Sheet (6)

Differentials

Solve the following equations.

I. *Separating the variables*

1 $x(y-3)\dfrac{dy}{dx} = 4y$

2 $(1+x^3)\dfrac{dy}{dx} = x^2y$, given that $x = 1$ when $y = 2$.

3 $x^3 + (y+1)^2\dfrac{dy}{dx} = 0$

4 $\cos y + (1+e^{-x})\sin y\dfrac{dy}{dx} = 0$, given that $y = \pi/4$ when $x = ($

5 $x^2(y+1) + y^2(x-1)\dfrac{dy}{dx} = 0$

II. *Homogeneous equations*

6 $(2y-x)\dfrac{dy}{dx} = 2x + y$, given that $y = 3$ when $x = 2$.

7 $(xy+y^2) + (x^2-xy)\dfrac{dy}{dx} = 0$

8 $(x^3+y^3) = 3xy^2\dfrac{dy}{dx}$

9 $y - 3x + (4y+3x)\dfrac{dy}{dx} = 0$

10 $(x^3+3xy^2)\dfrac{dy}{dx} = y^3 + 3x^2y$

III. *Integrating factor*

11 $x\dfrac{dy}{dx} - y = x^3 + 3x^2 - 2x$

12 $\dfrac{dy}{dx} + y\tan x = \sin x$

13 $x\dfrac{dy}{dx} - y = x^3\cos x$, given that $y = 0$ when $x = \pi$.

14 $(1+x^2)\dfrac{dy}{dx} + 3xy = 5x$, given that $y = 2$ when $x = 1$.

15 $\dfrac{dy}{dx} + y\cot x = 5e^{\cos x}$, given that $y = -4$ when $x = \pi/2$.

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IV. *Transformations*. Make the given substitutions and work in much the same way as for first-order homogeneous equations.

16 $(3x+3y-4)\dfrac{dy}{dx} = -(x+y)$ Put $x+y=v$

17 $(y-xy^2) = (x+x^2y)\dfrac{dy}{dx}$ Put $y=\dfrac{v}{x}$

18 $(x-y-1)+(4y+x-1)\dfrac{dy}{dx}=0$ Put $v=x-1$

19 $(3y-7x+7)+(7y-3x+3)\dfrac{dy}{dx}=0$ Put $v=x-1$

20 $y(xy+1)+x(1+xy+x^2y^2)\dfrac{dy}{dx}=0$ Put $y=\dfrac{v}{x}$

V. *Bernoulli's equation*

21 $\dfrac{dy}{dx}+y=xy^3$

22 $\dfrac{dy}{dx}+y=y^4e^x$

23 $2\dfrac{dy}{dx}+y=y^3(x-1)$

24 $\dfrac{dy}{dx}-2y\tan x=y^2\tan^2 x$

25 $\dfrac{dy}{dx}+y\tan x=y^3\sec^4 x$

VI. *Miscellaneous*. Choose the appropriate method in each case.

26 $(1-x^2)\dfrac{dy}{dx}=1+xy$

27 $xy\dfrac{dy}{dx}-(1+x)\sqrt{y^2-1}=0$

28 $(x^2-2xy+5y^2)=(x^2+2xy+y^2)\dfrac{dy}{dx}$

29 $\dfrac{dy}{dx}-y\cot x=y^2\sec^2 x$, given $y=-1$ when $x=\pi/4$.

30 $y+(x^2-4x)\dfrac{dy}{dx}=0$

VII. *Further examples*

31 Solve the equation $\dfrac{dy}{dx}-y\tan x=\cos x-2x\sin x$, given that $y=0$ when $x=\pi/6$.

32 Find the general solution of the equation

$$\frac{dy}{dx}=\frac{2xy+y^2}{x^2+2xy}.$$