Credit Rating Migration Risk and Business Cycles*

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Abstract

Basel III seeks to improve the financial sector's resilience to stress scenarios which calls for a reassessment of banks' credit risk models and, particularly, of their dependence on business cycles. This paper advocates a Mixture of Markov Chains (MMC) model to account for stochastic business cycle effects in credit rating migration risk. The MMC approach is more efficient and provides superior out-of-sample credit rating migration risk predictions at long horizons than a naïve approach that conditions deterministically on the business cycle phase. Banks using the MMC estimator would counter-cyclically increase capital by 6% during economic expansion and free up to 17% capital for lending during downturns relative to the naïve estimator. Thus the MMC estimator is well aligned with the Basel III macroprudential initiative to dampen procyclicality by reducing the recession-versus-expansion gap in capital buffers.

JEL classifications: C13; C41; G21; G28.

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1 INTRODUCTION

The Basel II Accord, issued in 2004 by the Basel Committee on Banking Supervision (BCBS), permitted banks to use internal models to calculate capital requirements. The old Basel I rules endorse a standardized risk-weighting approach to determine the capital needed for backing different assets. Under the internal ratings-based approach, encouraged by Basel II and Basel III, banks can use their "in-house" risk models to predict for each asset in their portfolio the corresponding probability of default (PD), exposure at default (EAD) and loss given default (LGD). The resulting numbers are then plugged into a formula that assigns a risk weight.[1]

Historical ratings are the main inputs to classical estimation of credit migration probabilities, of which default risk is a measure of special interest. The late 2000s global financial crisis (GFC), which started with the collapse of the US housing bubble, has prompted a lot of skepticism over agency ratings for not being informative enough on the credit quality of structured debt obligations and for lacking timeliness. A clear instance is the bipartisan investigation into the origins of the crisis that led to the Levin and Coburn (2011; p.1) report of the U.S. Senate stating that "the crisis was not a natural disaster, but the result of high risk, complex financial products; undisclosed conflicts of interest; and the failure of regulators, the credit rating agencies, and the market itself to rein in the excesses of Wall Street." Recent studies suggest though that rating actions convey new information to the marketplace and trigger capital restructuring. Hill and Faff (2010) and Faff et al. (2007) document significant causality running from credit rating events to, respectively, international equity markets and fund flows. In a model to explain credit default swap (CDS) prices, Batta (2011) finds that ratings are significant determinants of corporate credit risk and impound relevant accounting variables such as earnings and leverage. Kisgen (2009) documents that downgraded firms reduce leverage by about 1.5%-2% in the year following the rating change.

The recent GFC has also served as a stark reminder to the marketplace of the crucial role of systematic stress testing of financial institutions' portfolios, particularly, their lending books. In response to the regulatory deficiencies thus revealed, Basel III is seeking to achieve the broader macroprudential goal of protecting the banking sector from periods of excess credit growth by requesting longer horizon default probabilities, downturn loss-given-default measures and improved calibration of risk models (see BCBS (2011)). The calibration of models that translate credit ratings

[1] The expected likelihood that firms or sovereign borrowers will default in its contractual payments is a crucial input not only to the assignment of economic capital but also to other risk management applications such as portfolio risk analysis and the pricing of bonds or credit derivatives.

and/or market data into default probabilities has direct implications for the determination of the risk-adjusted capital (i.e., core Tier 1 capital ratio) that banks need to hold to back their loans and safeguard solvency. This paper proposes an approach to estimate credit rating migration risk that controls for the business-cycle evolution during the relevant time horizon in order to ensure adequate capital buffers both in good and bad times. The approach allows the default risk associated with a given credit rating to change as the economy moves through different points in the business cycle.

There is a body of research linking portfolio credit risk with macroeconomic factors showing, for instance, that default risk tends to increase during economic downturns. Figlewski et al. (2008) document that unemployment and real GDP growth are strongly correlated with default risk. Stefanescu et al. (2009) develop a Bayesian credit score model to capture the typical internal credit rating system of most major banks and show that macroeconomic covariates such as the S&P500 returns have good explanatory power. Thus point-in-time methodologies that account for business cycles should provide more realistic credit risk measures than through-the-cycle models that smooth out transitory fluctuations (perceived as random noise) in economic fundamentals.[2]

Some attempts have been made in the *classical* credit risk measurement literature to incorporate cyclicality. Nickell et al. (2000) subdivide the historical ratings into those observed in 'normal', 'peak' and 'trough' regimes according to real GDP growth and deploys a discrete time (cohort) estimator of migration risk separately on each subsample. This naïve approach to accommodating cyclicality is also deployed in several other studies although conditioning instead upon NBER-delineated expansion and recession phases: Jafry and Schuermann (2004) and Bangia et al. (2002) in a credit-portfolio stress testing context, Hanson and Schuermann (2006) in their statistical comparison of continuous time (hazard) versus discrete migration estimators, and Frydman and Schuermann (2008) in a Markov mixture estimator that allows for firm heterogeneity. In effect, the naïve estimator implicitly assumes that the current economic conditions prevail throughout the prediction time horizon of interest. In contrast, the estimator proposed in this paper relaxes this assumption by allowing the economy to evolve randomly between states of the business cycle during the risk horizon. The importance of doing so is implicitly underlined by the Basel III requirement to use long term data horizons of at least one year to estimate probabilities of default.

This paper contributes to the credit risk modeling literature as follows. It demonstrates the ad-

[2]Rating agencies tend to adopt a through-the-cycle approach seeking to provide stable risk assessments over the life of at least one business cycle. The survey by Cantor et al. (2007) reflects that whereas bond issuers tend to favor ratings stability, plan sponsors and fund managers/trustees have a stronger preference for point-in-time accuracy.

vantages of using a Mixture of Markov Chains (MMC) model to estimate rating migration risk that explicitly recognizes the stochastic evolution of the economy between phases of the business cycle. This is the first study that comprehensively evaluates a MMC estimator against a naïve counterpart that conditions deterministically on the current economic conditions by assuming that they prevail throughout the prediction time horizon, and against classical through-the-cycle estimators. It also departs from the aforementioned studies, which consider economic dynamics in credit risk modeling, by assessing the estimators in a strictly forward-looking sense. More specifically, we exploit a real-time leading indicator of business cycles based on a principal components methodology to generate out-of-sample predictions of credit migration risk. Overall the horse race of estimators is carried out in three complementary ways: a simulation analysis of their in-sample statistical properties with specific emphasis on accuracy, an out-of-sample forecasting exercise based on a range of (a)symmetric loss functions, and an economic Value-at-Risk (VaR) analysis drawing upon the CreditRisk+ risk management framework of Credit Suisse First Boston (CSFB (1997)).

To preview our key results, the in-sample analysis reveals efficiency gains in default risk measures derived from the MMC model and more so during economic contraction due to the paucity of ratings data. Acknowledging the risk that economic conditions randomly evolve over the risk horizon is shown to improve the accuracy of out-of-sample default probability predictions. This is clearly revealed through novel asymmetric loss functions that attach a relatively high penalty to the under(over)prediction of down(up)grade risk. Such accuracy gains of the MMC estimator vis-à-vis the naïve counterpart increase with the length of the forecast horizon. Both business cycle estimators make an important difference for economic capital attribution since they imply more prudent capital buffers than through-the-cycle estimators during contraction. However, the naïve cyclical approach suggests relatively high (low) contraction (expansion) risk-capital holdings. The MMC estimator of default risk implies about 17% less capital in downturns than the naïve estimator, which could be channeled into lending to stimulate the real economy, while the suggested capital during expansions exceeds by 6% that from the naïve estimator. The MMC estimator notably reduces the expansion-versus-recession gap in risk capital relative to the naïve counterpart and can be cast as an efficient way to perform stress testing. This is an important property because exaggerated cyclicality can fuel 'irrational exuberance' and deepen recessions by making lending too capital intensive, which is one of the main criticisms of Basel II (see Gordy and Howells (2004)). Thus, relative to its competitors, the MMC approach prescribes capital build-up in good times that banks can draw upon in bad times and so it is more aligned with the Basel III macroprudential initiative to dampen the procyclical transmission of risk and promote countercyclical capital buffers.

Section 2 below reviews the classical migration estimators. Section 3 presents the MMC hazard rate approach. The data and empirical results are outlined in Section 4, and Section 5 concludes.

2 CLASSICAL CREDIT MIGRATION ESTIMATORS

2.1 Rating process and transition probabilities

A *credit rating* is a financial indicator of an obligor's level of creditworthiness. Most firms issuing publicly traded debt are rated at least by one of the three major rating agencies, Moody's, Standard & Poor's (S&P), and Fitch Ratings.[3] Let the credit rating of a firm at time t be denoted $R(t) \in S = \{1, 2, \cdots, K\}$ where S is the rating space with 1 and $K-1$ representing, respectively, the best and worst credit quality; K represents default. For instance, the coarse S&P's rating system (AAA, AA, A, BBB, BB, B, CCC) together with the default state D imply $K = 8$. The rating definitions provided by the agencies are qualitative in nature which makes their mapping onto specific quantitative risk measures crucial; Appendix A provides summary S&P definitions.

The goal is to estimate the transition or migration probabilities over horizon $[t, t+\Delta t]$ denoted

$$\boldsymbol{Q}(\Delta t) \equiv \boldsymbol{Q}(t, t+\Delta t) = \begin{pmatrix} q_{11}(\Delta t) & q_{12}(\Delta t) & \cdots & q_{1K}(\Delta t) \\ q_{21}(\Delta t) & q_{22}(\Delta t) & \cdots & q_{2K}(\Delta t) \\ \vdots & \vdots & \ddots & \vdots \\ q_{K1}(\Delta t) & q_{K2}(\Delta t) & \cdots & q_{KK}(\Delta t) \end{pmatrix}, \tag{1}$$

where $q_{ij}(\Delta t) \equiv P[R(t+\Delta t) = j | R(t) = i] \geq 0, \forall i, j \in S$, is the chance that an obligor rated i at time t is assigned rating j at $t+\Delta t$, hence $\sum_{j=1}^{K} q_{ij}(\Delta t) = 1$. The K^{th} column contains the probabilities of default (PDs). Since default is treated as an "absorbing" state, $q_{Ki}(\Delta t) = 0$ and $q_{KK}(\Delta t) = 1$, implying that $R(t)$ will settle to the default steady-state in the limit as $\Delta t \to \infty$.

As noted earlier, two assumptions typically underlie the rating migration process:

A1) *Markovian behaviour*. The probability of transition to a future state j only depends on the

[3] Several papers examine the rating actions of the three agencies. For instance, Hill and Faff (2010) offer evidence of stronger stock market reaction to S&P's rating changes than to those of the other two agencies. Mählmann (2009) proposes a structural self-selection model to explain the decision by firms to seek a third 'optional' rating from Fitch based on their borrowing costs. Using survey research methods, Baker and Mansi (2002) find that issuers and institutional investors perceive corporate bond ratings from Moody's and S&P as more accurate than those of Fitch.

current state and is independent of the rating history. Formally,

$$P\left[R(t+\Delta t)=j|R(t),R(t-1),R(t-2),\cdots\right]=P[R(t+\Delta t)=j|R(t)] \qquad \forall j\in S.$$

and thus the current rating contains all relevant information to predict future ratings.[4]

A2) *Time homogeneity.* For a given risk horizon of interest, Δt, the transition probabilities are time-constant meaning that they only depend on Δt and thus there is a family of matrices

$$\boldsymbol{Q}(\Delta t)\equiv\boldsymbol{Q}(t,t+\Delta t)=\boldsymbol{Q}(t-k,t-k+\Delta t) \qquad \forall k.$$

A stochastic process satisfying A1 is called a Markovian process. If its state space is countable, it is called a *Markov chain* process. So a time-homogeneous Markov chain satisfies

$$P\left[R(t+\Delta t)=j|R(t)=i\right]=P\left[R(t-k+\Delta t)=j|R(t-k)=i\right].$$

Hence, the n-year migration matrix is given by the n^{th} power of the annual one, $\boldsymbol{Q}(n)=\boldsymbol{Q}(1)^n$.

Next we outline the two classical migration risk estimators known as *cohort* and *hazard rate* approaches. Both build on the Markov and time-homogeneity assumptions but differ mainly in that they are formulated in a discrete- and continuous-time framework, respectively.

2.2 Cohort or discrete multinomial approach

Let $N_i(t)$ denote the number of firms that start year t at rating i and $N_{ij}(t,t+1)$ the subset of them that have migrated to rating j by year-beginning $t+1$. Let the migration frequency be denoted $\hat{q}_{ij}(t,t+1)=\frac{N_{ij}(t,t+1)}{N_i(t)}$ in years $t=1,2,...,T$. Assuming a time-homogeneous Markov rating process, the maximum likelihood (ML) estimator of the one-year credit migration risk is

$$\hat{q}_{ij}\equiv\hat{q}_{ij}(1)=\sum_{t=1}^{T}w_i(t)\hat{q}_{ij}(t,t+1)=\frac{\sum_{t=1}^{T}N_{ij}(t,t+1)}{\sum_{t=1}^{T}N_i(t)}=\frac{N_{ij}}{N_i}, \tag{2}$$

where $w_i(t)=N_i(t)/\sum_{t=1}^{T}N_i(t)$ are yearly weights. Thus $\hat{q}_{ij}$ can be simply computed as the total number of annual migrations from i to j divided by the total number of obligors that were in grade

[4]Rating momentum or drift, first documented in Altman and Kao (1992) and Carty and Fons (1994), is a prime counterexample to Markovian dynamics. In the same vein, Lando and Skodeberg (2002) and Fuertes and Kalotychou (2007) show that a downgraded issuer is prone to further subsequent downgrades.

i at the start of any sample year. Time-homogeneity is called upon to obtain the n-year cohort migration matrix as $\hat{\boldsymbol{Q}}(n) = \hat{\boldsymbol{Q}}^n$ where $\hat{\boldsymbol{Q}}$ is obtained using (2). The major rating agencies routinely publish these migration estimates for horizons $\Delta t = 1, 2, ..., 10$ years.

One weakness of the cohort approach is that it neglects within-year rating transitions and rating duration information. For instance, if firm X is rated AA on 01/01/1992, A on 03/09/1992 and AA on 31/12/1992, the cohort estimator would consider its rating unchanged in year 1992. Thus this approach is very sensitive to data sparsity which is especially typical of transitions from top ratings to default. Another issue is that the discrete annual (or n-year) horizon of the baseline cohort estimator is too rigid as more flexibility is needed to price payoffs occurring at arbitrary points in time. These shortcomings call for a continuous-type credit migration estimator.

