Climate Change Mitigation: How Effective is Green Quantitative Easing?*

Raphael Abiry† Marien Ferdinandusse‡ Alexander Ludwig§
Carolin Nerlich¶

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Abstract

We develop a two sector integrated assessment model with incomplete markets to analyze the effectiveness of green quantitative easing in complementing fiscal policies for climate change mitigation. We model green quantitative easing through a given outstanding stock of bonds held by a monetary authority and its portfolio allocation between a clean (green) and a dirty (brown) sector of production. Our key research question is whether the monetary authority can effectively contribute to a reduction of global damages caused by carbon emissions. Our findings show that green quantitative easing does not lead to a perfect crowding out of capital and thus has real effects in the long-run. Since it only indirectly affects the allocation of production to dirty and clean technologies and since its overall economic size is relatively small, green quantitative easing is, however, a less effective climate change mitigating policy instrument than are carbon taxes. Our analysis also suggests that green quantitative easing might be a quantitatively important complement to fiscal policies if governments only insufficiently coordinate on implementing green fiscal policies.

Keywords: Climate Change; Integrated Assessment Model; 2-Sector Model; Green Quantitative Easing; Carbon Taxation

J.E.L. Classification Codes: E51; E62; Q54

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†Goethe University Frankfurt; raphael.abiry@gmail.com
‡European Central Bank; marien.ferdinandusse@ecb.europa.eu
§Goethe University Frankfurt & CEPR; mail@alexander-ludwig.com
¶European Central Bank; Carolin.Nerlich@ecb.europa.eu

1 Introduction

Climate change evoked by mankind will be one of the greatest global challenges in the next decades. Since pre-industrial times, global temperature has increased by more than 1.0 degrees of Celsius as a result of carbon and other greenhouse gas emissions. If this trend were to continue, extreme weather events would not only become more frequent - resulting in large macroeconomic costs - but one would also observe irreversible global environmental damages. To effectively reverse this trend, ambitious policy measures need to be adopted as the window of opportunity to act is closing rapidly. While there is broad consensus on the usefulness of carbon pricing as a policy tool to combat climate change, even though different views might exist about its optimal size, recently a vivid debate emerged on whether and how central banks should play a role in mitigating climate change.

This paper examines whether central banks can help to mitigate climate change through green quantitative easing. Our key research question is if a shift in a portfolio of bonds held by the central bank towards the clean sector of the economy can reduce climate change as measured by the global temperature increase and ensuing damage it causes, in comparison to, or in combination with, fiscal mitigation policies. In our setup, green quantitative easing refers to the portfolio allocation of a given outstanding stock of bonds held by the central bank, which is tilted towards bonds issued by a clean (green) sector of production over those issued by a dirty (carbon-emitting) sector of production.

Our main findings are that a portfolio reallocation by the central bank towards the clean sector of the economy can contribute to a reduction of climate change as measured by the global temperature increase. However, the maximum mitigation effects of such a policy are considerable smaller than those of a carbon tax, and while green quantitative easing complements a carbon tax, its marginal effects are smaller when combined with a carbon tax than in isolation. We therefore conclude that green quantitative easing may be an effective complementary policy instrument if governments fail to introduce a sizeable carbon tax or other measures of carbon pricing.

To address our research question, we develop a two-sector incomplete markets quantitative integrated assessment model where aggregate output is produced employing clean and dirty sectoral intermediate goods. Markets are incomplete for two reasons. First, there is income risk for households with specifics described next and, second, there exists a climate change externality leading to a reduction of total output.The model is calibrated to the world, which implies that both the fiscal and monetary authorities coordinate on the introduction of a global carbon tax, respectively green quantitative easing.

Intermediate goods in the economy in turn are produced using capital and labor, and either clean or dirty energy as inputs. Energy production itself takes place using a simple technology employing some exogenously growing technology level and labor as the only inputs. Dirty energy production leads to an accumulation of carbon in the atmosphere, which causes an increase of the global temperature leading to a damage to aggregate output, a standard production externality frequently employed in the climate change literature.

Households in the economy live until infinity and maximize their expected discounted life-time utility over consumption streams. Every household in the economy runs two intermediate goods firms in the two sectors by employing its own household capital and by hiring labor and energy on the respective labor and energy market. Since the return processes on capital in the two firms is stochastic, households are heterogenous whereby this heterogeneity is induced by different histories of return realizations. This return risk is idiosyncratic, thus there is no aggregate risk in the economy. Importantly, the shocks on the returns of the two capital stocks are independently distributed, both across sectors and over time. Households not only hire labor on the market for production, but also exogenously supply their own labor on the market and from this labor supply they earn a deterministic wage income. Given these income processes households solve a consumption savings problem and choose to allocate their savings between the two capital stocks as well as a risk-free bond which is assumed to be in zero net supply across households.

In this setup we first conduct a baseline experiment by simulating the transition of the laissez-faire competitive economy over the next decades—from 2020 to 2100—and compute the resulting temperature increase. We find that without policy intervention the global temperature would increase by about one degree of Celsius from 2020 to 2100, corresponding in our model to an about 2.7 degrees global temperature increase relative to pre-industrial levels.

We next consider three policy experiments. First, we model a fiscal authority, which sets a tax on dirty energy production commonly referred to as a carbon tax. Since there is no aggregate uncertainty, an equivalent policy in our model could be the introduction of carbon pricing through an emission trading scheme. We follow the existing literature and set an initial carbon tax in year 2020 at a level of 50 US dollars per gigaton of carbon emissions, which in our model corresponds to an ad valorem carbon tax of 6.6 percent. We hold this tax rate constant along the transition and find that with this tax rate in place the global temperature increase is reduced by 0.1 degrees of Celsius.

Second, we next consider a green quantitative easing experiment, which is modeled as a stylized scenario where we assume that a central bank representing the world pursue a portfolio reallocation of its capital—which is calibrated to 4 percent of the world capital stock and initially split equally across the two intermediate goods sectors—to the clean intermediate input. We thus consider a stylized strong green quantitative easing policy. Such a reallocation of the central bank portfolio towards the clean sector increases the capital stock employed for production in that sector relative to the capital stock in the dirty sector. This triggers a relative increase of labour demand in the clean sector and a relative expansion of output. In addition, within the dirty sector, the increased capital costs lead to greater use of labour in the production. Through these two mechanisms the central bank can thus influence relative production across the two sectors in the economy.

Thirdly, we combine both policies to see their effects in combination and determine whether they are substitutes or complements. We find that the reduction of the increase of global temperature through the introduction of a carbon tax of (initially) 50 US dollars is about 3-times larger than the maximum reduction that could be achieved through a plausible, but generously, calibrated green quantitative easing policy. Thus green quantitative easing is a much less effective policy instrument to mitigate the adverse effects of climate change than a carbon tax.

While green quantitative easing complements fiscal policy, i.e. green quantitative easing on top of a carbon tax will induce an additional reduction of the increase of global temperature, its marginal effects are lower than in isolation. The reason is that while the main effect of green quantitative easing, the reallocation of resources towards the green sector, still applies in combination with a carbon tax, the effect of increased capital costs on the allocation of resources within the dirty sector becomes less effective. This is because the increase in dirty capital costs through green quantitative easing and the increase in energy costs through the carbon tax partly negate each other. We therefore conclude that green quantitative easing may be an effective complementary policy instrument, if governments around the world fail to coordinate on introducing a sizeable carbon tax or equivalent carbon pricing through other policies.

Importantly, in our model a reallocation of capital by the monetary authority will only lead to a partial crowding out of private capital in the clean sector, i.e., only a partial reallocation of private capital from clean to dirty intermediate goods production. The reason for such a partial crowding out is the zero correlation assumption of returns so that out of optimal portfolio choice allocation perspectives it will not be optimal for households to fully reallocate their capital. While not explicitly modelled, this zero correlation assumption can be motivated by three (complementary) perspectives. First, our model starts from a situation in which the central bank in the past built up a portfolio of bonds, e.g. by engaging in quantitative easing, which is more consistent with the notion that bond purchases by the central bank can influence financial markets and less with a full reallocation of financial portfolios by private economic agents. Second, the decision of the central bank to engage in green quantitative easing can be thought of as a response to societal preferences that would also be expressed in the behaviour of private bondholders and argue against full reallocation. Finally, if the ongoing efforts to develop an informative green taxonomy will succeed, investors will be able to distinguish (climate-related) risks between the two categories of financial instruments, which speaks against a full reallocation of private capital.

In conclusion, we find that a carbon tax induced reduction of the increase of global temperature is about significantly larger than the reduction induced by green quantitative easing. While a portfolio reallocation by the monetary authority towards the clean sector of the economy can contribute to a reduction in climate change damages, this is a much less effective policy instrument than carbon taxation. However, green quantitative easing can usefully complement a carbon tax, in particular if governments only insufficiently coordinate on implementing green fiscal policies.

1.1 Relation to Existing Literature

The closest papers to our agenda are Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2019), on the one hand, and Ferrari and Landi (2020) and Benmir and Roman (2020) on the other hand. Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2019) study how public debt can be used to improve welfare of carbon taxation. They do so by embedding a quantitative overlapping generations model a la Auerbach and Kotlikoff in an integrated assessment model (IAM). Carbon taxation alone increases welfare of current young and future generations through mitigation climate change and thus lower damages that come with it. As these benefits quantitatively only materialize in the long-run these future benefits come at the costs of current market distortions. This leads to current older generations being harmed by carbon taxation as they do not live long enough to experience the benefits of climate change mitigation. Using debt financed transfers to the current old allows to transfer part of the future benefits to these generations, with young and future generations paying off the debt in the future. This generates a win-win situation where both young and old generations gain from carbon taxation.

Turning to quantitative easing (QE), Ferrari and Landi (2020) and Benmir and Roman (2020) study the impact of green QE on climate change mitigation. Both papers study climate policies along the business cycle by combining a climate model with a New Keynesian DSGE model with the financial accelerator framework of Gertler and Karadi (2011) and two production sectors, a green and brown one, where the latter emits green house gases. Through QE central banks acquire capital of both sectors. Green QE is understood as a tilting of the portfolio held by the central bank towards the green sector. Ferrari and Landi (2020) point out that absent additional frictions a balance-sheet neutral green QE has no effect on output nor on emissions. That is, keeping the central bank's balance sheet constant and only shifting the portfolio towards green assets has no effect on emissions. This is due to the no-arbitrage condition between green and brown assets. As green and brown assets are perfect substitutes, a rebalancing of the central bank's portfolio from brown to green assets is thus perfectly offset by a rebalancing of the portfolio of the private agents in the opposite direction. By introducing costly portfolio rebalancing for private agents Ferrari and Landi are able to break the perfect substitutability between green and brown assets. However, they find limited effects of green QE on climate change. The reason is that their perspective is on the business cycle horizon whereas climate change unfolds at longer horizons.

Besides considering the use of green QE to permanently lower emissions, it can also be used along business cylce swings. Ferrari and Landi (2020) find that an aggressive expansion of green QE (i.e. selling brown and buying green) during expansions is welfare improving. Related, Benmir, Jaccard, and Vermandel (2020) find that optimal carbon taxes should be pro-cyclical.

Our research also links to research in asset pricing and climate change. Most related is the work of Hambel, Kraft, and van der Ploeg (2020). They combine a climate model with two production sectors, green and brown, where the latter emits green house gases. Both sectors are subject to climate disaster shocks with an increasing likelihood the further climate change progresses. Private agents can invest in both sectors. The authors show that there is eventually a trade off between asset diversification and climate change mitigation. At the beginning when the green production sector is still small, both forces push up the share of the green sector and down the share of the brown sector. First, a declining brown sector decreases emissions thus lowering the costs associated with climate change. Second, increasing the portfolio share of green assets held diversifies the asset portfolio and thus improves the risk-return structure from the perspective of risk-averse agents. But the second force also slows the transition to a fully green economy, as increasing the portfolio share of green assets further beyond an equal portfolio between green and brown assets eventually decreases asset diversification.

