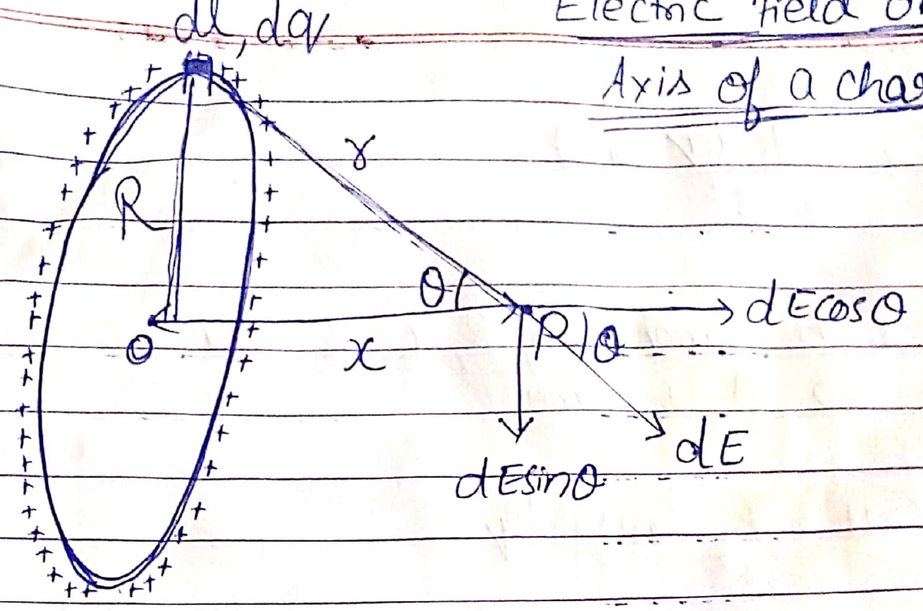
Class 12 Chapter 1 → Electric Charges and Fields 06

Lecture - 06

Electric Field on the Axis of a Charged Ring

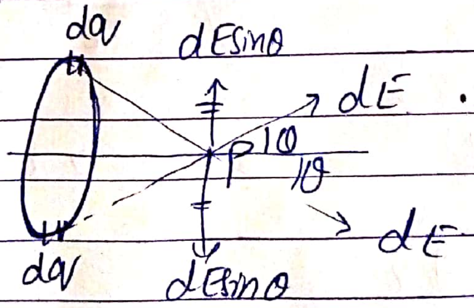


Charge 'Q' is uniformly distributed on a Ring of Radius 'R'. Find $\vec{E}$ at point 'P' on the axis of Ring, distance 'x' from centre of Ring.

Solution: we chose small element 'dq' and calculate $\vec{E}$ field due to 'dq'

$$\vec{dE} = \frac{K\,dq}{r^2}$$

Component dE sinθ will get cancelled out due to symmetry.



So $E_{net} = \int dE\cos\theta$

$$E_{net} = \int \frac{K\,dq}{r^2}\cos\theta$$

for each element r, R, x is same
$\Rightarrow \theta$ is same

$$E_{net} = \frac{K}{r^2}\cos\theta \int dq$$

$$\left[\int dq = Q\right]$$

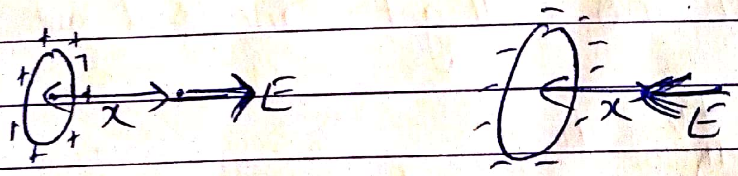
$$E_{net} = \frac{K}{r^2}\cos\theta\, Q$$

$$E_{net} = \frac{K}{r^2}\,\frac{x}{r}\,Q$$

$$E_{net} = \frac{KxQ}{r^3}$$

$$r^2 = R^2 + x^2$$
$$r = (R^2 + x^2)^{\frac{1}{2}}$$

$$\boxed{E_{net} = \frac{KQx}{(R^2+x^2)^{3/2}}}$$



if $x >>> R$ $\qquad x^2 + R^2 \longrightarrow x^2$

$$E_{net} = \frac{KQx}{x^3}$$

$$\boxed{E_{net} = \frac{KQ}{x^2}}$$ $\rightarrow$ Q behaves as point charge as x is large.

At centre $x = 0$, $E = 0$
$x \to \infty$ $E \to 0$

$$\boxed{E = \frac{KQ\,x}{(R^2+x^2)^{3/2}}}$$

Q) Find the value of x for which E is maximum. Calculate E_{max}. Draw a graph for E v/s x.

Sol: for $E \to max$

$$\frac{dE}{dx} = 0$$

$$KQ\left(\frac{x\,\frac{3}{2}(R^2+x^2)^{\frac{1}{2}}(2x) - (R^2+x^2)^{3/2}(1)}{(R^2+x^2)^3}\right) = 0$$

$$3x^2(R^2+x^2)^{\frac{1}{2}} - (R^2+x^2)^{3/2} = 0$$

$$(R^2+x^2)^{\frac{1}{2}}\left(3x^2 - (R^2+x^2)^1\right) = 0$$

$$\sqrt{R^2+x^2}\,(2x^2 - R^2) = 0$$

$\sqrt{R^2+x^2} = 0$ OR $2x^2 - R^2 = 0$

not possible

$$x^2 = \frac{R^2}{2}$$

$$\boxed{x = \pm\frac{R}{\sqrt{2}}}$$

$$E_{max} = \frac{KQx}{(R^2+x^2)^{3/2}}$$

$$x = \pm \frac{R}{\sqrt{2}}$$

$$= \frac{KQ\frac{R}{\sqrt{2}}}{\left(R^2+\frac{R^2}{2}\right)^{3/2}} = \frac{KQ\frac{R}{\sqrt{2}}}{\left(\sqrt{\frac{3R^2}{2}}\right)^3}$$

$$= \frac{KQ\frac{R}{\sqrt{2}}}{\frac{\sqrt{3}\times 3\ R^3}{\sqrt{2}\times 2}} = \frac{KQR\times 2}{\sqrt{3}\times 3\times R^3}$$

$$\boxed{E_{max} = \frac{2KQ}{3\sqrt{3}\,R^2}}$$

Graph of $\vec{E}$ v/s x

$x \to 0 \quad E \to 0$

$x \to \infty \quad E \to 0$

$x = \pm\frac{R}{\sqrt{2}} \quad E \to max$