Adam Christiansen
CS 360 Lab 5
Queuing Theory

Part 1: Queuing Theory

1. You are entering the Marriott Center to watch a basketball game. There is only one ticket line to purchase tickets. Each ticket purchase takes an average of 18 seconds. The average arrival rate is 3 persons/minute.
   Find the average length of queue and average waiting time in queue assuming M/M/1 queuing.

   n = 1
   mu (m) = 1/18 persons / sec
   lambda (l) = .05 persons/sec

   p = l/m = .05/(1/18) = 0.9
   average queue length = p^2/(1-p) = .9^2/ (1-.9) = 8.1 people
   average wait time = p/(m – l) = .9/((1/18)-(.05)) = 162 seconds

2. You are now in line to get into the arena. There are 3 operating turnstiles with one ticket-taker each. On average it takes 3 seconds for a ticket-taker to process your ticket and allow entry. The average arrival rate is 40 persons/minute.
   Find the average length of queue, average waiting time in queue assuming M/M/N queuing.
   What is the probability of having exactly 5 people in the system?

   n = 3
   m = 1/3 persons / sec
   l = 2/3 persons / sec

   p = l/m = (1/3) / (2/3) = 2
   P0 = 1/9
   average length of queue = ((P0*p^(n+1))/n!n)(1/(1-(p/n))^2) = .8889 people = Q
   average wait time = ((p+Q)/l) – (1/m) = 2.8889(3/2) – 3 = 1.333 seconds = w
   probability of exactly 5 people in system = P(5) = (p^5P0)/(n^(5-n)n!) = 0.06584

3. You are now inside the arena. They are passing out Cosmo the Cougar bobblehead dolls as a free giveaway. There is only one person passing these out and a line has formed behind her. It takes her exactly 6 seconds to hand out a doll and the arrival rate averages 9 people/minute.
   Find the average length of queue, average waiting time in queue, and average time spent in the system assuming M/D/1 queuing.

   n = 1
   m = 1/6 persons / sec
   l = 0.15 persons / sec

   p = l / m = (1/6)/.15 = 0.9
   average length of queue = p^2 / 2(1-p) = .9^2 / 2(.1) = 4.05 people
   average wait time = (1/(2m))(p/(1-p)) = (1 / (1/3))(.9/.1) = 27 seconds

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average time spent in system = (1/(2m))((2-p)/(1-p)) = (1/(1/3))((2-.9)/(1-.9)) = 33 seconds

Part 2: Queueing Analysis

We collected data on the service rate for two web servers, lighttpd and myServer, in order to find the average response time for both servers. To do this, we wrote a python script that had each server handle a single request (downloading file000.txt) and measured the time to do so. By measuring one request at a time we eliminated any delay that would be caused by queuing. We repeated this process 100 times and took the average to find both server's average response time. This data can be found in times.txt. Mu = 1/average resonse time. Next, we made a graph showing the theoretical response time for any utilization. Since utilization = lambda / mu, we calculated our x values by varying lambda from 0 to mu and dividing by mu. We calculated our y values using the average time in the system equation, t = 1 / (mu – lambda). We then made a plot of these values for both myServer (Fig. 1) and lighttpd server (Fig. 2) which can be seen below.

From these graphs, Fig. 1 and Fig. 2, we can see that as our lambda (arrival rate) approaches our mu (service rate) the time in the system increases exponentially. This is an expected result since the more clients who are making obviously implies that the wait time for a response increases. This is true for both myServer and the lighttpd server.

Part 3: Performance Evaluation

We wanted to test both servers to see how their actual response times compared with the theoretical response times we calculated earlier. To test the servers we used the provided workload generator python script to send 10 different loads at each server for 30 seconds. We then stored the output in several text files; for example, web-lighttpd-1.txt, web-myServer-1.txt refer to 1 client per second, for 30 seconds, accessing lighttpd server and myServer. After testing each server and storing the data we then made 2 graphs, one for each server. On each graph we plotted the theoretical line from part 2 and also box plots of the actual response time from the data we collected from the workload generator. See Fig. 3 and Fig. 4 for the actual response times for myServer and lighttpd server respectively. From these graphs, we can see that both servers followed the theoretical curve quite well, a gradual exponential curve upward as load size increased. Though the actual response times tend to be higher than the theoretical expected values, the box plots fit the curve nicely.

Conclusion:

After graphing the theoretical and actual response times for 2 different servers (Fig. 3 and Fig. 4), we can see that both servers perform at a very close rate to what we expected. Fig. 3 shows myServer which has a nice exponential curve upward as the load increases. Fig. 4 shows lighttpd server which also follows the same nice curve. From the data it is hard to tell if one of the servers more closely follows the expected response time. Perhaps lighttpd server does a little better but furhter testing would be necessary to fully determine.

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Fig. 1 Theoretical response time for myServer

