Exercise : Apply Breach & Bound

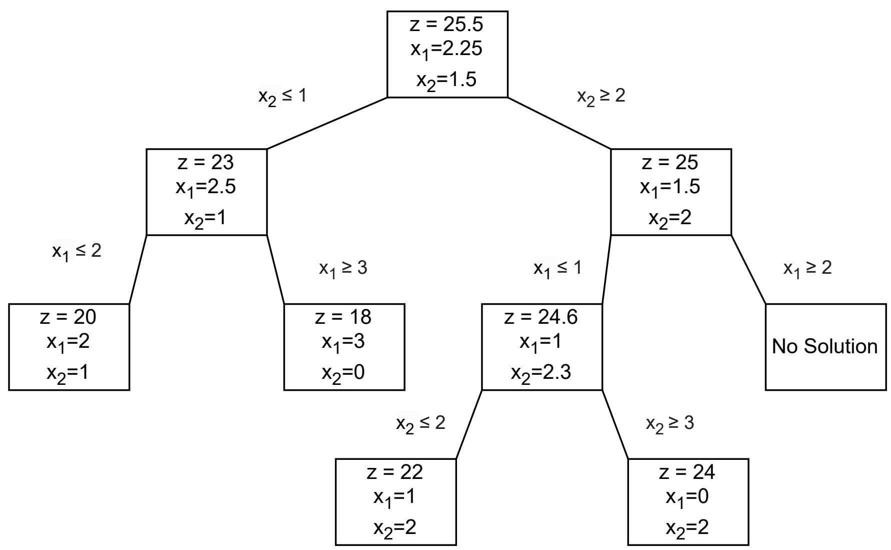
$$\max Z = 6x_1 + 8x_2 \qquad \min Z = x_1 - 2x_2$$

$$\begin{cases} 6x_1 + 3x_2 \leq 18 \\ 2x_1 + 3x_2 \leq 9 \\ x_1, x_2 \text{ are integers} \end{cases} \qquad \begin{cases} -4x_1 + 6x_2 \leq 9 \\ x_1 + x_2 \leq 4 \\ x_1, x_2 \text{ are integers} \end{cases}$$

