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%Section C1
%Math 250 MATLAB Assignment #5
%Question 1(a)
eigshow
% Question 1(a)(i)
%A has one positive eigenvalue
%Question 1(a)(ii)
%A has one negative eigenvalue
%Question 1(a)(iii)
%x is approximately 1 and A*x is approximately 1.5
%Question 1(b)
%Question 1(b)(i)
%There are no lines through zero that contain both Ax and x
%Question 1(b)(ii)
%We can conclude that A doesn't have any real eigenvectors or eigenvalues
%because Ax and x are always facing opposite sides
%Question 2
A = [1 3; 4 2]/4
A =
   0.2500    0.7500
   1.0000    0.5000
%Question 2(a)
syms t; I = eye(2); p = det(A - t*I)
p =
t^2 - (3*t)/4 - 5/8
% det(A) = (0.25*0.5)-(0.75*1) = 0.125 - 0.75 = -0.625 = -5/8
%Verified by hand calculation that constant term in polynomial is det(A)
%Question 2(b)
solve(p)
ans =
-1/2
5/4
%These values are approximately close to the estimates I made in Question 1,
%using the graphics
%Question 2(c)
A = [3 1; -2 4]/4
A =
   0.7500    0.2500
  -0.5000    1.0000
syms t; I = eye(2); p = det(A - t*I)
p =
t^2 - (7*t)/4 + 7/8
solve(p)
ans =
7/8 - (7^(1/2)*1i)/8
(7^(1/2)*1i)/8 + 7/8
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%The eigenvalues are not real numbers, which is why there were no lines
%through zero that contained both Ax and x
%Question 3
rand('seed', 8169)
%Question 3(a)
A = rmat(3,3), z = eig(A) - real(eig(A))
A =
    9     4     3
    6     6     8
    3     7     4
z =
    0
    0
    0
syms t; I = eye(3); p = det(A - t*I)
p =
- t^3 + 19*t^2 - 25*t - 216
figure; ezplot(p, [-10, 10]), grid
%Attached to assignment
%Approximate values for the three real roots of p(t) are: -2.5, 5, and 17
%Question 3(b)
[P D] = eig(A)
P =
   0.5429    0.7609    0.0568
   0.6754   -0.3925   -0.6995
   0.4991   -0.5167    0.7124
D =
  16.7348         0         0
        0    4.8995         0
        0         0   -2.6344
%The diagonal entries represent the eigenvalues of matrix A, which are close
%enough to the estimate made in Part A
p1 = P(:, 1), p2 = P(:, 2), p3 = P(:, 3)
p1 =
   0.5429
   0.6754
   0.4991
p2 =
   0.7609
  -0.3925
  -0.5167
p3 =
   0.0568
  -0.6995
   0.7124
A*p1 - D(1,1)*p1, A*p2 - D(2,2)*p2, A*p3 - D(3,3)*p3
ans =
  1.0e-14 *
   0.8882
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   0.8882
   0.3553
ans =
  1.0e-14 *
   0.5329
   0.0888
        0
ans =
  1.0e-15 *
  -0.1665
   0.2220
  -0.2220
%This tells me that the equation A = PDP^-1 (D = diagonal matrix & P is an
%invertible matrix) is only valid when
%the columns of P are a basis for R^n that consist of the eigenvectors of A
%and the diagonal entries of D are the eigenvalues that
%correspond to the respective columns of P. In this case, each column of P is
%an eigenvector of A, and each diagonal entry of D is eigenvalue that is
%consistent with the eigenvector
%Question 3(c)
P*D*inv(P)
ans =
   9.0000    4.0000    3.0000
   6.0000    6.0000    8.0000
   3.0000    7.0000    4.0000
A
A =
    9     4     3
    6     6     8
    3     7     4
%As seen above, A = P*D*inv(P)
P * D^5 * inv(P)
ans =
  1.0e+05 *
   4.6389    4.1112    3.6669
   5.7398    5.1279    4.5788