With respect to the question whether green and brown assets feature a positive or negative spread the literature is not univocal. Hambel, Kraft, and van der Ploeg (2020) show that within their model green assets feature higher risk premia than brown assets. Considering policy risk through unexpected increases in carbon pricing and regulation or ethical investment decisions it can also be motivated that brown / non-green assets should feature higher risk premia. And indeed the recent empirical literature finds lower risk premia for green assets, although these results are not unambiguous. In the following are examples for (i) stock returns, (ii) bond yields and (iii) loan spreads. Bolton and Kacperczyk (2020a) and Bolton and Kacperczyk (2020b) analyze the US and worldwide stock markets respectively and find a positive carbon premium, that is stock returns of carbon-intensive firms have higher risk premia (thus green stocks have lower risk premia) and that this carbon premium has been rising over the recent years. Kapraun and Scheins (2019) investigate a large dataset of government and corporate bonds. In the primary market they find that green bonds have lower yields than non-green bonds. However, in the secondary market this reverses and they find green bonds featuring higher yields. When separating between government and corporate bonds in the secondary market, they find small lower yields for green government bonds but higher yields for green corporate bonds, compared to their non-green alternatives. The authors find that this spread of green corporate bonds reduces when the green label is credible. Degryse, Goncharenko, Theunisz, and Vadazs (2020) investigate an international sample of syndicated loans and find that green firms borrow at significantly lower spreads.

The paper is organized as follows. Section 2 presents the model and Section 3 discusses the calibration. Section 4 presents our main results and Section 5 concludes the paper. Detailed derivations are contained in the appendix.

2 A Two-Sector Integrated Assessment Model

We develop a two sector world economy integrated assessment model with a monetary and a fiscal authority. Figure 1 provides an overview of the various sectors and entities in the economy, and Table 1 collects the main indices used throughout. The final consumption good is produced by a dirty and a clean intermediate goods sector, which itself uses capital, labor and energy as input. Labor is supplied by households and capital is supplied by households and a monetary authority. Energy is supplied by a dirty and a clean energy production sector, using labor supplied by households as input. We take the total capital stock of the monetary authority supplied to firms as given and thus the monetary authority solely faces a portfolio choice allocation problem and can thereby influence the production of clean and dirty intermediate inputs. Profits generated by the monetary authority flow to the fiscal authority which additionally raises revenue from households by consumption taxes and from dirty energy production by energy (carbon) taxes. Dirty energy production leads via its emissions to an accumulation of a carbon stock in the atmosphere which creates a temperature increase and with it causes a damage through a reduction of aggregate output. We now describe the main elements of the model in more detail.

Table 1: Indices

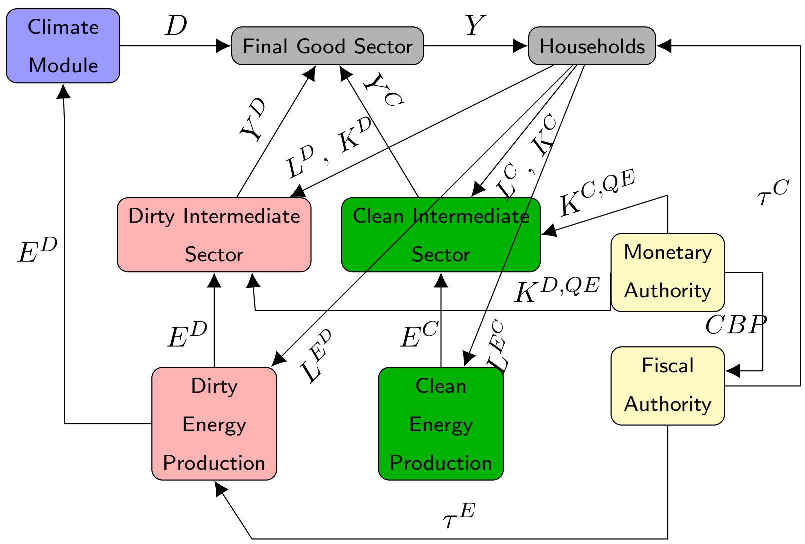
Index	Value	Interpretation
t	$t \in \{0, 1, \ldots, \infty\}$	Time
i	$i \in \{1, 2, \ldots, \infty\}$	Type
s	$s \in \{cl, di\}$	Sector (clean, dirty)
c	$c \in \{ra, sl\}$	Carbon Stocks (rapidly, slowly depreciating stock)

Notes: List of indices used in the integrated assessment model.

2.1 Time, Risk and Population Structure

Time in the model is discrete and runs from $t = 0, \ldots, \infty$. At time $t = 0$ a continuous distribution of infinitely lived representative agents index by $i \in \{1, 2, \ldots, \infty\}$ are born into the economy with total initial size $N_0 = 1$, which grows exogenously at time varying rate n_t. Each period

Figure 1: Overview of the 2 Sector Integrated Assessment Model



the heterogeneous households earn a deterministic labor income, stochastic returns on physical capital holdings from owning firms and risk-free returns from owning bonds.

2.2 Production

2.2.1 Final Good Production

The final output good Y_t in the economy is composed of two intermediate goods produced in a clean and a dirty sector Y_{ts}, $s \in \{cl, di\}$, and augmented according to a CES aggregator with substitution elasticity ε. At this outer layer of the production side we further assume an exogenous technology level Υ_t, which grows at the exogenous rate g. Additionally, there is a negative aggregation production externality D_t from air pollution which proportionally reduces aggregate output and thus

$$Y_t = (1 - D_t) \cdot \Upsilon_t \cdot \left(\sum_{s \in \{cl,di\}} \kappa_s Y_{ts}^{1-\frac{1}{\varepsilon}} \right)^{\frac{1}{1-\frac{1}{\varepsilon}}}, \tag{1}$$

where κ_s are the sectoral output shares with $\sum_{s\in\{cl,di\}} \kappa_s = 1$. The representative firm takes as given the final goods price p_t and the intermediate goods input prices p_{ts} and maximizes profits under perfect competition giving the intermediate goods demand

$$Y_{ts} = \left(\frac{\kappa_s}{\frac{p_{ts}}{p_t}} \right)^{\epsilon} ((1 - D_t) \cdot \Upsilon_t)^{\epsilon - 1} Y_t, \text{ for } s \in \{cl, di\}. \tag{2}$$

and the price index for the final good as

$$p_t = \frac{1}{(1-D_t)\Upsilon_t}\left(\sum_{s\in\{cl,di\}} \kappa_s^{\epsilon} p_{ts}^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}.$$

cf. Appendix A.1.

2.2.2 Intermediate Goods Production

Every household runs the two intermediate goods firms s by employing its own household capital k_{tis} and hiring labor l_{tis} and energy e_{tis} on the respective labor and energy market. Production of intermediates goods takes place according to a two-nests Cobb-Douglas technology with inner nest capital elasticity parameter α and outer nest energy elasticity $1-\gamma$. The value of the capital employed in production is subject to an idiosyncratic sector specific shock ζ_{tis} so that gross output is

$$y_{tis} = \psi_s\left[(k_{tis})^{\alpha}(\ell_{tis})^{1-\alpha}\right]^{\gamma}\cdot e_{tis}^{1-\gamma} + \zeta_{tis}k_{tis}, \tag{3}$$

where ψ_s is a technology level parameter. We assume that ζ_{tis} is i.i.d. with CDF $\Psi\left(0, \sigma_s^{\zeta^2}, \dots\right)$, with details on the shock distribution described in Appendix A.2. Households take as given the intermediate goods prices p_{ts}, wages, respectively the return on labor, r_t^l, energy prices p_{ts}^e and an exogenous depreciation rate on capital δ_s so that profits are

$$\pi_{ts} = p_{ts}\cdot y_{tis} - r_t^l\ell_{tis} - p_{ts}^e e_{tis} - \delta_s k_{tis}. \tag{4}$$

Assuming free entry and exit, profit maximization yields the demand for energy and labor as

$$e_{tis} = \Gamma(\psi_s,\alpha,\gamma)\cdot\frac{1-\gamma}{(1-\alpha)\gamma}\cdot\left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}}\cdot\left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma(1-\alpha)}{\alpha\gamma}}\cdot k_{tis} \tag{5a}$$

$$\ell_{tis} = \Gamma(\psi_s,\alpha,\gamma)\cdot\left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1}{\alpha}}\cdot\left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}}\cdot k_{tis}, \tag{5b}$$

where the constant $\Gamma(\psi_s,\alpha,\gamma)$ is

$$\Gamma(\psi_s,\alpha,\gamma) = \left[\psi_s(1-\gamma)\right]^{\frac{1-\gamma}{\alpha\gamma}}\cdot\left[\psi_s(1-\alpha)\gamma\right]^{\frac{1}{\alpha}}. \tag{6}$$

Using (5) in (3) we can rewrite output as

$$y_{tis} = \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{1}{(1-\alpha)\gamma} \cdot \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} \cdot k_{tis} + \zeta_{tis} \tag{7}$$

which is linearly increasing in k_{tis} and, using this in (4) gives profits as

$$\pi_{tis} = \left[\Gamma(\psi_s, \alpha, \gamma) \cdot \frac{\alpha}{(1-\alpha)} \cdot p_{ts} \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} - \delta_s + p_{ts}\zeta_{tis}\right] k_{tis},$$

which are proportional to k_{tis}. Defining the idiosyncratic return on capital as

$$r_{tis} = \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{\alpha}{(1-\alpha)} \cdot p_{ts} \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} - \delta_s + p_{ts}\zeta_{tis} \tag{8}$$

we can thus rewrite profits as

$$\pi_{tis}(k_{tis}) = r_{tis} \cdot k_{tis}. \tag{9}$$

2.2.3 Energy Production

Energy employed for production in the two intermediate goods sectors s is produced in two perfectly separated (across the two sectors) energy producing firms that employ labor L_{ts}^e and an a technology stock Υ_{ts}^e, which grows exogenously and deterministically at the time varying and sector specific rate g_{ts}. The energy production technology is linear and accordingly

$$E_{ts} = \Upsilon_{ts}^e L_{ts}^e.$$

Dirty energy production is subject to proportional carbon taxes $\tau_{ts=di}^e \geq 0$, whereas energy in the clean sector is untaxed, $\tau_{ts=cl}^e = 0$, and thus profits in the two energy producing firms are

$$\pi_{ts}^e = p_{ts}^e \left(1 - \tau_{ts}^e\right) \Upsilon_{ts}^e L_{ts}^e - r_t^l L_{ts}^e.$$

Assuming free entry and exit drives profits in the energy sector to zero and thus energy prices are given by

$$p_{ts}^e = \frac{r_t^l}{\left(1 - \tau_{ts}^e\right) \Upsilon_{ts}^e}. \tag{10}$$

2.3 Carbon Stock Accumulation, Temperature and the Damage Function

As in Golosov, Hassler, Krusell, and Tsyvinski (2014) and Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2019), the total carbon stock S_t in the atmosphere is composed of two stocks, a rapidly and a slowly depreciating stock, S_{tc} for $c \in \{ra, sl\}$, thus

$$S_t = \sum_{c\in\{ra,sl\}} S_{tc}$$

which accumulate through dirty energy emissions and feature persistence parameter ρ_c, where $1 > \rho_{c=sl} > \rho_{c=ra} > 0$, thus

$$S_{tc} = \phi_c \xi E_{ts=di} + \rho_c S_{t-1c} \tag{11}$$

where $\xi > 0$ and $\phi_c > 0$ and $\sum_{c\in\{ra,sl\}} \phi_c = 1$. Each unit of S_t leads to an increase of the global temparature according to

$$T_t = \lambda \frac{\log(S_t/S_{pre})}{\log(2)}, \tag{12}$$

where S_{pre} is the pre-industrial area carbon stock in the atmosphere, and $\lambda > 0$. The temperature increase in turn leads to the negative externality on aggregate output through the damage function

$$D_t = 1 - \frac{1}{1+\nu T_t^2}. \tag{13}$$

for $\nu > 0$.

