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EXTENDING THE MACHO SEARCH TO $\sim 10^6\ M_\odot$

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ABSTRACT

The search for a microlensing (changing light-curve) signature of massive compact halo objects (Machos) by the Macho Collaboration is currently believed to be sensitive in the range 10^{-7}–$10^2\ M_\odot$. Microlensing events from higher mass objects last longer than the 4 yr duration of the planned experiments and therefore, according to current beliefs, cannot be distinguished from long-term variables. In fact the signature of Machos in the range 10^2–$10^3\ M_\odot$ can be distinguished from background events by the annual modulation in light magnification induced by the Earth's motion. For Machos in the range 10^3–$10^6\ M_\odot$, *Hubble Space Telescope* (*HST*), or even ground-based measurements can resolve the split lensed images, thus confirming the lens interpretation of an event. If the *HST*'s optics were repaired, it could resolve images for Machos $\gtrsim 300\ M_\odot$. The lower mass limit can be reduced to $4 \times 10^{-9}\ M_\odot$ by conducting 1 month of rapid repeat observations of a single field. The standard view is that a Macho light curve yields only one physically relevant parameter, the time scale of the event. The time scale is a combination of the four parameters one would like to know: the mass, the distance, and the two components of transverse velocity of the Macho. I show that for masses 4–100 $M_\odot$, annual parallax oscillations in the light curve can be used to determine the transverse velocity. In the range 10^{-3}–$10^6\ M_\odot$ such measurements can be made using a small special-purpose satellite telescope. For masses 10^3–$10^6\ M_\odot$, one may determine all four Macho parameters by combining a number of techniques.

Subject headings: astrometry — Galaxy: halo — gravitational lensing — techniques: photometric

1. INTRODUCTION

Paczyński (1986) suggested that if the dark matter were in massive compact halo objects (Machos), it could be detected by the microlensing of stars in the Large Magellanic Cloud (LMC). The optical depth to microlensing, τ, is independent of the mass distribution of the Machos, but does depend somewhat on their spatial distribution. I will use a reasonably conservative estimate of $\tau \sim 5 \times 10^{-7}$ based on an isothermal halo truncated at 30 kpc (Griest 1991). Thus if N stars are observed, then $\sim N\tau$ of them should be microlensed at any given time. By definition, an object is said to be microlensed if its position relative to the lens, $\boldsymbol{\theta}_S$, lies inside the Einstein ring, θ_*. That is, $x < 1$, where

$$\boldsymbol{x} \equiv \frac{\boldsymbol{\theta}_S}{\theta_*}\,; \quad \theta_*^2 \equiv \frac{4GM}{Dc^2}\,; \quad D \equiv \frac{D_{\rm OL} D_{\rm OS}}{D_{\rm LS}}\,. \tag{1.1}$$

Here M is the mass of the lens and $D_{\rm OL}$, $D_{\rm LS}$, and $D_{\rm OS} \equiv D_{\rm LMC}$ are the distances between the observer, lens, and source. The magnification of the combined images, A, and its logarithmic gradient are given by

$$A(x) = \frac{x^2 + 2}{x(x^2 + 4)^{1/2}}\,; \quad \nabla \ln A = \frac{-8\hat{\boldsymbol{x}}}{x(x^2 + 2)(x^2 + 4)}\,. \tag{1.2}$$

A microlensing event is recognized not by the magnification alone, but by the characteristic light curve induced by the *changing* magnification as the lens moves across the line of sight. This light curve rises smoothly, peaks parabolically, and is time-reversal symmetric. It is also achromatic. These characteristics distinguish genuine microlenses from variable stars and other backgrounds.[1] If the time scale of a microlensing event, ω^{-1}, is longer than the duration of the observations, T, then the light curve becomes a less convincing signature of microlensing. Thus, even if the expected fragment of a light curve were clearly detected, it could not be conclusively distinguished from the backgrounds. Here,

$$\boldsymbol{\omega} \equiv \frac{\boldsymbol{v}}{D_{\rm OL}\,\theta_*}\,, \tag{1.3}$$

where $\boldsymbol{v}$ is the transverse velocity of the microlens relative to the line joining the Sun and the LMC source. The typical expected parameters entering equation (1.3) are $v \sim 200$ km s^{-1} (from the two-dimensional velocity dispersion characteristic of the Galactic potential) and $D_{\rm OL} \sim 10$ kpc (from the peak in the lensing cross section toward the LMC for an r^{-2} density profile). For these values, the time scale, $\omega^{-1} \sim 0.2(M/M_\odot)^{1/2}$ yr. It has therefore been believed that the $T = 4$ yr observations planned by the Macho Collaboration (Alcock

[1] Variable stars are the main anticipated background. The total fraction of stars which vary enough to be potentially confused with a lensing event is not well known but is estimated $\lesssim 10^{-4}$. (A systematic variable catalog will be an important secondary result of the experiment.) Thus, if none of the variables were excluded, they would overwhelm the fraction of lensing events ($\lesssim 10^{-6}$). On the other hand, if all of the variables were excluded, this would reduce the number of source stars only negligibly. Therefore, the proper strategy is to exclude all variables, even though in some cases it might be possible to recover a lensing signature of a variable by subtracting the measured intrinsic variation. Long-term variables are the main background for lensing events which last longer than the experiment. In this case, the intrinsic variation cannot be measured independently, and it is absolutely essential that these stars be rigorously excluded. Another source of background is measurement variations due to crowded fields and variable seeing. The problem of removing this background for long-term events is addressed in § 10.

et al. 1992) would be insensitive to Machos of Mass $M \gtrsim 100\ M_\odot$. This is an important limitation because Machos of mass up to $\sim 10^6\ M_\odot$ make plausible dark matter candidates (Lacey & Ostriker 1985).

