Groundwater Hydrology Equations

1. Porosity, n:

$$n = \frac{V_v}{V_t} = \frac{V_t - V_s}{V_t}$$

where V_v is the volume of voids, V_t is the total volume, and V_s is the volume of solids. $(0 < n < 1)$.

2. Effective porosity, n_e (specific yield, S_y):

$$n_e = \frac{V_m}{V_t}$$

where V_m is the volume of mobile water and V_t is the total volume.

3. Specific retention, S_r:

$$S_r = \frac{V_{im}}{V_t}$$

where V_{im} is the volume of immobile water and V_t is the total volume.

4. Relationship between porosity, n, specific yield, S_y, and specific retention, S_r:

$$n = S_y + S_r$$

5. Volumetric water content, θ (moisture content):

$$\theta = \frac{V_w}{V_t} \ ,$$

where V_w is the volume of water and V_t is the total volume.

6. Hydraulic head, h:

$$h = \frac{p}{\rho g} + z = \frac{p}{\gamma} + z$$

where z is the elevation of the point of interest above the datum, p is the fluid pressure at the point of interest, ρ is the fluid density, γ is the specific weight of the fluid, and g is acceleration due to gravity. The term $p/(\rho g)$ is the pressure head, h_p, which represents the height that water will rise in a piezometer above the point of interest. The term z is the elevation head, h_g, which represents the elevation of the point of interest above the datum ($z = 0$ at the datum).

7. Capillary pressure, p_c:

$$p_c = p_a - p$$

where p_a is air pressure and p is pressure of the water. In our applications, $p_a = 0$, so $p_c = -p = |p|$.

8. Capillary pressure head, h_c:

$$h_c = \frac{p_c}{\rho g} = \frac{p_c}{\gamma} = -h_p$$

where h_p is pressure head.

9. Effective water content, Θ:

$$\Theta = \frac{\theta - \theta_r}{n - \theta_r}$$

where θ is volumetric water content, n is porosity, and θ_r is residual water content.

10. van Genuchten model for water retention curve:

$$\begin{aligned} \Theta &= \left[1 + (\alpha |h_p|)^N\right]^{-M} \\ |h_p| &= \frac{1}{\alpha}\left[\Theta^{-1/M} - 1\right]^{1/N} \end{aligned}$$

where Θ is the effective water content, h_p is pressure head, and α, N, and M are parameters that depend on the soil type, with $M = 1 - 1/N$. Note that $|h_p| = h_c$, where h_c is the capillary pressure head.

11. Hydraulic gradient, ∇h:

$$\nabla h = \frac{\partial h}{\partial x}\vec{i} + \frac{\partial h}{\partial y}\vec{j} + \frac{\partial h}{\partial z}\vec{k}$$

where $\vec{i}$, $\vec{j}$, and $\vec{k}$ are unit vectors in the x-, y-, and z-directions, respectively.

12. In two dimensions, if head is measured at three points, and $h_1 = h(x_1, y_1)$, $h_2 = h(x_2, y_2)$, $h_3 = h(x_3, y_3)$, then

$$\begin{aligned} \frac{\partial h}{\partial x} &= \frac{(h_1 - h_2)(y_2 - y_3) - (h_2 - h_3)(y_1 - y_2)}{(x_1 - x_2)(y_2 - y_3) - (x_2 - x_3)(y_1 - y_2)} \\ \frac{\partial h}{\partial y} &= \frac{(h_1 - h_2)(x_2 - x_3) - (h_2 - h_3)(x_1 - x_2)}{(y_1 - y_2)(x_2 - x_3) - (y_2 - y_3)(x_1 - x_2)} \end{aligned}$$

13. Darcy's Law:

$$Q = -KA\frac{dh}{ds} = -KA\nabla h = KA\frac{dh}{dr}$$

where Q is the flow rate, K is the hydraulic conductivity, A is the cross-sectional area **perpendicular to flow**, h is head, s is flow direction, dh/ds is the hydraulic gradient between two points in the s-direction, ∇h is the general hydraulic gradient. The last equality is for radial flow, where r is the radial coordinate.

