6. oct. 2025 Lab 1

simona.cojocea@fmi.unibuc.ro.

→ 50 % examen scris (cică e greu prag: jumate din punctaj)
→ 30 % proiect
→ 20 % laborator (10% test; 10% activitate, ?% bonus) } max 50/100
↳ echipă 2/3/4 ↳ openbook (calculator fără net)

Introducem tripletul $(\Omega, \mathcal{F}, \mathbb{P})$

1) $\Omega \to$ nr stărilor (aruncarea cu un zar)
$\Omega = \{1, 2, 3, 4, 5, 6\}$

2) Aruncarea cu 2 zaruri

$\Omega = \{(1,1); (1,2); \dots (1,6) \dots (6,6)\}$

3) Aruncarea a n zaruri

$\Omega = \{(w_1, w_2, \dots w_n) \mid w_i = \overline{1,6}; i = 1, n\}$

Un eveniment = o mulțime de stări.

$A = \{1\}$
$B = \{2, 4, 6\}$

$\mathcal{F} \subseteq P(\Omega)$ → mulțimea tuturor evenimentelor posibile.
→ σ-algebră.

DEF: $\mathcal{F} \subseteq P(\Omega)$ se num σ-algebră dacă:

1) $\emptyset, \Omega \in \mathcal{F}$
2) $A \in \mathcal{F} \Rightarrow \overline{A} \in \mathcal{F}$ ($\overline{A} = A^c$)
3) $(A_n) \in \mathcal{F} \Rightarrow \bigcup_{n \in \mathbb{N}} A_n \in \mathcal{F}$

ex: $\Omega = \{a, b, c\}$

$\mathcal{P}(\Omega) = \{\emptyset; \{a\}; \{b\}; \{c\}; \{a,b\}; \{b,c\}; \{a,c\}; \{a,b,c\}\}$

$\mathcal{P}(\Omega)$ - Cea mai mare σ-algebră în rap. cu Ω

$\mathcal{F}_0 = \{\emptyset, \Omega\}$

$\mathcal{F} = \{\emptyset, \Omega, \{c\}, \{a,b\}\}$

Tema: generați toate σ-algebrele pt $\Omega = \{a, b, c, d\}$

13 oct 2025 Laborator 2

Prob

$(\Omega, \mathcal{F}, \mathbb{P})$

Exp: Gigel se află la o răsc. de drumuri, nu știe dacă să o ia la stg sau la dreapta

Dorim să dăm cu banu dar avem doar un zar.

2, 4, 6 → stânga

1, 3, 5 → dreapta

$\Omega = \{1, 2, \ldots, 6\}$

$\mathcal{F} = \{\emptyset, \Omega, \{1,3,5\}, \{2,4,6\}\}$

$\mathbb{P} : \mathcal{F} \to [0, 1]$

1) $\mathbb{P}(\Omega) = 1$

2) $(A_n)_n \subset \mathcal{F}$ cu $A_i \cap A_j = \emptyset$

$$\mathbb{P}\left(\bigcup_{n\in\mathbb{N}} A_n\right) = \sum_{n\in\mathbb{N}} \mathbb{P}(A_n)$$

20 octombrie lab/seminar 3

1. Dem că $0 \le P(A) \le 1, \forall A \in K$

Fie $B \in K$ a.î. $P(B) > 1$; $\bar{B} \in K$, $\bar{B} \neq 0$

$$P(\underbrace{B \cup \bar{B}}_{\Omega}) = \underbrace{P(B)}_{>1} + P(\bar{B})$$

$P(\Omega) = 1$ contradicție X

inc

2. $P(\Omega)=1 \iff \bar{P}(A\cup\bar{A})=1$; $\forall A\in K$

$\Updownarrow$

$P(A)+P(\bar{A})=1$

$P(\bar{A})=1-P(A)$

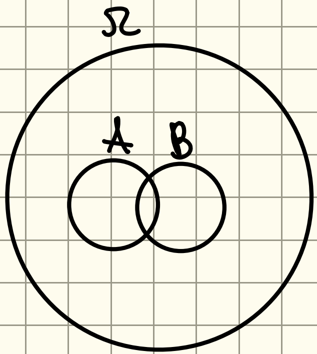
inc

3. $P(B)=P(A\cup(B\setminus A))=P(A)+P(B\setminus A)$

$P(B\setminus A) \geq 0$

$\Rightarrow P(B)\geq P(A)$

4.



$A\cup B = A\cup(B\setminus A)$

$P(A\cup B)=P(A\cup(B\setminus A))$

$=P(A)+P(B\setminus A)$

$=P(A)+P(B\cap\bar{A})$

$\mathbb{P}(A\cup B)=\mathbb{P}(A)+\mathbb{P}(B)-\mathbb{P}(A\cap B)$

$A\cup B = A\cup(B\setminus A) \Rightarrow \mathbb{P}(A\cup B)=\mathbb{P}(A)+\mathbb{P}(B\setminus A)$

$=\mathbb{P}(A)+\mathbb{P}(B)-\mathbb{P}(A\cap B)$

4. Probabilitatea condițională

Pt două evenimente A, B cu $P(A) > 0$ se def

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

am promovat

Obs: Notăm cu A ev. de a promova exam. și cu B evenimentul de a lua 10.

Un sist. complet de ev pe care-l mai numim și partiție a înseamnă $A_1, A_2, \dots A_n \in K$ cu prop:

$$\begin{cases} A_i \cap A_j \neq \emptyset & \forall i \neq j \\ \bigcup_{i=1}^{n} A_i = \Omega \end{cases}$$

FORMULA PROBABILITĂȚII TOTALE

$$\mathbb{P}(B) = \mathbb{P}\left(\bigcup_{i=1}^{n} A_i \cap B\right) = \sum_{i=1}^{n} \mathbb{P}(A_i \cap B) = \sum_{i=1}^{n} \mathbb{P}(B/A_i) \cdot \mathbb{P}(A_i)$$

incompatibile

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)}$$ nu e simetrică.

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}$$

ex: A = am luat 7
B = am luat nota cea mai mare din clasă.

$$\left.\begin{aligned} \mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)} &\Rightarrow \mathbb{P}(A \cap B) = \mathbb{P}(A|B) \cdot \mathbb{P}(B) \\ \mathbb{P}(B|A) = \frac{\mathbb{P}(B \cap A)}{\mathbb{P}(A)} &\Rightarrow \mathbb{P}(A \cup B) = \mathbb{P}(B|A) \cdot \mathbb{P}(A) \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \mathbb{P}(A|B) \cdot \mathbb{P}(B) = \mathbb{P}(B|A) \cdot \mathbb{P}(A)$$

$$\Downarrow$$

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A) \cdot \mathbb{P}(A)}{\mathbb{P}(B)}$$ Teorema lui BAYLIES.

