Transient Sprinkler Dynamics Update

JANUARY 2026

General Overview of Solution Presented

Goal: Fit tail of provided data to the ODE to obtain values for γ and ω :

$$\ddot{\phi} + 2\gamma\dot{\phi} + \omega^2\phi = 0. (1)$$

Problem with Previous Method:

- Tried to fit all four coefficients of ODE without good starting estimates for their values
- Experienced coefficient convergence instability based on starting estimates
- Fits to data were just ok

Solution Presented:

- Use data to provide good starting estimates for γ and ω
- Use the obtained estimates for γ and ω to analytically solve for ODE coefficients C_1 and C_2.
- Fit the data with starting estimates for γ, ω and analytically found C_1 and C_2.

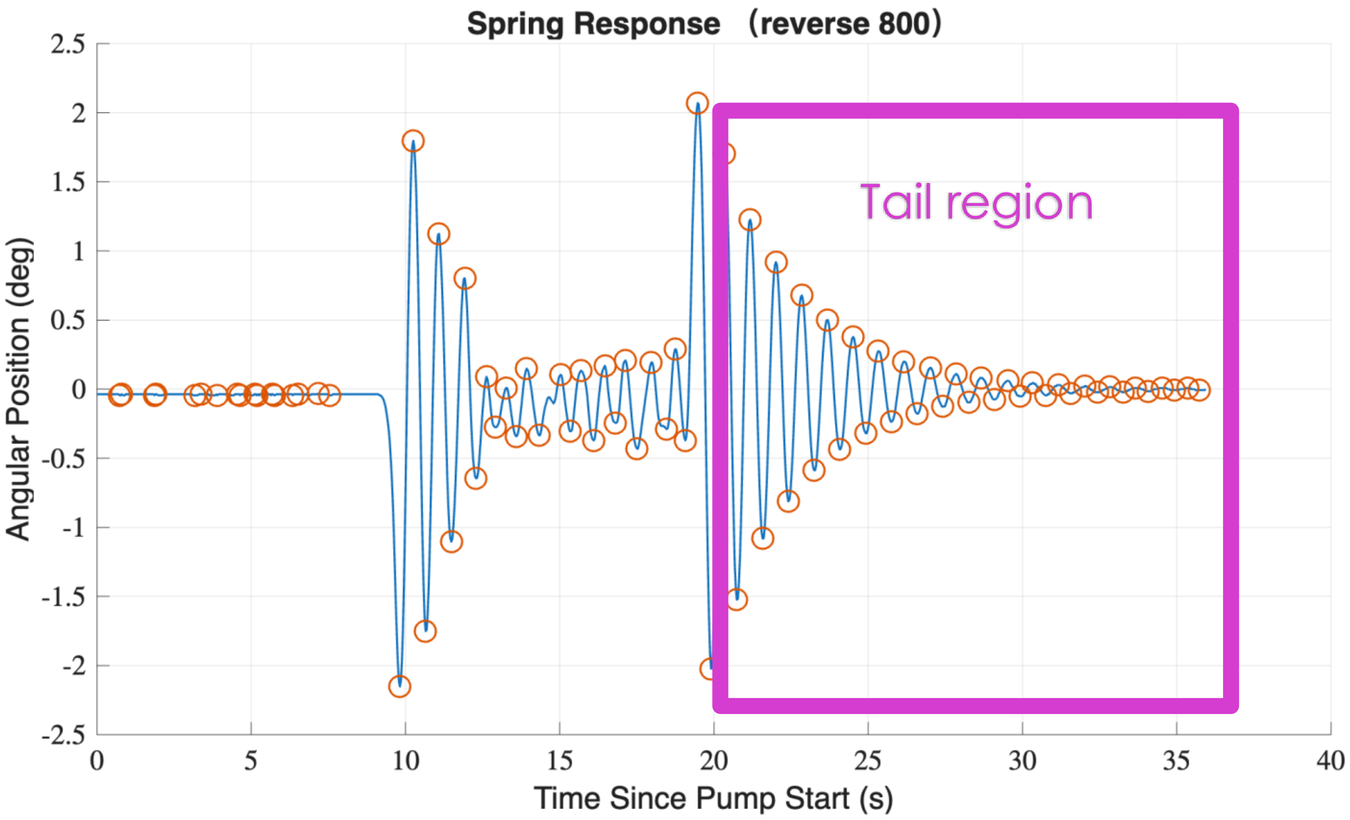
Necessary Equations

- General Solution to ODE:

