1. Matrices and Gaussian Elimination (矩陣與高斯消去法)

這章.我們主要是介紹矩陣及解線性聯立方程組.

1.1 Vectors and linear combinations (向量與線性組合)

Notation 1: $\mathbb{R}$ = the collection of all real numbers

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \left\{ \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} : a_1, a_2 \in \mathbb{R} \right\}$$

$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} : a_1, a_2, a_3 \in \mathbb{R} \right\}$$

$$\mathbb{R}^n = \left\{ \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} : a_1, a_2, \cdots, a_n \in \mathbb{R} \right\}$$: n-dimensional Euclidean space. (n維度歐氏空間)

$$a = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$: n-dimensional vector (n維度向量)

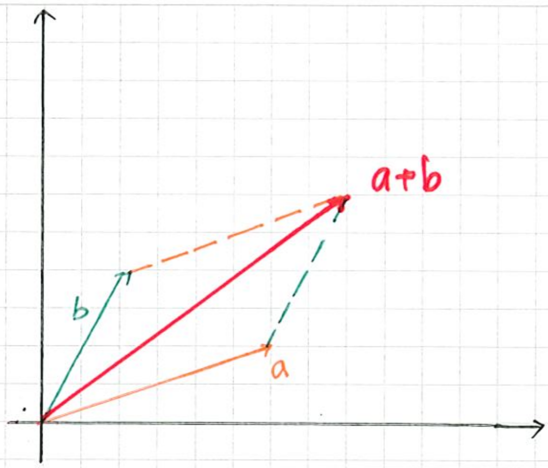
a_i : i th component (第i個分量) for $i = 1, 2, \cdots, n$.

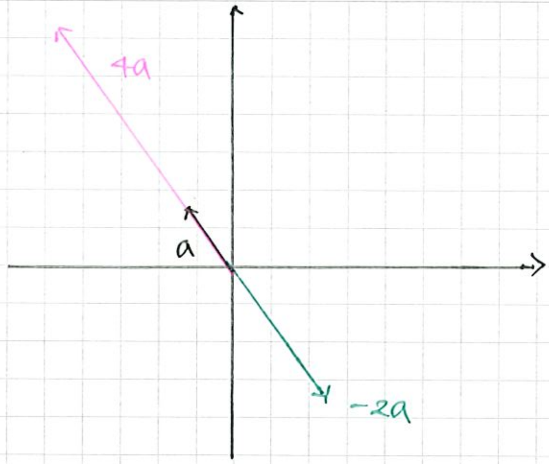
Notation 2: $a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \in \mathbb{R}^n$, $c \in \mathbb{R}$.

vector addition (向量加法): $a + b = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{bmatrix}$

scalar multiplication (常數乘法): $c \cdot a = \begin{bmatrix} c a_1 \\ c a_2 \\ \vdots \\ c a_n \end{bmatrix}$

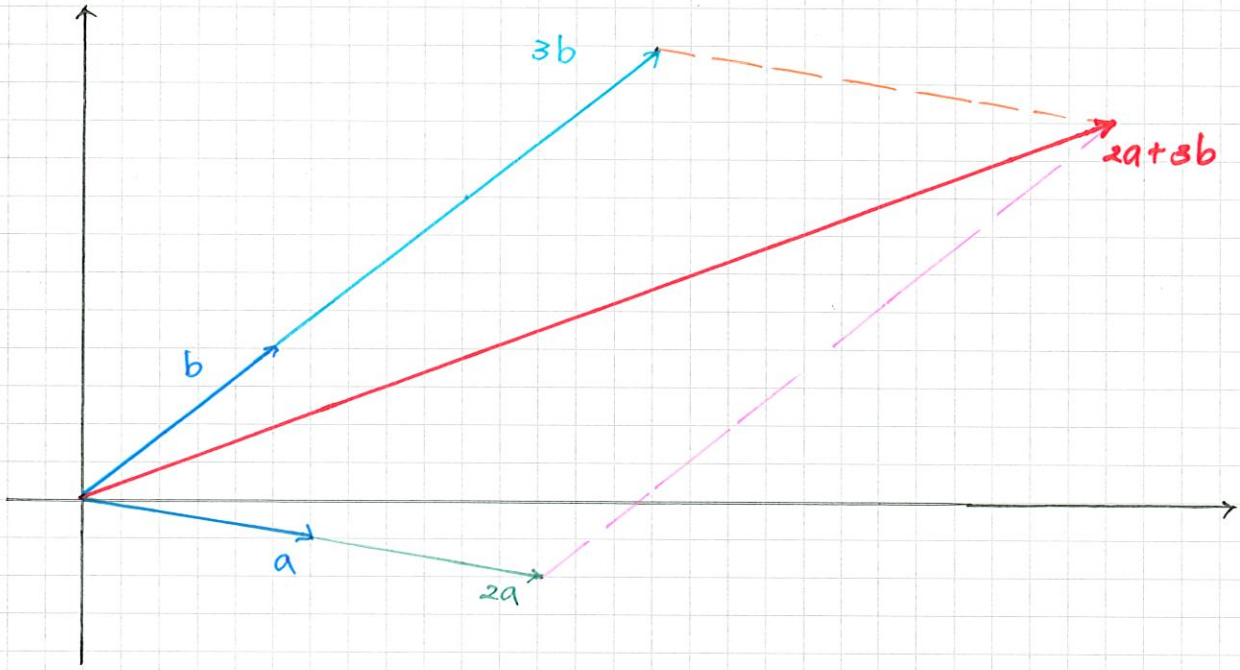
Remark 3: 用幾何圖形來看:





Definition 4: For $c_1, c_2 \in \mathbb{R}$, $c_1 a + c_2 b$ is a linear combination (線性組合) of a and b.

用圖形來看.



Example 5:

$$a = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \qquad b = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$a + b = \begin{pmatrix} 1+2 \\ 2+(-1) \\ 4+0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$$

$$a - b = \begin{pmatrix} 1-2 \\ 2-(-1) \\ 4-0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}.$$

$$3a+2b = \begin{pmatrix} 3 \\ 6 \\ 12 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 12 \end{pmatrix}.$$

$$4a-3b = \begin{pmatrix} 4 \\ 8 \\ 16 \end{pmatrix} - \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 11 \\ 16 \end{pmatrix}$$