

# PMARK: TOWARDS ROBUST AND DISTORTION-FREE SEMANTIC-LEVEL WATERMARKING WITH CHANNEL CONSTRAINTS

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## ABSTRACT

Semantic-level watermarking (SWM) for large language models (LLMs) enhances watermarking robustness against text modifications and paraphrasing attacks by treating the sentence as the fundamental unit. However, existing methods still lack strong theoretical guarantees of robustness, and reject-sampling-based generation often introduces significant distribution distortions compared with unwatermarked outputs. In this work, we introduce a new theoretical framework on SWM through the concept of proxy functions (PFs) – functions that map sentences to scalar values. Building on this framework, we propose PMARK, a simple yet powerful SWM method that estimates the PF median for the next sentence dynamically through sampling while enforcing multiple PF constraints (which we call channels) to strengthen watermark evidence. Equipped with solid theoretical guarantees, PMARK achieves the desired distortion-free property and improves the robustness against paraphrasing-style attacks. We also provide an empirically optimized version that further removes the requirement for dynamical median estimation for better sampling efficiency. Experimental results show that PMARK consistently outperforms existing SWM baselines in both text quality and robustness, offering a more effective paradigm for detecting machine-generated text. Our code will be released at [this URL](#).

## 1 INTRODUCTION

The rapid advancements of generative AI (GenAI) techniques (Achiam et al., 2023; Team et al., 2025; Rombach et al., 2022; Liu et al., 2024; Brooks et al., 2024) have transformed content creation across diverse fields (Mittal et al., 2024; Chen et al., 2025; Yang et al., 2024), raising significant concerns regarding the traceability of AI-generated text and copyright protection (Zhao et al., 2024; Wu et al., 2025). Watermarking (Kirchenbauer et al., 2023; Aaronson, 2023), which embeds distinctive patterns into generated content, has emerged as a critical solution to these challenges.

Token-level watermarking schemes for text generation have been widely studied. The popular Green-Red scheme (Kirchenbauer et al., 2023; Zhao et al., 2023a) biases token sampling toward a “green” subset, enabling watermark detection via statistical tests on the frequency of green tokens. However, Green-Red watermarking schemes are inherently *biased* (Hu et al., 2023), meaning that they deviate from the original sampling distribution of LLMs and may degrade text quality (Hu et al., 2023). Distortion-free methods such as Gumbel Watermarking (Aaronson, 2023) and PRC-based schemes (Christ & Gunn, 2024) have also been explored, typically associated with cryptographic techniques such as digital signatures (Rivest et al., 1978).

Unfortunately, token-level watermarks can be easily removed: an attacker can simply ask an unwatermarked model to rephrase the generated text while preserving most of its semantic information (Hou et al., 2023). To improve robustness against such attacks, semantic-level watermarking (SWM) approaches like SemStamp (Hou et al., 2023; 2024) treat a semantically complete sentence as the fundamental watermarking unit. These methods employ rejection sampling to ensure that generated sentences fall within a valid semantic region of the embedding space, analogous to the green list in Green-Red watermarking. Nonetheless, this split-and-reject paradigm inherits the distortion

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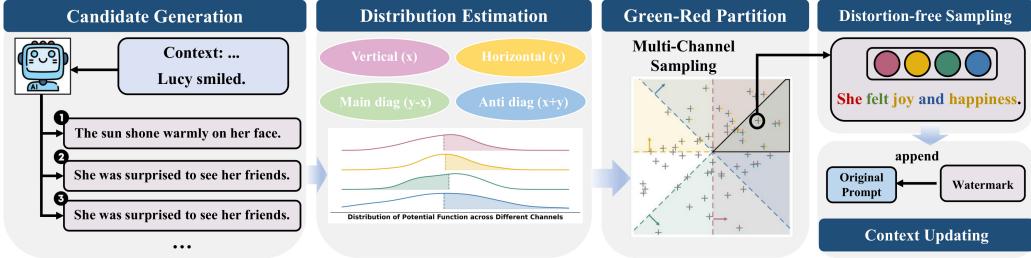


Figure 1: Illustration of PMARK pipeline in 2D space, with robustness enhanced by multi-channel constraints. Note that we use orthogonal pivots and distortion-free partition in practice.

drawback of Green-Red watermarking and also introduces additional weaknesses, such as sampling failures when all candidate sentences fall in invalid regions (Zhao et al., 2024).

In this work, we propose a new semantic-level watermarking scheme named PMARK with the distortion-free property. To achieve this goal, we first establish a solid framework for analyzing SWM schemes based on a core concept called the *proxy function* (PF). Within this framework, PMARK defines the PF of a sentence as the cosine similarity between its embedding and a pre-defined random vector. The key idea behind PMARK is that, given knowledge of certain statistical properties of the PF distribution, we can quantify the probability mass of the valid region and perform a sampling process with a strong theoretical guarantee of being distortion-free. We further incorporate multiple channel constraints to enhance the density of watermarking evidence, as illustrated in Figure 1. Leveraging the fact that random vectors in high-dimensional spaces are almost always orthogonal (Ruppert, 2004), we show that even when simply estimating the median PF as zero, PMARK still achieves high text quality with an efficient sampling process.

Empirically, our evaluation demonstrates that while PMARK preserves high text quality (PPL 4.37–4.71), it achieves significantly better robustness against paraphrasing attacks using GPT Paraphraser (OpenAI, 2022), with improvements of up to 14.8% and 44.6% over the prior best semantic-level and token-level watermarking schemes, respectively. In terms of sampling efficiency, the online version of PMARK requires only 20% of the resources (measured in token consumption) compared to prior SOTA semantic-level watermarking schemes, while the offline version exhibits even lower consumption, paving the way for real-world deployment of PMARK in practice.

#### Key contributions of this paper include:

- ① We propose a novel theoretical framework that unifies existing SWMs via the introduction of the proxy function, providing solid analytical foundations for performance evaluation.
- ② We identify that sparse watermarking evidence in SWMs negatively impacts adversarial robustness, and address this problem by introducing multiple channel constraints.
- ③ We introduce an online version of PMARK that achieves high robustness and is theoretically distortion free under mild prerequisites; we also present an offline version that reduces computational cost while maintaining low distortion.
- ④ We conduct comprehensive experiments across a variety of text watermarking tasks across multiple datasets and backbones, validating the effectiveness and efficiency of our method.

## 2 PRELIMINARIES

In this section, we delve into the problem of Semantic-level Watermarking (SWM), which treats semantically complete sentences as the fundamental unit. We begin by formalizing the problem and defining key concepts, inspired by Hu et al. (2023). See Appendix B for detailed proofs.

**Zero- and Multi-bit Watermarks.** We write  $\mathcal{M}(\pi) \rightarrow y$  to denote the process of sampling a response  $y$  from the model given an input prompt  $\pi$ . A watermark generation algorithm is denoted as  $\text{Generation}_k^{\mathcal{M}}(\pi) \rightarrow y$ , where  $k$  is the watermark generation key (Zhao et al., 2024). A detection algorithm determines whether a given text  $y$  is watermarked, denoted as  $\text{Detect}(y) \rightarrow \{\text{True}, \text{False}\}$ . In this work, we focus on *zero-bit* watermarks, which embed a single detectable signal (e.g., True or False) into text. In contrast, the concurrent **SAEMark** (Yu et al., 2025) investigates *multi-bit* watermarking, which embeds a message  $m \in \{0, 1\}^m$  into text (Lau et al., 2024). A detailed discussion of concurrent studies and related work can be found in Appendix C.

**Problem Setup.** Let  $\Sigma$  denote the vocabulary set of LLM. We define  $\Sigma^*$  as the set of all semantically complete sentences, including the null sentence of length zero. For any  $s \in \Sigma^*$ , let  $\bar{s}$  denote

its token sequence,  $\bar{s} = [x_1^s, \dots, x_{|s|}^s] = \text{tokenize}(s)$ .

At each generation step, the probability of producing the next token  $x_{n+1} \in \Sigma$  given the current context  $x_1, \dots, x_n$  is denoted by  $P_M(x_{n+1} | x_1, \dots, x_n)$ . The joint probability of generating a sequence of  $m$  tokens  $x_{n+1}, \dots, x_{n+m}$  is expressed as:

$$P_M(x_{n+1}, \dots, x_{n+m} | x_1, \dots, x_n) = \prod_{i=1}^m P_M(x_{n+i} | x_1, \dots, x_n, x_{n+1}, \dots, x_{n+i-1}). \quad (1)$$

In the context of SWM, we focus on the probability distribution of the next sentence  $s_{n+1} \in \Sigma^*$  given the preceding context  $s_1, \dots, s_n$ , denoted as:

$$P_M(s_{n+1} | s_1, \dots, s_n) = \prod_{i=1}^{|s|} P_M(x_i^s | \bar{s}_1, \dots, \bar{s}_n, x_1^s, \dots, x_{i-1}^s). \quad (2)$$

Here  $P_M(s) : \Sigma^* \rightarrow [0, 1]$  is a probability mass function over the countable set  $\Sigma^*$ .

For watermarking, we introduce a private key  $k$  drawn from a key space  $K$ , where  $k \in K$  is randomly sampled from a prior distribution  $P_K(k)$ . The watermarked output of the LLM follows a conditional distribution  $P_M^w(s_{n+1} | s_1, \dots, s_n; k)$ , which depends on the preceding context and  $k$ .

**Sampling-based Semantic-level Watermarking.** Directly modeling  $P_M(s_{n+1} | s_1, \dots, s_n)$  is generally intractable since  $\Sigma^*$  is infinite. A common approach adopted by SWM methods is to sample a set of i.i.d. candidate sentences  $x_1, \dots, x_N \sim P_M(s_{n+1} | s_1, \dots, s_n)$ , and then select a candidate  $Y$  that carries watermark evidence,  $Y_k = \text{SWM}_k(\{x_1, \dots, x_N\})$ . The distribution of the watermarked output is then defined as the distribution of  $Y$ :

$$Y_k \sim P_M^w(s_{n+1} | s_1, \dots, s_n; k). \quad (3)$$

**Definition 1 (Single-sentence Distortion-Free).** Given a context  $\pi = [s_1, \dots, s_n]$ , we say that the watermarked LLM distribution is single-sentence distortion-free relative to the original LLM  $P_M$  if, for all  $s_{n+1} \in \Sigma^*$ ,

$$P_M(s_{n+1} | \pi) = \sum_{k \in K} P_K(k) \cdot P_M^w(s_{n+1} | \pi; k). \quad (4)$$

This condition means that the marginal distribution of  $s_{n+1}$ , when averaged over all possible watermark keys  $k$ , is identical to its unwatermarked distribution under the original model.

**Definition 2** (Equivalence via Watermark Code Space). In most cases,  $P_K(k)$  is designed to induce uniform random sampling over a watermark code space  $E$ , where  $e_K \sim \text{Uniform}(E)$ . Under this construction, the right side of equation 4 can be equivalently rewritten as:

$$\sum_{k \in K} P_K(k) \cdot P_M^w(s_{n+1} | \pi; k) = \sum_{e \in E} P(e_K = e) \cdot P_M^w(s_{n+1} | \pi; e). \quad (5)$$

**Definition 3** (Probability Measure on  $\Sigma^*$ ). Let  $(\Sigma^*, \mathcal{G}, \mu_M)$  be a probability space, where  $\mathcal{G}$  is a  $\sigma$ -algebra over  $\Sigma^*$ , and  $\mu_M$  is the probability measure induced by the model  $P_M : \Sigma^* \rightarrow [0, 1]$ . For any measurable subset  $A \subseteq \Sigma^*$ , the measure  $\mu_M(A)$  is defined as

$$\mu_M(A) = \sum_{s \in A} P_M(s).$$

### 3 REVISITING SEMANTIC-LEVEL WATERMARKING

#### 3.1 PROXY FUNCTION

Section 2 discusses the sampling-based paradigm of SWM, which addresses the challenge of operating over the infinite set  $\Sigma^*$ . However, to enable rule-based selection among a candidate sentence set  $x_1, \dots, x_N \in \Sigma^*$ , SWM methods require a concrete mapping from the sentence space to the real numbers, since direct computation in the infinite textual space is generally intractable.

**Proxy Function.** To this end, we define a function  $\mathcal{F} : \Sigma^* \rightarrow \mathbb{R}$ , referred to as the *proxy function*, which assigns a real-valued score to each semantically complete sentence. Let  $U$  denote the range of the proxy function  $\mathcal{F}$ . We define a partition of the range  $U$  as  $\mathcal{P}(U)$  (see Definition 4). Each element  $u \in U$  belongs to exactly one subset  $A_i \in \mathcal{P}(U)$ , denoted as  $A_i = \mathcal{P}_u(U)$ .

We also define the inverse mapping of  $\mathcal{F}$ , denoted by  $\mathcal{F}^{-1} : U \rightarrow 2^{\Sigma^*}$ , where  $2^{\Sigma^*}$  is the power set of  $\Sigma^*$ . Each value  $u \in U$  thus maps to the set of sentences in  $\Sigma^*$  that share the same proxy value under  $\mathcal{F}$ , denoted as  $\mathcal{F}^{-1}(u)$ . Furthermore, the watermark code space is denoted as  $E \in 2^{\mathcal{P}(U)}$ , whose element  $e$  is selected by the watermarking scheme according to a prior distribution  $e_K \sim P_K(E)$  induced by the key  $P_K(k)$ .

### 3.2 ANALYSIS OF THE BASELINE

**Proxy Function of SemStamp.** We first focus on a simplified generation process of SemStamp, where the original margin constraint is removed for clarity. In SemStamp (Hou et al., 2023), the proxy function is defined as:

$$\mathcal{F}(s) = \text{LSH}(\mathcal{T}(s)) = [\text{LSH}_1(\mathcal{T}(s)) \| \cdots \| \text{LSH}_h(\mathcal{T}(s))], \quad (6)$$

where  $\text{LSH}_i(v) = \text{sign}(t_i \cdot v)$  (Indyk & Motwani, 1998; Charikar, 2002),  $\{t_i \mid i = 1, \dots, h\} \subset \mathbb{R}^d$  is a set of randomly initialized vectors,  $\mathcal{T}$  is a text encoder with  $\mathcal{T}(s) : \Sigma^* \rightarrow \mathbb{R}^d$ , and  $[ \cdot ]$  denotes the transformation from binary representation to its corresponding decimal value. Given that the range of  $\mathcal{F}$  is  $U = \{1, \dots, 2^h\}$ , the partition used in SemStamp is defined as  $\mathcal{P}(U) = \{\{1\}, \dots, \{2^h\}\}$ .

**Sampling-then-selecting Process.** We now reformulate the rejection-sampling process of SemStamp as a sampling-then-selecting process, without loss of generality:

- Given a context  $\pi$ , SemStamp samples  $N$  candidate sentences from the natural distribution:  $W = \{x_1, \dots, x_N\}$ ,  $x_i \sim P(s \mid \pi)$ ,  $i = 1, \dots, N$ .
- For a fixed green ratio  $\gamma$  such that  $\gamma|\mathcal{P}(U)| \in \mathbb{N}_+$ , the watermark code space is defined as  $E = \{e \in 2^{\mathcal{P}(U)} \mid |e| = \gamma|\mathcal{P}(U)|\}$ . The watermark code  $e_K$  is sampled uniformly at random from  $e_K \sim \text{Un}(E)$ , implying that each element of  $\mathcal{P}(U)$  is selected with probability  $\gamma$ . The *green region* of  $\Sigma^*$  is then defined as  $G = \{s \in \Sigma^* \mid \mathcal{F}(s) \in \bigcup e\}$ , while the set of valid candidates is  $V = \{x_i \mid \mathcal{F}(x_i) \in \bigcup e\}$ . We assume that  $V$  is always non-empty.
- A watermarked sentence  $Y$  is then uniformly selected from  $V$ , where the distribution of the watermarked output is  $Y \sim P_M^w(s \mid \pi)$ .

