1 Opration Research

1.1 What's Operations Research?

Definition

Operations Research (**OR**) is an interdisciplinary field that uses mathematical models, statistical analysis, and optimization techniques to help solve complex decision-making problems and achieve the most efficient outcomes. it involves the use of several key techniques, including:

- **Optimization**: Finding the best solution based on given criteria.
- **Mathematical Modeling**: Representing real-world scenarios through mathematical equations and models.
- **Statistical Analysis**: Using data to analyze and predict outcomes.
- **Simulation**: Testing strategies in a controlled model environment.

The goal of Operations Research is to offer data-driven insights and methods that lead to more informed, optimized decisions.

1.2 Origin of Operations Research

Origin

Operations Research (**OR**) originated during World War II, when the British government assembled a team of analysts, scientists, engineers, and military officers to study complex operational problems such as : air defense, bombing strategies, convoy routing, and other crucial military operations . Using mathematical models and data analysis, the team simulated various scenarios to predict outcomes and recommend optimal decisions. The success of these methods in improving military strategy inspired other nations to adopt similar approaches.This eventually led to the formalization of OR as a scientific discipline after the war.

1.3 Types of Problems Treated by OR

Types Of Problems

Operations Research (**OR**) focuses on solving real-world problems by finding the most optimal decisions.The types of problems OR addresses can generally be categorized as:

- **Maximization**: Achieving the highest possible value for an objective, such as maximizing profits, productivity, or efficiency.
  - Example: Maximizing a company's revenue by determining the most profitable product mix.
- **Minimization**: Reducing or minimizing undesirable factors, such as costs, time, or resource consumption.
  - Example: Minimizing the cost of materials in manufacturing while maintaining quality standards.
- **Optimization**: Finding the best possible solution from multiple alternatives, often involving both maximization and minimization aspects.
  - Example: Finding the shortest path in a transportation network or optimizing team roles in a project.

1.4 Algorithm Complexity

Algorithm Complexity

Algorithm complexity refers to the amount of computational resources an algorithm uses. These resources are typically categorized as:

- **Time Complexity**: The amount of time it takes for an algorithm to run, depending on the size of the input. It is commonly expressed using Big O notation (e.g., $O(n)$, $O(\log n)$), which describes the algorithm's growth rate as the input size increases.
- **Space Complexity**: The amount of memory or space an algorithm requires. This is influenced by the number and size of variables, data structures, and other memory-using elements.

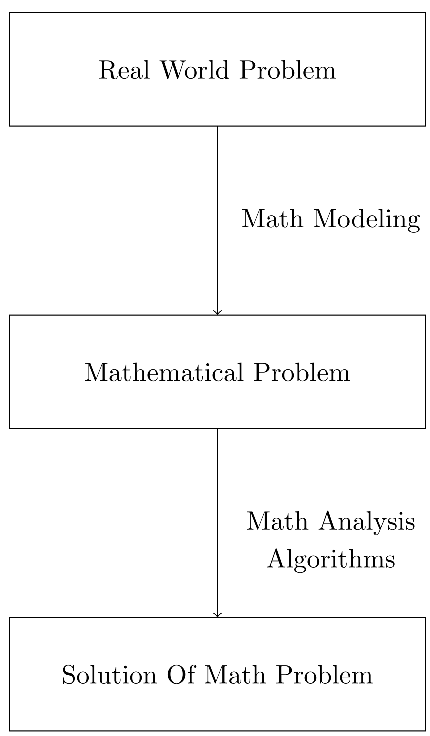
Minimizing both time and space complexity is crucial when developing efficient algorithms, as it leads to more optimal performance, especially for large-scale problems.

2 Linear Programming

2.1 What's Linear Programming ?

Definition

Linear programming is a sub-branch of optimization techniques. It involves modeling real-life problems as linear equations and inequalities and using specialized methods to find optimal solution(s), if they exist.



2.2 Models

2.2.1 Graph Model

This model is used when the objective function has two variables. It consists of converting all inequalities into equalities, drawing them as lines, and then identifying the feasible area where all conditions are met. We then sweep the objective function Z across the plot until we find the optimal solution(s).

Note

Solutions : When solving a linear program, the solution can be:

- **One or Multiple Optimal Solutions**: The feasible area is a polygon, and its vertex points are the possible optimal solutions.
- **Infinitely Many Solutions**: If the solution is unbounded, then the objective function will increase or decrease infinitely as we sweep the line.
- **No Solution**: If the feasible area is empty ($\{\emptyset\}$), it indicates that the system has contradictions.

