4.3 Applications
4.3.1 Powers and products: At and AB.
Lemma 29: >1, >2 >k: eigenvalues of A. Then
(1) λ_1^m , λ_2^m ,, λ_k^m : eigenvalues of A^m . (2) every eigenvector of A is also an eigenvector of A^m .
proof: Let v : eigenvector of A corresponding to the eigenvalue >
$A^{m}v = A^{m+1}(Av) = A^{m-1}(\lambda v) = \lambda A^{m-1}v = \lambda A^{m-2}(Av)$
$= \lambda A^{m-2}(\lambda v) = \lambda^2 A^{m-2}v = \dots = \lambda^m v$
1 mma 30: A: invertible => all of its eigenvalues = 0.
Moreover, the eigenvalue of $A^7 = \frac{1}{2}$
proof: A: invertible \implies det (A) + 0
⇒ o cannot be an eigenvalue of A.
Furthermore, $A v = \lambda v \implies v = A^{-}(\lambda v) = \lambda A^{-}v$
$\Rightarrow A^{-1} \circ = \frac{1}{2} \circ$
(⇒ A, A B) eigenvectors #[[B])
Lemma 31: A: diagonalizable $\Rightarrow A^m, A^{-1}$: diagonalizable.
proof: 這意力 Am in case.
A: diagonalizable \implies = eigenvector matrix Q and diagonal matrix
$A = QDQ^{-1}$

$\Rightarrow A^{m} = (QQQ^{-1})(QQQ^{-1}) \dots (QQQ^{-1})$ $= QQ (Q^{-1}Q) Q (Q^{-1}Q) Q \dots QQ^{-1}$ $= QQ Q^{m}Q^{-1}$		Date: / /
= Q D ^m O ⁻¹ and D ^m is a diagonal matrix. A ^m : diagonalizable. Remark 32: A, B have the same eigenvector O corresponding to the eigenvalues > and µ, respectively. i.e., Av = > 0. Bv = µv. Then C1) A + B has an eigenvalue > + M. 公1 AB has an eigenvalue > µ. 注意: 居 A, B " 為 有" 和 图 D eigenvector, 這 順 最 果 會 是 證 Example 33: Consider B - (1 2 2 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	$\Rightarrow A^{m} = (QDQ^{-1})(QDQ^{-1}) (QDQ^{-1})$	
and D ^m is a diagonal matrix. A ^m , diagonalizable. Remark 32: A, B have the same eigenvector U corresponding to the eigenvalues A and U, respectively. i.e., AU = AU. BU = UU. Then (1) A + B has an eigenvalue A + U. (2) AB has an eigenvalue A U. 章 是: 居 A, B "为 有" 初 园 D eigenvector, 這 個 張 果 會 是 顧 Example 33: Consider A: eigenvalue 5. Es = span { (1) } Cigenvalue -1. E ₁ = span { (1) } eigenvalue -1. E ₄ = span { (2) } eigenvalue -1. E ₄ = span { (1) } eigenvalue -1. E ₄ = span { (1) } eigenvalue -1. E ₄ = span { (1) } eigenvalue -1. E ₄ = span { (1) } eigenvalue -1. E ₄ = span { (2) } eigenvalue -1. E ₄ = spa	= QD(Q'Q)D(Q'Q)DDQ' = Q:	DD D 07
and D^m is a diagonal matrix. $\Rightarrow A^m$: diagonalizable. Remark 32: A. B. have the same eigenvector U corresponding to the eigenvalues A and A , respectively, i.e., $AU = AU$. $BU = AUU$. Then (1) A+B has an eigenvalue A + A . (2) AB has an eigenvalue A + A . $\Rightarrow A^m$: $A = A = A = A = A = A = A = A = A = A $		
Remark 32: A. B. have the same eigenvector V corresponding to the eigenvalues V and V respectively, i.e., V and V respectively, i.e., V and V respectively, i.e., V and V are V and V and V and V are V and V are V are V and V are V are V and V are V and V are V are V and V are V are V and V are V and V are V are V and V are V are V and V are V and V are V are V and V are V are V and V are V and V are V are V and V are V are V and V are V and V are V are V and V are V are V and V are V and V are V are V and V are V are V and V are V and V are V are V are V and V are V are V are V are V are V are V and V are V are V and V are V are V and V are V are V are V are V are V are V and V are V are V are V and V are V are V are V and V are V are V and V are V are V and V are V are V are V are V and V are V are V are V are V and V are V are V and V are V are V are V are V are V are V and V are V and V are V a		
Remark 32: A. B. have the same eigenvector U corresponding to the eigenvalues A and A , respectively. i.e., $AU = AU$. $BU = AU$. Then (1) $A + B$ has an eigenvalue $A + A$. (2) AB has an eigenvalue AA . $AB = \begin{pmatrix} A & B & B & B & B & B & B & B & B & B &$		
eigenvalues λ and μ , respectively, i.e., Au = λu . Bu = μu . Then (1) A + B has an eigenvalue $\lambda + \mu$. (2) AB has an eigenvalue $\lambda \mu$. 2 \mathbb{Z} \mathbb{Z} : \mathbb{Z} A, B " \mathbb{Z}	The alagonalizable.	
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(1) A+B has an eigenvalue $3+M$. (2) AB has an eigenvalue $2M$. 主意: E A, B "为有"和自 E eigenvector, 这 但 E 集會 E 的 E and E and E eigenvalue E eigenvalue E and E eigenvalue E eig	$A v = \lambda v$. $B v = \mu v$.	
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$\frac{3}{2}$	<u> </u>	
Example 33: Consider $A = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ $A : eigenvalue 5. E5 = span { \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}} $ $eigenvalue -1. E-1 = span { \begin{pmatrix} 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}} $ $eigenvalue -1. E-1 = span { \begin{pmatrix} 1 & 2 \\ 3 & 1 & 1 & 1 \end{pmatrix}} $ $eigenvalue -1. E-1 = span { \begin{pmatrix} 1 & 2 \\ 3 & 1 & 1 & 1 \end{pmatrix}} $ $A + B = \begin{pmatrix} 3 & 5 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1$		回話果會是儲
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