$$\Phi(t) = e^{-\gamma t}[C_1 cos(\Omega t) + C_2 sin(\Omega t)] \quad (2)$$

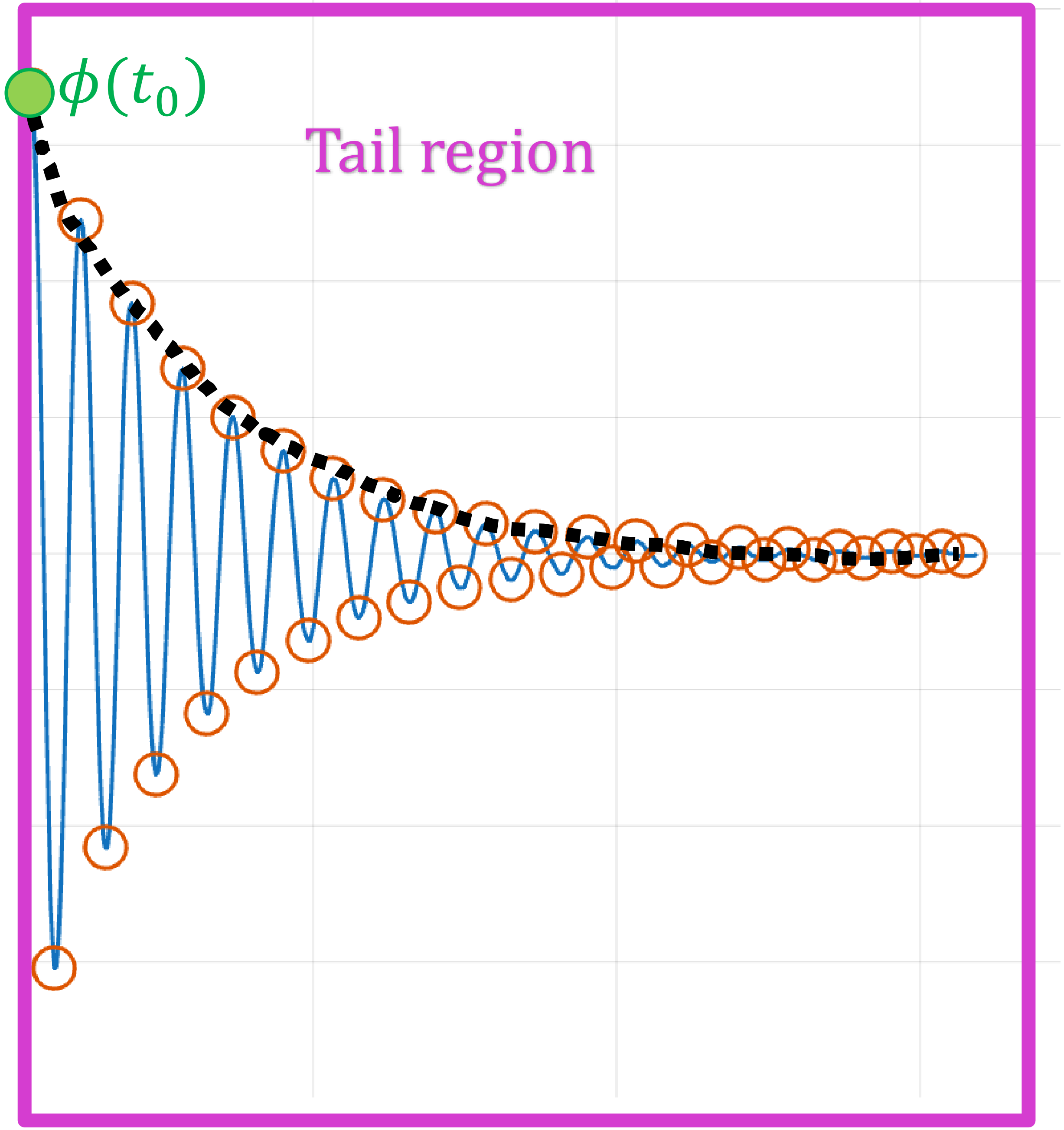
where $\Omega = \sqrt{\omega^2 - \gamma^2}$ and C_1, C_2 are constants.

