Transient Sprinkler Dynamics Update

JANUARY 2026

Necessary Equations

- Want to fit tail to the ODE:

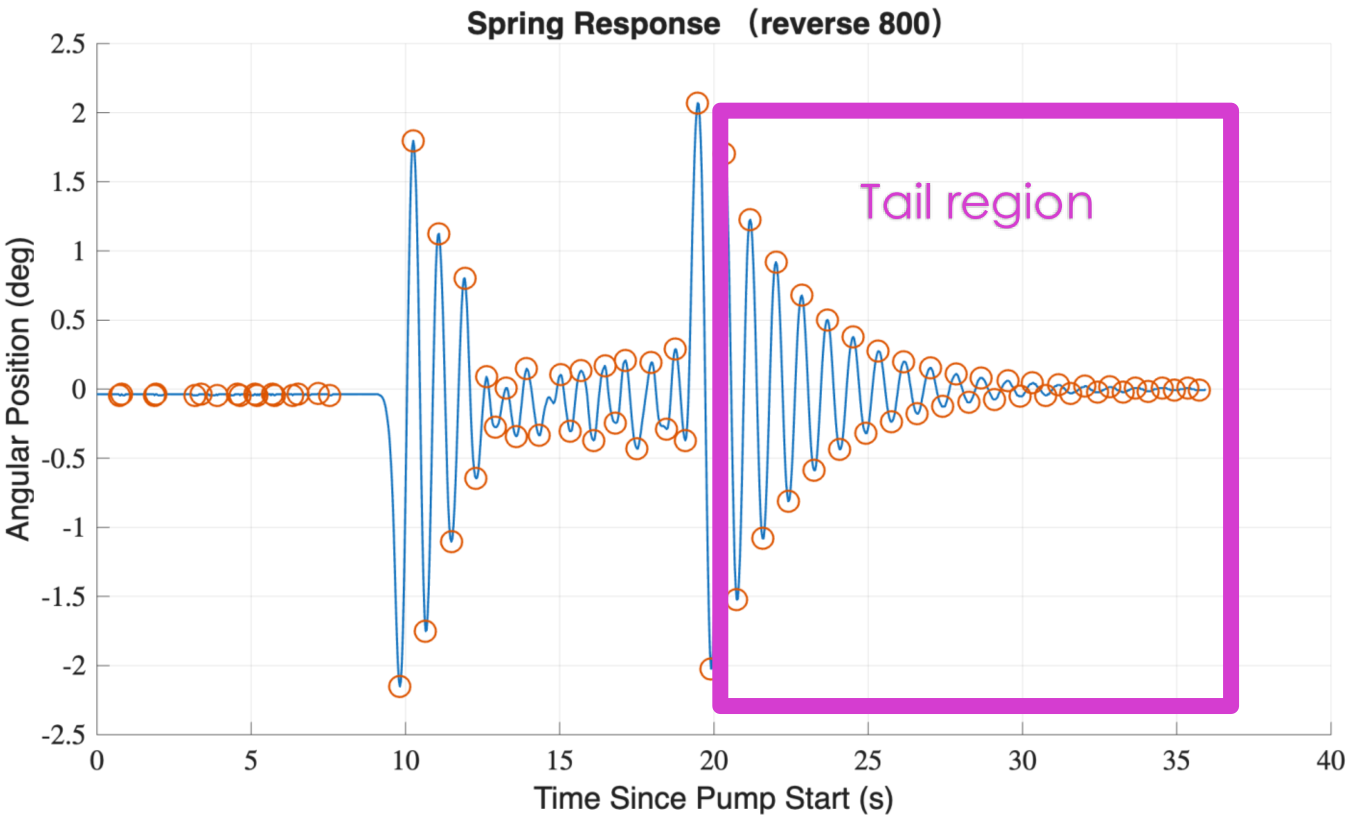
$$\ddot{\phi} + 2\gamma\dot{\phi} + \omega^2\phi = 0. \qquad (1)$$

