Q) $a_{n+1} - 2a_n = 1, \quad a_0 = 0, \quad n \geq 0$

$a_n - 2a_{n-1} = 1, \quad a_0 = 0, \quad n \geq 1$

$r - 2 = 1$

Pigeon hole Principle (Dirichlet's drawer principle)

If there are m pigeons with n pigeon holes, and $m > n$, there will be atleast one pigeon hole with two or more pigeons in it

Q) How many students must be there in a class to guarantee that two students got same marks in final marks, if exam is graded on a scale from 0 to 100 marks.

Ans) There are 101 possible marks. So if there are 102 students, we can gaurantee atleast two of them gets the same score

Generalised PHP

If there are N objects, placed into k boxes, then, there is atleast one box containing, $\lceil N/k \rceil$ objects in it.

Q) What is the minimum no. of students required in the class to be sure that, atleast six will recieve the same grade, if the available grades are ~~a, b,~~ A, B, C, D, F

Ans $\left\lceil \frac{x}{5} \right\rceil = 6$

$\Rightarrow$ min value of $x = 26$.

Q) How many no.s must be selected from the set $\{1, 2, 3, 4, 5, 6\}$ to gaurantee that atleast one pair of these no.s adds to 7.

Ans) $\left.\begin{matrix}(1, 6)\\(2, 5)\\(3, 4)\end{matrix}\right\}$ Consider these pairs as holes.

If we chose 4 elements then atleast one hole will have two elements.

Q) How many no.s must be selected from set of all odd no.s upto 15 to gaurantee that atleast one pair of these elements add to 16.

Ans $\{1, 3, 5, 7, 9, 11, 13, 15\}$

$(1, 15)$
$(3, 13)$
$(5, 11)$
$(7, 9)$

We need atleast 5 elements to guarentee that atleast on pair will add to 16.

Q) Prove that if 101 integers are selected from the set $S = \{1, 2, \ldots, 200\}$, then there are two integers such that, one divides the other.

Ans) Any integer a_i can be written as

$a_i = 2^{p_i} y_i$ where ~~p is a +ve inte~~ $p \geq 0$ and y is an odd integer.

$y_i < 200$.

In the range,

y_i can take 100 values.

So if we take 101 numbers, two of the odd no.s chosen will be same

atleast a pair

for ~~any~~ $a_i = 2^{P_i} y_i$ & $a_j = 2^{P_j} y_j \Rightarrow \frac{a_j}{a_i} = 2^{P_j - P_i}$

$y_i = y_j$

(i) $P_i < P_j$ $a_i | a_j$ (ii) $P_i > P_j$ $a_j | a_i$

Q. S.T any 8 +ve integers are chosen, two will have same remainder when divided by 7.

Ans).

Q) From the no.s 1 to 12 if 7 ~~student~~ no. are chosen, then two of them will add to 13.

Ans) (1, 12) (3, 10) (5, 8)

(2, 11) (4, 9) (6, 7)

Q) How many friends must you have to gaurentee that atleast 5 of them will have work days in same month.

Ans) $\left\lceil \frac{x}{12} \right\rceil = 5$

$\Rightarrow$ 49

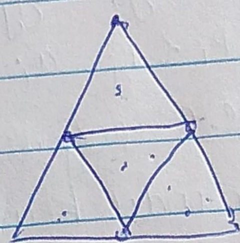
12×5

12×4

49

Q) Let T be an equilateral triangle whose sides are of ~~unit~~ 1 unit length. Show that if any 5 points are chosen, lying on or inside the Δ^{le}, then two of them must be no more than $1/2$ unit apart.

Ans). If we divide the the Δ^{le} as shown,



H.W.

Q1) Show that five points are selected in a square whose sides are one inch, atleast two of the points must be ~~$\sqrt{2}$ inches~~ not more than $\sqrt{2}$ inches apart.

Q2). Prove that if any 14 no.s from 1 to 25 are chosen, then one of them is a multiple of another.