- By picking a $\phi(t)$ such that $\dot{\phi}(t) = 0$ (at a peak/orange point) and given γ, ω, we can solve for C_1, C



Blue, solid curve is spring response of sprinkler. Orange, circular data points are where peaks occur (change in direction of curve).

Excellent data shown was provided by Kelly and Jesse, Dec. 2025



Finding γ estimate

- Black curve will follow form of
$$y(t) = \phi(t_0)e^{-\gamma t}.$$
- Then, we can normalize by dividing the curve by $\phi(t_0)$ and shift the curve to t $=$ 0 for proper fitting.
- This form is easy to fit in MATLAB.
- Retrieve $\gamma_{estimate}$ this way.

Finding ω estimate

- The period of this function occurs every other peak after $\phi(t_0)$.
- Then, for all $t_i \geq t_0$ where a peak occurs, the period estimate b_i is
$$b_i = t_{i+2} - t_i.$$
- Then, $\Omega_{estimate} = 2\pi/mean(b_i)$ and
$$\omega_{estimate} = \sqrt{\Omega_{estimate}^2 + \gamma_{estimate}^2}.$$

Results

Forward:
$\omega = 7.5324$
$\gamma = 0.36810$

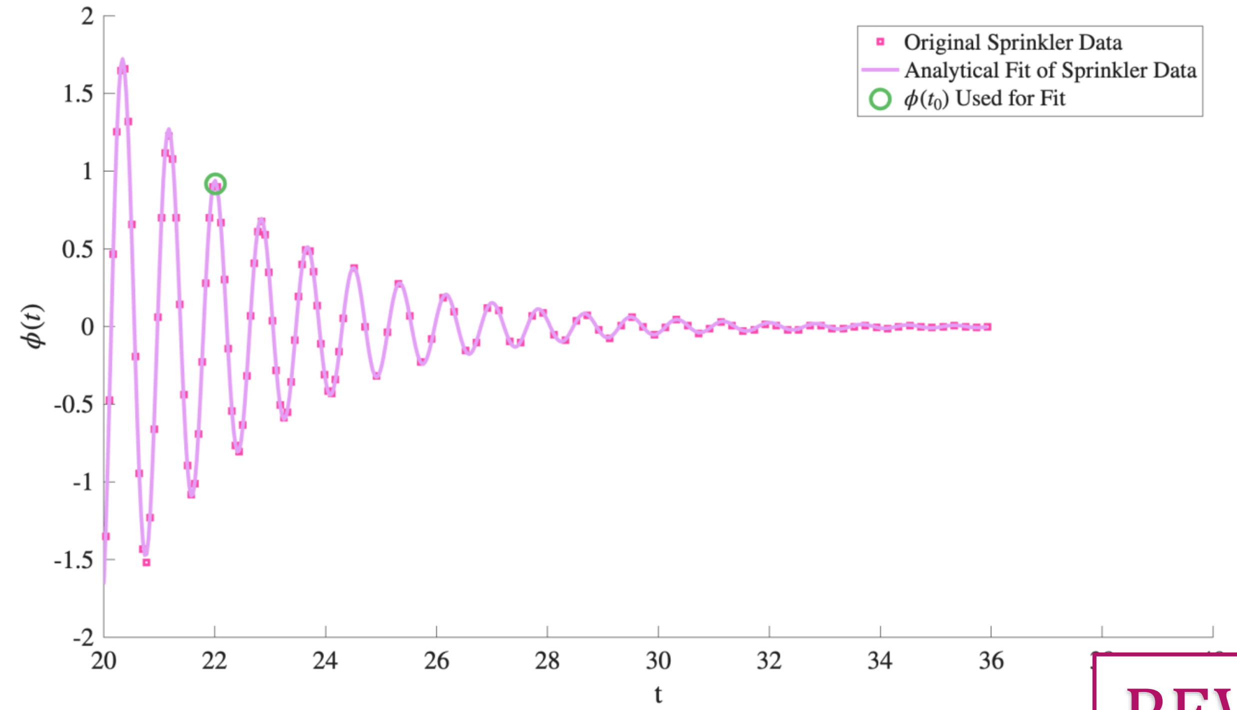
Reverse:
$\omega = 7.5515$
$\gamma = 0.36332$

