Assignment 5 Analysis

#this calculates the overall likelihood for the null hypothesis. It uses a mixture of a Poisson
#distribution with a lambda value of 10 and a Poisson distribution with a lambda value of 40.
#These two distributions/their values are combined to form the likelihood under the null
#hypothesis and the overall likelihood distribution.
v1 <- dpois(c(0:100), 20) /((dpois(c(0:100), 10) + dpois(c(0:100),40))/2)

#this calculates the likelihood of the data under the assumption that our data follows a Poisson
#distribution with a lambda value of 20. This represents the alternative hypothesis (as #defined
by the directions that it was a Poisson distribution of lambda 20).
alternative <- dpois(c(0:100),20)

<u>Using dpois() with changing likelihood ratios</u>

```
v1 <- dpois(c(0:100), 20) /((dpois(c(0:100), 10) + dpois(c(0:100),40))/2)

# order (by default this function will be ascending format)
o1 <- order(v1) o2 <- rev(o1)

# reverse the ordering so we can order in descending order
o2 <- rev(o1)

# this is subsetting the vector to just the first two elements - 14 gives sum the closet to 0.05
# 14 is the critical value for the v
v2 <-(o2[c(1:13)]-1) # this is too low
#v2 <-(o2[c(1:14)]-1) # just right so the cutoff ratio is 1:14
#v2 <-(o2[c(1:15)]-1) # this is too high
```

We used v1 <- dpois(c(0:100), 20) /((dpois(c(0:100), 10) + dpois(c(0:100),40))/2), ordered it using the order function, and reversed the order so we could work with values from greatest to smallest.

The order we received was as follows:

```
> o2
  [1]  23  22  24  21  25  20  26  19  27  18  28  17  29  16  30  15  31  14  32
 [20]  13  33  12  34  11  35  10  36   9  37   8  38   7  39   6  40   5  41   4
 [39]  42   3  43   2  44   1  45  46  47  48  49  50  51  52  53  54  55  56  57
 [58]  58  59  60  61  62  63  64  65  66  67  68  69  70  71  72  73  74  75  76
 [77]  77  78  79  80  81  82  83  84  85  86  87  88  89  90  91  92  93  94  95
 [96]  96  97  98  99 100 101
```

We continued to increase the likelihood ratio until we reached around the statistically significant target of the p value, 0.05.

```
> v2 <-(o2[c(1:13)]-1)
> sum((dpois(v2,10)+dpois(v2,40))/2)
[1] 0.03905688
> v2 <-(o2[c(1:14)]-1)
> sum((dpois(v2,10)+dpois(v2,40))/2)
[1] 0.05641766
> v2 <-(o2[c(1:15)]-1)
> sum((dpois(v2,10)+dpois(v2,40))/2)
[1] 0.06334246
```

We reach around a p value of 0.05641766 when we make the ratio 1:14. Going above this ratio (1:15) gives us a p value that is too high (0.06334246) and going below this ratio (1:13) gives us a p value that could be higher (0.03905688). Using this, we determine 1:14 to be our best ratio.

Comparing to test with .025 above poisson(10) and .025 below poisson(40)

```
# probability of being more extreme than the critical value under poisson of 10

#this is the critical value or upper tail/bound value for Poisson of lambda 10
critical_10 <- qpois(1-0.025,10)
#returns 17

#this is the probability of getting that critical value or higher for the null hypothesis
prob_critical10 <- 1-ppois(critical_10, 10)
#returns: 0.01427761

# this is the critical value or lower tail/bound value for poisson of lambda 40
critical_40 <- qpois(0.025,40)
#returns 28
```

# this is the probability of getting that critical value or higher under poisson of 40
prob_critical40 <- ppois(critical_40, 40)
#returns 0.02937958

```
> # represents critical value for poisson of lambda 10
> # probability of being more extreme than the critical value under poisson of 10
> critical_10 <- qpois(1-0.025,10)
> critical_10
[1] 17
> prob_critical10 <- 1-ppois(critical_10, 10)
> prob_critical10
[1] 0.01427761
> # represents critical value for poisson of lambda 40
> # probability of being more extreme than the critical value under poisson of 40
> critical_40 <- qpois(0.025,40)
> critical_40
[1] 28
> prob_critical40 <- ppois(critical_40, 40)
> prob_critical40
[1] 0.02937958
```

- Based on this, we can see the probability of getting a value of 18 or higher for the Poisson distribution with lambda equal to 10 is 0.01427761
- We can also see that the probability of getting a count of 27 or higher for the Poisson distribution with lambda equal to 40 is 0.02937958

Calculating the power of test

Power is defined as the sum of the probabilities under the alternative hypothesis, which is HA, where the likelihood ratio is greater than the cutoff ratio.

# summing the region over the alternative hypothesis - power of poisson with lambda 10
power <- sum(alternative[v2 >= critical_10])
power
#returns .8091

The value of 0.8091is close to 1, which means that the test is good at testing true effects, including a reliable test.

Conclusion

- The critical value closest to a p-value of 0.05 is 14. Thus, we can say that the value is in the expected range for the null hypothesis (which is the average of the Poisson distribution of Poisson(10) and Poisson(40).

- At the significance levels of 2.5% (since we are doing two distributions of Poisson(10) and Poisson(40), the values for the count are NOT statistically significant
- It shows us that the probability of observing values of 15 or greater would not be unusual for the null hypothesis.