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%Last 4 digits of RUID: 8169
%Section C1
%Math 250 MATLAB Lab Assignment #2
rand('seed', 8169)
%Question 1(a)
A = rmat(3, 5), rank(A(:,1:3))
A =
    9     4     3     7     8
    6     6     8     6     9
    3     7     4     5     5
ans =
    3
b = rvect(3), R = rref([A b])
b =
    9
    6
    5
R =
   1.0000         0         0    0.5741    0.6528    0.8704
        0    1.0000         0    0.5000    0.1250    0.5000
        0         0    1.0000   -0.0556    0.5417   -0.2778
S = R(:, 1:5)
S =
   1.0000         0         0    0.5741    0.6528
        0    1.0000         0    0.5000    0.1250
        0         0    1.0000   -0.0556    0.5417
%(i)
%Columns 1, 2, and 3 of S are pivot columns
%(ii)
%Rank(R) is 3 and Rank(A) is 3
%(iii)
%nullity of A is 2 (5 - 3)
%x4 and x5 are the free variables
%(iv)
%The equation Ax = b has a solution because the augmented matrix R is
%consistent (there is no zero row that equals a non-zero number)
%Question 1(b)
c = R(:,6)
c =
   0.8704
   0.5000
  -0.2778
x = [c; 0; 0]
x =
   0.8704
   0.5000
  -0.2778
        0
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        0
%(i)
A*x - b
ans =
  1.0e-14 *
  -0.1776
  -0.0888
        0
S*x - c
ans =
    0
    0
    0
%As shown above, Ax - b is (approximately also considered) equal to 0 and Sx -
%c is also 0
%(ii)
%In the equation A*x = b, x is created by solving for the basic variables (x1
%x2, x3) of matrix A in rref and making the free variables (x4 and x5) equal
%to 0. The value of this is equal to c. Ax = b implies that Ax - b = 0, which
%equals b - b = 0. The same principle can be applied to S*x - c = 0. The value
%of x remains constant, and S is matrix A in RREF. Hence, the equation S*x - c
%can be made into c - c, which equals 0. This is why the equations Ax - b and
%Sx - c both return the value of 0.
%Question 1(c)
u = [-S(:,4); 1; 0], v = [-S(:,5);0;1]
u =
  -0.5741
  -0.5000
   0.0556
   1.0000
        0
v =
  -0.6528
  -0.1250
  -0.5417
        0
   1.0000
%(i)
%Matrix S is matrix A in reduced row echelon form. From matrix S, we can see
%that variables x1, x2, and x3 are basic variables that you can solve for. The
%remaining variables, x4 and x5, are free variables. The general solution will
%be a linear combination of the free variables multiplied by a vector.
%For example, we will have x4 * (a vector) + x5 * (another vector), which will
%give us the general solution to Ax = 0.
S*u
ans =
    0
    0
    0
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A*u
ans =
     0
     0
     0
S*v
ans =
     0
     0
     0
A*v
ans =
     0
     0
     0
%We can confirm our above explanation with the following four calculations
%that are displayed above: A*u, A*v, S*u, S*v
s = rand(1), t = rand(1), y = s*u + t*v
s =
   0.9498
t =
   0.8405
y =
  -1.0939
  -0.5800
  -0.4025
   0.9498
   0.8405
%(ii)
%There are two properties that can be used to ensure Ay = 0
%The first one is that A(su) = s(Au). This can also be applied for: A(tv) =
%t(Av). Since A*u and A*v both equal 0 (from the calculations from the last
%part), we know that A(su)=s(Au) and A(tv)=t(Av) are both also equal to 0.
%The second property that can be used is A(u + v) = Au + Av. In this case, u
%can be replaced by s*u and v can be replaced by t*v but the property still
%applies. The edited property would be: A(su) + A(tv) = A(su + tv).
%A(su)=s(Au) and A(tv)=t(Av) are both equal to zero, hence A(su + tv) is also
%equal to 0. From this, we can also conclude that y = su + tv.
A*y
ans =
     0
     0
     0
%Question 1(d)
z = x + y
z =
  -0.2236
  -0.0800
  -0.6803
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   0.9498
   0.8405
%We have the property A(u + v) = Au + Av. In this case, we replace (u + v)
%with z, which is (x + y). Az = b --> A(x+y) = b --> Ax + Ay = b. We already
%know that Ax = b, hence we can transform the previous equation into: Ax + Ay
%= Ax. This allows us to find the value for Ay, which is 0. We then have the
%following equation: b + 0 = b, which simplifies to Az = b.