P1: $\sigma(\{a\}, \{b\}, \{c\}, \{d\})$

Temă: Probleme axiome → probleme lab 3-4.

Probleme câmp de evenimente

I 1) $A \cap B$

2) $A \cup B \cup C$

3) $A \cup B \cup C$

4) $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$

5) $A \cap C \cap \bar{B}$

6) $\underbrace{\bar{A} \cap \bar{B} \cap \bar{C}}_{\text{n-am niciun parazit}} \cup \underbrace{(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cdot \cup (\bar{A} \cap \bar{B} \cap C)}_{\text{am fix unul}} = D$

n-am niciun parazit

am fix unul.

7. $\overline{A \cap B \cap C}$

$\Downarrow$

$D \cup (A \cap B \cap \bar{C}) \cup (A \cup \bar{B} \cup C) \cup (\bar{A} \cap B \cap C)$

(6)

II a) $C \cap (A \cup B) = (C \cap A) \cup (C \cap B) = C \cup (C \cap B) = C$.

$C \subset A$

III b) $\bar{A} \cap \bar{B}$

a) evident nu.

24 oct temă

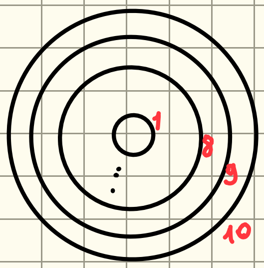
PS–Seminar 1

1, 2, 3 – la seminar

4. $\Omega = \{H, T\}$ o aruncare

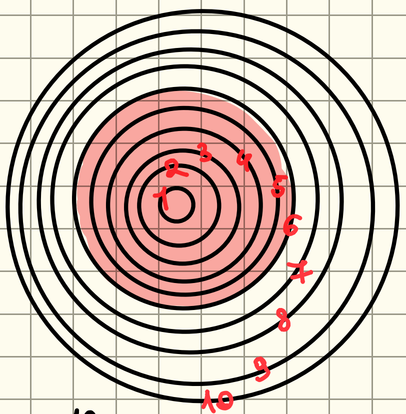
$\Omega = \{HH, HT, TH, TT\}$ 2 aruncări.

5.



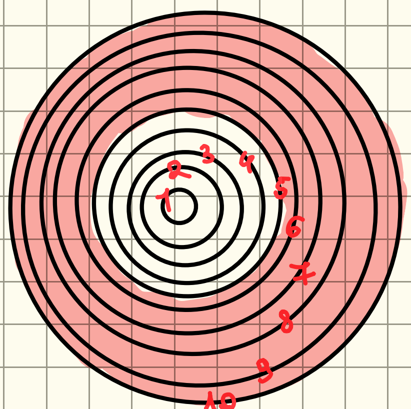
a) $A = \bigcup_{k=1}^{6} A_k = A_6$

ținta să lovească de la cercul 6 spre interior, oricare dintre primele $\overline{1,6}$ cercuri.



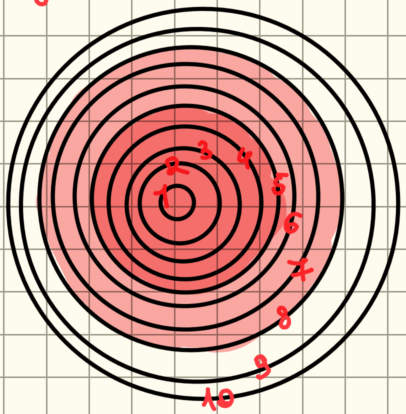
b) $B = \bigcap_{k=5}^{10} A_k = A_{10} \setminus A_5$

Intersecția cercurilor $\overline{5,10}$, cel puțin r_5 aproape de centru



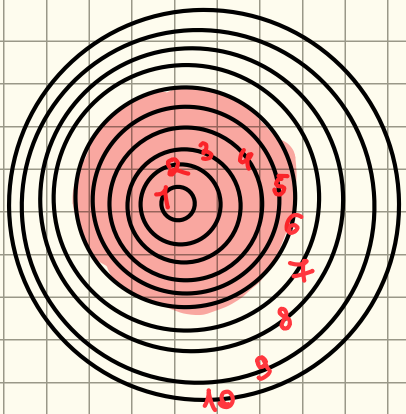
c) $C = \bigcup_{k=5}^{8} A_k = A_8$

Reuniunea cercurilor $\overline{5,8}$, dar $r_{\overline{1,4}} \subset r_5$, Oriunde dar maxim r_8



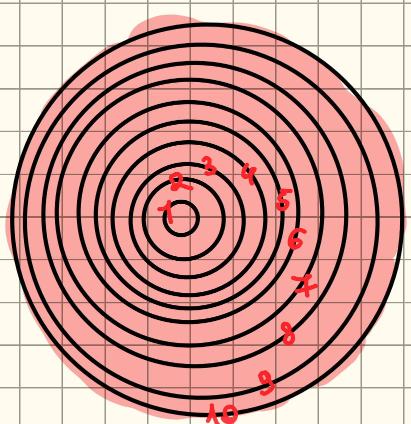
d) $D = \bigcap_{k=5}^{8} A_k = A_5$

Maxim r_5 spre exterior



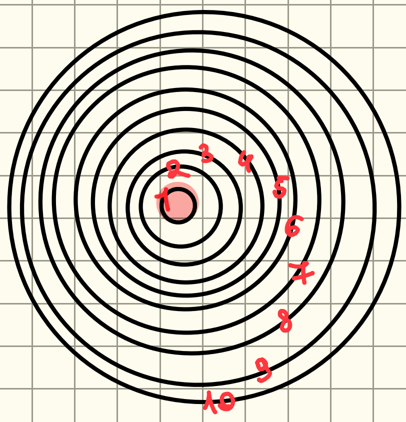
e) $F = \bigcup_{k=1}^{10} A_k = A_{10}$

Reuniunea tuturor cercurilor, adică tot cercul.



f) $F = \bigcap_{k=1}^{10} A_k = A_1$

Partea comună a tuturor cercurilor, A_1



Probleme – prob

I proprietăți axiomatice de bază.

