CIT Qiskit Fall Fest QC Hackathon

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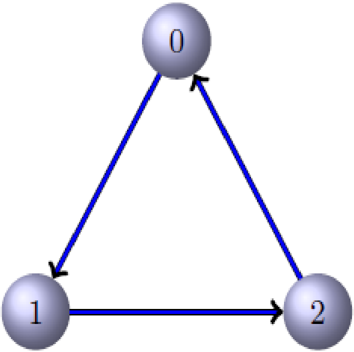
Oriented cycle graph on 3 vertices



A cycle graph which has direction in its edge is called oriented cycle graph.

Adjacency matrix

$$A = \begin{bmatrix} 0 & i & -i \\ -i & 0 & i \\ i & -i & 0 \end{bmatrix}$$

If there is an outgoing edge between two vertices u and v, we put the corresponding uv^{th} entry i, if incoming edge from u to v then entry of matrix will be $-i$, otherwise 0
So this will be a Hermitian matrix.

Transition Matrix in Continuous Time Quantum Walk

Definition

Let G be a graph with adjacency matrix A, that works as Hamiltonian(a Hermitian matrix) and t represents time. The transition matrix associated with G is

$$U(t) = exp(itA),$$

Perfect State Transfer

The graph G admits perfect state transfer from the vertex u to the vertex v at time t if there is a complex number λ with absolute value 1 such that,

$$U(t)e_u = \lambda e_v,$$

we use e_u to denote the unit norm vector whose uth entry is 1 and λ is called phase.

Or in other words if uv^{th} entry of matrix $U(t)$ is modulus unity.

Utilizing Spectral Decomposition Theorem

Suppose that the adjacency matrix A for any graph G has distinct eigenvalues $\theta_0 > \theta_1 > \ldots > \theta_d$ By the Spectral Decomposition Theorem , we have

$$A = \sum_{r=0}^{d} \theta_r E_r$$

Important consequence of the theorem is,

$$U(t) = exp(itA),$$

$$U(t) = \sum_{r=0}^{d} e^{it\theta_r} E_r$$

Now U(t) for oriented cycle graph on 3 vertices

Eigenvalues for oriented C3 are :$\{\theta_1, \theta_2, \theta_3\} = \{-\sqrt{3}, \sqrt{3}, 0\ \}$.

And corresponding eigenvectors are $X1 = \begin{bmatrix} i(\sqrt{3}+i)/2 \\ i(-\sqrt{3}+i)/2 \\ 1 \end{bmatrix}$,

$X2 = \begin{bmatrix} i(-\sqrt{3}+i)/2 \\ i(\sqrt{3}+i)/2 \\ 1 \end{bmatrix}$, $X3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$,

Corresponding projection matrices E_r and evolution matrix $U(t)$

$$E1 = X1 \times X1^*, E2 = X2 \times X2^*, E3 = X3 \times X3^*$$

where $X1^*$ is the conjugate transpose of eigenvector $X1$,

So evolution operator/transition matrix

$$U(t) = e^{it\theta_1}E_1 + e^{it\theta_2}E_2 + e^{it\theta_3}E_3$$

Final evolution operator for oriented C3

$$U(t) = 1/3 \begin{bmatrix} 1 + 2\cos(t\sqrt{(3)} & 1 + 2\cos(4\pi/3 + t\sqrt{(3)}) & 1 + 2\cos(2\pi/3 + t\sqrt{(} \\ 2\cos(2\pi/3 + t\sqrt{(3)}) & 1 + 2\cos(t\sqrt{(3)} & 1 + 2\cos(4\pi/3 + t\sqrt{(} \\ 1 + 2\cos(4\pi/3 + t\sqrt{(3)}) & 1 + 2\cos(2\pi/3 + t\sqrt{(3)}) & 1 + 2\cos(t\sqrt{(3)} \end{bmatrix}$$

U(t) at different times

$$U(\frac{2\pi}{3\sqrt{3}}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$U(\frac{4\pi}{3\sqrt{3}}) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$U(\frac{2\pi}{\sqrt{3}}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

we can see that at time $\frac{2\pi}{3\sqrt{3}}$ we have **perfect state transfer** from vertex 0 to vertex 1 (as entry 1st row 2nd column is 1) and at time $\frac{4\pi}{3\sqrt{3}}$ and we have **perfect state transfer** from vertex 0 to vertex 2 (as entry 1st row 3rd column entry is 1). Therefore, vertex 0 has perfect state transfer between more than two vertices.