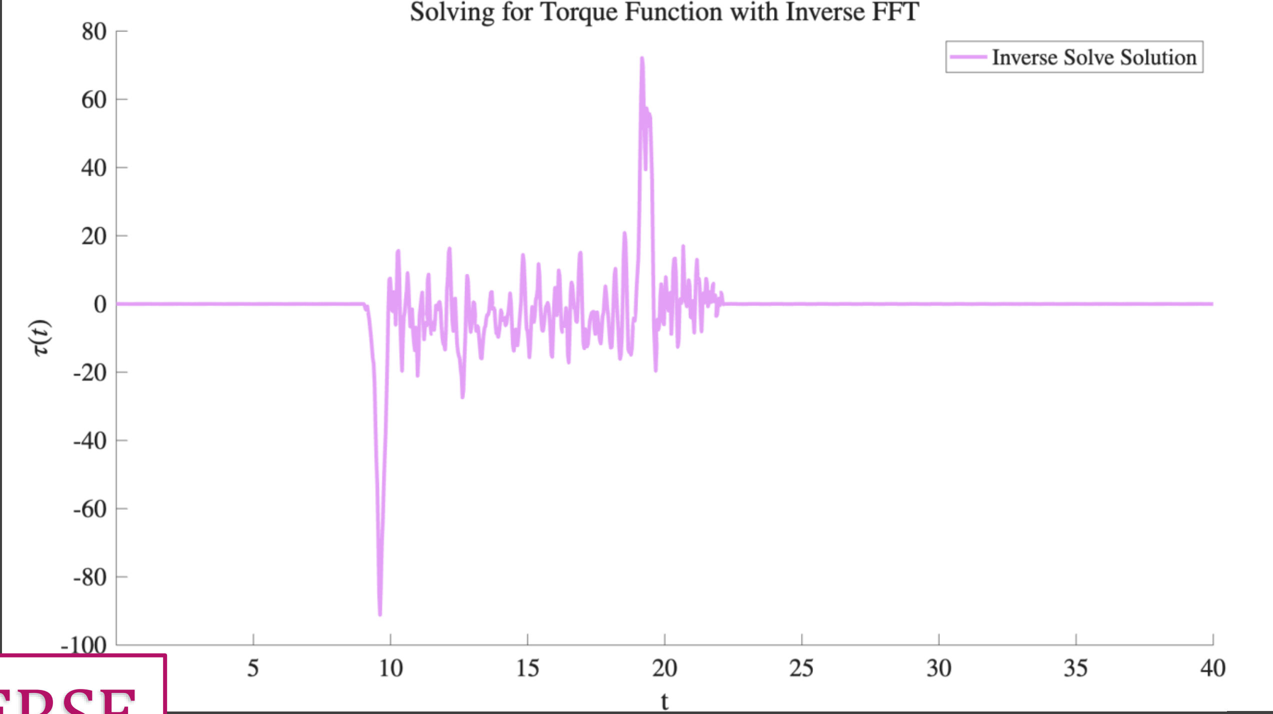
Overall:
$\omega_{average} = 7.5419$
$\gamma_{average} = 0.36571$

