Chapter 1: Introduction

1 Numerical Algorithm (Numerical Method)

Numerical Algorithm

A Numerical Algorithm provides an approximate solution and is used to solve problems involving large amounts of data. The algorithm is based on a well-defined iterative sequence, starting with an initial solution that progressively converges toward the desired result with each iteration.

$$\begin{cases} (U_n) & n \geq 0 \\ S_0 & \text{Initial Solution (Starting Point)} \end{cases}$$

2 Convergence Speed (Order of Convergence)

Convergence Speed

The number of iterations required to find the solution we are looking for:

- Linear Order: 1 (slow)
- Quadratic Order: 2 (faster)
- $>>$ 2 (very fast)

3 Interpolation

Interpolation

Estimates the value between two known points of a function allowing for a smoother representation of the function's behavior.

4 Approximation

Approximation

Approximates the formula of a function from a set of values, with the objective of finding a simpler function that represents the general trend of the data, even if it doesn't pass through every point exactly.

5 Error

Error

An error represents the difference between the actual solution and the computed result. It indicates how far we are from the true solution. There are two cases:

- **Evaluation**: We know the exact solution, so we can directly calculate the error:

$$\boxed{E_r = |\overline{x} - x_{\text{app}}|}$$

- **Estimation**: We don't know the exact solution, so we only have an estimate of the error, based on the output of the algorithm:

$$\boxed{E_r = |\overline{x} - x_{\text{app}}| \leq \text{Algo}}$$

Where:

- E_r : The error value.
- $\overline{x}$: The exact solution.
- x_{app} : The approximate solution.
- Algo : The error value found by the algorithm.

6 Optimization

Optimization

Optimization in numerical algorithms refers to two things:

- **Error**: We aim to minimize the error in order to achieve the most accurate approximate solution.
- **Convergence Speed**: The higher the order of convergence, the less time the algorithm will take to converge to the solution we are looking for.