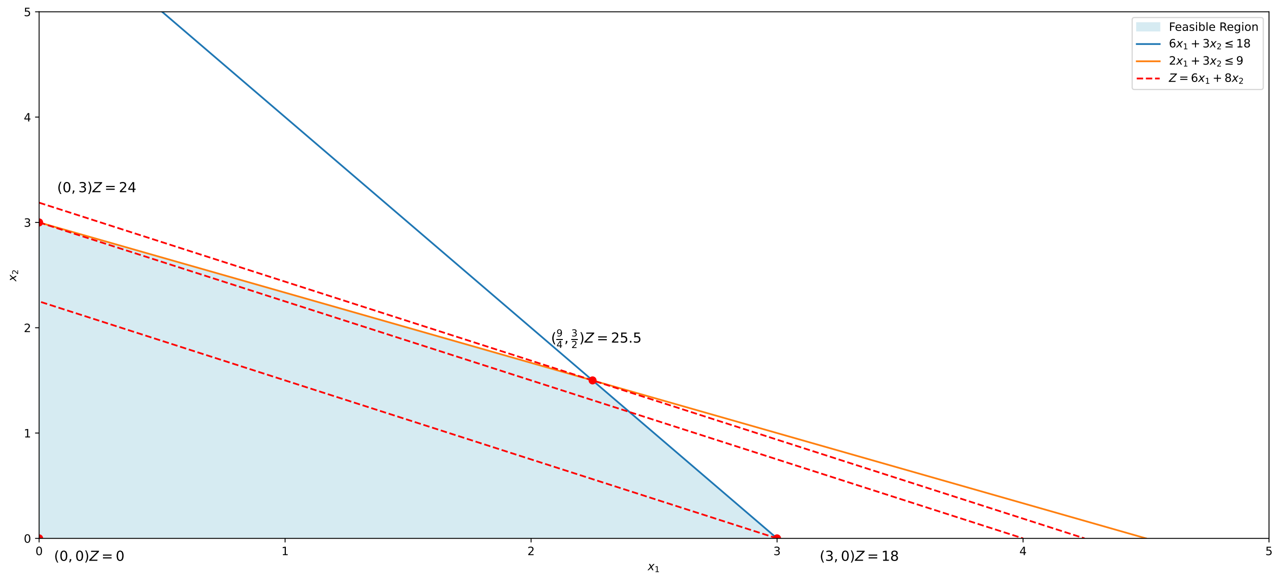
Solution

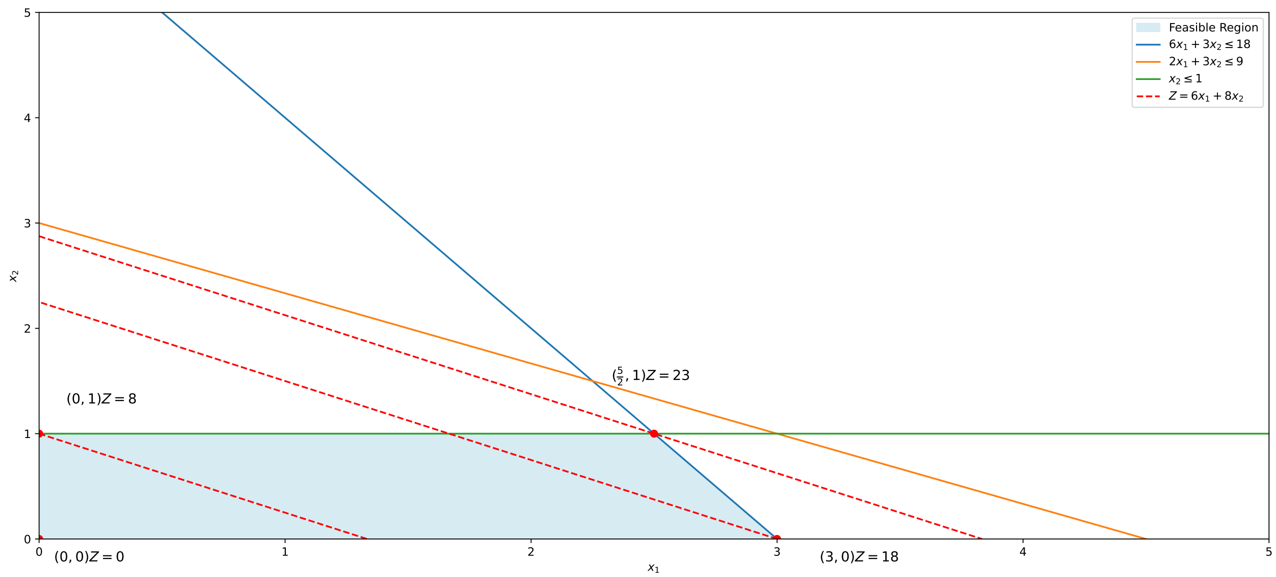
$$\max Z = 6x_1 + 8x_2$$

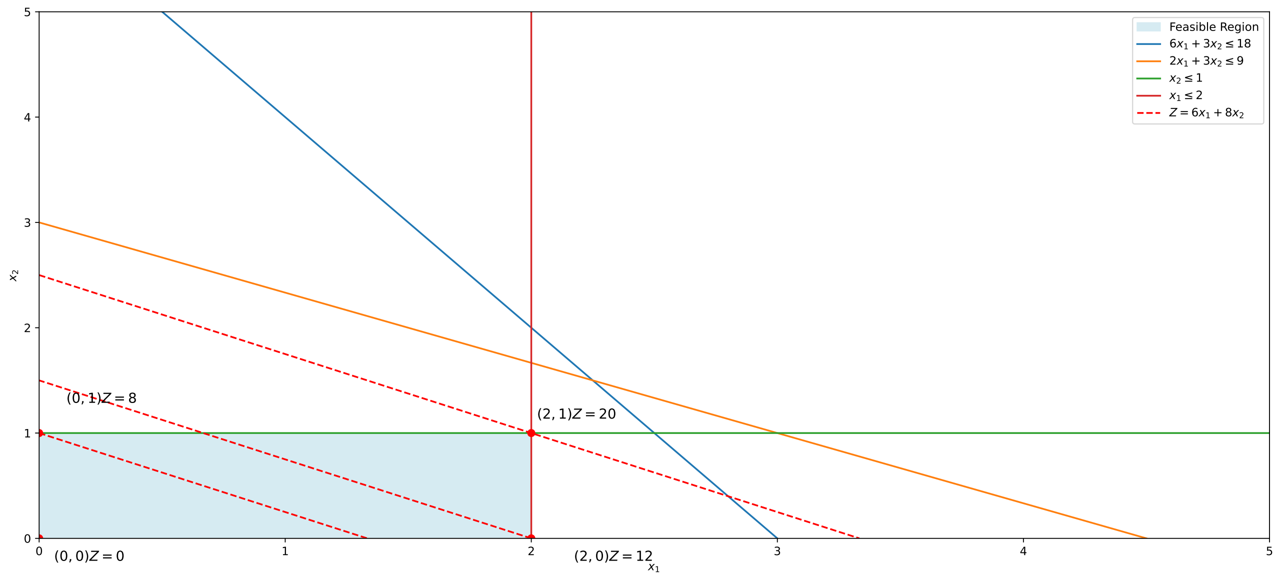
$$\begin{cases} 6x_1 + 3x_2 \leq 18 \\ 2x_1 + 3x_2 \leq 9 \\ x_1, x_2 \text{ are integers} \end{cases}$$