$T_d = 2.7344$

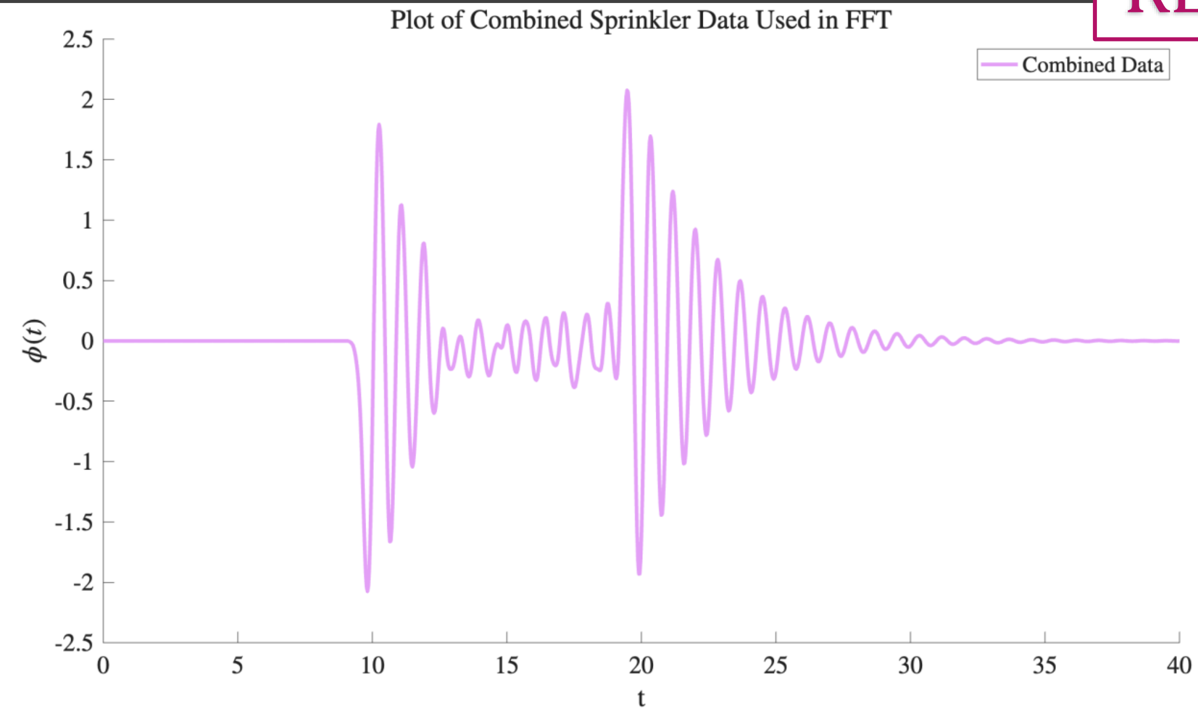
$T_0 = 0.83310$

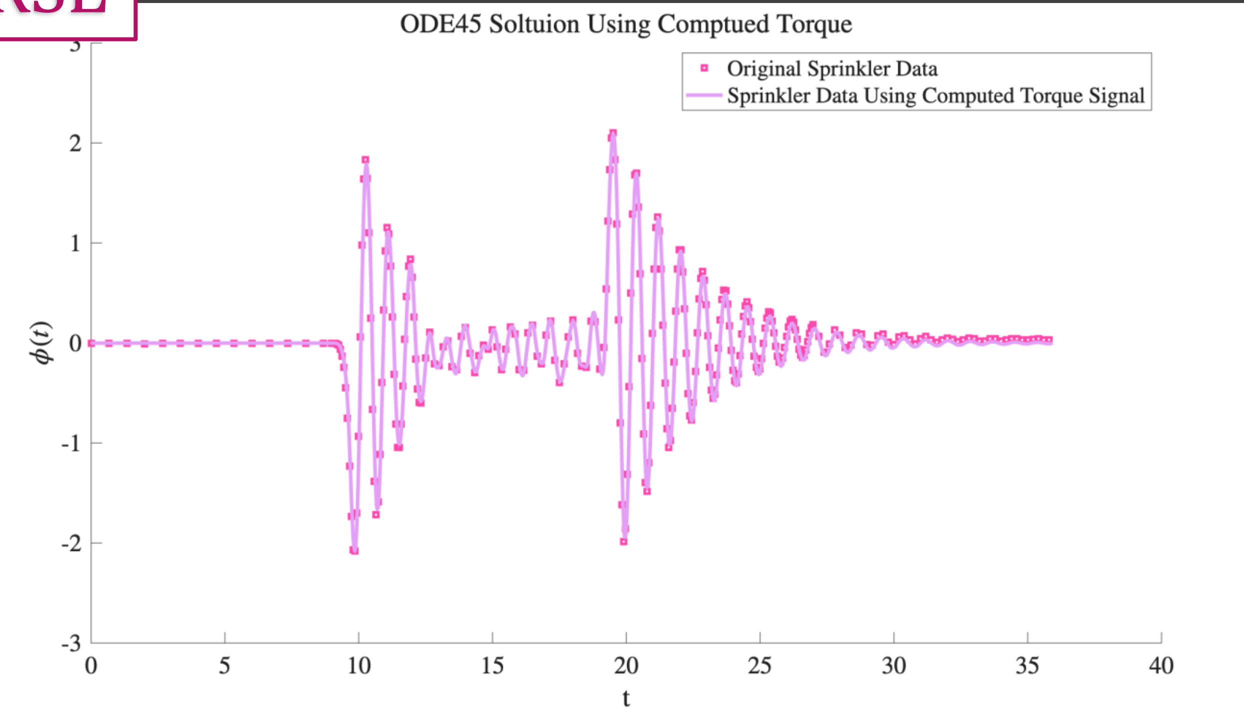
- With $\gamma_{estimate}, \omega_{estimate}$, we obtain estimates for C_1, C_2 by analytically solving equation (2) with $\phi(t_0)$.
- Then, we can use those estimates as starting points for a fitting function to the tail data.
- We can estimate the decay timescale, $T_d = 1/\gamma$ and the natural, undamped period $T_0 = 2\pi/\omega$ using the average of our results for ω, γ from forward and reverse cases.

