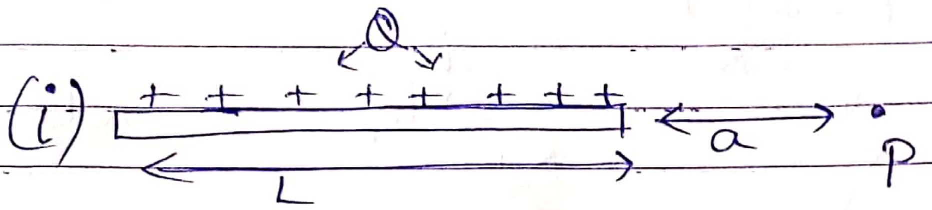
Class 12 Chapter-1 Lecture-6

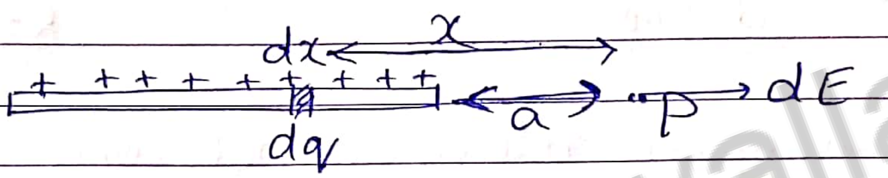
Electric charges and Fields 06

Electric Field due to Continous charge Distribution.

Linear charge distribution $\lambda = \frac{Q}{L}$

(i) 

Find $\vec{E}$ at P if Q is distributed uniformly over rod (L).



$\lambda = \frac{Q}{L}$

$dq = \lambda dx$

$\left(\lambda = \frac{dq}{dx}\right)$

$\vec{dE}$ due to small charge dq

$$\vec{dE} = \frac{K\,dq}{x^2}$$

$$E_{net} = \int dE = \int \frac{K\,dq}{x^2} = K\int_a^{a+L} \frac{dq}{x^2}$$

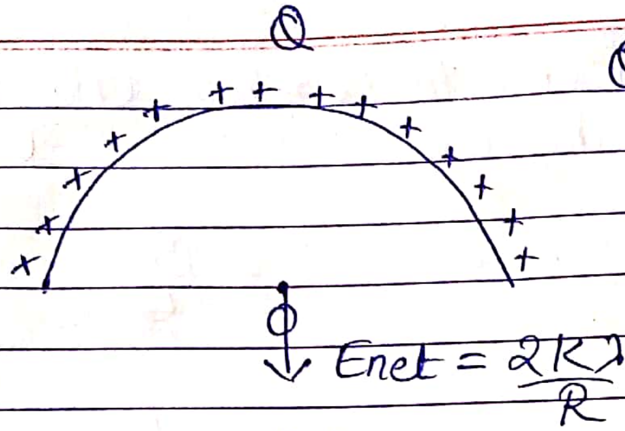
$$= K\int_a^{a+L} \frac{\lambda dx}{x^2} = K\lambda\left[-\frac{1}{x}\right]_a^{a+L}$$

$$= K\lambda\left[-\frac{1}{a+L} + \frac{1}{a}\right]$$

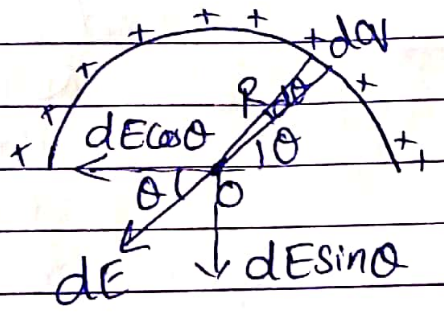
$$\vec{E}_{net} = \frac{KQ}{L}\left[\frac{1}{a} - \frac{1}{a+L}\right]$$

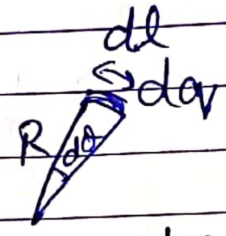
Note: $\vec{E}$ field on the axis of Ring $\Rightarrow$ Next Lecture.

(ii)



Q is distributed uniformly over a semi circular wire (radius R). Find $\vec{E}_{net}$ at O.