- General Solution to ODE:

$$\Phi(t) = e^{-\gamma t}[C_1 cos(\Omega t) + C_2 sin(\Omega t)], \ (2)$$

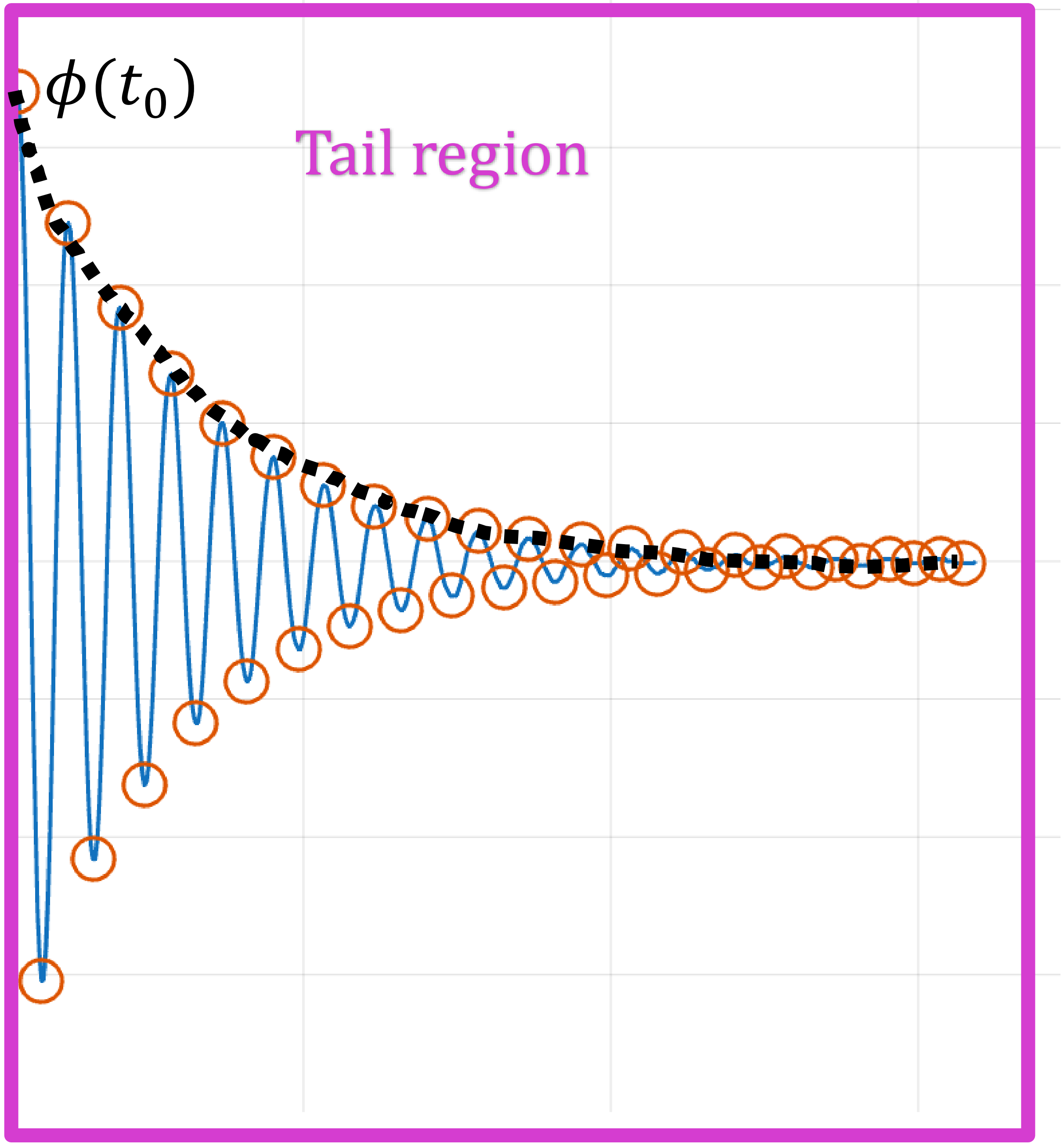
where $\Omega = \sqrt{\omega^2 - \gamma^2}$ and C_1, C_2 are constants.

- By picking a $\phi(t)$ such that $\dot{\phi}(t) = 0$ (at a peak/orange point) and given γ, ω, we can solve for C_1, C_2.



Blue, solid curve is spring response of sprinkler. Orange, circular data points are where peaks occur (change in direction of curve).