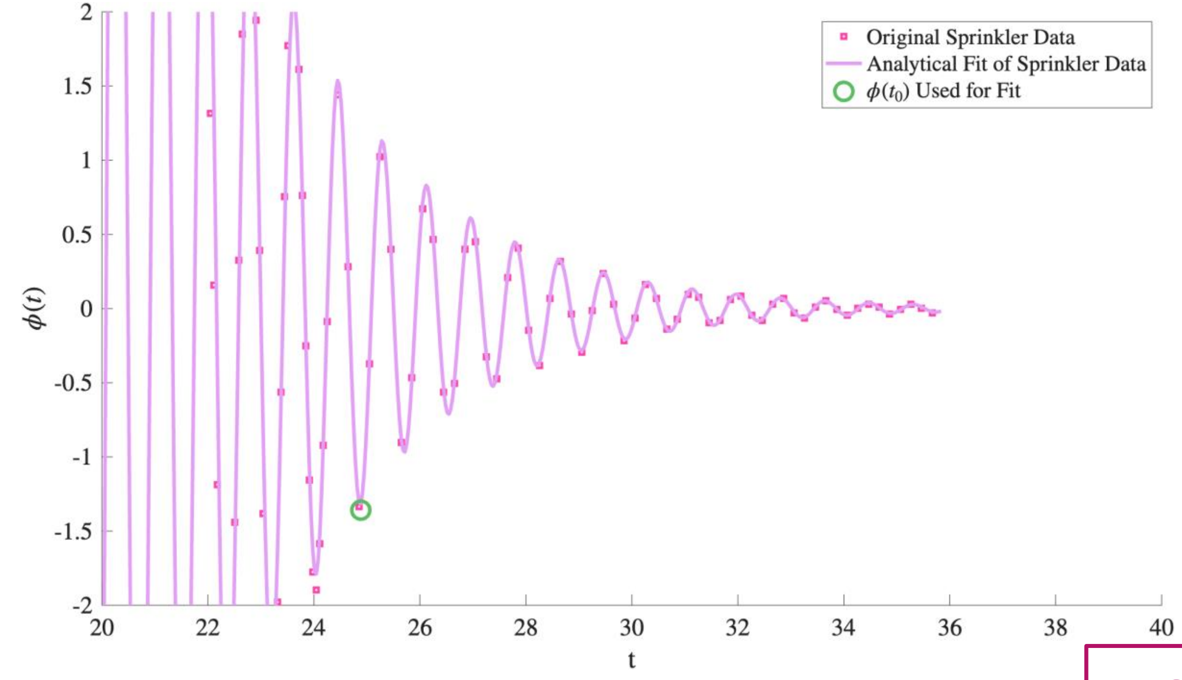

Original Sprinkler Data
Analytical Fit of Sprinkler Data
$\phi(t_0)$ Used for Fit
$\phi(t)$
t