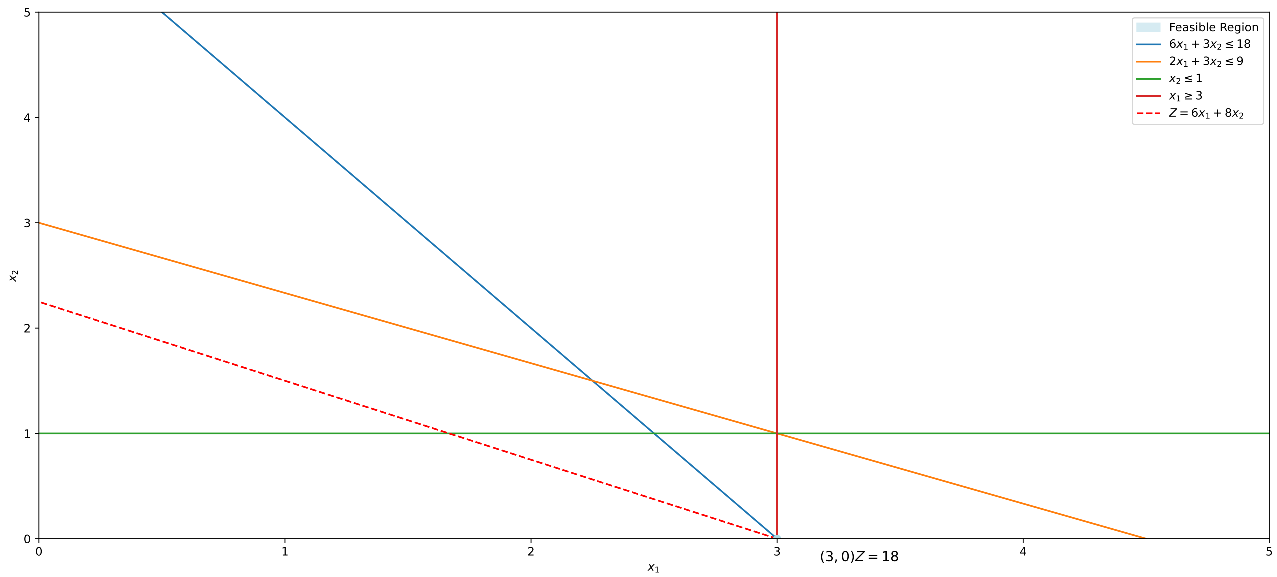


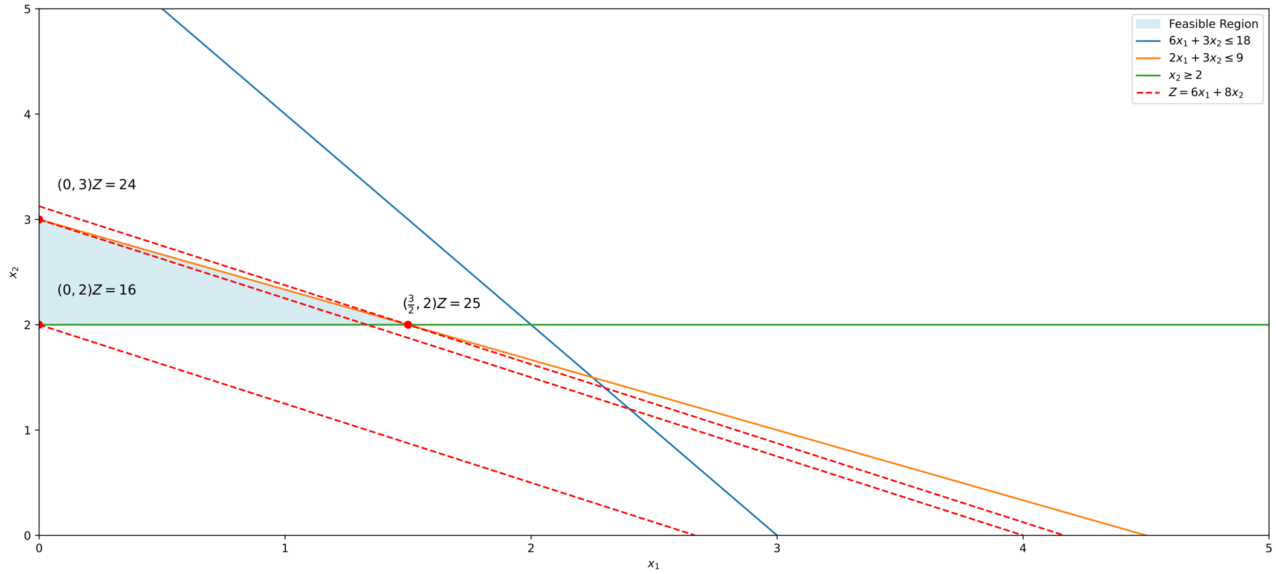
Optimal Solution is $\boxed{(x_1, x_2) = (0, 2)}$

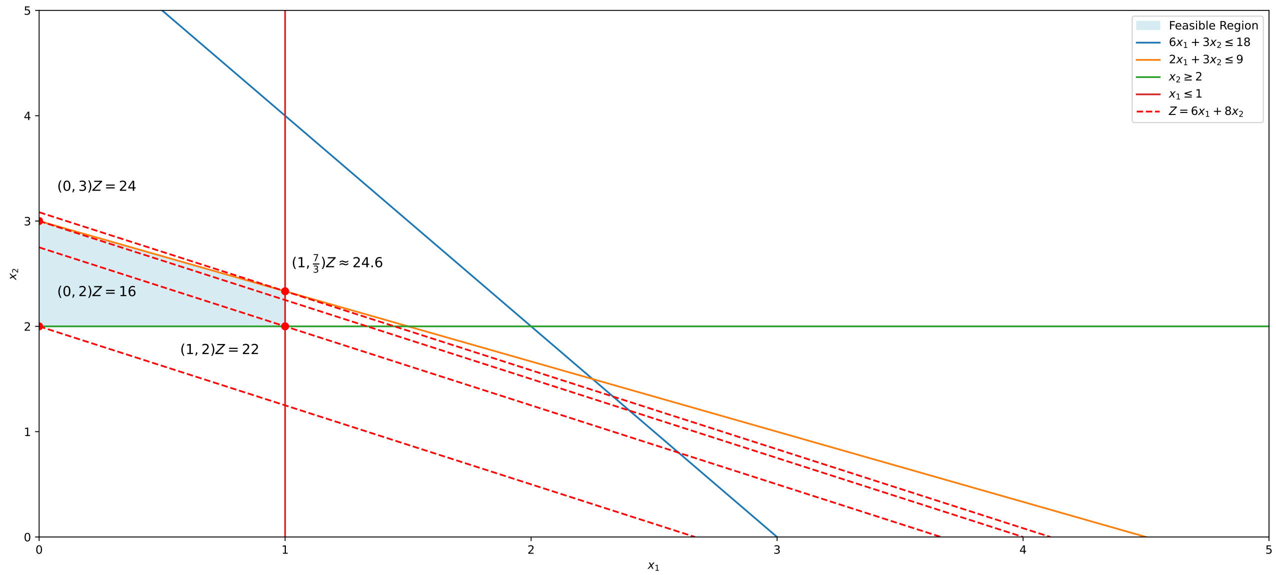

Feasible Region
$6x_1 + 3x_2 \leq 18$
$2x_1 + 3x_2 \leq 9$
$Z = 6x_1 + 8x_2$
$(0, 3)Z = 24$
$(\frac{9}{4}, \frac{3}{2})Z = 25.5$
$(0, 0)Z = 0$
$(3, 0)Z = 18$
x_1
x_2