2.3 Hazard-rate or duration approach

Let the *transition intensity* or *generator* matrix of a continuous Markov chain be denoted $\boldsymbol{\Lambda}_t \equiv \{\lambda_{ij}(t)\}_{ij \in S}$ where $\lambda_{ij}(t)_{i \neq j}$ is the *hazard rate* function or intensity representing the instantaneous transition rate at time t; the diagonal entries are given by $\lambda_{ii}(t) \equiv \lambda_i(t) = -\sum_{j \neq i} \lambda_{ij}(t)$. The probability of migration from rating i to j over an arbitrarily small time horizon τ is given by

$$P[R(t+\tau) = j \mid R(t) = i] = \lambda_{ij}(t)\tau \qquad \text{for } i \neq j$$

Under time-homogeneity it follows that $\lambda_{ij}(t) = \lambda_{ij}$ and its ML estimator is

$$\hat{\lambda}_{ij} = \frac{N_{ij}(0,T)}{\int_0^T Y_i(s)ds} \qquad \text{for } i \neq j \tag{3}$$

where $N_{ij}(0,T)$ is the total number of transitions from i to j observed over the sample period, $Y_i(s)$ is the number of firms rated i at time s and thus $D_i \equiv \int_0^T Y_i(s)ds$ gives the rating *duration* or total time spent in state i by all sampled obligors. The transition risk matrix estimator is

$$\hat{\boldsymbol{Q}}(\Delta t) \equiv \hat{\boldsymbol{Q}}(t + \Delta_t) = e^{(\Delta t)\hat{\boldsymbol{\Lambda}}} \tag{4}$$

where the matrix exponential can be obtained via a Taylor-series expansion, $e^{(\Delta t)\hat{\boldsymbol{\Lambda}}} = \sum_{k=0}^{\infty} \frac{[(\Delta t)\hat{\boldsymbol{\Lambda}}]^k}{k!}$.

One appealing feature of (4) is its flexibility to measure credit migration risk over any arbitrary time horizon, Δt. Moreover, it exploits rating transitions that occur at any point in the sample as well as rating duration information. For illustration, with reference to the example at the end of Section 2.2, the hazard-rate estimator exploits the intermediate within-year migrations to/from A through the transition intensities as expressed in (3). Furthermore, suppose that the transition $AAA \to D$ is not observed but there are transitions $AAA \to BB$ and $BB \to D$. By contrast with the cohort measure, the hazard-rate PD estimate for AAA-rated bonds is non-zero, albeit small, in line with economy theory since no bond is default free.[5]

3 BUSINESS CYCLES AND CREDIT MIGRATION RISK

In this section we present a Mixture of Markov Chains (MMC) estimator of credit migration risk that accounts for the current (time t) economic phase while acknowledging the stochastic business-cycle evolution over the migration horizon of interest $(t, t+\Delta t)$. It mixes two time-homogeneous Markov chains, one that models the ratings process and another that models the business cycle process. We focus the exposition and ensuing analysis on two phases, expansion (E) and contraction (C), but the estimator can be readily extended to more phases; Appendix B presents the 3-regime case.[6] Let the following matrix characterize the *economic* evolution over a one-period horizon

$$\mathbb{S}(1) \equiv \mathbb{S}(t, t+1) = \begin{pmatrix} \theta & 1-\theta \\ 1-\phi & \phi \end{pmatrix}, \tag{5}$$

where θ is the probability that the next phase at $t+1$ is an expansion conditional on the time t phase being an expansion, and $(1-\theta)$ is the probability of switching to a contraction. The parameters in $\mathbb{S}$ are treated as exogenous and obtained in the spirit of the *hazard-rate* approach via the corresponding transition intensity matrix $\mathbf{\Lambda}_{\mathbb{S}}$. For instance, if the baseline 1-period window is one quarter, the transition intensity $\hat{\lambda}_{E,C}$ can be computed as the number of $E \to C$ transitions over the entire sample divided by the total duration of expansion phases in months; likewise for $\hat{\lambda}_{C,E}$. The quarterly regime-switching matrix (5), simply called $\mathbb{S}$, can then be estimated as $\hat{\mathbb{S}} = e^{3\hat{\mathbf{\Lambda}}_{\mathbb{S}}}$. Figure 1 characterizes for current (time t) expansion the subsequent unfolding of the economy, or business-cycle dynamics, as a binomial tree. Within each economic phase, expansion or contraction, the

[5] This is an important aspect implicitly recognized, for instance, by the S&P definitions which state "For example, a corporate bond that is rated AA is viewed by S&P as having a higher credit quality than a corporate bond with a BBB rating. But the AA rating isn't a guarantee that it will not default, only that, in our opinion, it is less likely to default than the BBB bond." (Source: www.standardandpoors.com).

[6] One could define three regimes as expansion, 'mild' recession and 'severe' recession, where mild and severe are qualified in terms of the time-length or severity measured, say, as the percentage decrease in real GDP growth. Or one might identify 'above', 'below' and 'full' capacity phases using the Hodrick-Prescott filtered real GDP.

ratings evolution follows another time-homogeneous Markov chain characterized, respectively, by the conditional transition matrices $\boldsymbol{Q}_E$ and $\boldsymbol{Q}_C$. These two matrices are estimated by splitting the observed ratings into two subsets according to whether they have been observed during expansion or contraction, and deploying the hazard-rate estimator (4) separately on each.

The *mixture* process is characterized by the following one-period transition matrix

$$\mathbb{M}(t,t+1) = \begin{pmatrix} \boldsymbol{M}_1 & \boldsymbol{M}_2 \\ \boldsymbol{M}_3 & \boldsymbol{M}_4 \end{pmatrix} \equiv \left(\begin{array}{cc|c|c} & & \text{Expansion} & \text{Contraction} \\ & & \text{AAA} \ \cdots \ \text{CCC} \ \ \text{D} & \text{AAA} \ \cdots \ \text{CCC} \ \ \text{D} \\ \hline \text{Expansion} & \begin{matrix} \text{AAA} \\ \vdots \\ \text{CCC} \\ \text{D} \end{matrix} & \theta \boldsymbol{Q}_E & (1-\theta)\boldsymbol{Q}_C \\ \hline \text{Contraction} & \begin{matrix} \text{AAA} \\ \vdots \\ \text{CCC} \\ \text{D} \end{matrix} & (1-\phi)\boldsymbol{Q}_E & \phi \boldsymbol{Q}_C \end{array} \right) \tag{6}$$

all entries in (6), simply called $\mathbb{M} \equiv \mathbb{M}(1)$, are non-negative and each row sums to 1. The probability that an i graded obligor at current time t, pertaining to economic expansion, is downgraded to j at $t+1$, also in expansion, is given by $\theta q_{i,j,E}$. Given current expansion, $E(t)$, the one-period MMC transition matrix, $\mathbb{Q}_E(1) \equiv \mathbb{Q}_E(t,t+1)$, is calculated by adding the transition probabilities associated to the two possible business-cycle pathways, $E(t) \rightarrow E(t+1)$ and $E(t) \rightarrow C(t+1)$, as

$$\mathbb{Q}_E(1) \equiv \boldsymbol{M}_1 + \boldsymbol{M}_2 = \mathbb{X} = \theta \boldsymbol{Q}_E + (1-\theta)\boldsymbol{Q}_C. \tag{7}$$

For a two-period horizon, as Figure 1 illustrates, there are four possible business-cycle pathways which result in the MMC rating transition matrix for current expansion

$$\begin{aligned} \mathbb{Q}_E(2) \equiv \boldsymbol{M}_1 \mathbb{X} + \boldsymbol{M}_2 \mathbb{Y} = \mathbb{F}\mathbb{L}' = & \\ & \theta \boldsymbol{Q}_E \theta \boldsymbol{Q}_E + \theta \boldsymbol{Q}_E (1-\theta)\boldsymbol{Q}_C + (1-\theta)\boldsymbol{Q}_C (1-\phi)\boldsymbol{Q}_E + (1-\theta)\boldsymbol{Q}_C \phi \boldsymbol{Q}_C \end{aligned} \tag{8}$$

where $\mathbb{Y} \equiv \mathbb{Q}_C(1) \equiv \boldsymbol{M}_3 + \boldsymbol{M}_4 = (1-\phi)\boldsymbol{Q}_E + \phi \boldsymbol{Q}_C$ is the one-period MMC transition matrix conditional on current contraction, $\mathbb{F} \equiv (\boldsymbol{M}_1\ \boldsymbol{M}_2)$, and $\mathbb{L}'$ is the transpose of $(\mathbb{X}\ \mathbb{Y})$. Over an n-period horizon, $\Delta t \equiv n$, the ratings evolution will have 2^n unique pathways but the MMC migration matrix for current expansion can be fashioned in an elegant closed-from solution as

$$\mathbb{Q}_E(n) \equiv \mathbb{F}\mathbb{M}^{n-2}\mathbb{L}'. \tag{9}$$

The MMC migration matrix for current contraction, denoted $\mathbb{Q}_C(n)$, is derived similarly by defining instead $\mathbb{F} \equiv (\boldsymbol{M}_3\ \boldsymbol{M}_4)$ in (9). The MMC approach implicitly addresses to some extent two other issues beyond time-heterogeneity. One is cross-sectional dependence in default rates across obligors due to common macroeconomic conditions or systematic risk; the other is serial dependence (e.g., ratings drift or non-Markovian behaviour) induced by the cyclical behaviour of latent macro factors. A simpler way to embed business cycle effects in credit migration risk models is to assume a deterministic business-cycle evolution, namely, that the economy remains in the same (as the current or time t) phase throughout the horizon of interest t to $t+n$.[7] This naïve estimator emerges as a particular case of the MMC hazard-rate estimator (9) by conceptualizing $\mathbb{S}$ as the identity matrix, i.e. assuming $\theta = \phi = 1$ in (5) which gives $\mathbb{Q}_E(1) = \boldsymbol{Q}_E$ and $\mathbb{Q}_E(n) = \boldsymbol{Q}_E^n$; likewise for $\mathbb{Q}_C(n)$.

4 EMPIRICAL ANALYSIS

4.1 Data description

Our sample contains 7,514 US corporate bond rating histories over the 26-year period 01/01/1981 to 31/12/2006 from the S&P CreditPro 7.7 database. Like Altman and Kao (1992) inter alios, we track the ratings of individual bond issues in order to increase the number of observed migrations.[8] Among the debt issues sampled, mainly from large corporations, 2,218 are industrials, 1,677 utilities and 1,494 financials. Figure 2 illustrates that the representation of financials over our sample period initially experienced a gradual increase at the expense of industrials and utilities but the relative proportions have remained roughly steady for over half of the sample.[9]

[7] Cyclicality is accounted for in this manner by Bangia et al. (2002) using the cohort estimator, Jafry and Schuermann (2004) using the hazard-rate estimator, Hanson and Schuermann (2006) using both estimators and Frydman and Schuermann (2008) using a Markov mixture estimator designed to capture firm heterogeneity.

[8] S&P maps individual issue ratings into issuer ratings through the implied long-term senior unsecured rating.

[9] Ratings for sovereigns and municipals are not included. The *industrial* sector amalgamates aerospace, automotive, capital goods, metal/forest and building products, homebuilders, healthcare, chemicals, high technology, computers and office equipment firms. Energy and natural resources, transportation and telecommunication companies are included in the *utility* sector. The *financial* sector comprises financial institutions and insurance firms. Other sectors include consumer service (miscellaneous retailers) and leisure time/media firms.

The S&P rating scale comprises 22 fine categories but they are typically shrunk into a coarse rating system which excludes the +/- modifiers and has become the industry standard (8 rating categories plus default) .[10] Each bond issue has experienced more than 3 rating transitions over the sample period, a total of 1,166 bonds finally default and there are 4,202 *Not Rated* (NR) assignments in total.[11] Transitions to NR may be due to debt expiration, calling of the debt or failure to pay the required fee to the rating agency. Following Altman and Kao (1992), Carty and Fons (1994) and Frydman and Schuermann (2008) inter alios, we keep NR as another "rating" category in the sample. However, since rating migrations from/to NR do not provide any information about the obligor's credit quality, they are not counted as upgrades or downgrades. By incorporating new issues in the sample and discarding existing ones after default, we allow the cross-section to vary over time. These considerations help us to increase the sample size for each transition.

The ratings are allocated into two categories, respectively, those observed during economic expansion and contraction (or recession). Recession is conceptualized by the National Bureau of Economic Research (NBER) as a significant decline in US economic activity lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales.[12] There are three official periods of recession in our sample as identified by the NBER dating: 1) the early 1980s recession linked to the hike in oil prices after the late 70s energy crisis, 2) the early 1990s recession characterized by decreases in industrial production and manufacturing-trade sales, 3) the early 2000s recession following the collapse of the dotcom bubble, the 9/11 terrorist attacks and accounting scandals like Enron. These three economic stress episodes represent overall merely 11% of quarters in our 26-year sample period.

Table 1 reports for the entire sample and for the NBER expansion and contraction subsamples: *i*) total rating assignments, *ii*) rating duration in quarters, and *iii*) proportion of up/downgrades. Most of the assignments are to the intermediate A, BBB, BB or B ratings. A much lower percentage of the speculative or non-investment grade (i.e., BB or below) rating assignments and durations pertain to financials than to industrials/utilities whereas the opposite is true for the top-quality ratings; this maybe because it is very difficult to keep banks operating when a low credit rating has damaged customers' trust.[13] Downgrades are more likely than upgrades particulary in contraction.

[10]The coarse classification implies, for instance, that CCC+, CCC, CCC-, CC and C ratings are grouped as CCC.

[11]Among the 3,933 issues whose rating is withdrawn (NR) at some point, 802 are re-rated, of which 285 finally default; 251 bond rating histories have at least 2 different NR episodes. Only 5 of the 1,166 issues that enter default are rated again but, since the default state is conceptualized as absorbing, we discard their post-default rating history.

[12]The NBER Business Cycle Dating Committee has since 1978 delineated peak/trough months of economic activity. We adopt the first day of the peak/trough month as the business cycle turning point. See www.nber.com/cycles.

[13]For instance, 10.41% of the AA assignments correspond to industrials, 24.32% to utilities and 54.93% to financials.