$\boxed{dl = Rd\theta}$

$$dE = \frac{Kdq}{R^2}$$

dE cosθ component will cancel out with symmetrically opposited dq.

$$E_{net} = \int dE\sin\theta = \int \frac{Kdq\sin\theta}{R^2}$$

$$= \frac{K}{R^2}\int dq\sin\theta$$

$$\lambda = \frac{Q}{L} = \frac{Q}{\pi R}$$

$$= \frac{K}{R^2}\int_0^{\pi} \lambda R d\theta \sin\theta$$

$$dq = \lambda dl$$
$$dq = \lambda R d\theta$$

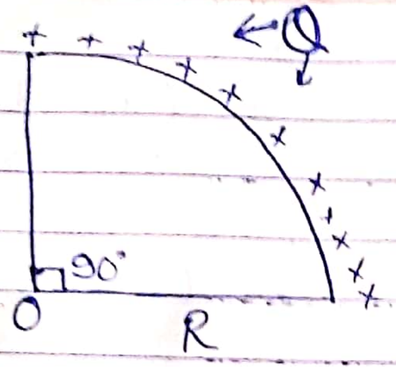
$$= \frac{K}{R^2}\lambda R\int_0^{\pi}\sin\theta\, d\theta$$

$$= \frac{K}{R^2}\times\lambda R\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{K\lambda}{R}\left[-\cos\pi + \cos 0^\circ\right]$$

$$= \frac{2K\lambda}{R} = \frac{2KQ}{\pi R^2}$$

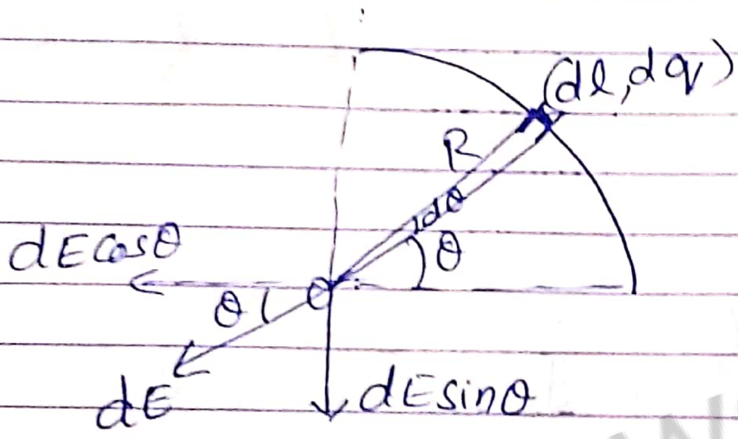
(iii)



Find E_{net} at O

$$\lambda = \frac{Q}{L} = \frac{Q}{\frac{\pi R}{2}}$$

$$= \frac{2Q}{\pi R}$$



dl, dq

$$dE = \frac{K\,dq}{R^2}$$

$$dl = R\,d\theta$$
$$dq = \lambda\,dl$$
$$\left(\lambda = \frac{dq}{dl}\right)$$