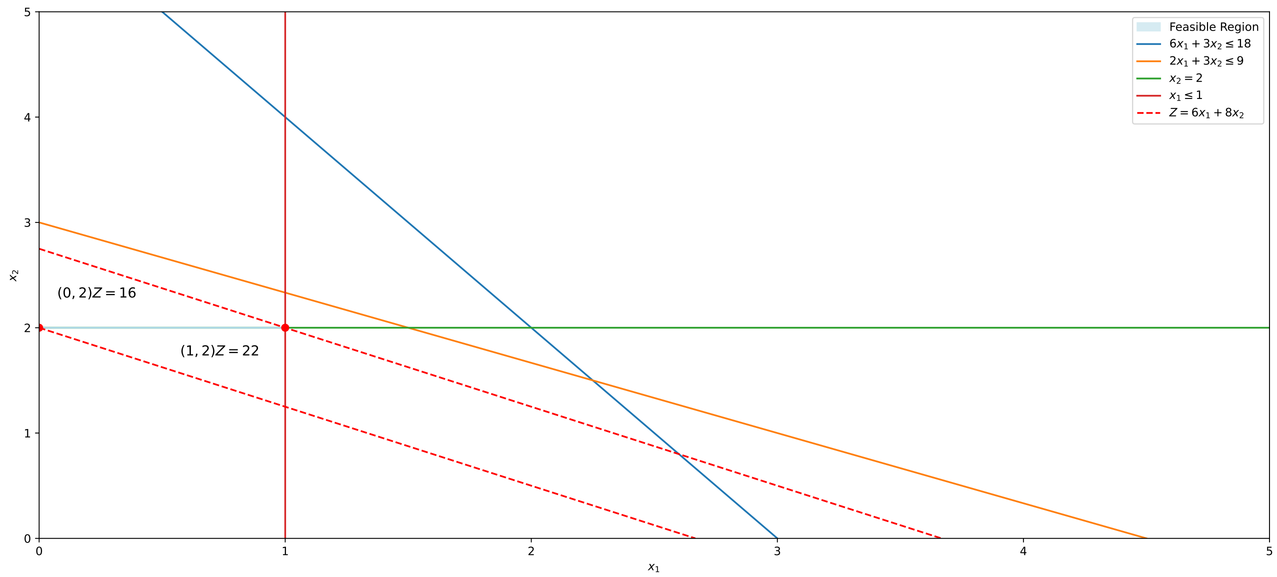

Feasible Region
$6x_1 + 3x_2 \leq 18$
$2x_1 + 3x_2 \leq 9$
$x_2 \leq 1$
$Z = 6x_1 + 8x_2$
$(\frac{5}{2}, 1)Z = 23$
$(0, 1)Z = 8$
$(0, 0)Z = 0$
$(3, 0)Z = 18$
x_1
x_2

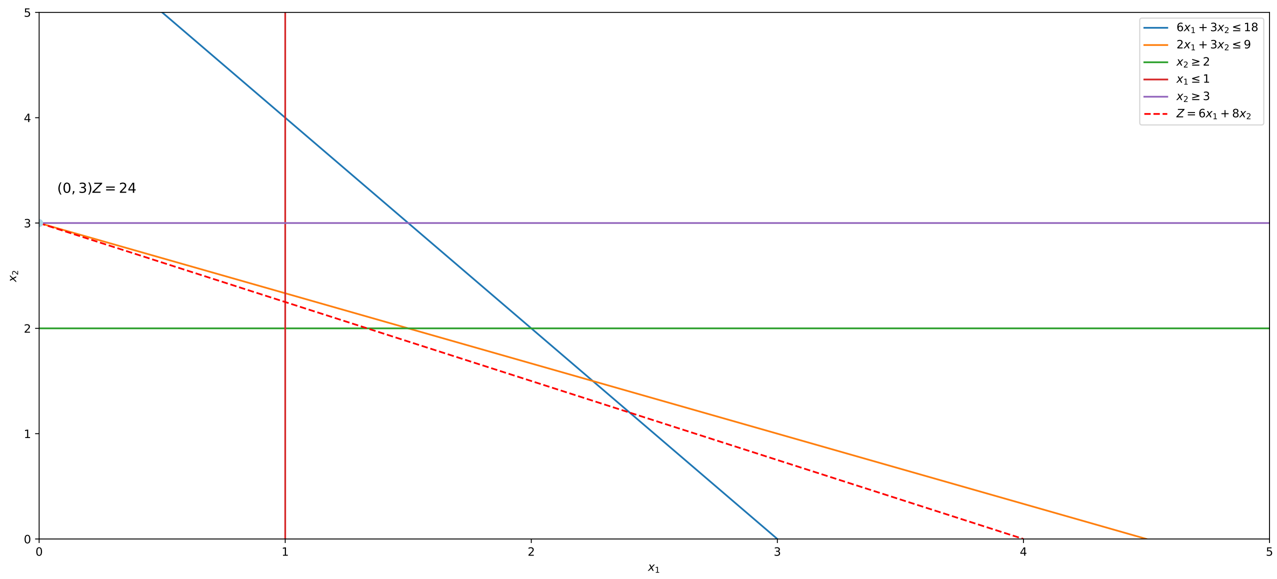

Feasible Region
$6x_1 + 3x_2 \leq 18$
$2x_1 + 3x_2 \leq 9$
$x_2 \leq 1$
$x_1 \leq 2$
$Z = 6x_1 + 8x_2$
$(0, 1) Z = 8$
$(2, 1) Z = 20$
$(0, 0) Z = 0$
$(2, 0) Z = 12$
x_1
x_2

