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Analysis of Iterative Multi-User Detection by Information Theory

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Supervisors

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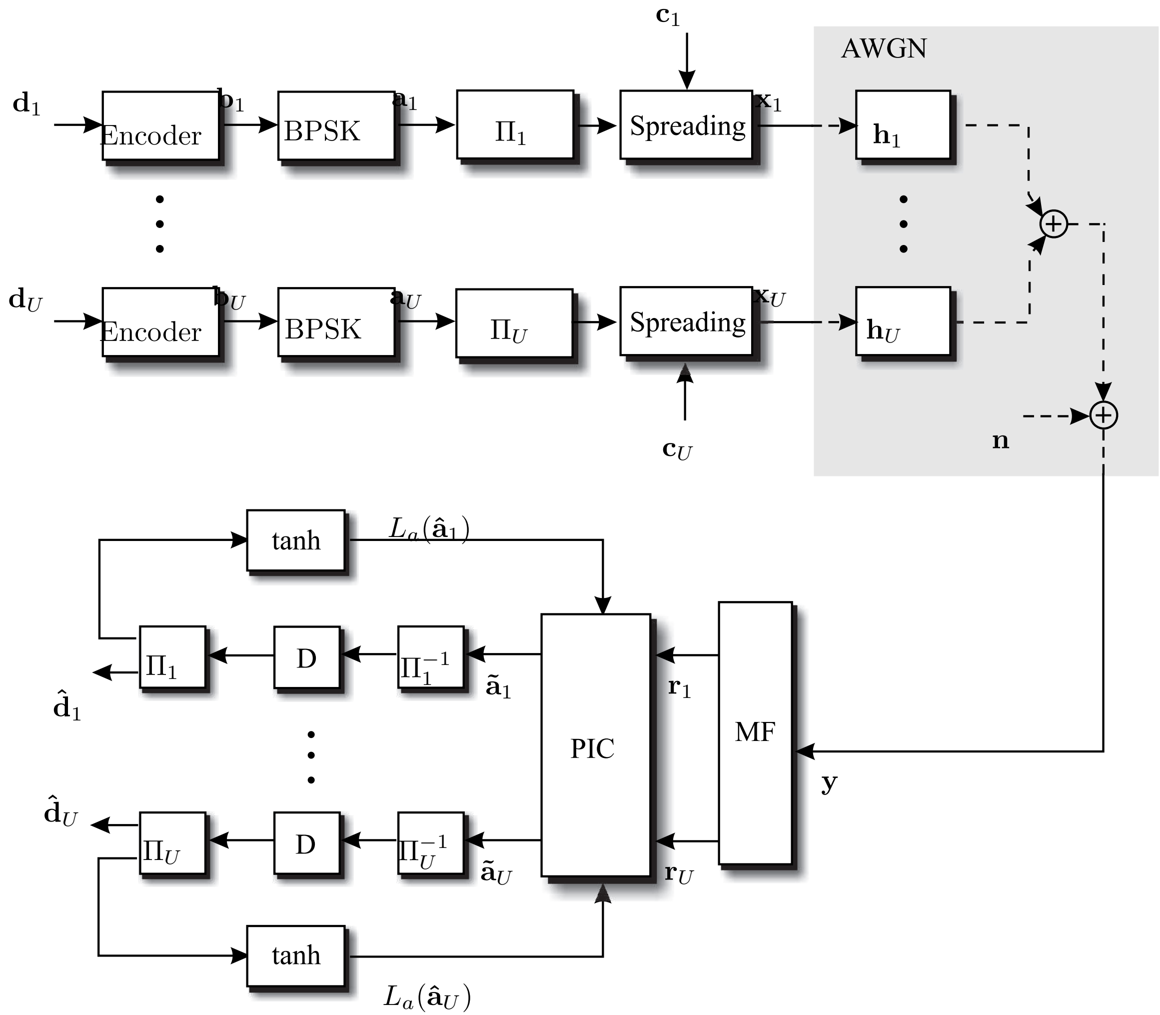
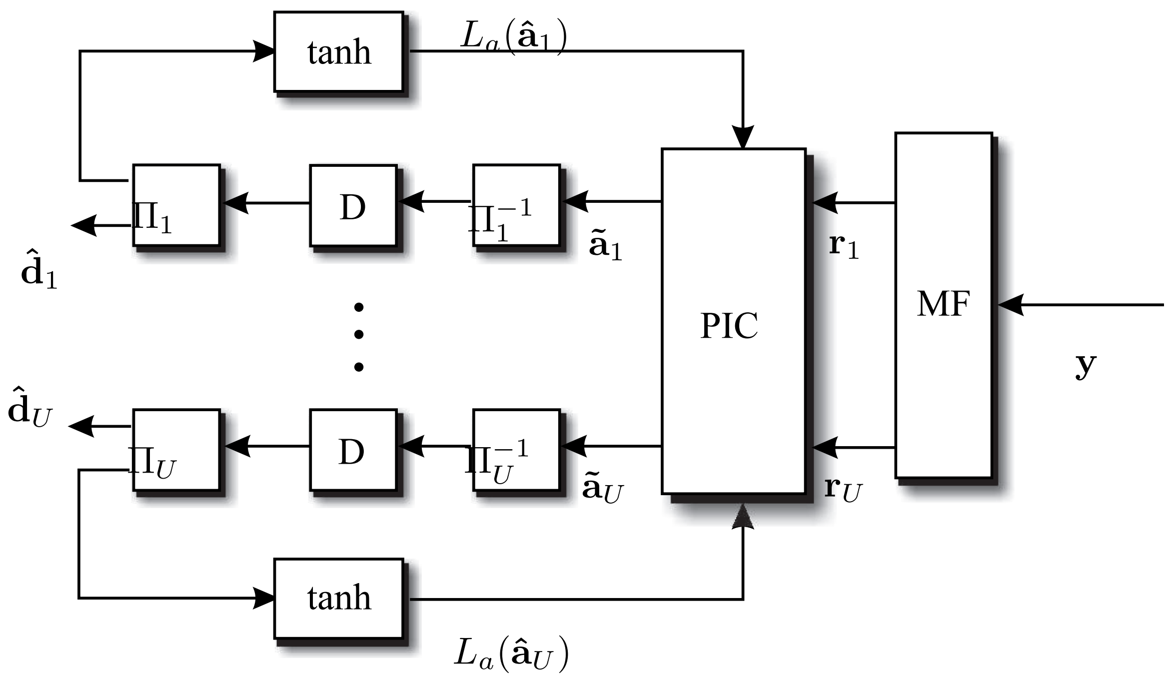
Multi User Communication Model



- Let u be the user index with $1 \leq u \leq U$
- Information bits: $\mathbf{d}_u = [d_u[1], \ldots, d_u[K]]$ where, $d_u[k] \in \{0, 1\}$ and $1 \leq k \leq K$.
- Coded bits: $\mathbf{b}_u = [b_u[1], \ldots, b_u[N]]$ where, $b_u[n] \in \{0, 1\}$ and $1 \leq n \leq N$.
- Coded rate $R = K/N$.
- Modulated Symbols: $\mathbf{a}_u = [a_u[1], \ldots, a_u[N]]$. where, $a_u[n] \in \{+1, -1\}$ and $1 \leq n \leq N$.
- User specific Interleavers Π_u of length L_π.
- Spreading: DS-CDMA with N_s as spreading factor and $\beta = U/N_s$.
- Spreading sequence $\mathbf{c}_u = [c_u[1], \ldots, c_u[N_s]]$ where, $c_u[\ell] \in \{+1, -1\}$ and $1 \leq \ell \leq N_s$.
- Spreaded seq $\mathbf{x}_u$ is of length $(N \cdot N_s)$ x 1
- $\mathbf{h}_u$: Channel impulse response of user u.

Receiver Section



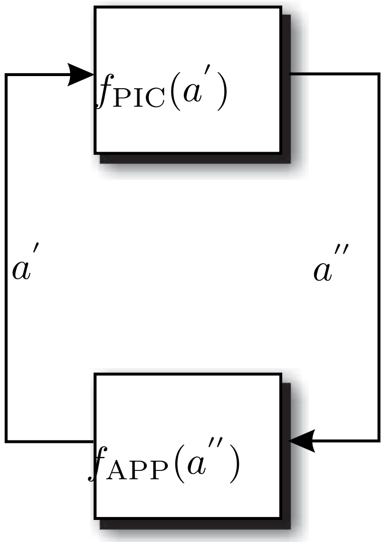
- Received vector $\mathbf{y} = \mathbf{H} \cdot \mathbf{x} + \mathbf{n} = \mathbf{S} \cdot \mathbf{a} + \mathbf{n}$
- $\mathbf{H} = [\mathbf{T}_{\mathbf{h}_1} \ldots \mathbf{T}_{\mathbf{h}_U}]$ and $\mathbf{S}$ is the Signature of all the users.
- $\mathbf{n}$: AWGN with zero mean and variance σ_n^2.
- MF: $\mathbf{r} = \mathbf{S}^H \cdot \mathbf{y} = \mathbf{S}^H(\mathbf{S} \cdot \mathbf{a} + \mathbf{n}) = \mathbf{R} \cdot \mathbf{a} + \tilde{\mathbf{n}}$
- $\tilde{\mathbf{a}}_u = 4\frac{E_s}{N_0} \cdot \left[\mathbf{r}_u - \sum_{\substack{v=1 \\ v \neq u}}^{U} \tanh(L_a(\hat{\mathbf{a}}_v)/2) \right]$
- D: Soft-Output Decoder e.g., Max-Log-Max delivers LLR's $L(\tilde{\mathbf{a}}_u)$ of the coded bits.

$\Longrightarrow$ How to analyze such a system ?