14. Darcy's Law in terms of specific discharge:

$$\vec{q} = -K\frac{dh}{ds} = -K\nabla h$$

where $\vec{q}$ is the specific discharge or Darcy velocity.

15. Darcy's Law in terms of groundwater velocity:

$$\vec{v} = -\frac{K}{n_m}\frac{dh}{ds} = -\frac{K}{n_m}\nabla h$$

where $\vec{v}$ is the groundwater velocity or average linear velocity and n_m is the mobile porosity porosity.

16. Hydraulic conductivity, K:

$$K = \frac{k\rho_w g}{\mu} = \frac{kg}{\nu}$$

where k is permeability, ρ_w is fluid density, g is acceleration due to gravity, μ is fluid viscosity, and $\nu = \mu/\rho_w$.

17. Average hydraulic conductivity for flow parallel to bedding:

$$\bar{K}_{\parallel} = \frac{\sum_i K_i d_i}{\sum_i d_i}$$

where K_i is the hydraulic conductivity of layer i and d_i is the thickness of layer i.

18. Average hydraulic conductivity for flow perpendicular to bedding:

$$\bar{K}_{\perp} = \frac{\sum_i d_i}{\sum_i \frac{d_i}{K_i}}$$

where K_i is the hydraulic conductivity of layer i and d_i is the thickness of layer i.

19. Tangent law:

$$\frac{K_1}{K_2} = \frac{\tan\alpha_1}{\tan\alpha_2}$$

where α_1 and α_2 are the angles between the line perpendicular to the layering and the flow direction in Layers 1 and 2, respectively.

20. Darcy's Law in three-dimensions:

$$\begin{aligned} q_x &= -K_{xx}\frac{\partial h}{\partial x} - K_{xy}\frac{\partial h}{\partial y} - K_{xz}\frac{\partial h}{\partial z} \\ q_y &= -K_{yx}\frac{\partial h}{\partial x} - K_{yy}\frac{\partial h}{\partial y} - K_{yz}\frac{\partial h}{\partial z} \\ q_z &= -K_{zx}\frac{\partial h}{\partial x} - K_{zy}\frac{\partial h}{\partial y} - K_{zz}\frac{\partial h}{\partial z} \end{aligned}$$

where q_x, q_y, and q_z are the components of specific discharge in the x-, y-, and z-directions, respectively, and K_{ij} is the hydraulic conductivity in the i-direction due to a gradient in the j-direction, where $i, j = x, y, z$.

21. Darcy's Law in matrix form:

$$\vec{q} = \begin{bmatrix} q_x \\ q_y \\ q_x \end{bmatrix} = -\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \begin{bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{bmatrix}$$

or in short-hand as

$$\vec{q} = -\underline{\underline{K}}\nabla h$$

where

$$\underline{\underline{K}} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

$\underline{\underline{K}}$ is a symmetric matrix (i.e., $K_{xy} = K_{yx}$, $K_{xz} = K_{zx}$, and $K_{yz} = K_{zy}$).

22. Components of $\underline{\underline{K}}$ in two dimensions:

$$\begin{aligned} K_{xx} &= K_{\parallel}\cos^2\theta + K_{\perp}\sin^2\theta \\ K_{xy} &= K_{yx} = (K_{\parallel} - K_{\perp})\sin\theta\cos\theta \\ K_{yy} &= K_{\perp}\cos^2\theta + K_{\parallel}\sin^2\theta \end{aligned}$$

23. Relative hydraulic conductivity:

$$K_r(\Theta) = \frac{K(\Theta)}{K_s}$$

where $K_r(\Theta)$ is the relative hydraulic conductivity as a function of effective water content, Θ, $K(\Theta)$ is the actual hydraulic conductivity as a function of Θ, and K_s is the saturated hydraulic conductivity.

24. van Genuchten model for hydraulic conductivity:

$$K_r(\Theta) = \sqrt{\Theta}\left[1 - \left(1 - \Theta^{1/M}\right)^M\right]^2$$

where $K_r(\Theta)$ is relative hydraulic conductivity as a function of effective water content, Θ, (which is a function of pressure head, h_p), and M is a parameter that depends on soil type.