For masses $M \lesssim 10^{-7}\ M_\odot$, the Einstein ring falls below the angular extent of an LMC star. The maximum magnification of an A or a B star (assumed to have a radius $R \sim 3\ R_\odot$) is then

$$A_{\max} = \left(1 + \frac{16GM}{c^2}\frac{D_{\rm LS}D_{\rm OS}}{R^2 D_{\rm OL}}\right)^{1/2} \sim 1 + 1.5\,\frac{M}{10^{-7}\ M_\odot} \quad (1.4)$$

for a lens at $D_{\rm OL} \sim 10$ kpc. This small maximum magnification then sets the lower limit of sensitivity.

If either of the two observation programs currently being undertaken (Alcock et al. 1992; Aubourg et al. 1991) do detect microlenses, it will be a spectacular success: at least one component of the dark matter will have been basically identified. However, there will remain the problem of sorting out the mass, velocity, and spatial distributions of these objects. Measurement of these distributions will be crucial to pinpointing the nature of the dark matter and to understanding the origin and the structure of the Galaxy. It will not be easy to disentangle these three distributions from the catalog of measured lensing events. Of the four Macho parameters that one might hope to measure, the mass, M, the distance, $D_{\rm OL}$, and the two components of the transverse velocity, $\boldsymbol{v}$, only a specific combination, the time scale,

$$\omega^{-1} = \frac{[4GMD_{\rm OL}(1 - D_{\rm OL}/D_{\rm LMC})]^{1/2}}{vc}\,, \quad (1.5)$$

can be measured. For any particular event, it is impossible to say anything about the parameters individually. The hope is that with a sufficiently large number of events one could make a statistical statement about the Macho mass, velocity, and spatial distributions. However, the total expected number of events, $N_{\rm events}$, is only

$$N_{\rm events} \sim N\tau\left(1 + \frac{2}{\pi}\,\omega T\right) \sim 5\left[1 + \left(\frac{M}{200\ M_\odot}\right)^{-1/2}\right], \quad (1.6)$$

for an assumed $N \sim 10^7$ stars observed. The first term comes from the stars which initially lie within an Einstein ring of some lens, and the second comes from the stars which pass into an Einstein ring after the observations commence. Thus if the Machos prove to be relatively heavy, the statistics will be poor.

In this paper, I remove several of these limitations. In § 2, I show that candidate lensing events can be identified provided $M \lesssim 10^6\ M_\odot$. In §§ 3 and 4, I show that these candidates can be distinguished from variable stars on the basis of their annual parallax-induced modulation, or by resolving their split images. In §§ 4–8, I discuss several methods for determining additional parameters of Machos after they have been initially identified. As mentiond above, such measurements would help pinpoint the nature of the dark matter and enhance our understanding of the origin and structure of the Galaxy. For Machos 10^3–$10^6\ M_\odot$, one may determine three parameters, $\boldsymbol{\omega}$ (two components) and θ_*, by combining the basic time-series observations with high-resolution imaging (§ 4). For masses 4–100 $M_\odot$, parallax-induced oscillations can be used to measure two additional parameters, effectively the two components of transverse velocity (§ 5). This technique can be extended to the range 10^{-3}–$10^6\ M_\odot$ by using a relatively small, space-based telescope (§ 6). For masses, 10^3–$10^6\ M_\odot$, all four parameters can be measured by combining space-based

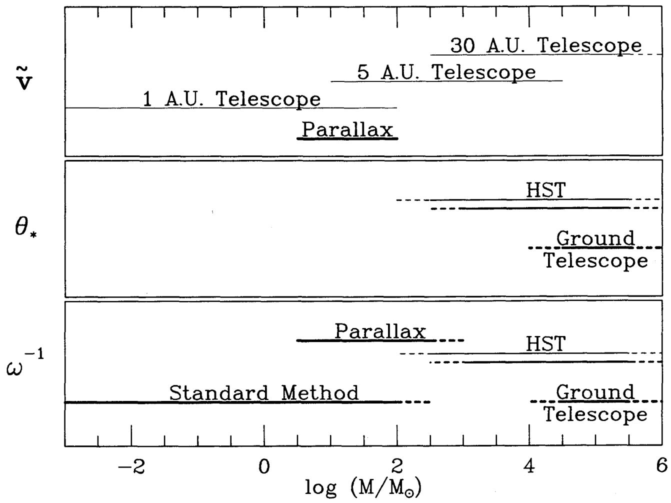

FIG. 1—Summary of results for Machos 10^{-3}–$10^6\ M_\odot$. Using the standard method of light curve analysis, only Machos $\lesssim 100\ M_\odot$ can be detected, and only one parameter (the time scale, ω^{-1}, eq. [1.5]) can be measured. Using the parallax method and ground and *HST* measurements (with or without repairs), these detections and time-scale measurements can be extended to $\sim 10^6\ M_\odot$ (§§ 3 and 4). For heavier Machos, the same ground and *HST* measurements yield a second parameter (the Einstein radius, θ_*, eq. [1.1]). Two additional parameters (the reduced transverse velocity, $\tilde{\boldsymbol{v}}$, eq. [5.1]) can be found directly from the observations for Machos 4–100 $M_\odot$ (§ 5). For other masses, $\tilde{\boldsymbol{v}}$ can be determined from observations by a satellite telescope in solar orbit (§§ 6 and 7). Line thickness represents the relative accessibility of various data. The "Standard Method," "Parallax," and "Ground Telescope" data can all be recovered from the basic experiment or using readily available instruments. At the other extreme, a solar-orbit telescope would be a major undertaking. Dashed extensions of solid lines show mass ranges where a given method goes from being robust to marginal.

observations with high-resolution imaging (§ 7). If any of the lensed stars are found to be double spectroscopic binaries, these may be used to measure or constrain the mass and distance of the lensing Macho (§ 8). In § 9, I show that, in spite of equation (1.4), one may detect microlenses as small as $4 \times 10^{-9}\ M_\odot$ by using rapidly repeated observations. Thus the effective range of current observation programs is potentially more than 14 orders of magnitude. In § 10, I discuss some possible systematic effects. In § 11, I briefly consider parallax measurement of objects in the direction of the Galactic bulge, and in § 12, I summarize my conclusions.