Single-Parameter Dynamical Model

Iterative Multiuser Joint Decoding: Unified Framework and Asymptotic Analysis

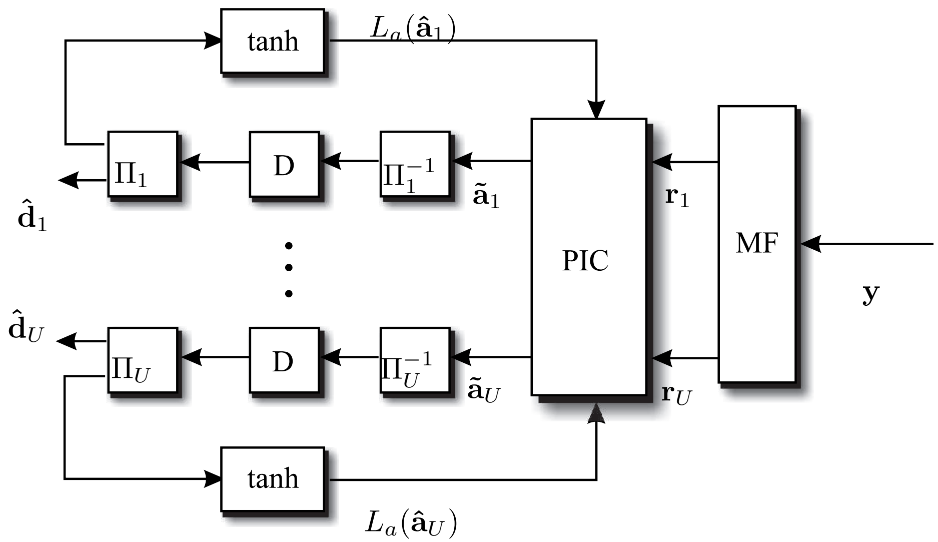
(Joseph Boutros and Giuseppe Caire)



- Behavior of iterative multiuser joint decoding is approximately characterized by the stable fixed points of a simple one-dimensional non-linear dynamical systems, when making Gaussian approximation of the decoder soft-outputs.
- a is a single-parameter which describes the system of whole.
- With a as Mutual Information $\Longrightarrow$ EXIT Charts.
- With a as Multi-User Interference $\Longrightarrow$ MUE.
- With a as Variance of the estimation error $\Longrightarrow$ VTC.

EXIT Charts

- Extrinsic Information Transfer Characteristic Charts are obtained by plotting the exchange of Mutual-Information between the components of a iterative multiuser joint decoder.



- Let x be the transmitted signal and y be the received signal and the mutual information between x and y

- $I(x;y) = 1 + \frac{1}{2} \cdot \sum\limits_{d=\pm1} \int\limits_{-\infty}^{\infty} p(y|x=d)$

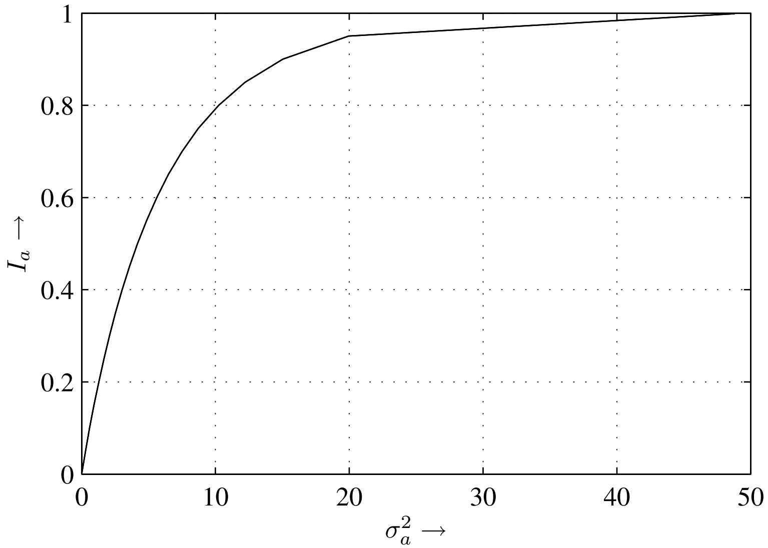
$$\text{x} \log \frac{p(y|x=d)}{p(y|x=+1)+p(y|x=-1)} \cdot dy$$

- ten's approach to model a-priori LLR's $L_a(\hat{\mathbf{a}}_u)$.

- $L_a(\hat{\mathbf{a}}_u) = \bar{n}_u b_u + n_u$ where, n_u WGN with variance σ_a^2 and the mean of $L_a(\hat{\mathbf{a}}_u)$ is $\bar{n}_u = \sigma_a^2/2$

- $I(L_a(\hat{\mathbf{a}}_u);\mathbf{a}) = 1 - \frac{1}{\sqrt{2\pi\sigma_a^2}} \cdot \int\limits_{-\infty}^{+\infty} e^{-\frac{(\xi-\sigma_a^2/2)}{2\sigma_a^2}} \log(1+e^{-\xi}) d\xi.$

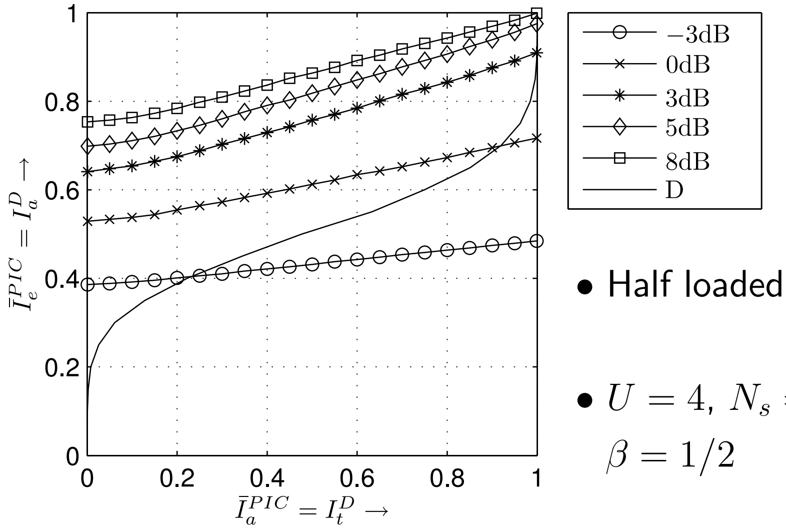
EXIT Charts



Mutual Information Vs σ_a^2

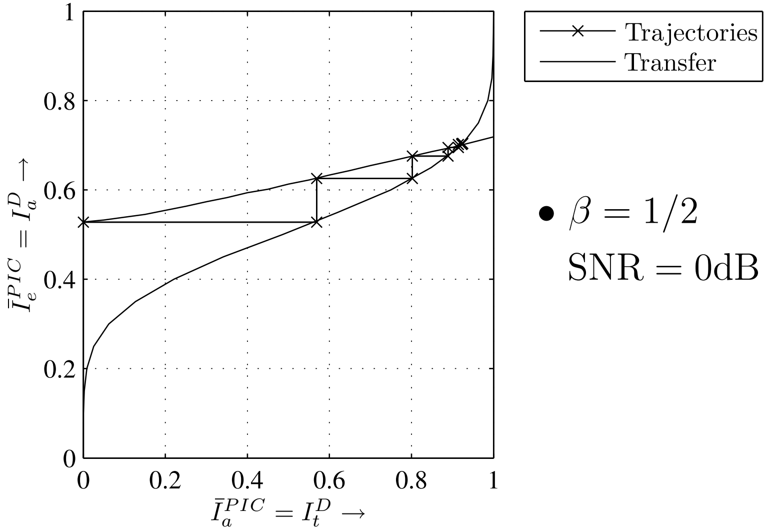
- Mutual information at the input and the output of the PIC and the Decoder.
  - $I_{a,u}^{PIC} = I(L_a(\hat{\mathbf{a}}_u); \mathbf{a}_u)$
  - $I_{e,u}^{PIC} = I((L_a(\hat{\mathbf{a}}_u|\mathbf{r}) - L_a(\hat{\mathbf{a}}_u)); \mathbf{a}_u)$
  - $I_{a,u}^{D} = I_{e,u}^{PIC}$
  - $I_{t,u}^{D} = I_{a,u}^{PIC}$

- $\bar{I}_e^{PIC} = \frac{1}{U} \cdot \sum_{u=1}^{U} I_{e,u}^{PIC}$
- Mutual Information depends on SNR.
- $\bar{I}_e^{PIC} = 1$ interference is totally suppressed, reaches to single-user AWGN system.