25. Darcy's law for one-dimensional vertical flow in the unsaturated zone:

$$q \quad = \quad -K_r(\Theta)K_s\frac{dh}{dz} = -K_r(\Theta)K_s\left[\frac{dh_p}{dz} + 1\right] \; ,$$

where q is specific discharge, $K_r(\Theta)$ is the relative hydraulic conductivity as a function of effective water saturation, Θ, and K_s is saturated hydraulic conductivity.

26. Groundwater velocity in the unsaturated zone:

$$v = \frac{q}{\theta} \; .$$

27. Hydraulic conductivity from a constant-head permeameter:

$$K = \frac{V L}{A\,h\,t} = \frac{Q L}{A\,h}$$

where V is the volume of water flowing through the permeameter in an amount of time t, L is the length of the sample, A is the cross-sectional area of the permeameter chamber, h is the head in the constant head tank relative to the datum, and Q is the volumetric flow rate.

28. Hydraulic conductivity from a falling-head permeameter:

$$K = \frac{aL}{At}\ln\left(\frac{h_o}{h_1}\right)$$

where a is the cross-sectional area of the tube, A is the cross-sectional area of the permeameter chamber, L is the length of the sample, h_o is the initial head in the tube, and h_1 is the head in the tube at time t.

29. Drawdown, s:

$$s = h_o - h$$

where h_o is initial head and h is head at an arbitrary time.

30. Hvorslev slug test equation:

$$\ln\left(\frac{s_o}{s}\right) = \frac{K}{Fd^2}t$$

where s_o is the drawdown at the time that the slug is removed, s is the drawdown at time t, K is the radial hydraulic conductivity, d is the diameter of the well casing where the water level is observed, and F is a shape factor that depends on the well geometry.

31. Specific yield, S_y:

$$S_y = \frac{V_{\text{drained}}}{A\,\Delta h}$$

where V_{drained} is the volume of water removed, A is the plan area, and Δh is the change in head.

32. Specific storage, S_s:

$$S_s = \rho g(\alpha + n\beta_w) = \frac{V_{\text{drained}}}{V_t\,\Delta h}$$

where ρ is density of the fluid, g is acceleration due to gravity, α is the compressibility of the rock matrix, n is porosity, β_w is compressibility of water, V_{drained} is the volume of water removed, V_t is the total aquifer volume, and Δh is the change in head.

33. Storage coefficient, S:

$$S = S_s b = \rho g(\alpha + n\beta_w)b = \frac{V_{\text{drained}}}{A\,\Delta h}$$

where S_s is specific storage, b is aquifer thickness, A is plan area of aquifer, and other notation is defined in item 32.

34. Flow equation for confined aquifer with axes aligned with principal direction of anisotropy:

$$S_s\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)$$

35. Flow equation for confined aquifer; isotropic aquifer:

$$S_s\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial h}{\partial z}\right)$$

36. Flow equation for confined aquifer with axes aligned with principal direction of anisotropy; homogeneous aquifer:

$$S_s \frac{\partial h}{\partial t} = K_{xx} \frac{\partial^2 h}{\partial x^2} + K_{yy} \frac{\partial^2 h}{\partial y^2} + K_{zz} \frac{\partial^2 h}{\partial z^2}$$

37. Flow equation for confined aquifer with axes aligned with principal direction of anisotropy; steady flow:

$$0 = \frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)$$

38. Flow equation for confined aquifer with axes aligned with principal direction of anisotropy; essentially horizontal flow:

$$S\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial y}\right) \ ,$$

where $S = S_s b$ is the storativity, $T = Kb$ is the transmissivity, and the aquifer thickness is b.