Many of the results of this paper are summarized in Figure 1.

2. MICROLENS CANDIDATES $M \gtrsim 100\ M_\odot$

The limit of sensitivity of the present observation programs is generally believed to be $M \lesssim 100\ M_\odot$. Above this mass, the Einstein ring is so large that a microlensing event takes longer than the planned 4 yr span of the observations. This has two bad effects. First, it means that the only microlensing events which take place during the observations are the $N\tau \sim 5$ which are in the process of occurring as the observations commence. Second, only a partial light curve is available. Indeed, as the Machos get heavier, only a straight-line segment of the light curve might be seen. Since the background of long-term variables is unknown, there has appeared to be no way of distinguishing these five or so events from the background.

In this section, I show that microlens candidates $\lesssim 10^6\ M_\odot$ can be distinguished from random noise. In §§ 3 and 4, I show that these candidates can then be "confirmed," i.e., distinguished from variable stars, on the basis of parallax measurements and/or high-resolution imaging.

The task of identifying candidates becomes progressively more difficult as the Machos get heavier, and so the characteristic time, ω^{-1}, gets longer. To establish the highest detectable mass, I therefore work in the limit of very long events, $\omega^{-1} \gg 4$ yr. In this limit, perturbations due to the Earth's orbit are completely negligible and the equation for the lensing parameter, $\boldsymbol{x}$, can be written,

$$\boldsymbol{x}_{\odot}(t) = \boldsymbol{\beta} + \boldsymbol{\omega}(t - t_0) , \tag{2.1}$$

where t_0 is the closest approach, $\boldsymbol{\omega}$ is the vector event rate (eq. [1.3]), and $\boldsymbol{\beta}$ is the normalized impact parameter,

$$\boldsymbol{\beta} \equiv \frac{\boldsymbol{b}}{D_{\mathrm{OL}}\,\theta_*} . \tag{2.2}$$

Here $\boldsymbol{b}$ is the minimum physical distance between the Macho and the source-Sun line of sight. The light curve, $m(t)$, is given by

$$m(t) = -2.5 \log A[x(t)] + m_s , \tag{2.3}$$

where m_s is the unmagnified stellar magnitude. For $\omega^{-1} \gg 4$ yr, $m(t)$ can be approximated as a straight line

$$m(t) = m(t_c) - \frac{2.5}{\ln 10}(t - t_c)\boldsymbol{\omega} \cdot \boldsymbol{\nabla} \ln A(x_c) , \tag{2.4}$$

where x_c and t_c are evaluated at the midpoint of the observations. The intrinsic variance of the light curve is then

$$\langle m(t)^2 \rangle - \langle m(t) \rangle^2 = \frac{1}{12}\left(\frac{2.5}{\ln 10}\right)^2 [T\boldsymbol{\omega} \cdot \boldsymbol{\nabla} \ln A(x_c)]^2 , \tag{2.5}$$

where T is the length of observations and where the logarithmic gradient is given by equation (1.2). If the measurements are made at a rate $\dot{N}_{\mathrm{obs}}$ and with an accuracy σ_m, then the random variance in the measurement is $\sigma_m^2/(\dot{N}_{\mathrm{obs}}\,T)$. Hence, the slope of the light curve can be measured with fractional accuracy Q^{-1}, where

$$Q = \frac{(2.5/\ln 10)T\,|\boldsymbol{\omega} \cdot \boldsymbol{\nabla} \ln A(x_c)|/12^{1/2}}{\sigma_m/(\dot{N}_{\mathrm{obs}}\,T)^{1/2}} . \tag{2.6}$$

What minimum value of Q is required to select candidates while screening out spurious events generated by random noise? To address this and all future quantitative questions, I assume that the preliminary estimates ($N = 10^7$ stars observed; 55% good weather; 8 months per year; 4 yr of observations; two bands; $\sigma_m = 0.08$ [D. Bennet, K. Freeman, K. Griest, private communications]) regarding the Macho Collaboration experiment (Alcock et al. 1992) will prove accurate.[2] Suppose that only events with $Q > Q_{\min}$ are considered candidates. The probability of random fluctuations generating this large a signal is $\sim\exp(-Q_{\min}^2/2)(2/\pi)^{1/2}/Q_{\min}$ or 6×10^{-7} for $Q_{\min} = 5$, and hence there would be about six spurious events for $N = 10^7$ stars observed. (Note that the individual magnitude measurement variations are far from Gaussian, but their joint effect is Gaussian by the central limit theorem.) This is an acceptable level for a first cut and is in any event likely to be exceeded by nonrandom backgrounds. Using $Q_{\min} = 5$, and $\dot{N}_{\mathrm{obs}} = 265\ \mathrm{yr}^{-1}$, I find from equations (1.2) and (2.6) that a candidate will be recognized provided that

$$\frac{x}{\hat{\boldsymbol{\omega}} \cdot \hat{\boldsymbol{x}}}\left(1 + \frac{x^2}{4}\right)\left(1 + \frac{x^2}{2}\right) < 100\omega \text{ yr} . \tag{2.7}$$

