```
%Raashi Maheshwari
%8169
%Section C1
%Math 250 MATLAB Lab Assignment
rand('seed', 8169)
%Question 1
u = rvect(2), v = rvect(2)
u =
    9
    6
v =
    3
    4
rank([u, v])
ans =
    2
%Question 1(a)
%The triangle inequality is  ||u + v|| <= ||u|| + ||v||
%In words, its saying that the norm of (u + v) is less than the norm of u plus
%the norm of v
norm(u)
ans =
  10.8167
norm(v)
ans =
    5
norm (u+v)
ans =
  15.6205
%As seen above, the norm of (u+v) is less than the norm of u + the norm of v
%15.6205 < 15.8167
%Question 1(b)
%The Cauchy-Schwarz inequality states: |u * v| <= ||u|| * ||v||
%In words, it says that the dot product of (u*v) is less than the dot product
%of the norm of u and the norm of v
norm(u)' * norm(v)
ans =
  54.0833
abs(u'*v)
ans =
   51
%as seen above, the absolute value of u*v is less than the norm of u * norm of
%v (51 < 54.0833)
%Question 1(c)
w = ((u'*v)/(v'*v))*v
w =
   6.1200
   8.1600
z = u - w
```

```
z =
   2.8800
  -2.1600
z'*v
ans =
    0
%As seen above, the dot product is zero, hence we can conclude that the vectrs
%are orthogonal
%Question 1(d)
P = v*inv(v'*v)*v'
P =
   0.3600    0.4800
   0.4800    0.6400
%P would be a 2*2 matrix because when you multiply V (2*1 matrix) by its
%transpose (1*2), you would end up with a 2*2 matrix.
P*u
ans =
   6.1200
   8.1600
w
w =
   6.1200
   8.1600
%As seen above, P*u is the vector w
%Question 2
u1 = rvect(3), u2 = rvect(3), u3 = rvect(3)
u1 =
    6
    7
    3
u2 =
    8
    4
    7
u3 =
    6
    5
    8
rank([u1, u2, u3])
ans =
    3
%Question 2(a)
r = 0:0.05:1; hold on
plot3(r*u1(1),r*u1(2),r*u1(3), 'r:')
plot3(r*u2(1), r*u2(2), r*u2(3), 'g:')
plot3(r*u3(1), r*u3(2), r*u3(3), 'b:')
%As shown by the graph, u1, u2, and u3 are not orthogonal since they are not
%perpendicular to each other
%Question 2(b)
```

```
v1 = u1
v1 =
     6
     7
     3
P1 = v1*inv(v1'*v1)*v1', v2 = u2 - P1*u2
P1 =
   0.3830    0.4468    0.1915
   0.4468    0.5213    0.2234
   0.1915    0.2234    0.0957
v2 =
   1.8085
  -3.2234
   3.9043
v1'*v2
ans =
 -1.7764e-15
%The dot product is approximately 0, which means that v1 and v2 are mutually
%orthogonal
plot3(r*v2(1), r*v2(2), r*v2(3), 'g-.')
P2 = v2*inv(v2'*v2)*v2', v3 = u3 - P1*u3 - P2*u3
P2 =
   0.1132   -0.2017    0.2443
  -0.2017    0.3595   -0.4354
   0.2443   -0.4354    0.5274
v3 =
  -1.6886
   0.8215
   1.4604
V3’ * v1
ans =
    1.0000e-04
%The dot product is approximately 0, which means that v3 is orthogonal to the
%vectors v1 and v2
plot3(r*v3(1), r*v3(2), r*v3(3), 'b-.')
%Question 2(c)
w1 = v1/norm(v1), w2 = v2/norm(v2), w3 = v3/norm(v3)
w1 =
   0.6189
   0.7220
   0.3094
w2 =
   0.3364
  -0.5996
   0.7262
w3 =
  -0.7098
   0.3453
   0.6139
```

```
Q = [w1, w2, w3]
Q =
   0.6189    0.3364   -0.7098
   0.7220   -0.5996    0.3453
   0.3094    0.7262    0.6139
%Question 2(c) (i)
%Q transpose * Q would return an identity matrix, where the entries of the
%answer would be the dot products of Q*T and Q. Q*(Q*T) would mean that the
%entries would be zero because the vectors are orthogonal, which means that
%the dot product would be 0. However, the entries for Q and Q*T would be 1 as
%they are the vectors multiplied by themselves. This proves the property that
%if a square matrix is an orthogonal matrix where all the columns are
%orthonormal, then (Q*T)*Q would give you the identity matrix as your answer.
