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%Raashi Maheshwari
%Last 4 of RUID: 8169
%Section C1
%Math 250 MATLAB Lab Assignment #3
rand('seed',8169)
%Question 1(a)
A = rand(3), A(1,1), det(A(1:2,1:2)), det(A)
A =
   0.9531    0.4286    0.3445
   0.6918    0.6588    0.8738
   0.3798    0.7423    0.4270
ans =
   0.9531
ans =
   0.3314
ans =
  -0.2438
U = A
U =
   0.9531    0.4286    0.3445
   0.6918    0.6588    0.8738
   0.3798    0.7423    0.4270
col1
L1 =
   1.0000         0         0
  -0.7258    1.0000         0
  -0.3985         0    1.0000
U = L1*U
U =
   0.9531    0.4286    0.3445
        0    0.3477    0.6237
        0    0.5715    0.2897
%Multiplying by L1 changes U from the previous one to the new U
%L1 started out as a 3*3 identity matrix, but its row 2 and row 3 were altered
%using row operations
%For row 2 for L1, we took the value of U(2,1) and divided it by U(1,1). We
%then multiplied that result by row 1 of the L1 identity matrix.
%We then took that whole value and subtracted it from row 2 of the L1 identity
%matrix.
%We repeated a similar set of steps for row 2, where we took the value of
%U(3,1) and divided it by U(1,1). We then multiplied that result by row 1 of
the %L1 identity matrix
%We then took that whole value and subtracted it from row 3 of the L1 identity
%matrix and this value/process was used to replace the numbers for Row 3 of L1
%This process transformed L1 from an identity matrix to a different matrix.
%When this was multiplied by U, the first row of matrix U stayed intact
%however, the first column beneath the main diagonal of matrix U has zeros and
%the remaining values below row 1 also changed.
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%In short, this was similar to doing row operations on U to get zeros under the
%leading entry of row 1 we did row operations by multiplying the first row by
%a constant, subtracting the second row, and setting that equal to the second
%row
%For the third row, we multiplied the third row by a scalar/constant and
%subtracted the third row, which was used to replace the third row
%Question 1(b)
col2
L2 =
   1.0000         0         0
        0    1.0000         0
        0   -1.6439    1.0000
U = L2*U
U =
   0.9531    0.4286    0.3445
        0    0.3477    0.6237
        0         0   -0.7356
%This process was similar to the previous process. We started off with L2
%being a 3*3 identity matrix that was then transformed into a different
%matrix.
%We took the value from U(3,2) and divided it by the value at U(2,2). We then
%took the result from the division and multiplied it by row 2 of the identity
%matrix.
%Lastly, we subtracted that value from the values present in row 3. These
%values were used to replace row 3 of the identity matrix L2
%When U was multiplied by L2, this kept the first and second rows intact, but
%changed the values of the third row because we did row two multiplied by a
%scalar subtracted from the third row, which replaced the third row
%It made the value beneath the main diagonal all 0 (the value underneath U(2,2)
), and also changed the value of the element at U(3,3)
L2*L1*A
ans =
   0.9531    0.4286    0.3445
        0    0.3477    0.6237
   0.0000    0.0000   -0.7356
U
U =
   0.9531    0.4286    0.3445
        0    0.3477    0.6237
        0         0   -0.7356
%The values of U and L2*L1*A match (as shown above), hence we can conclude
%that U = L2*L1*A
%Question 1(c)
inv(L1), inv(L2), L = inv(L1)*inv(L2)
ans =
   1.0000         0         0
