1 Opration Research

1.1 What's Operations Research?

Definition

Operations Research (**OR**) is an interdisciplinary field that uses mathematical models, statistical analysis, and optimization techniques to help solve complex decision-making problems and achieve the most efficient outcomes. it involves the use of several key techniques, including:

- **Optimization**: Finding the best solution based on given criteria.
- **Mathematical Modeling**: Representing real-world scenarios through mathematical equations and models.
- **Statistical Analysis**: Using data to analyze and predict outcomes.
- **Simulation**: Testing strategies in a controlled model environment.

The goal of Operations Research is to offer data-driven insights and methods that lead to more informed, optimized decisions.

1.2 Origin of Operations Research

Origin

Operations Research (**OR**) originated during World War II, when the British government assembled a team of analysts, scientists, engineers, and military officers to study complex operational problems such as : air defense, bombing strategies, convoy routing, and other crucial military operations . Using mathematical models and data analysis, the team simulated various scenarios to predict outcomes and recommend optimal decisions. The success of these methods in improving military strategy inspired other nations to adopt similar approaches.This eventually led to the formalization of OR as a scientific discipline after the war.

1.3 Types of Problems Treated by OR

Types of Problems

Operations Research (**OR**) focuses on solving real-world problems by finding the most optimal decisions.The types of problems OR addresses can generally be categorized as:

- **Maximization**: Achieving the highest possible value for an objective, such as maximizing profits, productivity, or efficiency.
  - Example: Maximizing a company's revenue by determining the most profitable product mix.
- **Minimization**: Reducing or minimizing undesirable factors, such as costs, time, or resource consumption.
  - Example: Minimizing the cost of materials in manufacturing while maintaining quality standards.
- **Optimization**: Finding the best possible solution from multiple alternatives, often involving both maximization and minimization aspects.
  - Example: Finding the shortest path in a transportation network or optimizing team roles in a project.

1.4 Algorithm Complexity

Algorithm Complexity

Algorithm complexity refers to the amount of computational resources an algorithm uses. These resources are typically categorized as:

- **Time Complexity**: The amount of time it takes for an algorithm to run, depending on the size of the input. It is commonly expressed using Big O notation (e.g., $O(n)$, $O(\log n)$), which describes the algorithm's growth rate as the input size increases.
- **Space Complexity**: The amount of memory or space an algorithm requires. This is influenced by the number and size of variables, data structures, and other memory-using elements.

Minimizing both time and space complexity is crucial when developing efficient algorithms, as it leads to more optimal performance, especially for large-scale problems.

2 Linear Programming

2.1 What's Linear Programming ?

Definition

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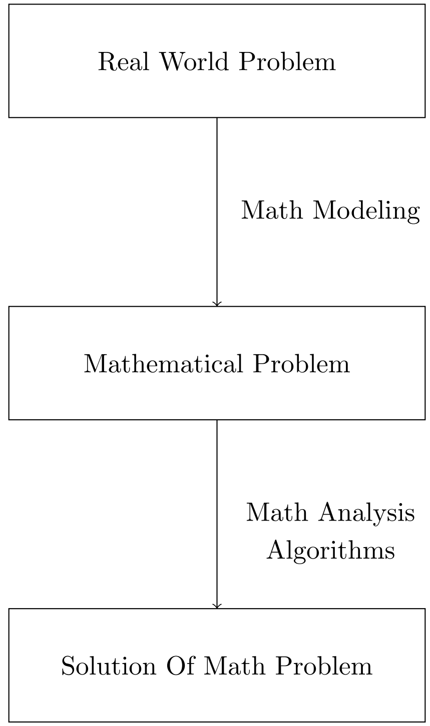


2.2 Modeling

Example : Diet Problem

The Goal is to minimize food cost but to meet the minimum daily nutrition requirement

Food	Units	Protein	Vit c	Iron	Price
Apples	1 med	0.4	6	0.4	8
Banana	1 med	1.2	10	0.6	10

<u>Variables Definition:</u>

Let x_1 be the number of Daily Unit Appels.

Let x_2 be the number of Daily Unit Banana.

<u>Constraint:</u>

$$\begin{cases} \forall x_1, x_2 \geq 0 \quad \text{(Non-negative number of food item) ...C1} \\ 0.4x_1 + 1.2x_2 \geq 70 \quad \text{(Minimum Protein Daily) ...C2} \\ 6x_1 + 10x_2 \geq 50 \quad \text{(Minimum Vitamine c Daily) ...C3} \\ 0.4x_1 + 0.6x_2 \geq 12 \quad \text{(Minimum Iron Daily) ...C4} \end{cases}$$

Objectif Function

$$f(x) = 8x_1 + 10x_2$$

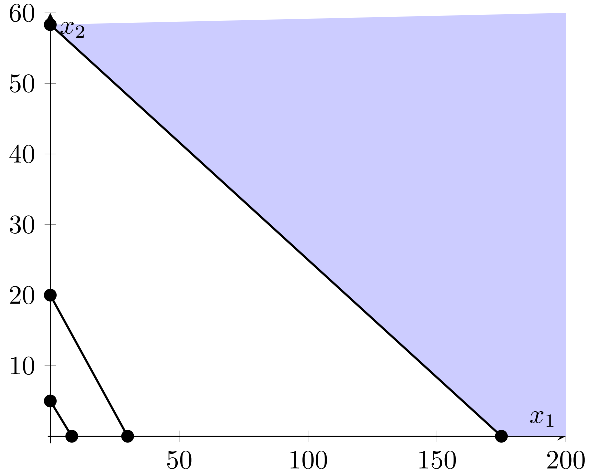
The goal is to minimize food cost by maximizing $f(x)$, while meeting the minimum daily nutrition .

<u>Problem :</u> Find the minimum of $f(x_1, x_2)$ subject to the contraints

2.3 Graph Model

This model is used when the objective fucntion has 2 variables consist into turning all the constraint into lines and then finding the feasible region and then to search for min or max of the objective function

Feasible Region :


60
50
40
30
20
10
x_2
x_1
50
100
150
200