1–5 ⇒ Seminar.

6. $\Omega = A \cup \bar{A}$, disjunctă

$\Rightarrow P(\Omega) = 1$

$\Downarrow$ disj

$P(A) + P(\bar{A}) = 1$

$P(\bar{A}) = 1 - P(A)$

7. $P(A \setminus B) = P(A) - P(A \cap B)$

$A = \underbrace{(A \cap B)}_{\text{disj}} \cup \underbrace{(A \setminus B)}_{\text{disj}}$

$P(A) = P(A \cap B) + P(A \setminus B)$

$P(A \setminus B) = P(A) - P(A \cap B)$

8. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$(A \cap C) \cup (B \cap C)$

$\Uparrow$

$P((A \cup B) \cup C) = P(A \cup B) + P(C) - P((A \cup B) \cap C)$

$= P(A) + P(B) - P(A \cap B) - (P(A \cap C) + P(B \cap C)$

$+ P(C)$

??

9. A, B incompatibile $\Rightarrow A \cap B = \emptyset$

$$P(A \cap B) = 0.$$

$$P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}_{0} = P(A) + P(B)$$

10. $A_1 \subset A_2 \subset A_3 \dots \Leftrightarrow P(\bigcup_1^{n} A_n) = \lim_{n \to \infty} P(A_n)$

$\Leftrightarrow P(A_n) \nearrow \lim P(A_n)$

Fie $B_1 = A_1$

$B_n = A_n \setminus A_{n-1}$

B_i, $i = \overline{1,n}$ disjuncte

$$\Rightarrow \bigcup_1^n A_n = \bigcup_1^n B_n$$

$$P(\bigcup_1^n A_n) = \sum_{n=1}^{\infty} P(B_n) = P(A_n)$$

II Probleme Numerice elementare.

11. $P(A) = 0,2$
$P(B) = 0,5$
$P(A \cap B) = 0,1$

$P(A \cup B) = ?$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0,2 + 0,5 - 0,1$$
$$= 0,6.$$

12. $P(A) = 0,8$

$$P(\bar{A}) = 1 - P(A)$$
$$= 1 - 0,8$$
$$= 0,2.$$

13. $P(A) = 0,3$

$P(A \cup B) = 0,7$

$P(A \cap B) = 0,1$

$P(B)=?$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$0,7=0,3+P(B)-0,1$

$P(B)=0,5.$

14. $P(A)=0,5$

$P(B)=0,6$

$P(A\cap B)=0,4$

$P(A\setminus B)=?$

$P(A\setminus B)=P(A)-P(A\cap B)$

$=0,5-0,4$

$=0,1.$

15. A,B – compatibile

$P(A\cup B)<P(A)+P(B)$

A,B - comp $\Rightarrow P(A\cap B)>0$

$P(A\cap B)-P(A)-P(B)>-P(A)-P(B)$

$P(A\cup B)<P(A)+P(B)$

16. $\left.\begin{array}{l} P(A)=0,4 \\ P(B)=0,5 \\ P(A\cap B)=0,2 \end{array}\right\} \Rightarrow P(A\cup B)=0,4+0,5-0,2=0,7.$

$P(A\cup B)\leq P(A)+P(B)$

$0,7\leq 0,4+0,5$

$0,7\leq 0,9$ ✓

17. A,B independente

$\Rightarrow P(A\cap B)=P(A)\cdot P(B)$?

$\underbrace{P(A|B)}_{=P(A)}=\dfrac{P(A\cap B)}{P(B)}$

indep

$P(A \cap B) = P(A) \cdot P(B)$.

18. $A \subset B$

$P(B \setminus A) = P(B) - P(A)$?

$B = B \setminus A \cup A$ disjuncte

$P(B) = P(B \setminus A) + P(A)$

$P(B \setminus A) = P(B) - P(A)$

19. $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

$P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

20. Seminar

III Aditivitate și inegalități

21. $A_1, A_2, \ldots, A_n$ disjuncte

$B_n: \ P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$

1. Verificare: $B_1: P(A_1) = P(A_1)$

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

dar A_1, A_2 disjuncte $\Rightarrow P(A_1 \cap A_2) = 0$.

$\Rightarrow P(A_1 \cup A_2) = P(A_1) + P(A_2)$

21. $B_k \to B_{k+1}$

$P(A_k \cup A_{k+1}) = P(A_k) + P(A_{k+1}) - P(A_k \cap A_{k+1})$; A_k, A_{k+1} disjuncte.

$= P(A_k) + P(A_{k+1}) - 0$ $\Rightarrow P(A_k \cap A_{k+1}) = 0$

$= P(A_k) + P(A_{k+1})$

$= P(A_1) + \ldots + P(A_k) + P(A_{k+1}) = \sum_{i=1}^{k+1} P(A_i)$

$\Rightarrow$ Rezultă prin metoda inducției matematice că B_n adevărată.

22. $A_1, A_2, \ldots, A_n$

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

1. $B_n : P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$

2. Verificare B_1: $P(A_1) \leq P(A_1)$

$B_2 : P(A_1 \cap A_2) \geq 0$

$-P(A_1 \cap A_2) \leq 0$

$P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

22. $P_k \to P_{k+1}$.

$P(A_k \cap A_{k+1}) \geq 0$

$-P(A_k \cap A_{k+1}) \leq 0$.

$P(A_k) + P(A_{k+1}) - P(A_k \cap A_{k+1}) \leq P(A_k) + P(A_{k+1})$

$P(A_k \cup A_{k+1}) \leq \sum_{i=1}^{k+1} P(A_{k+1})$ $\qquad P(A_1) + \ldots P(A_k)$

$\Rightarrow$ Rezultă prin metoda inducției matematice că B_n adevărată.

23. $\lim_{n \to \infty} P(A_n)$; dacă $A_n \downarrow \emptyset$

0? dar cum???