Figure 3 reports the quarterly evolution of upgrades, downgrades, and defaults over the sample period. Shaded areas are NBER contraction quarters. The graph confirms business cycle time-heterogeneity in the rating migration process by illustrating that the number of downgrades (defaults) rises in contractions. The quarterly regime-switching matrix $\hat{\mathbb{S}}$ estimated via transition intensities (with duration in months) has off-diagonal entries $1-\theta = 0.0276$ and $1-\phi = 0.241$. Hence, if the economy is currently in expansion, the probability that it enters contraction over the next quarter is 2.8% and the probability that it switches from contraction to expansion is 24.1%.

4.2 Cohort and hazard-rate migration risk

The finest sampling interval adopted for tracking the ratings evolution is one quarter so as to match the window length of the expansion/contraction switching probability matrix $\mathbb{S}$. Hence, in the notation of Sections 2 and 3, the one-year horizon for the migration matrices corresponds to $\Delta t \equiv n = 4$, the two-year horizon to $\Delta t \equiv n = 8$ and so forth. Thus the cohort estimator (2) is deployed on a quarterly basis, giving $\hat{\boldsymbol{Q}}$, and the one-year migration matrix is computed as $\hat{\boldsymbol{Q}}(4) = \hat{\boldsymbol{Q}}^4$. In the hazard-rate framework, first, we deploy (3) to obtain the intensities or entries of $\hat{\boldsymbol{\Lambda}}$, defined as rating transitions per quarter, and the one-year transition matrix is given by $\hat{\boldsymbol{Q}}(4) = e^{4\hat{\boldsymbol{\Lambda}}}$. Table 2 reports the one-year cohort and hazard rate migration risk measures.[14]

As expected, both credit migration matrices are diagonally dominant implying relatively large ratings stability over a one-year horizon. However, the diagonal entries are smaller for *speculative grade* ratings than for *investment grade* ones, confirming that low ratings are more volatile. Unsurprisingly also, the default likelihood increases monotonically as credit quality deteriorates. Another commonality across the two matrices is that the immediate off-diagonal elements are generally larger for downgrades than upgrades, e.g. the cohort probability for a *BB* issuer to attain *BBB* next year is 5% whilst the chance of being downgraded to *B* is 7%. The estimates essentially confirm the stylized row monotonicity in rating migrations, i.e. the migration likelihood generally decreases the further away from the diagonal, reflecting the typical practice by S&P (and other rating agencies) of changing ratings in one-notch steps. These findings are in line with the evidence in Nickell et al. (2000), Bangia et al. (2002), Lando and Skodeberg (2002), and Fuertes and Kalotychou (2007).

The hazard-rate approach overcomes the cohort estimation pitfall of producing zero default risk

In contrast, 39.66% of the B assignments correspond to industrials, 17.83% to utilities and 5.41% to financials.

[14] In order to identify an appropriate truncation lag k for the Taylor-series expansion $e^{(\Delta t)\hat{\boldsymbol{\Lambda}}} \equiv \sum_{k=0}^{\infty} \frac{[4\hat{\boldsymbol{\Lambda}}]^k}{k!}$ in (4), we follow Löffler and Posch (2007) and compute the sum of the squared elements of the kth summand. If this is smaller than 10^{-320} the series is truncated at that k, otherwise one more summand is added.

measures for AAA bonds in the absence of $AAA \to D$ sample migrations. More specifically, the hazard-rate estimator suggests a non-zero, albeit small, PD at 1.5bp for AAA bonds. The estimated PD for CCC bonds offers also an interesting contrast: 28% (cohort) and 41% (hazard). On the other hand, the cohort approach overestimates (relative to the hazard-rate method) the default probabilities for middle ratings which is in line with the results in Jafry and Schuermann (2004), Hanson and Schuermann (2006) and Fuertes and Kalotychou (2007). For instance, the cohort PD estimate for BBB bonds is 23.7bp whereas the hazard-rate PD is 9.9bp. This may be because longer durations in the middle 'stepping stone' ratings (of which downgrade drift is largely responsible) reduce the transition intensities and, in turn, the hazard-rate transition probability estimates.

4.3 Business cycle-adjusted migration risk

We now compare the term structure of default probabilities implied by the naïve and MMC business-cycle estimators. The results are set out in Table 3. To ease the comparison, Panel I reports also the estimates from the classical hazard-rate approach. Panels II and III pertain, respectively, to the naïve expansion and contraction matrices $\hat{\boldsymbol{Q}}_E(n)$ and $\hat{\boldsymbol{Q}}_C(n)$ obtained as follows. The two intensity matrices $\hat{\Lambda}_E$ and $\hat{\Lambda}_C$ with rating duration in quarters are computed, respectively, by deploying (3) on the expansion and contraction rating subsamples. The naïve business-cycle transition risk for 1-year horizon is then given by $\hat{\boldsymbol{Q}}_E(4) = e^{4\hat{\boldsymbol{\Lambda}}_E}$ and $\hat{\boldsymbol{Q}}_C(4) = e^{4\hat{\boldsymbol{\Lambda}}_C}$. Panels IV and V show, respectively, the MMC migration matrices $\hat{\mathbb{Q}}_E(n)$ and $\hat{\mathbb{Q}}_C(n)$ obtained using as inputs the quarterly matrices $\hat{\mathbb{S}}$, $\hat{\boldsymbol{Q}}_E \equiv \hat{\boldsymbol{Q}}_E(1) = e^{\hat{\boldsymbol{\Lambda}}_E}$ and $\hat{\boldsymbol{Q}}_C \equiv \hat{\boldsymbol{Q}}_C(1) = e^{\hat{\boldsymbol{\Lambda}}_C}$. Thus the 1-year PDs for current expansion in Panel IV are obtained from $\hat{\mathbb{Q}}_E(4)$, i.e. formulae (9) is deployed for $n = 4$ quarters.

Both the naïve and MMC business-cycle estimators yield much higher default risk in current contraction than in expansion. Intuitively, this means that a firm operating in a contracted economy at time t is more likely to default over the horizon $(t, t + \Delta t)$ than another similarly rated firm currently in expansion ceteris paribus. Moreover, accounting for business cycles increases the default risk estimates relative to the baseline hazard-rate approach, particularly, in contraction. For example, the 1-year PD for a CCC issuer is 40.913% according to the classical hazard-rate estimator, and increases to 54.988% (65.135%) with the MMC (naïve) contraction estimator.

It is worth emphasizing that the term "naïve" for the estimates shown in Panels II-III refers to the implicit assumption that the economy remains in the same time t conditions, expansion or contraction, throughout the horizon $(t, t+\Delta t)$. In reality, the economy randomly alternates between business-cycle phases over time. By contrast, the MMC estimator not only takes into account the time t conditions but also acknowledges the stochastic evolution of the business cycle. A comparison of Panels II and IV reveals that the expansion PDs are too low (i.e., over-optimistic) for the naïve estimator relative to the MMC estimator, and vice versa (i.e. over-conservative) in contraction. The 1-year naïve PD estimates for AAA and AA when the economy is currently in expansion are, respectively, 1.7bp and 2.3bp and increase to 4.6bp and 10.8bp with the MMC estimator; the latter, but not the former, are in line with the 3bp floor imposed by the Basel II Accord on any PD estimate. Similarly for contraction, the 1-year PDs for AAA and AA are 29.7bp and 77.5bp according to the MMC approach, halving those implied by the naïve one. These contrasting results from the two cyclical estimators, MMC and naïve, are driven by the deterministic nature of the latter which rules out the chance of economic regime-switching over the horizon of interest; thus the naïve PDs are, by construction, too high (low) for current contraction (expansion). The gap between the naïve and MMC estimates is visibly larger in contraction than in expansion. Historically the US economy has stayed longer in expansion which, in turn, implies a smaller probability of switching from expansion to contraction than vice versa (respectively, $1-\hat{\theta} = 2.8\%$ and $1-\hat{\phi} = 24.1\%$, over the entire sample period). Since the MMC estimator collapses to the naïve one for $1-\theta = 1-\phi = 0$ in (5), effectively, the gap between the two should be narrower when conditioning on current expansion.

Figure 4 plots default risk estimates for CCC-rated issues over a time horizon of up to 30 years. The discrepancy between the expansion and contraction PDs from the MMC approach gradually starts to narrows down for large time horizons. This is intuitively plausible since, as time passes, the effect of the current economic regime starts to dilute and the permanent component of default risk outweighs the temporary variations. This matches the evidence in Galbraith and Tkacz (2007) and Shcherbakova (2008) that the additional information content of models conditional on macroeconomic variables tends to decline as the forecast horizon increases. By contrast, the gap between the expansion and contraction PDs implied by the naïve estimator does not dampen over time, in line with the fact that this simple approach implicitly assumes that the economy stays put (i.e., the current state prevails) throughout the estimation horizon. The upshot is that the expansion-versus-contraction PD differential implied by the naïve estimator is inflated relative to that of the MMC estimator and more so as the time horizon lengthens.

Table 4 reports the 1-year default risk estimates for B and CCC corporate bonds from existing studies in the literature together with ours. Although the data sources and time spans differ, two common findings across studies that deploy the *classical* estimators (top panel) are: i) the default risk for CCC-rated issues suggested by cohort estimates is typically lower than that implied by hazard-rate ones, ii) for B-rated issues, however, the cohort estimator yields relatively higher PDs than the hazard-rate estimator because of relatively longer durations (time spent) in this rating. Studies that incorporate *business cycle* information into the migration risk estimation (bottom panel), consistently suggest lower annual default risk in expansion than contraction.

Table 5 shows the entire 1-year migration matrix from the two business-cycle approaches, naïve and MMC. The upper off-diagonal entries suggest that the chance of a downgrade (to either neighbour or extreme ratings) is higher if the current phase is contraction. The diagonal entries are larger for expansion than contraction in line with a rise in ratings volatility when economic conditions deteriorate. Overall, the contraction-versus-expansion gap in migration risk suggested by the naïve estimator is magnified relative to that implied by the MMC estimator.

Finally, we examine the robustness of the results per sector. Table 6 presents the term structure of PDs for CCC bond issues from the classical hazard-rate model and the MMC model separately for industrials, utilities and financials. The default risk in the utility and financial sectors is lower than that in the overall economy and vice versa for the industrial sector. Sector by sector, the PDs appear again underestimated if the business cycle is ignored, particularly, in contraction. The long term structure of PDs for CCC-rated issues in each of the three sectors (unreported, to preserve space) is qualitatively similar to that shown in Figure 4 confirming that the differences between the MMC default risk in contraction and expansion trail off in the limit as $\Delta t \to \infty$. A feature of the diagonal entries in the sectoral MMC matrices is that at low credit quality levels (BBB and below), ratings volatility is higher for financials than industrials/utilities whereas the opposite holds at top credit quality levels (A and above). This pattern is common across expansion and contraction phases although somewhat stronger for the latter, e.g. the probability of staying in rating B is 11.2 (expansion) and 12.5 (contraction) percentage points higher for industrials than financials; Appendix C reports the sectoral 1-year MMC contraction matrices. The fact that lower-rated financials appear relatively more volatile than industrials/utilities, mirrored in more frequent upgrades, may be linked to bank bail-outs or 'gambling for resurrection' strategies characterized by excessive risk-taking influenced by moral hazard; see Goodhart (2006). Through explicit or implicit deposit insurance, banks on the road to insolvency can disguise the problem by aggressively raising money through unsustainable high interest rates.

4.4 Empirical Distribution of Default Rates

We now analyze the small-sample properties of the default risk estimators. Relative accuracy is gauged by comparing confidence intervals around the PD estimates as a way of quantifying estimation error or noise. In order to assess the statistical significance of differences in the default risk measures, the PDs are re-estimated M times using bootstrap (artificial) ratings samples.[15] Each bootstrap sample has the same number of obligors (cross-section size N) as the original dataset and is obtained as follows: a bond's entire rating history is randomly drawn with replacement so as to preserve the serial (e.g. business cycle) dependence in ratings; N random draws are thus made. This process is repeated $M = 1,000$ times.[16] This non-parametric bootstrap approach where the unit of resampling is a realized bond-history is advocated by Hanson and Schuermann (2006) and Löffler and Posch (2007) to circumvent having to choose a data generating process for the ratings.

The bootstrap simulation results for the classical cohort and hazard-rate PDs over a 1-year horizon are set out in the top left panel of Table 7. The mean and standard deviation of the PD estimates over replications are given, first, followed by the 95% confidence interval and the interval length. The bootstrapped intervals for hazard-rate PDs are much tighter than those for cohort PDs, especially with top ratings (e.g., about 16 times tighter for rating A). Our findings are in line with the studies by Christensen et al. (2004) from Moody's 1987-1991 firm ratings, Jafry and Schuermann (2004) from S&P's 1981-2002 firm ratings and Fuertes and Kalotychou (2007) from Moody's 1981-2004 sovereign ratings, in suggesting that the hazard-rate estimator is more efficient (accurate) than the cohort one. Those studies opt instead for a parametric bootstrap that uses a fitted Markov process as the basis for generating artificial rating histories.

We further investigate the impact of controlling for cyclicality on the accuracy of the PD estimates. We first focus on the naïve estimator. The top middle and right panels of Table 7 show that the mean naïve PD over bootstrap replications is markedly higher for current recession than for contraction. The confidence intervals of PDs in economic recession are around 16 to 80 times wider (investment grades) and 2 to 14 times wider (junk grades) than in expansion. This accuracy

[15] A simple approach to calculate analytical confidence intervals for the discrete cohort PDs, which many studies such as Nickell et al. (2000) and Christensen et al. (2004) adopt, is through the binomial distribution under the simplifying assumption that the ratings are independent over time and across obligors. However, as discussed earlier, this is not a realistic assumption. To sidestep it and in order to make the comparison across estimators more informative we employ bootstrapped confidence sets throughout.

[16] This M choice follows from the fact that there is no evidence of non-normality in the empirical distribution of PDs (e.g., the baseline hazard-rate PD density for BBB-rated bonds exhibits small skewness and kurtosis at 0.290 and 3.017, ranging between 0.071-0.910 and 2.698-3.458, respectively, for all ratings). Efron and Tibshirani (1997) show that $M = 1,000$ bootstrap replications are sufficient to obtain a good approximation in this context.

loss stems from the relatively sparse set of rating migrations observed in contraction periods since the sample entails 976 contraction days versus 8,520 expansion days.