Direction of Increase/Decrease:

- **Both Positive** ($a > 0$, $b > 0$): Since both coefficients are positive, Z increases as x_1 and x_2 increase, and decreases as they decrease. The direction of increase is towards the right, and the direction of decrease is towards the left.
- **Both Negative** ($a < 0$, $b < 0$): The opposite of the positive case. Here, Z increases as x_1 and x_2 decrease, and decreases as they increase. The direction of increase is towards the left, while the direction of decrease is towards the right.
- **Different Signs** (a and b have opposite signs): The direction is determined by the coefficient with the larger absolute value, $\max(|a|, |b|)$. If this coefficient is positive, the direction of increase follows the same pattern as when both coefficients are positive. If this coefficient is negative, the direction of increase follows the pattern for both negative coefficients.

Example1 : Diet Problem

The Goal is to minimize food cost but to meet the minimum daily nutrition requirement

Food	Units	Protein	Vit c	Iron	Price
Apples	1 med	0.4	6	0.4	8
Banana	1 med	1.2	10	0.6	10

Variables Definition:

Let x_1 be the number of Daily Unit Appels.

Let x_2 be the number of Daily Unit Banana.

Constraint:

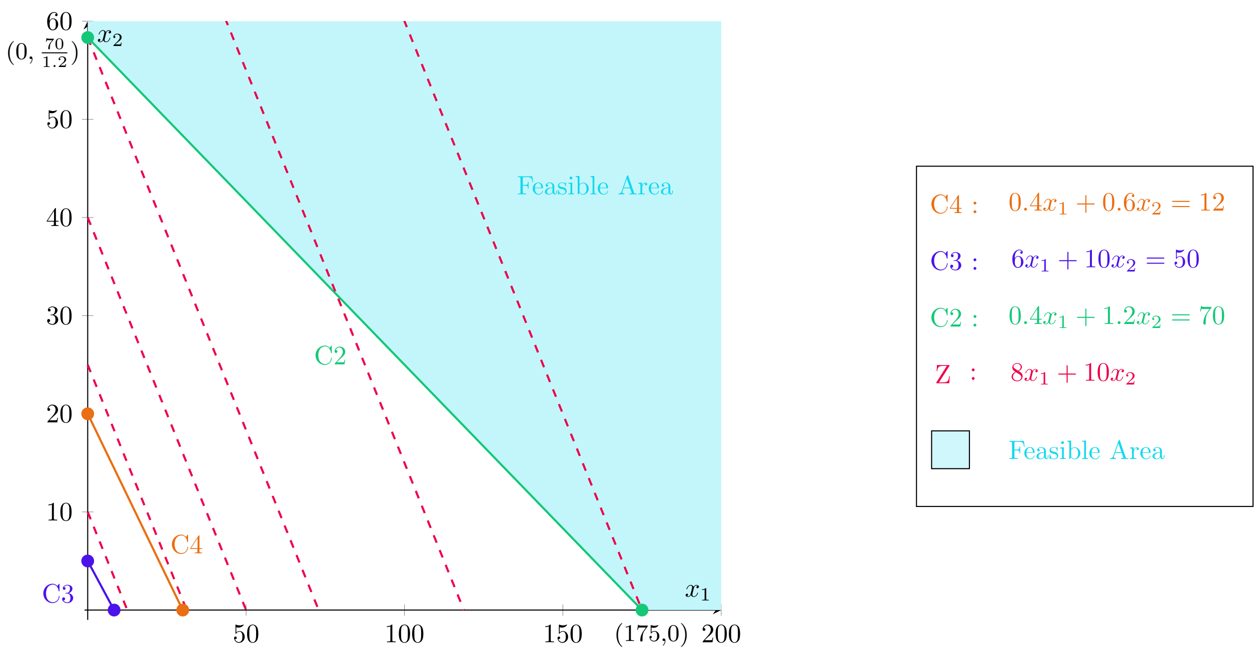
$$\begin{cases} \forall x_1, x_2 \geq 0 \quad \text{(Non-negative number of food item) ...C1} \\ 0.4x_1 + 1.2x_2 \geq 70 \quad \text{(Minimum Protein Daily) ...C2} \\ 6x_1 + 10x_2 \geq 50 \quad \text{(Minimum Vitamine c Daily) ...C3} \\ 0.4x_1 + 0.6x_2 \geq 12 \quad \text{(Minimum Iron Daily) ...C4} \end{cases}$$

Objective Function

$$f(x_i) = Z = 8x_1 + 10x_2$$

The goal is to minimize food cost by minmizing $f(x_i)$, while meeting the minimum daily nutrition .

Problem : Find the minimum of $f(x_i)$ subject to the contraints



Possible Solutions	$(0,\frac{70}{1.2})$	(175,0)
Objective Function	$\frac{700}{1.2} \approx 583.33$	1400

Solution

The blue area in the plot represents the feasible region, so the optimal solution(s) must be within this area. Since the objective function Z increases towards the right (due to both coefficients being positive) and we want to minimize Z, we need to sweep the objective function line towards the feasible area , and the first intresection between the objective function and one of the vertex points is the min the optimal solution . Therefore,the optimal solution is $(0, \frac{70}{1.2})$ with $Z = 8 \times 0 + 10 \times \frac{70}{1.2} \approx 583.33$.

