1.3 Matrix notation and matrix multiplication.

1.3.1 Matrix notation.

這節我們要定義矩陣的符號.

Definition 21: (1) An $m \times n$ matrix is a rectangle array of the form

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} = (a_{ij})_{\substack{1 \le i \le m \\ 1 \le j \le n}}.$$

The entries $a_{i1}, a_{i2}, \cdots, a_{in}$ 為 the ith row (第 i 列) of the matrix

The entries $a_{1j}, a_{2j}, \cdots, a_{mj}$ 為 the jth column (第 j 行) of the matrix.

也可以將 row 與 column 寫成向量形式.

(2) Two matrices A and B are equal if

$$a_{ij} = b_{ij} \quad \forall\ 1 \le i \le m,\ 1 \le j \le n.$$

(3) 若矩陣的 row 的個數 = column 的個數, i.e., $m = n$,

矩陣稱為 square matrix (方陣)

Notation 22: 所有 $m \times n$ 矩陣, entries 均為實數, 所形成的集合, 記為 $M_{m \times n}(\mathbb{R})$, i.e.,

$$M_{m \times n}(\mathbb{R}) = \{ A = (a_{ij})_{1 \le i \le m,\ 1 \le j \le n} : a_{ij} \in \mathbb{R}\ \forall\ i, j \}$$