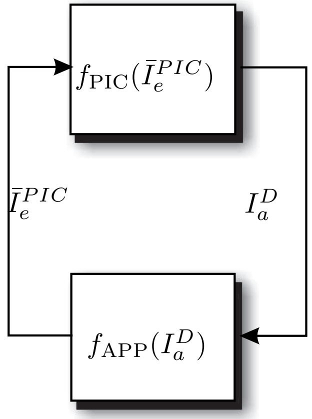


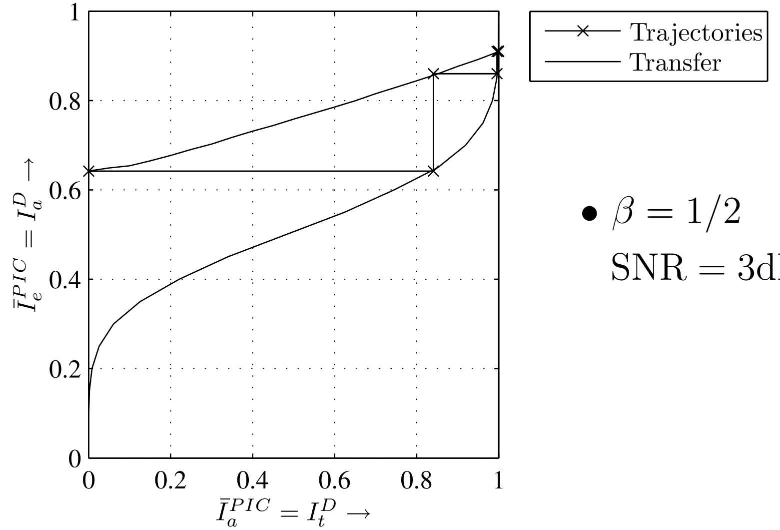
- Half loaded System
- $U = 4,\ N_s = 8$ and $\beta = 1/2$

EXIT Charts

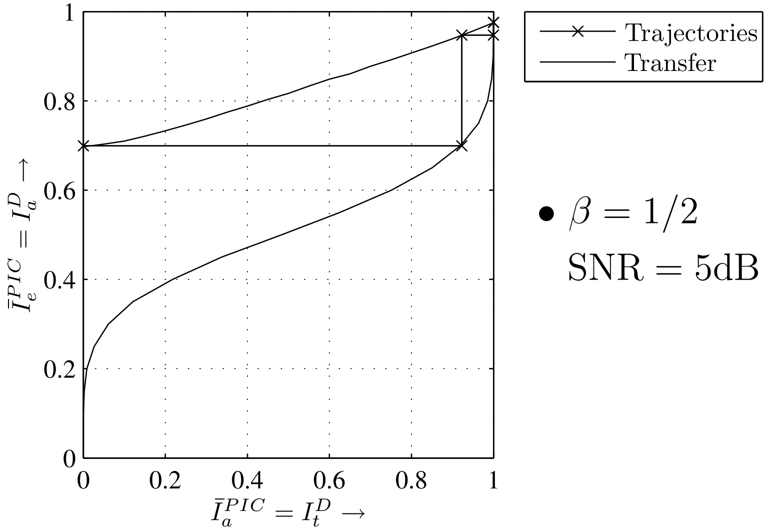


- $\beta = 1/2$
  SNR = 0dB

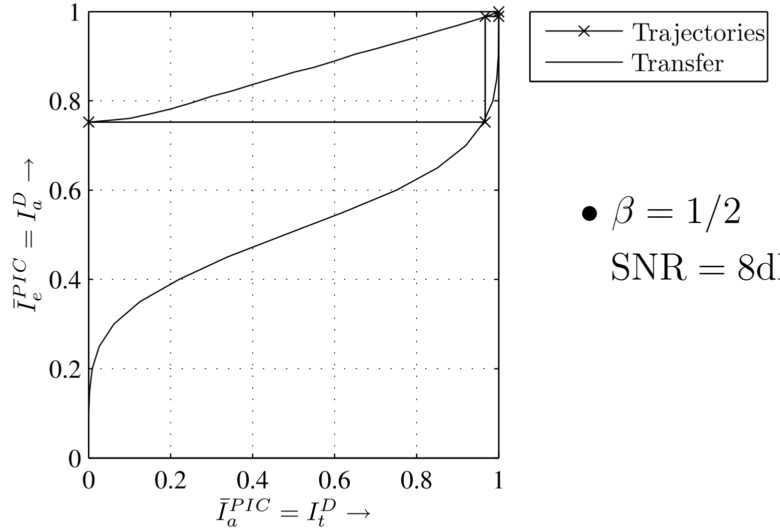




- $\beta = 1/2$
  SNR = 3dB

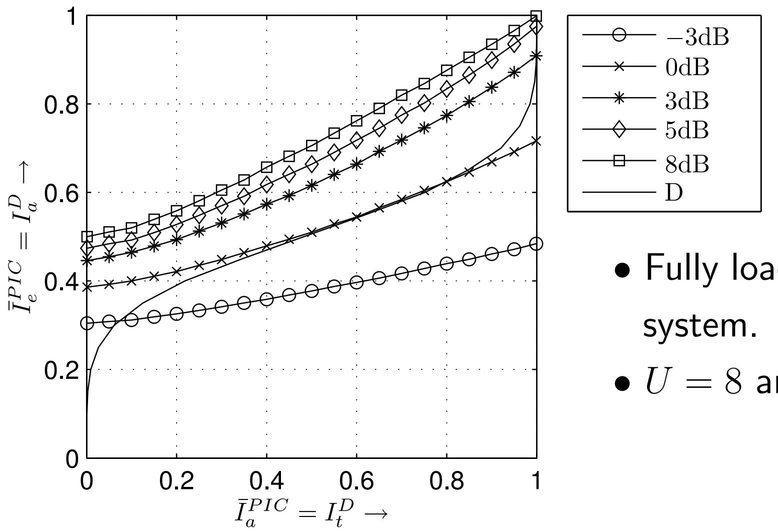


- $\beta = 1/2$
  SNR = 5dB

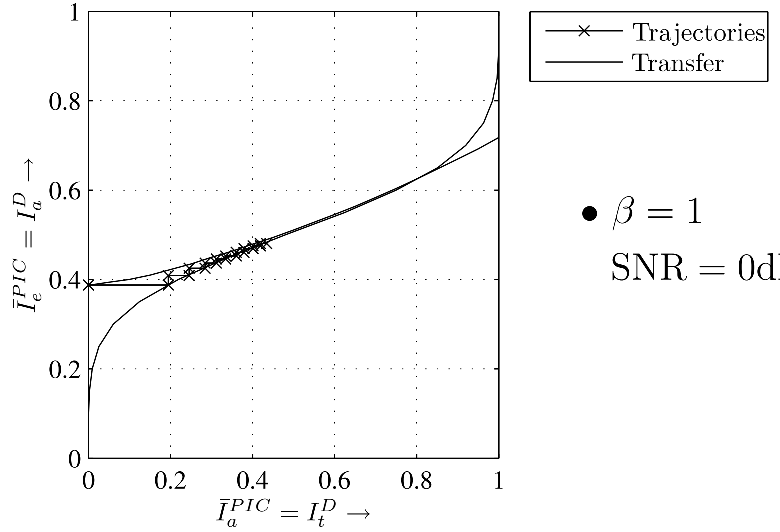


- $\beta = 1/2$
  SNR = 8dB

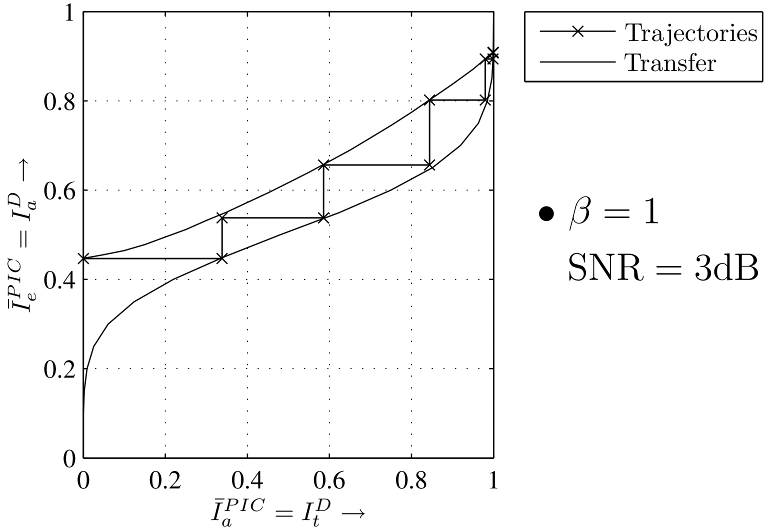
EXIT Charts



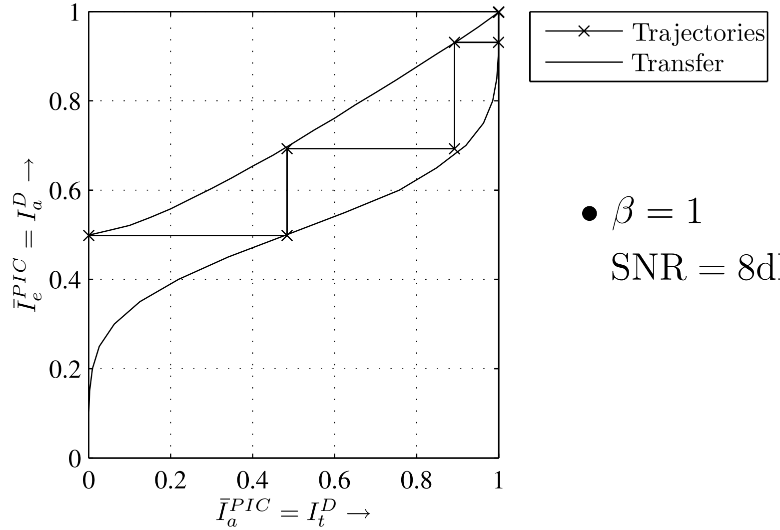
- Fully loaded system.
- $U = 8$ and $N_s = 8$



