Designs with more than one blocking system: the conflict between valid randomization and latinization

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EUCARPIA, Edinburgh, September 2025

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They took the view that if Rows and Columns are two necessary blocking systems, and each intersection of a Row and a Column is called a Cell, then Cells must also be included as a blocking system.
Moreover, they did not regard the variability between different plots as given by a simple equation. They said that a method of randomization was valid if, when averaged over the outcomes of all possible randomizations, the expectation of the mean square for a given treatment effect is the same as the expectation of the mean square for the residual that it will be compared with, in the case that that treatment effect is zero.

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I will try to explain the two approaches, and then give examples where they differ.

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Unfortunately, this mistake is still made today,
especially in clinical trials.

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D	C	E	A	B
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(Or maybe he just loved Latin squares so much that he wanted an excuse to do that enumeration?)

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He showed that it is sufficient to start with one Latin square of order v, then randomize rows using a random permutation from S_v, and then randomize columns by using an independently chosen random permutation from S_v.

More systems of blocks

A semi-Latin square is an extension of the idea of Latin square. Now there are n^2k plots, which are arranged in a $n \times n$ array of cells, where each cell has k plots. There are nk treatments, each allocated to one plot in each row and one plot in each column.

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A E I	B F J	C G K	D H L
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C F L	D E K	A H J	B G I
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In his 1935 paper on "Complex experiments", Yates said that this arrangement "stands condemned", because he saw data being analysed without including cells as a blocking factor. They essentially form a system of incomplete blocks (which means that each cell contains less than all the treatments). Once people started to include the cells in the data analysis, he became enthusiastic about of the use of semi-Latin squares.

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2. Each system of blocks has all its blocks of the same size.
3. If Rows and Columns are two systems of blocks then they are orthogonal to each other.
   (I will not go into the technicalities of this definition here, but just say that it is satisfied in all of my examples.)

Valid randomization

The linear model that we assume here is not the simple text-book one. We assume that for each treatment i there is a (usually fixed) effect τ_i and that there is a number (maybe fixed, maybe random) ζ_ω such that if treatment i is applied to plot ω then the response is $\tau_i + \zeta_\omega$.

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Of course, this definition assumes that we are using the appropriate error mean square.

Grundy and Healy, continued

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They succeeded in finding a suitable starting array.

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2. Create a catalogue of rows of length 9 which includes no undesirable ones but has the property that, for every pair of plots, there are precisely one quarter of the rows in which they have the same treatment. (If A occurs in plot 1, then it also occurs in 2 of the remaining 8 plots, so the number of rows in the catalogue must be a multiple of 4.)

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3. Choose one row from the catalogue at random.
4. Randomly allocate the 3 actual treatments to the letters A, B, C.

Example of using Youden's approach

Choose a row at random from the list below.

$$\begin{array}{ccccccccc}
A & B & A & C & A & C & C & B & B \\
A & A & B & B & C & A & C & B & C \\
A & B & B & A & C & C & B & C & A \\
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\end{array}$$

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Then randomly allocate the three actual treatments to the letters A, B, C.

Connection between block systems and valid randomization

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In a 1987 paper, Bailey and Rowley proved that if the block system contains both rows and columns then there cannot be a valid method of randomization unless the block system also includes cells.

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My approach specifies when $\rho_{\alpha\beta}$ must be equal to $\rho_{\gamma\delta}$ but it does not insist that the correlations are non-negative.

For example, if the experiment is conducted in a greenhouse, with plants of several different varieties in each pot, then competition for resources may well lead to negative correlations in outcomes per plant in the same pot.

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3. If Rows and Columns are two necessary blocking systems, and each intersection of a Row and a Column is called a Cell, then Cells must also be included as a blocking system (unless each cell is an individual plot).

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2. If any system of blocks is included in the design of the experiment, it must also be included in the data analysis.
3. If Rows and Columns are two necessary blocking systems, and each intersection of a Row and a Column is called a Cell, then Cells must also be included as a blocking system (unless each cell is an individual plot).
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Each of those aligns well with Yates's views in his comments in his book *Experimental Design: Selected Papers*.