2.4 Fiscal and Monetary Authorities

The model features a fiscal and a monetary authority. The fiscal authority levies Carbon taxes at rate $\tau^e_{ts=di} \geq 0$ and receives profits from the monetary authority π^m_t. These sources of income are distributed to households in the form of subsidies on consumption, $\tau^c_t \leq 0$ and thus each period the fiscal authority features a balanced budget of

$$\tau^e_{ts=di} E_{ts=di} + \pi^m_t + \tau^c_t C_t = 0. \tag{14}$$

The monetary authority in turn holds an exogenous amount of capital K^m_t in the economy which may be growing at exogenous time varying rate $g^m_t \geq 0$. This capital is exogenously split across the two capital stocks in the intermediate goods production sectors, thus

$$K_t^m = \sum_{s\in\{cl,di\}} K_{ts}^m.$$

The monetary authority earns the average marginal products in the two sectors and its profits are thus

$$\pi_t^m = \sum_{s\in\{cl,di\}} \mathbb{E}[r_{ts}]K_{ts}^m.$$

2.5 Households

2.5.1 Preferences

Each household i at time t has Epstein-Zin-Weil (Epstein and Zin 1989; Epstein and Zin 1991; Weil 1989) recursive preferences u_{ti} over consumption c_{ti} and continuation utility u_{t+1i} which is discounted at factor $\beta \in (0,1)$ and features risk aversion parameterized by θ and resistance to intertemporal substitution υ. Thus, preferences are given by

$$u_{ti} = \left[c_{ti}^{1-\upsilon} + \beta \cdot \left(\mathbb{E}[u_{t+1i}^{1-\theta}]\right)^{\frac{1-\upsilon}{1-\theta}}\right]^{\frac{1}{1-\upsilon}}, \tag{15}$$

where $\mathbb{E}$ is an expectations operator with expectations taken with respect to idiosyncratic shocks to the return on physical capital.

2.5.2 Endowments

Household operate the two intermediate goods firms. Accordingly, household i enters into model period t with capital stocks k_{tis} in the two firms and earns in the current period stochastic profits generated from production in those firms π_{tis}. Households also earn a deterministic labor income $r_t^l \ell_t$ where r_t^l denotes the wage rate on the exogenous labor endowment ℓ_t, which is the same for all households. Furthermore, households enter the period with bond holdings b_{ti}, which are in zero net supply across all households and earn a risk-free return r_t^f. The household spends its income from these sources on consumption of the final good c_{ti}—which has price p_t and is taxed, respectively subsidized, at rate τ_t^c—, on savings in the two capital goods k_{t+1is} as well as on risk free bond purchases b_{t+1i}. Thus the dynamic household budget constraint reads as

$$\sum_{s\in\{c,d\}} k_{t+1is} + b_{t+1i} + (1+\tau_t^c)p_t c_{ti} = \sum_{s\in\{c,d\}} k_{tis}\left(1+r_{tis}\right) + r_t^l \ell_t$$

where $r_{tjs} = \frac{\pi_{tis}}{k_{tis}}$ is the stochastic return on capital in sector s.

2.5.3 Analysis of the Household Problem

Conditional on the aggregate law of motion of the economy, i.e., for given prices, wages, interest rates and taxes, the household model permits a closed form solution. To derive it, first rewrite the budget constraint in terms of cash-on-hand

$$x_{ti} = \sum_{s\in\{c,d\}} k_{tis}\left(1 + r_{tis}\right) + r_t^l \ell_t$$

to get

$$\sum_{s\in\{c,d\}} k_{t+1is} + b_{t+1i} = x_{ti} - (1+\tau_t^c)p_t c_{ti}.$$

Next, define the portfolio shares as shares invested in the respective asset as a function of total savings $x_{ti} - (1-\tau_t^c)c_{ti}$ as

$$\alpha_{tis} = \frac{k_{t+1is}}{x_{ti} - (1+\tau_t^c)p_t c_{ti}}, \qquad 1 - \sum_{s\in\{cl,di\}} \alpha_{tis} = \frac{b_{t+1is}}{x_{ti} - (1+\tau_t^c)p_t c_{ti}}$$

to note that

$$x_{t+1i} = \sum_{s\in\{c,d\}} \left(1 + r_{t+1}^f + \alpha_{tis}\left(r_{t+1is} - r_{t+1}^f\right)\right)\left(x_{ti} - (1+\tau_t^c)p_t c_{ti}\right) + r_{t+1}^l \ell_{t+1}. \tag{16}$$

Next, denote by h_t the human capital wealth of a household at date t, which is the discounted sum of future labor income

$$h_t = \sum_{j=0}^{\infty} r_{t+1+j}^l \ell_{t+1+j} \prod_{k=0}^{j}\left(1 + r_{t+k+1}^f\right)^{-1}$$

which thus obeys the human capital accumulation equation

$$h_{t+1} = h_t(1 + r_{t+1}^f) - r_{t+1}^l \ell_{t+1} \tag{17}$$

Finally, define total wealth of the household as

$$w_{ti} = x_{ti} + h_t$$

and take the sum of (16) and (17) to get

$$w_{t+1i} = \left(w_{ti} - (1+\tau^c_t)p_t c_{ti}\right) R^p_{t+1i}\left(\{\hat{\alpha}_{tis}\}_{s\in\{cl,di\}}\right), \tag{18}$$

where

$$R^p_{t+1i}\left(\{\hat{\alpha}_{tis}\}_{s\in\{cl,di\}}\right) = 1 + r^f_{t+1} + \sum_{s\in\{cl,di\}} \hat{\alpha}_{tis}\left(r_{t+1is} - r^f_{t+1}\right)$$

is a portfolio return on total savings $w_{ti} - (1-\tau^c_t)c_{ti}$ and where

$$\hat{\alpha}_{tis} = \frac{k_{t+1is}}{w_{ti} - (1+\tau^c_t)p_t c_{ti}}, \qquad 1 - \sum_{s\in\{cl,di\}} \hat{\alpha}_{tis} = \frac{b_{t+1is}}{w_{ti} - (1+\tau^c_t)p_t c_{ti}}$$

are the portfolio investments in the respective asset in relation to total savings.

Maximization of (15) subject to the resource constraint (18) gives rise to optimal decisions in terms of consumption policy functions and portfolio allocation decisions as stated in the next proposition, which we formally prove in Appendix A.3:

Proposition 1. • *Consumption policy functions are linear functions of total wealth*

$$c_{ti} = m_t w_{ti}$$

where the marginal propensities to consume are

$$m_t = \frac{\Theta\left(p_t, p_{t+1}, \tau^c_t, \tau^c_{t+1}, R^p_{t+1}\left(\{\hat{\alpha}_{ts}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) m_{t+1i}}{1 + (1+\tau^c_t)\Theta\left(p_t, p_{t+1}, \tau^c_t, \tau^c_{t+1}, R^p_{t+1}\left(\{\hat{\alpha}_{ts}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) m_{t+1i}}, \tag{19}$$

where

$$\Theta\left(p_t, p_{t+1}, \tau^c_t, \tau^c_{t+1}, R^p_{t+1}\left(\{\hat{\alpha}_{ts}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) = \left(\beta \frac{p_t(1+\tau^c_t)}{p_{t+1}(1+\tau^c_{t+1})}\left(\mathbb{E}_t\left[R^p_{t+1}\left(\{\hat{\alpha}^*_{ts}\}_{s\in\{cl,di\}}\right)^{1-\theta}\right]\right)^{\frac{1-\upsilon}{1-\theta}}\right)^{-\frac{1}{\upsilon}}$$

• *The optimal portfolio shares are given by*

$$\hat{\alpha}^*_{ts} \approx \frac{\ln(1+\mathbb{E}\left[r_{t+1s}\right]) - \ln(1+r^f_{t+1})}{\theta \cdot Var(\ln(1+r_{t+1s}))}, \tag{20}$$

and thus the marginal propensities to consume out of total wealth and the optimal portfolio shares in t, s are the same for all i, $m_{ti} = m_t$, $\hat{\alpha}_{tis} = \hat{\alpha}_{ts}$.

Linearity of policy functions in total wealth and identical marginal propensities to consume in any t, s across all households is a very convenient property of the model as it simplifies the aggregation to the effect that we only need to keep track in the mean decisions and not their distribution.

2.6 Definition of Equilibrium

We define the equilibrium in this economy sequentially. By the result in Proposition 1 we do not need to keep track of the distribution of heterogenous households and thus household specific variables are not indexed by i and it is understood that the household variables in the new formal equilibrium definition indexed by t, respectively by t and s, refer to average allocations.

Definition 1. *Given an initial total wealth level w_0, initial carbon stocks $\{S_{0c}\}_{c\in\{ra,sl\}}$, a sequence of technology levels and of the population $\{\Upsilon_t, \{\Upsilon^e_{ts}\}_{s\in\{cl,di\}}, N_t\}_{t=0}^{\infty}$ and a sequence of policy parameters $\{\tau^c_t, \tau^e_{ts=di}, \{K^m_{ts}\}_{s\in\{cl,di\}}\}_{t=0}^{\infty}$, a competitive equilibrium is an allocation $\{\{E_{ts}, K_{ts}, L_{ts}, Y_{ts}, \hat{\alpha}_{ts}, \}_{s\in\{cl,di\}}$ a sequence of prices $\{\{p_{ts}, p^e_{ts}, r_{ts}\}_{s\in\{cl,di\}}, r^l_t\}_{t=0}^{\infty}$ and a sequence of profits $\{\{\pi_{ts}\}_{s\in\{cl,di\}}, \pi^m_t\}_{t=0}^{\infty}$ such that*

1. *given prices $\{\{p_{ts}, p^e_{ts}, r_{ts}\}_{s\in\{cl,di\}}, r^l_t\}_{t=0}^{\infty}$ and policies $\{\tau^c_t, \tau^e_{ts=di}, \{K^m_{ts}\}_{s\in\{cl,di\}}\}_{t=0}^{\infty}$ households behave optimally with resulting optimal policy functions choices $c_t, \hat{\alpha}_{ts}, w_{t+1}$ as characterized in Proposition 1.*

2. *prices satisfy (5),(10) and*

$$r_{ts} = \int r_{tis} di$$

   where r_{tis} is given in (8);

3. *the government budget constraint (14) holds in all $t \geq 0$;*

4. *the sequence of carbon stocks, global temperature and global damage $\{\{S_{tc}\}_{c\in\{ra,sl\}}, T_t, D_t\}_{t=0}^{\infty}$ evolve according to (11)–(13);*

5. *markets clear:*

$$L_t = N_t \ell_t \tag{21a}$$

$$K_{ts} = N_t \int k_{tsi} di, \text{ for } s \in \{cl, di\} \tag{21b}$$

$$L_{ts} = N_t \int \ell_{tsi} di = N_t \cdot \Gamma(\psi_s, \alpha, \gamma) \cdot \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} \cdot K_{ts}, \text{ for } s \in \{cl, di\} \tag{21c}$$

$$E_{ts} = N_t \int e_{tsi} di = N_t \cdot \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{1-\gamma}{(1-\alpha)\gamma} \cdot \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma(1-\alpha)}{\alpha\gamma}} \cdot K_{ts}, \text{ for } s \in \{cl, d \tag{21d}$$

$$Y_{ts} = N_t \int y_{t,j,i}^X di = N_t \cdot \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{1}{(1-\alpha)\gamma} \cdot \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} \cdot K_{ts}, \text{ for } s \in \{cl, di\} \tag{21e}$$

where $\Gamma(\psi_s, \alpha, \gamma)$ is given in (6).