- $\beta = 1$
  $\mathrm{SNR} = 0\mathrm{dB}$

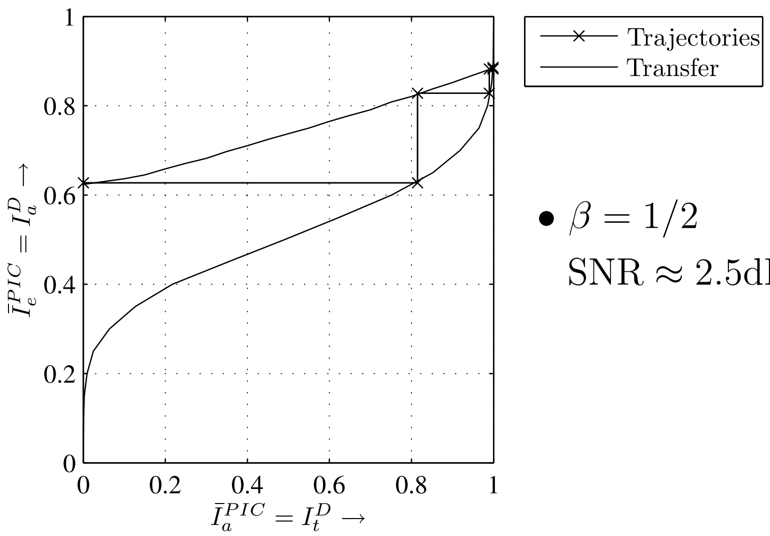


- $\beta = 1$
  $\mathrm{SNR} = 3\mathrm{dB}$

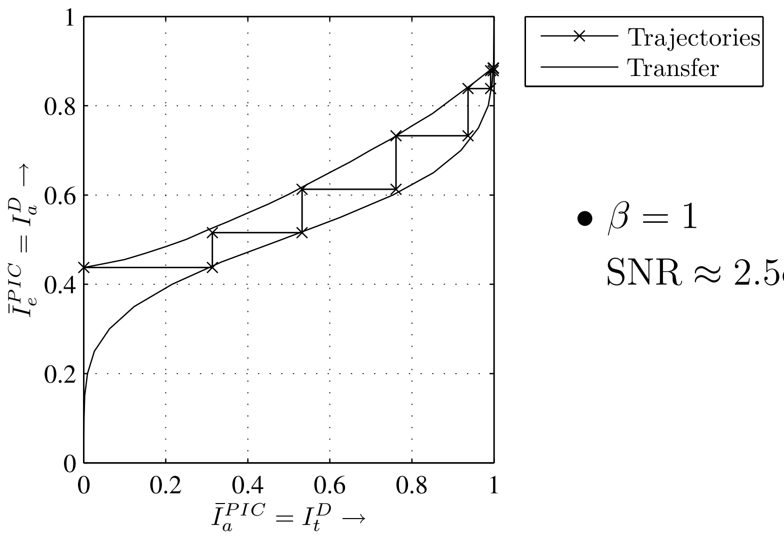


- $\beta = 1$
  $\mathrm{SNR} = 8\mathrm{dB}$

Concluding remarks on EXIT Charts



- $\beta = 1/2$
  SNR $\approx$ 2.5dB

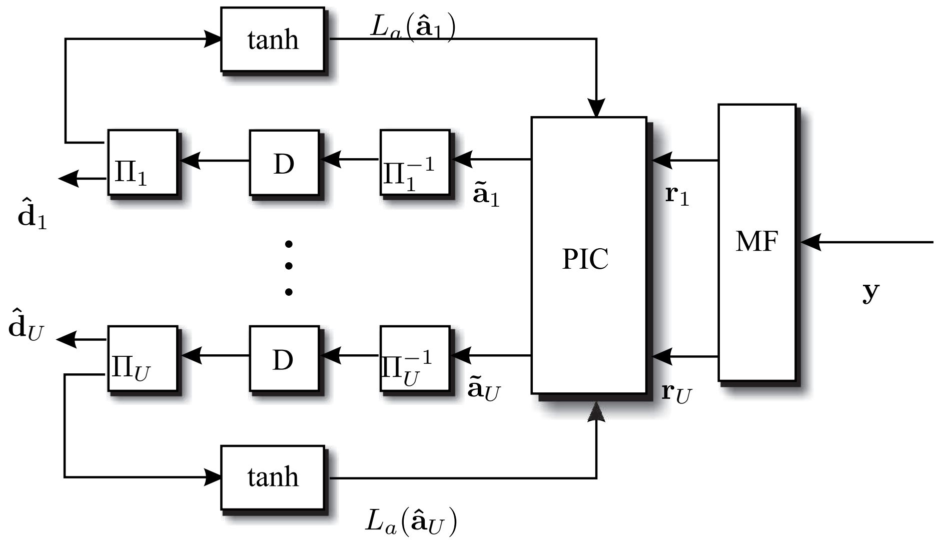


- $\beta = 1$
  SNR $\approx$ 2.5dB

- EXIT charts characterize the system very well, since prediction curves are so tight.

- Convergence properties of the system can be studied very well.

Multi User Efficiency



- $\eta = \frac{\text{SINR}}{\text{SNR}} = \frac{2\sigma_d^2/(\sigma_n^2 + \sigma_{\text{MUI}}^2)}{2\sigma_d^2/\sigma_n^2}$

- σ_d^2 Variance of the desired signal.

- σ_{MUI}^2 Variance of the remaining MUI after cancellation.

$\Longrightarrow \sigma_{\text{MUI}}^2 = \sigma_d^2 \cdot \mu(U-1)/N_s$, where $\mu = \text{E}\{|L_a(\hat{\mathbf{a}}) - \mathbf{a}|^2\}$

- σ_n^2 Variance of the noise.

$$\Longrightarrow \eta = \frac{1}{1 + \beta\mu E_s/N_0}$$

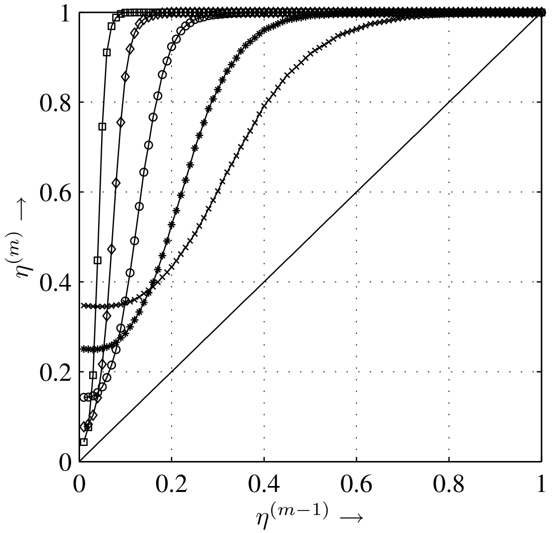
- $\eta^{(1)} = 1/(1 + \beta E_s/N_0)$

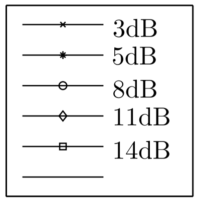
- The predetermination of $\mu^{(m)} = g(\text{SINR}) = g(\eta^{(m-1)}\text{SNR})$

- Relating everything we obtain $\eta^{(m)} = f(\eta^{(m-1)})$

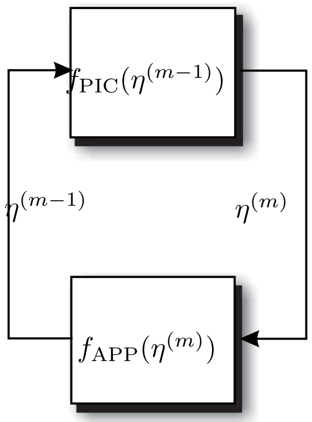
- Hence, single-parameter dynamical model is obtained.

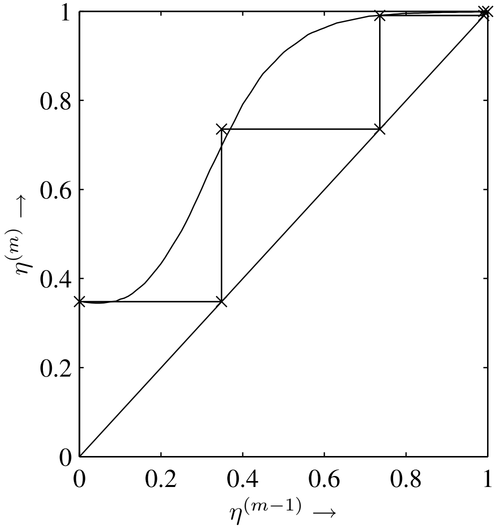
Multi User Efficiency

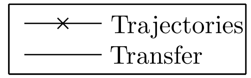




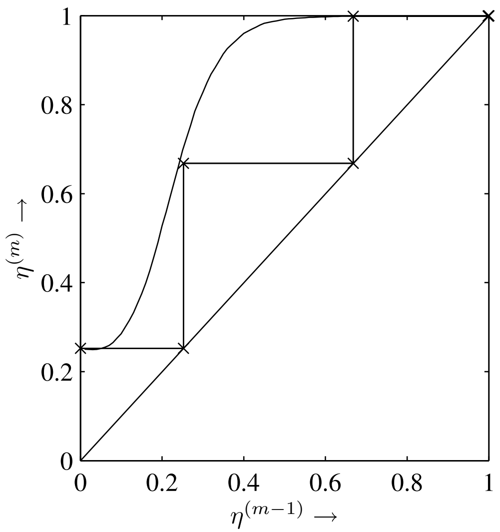
- $U = 16$
  $N_s = 16$
  $\beta = 1$

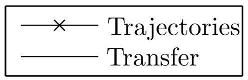




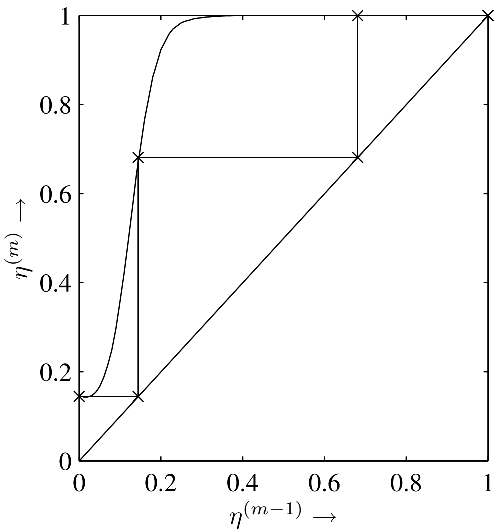


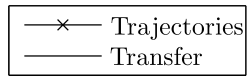
- $\beta = 1$
  $\mathrm{SNR} = 3\mathrm{dB}$





