Mate1009 Algebra — Lecture 2
Homework

17 February 2021

Problem 1. Solve the following systems of linear equations by reducing the augmented matrix to reduced row-echelon form:

(a)
$$\begin{aligned} x+y+z &= 2 \\ 2x+3y-z &= 8 \\ x-y-z &= -8 \end{aligned}$$

(b)
$$\begin{aligned} x_1+x_2-x_3+2x_4 &= 10 \\ 3x_1-x_2+7x_3+4x_4 &= 1 \\ -5x_1+3x_2-15x_3-6x_4 &= 9 \end{aligned}$$

(c)
$$\begin{aligned} 3x-y+7z &= 0 \\ 2x-y+4z &= \tfrac{1}{2} \\ x-y+z &= 1 \\ 6x-4y+10z &= 3 \end{aligned}$$

(d)
$$\begin{aligned} 2x_2+3x_3-4x_4 &= 1 \\ 2x_3+3x_4 &= 4 \\ 2x_1+2x_2-5x_3+2x_4 &= 4 \\ 2x_1-6x_3+0x_4 &= 7 \end{aligned}$$

Problem 2. Show that the following system is consistent if and only if $c = 2a-3b$ and solve the system in this case.

$$\begin{aligned} 2x-y+3z &= a \\ 3x+y-5z &= b \\ -5x-5y+21z &= c \end{aligned}$$