore, (A3) is a necessary condition for the stability of (A5) and (A9). But then (A5) and (A9) are transformed versions of (A1) and (A2). Therefore (A3) is a necessary condition for the stability of the system (A1) and (A2). This completes the proof of Proposition 10.

APPENDIX 2

Proof of Proposition 6: Let the distribution function of A_1 be denoted by $G_1(A_1, \emptyset)$ such that a larger value of the parameter $\emptyset$ signifies a first-order stochastic improvement. That is: $\partial G_1/\partial\emptyset \leq 0$, where the strict inequality holds for at least some values of A_1, and where the end-points of the distribution of A_1 are fixed. From (22) and from a standard result on stochastic dominance, then $F_\emptyset > 0$ if: $dB/dA_1 \geq 0$ and the strict inequality holds for at least some values of t, and for some values of A_1 where $dG_1/d\emptyset < 0$. Next, note from (11a), (11b) and (20) that: $dR/dA_1 \leq 0$, and $dB/dR < 0$. Also, to avoid unnecessary details, I assume that dR/dA_1 is strictly negative for some values of t, and for some values of A_1 where $dG_1/d\emptyset$ is strictly negative. Therefore $F_\emptyset > 0$, and the result concerning the effect of a change in $\emptyset$ on C and c follows from (26). Analogous proofs apply for the results concerning the stochastic changes in the distributions of A_2, a_1 and a_2.

Derivation of Expressions (27) and (28): If t is treated as a continuous variable, then the distinction between the reservation level and reservation score (that is between $\bar{R}$ and R) can be ignored. Next, a normal approximation of the binomial distribution function (20) is:

$$\text{(A15)} \qquad B = 1 - N[(\bar{R} - tMc)\{tMc(1 - c)\}^{-1/2}],$$

where $\bar{R}$ is given by (11a) and N[ ] represents the distribution function for the unit normal variate. The derivative of B with respect to t, from (A15), can be rearranged to yield:

$$\text{(A16)} \qquad \mathrm{sgn}(dB/dt) = \mathrm{sgn}[tM(c - U) + A^*],$$

where $A^* \equiv (A_1 + A_2)U - A_1$ denotes the initial reservation level. Now from the definition of A^* and from (8) and (11a) it follows that the youngest generations of bureaucrats are indifferent between being corrupt or not corrupt if $A^* \approx 0$. The result (27) then is obtained from (A16) and (21). The derivation of (28) follows similar lines.

Proof of Proposition 9: Treating M as a continuous variable and using the approximation (A15), one obtains

$$\text{(A17)} \qquad \mathrm{sgn}(dB/dM) = \mathrm{sgn}[tM(c - U) + A^*],$$

where A^* represents (as before) the initial reservation level. Therefore: $dB/dM \gtrless 0$ if $c \gtrless U$, and $A^* \approx 0$. The proposition follows by using (22) and (26).

FOOTNOTES

1. See Lewis (1955, Chs. III and VII) for a pioneering discussion. For related Marxist views, see Melotti (1977).

2. Dynamic behavioral externalities of the kind described here may play a fundamental economic role in determining other societal phenomena as well. But, as one would expect, the motivation of the analysis as well as the formalization would be quite different depending on what is being studied. See my 1985 and 1987 papers respectively for analyses of societal honesty and of criminal behavior. It should also be noted here that several earlier researchers have alluded to the idea of sequential causation between individuals' beliefs and economic behavior. Pareto's (1916) discussion is one of the earliest; for a summary of some of the views of Pareto, see Samuels (1974, pp. 111-123).

3. An example in a related but different context is the Gallup Poll on how citizens rate the honesty and ethical standards of local political officeholders in the United States [see, Gallup (1976-86)]. This survey, which has now been repeated for several years, shows substantial variance in the cross-section of responses, and this variance remains substantial even among respondents who are homogeneous with respect to characteristics such as income, occupation, race and sex [see, Vol. 2 (1972-76), pp. 823-850]. I am not aware of a routinely conducted survey of how citizens in an LDC rate the extent of bureaucratic corruption, but I expect that significant variance in responses would be observed in such a survey.