Solving for Torque Function with Inverse FFT
Inverse Solve Solution
$\tau(t)$
t

REVERSE


Plot of Combined Sprinkler Data Used in FFT
Combined Data
$\phi(t)$
t


ODE45 Soltuion Using Comptued Torque
Original Sprinkler Data
Sprinkler Data Using Computed Torque Signal
$\phi(t)$
t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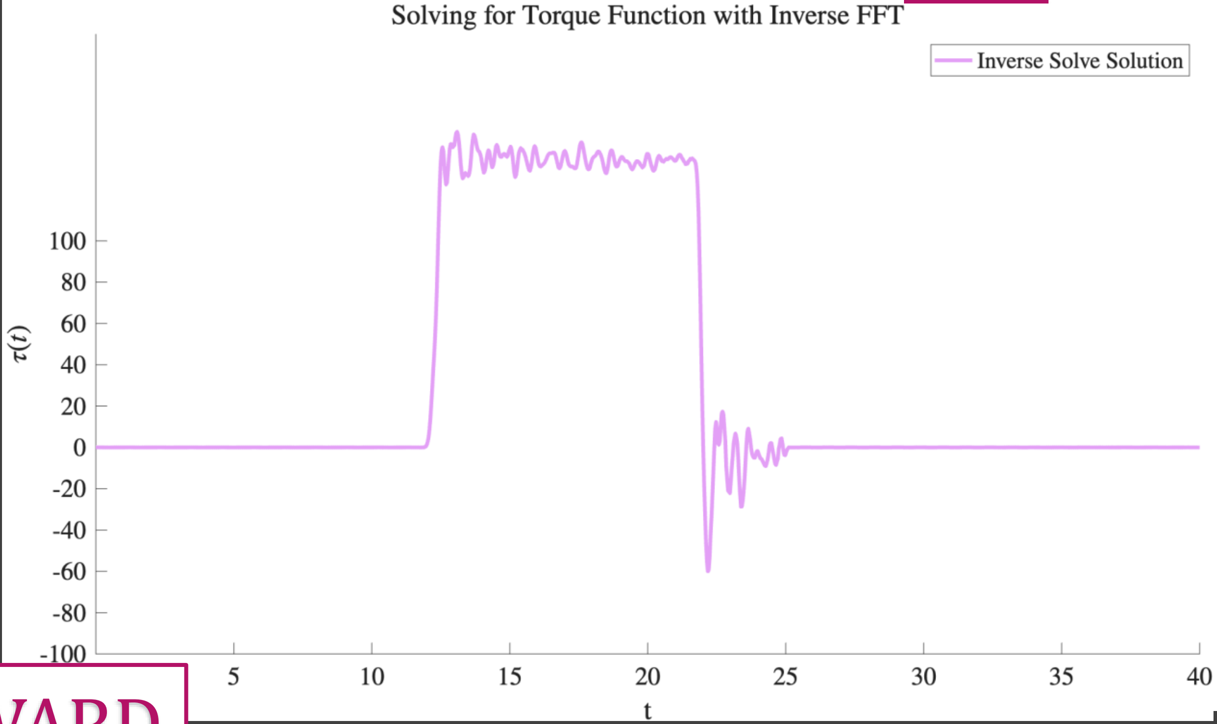
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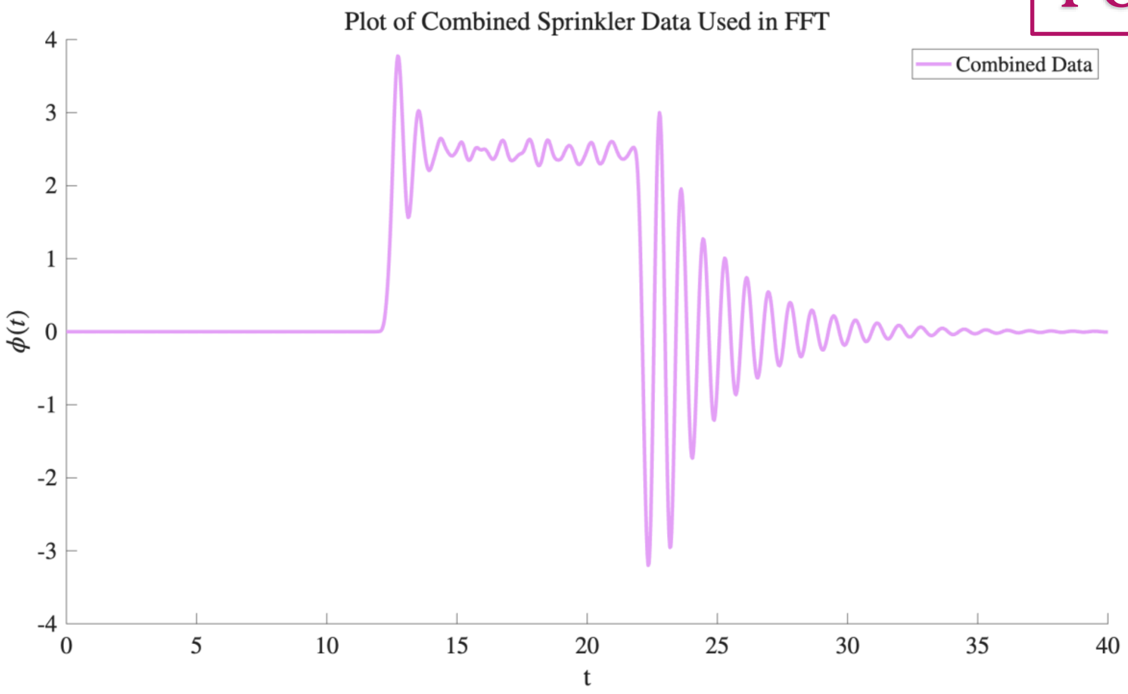
Inverse Solve Solution
$\tau(t)$
t