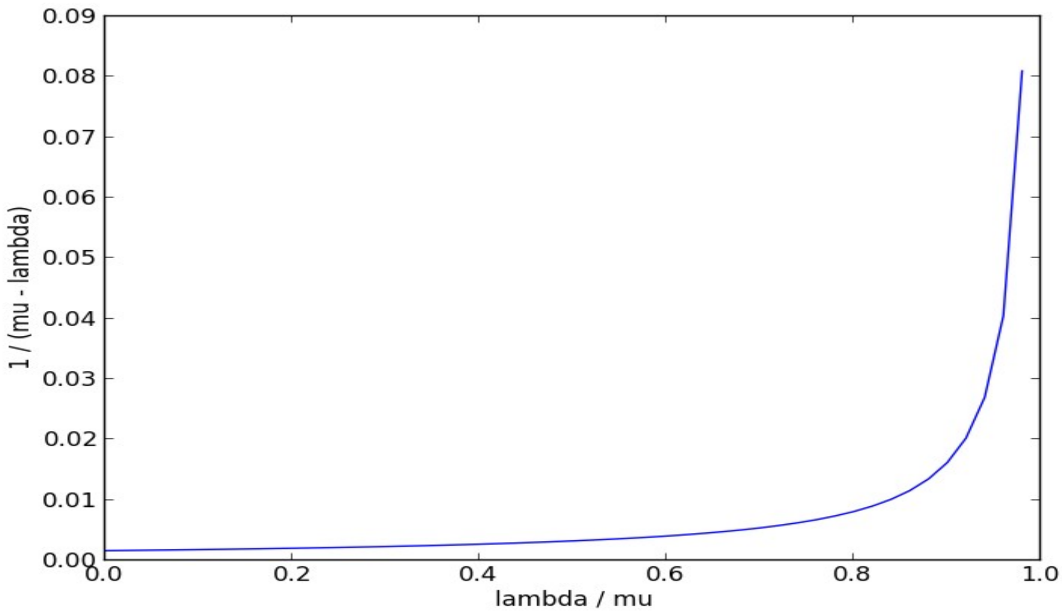

Fig. 2 Theoretical response time for lighttpd server

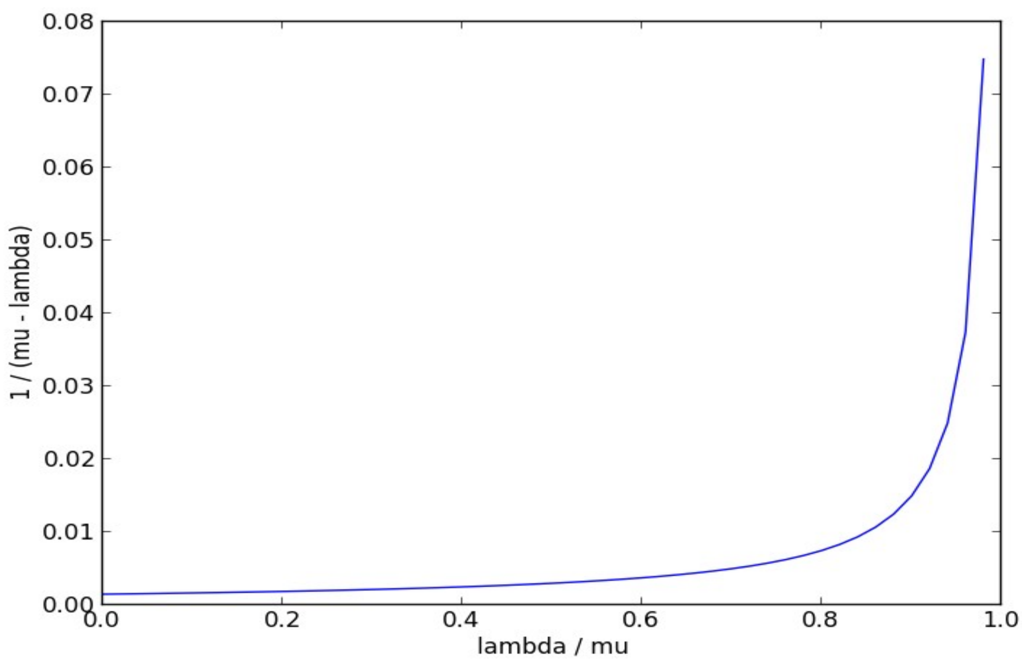

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Fig. 3 Actual response time for myServer

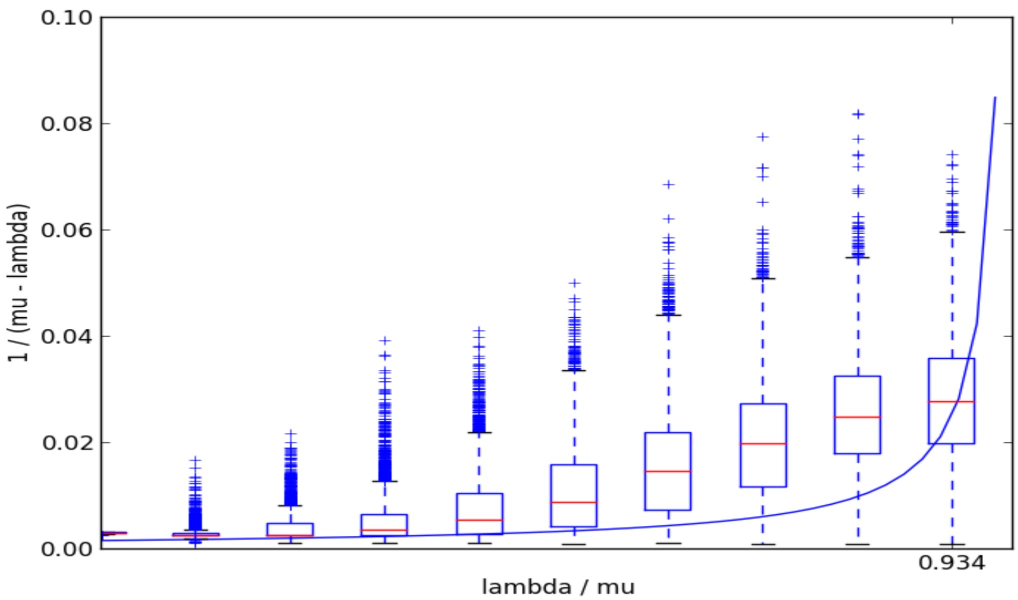

Fig. 4 Actual response time for lighttpd server