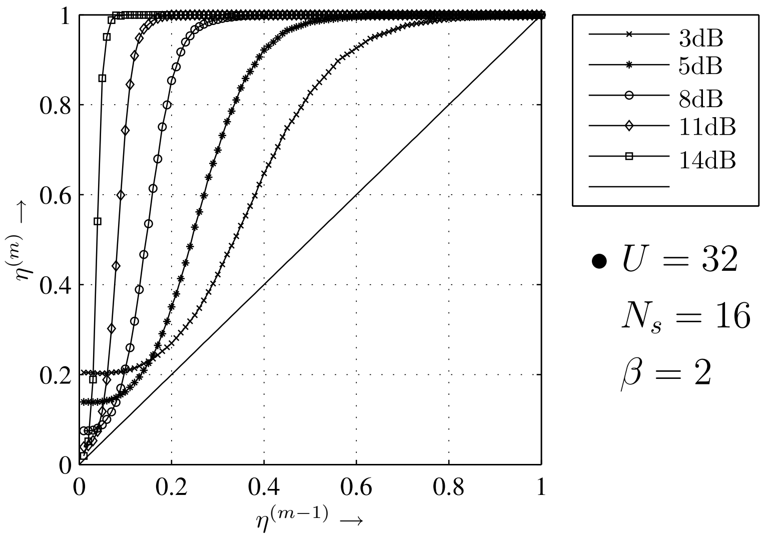
- $\beta = 1$
  $\mathrm{SNR} = 5\mathrm{dB}$



