Dynamic Interventions for Networked Contagions

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Problem

- We study designing dynamic intervention policies for **minimizing networked defaults** in financial networks under the Eisenberg-Noe model.
- Our framework can be used to tackle problems regarding **general dynamic allocations subject to contagion**.

Setting: We consider a financial network of n entities with *liabilities* to one another (in the form of a graph) where nodes can be solvent – i.e. pay all of their debts – or default in which case they proportionaly pay off their debts.

The financial network **evolves over time** and unmet debts are getting carried-over to the next round. Most existing works (e.g. [4, 2, 3]) consider a **static** (one-shot) setting.

Applications: Our framework can be used **beyond** designing dynamic allocation poilicies in financial networks. More specifically, a dynamic supply-demand network that experiences shocks and resources are to be allocated can be modeled with our framework.

Examples: ridesharing, resource allocation (e.g. CPUs) in computer networks, financial networks, ad placement, influence maximization

Setup

- **Design:** We consider a dynamic version of the Eisenberg-Noe [1] model of financial network liabilities, and use this to study the design of external intervention policies.
- We formulate the dynamic contagion problem as a Markov Decision Process (MDP) evolving in an uncertain environment.
- **Financial Environment:** We have a set $[n] = \{1, \dots, n\}$ of nodes and the system evolves in T rounds. Each node has
  - Instantaneous assets $c(t) \ge 0$.
  - Instantaneous external liabilities $b(t) > 0$.
  - Instantaneous internal liabilities $\{\ell_{ij}(t)\}_{j \in [n]}$ towards other nodes.

  These assets and liabilities evolve as a Markov Chain $u(t) = (b(t), c(t), \ell(t))$.
- **Liabilities:** Each node clears (pays-off) $\tilde{P}_i(t-1) \in [0, P_i(t-1)]$ liabilities from the previous round. The total liabilities from i to j at round t are

$$p_{ij}(t) = \underbrace{\ell_{ij}(t)}_{\text{instantaneous liabilities}} + \underbrace{p_{ij}(t-1)\left(1 - \frac{\tilde{P}_i(t-1)}{P_i(t-1)}\right)}_{\text{liabilities accrued from previous round}}$$

  and the total liabilities of each node are $P_i(t) = b_i(t) + \sum_{j \in [n]} \ell_{ij}(t) + (P_i(t-1) - \tilde{P}_i(t-1)) > 0$.
  The relative liabilities between i and j at time t are given as $a_{ij}(t) = \frac{p_{ij}(t)}{P_i(t)}$, and the relative liability matrix equals $A(t) = \{a_{ij}(t)\}_{i,j \in [n]}$.
- The financial connectivity of each node is $\beta_i(t) = \sum_{j \in [n]} a_{ij}(t) < 1$.

Interventions

Fractional interventions: A planner has a budget B (which replenishes at each round) and funds every node with $z_i(t) \in [0, L_i]$ resources, where $L_i \ge 0$. The system responds with a fixed point

$$\tilde{P}(t) = P(t) \wedge [A^T(t)\tilde{P}(t) + c(t) + z(t)] \tag{1}$$

This sequence fixed points is unique because of the assumption $\beta_i(t) < 1$ for all $i \in [n], t \in [T]$.

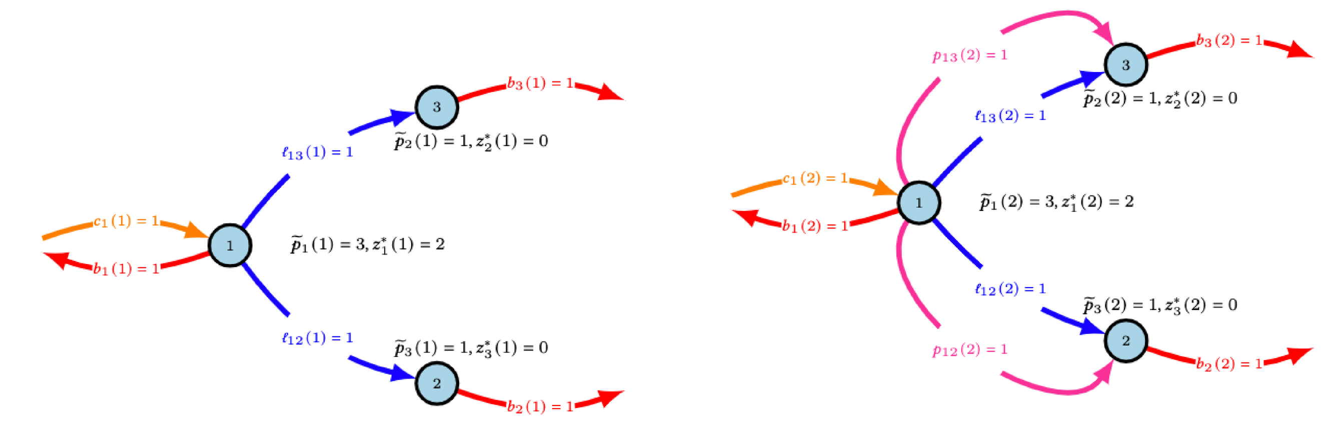
The planner observes a reward $R(t) = 1^T \tilde{P}(t)$. The objective of the planner is to find the optimal policy such that