39. Flow equation for unconfined aquifer; essentially horizontal flow (Dupuit assumption), principal axes aligned with coordinate axes:

$$S_y\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_{xx}(h-\zeta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}(h-\zeta)\frac{\partial h}{\partial y}\right)$$

40. Flow equation for unconfined aquifer; essentially horizontal flow (Dupuit assumption); homogeneous aquifer; principal directions of anisotropy aligned with coordinate axes:

$$S_y\frac{\partial h}{\partial t} = K_{xx}\frac{\partial}{\partial x}\left((h-\zeta)\frac{\partial h}{\partial x}\right) + K_{yy}\frac{\partial}{\partial y}\left((h-\zeta)\frac{\partial h}{\partial y}\right)$$

41. Flow equation for unconfined aquifer; essentially horizontal flow (Dupuit assumption); isotropic aquifer:

$$S_y\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K(h-\zeta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K(h-\zeta)\frac{\partial h}{\partial y}\right)$$

42. Flow equation for unconfined aquifer; essentially horizontal flow (Dupuit assumption); steady flow; principal directions of anisotropy aligned with coordinate axes:

$$0 = \frac{\partial}{\partial x}\left(K_{xx}(h-\zeta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}(h-\zeta)\frac{\partial h}{\partial y}\right)$$

43. Flow equation for unconfined aquifer with recharge; essentially horizontal flow:

$$S_y\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K_{xx}(h-\zeta)\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy}(h-\zeta)\frac{\partial h}{\partial y}\right) + N$$

where N is the recharge rate.

44. Groundwater flow equation in radial coordinates for an isotropic, confined aquifer with essentially horizontal flow and recharge:

$$S\frac{\partial h}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rT\frac{\partial h}{\partial r}\right) + N,$$

where S is the storage coefficient, T is transmissivity, h is head, t is time, r is radial distance, and N is recharge rate.

45. Groundwater flow equation in radial coordinates for an isotropic, unconfined aquifer with essentially horizontal flow and recharge:

$$S_y\frac{\partial h}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rKh\frac{\partial h}{\partial r}\right) + N,$$

where S_y is specific yield, K is hydrauilc conductivity, h is head, t is time, r is radial distance, and N is recharge rate.

46. Groundwater flow equation in radial coordinates for a homogeneous, isotropic, confined aquifer with essentially horizontal flow:

$$\frac{S}{T}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right),$$

where S is the storage coefficient, T is transmissivity, h is head, t is time, and r is radial distance.

47. Theis equation:

$$s = h_o - h = \frac{Q}{4\pi T}W(u),$$

where s is drawdown, h_o is initial head, Q is the pumping rate ($Q > 0$ for pumping), T is transmissivity, $u = r^2S/(4Tt)$, S is storage coefficient, t is time, and $W(u)$ is the well function (exponential integral) given by

$$\begin{aligned} W(u) &= \int_u^\infty \frac{e^{-a}}{a}\,da \\ &= -0.5772 - \ln u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} \pm \ldots \end{aligned}$$

48. Cooper-Jacob approximation:

$$s = h_o - h = \frac{2.3Q}{4\pi T}\log\left(\frac{2.25Tt}{r^2S}\right).$$

Using parameters of the best-fit straight line from a semilog plot of drawdown vs. time, the aquifer parameters are given by $T = 2.3Q/(4\pi\Delta s)$ and $S = 2.25Tt_0/r^2$, where Δs is drawdown over one log cycle and t_0 is the time axis intercept of the straight line.

49. Neuman solution for drawdown in a phreatic aquifer at early and late times, but not during the transitional period:

$$s = \frac{Q}{4\pi T} W(u) \ ,$$

where $u = r^2 S/(4Tt)$ for early time and $u = r^2 S_y/(4Tt)$ for late time.

50. Hantush-Jacob equation for drawdown in a leaky aquifer with no aquitard storage:

$$s = h_o - h = \frac{Q}{4\pi T} W'(u, r/B),$$

where $B = \sqrt{Tb'/K'}$ and b' and K' are the aquitard thickness and hydraulic conductivity, respectively.

51. Drawdown due to pumping in multiple wells:

$$s = h_o - h = \sum \frac{Q_i}{4\pi T} W(u_i),$$

where $u_i = r_i^2 S/(4Tt)$.

52. Drawdown due to pumping near an impermeable boundary:

$$s = h_o - h = \frac{Q}{4\pi T} \left[W\left(\frac{r^2 S}{4Tt}\right) + W\left(\frac{r'^2 S}{4Tt}\right)\right],$$

where r and r' are the distances from the point of interest to the real and image wells, respectively.