A*z - b
ans =
    0
    0
    0
%The calculations match the idea that Az - b is (approximately) 0
%Question 2
u1 = rvect(3), u2 = rvect(3), u3 = rvect(3), u4 = rvect(3)
u1 =
    5
    6
    4
u2 =
    5
    1
    8
u3 =
    5
    3
    6
u4 =
    7
    4
    2
%Question 2(a)
A = [u1, u2, u3], rref(A)
A =
    5     5     5
    6     1     3
    4     8     6
ans =
    1     0     0
    0     1     0
    0     0     1
%(i)
%Ax = 0 does not have any free variables
%(ii)
%Since there are no free variables (nullity = 0), the Set S is linearly
%independent
%Question 2(b)
B = [u1 u2 u3 u4]
B =
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     5     5     5     7
     6     1     3     4
     4     8     6     2
rref(B)
ans =
   1.0000         0         0   -6.6000
        0    1.0000         0   -9.8000
        0         0    1.0000   17.8000
%(i)
%There is one free variable (x4) in the equation Bx = 0
%(ii)
%The Set T is linearly dependent since there are free variables (there are 4
%columns but only 3 of them are pivot columns)
%Question 2(c)
v = rand(1)*u1 + rand(1)*u2
v =
   2.4546
   2.0888
   2.6491
%v is a linear combination of u1 and u2, which means there is a value for
%c1*u1 + c2*u2 + c3*u3, etc. where c1, c2, and c3 will be equal to non-zero
%numbers. Linear independent sets do not have any free variables, however, set
%U will have free variables, hence it will not be linearly independent, but
%will be linearly dependent
U = [u1 u2 v]
U =
   5.0000    5.0000    2.4546
   6.0000    1.0000    2.0888
   4.0000    8.0000    2.6491
rref(U)
ans =
   1.0000         0    0.3196
        0    1.0000    0.1713
        0         0         0
%Hence, as shown by the calculation above, there is a free variable (x3) in U,
%which shows that the set U is linearly dependent.
%Question 3
A = rmat(2,3), B = rmat(3,4), C = rmat(4,3), v = rvect(4)
A =
     0     6     9
     8     3     0
B =
     4     7     4     7
     3     1     6     2
     7     4     7     6
C =
     6     9     7
     1     3     2
     1     5     5
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    4     2     6
v =
    8
    9
    5
    1
%Question 3(a)
u = B*v, A*u, D = A*B, D*v
u =
  122
   65
  133
ans =
       1587
       1171
D =
   81    42    99    66
   41    59    50    62
ans =
       1587
       1171
%The above answers match, we can see the associativity of matrix
%multiplication
%We can verify the associativity of matrix multiplication by doing A(BC) =
%(AB)C, shown below
A*(B*C)
ans =
        891        1482        1542
        603         920        1027
(A*B)*C
ans =
        891        1482        1542
        603         920        1027
%The above answers match, showing the associativity of matrix multiplication
%Question 3(b)
A = [0 1; 0 0], B = [0 0; 1 0], C = [0 1; 1 0]
A =
    0     1
    0     0
B =
    0     0
    1     0
C =
    0     1
    1     0
%(i)
A*B
ans =
    1     0
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```
    0     0
B*A
ans =
    0     0
    0     1
%A*B and B*A are not equal
(A + B)*(A+B)
ans =
    1     0
    0     1
(A*A) + 2*A*B + (B*B)
ans =
    2     0
    0     0
%(A+B)^2 is not equal to A^2 + 2AB + B^2
%(ii)
A*A
ans =
    0     0
    0     0
%If A was a number instead of a matrix, then A*A would be the same as A^2.
%However, this doesn’t apply to a matrix for 2 reasons: (1) the matrix would
%always have to be square, which is not always the case in matrix
%multiplication and (2) Multiplying a matrix by itself doesn’t mean that
%you’re squaring each value of the matrix.
%(iii)
A*C
ans =
    1     0
    0     0
A*B
ans =
    1     0
    0     0
%A*B and A*C return the same answer
%If A, B, C were numbers with A not equal to 0, we can conclude that the value
%of B is equal to the value of C (B = C). However, we cannot make this
%conclusion for matrices, regardless if we get the same answer for A*B and
%A*C. We can see this example illustrated above since we can already see that
%matrix B and matrix C are not equivalent.