FORWARD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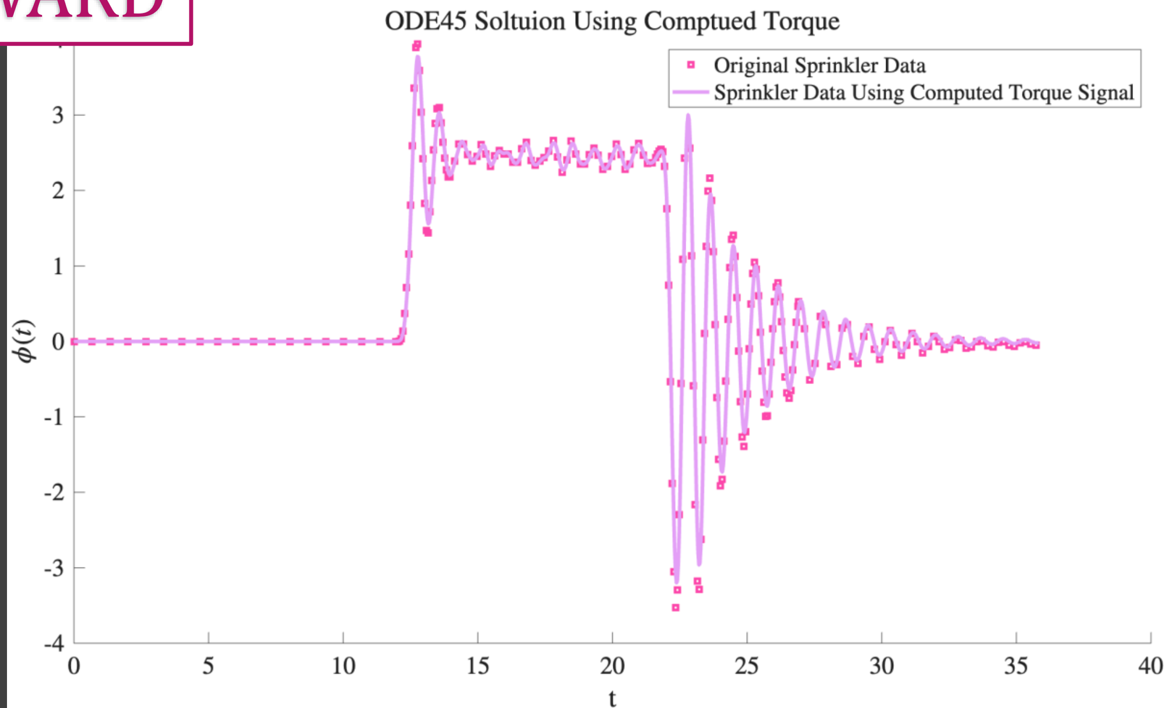
Combined Data
$\phi(t)$
t


ODE45 Soltuion Using Comptued Torque
Original Sprinkler Data
Sprinkler Data Using Computed Torque Signal
$\phi(t)$
t

Error Analysis

Forward

- norm two error between data and analytical fit: 4.3667e-02
- norm two error from smoothing phi: 4.5624e-02
- norm two error of original data and output using solved torque: 1.0750e-01

Reverse

- norm two error of data and analytical fit: 2.7083e-02
- norm two error from smoothing phi: 2.8165e-02
- norm two error of original data and output using solved torque: 2.2769e-01