Example2 : Blending Model

similar to the previous example , this time we have a farm that needs to feed their chicken , there are 2 feeds , we need to minimize cost of the feeds while meeting the minimum nutrient requirement

Feed	Nut_A	Nut_B	Nut_C	Cost
1	3	7	3	10
2	2	2	6	4
Min Require	60	84	72	

Variables Definition:

Let x_1 be the number of Feed 1.

Let x_2 be the number of Feed 2.

Constraint:

$$\begin{cases} \forall x_1, x_2 \geq 0 & \text{(Non-negative number of feeds) ...C1} \\ 3x_1 + 2x_2 \geq 60 & \text{(Minimum Nut}_A\text{ Daily) ...C2} \\ 7x_1 + 2x_2 \geq 84 & \text{(Minimum Nut}_B\text{ Daily) ...C3} \\ 3x_1 + 6x_2 \geq 72 & \text{(Minimum Nut}_C\text{ Daily) ...C4} \end{cases}$$

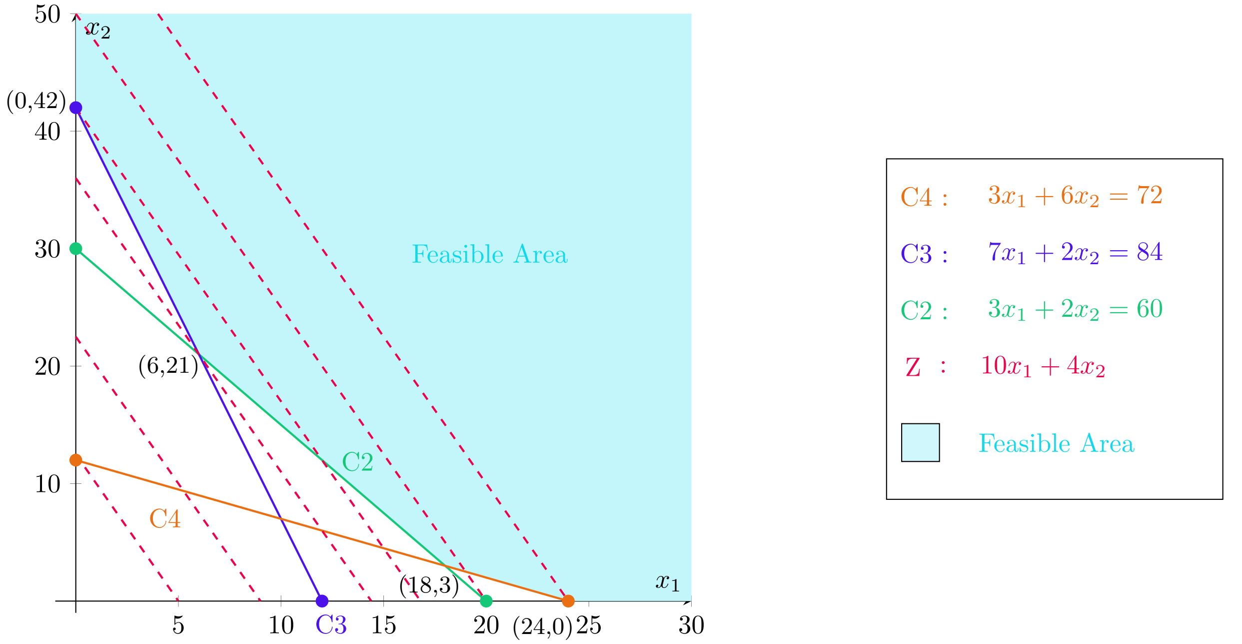
Objective Function

$$f(x_i) = Z = 10x_1 + 4x_2$$

The goal is to minimize feeds cost by minmizing $f(x_i)$, while meeting the minimum daily nutrition .

Problem : Find the minimum of $f(x_i)$ subject to the contraints

Possible Solutions	(0,42)	(6,21)	(18,3)	(24,0)
Objective Function	168	144	192	240



Solution

The blue area in the plot represents the feasible region, so the optimal solution(s) must be within this area. Since the objective function Z increases towards the right (due to both coefficients being positive) and we want to minimize Z, we need to sweep the objective function line towards the feasible area , and the first intresection between the objective function and one of the vertex points is the min the optimal solution . Therefore,the optimal solution is $(6, 21)$ with $Z = 10 \times 6 + 4 \times 21 = 144$.