From this equation, it is clear that most events with time scales $\lesssim 100$ yr will become candidates. For longer time scales, only events with low radius, $x < x_{\max}$, will be recognized:

$$x_{\max} \sim 0.47\left(\frac{M}{10^6\,M_{\odot}}\frac{D}{10\text{ kpc}}\right)^{-1/2}\frac{|\tilde{v}_r|}{200\text{ km s}^{-1}} , \tag{2.8}$$

where $\tilde{v}_r$ is the component of $\tilde{\boldsymbol{v}}$ along the radius of the Einstein ring. Thus, potential candidates begin to be lost due to low radial velocities ($\tilde{v}_r \lesssim 200$ km s^{-1}) and high radial positions ($x \sim 1$) for masses $\gtrsim 10^5\,M_{\odot}$. For $M \gtrsim 10^6\,M_{\odot}$ very few candidates will be detected. However, Machos of mass $M > 10^6\,M_{\odot}$ are ruled out by the stability of the Galactic disk (Lacey & Ostriker 1985).

I emphasize that in arriving at these estimates, I have incorporated the expected experimental parameters. These enter through Q in equation (2.6). If Q were 33% smaller (say, because the true errors were $\sigma_m = 0.12$ rather than 0.08), then the prefactor in equation (2.8) would likewise be 33% smaller.

3. PARALLAX CONFIRMATIONS

Some canditates may turn out to be microlensing events, but others will be long-term variable stars. One way of distinguishing the two is that a light curve caused by lensing will oscillate on annual scales due to motion of the Earth. The lensing parameter at the position of the Earth, $\boldsymbol{x}$, is perturbed slightly from the one at the Sun, $\boldsymbol{x}_{\odot}(t) = \boldsymbol{\beta} + \boldsymbol{\omega}(t - t_0)$ (eq. [2.1]). Its equation is therefore

$$\boldsymbol{x}(t) = \boldsymbol{x}_{\odot}(t) + \epsilon\{\hat{\boldsymbol{\beta}} \cos[\Omega(t - t_0) + \phi] + \Lambda\hat{\boldsymbol{\omega}} \sin[\Omega(t - t_0) + \phi]\} ; \quad \epsilon \equiv \frac{a_{\oplus}}{\tilde{r}_e} , \tag{3.1}$$

where $a_{\oplus}$ is an astronomical unit, $\Omega = 2\pi$/yr, ϕ is the phase of the Earth's orbit relative to $\hat{\boldsymbol{\beta}}$ at $t = t_0$, and $\tilde{r}_e$ is the "reduced" Einstein ring radius,

$$\tilde{r}_e \equiv \theta_* D = \left(\frac{4GMD}{c^2}\right)^{1/2} . \tag{3.2}$$

The parity, $\Lambda \equiv -\hat{\boldsymbol{j}} \cdot (\hat{\boldsymbol{\beta}} \times \hat{\boldsymbol{\omega}}) = \pm 1$, where $\hat{\boldsymbol{j}}$ is the unit vector toward the North eclitic pole. (In writing eq.[3.1.]), I have for simplicity assumed that the parallax ellipse is a circle, i.e., that the LMC lies exactly at the South ecliptic pole. In fact, the LMC lies $\sim 3°$ from the pole, so that the axis ratio of the ellipse is ~ 0.999.) The light curve is obtained by substituting equation (3.1) into equation (1.2). A sample light curve is shown in Figure 2, both with (*dashes*) and without (*solid*) correction for the Earth's motion. The light curve has parameters $\beta = 0.2$, $\omega^{-1} = 6$ yr, $\phi = 20°$, $\epsilon \sim 0.0032$, and $D_{\mathrm{OL}} = 10$ kpc, which correspond to $v \sim 200$ km s^{-1} and $M \sim 10^3\,M_{\odot}$.

[2] Actually, the magnitude error σ_m will vary from star to star, and from night to night for a given star. In the interest of simplicity, I use in this paper a single value for the errors. With regard to the night-by-night variations, this is not actually a problem. The night-by-night variations are due primarily to the seeing and to the phase of the Moon. Hence, there should be little long-term systematic variation in the errors. This means that for each star the quantity $\dot{N}_{\mathrm{obs}}\,T\sigma_m^{-2}$, which appears in many formulae, can simply be replaced by $\sum_i \sigma_{m,i}^{-2}$, where $\sigma_{m,i}$ is the error of the ith observation. The star-by-star variations are a function primarily of magnitude, but also of crowding. These do complicate the analysis. Roughly 3 million stars will have $\sigma_m < 0.05$, while some of the stars may have errors as large as 15%. Within the full group of $N \sim 10^7$ stars, one may define subgroups, each with approximately the same error. The sensitivity of the survey is then determined by the sum of the sensitivities of the subgroups. However, the roughness of the preliminary estimates of the error distribution makes such a detailed approach unwarranted at the present time.

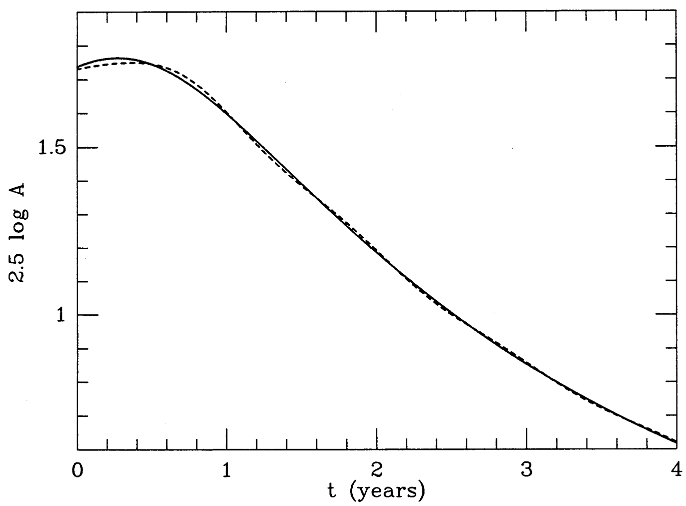

FIG. 2.—Magnification in magnitudes for a lensing event shown both with (*dashes*) and without (*solid*) correction for the Earth's motion. The lens is assumed to have mass $M \sim 10^3\ M_\odot$, distance $D_{OL} \sim 10$ kpc, and transverse speed $v \sim 200\ \mathrm{km\ s^{-1}}$. At maximum magnification, ($x_\odot = \beta = 0.2$) the phase of the Earth's orbit is offset by 20° relative to the projected lens-Sun vector. The Einstein ring is $\sim$300 AU.