24. $P(A) = 0{,}7$
$P(B) = 0{,}5$
$P(C) = 0{,}4$

$$P(A \cup B \cup C) = \underbrace{P(A) + P(B) + P(C)}_{1{,}6 \leq 1} - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

25. $P(A_1 \cup A_2 \cup A_3)$

$P(A_1 \cap A_2) \geq 0$

$-P(A_1 \cap A_2) \leq 0$

$P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$

$P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

$P((A_1 \cup A_2) \cap A_3) \geq 0$

$-P((A_1 \cup A_2) \cap A_3) \leq 0$

$P(A_1 \cup A_2) + P(A_3) - P((A_1 \cup A_2) \cap A_3) \leq P(A_1 \cup A_2) + P(A_3)$

$P(A_1 \cup A_2 \cup A_3) \leq P(A_1 \cup A_2) + P(A_3)$

$P(A_1 \cup A_2 \cup A_3) - P(A_3) \leq P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

$P(A_1 \cup A_2 \cup A_3) \leq P(A_1) + P(A_2) + P(A_3)$

26. ex 22

27. ex: $P(A) = 0.3$

$P(B) = 0.4$

$P(C) = 0.4$

$P(A \cap B) = 0$

$P(B \cap C) = 0.1$

$P(C \cap A) = 0$.

$P(A \cap B \cap C) = 0$.

$P(A \cup B \cup C) = 1$.

$P(A \cup B \cup C) \leq 0.3 + 0.4 + 0.4 - 0.1$

≤ 1.

28. $A =$ "nr par"

$2 \leq 2k \leq 100, k \in \mathbb{Z}$

$1 \leq k \leq 50 \Rightarrow 50 - 1 + 1 = 50$ nr.

$P(A) = 0.5$

$B =$ „multiplu de 5"

$5 \le 5k \le 100;\ k \in \mathbb{Z}$
$1 \le k \le 20 \Rightarrow 20-1+1 = 20$ nr

$\Rightarrow P(B) = 0.2$.

$P(A \cup B) \le 0.2 + 0.5 = 0.7$.

Verificare

$A \cup B =$ " multiplu de 10

$10 \le k \cdot 10 \le 100,\ k \in \mathbb{Z}$
$1 \le k \le 10 \Rightarrow 10$ nr.
$\Rightarrow$

$P(A \cup B) = 0.1 \le 0.7$. ADEVĂRAT.

29. A, B, C incompatibile

$\Rightarrow A \cap B = \emptyset \Rightarrow P(A \cap B) = 0$
$A \cap C = \emptyset \Rightarrow P(A \cap C) = 0$
$B \cap C = \emptyset \Rightarrow P(B \cap C) = 0$
$A \cap B \cap C = \emptyset \Rightarrow P(A \cap B \cap C) = 0$

$\Rightarrow P(\cup A \cup B \cup C) = \sum P(A_i)$.

30. A, B, C complementare

$\Rightarrow A \cup B \cup C = \Omega$

$\Rightarrow P(A \cup B \cup C) = 1$.

31. $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

$$\left.\begin{array}{l} P(A) = 0,4 \\ P(A \cap B) = 0,2 \end{array}\right\} \Rightarrow P(B|A) = \frac{0,2}{0,4} = \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}.$$

32. $P(A) = 0,6$
$P(B) = 0,3$
$P(A \cap B) = 0,18$.

$$P(B|A) = \frac{0,18}{0,6} = \frac{\frac{18}{100\cancel{0}}}{\frac{6}{10\cancel{0}}} = \frac{18}{60} = 0.3$$

$$P(A|B)=\frac{0,18}{0.3}=\frac{18}{30}=\frac{3}{5}=0,6$$

33. $P(A)=0.3$

$P(A\cup B)=0.6$

$P(A\cap B)=0.15$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$0.6=0.3+P(B)-0.15$

$P(B)=0.45.$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0,15}{0,45}=\frac{1}{3}=0.(3)$$

$$P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{0,15}{0,3}=\frac{1}{2}=0.5$$

34. $P(B|A)=\dfrac{P(A\cap B)}{P(A)}\ \Big|\cdot P(A)$

$P(A\cap B)=P(B|A)\cdot P(A)$?

35. $P(A)=0.8$

$P(B|A)=0.75.$

$P(A\cap B)=?$

$$P(A\cap B)=0.8\cdot 0.75=\frac{8}{10}\cdot\frac{75}{100}=\frac{4}{5}\cdot\frac{3}{4}=\frac{3}{5}.$$

36. A, B, C

$P(\underbrace{A\cap B}_{D}\cap C)=P(A)\cdot P(B|A)P(C|A\cap B)$

$P(D\cap C)=P(D)\cdot P(C|D)$

$P(D\cap C)=\underbrace{P(A\cap B)}_{D}\cdot P(C|A\cap B)$

$P(D\cap C)=P(A)\cdot P(B|A)\cdot P(C|A\cap B)$

37. 60% au trecut la examen $= A$

70% au trecut la cel practic $= B$.

$P(A) = 60\% = 0.6$
$P(B|A) = 70\% = 0.7$

$P(A \cap B) = P(A) \cdot P(B|A) = 0.6 \cdot 0.7 = 0.42$.

$\downarrow$

cei care au trecut la ambele

38. $\left.\begin{array}{l} P(U_1) = 0.4 \\ P(U_2) = 0.6 \end{array}\right\} \Rightarrow P(U_1) + P(U_2) = 1 \xrightarrow{\text{formula prob totale}}$

$P(R|U_1) = 0.3$
$P(R|U_2) = 0.1$ $\Rightarrow P(R) = P(U_1) \cdot P(R|U_1) + P(U_2) \cdot P(R|U_2)$

$P(R) = ?$

$= 0.4 \cdot 0.3 + 0.6 \cdot 0.1$

$= 0.12 + 0.06$

$= 0.18$.

39. $P(U_1|R) = ?$

$$P(U_1|R) = \frac{P(U_1 \cap R)}{P(R)} = \frac{P(U_1) \cdot P(R|U_1)}{P(R} = \frac{0.4 \cdot 0.3}{0.18}$$

$$= \frac{0.12}{0.18} = \frac{12}{18} = \frac{2}{3} = 0,(6)$$

40. Inducție

1: $B_n: P(\bigcap_{i=1}^{n} A_i) = P(A_1) P(A_2|A_1) \ldots P(A_n | A_1 \cap \ldots A_{n-1})$

2: B_1: $P(A_1 \cap A_2) = P(A_1) \cdot P(A_2|A_1)$ Adevărat.

3: P_k - adev

$P_k \Rightarrow P_{k+1}$: $P((A_1 \cap A_2 \ldots A_k) \cap A_{k+1}) = P(A_k) \cdot P(A_{k+1} | A_k)$

$= P(A_1) \cdot P(A_2|A_1) \cdot \ldots \cdot P(A_{k+1}|A_1 \cap \ldots$
$\ldots A_k)$ ADEVĂRAT.