$$\max_{z(t)} \sum_{t \in [T]} 1^T \tilde{P}(t) \ \text{ s.t. } \ \tilde{P}(t) \ge 0, \ \tilde{P}(t) = P(t) \wedge [A^T(t)\tilde{P}(t) + c(t) + z(t)], \ z(t) \in [0, L], \ 1^T z(t) \le B \tag{2}$$

Discrete interventions: The interventions can also be **discrete**, i.e. each node i can get resources $z_i(t) \in \{0, 1, 2, \dots, L_i\}$ for some $L_i \in \mathbb{N}$. The optimization problem remains the same with the only change that now interventions are discrete.

Ridesharing Example

- Environment
  - Vertices: neighborhoods of a borough (e.g. Manhattan)
  - Outside network: other boroughs
  - $\ell_{ij}(t)$ = # of rides requested from i to j
  - $b_i(t)$ = # of rides requested from outside boroughs
  - $c_i(t)$ = # of incoming rides & shocks (e.g. traffic jams)
- Allocations
  - $z(t)$ = # allocated vehicles at each neighborhood
  - L = max # of vehicles that can be allocated in a neighborhood
  - B = total # of vehicles
- $\tilde{P}_i(t)$ = # of vehicles relocated from neighborhood i



(a) $t = 1$: $P(1) = (3, 1, 1)^T, R(1) = 5$ (b) $t = 2$: $P(2) = (3, 1, 1)^T, R(2) = 5$

Figure 1. Example.

Algorithms & Theoretical Results

Fractional interventions: We prove that the optimization problem defined in (2) can be efficiently solved by sampling realizations of the financial environment $u(t)$ and solving T Linear Programs (LPs) for each sample path.

Discrete interventions: The discrete allocation problem is NP-Hard even for the static setting as proven in [4]. For the dynamic intervention scenario, we provide an approximation algorithm (SOL) based on solving the fractional LPs and randomized rounding of the solutions such that the value function V_{SOL} satisfies $\mathbb{E}[V_{SOL}] \ge \left(1 - \sup_{u(1:T) \in \mathcal{U}} \max_{i \in [n], t \in [T], u \in} \beta_i(t)\right) \cdot \mathbb{E}[V_{OPT}]$ where $\mathbb{E}[V_{OPT}]$ is the expected value function of the optimal policy.

Fairness

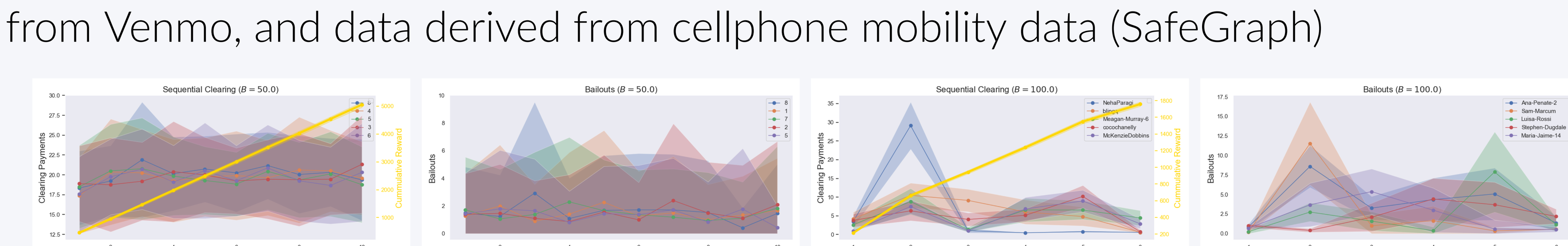
We can equitably distribute resources by constraining the **generalized Gini coefficient** measured on a graph sequence $\{H_t\}_{t \in [T]}$ (which could be the complete graph, the financial network G, etc.)

$$\text{GC}(t; H_t) = \frac{\sum_{(i,j) \in E(H_t)} w_{ij}(t)|z_i(t) - z_j(t)|}{\sum_{i \in [n], j \in [n]} z_i(t)(w_{ij}(t) + w_{ji}(t))} \le g(t) \quad \text{for some sequence } g(t) \in [0, 1]$$

We can extend the sequence of LPs of (2) to solve the clearing problem subject to fairness constraints.