3 Calibration

3.1 Overview of Calibration

We calibrate the model by fixing some parameters exogenously (first stage parameters) and by calibrating others to match certain moment in an initial steady state year, which we pick to be year 2000. While the latter set of parameters are calibrated jointly, for clarity of identification of the parameter values we relate each parameter with a specific target. Tables 2 and 3 provide an overview of all first- and second-stage parameters and the subsequent sections provide the details of the calibration by sector in the economy.

3.2 Population and Labor Supply

The exogenous population growth rate n_t is calibrated from the population growth rate information (and projections) provided in the World Population Prospects of the United Nations. The initial year 2000 population growth rate is $n_0 = 0.0127$ and population growth shrinks gradually to reach zero growth by year 2100 thus $n_t = 0$, for all $t > 100$. The aggregate amount of labor supply L_t, is assumed to grow at the same rate as the population. We calibrate the initial aggregate labor supply L_0 endogenously so that the model generates an output of energy in the dirty sector in the initial equilibrium of 30 gigatons of C02.

Table 2: Calibration: First Stage Parameters

Parameter	Value	Target (Source)
Population and Labor supply		
Initial population growth rate n_0	0.0127	Data moment (United Nations)
Final good technology		
Elast. of subst. ε	26	Elasticity of energy subst.
Intermediate good technology		
Non-energy share: γ	0.96	Kotlikoff et al. (2019)
Capital share: α	0.33	Standard value
Climate Module		
Initial carbon stock: S_0	800 Gt	
Pre-industrial carbon stock: S_{pre}	581 Gt	
Stock 1 share: ϕ_s	[0.5,0.5]	
Emission share in atmosphere: ξ	0.4	
Carbon stock persistence: ρ_c	$\rho_c = [0.996, 0.999], c \in \{ra, sl\}$	
Temp. increase with S: λ	3	
Temperature to damage: ν	0.0028388	
Central bank portfolio		
Capital holdings $K^m_{0s}, s \in \{cl, di\}$	$[6245, 8928]$	$K^m_{0s}/K_{0s} = 4\%, s \in \{cl, di\}$
Preferences		
Elasticity inter-temp. substit., $1/\upsilon$	0.5	Standard value

Notes: Calibration in the baseline model. First stage parameters calibrated with reference to other studies or without using the model. Steady state year is year 2000.

Table 3: Calibration: Second Stage Parameters

Parameter	Value	Moment
Population and Labor supply		
L_0	$6.0e{+}5$	$E_{0s=di} = 30$ GtCO2
Final good technology		
Interm. good weight $\kappa_s, s \in \{cl, di\}$	$0.42, 0.58$	$E_{0s=di}/E_{0s=cl} = 4$
Growth rate final good TFP, g	0.0103	$(\frac{Y_{100}}{L_{100}}/\frac{Y_{30}}{L_{30}})^{\frac{1}{70}} - 1 = 1.55\%$
Intermediate good technology		
Expected depreciation rate: $\delta_s, s \in \{cl, di\}$	$0.015, 0.087$	$\mathbb{E}[r_{0s}] = 6.94\%, , s \in \{cl, di\}$
Std. of depreciation shock: $\sigma^{\zeta}_s, s \in \{cl, di\}$	$0.034, 0.024$	$\sigma^{r_{0s}} = 8.4\%, s \in \{cl, di\}$ (std. of capital returns)
Energy production technology		
Clean productivity factor, $\Upsilon^e_{0s=cl}$	$2.1e{-}4$	$p^e_{0s=cl} = 810$ USD/tCe
Dirty productivity factor, $\Upsilon^e_{0s=di}$	$3.2e{-}4$	$p^e_{0s=di} = 540$ USD/tCe
Growth rate clean prod. fact., $g^e_{s=cl}$	0.02	$(p^e_{100s=di}/p^e_{30s=cl})^{\frac{1}{70}} - 1 = 0.44\%$
Preferences		
Time discount factor: β	0.998	$K/Y = 3.0$
Relative risk aversion: θ	57.9	$r^f = 2.9\%$

Notes: Calibration in the baseline model. Second stage parameters calibrated endogenously by matching of moments. Steady state year is year 2000.

3.3 Production

Final Good Production We take an indirect approach to the calibration of the parameter governing the elasticity of final output in the two goods, Y_{ts}, $s \in \{cl, di\}$ in equation (1). In our model with the two separate firms for energy production there is no direct parameter that would govern the demand elasticity for energy, which we in turn refer to as the percent change in the ratio of dirty to clean energy demand $\frac{E_{ts=d}}{E_{ts=c}}$ in response to a percent change of relative prices $\frac{p^e_{ts=d}}{p^e_{ts=c}}$ denoted as $\eta_{\frac{E_{ts=d}}{E_{ts=c}},\frac{p^e_{ts=d}}{p^e_{ts=c}}}$. This elasticity is a key statistic in our model for the response of energy use induced by exogenous price changes through carbon taxes. According to Papageorgiou, Saam, and Schulte (2017) the energy demand elasticity is about 2-3, where the lower value refers to the electricity-generating sector and values close to 3 are in nonenergy industries. We take the lower value as target for the calibration of parameter ε. In appendix B.1 we derive that locally—i.e., holding constant the (expected) marginal remuneration of capital $\mathbb{E}r_{ts}$, $s \in \{cl, di\}$ and labor r^l_t—the energy demand elasticity is given by

$$\eta_{\frac{E_{ts=di}}{E_{ts=cl}},\frac{p^e_{ts=di}}{p^e_{ts=cl}}} = \varepsilon \cdot (1-\gamma) + \gamma$$

which we invert to calibrate ε for given target $\eta_{\frac{E_{ts=di}}{E_{ts=cl}},\frac{p^e_{ts=di}}{p^e_{ts=cl}}}$ and a given γ.

The relative weights on the two goods in (1), $\kappa_s, s \in \{cl, di\}$ are calibrated such that (i) we normalize $\kappa_{s=di} = 1 - \kappa_{s=cl}$ and (ii) match the ratio of energy output in the two sectors of $\frac{E_{0s=di}}{E_{0s=cl}} = 4$ giving $\kappa_{s=cl} = 0.42$ and thus $\kappa_{s=di} = 0.58$. The final good output growth rate g is calibrated to generate a total annual output growth of 1.5%, giving $g = 0.0103$

Intermediate Goods Production We set the production elasticity of capital employed in the intermediate goods sectors, cf. equation (3), exogenously to $\alpha = 0.33$, corresponding to standard estimates of capital elasticities in production. The output elasticity parameter of non-energy inputs γ is set to 0.96, following Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2019). The technology levels in both sectors are normalized to $\psi_s = 1$.

The average depreciation rates $\delta_s, s \in \{c, d\}$ and the standard deviation of depreciations shocks $\sigma^\zeta_s, s \in \{c, d\}$ are calibrated to yield expected average returns of 6.94% in both sectors and a standard deviation of expected returns of returns 8.4%, based on empirical estimates of Piazzesi, Schneider, and Tuzel (2007). This gives $\delta_s = [0.015, 0.087], s \in \{cl, di\}$

Energy Production Recall from equation (10) that energy prices are inversely proportional to the technology level in the energy sector. Based on this relationship we calibrate the technology parameters $\Upsilon^e_{0s}, s \in \{cl, di\}$ to match the absolute price levels[1] in the two sectors per ton of car-

[1]Recall that final consumption is the numeraire good in the economy so that these absolute price levels are equal to the relative prices in units of the final consumption good.

bon emission (tCe) of USD 810, respectively 540 , which requires $\Upsilon^e_{0s=cl} = 2.1e{-}4$ and $\Upsilon^e_{0s=di} = 3.2e{-}4$. We further hold constant the technology level in the dirty energy sector $\Upsilon^e_{ts=di} = \Upsilon^e_{0s=di}$, for all $t > 0$ and calibrate a growth rate $g^e_{s=cl}$ so that $\Upsilon^e_{ts=cl} = \Upsilon^e_{t-1s=cl}(1 + g^e_{s=cl})$ such that energy prices fall by 0.44% on average over the 70 years between year 2030 and 2100. This calibration is based on Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2019), who in turn refer to Nordhaus (2017).

3.4 Carbon Stock Accumulation, Temperature and the Damage Function

The calibration of the climate module follows Golosov, Hassler, Krusell, and Tsyvinski (2014), Kotlikoff, Kubler, Polbin, and Scheidegger (2021) and Van Der Ploeg and Rezai (2021).

Carbon Stock The initial carbon stock in the atmosphere is set to $S_0 = 800$ gigatons, where $S_{0c=sl} = 684$ and $S_{0c=ra} = 116$. As to the dynamics of the two carbon stocks in equation 11 we assume that 40% of dirty energy output leads to an accumulation of the carbon stocks and thus $\xi = 0.4$, which is split up equally across the two stocks, thus $\phi_c = 0.5, c \in \{sl, ra\}$. The slow decumulating carbon stock features a persistence of $\rho_{c=sl} = 0.999$, and the rapidly decumulating of $\rho_{c=ra} = 0.995$.

Temperature and Damage Function We calibrate the temperature function in (12) by setting $\lambda = 3$ and $S_{pre} = 581$, and the damage function in (13) by letting $\nu = 0.0028388$.

3.5 Fiscal and Monetary Authorities

Monetary Authority The central bank portfolio is calibrated such that in the initial steady state 4% of capital in both sectors is held directly by the central bank, i.e., $\frac{K^m_{0s}}{K_s} = 0.04$ for $s \in \{cl, di\}$. Given the endogenously determined sizes of the two sectors in the economy, this requires $K^m_{0s=cl} = 6245$ and $K^m_{0s=di} = 8928$.

Fiscal Authority In the initial steady state equilibrium, the fiscal authority is inactive, thus $\tau^e_{ts} = \tau^c_t = 0$.

3.6 Household Preferences

The elasticity of inter-temporal substitution $1/\upsilon = 0.5$, corresponding to the standard estimate in the literature. The remaining household preference parameters are calibrated endogenously to match a capital output ratio of 3 by choice of the discount factor, which gives $\beta = 0.998$, and a risk-free rate of return of 2.9% by choice of the coefficient of risk aversion which requires $\theta = 57.9$. This high value is not surprising because shocks in our model are assumed to be distributed as log-normal (thus, there are no extreme events), and there are no additional income shocks (no background risks) for households.

