#Past Questions

Q. No. 5 -2021.Find the optimal solution for the following transportation problem using any methods.

1	2	1	4	30
3	3	2	1	50
4	2	5	9	20
20	40	30	9	

Solution:

Here Total Demand = 99 is less than Total Supply = 100. So We add a dummy demand constraint with 0 unit cost and with allocation 1.
Now, The modified table is

	D_1	D_2	D_3	D_4	D_{dummy}	Supply
S_1	1	2	1	4	0	30
S_2	3	3	2	1	0	50
S_3	4	2	5	9	0	20
Demand	20	40	30	9	1	

Obtaining Initial feasible solution using VAM

	D_1	D_2	D_3	D_4	D_{dummy}	Supply	Row Penalty
S_1	1(20)	2	1(10)	4	0	~~30~~ 10	1 | 1 | 0 | 1 | 1
S_2	3	3(20)	2(20)	1(9)	0(1)	~~50~~ ~~41~~ 40	1 | 2 | 1 | 1 | 1
S_3	4	2(20)	5	9	0	~~20~~	2 | 2 | 2 | 3 | --
Demand	~~20~~	~~40~~ 20	~~30~~ 20	~~9~~	~~1~~		
Column Penalty	2 2 2 -- --	0 0 0 0 1	1 1 1 1 1	3 -- -- -- --	0 0 -- -- --		

The minimum total transportation cost $=1\times20+1\times10+3\times20+2\times20+1\times9+0\times1+2\times20=179$

Here, the number of allocated cells = 7 is equal to m + n - 1 = 3 + 5 - 1 = 7
∴ This solution is non-degenerate

Optimality test using modi method...

Find u_i and v_j for all occupied cells(i,j), where $c_{ij}=u_i+v_j$

	D_1	D_2	D_3	D_4	D_{dummy}	u_i
S_1	1 **(20)**	2	1 **(10)**	4	0	$u_1=-1$
S_2	3	3 **(20)**	2 **(20)**	1 **(9)**	0 **(1)**	$u_2=0$
S_3	4	2 **(20)**	5	9	0	$u_3=-1$
v_j	$v_1=2$	$v_2=3$	$v_3=2$	$v_4=1$	$v_5=0$	

Find d_{ij} for all unoccupied cells(i,j), where $d_{ij}=c_{ij}-\left(u_i+v_j\right)$

$$d_{12} = c_{12} - \left(u_1 + v_2\right) = 2 - (-1 + 3) = 0$$

$$d_{14} = c_{14} - \left(u_1 + v_4\right) = 4 - (-1 + 1) = 4$$

$$d_{15} = c_{15} - \left(u_1 + v_5\right) = 0 - (-1 + 0) = 1$$

$$d_{21} = c_{21} - \left(u_2 + v_1\right) = 3 - (0 + 2) = 1$$

$$d_{31} = c_{31} - \left(u_3 + v_1\right) = 4 - (-1 + 2) = 3$$

$$d_{33} = c_{33} - \left(u_3 + v_3\right) = 5 - (-1 + 2) = 4$$

$$d_{34} = c_{34} - \left(u_3 + v_4\right) = 9 - (-1 + 1) = 9$$

$$d_{35} = c_{35} - \left(u_3 + v_5\right) = 0 - (-1 + 0) = 1$$

	D_1	D_2	D_3	D_4	D_{dummy}	u_i
S_1	1 **(20)**	2 [0]	1 **(10)**	4 [4]	0 [1]	$u_1=-1$
S_2	3 [1]	3 **(20)**	2 **(20)**	1 **(9)**	0 **(1)**	$u_2=0$
S_3	4 [3]	2 **(20)**	5 [4]	9 [9]	0 [1]	$u_3=-1$
v_j	$v_1=2$	$v_2=3$	$v_3=2$	$v_4=1$	$v_5=0$	

Since all $d_{ij} \geq 0$.So final optimal solution is arrived.