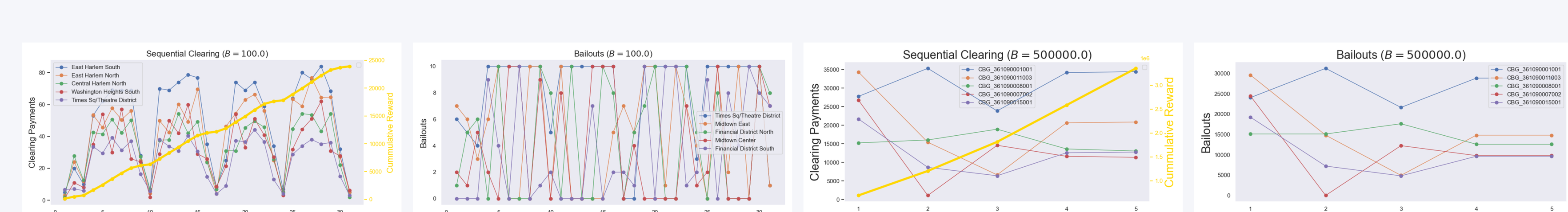
Experiments

Datasets: We experiment with a variety of results: synthetic data, ridesharing data, data from Venmo, and data derived from cellphone mobility data (SafeGraph)



(a) Payments ($L_i = B = 50$) (b) Interventions ($L_i = B = 50$) (c) Payments ($L_i = B = 100$) (d) Interventions ($L_i = B = 100$)

Figure 2. Fractional Interventions for Synthetic Core-periphery Data (a-b) and Venmo (c-d).

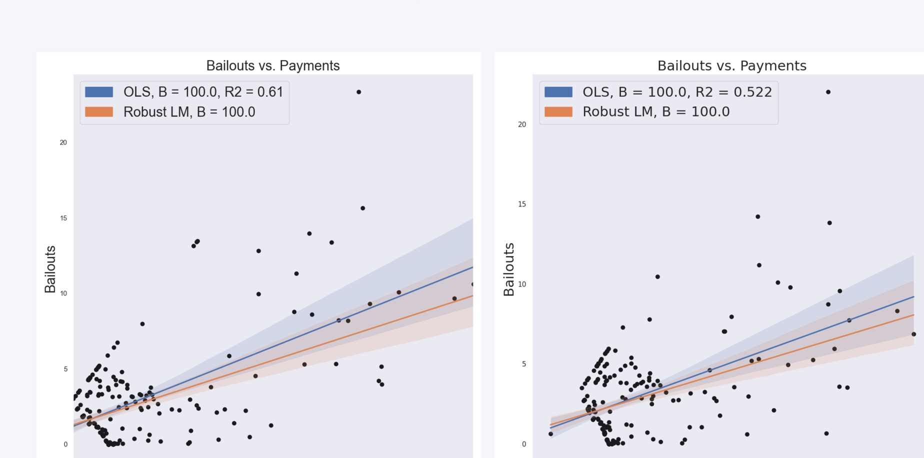


(a) Payments ($B = 100, L_i = 10$) (b) Interventions ($B = 100, L_i = 10$) (c) Payments ($B = 500K, L$ custom) (d) Interventions ($B = 500K, L$ custom)

Figure 3. Discrete Interventions for TLC FHV Data (a-b) and SafeGraph Data (c-d).

Fairness Constraint	Synthetic	TLC	Venmo	Safegraph
Budget B	50	100	50	500K
Spatial GC ($w_{ij}(t) = a_{ij}(t)$)	1.001	1.007	1.019	1.037
Standard GC ($w_{ij}(t) = 1\{i \ne j\}$)	1.011	1.009	1.017	1.102

Table 1. Price of Fairness. We set $g(t) = 0.5$.



(a) No Fairness (b) Spatial GC Fairness ($g(t) = 0.5$)

Figure 4. Correlation between the total payments of nodes and the total interventions received. We use $L = B \cdot 1$.

[1] Larry Eisenberg and Thomas H Noe. Systemic risk in financial systems. *Management Science*, 47(2):236–249, 2001.

[2] Paul Glasserman and H Peyton Young. How likely is contagion in financial networks? *Journal of Banking & Finance*, 50:383–399, 2015.

[3] Ariah Klages-Mundt and Andreea Minca. Optimal intervention in economic networks using influence maximization methods. *European Journal of Operational Research*, 300(3):1136–1148, 2022.

[4] Marios Papachristou and Jon Kleinberg. Allocating stimulus checks in times of crisis. In *Proceedings of the Web Conference 2022*, 2022.

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