4. For an analysis of how legal and administrative structure and sanctions influence a bureaucrat's incentive to be corrupt, see Rose-Ackerman (1978). For a descriptive taxonomy of bribery, see Reisman (1979).

5. I do not however consider the case in which infinite punishment is imposed on a corrupt bureaucrat (with some probability of conviction) because such practices are not typically observed. Whether the absence of such punishments is due to the costs of erroneous conviction or due to other reasons is not a relevant issue for the present positive analysis.

6. The distribution of initial beliefs across individuals is assumed to be an exogenously specified feature of the economy. I do not go into the origins of initial beliefs.

7. These pay-offs can be further specialized to the case where a citizen who does not cheat obtains the same utility whether he encounters a corrupt or a not corrupt bureaucrat. That is, $u^{10} = u^{11}$. But this special case may not represent the oft-observed situations in which a corrupt bureaucrat extorts resources from even those citizens who have not cheated.

8. To derive (1), note that the utility from cheating is $\hat{C}u^{00} + (1 - \hat{C})u^{01}$ and the utility from not cheating is $\hat{C}u^{10} + (1 - \hat{C})u^{11}$. The former is no smaller than the latter if (1) is satisfied. For simplicity, it is assumed in (1) that an individual chooses to cheat if he is indifferent between cheating and not cheating. The analysis remains unchanged if the opposite assumption

is made.

9. The derivation of (3) is as follows. Let $dh(\varepsilon;\ a_1,\ a_2)$ denote the probability density which a citizen associates in the beginning of life with the level of corruption ε, where $1 \geq \varepsilon \geq 0$, and where $h(\varepsilon;\ a_1,\ a_2)$ is the distribution function for beta distribution with parameters a_1 and a_2. Now, whether or not this citizen encounters a corrupt bureaucrat during any single period can be viewed as the outcome of a Bernoulli trial. Therefore, using a standard result in Bayesian statistics (see, Rao, 1973, p. 335), the posterior beliefs of a citizen who has encountered $s(t)$ corrupt bureaucrats during the past t periods can be represented by the distribution function $h(\varepsilon;\ a_1 + s(t),\ a_2 + t - s(t))$. The mean estimate of the level of corruption is then $\int \varepsilon dh(\varepsilon;\ a_1 + s(t),\ a_2 + t - s(t))$ which equals $(a_1 + s(t))/(a_1 + a_2 + t)$.

10. Once again, this assumption is not only reasonable but it also allows us to abstract from those cases where a bureaucrat's choice is entirely insensitive to his beliefs.

11. The proof is as follows. From (2) and (5a), $s(t) \geq (a_1 + a_2 + t)u - a_1$ for a citizen who had chosen to cheat in the last period. Also, $s(t+1) = s(t) + 1$ if this citizen encountered a corrupt bureaucrat in the last period. Since $1 > u$, the preceding two expressions yield: $s(t+1) > (a_1 + a_2 + t + 1)u - a_1$. From (2) and (5a), therefore, this citizen chooses to cheat in the current period as well.

12. Let the parameters A_1^j and A_2^j denote the initial beliefs of two bureaucrats represented by j = 1, 2. Now suppose bureaucrat 1 is initially corrupt and bureaucrat 2 is not. Then substitution of $t = S(t) = 0$ into (8) and (11a) yields: $[(A_1^2 + A_2^2)U - A_1^2] > 0 \geq [(A_1^1 + A_2^1)U - A_1^1]$. Adding tMU to all terms in the preceding expression and using (11a) and (11b) one obtains the result: $R(t, U, M, A_1^2, A_2^2) \geq R(t, U, M, A_1^1, A_2^1)$. An analogous result holds concerning the effects of initial beliefs of different citizens on their reservation scores.