The minimum total transportation cost $=1\times20+1\times10+3\times20+2\times20+1\times9+0\times1+2\times20=179$

Q. Given the transportation framework, find the optimal transportation cost. – 2023

From↓ | To→	A	B	C	Plant Capacity
W	40	80	80	55
X	160	-	160	25
Y	80	160	240	35
Requirement	35	45	35	115

Soln

1) obtaining initial feasible solution from VAM

	A	B	C	Sup.	Row penalty	
W	40	80 (45)	80 (10)	55	40	0
~~X~~	160	-	160 (25)	~~25~~ 0	0	160
~~Y~~	80 (35)	160	240	~~35~~ 0	80	
Dmd.	~~35~~ 0	45	~~35~~ 10	115		
Column	40	80	80			
penalty		80	80			

The min. Total TC = $80 \times 45 + 80 \times 10 + 160 \times 25 + 80 \times 35 = 11200$

Here, no. of occupied cell = 4 which is less than $m+n-1 = 3+3-1 = 5$

∴ The solution is degenerate.

To resolve it, we add $\varepsilon \approx 0$ to unoccupied cell having least cost where loop cannot be formed (i.e. cell (1,1))

2) Testing optimality using MODI method

	A	B	C	
W	40 (ε)	80 (45)	80 (10)	$u_1 = 0$
X	160 [120]	-	160 (25)	$u_2 = 80$
Y	80 (35)	160 [120]	240 [120]	$u_3 = 40$
	$v_1 = 40$	$v_2 = 80$	$v_3 = 80$	

For Allocate cell

- Find u_i & v_j using $u_i + v_j = c_{ij}$

For unallocated cell

- Find $d_{ij} = c_{ij} - (u_i + v_j)$
- $d_{21} = 160 - 120 = 40$
- $d_{32} = 160 - 120 = 40$
- $d_{33} = 240 - 120 = 120$

Since, all $d_{ij} \geq 0$.

So, the solution is optimal.

∴ Total min. TC = $40 \times 0 + 80 \times 45 + 80 \times 10 + 80 \times 35 + 160 \times 25$

= Rs. 11200

Q. Write Hungarian algorithm to solve the assignment problem. -2021/2023

Algorithm:

1. Check whether the problem is balanced or not. If not make it balanced by adding dummy rows/columns.
2. **Row reduction:** Subtract minimum value from each row.

3. **Column reduction:** Subtract minimum value from each column.
4. **Cover all zeros** with minimum number of horizontal/vertical lines.
    - Check no. of zero in 1st row , if there is only one zero make a box over it and draw a vertical line through the column and if there is more than one zero skip that row and go to next row.
    - Repeat this process to all the rows and column and draw lines accordingly(through column for row and through row for column) until all zeros are covered.
5. If number of lines(allocations) = number of rows/columns, **optimal assignment** found.
6. If not, adjust matrix:
    - Subtract smallest uncovered value from uncovered elements.
    - Add it to the elements covered twice.
    - Repeat step 4.

Q. A city corporation has decided to carry out Maintenance work in the city. There are 4 tasks to be completed using 5 contractors . Which contractors should be assigned those tasks? Solve the assignment problem to get best result. -2023 long

Contractor	Task 1	Task 2	Task 3	Task 4
A	19	24	29	25
B	17	27	30	29
C	19	28	31	28
D	20	12	28	29
E	20	25	31	26

Soln

Here, no. of rows = 5, no. of column = 4. so the problem is unbalanced. To make it balanced, we add dummy Column/Task (Task 5)

Tasks / Contractor	1	2	3	4	5
A	19	24	29	25	0
B	17	27	30	29	0
C	19	28	31	28	0
D	20	12	28	29	0
E	20	25	31	26	0

⇒ Column Reduction:

	1	2	3	4	5
A	2	12	1	$\boxed{0}$	0
B	$\boxed{0}$	15	2	4	0
C	2	16	3	3	$\boxed{0}$
D	3	$\boxed{0}$	0	4	0
E	3	13	3	1	0

Since, min. element of each row is '0'. so row reduction is not necessary.