If The Prices Changed

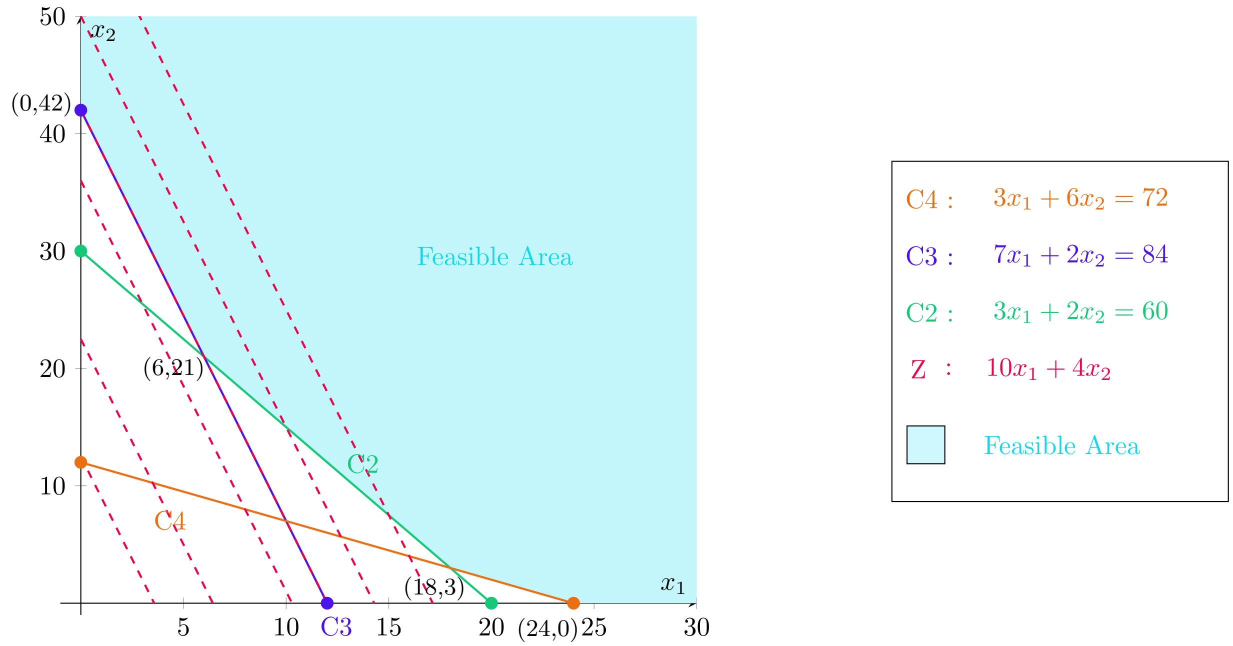
the price of feed1 is now 14 and feed2 remains the same 4 , the constraints don't change meaning the feasible region also stays the same and the possible solutions (coordinates of the vertex points of the polygon) only thing that changes is the objective function

New Objective Function

$$f(x_i) = Z = 14x_1 + 4x_2$$

The goal is to minimize feeds cost by minmizing $f(x_i)$, while meeting the minimum daily nutrition .

Possible Solutions	(0,42)	(6,21)	(18,3)	(24,0)
Objective Function	168	168	264	336



Observation

- The value of the minimum or maximum of the objective function is unique, but there can be multiple coordinate solutions.
- The line through (0, 42) and (6, 21) represents a contour line, indicating that the value of Z is constant along this line.
- If P_1 and P_2 are both optimal solutions, they must be adjacent boundary corners of the feasible region.
- When there are multiple solutions, they form a segment of coordinates on the boundary (edge) of the feasible region, where each coordinate in that segment represents an optimal solution. In linear programming, we typically focus on the vertex points.

Note

Difference Between Multiple & Infinite Solutions:

One might wonder about the difference between these two terms since even multiple solutions have an infinite number of solutions. The difference is that in infinite solutions, the solution is unbounded, unlike multiple solutions, which have their solutions on a finite segment. But as mentioned before, even though multiple solutions have infinite solutions, we focus only on the boundary corner points (vertex points).

Example :

Minimize $f(x, y) = x + y$

Subject To :

$$\begin{cases} x + y \geq 10 \\ x + y \leq 9 \\ x, y \geq 0 \end{cases}$$

Solution :

the system clearly contains a contradiction so there is no solution

Example :

Maximize $f(x, y) = x + y$

Subject To :

$$\begin{cases} 2x + y \geq 9 \\ x + 3y \geq 10 \\ x, y \geq 0 \end{cases}$$

Solution :

As $x, y \to +\infty$, $f \to +\infty$, solution unbounded

Example :

There is an industry that creates two types of boats, A and B. We want to maximize the profit while adhering to the limitations of resources and labor.

Boat	Aluminum (lb)	Machine Time (min)	Labor (hr)	Profit ($)
A	50	6	3	50
B	30	5	5	60
max	2000	300	200	

Variables Definition:

Let x_1 be the number of boat A.

Let x_2 be the number of boat B.