Turning now attention to the MMC estimator, the mean PD over bootstrap replications is higher if the current phase is recession than if it is expansion, as with the naïve cyclical estimator. The confidence bands for the MMC default rates are wider in contraction than in expansion but notably less so than with the naïve estimator; around 5.9-7.4 times wider for investment grades and 1.6-4.5 times wider for junk grades. The PDs from the MMC estimator for current contraction are notably more accurate than those from the naïve one; the confidence band length is halved.[17] Such efficiency gain is not observed in expansion which is not surprising given that the expansion ratings subsample is much richer than the contraction one. The clear improvement in accuracy of the MMC estimator relative to the naïve one in contraction rationalizes the large gap between the MMC and naïve estimates shown previously in contraction also (c.f., Table 3 and Figure 4).

The bottom half of Table 7 summarizes to the distribution of the PD differential. For current contraction, the hazard-rate default risk estimates are significantly understated relative to the MMC ones. The question of whether the two business cycle-adjusted models yield statistically different PDs is addressed in the bottom (middle and right) panels of Table 7. In line with the findings in Section 4.3, the 95% confidence bands suggest that the naïve approach conditional on current expansion (contraction) significantly under(over)estimates the PD relative to the MMC approach.

One important message from this simulation analysis is that risk management practices for economic capital attribution that build upon through-the-cycle (i.e., classical cohort or hazard) default risk estimates or upon those obtained by simply splitting the sample into contraction and expansion ratings (i.e., naïve business-cycle approach) can suffer from various, bias and inefficiency, distortions especially in economic stress scenarios. This issue is further investigated in Section 4.6.

4.5 Out-of-Sample Forecast Evaluation

In this section we conduct an out-of-sample prediction exercise to shed further light on the relative merits of the MMC estimator. We choose as evaluation or holdout period the last eight sample years

[17] The simulation results suggest that conditional on current expansion there is a gain in accuracy, albeit overall very modest, in the naïve versus the MMC estimator. This maybe because the latter estimator is less parsimonious (more parameters) since it additionally controls for the fact that the business cycle is stochastically evolving over time through the switching matrix $\mathbb{S}$. The naïve estimator instead assumes that the economy remains in the same economic phase throughout the migration horizon of interest. When the current phase is expansion, this assumption is relatively mild since historically expansion has been more pervasive than contraction. Thus over the entire sample the estimated probability of being in expansion over the next quarter given current expansion is 97.2%.

(1999-2006) which amounts to one third of the data. This is a sensible choice since, as illustrated in Figure 3, it comprises an entire business cycle with both expansion and contraction phases. Akin to Frydman and Schuermann (2008), Koopman et al. (2008) and Stefanescu et al. (2009), we consider recursive estimation windows such that one year of ratings data is added at each iteration, i.e. 1981-1998, 1981-1999 and so forth. Using the ratings information in each window, 1- to 3-year migration matrix predictions are obtained according to the risk models entertained in the paper. For the sake of simplicity, most of the methodological discussion focuses on the 1-year horizon.

In order to deploy the naïve and MMC business-cycle estimators for prediction purposes, the forecaster needs to acknowledge the prevailing economic conditions at the current time point, namely, at the beginning (end) of the forecast horizon (estimation window) referred to as time t. In the first iteration, the economic conditions on year-end 1998 (i.e., last quarter of estimation window) are taken as the time t regime and, accordingly, a forecast is generated conditional either on current expansion or contraction for the migration risk over the subsequent 1-year horizon ending at $t+1$ (i.e., first out-of-sample year 1999) and so on.

With the naïve approach, the ratings in a given estimation window ending at t are classified as 'expansion' or 'contraction', and two distinct forecasts are generated for the migration risk over the horizon (t, $t+1$) which apply, respectively, when the time t economic phase is either expansion or contraction. Thus the prevailing economic conditions on year-end 1998 determine how the future migration risk over 1999 is forecasted, and so on. Likewise with the MMC approach but, in contrast with the naïve one, it does not assume that the current economic phase remains over the entire forecast horizon. Instead the MMC estimator uses as input the time-varying (i.e., recursively estimated) regime-switching matrix $\mathbb{S}$, equation (5), that governs the stochastic business-cycle evolution. To illustrate, since expansion prevails at the end of the first estimation window (1998:Q4), the corresponding 1-year-ahead transition risk matrix forecast incorporates the prediction that the economy will remain in expansion at the end of 1999:Q1 with probability $\hat{\theta}_t$ and will switch to contraction with probability $(1-\hat{\theta}_t)$; and so forth over the remaining quarters of the first out-of-sample year 1999 according to the Markov chain portrayed in Figure 1.

Our forecasts are out-of-sample in the sense that, say, the prediction of credit migration risk over 1999 is based on data up to year-end 1998. But in order for the predictions from the naïve and MMC business cycle estimators to be strictly forward-looking, we need real-time identification of the prevailing economic conditions at the point the forecasts are made (time t). For this purpose, we utilize the Chicago Fed National Activity Index (CFNAI) or, more specifically, its three-month moving average release denoted CFNAI-MA3. A practical problem with the NBER-dating employed in our in-sample analysis, and in several related studies,[18] is that the announcement of a peak or trough (turning point) usually occurs many months after the event. Therefore, the NBER-dating cannot be relied upon to identify the current economic phase in a real-time framework. By contrast, the CFNAI-MA3, which is released (toward the end of) each calendar month, has been designed as an objective timely indicator of economic conditions.[19]

We employ the real-time history of the CFNAI-MA3 to label the end (i.e., final month) of each estimation window or current time t as expansion or contraction according to the 'official' threshold rule: i) a CFNAI-MA3 value below -0.7 after a period of economic expansion signals that a recession has begun, ii) conversely, a CFNAI-MA3 value above -0.7 after a period of economic recession is taken as suggestive that a recession has ended.[20] Several studies have shown that this index matches remarkably well the NBER-designated business cycles and can be used to obtain good forecasts of inflation and of overall economic activity; see Brave and Butters (2010), Evans et al. (2002) and Stock and Watson (1999). Figure 3 plots the real-time history of the CFNAI-MA3 and illustrates that the -0.7 threshold rule yields a timely classification of expansions and recessions that is virtually identical to the lagged official NBER chronology over our sample period.

Two distinct forecast *evaluation* approaches are adopted. First, the migration risk predictions are compared with the 'true' migration risk. A practical difficulty, also common to the volatility forecasting literature, is that the variable being forecasted is unobserved (latent) and a proxy is needed. Stefanescu et al. (2009) proxy the true default risk by the observed yearly default frequencies (i.e., obtained by deploying the discrete cohort estimator over each of the holdout sample years) but they acknowledge a deficiency of this proxy, namely, since top-rated bonds have experienced no direct default over the sample period, their true default risk is unrealistically set to zero. Koopman

[18]For instance, see Bangia et al. (2002), Jafry and Schuermann (2004), Hanson and Schuermann (2006) and Frydman and Schuermann (2008). The real GDP growth criterion employed in Nickell et al. (2000) to classify the ratings sample into 'normal', 'peak' and 'trough' suffers from the same time delay problem.

[19]The CFNAI was developed by Stock and Watson (1999) for inflation forecasting purposes. It is the first principal component (i.e., a weighted average) of 85 inflation-adjusted economic indicators drawn from four broad categories: 1) production and income (23 series), 2) (un)employment and hours (24 series), 3) sales, orders, and inventories (23 series), 4) personal consumption and housing (15 series). It is based on projections for about 1/3 of the 85 series and therefore its real-time release is subject to subsequent revisions; however, due to its weighted-average nature the revision changes are far smaller than those of the individual series. The CFNAI has been successfully adopted as macroeconomic covariate in the credit rating model of Stefanescu et al. (2009) inter alios.

[20]See www.chicagofed.org/cfnai. Berge and Jordà (2009) develop a routine using a receiver operating characteristics (ROC) curve that yields -0.8 as alternative threshold rule which places equal weight on avoiding misclassifying a recession month as a non-recession month and a non-recession month as a recession month. The -0.7 threshold put forward by Chicago Fed researchers places marginally more weight on the second type of error.

et al. (2008) adopt instead a hazard-rate type proxy for the true default risk by deploying the non-parametric Aalen-Johansen estimator. In this same spirit, we deploy the continuous hazard-rate estimator (4) over each out-of-sample year (biennium or triennium) sequentially and the resulting measures are taken as true 1-year (2-year or 3-year) migration risk denoted generically $\boldsymbol{Q}_{t+1}$.[21]

Several criteria are adopted to compare the $K \times K$ migration risk matrix predictions ($\hat{\boldsymbol{Q}}_{t+1}$) and the 'true' migration risk ($\boldsymbol{Q}_{t+1}$): the $\mathbb{L}^1$ and $\mathbb{L}^2$ Euclidean distances and asymmetric extensions thereof, and a singular value decomposition (SVD) measure. For a given out-of-sample year (biennium or triennium) denoted $t+1$ the element-by-element forecast error is given by $\widehat{e}_{i,j,t+1} = \hat{q}_{i,j,t+1} - q_{i,j,t+1}$ and the Euclidean distance metrics are computed as

$$MAE_{\mathbb{L}^1} \equiv \frac{1}{K^2} \sum_{i=1}^{K} \sum_{j=1}^{K} |\hat{e}_{i,j,t+1}|, \tag{10}$$

and its counterpart $MSE_{\mathbb{L}^2}$ that replaces the absolute errors by squared errors. Two novel asymmetric criteria are considered in the present context to allow for asymmetry in up/downgrades regarding the losses associated with over/underpredictions. A prudential view on capital requirements may imply that, from the point of view of regulators, underpredicting the probability of a downgrade is more worrisome than overpredicting it; likewise, overpredictions regarding the probability of upgrades (or of ratings stability) are less desirable than underpredictions. Accordingly, we segment the transition matrix as: I) upper diagonal elements (i.e., downgrades), and II) lower (upgrades) and diagonal (stability) elements. Since all absolute forecast errors are less than unity, by taking their square root a heavier penalty is placed on them, i.e. $\sqrt{|\hat{e}_{t+1}|} > |\hat{e}_{t+1}|$. We extend the Mean Mixed Error (MME) loss function in Brailsford and Faff (1996) so that underprediction (U) and overprediction (O) errors corresponding to downgrades enter the loss function, respectively, in square root and absolute form; and vice versa for upgrades/no rating changes, as follows

$$MME = \frac{1}{K^2} \left[\sum_{t,i<j} \sqrt{|\hat{e}^U_{i,j,t+1}|} + \sum_{t,i<j} |\hat{e}^O_{i,j,t+1}| + \sum_{t,i \geq j} \sqrt{|\hat{e}^O_{i,j,t+1}|} + \sum_{t,i \geq j} |\hat{e}^U_{i,j,t+1}| \right]. \tag{11}$$

[21] Although the Aalen-Johansen (AJ) estimator allows for time-heterogeneity, several studies have shown that for large (cross-section) samples it does not produce significantly different estimates nor efficiency gains relative to the classical hazard-rate approach over short horizons, say, one to three years; the difference is much less significant than the one between the cohort and hazard-rate estimators (see Lando and Skodeberg (2002), Jafry and Schuermann (2004) and Fuertes and Kalotychou (2007)). Jafry and Schuermann (2004) illustrate that whether the hazard-rate estimator or the AJ approach is utilized makes little difference from the point of view of 1-year risk capital attribution. Since our cross-section is very large and we deploy the hazard-rate estimator over each out-of-sample year or biennium/triennium (with rating durations in quarters) the resulting 'true' migration risk matrices should be at worse as trustworthy as those obtained from the computationally rather expensive AJ estimator.

Moreover, the $MSE_{\mathbb{L}^2}$ criterion lends itself to an asymmetric extension (with a specific focus on rating mobility) that we put forward where all the errors enter squared but underpredictions are weighted more heavily than overpredictions for downgrades; and vice versa for upgrades. Formally,

$$MSE^{asy}_{\mathbb{L}^2} = \left[w^U_I \sum_{t,i<j} (\hat{e}^U_{i,j,t+1})^2 + w^O_I \sum_{t,i<j} (\hat{e}^O_{i,j,t+1})^2 \right] + \left[w^U_{II} \sum_{t,i>j} (\hat{e}^U_{i,j,t+1})^2 + w^O_{II} \sum_{t,i>j} (\hat{e}^O_{i,j,t+1})^2 \right] \tag{12}$$

with $w^U_I > w^O_I$, $w^U_{II} < w^O_{II}$ and $w^U_I + w^O_I + w^U_{II} + w^O_{II} = 1$. We consider three weight combinations: $(\frac{3}{10}, \frac{2}{10}, \frac{2}{10}, \frac{3}{10})$, $(\frac{2}{6}, \frac{1}{6}, \frac{1}{6}, \frac{2}{6})$ and $(\frac{4}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10})$. The latter case amounts to assuming a larger loss differential between underpredictions and overpredictions for downgrades. Finally, we deploy the average of singular values metric proposed by Jafry and Schuermann (2004) defined as

$$SVD \equiv \frac{1}{K} \sum_{i=1}^{K} \sqrt{\lambda_{i,t+1} \left(\tilde{\boldsymbol{Q}}'_{t+1} \tilde{\boldsymbol{Q}}_{t+1} \right)}, \tag{13}$$

where $\lambda_{i,t+1}(\mathbf{z})$ is the ith eigenvalue of $\mathbf{z}$, and $\tilde{\boldsymbol{Q}}_{t+1} \equiv \boldsymbol{Q}_{t+1} - \boldsymbol{I}$, with $\boldsymbol{I}$ denoting the identity matrix.[22] In this context, the prediction error $|\hat{e}_{t+1}|$ is defined as the absolute difference between the above SVD formulae deployed on the forecasted migration risk matrix and on the 'true' migration matrix. Each of the forecast error metrics is calculated as outlined above over every out-of-sample year $t+1$ and then averaged out over $t = 1, 2, ..., 8$; likewise, over the 2- and 3-year periods.

Our second forecast evaluation approach, following Frydman and Schuermann (2008), circumvents the difficulty of having to proxy the true migration risk. Forecast ability is gauged by subtracting the forecasted probability of each rating transition realized by the end of each out-of-sample year (biennium or triennium) from 1. For instance, take a corporate bond which is rated BBB at the end of the first estimation window (i.e. year-end 1998) and remains rated BBB at the end of the first out-of-sample year (i.e. year-end 1999) and a second BBB bond that was instead rated AA at year-end 1999. Suppose that the 1-year migration risk matrix forecast for 1999 gives $\hat{p}_{BBB,t+1}$=0.93 as diagonal entry corresponding to BBB and $\hat{p}_{BBB,AA,t+1} = 0.42$. Hence, the forecast error for the first bond is small, namely, $1 - 0.93$, but relatively large for the second bond, $1 - 0.42$. We deploy this approach and summarize the resulting error over all corporate bonds using mean absolute and mean square error metrics subsequently denoted, respectively, MAE_{1-p} and MSE_{1-p}.

