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Probability theory and mathematical statistics:

Conditional probability — Practice

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Two dice are rolled. What's the conditional probability that both dice are 6's if it's known that the sum of points is divisible by 5?

An urn has 5 red balls and 8 blue balls. Two balls are taken out, Let event A occur when the first ball is red and B when the second ball is red. Compute $P(B|A)$.

Let $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $A = \{\omega_1, \omega_3\}$, $B = \{\omega_2, \omega_3\}$, $p(\omega_1) = 0,2$, $p(\omega_2) = 0,2$, $p(\omega_3) = 0,6$. Calculate $P(A|B)$.

(NB: *Discrete probability space was introduced in Lecture 4.*)

One card is taken from a pile of 52 cards. Let event A occur when an ace is taken out, event B when a card of spades is taken out. Compute $P(A|B)$ and $P(B|A)$.

Three random numbers are chosen without replacement from the set $\{1, 2, \dots, N\}$. What's the conditional probability that the third number is between the first and the second one if it is known that the first number is smaller than the second one.

There is a set of 100 cards with numbers $00, 01, \dots, 99$. A card is taken at random from the set. Let h_1 and h_2 denote the sum and the product of the digits of a chosen card. Compute $P(h_1 = i | h_2 = 0)$, $i = 0, 2, \dots, 18$.

A random point $\omega = (x, y)$ is chosen in a square

$$G = \{(x, y) : 0 \leq x, y \leq 1\}.$$

Let

$$A_1 = \left\{x \leq \frac{1}{2}\right\} \subset G, \quad A_2 = \left\{y \leq \frac{1}{2}\right\} \subset G,$$

$$A_3 = \left\{\left(x - \frac{1}{2}\right)\left(y - \frac{1}{2}\right) < 0\right\} \subset G$$

Compute $P(A_1|A_2)$, $P(A_3|A_1)$, $P(A_3|A_1 \cap A_2)$.

One die and one coin are tossed together. Let $A = \{\text{Head appeared}\}$, $B = \{\text{Even number of points appeared}\}$. Are A and B dependent?

Out of 100 students, 50 speak English, 40 speak French, 35 speak German; 20 speak English and French, 8 speak English and German, 10 speak French and German; 5 speak all three languages. One student is taken out at random. Consider three events: the student speaks English, the student speaks French, the student speaks German. Find all pairs of independent events. Are all three events independent?