Data shown was provided by Kelly Yu, Dec. 2025



Finding γ estimate

- Black curve will follow form of
$$y(t) = \phi(t_0)e^{-\gamma t}.$$
- Then, we can normalize by dividing the curve by $\phi(t_0)$ and shift the curve to $t = 0$.
- This form is easy to fit in MATLAB.
- Retrieve $\gamma_{estimate}$ this way.

Finding ω estimate

- The period of this function occurs every positive peak after $\phi(t_0)$.
- Then, for all $t_i \geq t_0$ where a peak occurs, the period estimate b_i is
$$b_i = t_{i+2} - t_i.$$
- Then, $\Omega_{estimate} = 2\pi/mean(b_i)$ and
$$\omega_{estimate} = \sqrt{\Omega_{estimate}^2 + \gamma_{estimate}^2}.$$

Results

Forward:
$\omega = 7.5324$
$\gamma = 0.3681$

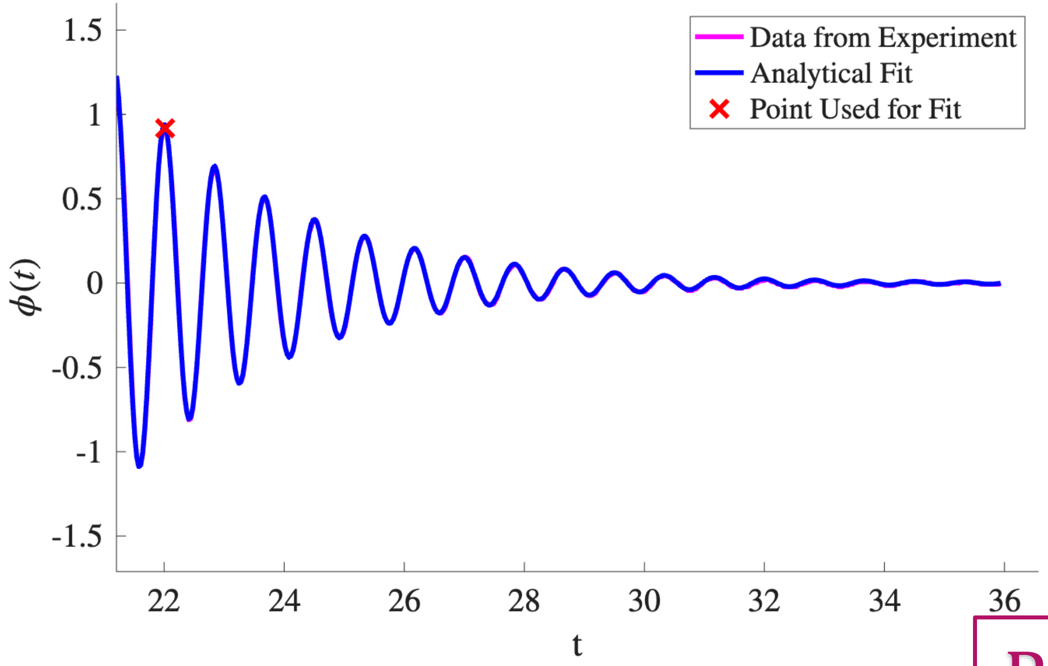
Reverse:
$\omega = 7.5515$
$\gamma = 0.3633$