The average forecast errors are summarized in Table 8 as percentage reduction relative to a

[22] By subtracting the identity matrix, the resulting migration matrix reflects just mobility, that is, the focus of the SVD metric (like $MSE^{asy}_{\mathbb{L}^1}$ which is computed from off-diagonal elements) is the dynamic part of $\boldsymbol{Q}_{t+1}$.

benchmark. A general message that comes across is that the naïve business-cycle estimator provides (from very little to) no forecast gains vis-$\grave{a}$-vis the classical hazard-rate estimator. By contrast, the MMC business-cycle estimator entails improvements in out-of-sample forecasting performance relative to both benchmarks, hazard-rate and naïve cyclical estimator, and across all forecast horizons. For instance, in terms of Mean Mixed Error (MME), the 1-year MMC migration risk estimator affords a forecast error reduction of 4% vis-à-vis the naïve cyclical estimator and 5.48% vis-à-vis the through-the-cycle hazard rate estimator. Overall, with all criteria and horizons, the MMC estimator provides forecast improvements relative to the naïve counterpart ranging from 13.82% (SVD) to 3.97% (MAE_{1-p}) for the 1-year horizon, and from 59.35% ($MSE_{\mathbb{L}^2}$) to 2.72% (MAE_{1-p}) for the 3-year. The out-of-sample forecast error reduction of the MMC cyclical estimator relative to the hazard-rate benchmark falls between 0.58% (MAE_{1-p}) and 12.34% (SVD) over the 1- to 3-year horizons. The improvements in forecast accuracy afforded by the MMC estimator relative to the through-the-cycle hazard rate benchmark are generally more sizeable on the basis of asymmetric loss functions than with the symmetric ones (e.g. $MSE^{asy}_{\mathbb{L}^2}$ versus $MSE_{\mathbb{L}^2}$). Thus conditioning on the economic state becomes even more relevant according to novel "regulatory" oriented asymmetric loss functions that attach a heavier penalty to underpredictions of rating downgrade risk than to overpredictions and vice versa for upgrades.

The percentage forecast error reduction of the MMC estimator relative to the naïve counterpart is more noticeable as the horizon of interest increases, for instance, it more than trebles from 11.17% (1-year) to 34.81% (3-year) with the $MAE_{\mathbb{L}^1}$ loss function.[23] The intuition behind this pattern is that the naïve estimator's implicit assumption that the current economic conditions prevail over the entire forecast horizon becomes less innocuous as the latter lengthens. This is important in the light of the new Basel III Accord (under preparation) which states as one of its goals to increase the mandatory time horizon for the estimation of default risk.[24]

4.6 Economic Relevance: Risk Capital Attribution

The Basel Committee requires banks to hold sufficient Tier 1 and Tier 2 capital to cover unexpected credit losses over a 1-year horizon at the 99.9% confidence level. Accordingly, a bank failure should

[23]The reported $MSE^{asy}_{\mathbb{L}^2}$ are for weights ($\frac{4}{10}$, $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$) but qualitatively similar results are obtained for the other two sets of weights, e.g. the forecast error reduction of the MMC versus naïve estimator is 5.08% (1-year), 10.36% (2-year) and 17.92% (3-year) for ($\frac{2}{6}$, $\frac{1}{6}$, $\frac{1}{6}$, $\frac{2}{6}$). Moreover, when the focus is exclusively on default risk the same pattern is observed, e.g. the $MSE_{\mathbb{L}^2}$ criteria illustrates forecast gains of MMC versus naïve of 5.54% (1-year), 14.24% (2-year) and 21.08% (3-year).

[24]For a detailed exposition of the Basel III regulatory framework see www.bis.org/bcbs/basel3.htm.

be observed only once in a thousand years. The mean of the credit loss distribution is the expected loss associated with all possible changes in credit quality over a target horizon which is covered by credit reserves, while Value-at-Risk (VaR) is the quantile of the credit loss distribution which will not be exceeded at a given probability level α. The unexpected loss, or discrepancy between the α-th VaR measure and the expected credit loss, defines economic capital at confidence level α.[25] We now compare the various rating migration measures through the lens of risk capital attribution.

To this end, we utilize the popular CreditRisk$^+$ model to derive the portfolio default loss distribution over a 1-year horizon.[26] In this model, obligors are classified into n independent sectors; a 'sector' is a group of obligors under the influence of a common systematic factor which induces default correlations. Each factor $k = 1, ..., n$ is assumed to be Gamma distributed with mean default intensity $\lambda_k = \sum_j \theta_{jk}\lambda_j$ where $\lambda_j = -log(1 - \overline{PD}_j)$ and volatility of default intensity $\sigma_k = \sum_j \sigma_{PD_j}$ where $\overline{PD}_j$ and σ_{PD_j} are, respectively, the mean and standard deviation of the jth obligor default probability; each of the sector weights θ_{jk} represents the extent to which sector k influences obligor j, so that $\sum_k \theta_{jk} = 1$.[27] Hence, the model inputs needed to derive the closed-form distribution of the portfolio losses are, for each obligor: the sector weights (θ_{jk}), the loss given default (LGD), the exposure at default (EAD), the mean probability of default ($\overline{PD}$) and its volatility (σ_{PD}).

We build a fictitious credit portfolio of 100 bonds and assign to each a random initial rating j uniformly drawn from the space $S = \{AAA, ..., CCC\}$. $\overline{PD}_j$ and σ_{PD_j} are taken from the bootstrap distributions set out in Table 7. The EAD for each bond is randomly drawn from a uniform distribution with range \$1 to \$1m summing to a total of \$50,030,818 for the portfolio. All obligors pertain to the same sector (thus $n = 1$ and $\theta_{jk} = \theta_j = 1$ for all j) and have full LGD. Table 9 reports the economic risk capital estimates at confidence levels $\alpha = \{99.0\%, 99.9\%\}$. The risk capital suggested by the baseline hazard rate estimator at \$6.64m (99.0% level) and \$8.25m (99.9% level) is about 15% larger than that suggested by the less efficient cohort estimator; this confirms the earlier evidence in Jafry and Schuermann (2004) that the choice between a discrete-time or continuous-time estimator can matter substantially for economic capital assessment.

[25] The bank's risk manager faces the task of justifying that the estimated economic capital (based on default risk estimates) reflects the actual level of credit risk the institution is taking and to present evidence to the regulators. VaR backtesting has become standard in this regard. Regulatory capital acts as a constraint for banks in the sense that target capital ratios usually exceed regulatory capital, the so called "headroom", for strategic reasons (e.g. to be able to take advantage of growth opportunities), operational reasons (e.g. to avoid the direct and indirect costs of having to raise capital at short notice) and to mitigate regulatory intervention; see Francis and Osborne (2011).

[26] For details see http://www.csfb.com/institutional/research/assets/creditrisk.zip

[27] CreditRisk$^+$ classifies the obligors in each sector into $i = 1, ..., m(k)$ sub-portfolios or bands of similar exposure at default. The distribution of the number of default events in each exposure band is treated as Poisson with mean equal to the expected number of defaults in each sub-portfolio over one year. The default loss distribution for each sector is thus obtained by aggregating with weights θ_{ik} the individual sub-portfolio loss distributions.

At the 99.9% level, the naïve business-cycle estimator suggests a risk capital of \$8.19m in expansion, a modest 0.7% decrease versus the classical hazard-rate estimator; by contrast, there is a dramatic 70% increase for the required risk capital in contraction. This asymmetry arises because the naïve estimator implicitly assumes that the same current economic conditions stay during the 1-year horizon. Historically, expansions have been more pervasive than contractions and so the gap between the hazard-rate and naïve estimator is plausibly very modest in expansion. Although the naïve estimator we deploy is of continuous-time (hazard rate) type whereas that in Bangia et al. (2002) builds on the discrete-time (cohort) framework, our analyses concur in suggesting that classical through-the-cycle approaches can greatly underestimate economic capital in contractions.

The 99.9% economic capital suggested by the MMC estimator in expansion (contraction) at \$8.69m (\$11.42m) represents an increase (decrease) of 6% (17%) vis-à-vis the naïve counterpart measure. The contraction risk capital is 1.7 times that in expansion according to the naïve estimator but only 1.3 times larger according to the MMC estimator. Thus the naïve estimator underestimates risk capital in expansion and substantially overestimates it in recession. In times of economic stress banks could free up to 17% of capital by opting for the MMC business-cycle approach instead of the naïve counterpart. This may have important macroeconomic implications since holding a large capital buffer is costly for banks and impairs their ability to grant credit. Excessive cyclicality in risk capital − a grievance of the naïve cyclical estimator − may materialize, unfortunately, in less lending during a downturn or a "credit crunch" period which could further aggravate the economic conditions. The more contained cyclicality of the capital requirements associated with the MMC estimator vis-à-vis the naïve counterpart is attractive in the context of another of the new Basel III reforms which seeks to dampen the procyclical amplification of financial shocks.

5 CONCLUSIONS

The Basel Committee on Banking Supervision published in 2004 the Basel II Accord that allows banks to use internal ratings-based models in deriving loan loss distributions and credit risk-weights for their assets. Given that regulatory measures of financial strength such as the Tier 1 capital ratio are expressed as core capital to total risk-weighted assets, the choice of approach for estimating credit migration risk is a key determinant of the capital that banks hold against unexpected losses. This paper enriches the literature by rigorously assessing the merit of accounting for economic conditions in credit risk measurement. We advocate a Mixture of Markov Chains (MMC) estimator of rating migration risk which explicitly recognizes the stochastic business cycle. A particular case of the MMC estimator is the de facto naïve cyclical approach that conditions deterministically on economic phases by assuming that the same conditions prevail throughout the prediction horizon. We compare the MMC estimator with the naïve cyclical counterpart and with classical through-the-cycle estimators in three different frameworks. One is purely statistical and uses simulations to assess the estimators' in-sample properties with emphasis on accuracy. The second is a forward-looking framework that evaluates credit risk forecasts using conventional and novel (a)symmetric loss functions. Third, we confront the capital requirements implied by the different estimators. The analysis is based on a 26-year sample of Standard & Poor's US corporate bond ratings.

Ignoring business cycles significantly understates default risk during economic contraction. The MMC approach yields more reliable default risk measures than the naïve cyclical estimator, especially in contraction. The same conclusions are reached when the analysis is conducted at the sectoral level. In terms of out-of-sample prediction, the performance of the MMC estimator is superior to that of the naïve counterpart and this is clearly revealed through novel asymmetric loss functions which attach a relatively heavy penalty to under(over)predictions of down(up)grade risk. These forecast accuracy gains become more prominent as the time horizon lengthens.

An application to economic capital attribution via the CreditRisk+ model suggests that the buffers prescribed by the MMC and naïve cyclical approaches are higher than those from classical through-the-cycle estimators, particularly in economic contraction. However, default risk during contraction (expansion) is statistically and economically overestimated (underestimated) by the naïve cyclical approach relative to the MMC approach and more so for longer prediction horizons. The MMC estimator here proposed, which can be seen as a way to perform stress testing, prescribes about 17% less capital holdings during downturns and 6% more capital in expansions than the naïve counterpart. The excess cyclicality in capital requirements associated to the naïve model would make lending very costly for banks in troubled times imposing too great a cost on economic growth and potentially aggravating a contraction. Our analysis has important implications for the ongoing financial regulatory reforms. The properties of the MMC estimator here documented become quite relevant in the light of the Basel III initiatives to lengthen the time horizon over which to measure credit risk, and to promote countercyclical capital buffers in order to dampen procyclicality. Thus by adopting more sophisticated models that account for the stochastic business cycle, the banking system can serve better as shock absorber instead of transmitter of risk to the broader economy.

APPENDIX

A Standard & Poor's Rating Definitions

Long-Term Issue Credit Ratings. Long-term ratings assigned to obligations with an original maturity above 365 days which are based, in varying degrees, on S&P's analysis of the following considerations: i) Likelihood of payment—capacity and willingness of the obligor to meet its financial commitment on an obligation in accordance with the terms of the obligation; ii) Nature and provisions of the obligation; iii) Protection afforded by, and relative position of, the obligation in the event of bankruptcy, reorganization, or other arrangement under the laws of bankruptcy and other laws affecting creditors' rights. The general meaning of the main rating categories is:

AAA. The obligor has a extremely strong capacity to meet its financial commitment on the obligation. Highest rating.

AA . Very strong capacity to meet financial commitments.

A. Strong capacity to meet financial commitments, but somewhat susceptible to adverse economic conditions and changes in circumstances.

BBB. Adequate capacity to meet financial commitments, but more subject to adverse economic conditions.

BB. Less vulnerable in the near-term but faces major ongoing uncertainties to adverse business, financial and economic conditions.

B. More vulnerable to adverse business, financial and economic conditions but currently has the capacity to meet financial commitments.

CCC. Currently vulnerable and dependent on favorable business, financial and economic conditions to meet financial commitments.

CC. Obligation currently highly vulnerable to non-payment.

C. Obligations that have payment arrears allowed by the terms of the documents, or obligations of an issuer that is the subject of a bankruptcy petition or similar action which have not experienced a payment default.

D. Payments of an obligation are not made on the date due even if the applicable grace period has not expired, unless S&P's believes that such payments will be made during such grace period.

(*Source: Standard & Poor's.*)

B Three-Regime MMC Estimator

The ratings sample is divided into 3 subsamples referred to as expansion, contraction and "normal" (intermediate) state. For each subsample, a one-year rating migration matrix denoted, respectively, $\boldsymbol{Q}_E$, $\boldsymbol{Q}_C$ and $\boldsymbol{Q}_N$, is calculated using the continuous hazard rate estimator (4).

