Windowed SPP SDM: (20240628)

(complete data likelihood)

$$[\{\underline{s}_{ij}\},\{n_j\}|\beta_0,\underline{\beta}] = \frac{\prod_{j=1}^{J}\prod_{i=1}^{n_j}\lambda(\underline{s}_{ij})}{n!\,\exp\left(\int_{\cup_j S_j}\lambda(\underline{s})d\underline{s}\right)}$$

for $\lambda(\underline{s}) = e^{\beta_0 + \underline{x}'(\underline{s})\underline{\beta}}$ and $\underline{s}_{ij} \in S_j$

for $j = 1,\ldots,J$ and $i = 1,\ldots,n_j$ (and $n = \sum_{j=1}^{J} n_j$)

In the case where $n_j = 0$, there are no obs. points and thus

$$[\{\underline{s}_{ij}\},\{n_j\}|\beta_0,\underline{\beta}] = \frac{\prod_{\{\forall j,\, n_j>0\}}\prod_{i=1}^{n_j}\lambda(\underline{s}_{ij})}{n!\,\exp\left(\int_{\cup_j S_j}\lambda(\underline{s})d\underline{s}\right)}$$

Then the full-conditional distribution for the # of unobs. points: $[N_0|\cdot] = \text{Pois}(\lambda_0)$

s.t. $\lambda_0 = \int_{S_0}\lambda(\underline{s})d\underline{s}$ for $S_0 = S \setminus \bigcup_{j=1}^{J} S_j$

(assume $\bigcap_{j=1}^{J} S_j = 0$ for now)

and $N = n + N_0$ is total abundance in $S = \bigcup_{j=0}^{J} S_j$

Note: Use composition sampling to get $N_0^{(k)}$ from

$$[N_0|\{\underline{s}_{ij}\},\{n_j\}] = \iint [N_0|\cdot][\beta_0,\underline{\beta}|\{\underline{s}_{ij}\},\{n_j\}]\,d\beta_0\,d\underline{\beta}$$

Recursive Bayes procedure:

Note that: $[\{\underline{s}_{ij}\}, \{n_j\} \mid \beta_0, \underline{\beta}] = [\{\underline{s}_{ij}\} \mid \underbrace{\beta_0}_{\beta_0 \text{ cancels here}}, \underline{\beta}, \{n_j\}][\underbrace{\{n_j\}}_{n} \mid \beta_0, \underline{\beta}]$

1.) Fit model using cond. likelihood:

$$[\{\underline{s}_{ij}\} \mid \beta_0, \underline{\beta}, \{n_j\}] = \frac{\prod_{\forall j, n_j > 0} \prod_{i=1}^{n_j} \lambda(\underline{s}_{ij})}{\left(\int_{\cup S_j} \lambda(\underline{s}) d\underline{s}\right)^n}$$

Note: could use logistic or Poisson reg. approach.

Also, sample $\beta_0^{(k)} \sim [\beta_0]$, $k = 1, \ldots, K$

2.) propose $\{\beta_0^{(*)}, \underline{\beta}^{(*)}\}$ randomly (w/ replacement) from 1st stage
Then update w/ M-H ratio:

$$mh = \frac{[n \mid \beta_0^{(*)}, \underline{\beta}^{(*)}]}{[n \mid \underbrace{\beta_0^{(k-1)}, \underline{\beta}^{(k-1)}}_{\int_{\cup S_j} \lambda(\underline{s}) d\underline{s}}]}$$

These can be computed in parallel before stage 2!

3.) sample for $k = 1, \ldots, K$ $N_0^{(k)} \sim \text{Pois}\left(\int_{S_0} \lambda^{(k)}(\underline{s}) d\underline{s}\right)$ for $k = 1, \ldots, K$

(These can also be sampled in parallel).