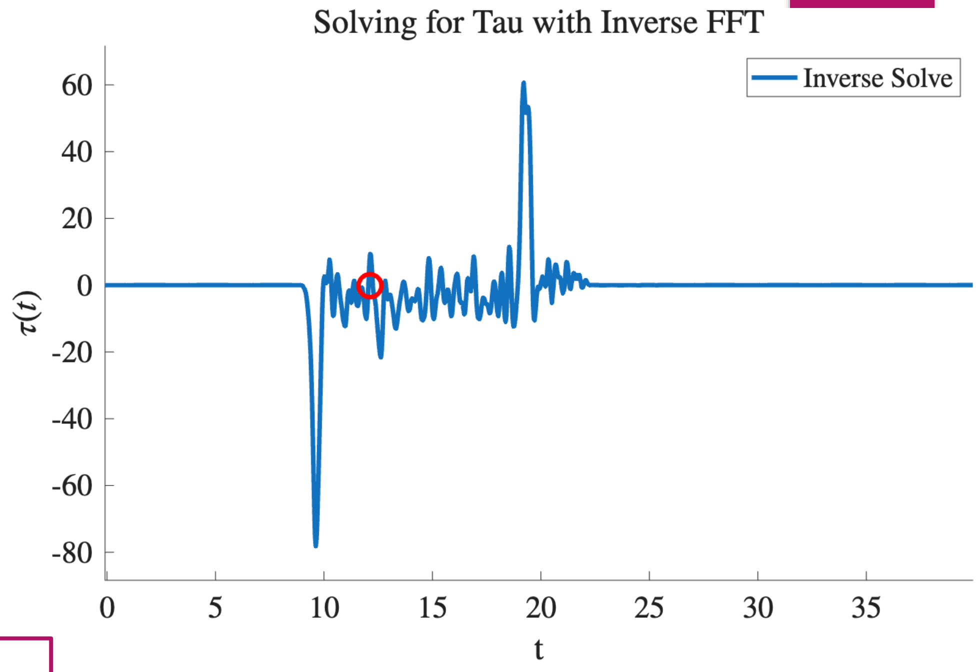
Overall:

$T_d = 2.7345$

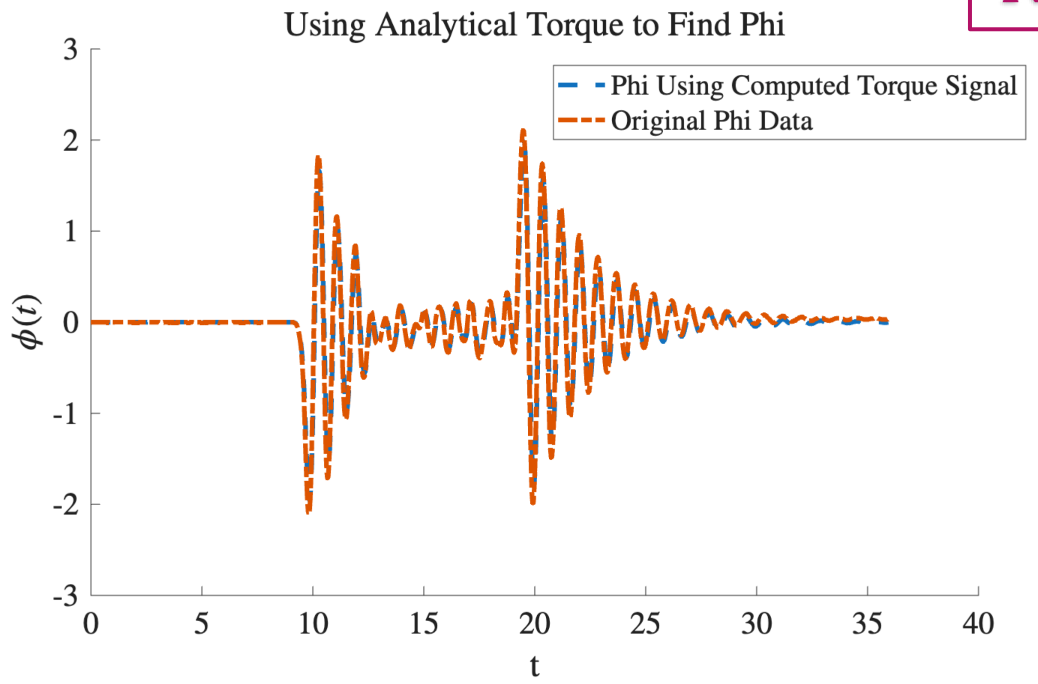
$T_0 = 0.83310$

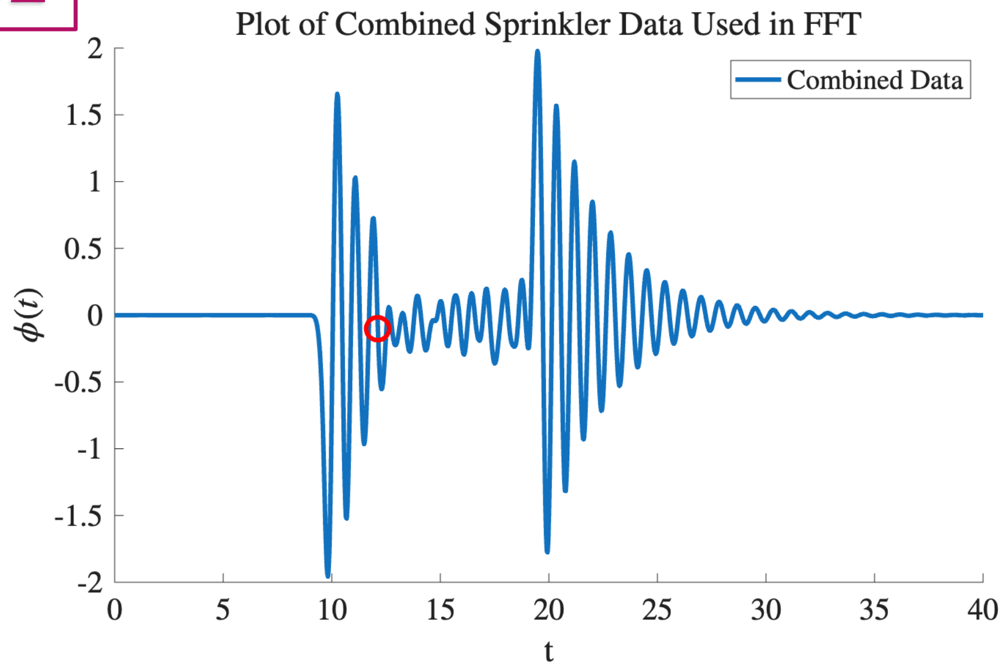
- With $\gamma_{estimate}, \omega_{estimate}$, we obtain estimates for C_1, C_2 by analytically solving equation (2) with $\phi(t_0)$.
- Then, we can use those estimates as starting points for a fitting function to the tail data.
- We can estimate the decay timescale, $T_d = 1/\gamma$ and the natural, undamped period $T_0 = 2\pi/\omega$ using the average of our results for ω, γ from forward and reverse cases.

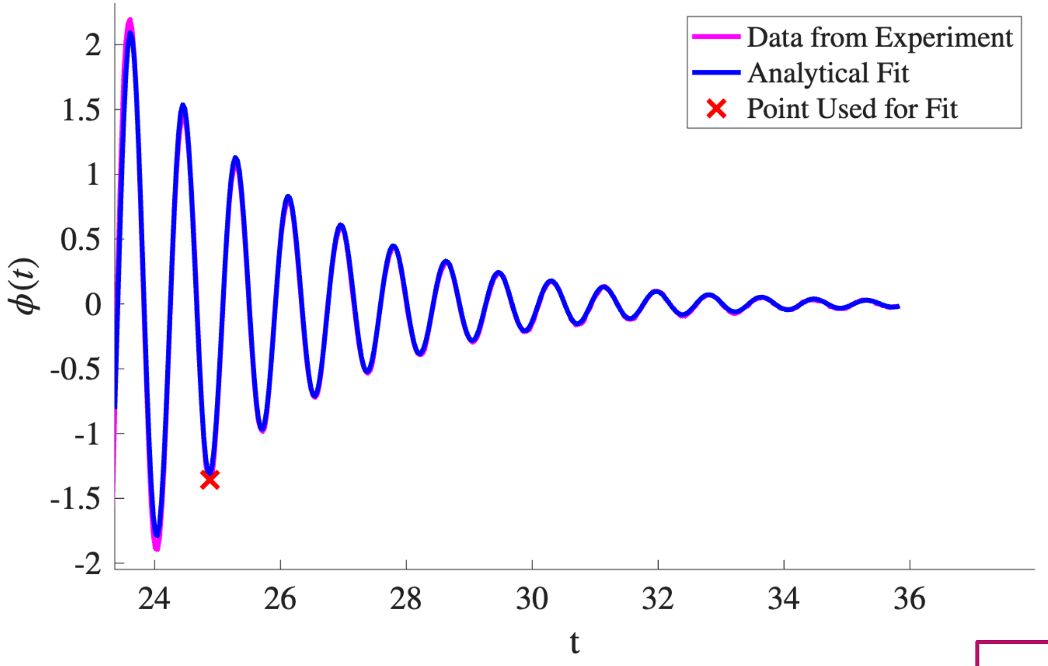

Data from Experiment
Analytical Fit
Point Used for Fit
$\phi(t)$
t


Solving for Tau with Inverse FFT
Inverse Solve
$\tau(t)$
t

REVERSE


Using Analytical Torque to Find Phi
Phi Using Computed Torque Signal
Original Phi Data
$\phi(t)$
t