- The regime-switching matrix is $\mathbb{S} = \begin{pmatrix} \alpha & \beta & (1-\alpha-\beta) \\ \delta & \mu & (1-\delta-\mu) \\ \eta & (1-\eta-\varepsilon) & \varepsilon \end{pmatrix}$

- Assuming that the initial state is economic expansion, the mixture matrix is

$$\mathbb{M} = \begin{pmatrix} \boldsymbol{M}_1 & \boldsymbol{M}_2 & \boldsymbol{M}_3 \\ \boldsymbol{M}_4 & \boldsymbol{M}_5 & \boldsymbol{M}_6 \\ \boldsymbol{M}_7 & \boldsymbol{M}_8 & \boldsymbol{M}_9 \end{pmatrix} = \begin{pmatrix} \alpha\boldsymbol{Q}_E & \beta\boldsymbol{Q}_N & (1-\alpha-\beta)\boldsymbol{Q}_C \\ \delta\boldsymbol{Q}_E & \mu\boldsymbol{Q}_N & (1-\delta-\mu)\boldsymbol{Q}_C \\ \eta\boldsymbol{Q}_E & (1-\eta-\varepsilon)\boldsymbol{Q}_N & \varepsilon\boldsymbol{Q}_C \end{pmatrix}$$

- Let $\mathbb{X}$, $\mathbb{Y}$, $\mathbb{Z}$, $\mathbb{F}$, and $\mathbb{L}'$ be defined as

  $\mathbb{X} \equiv \boldsymbol{M}_1 + \boldsymbol{M}_2 + \boldsymbol{M}_3 = \alpha\boldsymbol{Q}_E + \beta\boldsymbol{Q}_N + (1-\alpha-\beta)\boldsymbol{Q}_C$

  $\mathbb{Y} \equiv \boldsymbol{M}_4 + \boldsymbol{M}_5 + \boldsymbol{M}_6 = \delta\boldsymbol{Q}_E + \mu\boldsymbol{Q}_N + (1-\delta-\mu)\boldsymbol{Q}_C$

  $\mathbb{Z} \equiv \boldsymbol{M}_7 + \boldsymbol{M}_8 + \boldsymbol{M}_9 = \eta\boldsymbol{Q}_E + (1-\eta-\varepsilon)\boldsymbol{Q}_N + \varepsilon\boldsymbol{Q}_C$

  $\mathbb{F} \equiv (\boldsymbol{M}_1\ \boldsymbol{M}_2\ \boldsymbol{M}_3)$ and $\mathbb{L}' = (\mathbb{X}\ \mathbb{Y}\ \mathbb{Z})'$

- The Markov-switching credit migration matrix over different time horizons is

  1-year horizon: $\mathbb{Q}_E(1) \equiv \alpha\boldsymbol{Q}_E + \beta\boldsymbol{Q}_N + (1-\alpha-\beta)\boldsymbol{Q}_C = \mathbb{X}$

  2-year horizon: $\mathbb{Q}_E(2) \equiv \boldsymbol{M}_1\mathbb{X} + \boldsymbol{M}_2\mathbb{Y} + \boldsymbol{M}_3\mathbb{Z} = \mathbb{F}\mathbb{L}'$

  3-year horizon: $\mathbb{Q}_E(3) \equiv \mathbb{F}\mathbb{M}\mathbb{L}'$

  4-year horizon: $\mathbb{Q}_E(4) \equiv \mathbb{F}\mathbb{M}^2\mathbb{L}'$

  $\vdots$

  n-year horizon: $\mathbb{Q}_E(n) \equiv \mathbb{F}\mathbb{M}^{n-2}\mathbb{L}'$

  If the initial state is "normal" then $\mathbb{F} \equiv (\boldsymbol{M}_4\ \boldsymbol{M}_5\ \boldsymbol{M}_6)$ whereas for initial economic contraction $\mathbb{F} \equiv (\boldsymbol{M}_7\ \boldsymbol{M}_8\ \boldsymbol{M}_9)$.

C Sectoral MMC Migration Risk During Contraction

	AAA	AA	A	BBB	BB	B	CCC	D	NR
Industrials									
AAA	68.252	13.577	6.576	0.939	0.611	0.103	0.008	0.253	9.682
AA	0.683	64.800	15.044	3.513	0.645	0.198	0.015	0.520	14.581
A	0.048	1.183	**65.677**	10.093	2.771	0.709	0.041	1.718	17.759
BBB	0.011	0.558	4.429	66.024	9.331	1.977	0.125	2.232	15.312
BB	0.009	0.181	0.816	4.638	61.760	8.825	0.530	4.046	19.192
B	0.009	0.030	0.232	0.963	3.689	59.774	3.561	10.091	21.652
CCC	0.004	0.011	0.080	1.008	1.724	4.764	24.978	56.869	11.316
D	0	0	0	0	0	0	0	100	0
NR	0.058	0.022	0.298	0.637	0.851	0.918	0.097	3.040	94.078
Utilities									
AAA	**64.985**	6.120	5.801	3.068	0.144	0.027	0.002	0.197	19.655
AA	0.213	**59.639**	15.799	4.024	0.696	0.053	0.004	0.292	19.281
A	0.016	2.888	70.400	11.524	0.734	0.118	0.009	0.719	13.592
BBB	0.022	0.528	7.658	72.683	3.385	0.577	0.066	0.617	14.284
BB	0.045	0.110	2.022	10.859	59.339	4.690	0.468	5.068	17.398
B	0.003	0.043	0.604	1.750	4.133	59.519	5.777	12.754	15.418
CCC	0.001	0.008	0.573	0.269	0.364	6.424	30.077	52.485	9.799
D	0	0	0	0	0	0	0	100	0
NR	0.006	0.035	0.586	1.007	0.133	0.104	0.009	1.242	96.878
Financials									
AAA	74.818	12.738	3.019	0.396	0.054	0.010	0.002	0.149	8.814
AA	1.192	72.759	11.638	2.181	0.159	0.024	0.014	0.985	11.047
A	0.049	3.215	73.056	6.807	0.625	0.092	0.016	0.509	15.631
BBB	0.018	1.410	5.207	**65.253**	6.465	0.856	0.200	1.733	18.857
BB	0.008	0.394	2.114	6.102	**46.146**	8.773	2.145	8.027	26.292
B	0.007	0.307	2.556	3.922	5.510	**47.284**	7.094	13.374	19.947
CCC	0.005	0.097	0.077	0.110	0.980	2.526	**24.224**	53.375	15.852
D	0	0	0	0	0	0	0	100	0
NR	0.049	0.844	0.569	0.289	0.041	0.110	0.016	0.728	97.356

This table reports the probability that an industrial, utility or financial firm rated at time t, as indicated in the first column, is rated at time $t + \Delta t$ where $\Delta t = 1$ year, as indicated in the first row. NR denotes Not Rated status. D indicates default. The MMC estimator assumes that the economy is in contraction at time t and evolves stochastically over $(t, t + \Delta t)$. The counterpart expansion matrices are qualitatively similar across the three sectors.

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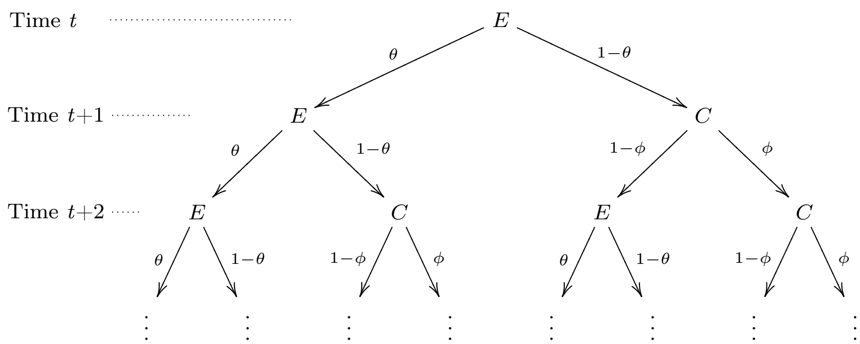
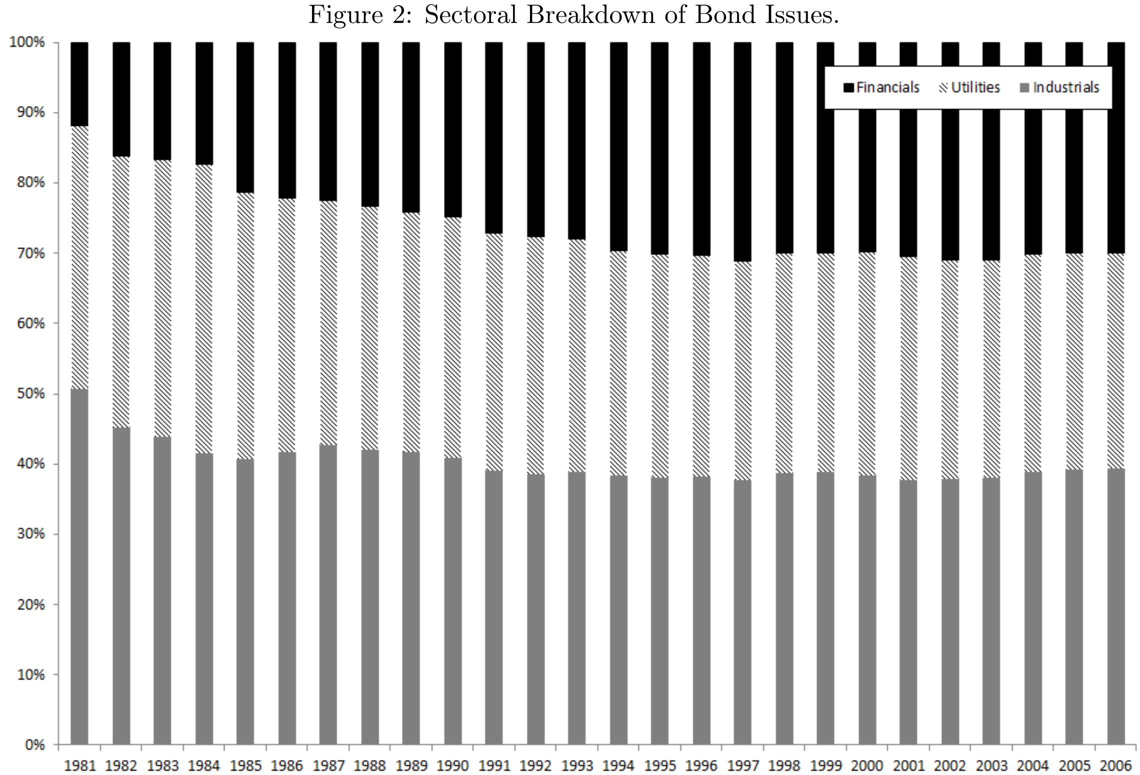
Figure 1: Business-Cycle Dynamics.



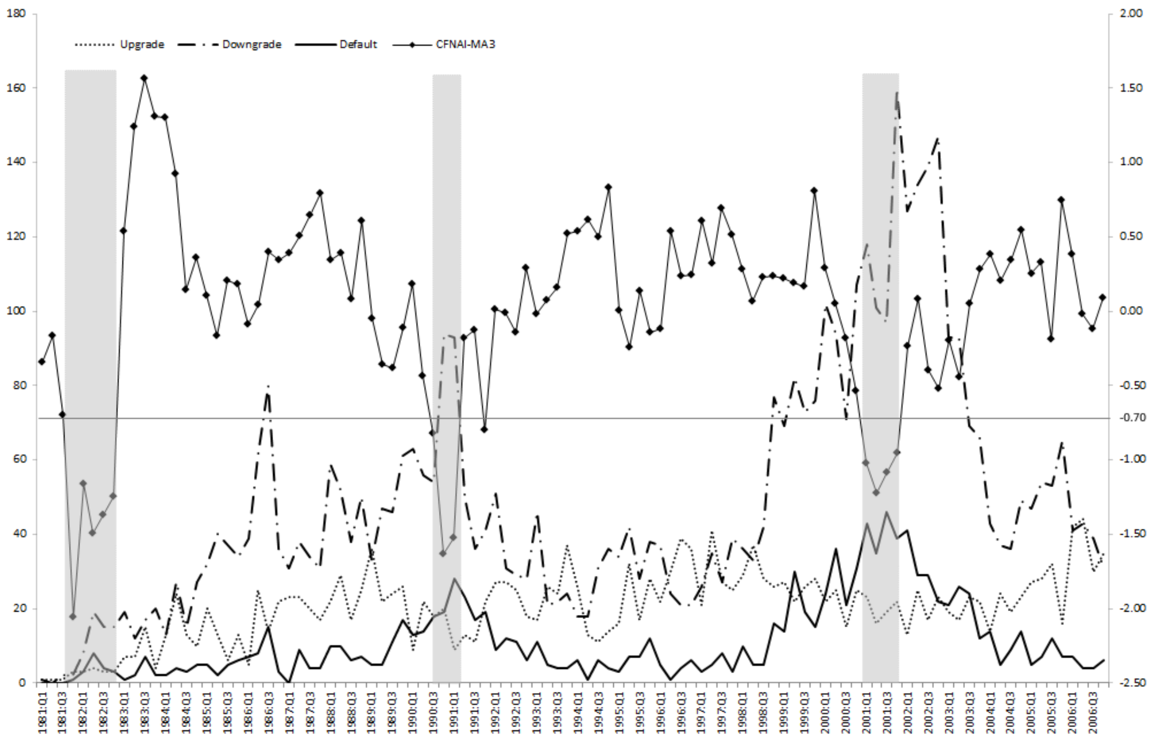
Notes: This figure characterizes for current expansion at time t the subsequent unfolding of the economy, or business-cycle dynamics, as a binomial tree. E denotes expansion and C denotes contraction. The parameter $\theta(\phi)$ denotes the probability that the next phase at $t+j$ is an expansion (contraction) conditional on the time $t+j-1$ phase being an expansion (contraction).

Figure 2: Sectoral Breakdown of Bond Issues.



