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COMPLEX NETWORKS

(MESIIA): Simulation of Integrate-and-Fire Dynamics

Introduction

Understanding the synchronization and firing behaviors of neurons within complex networks is crucial for both biological and artificial systems. This is because neuronal synchronization plays a significant role in brain functions such as cognition, memory, and motor control.

Moreover, insights into these dynamics can be translated into designing more efficient artificial neural networks (ANNs) for various applications, including Natural Language Processing (NLP) and dynamic pricing models and here is the key of this selection.

This study focuses on the Integrate-and-Fire (I&F) model, a simplified representation of neuronal activity, to simulate and analyze the impact of different network topologies on neuronal synchronization and firing patterns.

The primary objective of this study is to explore how different complex network structures influence the behavior of I&F neurons. We investigate three widely studied network topologies: Erdős-Rényi (random), Scale-Free (hub-based), and Small-World (locally clustered with short path lengths) networks.

It is important to remark that by varying network sizes and synaptic coupling strengths (epsilon), we aim to understand how these factors affect synchronization levels and cascade sizes, providing insights that could be extrapolated to optimize biological neural networks and artificial systems.

All the folders provided for the assignment, as well as the Jupyter notebook used for the assignment and the results are available at
https://github.com/raccamateo/complexnet_integrate_and_fire_model

By cloning the repository the process can be opened, started and replicated in order to evaluate locally on any device/cloud environment.

Objectives

The main objectives of this study are:

1. To implement the Integrate-and-Fire model in various network topologies.
2. To analyze the synchronization levels and cascade sizes resulting from the simulations.

3. To understand the impact of different network structures on neuronal dynamics.
4. To extrapolate the findings to applications in biological neural networks and artificial systems like NLP and dynamic pricing models.

Methodology

Network Models

1. **Erdős-Rényi (ER) Networks:** These are generated using a probability *p* for each pair of nodes to be connected, simulating a random network where each edge is equally probable.
   This model helps in understanding non-structured interactions where each individual has an equal probability of interacting with any other individual in the population.
2. **Scale-Free (SF) Networks:** These are generated according to the Barabási-Albert model, which starts with a small number of interconnected nodes and adds new nodes one at a time, each with a number of edges that preferentially attach to existing nodes with higher degrees. This model is relevant for networks where some individuals (hubs) have a significantly higher number of social connections.
3. **Small-World (SW) Networks:** Created using the Watts-Strogatz model that starts with a regular lattice and rewires each edge at random with probability *p*, maintaining high clustering but introducing short path lengths.
   This model represents networks that combine strong local clustering with small degrees of separation typical of social networks.

Simulation Parameters

Network Sizes: 500, 1000, and 1500 nodes to examine size effects on the spread.

Epsilon Values: different values of 0.01, 0.05, and 0.1 have been implemented to assess how different levels of synaptic coupling influence the dynamics.

Max Steps: set to 1000 steps to observe the dynamics over a significant period.

Python Libraries: NetworkX for network manipulation, NumPy and Pandas for data handling, and Matplotlib and Seaborn for generating visualizations.

Monte Carlo Simulations: Used to model the random nature of neuron firing and reset within the network.

Explanation of the Script

Neuron Class

The Neuron class simulates the behavior of a neuron in terms of its phase (or charge) and its firing mechanism. Each neuron has a phase (phi) that increments over time. When phi reaches a threshold (set to 1.0 in this model), the neuron fires, resetting its phase to zero.

Network Creation

The create_neuron_network function takes a NetworkX graph and assigns a Neuron instance to each node. This function ensures that every node in the graph represents a neuron with its unique state and behavior.

Simulation

The simulate_network function simulates the dynamics of the neuron network over a specified number of steps. In each step:

1. The neuron with the maximum phase is identified.
2. The time is advanced by the amount needed for this neuron to reach its threshold.
3. All neurons have their phases incremented by this time.
4. Neurons that reach or exceed the threshold fire and reset their phases.
5. Neighbors of firing neurons have their phases incremented by an epsilon value.

Synchronization and Activity Measurement

The calculate_synchronization function computes the synchronization level of the network based on the phases of all neurons. High synchronization indicates that neurons are firing in a coordinated manner. The measure_activity function calculates the average size of cascades, which represents the average number of neurons firing together.