**Distortionary Distribution of Watermarked Output.** Fix a context  $\pi$  and let  $\mathcal{F} : \Sigma^* \rightarrow U$  be the proxy function with a finite range  $U$  of size  $M \triangleq |U|$ . For  $\forall u \in U$ , define its natural mass:

$$q(u) \triangleq \mu_M(\mathcal{F}^{-1}(u)) = \sum_{t \in \mathcal{F}^{-1}(u)} P_M(t \mid \pi), \quad \sum_{u \in U} q(u) = 1. \quad (7)$$

Fix a green ratio  $\gamma \in (0, 1)$  with  $m = \gamma M \in \mathbb{N}_+$ . The code space consists of all size- $m$  subsets  $S \subseteq U$ , each chosen uniformly at random. For any  $S \subseteq U$ , define the green mass  $q(S) \triangleq \sum_{v \in S} q(v)$ .

**Lemma 1** (Probability Scaling of Green Region). *For any fixed green set  $S \subseteq U$  and any  $N \geq 1$ , the distribution of the output sentence  $Y$  is independent of  $N$  and equals the natural distribution conditioned on  $\mathcal{F} \in S$ . Specifically, for any  $s \in \Sigma^*$  with  $u = \mathcal{F}(s)$ ,*

$$P(Y = s \mid \pi, S) = \begin{cases} \frac{P_M(s \mid \pi)}{q(S)}, & u \in S, \\ 0, & u \notin S. \end{cases} \quad (8)$$

Averaging over the uniformly random choice of green set  $S$  yields a closed-form expression:

**Theorem 2** (Closed-form of Watermarked PMF). *For any  $s \in \Sigma^*$  with  $u = \mathcal{F}(s) \in U$ ,*

$$P_M^w(s \mid \pi) = P_M(s \mid \pi) \cdot \frac{1}{\binom{M}{m}} \sum_{\substack{S \subseteq U \\ |S|=m, u \in S}} \frac{1}{q(S)}. \quad (9)$$

where  $q(u)$  is defined in equation 7, and  $M, m, u, U, \Sigma^*, \mathcal{F}, \mathcal{F}^{-1}$  are known quantities.

**Corollary 2.1.**  $P_M^w(s \mid \pi)$  is distortion-free if and only if  $q(u) = \frac{1}{M}$  for all  $u \in U$ .

However, the condition in Corollary 2.1 is not guaranteed in SemStamp, whose embedding space is randomly partitioned. Similar conclusions apply to other SWM methods such as k-SemStamp and SimMark, with further analysis provided in Appendix D and E.

### 3.3 SINGLE-CHANNEL DISTORTION-FREE SAMPLING

In this section, we present a toy example of a distortion-free sampling scheme for next-sentence generation, serving as a foundation for subsequent sections.

**Proxy Function via Pivot Vector.** Given a text encoder  $\mathcal{T} : \Sigma^* \rightarrow \mathbb{R}^d$  and a fixed pivot vector  $v \in \mathbb{R}^d$ , we define the proxy function as  $\mathcal{F}_v(s) = \langle v, \mathcal{T}(s) \rangle = \frac{v \cdot \mathcal{T}(s)}{\|v\| \cdot \|\mathcal{T}(s)\|}$ , which denotes the cosine similarity between the pivot vector and the encoded sentence. The range of  $\mathcal{F}_v$  is  $[-1, 1]$ .

<b>Algorithm 1:</b> PMARK Online Generation	<b>Algorithm 2: PMARK Online Detection</b>
<p><b>Input :</b> LLM <math>M</math>; prompt <math>s^{(0)}</math>;  encoder <math>\mathcal{T}</math>; <math>T</math>; <math>b</math>;  random seeds <math>R = \{r_{(t,j)}\}</math>;  sample budget <math>N</math>;</p> <p><b>Output:</b> <math>s^{(1)}, \dots, s^{(T)}</math></p> <pre> 1 Initialize <math>b</math> orthogonal pivots     <math>v^{(1)}, \dots, v^{(b)}</math>; 2 for <math>t \leftarrow 1</math> to <math>T</math> do 3     Sample <math>W^{(t)} = \{x^{(1)}, \dots, x^{(N)}\}</math>         with <math>x^{(i)} \sim P_M(s   s^{(0:t-1)})</math>; 4     <math>V^{(0)} \leftarrow W^{(t)}</math>; 5     for <math>j \leftarrow 1</math> to <math>b</math> do 6         <math>\mathcal{F}_j(s) = \langle v^{(j)}, \mathcal{T}(s) \rangle</math>; 7         <math>V^{(j)} \leftarrow</math>         <math>\mathcal{D}(V^{(j-1)}   r_{(t,j)}, \mathcal{F}_j)</math> on         channel <math>v^{(j)}</math>; 8     end 9     Select <math>s^{(t)} \sim \text{Un}(V^{(b)})</math>; 10 end 11 return <math>s^{(1)}, \dots, s^{(T)}</math>;</pre>	<p><b>Input :</b> <math>S = [s^{(0)}, \dots, s^{(T)}]; M; \mathcal{T}; v^{(1\dots b)}; R = \{r_{(t,j)}\}; N; \alpha</math>; threshold <math>\delta</math>; Smooth Factor <math>K</math></p> <p><b>Output:</b> True or False</p> <pre> 1 <math>N_g \leftarrow 0</math>; <math>N_{\text{total}} \leftarrow b \cdot T</math>; 2 for <math>t \leftarrow 1</math> to <math>T</math> do 3     Resample <math>W_t = \{x^{(1)}, \dots, x^{(N)}\}</math> with         <math>x^{(i)} \sim P_M(s   s^{(0:t-1)})</math>; 4     for <math>j \leftarrow 1</math> to <math>b</math> do 5         Define <math>\mathcal{F}_j(s) = \langle v^{(j)}, \mathcal{T}(s) \rangle</math>;         <math>\hat{m}_{(t,j)} \leftarrow \text{HMDmedian}(\{\mathcal{F}_j(x^{(i)})\}_{i=1}^N)</math>;         <math>x_{(t,j)} \leftarrow \mathcal{F}_j(s^{(t)})</math>; 6         if <math>r_{(t,j)} = 1</math> and <math>x_{(t,j)} &gt; \hat{m}_{(t,j)} - \delta</math> then 7             <math>c_{(t,j)} \leftarrow 1</math> 8         else if <math>r_{(t,j)} = 0</math> and <math>x_{(t,j)} &lt; \hat{m}_{(t,j)} + \delta</math> then 9             <math>c_{(t,j)} \leftarrow 1</math> 10        else 11            <math>c_{(t,j)} \leftarrow \exp(-K x_{(t,j)} - \hat{m}_{(t,j)} )</math> 12        end 13        <math>N_g \leftarrow N_g + c_{(t,j)}</math>; 14    end 15 end 16 end 17 <math>z \leftarrow \frac{ N_g - 0.5 N_{\text{total}} }{\sqrt{0.25 N_{\text{total}}}}</math>; return (<math>z &gt; z_\alpha</math>);</pre>

Figure 2: PMARK Online Watermarking. **Left:** Multi-channel constrained generation; **Right:** Detection via soft- $z$ -test.

**Median-based Sampling.** Given a context  $\pi$ , we sample  $N$  i.i.d. candidate sentences from the natural distribution:  $W = \{x_1, \dots, x_N\}, x_i \sim P_M(s | \pi)$ , where  $N$  is an even integer. The proxy function  $\mathcal{F}_v$  assigns each candidate a scalar score  $F = \{f_i = \mathcal{F}_v(x_i) \mid i = 1, \dots, N\}$ . Let  $m_v$  be the median of  $F$ . We partition  $W$  into two equal-sized subsets:

$$F_{\text{upper}} = \{x_i \mid f_i \geq m_v\}, \quad F_{\text{lower}} = \{x_i \mid f_i < m_v\}, \quad (10)$$

with ties broken randomly to ensure balance.

Define the watermark code space as  $E = \{F_{\text{upper}}, F_{\text{lower}}\}$ . A random seed  $k \in \{0, 1\}$  is sampled uniformly, and the selected code is  $e_K = \begin{cases} F_{\text{upper}}, & \text{if } k = 1, \\ F_{\text{lower}}, & \text{if } k = 0. \end{cases}$

The valid sentence set is  $V = e_K$ , and the next sentence is sampled uniformly from  $V$ :  $Y \sim \text{Un}(V)$ . We denote the operation that selects the half-partition  $V$  from the whole set  $W$  based on random seed  $k$  and proxy function  $\mathcal{F}_v$  as  $V = \mathcal{D}(W | k, \mathcal{F}_v), |V| = |W|/2$ .

**Theorem 3** (Distortion-free on a Single Channel). *For any context  $\pi$ , sample  $N$  i.i.d. candidates  $W = \{x_1, \dots, x_N\}$  from  $P_M(\cdot | \pi)$ , and form half-sets  $e_0, e_1$  by the median rule. Draw  $k \sim \text{Unif}\{0, 1\}$  and then sample  $Y$  uniformly from  $e_k$ . Then  $P_M^w(s | \pi) = P_M(s | \pi)$ .*

*Proof.* For any  $x_i \in W$ , we have

$$P(Y = x_i | W) = \sum_{k \in \{0, 1\}} P(k)P(Y = x_i | W, k) = \frac{1}{2} \left( \frac{2}{N} \mathbf{1}\{x_i \in e_0\} + \frac{2}{N} \mathbf{1}\{x_i \in e_1\} \right) = \frac{1}{N}.$$

Averaging over the randomness of  $W$  yields

$$P_M^w(s | \pi) = \sum_{i=1}^N P(x_i = s)P(Y = x_i | W) = P_M(s | \pi),$$

which proves that the single-channel sampling method above is distortion-free.  $\square$

## 4 PROPOSED METHOD

### 4.1 MULTI-CHANNEL CONSTRAINED GENERATION (ONLINE)

**Single-Channel Robustness.** While Section 3.3 introduces a simple distortion-free sampling strategy for SWM, it remains vulnerable to paraphrasing attacks.

**Theorem 4** (Semantic Robustness on Single Channel). *Let  $s$  be a watermarked sentence and  $v \in \mathbb{R}^d$  the pivot vector. Suppose an attacker  $\mathcal{A} : \Sigma^* \rightarrow \Sigma^*$  modifies  $s$  such that  $1 - \langle \mathcal{T}(\mathcal{A}(s)), \mathcal{T}(s) \rangle \leq d$ . Then the probability that the watermark evidence of  $s$  is removed by such an attack is bounded by*

$$P_{\text{rm}} \leq \mu_M \left( s \in \Sigma^* \mid \mathcal{F}_v(s) \in [m_v - \sqrt{2d}, m_v + \sqrt{2d}] \right). \quad (11)$$

The robustness of the entire watermarked text  $S = [s_1, \dots, s_n]$  depends on the specific detection scheme, with a detailed analysis available in [Giboulot & Furon \(2024\)](#). Importantly, compared with token-level baselines where each token contributes one bit of watermark evidence, the evidence density of SWM in the single-channel setting is extremely sparse. This sparsity results in a significant degradation in detectability under adversarial attacks. To address this limitation, we extend the paradigm to incorporate multiple channel constraints.

**Multi-Channel Sampling.** Given a pre-defined set of  $b$  orthogonal vectors  $v_1, \dots, v_b$ , we refer to each pivot as a *channel*. The proxy function defined on  $v_i$  is  $\mathcal{F}_{v_i}(s) = \langle v_i, \mathcal{T}(s) \rangle$ , also denoted as  $\mathcal{F}_i$  for simplicity. Specifically, these pivots can be generated from the QR decomposition of a matrix. Since this approach relies on dynamic median estimation, we refer to it as the online PMARK.

1. Given a context  $\pi$ , PMARK samples  $N$  candidate sentences from the natural distribution:  $W = \{x_1, \dots, x_N\}$ ,  $x_i \sim P(s \mid \pi)$ ,  $i = 1, \dots, N$ .
2. For  $i = 1$  to  $b$ , update  $V_i = \mathcal{D}(V_{i-1} \mid k_i, \mathcal{F}_i)$ , where  $V_0 = W$  and  $k_i \sim \text{Un}(\{0, 1\})$ .
3. A watermarked sentence  $Y$  is then uniformly sampled from  $V_b$ , so that the distribution of the watermarked output is  $Y \sim P_M^w(s \mid \pi)$ .

**Remark 1** (Multi-channel Distortion-free). *It is straightforward to verify that such multi-channel sampling is distortion-free, since for  $\forall x_j \in V_i$ ,  $P(x_j \in V_{i+1}) = \frac{1}{2}$  and  $P(Y = x \mid x \in W) = 1/N$ .*

The complete process for generating watermarked texts is described in Algorithm 1.

#### 4.2 SOFT- $z$ -TEST DETECTION (ONLINE)

We now introduce a robust detection scheme for text watermarked with the online PMARK.

1. **Sentence Segmentation.** The input text is segmented into sentences  $S = [s_0, s_1, \dots, s_T]$  to enable subsequent processing.
2. **Median Reconstruction via Sampling.** For each sentence  $s_t$  ( $1 \leq t \leq T$ ), resample  $W'_t = \{x_1, \dots, x_N\}$  i.i.d. from  $P_M(s_t \mid s_{0:t-1})$ .
3. **Online Median Estimation.** For each pivot vector  $v_j$  ( $1 \leq j \leq b$ ), estimate the median  $m'_{(t,j)}$  of the values  $\mathcal{F}_j(W'_t)$  using the Harrell–Davis estimator ([Harrell & Davis, 1982](#)).
4. **Smooth Counting of Watermark Evidence.** To mitigate discrepancies between the medians estimated during generation and detection, we introduce a smooth counting mechanism inspired by ([Dabiriaghdam & Wang, 2025](#)). Specifically, given a margin threshold  $\delta > 0$  and a smoothing factor  $K > 0$ , the watermark evidence contributed by  $s_t$  along axis  $v_j$  is defined as

$$c_{(t,j)} = \begin{cases} 1, & \text{if } (r_{(t,j)} = 1 \wedge \mathcal{F}_j(s_t) > m'_{(t,j)} - \delta) \vee (r_{(t,j)} = 0 \wedge \mathcal{F}_j(s_t) < m'_{(t,j)} + \delta), \\ e^{-K|\mathcal{F}_j(s_t) - m'_{(t,j)}|}, & \text{otherwise.} \end{cases}$$

5. **Robust Detection via Soft- $z$ -Test.** Finally, to determine whether the input text was generated by PMARK, we apply a soft-count-based  $z$ -test. The test statistic is computed as  $z = \frac{\sum_{t=1}^T \sum_{j=1}^b c_{(t,j)} - 0.5bT}{\sqrt{bT \cdot 0.5 \cdot 0.5}}$ , where  $b \cdot T$  is the total number of potential watermark bits across channels.

The complete description of the online detection scheme is provided in Algorithm 2.

#### 4.3 OFFLINE PARTITION WITH A PRIOR THRESHOLD

**Concentration of Measure.** In practice, we find the range of  $\mathcal{F}$  is confined to a small interval around zero,  $[-\epsilon, \epsilon]$  with  $\epsilon \leq 0.08$  in most cases. This observation is consistent with Lemma 5:

**Lemma 5** (Orthogonality). *Let  $x, y$  be random vectors drawn uniformly from the unit sphere  $S^{d-1} \subset \mathbb{R}^d$ . Then the angle  $\theta$  between them has density*

$$p_d(\theta) = \frac{\Gamma(\frac{d}{2})}{\Gamma(\frac{d-1}{2}) \sqrt{\pi}} \sin^{d-2} \theta \quad (\theta \in [0, \pi]). \quad (12)$$

Moreover, we observe a concentration phenomenon whereby the median  $m$  remains close to zero. Building on this intuition and supporting empirical results, we use zero as an effective prior for the median in the offline PMARK. Experimental results in Section 5 demonstrate the effectiveness of this approximation. Nevertheless, we leave the strict proof of the theoretical error for future work.