   0.7258    1.0000         0
   0.3985         0    1.0000
ans =
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   1.0000         0         0
        0    1.0000         0
        0    1.6439    1.0000
L =
   1.0000         0         0
   0.7258    1.0000         0
   0.3985    1.6439    1.0000
%Column 1 of L is the same as column 1 of inv(L1) and column 2 of L is the
%same as column 2 of inv(L2)
L*U
ans =
   0.9531    0.4286    0.3445
   0.6918    0.6588    0.8738
   0.3798    0.7423    0.4270
A
A =
   0.9531    0.4286    0.3445
   0.6918    0.6588    0.8738
   0.3798    0.7423    0.4270
%As you can see above, L*U gives you matrix A as the answer, hence we can
%conclude that A = L*U
%Question 2(a)
%The formula for the inverse matrix inv(L) using matrices L1 and L2 is:
% inv(L) = inv(inv(L1)*inv(L2)) = inv(inv(L2)*inv(L1)) = L1*L2
inv(L)
ans =
   1.0000         0         0
  -0.7258    1.0000         0
   0.7946   -1.6439    1.0000
inv(U)
ans =
   1.0492   -1.2935   -0.6054
        0    2.8763    2.4388
        0         0   -1.3594
%Question 2(b)
b = rvect(3)
b =
    9
    1
    9
c = inv(L)*b
c =
   9.0000
  -5.5322
  14.5075
x = inv(U)*c
x =
   7.8159
  19.4686
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 -19.7215
A*x
ans =
   8.9995
   1.0003
   8.9989
%As you can see above, A*x equals b
%Question 3
A = rand(500) ; b = rand(500,1); [L U] = lu(A);
%Question 3(a)
tic; R = rref([A b]); y = R(:,501); toc
Elapsed time is 1.231501 seconds.
rreftime = 1.231501
rreftime =
   1.2315
%Question 3(b)
tic; c = inv(L)*b; x = inv(U)*c; toc
Elapsed time is 0.024202 seconds.
lutime = 0.024202
lutime =
   0.0242
norm(x-y)
ans =
  1.5027e-12
%Question 3(c)
%(i)
%The theoretical ratio rreftime/lutime when n = 500 is 500 is:
%(2cn^3/3)/(2cn^2) = n/3 = 500/3 which is approximately 166.667
%(ii)
%The observed rreftime/lutime is:
rreftime/lutime
ans =
  50.8843
%Question 4
a = rmat(4,4)
a =
    6     5     7     8
    8     2     4     5
    2     5     8     1
    4     2     7     4
c = cofactor(a)
c =
  73.0000  -94.0000   56.0000 -124.0000
-151.0000  -22.0000   40.0000   92.0000
 -33.0000 -122.0000   -8.0000  108.0000
  51.0000  246.0000 -160.0000  -52.0000
a(1,1)*c(1,1) + a(1,2)*c(1,2) + a(1,3)*c(1,3) + a(1,4)*c(1,4)
ans =
 -632
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a(2,1)*c(2,1) + a(2,2)*c(2,2) + a(2,3)*c(2,3) + a(2,4)*c(2,4)
ans =
-632.0000
a(1,3)*c(1,3) + a(2,3)*c(2,3) + a(3,3)*c(3,3) + a(4,3)*c(4,3)
ans =
 -632
a(1,4)*c(1,4) + a(2,4)*c(2,4) + a(3,4)*c(3,4) + a(4,4)*c(4,4)
ans =
 -632
%All the sums return the same number because the first two calculations are
%the co-factor expansion of rows 1 and 2
%The 3rd and 4th calculations provide the same result because the co-factor
%expansion of the rows of a matrix is the same as the co-factor expansion of
%its column
%which we know because the transpose of the rows of a matrix is equal to the
%column of a matrix
det(a)
ans =
 -632
%Question 4(b)
A = rmat(5,5), U = triu(A)
A =
    9     5     7     0     6
    7     8     1     6     4
    0     9     8     4     1
    3     8     2     6     6
    6     6     3     0     1
U =
    9     5     7     0     6
    0     8     1     6     4
    0     0     8     4     1
    0     0     0     6     6
    0     0     0     0     1
A(1,1)*A(2,2)*A(3,3)*A(4,4)*A(5,5)
ans =
       3456
U(1,1)*U(2,2)*U(3,3)*U(4,4)*U(5,5)
ans =
       3456
%Since U is the upper triangular matrix of A, we can use the rule for the
determinants of triangular matrices.