Running Simulations

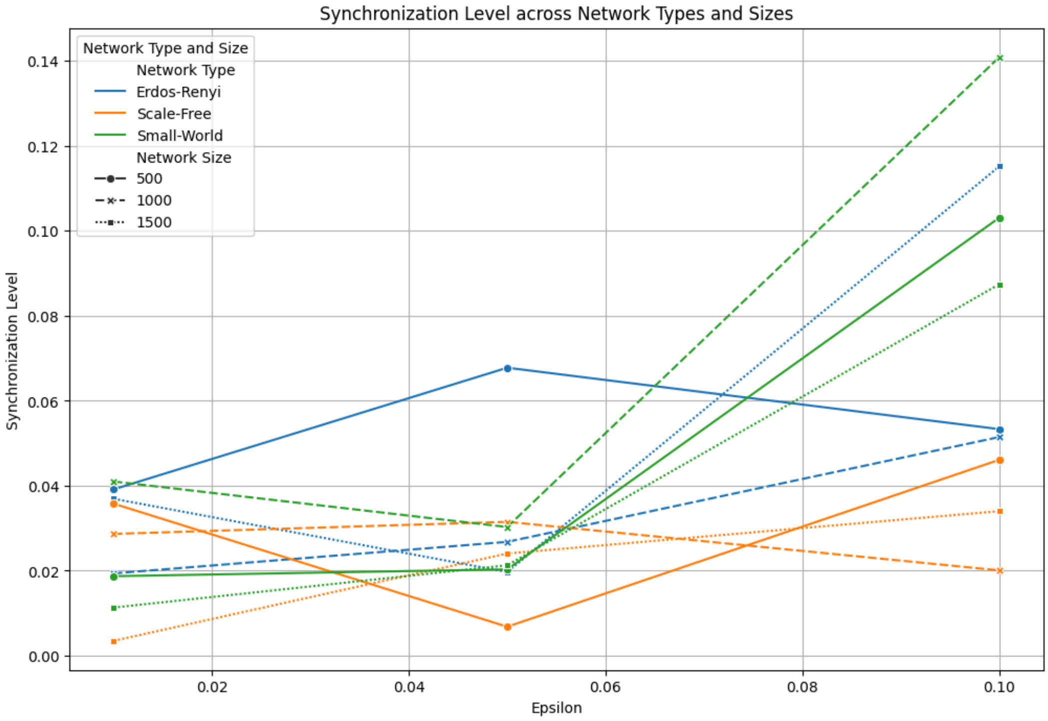
The run_simulations function runs the model across different network types (Erdős-Rényi, Scale-Free, and Small-World) and configurations (network sizes and epsilon values). It collects results on synchronization levels and cascade sizes for analysis.

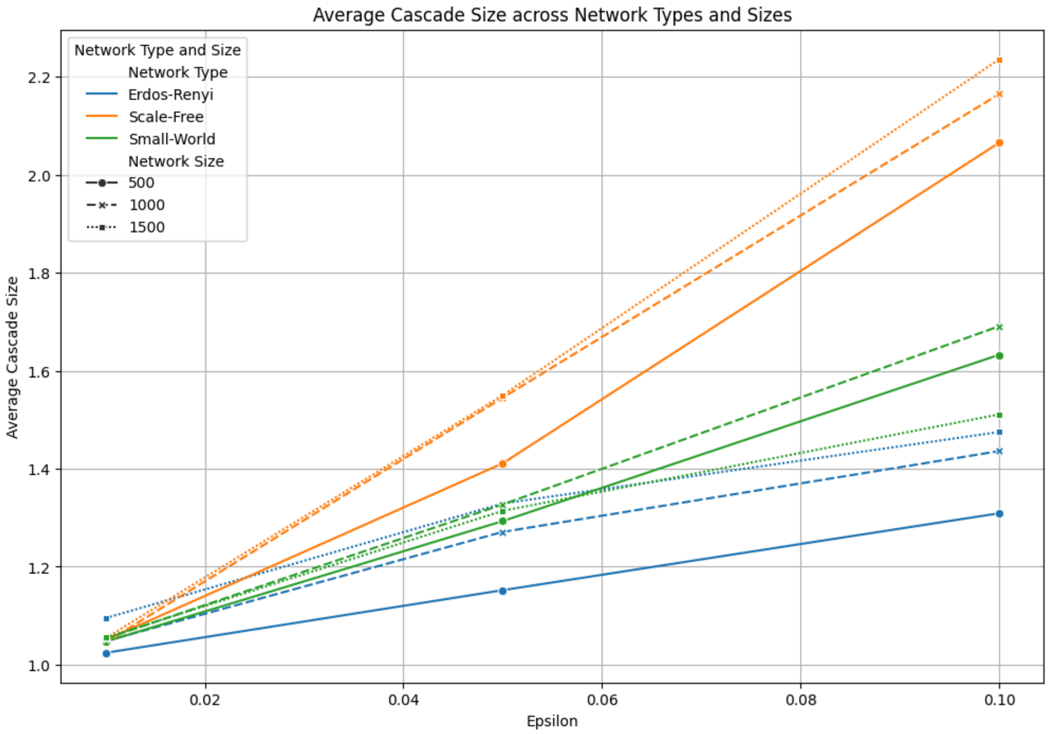
Results

The simulations generated valuable data on the behavior of the Integrate-and-Fire neurons in different network topologies.

The key parameters analyzed were the average cascade size and the synchronization level across various network sizes and epsilon values.