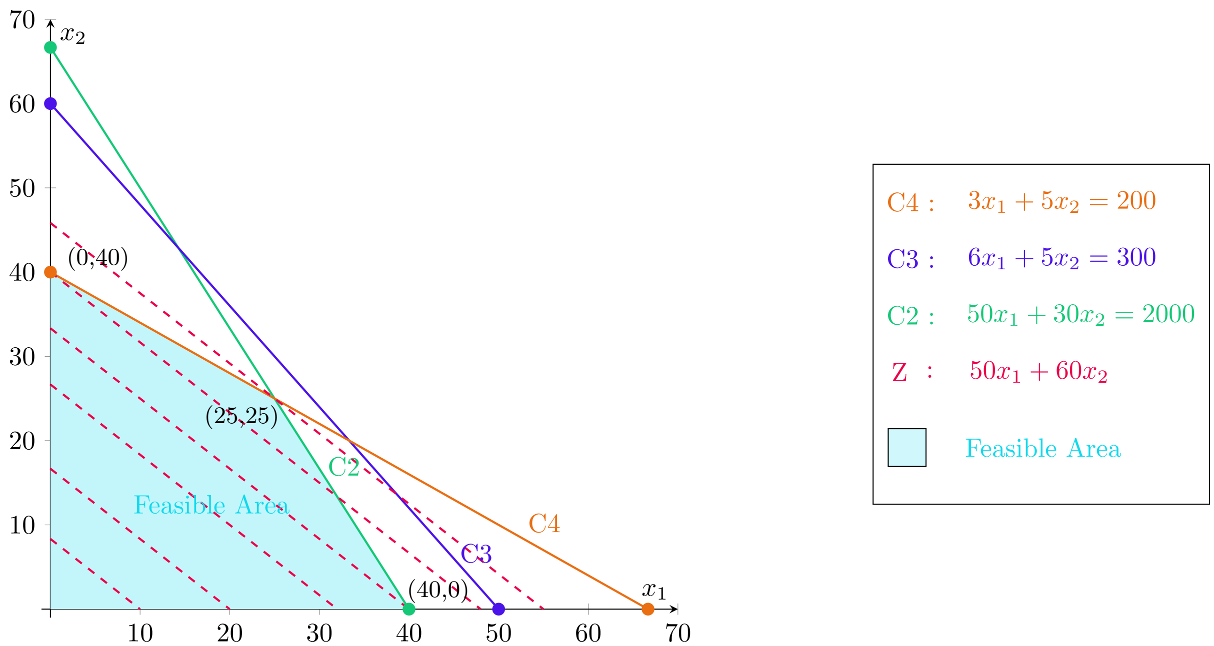
Constraint:

$$\begin{cases} \forall x_1, x_2 \geq 0 & \text{(Non-negative number of boats) ...C1} \\ 50x_1 + 30x_2 \leq 2000 & \text{(Maximum Aluminium (lb)) ...C2} \\ 6x_1 + 5x_2 \leq 300 & \text{(Maximum Machine Time(min)) ...C3} \\ 3x_1 + 5x_2 \leq 200 & \text{(Maximum Labor(h)) ...C4} \end{cases}$$

Objective Function

$$f(x_i) = Z = 50x_1 + 60x_2$$

The goal is to maximize the profits by maximizing $f(x_i)$, while adhering to the ressource limitations .



Possible Solutions	(0,40)	(25,25)	(40,0)
Objective Function	2400	2750	2000

Solution

The blue area in the plot represents the feasible region, so the optimal solution(s) must be within this area. Since the objective function Z increases towards the right (due to both coefficients being positive) and we want to maximize Z, we need to sweep the objective function line towards the right , and the last intresection between the objective function and one of the vertex points of the feasible region is the max the optimal solution . Therefore,the optimal solution is $(25, 25)$ with $Z = 50 \times 25 + 60 \times 25 = 2750$.

Example :

An Industry produce two products , P1 and P2 ,they want to minimize the costs of the ressources needed to create the product : raw material M_A(lb) & M_B(lb) and the needed labour (hr) while adhering the constraint set upon the ressources and minimum production

Process	Labour(hr)	M_A(lb)	M_B(lb)		P1	P2
1	20	160	30		35	55
2	30	100	35		45	42
3	10	200	60		70	0
4	25	75	80		0	90
max	1000	8000	4000	min	2100	1800
price	fixed salary	3$/lb	7$/lb			

Variables Definition:

Let x_1 be the process 1.

Let x_2 be the process 2.

Let x_3 be the process 3.

Let x_4 be the process 4.

Constraint:

$$\begin{cases} \forall x_1, x_2, x_3, x_4 \geq 0 & \text{(Non-negative number of processes) ...C1} \\ 20x_1 + 30x_2 + 10x_3 + 4x_4 \leq 1000 & \text{(Maximum Labor(hr)) ...C2} \\ 160x_1 + 100x_2 + 200x_3 + 75x_3 \leq 8000 & \text{(Maximum M}_A\text{(lb)) ...C3} \\ 30x_1 + 35x_2 + 60x_3 + 80x_4 \leq 4000 & \text{(Maximum M}_B\text{(lb)) ...C4} \\ 35x_1 + 45x_2 + 70x_3 \geq 2100 & \text{(Minimum P1) ...C5} \\ 55x_1 + 42x_2 + 90x_4 \geq 1800 & \text{(Minimum P2) ...C6} \end{cases}$$

Objective Function

$$f(x_i) = Z = 3 \times (160x_1 + 100x_2 + 200x_3 + 75x_3) + 4 \times (30x_1 + 35x_2 + 60x_3 + 80x_4) = 690x_1 + 545x_2 + 1070x_3 + 785x_4$$

The goal is to minimize the cost by minimizing $f(x_i)$, while adhering to the ressource limitations and minimum production of P1 & P2.

