

# Chapter 1: Introduction

## 1 Numerical Analysis

### Definition

Numerical analysis is a **field of study** focused on developing and analyzing methods for solving mathematical problems that are difficult or impossible to solve exactly using traditional algebraic techniques. It involves studying the **accuracy, stability, and efficiency** of numerical methods.

## 2 Numerical Algorithm (Numerical Method)

### Numerical Algorithm

Numerical Algorithms are practical techniques used to perform **numerical computations**, providing an approximate solution and is used to solve problems involving large amounts of data. The algorithm is based on a well-defined iterative sequence, starting with an initial solution that progressively converges toward the desired result with each iteration.

$$\begin{cases} (U_n) & n \geq 0 \\ S_0 & \text{Initial Solution (Starting Point)} \end{cases}$$

## 3 Convergence Speed (Order of Convergence)

### Convergence Speed

The number of iterations required to find the solution we are looking for:

- Linear Order: 1 (slow)
- Quadratic Order: 2 (faster)
- $>> 2$  (very fast)

## 4 Interpolation

### Interpolation

Estimates the value between two known points of a function allowing for a smoother representation of the function's behavior.

## 5 Approximation

### Approximation

Approximates the formula of a function from a set of values, with the objective of finding a simpler function that represents the general trend of the data, even if it doesn't pass through every point exactly.

## 6 Error

### Error

An error represents the difference between the actual solution and the computed result. It indicates how far we are from the true solution. There are two cases:

- **Evaluation:** We know the exact solution, so we can directly calculate the error:

$$E_r = |\bar{x} - x_{\text{app}}|$$

- **Estimation:** We don't know the exact solution, so we only have an estimate of the error, based on the output of the algorithm:

$$E_r = |\bar{x} - x_{\text{app}}| \leq \text{Algo}$$

Where:

- $E_r$  : The error value.
- $\bar{x}$  : The exact solution.
- $x_{\text{app}}$  : The approximate solution.
- Algo : The error value found by the algorithm.

## 7 Optimization

### Optimization

Optimization in numerical algorithms refers to two things:

- **Error:** We aim to minimize the error in order to achieve the most accurate approximate solution.
- **Convergence Speed:** The higher the order of convergence, the less time the algorithm will take to converge to the solution we are looking for.