53. Drawdown due to pumping near a constant head boundary:

$$s = h_o - h = \frac{Q}{4\pi T} \left[W\left(\frac{r^2 S}{4Tt}\right) - W\left(\frac{r'^2 S}{4Tt}\right)\right],$$

where r and r' are the distances from the point of interest to the real and image wells, respectively.

54. Drawdown due to variable pumping rates in one well:

$$s = h_o - h = \frac{Q_o}{4\pi T} W(u) + \sum \frac{\Delta Q_i}{4\pi T} W(u_i),$$

where $\Delta Q_i = Q_i - Q_{i-1}$ and

$$u_i = \begin{cases} 0 & t \leq t_i \\ \frac{r^2 S}{4T[t-t_i]} & t > t_i \end{cases}$$

55. Maximum width of a single-well capture zone:

$$y_{\max} = \frac{Q}{2Ti},$$

where Q is the pumping rate, T is transmissivity, and i is the hydraulic gradient under ambient conditions.

56. Location of stagnation point of a single-well capture zone:

$$x_o = \frac{-Q}{2\pi Ti} \qquad y_o = 0$$

and where the coordinates of the well are $(0, 0)$.

57. Coordinates of the boundary of a single-well capture zone:

$$x = \frac{-y}{\tan(2\pi Tiy/Q)},$$

where tan() is in radians and the coordinates of the well are $(0, 0)$.

58. Maximum width of a dipole capture zone:

$$y_{\max} = \frac{Q}{\pi Ti}\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{y_{\max}}{L}\right)\right],$$

where Q is the pumping rate, i is the hydraulic gradient under ambient conditions, T is transmissivity, L is half the distance between the two wells, and the angle of the inverse tangent is in radians. Note this this equation must be solved iteratively. A good starting guess is $y_{\max} = x_o - L$.

59. Location of the stagnation points of a dipole capture zone:

$$x_o = \pm\sqrt{L^2 + \frac{QL}{\pi Ti}} \qquad y_o = 0$$

where the well locations are $(\pm L, 0)$.

60. Coordinates of the boundary of a dipole capture zone:

$$x = \pm\sqrt{L^2\left(1 + \frac{1}{\beta^2}\right) - \left(y - \frac{L}{\beta}\right)^2}$$

for $y \neq 0$, where $\beta = \tan(2\pi Tiy/Q)$, tan() is in radians, the well locations are $(\pm L, 0)$, and $x = x_o$ for $y = 0$.

61. Advective mass flux:

$$f_a = qC = n_m v C$$

where q is specific discharge, C is volume concentration (mass of solute per volume of water), n_m is mobile porosity, and v is groundwater velocity.

62. Dispersive mass flux (for velocity in x direction only):

$$\begin{aligned} f_{dx} &= -n_m D_L \frac{\partial C}{\partial x} \\ f_{dy} &= -n_m D_T \frac{\partial C}{\partial y} \\ f_{dz} &= -n_m D_{TV} \frac{\partial C}{\partial z}, \end{aligned}$$

where D_L, D_T, and D_{TV} are the longitudinal, transverse, and transverse vertical dispersion coefficients, respectively.

63. Diffusive mass flux:

$$f_m = -n_m \tau d_m \nabla C,$$

where τ is tortuosity ($\tau \approx n_m^{1/3}$) and d_m is the molecular diffusion coefficient in free water.

64. First-order decay rate:

$$\lambda = \frac{\ln 2}{t_{1/2}} \ ,$$

where $t_{1/2}$ is the half-life.

65. Linear sorption isotherm:

$$\overline{C} = K_d C \ ,$$

where $\overline{C}$ is the chemical concentration in the sorbed phase [M solute/M solid], K_d is the distribution coefficient [volume/mass], and C is the aqueous concentration.

66. Retardation coefficient, R:

$$R = 1 + \frac{\rho_b K_d}{n_m}$$

where ρ_b is bulk density of the aquifer material.