V. PROBABILITATEA TOTALĂ și TEOREMA LUI BAYES

41. $(A_1, A_2, \ldots, A_n)$ Sistem complet evenimente.

$$P(B) = \sum_{i=1}^{n} P(A_i) P(B|A_i)$$

$A_i \cap A_j = \emptyset$, $i \neq j$

$\bigcup_{i=1}^{n} A_i = \Omega$ $\qquad$ A_i - partiții.

$$P(B) = P\left(\bigcup_{i=1}^{n} (B \cap A_i)\right) = \sum_{i=1}^{n} P(B \cap A_i)$$

dar $P(B \cap A_i) = P(A_i) \cdot P(B|A_i)$

$$\Rightarrow P(B) = \sum_{i=1}^{n} P(A_i) \cdot P(B|A_i)$$

42. $P(U_1) = 0.2$
$P(U_2) = 0.5$
$P(U_3) = 0.3$

$P(B|U_i) = 0,9; 0,7; 0,2$

$P(B) = 0.2 \cdot 0.9 + 0.5 \cdot 0.7 + 0.3 \cdot 0.2$

$= 0.18 + 0.35 + 0.06$

$= 0.59$.

43. $P(U_2|B) = ?$

$$P(U_2|B) = \frac{P(U_2 \cap B)}{P(B)} = \frac{P(U_2) \cdot P(B|U_2)}{P(B)}$$

$$= \frac{0.5 \cdot 0.7}{0.59} = \frac{35}{59}.$$

44. ?

45. B: evenim în care pers e bolnavă

T^+: evenimentul să fie pozitiv

?

27 octombrie 2025 seminar 5

2. A_i = "evenimentul ca zarul i să cadă pe fața i"

$$P(A') = \underbrace{P\left(\bigcap_{i=1}^{6} A_i\right)}_{\text{independente}} = \prod_{i=1}^{6} P(A_i) = \frac{1}{6^6}$$

Cerința inițială.

a) $P(A) = P(A') \cdot 6! = \dfrac{5!}{6^5}$

b) B = "cel puțin o dată nr 6"

B_i = "evenimentul ca zarul i să obțină 6"

$$\underbrace{P(B)}_{\text{ind}} = 1 - P(\bar{B}) = 1 - P\left(\bigcap_{i=1}^{6} \underbrace{\bar{B_i}}_{\text{ind}}\right)$$

$$= 1 - \prod_{i=1}^{6} P(\bar{B_i})$$

$$= 1 - \left(\frac{5}{6}\right)^6$$

c) C_i = "evenimentul de a obține nr impar la extragerea i)

$P(C_i) = \frac{1}{2}$

$$P(C) = 1 - P(\bar{C}) = 1 - P\left(\bigcap_{i=1}^{6} \underbrace{\bar{C_i}}_{\text{indep}}\right)$$

$$= 1 - \frac{1}{2^6}$$

d) să pice o singură dată nr 1

D_i = „evenimentul ca pe poz i să pice 1"

$$\mathbb{P}(D) = \mathbb{P}\Big(\underbrace{D_1 \cap \bigcap_{i=2}^{6} \bar{D_i}}_{\text{inc}} \cup \underbrace{D_2 \cap \bigcap_{\substack{i=1 \\ i \neq 2}}^{6} \bar{D_i}}_{\text{inc}} \cup \ldots \cup \underbrace{D_6 \cap \bigcap_{i=1}^{5} \bar{D_i}}_{\text{inc}}$$

disj două câte două

$$\mathbb{P}(D) = \sum_{i=1}^{6} \mathbb{P}(\underbrace{D_i}_{ind} \cap \underbrace{\bigcap_{\substack{j=1 \\ j\neq i}}^{6} \overline{D_j}}_{ind})$$

$$= \sum_{i=1}^{6} \mathbb{P}(D_i) \cdot \prod_{\substack{j=1 \\ i\neq j}}^{6} \mathbb{P}(\overline{D_j})$$

$$= \sum_{i=1}^{6} \frac{1}{6} \left(\frac{5}{6}\right)^5$$

$$= \cancel{6} \cdot \frac{1}{\cancel{6}} \left(\frac{5}{6}\right)^5 = \left(\frac{5}{6}\right)^5 .$$

e) $E = \overline{C_1} \cap \overline{C_2} \cap C_3 \cap C_4 \cap C_5 \cap C_6 \ldots$

$$E = \bigcup_{i=1}^{6} (\overline{C_i} \cap \overline{C_j} \cap \bigcap_{\substack{k=1 \\ k\neq i \\ k\neq j}}^{6} C_k) \quad j = \overline{1,6},\ j\neq i,\ i<j.$$

$$P(E) = P(\bigcup_{i=1}^{6} (\overline{C_i} \cap \overline{C_j} \cap \bigcup_{k=1}^{6} C_k)),\ j = \overline{1,6}\quad i<j.$$

$$= \sum_{i=1}^{6} \mathbb{P}(\overline{C_i} \cap C_j \cap \bigcap_{k=1} C_k),\ j = \overline{1,6},\ i<j .$$

$$= \sum_{i=1}^{6} \mathbb{P}(\overline{C_i}) \cdot \mathbb{P}(C_j) \cdot \prod_{k=1}^{6} \mathbb{P}(C_k)\ ;\ j = 1,6,\ i<j .$$

$$= 6 \left(\frac{1}{2}\right)^6 .$$

10. F_1, F_2, F_3

F_1	F_2	F_3
2%	4%	5%
30	50	20

Notăm cu D un calc se defectează.