The light curve in Figure 2 looks suspiciously like a lensing event. Nevertheless, because the beginning of the "event" comes before the observations, symmetry cannot be used to confirm it as a genuine lensing event. Can the event be confirmed from its annual oscillations?

The answer to this question depends in part on the background. The size of the background can be estimated from the data because one knows a priori that there cannot be more than a dozen or so long-term lenses. If the background is low, the threshold for a "confirmation" can be modest, say 3 σ. If the background is high, one would have to be more conservative.

The amplitude of the sinusoidal oscillations induced by the Earth's motion can be found by substituting equation (3.1) into equation (1.2). The amplitude is: $(2.5/\ln 10)(a_\oplus/\tilde{r}_e)\nabla \ln A(x)$. Since the event is longer than the observations, it is appropriate to estimate the average amplitude from its value at the midpoint of observations, $x_c = x(t_c)$. One knows the period of this oscillation a priori, but not the phase. Hence, the signal-to-noise ratio, Q', is given by

$$Q' = \frac{(2.5/\ln 10)(a_\oplus/\tilde{r}_e)\,|\nabla \ln A(x)|/2}{\sigma_m/(\dot{N}_{\rm obs}\,T)^{1/2}}\,. \qquad (3.3)$$

This leads to a condition on x_c for marginal confirmation of

$$x_c\left(1+\frac{x_c^2}{4}\right)\left(1+\frac{x_c^2}{2}\right) < 0.26\,\frac{3}{Q'_{\rm min}}\left(\frac{M}{10^3\ M_\odot}\,\frac{D}{10\ \mathrm{Kpc}}\right)^{-1/2}, \qquad (3.4)$$

where $Q'_{\rm min}$ is the minimum signal to noise. Thus, although Machos with $M \lesssim 10^5\ M_\odot$ can easily become candidates, those with $M \gtrsim 10^3\ M_\odot$ can be confirmed only if they happen to lie relatively close to the center of the Einstein ring.

In order to estimate the number of confirmable events, I therefore now investigate the distribution of lensed stars, $n(x)$. Let us say that the survey has a magnitude threshold of $m_{\rm max}$, or apparent luminosity threshold, $L_{\rm min}$. If we restrict consideration to those stars whose true luminosity is above this limit, then $n(x) = 2xN\tau$, because a star is equally likely to be found in any part of the Einstin ring area. However, taking account of the fact that the lensing candidate is magnified above its true luminosity, the distribution is

$$n(x) = 2xN\tau\,\frac{\Phi[L > L_{\rm min}/A(x)]}{\Phi(L > L_{\rm min})}\,, \qquad (3.5)$$

where Φ is the cumulative surface density of stars as a function of luminosity. To estimate the enhancement ratio in equation (3.5), I use the Galaxy present luminosity function (Bahcall & Soneira 1980), from which I find $\Phi(L > L_0) \propto L_0^{-2/3}$ for $1 \lesssim M_V \lesssim 4$. I infer

$$n(x) \sim 2xN\tau A(x)^{2/3} \qquad (3.6)$$

for the distribution. Thus, the total number of lensed stars is $1.6N\tau$, rather than the naive estimate, $N\tau$. The integrated number of events interior to $x_{\rm max}$ is

$$\int_0^{x_{\rm max}} dx\, n(x) \sim 1.5N\tau x_{\rm max}^{4/3} \quad (x_{\rm max} \lesssim 1)\,. \qquad (3.7)$$

I return now to equation (3.4). If $N = 10^7$ and $\tau = 5 \times 10^{-7}$, the expected number of stars with $x < 0.26$ is $\sim$1.25 for $M \sim 10^3\ M_\odot$. According to equation (2.7), these should almost certainly have been singled out as candidates. Therefore, at $M \sim 10^3\ M_\odot$, there is a reasonable probability of at least one confirmed lens. (Fig. 2 is an example of a marginally confirmable, i.e., 3 σ event.) For $M \sim 10^4\ M_\odot$, the expected number falls $\ll 1$, making confirmation by parallaxes much more problematical.

4. HIGH-RESOLUTION CONFIRMATIONS

However, there is another route to confirmation for masses 10^3–$10^6\ M_\odot$: the *Hubble Space Telescope* (*HST*). If the candidate is a lens, there wil be two images, split by $\Delta\theta$,

$$\Delta\theta = (4 + x^2)^{1/2}\theta_* \gtrsim 0''.06\left(\frac{M}{10^3\ M_\odot}\right)^{1/2}\left(\frac{D}{10\ \mathrm{kpc}}\right)^{-1/2}, \qquad (4.1)$$

and these will have an amplitude ratio, A_1/A_2,

$$\frac{A_1}{A_2} = \left(\frac{x + \sqrt{x^2+4}}{2}\right)^4; \quad x = \left(\frac{A_1}{A_2}\right)^{1/4} - \left(\frac{A_1}{A_2}\right)^{-1/4}. \qquad (4.2)$$

That is, the images will differ by $\sim$2.1x magnitudes for any position x within the Einstein ring. The core of an image taken with the Faint Object Camera (f/96) on the *HST* has a full-width half-maximum of 0''.07 even with the spherical aberration (Paresce et al. 1991) and would be reduced to $\sim$0''.038 if the optics were repaired. Hence, at $M \sim 10^3\ M_\odot$, the split images could probably be marginally resolved for $x \lesssim 0.6$ and fully resolved for somewhat smaller x. Consequently, there should be about four detectable Machos at $10^3\ M_\odot$. At higher masses, larger values of x would be resolvable.