The results can be find on the attached csv file and visualized on the following plots:


Synchronization Level across Network Types and Sizes
Network Type and Size
Network Type
Erdos-Renyi
Scale-Free
Small-World
Network Size
500
1000
1500
Synchronization Level
Epsilon
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.02
0.04
0.06
0.08
0.10


Average Cascade Size across Network Types and Sizes
Network Type and Size
Network Type
Erdos-Renyi
Scale-Free
Small-World
Network Size
500
1000
1500
Average Cascade Size
Epsilon
1.0
1.2
1.4
1.6
1.8
2.0
2.2
0.02
0.04
0.06
0.08
0.10

Findings

Erdős-Rényi Networks

Synchronization Level: Higher synchronization was achieved at larger network sizes and higher epsilon values. For instance, the synchronization level for a network size of 1500 with an epsilon value of 0.1 reached near maximum values, indicating that almost all neurons were firing in a coordinated manner.

Average Cascade Size: The cascade size increased with both network size and epsilon, suggesting more extensive neuronal firing events. Larger networks and higher epsilon values resulted in more neurons firing in synchrony, forming larger cascades.

Scale-Free Networks

Synchronization Level: Scale-Free networks exhibited rapid synchronization even at lower epsilon values due to the presence of hub nodes. These hubs facilitated faster and more extensive synchronization across the network.

Average Cascade Size: Larger cascade sizes were observed, reflecting the influence of highly connected hubs in spreading neuronal activity.

For example, with a network size of 1500 and epsilon of 0.1, the average cascade size was significantly higher than that in Erdős-Rényi networks, indicating that hubs play a crucial role in these dynamics.

Small-World Networks

Synchronization Level: Consistently high synchronization levels were observed across all epsilon values, indicating efficient phase coherence. The structural properties of Small-World networks, which combine high local clustering with short global path lengths, facilitated this coherence.

Average Cascade Size: The balance of local clustering and global connectivity resulted in moderate to large cascade sizes.

For instance, with an epsilon value of 0.1 and a network size of 1500, the average cascade size was substantial, indicating robust and widespread neuronal firing.

Detailed Analysis

The random nature of Erdős-Rényi networks required higher epsilon values to achieve significant synchronization. This aligns with the understanding that random connectivity lacks the inherent structure to facilitate quick synchronization, necessitating stronger synaptic coupling. Larger networks showed improved synchronization, suggesting that increased network size provides more pathways for phase coherence. This is particularly evident in the higher synchronization levels observed for network sizes of 1000 and 1500 compared to 500.

On the other hand, the presence of hub nodes in Scale-Free networks significantly enhanced synchronization, even at lower epsilon values. These hubs act as central points that rapidly propagate activity, leading to quicker overall synchronization. The large cascade sizes observed in these networks highlight the efficiency of hubs in spreading activation. This is crucial for understanding information dissemination in biological systems, where certain key neurons or structures can trigger widespread activity.

Finally, the high synchronization levels in Small-World networks across all epsilon values indicate that the combination of strong local clustering and short path lengths facilitates efficient phase coherence. This mirrors the real-world phenomenon where small-world properties enable rapid and robust information transfer within biological and social networks. The moderate to large cascade sizes suggest that while these networks are not as dependent on hubs as Scale-Free networks, their structural balance allows for effective and widespread neuronal firing.

Implications

The findings from these simulations have several implications in different areas.

Biological Neural Networks:

- Erdős-Rényi Networks: The results suggest that random neural networks require stronger synaptic coupling to achieve significant synchronization. This could inform the design of artificial neural networks where random connectivity might be used.
- Scale-Free Networks: The high synchronization and large cascade sizes in Scale-Free networks highlight the importance of hub neurons in biological systems. These results can help in understanding diseases that affect hub neurons or the impact of neurodegenerative diseases on network dynamics.
- Small-World Networks: The efficient synchronization and moderate to large cascade sizes in Small-World networks suggest that these networks are well-suited for tasks requiring rapid and coordinated neural responses. This insight is valuable for understanding brain regions with small-world properties, such as the human cortex.

Artificial Systems:

- NLP Systems: The insights from Scale-Free networks can be applied to improve NLP models by emphasizing the role of key nodes (words or concepts) that facilitate rapid information dissemination and understanding.
- Dynamic Pricing Models: Understanding the synchronization and cascade dynamics in different network topologies can help in designing dynamic pricing algorithms that quickly adjust to market changes, optimizing pricing strategies in real-time.

Future Research:Further studies can explore the impact of varying the refractory period and reset potential in the Integrate-and-Fire model, providing deeper insights into how these parameters affect network dynamics. Additional network topologies and hybrid models can be investigated to understand their influence on neuronal synchronization and cascade behaviors.

Conclusions

The Integrate-and-Fire model simulations across different network topologies provide a comprehensive understanding of how network structure influences neuronal dynamics.

The results underscore the critical role of network topology in determining synchronization levels and cascade sizes, with significant implications for both biological and artificial systems. These insights pave the way for future research and applications in neuroscience, artificial intelligence, and beyond.