**Offline Watermarking with a Prior-Median Assumption.** Accordingly, we adopt zero as the prior median during both generation and detection, while maintaining the same watermarking rate  $\gamma$  for the  $z$ -test. For a candidate sentence  $x$ , define the binary signal vector

$$\text{Sig}(x) = [[f_1(x) > 0], \dots, [f_b(x) > 0]], \quad (13)$$

and let  $r \in \{0, 1\}^b$  denote the random channel seeds. The total watermark evidence contributed by  $x$  is

$$E(x) = \sum_{j=1}^b (\text{Sig}(x)(j) \wedge r(j)). \quad (14)$$

During generation, we select the sentence that maximizes this watermark evidence. This avoids the computational overhead of repeated sampling and the need to access the original prompt during detection by eliminating dynamical estimation. The complete algorithm is provided in Appendix F.

## 5 EXPERIMENTS

In this section, we conduct extensive experiments to answer the following research questions: **(RQ1)** How does PMARK perform under various attacks? **(RQ2)** Can PMARK enable high-quality text generation in practice? **(RQ3)** What is the computational cost of PMARK compared with other SWM baselines? **(RQ4)** How sensitive is PMARK to its key components or parameters?

### 5.1 EXPERIMENT SETUP

**Dataset and Baselines.** Following previous work (Hou et al., 2023; Wang et al., 2025), we evaluate the performance of PMARK using 500 samples from the C4 (Raffel et al., 2020) and BOOKSUM (Kryściński et al., 2021) datasets separately, with OPT-1.3B (Zhang et al., 2022) and Mistral-7B-v0.1 (Jiang et al., 2023) as backbone models. Our baselines are based on the official implementation of MarkLLM (Pan et al., 2024), including token-level watermarking methods KGW (Kirchenbauer et al., 2023), UPV (Liu et al., 2023a), MorphMark (Wang et al., 2025), SIR (Liu et al., 2023b), EXP (Aaronson, 2023), EXPGumbel (Aaronson, 2023), SynthID (Dathathri et al., 2024), as well as semantic-level baselines SemStamp (Hou et al., 2023), k-SemStamp (Hou et al., 2024), and SimMark (Dabiriaghdam & Wang, 2025). For the PMARK generation process, we set the number of channels to 4 with a sample budget  $N = 64$ . The hyperparameters of the soft- $z$ -test during detection are set as  $K = 150$  and  $\delta = 0.001$ . More implementation details can be found in Appendix G.1.

**Metrics.** Following previous work, we use perplexity (PPL) to measure the quality of generated text, computed using LLaMA-2-7B (Touvron et al., 2023). We also assess watermark effectiveness in terms of detectability (TPR@1%, AUC) and robustness under sentence-level attacks (Paraphrase Attack by Parrot (Sadasivan et al., 2023), GPT-3.5-turbo (OpenAI, 2022), and Back-Translation Attack by LLAMA-3.1-8B (Pan et al., 2024; Dubey et al., 2024)). Word-level attacks (Pan et al., 2024) such as word deletion and synonym substitution at rates 0.05, 0.15, and 0.30 are also considered. Refer to Appendix G.2 for a more detailed discussion of these attacks.

### 5.2 ROBUSTNESS AND TEXT QUALITY (RQ1 & RQ2)

To answer **RQ1**, we comprehensively compare PMARK with seven widely used token-level watermarks and three semantic-level watermarks under various attacks. Table 1 presents comprehensive results of watermarking methods under paraphrase attacks, with Pegasus (Pu et al., 2023), Parrot (Sadasivan et al., 2023), and GPT-3.5-turbo (OpenAI, 2022) as different paraphrasers. Figure 3 illustrates the trade-off between text quality and robustness under back-translation attacks, while Figure 4 shows the results of various SWM methods under word-level attacks. We provide the following observations (**Obs.**):

**Obs. ① PMARK demonstrates superior semantic robustness across different backbones and benchmarks.** As shown in Table 1, PMARK exhibits strong robustness to paraphrase attacks. Specifically, the TP@FP1% of online PMARK remains above 93.0% across different paraphrasers, substantially outperforming the baselines. For Mistral-7B, the TP@FP1% on BOOKSUM and C4 after GPT paraphrasing reaches 96.8 and 95.2, exceeding the best SWM baseline SemStamp by 20.8% and 26.0% and the best TWM baseline SIR by 55.6% and 31.8%. The offline PMARK

Table 1: Overall results for baseline methods and PMARK on OPT-1.3B and Mistral-7B across the BOOKSUM and C4 benchmarks. **Doc-P** denotes Paraphrase Attack, while Pegasus, Parrot, GPT are different paraphrasers. For each attack, we report TP@FP=1%, TP@FP=5%, and ROC-AUC from left to right. **Bold** denotes the best result, and underlined denotes the second-best.

Method	BOOKSUM				C4			
	No Attack↑	Doc-P (Pegasus)↑	Doc-P (Parrot)↑	Doc-P (GPT)↑	No Attack↑	Doc-P (Pegasus)↑	Doc-P (Parrot)↑	Doc-P (GPT)↑
OPT-1.3B								
EXP (2023)	98.8/99.0/99.3	4.2/14.6/52.5	5.4/16.0/55.2	4.4/13.4/53.0	99.0/99.6/99.8	54.6/68.2/86.9	42.4/59.6/84.6	21.2/37.2/73.3
EXPGumbel (2023)	99.4/99.4/99.4	12.4/20.2/50.6	13.0/23.2/54.4	98.6/98.6/99.3	75.2/84.6/91.8	74.6/81.0/92.8	55.6/66.0/86.0	
KGW (2023)	100.0/100.0/100.0	1.4/4.6/56.8	1.4/5.9/55.7	6.0/4.2/54.6	100.0/100.0/100.0	89.3/97.0/99.2	76.2/91.4/98.1	51.4/78.5/95.0
SIR (2023b)	99.6/100.0/99.8	84.2/93.8/97.5	80.6/93.8/98.1	49.8/82.8/95.9	99.8/100.0/99.9	91.6/94.6/98.7	83.4/90.6/97.6	74.2/88.6/97.7
UPV (2023a)	100.0/100.0/100.0	90.0/95.6/98.7	89.2/98.2/99.3	73.6/91.0/98.4	100.0/100.0/100.0	90.6/95.4/98.3	78.7/88.4/97.4	80.9/91.3/98.2
SynthID (2024)	100.0/100.0/99.9	38.2/56.6/86.9	25.8/43.6/83.8	1.8/14.6/65.7	100.0/100.0/99.9	45.4/66.2/89.5	26.4/49.4/82.7	6.4/21.2/67.6
MorphMark (2025)	100.0/100.0/100.0	1.8/7.7/55.2	1.2/6.1/54.6	0.8/4.2/51.1	100.0/100.0/100.0	78.6/93.0/97.8	70.2/87.1/96.8	46.7/76.2/93.2
SemStamp (2023)	97.7/98.8/99.4	89.0/93.2/97.3	90.7/93.0/97.5	79.6/85.4/94.2	94.6/96.1/98.9	85.8/91.0/96.9	84.4/89.3/96.2	73.5/81.4/93.0
k-SemStamp (2024)	99.6/100.0/99.9	74.5/84.6/97.4	72.3/86.6/97.5	62.1/72.9/95.2	100.0/100.0/99.9	76.6/89.2/97.1	74.3/87.0/97.5	62.9/78.6/94.1
SimMark (2025)	88.2/94.0/98.8	23.4/42.8/84.6	29.0/45.6/87.3	22.0/40.4/83.6	77.6/94.2/98.5	11.6/35.8/82.9	13.6/39.2/84.5	16.0/44.8/86.5
Ours(Online)	99.8/99.8/99.9	<b>97.4/99.0/99.8</b>	<b>96.8/99.0/99.8</b>	<b>95.4/99.0/99.6</b>	100.0/100.0/99.9	<b>96.2/99.6/99.8</b>	<b>97.2/99.0/99.8</b>	<b>97.8/99.6/99.8</b>
Ours(Offline)	99.4/99.6/99.8	93.2/98.2/99.5	95.2/98.6/99.5	94.2/98.8/99.6	98.0/99.0/99.8	91.7/94.4/98.8	91.4/95.2/99.0	92.6/96.2/99.1
Mistral-7B								
EXP (2023)	99.8/99.8/99.9	36.4/53.0/85.8	47.2/61.8/89.1	10.4/22.0/71.7	99.2/99.2/99.5	44.4/60.4/85.7	34.2/54.8/84.2	17.4/29.8/72.1
EXPGumbel (2023)	99.6/99.6/99.7	65.6/77.2/90.8	78.8/85.2/92.8	38.4/53.2/81.9	98.6/98.6/99.3	71.2/82.0/92.5	66.2/77.2/90.8	40.2/55.4/82.5
KGW (2023)	100.0/100.0/100.0	84.2/95.4/98.7	90.2/97.3/99.2	35.8/68.9/92.1	100.0/100.0/100.0	85.6/95.5/98.8	79.2/93.7/98.8	54.2/76.7/95.6
SIR (2023b)	100.0/100.0/99.9	85.0/92.6/98.0	88.0/92.0/98.2	41.2/71.4/93.6	100.0/100.0/99.9	87.6/93.4/98.1	83.2/89.6/97.8	63.4/80.8/96.4
UPV (2023a)	99.6/100.0/100.0	72.6/96.2/98.5	73.0/96.9/99.0	33.4/76.2/94.6	99.4/99.8/99.9	71.4/91.4/97.7	61.9/85.4/97.5	34.9/68.6/93.4
SynthID (2024)	100.0/100.0/100.0	30.2/49.2/83.5	28.2/48.4/85.5	3.0/13.0/65.0	99.8/98.9/99.8	47.0/56.0/87.6	31.4/41.4/80.7	7.4/17.2/68.2
MorphMark (2025)	99.8/100.0/100.0	68.2/88.9/97.2	77.4/93.3/98.4	24.6/52.8/87.7	98.6/100.0/100.0	74.8/89.4/97.5	70.0/86.4/97.4	35.2/59.7/91.4
SemStamp (2023)	97.7/98.4/99.5	88.1/92.8/97.8	92.4/95.2/98.1	76.0/83.0/94.5	92.5/95.8/98.3	80.9/87.9/95.9	77.9/86.3/95.7	69.2/80.0/92.4
k-SemStamp (2024)	99.0/99.0/99.7	45.5/64.1/92.4	53.9/70.0/94.8	43.5/61.9/91.6	100.0/100.0/99.9	49.2/68.4/93.2	54.8/71.1/93.7	43.9/61.2/89.2
SimMark (2025)	78.0/89.4/97.9	19.8/40.6/83.6	28.8/46.2/85.9	23.4/45.0/84.9	70.8/89.6/97.9	12.4/31.2/81.5	14.6/37.3/84.4	22.6/47.9/86.4
Ours(Online)	100.0/100.0/99.9	<b>94.4/98.2/99.6</b>	<b>96.6/98.8/99.7</b>	<b>96.8/99.0/99.7</b>	100.0/100.0/99.9	<b>93.0/96.8/99.4</b>	<b>93.6/97.6/99.4</b>	<b>95.2/98.8/99.7</b>
Ours(Offline)	99.4/100.0/99.9	91.4/97.4/99.3	94.6/97.8/99.6	94.4/98.6/99.6	99.7/99.8/99.9	90.8/95.8/99.1	91.3/95.8/99.3	92.0/95.2/99.3

also shows strong robustness to paraphrase attacks, with TP@FP1% remaining above 90.8%, consistently ranking second to the online PMARK. A similar conclusion can be drawn from Figure 3, where both online and offline PMARK excel in TP@FP1% under back-translation attack.

**Obs. ② PMARK achieves superior text generation quality.** As shown in Figure 3, the online version of PMARK attains the highest text quality on the BOOKSUM benchmark, with perplexity lower than the best baseline EXP (Aaronson, 2023) by nearly 0.7 on Mistral-7B. Among prior SWMs, k-SemStamp (Hou et al., 2024) achieves the lowest PPL (around 5.0) on BOOKSUM, yet it still underperforms PMARK by a clear margin. We observe similar trends on the C4 dataset: both the online and offline variants of PMARK deliver the highest-quality watermarked text. Similarly, EXP and k-SemStamp remain the strongest token-level and semantic-level baselines, respectively. Additional results on robustness and text quality are provided in Appendix H.1 and H.2.

**Obs. ③ Previous SWM methods are vulnerable to word-level attacks.** We also evaluate PMARK under word-level deletion and synonym-substitution attacks with Mistral-7B on BOOKSUM, as shown in Figure 4. Surprisingly, existing SWMs remain vulnerable to these token-level attacks because their watermark evidence is relatively sparse. As a result, even the reversal of one-bit watermark evidence can cause a large deviation in the  $z$  statistic. In our work, we address this problem efficiently by applying multiple channel constraints to each sentence, rather than simply increasing generation length. Consequently, our method achieves state-of-the-art robustness: in online mode, TP@FP1%

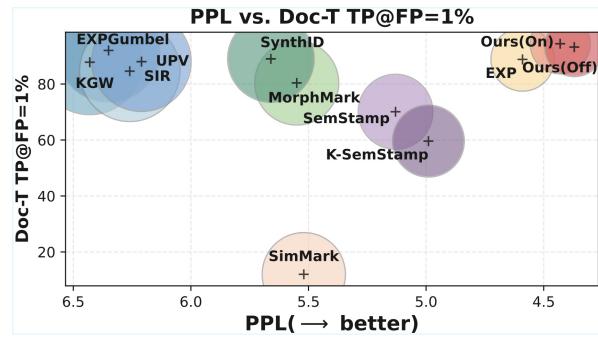


Figure 3: Results of Mistral-7B on the C4 dataset. Smaller bubbles denote lower PPL.

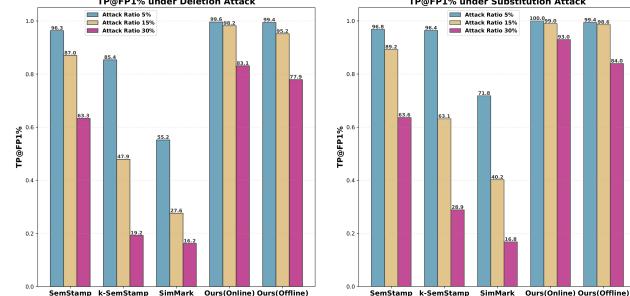


Figure 4: TP@FP1% under Word-D and Word-S attacks. The figure consists of two bar charts. The left chart is titled 'TP@FP1% under Deletion Attack' and the right is 'TP@FP1% under Substitution Attack'. Both charts show TP@FP1% (Y-axis, 0.0-1.0) for five methods: SemStamp, k-SemStamp, SimMark, Ours(Online), and Ours(Offline). The legend indicates attack ratios: 5% (blue), 15% (orange), and 30% (pink). In both cases, Ours(Offline) shows the highest robustness, followed by Ours(Online), then the other methods.

reaches 98.2% and 99.0% under 15% word deletion and synonym substitution, respectively; in offline mode, the corresponding TP@FP1% values are 95.2% and 98.6%.

### 5.3 COMPUTATION CONSUMPTION (RQ3)

**Obs. ④ Evidence density is critical for SWM detectability.** Table 2

confirms our understanding of watermark evidence and sampling efficiency in SWM, evaluated with OPT-1.3B on BOOKSUM. Specifically, online

PMARK exhibits a high TP@FP% of 98% with an extremely limited sampling budget  $N = 8$ , supported by 3 channels. This suggests that dense watermark evidence is necessary for reducing sampling budget. We also note that offline PMARK requires a budget of 16 to achieve practical detectability, which we attribute to the high risk of prior deviation caused by a limited budget. While Bayesian models can improve accuracy in such probability estimation as shown in SynthID, we only present the naive results here and leave related improvements for future work.

