1

$\{x, y\}$

Properties of glb and lub

(i) $glb(x, y) \leq x$ & $glb(x, y) \leq y$.

(ie) $xy \leq x$, $xy \leq y$.

(ii) $m \leq x$ & $m \leq y$

ie meet $xy$ is $x \wedge y$.

$\Rightarrow m \leq glb(x, y)$

$m \leq xy$.

(iii) $x \leq lub(x, y)$

$y \leq lub(x, y)$

$x+y$ represents $x \vee y$.

$\Rightarrow x \leq x+y$

$\Rightarrow y \leq x+y$

(iv) $x \leq m$ & $y \leq m$.

$\cancel{x+y \leq m}$

$lub(x, y) \leq m$

$x+y \leq m$.

Properties of lattices

(i) $x+x = x$

$x \cdot x = x$.

ie) $lub(x, x) = glb(x, x) = x$.

(ii) $x+y = y+x$

$x \cdot y = y \cdot x$

ie, glb and lub are commutative

glb $\le y$ & glb $\le z$

(iii) $x+(y+z) = (x+y)+z$

$x.(y.z) = (x.y).z$

Let $w = x.(y.z)$

$w \le x \qquad w \le yz$

Let $b = y.z$

$w \le x \qquad b \le y$ & $b \le z$

$w \le x \qquad w \le y \qquad w \le z$

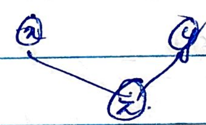
Take the other

Take dual.

**Absorption**

$x + xy = x$

$x.(x+y) = x$



**Proof**

## Special Lattices

$(A, R)$

(i) Complete lattice

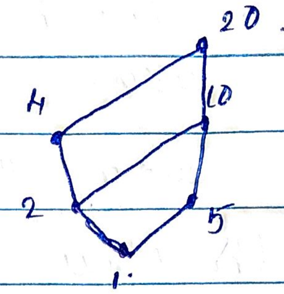
If for every subset of lattice set, has a lub and glb, then it is called a complete lattice.

$x \subseteq$ $x \leq m$

A lattice is called complete, if each of its non empty subsets will have a lub and glb. The greatest element of a lattice, if its exists is $I$ (unit element) and the ~~the~~ least element is 0 (zero element).

eg) $\{D_{20}, \mid\}$, $(P(S), \subseteq)$.

$D_{20} = \{1, 2, 4, 5, 10, 20\}$




(ii) Bounded lattice

A lattice is said to be bounded if it has a greatest element $I$ and least element 0. The elements ~~any~~ $I$ and 0 are called bounds.

eg) $(P(A), \subseteq)$ $\quad I = A, \; 0 = \phi$

Let $A = \{a_1, a_2, a_3, a_4, \ldots a_n\}$

$I = a_1 + a_2 + a_3 + \ldots a_n$

$0 = a_1 a_2 a_3 a_4 \ldots a_n$.

If $a \in A$

$a + 0 = a$

$a + I = I$

$a . 0 = 0$

$a . I = a$.

## Compliment of a lattice

For a bounded lattice $b$ is $a^c$ if

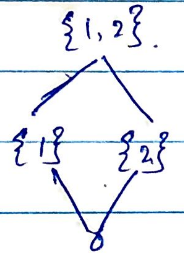
$a + b = I$

$a \cdot b = 0$

Complimit is represented as $a'$.
compliment is symmetric.

A lattice is said to be complemented, if every element in lattice has atleast one compliment.

$(P(A), \subseteq)$ Let $A = \{1, 2\}$.




$\forall\ B \in P(A)$.

~~B~~ We need a set

$B + B^c = \{1, 2\}$.

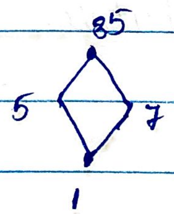
$B \cdot B^c = \phi$

Here $\{1\}$ is the compliment of $\{2\}$. and vice versa.
$\phi$ is the compliment of $\{1,2\}$ and vice verse.

So, we have compliment for every element.
ie it is ~~a~~ complemented.

Consider $B = (\{1, 5, 7, 35\}, |)$, Is it complemented?

~~ϕ~~ Ans)




$b + b' = 35$

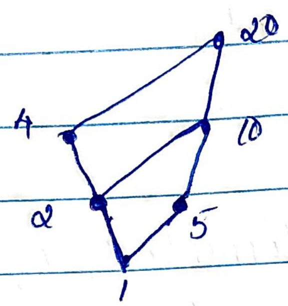
$b \cdot b' = 1$

$5' = 7,\ 7' = 5$
$1' = 35,\ 35' = 1$

ie, every element has a compliment.

Consider $(D_{20}, |)$.

Ans. $D_{20} = \{1, 2, 4, \cancel{...}, 5, 10, 20\}$




$b + b' = 20$

$b b' = 1$

~~$1' = 20$, $2' = 10$~~ 2 doesn't have a complement.

## Distributive lattice

If for $(A, \cdot, +)$, if,
mult join.

$a + (b \cdot c) = a + b \cdot a + c$.

$a \cdot (b + c) = a \cdot b + a \cdot c$ ~~min(b,c)~~

eg) $(P(A), \cdot, +)$.

Here

Q) Show that $(Z^+, \leq)$ is a distributive lattice, where ~~...~~ for any $a, b \in$ lattice, $a + b = \max(a, b)$
$a \cdot b = \min(a, b)$. Check if its distributive

Ans), $a \cdot (b + c) = a \cdot \max(b, c)$

$= \min(a, \max(b, c))$