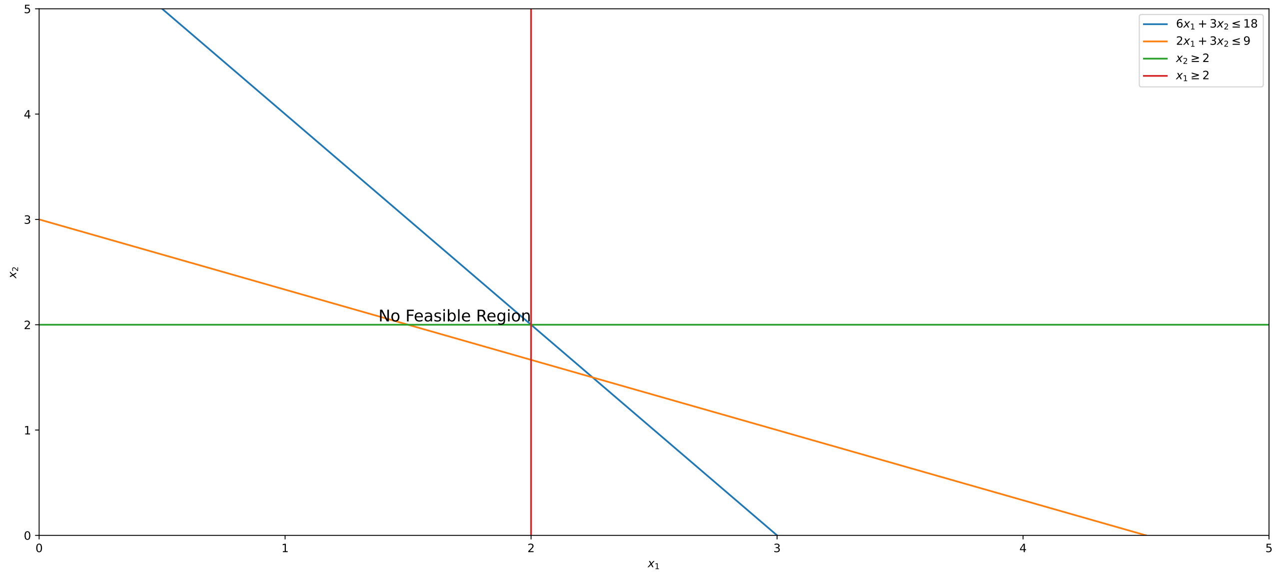

Feasible Region
$6x_1 + 3x_2 \leq 18$
$2x_1 + 3x_2 \leq 9$
$x_2 \leq 1$
$x_1 \geq 3$
$Z = 6x_1 + 8x_2$
$(3, 0) Z = 18$
x_1
x_2


Feasible Region
$6x_1 + 3x_2 \leq 18$
$2x_1 + 3x_2 \leq 9$
$x_2 \geq 2$
$Z = 6x_1 + 8x_2$
$(0,3) Z = 24$
$(0,2) Z = 16$
$(\frac{3}{2}, 2) Z = 25$
x_1
x_2


Feasible Region
$6x_1 + 3x_2 \leq 18$
$2x_1 + 3x_2 \leq 9$
$x_2 \geq 2$
$x_1 \leq 1$
$Z = 6x_1 + 8x_2$
$(0,3) Z = 24$
$(1, \frac{7}{3}) Z \approx 24.6$
$(0,2) Z = 16$
$(1,2) Z = 22$
x_1
x_2


Feasible Region
$6x_1 + 3x_2 \leq 18$
$2x_1 + 3x_2 \leq 9$
$x_2 = 2$
$x_1 \leq 1$
$Z = 6x_1 + 8x_2$
$(0, 2)Z = 16$
$(1, 2)Z = 22$
x_1
x_2

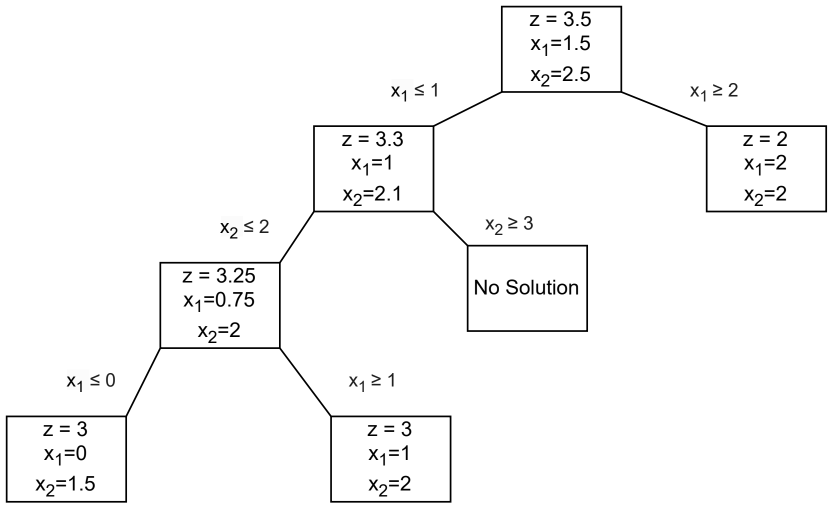

$6x_1 + 3x_2 \leq 18$
$2x_1 + 3x_2 \leq 9$
$x_2 \geq 2$
$x_1 \leq 1$
$x_2 \geq 3$
$Z = 6x_1 + 8x_2$
$(0, 3)Z = 24$
x_1
x_2



$6x_1 + 3x_2 \leq 18$
$2x_1 + 3x_2 \leq 9$
$x_2 \geq 2$
$x_1 \geq 2$
No Feasible Region
x_1
x_2

$$\min\ Z = x_1 - 2x_2 \quad \Longrightarrow \quad \max\ -Z = -x_1 + 2x_2$$

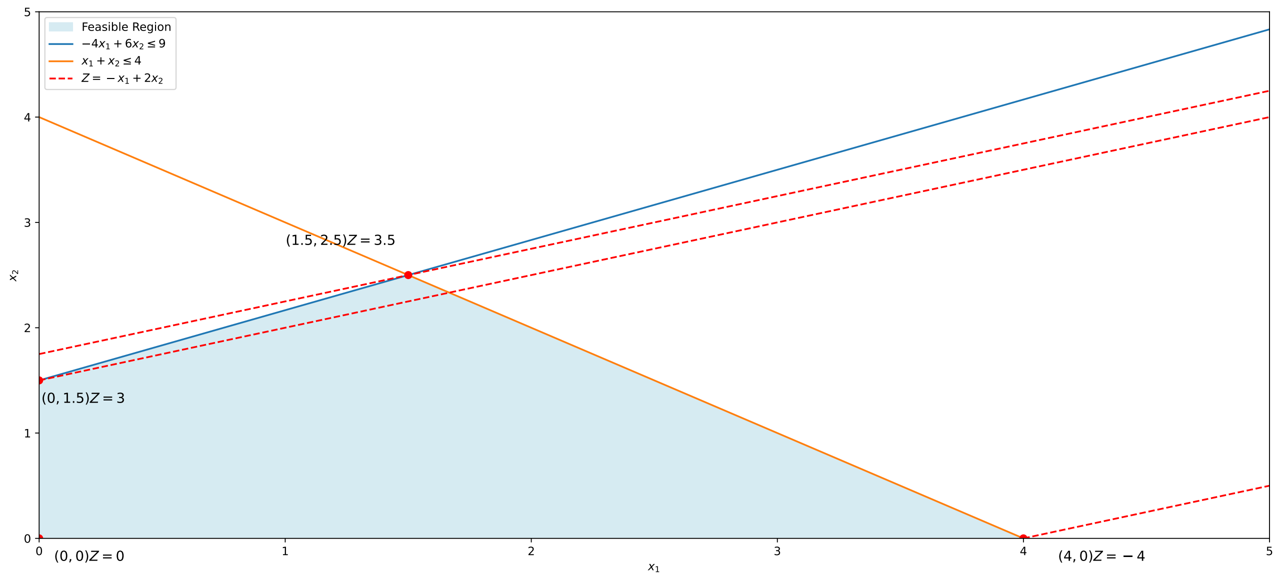
$$\begin{cases} 6x_1 + 3x_2 \leq 18 \\ 2x_1 + 3x_2 \leq 9 \\ x_1, x_2 \text{ are integers} \end{cases}$$