$$E_x = \int dE\cos\theta$$

$$= \int \frac{K\,dq}{R^2}\cos\theta$$

$$= \frac{K}{R^2}\int dq\cos\theta$$

$$= \frac{K}{R^2}\int \lambda\,dl\cos\theta$$

$$= \frac{K}{R^2}\int \lambda R\,d\theta\cos\theta$$

$$= \frac{K\lambda}{R}\int_0^{\pi/2}\cos\theta\,d\theta$$

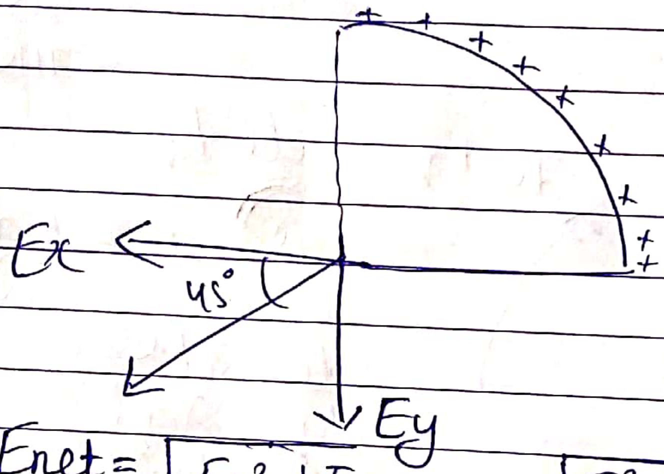
$$E_y = \int dE\sin\theta$$

$$=$$

$$= \frac{K\lambda}{R}\int_0^{\pi/2}\sin\theta\,d\theta$$

$$E_x = \frac{K\lambda}{R}\left[\sin\theta\right]_0^{\pi/2} \qquad E_y = \frac{K\lambda}{R}\left[-\cos\theta\right]_0^{\pi/2}$$

$$E_x = \frac{K\lambda}{R} \qquad E_y = \frac{K\lambda}{R}$$

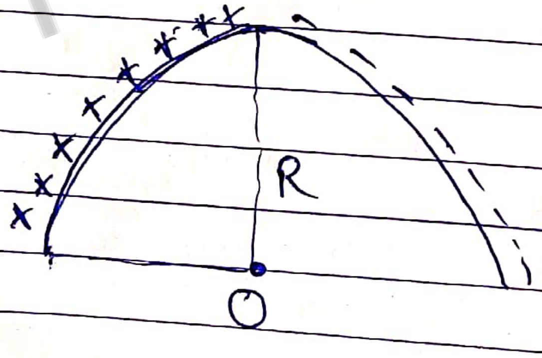


$E_x = E_y = E$

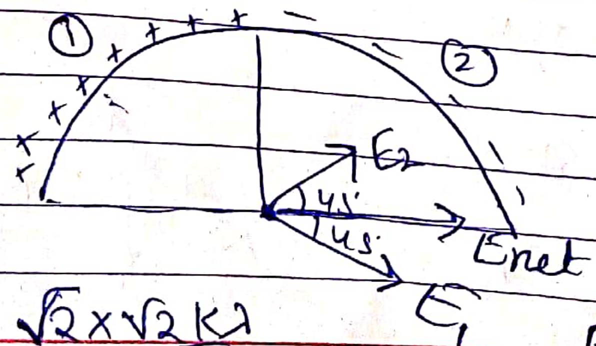
$$E_{net} = \sqrt{E_x^2 + E_y^2} = \sqrt{E^2+E^2} = E\sqrt{2}$$

$$\boxed{E_{net} = \sqrt{2}\,\frac{K\lambda}{R}} = \frac{2\sqrt{2}\,KQ}{\pi R^2} \qquad \left\{\lambda = \frac{2Q}{\pi R}\right]$$

(iv)



Find net $\vec{E}$ at O if 'λ' is linear charge density

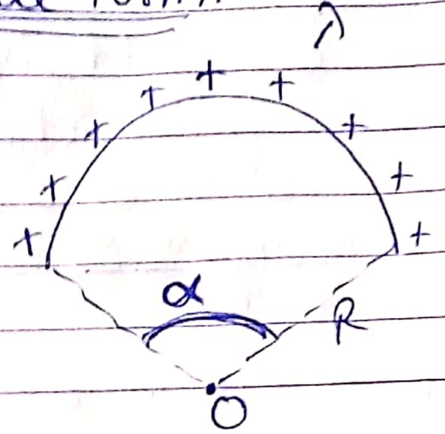


$$\left.\begin{aligned} E_1 &= \frac{\sqrt{2}K\lambda}{R} \\ E_2 &= \frac{\sqrt{2}K\lambda}{R} \end{aligned}\right\} = E$$