A_i un calc ales provine de la F_i, $i=\overline{1,3}$

$\mathbb{P}(A_1) = \frac{3}{10} = 0(3)$ $\qquad$ $\mathbb{P}(D|A_1) = 2\% = 0,02$

$\mathbb{P}(A_2) = \frac{50}{100} = 0.5$ $\qquad$ $\mathbb{P}(D|A_2) = 0.04$

$\mathbb{P}(A_3) = \frac{20}{100} = 0.2$ $\qquad$ $\mathbb{P}(D|A_3) = 0.05$

a) Pt că a sist complet

$$\mathbb{P}(D) = \sum_{i=1}^{3} (D|A_i)\, \mathbb{P}(A_i) = 0.02 \cdot 0.3 + 0.04 \cdot 0.5 + 0.05 \cdot 0.2 =$$

b)

c) $$\mathbb{P}(D | A_1 \cup A_3) = \frac{P(D \cap (A_1 \cup A_3))}{P(A_1 \cup A_3)} = \frac{P((D \cap A_1) \cup (D \cap A_3))}{P(A_1) + P(A_3)}$$

$$= \frac{P(D \cap A_1) + P(D \cap A_3)}{P(A_1) + P(A_3)}$$

$$= \frac{P(D|A_1) \cdot P(A_1) + P(D|A_3) \cdot P(A_3)}{P(A_1) + P(A_3)}$$

d) $$P(A_1 \cup A_2 | \overline{D}) = \frac{P((A_1 \cup A_2) \cap \overline{D})}{\underbrace{P(\overline{D})}_{1-D}}$$

$$= \frac{P(A_1 \cap \overline{D}) + P(A_2 \cap \overline{D})}{1 - P(D)}$$

$$= \frac{P(\overline{D}|A_1) \cdot P(A_1) + P(\overline{D}|A_2)}{1 - P(D)}$$

3 noiembrie 2025 Seminar 6.

Câte nr de 3 cifre se pot forma cu cifre din $\{2 \dots 7\}$

6^3

11. – calc prob cu even.

D_i = "evenimentul să se defecteze componenta i"

$\mathbb{P}(D_1) = 0.075$

$\mathbb{P}(D_2) = 0.09$

$\mathbb{P}(D_3) = 0.082$

F = "evenimentul că agregatul funcționează"

$$\mathbb{P}(F) = \mathbb{P}(\overline{D_1} \cap \overline{D_2} \cap \overline{D_3})$$

Nu putem presupune indep $\Rightarrow$ nu putem calcula direct.

$$= \mathbb{P}(\overline{D_1 \cup D_2 \cup D_3}) = 1 - \mathbb{P}(D_1 \cup D_2 \cup D_3)$$

Inegalitatea lui Boole

$$\mathbb{P}(D_1 \cup D_2 \cup D_3) \leq \mathbb{P}(D_1) + \mathbb{P}(D_2) + \mathbb{P}(D_3)$$

$$1 - \mathbb{P}(D_1 \cup D_2 \cup D_3) \geq -(\mathbb{P}(D_1) + \mathbb{P}(D_2) + \mathbb{P}(D_3)) + 1$$

$$\geq 1 - 0.075 - 0.09 - 0.082 = 0.753$$

$$\Rightarrow \min(\mathbb{P}(F)) = 0.753.$$

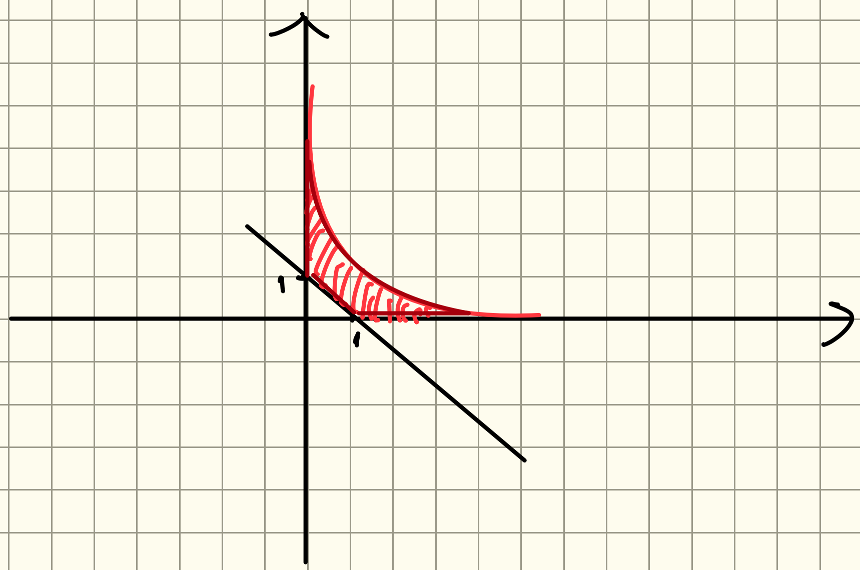
$$\mathbb{P}(F) = \mathbb{P}\left(\bigcup_{i=1}^{3} \overline{D_i}\right)$$

În ipoteza de defectare independentă

$$\mathbb{P}\left(\bigcap_{i=1}^{3} \overline{D_i}\right) = \prod_{i=1}^{3} \mathbb{P}(\overline{D_i}) = 0.925 \cdot 0.91 \cdot 0.918 = 0.772.$$

$$\mathbb{P}(D_1 \cap (D_2 \cap D_3)) = \mathbb{P}(D_2 \cap D_3 \mid D_1) \cdot \mathbb{P}(D_1)$$

13. $\begin{cases} x+y \leq 1 \rightsquigarrow & x+y=1 \Rightarrow y = 1-x \\ x \cdot y \leq \frac{2}{9} \rightsquigarrow & x \cdot y = \frac{2}{9} \Rightarrow y = \frac{2}{9} \cdot \frac{1}{x} \end{cases}$



Dintr-un lot ce conține 4 piese coresp și 3 piese defecte se extrag simultan 3 piese. Fie X variabila aleatoare ce indică nr de piese corespunzătoare obținute în cele 3 extrageri.

Determinați:

a) Repartiția v.a. X.

$$X = \begin{pmatrix} 0 & 1 & 2 & 3 \\ p_0 & p_1 & p_2 & p_3 \end{pmatrix}$$

Pentru calc. prob vom fol Schema cu

$N = 7$

$N_1 = 4$

$n = 3$ (nr de extrageri)

$$p_k = \frac{C_4^k \cdot C_3^{3-k}}{C_7^3} \qquad k = \overline{0,3}$$

$$p_0 = \frac{C_4^0 \cdot C_3^3}{C_7^3} = \frac{1 \cdot 1}{35} = \frac{1}{35}$$

$$p_1 = \frac{12}{35}$$

$$p_2 = \frac{18}{35}$$

$$p_3 = \frac{4}{35}$$

$$\begin{array}{cccc} & 0{,}7 & & 2.8 \end{array}$$

$$X = \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{35} & \frac{12}{35} & \frac{18}{35} & \frac{4}{35} \end{pmatrix}$$