Thus, with the Macho Collaboration parameters I have adopted, there is marginal overlap near $M \sim 10^3\ M_\odot$ between the two methods of confirmation. If the Macho Collaboration parameters prove less favorable than anticipated, the gap could still be filled by a repaired *HST*, which could reach $\sim$300 $M_\odot$ or perhaps a bit lower.

The resolved images can be used to measure the time scale of the event, even if this time scale is very long compared to the observations. First one would determine x from the difference of the magnitudes of the two images (eq. [4.2]).

Hence, $\nabla \ln A(x)$ would be known. The rate $dm/dt = (2.5/\ln 10)\boldsymbol{\omega} \cdot \nabla \ln A(x)$ would have been measured to fractional accuracy, $Q^{-1} \lesssim 20\%$, as part of selecting the star as a lensing candidate (see eq. [2.6]). Thus, ω_r, the component of $\boldsymbol{\omega}$ in the lens radial direction could be determined.

Next, ω_t, the tangential component of $\boldsymbol{\omega}$ could be determined by a proper motion study of the images. The images would rotate on the sky at an angular rate ω_t/x. They would therefore move relative to one another in the tangential direction at a rate $\omega_t \Delta\theta/x$. By measuring this rate, one would know ω_t. The rate depends on the tangential velocity, v_t, but not directly on the time scale of the event,

$$\frac{\omega_t \Delta\theta}{x} = \frac{(4 + x^2)^{1/2}}{x} \frac{v_t}{D_{\rm OL}}$$
$$\gtrsim 0\rlap{.}''009 \text{ yr}^{-1} x^{-1}\left(\frac{v_t}{200 \text{ km s}^{-1}}\right)\left(\frac{D_{\rm OL}}{10 \text{ kpc}}\right)^{-1} . \quad (4.3)$$

This is an order of magnitude faster than the relative proper motions of LMC stars. Thus, within a few years, one could measure the time scale of even the heaviest-Macho events, even those lasting $\lesssim$200 yr.

In addition, resolution of the split image would immediately yield the Einstein radius, θ_*, from the magnification ratio, the image separation, and equations (4.1) and (4.2). That is, for these heavy Machos, it is relatively easy to obtain three independent parameters, $\boldsymbol{\omega}$ and θ_*.

In this discussion, I have implicitly assumed that resolution of a companion image would establish that the candidate was not a variable star. Several arguments support this assumption. First, most variables are likely to vary in color as well as magnitude and so could be excluded as Macho candidates. Second, assuming $\sim 2.5 \times 10^5$ stars per square degree, the probability of a chance association of a second star of comparable magnitude and any color is $\sim 0.25(\Delta\theta/2'')^2$. The probability that the companion would have identical color is substantially smaller. However, if all these tests failed to be convincing, the reality of the lens could be established by the relative proper motion of the images over $\sim$2 yr, as I discussed above. In addition, if the image splitting were $\gtrsim$4 times the resolution (i.e., $\gtrsim 0\rlap{.}''3$ for the present *HST*), then the lens could be confirmed by the "texture" of a high-resolution image (Turner, Wardle, & Schneider 1990).

As a practical matter, *HST* observations are not generally scheduled immediately after the need for them is recognized. In the present case, the relative urgency of the scheduling is set by the time scale of the lensing events $\omega^{-1} \sim 6 \text{ yr } (M/10^3 M_\odot)^{1/2}$. To measure proper motions, at least two observations separated by a few years are required. Thus, it would be prudent to schedule the initial observations within $\sim$1 yr of the time the event was recognized.

According to equations (2.8) and (3.6), the upper limit for candidate recognition is $\sim 10^6 M_\odot$, where the number of candidates drops to a few. Any such candidates could be confirmed by ground-based resolution of their $\sim 2''$ splittings. Indeed, each image would appear in the program as a separate "star," assuming that the fainter image was brighter than the survey magnitude limit. This is actually quite likely, because for lenses $\sim 10^6 M_\odot$, only those with $x \gtrsim 0.5$ can become candidates in the first place, so that the images differ by $\lesssim$1 mag. Changing light curves in two nearby "stars" with identical colors would be a strong signature of lensing. Further, the time derivatives of two curves should be related in a definite way. That is, from the magnification ratio and equation (4.2) one can infer x. One can then infer a value of dx/dt from the time derivative of each curve separately. These two values should agree.

5. PROPER-MOTION MEASUREMENT BY PARALLAXES

If Machos are discovered, one would like to learn as much as possible about each individual object, in particular, its mass, M, distance, $D_{\rm OL}$, and transverse velocity, $\boldsymbol{v}$. As I discussed in the Introduction, however, the basic light curve reveals only a specific combination of these parameters, the time scale ω^{-1} (see eq. [1.5]). I now turn to the problem of determining some or all of the additional parameters. In § 4, I showed that for $M \gtrsim 10^3 M_\odot$, one could determine three parameters from high-resolution images. In this section, I show that for masses $4 M_\odot \lesssim M \lesssim 100 M_\odot$, the "reduced velocity," $\tilde{\boldsymbol{v}} = (D_{\rm OS}/D_{\rm LS})\boldsymbol{v}$, can be determined directly from the observations. That is, three of the four parameters can also be measured. In § 6, I will show that by augmenting the ground-based observations with those of a satellite telescope, one could determine these three parameters over the range, $10^{-3} M_\odot \lesssim M \lesssim 10^3 M_\odot$. All four parameters could be determined for $10^3 M_\odot \lesssim M \lesssim 10^6 M_\odot$, as I discuss in § 7. Finally, in § 8 I show that if the lensed star happens to be a double spectroscopic binary of an appropriate period, all four parameters can also be measured.