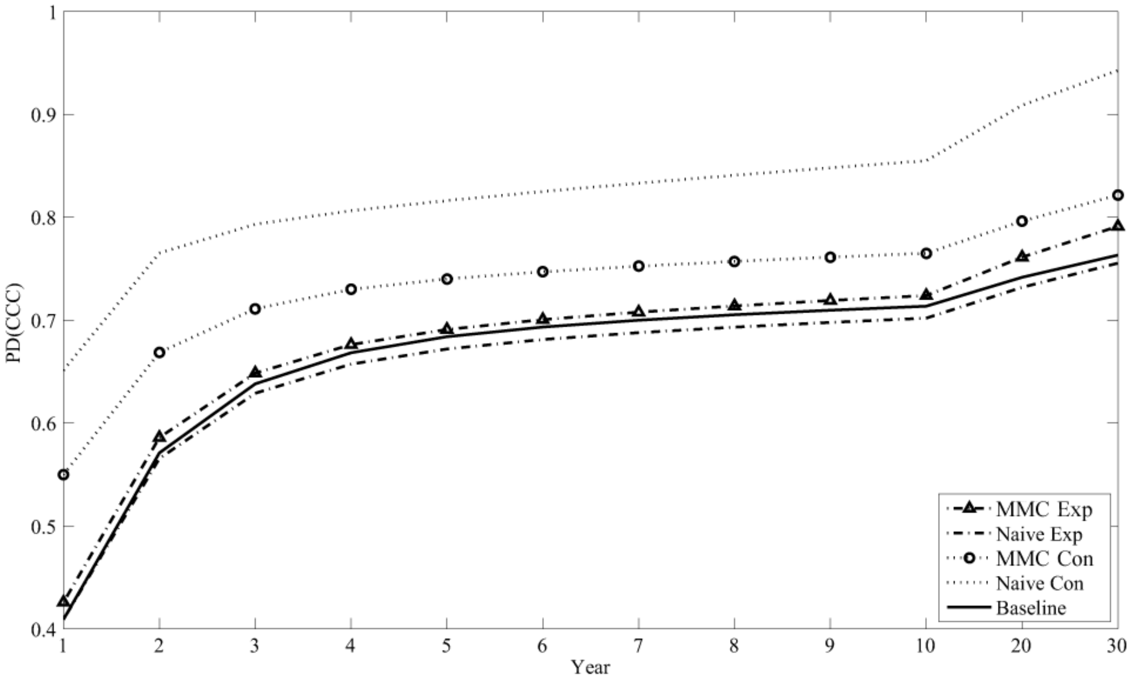
Notes: This figure plots the relative proportion of industrial, utility and financial bond issues rated by Standard & Poor's on each year from 01/01/1981 to 31/12/2006.

Figure 3: Upgrades, Downgrades, Defaults and Overall Economic Activity.



Notes: This figure shows the quarterly number of S&P's corporate bond rating upgrades, downgrades and defaults alongside the real-time history of the Chicago Fed National Activity Index as a 3-month moving average (CFNAI-MA3). Shaded areas indicate official NBER-delineated contraction episodes.

Figure 4: Term Structure of Default Probabilities for CCC bonds.



Notes: This figure shows the default probability for CCC bonds over a time horizon from 1 to 30 years. The five lines plotted correspond, as indicated in the legend, to the classical hazard-rate estimator that ignores business cyclicality (Baseline), the naïve estimator conditional on current contraction (Naive Con) and expansion (Naive Exp), and the MMC estimator conditional on current contraction (MMC Con) and expansion (MMC Exp).

Table 1: Summary Statistics of US Corporate Ratings.

						Expansion				***Contraction***			
State i	N_i	$N_{i\to j, i\neq j}$	D_i	$N_{i,u}\%$	$N_{i,d}\%$	N_i^E	D_i^E	$N_{i,u}^E\%$	$N_{i,d}^E\%$	N_i^C	D_i^C	$N_{i,u}^C\%$	$N_{i,d}^C\%$
AAA	240	185	6,792.55	0	100.00	224	6,356.00	0	100.00	16	435.69	0	100.00
AA	1,480	825	23,592.24	6.42	93.58	1,346	21,924.52	6.80	93.20	134	1,664.96	3.28	96.72
A	3,777	1,749	52,324.63	20.12	79.88	3,409	48,794.15	20.91	79.09	368	3,523.14	11.11	88.89
BBB	4,469	2,067	49,441.55	39.89	60.11	4,076	46,035.30	41.53	58.47	393	3,397.91	21.70	78.30
BB	4,105	2,313	35,443.58	36.36	63.64	3,859	33,106.71	37.15	62.85	246	2,331.50	27.78	72.22
B	5,418	3,004	41,775.67	36.34	63.66	5,144	38,863.57	38.90	61.10	274	2,965.03	14.44	85.56
CCC	1,405	970	4,055.99	19.50	80.50	1,245	3,688.46	21.75	78.25	160	358.89	5.45	94.55
D	1,166	0	45,943.63	–	–	987	43,147.33	–	–	179	2,787.09	–	–
NR	4,202	802	143,291.98	–	–	3,941	133,925.84	–	–	261	9,287.68	–	–
Total	26,262	11,915	402,661.81	29.58	70.42	24,231	375,841.88	31.12	68.88	2,031	26,751.89	15.01	84.99

Notes: This table presents summary statistics for the sample of S&P's rating events from 01/01/1981 to 31/12/2006 employed in our subsequent analysis. N_i denotes the total assignments to state i that represents either a rating, default or NR. $N_{i\to j, i\neq j}$ is the total number of migrations from state i to state j. D_i is the sum of quarters spent in rating i across all obligors. $N_{i,u}$ and $N_{i,d}$ are, respectively, the proportion of upgrades and downgrades relative to total migrations from rating i. E and C denote, respectively, expansion and contraction phases according to NBER chronology. Full details are given in Section 4.1 of the paper.

Table 2: One-Year Rating Transition Risk.

	AAA	AA	A	BBB	BB	B	CCC	D	NR
				Panel I: *Discrete Cohort estimator*					
Transition probabilities									
AAA	89.861	5.910	0.348	0.058	0.174	0.000	0.000	0.000	3.650
AA	0.551	86.748	7.604	0.635	0.050	0.167	0.033	0.017	4.195
A	0.053	1.542	87.686	5.551	0.481	0.173	0.038	0.075	4.400
BBB	0.008	0.150	3.616	85.164	4.177	0.671	0.118	0.237	5.859
BB	0.033	0.066	0.231	5.504	77.456	6.987	0.703	0.989	8.842
B	0.000	0.056	0.139	0.250	4.971	75.693	3.951	4.674	10.266
CCC	0.000	0.000	0.380	0.285	1.139	8.444	48.767	27.799	13.188
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	0.019	0.061	0.147	0.270	0.275	0.318	0.011	0.591	98.308
				Panel II: *Continuous Hazard-Rate estimator*					
Transition intensities									
AAA	-0.109	0.061	0.005	0.001	0.001	0.000	0.000	0.000	0.042
AA	0.006	-0.140	0.085	0.005	0.001	0.001	0.000	0.000	0.042
A	0.001	0.017	-0.134	0.064	0.004	0.002	0.000	0.000	0.048
BBB	0.000	0.002	0.040	-0.168	0.057	0.005	0.001	0.001	0.064
BB	0.000	0.001	0.003	0.057	-0.262	0.099	0.006	0.002	0.093
B	0.000	0.001	0.002	0.004	0.057	-0.288	0.086	0.023	0.117
CCC	0.000	0.000	0.005	0.007	0.011	0.131	-0.960	0.634	0.172
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
NR	0.000	0.001	0.002	0.003	0.004	0.005	0.000	0.007	-0.023
Transition probabilities									
AAA	89.669	5.376	0.651	0.093	0.110	0.017	0.001	0.015	4.067
AA	0.559	86.994	7.416	0.695	0.082	0.080	0.013	0.020	4.131
A	0.067	1.467	87.630	5.525	0.466	0.162	0.008	0.022	4.654
BBB	0.017	0.180	3.429	84.815	4.601	0.630	0.063	0.099	6.166
BB	0.030	0.078	0.365	4.650	77.335	7.617	0.573	0.520	8.831
B	0.002	0.054	0.174	0.446	4.369	75.565	4.725	3.874	10.789
CCC	0.002	0.011	0.327	0.477	0.871	7.226	38.563	40.980	11.541
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	0.024	0.078	0.199	0.318	0.366	0.438	0.024	0.710	97.842

Notes: This table presents the rating transition probabilities in percentage points obtained from the discrete-time cohort estimator (Panel I) and continuous hazard-rate estimator (Panel II, second exhibit). The first exhibit of Panel II reports the transition intensity or generator matrix that represents the instantaneous rate of transition from one rating i to another rating $j (i \neq j)$ expressed as number of rating transitions per quarter. Full details of the computations are given in Sections 2.2 and 2.3 of the paper.

Table 3: Term Structure of Default Probabilities.

	1 Year	2 Year	3 Year	4 Year	5 Year
			Panel I: *Baseline Hazard-Rate estimator*		
AAA	0.015	0.060	0.136	0.242	0.379
AA	0.020	0.079	0.174	0.304	0.470
A	0.022	0.091	0.211	0.383	0.606
BBB	0.099	0.300	0.604	1.006	1.494
BB	0.520	1.513	2.825	4.311	5.863
B	3.874	8.811	13.441	17.396	20.678
CCC	40.980	57.096	63.804	66.829	68.386
			Panel II: *Naïve Expansion estimator*		
AAA	0.017	0.069	0.155	0.276	0.432
AA	0.023	0.091	0.200	0.351	0.542
A	0.034	0.123	0.272	0.480	0.747
BBB	0.132	0.385	0.755	1.229	1.793
BB	0.666	1.828	3.300	4.922	6.586
B	4.314	9.545	14.301	18.287	21.547
CCC	40.903	56.554	62.886	65.719	67.196
			Panel III: *Naïve Contraction estimator*		
AAA	0.626	2.292	4.709	7.635	10.875
AA	1.506	3.898	6.883	10.224	13.741
A	1.938	4.736	8.044	11.611	15.268
BBB	2.617	6.191	10.138	14.139	18.044
BB	8.700	15.982	21.805	26.539	30.540
B	18.015	30.343	37.819	42.685	46.243
CCC	65.135	76.518	79.317	80.630	81.618
			Panel IV: *MMC Expansion estimator*		
AAA	0.046	0.196	0.438	0.758	1.150
AA	0.108	0.358	0.710	1.143	1.651
A	0.145	0.463	0.906	1.450	2.081
BBB	0.282	0.843	1.598	2.494	3.495
BB	1.185	3.128	5.356	7.639	9.859
B	5.217	11.559	17.126	21.646	25.255
CCC	42.581	58.587	64.844	67.613	69.089
			Panel V: *MMC Contraction estimator*		
AAA	0.297	0.713	1.133	1.585	2.089
AA	0.775	1.387	1.942	2.518	3.143
A	1.013	1.750	2.408	3.090	3.827
BBB	1.404	2.437	3.381	4.362	5.406
BB	5.018	7.668	9.844	11.898	13.856
B	11.770	18.389	23.129	26.821	29.764
CCC	54.988	66.863	71.105	72.984	74.010

Notes: This table reports estimates of the probability that a firm currently rated as indicated in the first column (from AAA to CCC) enters default over a time horizon $(t, t + \Delta_t)$ from 1 to 5 years. Panel I corresponds to the hazard-rate estimator. Panels II and IV correspond, respectively, to the naïve and Mixture of Markov Chains (MMC) estimator conditional on current expansion. Panels III and V correspond, respectively, to the naïve and MMC estimator conditional on current contraction. Full details of the computations are given in Section 2.3 and 3 of the paper. All figures are in percentage points.

Table 4: One-Year Corporate Default Risk Estimates from Existing Studies.

Panel A: *Classical estimators: time-homogeneity*

Estimator	Nickell et al. (2000)	Bangia et al. (2002)	Lando & Skodeberg (2002)	Jafry & Schuermann (2004)	Hanson & Schuermann (2006)	Christensen et al. (2004)	Ours
Cohort	6.90; 20.6	5.45; 23.69	5.00; 20.48	14.61; 30.92	6.52; 28.54	—	4.67; 27.80
Hazard rate	—	—	4.40; 37.60	13.56; 44.02	4.70; 42.49	4.82; 43.54	3.87; 40.98
Sample period	12/70-12/97	01/81-12/98	01/88-12/98	01/81-12/02	01/81-12/02	01/87-12/91	01/91-12/06
Data source	Moody's	S&P	S&P	S&P	S&P	Moody's	S&P

Panel B: *Estimators controlling for economic cyclicality*

	Nickell et al. (2000)	Bangia et al. (2002)	Present study: Naïve	Present study: MMC
Expansion	4.5; 22.4	3.90; 27.16	4.31; 40.90	5.22; 42.58
Contraction	9.4; 23.5	8.16; 42.58	18.02; 65.14	11.77; 54.99
Sample period	12/70-12/97	01/81-12/98	01/91-12/06	
Data source	Moody's	S&P	S&P	

Notes: This table summarizes existing default risk estimates for B and CCC corporate bonds over a 1-year horizon. The two default probabilities shown in each case are for B-rated bonds and CCC-rated bonds, $(P_{B,D}; P_{CCC,D})$, in percentage points. The reported figures in Panel B from Nickell et al. (2000) and Bangia et al. (2002) are for the naïve cyclical approach that deploys the cohort estimator separately on 'expansion' and 'contraction' ratings. Nickell et al. (2000) categorize the business cycle phases as 'peak', 'normal' and 'trough'; the reported figures are for the former and the latter phases, respectively. Bangia et al. (2002) report quarterly transition probabilities that we have annualized.

Table 5: Rating Transition Matrices Conditional on the Economic State.