Example :

Let's take the same problem as before but allow overtime max 200hr with additional salary \$30/hr what's the new LP problem ?

Variables Definition:

Let x_1 be the process 1.

Let x_2 be the process 2.

Let x_3 be the process 3.

Let x_4 be the process 4.

Let y be the number of overtime hour

Constraint:

$$\begin{cases} \forall x_1, x_2, x_3, x_4 \geq 0 & \text{(Non-negative number of processes) ...C1} \\ 20x_1 + 30x_2 + 10x_3 + 4x_4 \leq 1000 + y & \text{(Maximum Labor(hr)) ...C2} \\ y \leq 200 & \text{(Maximum Overtime(hr)) ...C3} \\ y \geq 0 & \text{(Overtime cant be negative) ...C4} \\ 160x_1 + 100x_2 + 200x_3 + 75x_3 \leq 8000 & \text{(Maximum M}_A\text{(lb)) ...C5} \\ 30x_1 + 35x_2 + 60x_3 + 80x_4 \leq 4000 & \text{(Maximum M}_B\text{(lb)) ...C6} \\ 35x_1 + 45x_2 + 70x_3 \geq 2100 & \text{(Minimum P1) ...C7} \\ 55x_1 + 42x_2 + 90x_4 \geq 1800 & \text{(Minimum P2) ...C8} \end{cases}$$

Objective Function

$$g(x_i) = Z = f(x_i) + 30y = 690x_1 + 545x_2 + 1070x_3 + 785x_4 + 30y$$

The goal is to minimize the cost by minimizing $g(x_i)$, while adhering to the ressource limitations and minimum production of P1 & P2.

Example :

The transportation model setting , good at location A_i, transport to B_j cost from A_i Tp B_j, the goal is to find a schedule to minimize the cost At location A_1 we have 200 units , location A_2 we have 300 units , the will be distriuted to B_1 ,B_2, B_3 :
B_1 = 100units ,B_2 = 200units , B_3 = 200units

Transportation cost from A_i to B_j :

	B_1	B_2	B_3
A_1	11	12	13
A_2	21	22	23

Variables Definition:

Let x_{ij} be the number of transported Units from A_i to B_j.

Constraint:

$$\begin{cases} \forall x_{ij} \geq 0 & \text{(Non-negative number of units) ...C1} \\ i = \{1,2\} \quad , \quad j = \{1,2,3\} \\ x_{11} + x_{12} + x_{13} = 200 & \text{(Maximum Capacity of A}_1\text{) ...C2} \\ x_{21} + x_{22} + x_{23} = 300 & \text{(Maximum Capacity of A}_2\text{)...C3} \\ x_{11} + x_{21} = 100 & \text{(Maximum Capacity of B}_1\text{)...C4} \\ x_{12} + x_{22} = 200 & \text{(Maximum Capacity of B}_2\text{)...C5} \\ x_{13} + x_{23} = 200 & \text{(Maximum Capacity of B}_3\text{)...C6} \end{cases}$$

Objective Function

$$f(x_i) + 30y = 11x_{11} + 12x_{12} + 13x_{13} + 21x_{21} + 22x_{22} + 23x_{23}$$

The goal is to minimize the cost of transport by minimizing $f(x_i)$, while adhering to the capacity limitation

Note

Constraint can be : $=, \leq, \geq, >, <$
We can rewrite any constraint into a **Standard Form**

2.3 Standard Form

The standard form is a structured way of representing LP problems. It helps to standardize diverse problems, making them easier to analyze and solve using models like the Simplex Method and primal-dual relationships. Converting problems to standard form ensures uniformity, simplifying both theoretical understanding and computational implementation.

$$\text{Maximize } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

Subject to:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

$$x_1, x_2, \ldots, x_n \geq 0 \quad b_1, b_2, \ldots, b_m \geq 0$$

2.3.1 How To Transition To Standard Form

Minimization Problem

In the standard form, all linear programming problems must be expressed as maximization problems. If we are given a minimization problem, we can convert it by multiplying the objective function by -1, effectively turning it into a maximization problem.