4 The Quantitative Relevance of Quantitative Easing

4.1 Thought Experiments

Taking as given exogenous dynamics of population and technology, we compute transitions under alternative fiscal and monetary policy scenarios. Throughout, we compute transitions over 500 model periods, starting in year 2000 with an initial steady state. We treat the first 20 years as a phase-in period and show results until 2100, that is overall we will display output for the next 80 years form 2020-2100. We first, conduct a *baseline experiment*, where all policies are held constant at their 2000 values. Next, we consider a *carbon tax scenario* where the carbon tax is assumed to increase. Revenue from carbon taxation is redistributed to households through consumption subsidies. As our second policy scenario, we contrast the carbon tax scenario with a *green quantitative easing scenario* according to which the portfolio composition of the central bank changes such that it reshuffles all of its capital holdings towards the brown sector. We also assume that the capital stock of the central bank and thereby the magnitude of green quantitative easing grow exogenously at the growth rate of GDP. Our economy does not feature aggregate risk and thus there are no recessions, which in an economy with non-standard monetary policy would endogenously lead to repeated monetary policy interventions through quantitative easing policies. Therefore, our assumption of an exogenous growth in the size of central bank asset holdings in the economy should be interpreted as approximating a real world economy in which quantitative easing policies take place with a certain regularity. Also in regard to other assumption on calibration we thus give green quantitative easing a maximum possible role. These additional assumptions are, first, that the central banks portfolio is large in the world economy, second, that all countries in the world execute quantitative easing policies, third, that clean and dirty asset returns are uncorrelated so that the quantitative easing policy will not lead to a full crowding out of private clean capital investments and fourth, that the there is a very high elasticity of substitution between clean and dirty intermediate inputs in the final production so that the already relatively large changes induced by green quantitative easing policies do have relatively mild price effects only, which is another reason for the modest crowding out.

Our results on the *green quantitative easing* experiment should therefore not be interpreted as providing a realistic quantitative assessment of real world green quantitative easing policies. We rather ask a hypothetical *what if* question, i.e., within the structure of our model we evaluate the maximum climate change mitigating potency of such policies.[2] Finally, as a *full policy scenario*

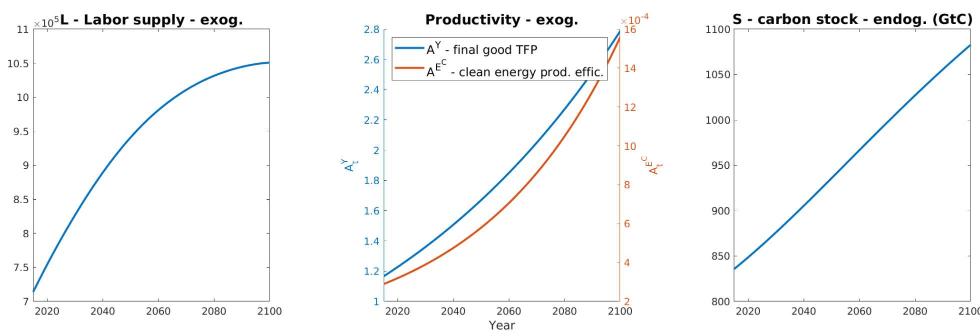
[2] However, note that within the structure of our model there is no role for a triggering mechanism of a form that private investors may follow the example of the central bank. If such a mechanism is at work in the real world, then we underestimate the role of green quantitative easing policies.

we consider both instruments jointly and thereby investigate whether both policies are substitutes or complements in mitigating the adverse effects of climate change.

4.2 Baseline Results

Driving Forces We start by displaying in Figure 2 the driving forces in the model over the next 80 years, from year 2020 to year 2100. Panel (a) shows aggregate efficiency weighted labor supply, which features a gradually decreasing growth rate and is thus hump-shaped over the next 80 years. This reflects the increase in the world population from 7.8 Billion in year 2020 to about 10.9 Billion people in year 2100 according to the median variant of the UN projections. Panel (b) displays the technology level in the clean sector $\Upsilon_{ts=cl}$. Due to the strong growth assumption relative to the zero growth in the dirty energy sector, the time path of the technological level in the energy sector is more strongly convex than technological progress in production of the final good.

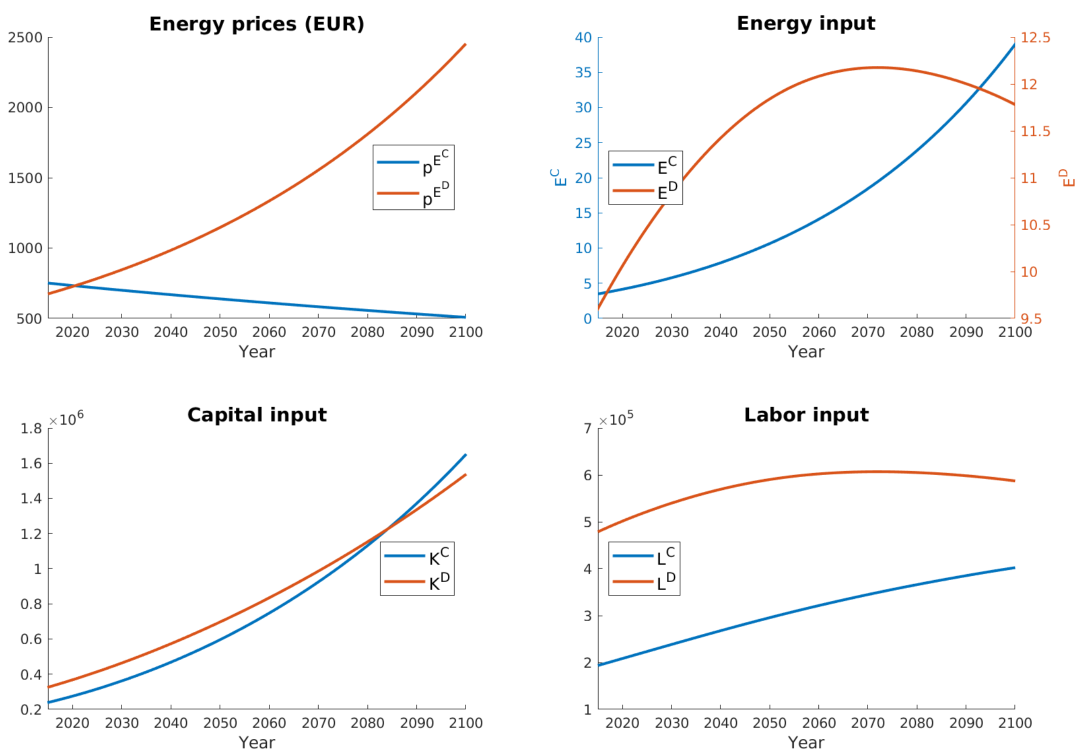
Figure 2: Baseline: Driving forces



Notes: Exogenous driving forces: Aggregate labor supply in panel (a) and technology levels in final goods sector, Υ_t, and clean energy sector, $\Upsilon^e_{ts=cl}$ in panel (b).

Intermediate Goods Production Figure displays prices in the clean and the dirty energy sector in panel (a). The increasing prices of dirty energy and the falling price of clean energy is a consequence of the exogenously assumed increase of relative productivity in the clean energy sector. With regard to the ensuing dirty energy production and thus dirty emissions, two mechanisms are at work in the model. On the one hand, demand for goods through population growth and technological progress in the final goods sector will lead to an increase of harmful emissions, $E_{ts=di}$. On the other hand, the technological progress in the clean sector $\Upsilon_{ts=cl}$ by increasing the relative price of dirty intermediate goods leads to a substitution of intermediate goods production towards the clean sector. Two forces lead to this substitution. The one is a reduction of demand for dirty energy in the intermediate goods sector. The second is a substitution towards clean intermediate goods in the production of the final good. As a consequence, dirty energy production $E_{ts=di}$ (and thus emissions) shown in Panel (b) of the figure is increasing until it peaks in year 2070 at a level that is about 20% higher than the dirty energy production in year 2020. In year 2100 dirty energy production is still about 18% higher than in year 2020. Since the clean intermediate goods sector expands relative to the dirty sector, the aggregate input factors capital and labor in the economy are increasingly employed in the production of clean intermediate goods, cf. panels (c) and (d) of the figure.

Figure 3: Baseline: Intermediate Production Inputs

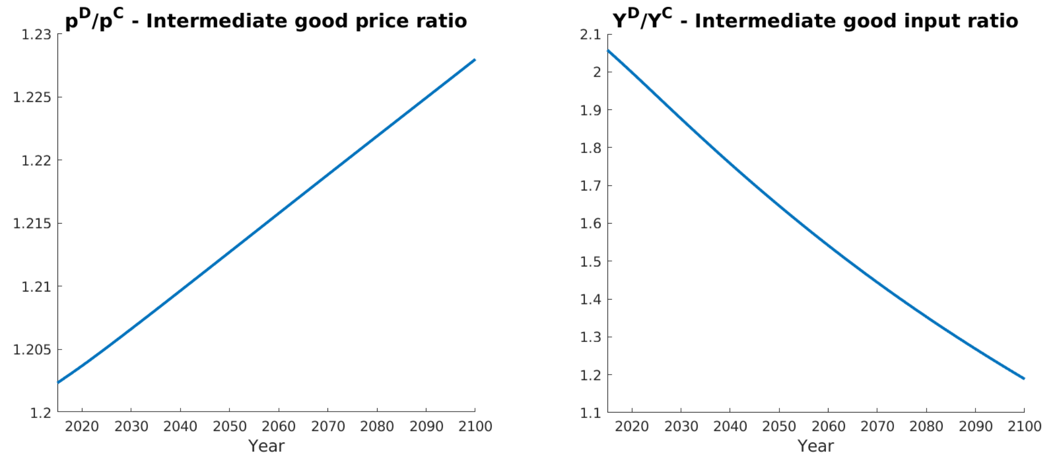


Notes: Intermediate production inputs: dirty energy emissions $E_{ts=di}$ in panel (a), carbon stocks S_t, $\{S_{tc}\}_{c\in\{ra,sl\}}$ in panel (b), world temperature T_t in degree Celsius in panel (c), and aggregate damage D_t (in percent) in panel (d).

Overall, these dynamic adjustments lead to an increase of the relative price of dirty intermediate inputs $\frac{p_{ts=di}}{p_{ts=cl}}$ by 2.5% and an reduction of relative output $\frac{y_{ts=di}}{y_{ts=cl}}$ by almost 43%, cf. Figure 4.

Climate Implications The implications of the above shown gradual substitution towards cleaner intermediate goods production for the global climate are shown in Figure 5, where Panel (a) again shows the emissions of the dirty sector $E_{ts=di}$. Panel (b) displays the resulting time paths of the

Figure 4: Baseline: Intermediate Production Inputs



Notes: Intermediate production: relative price of dirty to clean goods, $\frac{p_{ts=di}}{p_{ts=cl}}$ in panel (a), and relative intermediate goods output, $\frac{y_{ts=di}}{y_{ts=cl}}$ in panel (b).

carbon stocks that accumulate as a consequence of these emissions according to the calibrated process described in (11). By year 2100 the total carbon stock will have increased by about 27% relative to its year 2020 level. This leads to an increase of global temperature as shown in Panel (c). According to our model, the year 2020 temperature level is about 1.6 degrees Celsius above the pre-industrial level, and it will increase to about 2.7 degrees, an increase over 80 years by 1.1 degrees, or 0.013 degree per year. The resulting damage, shown in Panel (d) of the figure in terms of a percent output loss increases by 1.2%p, from 0.8% in 2020 to 2% in 2100.