%Since U is an upper triangular matrix, the determinant of U is equal to the
product of its diagonal entries.
%As shown above, that is what we calculated, hence we can conclude that the
det(U) is 3456
%However, we cannot make this conclusion for matrix A, since matrix A had to
%go through row operations to be transformed into the upper triangular
%matrix U
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%The determinant of A also went through changes, hence the determinant would
%not be the same as the determinant would be for Matrix U
%Along with that, matrix A is not an upper or lower triangular matrix, hence
%you cannot just multiply the diagonal entries to get the determinant
det(A)
ans =
  8.6300e+03
det(U)
ans =
        3456
%As you can see above, the determinant of U is the same as it would be if you
%were multiplying the diagonal entries together
%However, for A, the determinant is different than what you would get if you
%multiply the diagonal entries of the matrix together
%Question 4(c)
A = rmat(5,5)
A =
    3     0     0     7     2
    2     8     3     8     1
    4     2     0     0     4
    6     7     9     7     9
    5     9     7     5     4
B = A; B(2,:) = A(1,:); B(1,:) = A(2,:)
B =
    2     8     3     8     1
    3     0     0     7     2
    4     2     0     0     4
    6     7     9     7     9
    5     9     7     5     4
%Question (i)
%Since we are exchanging the rows of matrix A and matrix B, the det(B) =
-det(A)
det(A)
ans =
 -4.9020e+03
det(B)
ans =
  4.9020e+03
%As seen above, det B = - det A
%(det B = -(-4.9020e+03)) hence det(B) = 4.9020e+03
C = A; C(2,:) = A(2,:) + 10*A(1,:)
C =
    3     0     0     7     2
   32     8     3    78    21
    4     2     0     0     4
    6     7     9     7     9
    5     9     7     5     4
%Question (ii)
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%Since we are multiplying row1 of matrix A by 10 and adding it to row2 of
%matrix A to replace row2 of matrix C, we can conclude that the det(C) = det(A)
det(A)
ans =
 -4.9020e+03
det(C)
ans =
 -4.9020e+03
%As you can see above, the relationship between det(A) and det(C) is that they
%are the same
D = A; D(1,:) = 10*A(1,:)
D =
   30     0     0    70    20
    2     8     3     8     1
    4     2     0     0     4
    6     7     9     7     9
    5     9     7     5     4
%Question (iii)
%The relationship between det(A), det(D), and det(10*A) is that 10*det(A) =
%det(D) = det(10*A)*(1/10000)
%This can be seen verified below
det(A)
ans =
 -4.9020e+03
det(D)
ans =
 -4.9020e+04
det(A*10)
ans =
   -4.9020e+08
%Question 4(d)
A = rmat(5,5)
A =
    1     8     3     4     5
    0     3     7     1     1
    5     3     9     4     7
    3     3     0     7     1
    8     1     4     7     6
A(1,1) = 0; A(2,1) = 0
A =
    0     8     3     4     5
    0     3     7     1     1
    5     3     9     4     7
    3     3     0     7     1
    8     1     4     7     6
[P, L, U, sign] = splu(A)
P =
    0     0     1     0     0
    0     1     0     0     0
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    1     0     0     0     0
    0     0     0     1     0
    0     0     0     0     1
L =
   1.0000         0         0         0         0
        0    1.0000         0         0         0
        0    2.6667    1.0000         0         0
   0.6000    0.4000    0.5234    1.0000         0
   1.6000   -1.2667    0.0979    0.4957    1.0000
U =
   5.0000    3.0000    9.0000    4.0000    7.0000
        0    3.0000    7.0000    1.0000    1.0000
        0         0  -15.6667    1.3333    2.3333
        0         0         0    3.5021   -4.8213
        0         0         0         0   -1.7716
sign =