13. Such a steady-state is a stylized depiction of the economy and, as will become evident, the main value of this depiction is that it helps us study the properties of the economy. In reality, an economy need not arrive at such a steady-state, specially if the dynamic path of the economy is subject to perturbations by random influences. See Samuelson (1979, Ch. XI) for an extensive discussion of these issues.

14. The function B(r, t, C), which is used extensively below, is assumed to possess the following conventional properties. If $1 > C > 0$, then: (i) B is given by the right hand side of (16) if $t \geq r > 0$, (ii) $B = 1$ if $r \leq 0$, and (iii) $B = 0$ if $r > t$. If $C = 0$, then: $B = 1$ if $r \leq 0$, and $B = 0$ if $r > 0$. If $C = 1$, then: $B = 1$ if $r \leq t$, and $B = 0$ if $r > t$. The above properties assume that $t > 0$. If $t = 0$, then: $B = 1$ if $r \leq 0$, and $B = 0$ if $r > 0$. Using the last set of properties and the expressions (1) and (4), it can be verified that (15) is a special case of (17), to be derived below, when $t = 0$.

15. As we had indicated earlier, this paper abstracts from normative or welfare analysis. It might be useful to note, however, that different steady-states are in general not comparable on the Pareto basis. Moreover, even if a stronger criterion (such as a social welfare function) for comparison across steady-states were to be employed, the present model does not support any reason to believe that the economy would arrive at the 'optimal' steady-state. The last conclusion is analogous to that of Field (1981) and Basu et al. (1987) in the context of social customs.

16. To establish part (a), note from (22) and from the properties of the function B described in footnote 14 that: if $C^1 = 0$ then $C = 0$ only if $R(t, U, M, A_1, A_2) > tM$ for all $t > 0$. From (11a) and (11b), this can happen only if $\bar{R}(t) = (A_1 + A_2 + tM)U - A_1 > tM$ for all $t > 0$. But since $\bar{R}(t) - tM$ is linearly decreasing in t, the preceding condition can not be satisfied if t is sufficiently large. The proofs of other parts of Proposition 3 are similar.

17. To establish the sign of $dC/d\theta$ in (25), it is also required that $f_C > 0$. But the latter condition is automatically satisfied from the definition of f in (18). Similarly, the sign of $dc/d\emptyset$ in (26) requires that $F_c > 0$, which is satisfied by the definition of F in (22). It is assumed throughout that the values of c and C are in the interior; that is, they are larger than zero but smaller than one.

18. A change in the extent and the nature of government intervention may also alter the patterns of interactions between citizens and bureaucrats. The effects of such changes are analyzed below.

19. For a derivation of expression (27), and of expression (28) below, see Appendix 2.

20. To obtain (29), denote the function F in (22) as F(c, T). Expression (26) then indicates that an increase in T, from T to T + 1, increases c as well as C if F(c, T + 1) − F(c, T) is positive. The result (29) then follows by recalling from (22) that $F(c, T) = \sum_{t=0}^{T-1} c^{t+1}/T$. The result (30) is obtained similarly from (18) and (25).

21. While the present paper is based on a model of Bayesian learning, this aspect is not essential. Similar analysis applies with other kinds of systematic learning. For instance, the qualitative features of the analysis remain unaltered so long as a citizen who has encountered a larger number of corrupt bureaucrats in the past believes that the probability of encountering a corrupt bureaucrat in the current period is larger.

22. Note also that these intertemporal learning externalities are quite different (but not mutually exclusive) from some other economic forces which have been viewed in the literature as sources of specific types of societal behavior. Particularly important are the analyses in which reputation, sanctions and other strategic aspects of an individual's behavior are central; see Akerlof (1976, 1980).

23. Such an explicit process should be contrasted with some other contexts where multiple equilibria have been emphasized but where the historical process bringing the economy to one or the other equilibrium is implicit or unspecified. See Schelling (1978, Ch. 7) in the

context of models of norms, and Diamond (1987) in the context of models of credit market.

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