: Se exclud reciproc ⇒ inc

b) Prob ca cel puțin două piese să fie coresp

$$\mathbb{P}(X \geq 2) = \frac{18}{35} + \frac{4}{35} = \frac{22}{35}$$

$$\mathbb{P}(0.7 \leq X \leq 2.8) = \frac{12}{35} + \frac{18}{35} = \frac{30}{35}.$$

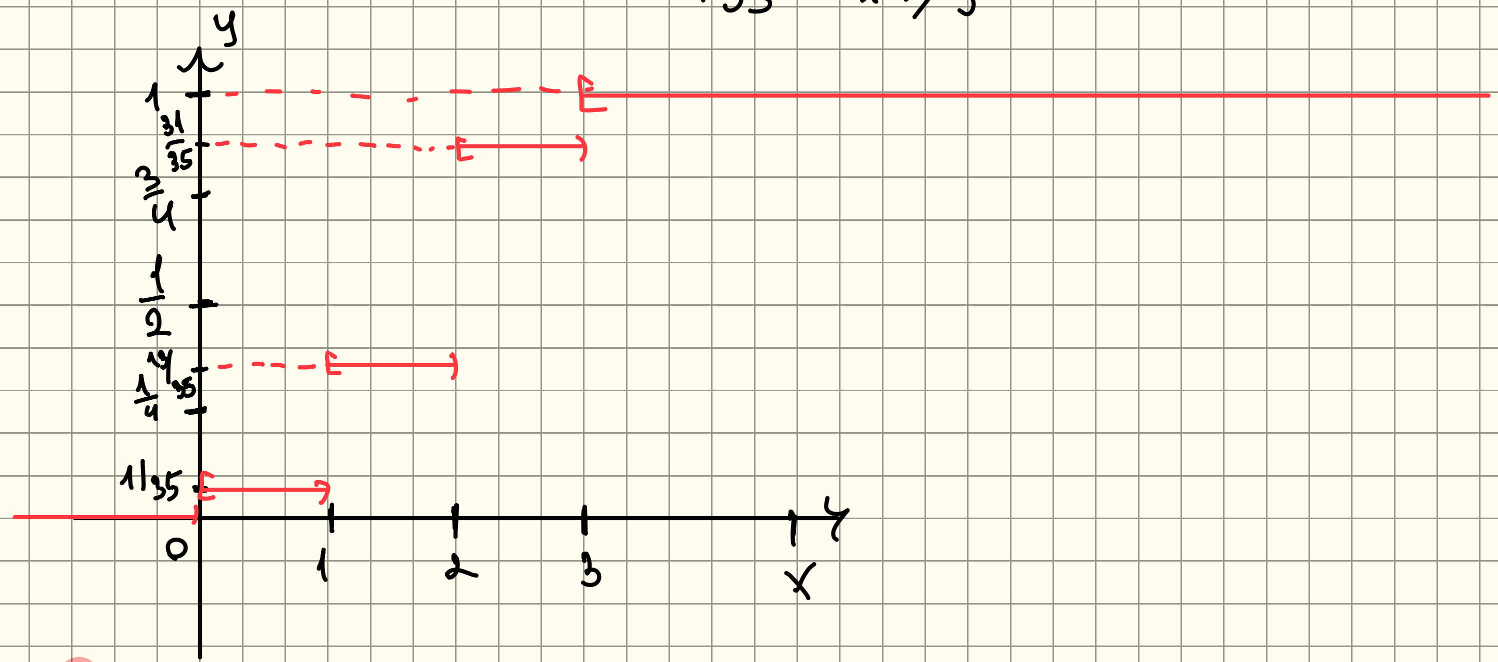
c) $\mathbb{P}(\underbrace{X>1.4}_{A} \mid \underbrace{X<3}_{B}) = \dfrac{\mathbb{P}((X>1.4)\cap(X<3))}{\mathbb{P}(X<3}$

$= \dfrac{18}{31}$

$$X = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1/35 & 12/35 & 18/35 & 4/35 \end{pmatrix}$$

c) $F(x) = \mathbb{P}(X \leq x) = \begin{cases} 0; & x<0 \\ \frac{1}{35}; & 0 \leq x<1 \\ 13/35 & 1 \leq x<2 \\ 31/35 & 2 \leq x<3 \\ 1/35 & x \geq 3 \end{cases}$

$x \in \mathbb{R}$



$\mathbb{P}(a<X\leq b) = F(b)-F(a)$

$\mathbb{P}(1<X<3) = (F(3)-F(1)) - \mathbb{P}(X=3)$

$= \left(1-\frac{13}{35}\right) - \frac{4}{35}$

$= \frac{18}{35}$

$\mathbb{P}(0\leq X<2) = (F(2)-F(0)) - \mathbb{P}(X=2) + \mathbb{P}(X=0)$

$= \left(\frac{31}{35} - \cancel{\frac{1}{35}}\right) - \frac{18}{35} + \cancel{\frac{1}{35}}$

$$= \frac{13}{35}$$

$$X: \begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{35} & \frac{12}{35} & \frac{18}{35} & \frac{4}{35} \end{pmatrix}$$

$$\mathbb{E}(X) = \sum_{i=0}^{3} x_i \cdot p_i = 0 \cdot \frac{1}{35} + 1 \cdot \frac{12}{35} + 2 \cdot \frac{18}{35} + 3 \cdot \frac{4}{35} = \frac{12}{7} \in (1,2)$$

$$Var(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = \left(0 \cdot \frac{1}{35} + 1 \cdot \frac{12}{35} + 4 \cdot \frac{18}{35} + 9 \cdot \frac{4}{35}\right) - \frac{144}{49}$$

$$X^2: \begin{pmatrix} 0 & 1 & 4 & 9 \\ \frac{1}{35} & \frac{12}{35} & \frac{18}{35} & \frac{4}{35} \end{pmatrix} = \frac{12+72+36}{35} - \frac{144}{49} = \frac{120}{35} - \frac{144}{49} = \frac{24}{49}$$

2. $X_1: \begin{pmatrix} -3 & 3 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

$X_2: \begin{pmatrix} -4 & 3 & 6 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \end{pmatrix} \Rightarrow X_2^2: \begin{pmatrix} 9 & 16 & 36 \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}$

$\mathbb{E}(X_1) = 0$

$\mathbb{E}(X_2) = 0$

$Var(X) = 9 - 0 = 9$ $\quad$ $Var(X) = 17 - 0 = 17$

8 decembrie seminar / lab 9

Necorelarea a două v.a nu implică independența deoarece necorelarea înseamnă că nu există o legătură liniară între două variabile (dar poate exista una de orice altfel). Așadar necorelarea e un ev mai slab decât independența.