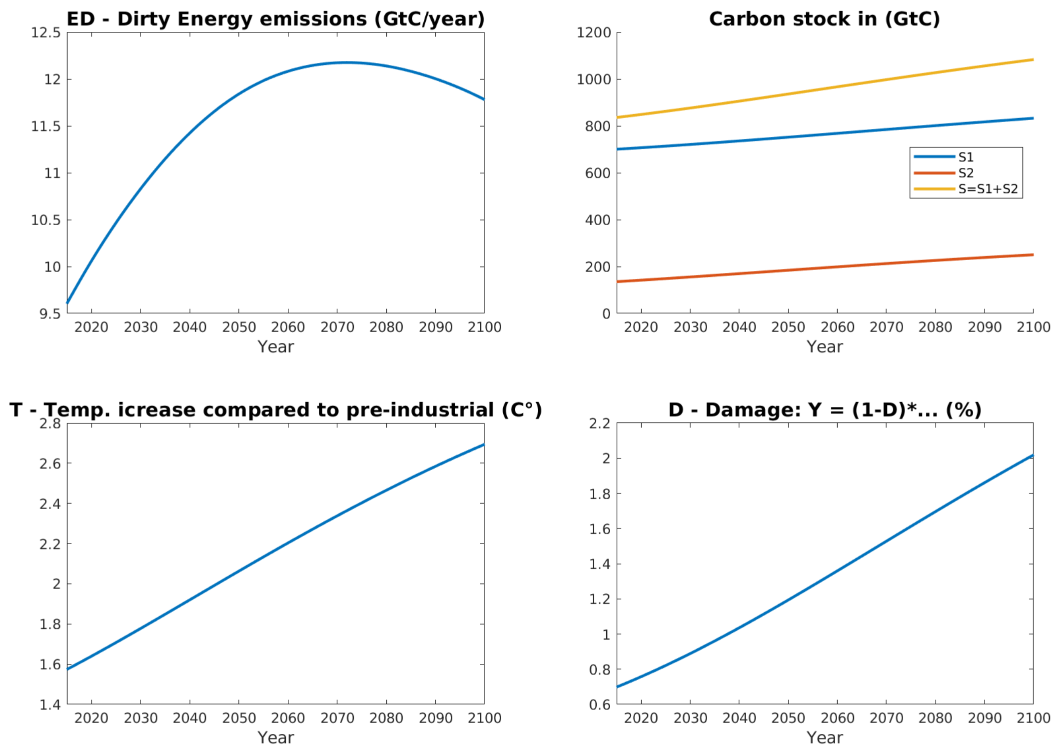
4.3 Policy Reform Scenarios

We now turn to the two policy reform scenarios, the introduction of carbon tax and a portfolio shift of the capital holdings by the monetary authority. We first study both scenarios in isolation before turning to a joint analysis.

Figure 6 shows the time path of the absolute amount of the carbon tax in Panel (a). We assume it is introduced in year 2020 at a level of 50 US dollars per gigaton of carbon emissions. Since the year 2020 price of dirty energy in our model is at 750 USD this corresponds to a tax rate of 6.6%. We hold this tax rate constant along the transition, $\tau_{ts=di} = 0.066$ which implies that the absolute amount of carbon taxation increases at the growth rate of the dirty energy price $p^e_{ts=di}$. By 2100 the absolute carbon tax reaches almost 180 USD.

Panel (b) of the same figure show the capital holdings of the monetary authority in the two sectors. We assume that in year 2020 there is a full shift towards capital holding in the clean intermediate goods sector. While this is, of course, an extreme assumption, it enables

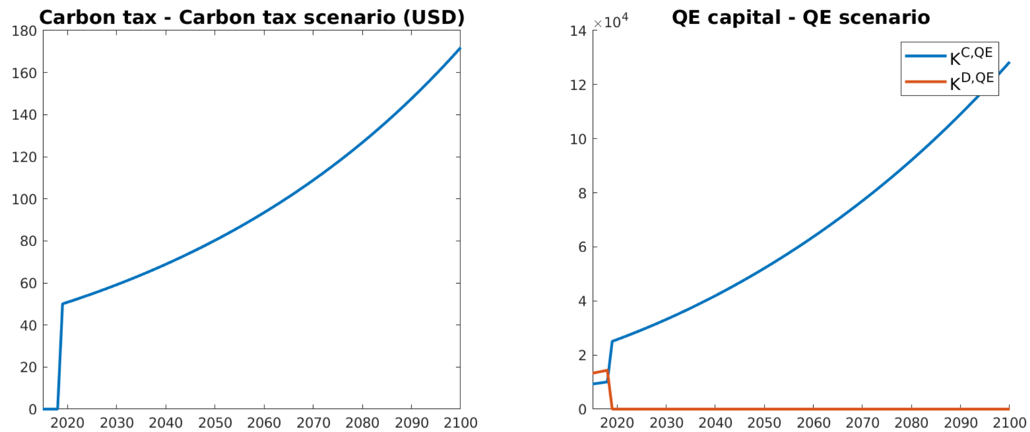
Figure 5: Baseline: Climate Variables



Notes: Climate variables: dirty energy emissions $E_{ts=di}$ in panel (a), carbon stocks $S_t, \{S_{tc}\}_{c\in\{ra,sl\}}$ in panel (b), world temperature T_t in degree Celsius in panel (c), and aggregate damage D_t (in percent) in panel (d).

us to investigate the effects of quantitative easing on climate change assuming a (hypothetical) situation where quantitative easing is at its maximum potency.

Figure 6: Reforms: Carbon Taxation and Portfolio Reallocation

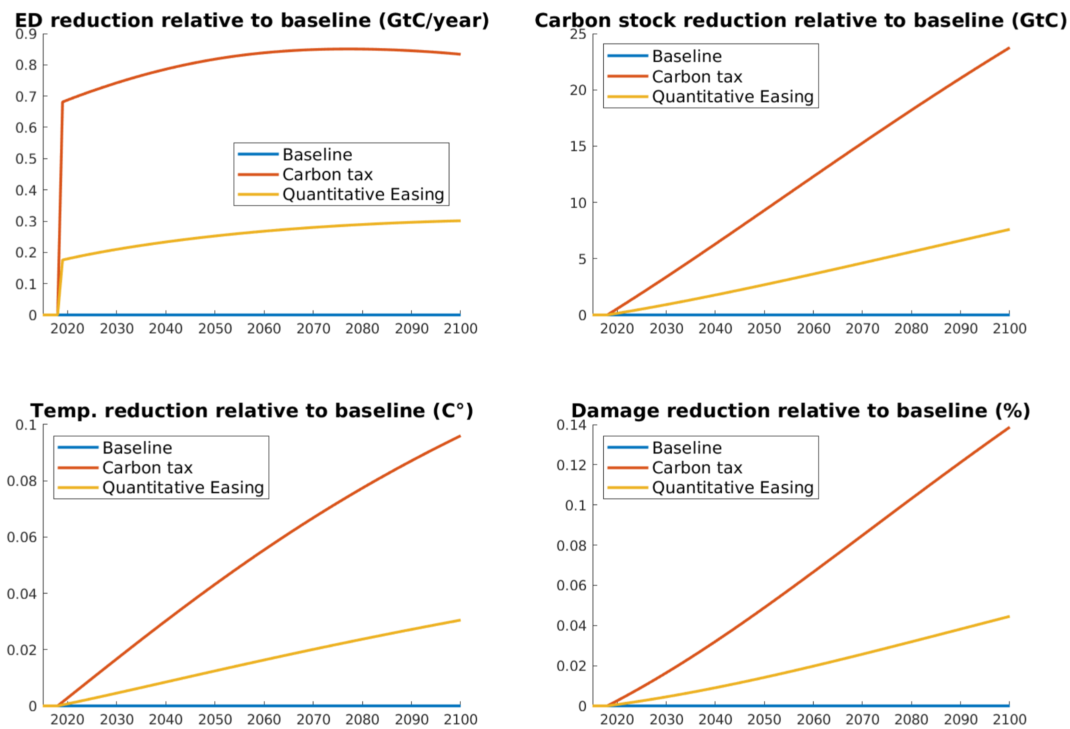


Notes: Policy reforms: carbon tax (in US dollars) in panel (a) and portfolio allocation of monetary authority in panel (b).

Figure 7 shows the key outcome variables of our experiments, in terms of changes relative to the baseline path. Turning to the reduction of global temperature we observe from panels (c) that the global temperature reduction in the carbon tax experiment is about 3.3 times larger than through quantitative easing. Carbon taxes through changing the relative price of dirty energy lead to a reduction of dirty energy production and thus a reduction of the increase in the global temperature through two mechanisms. First, the price increase leads to a substitution of dirty energy through clean energy in the production of intermediate goods, thus more clean intermediate goods are produced (supply side mechanism). Second, the price increase of dirty energy $p^e_{ts=di}$ increases the price of the dirty intermediate good $p_{ts=di}$ which leads to a substitution in the production of the final output away from dirty intermediate towards clean intermediate goods (demand side mechanism).

The portfolio reallocation of the monetary authority, in contrast, has a theoretically ambiguous effect on dirty energy demand. First, on impact, i.e., holding factor prices constant, a reduction of capital employed for production in the dirty intermediate goods sector and a simultaneous increase of capital in the dirty intermediate goods sector increases the marginal return on capital in the dirty and decreases it in the clean energy sector. This leads to an adjustment of private capital, which is reallocated from clean to dirty intermediate production and thus the portfolio reallocation by the monetary authority leads to a partial crowding out of private capital in the clean intermediate production. Also, the increased rate of return on capital in the dirty intermediate goods production increases capital costs for the intermediate goods firms leading to a substitution towards energy and labor employed in production. Thus, while output is reduced by the portfolio reallocation which also reduces energy demand in the dirty intermediate goods sector, this reduction in energy demand is partially muted by a substitution towards energy in production. Additionally, the increased capital costs of the firm leads to an increase of the intermediate goods price $p_{ts=di}$ which induces a substitution in the production of the final good towards the clean intermediate input and through this channel reduces the demand for energy. Quantitatively, it turns out that the energy demand reducing mechanisms dominate.

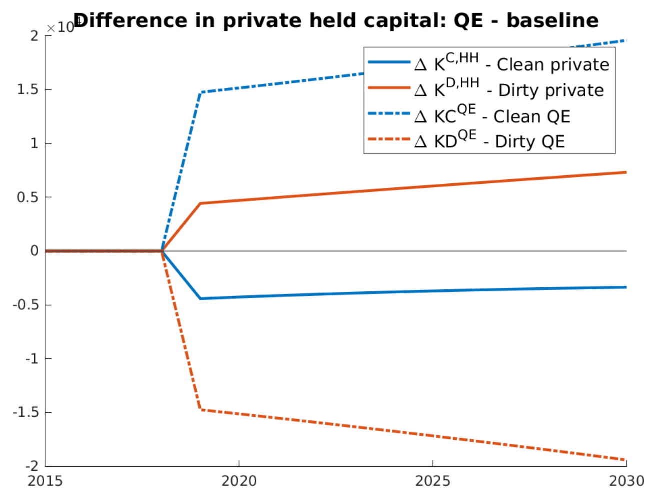
Figure 7: Reforms: Climate Variables



Notes: Policy reforms: dirty energy reduction relative to baseline in gigatons of carbon in panel (a), carbon stock reduction relative to baseline in gigatons of carbon in panel (b), temperature reduction in degrees of Celsius in panel (c), and damage reduction in percent in panel (d).

One key feature of the calibration of our two sector two physical assets model is the assumed zero correlation of the idiosyncratic returns across the two sectors. It implies that, out of portfolio choice motives holding capital in both sectors will provide a hedge to financial investors. This explains why quantitative easing in our model is not neutral: a portfolio reallocation by the central bank towards the clean sector does not induce a perfect crowding out of private capital in the clean sector, but leads to a partial crowding out only. To illustrate the extend of this partial crowding out in our model, Figure 8 shows the allocation of capital in both sectors, by the monetary authority as dashed lines and by the private sector as solid lines. As a consequence of the portfolio reallocation, capital holding by the monetary authority in the clean sector increase and in the dirty sector they decrease. In response to this private investors hold less capital in the clean sector, but this crowding out effect is much smaller than the additional capital held by central banks in the clean sector. Likewise, the substitution of private investors into dirty capital holdings is smaller than the reduction of capital in the dirty intermediate goods sector through the monetary authority. Thus the net effect of capital holdings in the clean intermediate goods sector is positive, and it is negative in the dirty intermediate goods sector.

Figure 8: Quantitative Easing: Why no crowding out?