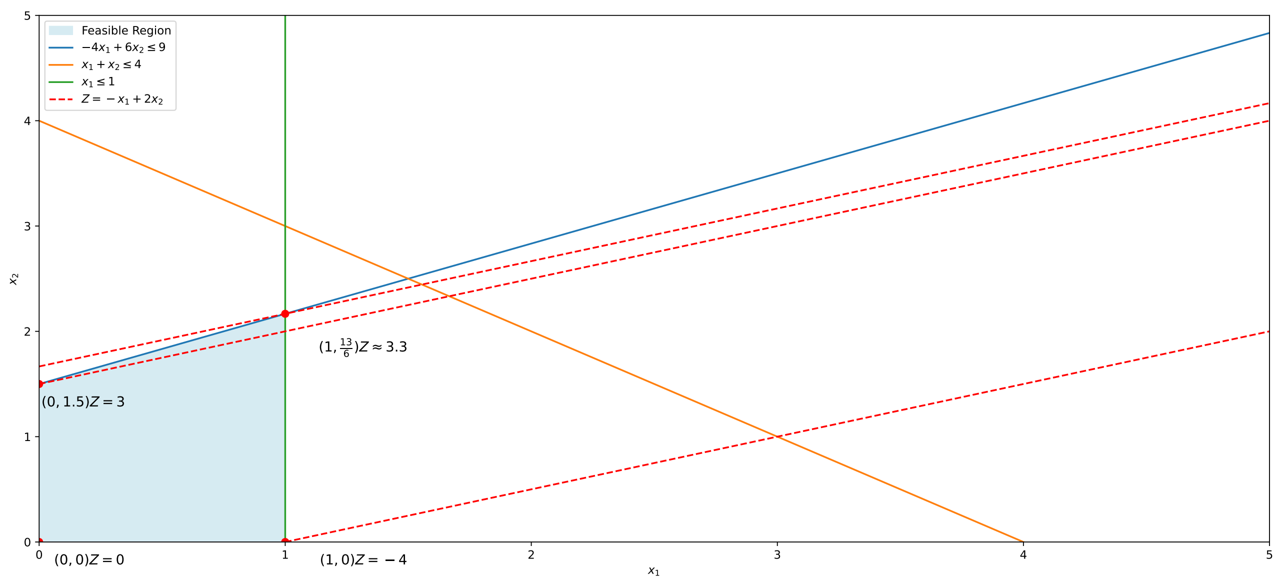
Optimal Solution is $\boxed{(x_1, x_2) = (1, 2)}$

Note

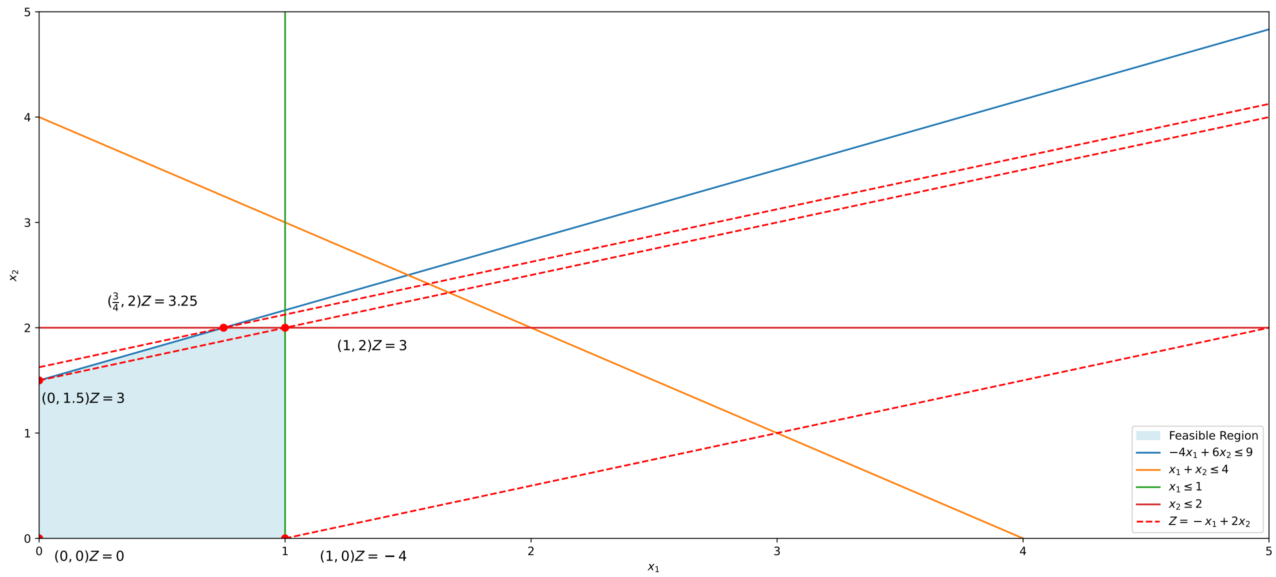
We did not branch the node $(Z = 3, x_1 = 0, x_2 = 1.5)$ because another node at the same level $(Z = 3, x_1 = 1, x_2 = 2)$ was already completed. Continuing to branch the first node would only cause Z to keep decreasing. Hence, we pruned it.