5. $X: \begin{pmatrix} -2 & 1 \\ 0.4 & 0.6 \end{pmatrix}$

$Y: \begin{pmatrix} -1 & 3 \\ 0.3 & 0.7 \end{pmatrix}$

$K = \mathbb{P}(X=-2, Y=3)$

a) Repartiția Comună !

$X \backslash Y$	-1	3	
-2	$0.4-K$	K	0.4
1	$K-0.1$	$0.7-K$	0.6
	0.3	0.7	1

b) K a.î. cele 2 variabile necorelate

X, Y necorelate $\Longleftrightarrow \rho(X,Y)=0 \Longleftrightarrow cov(X,Y)=0$

$\rho(X,Y) = \dfrac{cov(X,Y)}{\sqrt{var(X)\, var\, Y}}$

$\mathbb{E}(X\cdot Y) - \mathbb{E}(X)\mathbb{E}(Y) = 0.$

$\mathbb{E}(X) = -2\cdot 0{,}4 + 1\cdot 0{,}6$

$= -2\cdot\frac{2}{5} + 1\cdot\frac{3}{5} = -\frac{4}{5}+\frac{3}{5} = -\frac{1}{5} = -0.2$

$\mathbb{E}(Y) = 1.8.$

$\mathbb{E}(X\cdot Y) = ?$

$X\cdot Y = \begin{pmatrix} 2 & -6 & -1 & 3 \\ 0.4-K & K & K-0.1 & 0.7-K \end{pmatrix}$

$\mathbb{E}(X\cdot Y) = 2(0.4-K) - 6K - (K-0.1) + 3(0.7-K)$

$= 0.8 - 2K - 6K - K + 0{,}1 + 2.1 - 3K$

$= 3 - 12K$

$$\operatorname{cov}(X\cdot Y) = 3-12K-(-0{,}2)(1.8) = 3-12K+0.36 =$$
$$= 3.36-12K$$

$$\operatorname{cov}(X\cdot Y) = 0 \Rightarrow 3.36-12K = 0$$
$$12K = 3.36$$
$$K = \frac{3.36}{12} = 0.28 \quad \checkmark$$

Cond: $0 \le 0{,}4-K \le \min\{0.4,\ 0.3\}$
$0 \le 0{,}7-K \le \min\{0.6,\ 0.7\}$ $\Leftrightarrow 0.1 \le K \le 0.4$
$0 \le K-0.1 \le \min\{0.3,\ 0.6\}$
$0 \le K \le \min\{0.4,\ 0.7\}$

$K = 0.28$

X \ Y	-1	3	
-2	0.12	0.28	0.4
1	0.18	0.42	0.6
	0.3	0.7	1

linie coloană

$\pi_{11} = 0{,}12 = 0{,}3 \cdot 0{,}4 = p_1 \cdot q_1$ ✓
$\pi_{12} = 0{,}28 = 0{,}4 \cdot 0{,}7 = p_1 \cdot q_2$ ✓ $\Rightarrow$ X, Y <u>Independente</u>
$\pi_{21} = 0.18 = 0{,}3 \cdot 0{,}6 = p_2 \cdot q_1$ ✓
$\pi_{22} = 0.42 = 0{,}6 \cdot 0.7 = p_2\, q_2$ ✓

9.

$x \backslash y$	-2	-1	0	1	2	p
-1	$1/10$	$1/50$	$3/50$	$1/50$	$1/10$	$15/50$
0	$1/50$	$3/25$	$2/50$	$3/25$	$2/50$	$18/50$
1	$2/25$	$1/50$	$7/50$	$1/50$	$2/25$	$17/50$
q	$11/50$	$8/50$	$6/25$	$8/50$	$11/50$	1

a) Repartiții Marginale:

$$X: \begin{pmatrix} -1 & 0 & 1 \\ 15/50 & 18/50 & 17/50 \end{pmatrix}$$

$$Y: \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 11/50 & 8/50 & 9/25 & 8/50 & 11/50 \end{pmatrix}$$

$$X^2: \begin{pmatrix} 0 & 1 \\ 18/50 & 32/50 \end{pmatrix}$$

$$Y^2: \begin{pmatrix} 0 & 1 & 4 \\ 6/25 & 16/50 & 22/50 \end{pmatrix}$$

b) Coeficientul de corelație.

$$cov(X;Y) = \mathbb{E}[X \cdot Y] - \mathbb{E}[X] \cdot \mathbb{E}[Y]$$

$$\mathbb{E}[X] = \frac{2}{50}$$

$$\mathbb{E}[X^2] = \frac{32}{50} = \frac{16}{25} =$$

$$var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2 = \frac{32}{50} - \frac{4}{\quad} =$$

$$\mathbb{E}[Y] = -\frac{22}{50} - \frac{8}{50} + \frac{8}{50} + \frac{22}{50} = 0.$$

$$\mathbb{E}[Y^2] = \frac{104}{50}$$

$$var(Y) = \mathbb{E}[Y^2] - (\mathbb{E}[Y])^2 = \frac{104}{50} - 0$$

$$\mathbb{E}(XY) = 2 \cdot \frac{1}{10} + \cancel{\frac{1}{50}} - \cancel{\frac{1}{50}} - 2 \cdot \frac{1}{10}$$

$$-\cancel{2 \cdot \frac{2}{25}} - \cancel{\frac{1}{50}} + \cancel{\frac{1}{50}} + \cancel{2 \cdot \frac{2}{25}}$$

$$= \cancel{\frac{1}{5}} - \cancel{\frac{1}{50}} = 0$$

$$cov(X,Y) = 0 - 0 = 0$$

$$\rho(X;Y) = 0.$$

d) $Var(3X+5) = Var(3X) = 9\,Var(X) = 9 \cdot \frac{1596}{2350}$.

const dispare

iese în față la pătrat

$P(X<1;\ Y>0) = \frac{1}{50} + \frac{2}{50} + \frac{1}{10} + \frac{3}{25} = \frac{14}{50} = 0.28$

c) $X \mid Y=0 \quad : \begin{pmatrix} -1 & 0 & 1 \\ 1/4 & 1/6 & 7/12 \end{pmatrix}$

$Y \mid X=1 \quad : \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 4/17 & 7/17 & 7/17 & 7/17 & 4/17 \end{pmatrix}$