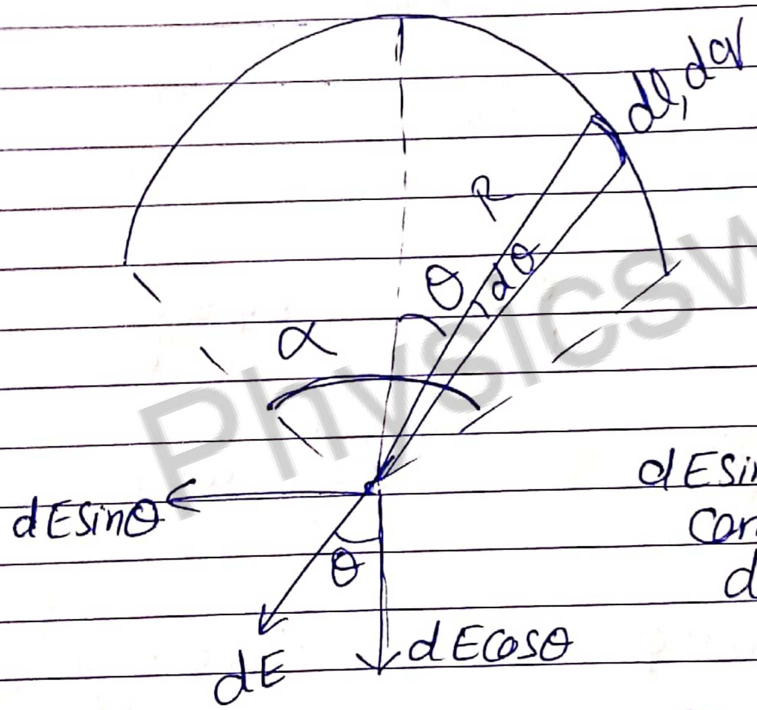
$$E_{net} = \sqrt{2}\times\sqrt{2}\frac{K\lambda}{R} = \frac{2K\lambda}{R}$$

$$E_{net} = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos 90} = \sqrt{2}\,E$$

General Form:

(v) 

Find $\vec{E}_{net}$ at O due to arc of Radius R and linear charge density λ.



$\lambda = \frac{dq}{dl}$

$dq = \lambda\, dl$

$dq = \lambda R\, d\theta$

dE sinθ will get cancelled due to opposite dq

$$E_{net} = \int dE\cos\theta = \int \frac{k\,dq}{R^2}\cos\theta$$

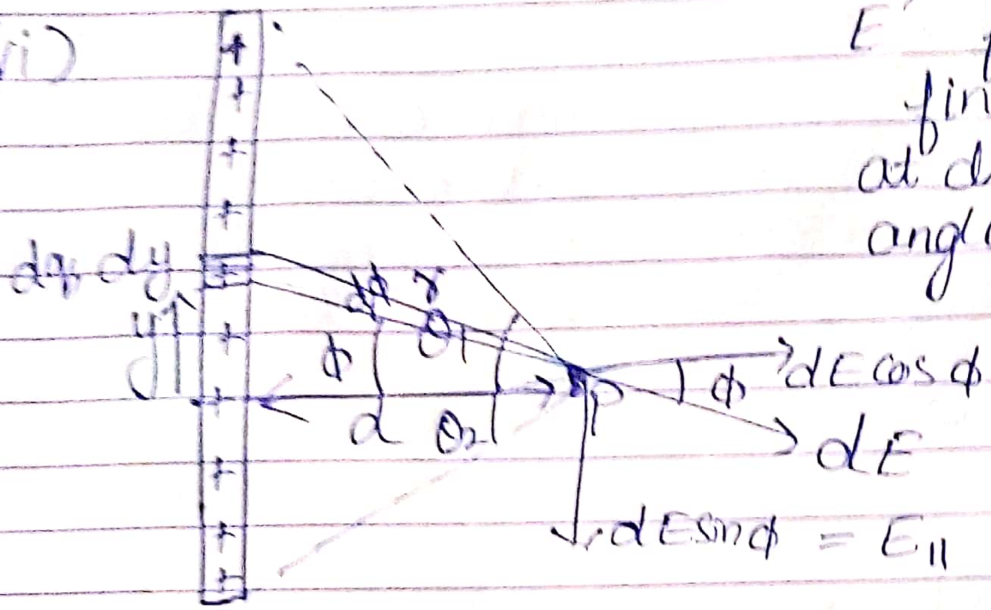
$$= \int \frac{k\lambda R\, d\theta \cos\theta}{R^2} = \frac{k\lambda}{R}\int_{-\alpha/2}^{+\alpha/2} \cos\theta\, d\theta$$

$$E_{net} = \frac{k\lambda}{R}\left[\sin\theta\right]_{-\alpha/2}^{\alpha/2} = \frac{2k\lambda}{R}\sin\frac{\alpha}{2}$$

For semicircle $\alpha = 180°$, For quad $\alpha = 90°$

$E = \frac{2k\lambda}{R}$, $E = \frac{\sqrt{2}k\lambda}{R}$

(vi)



$\vec{E}$ field due to a finite charged Rod at distance d from it angles θ_1 & θ_2 given.