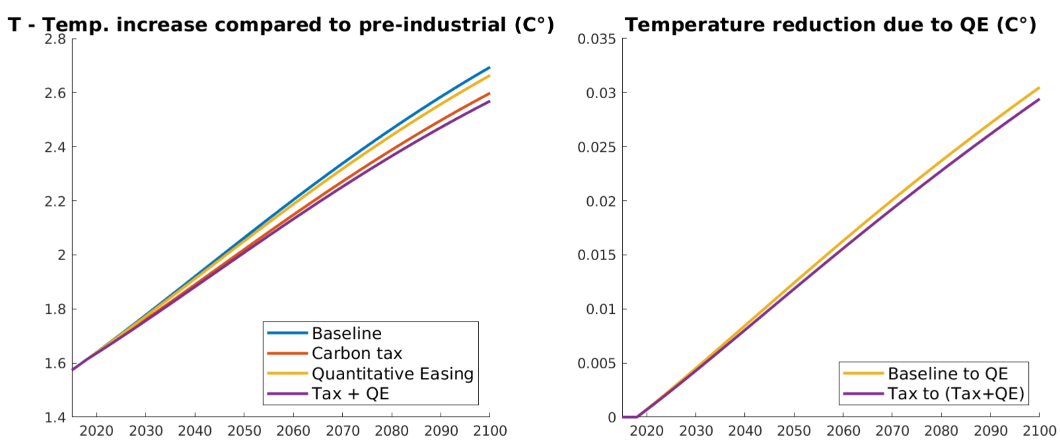
Notes: Policy reforms: Difference in dirty and clean sector capital holdings by private households and the monetary authority.

From the above analysis we have seen that quantitative easing has a much milder effect on key climate variables such as the global temperature increase than carbon taxation. We can thus conclude that relative to carbon taxation green quantitative easing is a relatively inefficient instrument to mitigate the adverse societal effects of climate change, at least as long as governments around the world pursue a path of introducing effective carbon taxes at a relatively large scale, as we have assumed. If not, then of course green quantitative easing plays a more important role.

A closely related question is whether it does harm, i.e., whether it could potentially reduce the effectiveness of carbon taxes. We therefore next consider both instruments jointly with results displayed in Figure 9. This shows that green quantitative easing has an additional climate change mitigating effect, and thus the concern is not warranted, see Panel (a) of the figure.

The next, and related, question is whether there is a positive interaction of both policy instruments in the sense that the climate change mitigating impact of carbon taxes is magnified when green quantitative easing is simultaneously at work. Our results in Panel (b) of the figure suggest that this also not the case. There we show the effects of quantitative easing, first, in isolation as above. This leads to a reduction of the global temperature by 0.03 degrees Celsius. We next compute the differential effects on the global temperature of a policy reform with positive carbon taxes plus quantitative easing net of the effects of a policy reform where carbon taxes are implemented in isolation. This leads to a lower increase of the global temperature by 0.028 degrees of Celsius and thus there is no positive interaction effect. The reason for the absence of a positive interaction effect is the substitution of input factors due to changes in the costs structure in production. Specifically, on the one hand, the carbon tax increases the costs of energy, leading dirty firms to substitute some energy with labour and capital. On the other hand, green quantitative easing by increasing the costs of capital leads to a substitution of some capital with labour and energy. In combination these effects partially offset each other.

Figure 9: Temperature Reduction with Both Policy Instruments



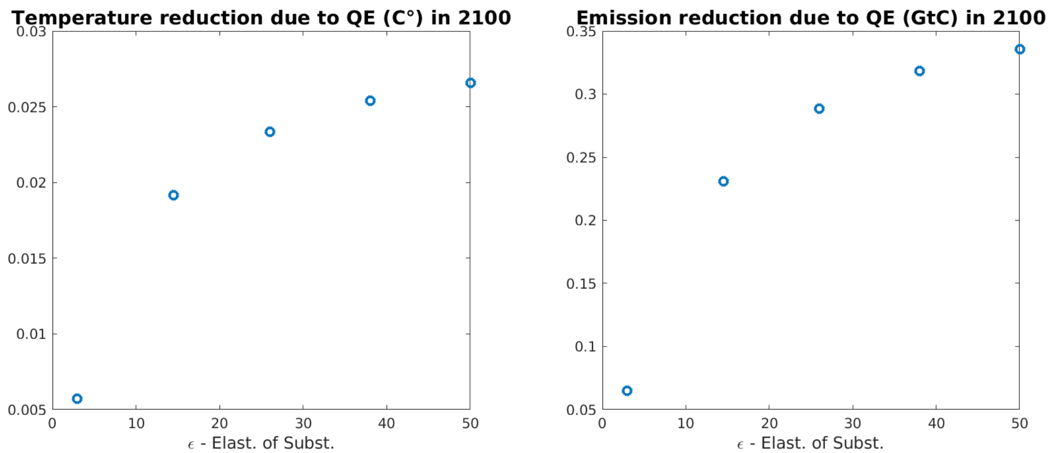
Notes: Policy reforms: Temperature compared to pre-industrial in degrees of Celsius in panel (a) for baseline, carbon tax, quantitative easing and carbon tax plus quantitative easing; panel (b) shows the temperature reduction from baseline to quantitative easing and from the carbon tax to the carbon tax plus quantitative easing.

4.4 Sensitivity Analysis

A key parameter in our model is the elasticity of final output in the two intermediate goods. As described above our baseline results assume a very high elasticity which we compute from the model by targeting the price elasticity of dirty energy input. Figure 10 shows results for lower values of the substitution elasticity than our baseline calibration of $\varepsilon = 26$. We see that the effects are monotonically increasing and concave in that parameter, which is quite intuitive as it governs the extend of substitution in production and hence the demand side mechanism described above.

Do our results with such a high elasticity of substitution of final goods give realistic orders of magnitude? We would argue that they do for the following reasons: First, we calibrate it by explicitly targeting price elasticities of energy demand. Second, ours is a long-run question and it is reasonable that elasticities of substitution across goods are close to perfect in the long-run.

Figure 10: Temperature Reduction under Green Quantitative Easing: The Role of the Substitution Elasticity



Notes: Policy reforms: temperature reduction for different values of the substitution elasticity ε in the production of the final good.

5 Concluding Discussion

We develop and calibrate a two-sector two-goods integrated assessment model to study the roles of green quantitative easing and carbon taxation for mitigating climate change. Green quantitative easing is modelled through an exogenous portfolio reallocation by the monetary authority. A key element of our model is differential risky returns in the two sectors so that this exogenous reallocation of capital does not lead to a perfect crowding out of private capital employed for production in the green sector. We consider a very strong quantitative easing experiments by assuming a complete reallocation of capital from a 50-50 split between dirty and clean towards the clean sector. As our baseline carbon tax scenario we consider an introduction of a carbon tax of 50 US dollars per gigaton of carbon emissions which grows exogenously at the price of carbon emissions such that the ad valorem tax stays constant.

Despite the rather extreme assumptions on green quantitative easing the effects of the policy on the global temperature increase are much milder—by a factor of about 3—than the carbon tax. We also find that pursuing a green quantitative easing policy on top of the introduction of the carbon tax leads to an additional climate change mitigation. Thus, while the effects of green quantitative easing are rather mild, they still have positive effects in a world where fiscal policy instruments are in place. On the other hand, we also do not find positive interaction effects. A green quantitative easing policy has a larger effect if used in isolation than when it comes on top of a carbon tax already in place.

In our analysis, we treat the amount of capital held by the monetary authority as given, respectively assume that it grows with the size of the economy. We thus implicitly assume—and in fact model—a persistent low interest rate environment so that (not explicitly modeled) repeated recessions call for an increase in the capital stock held by the monetary authority. We leave for future research an extension of our model towards an endogenously adjusting capital stock held by the central bank by quantitative easing policies, which requires extending our model by adding aggregate shocks and an explicit role for conventional and non-conventional monetary policy in recessions.

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A Analytical Derivations and Proofs

A.1 Intermediate Goods Demand

The final representative firm operates under perfect competition maximizing

$$\begin{aligned}&\max_{\{Y_{ts}\}_{s\in\{cl,di\}}}\left\{p_tY_t-\sum_{s\in\{cl,di\}}p_{ts}Y_{ts}\right\}\\&=\max_{\{Y_{ts}\}_{s\in\{cl,di\}}}\left\{p_t\cdot(1-D_t)\cdot\Upsilon_t\cdot\left(\sum_{s\in\{cl,di\}}\kappa_sY_{ts}^{1-\frac{1}{\varepsilon}}\right)^{\frac{1}{1-\frac{1}{\varepsilon}}}-\sum_{s\in\{cl,di\}}p_{ts}Y_{ts}\right\}\end{aligned}$$

which gives the price of intermediate good s as

$$\frac{p_{ts}}{p_t}=\kappa_s\left((1-D_t)\cdot\Upsilon_t\right)^{\varepsilon-1}\left(\frac{Y_t}{Y_{ts}}\right)^{\varepsilon},\ \text{for } s\in\{cl,di\}.$$

and thus the intermediate goods demand

$$Y_{ts}=\left(\frac{\kappa_s}{\frac{p_{ts}}{p_t}}\right)^{\varepsilon}\left((1-D_t)\cdot\Upsilon_t\right)^{\varepsilon-1}Y_t,\ \text{for } s\in\{cl,di\}.$$

and the price of the final good as

$$p_t=\frac{1}{(1-D_t)\Upsilon_t}\left(\sum_{s\in\{cl,di\}}\kappa_s^{\varepsilon}p_{ts}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}.$$

A.2 The Shock Distribution

The distribution of ζ_{tis}, Ψ is defined implicitly via the distribution of the gross return on capital. The gross return on capital is assumed to be log-normally distributed with

$$\log(1+r_{tis})\sim\mathcal{N}\left(\log(1+\mathbb{E}r_{ts})-\frac{\left(\sigma_s^{\zeta}\right)^2}{2},\left(\sigma_s^{\zeta}\right)^2\right),$$

where

$$\mathbb{E}r_{ts}=\int\mathbb{E}r_{tis}di=\Gamma(\psi_s,\alpha,\gamma)\cdot\frac{\alpha}{(1-\alpha)}\cdot p_{ts}\left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}}\cdot\left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}}-\delta_s$$

is the average marginal profit from an additional unit of capital. Given this distributional assumption we get $Var\left(r_{tis}\right) = (1+\mathbb{E}r_{ts})^2 \cdot \exp\left(\left(\sigma_s^{\zeta}\right)^2 - 1\right)$.

A.3 Proof of Proposition 1

The proof is by guess and verify using the method of undetermined coefficients. We start by showing linearity of policy functions in total wealth, which differs across all i through optimal portfolio shares $\hat{\alpha}^*_{tis}$. In a second step we show that $\hat{\alpha}^*_{tis} = \hat{\alpha}^*_{ts}$ for all i and thereby that $m^*_{tis} = m^*_{ts}$ for all i.

Proof. 1. Claims: The consumption policy function in each period t for household i is

$$c(w_{ti}) = m_{ti} w_{ti}$$

for some m_{ti} and the associated value function is

$$U(w_{ti}) = \varrho_{ti} w_{ti}$$

for some ϱ_{ti}.