**Obs. ⑤ PMARK achieves the best trade-off between token efficiency and performance among SWM methods.** To further verify the efficiency of

PMARK, we conducted additional experiments on token efficiency across existing SWM methods at the same detectability level, as shown in Table 3. While SimMark achieves the lowest token consumption during watermark generation, it performs poorly without well-tuned hyperparameters, as shown in Table 1. Similarly, k-SemStamp requires fewer tokens than online PMARK, but its robustness to paraphrasing is limited. In contrast, SemStamp exhibits the highest robustness to semantic attacks among existing SWMs, but at the cost of an impractical 1694.4 tokens per sentence. As a result, we argue that PMARK achieves the best trade-off among SWM methods: the offline mode requires fewer tokens than k-SemStamp while maintaining competitive robustness, and the online mode achieves the highest robustness with only 20% additional token consumption.

### 5.4 SENSITIVITY ANALYSIS (RQ4)

**Obs. ⑥ Soft- $z$ -test boosts the detection accuracy of online PMARK.**

We present the parameter sensitivity analysis of two hyperparameters in Figure 5. Results of Doc-P(GPT) are also included for comparison. While the soft- $z$ -test yields a stable improvement in robustness compared with the naive  $z$ -test ( $K = +\infty, \delta = 0$ ), we find it is more beneficial for online PMARK, where increases of 11.9% and 6.4% in attacked TP@FP1% are observed on BOOKSUM and C4, respectively. More results and cases can be found in Appendix H.4.

Table 2: TP@FP1% across different sample budgets and channel numbers in online/offline modes.

$N/b$	b=1	b=2	b=3	b=4
$N = 8$	81.0/85.0	97.0/47.0	98.0/83.0	-/-
$N = 16$	84.0/93.0	100.0/90.0	100.0/96.0	100.0/93.0
$N = 32$	97.0/99.0	100.0/98.0	100.0/98.0	100.0/98.0
$N = 64$	99.0/95.0	100.0/98.0	100.0/99.0	100.0/100.0

Table 3: Token efficiency of SWM methods.  $T_s$  denotes average token consumption per sentence, while  $T_t$  denotes consumption per token.

Method	k-Sem	SimMark	Sem	Ours(On)	Ours(Off)
$T_s$	246.9	186.7	1694.4	315.8	239.7
$T_t$	13.3	8.1	99.3	16.0	12.1

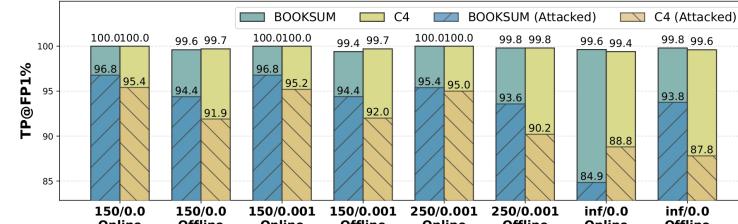


Figure 5: Performance of Mistral-7B under different hyperparameter settings ( $K = 150, 250, +\infty$  and  $\delta = 0, 0.001$ ).

## 6 CONCLUSION

The semantics of the next sentence is an abstract and complex concept. To effectively estimate it indirectly, a proxy function that maps sentences from textual space to a scalar value is often used, either implicitly or explicitly. Building on this insight, we present a unified perspective on existing SWMs and propose a novel distortion-free paradigm based on median estimation. Furthermore, we identify that the vulnerability of current SWMs to adversarial attacks arises from the sparsity of watermark evidence. To address this issue, we introduce multi-channel constraints to enhance robustness. To the best of our knowledge, this is the first work to provide a unified theoretical

framework for SWMs, the first distortion-free SWM approach, and the first to highlight and realize dense watermark evidence in SWM. Experimental results demonstrate that PMARK consistently outperforms existing SWM baselines in both text quality and robustness, offering a more reliable and effective paradigm for semantic-level watermarking.

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## A USE OF LLMs

We used AI assistants for two purposes: (1) generating routine code and boilerplate functions, which were subsequently reviewed and debugged by humans, and (2) performing grammatical review and sentence-level editing of the manuscript. The research methodology, findings, and analysis were independently proposed and conducted.

## B DETAILED DEFINITION AND ADDITIONAL PROOFS

**Lemma 6** (Countable). *Let  $\Sigma$  be a finite tokenizer vocabulary, and let  $\Sigma^*$  denote the set of all semantically complete sentences (including the null sentence) represented as finite token sequences over  $\Sigma$ . Then  $\Sigma^*$  is countable.*

*Proof.* Since  $\Sigma$  is finite, fix a bijection  $e : \Sigma \rightarrow \mathbb{N}_{\geq 1}$  (i.e. the index on the tokenizer). Let  $(p_k)_{k \geq 1}$  be the increasing sequence of prime numbers. Define  $C : \Sigma^{<\omega} \rightarrow \mathbb{N}$  by

$$C(s) := \begin{cases} 1, & s = \emptyset, \\ \prod_{k=1}^{|s|} p_k^{e(x_k)+1}, & s = (x_1, \dots, x_{|s|}) \in \Sigma^{<\omega}. \end{cases} \quad (15)$$

If  $C(s) = C(t)$ , the Fundamental Theorem of Arithmetic forces the exponent of each  $p_k$  to coincide on both sides. Hence  $|s| = |t|$  and  $e(x_k) = e(y_k)$  for all  $k$ , which implies  $x_k = y_k$  by bijectivity of  $e$ , so  $s = t$ . Therefore  $\Sigma^{<\omega}$  injects into  $\mathbb{N}$  and is countable. Since  $\Sigma^* \subseteq \Sigma^{<\omega}$ , it is countable as well.  $\square$

**Definition 4** (Partition of a Set). *Let  $U$  be a non-empty set. A partition  $\mathcal{P}(U)$  of  $U$  is a collection of non-empty, pairwise disjoint subsets of  $U$  whose union equals  $U$ . Formally,*

$$\mathcal{P}(U) = \{A_1, \dots, A_n\}$$

such that:

- $A_i \neq \emptyset$  for all  $i$ ,
- $A_i \cap A_j = \emptyset$  for all  $i \neq j$ ,
- $\bigcup_{i=1}^n A_i = U$ .

Each element  $u \in U$  belongs to exactly one subset  $A_i \in \mathcal{P}(U)$ , which we denote as  $\mathcal{P}_u(U)$  in the main body.

**Proof of Lemma 1.** Here we suppose that  $q(S) > 0$  in practice, meaning that reject-sampling will always succeed when  $N$  is large enough.

*Proof.* Let  $\mathcal{F} : \Sigma^* \rightarrow U$  be the proxy with finite range  $U$ , and fix a green set  $S \subseteq U$ . For  $u \in U$ ,

$$q(u) \triangleq \sum_{t \in \mathcal{F}^{-1}(u)} P_M(t \mid \boldsymbol{\pi}), \quad q(S) \triangleq \sum_{\mathcal{F}(t) \in S} P_M(t \mid \boldsymbol{\pi}). \quad (16)$$

Hence

$$\Pr(Y = s \mid \boldsymbol{\pi}, S) = \sum_{k=0}^{\infty} (1 - q(S))^k P_M(s \mid \boldsymbol{\pi}) = \frac{P_M(s \mid \boldsymbol{\pi})}{q(S)} \quad \text{for } \forall \mathcal{F}(s) \in S. \quad (17)$$

$\square$

**Proof of Theorem 2.** Here the watermark code space is  $E = \{S \mid S \subseteq U, |S| = m\}$ , where  $|U| = M$ ,  $m = \gamma M$  and  $|E| = \binom{M}{m}$ .

*Proof.* Based on equation 5 we have

$$P_M^w(s \mid \boldsymbol{\pi}) = \sum_{e_K \sim \text{Un}(E)} P(Y = s \mid \boldsymbol{\pi}, e_K) P(e_K) = \frac{1}{\binom{M}{m}} \sum_{S \in E} P(Y = s \mid \boldsymbol{\pi}, S). \quad (18)$$

From Lemma 1,

$$P(Y = s \mid \boldsymbol{\pi}, S) = \begin{cases} \frac{P_M(s \mid \boldsymbol{\pi})}{q(S)}, & u \in S, \\ 0, & u \notin S, \end{cases} \quad (19)$$

therefore only sets  $S$  containing  $u = \mathcal{F}(s)$  contribute:

$$P_M^w(s | \boldsymbol{\pi}) = \frac{1}{\binom{M}{m}} \sum_{\substack{S \subseteq E \\ u \in S}} \frac{P_M(s | \boldsymbol{\pi})}{q(S)} = P_M(s | \boldsymbol{\pi}) \cdot \frac{1}{\binom{M}{m}} \sum_{\substack{S \subseteq U \\ |S|=m, u \in S}} \frac{1}{q(S)}. \quad (20)$$

which is exactly equation 9.  $\square$

**Proof of Corollary 2.1.** Denote equation 9 as

$$\begin{aligned} P_M^w(s | \boldsymbol{\pi}) &= P_M(s | \boldsymbol{\pi}) A(u), \\ \text{where } A(u) &\triangleq \frac{1}{\binom{M}{m}} \sum_{\substack{S \subseteq U \\ |S|=m, u \in S}} \frac{1}{\sum_{v \in S} q(v)}. \end{aligned} \quad (\star)$$

So  $P_M^w(\cdot | \boldsymbol{\pi})$  is distortion-free if and only if  $A(u) \equiv 1$  for all  $u \in U$ .

*Proof.*

*Sufficiency.* If  $q(u) \equiv 1/M$  for all  $u$ , then for every  $S$ ,  $q(S) = \sum_{v \in S} q(v) = m/M = \gamma$ . Also  $\Pr(u \in S) = \frac{\binom{M-1}{m-1}}{\binom{M}{m}} = \frac{m}{M} = \gamma$ . Hence

$$A(u) = \frac{m}{M} \cdot \frac{1}{\gamma} = 1,$$

which means

$$P_M^w(s | \boldsymbol{\pi}) = P_M(s | \boldsymbol{\pi}).$$

*Necessity.* Assume distortion-free, so  $A(u) = 1$  for all  $u$ . Fix two indices  $u \neq v$ . Write the difference using  $(\star)$ . Denote  $T \subseteq U \setminus \{u, v\}$ ,  $|T| = m - 1$ , as a set containing both  $u$  and  $v$  will be neutralized in the difference, thus

$$A(u) - A(v) = \frac{1}{\binom{M}{m}} \sum_{\substack{T \subseteq U \setminus \{u, v\} \\ |T|=m-1}} \left( \frac{1}{q(u) + \sum_{w \in T} q(w)} - \frac{1}{q(v) + \sum_{w \in T} q(w)} \right).$$

Since  $f(x) = \frac{1}{x+a}$  is a strictly decreasing function for any fixed  $a > 0$ , therefore

- If  $q(u) > q(v)$ , then every term in the sum is negative and hence  $A(u) < A(v)$ .
- If  $q(u) < q(v)$ , then  $A(u) > A(v)$ .

Distortion-free requires  $A(u) \equiv A(v)$ , hence we must have  $q(u) = q(v)$ . Since  $u, v$  were arbitrary, all  $q(\cdot)$  are equal, meaning that  $q(u) = \frac{1}{M}$  for every  $u \in U$ .  $\square$

**Proof of Theorem 4.** Let  $\vec{x} = \frac{\mathcal{T}(s)}{\|\mathcal{T}(s)\|}$ ,  $\vec{x}' = \frac{\mathcal{T}(\mathcal{A}(s))}{\|\mathcal{T}(\mathcal{A}(s))\|}$ , and  $\vec{v} = \frac{v}{\|v\|}$ . Then the proxy is  $f = \langle \vec{v}, \vec{x} \rangle \in [-1, 1]$  and  $f' = \langle \vec{v}, \vec{x}' \rangle \in [-1, 1]$ .

*Proof.* For unit vectors,  $\|\vec{x}' - \vec{x}\|^2 = 2 - 2\langle \vec{x}', \vec{x} \rangle \leq 2d$ , hence

$$|f' - f| = |\langle \vec{v}, \vec{x}' - \vec{x} \rangle| \leq \|\vec{v}\| \|\vec{x}' - \vec{x}\| \leq \sqrt{2d}.$$

A flip is possible only if the original score sits within the  $\sqrt{2d}$  band around the median:

$$|f - m_v| \leq |f' - f| \leq \sqrt{2d} \implies f \in [m_v - \sqrt{2d}, m_v + \sqrt{2d}].$$

Therefore,

$$P_{\text{rm}} \leq \mu_M \left( s \in \Sigma^* \mid \mathcal{F}_v(s) \in [m_v - \sqrt{2d}, m_v + \sqrt{2d}] \right).$$

$\square$

**Proof of Lemma 5.**

*Proof.* Without loss of generality, fix  $x = e_1 = (1, 0, \dots, 0)$ . Draw  $y$  uniformly on  $S^{d-1}$ . The angle  $\theta$  between  $x$  and  $y$  satisfies  $\cos \theta = x \cdot y = g(y)$ . For  $t \in (-1, 1)$ , the set  $\{y \in S^{d-1} : g(y) = t\}$  is a  $(d-2)$ -sphere of radius  $\sqrt{1-t^2}$ . Its  $(d-2)$ -dimensional surface area is

$$A(t) = A(S^{d-2})(1-t^2)^{\frac{d-3}{2}},$$

where  $A(\cdot)$  denotes the surface area,  $A(S^k) = \frac{2\pi^{(k+1)/2}}{\Gamma((k+1)/2)}$ . Hence the probability that  $T \in [t, t+dt]$  is proportional to that area times  $dt$ .

$$f_T(t) = \frac{A(S^{d-2})}{A(S^{d-1})} (1-t^2)^{\frac{d-3}{2}} = \frac{\Gamma(\frac{d}{2})}{\Gamma(\frac{d-1}{2}) \sqrt{\pi}} (1-t^2)^{\frac{d-3}{2}}, \quad t \in [-1, 1].$$

With  $t = \cos \theta$ ,  $dt = -\sin \theta d\theta$ , and  $1-t^2 = \sin^2 \theta$ . Therefore

$$p_d(\theta) = f_T(\cos \theta) \sin \theta = \frac{\Gamma(\frac{d}{2})}{\Gamma(\frac{d-1}{2}) \sqrt{\pi}} \sin^{d-2} \theta, \quad \theta \in [0, \pi].$$

□

## C ADDITIONAL RELATED WORK

Text watermarking techniques for large language models can be broadly categorized into two main paradigms: token-level methods and sentence-level methods.

### C.1 TOKEN-LEVEL TEXT WATERMARKING

Token-level watermarking, the more traditional approach, embeds signals by manipulating the probability distribution of tokens during the decoding process. A foundational method in this category partitions the vocabulary into “green” and “red” lists, boosting the probability of green-listed tokens to create a detectable statistical bias (Kirchenbauer et al., 2023). Numerous subsequent works have built upon this concept. For instance, some methods focus on enhancing security and text quality by adaptively watermarking only high-entropy tokens (Liu & Bu, 2024; Lu et al., 2024), while others dynamically adjust watermark strength to balance effectiveness and quality (Wang et al., 2025). To improve robustness against spoofing attacks, researchers have explored contrastive representation learning to generate semantic-aware token lists (An et al., 2025). Other variations introduce signals in different domains, such as sinusoidal perturbations in the token probability vector to guard against model distillation (Zhao et al., 2023b), or frequency-based signals detectable via STFT (Xu et al., 2024b).