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Such designs are usually called semi-Latin squares today.

Possible bad outcome of randomization

Williams (1986) pointed out that one possible outcome of starting with Harshbarger and Davis's design and then randomizing Sets, Rows and then Plots within each Set–Row combination could be the one shown here.

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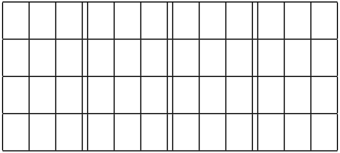
A	G	E

In an agricultural field trial it is likely that each column consists of contiguous plots, in which case this design is not desirable.

Include another blocking system

Williams (1986) proposed avoiding this problem by including Columns within Sets as another blocking system. Here is an example.



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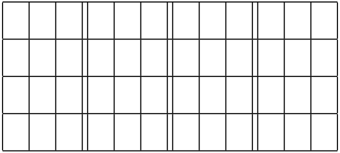

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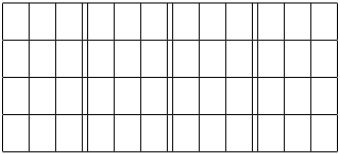

Now the data analysis has to take account of Sets, Rows, Blocks and Columns.

This paper also proposes a generalization of Latinization.
This is satisfied by a given system of blocks if
there is some integer λ such that each block contains each treatment either λ or $\lambda + 1$ times.

Terminology

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I prefer not to do this, for the following reasons.

1. I think that the blocking systems are defined before the decision is made as to how to allocate treatments to plots.
2. In some cases, such as a Latin square, there may be more than one system of blocks (for example, Rows and Columns) where each treatment occurs once in each relevant block. We cannot call both of these “replicates”.

Extending the concept of Latinizing

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Rep 1	N	K	A	V	E	M
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	H	J	B	U	S	G
Rep 2	T	G	H	X	D	N
	M	A	R	O	C	Q
	L	S	I	F	B	P
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Here $r = 2$, $s = 6$, $k = 4$ and $t = 2$.

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Here $r = 2$, $s = 6$, $k = 4$ and $t = 2$.

The rule is that no treatment occurs more than once in any group.

Extending the concept of Latinizing, continued

Rep 1	*N*	*K*	*A*	*V*	*E*	*M*
	X	*P*	*T*	*D*	*I*	*R*
	C	*F*	*Q*	*W*	*L*	*O*
	H	*J*	*B*	*U*	*S*	*G*
Rep 2	*T*	*G*	*H*	*X*	*D*	*N*
	M	*A*	*R*	*O*	*C*	*Q*
	L	*S*	*I*	*F*	*B*	*P*
	U	*V*	*J*	*E*	*K*	*W*

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	X	P	T	D	I	R
	C	F	Q	W	L	O
	H	J	B	U	S	G
Rep 2	T	G	H	X	D	N
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The paper does not mention how Long Blocks or Groups will be used in the data analysis.

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So it seems that there can be no valid system of randomization.

Unequal block sizes

Patterson and Williams (1976) introduced a class of resolvable incomplete-block designs which they called α-designs.

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Some of these have blocks with unequal sizes.
In such cases there can be no valid randomization.

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A Positive Note

After I had finished the previous slides, I re-read the paper "Beyond Latin Squares: A Brief Tour of Row–Column Designs" by Hans-Peter Piepho, Emlyn R. Williams and Volker Michel, published in *Agronomy Journal*, **107**, (2015), 2263–2270.

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Last week I was at the XIII Working Seminar on Variety Testing at COBORU. In her talk, Karen Wolf mentioned the principle "Analyse as randomized" citing the paper
"Analyze as randomized—why dropping block effects in randomized experiments is a bad idea"
by J. Frey, J. Hartung, J. Ogutu and H.-P. Piepho,
published in *Agronomy Journal*, **116**, (2024), 1371–1381.

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3. "What are the blocks in this experiment?"

   "We do not know: we just copied the design and layout from someone else's experiment that they did last year. We have no idea what they mean by 'blocks'."

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I do hope that some of the people in this room can explain to experimenters how to improve their approach.