   4.2362    3.7930    3.3854
A^5
ans =
 1.0e+05 *
   4.6389    4.1112    3.6669
   5.7398    5.1279    4.5788
   4.2362    3.7930    3.3854
%As seen above: A^5 = P * D^5 * inv(P)
P * D^10 * inv(P)
ans =
  1.0e+11 *
   6.0650    5.4061    4.8249
   7.5456    6.7260    6.0028
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   5.5764    4.9707    4.4362
A^10
ans =
  1.0e+11 *
   6.0650    5.4061    4.8249
   7.5456    6.7260    6.0028
   5.5764    4.9707    4.4362
%As seen above: A^10 = P * D^10 * inv(P)
%Question 4
%Question 4(a)
A = eye(2); B = rand(2)
B =
   0.7102    0.5386
   0.6764    0.8745
A(:, 1) = B(:, 1)/sum(B(:,1)); A(:,2) = B(:,2)/sum(B(:,2))
A =
   0.5122    0.3811
   0.4878    0.6189
[1 1]*A
ans =
   1.0000    1.0000
%[1 1]*A = [(A(1,1) + A(2,1)) (A(2,1)+A*2,2)]
%The sum of the above equation would equal 1, hence we can say that A is a
%transition matrix
%Question 4(b)
u = nulbasis(A - eye(2)), v = u/sum(u)
u =
   0.7813
   1.0000
v =
   0.4386
   0.5614
A*v
ans =
   0.4386
   0.5614
v
v =
   0.4386
   0.5614
%As seen above A*v = v
figure; plot([0, v(1)], [0, v(2)]), hold on
%Question 4(c)
w = rand(2,1), p = w/sum(w)
w =
   0.9984
   0.5439
p =
   0.6473
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   0.3527
p = A*p, plot([0, p(1)], [0, p(2)], ':'), hold on
p =
   0.4660
   0.5340
p = A^2*p, plot([0, p(1)], [0, p(2)], ':'), hold on
p =
   0.4391
   0.5609
p = A^3*p, plot([0, p(1)], [0, p(2)], ':'), hold on
p =
   0.4386
   0.5614
%As seen above, this matches vector v
%Question 5
A=[0.6, 0.1, 0.5; 0.2,0.7,0.1; 0.2,0.2,0.4]
A =
   0.6000    0.1000    0.5000
   0.2000    0.7000    0.1000
   0.2000    0.2000    0.4000
u = [1 1 1]
u =
    1     1     1
u * A
ans =
   1.0000    1.0000    1.0000
%each column of A sums to 1
%Question 5(b)
p = [0.5;0.3;0.2]
p =
   0.5000
   0.3000
   0.2000
u * p
ans =
    1
%Entries of p sum to 1
A * p
ans =
   0.4300
   0.3300
   0.2400
%43% of people live in the city, 33% of people live in the suburbs, and %24 of
people live in the country after one year
(A^2) * p
ans =
   0.4110
   0.3410
   0.2480
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%41.1% of people live in the city, 34.1% of people live in the suburbs, and
%24.8% of people live in the country after two years
(A^3) * p
ans =
   0.4047
   0.3457
   0.2496
%40.47% of people live in the city, 34.57% of people live in the suburbs, and
%24.96 of people live in the country after three years
(A^5) * p
ans =
   0.4011
   0.3490
   0.2500
%40.11% of people live in the city, 34.90% of people live in the suburbs, and
%25% of people live in the country after five year
(A^8) * p
ans =
   0.4001
   0.3499
   0.2500
%40.01% of people live in the city, 34.99% of people live in the suburbs, and
%25% of people live in the country after eight year
%Question 5(c)
u = nulbasis(A - eye(3)), v = u/sum(u)
u =
   1.6000
   1.4000
   1.0000
v =
   0.4000
   0.3500
   0.2500
%Vector v and the population distribution vector in part B are almost the same
%hence after eight years, the population will be approximately the same.
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