Figure 3 displays a light curve which is reasonably contained within the period of observations. It is shown both with (*dashes*) and without (*solid*) correction for the Earth's motion. The light curve has parameters $\beta = 0.4$, $\omega^{-1} = 2$ yr, $\phi = 60°$, $\epsilon \sim 0.01$, and $D_{\rm OL} = 10$ kpc, which correspond to $v \sim 200$ km s^{-1} and $M \sim 10^2 M_\odot$.

It is apparent from Figure 3 that parallax induces a relatively minor perturbation on the Sun-based light curve. This suggests that, at least conceptually, the analysis of the light profile might be divided into three steps. First determine the four parameters which specify $x_\odot(t)$, (β, ω, t_0, and the unmagnified stellar magnitude, see eqs. [2.1] and [3.1]). These measurements will be quite accurate because the signal is so large.

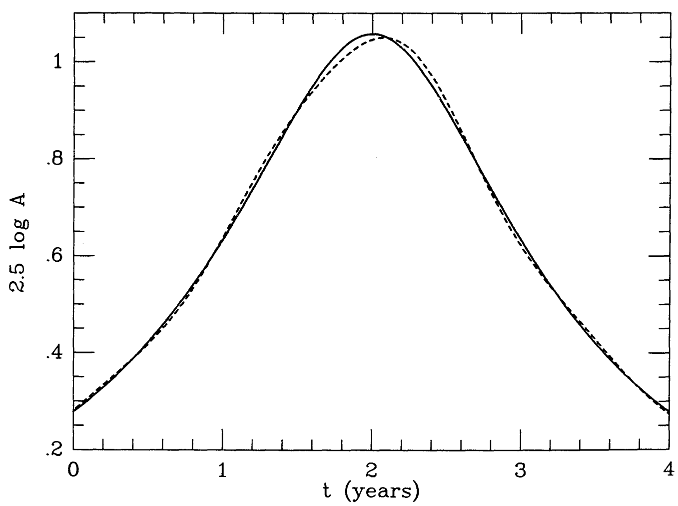

FIG. 3.—Magnification in magnitudes for a lensing event shown both with (*dashes*) and without (*solid*) correction for the Earth's motion. The lens is assumed to have mass $M \sim 10^2 M_\odot$, distance $D_{\rm OL} \sim 10$ kpc, and transverse speed $v \sim 200$ km s^{-1}. At maximum magnification ($x = \beta = 0.4$), the phase of the Earth's orbit is offset by 60° relative to the projected lens-Sun vector. The Einstein ring is 100 AU.

Second, for each value of $\Lambda = \pm 1$, find the best fits and uncertainties of the amplitude (ϵ) and phase (ϕ) of the parallax correction. The fractional uncertainties of these two quantities will be much larger than those of the first four. From ϵ and ω, one may determine the magnitude of the "reduced velocity," $\tilde{\boldsymbol{v}}$,

$$\tilde{v} = \frac{a_\oplus \omega}{\epsilon}\ ; \quad \tilde{\boldsymbol{v}} \equiv \frac{D_{OS}}{D_{LS}} \boldsymbol{v}\ . \tag{5.1}$$

From t_0 and ϕ, one may then determine the direction of $\tilde{\boldsymbol{v}}$, up to the overall ambiguity of sign implied by the parity, Λ. Third, one may try to resolve the sign of Λ, if possible. In practice, of course, one would fit all seven parameters simultaneously. I have numerically calculated the six-dimensional covariance matrices for each of several hundred simulated lensing events, and I find the analytic approach outlined above gives a good estimate of the uncertainties.

I therefore proceed by assuming that β, ω, t_0, and the magnitude normalization are perfectly known from the analysis of the overall light curve, and I assess the uncertainty in the determination of ϵ and ϕ (and hence $\tilde{\boldsymbol{v}}$) from the residuals of the Earth-based light curve (eq. [3.1]) relative to the Sun-based curve, (eq. [2.1]). I further assume that the lensing event is long compared to a year, but short compared to the span of observations. (In view of the 4 yr span, this may seem like a questionable assumption, but it is justified by numerical work). Next, I assume that the magnitude measurement error, σ_m, is constant (although actually it should fall somewhat near the peak of the curve) and that the observations are taken uniformly at a rate $\dot{N}_{obs}$. (The actual observations will be interrupted for $\sim$4 months per year, but this should induce a modest correction in the estimates except when the hiatus contains the peak of the curve.) Finally, I assume that the errors in ϕ and ϵ are uncorrelated (again justified by numerical work).