67. Advection Dispersion Reaction Equation (ADRE) for transport of a solute in a two-dimensional porous medium with groundwater flow in the x direction:

$$R\frac{\partial C}{\partial t} = -\frac{\partial}{\partial x}(v_x C) + \frac{\partial}{\partial x}\left(D_L \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_T \frac{\partial C}{\partial y}\right) - R\lambda C \ .$$

68. Solution to the 1-D ADRE for an instantaneous point source of a solute in an infinite domain:

$$C(x,t) = \frac{M}{An_m}\frac{1}{\sqrt{4\pi D_L Rt}}\exp\left\{-\frac{(R[x-x_o]-v_x t)^2}{4D_L Rt}\right\}\exp\{-\lambda t\} \ ,$$

where M is the source mass, x_o is the source location, n_m is mobile porosity, and A is the cross-sectional area of the one-dimensional aquifer.

69. Solution to the 2-D ADRE for an instantaneous point source of a solute in an infinite domain:

$$C(x,y,t) = \frac{M}{bn_m}\frac{\exp\{-\lambda t\}}{4\pi t\sqrt{D_L D_T}}\exp\left\{-\frac{(R[x-x_o]-v_x t)^2}{4D_L Rt}\right\}\exp\left\{-\frac{(R[y-y_o])^2}{4D_T Rt}\right\} \ ,$$

where M is the source mass, (x_o, y_o) is the source location, n_m is mobile porosity, b is the aquifer thickness, and D_L and D_T are the longitudinal and transverse dispersion coefficients. Groundwater is assumed to be flowing in the x direction only.

70. Mass of aqueous solute:

$$M_A = \int_x\int_y\int_z n_m C(x,y,z)\,dz\,dy\,dx$$

where $C(x,y,z)$ is the aqueous concentration as a function of x, y, and z, and n_m is mobile porosity.

71. Mass of sorbed-phase solute:

$$M_S = \int_x\int_y\int_z \rho_b\overline{C}(x,y,z)\,dz\,dy\,dx$$

where $\overline{C}(x,y,z)$ is the sorbed-phase concentration as a function of x, y, and z, and ρ_b is bulk density.

72. Relationship between plume size and dispersion coefficient for a conservative solute, i.e., $R=1, \lambda=0$:

$$D_L = \frac{\Gamma_x^2}{11.1t} \qquad\qquad D_T = \frac{\Gamma_y^2}{11.1t} \ ,$$

where Γ_x and Γ_y are the width of the plume in the x and y directions, respectively, between the two points where concentration is $0.5C_{\max}$, $C_{\max}$ is the peak concentration of the plume at time t, and D_L and D_T are the longitudinal and transverse dispersion coefficients, respectively.

73. Solution to the 1-D ADRE for a continuous source of contamination at $x=0$ with linear equilibrium sorption and first-order decay:

$$C(x,t) \approx \frac{C_o}{2}\exp\left(-\frac{\lambda Rx}{v_x}\right)\left[\operatorname{erfc}\left(\frac{Rx-v_x t}{\sqrt{4D_L Rt}}\right) - \exp\left\{\frac{v_x x}{D_L}\right\}\operatorname{erfc}\left(\frac{Rx+v_x t}{\sqrt{4D_L Rt}}\right)\right] \ .$$

74. Solution to the 2-D ADRE for a continuous source of contamination at $x = 0$ with linear equilibrium sorption and first-order decay:

$$C(x,y,t) \approx \frac{C_o}{2}\exp\left\{-\frac{\lambda R x}{v_x}\right\}\left[\mathrm{erfc}\left(\frac{Rx - v_x t}{\sqrt{4D_L R t}}\right) - \exp\left\{\frac{v_x x}{D_L}\right\}\mathrm{erfc}\left(\frac{Rx + v_x t}{\sqrt{4D_L R t}}\right)\right]\mathrm{erfc}\left(\frac{Ry}{\sqrt{4D_T R t}}\right)$$

$$\lim_{t\to\infty} C(x,y,t) = C_o \exp\left\{\frac{x}{2D_L}\left(v_x - \sqrt{v_x^2 + 4\lambda D_L R}\right)\right\}\exp\left\{-y\sqrt{\frac{\lambda R}{D_T}}\right\}.$$