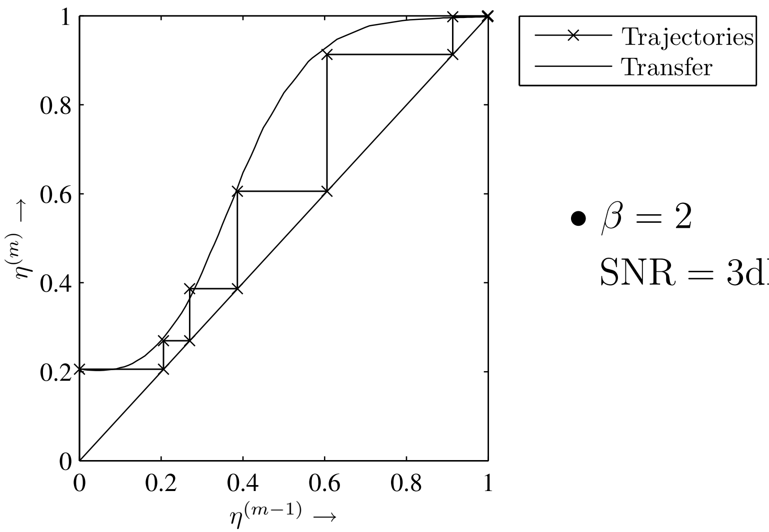


- $\beta = 1$
  $\mathrm{SNR} = 8\mathrm{dB}$

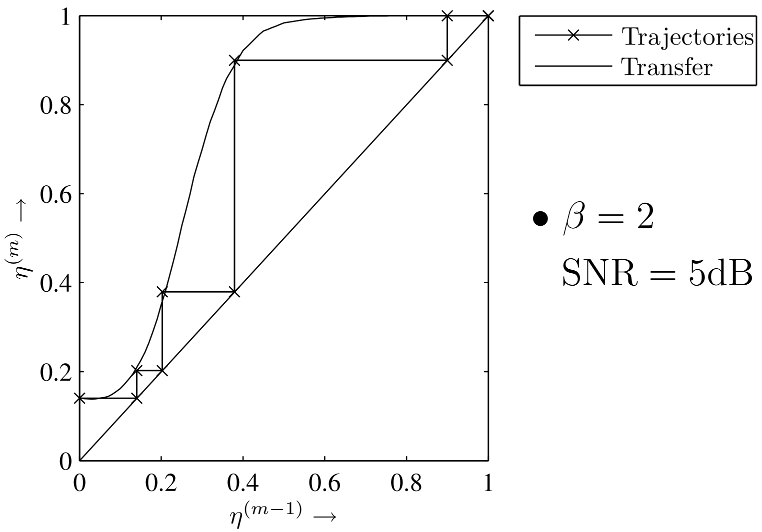
Multi User Efficiency



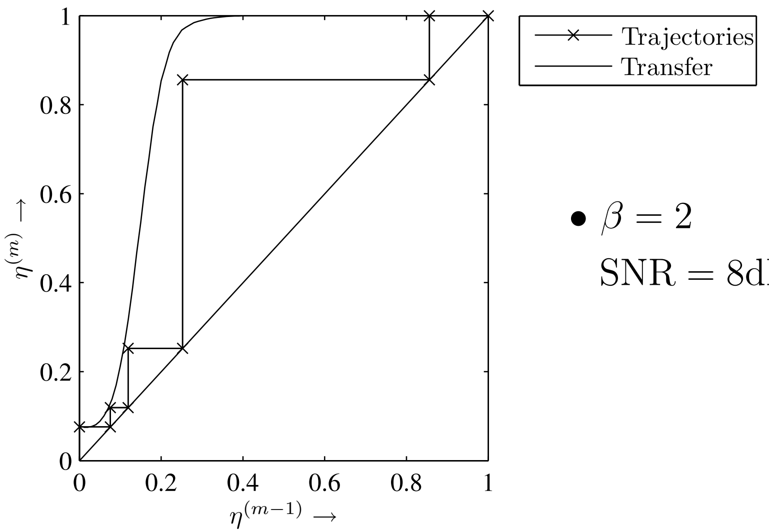
- $U = 32$
  $N_s = 16$
  $\beta = 2$



- $\beta = 2$
  $\mathrm{SNR} = 3\mathrm{dB}$

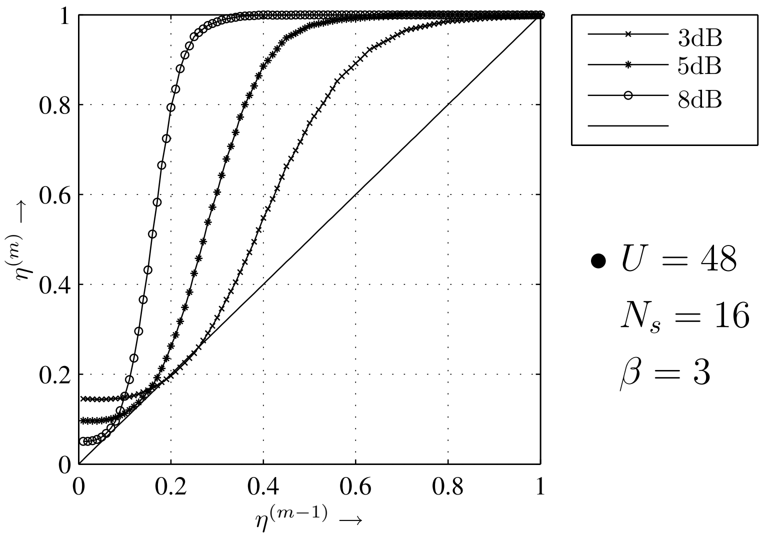


- $\beta = 2$
  $\mathrm{SNR} = 5\mathrm{dB}$



- $\beta = 2$
  $\mathrm{SNR} = 8\mathrm{dB}$

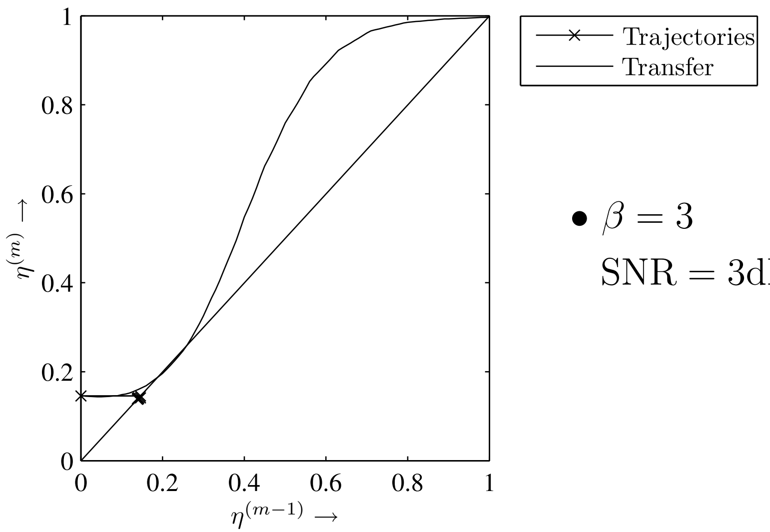
Multi User Efficiency



- $U = 48$

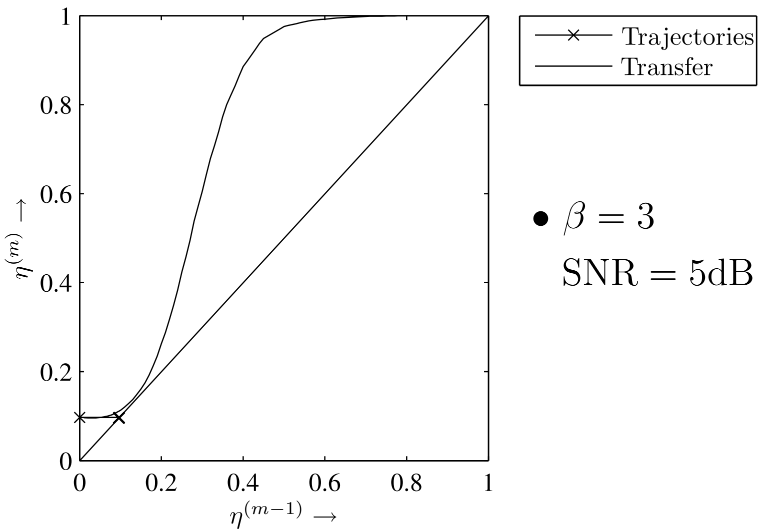
  $N_s = 16$

  $\beta = 3$



- $\beta = 3$

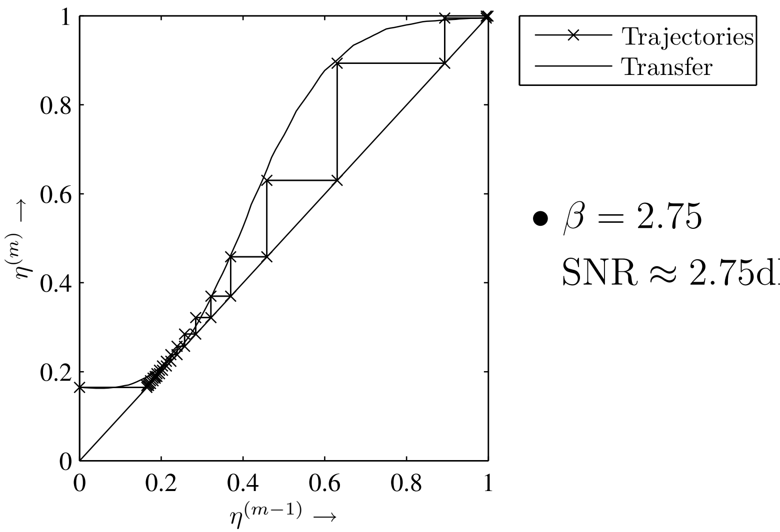
  $\mathrm{SNR} = 3\mathrm{dB}$



- $\beta = 3$

  $\mathrm{SNR} = 5\mathrm{dB}$

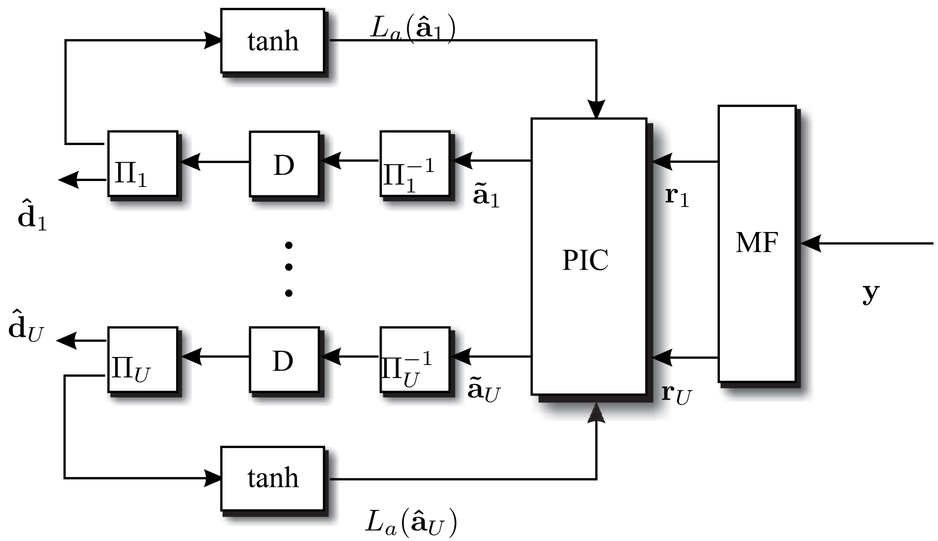
Multi User Efficiency

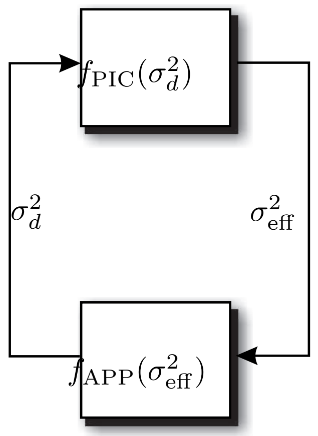


- $\beta = 2.75$
  SNR ≈ 2.75dB

- Minimum SNR required at the load $\beta = 2.75$ to reach the convergence.
- MUE is one of the analysis tool gives very tight predictions curves.
- Can characterize the system very well.

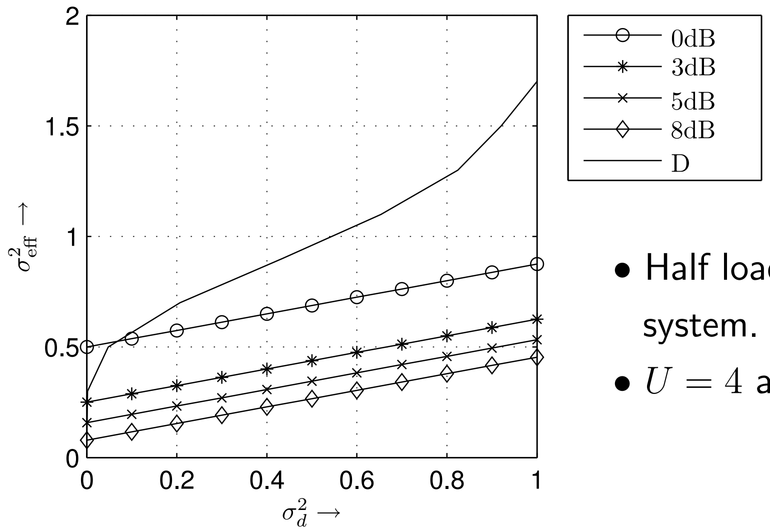
Variance Transfer Characteristics



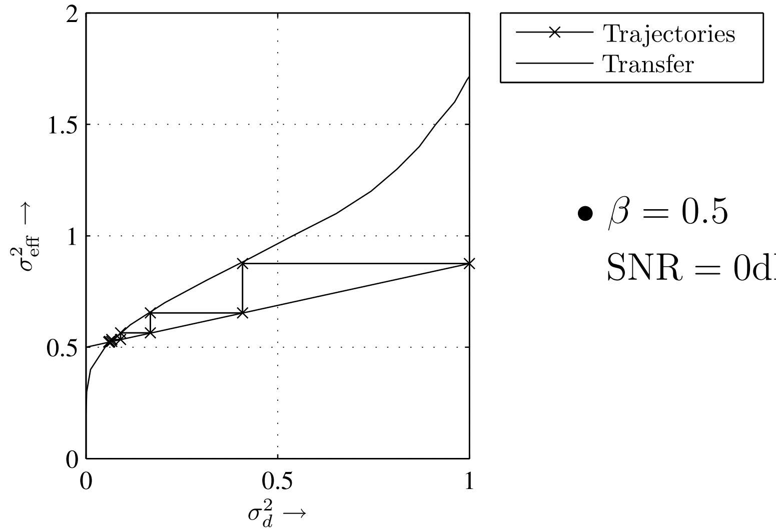


- VTC: Variance of an estimation error is exchanged between the component.
- Estimation error: $\gamma = \sum_{\substack{m=1\\(m\neq u)}}^{U} (a_m[n] - \tilde{a}_m[n]) s_m[\ell]^H s_m[\ell] + \tilde{\mathbf{n}}$
- $\mathrm{E}[\gamma] = 0$
- $\mathrm{E}[\gamma^2] = \sum_{\substack{m=1\\(m\neq u)}}^{U} \mathrm{E}\left[(a_m[n] - \tilde{a}_m[n])^2\right] \mathrm{E}\left[(s_m[\ell]^H s_m[\ell])\right] + \sigma_n^2$
- With $P(a = 1/\sqrt{N}) = P(a = -1/\sqrt{N}) = 1/2$,
  $\mathrm{E}\left[(s_m[\ell]^H s_m[\ell])\right] = 1/N$ and $\mathrm{E}\left[(a_m[n] - \tilde{a}_m[n])^2\right] = \sigma_d^2$
- $\sigma_{\mathrm{eff}}^2 = \dfrac{K-1}{N}\sigma_d^2 + \sigma_n^2$

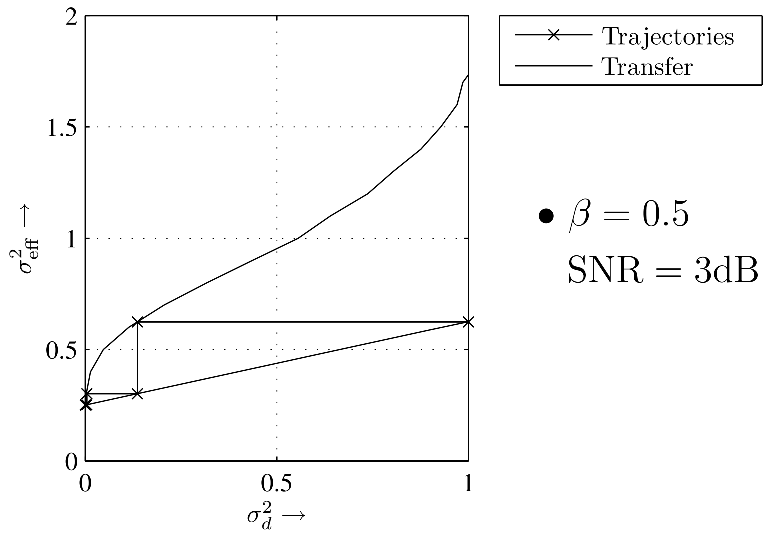
Variance Transfer Characteristics



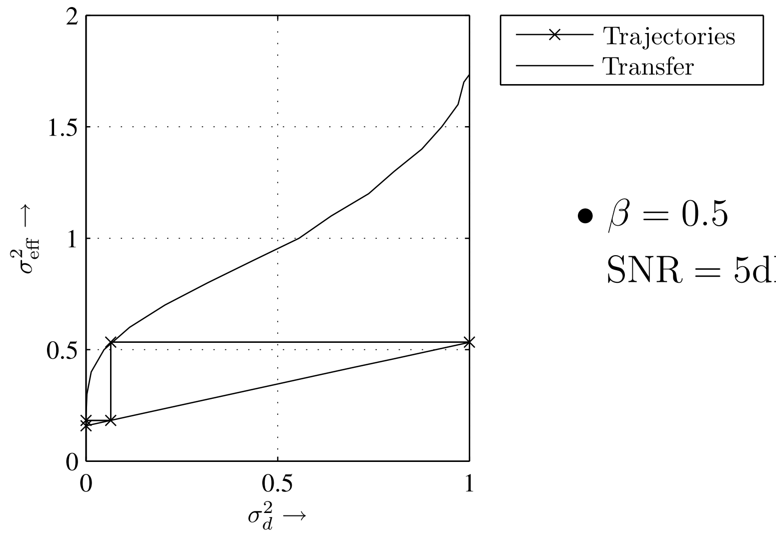
- Half loaded system.
- $U = 4$ and $N_s = 8$