Multi-bit watermarking has also been a significant focus, with techniques employing error-correction codes (Qu et al., 2025), probability balancing (Wang et al., 2023), and majority-bit-based list construction (Xu et al., 2025) to embed more complex information. Furthermore, some approaches frame watermarking as a problem of hypothesis testing, optimizing detection accuracy by coupling output tokens with pseudo-random generators (Huang et al., 2023). Despite their diversity and ingenuity, a fundamental limitation shared by nearly all token-level methods is their vulnerability to semantic-level attacks. Since the watermark is tied to specific token choices, paraphrasing or other meaning-preserving modifications can easily disrupt or erase the embedded signal.

### C.2 SENTENCE-LEVEL TEXT WATERMARKING

To address the vulnerability of token-level methods to paraphrasing, sentence-level watermarking (SWM) has emerged as a more robust paradigm. These methods treat entire sentences as the fundamental unit for embedding information. A pioneering work in this area, SEMSTAMP (Hou et al., 2023), partitions the semantic embedding space using locality-sensitive hashing and employs rejection sampling to ensure that generated sentences fall into a pre-defined “valid” semantic region. This approach significantly enhances robustness against paraphrastic attacks.

Building on this idea, subsequent methods have explored various ways to define and select valid semantic regions. For example, SimMark (Dabiriaghdam & Wang, 2025) uses sentence embedding similarity, PersonaMark (Zhang et al., 2024) hashes sentence structures for personalization, and CoheMark (Zhang et al., 2025) leverages fuzzy c-means clustering and inter-sentence cohesion. Other approaches utilize advanced feature extractors such as sparse autoencoders (Yu et al., 2025) or fine-tuned LLM-based paraphrasers (Xu et al., 2024a) to embed multi-bit watermarks. However, a persistent challenge for most existing SWM techniques is their reliance on rejection sampling. Although effective for ensuring semantic validity, this mechanism often introduces significant distortion to the original text distribution, degrades text quality, and risks sampling failure when no valid candidates are found within a reasonable number of attempts. Furthermore, many of these methods lack rigorous theoretical guarantees for their robustness and distortion properties, a critical gap that our work aims to address.

**Concurrent Work.** A concurrent work is SAEMark (Yu et al., 2025), which introduces a concept analogous to the proxy function used in our framework, referred to as a “feature extractor.” While both approaches share some conceptual similarities, PMARK offers notable advantages in several respects. (1) *Task setting*. SAEMark targets multi-bit watermarking that embeds additional information into the generated text, whereas PMARK is designed for single-bit watermarking. (2) *Text quality*. SAEMark selects sentences based on proximity to a target threshold, which may result in distortion. (3) *Robustness*. SAEMark embeds watermark signals on a per-sentence basis, which has been shown to be vulnerable to certain attacks in their original work, while PMARK leverages multi-channel constraints to significantly improve resistance to such threats. (4) *Flexibility*. PMARK allows for greater adaptability through customizable settings such as the pivot vector and the number of channels, which are more difficult to implement effectively within the SAEMark framework.

## D ADDITIONAL DISCUSSION

**Random Seeds in SWMs.** In previous SWM schemes such as SemStamp and k-SemStamp, the random seed for  $s_n$  is extracted from the proxy value of the previous sentence  $s_{n-1}$ , as shown in Appendix E. However, this approach may not be truly random, since consecutive sentences often relate to the same topic (Nemecek et al., 2024). Moreover, SimMark (Dabiriaghdam & Wang, 2025) eliminates the use of a random seed entirely, relying instead on a fixed green region. These approaches can reduce the randomness of the key  $k$  and thus break the assumption in 5. In this work, we use pre-defined random seeds that are de facto bound to specific positions  $t$ . This partially addresses the problem but leaves the challenge of achieving  $n$ -shot undetectability (Hu et al., 2023) unresolved.

**Definition 5** (Sentence-level  $n$ -shot-undetectable). *For a fixed context  $\pi$ , we say that a watermarked LLM distribution is  $n$ -shot-undetectable compared to the original LLM if, for all  $s \in \Sigma^*$ ,*

$$\sum_{i=1}^n P_M(s | \pi) = \sum_{k \in K} P(K) \sum_{i=1}^n P_M^w(s | \pi; k).$$

Imagine a user asking the model the same question  $n$  times. An unwatermarked model or  $n$ -shot-undetectable scheme would generate answers  $x_i, i = 1, \dots, n$  i.i.d. from  $P_M(s | \pi)$ , while watermarking schemes such as SemStamp would always generate answers based on the same  $k$ , since the context is identical across queries. A stronger notion,  *$n$ -sequence distortion-free*, is proposed in SynthID, which removes the requirement of identical contexts in Definition 5.

**Definition 6** (Sentence-level  $n$ -sequence Distortion-free). *For a sequence of  $n$  prompts  $\pi_1, \dots, \pi_n$  and a sequence of  $n$  responses  $y_1, \dots, y_n \in \Sigma^*$ , we say that a watermarked LLM distribution is  $n$ -sequence distortion-free if, for all  $s \in \Sigma^*$ ,*

$$\sum_{i=1}^n P_M(y_i | \pi_i) = \sum_{k \in K} P(K) \sum_{i=1}^n P_M^w(y_i | \pi_i; k; (\pi_1, y_1), \dots, (\pi_{i-1}, y_{i-1})).$$

In token-level watermarking methods such as Unbiased (Hu et al., 2023) and SynthID (Dathathri et al., 2024), this is achieved by skipping previously seen contexts and directly outputting unwatermarked tokens.

However, in semantic-level watermark this is difficult to implement. Using the proxy of the previous sentence like SemStamp reintroduces the problem of reduced randomness in  $K$ . Furthermore, mixing SWM with token-level random seed generators may increase vulnerability to paraphrase attacks. Therefore, we consider semantic-level dynamic random seed generation to be a challenging open problem and leave it for future work. In this work, we directly adopt pre-defined random seeds following Christ et al. (2024) to achieve single-shot distortion-freeness.

## E ANALYSIS OF EXISTING SWM METHODS

In Section 2, we presented a brief analysis of existing SWM methods, whose core lies in the proxy functions they employ.

**SemStamp.** Specifically, the proxy function of SemStamp can be defined as

$$\mathcal{F}_{\text{SemStamp}} = \text{LSH}(\mathcal{T}(s)) = [\text{LSH}_1(\mathcal{T}(s)) \parallel \dots \parallel \text{LSH}_n(\mathcal{T}(s))],$$

where  $\text{LSH}_i(v) = \text{sign}(u_i^\top v)$  (Indyk & Motwani, 1998; Charikar, 2002),  $\{u_i\}_{i=1}^n \subset \mathbb{R}^d$  is a set of randomly initialized vectors, and  $\mathcal{E}$  is a text encoder with  $\mathcal{T}(s) \in \mathbb{R}^d$ . The Green Region  $G_t$  for

each sentence position  $t$  is then defined as

$$G_t = U'[:, \gamma \cdot 2^d], \quad U' = \text{Shuffle}(U; r_t), \quad r_t = q \cdot \mathcal{F}(s_{t-1}),$$

where  $\text{Shuffle}(U; r)$  denotes shuffling the ordered list  $U$  with random seed  $r$ , and  $q$  is a large prime.

**k-SemStamp.** For k-SemStamp (Hou et al., 2023),

$$\mathcal{F}_{\text{k-SemStamp}} = \operatorname{argmax}_j \frac{u_j^\top \mathcal{T}(s)}{\|u_j\| \cdot \|\mathcal{T}(s)\|},$$

where  $\{u_j\}$  are cluster centers obtained via k-means clustering on the entire dataset. Similar to SemStamp (Hou et al., 2023), the Green Region  $G_t$  is given by

$$G_t = U'[:, \gamma \cdot 2^d], \quad U' = \text{Shuffle}(U; r_t), \quad r_t = q \cdot \mathcal{F}(s_{t-1}).$$

**SimMark.** For SimMark (Dabiriaghdam & Wang, 2025),

$$\mathcal{F}_{\text{SimMark}}(s_t) = \frac{\mathcal{T}(s_{t-1})^\top \mathcal{T}(s_t)}{\|\mathcal{T}(s_{t-1})\| \cdot \|\mathcal{T}(s_t)\|},$$

where  $s_{t-1}$  is the previous sentence. A fixed interval is then used as the Green Region. Under the cosine similarity used above,  $G_t$  is always

$$G_t = [0.68, 0.76].$$

Such a pre-defined threshold carries a high risk that no valid candidate sentence has a proxy lying within the Green Region. Moreover, these carefully tuned hyperparameters may perform poorly in other settings, as shown in Table 1.

## F OFFLINE ALGORITHM

To describe the generation and detection process of the offline version of PMARK more precisely, we illustrate it in pseudocode, as shown in Figure 6.

## G IMPLEMENTATION DETAILS

### G.1 BASELINES

**MarkLLM.** We use the official implementation of MarkLLM (Pan et al., 2024)<sup>1</sup> to reproduce the results of various token-level baselines, including KGW (Kirchenbauer et al., 2023), UPV (Liu et al., 2023a), MorphMark (Wang et al., 2025), SIR (Liu et al., 2023b), EXP (Aaronson, 2023), EXPGumbel (Aaronson, 2023), and SynthID (Dathathri et al., 2024). During generation, we set `max_new_tokens` to 256, comparable to the generation length used by semantic-level methods. The temperature and `top_p` are fixed at 0.7 and 0.95, respectively, across all experiments to ensure fair comparison. In addition, for baselines that depend on external networks (e.g., SIR (Liu et al., 2023b) and UPV (Liu et al., 2023a)), we use the official weights provided by MarkLLM.

**k-SemStamp.** We adapt the official k-SemStamp implementation from MarkLLM to align it with other semantic-level methods. Specifically, we set `max_new_sentences` to 12 (instead of a token count), which is applied consistently across all semantic-level baselines. We set `max_trials` to the default value of 100 for SemStamp, k-SemStamp, and SimMark. For the embedding model, we use the fine-tuned `all-mpnet-base-v2` (Reimers & Gurevych, 2019)<sup>2</sup> provided by Hou et al. (2024). We also employ the k-means centroid weights<sup>3</sup> released by Hou et al. (2024), trained on the C4 (Raffel et al., 2020) and BookSum (Kryściński et al., 2021) datasets. For the backbone generation models OPT-1.3B (Zhang et al., 2022)<sup>4</sup> and Mistral-7B-v0.1 (Jiang et al., 2023)<sup>5</sup>, we use the original Hugging Face versions to ensure fair comparison with all other methods. All other hyperparameters are kept at their default settings in the k-SemStamp implementation of MarkLLM. For all SWM experiments, we assume that sentence segmentation is consistent during both generation and detection, such that we evaluate only the intrinsic capability of the SWM method.

**SemStamp.** For SemStamp (Hou et al., 2023), we use the authors’ official implementation, setting `temperature` = 0.7, `top_p` = 0.95, and `max_new_sentences` = 12 to match the other semantic-level methods. We also use the fine-tuned embedding model and the original generation model to ensure fair comparison.

<sup>1</sup><https://github.com/THU-BPM/MarkLLM>

<sup>2</sup><https://huggingface.co/AbeHou/SemStamp-c4-sbert>

<sup>3</sup><https://github.com/abehou/SemStamp>

<sup>4</sup><https://huggingface.co/facebook/opt-1.3b>

<sup>5</sup><https://huggingface.co/mistralai/Mistral-7B-v0.1>

Table 4: Overall results for baseline methods and PMARK on OPT-1.3B and Mistral-7B on BOOKSUM. **Doc-T** denotes back-translation attack by LLama-3.1-8B, while Word-D and Word-S are word-level attacks under different ratios.

