1 Exercise 1: Part-Time Work

Adem works two part-time jobs, E1 and E2, and wants to limit his total working hours to no more than 12 per week. For each hour worked at E1, he requires 2 hours of preparation, and for each hour worked at E2, he requires 1 hour of preparation. Adem cannot spend more than 16 hours preparing each week. Given that he earns 40 MU per hour at E1 and 30 MU per hour at E2, how many hours should he work at each job to maximize his total income?
MU = Monetary Unit

Solution:

Variables Definition:

Let x_1 be the number of hours worked in E_1.

Let x_2 be the number of hours worked in E_2.

Constraint:

$$\begin{cases} \forall x_1, x_2 \geq 0 \quad \text{(Non-negative work hours)} \\ x_1 + x_2 \leq 12 \quad \text{(Work limit of 12 hours per week)} \\ 2x_1 + x_2 \leq 16 \quad \text{(Preparation time limit of 16 hours)} \end{cases}$$

Objectif Function

$$f(x_1, x_2) = 40x_1 + 30x_2$$

The goal is to maximize Adem's income by maximizing $f(x_1, x_2)$, while adhering to the work and preparation time constraints.

Exercise 2: Optimal Production

A food company has 1000 kilos of African coffee, 2000 kilos of Brazilian coffee, and 500 kilos of Colombian coffee. The company produces two types of coffee:

- **Type 1**: A blend of three parts African coffee for one part of each of the two others (Brazilian and Colombian), sold at 10 UM per kilo.
- **Type 2**: A blend of three parts Brazilian coffee for one part Colombian coffee, sold at 8 UM per kilo.

The problem is to determine how much coffee of each type the company should produce in order to maximize its profit. Model this problem.

Solution:

Variables Definition:

Let x_1 be Type 1 coffee.

Let x_2 be Type 2 coffee.

Constraint:

$$\begin{cases} \forall x_1, x_2 \geq 0 \quad \text{(Non-negative number of type of coffee)} \\ \\ \frac{3}{5}x_1 \leq 1000 \quad \text{(Total number of African coffee)} \\ \\ \frac{1}{5}x_1 + \frac{3}{4}x_2 \leq 2000 \quad \text{(Total number of Brazilian coffee)} \\ \\ \frac{1}{5}x_1 + \frac{1}{4}x_2 \leq 500 \quad \text{(Total number of Colombian coffee)} \end{cases}$$

Objectif Function

$$f(x_1, x_2) = 10x_1 + 8x_2$$

The goal is to maximize The Company income by maximizing $f(x_1, x_2)$, while adhering to the limit of each coffe type constraints (Af,Col,Br).

Exercise 3: Transportation Problem

A company needs to transport goods from two warehouses (E1, E2) to three destinations (D1, D2, D3). The transportation costs to move one unit of product between the warehouses and the destinations are as follows:

	D1	D2	D3
E1	4	6	8
E2	5	3	7

The capacities of the warehouses and the demands of the destinations are as follows:

- Capacity of E1: 100 units
- Capacity of E2: 120 units
- Demand of D1: 90 units
- Demand of D2: 80 units

- Demand of D3: 50 units

Formulate a linear program to establish an optimal distribution plan.

Solution:

Method 1:

Variables Definition:

Let x_1 be number of goods transported from E_1 ->D_1.

Let x_2 be number of goods transported from E_1 ->D_2.

Let x_3 be number of goods transported from E_1 ->D_3.

Let x_4 be number of goods transported from E_2 ->D_1.

Let x_5 be number of goods transported from E_2 ->D_2.

Let x_6 be number of goods transported from E_2 ->D_3.

Constraint:

$$\begin{cases} \forall x_i \geq 0 \quad \text{(Non-negative number of type of goods)} \\ x_1 + x_2 + x_3 \leq 100 \quad (\text{Capacity of E}_1) \\ x_4 + x_5 + x_6 \leq 120 \quad (\text{Capacity of E}_2) \\ x_1 + x_4 \leq 90 \quad (\text{Demand of D}_1) \\ x_2 + x_5 \leq 80 \quad (\text{Demand of D}_2) \\ x_3 + x_6 \leq 50 \quad (\text{Demand of D}_3) \end{cases}$$

Method 2:

Variables Definition:

Let x_{ij} be a matrix of [2,3] where each cell x_{ij} represent number of goods transported from E_i ->D_j.

Constraint:

$$\begin{cases} \forall x_{ij} \geq 0 \quad \text{(Non-negative number of type of goods)} \\ x_{11} + x_{12} + x_{13} \leq 100 \quad \text{(Capacity of E1)} \\ x_{21} + x_{22} + x_{23} \leq 120 \quad \text{(Capacity of E2)} \\ x_{11} + x_{21} \leq 90 \quad \text{(Demand of D1)} \\ x_{12} + x_{22} \leq 80 \quad \text{(Demand of D2)} \\ x_{13} + x_{23} \leq 50 \quad \text{(Demand of D3)} \end{cases}$$

Objective Function

$$f(x_{ij}) = 4x_{11} + 6x_{12} + 8x_{13} + 5x_{21} + 3x_{22} + 7x_{23}$$

The goal is to minimize the transportation cost by minimizing $f(x_{ij})$, while respecting the capacity constraints of each warehouse and the demand at each destination.

Exercise 4: Stock Optimization

A company must decide how many tons of two raw materials (M1 & M2) to purchase in order to manufacture a product. Each ton of M1 costs 200 UM, and each ton of M2 costs 300 UM. For each finished product, the company needs 3 tons of M1 and 2 tons of M2. The company has a budget of 30 000 UM and must produce at least 50 tons of the product.
Formulate this problem as a linear program to optimize expenses while respecting production constraints.

Solution:

Variables Definition:

Let x_1 be number of tons of M1.

Let x_2 be number of tons of M2.

Constraint:

$$\begin{cases} \forall x_1, x_2 \geq 0 \quad \text{(Non-negative number of tons of materials)} \\ 200x_1 + 300x_2 \leq 30000 \quad \text{(Budget Contraint)} \\ x_1 \geq 150 \quad \text{(At least 50 products from M1)} \\ x_2 \geq 100 \quad \text{(At least 50 products from M2)} \end{cases}$$

Objectif Function

$$f(x_1, x_2) = 200x_1 + 300x_2$$

The goal is to minimize The Company cost of production by minimizing $f(x_1, x_2)$, while adhering to the limit of the budget and the min number of products.