Plot of Combined Sprinkler Data Used in FFT
Combined Data
$\phi(t)$
t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Analytical Fit
Point Used for Fit
$\phi(t)$
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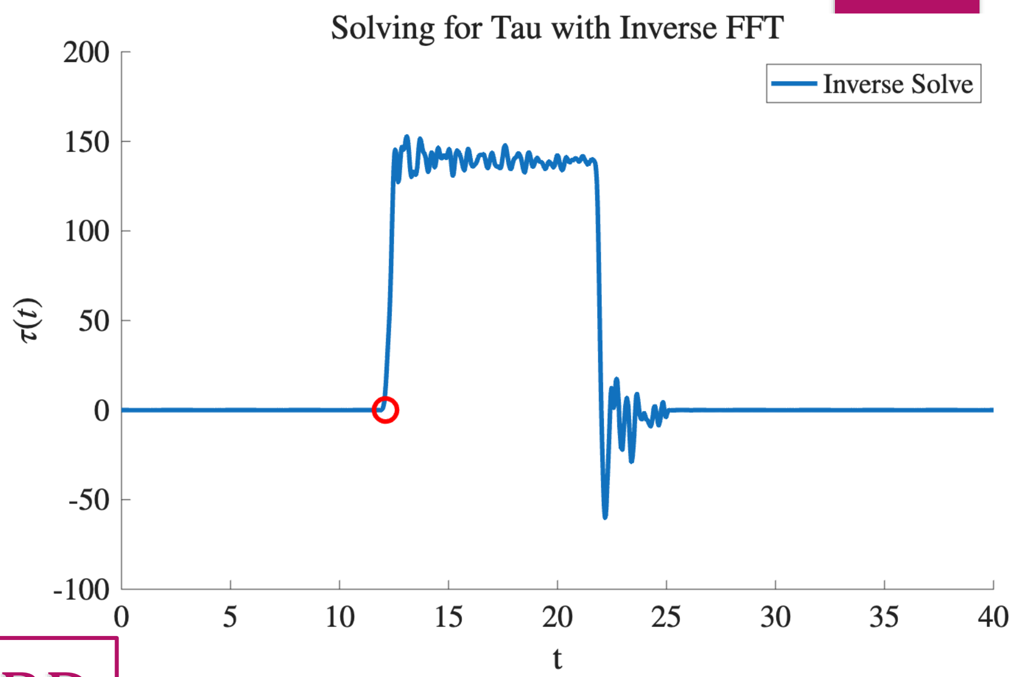
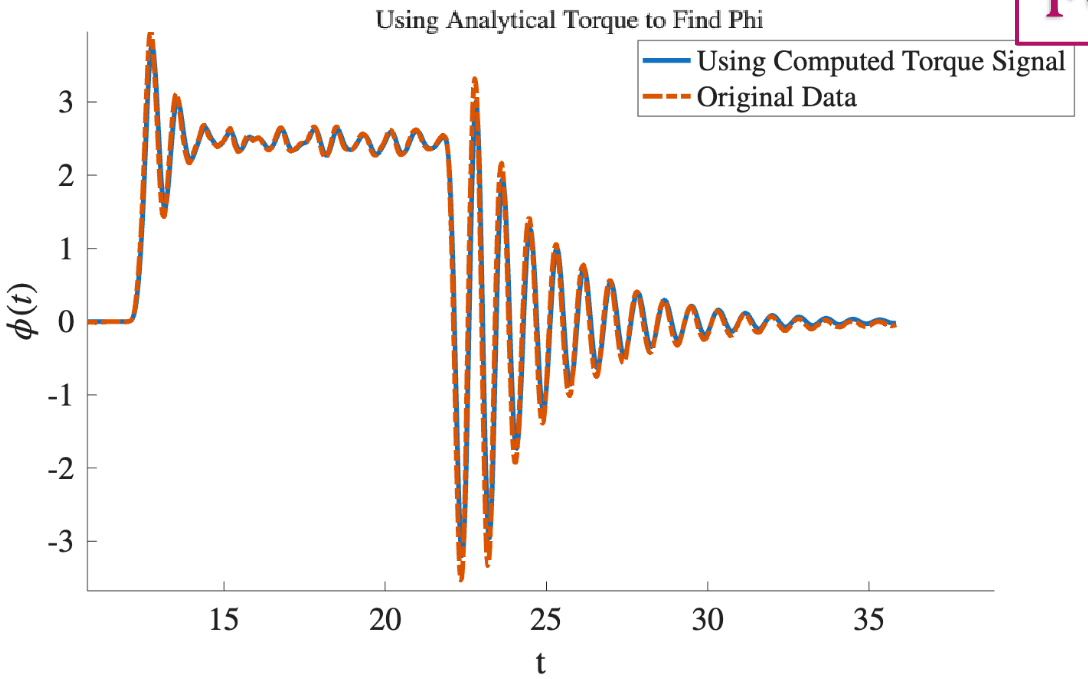
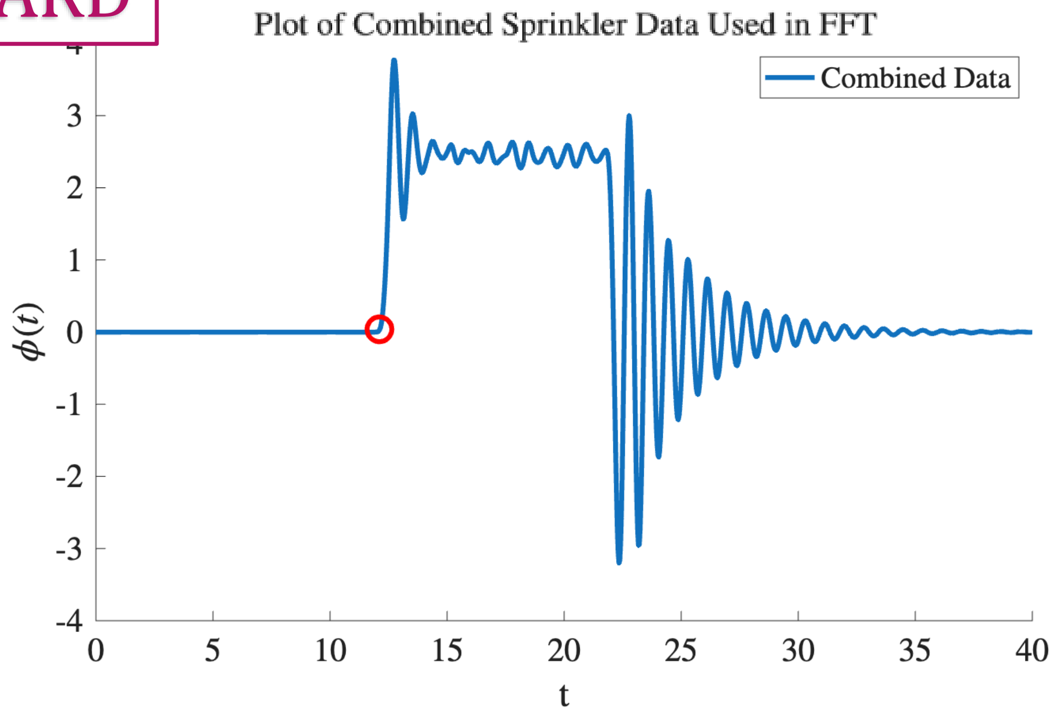

Solving for Tau with Inverse FFT
Inverse Solve
$\tau(t)$
t

FORWARD


Using Analytical Torque to Find Phi
Using Computed Torque Signal
Original Data
$\phi(t)$
t


Plot of Combined Sprinkler Data Used in FFT
Combined Data
$\phi(t)$
t

Error Analysis

Forward

- norm two error between data and analytical fit: 4.366702e-02
- norm two error from smoothing phi: 4.562400e-02
- norm two error of original data and output using solved torque: 1.075018e-01

Reverse

- norm two error of data and analytical fit: 2.708292e-02
- norm two error from smoothing y: 2.816452e-02
- norm two error of original data and output using solved torque: 2.276942e-01