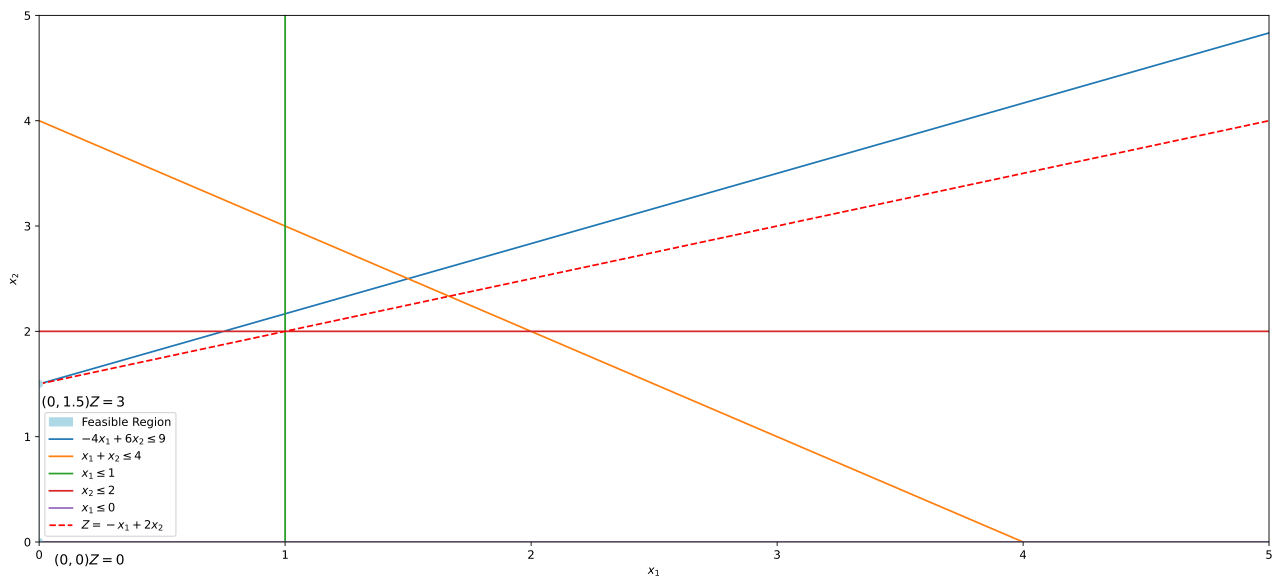
Feasible Region
$-4x_1 + 6x_2 \leq 9$
$x_1 + x_2 \leq 4$
$Z = -x_1 + 2x_2$
$(1.5, 2.5)Z = 3.5$
$(0, 1.5)Z = 3$
$(0, 0)Z = 0$
$(4, 0)Z = -4$
x_1
x_2



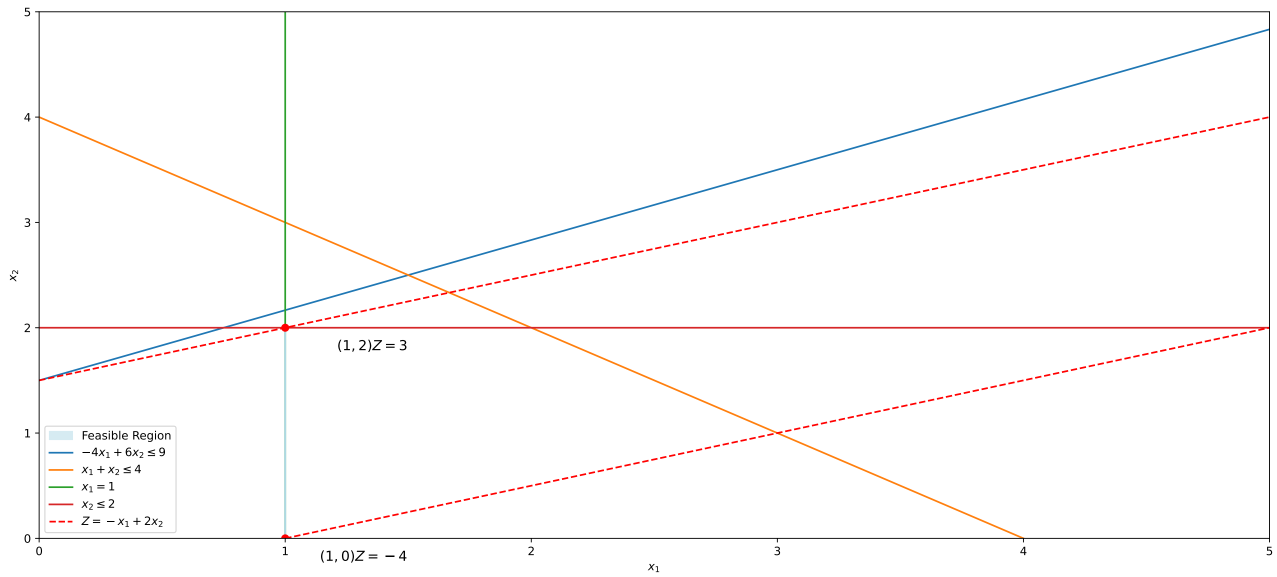
Feasible Region
$-4x_1 + 6x_2 \leq 9$
$x_1 + x_2 \leq 4$
$x_1 \leq 1$
$Z = -x_1 + 2x_2$
$(1, \frac{13}{6})Z \approx 3.3$
$(0, 1.5)Z = 3$
$(0, 0)Z = 0$
$(1, 0)Z = -4$
x_1
x_2

