N.I. Lobachevsky State University
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Probability theory and mathematical statistics:

Classical probability — Practice

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Solution

$$\mathrm{P}(A) = 1/10$$

There are 5 red balls, 5 blue balls and 10 green balls in a box. You pick one ball. What's the probability it's either blue or green (event A)?

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Solution Enumerate all balls, so that red balls are 1–5, blue balls are 6–10, green balls are 11–20.

$$\Omega = \{1, 2, \ldots, 20\}, \quad A = \{6, 7, \ldots, 20\}, \quad \mathrm{P}(A) = 15/20 = 3/4$$

An integer between 1 and 1 000 000 is picked at random. What's the probability the number is divisible by 17?

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Solution There are $\left[\frac{1\,000\,000}{17}\right] = 58823$ integers dividable by 17 among $1, 2, \ldots, 1\,000\,000$. $\mathrm{P}(A) = 58823/1\,000\,000 = 0{,}058823$.

A student has learned by rote 20 questions out of 30 for an exam. What's the probability for him to take favorable question if he's the first in a waiting line?

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Solution

$$20/30 = 2/3$$

If two dice are thrown, what's the probability that the throw will be greater than 8?

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Solution $N(A) = 10$, $N(\Omega) = 36$. $\mathrm{P}(A) = 10/36$

Three men get into an elevator of a 10-floored building at the ground floor. What is the probability that the first man quits on 4th floor, the second man quits on 8 floor and the third man quits on 9th floor (event A)? What is the probability for them to quit on the same floor (event B)? On different floors (event C)?

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Solution

$$\Omega = \{(x_1, x_2, x_3) : x_1, x_2, x_3 = 1, 2, \ldots, 9\}, \quad N(\Omega) = 9^3 = \bar{A}_9^3$$

$$A = \{(4, 8, 9)\}, \quad N(A) = 1, \quad \mathrm{P}(A) = 1/729$$

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Solution

$$\Omega = \{(x_1, x_2, x_3) : x_1, x_2, x_3 = 1, 2, \ldots, 9\}, \quad N(\Omega) = 9^3 = \bar{A}_9^3$$

$$B = \{(x_1, x_1, x_1) : x_1 = 1, 2, \ldots, 9\},$$

$$N(B) = 9 \cdot 1 \cdot 1 \quad \mathrm{P}(B) = \frac{9 \cdot 1 \cdot 1}{9^3}$$

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Solution

$$\Omega = \{(x_1, x_2, x_3) : x_1, x_2, x_3 = 1, 2, \ldots, 9\}, \quad N(\Omega) = 9^3 = \bar{A}_9^3$$

$$C = \{(x_1, x_2, x_3) : x_1 \neq x_2, x_1 \neq x_3, x_2 \neq x_3\},$$

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Solution $N(\Omega) = l^k$, $N(A) = l$, $\mathrm{P}(A) = 1/l^{k-1}$.

One black rook and one white rook are placed on a chess board. What's the probability they don't attack each other?

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Solution $N(\Omega) = 64 \cdot 63$, $N(A) = 64 \cdot 49$, $\mathrm{P}(A) = 49/63$.

k users choose in turn between l mirrors to download a piece of Open Source Software ($k \leqslant l$). Each mirror supports only one connection. What's the probability that mirrors $1, 2, \ldots, k$ will be chosen?

k users choose in turn between l mirrors to download a piece of Open Source Software ($k \leqslant l$). Each mirror supports only one connection. What's the probability that mirrors $1, 2, \ldots, k$ will be chosen?

Solution Let x_i be the mirror number chosen by i-th user,
$\Omega = \{(x_1, x_2, \ldots, x_k) : x_i = 1, 2, \ldots, l,\ x_i \neq x_i \text{ if } i \neq j\}$,
$N(\Omega) = l(l-1)\cdots(l-k+1)$,
$A = \{(x_1, x_2, \ldots, x_k) : x_i = 1, 2, \ldots, k,\ x_i \neq x_i \text{ if } i \neq j\}$, $N(A) = k!$.

$$\mathrm{P}(A) = \frac{k!}{l(l-1)\cdots(l-k+1)} = \frac{1}{C_l^k}$$

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Solution

Assume the student knows questions 1–20, let x_1 be the question number taken by the first student in the waitnig line, x_2 be the question number taken by the second student in the waiting line, x_3 be the question number taken by our student. Then $\omega = (x_1, x_2, x_3)$, $N(\Omega) = 30 \cdot 29 \cdot 28$, $A = \{(x_1, x_2, x_3) : x_2 = 1, 2, \ldots, 20\}$, $N(A) = 20 \cdot 29 \cdot 28$, $\mathrm{P}(A) = 2/3$.

Four letters are placed in four envelopes. What's the probability each letter is placed into proper envelope?

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Solution Enumerate envelopes. Denote by x_1 the number of envelope which has the first letter, x_2 the number of envelope which has the second letter, x_3 the number of envelope which has the third letter, x_4 the number of envelope which has the fourth letter.
$\Omega = \{(x_1, x_2, x_3, x_4), x_i = 1, 2, 3, 4\}$, $N(\Omega) = 4! = 24$,
$A = \{(1, 2, 3, 4)\}$, $\mathrm{P}(A) = 1/24$.

There are a red balls and b blue balls in an urn. Two balls are taken out simultaneously. What's the probabiltiy to have two balls of the same color (event A)? What's the probability to have balls of different colors (event B)?

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Solution Enumerate red balls from 1 to a, blue balls from $a+1$ to $a+b$. Elementary outcome is a combination $\omega = \{x_1, x_2\}$, $N(\Omega) = C_{a+b}^2$, $N(A) = C_a^2 + C_b^2$, $N(B) = ab$.

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Solution

$$\mathrm{P}(A) = \frac{5 \cdot C_{45}^{6}}{C_{50}^{7}} \approx 0{,}40772$$