Method	No Attack <sup>†</sup>	Doc-P (Pegasus) <sup>‡</sup>	Doc-P (Parrot) <sup>‡</sup>	Doc-P (GPT) <sup>†</sup>	Doc-T <sup>§</sup>	Word-D(0.05) <sup>¶</sup>	Word-D(0.15) <sup>¶</sup>	Word-D(0.30) <sup>¶</sup>	Word-S(0.05) <sup>¶</sup>	Word-S(0.15) <sup>¶</sup>	Word-S(0.30) <sup>¶</sup>
OPT-1.3B											
EXP (2023)	98.8/99.0/99.3	4.2/14.6/52.5	5.4/16.0/55.2	4.4/13.4/53.0	5.4/17.8/53.8	3.8/14.2/56.5	2.2/11.0/56.0	1.6/7.2/54.3	6.0/17.2/57.1	2.4/14.4/55.5	3.2/11.4/54.4
EXP/Gumbel (2023)	99.4/99.4/499.4	12.4/20.2/52.6	12.4/20.2/52.2	13.0/23.2/52.4	21.4/20.6/55.4	23.8/32.6/55.4	20.4/27.6/57.2	13.4/34.0/58.1	19.8/28.6/56.3	16.4/24.6/52.1	
KGW (2023)	100.0/100.0/100.0	1.4/10.6/52.5	1.4/5.9/52.2	0.6/10.0/52.5	1.4/10.0/52.5	1.4/20.7/58.5	1.2/17.8/58.5	0.6/10.0/52.5	1.2/17.8/58.5	1.4/12.2/52.9	
SIR (2023)	99.8/100.0/99.8	82.9/93.9/97.5	80.8/93.8/99.1	83.8/82.8/99.9	77.4/83.9/99.5	96.2/98.5/99.6	86.0/93.8/99.5	62.4/76.6/99.0	98.4/100.0/99.8	92.4/99.4/99.5	69.4/99.6/99.5
UPV (2023a)	100.0/100.0/0.100.0	90.0/95.6/69.8	77.8/20.9/29.9	73.6/91.0/98.0	98.6/99.6/69.7	100.0/100.0/100.0	98.7/100.0/100.0	100.0/100.0/100.0	100.0/100.0/100.0	100.0/100.0/100.0	99.4/99.6/99.8
SynthID (2024)	100.0/100.0/0.099.9	38.2/56.6/86.9	25.8/13.6/63.8	1.8/14.6/65.7	96.0/97.0/99.2	99.2/100.0/0.099.9	95.4/98.0/0.099.5	56.0/77.4/94.5	99.6/100.0/0.099.9	95.6/98.5/99.9	54.8/76.0/94.5
MorphMark (2025)	100.0/100.0/0.100.0	1.8/7.7/55.2	1.2/6.1/54.6	0.8/4.2/51.1	5.0/13.3/60.0	1.8/7.3/57.9	1.2/7.2/56.7	1.2/7.5/57.0	2.0/9.2/58.7	3.4/10.5/60.0	1.0/8.2/61.6
SemStamp (2023)	97.7/98.8/89.4	89.0/93.2/97.3	79.6/85.4/97.5	87.8/92.8/97.7	97.6/98.2/99.3	89.4/94.4/98.4	67.5/78.0/89.5	97.7/98.6/99.5	93.2/96.8/99.1	78.1/86.6/96.4	
KSESTAMP (2024)	99.6/100.0/0.099.9	74.5/84.6/69.7	72.3/86.6/69.7	62.1/72.9/95.2	72.3/84.2/97.2	93.8/97.0/99.6	64.7/76.8/96.3	29.1/44.3/86.7	99.2/99.8/99.9	81.8/90.6/98.5	44.1/59.3/92.5
SimMark (2025)	88.2/94.8/09.8	23.4/42.8/84.6	29.0/45.6/87.3	22.0/40.4/83.6	22.2/31.8/76.7	68.4/82.4/97.0	39.8/56.8/91.1	11.8/26.6/79.7	80.4/93.4/98.4	45.8/63.0/92.2	19.6/34.0/79.5
Ours(Online)	99.8/99.8/89.9	<b>97.4/99.0/99.8</b>	<b>96.8/99.8/99.8</b>	<b>95.4/99.0/99.8</b>	<b>93.2/96.8/99.2</b>	<b>99.8/99.8/99.9</b>	<b>96.0/97.8/99.9</b>	<b>86.2/94.4/98.7</b>	<b>99.8/99.8/99.9</b>	<b>98.2/99.0/99.8</b>	<b>90.8/95.2/99.1</b>
Ours(Offline)	99.4/99.6/69.8	<b>93.2/98.2/99.5</b>	<b>95.2/98.6/69.5</b>	<b>94.2/98.8/99.6</b>	<b>96.4/98.8/99.6</b>	<b>98.6/99.4/49.8</b>	<b>98.0/99.0/99.7</b>	<b>86.7/96.0/99.0</b>	<b>99.4/99.6/69.9</b>	<b>98.4/99.4/99.8</b>	<b>87.0/96.6/99.4</b>
Mistral-7B											
EXP (2023)	99.8/99.8/89.9	36.4/53.0/85.8	47.2/61.8/89.1	10.4/22.0/71.7	93.6/95.4/97.7	98.4/98.8/99.4	95.6/97.4/99.2	76.8/86.6/96.6	98.6/98.6/99.5	97.2/98.2/99.1	82.2/92.0/97.6
EXP/Gumbel (2023)	99.6/99.6/69.7	65.6/77.2/90.8	78.8/85.2/92.8	38.4/53.2/81.9	95.6/96.4/98.0	98.6/99.4/49.9	98.2/98.0/99.1	98.6/99.2/99.5	98.2/98.4/99.2	95.0/97.4/99.1	
KGW (2023)	100.0/100.0/0.100.0	84.2/95.4/98.7	90.2/97.3/99.2	35.8/68.6/99.2	93.0/96.8/89.8	<b>100.0/100.0/100.0</b>	<b>99.8/100.0/100.0</b>	<b>99.0/99.7/99.9</b>	<b>100.0/100.0/100.0</b>	<b>100.0/100.0/100.0</b>	<b>99.6/99.9/99.9</b>
SIR (2023b)	100.0/100.0/0.100.0	85.0/92.6/69.8	80.8/92.0/98.2	41.2/71.4/93.8	81.2/91.0/96.9	90.0/95.6/69.8	72.4/81.2/93.5	96.4/98.6/99.7	99.6/100.0/0.100.0	99.6/100.0/0.100.0	
UPV (2023a)	99.6/100.0/100.0	72.6/96.6/29.8	73.0/96.6/99.0	33.4/76.2/29.4	92.4/97.0/99.5	<b>98.8/100.0/100.0</b>	<b>99.6/100.0/100.0</b>	<b>96.6/99.8/99.5</b>	<b>98.8/100.0/100.0</b>	<b>99.6/100.0/100.0</b>	<b>98.8/99.8/99.9</b>
SynthID (2024)	100.0/100.0/0.100.0	30.4/49.8/84.0	38.4/48.8/84.5	24.6/52.8/86.7	95.0/99.4/99.4	99.0/99.8/99.8	64.4/80.8/99.7	99.6/99.9/99.6	99.6/100.0/100.0	97.4/98.6/99.6	53.2/74.4/94.5
MorphMark (2025)	99.8/99.8/89.9	38.2/49.8/87.2	77.4/80.8/89.0	24.6/52.8/86.7	95.0/99.4/99.4	99.0/99.8/99.8	64.4/80.8/99.7	99.6/99.9/99.6	100.0/100.0/100.0	97.4/98.6/99.6	
SemStamp (2023)	97.7/98.8/89.5	88.1/92.8/89.7	92.4/95.2/29.1	76.0/83.0/94.5	87.1/94.2/29.8	96.3/98.6/99.4	87.9/94.2/98.6	96.7/98.4/99.4	98.4/95.0/98.7	64.4/77.6/94.6	
KSESTAMP (2024)	99.0/99.0/0.097.7	45.5/64.1/192.4	53.9/70.3/94.8	43.5/61.9/191.6	67.3/79.8/95.9	85.4/92.8/93.9	47.9/64.5/59.3	19.2/33.2/80.0	96.4/97.8/89.5	63.1/80.0/96.2	28.9/47.3/88.8
SimMark (2025)	78.0/89.4/49.7	19.8/40.6/83.6	28.8/46.2/85.9	23.4/45.0/84.9	21.0/35.0/74.3	54.6/77.0/95.2	27.6/64.8/28.4	16.4/32.6/78.7	70.6/84.6/69.7	40.4/61.2/91.3	17.4/32.6/78.9
Ours(Online)	100.0/100.0/0.099.9	<b>94.4/98.2/99.6</b>	<b>96.6/98.8/99.7</b>	<b>94.4/97.4/99.5</b>	<b>99.6/99.8/99.9</b>	<b>98.2/100.0/99.9</b>	<b>82.9/93.2/98.7</b>	<b>100.0/100.0/99.9</b>	<b>99.0/100.0/99.9</b>	<b>93.6/97.8/99.9</b>	
Ours(Offline)	99.4/100.0/0.099.9	<u>91.4/97.4/99.3</u>	<u>94.6/97.8/99.6</u>	<u>94.4/98.6/99.6</u>	<u>95.8/99.2/99.7</u>	<u>99.4/100.0/0.099.9</u>	<u>95.4/99.4/99.8</u>	<u>78.9/95.8/98.6</u>	<u>99.4/99.8/99.9</u>	<u>98.6/99.6/99.8</u>	<u>84.9/95.2/98.8</u>

**SimMark.** We use the official implementation of SimMark ([Dabiriaghdam & Wang, 2025](#))<sup>6</sup>. We adopt cosine distance with the recommended validity interval [0.68, 0.76]. Additionally, we use the recommended embedding model `instructor-large` ([Su et al., 2023](#))<sup>7</sup> and the original generation models OPT-1.3B and Mistral-7B-v0.1 to maintain comparability with other methods. The factor  $K$  is set to 250 for the soft- $z$  test, following the original paper.

**PMARK.** During generation, we use the original `all-mpnet-base-v2` embedding model ([Reimers & Gurevych, 2019](#))<sup>8</sup> without fine-tuning. Pivot vectors are generated via QR decomposition of a Gaussian random matrix ([Trefethen & Bau, 2022](#)) to ensure orthogonality. We set temperature = 0.7, top-p = 0.95, and max\_new\_sentences = 12, consistent with other semantic-level methods. The sample budget is  $N = 64$ , and the number of channels is  $b = 4$ . For detection, we set the numerical threshold  $\delta = 0.001$  and the smoothing factor  $K = 150$ .

## G.2 ADVERSARIAL ATTACKS

**Doc-P.** We conduct paraphrase attacks following the official SemStamp implementation [Hou et al. \(2023; 2024\)](#). Specifically, for the paraphrasers Pegasus ([Pu et al., 2023](#))<sup>9</sup> and Parrot ([Sadasivan et al., 2023](#))<sup>10</sup>, we adopt the bigram-style attack proposed by SemStamp [Hou et al. \(2023\)](#), which selects paraphrases with minimal bigram overlap relative to the source. For the GPT ([OpenAI, 2022](#)) paraphrasing attack, we use the prompt provided by MarkLLM ([Pan et al., 2024](#)). All attacks are applied at the sentence level across all methods and settings to simulate real-world scenarios.

**Doc-T.** We implement the back-translation attack following MarkLLM. Specifically, we use Llama-3.1-8B ([Dubey et al., 2024](#))<sup>11</sup> to translate watermarked English sentences into Spanish and then back into English in an attempt to remove watermark evidence. However, as noted in Section 5.2, this attack can be biased and unstable in some cases.

**Word-D and Word-S.** We implement word deletion (Word-D) and synonym substitution (Word-S) attacks following the official MarkLLM implementation. We set the attack ratios to 5%, 15%, and 30% to evaluate the overall robustness of our method. Additional experimental results are reported in Appendix H.

## H SUPPLEMENTED EXPERIMENTAL RESULTS

### H.1 ROBUSTNESS TO ADVERSARIAL ATTACKS

We evaluate the robustness of existing watermarking methods under various attacks, with results shown in Table 4 and Table 5.

<sup>6</sup><https://github.com/DabiriAghdam/SimMark>

<sup>7</sup><https://huggingface.co/hkunlp/instructor-large>

<sup>8</sup><https://huggingface.co/sentence-transformers/all-mpnet-base-v2>

<sup>9</sup>[https://huggingface.co/tuner007/pegasus\\_paraphrase](https://huggingface.co/tuner007/pegasus_paraphrase)

<sup>10</sup>[https://github.com/PrithivirajDamodaran/Parrot\\_Paraphraser](https://github.com/PrithivirajDamodaran/Parrot_Paraphraser)

<sup>11</sup><https://huggingface.co/meta-llama/Llama-3.1-8B>

**Algorithm 3: PMARK Offline Generation**

**Input:**  $M; s^{(0)}; \mathcal{T}; T; b; v^{(1)}, \dots, v^{(b)}; R = \{r_{(t,j)}\}; N$   
**Output:**  $s^{(1)}, \dots, s^{(T)}$   
**for**  $t \leftarrow 1$  **to**  $T$  **do**  
  **for**  $j \leftarrow 1$  **to**  $b$  **do**  
     $\mathcal{F}_j(s) \leftarrow \langle v^{(j)}, \mathcal{T}(s) \rangle$   
  **end**  
   $u_{\max} \leftarrow -\infty; B \leftarrow \emptyset$  **for**  $i \leftarrow 1$  **to**  $N$  **do**  
    Sample  $x^{(i)} \sim P_M(s | s^{(0:t-1)})$   
     $\text{Sig}(x^{(i)}) \leftarrow [\mathcal{F}_j(x^{(i)}) > 0]_{j=1}^b$   
     $u^{(i)} \leftarrow \sum_{j=1}^b \text{Sig}(x^{(i)})(j)$  **if**  $u^{(i)} = b$  **then**  
       $s^{(t)} \leftarrow x^{(i)}$   
      **continue**  
    **end**  
    **if**  $u^{(i)} > u_{\max}$  **then**  
       $u_{\max} \leftarrow u^{(i)}; B \leftarrow \{x^{(i)}\}$   
    **end**  
    **else if**  $u^{(i)} = u_{\max}$  **then**  
       $B \leftarrow B \cup \{x^{(i)}\}$   
    **end**  
  **end**  
  Select  $s^{(t)} \sim \text{Un}(B)$   
**end**  
**return**  $s^{(1)}, \dots, s^{(T)}$

**Algorithm 4: PMARK Offline Detection**

**Input:**  $S = [s^{(0)}, \dots, s^{(T)}]; \mathcal{T}; v^{(1\dots b)}; R = \{r_{(t,j)}\}; \delta; K; \alpha$   
**Output:** True or False  
**for**  $j \leftarrow 1$  **to**  $b$  **do**  
  Define  $\mathcal{F}_j(s) = \langle v^{(j)}, \mathcal{T}(s) \rangle$   
**end**  
 $N_g \leftarrow 0; N_{\text{total}} \leftarrow b \cdot T$   
**for**  $t \leftarrow 1$  **to**  $T$  **do**  
  **for**  $j \leftarrow 1$  **to**  $b$  **do**  
     $x_{(t,j)} \leftarrow \mathcal{F}_j(s^{(t)}); m_{(j)} \leftarrow 0$   
    **if**  $r_{(t,j)} = 1$  **and**  
       $x_{(t,j)} > m_{(j)} - \delta$  **then**  
        $c_{(t,j)} \leftarrow 1$   
    **else if**  $r_{(t,j)} = 0$  **and**  
       $x_{(t,j)} < m_{(j)} + \delta$  **then**  
        $c_{(t,j)} \leftarrow 1$   
    **else**  
       $c_{(t,j)} \leftarrow \exp(-K|x_{(t,j)} - m_{(j)}|)$   
    **end**  
     $N_g \leftarrow N_g + c_{(t,j)}$   
  **end**  
**end**  
 $z \leftarrow \frac{|N_g - 0.5 N_{\text{total}}|}{\sqrt{0.25 N_{\text{total}}}}$ ; **return** ( $z > z_{\alpha}$ )

Figure 6: PMARK Offline Watermarking: Left: Generation using fixed prior median = 0; Right: Detection using soft z-test.

Table 5: Overall results for baseline methods and PMARK on OPT-1.3B and Mistral-7B on C4.