Here, no. of allocation = 4, no. of rows = 5, so the solution is not optimal

	1	2	3	4	5
A	2	12	1	$\boxed{0}$	1
B	$\boxed{0}$	15	2	4	1
C	1	15	2	2	$\boxed{0}$
D	3	$\boxed{0}$	0	4	1
E	2	12	2	0	0
	2			1	3

2

→ Not optimal

⇒

	1	2	3	4	5
A	2	11	$\boxed{0}$	0	1
B	$\boxed{0}$	14	1	4	1
C	1	14	1	2	$\boxed{0}$
D	4	$\boxed{0}$	0	5	2
E	2	11	1	$\boxed{0}$	0

→ optimal

Optimal solution is

Contractor	Task	Cost
A	3	29
B	1	17
C	5	0
D	2	12
E	4	26
Total min. cost		84

Q. Carew's machine shop has 4 machines on which 3 jobs have to be done. Each job can be assigned to one and only one machine. The cost(in Rs) of each job on each machine is given below:

	Machines			
Job	**P**	**Q**	**R**	**S**
A	45	60	70	80
B	20	32	42	47
C	25	37	47	55

Req: What are the job assignments which will minimize total cost?

Soln

Here, no. of rows = 3, no. of column = 4, so the given problem is unbalanced. To make it balance we add dummy row/Job (D)

	P	Q	R	S
A	45	60	70	80
B	20	32	42	47
C	25	37	47	55
D	0	0	0	0

⇒ Row Reduction:

	P	Q	R	S
A	$\boxed{0}$	15	25	35
B	0	12	22	27
C	0	12	22	30
D	0	$\boxed{0}$	0	0

→ not optimal

	P	Q	R	S
A	$\boxed{0}$	3	13	23
B	0	$\boxed{0}$	10	15
C	0	0	10	18
D	12	0	$\boxed{0}$	0

→ not optimal

⇒

	P	Q	R	S	
A	$\boxed{0}$	3	3	13	
B	0	$\boxed{0}$	0	5	
C	0	0	$\boxed{0}$	8	
D	22	10	0	$\boxed{0}$	2
	1	3	4		

Here, Alternate soln exists betwn B & C

Optimal solution is

Job	Machines	Cost
A	P	45
B	Q	32
C	R	47
D	S	0
Total min. cost		124

11. Jack Evan owns several trucks used to haul crushed stone to road project iun the country. The road contractor for whom jack hauls, N Teer, has given jack this schedule for next week:

Project	Requirement per week	Plant	Available per week
A	50	W	45
B	75	X	60
C	50	Y	60

Jack figures his cost from the crushing plant to each of the road projects to be these:

Cost information (in RS)

To /From	A	B	C
W	4	8	3
X	6	7	9
Y	8	2	5

Req: Compute Jack's optimal hauling schedule for next week and his transportation cost.

Solution:

Problem Table is

	A	B	C	Supply
W	4	8	3	45
X	6	7	9	60
Y	8	2	5	60
Demand	50	75	50	

Here Total Demand = 175 is greater than Total Supply = 165. So, We add a dummy supply constraint with 0 unit cost and with allocation 10.
Now, The modified table is

	A	B	C	Supply
W	4	8	3	45
X	6	7	9	60
Y	8	2	5	60
S_{dummy}	0	0	0	10
Demand	50	75	50	

1. Obtaining Initial feasible solution using VAM

	A	B	C	Supply	Row Penalty
W	4	8	3**(45)**	45	1 | 1 | 1
X	6**(40)**	7**(15)**	9**(5)**	60	1 | 1 | 1
Y	8	2**(60)**	5	60	3 | 3 | --
S_{dummy}	0**(10)**	0	0	10	0 | -- | --
Demand	50	75	50		
Column Penalty	4 2 2	2 5 1	3 2 6		

The minimum total transportation cost $=3 \times 45+6 \times 40+7 \times 15+9 \times 5+2 \times 60+0 \times 10=645$
Here, the number of allocated cells = 6 is equal to m + n - 1 = 4 + 3 - 1 = 6
∴ This solution is non-degenerate