2. Induction step: In any period t we get under the induction claim, writing $U(w_{ti}) = \varrho_{ti} w_{ti}$

$$U(w_{ti}) = \max_{c_{ti},\hat{\alpha}_{ti}} \left\{ \left(c_{ti}^{1-\upsilon} + \beta \left(\mathbb{E}_t \left[\left(\varrho_{t+1i} w_{t+1i} \right)^{1-\theta} \right] \right)^{\frac{1-\upsilon}{1-\theta}} \right)^{\frac{1}{1-\upsilon}} \right\}.$$

Using the resource constraint we get

$$\begin{aligned} U_{ti}(w_{ti}) &= \max_{c_{ti},\hat{\alpha}_{ti},w_{t+1i}} \left\{ \left(c_{ti}^{1-\upsilon} + \beta \left(\mathbb{E}_t \left[\left(\varrho_{t+1i} \left(w_{ti} - (1+\tau_t^c) p_t c_{ti} \right) R^p_{t+1i} \left(\{\hat{\alpha}^*_{tis}\}_{s\in\{cl,di\}} \right) \right)^{1-\theta} \right] \right)^{\frac{1-\upsilon}{1-\theta}} \right. \\ &= \max_{c_{ti},\hat{\alpha}_{ti}} \left\{ \left(c_{ti}^{1-\upsilon} + \beta \left(w_{ti} - (1+\tau_t^c) p_t c_{ti} \right)^{1-\upsilon} \left(\mathbb{E}_t \left[\left(\varrho_{t+1i} R^p_{t+1i} \left(\{\hat{\alpha}^*_{tis}\}_{s\in\{cl,di\}} \right) \right)^{1-\theta} \right] \right)^{\frac{1-\upsilon}{1-\theta}} \right) \right. \\ &= \max_{c_{ti},\hat{\alpha}_{ti}} \left\{ \left(c_{ti}^{1-\upsilon} + \beta \left(w_{ti} - (1+\tau_t^c) p_t c_{ti} \right)^{1-\upsilon} \Lambda_{t+1i} \right)^{\frac{1}{1-\upsilon}} \right\} \end{aligned}$$

where $\Lambda_{t+1i} \equiv \left(\mathbb{E}_t \left[\left(\varrho_{t+1i} R^p_{t+1} \left(\{\hat{\alpha}^*_{tis}\}_{s\in\{cl,di\}} \right) \right)^{1-\theta} \right] \right)^{\frac{1-\upsilon}{1-\theta}}$.

Take the first-order condition w.r.t c_{ti} to obtain

$$\begin{aligned} & c_{ti}^{-\upsilon} = \beta \left(w_{ti} - (1+\tau_t^c) p_t c_{ti} \right)^{-\upsilon} (1+\tau_t^c) p_t \Lambda_{t+1i} \\ \Leftrightarrow \qquad & c_{ti} = \left(w_{ti} - (1+\tau_t^c) p_t c_{ti} \right) \Xi_{t+1i} \end{aligned}$$

for

$$\Xi_{t+1i} = \left(\beta(1+\tau_t^c)p_t\Lambda_{t+1i}\right)^{-\frac{1}{v}},$$

and thus

$$c_{ti} = m_{ti}w_{ti}$$

where

$$m_{ti} = \frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}$$

Use this back in the objective to get

$$\begin{aligned}
U(w_{ti}) &= \left((m_{ti}w_{ti})^{1-v} + \beta\left(\mathbb{E}_t\left[\left(\varrho_{t+1i}(1-(1+\tau_t^c)p_t m_{ti})w_{ti}R^p_{t+1i}\left(\{\hat{\alpha}^*_{tis}\}_{s\in\{cl,di\}}\right)\right)^{1-\theta}\right]\right)^{\frac{1-v}{1-\theta}}\right)^{\frac{1}{1-v}} \\
&= \left((m_{ti})^{1-v} + \beta(1-(1+\tau_t^c)p_t m_{ti})^{1-v}\Lambda_{t+1i}\right)^{\frac{1}{1-v}} w_t \\
&= \left(\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-v} + \frac{\Xi_{t+1i}^{-v}}{(1+\tau_t^c)p_t}\left(\frac{1}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-v}\right)^{\frac{1}{1-v}} w_t \\
&= \left(\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-v}\left(1+\frac{1}{(1+\tau_t^c)p_t\Xi_{t+1i}}\right)\right)^{\frac{1}{1-v}} w_t \\
&= \left(\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-v}\frac{1+(1+\tau_t^c)p_t\Xi_{t+1i}}{(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{\frac{1}{1-v}} w_t \\
&= \left(\frac{1}{(1+\tau_t^c)p_t}\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-v}\frac{1+(1+\tau_t^c)p_t\Xi_{t+1i}}{\Xi_{t+1i}}\right)^{\frac{1}{1-v}} w_t \\
&= \left(\frac{1}{(1+\tau_t^c)p_t}\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{-v}\right)^{\frac{1}{1-v}} w_t \\
&= \left(\frac{1}{(1+\tau_t^c)p_t}m_{ti}^{-v}\right)^{\frac{1}{1-v}} w_t.
\end{aligned}$$

We therefore get

$$\varrho_{ti} = \left(\frac{1}{(1+\tau_t^c)p_t}m_{ti}^{-v}\right)^{\frac{1}{1-v}},$$

which is non-stochastic, and we can accordingly rewrite Λ_{t+1i} as

$$\Lambda_{t+1i} \equiv \frac{1}{(1+\tau^c_{t+1})p_{t+1}} m^{-\upsilon}_{t+1i} \left(\mathbb{E}_t \left[R^p_{t+1}\left(\{\hat{\alpha}^*_{tis}\}_{s\in\{cl,di\}}\right)^{1-\theta}\right]\right)^{\frac{1-\upsilon}{1-\theta}}$$

and thus

$$\begin{aligned}\Xi_{t+1i} &= \left(\beta\frac{(1+\tau^c_t)p_t}{(1+\tau^c_{t+1})p_{t+1}} \left(\mathbb{E}_t \left[R^p_{t+1}\left(\{\hat{\alpha}^*_{tis}\}_{s\in\{cl,di\}}\right)^{1-\theta}\right]\right)^{\frac{1-\upsilon}{1-\theta}}\right)^{-\frac{1}{\upsilon}} m_{t+1i} \\ &= \Theta\left(p_t, p_{t+1}, \tau^c_t, \tau^c_{t+1}, R^p_{t+1}\left(\{\hat{\alpha}^*_{tis}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) m_{t+1i}\end{aligned}$$

and thus

$$m_{ti} = \frac{\Theta\left(p_t, p_{t+1}, \tau^c_t, \tau^c_{t+1}, R^p_{t+1}\left(\{\hat{\alpha}_{tis}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) m_{t+1i}}{1+(1+\tau^c_t)\Theta\left(p_t, p_{t+1}, \tau^c_t, \tau^c_{t+1}, R^p_{t+1}\left(\{\hat{\alpha}_{tis}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) m_{t+1i}}.$$

3. Finally, from the FOC w.r.t. $\hat{\alpha}_{tis}$ we get

$$\frac{\partial\mathbb{E}_t \left[R^p_{t+1}\left(\{\hat{\alpha}_{tis}\}_{s\in\{cl,di\}}\right)^{1-\theta}\right]}{\partial\hat{\alpha}_{tis}} = 0$$

and we thus get $\hat{\alpha}^*_{tis} = \hat{\alpha}^*_{ts}$ for all i, which implies that $m_{tis} = m_{ts}$ for all i. Assuming that $R^p_{t+1}\left(\{\hat{\alpha}_{tis}\}_{s\in\{cl,di\}}\right)$ is distributed as log-normal we get as an approximation applying results in Campbell and Viceira (2002) that under the assumed cross-sectional independence of the returns

$$\hat{\alpha}^*_{ts} \approx \frac{\ln(1+\mathbb{E}\left[r_{t+1s}\right]) - \ln(1+r^f_{t+1})}{\theta\cdot Var(\ln(1+r_{t+1s}))},$$

□

B Calibration Appendix

B.1 Output Elasticity ε

Start from equation (8) and integrate out across all i to get using $\mathbb{E}[\zeta_{tis}] = 0$ that

$$\mathbb{E}r_{ts} = \Gamma(\psi_s, \alpha, \gamma)\cdot\frac{\alpha}{(1-\alpha)}\cdot p_{ts}\left(\frac{r^l_t}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}}\cdot\left(\frac{p^e_{ts}}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} - \delta_s$$

from which we get

$$p_{ts} = \left(\frac{1-\alpha}{\alpha\Gamma(\psi_s,\alpha,\gamma)}\right)^{\alpha\gamma} \cdot \left(\mathbb{E}r_{ts}+\delta_s\right)^{\alpha\gamma} r_t^{l\,(1-\alpha)\gamma} p_{ts}^{e\;1-\gamma} \tag{22}$$

and thus

$$\frac{p_{ts=cl}}{p_{ts=di}} = \left(\frac{\mathbb{E}r_{ts=cl}+\delta_{s=cl}}{\mathbb{E}r_{ts=di}+\delta_{s=di}}\right)^{\alpha\gamma} \left(\frac{p^e_{ts=cl}}{p^e_{ts=di}}\right)^{1-\gamma}. \tag{23}$$

From the demand for intermediate goods by the final firm (2) we get the intermediate goods demand ratio

$$\frac{Y_{ts=di}}{Y_{ts=cl}} = \left(\frac{\kappa_{s=di}p_{ts=cl}}{\kappa_{s=cl}p_{ts=di}}\right)^{\varepsilon}. \tag{24}$$

Using (23) in the above we obtain

$$\frac{Y_{ts=di}}{Y_{ts=cl}} = \Xi\left(\left\{\mathbb{E}r_{ts},\delta_s,\kappa_s\right\}_{s\in\{cl,di\}}\right)\left(\frac{p^e_{ts=cl}}{p^e_{ts=di}}\right)^{\varepsilon(1-\gamma)} \tag{25}$$

for some time varying $\Xi\left(\left\{\mathbb{E}r_{ts},\delta_s,\kappa_s\right\}_{s\in\{cl,di\}}\right)$.

Next, on the supply side for intermediate goods, we get from (21d) and (21e)

$$Y_{ts} = \frac{1}{1-\gamma}\frac{p^e_{ts}}{p_{ts}}E_{ts}$$

and using (22) in the above we obtain

$$\frac{Y_{ts=di}}{Y_{ts=cl}} = \Lambda\left(\alpha,\gamma,\left\{\mathbb{E}r_{ts},\delta_s,\Gamma(\psi_s,\alpha,\gamma)\right\}_{s\in\{cl,di\}},r_t^l\right)\frac{p^e_{ts=cl}}{p^e_{ts=di}}^{-\gamma}\frac{E_{ts=di}}{E_{ts=cl}} \tag{26}$$

for some time varying $\Lambda\left(\alpha,\gamma,\left\{\mathbb{E}r_{ts},\delta_s,\Gamma(\psi_s,\alpha,\gamma)\right\}_{s\in\{cl,di\}},r_t^l\right)$.

Combining the intermediate goods demand and supply side, i.e., equations (25) and (26), we thus get

$$\frac{E_{ts=di}}{E_{ts=cl}} = \frac{\Lambda\left(\alpha,\gamma,\left\{\mathbb{E}r_{ts},\delta_s,\Gamma(\psi_s,\alpha,\gamma)\right\}_{s\in\{cl,di\}},r_t^l\right)}{\Xi\left(\left\{\mathbb{E}r_{ts},\delta_s,\kappa_s\right\}_{s\in\{cl,di\}}\right)}\left(\frac{p^e_{ts=cl}}{p^e_{ts=di}}\right)^{\varepsilon(1-\gamma)+\gamma}. \tag{27}$$

Holding constant the (expected) returns $\{\mathbb{E}r_{ts}\}_{s\in\{cl,di\}}, r_t^l$ we thus find that the energy demand elasticity is given by

$$\eta_{\frac{E_{ts=di}}{E_{ts=cl}},\frac{p^e_{ts=di}}{p^e_{ts=cl}}} = \varepsilon\cdot(1-\gamma)+\gamma.$$