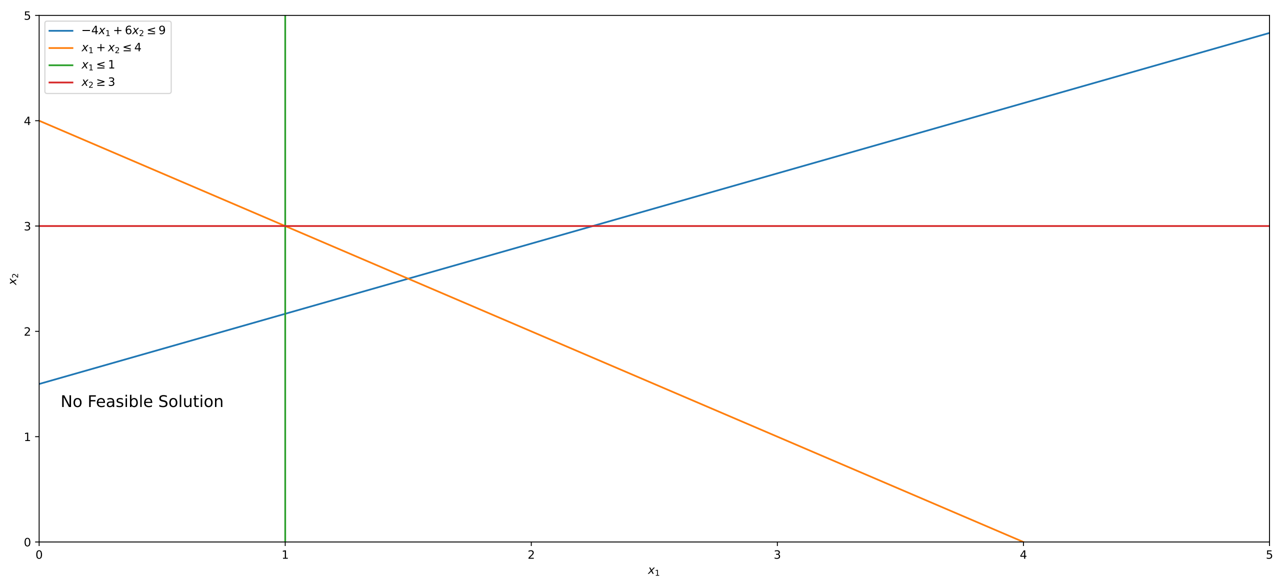


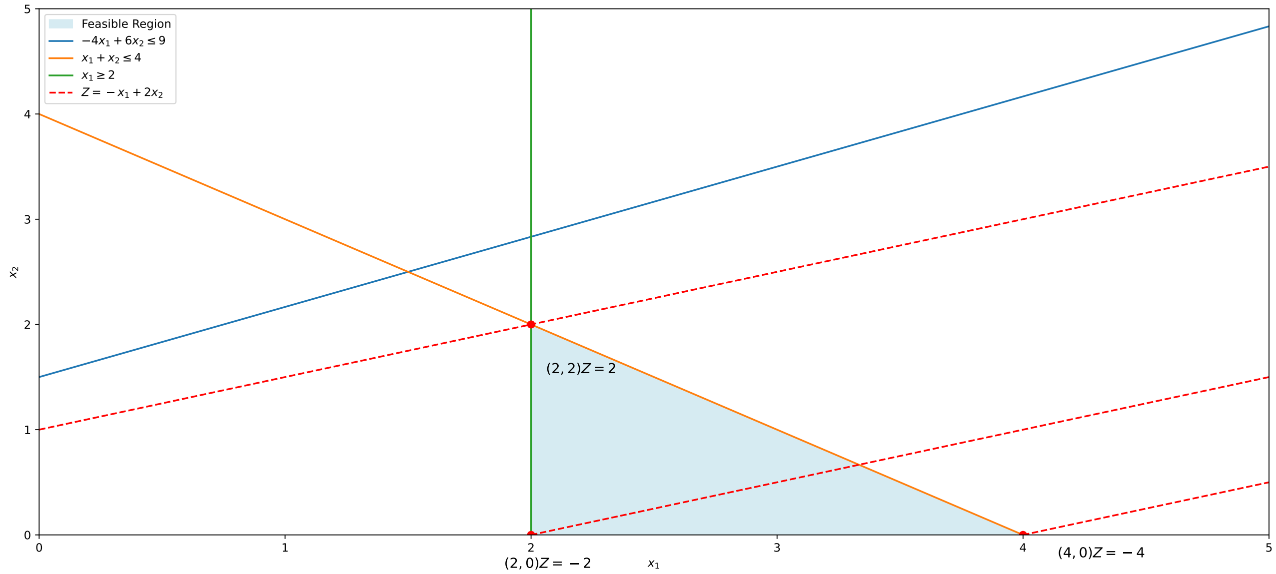
(3/4, 2)Z = 3.25
(1, 2)Z = 3
(0, 1.5)Z = 3
(0, 0)Z = 0
(1, 0)Z = −4
Feasible Region
−4x1 + 6x2 ≤ 9
x1 + x2 ≤ 4
x1 ≤ 1
x2 ≤ 2
Z = −x1 + 2x2
x1
x2



(0, 1.5)Z = 3
(0, 0)Z = 0
Feasible Region
−4x1 + 6x2 ≤ 9
x1 + x2 ≤ 4
x1 ≤ 1
x2 ≤ 2
x1 ≤ 0
Z = −x1 + 2x2
x1
x2


Feasible Region
$-4x_1 + 6x_2 \leq 9$
$x_1 + x_2 \leq 4$
$x_1 = 1$
$x_2 \leq 2$
$Z = -x_1 + 2x_2$
$(1, 2) Z = 3$
$(1, 0) Z = -4$
x_1
x_2


$-4x_1 + 6x_2 \leq 9$
$x_1 + x_2 \leq 4$
$x_1 \leq 1$
$x_2 \geq 3$
No Feasible Solution
x_1
x_2


Feasible Region
$-4x_1 + 6x_2 \leq 9$
$x_1 + x_2 \leq 4$
$x_1 \geq 2$
$Z = -x_1 + 2x_2$
$(2, 2) Z = 2$
$(2, 0) Z = -2$
$(4, 0) Z = -4$
x_1
x_2
0
1
2
3
4
5