- $\beta = 0.5$
  $\mathrm{SNR} = 0\mathrm{dB}$

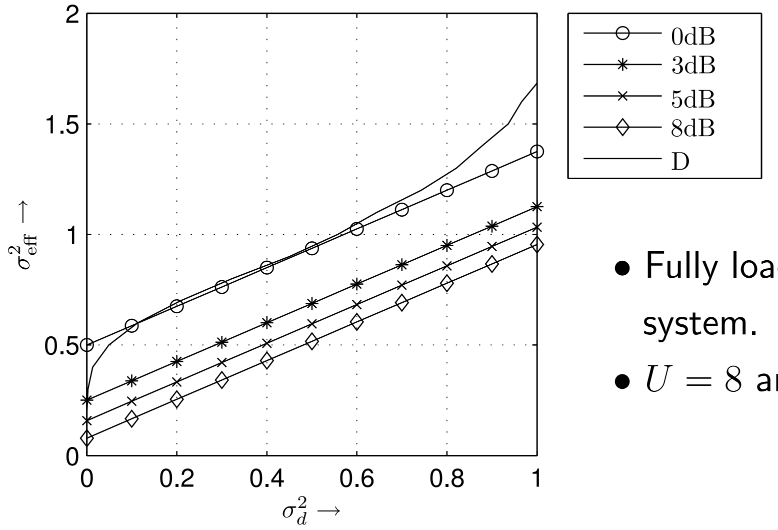


- $\beta = 0.5$
  $\mathrm{SNR} = 3\mathrm{dB}$

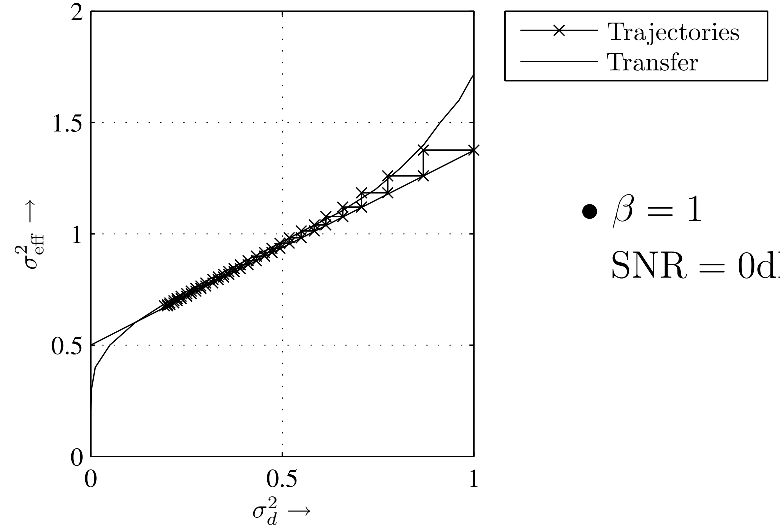


- $\beta = 0.5$
  $\mathrm{SNR} = 5\mathrm{dB}$

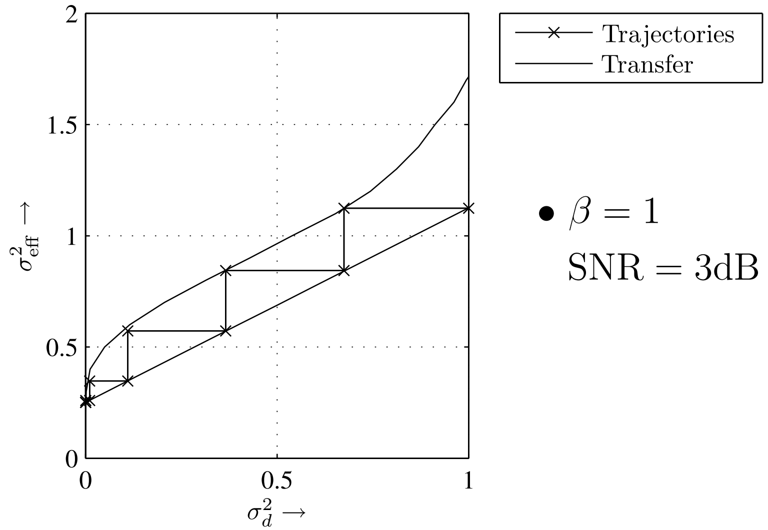
Variance Transfer Characteristics



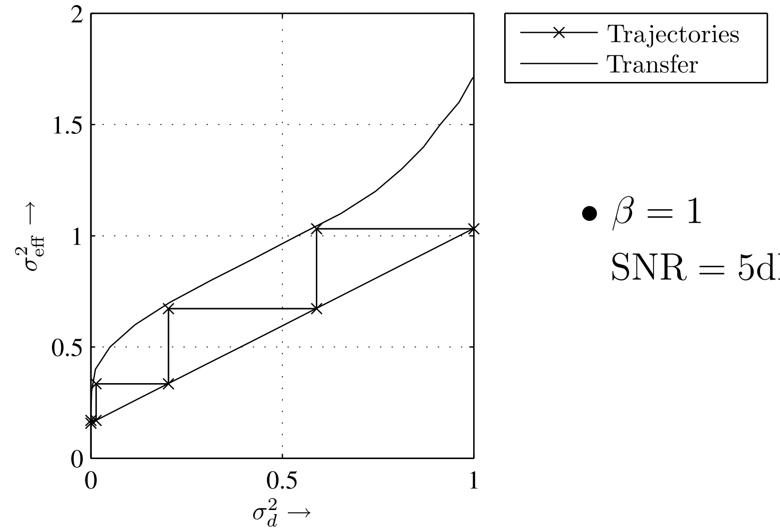
- Fully loaded system.
- $U = 8$ and $N_s = 8$