2. Optimality test using modi method...
Allocation Table is

	A	B	C	Supply
W	4	8	3 **(45)**	45
X	6 **(40)**	7 **(15)**	9 **(5)**	60
Y	8	2 **(60)**	5	60
S_{dummy}	0 **(10)**	0	0	10
Demand	50	75	50	

I. Find u_i and v_j for all occupied cells(i,j), where $c_{ij}=u_i+v_j$

	A	*B*	*C*	*ui*
W	4	8	3 **(45)**	$u_1=-6$
X	6 **(40)**	7 **(15)**	9 **(5)**	$u_2=0$
Y	8	2 **(60)**	5	$u_3=-5$
Sdummy	0 **(10)**	0	0	$u_4=-6$
vj	$v_1=6$	$v_2=7$	$v_3=9$	

II. Find d_{ij} for all unoccupied cells(i,j), where $d_{ij}=c_{ij}-\left(u_i+v_j\right)$

$$d_{11} = c_{11} - \left(u_1 + v_1\right) = 4 - (-6+6) = 4$$

$$d_{12} = c_{12} - \left(u_1 + v_2\right) = 8 - (-6+7) = 7$$

$$d_{31} = c_{31} - \left(u_3 + v_1\right) = 8 - (-5+6) = 7$$

$$d_{33} = c_{33} - \left(u_3 + v_3\right) = 5 - (-5+9) = 1$$

$$d_{42} = c_{42} - \left(u_4 + v_2\right) = 0 - (-6+7) = -1$$

$$d_{43} = c_{43} - \left(u_4 + v_3\right) = 0 - (-6+9) = -3$$

	A	*B*	*C*	*ui*
W	4 [4]	8 [7]	3 **(45)**	$u_1=-6$
X	6 **(40)**	7 **(15)**	9 **(5)**	$u_2=0$
Y	8 [7]	2 **(60)**	5 [1]	$u_3=-5$
Sdummy	0 **(10)**	0 [-1]	0 [-3]	$u_4=-6$
vj	$v_1=6$	$v_2=7$	$v_3=9$	

iii. Now choose the most negative value from all d_{ij} (opportunity cost) = d_{43} = [-3]

and draw a closed path from *SdummyC (cell(4,3))*.

Closed path is $SdummyC \to SdummyA \to XA \to XC$

Closed path and plus/minus sign allocation...

	A	*B*	*C*	*ui*
W	4 [4]	8 [7]	3 **(45)**	$u_1=-6$
X	6 **(40)** **(+)**	7 **(15)**	9 **(5)** **(-)**	$u_2=0$
Y	8 [7]	2 **(60)**	5 [1]	$u_3=-5$
Sdummy	0 **(10)** **(-)**	0 [-1]	0 [-3] **(+)**	$u_4=-6$
vj	$v_1=6$	$v_2=7$	$v_3=9$	

iv. Minimum allocated value among all negative position **(-)** on closed path = 5
Subtract 5 from all **(-)** and Add it to all **(+)**

	A	B	C
W	4	8	3 **(45)**
X	6 **(45)**	7 **(15)**	9
Y	8	2 **(60)**	5
$Sdummy$	0 **(5)**	0	0 **(5)**

v. Repeat the step i-iv, until an optimal solution is obtained.

I. Find u_i and v_j for all occupied cells(i,j), where $c_{ij}=u_i+v_j$

	A	B	C	ui
W	4	8	3 **(45)**	$u_1=-3$
X	6 **(45)**	7 **(15)**	9	$u_2=0$
Y	8	2 **(60)**	5	$u_3=-5$
$Sdummy$	0 **(5)**	0	0 **(5)**	$u_4=-6$
vj	$v_1=6$	$v_2=7$	$v_3=6$	