   -1
P*A
ans =
    5     3     9     4     7
    0     3     7     1     1
    0     8     3     4     5
    3     3     0     7     1
    8     1     4     7     6
L*U
ans =
   5.0000    3.0000    9.0000    4.0000    7.0000
        0    3.0000    7.0000    1.0000    1.0000
        0    8.0000    3.0000    4.0000    5.0000
   3.0000    3.0000         0    7.0000    1.0000
   8.0000    1.0000    4.0000    7.0000    6.0000
%As seen above, P*A = L*U
%Question (i)
%det(P) is -1 because if you calculate the determinant, you will get that all
%the terms except one of them is equal to 0, since there is only 1 non-zero
%entry in each row. This matches with MATLAB’s value of sign
%Since that non-zero entry is 1 but you multiply by (-1)^i+j, the determinant
always comes out to be -1
%This can be checked with the value of sign, which is -1
%Along with that, P represents the number of row exchanges used to get the
%value of 0 at A(1,1). Since there is only one row exchange involved in making
%A(1,1) = 0, det(P) = -1
%Question (ii)
%det(L) is 1 because L is a lower triangular matrix, hence you can multiply the
diagonal entries together to get the value of the determinant
%Since the values of all the diagonal entries is 1, the det(L) is also equal
%to 1
det(L)
ans =
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    1
%As shown above, the det(L) = 1
%Question (iii)
%The relationship of the det(A) and det(U) is that det(A) = - det(U) because
%of the  multiplicative property of determinants which says
%det(U) = det(P)*det(A) = -1 which says that -1*det(A) = det(U) which shows
%that det(A) = - det(U)
ans =
 -1.4580e+03
det(U)
ans =
  1.4580e+03
%The relationship between det(A) and det(U) can be seen verified above
%Question 5
H = house; plot2d(H), hold on
%Question 5(a)
t = pi/6; Q = [cos(t), -sin(t); sin(t), cos(t)]
Q =
   0.8660   -0.5000
   0.5000    0.8660
plot2d(Q*H)
%(i) The house has rotated/tilted to the left (counterclockwise) by pi/6
%(ii) det(Q) is 0.999956
%This tells us that the area inside the transformed house is the same as the
area of the original house
t = -pi/3; Q = [cos(t), -sin(t); sin(t), cos(t)]
Q =
   0.5000    0.8660
  -0.8660    0.5000
plot2d(Q*H)
%This tilted/rotated the house to the right/clockwise by pi/3
%det(Q) = 0.999956 which tells us that the area inside the transformed house is
still the same, as rotating doesn't affect/change the original house
%Question 5(b)
H = house; plot2d(H), hold on
r = 0.9; D = [r, 0; 0, 1/r]
D =
   0.9000         0
        0    1.1111
plot2d(D*H)
%This made the house longer lengthwise (it got taller in a way/stretched from
the top and bottom)/stretched vertically
%det(D) = 0.9999 which shows the area of the transformed house is the same as
the area of the original house
r = 0.8; D = [r, 0; 0, 1/r]
D =
   0.8000         0
        0    1.2500
plot2d(D*H)
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%This made the house even longer lengthwise (it got taller than the previous
part)/stretched vertically
%det(D) = 1
%The area inside the transformed house is the same as the area of the original
house
%Question 5(c)
H = house; plot2d(H), hold on
t = 1/2; T = [1, t; 0, 1]
T =
   1.0000    0.5000
        0    1.0000
plot2d(T*H)
%(i) The house has been slanted to the right
%(ii) det(T) = 1. This tells me that the area inside the transformed house
still remains the same
t = -1/2; T = [1, t; 0, 1]
T =
   1.0000   -0.5000
        0    1.0000
plot2d(T*H)
%(i) The house has been slanted to the left
%(ii) det(T) = 1. This tells me that the area inside the transformed house
still remains the same as the area inside the original house
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