Method	No Attack↑	Doc-P (Pegasus)	Doc-P (Parrot)	Doc-P (GPT)*	Doc-T*	Word-D(0.05)*	Word-D(0.15)*↑	Word-D(0.30)*↑	Word-S(0.05)*↑	Word-S(0.15)*↑	Word-S(0.30)*↑
	OPT-1.3B										
EXP (2023)	99.009/0.009.8	54.6/65.2/86.9	42.4/59.6/84.6	21.2/37.2/73.3	90.6/93.4/96.9	98.0/95.2/99.5	96.0/97.8/99.2	72.0/85.6/95.8	98.4/95.8/99.6	96.5/98.2/99.5	80.4/89.4/95.9
EXP-Gumbel (2023)	98.6/98.6/0.009.3	75.2/84.6/80.3	74.4/86.1/92.8	55.0/66.0/86.0	94.2/95.5/96.5	97.2/97.2/99.0	95.0/95.8/98.1	97.8/93.3/99.1	96.5/97.4/99.3	94.4/95.4/99.3	84.0/89.4/95.9
KGW (2023)	100.0/100.0/0.000.0	89.3/97.0/0.009.2	76.2/91.4/98.1	51.4/78.5/95.0	90.5/94.6/97.3	100.0/100.0/100.0	100.0/100.0/100.0	5.5/100.0/100.0	100.0/100.0/100.0	100.0/100.0/100.0	98.5/99.8/99.9
SIR (2023)	98.8/100.0/0.009.0	91.6/94.6/0.007.8	82.4/90.6/97.5	71.2/88.7/95.7	82.8/91.4/94.9	98.6/99.4/99.9	99.0/98.2/99.8	79.0/88.6/98.2	88.8/90.8/99.8	94.4/96.8/99.2	77.0/86.2/95.8
UPV (2023)	100.0/100.0/0.000.0	90.6/95.4/98.3	78.1/88.4/97.4	80.9/91.3/98.2	98.2/98.8/99.5	100.0/100.0/100.0	100.0/100.0/100.0	100.0/100.0/100.0	100.0/100.0/100.0	100.0/100.0/100.0	99.9/100.0/99.9
SynhID (2024)	100.0/100.0/0.009.9	45.4/56.2/289.5	26.4/49.4/82.7	6.4/21.2/67.6	90.2/98.7/99.5	100.0/100.0/0.009.9	93.4/99.0/0.009.7	46.8/73.0/92.9	100.0/100.0/0.009.9	96.6/99.6/99.7	49.4/76.4/94.8
MorphMark (2025)	100.0/100.0/0.000.0	78.6/93.0/0.007.8	70.2/87.1/106.8	46.7/76.2/93.2	87.0/92.4/97.2	100.0/100.0/100.0	99.9/100.0/100.0	93.2/98.2/99.7	100.0/100.0/100.0	94.4/100.0/99.9	94.9/99.8/99.9
SemStamp (2023)	94.6/96.1/0.008.9	85.8/89.0/0.006.9	84.4/89.3/96.2	73.5/81.4/93.0	80.0/85.7/95.2	91.7/94.7/98.5	82.8/90.0/0.007.0	57.8/71.4/90.0	93.1/96.5/98.9	86.7/91.6/97.4	61.6/75.0/92.2
KSEMSTAMP (2024)	100.0/100.0/0.009.9	76.6/89.2/0.007.1	74.3/87.0/0.007.5	62.9/78.6/94.1	62.4/78.0/93.8	95.9/98.8/99.5	69.2/84.6/95.9	35.2/53.2/85.8	98.9/99.8/99.9	79.8/90.2/97.7	44.4/63.0/90.7
SimMark (2025)	77.6/94.2/0.008.9	11.6/35.8/82.9	13.6/39.2/84.5	16.0/44.8/86.5	11.0/26.8/75.3	52.2/83.4/96.6	27.4/57.0/87.5	8.8/27.4/90.7	66.6/68.2/97.5	30.0/60.0/91.4	9.8/28.2/77.7
Ours(Online)	100.0/100.0/0.009.9	<b>96.2/99.6/99.9</b>	<b>97.2/99.0/99.8</b>	<b>97.8/99.6/99.9</b>	<b>95.2/98.0/0.009.5</b>	<b>99.2/99.8/99.9</b>	<b>97.2/98.4/99.7</b>	<b>85.6/94.2/98.7</b>	<b>100.0/100.0/0.009.9</b>	<b>98.4/99.6/99.9</b>	<b>87.6/96.0/99.2</b>
Ours(Offline)	98.0/99.8/0.009.8	91.7/94.4/98.8	91.4/95.2/99.0	92.6/96.2/299.1	94.7/97.0/0.009.4	97.9/99.0/0.009.7	95.3/98.0/0.009.5	84.5/91.6/98.1	97.8/98.8/99.7	95.1/98.4/99.4	88.2/93.8/98.5
Mistral-7B											
EXP (2023)	99.2/99.2/299.5	44.4/60.4/85.7	34.2/54.8/84.2	17.4/29.8/72.1	88.8/92.4/97.6	98.2/98.8/99.4	93.2/96.0/0.009.8	63.0/75.6/94.2	98.2/99.0/0.009.4	95.6/98.6/99.3	74.0/85.2/96.5
EXP-Gumbel (2023)	98.6/98.6/0.009.3	71.2/82.0/92.5	66.2/77.2/90.8	40.2/55.4/82.5	92.0/93.8/97.2	98.2/98.4/99.0	97.2/98.6/99.0	91.4/95.8/98.3	98.2/99.2/99.0	92.6/96.4/98.4	74.0/85.2/96.5
KGW (2023)	100.0/100.0/0.000.0	85.6/95.5/0.009.8	79.2/93.7/98.8	54.2/76.7/95.6	92.0/93.2/99.5	100.0/100.0/100.0	100.0/100.0/100.0	99.4/99.9/99.9	100.0/100.0/100.0	99.0/100.0/99.9	94.0/98.0/99.9
UPV (2023)	99.4/99.8/0.009.9	87.9/93.0/0.007.8	80.8/89.4/98.8	83.3/91.4/98.7	84.0/93.5/98.7	95.2/97.8/99.5	75.7/87.0/0.009.8	94.4/97.8/99.5	94.4/97.8/99.5	94.4/97.8/99.5	74.0/85.2/96.5
SynhID (2024)	99.8/99.8/0.009.8	47.0/56.0/86.7	11.4/91.4/97.7	51.0/98.5/97.5	34.0/95.6/93.4	88.0/95.6/98.9	100.0/100.0/100.0	2/100.0/0.009.9	7/100.0/0.009.9	92.8/100.0/0.009.9	92.4/100.0/0.009.9
MorphMark (2025)	98.6/98.0/0.009.0	74.8/89.4/97.5	70.0/86.4/97.4	74.7/89.2/91.4	80.4/88.4/96.5	94.4/100.0/100.0	94.0/99.8/99.9	57.2/70.4/93.5	99.4/99.8/99.9	92.6/98.4/99.7	51.8/71.4/95.0
SemStamp (2023)	92.5/95.8/0.008.3	80.9/87.9/95.9	77.9/86.3/95.7	69.2/80.0/92.4	70.1/82.4/94.6	89.3/93.6/97.8	82.0/88.8/96.3	53.0/68.1/90.8	92.3/94.9/98.2	82.1/88.9/96.8	57.8/73.9/91.5
KSEMSTAMP (2024)	100.0/100.0/0.009.9	49.2/68.4/93.2	54.8/71.6/93.2	43.9/61.2/89.2	59.6/75.8/94.6	89.7/95.6/99.2	53.7/73.2/94.2	20.8/40.6/81.8	97.2/99.4/99.7	66.2/81.8/96.5	31.0/48.0/86.6
SimMark (2025)	70.8/89.6/0.007.9	12.4/31.2/81.5	14.6/37.3/84.4	12.6/47.9/86.4	12.0/22.9/67.9	50.8/76.8/95.8	27.0/51.9/89.2	13.8/31.6/80.7	64.6/68.5/97.2	28.6/54.7/90.5	10.8/25.6/74.1
Ours(Online)	100.0/100.0/0.009.9	<b>93.0/96.8/99.4</b>	<b>93.6/97.6/99.4</b>	<b>95.2/98.8/99.7</b>	<b>93.6/97.2/99.2</b>	<b>100.0/100.0/0.009.9</b>	<b>98.2/99.4/99.7</b>	<b>85.8/93.0/0.009.6</b>	<b>100.0/100.0/0.009.9</b>	<b>98.6/100.0/0.009.9</b>	<b>89.4/96.4/99.4</b>
Ours(Offline)	99.7/99.8/0.009.9	90.8/95.8/99.1	91.3/95.8/99.3	92.0/95.2/99.3	<b>94.0/97.2/99.5</b>	99.2/99.8/0.009.9	95.2/98.4/99.7	84.1/93.6/98.9	99.6/99.8/0.009.9	97.1/98.6/99.7	87.1/94.6/98.5

## H.2 QUALITY OF WATERMARKED TEXT

Similar to Figure 3, we present additional results about the perplexity of existing methods across different benchmarks and backbones in Figure 7, 8 and 9.

## H.3 SENSITIVITY OF HYPERPARAMETER

We present additional hyperparameter analysis results for OPT-1.3B in Figure 10, where similar conclusions can be deduced.

## H.4 CASE STUDY

We illustrate various cases of SemStamp and PMARK across different backbones and datasets. Compared with the baseline, PMARK demonstrates superior text quality and robustness, producing more diverse sentences and retaining watermark evidence even after paraphrase attacks.

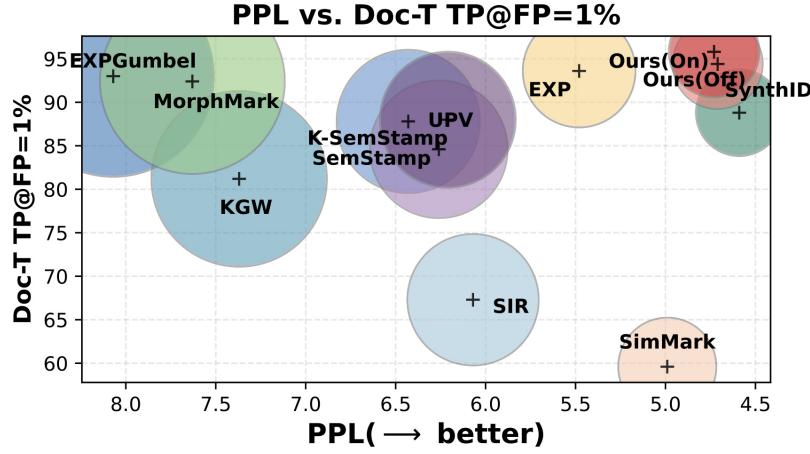


Figure 7: Mistral-7B PPL on BOOKSUM.

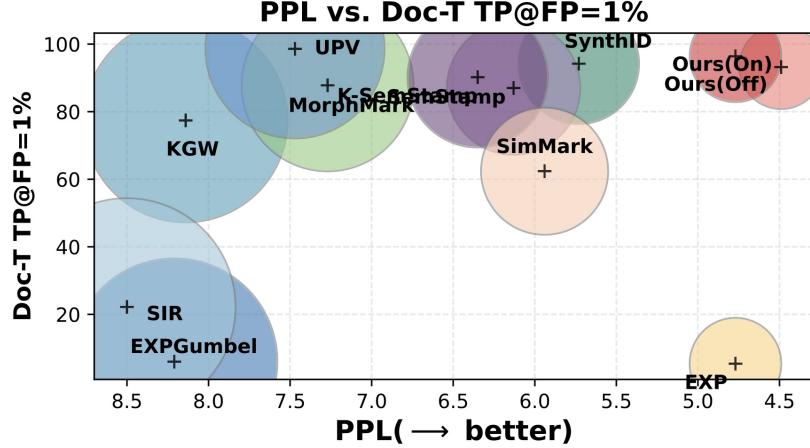


Figure 8: OPT-1.3B PPL on BOOKSUM.

## I BROADER IMPACTS AND LIMITATIONS

As a pioneering study on distortion-free and robust semantic-level watermarking, PMARK paves the way for more secure and powerful solutions for copyright protection and content attribution in the AI industry. However, several limitations remain. (1) For simplicity, we consider only fixed random seeds in this work, although more flexible techniques such as sliding windows and pseudo-random error-correcting codes could enhance adaptability. (2) While the sampling-based generation paradigm avoids the sampling failures of prior methods and provides an indirect estimate of the next-sentence distribution—thereby reducing distortion—the required sampling budget still needs to be further reduced for practical deployment. (3) Current SWM methods, including PMARK, operate at the sentence level, which depends on consistent sentence segmentation between the generation and detection processes. Future research should explore more advanced SWM methods that eliminate this reliance and embed watermark evidence directly into the semantics of text, potentially enabling more sophisticated capabilities such as detecting whether specific ideas originate from LLMs.

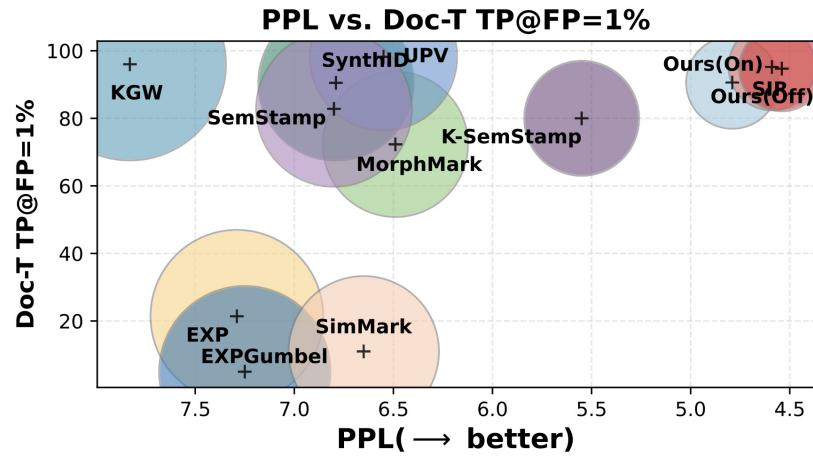


Figure 9: OPT-1.3B PPL on C4.

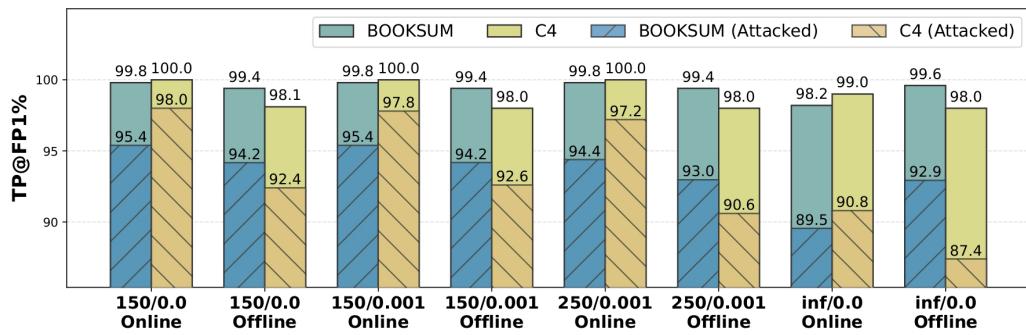


Figure 10: Performance comparison of OPT-1.3B under different hyperparameter settings.

<p>Peggotty and David prepare to depart for the Spenlows' home.</p> <p>The Dirty Fingers lead a young man from the city to the Dover cliffs.</p> <p>The man sees a vision of his love, who is about to marry another man, and commits suicide.</p> <p>David and Peggotty arrive at the Spenlows' house, where they are welcomed by Dora and her aunt, Miss Clarissa.</p> <p>The sisters and Peggotty are all "greatly affected" at the sight of one another, and Lavinia and Clarissa are "as ha.....</p> <p>David and Peggotty are invited in, and they all sit down to tea.</p> <p>Peggotty, who has been too overcome with joy to speak suddenly begins to weep, and Lavinia and Clarissa embr.....</p> <p>Peggotty then tells the sisters all about the events of the last few days, including the recent discovery of David's paren.....</p> <p>The sisters are "very much affected" at this news, and Clarissa promises to help David get back to London.</p> <p>David's meeting with the Spenlows is yet another example of the way in which people who have been separated from .....</p> <p>Peggotty's joy at seeing her sisters again, David's joy at learning his parentage, and the Spenlows' joy at seeing .....</p>	<p>David arrives at the Spenlow house, and is shown to a room with a view of the river.</p> <p>Miss Lavinia enters and they speak of Dora's recent illness.</p> <p>David relates the story of his marriage to Dora, and Lavinia urges him to accompany Dora to Switzerland.</p> <p>David says he cannot, because he does not have the money to go.</p> <p>Miss Lavinia is Dora's aunt, and Miss Clarissa is Dora's other aunt.</p> <p>They are unmarried, and live together.</p> <p>Analysis: David's marriage to Dora and his subsequent inability to provide for her financially are symbolic of his failure to.....</p> <p>David's financial difficulties, his lack of employment, and his inability to make money are representative of a larger pr.....</p> <p>He is not mature enough to provide for himself, much less for a wife.</p> <p>He has no skills and no prospects.</p> <p>Get the entire David Copperfield LitChart as a printable PDF.</p> <p>Topics of Discussion: David Copperfield is a story of a young boy's life, told by David Copperfield himself.</p>	<p>He also receives a letter from Dora, who is now living in a boarding-house in Southampton, and is in need of money.</p> <p>David decides to take his friend Steerforth to call on Miss Spenlow, in order to arrange the introduction of Steerforth .....</p> <p>Steerforth's visit to Miss Spenlow is very successful.</p> <p>He manages to charm Miss Lavinia, and also to prevent Miss Clarissa from expressing any reservations about David's .....</p> <p>The next day, David goes to visit Dora with Steerforth.</p> <p>David is overwhelmed by the sight of Dora, whom he had not seen for several years.</p> <p>She is now a young woman, and David feels as if he has only known her as a child.</p> <p>Her is also won over by the changes in Steerforth, who has now become a dashing young man.</p> <p>After spending some time with Dora, David and Steerforth go for a walk, during which Steerforth encourages David to .....</p> <p>David returns to his lodgings, where he receives a visit from Mrs. Steerforth.</p> <p>She tells him that she has received a letter from Steerforth's aunt, Mrs. Steerforth, requesting David to visit her in the .....</p> <p>David agrees to go, but he is reluctant to leave London for the country, as he has business there.</p>
(a) SemStamp without attack.	(b) Offline PMARK without attack.	(c) Online PMARK without attack.
<p>Peggotty and David get ready to leave for the Spenlows' residence.</p> <p>The Dirty Fingers guided a young man from the city to the cliffs of Dover.</p> <p>The man witnesses a vision of his beloved, who is on the verge of marrying someone else, and tragically takes his own life.</p> <p>David and Peggotty reach the Spenlows' residence and are greeted by Lavinia, a petite and elegant individual. Lavinia .....</p> <p>The sisters and Peggotty were deeply moved upon seeing each other, while Lavinia and Clarissa were overjoyed like .....</p> <p>David and Peggotty were welcomed inside, and they all gather around for tea.</p> <p>Peggotty, overwhelmed with joy, is unable to speak and starts to cry, prompting David to leave to comfort .....</p> <p>Peggotty is eager to inform the sisters about the recent events of the past few days, which also includes the recent .....</p> <p>The sisters are deeply impacted by this news, and Clarissa insists that David must accompany them back to London.</p> <p>David's encounter with the Spenlows illustrates how individuals who have been apart are reunited through .....</p> <p>The sisters are overjoyed when Peggotty reunites with her sisters. David discovers his true parentage, and the Spenlows are delighted .....</p>	<p>David reaches the Spenlow residence and is led to a room that overlooks the river.</p> <p>Miss Lavinia arrives and they discuss Dora's recent sickness.</p> <p>David tells the tale of his wedding to Dora, and Lavinia encourages him to go with Dora to Switzerland.</p> <p>David mentioned that he is unable to go because he lacks the necessary funds.</p> <p>Miss Lavinia is Dora's aunt, and Miss Clarissa is her other aunt.</p> <p>They are not married and reside together.</p> <p>Interpretation: David's marriage to Dora and his subsequent failure to support her financially represent his struggle to.....</p> <p>David's financial difficulties, unemployment, and struggle to.....</p> <p>His personal income are indicative of a broader issue.</p> <p>He is forced to turn to the community to take care of himself, let alone support a spouse.</p> <p>He lacks skills and opportunities for the future.</p> <p>Obtain the complete David Copperfield LitChart in a downloadable PDF format.</p> <p>Discussion Topics: "David Copperfield" is a narrative recounting the life of a young boy, as narrated by the pro .....</p>	<p>He also gets a letter from Dora, who currently resides in a boarding house in Southampton, requesting financial ass .....</p> <p>David chooses to bring his friend Steerforth along to visit Miss Spenlow, with the intention of introducing Steerforth .....</p> <p>Steerforth's meeting with Miss Spenlow goes extremely well.</p> <p>He successfully wins over Miss Lavinia and ensures that Miss Clarissa refrains from voicing any concerns about David's .....</p> <p>The following day, David visits Dora accompanied by Steerforth.</p> <p>David was taken aback by the sight of Dora, whom he hadn't seen for several years.</p> <p>She has grown into a young woman, and David feels like he has only ever known her as a child.</p> <p>He is also impressed by the transformation in Steerforth, who has blossomed into a charming young man.</p> <p>After spending some time with Dora, David and Steerforth take a stroll. During their walk, Steerforth urges David to pursue Creekle.</p> <p>She informs him that she has received a letter from Steerforth's aunt, Mrs. Steerforth, asking David to come .....</p> <p>David reluctantly agrees to go to the country, as he has business in London and is hesitant to leave the city.</p>
(d) SemStamp paraphrased by GPT.	(e) Online PMARK paraphrased by GPT.	(f) Offline PMARK paraphrased by GPT.