%transpose(Q)*Q. This can be seen verified using MATLAB below
ans =
   1.0000   -0.0000    0.0000
  -0.0000    1.0000    0.0000
   0.0000    0.0000    1.0000
%Question 2(c) (ii)
%The inverse matrix (Q^-1) would be the transpose of Q. This can be seen
%verified using MATLAB below
inv(Q)
ans =
   0.6189    0.7220    0.3094
   0.3364   -0.5996    0.7262
  -0.7098    0.3453    0.6139
%Question 2(d)
A = [u1, u2, u3], R = Q'*A
A =
    6     8     6
    7     4     5
    3     7     8
R =
   9.6954   10.0048    9.7985
  -0.0000    5.3763    4.8301
   0.0000    0.0000    2.3789
Q*R
ans =
   6.0000    8.0000    6.0000
   7.0000    4.0000    5.0000
   3.0000    7.0000    8.0000
%As verified above, A = Q*R
%Question 2(c) (iii)
%In order for R to be an upper triangular matrix, the following spots of the
%matrix have to be zero: (2,1), (3,1), (3,2). For this to be true, the dot
%products of w2 and u1 have to be zero, and the dot product of w3 and u2 has
%to be 0, which are both true. Hence the entries in (2,1), (3,1), (3,2) of
%matrix R are 0, which makes R an upper triangular matrix
%Question 3
```

```
a1 = rvect(5); a2 = rvect(5); a3 = rvect(5); A = [a1, a2, a3]
A =
    1     5     8
    8     1     2
    6     1     8
    0     6     4
    7     1     1
rank(A)
ans =
    3
%Question 3(a)
Q = grams(A); w1 = Q(:,1), w2 = Q(:,2), w3 = Q(:,3)
w1 =
   0.0816
   0.6532
   0.4899
        0
   0.5715
w2 =
   0.6258
  -0.0501
  -0.0052
   0.7779
  -0.0277
w3 =
   0.3759
  -0.2729
   0.7341
  -0.3283
  -0.3710
Q'*Q
ans =
   1.0000   -0.0000    0.0000
  -0.0000    1.0000    0.0000
   0.0000    0.0000    1.0000
%Since the answer to Q'*Q is an identity matrix, we can conclude that the set
%{w1, w2, w3} is an orthonormal set of vectors
%Question 3(b)
P = w1*w1' + w2*w2' + w3*w3'
P =
   0.5395   -0.0806    0.3127    0.3634   -0.1101
  -0.0806    0.5037    0.1199    0.0506    0.4760
   0.3127    0.1199    0.7789   -0.2450    0.0078
   0.3634    0.0506   -0.2450    0.7129    0.1003
  -0.1101    0.4760    0.0078    0.1003    0.4651
v = rvect(5)
v =
    1
    2
```

```
     4
     9
     9
w = P*v
w =
   3.9087
   6.1455
   1.5334
   6.8028
   5.9610
z = v - w
z =
  -2.9087
  -4.1455
   2.4666
   2.1972
   3.0390
P*w
ans =
   3.9087
   6.1455
   1.5334
   6.8028
   5.9610
%As seen above P*w = w
P*z
ans =
  1.0e-04 *
  -0.0187
  -0.1403
   0.2272
  -0.3031
  -0.1745
%As seen above P*z is approximately 0
%Question 3(c)
PW = A*inv(A'*A)*A'
PW =
   0.5395   -0.0806    0.3127    0.3634   -0.1101
  -0.0806    0.5037    0.1199    0.0506    0.4760
   0.3127    0.1199    0.7789   -0.2450    0.0078
   0.3634    0.0506   -0.2450    0.7129    0.1003
  -0.1101    0.4760    0.0078    0.1003    0.4651
norm(PW-P)
ans =
  4.6353e-16
%As seen above, norm(PW - p) is approximately 0
```