- $\beta = 1$
  $\mathrm{SNR} = 0\mathrm{dB}$

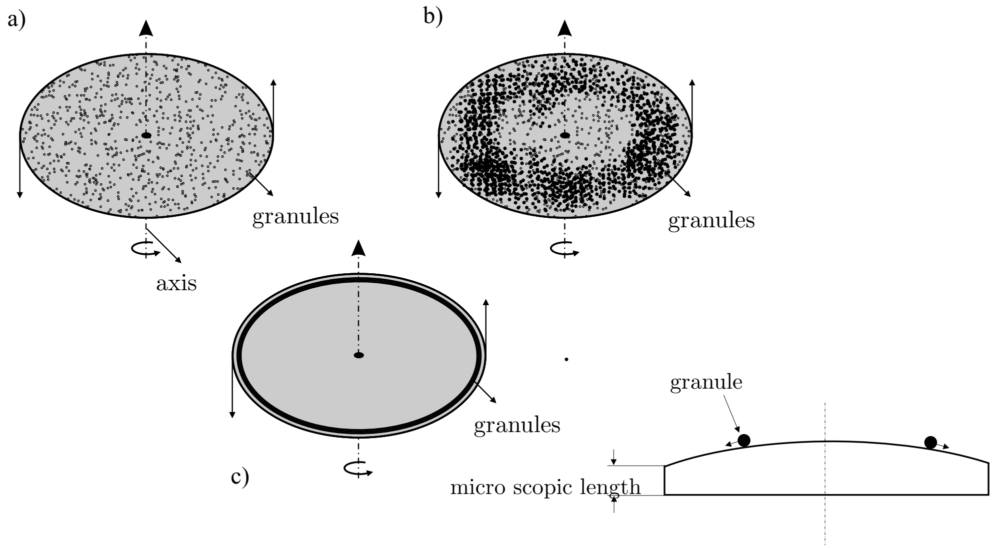


- $\beta = 1$
  $\mathrm{SNR} = 3\mathrm{dB}$

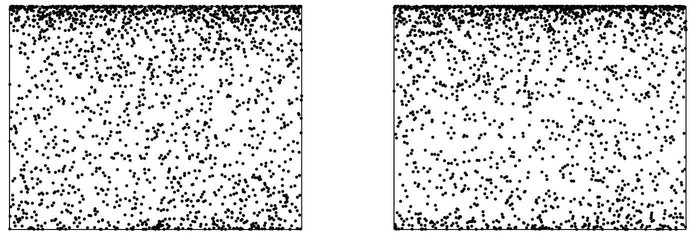


- $\beta = 1$
  $\mathrm{SNR} = 5\mathrm{dB}$

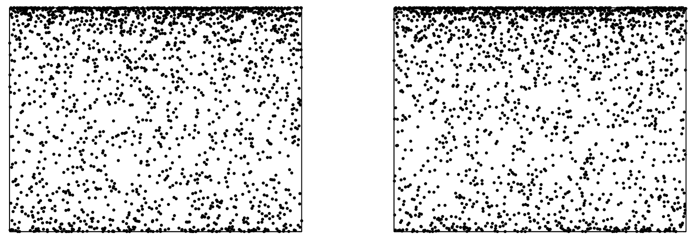
Cyclic Behavior and Circular Disk

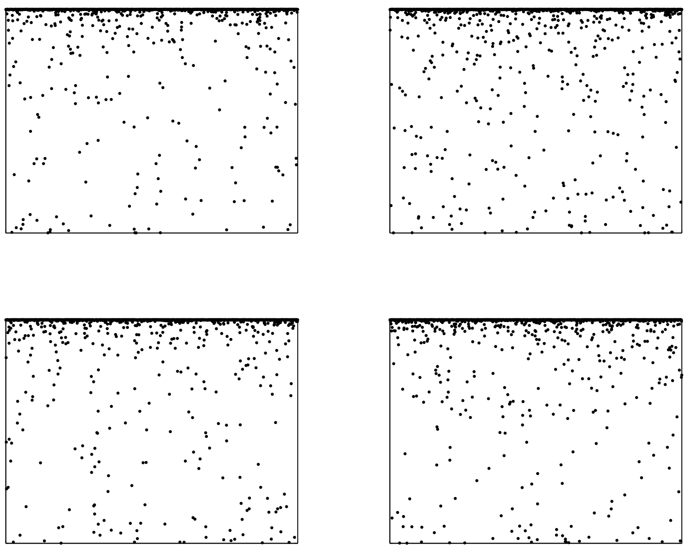


Extrinsic Information

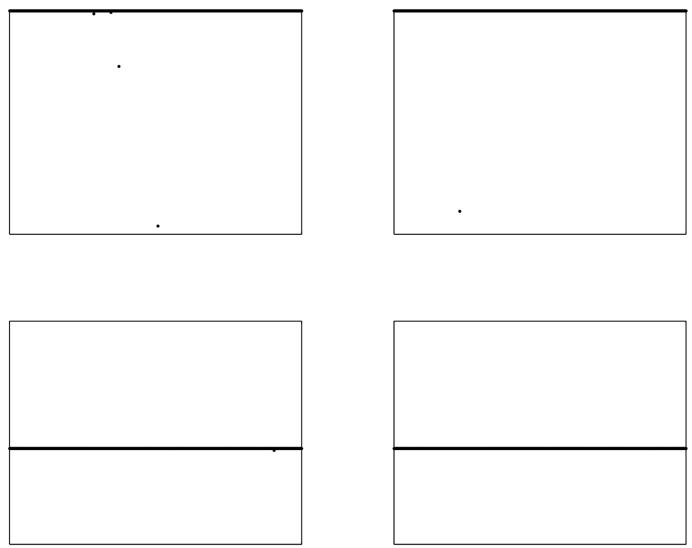


Start



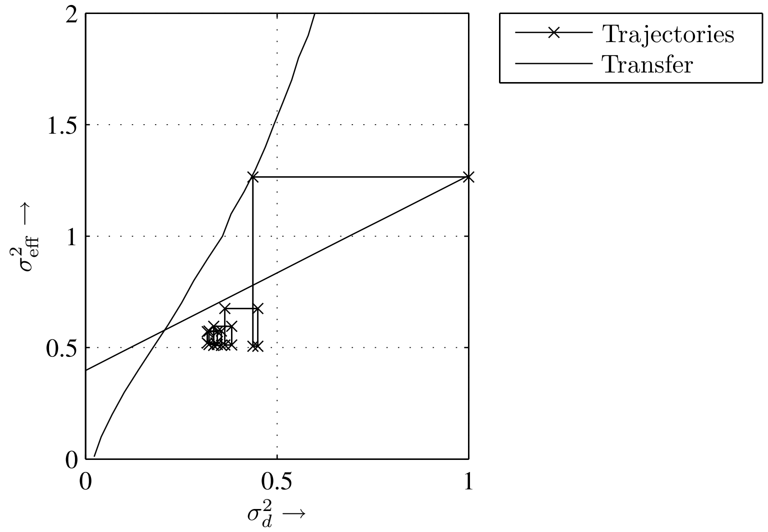


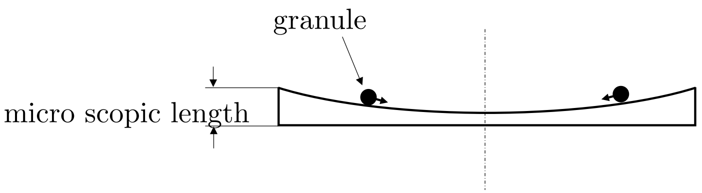
Middle



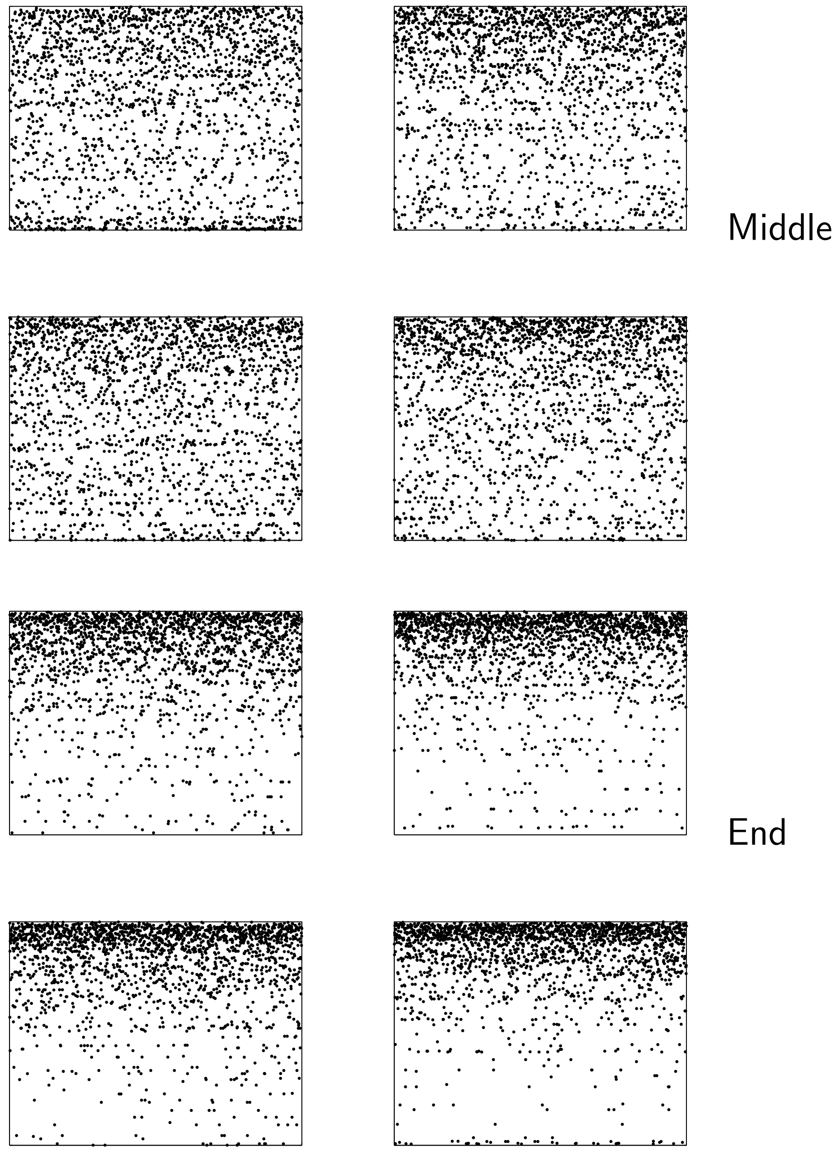
End

Cyclic Behavior and Circular Disk

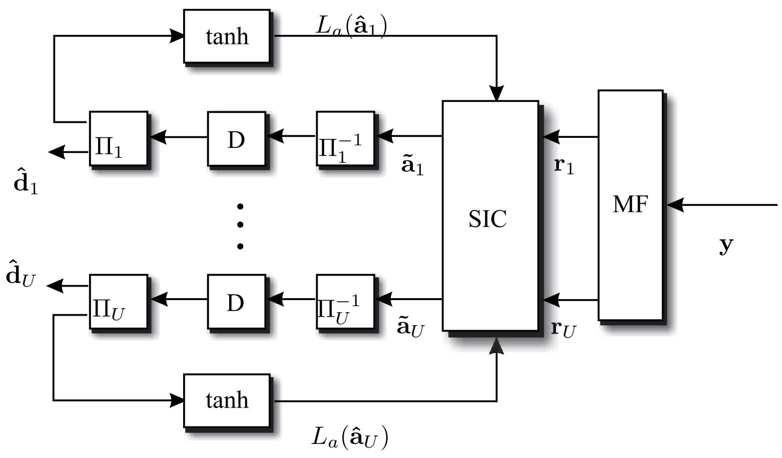




Total Information



Analysis of Successive Interference Cancellation



- By successive treatment all users cannot be treated as same, as in case of PIC
- Hence averaging among all users is not possible.
- Due to this convergence properties cannot be plotted in 2-D plot.

$\Longrightarrow$ N-dimensional problem.

Conclusion

- Convergence properties of an Iterative Multi-User detector was studied.
- Various Analysis tools was proposed and studied in detail.
- Circular disk was introduced to understand the cyclic behavior.
- Analysis of SIC became infeasible, since it poses N-dimensional problem.