Figure 11: Cases from Mistral-7B on BOOKSUM dataset. •, ●, ■ and □ indicate valid watermark evidence is detected with the transparency denoting evidence strength, while ● indicates that no valid evidence detected in this channel.

<p>It is difficult to put into words the effect these three men had on my life.</p> <p>When I was a boy, I watched them fight against the right with dignity and determination, with its Royal .....</p> <p>They had the hope of the working class, and it was an honour to be able to call them my friends.</p> <p>They were not perfect.</p> <p>The great Labour Prime Minister, Harold Wilson, said that he could not be a socialist because he was not perfect.</p> <p>These three men were not perfect, but they were the best we had, and we will never see their like again.</p> <p>They stood for fairness and social justice, and they put the interests of working people before those of the rich.</p> <p>They all went through terrible times, but they never gave up.</p> <p>They believed in a better society, and they were prepared to fight for it.</p> <p>They were the greatest British politicians of my lifetime, and I shall miss them all.</p> <p>But I shall never forget their courage and their determination to fight against all the odds to make Britain a better and .....</p>	<p>Labour's former deputy leader Tom Watson has written a book which is a fascinating insight into life in the Commons.</p> <p>He was a member of the Labour Party's ruling NEC and the party's deputy chair.</p> <p>He has now left to concentrate on his family.</p> <p>I feel he is a man of principle, who has been smeared by his former boss Jeremy Corbyn, and was a victim of his own .....</p> <p>Watson was the youngest MP in the Commons and the first to have .....</p> <p>He had a humble start as a Labour councillor in West Bromwich and a trade union official.</p> <p>He was 29 when he became an MP, working for the people he represented as a true MP and not as a self-serving career .....</p> <p>He is a devout Christian and was a member of the Labour Friends of Israel, a group that was unfairly criticized by s .....</p> <p>His new book Dangerous Games is a detailed account of his life in Parliament, as he was a victim of the anti-Semites .....</p> <p>He is a passionate defender of the Jewish people and had to leave Parliament because of the vitriol he received from .....</p> <p>Watson has always been a thorn in the side of the hard left, which was keen to change Labour Party to make it an .....</p> <p>Watson has been a strong defender of Israel and a friend to the Jewish people.</p>	<p>I'm not sure I've ever read an article by John Rentoul that I haven't disagreed with, but this time he has gone too far.</p> <p>He is determined to resurrect the political career of George Galloway, an MP whose election to the Commons in 2012 .....</p> <p>The current Labour MP for Bradford West, Naz Shah, is being investigated by the Parliamentary standards watchdog to .....</p> <p>She has denied any wrongdoing.</p> <p>Rentoul claimed that Shah's election campaign had been dogged by allegations of fraud, and that she had been un .....</p> <p>We have not been told whether the police investigation is still ongoing or whether it has been dropped.</p> <p>Shah has said she has done nothing wrong and that the investigation was triggered by complaints from a Tory c .....</p> <p>Rentoul called for Shah to be expelled from Labour and for the Labour Party to accept her as a candidate to stand as a Lab .....</p> <p>Galloway would be a more attractive candidate for Labour in Bradford West than Shah, Rentoul said.</p> <p>The first reason given is that Shah has a criminal conviction for benefit fraud, for which she served a suspended sent .....</p> <p>A suspended sentence is not a criminal conviction.</p> <p>The second reason given is that she is under investigation by the police.</p>
(a) SemStamp without attack.	(b) Offline PMARK without attack.	(c) Online PMARK without attack.
<p>Describing the impact these three men had on my life is a challenging task.</p> <p>As a young boy, I witnessed their battle against the conservative, prejudiced, and traditional establishment, .....</p> <p>They were seen as the beacon of hope for the working class, and I felt privileged to consider them my friends.</p> <p>They were not flawless.</p> <p>The renowned Labour Prime Minister, Harold Wilson, once stated that he was a good man, but not a great one.</p> <p>These three individuals were not flawless, but they were the finest among us, and their kind will never be seen again.</p> <p>They advocated for fairness and social justice, prioritizing the needs of working individuals over the wealthy.</p> <p>Despite facing difficult circumstances, they persevered with determination and resilience.</p> <p>They had faith in creating a more just society and were willing to take action to achieve it.</p> <p>They were the most influential British politicians in my lifetime, and I will dearly miss each one of them.</p> <p>I will always remember their bravery and unwavering resolve to overcome challenges and create a more just and imp .....</p>	<p>Tom Watson, the ex-deputy leader of the Labour Party, has authored a book which offers a unique perspective on the Labour NEC and the party's position of deputy chair.</p> <p>He has departed to focus on his family now.</p> <p>I believe he is a principled individual who was unfairly targeted by a group of right-wing journalists and politicians.</p> <p>Watson was the youngest Member of Parliament in the House of Commons and made history as the first MP to have .....</p> <p>He began his career humbly as a Labour councillor in West Bromwich and a trade union official.</p> <p>At the age of 29, he entered Parliament as a Member of Parliament, dedicated to serving his constituents with .....</p> <p>He is a committed Christian and was a member of the Labour Friends of Israel, a group that was unfairly criticized by s .....</p> <p>His latest book, "Dangerous Games," provides a comprehensive analysis of the political landscape in the UK.</p> <p>He is a fervent advocate for the Jewish community and was compelled to resign from Parliament due to the hostility .....</p> <p>Watson has consistently been a source of contention for the far left, who sought to transform the Labour Party into an .....</p> <p>Watson has consistently supported Israel and has been a loyal ally to the Jewish community.</p>	<p>I'm not sure I've ever come across an article by John Rentoul that I haven't disagreed with, but this time he has gone too far.</p> <p>He is committed to reviving the political trajectory of George Galloway, a Member of Parliament whose entry into the .....</p> <p>The Parliamentary standards watchdog is investigating Naz Shah, the current Labour MP for Bradford West, for acc .....</p> <p>She has refuted any allegations of misconduct.</p> <p>Rentoul alleged that Shah's election campaign had faced accusations of fraud and that she was being investigated .....</p> <p>It is unclear whether the police investigation is ongoing or if it has been dropped.</p> <p>Shah maintains her innocence, stating that she has not committed any wrongdoing and that the investigation w .....</p> <p>Rentoul suggested that Shah should be removed from the Labour party and proposed that the party should invite G .....</p> <p>Rentoul States that Galloway would be a more appealing candidate than Shah, who has been suspended from the party.</p> <p>Shah's first reason for concern is her criminal conviction for benefit fraud, resulting in a suspended sentence.</p> <p>A suspended sentence does not result in a criminal conviction.</p> <p>The second reason provided is that she is currently being investigated by the police.</p>
(d) SemStamp paraphrased by GPT.	(e) Online PMARK paraphrased by GPT.	(f) Offline PMARK paraphrased by GPT.

Figure 12: Cases from Mistral-7B on C4 dataset.

<p>He tells her he loves her and that he'll always be there for her.</p> <p>CastProductionEdgar's letter was written on the day of his son's death.</p> <p>He gives his son's name, James, and his address, 30 Barrington Street, London W1H 1EN.</p> <p>He also tells Isabella that he'll be there for her at all times.</p> <p>He tells her he'll always be there for her and that he loves her.</p> <p>His letter was written on the day of his son's death.</p> <p>After James' death, Edgar goes to the Tower of London and writes a letter to Isabella.</p> <p>He tells her he'll always be there for her and that he loves her.</p> <p>His letter was written on the day of his son's death.</p>	<p>Edgar's death, the letter, and the letter's return to Edgar are all contained in the first three pages of the book.</p> <p>In the first chapter, Edgar and Isabella walk to the cemetery. In the following chapter, Edgar and Isabella attend the funeral.</p> <p>Edgar and Isabella's only interaction is in the chapter, "An Ending."</p> <p>Edgar tells Isabella of his plans to write a book about his life and is upset to find out that the book he's going to be a ...</p> <p>In the chapter, "A Note From the Author," Edgar tells Isabella that he has written a letter announcing his death.</p> <p>He also shows her the letter, and she writes a letter to him.</p> <p>The events of the book take place over a period of several months.</p> <p>In the final chapter, Edgar and Isabella discuss their plans to move to Paris together.</p> <p>Isabella's deathIsabella's death is a major plot point in the book.</p> <p>Edgar is determined to write a book about his life.</p> <p>He and Isabella have been dating for two years.</p>	<p>The letter is not published, but is seen by Kate when she returns to the house after the show.</p> <p>CastProductionDevelopmentThe show was commissioned by the BBC on 5 July 2012.</p> <p>It was announced that it would be filmed in the town of Eccles, Lancashire, England, and will be set in 17th ce .....</p> <p>The show's first season began on 14 July 2013.</p> <p>The show was broadcast in the UK on BBC1 and on BBC Two in the UK.</p> <p>On 29 September 2013, it was announced that the show had been renewed for a second season.</p> <p>The second season premiered on 18 March 2014.</p> <p>SynopsisThe series begins with the death of Edward IV and the coronation of his young son, Edward V ....</p> <p>The two sisters go to live with the Earl's uncle, Lord Grantham, who is now in charge of the household at court .....</p> <p>The baby is named Henry, and it is revealed that Anne has been secretly married to Lord Grantham's cousin, George .....</p> <p>During the winter, the sisters plan to have a regency period, leading to Henry and Anne moving into a new home at the Dower House.</p> <p>Henry is named after his grandfather, Henry VII, and Anne is named after his grandmother, Anne Boleyn.</p>
<p>(a) SemStamp without attack. (b) Offline PMARK without attack. (c) Online PMARK without attack.</p>	<p>(d) SemStamp paraphrased by GPT.</p>	<p>(e) Online PMARK paraphrased by GPT.</p>
<p>(f) Offline PMARK paraphrased by GPT.</p>	<p>(g) Online PMARK paraphrased by GPT.</p>	<p>(h) Offline PMARK paraphrased by GPT.</p>

Figure 13: Cases from OPT-1.3B on BOOKSUM dataset.

<p>Google has announced a new ad format that lets you watch live video from YouTube without being tied to a specific channel.</p> <p>"The idea is to get you to stay on YouTube longer," YouTube product manager Alex Lai told Mashable.</p> <p>The new ads also include a "what you're watching" section that shows a quick list of what you're currently watching.</p> <p>The idea is to get you to stay on YouTube longer.</p> <p>The new ads also include a "what you're watching" section that shows a quick list of what you're currently watching.</p> <p>According to Google's Alex Lai, the company is trying to "get you to stay on YouTube longer."</p> <p>The new ads also include a "what you're watching" section that shows a quick list of what you're currently watching.</p> <p>According to Google's Alex Lai, the company is trying to "get you to stay on YouTube longer."</p> <p>The new ads also include a "what you're watching" section that shows a quick list of what you're currently watching.</p>	<p>It's the latest push by Google to make its services more immersive and connect with the internet's users.</p> <p>Google launched the new service, called YouTube TV, on Friday in the U.S. and Canada.</p> <p>The company has been testing this feature for a while now.</p> <p>Google claims that the new service is easier to use than its original YouTube TV, which required you to download the app.</p> <p>Google's streaming video service, YouTube TV, launched earlier this year with some TV channels.</p> <p>It's unclear if Google is planning to expand the service to more countries.</p> <p>After years of building a streaming video service, Google is now ready to compete with other services in the space.</p> <p>YouTube TV is available in the U.S. and Canada, with plans to expand to more countries in the future.</p> <p>But Google is likely to talk about when the service will go live in other countries.</p> <p>The service uses a combination of hardware and software to make it easy to stream from your computer or laptop.</p> <p>It's available in the U.S. and Canada, and Google is planning to expand to more countries in the future.</p> <p>The new service uses a combination of hardware and software to make it easy to stream from your computer or laptop.</p>	<p>They can be accessed by clicking the "Watch on TV" button at the top of the search results page, or by swiping left on the screen.</p> <p>YouTube is testing the feature with a small group of users in the United States and Australia, and says it will be rolled out to more users.</p> <p>The new feature is a welcome addition, but it's not the only one.</p> <p>Google is also experimenting with the idea of bringing video from YouTube to the TV with YouTube TV.</p> <p>And on the video front, Google is experimenting with making its own video streaming service, YouTube TV, available to more users.</p> <p>Google's new ads also add a layer of privacy to the search experience.</p> <p>With Google's search ads, you can choose to allow advertisers to target you based on your viewing history.</p> <p>You can also opt out of advertising for specific categories, such as sports, health, and movies and television shows.</p> <p>Google's search ads let you opt out of ads based on your search history by default, which is a first for search.</p> <p>The company also offers advertisers a wider range of advertising options, including a "pay for results" option.</p> <p>You can opt out of the pay for results option if you don't want to see ads for your favorite shows.</p> <p>Google's search ads let you play games directly from your browser.</p>
<p>(a) SemStamp without attack.</p>	<p>(b) Offline PMARK without attack.</p>	<p>(c) Online PMARK without attack.</p>
<p>(d) SemStamp paraphrased by GPT.</p>	<p>(e) Online PMARK paraphrased by GPT.</p>	<p>(f) Offline PMARK paraphrased by GPT.</p>

Figure 14: Cases from OPT-1.3B on C4 dataset.