λ = linear charge density

$$dE = \frac{K\,dq}{r^2}$$

$$= \frac{K\lambda\,dy}{r^2}$$

$$\cos\phi = \frac{d}{r}$$

$$r = \frac{d}{\cos\phi}$$

$$= \frac{K\lambda\,dy}{\frac{d^2}{\cos^2\phi}}$$

$$= \frac{K\lambda\,dy\cos^2\phi}{d^2}$$

$$\tan\phi = \frac{y}{d}$$

$$y = d\tan\phi$$

$$= \frac{K\lambda\,d\sec^2\phi\,d\phi\,\cos^2\phi}{d^2}$$

$$\frac{dy}{d\phi} = d\sec^2\phi$$

$$dE = \frac{K\lambda}{d}\,d\phi$$

$$dy = d\sec^2\phi\,d\phi$$

$$E_\perp = \int dE\cos\phi = \int_{-\theta_2}^{\theta_1} \frac{K\lambda}{d}\, d\phi \cos\phi$$

$$= \frac{K\lambda}{d}\int_{-\theta_2}^{\theta_1} \cos\phi \, d\phi$$

$$= \frac{K\lambda}{d}\Big[\sin\phi\Big]_{-\theta_2}^{\theta_1}$$

$$\boxed{E_\perp = \frac{K\lambda}{d}\left[\sin\theta_1 + \sin\theta_2\right]}$$

$$E_\parallel = \int dE\sin\phi = \int_{-\theta_2}^{\theta_1} \frac{K\lambda}{d}\, d\phi \sin\phi$$

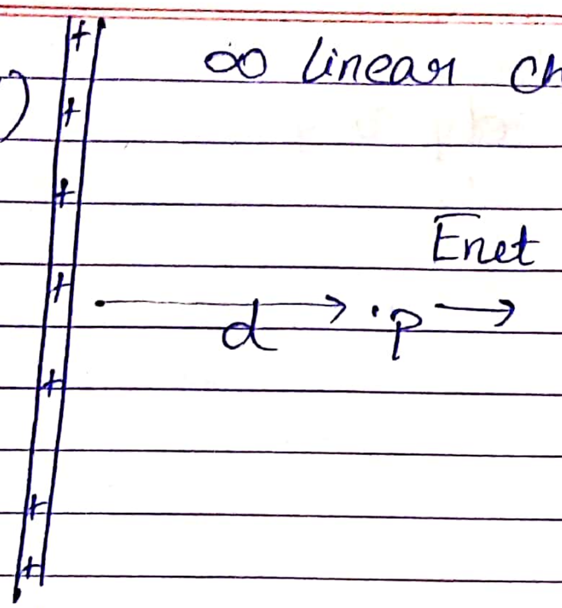
$$= \frac{K\lambda}{d}\Big[-\cos\phi\Big]_{-\theta_2}^{\theta_1}$$

$$= \frac{K\lambda}{d}\left[-\cos\theta_1 + \cos\theta_2\right]$$

$$\boxed{E_\parallel = \frac{K\lambda}{d}\left[\cos\theta_2 - \cos\theta_1\right]}$$

$$E_{net} = \sqrt{(E_\perp)^2 + (E_\parallel)^2}$$

(vii) ∞ linear charge density $\theta_1 = \theta_2 = 90^\circ$



$$E_{\perp} = \frac{K\lambda}{d}\left[\sin 90 + \sin 90\right]$$

$$E_{\perp} = \frac{2K\lambda}{d}$$

$$E_{\parallel} = \frac{K\lambda}{d}\left[\cos 90 - \cos 90\right]$$

$$E_{\parallel} = 0$$

$$\boxed{E_{net} = \frac{2K\lambda}{d}}$$