Example

$$\text{Minimize } f(x_i) = 3x_1 + 2x_2$$

is equivalent to:

$$\text{Maximize } g(x_i) = -f(x_i) = -3x_1 - 2x_2$$

Constraints

In the standard form , all constraints are equalities , if a contraint is not an equality we have to add or substract new positive variables called slack variables

$\leq$ Constraint

$$x \leq b \Leftrightarrow x + s_1 = b \quad \text{with} \quad s_1 \geq 0$$

$\geq$ Constraint

$$x \geq b \Leftrightarrow x - s_1 = b \quad \text{with} \quad s_1 \geq 0$$

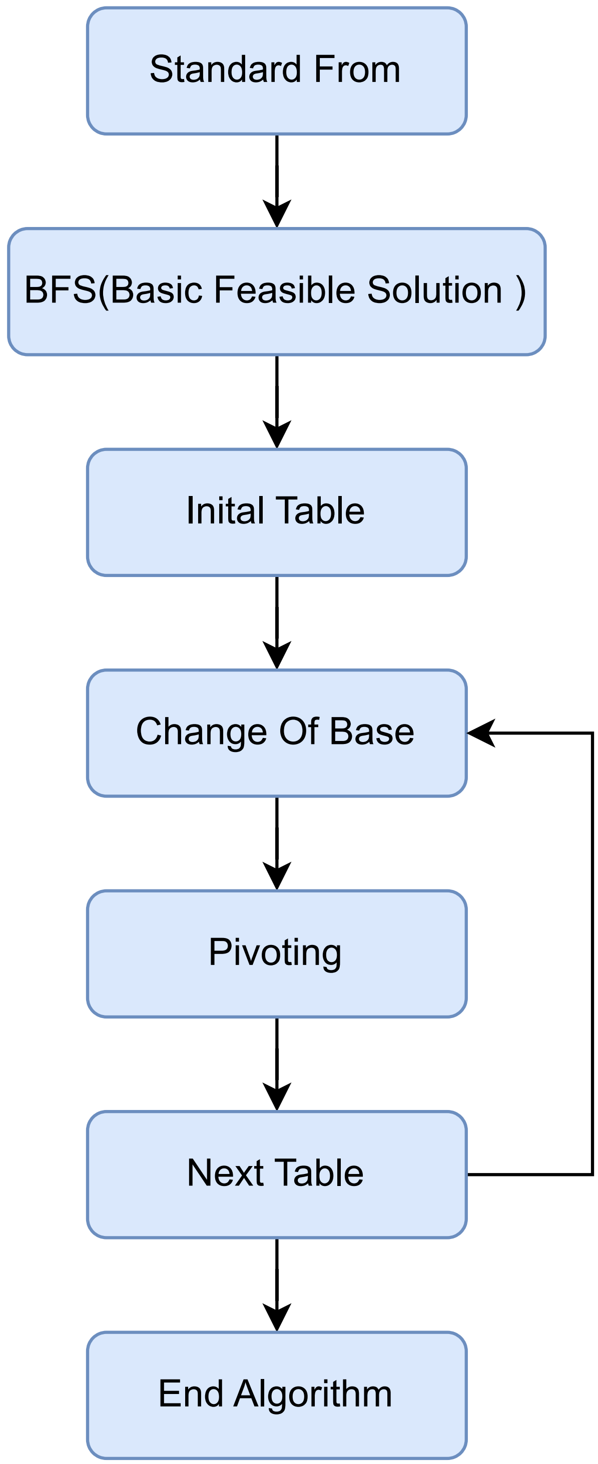
Example

$$\begin{cases} x_1 + x_2 + x_3 \leq 100 \\ x_1 - x_2 \geq 5 \\ x_1, x_2, x_3 \geq 0 \end{cases} \implies \begin{cases} x_1 + x_2 + x_3 + s_1 = 100 \\ x_1 - x_2 - s_2 = 5 \\ x_1, x_2, x_3, s_1, s_2 \geq 0 \end{cases}$$

2.4 Simplex Method

The Simplex method, an algebraic iterative algorithm developed by B. Dantzig in 1948, allows us to solve linear programming (LP) problems even those involving more than two variables. The principle of the method is to start from an initial feasible solution and iteratively move along the vertices of the polyhedron (feasible area) until the optimal solution is reached.

Overview Of Simplexe Steps



2.4.1 BFS

Basic Feasible Solution

After converting the **LP** problem into standard form, the next step is to find a basic feasible solution. To achieve this, set all decision variables to 0 and assign the slack variables the values of the right-hand side (**RHS**) of the constraints.

If any slack variable equals −**RHS**, an artificial variable must be added to satisfy the non-negativity constraint for slack variables. In this case, the slack variable is set to 0, and the artificial variable is assigned the RHS value.

For example:

$$x_1 + x_2 - s_1 = 10 \quad \Longrightarrow \quad x_1 + x_2 - s_1 + t_1 = 10$$

where $t_1 \geq 0$ is the artificial variable added to the equation.

Coefficient Of Objective Function

The coefficient of artificial variables in the objective function is $-M$, where M is a very large number, ensuring that artificial variables are driven out of the basis during the optimization process.

The coefficient of slack variables is 0.