	AAA	AA	A	BBB	BB	B	CCC	D	NR
				Panel I: *Naïve Expansion estimator*					
AAA	89.030	5.683	0.704	0.102	0.117	0.019	0.001	0.017	4.327
AA	0.595	86.196	7.827	0.767	0.076	0.095	0.014	0.023	4.407
A	0.071	1.555	86.912	5.786	0.517	0.188	0.009	0.034	4.927
BBB	0.019	0.193	3.618	83.970	4.695	0.683	0.077	0.132	6.614
BB	0.032	0.083	0.384	4.894	76.165	7.828	0.589	0.666	9.359
B	0.003	0.050	0.202	0.467	4.606	74.220	4.673	4.314	11.465
CCC	0.002	0.013	0.354	0.516	0.949	7.650	37.210	40.903	12.403
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	0.026	0.083	0.207	0.338	0.386	0.463	0.028	0.751	97.719
				Panel II: *Naïve Contraction estimator*					
AAA	56.194	14.169	6.214	2.091	0.301	0.106	0.012	0.626	20.286
AA	0.785	49.686	18.436	4.841	0.995	0.222	0.027	1.506	23.503
A	0.155	3.242	56.628	12.161	1.950	0.621	0.081	1.938	23.224
BBB	0.019	1.048	6.874	58.507	6.993	1.365	0.113	2.617	22.463
BB	0.009	0.228	1.970	6.205	46.317	6.926	0.865	8.700	28.780
B	0.007	0.137	0.643	1.803	3.136	44.912	4.201	18.015	27.147
CCC	0.004	0.036	0.110	0.557	0.811	2.275	15.696	65.135	15.376
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	0.034	0.260	0.540	1.014	0.677	0.560	0.068	4.299	92.549
				Panel III: *MMC Expansion estimator*					
AAA	86.809	6.340	1.071	0.226	0.124	0.023	0.002	0.046	5.359
AA	0.611	83.679	8.647	1.037	0.135	0.101	0.015	0.108	5.668
A	0.077	1.684	84.858	6.259	0.613	0.217	0.014	0.145	6.133
BBB	0.018	0.254	3.854	82.264	4.872	0.730	0.078	0.282	7.649
BB	0.030	0.092	0.497	4.995	74.129	7.782	0.614	1.185	10.676
B	0.003	0.055	0.232	0.563	4.504	72.223	4.661	5.217	12.541
CCC	0.002	0.014	0.337	0.523	0.943	7.276	35.661	42.581	12.661
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	0.026	0.095	0.228	0.383	0.405	0.469	0.030	0.973	97.390
				Panel IV: *MMC Contraction estimator*					
AAA	70.961	10.697	3.716	1.156	0.203	0.067	0.007	0.297	12.896
AA	0.711	65.902	14.057	2.997	0.577	0.163	0.019	0.775	14.798
A	0.120	2.548	70.218	9.425	1.313	0.435	0.048	1.013	14.880
BBB	0.019	0.677	5.499	70.022	6.051	1.082	0.095	1.404	15.152
BB	0.019	0.157	1.285	5.652	59.671	7.397	0.743	5.018	20.058
B	0.005	0.096	0.448	1.221	3.795	58.059	4.461	11.770	20.144
CCC	0.003	0.023	0.193	0.531	0.872	4.410	24.946	54.988	14.032
D	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000	0.000
NR	0.030	0.181	0.387	0.705	0.549	0.524	0.049	2.627	94.948

Notes: This table presents the rating transition probabilities obtained using the naïve business-cycle estimator when the current economic phase is expansion (Panel I) or contraction (Panel II), and the MMC business-cycle estimator when the current economic phase is expansion (Panel III) or contraction (Panel IV). Full details of the computations are given in Section 3. All figures are in percentage points.

Table 6: Default Rates for CCC Bonds per Sector.

	1 Year	2 Year	3 Year	4 Year	5 Year
			Baseline Hazard-Rate estimator		
Industrial	42.257	57.759	63.789	66.413	67.769
Utility	39.403	55.910	63.158	66.593	68.406
Financial	37.699	53.852	61.111	64.612	66.468
Overall	40.913	57.096	63.804	66.829	68.386
			MMC Expansion estimator		
Industrial	43.513	58.817	64.515	67.008	68.382
Utility	40.974	57.582	64.544	67.752	69.444
Financial	41.020	56.982	63.405	66.264	67.722
Overall	42.581	58.587	64.844	67.613	69.089
			MMC Contraction estimator		
Industrial	56.869	68.708	72.678	74.380	75.322
Utility	52.485	66.363	71.708	74.154	75.456
Financial	55.375	66.695	70.738	72.480	73.355
Overall	54.988	66.863	71.105	72.984	74.010

Notes: This table provides estimates of the probability that a firm currently rated CCC enters default over a time horizon $(t, t + \Delta_t)$ from 1 to 5 years. The top panel corresponds to the hazard-rate estimator that ignores business cycles. The mid and bottom panels pertain, respectively, to the MMC estimator conditional on current expansion and contraction. The labels in the first column indicate that the estimates are based on the rating histories of firms either in the industrial sector, utility sector, financial sector or the entire sample.

Table 7: Empirical Distribution of One-Year Default Risk Estimates.

	Cohort estimator				*Naïve Expansion estimator*				*Naïve Contraction estimator*			
	Mean	St. Dev.	(Lower, Upper)	Length	Mean	St. Dev.	(Lower, Upper)	Length	Mean	St. Dev.	(Lower, Upper)	Length
AAA	0	0	(0.000, 0.000)	0.000	0.017	0.002	(0.013, 0.022)	0.008	0.633	0.103	(0.447, 0.847)	0.400
AA	0.016	0.018	(0.000, 0.068)	0.068	0.023	0.004	(0.017, 0.034)	0.017	1.529	0.351	(0.917, 2.259)	1.342
A	0.075	0.023	(0.032, 0.128)	0.096	0.034	0.008	(0.024, 0.051)	0.028	1.920	0.300	(1.374, 2.559)	1.184
BBB	0.239	0.046	(0.174, 0.335)	0.161	0.132	0.023	(0.090, 0.177)	0.087	2.629	0.371	(1.956, 3.398)	1.443
BB	0.983	0.102	(0.824, 1.203)	0.379	0.647	0.062	(0.531, 0.770)	0.239	8.764	0.851	(7.139, 10.467)	3.328
B	4.692	0.201	(4.216, 5.122)	0.906	4.019	0.177	(3.686, 4.375)	0.689	18.047	1.025	(16.198, 20.052)	3.854
CCC	28.054	1.615	(25.066, 31.477)	6.411	41.002	1.606	(37.974, 44.340)	6.366	65.215	3.306	(58.978, 71.613)	12.635
	Hazard-Rate estimator				*MMC Expansion estimator*				*MMC Contraction estimator*			
AAA	0.015	0.002	(0.011, 0.019)	0.007	0.046	0.006	(0.040, 0.054)	0.014	0.300	0.048	(0.212, 0.400)	0.188
AA	0.020	0.004	(0.014, 0.030)	0.015	0.110	0.024	(0.081, 0.143)	0.062	0.788	0.196	(0.446, 1.196)	0.750
A	0.022	0.001	(0.019, 0.025)	0.006	0.144	0.021	(0.120, 0.175)	0.055	1.002	0.165	(0.707, 1.349)	0.641
BBB	0.098	0.018	(0.065, 0.138)	0.072	0.283	0.032	(0.225, 0.350)	0.125	1.410	0.204	(1.039, 1.843)	0.804
BB	0.519	0.052	(0.425, 0.631)	0.206	1.191	0.088	(1.016, 1.350)	0.334	5.055	0.487	(4.127, 6.024)	1.896
B	3.865	0.160	(3.559, 4.171)	0.612	5.225	0.203	(4.839, 5.615)	0.776	11.790	0.611	(10.667, 12.994)	2.327
CCC	41.015	1.562	(38.086, 44.366)	6.281	42.682	1.589	(39.733, 46.105)	6.372	55.091	2.367	(50.613, 59.402)	8.789

	Hazard-Rate vs Naïve Expansion				*Hazard-Rate vs Naïve Contraction*				*Hazard-Rate vs MMC Expansion*			
	Mean	St. Dev.	(Lower, Upper)	H0: $\Delta = 0$	Mean	St. Dev.	(Lower, Upper)	H0: $\Delta = 0$	Mean	St. Dev.	(Lower, Upper)	H0: $\Delta = 0$
AAA	-0.002	0.001	(-0.004, 0.000)	Not Reject	-0.618	0.102	(-0.817, -0.418)	Reject	-0.031	0.005	(-0.041, -0.022)	Reject
AA	-0.003	0.002	(-0.006, 0.000)	Not Reject	-1.509	0.352	(-2.198, -0.820)	Reject	-0.089	0.024	(-0.135, -0.043)	Reject
A	-0.012	0.007	(-0.027, 0.003)	Not Reject	-1.898	0.300	(-2.485, -1.310)	Reject	-0.122	0.021	(-0.163, -0.081)	Reject
BBB	-0.033	0.020	(-0.072, 0.006)	Not Reject	-2.531	0.370	(-3.256, -1.805)	Reject	-0.184	0.027	(-0.237, -0.132)	Reject
BB	-0.128	0.046	(-0.237, 0.002)	Not Reject	-8.245	0.841	(-9.893, -6.598)	Reject	-0.672	0.062	(-0.794, -0.550)	Reject
B	-0.154	0.081	(-0.312, 0.008)	Not Reject	-14.182	0.970	(-16.084, -12.281)	Reject	-1.360	0.079	(-1.514, -1.205)	Reject
CCC	0.014	0.347	(-0.667, 0.695)	Not Reject	-24.199	2.927	(-29.936, -18.463)	Reject	-1.667	0.273	(-2.202, -1.132)	Reject
	Hazard-Rate vs MMC Contraction				*Expansion: Naïve vs MMC*				*Contraction: Naïve vs MMC*			
AAA	-0.285	0.047	(-0.378, -0.192)	Reject	-0.029	0.005	(-0.039, -0.020)	Reject	0.333	0.054	(0.226, 0.439)	Reject
AA	-0.768	0.196	(-1.152, -0.384)	Reject	-0.086	0.024	(-0.132, -0.040)	Reject	0.741	0.156	(0.435, 1.047)	Reject
A	-0.980	0.165	(-1.304, -0.657)	Reject	-0.110	0.020	(-0.149, -0.071)	Reject	0.918	0.135	(0.653, 1.182)	Reject
BBB	-1.312	0.203	(-1.709, -0.915)	Reject	-0.151	0.024	(-0.199, -0.104)	Reject	1.219	0.168	(0.889, 1.548)	Reject
BB	-4.536	0.475	(-5.467, -3.605)	Reject	-0.524	0.056	(-0.634, -0.414)	Reject	3.709	0.366	(2.992, 4.427)	Reject
B	-7.925	0.548	(-8.999, -6.851)	Reject	-0.906	0.064	(-1.032, -0.780)	Reject	6.257	0.424	(5.426, 7.088)	Reject
CCC	-14.075	1.762	(-17.530, -10.621)	Reject	-1.681	0.226	(-2.124, -1.237)	Reject	10.124	1.174	(7.824, 12.424)	Reject

Notes: This table reports the mean, standard deviation, and 95% confidence interval (upper limit, lower limits and length) of the 1-year default risk estimates (top panels) and their differential (bottom panels) obtained over $M = 1,000$ bootstrap replications. $H_0 : \Delta = 0$ denotes the null hypothesis that the difference in PDs obtained from estimators i and j ($\Delta \equiv \widehat{PD}^i - \widehat{PD}^j$) is negligible. All numbers are in percentage points.

Table 8: Out-of-Sample Forecast Errors.

Estimator	Benchmark	$MAE_{\mathbb{L}^1}$	$MSE_{\mathbb{L}^2}$	MME	$MSE_{\mathbb{L}^2}^{asy}$	SVD	MAE_{1-p}	MSE_{1-p}
					1-year horizon			
Naïve cyclical	Hazard rate	-8.50	-55.98	1.54	2.96	-1.71	-3.37	-9.41
MMC cyclical	Hazard rate	3.63	8.77	5.48	8.59	12.34	0.74	1.56
	Naïve cyclical	11.17	41.51	4.00	5.80	13.82	3.97	10.03
					2-year horizon			
Naïve cyclical	Hazard rate	-40.75	-106.72	-3.64	-3.75	-64.65	-2.75	-20.91
MMC cyclical	Hazard rate	2.60	2.53	3.55	8.13	1.11	0.63	0.93
	Naïve cyclical	30.80	52.85	6.93	11.45	39.94	3.29	18.06
					3-year horizon			
Naïve cyclical	Hazard rate	-51.01	-144.53	-8.09	-11.56	-84.20	-2.19	-24.66
MMC cyclical	Hazard rate	1.55	0.59	3.85	8.92	0.90	0.58	0.80
	Naïve cyclical	34.81	59.35	11.05	18.36	46.20	2.72	20.43

Notes: This table shows the percentage reduction in the average out-of-sample forecast error of each estimator (column 1) vis-à-vis a benchmark (col. 2); positive numbers denote a decrease in average forecast error. $MAE_{\mathbb{L}^1}$, MME, $MSE_{\mathbb{L}^2}^{asy}$ and SVD, as formalized in (10), (11), (12) and (13), respectively, are forecast error metrics that require 'true' rating migration risk. $MSE_{\mathbb{L}^2}$ is the squared error version of $MAE_{\mathbb{L}^1}$. The 1-year, 2-year and 3-year migration risk matrices obtained by applying the continuous time hazard-rate estimator over each out-of-sample year (biennium or triennium) are taken as 'true' migration risk. MAE_{1-p} and MSE_{1-p} are Frydmann and Schuermann (2008) evaluation metrics that are based on the forecasted probabilities of each actual transaction over the out-of-sample period and do not require a 'true' migration risk proxy. Naïve cyclical and MMC cyclical are the two estimators that account for the current economic conditions. Hazard rate is the continuous through-the-cycle estimator. $MSE_{\mathbb{L}^2}^{asy}$ is based on weights $(\frac{4}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10})$. The out-of-sample period are the last 8 sample years.

Table 9: Economic Capital Attribution for One-Year Risk Horizon.

	Cohort	*Hazard*	*Naïve(Exp)*	*Naïve(Con)*	*MMC(Exp)*	*MMC(Con)*
99.0% level	\$5,467,943	\$6,644,600	\$6,600,314	\$11,589,362	\$7,038,285	\$9,469,067
99.9% level	\$6,918,574	\$8,250,077	\$8,193,990	\$13,774,556	\$8,693,018	\$11,416,857
		$\frac{Cohort}{Hazard}$	$\frac{Naïve(Exp)}{Hazard}$	$\frac{Naïve(Con)}{Hazard}$	$\frac{MMC(Exp)}{Hazard}$	$\frac{MMC(Con)}{Hazard}$
99.0% level	–	82.29%	99.33%	174.42%	105.92%	142.51%
99.9% level	–	83.86%	99.32%	166.96%	105.37%	138.38%
			$\frac{MMC(Exp)}{Naïve(Exp)}$	$\frac{MMC(Con)}{Naïve(Con)}$	$\frac{Naïve(Con)}{Naïve(Exp)}$	$\frac{MMC(Con)}{MMC(Exp)}$
99.0% level	–	–	106.64%	81.70%	175.59%	134.54%
99.9% level	–	–	106.09%	82.88%	168.11%	131.33%

Notes: This table shows the capital requirements implied by the rating migration risk measures obtained from the two classical through-the-cycle estimators (cohort, hazard-rate) and the two cyclical estimators (naïve and MMC) which are inputs to the CreditRisk+ portfolio model of Credit Suisse First Boston (CSFB, 1997). The hypothetical credit portfolio is made up of 100 bonds with random exposure at default (EAD) ranging from \$1 to \$1m, and random initial rating. Cohort and Hazard refers to the two classical through-the-cycle estimators. *Exp* denotes current economic expansion and *Con* denotes current economic contraction. Rows 1 and 2 report the implied capital allocation levels in US\$. Rows 3 to 6 report relative capital allocation levels.