II. Find d_{ij} for all unoccupied cells(i,j),

where $d_{ij}=c_{ij}-\left(u_i+v_j\right)$

$$d_{11}=c_{11}-\left(u_1+v_1\right)=4-(-3+6)=1$$

$$d_{12}=c_{12}-\left(u_1+v_2\right)=8-(-3+7)=4$$

$$d_{23}=c_{23}-\left(u_2+v_3\right)=9-(0+6)=3$$

$$d_{31}=c_{31}-\left(u_3+v_1\right)=8-(-5+6)=7$$

$$d_{33}=c_{33}-\left(u_3+v_3\right)=5-(-5+6)=4$$

$$d_{42}=c_{42}-\left(u_4+v_2\right)=0-(-6+7)=-1$$

	A	B	C	ui
W	4 [1]	8 [4]	3 **(45)**	$u_1=-3$
X	6 **(45)**	7 **(15)**	9 [3]	$u_2=0$
Y	8 [7]	2 **(60)**	5 [4]	$u_3=-5$
$Sdummy$	0 **(5)**	0 [-1]	0 **(5)**	$u_4=-6$
vj	$v_1=6$	$v_2=7$	$v_3=6$	

III. Again, $d_{ij} \le 0$ at cell (4,2) i.e. d_{42} = [-1] . So, draw a closed path from cell(4,2) $SdummyB$.
Closed path is $SdummyB \to SdummyA \to XA \to XB$

	A	B	C	u_i
W	4 [1]	8 [4]	3 **(45)**	u_1=-3
X	6 **(45)** **(+)**	7 **(15)** **(-)**	9 [3]	u_2=0
Y	8 [7]	2 **(60)**	5 [4]	u_3=-5
S_{dummy}	0 **(5)** **(-)**	0 [-1] **(+)**	0 **(5)**	u_4=-6
v_j	v_1=6	v_2=7	v_3=6	

IV. Minimum allocated value among all negative position **(-)** on closed path = 5

Subtract 5 from all **(-)** and Add it to all **(+)**

	A	B	C	Supply
W	4	8	3 **(45)**	45
X	6 **(50)**	7 **(10)**	9	60
Y	8	2 **(60)**	5	60
S_{dummy}	0	0 **(5)**	0 **(5)**	10
Demand	50	75	50	

v. Repeat the step i-iv, until an optimal solution is obtained.

i. Find u_i and v_j for all occupied cells(i,j), where $c_{ij}=u_i+v_j$

	A	B	C	u_i
W	4	8	3 **(45)**	u_1=3
X	6 **(50)**	7 **(10)**	9	u_2=7
Y	8	2 **(60)**	5	u_3=2
S_{dummy}	0	0 **(5)**	0 **(5)**	u_4=0
v_j	v_1=-1	v_2=0	v_3=0	

ii. Find d_{ij} for all unoccupied cells(i,j), where $d_{ij}=c_{ij}-\left(u_i+v_j\right)$

	A	B	C	Supply	u_i
W	4 [2]	8 [5]	3 **(45)**	45	$u_1=3$
X	6 **(50)**	7 **(10)**	9 [2]	60	$u_2=7$
Y	8 [7]	2 **(60)**	5 [3]	60	$u_3=2$
S_{dummy}	0 [1]	0 **(5)**	0 **(5)**	10	$u_4=0$
Demand	50	75	50		
v_j	$v_1=-1$	$v_2=0$	$v_3=0$		

$$d_{11} = c_{11} - (u_1 + v_1) = 4 - (3 - 1) = 2$$

$$d_{12} = c_{12} - (u_1 + v_2) = 8 - (3 + 0) = 5$$

$$d_{23} = c_{23} - (u_2 + v_3) = 9 - (7 + 0) = 2$$

$$d_{31} = c_{31} - (u_3 + v_1) = 8 - (2 - 1) = 7$$

$$d_{33} = c_{33} - (u_3 + v_3) = 5 - (2 + 0) = 3$$

$$d_{41} = c_{41} - (u_4 + v_1) = 0 - (0 - 1) = 1$$

Since all $d_{ij} \geq 0$. So final optimal solution is arrived.

∴The minimum total transportation cost $=3\times45+6\times50+7\times10+2\times60+0\times5+0\times5=625$