Basic/Non-Basic

In the context of the Basic Feasible Solution (BFS), the variables that are non-zero are referred to as **basic variables**, while the remaining variables are called **non-basic variables**.

2.4.2 Initial Table

Create Initial Table

$$\text{Maximize } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

Subject to:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

$$x_1, x_2, \ldots, x_n \geq 0 \quad b_1, b_2, \ldots, b_m \geq 0$$

The notation for vectors and matrices is as follows:

$$X = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}, \quad C = \begin{bmatrix} c_1 & c_2 & \dots & c_n \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_m \end{bmatrix}, \quad C_B = \begin{bmatrix} cb_1 \\ cb_2 \\ \vdots \\ cb_m \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$$

$$Z_1 = C_B \cdot b \quad \text{and} \quad Z_i = C_B \cdot A(:, i-1) \quad \text{for} \quad i \geq 2$$

Where:

- n is the number of variables.
- m is the number of constraints.
- b is the vector of right-hand side (RHS) values.
- X is the vector of variables.
- C is the vector of coefficients in the objective function.
- A is the matrix of coefficients in the constraints.
- $A(:, i-1)$ is submatrix of A all rows of column$_{i-1}$
- B is the vector of basic variables.
- C_B is the vector of the coefficients of the objective function for the basic variables.
- Z_1 is the value of the objective function.

Reminder: Dot Product

$$U = \begin{bmatrix} u_1 & u_2 & \dots & u_m \end{bmatrix}, \quad V = \begin{bmatrix} v_1 & v_2 & \dots & v_m \end{bmatrix}$$

$$U \cdot V = u_1.v_1 + u_2.v_2 + \cdots + u_m.v_m$$

	C_i		c_1	c_2	....	c_n
C_B	B	b	x_1	x_2	....	x_n
cb_1	B_1	b_1	a_{11}	a_{12}	....	a_{1n}
cb_2	B_2	b_2	a_{21}	a_{22}	....	a_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	....	$\vdots$
cb_m	B_m	b_m	a_{m1}	a_{m2}	....	a_{mn}
	Z_i	Z_1	Z_2	Z_3	....	Z_{n+1}
	$C_i - Z_i$		$C_1 - Z_2$	$C_2 - Z_3$	....	$C_n - Z_{n+1}$

Legends

- Matrix A coefficients of the constraints
- Vector X all variables
- Vector B basic variables
- Vector C all coefficients of the objective function
- Vector C_B basic coefficients of the objective function
- Vector b RHS of constraints
- Vector Z

2.4.3 Change of Basis

Changing the Basis

To change the basis, follow these steps:

1. Choose a non-basic variable to enter the basis.
2. Choose a basic variable to leave the basis.

Choose Variable to Enter the Basis

Select the column where $C_i - Z_{i+1} > 0$ and is at its maximum. This column is called the **pivoting column**, and x_i is the variable that will enter the basis.

Choose Variable to Leave the Basis

Select the line where $a_{ji} > 0$ and $\frac{b_j}{a_{ji}}$ is at its minimum (where i is the pivoting column). This line is called the **pivoting line**, and B_i is the basic variable that will leave the basis.

Note

The intersection of the pivoting column and the pivoting line is called the **pivot** which is a_{ji}.

	C_i		c_1	c_2	....	c_n
C_B	B	b	x_1	x_2	....	x_n
cb_1	B_1	b_1	a_{11}	a_{12}	....	a_{1n}
cb_2	B_2	b_2	a_{21}	a_{22}	....	a_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	....	$\vdots$
cb_m	B_m	b_m	a_{m1}	a_{m2}	....	a_{mn}
	Z_i	Z_1	Z_2	Z_3	....	Z_{n+1}
	$C_i - Z_i$		$C_1 - Z_2$	$C_2 - Z_3$	....	$C_n - Z_{n+1}$

- In this case the max of $C_i - Z_{i+1} > 0$ is $C_2 - Z_3$ i $= 2$ so x_2 will be the variable to enter the basis.
- The min of $\frac{b_j}{a_{ji}}$ with $a_{ji} > 0$ is $\frac{b_1}{a_{12}}$ j $= 1$ so B_1 will leave the basis
- The intersection between the pivoting line and pivoting column is a_{12}

2.4.4 Pivoting

Pivoting

To get the new table, follow these steps:

1. Replace B_j with x_i.
2. Divide all the elements of the pivoting line by the pivot value (excluding C_B and B).
3. Set all cells above or below the pivot to 0.
4. Fill the remaining cells using the rectangle rule:
$$a = a' - \frac{b \times c}{\text{pivot}}$$
where:
   - a: New value of the cell.
   - a': Old value of the cell.
   - b: Intersection of the cell's row with the pivoting column.
   - c: Intersection of the cell's column with the pivoting row.

Note

- If a cell in the pivoting column is 0 ($b = 0$), the entire row intersecting with it stays the same.
- If a cell in the pivoting line is 0 ($c = 0$), the entire column intersecting with it stays the same.