%Question 4
%(i)
%Scanned picture submitted on Canvas
A = [0 0 0 1 0 1; 1 0 1 1 0 0; 0 1 0 1 0 0; 1 0 1 0 0 0; 1 1 1 0 0 1; 0 0 1 1 0
0]
A =
    0     0     0     1     0     1
    1     0     1     1     0     0
    0     1     0     1     0     0
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```
    1     0     1     0     0     0
    1     1     1     0     0     1
    0     0     1     1     0     0
%(ii)
%From the picture, we can see that if a person does not have an arrow pointing
%to them from another person, they will not be able to receive messages from
%that person
%The following fall under that category and can be deducted using the
%submitted picture:
%Person 1 cannot receive messages from Person 3 and Person 6
%Person 2 cannot receive messages from People 1, 4, and 6
%Person 3 cannot receive messages from Person 1
%Person 4 cannot receive messages from Person 5
%Person 5 cannot receive messages from People 1, 2, 3, 4, and 6 (in other
%words, person 5 cannot receive messages from anyone)
%Person 6 cannot receive messages from People 2, 3, 4
%The image uploaded on Canvas is color-coded, which makes it easier to
%identify who can send and receive messages from what people
%Looking at the matrix, we see that each column corresponds to a person (from
%1 to 6) and each row also corresponds of a person (1 to 6)
%Going down the columns for each person, if there is a 0 there (except that
%own person's number), we know that that person (column number) cannot receive
%messages from the person corresponding to the row number
%For example, in column 1, row 3 has an empty spot which shows that person 1
%cannot receive a message from person 3.
%Another example is in column 4, row 5 is empty, which shows that person 4
%cannot receive a message from person 5
%Hence the column corresponds to a person, and the row corresponds to a person
%as well. If there is a zero at that value, we know that the person (denoted
%by the column number) cannot receive a message from the other person (denoted
%by the row number)
%(iii)
A
A =
    0     0     0     1     0     1
    1     0     1     1     0     0
    0     1     0     1     0     0
    1     0     1     0     0     0
    1     1     1     0     0     1
    0     0     1     1     0     0
A*A
ans =
    1     0     2     1     0     0
    1     1     1     2     0     1
    2     0     2     1     0     0
    0     1     0     2     0     1
    1     1     2     4     0     1
    1     1     1     1     0     0
A*A*A
```

```
ans =
     1     2     1     3     0     1
     3     1     4     4     0     1
     1     2     1     4     0     2
     3     0     4     2     0     0
     5     2     6     5     0     1
     2     1     2     3     0     1
A*A*A*A
ans =
     5     1     6     5     0     1
     5     4     6     9     0     3
     6     1     8     6     0     1
     2     4     2     7     0     3
     7     6     8    14     0     5
     4     2     5     6     0     2
%The entry in (1,4) of the answer matrices (A, A^2, A^3, A^4) gives you the
%number of ways that Person 1 can send a message to person 4 in the different
%stages
%Stage 1 corresponds to Matrix A, where the (1,4) entry has the value 1, hence
%Person 1 can only send Person 4 a message one way in Stage 1
%Stage 2 corresponds to Matrix A*A, where the (1,4) entry has the value 1,
%hence Person 1 can only send Person 4 a message one way in Stage 2
%Stage 3 corresponds to Matrix A*A*A, where the (1,4) entry has the value 3,
%hence Person 1 can send Person 4 a message three ways in Stage 3
%Stage 4 corresponds to Matrix A*A*A*A, where the (1,4) entry has the value 5,
%hence Person 1 can send Person 4 a message five ways in Stage 4
%(iv)
B = A + (A*A) + (A*A*A) + (A*A*A*A)
B =
     7     3     9    10     0     3
    10     6    12    16     0     5
     9     4    11    12     0     3
     6     5     7    11     0     4
    14    10    17    23     0     8
     7     4     9    11     0     3
%Using the formula given to us in part h, we can create a matrix B which will
%allow us to find the number of ways in which person i can send a message to
%person j in at most m stages
%More specifically, the (3,4) entry of matrix B will allow us to find the
%number of ways Person 3 can send a message to Person 4 in at most 4 stages
%As we can see above, the (3,4) entry of matrix B is 12, which tells us that
%there are 12 ways Person 3 can send Person 4 a message in at most 4 stages
```