Suppose then that the lens is observed continuously for a year. Since the Earth's motion is a small perturbation on that of the lens, the magnification of the star in magnitudes, $m(t)$, may be Taylor expanded (eq. [1.2]),

$$\begin{aligned} m(t) - m_s &= m_0(t) + \Delta m(t) \\ &= -2.5 \log A[x_\odot(t)] - \frac{2.5}{\ln 10} \epsilon \hat{\boldsymbol{a}}_\oplus \cdot \nabla \ln A[x_\odot(t)]\ , \end{aligned} \tag{5.2}$$

where $\hat{\boldsymbol{a}}_\oplus$ is the direction of the Sun-Earth vector. Now, by assumption $m_0(t)$ is known. For purposes of estimating the error in ϵ, the angle between $\hat{\boldsymbol{a}}_\oplus$ and $\boldsymbol{x}$ may also be treated as known since ϵ and ϕ are independent. Moreover, by the assumption that the event is long compared to a year, the velocity of this angle is nearly uniform. Hence, the variance in the estimate of ϵ from this one year of observations centered on time t_i is

$$\mathrm{var}_{t_i}(\epsilon) = \frac{\sigma_m^2}{\dot{N}_{obs}\,\mathrm{yr}} \langle \cos^2 \Omega t' \rangle^{-1} \left\{ \frac{2.5}{\ln 10} \nabla \ln A[x_\odot(t_i)] \right\}^{-2}\ , \tag{5.3}$$

where $\langle \cos^2 \Omega t' \rangle = \frac{1}{2}$ is the normalized mean square amplitude of the perturbation. Combining the measurements over several years to obtain the net variance of ϵ gives

$$\mathrm{var}(\epsilon)^{-1} = \sum_i \mathrm{var}_{t_i}(\epsilon)^{-1} \to \frac{(2.5/\ln 10)^2 \int dt \{\nabla \ln A[x(t)]\}^2}{2\sigma_m^2/\dot{N}_{obs}}\ ;$$
$$x(t) \equiv (\beta^2 + \omega^2 t^2)^{1/2}\ . \tag{5.4}$$

Using the assumption that the event is short compared to the observation span, I take the integration limits to infinity and find $\sigma_\epsilon \equiv \mathrm{var}(\epsilon)^{1/2}$,

$$\sigma_e = \frac{\ln 10}{2.5} \sqrt{\frac{2}{\pi}} \left[\frac{\dot{N}_{obs}}{\omega} g(\beta) \right]^{-1/2} \sigma_m\ ;$$
$$g(\beta) \equiv \frac{1}{\beta} - \frac{5\beta^2 + 22}{(\beta^2 + 4)^{3/2}} + \frac{4\beta^2 + 4}{(\beta^2 + 2)^{3/2}}\ . \tag{5.5}$$

Finally, this may be converted into an error estimate for $\tilde{v}^{-1}$, using the relation $\tilde{v} = a_\oplus \omega/\epsilon$,

$$\tilde{v}\sigma(\tilde{v}^{-1}) = 0.74 \left[\frac{\dot{N}_{obs}}{\omega} g(\beta) \right]^{-1/2} \frac{\tilde{r}_e}{a_\oplus} \sigma_m\ . \tag{5.6}$$

Equation (5.6) is almost exactly what one would guess naively for the uncertainty in $\ln \tilde{v}$.The number of effective observations is $\dot{N}_{obs}/\omega$ The leverage of each measurement is the Earth's orbital radius divided by the reduced Einstein ring radius, $\tilde{r}_e$. This leverage is enhanced by a factor $\sim\beta^{-1}$ (see eq. [1.2]), but only for a fraction $\sim\beta$ of the effective observations, yielding a net enhancement $\sim\beta^{-1/2}$. Note that $g(\beta) \to \beta^{-1}$ as $\beta \to 0$.

A similar analysis leads to an estimate of the uncertainty in ϕ, $\sigma_\phi = \tilde{v}\sigma(\tilde{v}^{-1})$,

$$\sigma_\phi = 0.74 \left[\frac{\dot{N}_{obs}}{\omega} g(\beta) \right]^{-1/2} \frac{\tilde{r}_e}{a_\oplus} \sigma_m\ . \tag{5.7}$$

Numerical simulations of 4 yr of observations show that these estimates generally tend to underestimate the uncertainties by $\lesssim$20%–30% for 1 yr $\lesssim \omega^{-1} \lesssim$ 4 yr. As one would expect, however, the situation gets substantially worse for cases where $\beta\omega^{-1} \ll 1$ yr, because the Earth's orbit is not fully sampled during maximum magnification. Factor of 2 errors also appear as $\beta\omega^{-1} \gtrsim 4$ yr, because the formal integral samples significantly more of the light curve than do the finite observations. Outside the range 1–4 yr, one or the other of these problems overwhelms the calculation at all β. I conclude that equation (5.6) provides a good guide to the uncertainties, provided that it is used sensibly.

Are the expected data good enough to measure proper motions? To address this question, I continue to assume that the preliminary estimate ($N = 10^7$ stars observed; 55% good weather; 8 months per year; 4 yr of observations; two bands; $\sigma_m = 0.08$) regarding the Macho Collaboration experiment will prove accurate. These estimates imply that

$$\tilde{v}\sigma(\tilde{v}^{-1}) \sim 0.13[g(\beta)]^{-1/2} \left(\frac{M}{10\,M_\odot} \frac{D}{10\,\mathrm{kpc}} \right)^{1/4} \left(\frac{\tilde{v}}{200\,\mathrm{km\,s^{-1}}} \right)^{1/2}\ . \tag{5.8}$$

Thus, measurements as accurate as 10%–20% are possible, especially for Machos in the range of 10 solar masses, or those with small impact parameters or slow speeds.

Unfortunately, it appears that it will not be possible to resolve the directional ambiguity of $\Lambda = \pm 1$ on the basis of the expected data. Light curves from prograde Machos ($\Lambda = 1$) differ from those of retrograde Machos ($\Lambda = -1$) of the same phase, in that their positive deviations last longer by $O(v_\oplus/\tilde{v})$, where $v_\oplus = 30$ km s^{-1} is the Earth's orbital speed. Thus, the uncertainties in discriminating $\Lambda = \pm 1$ are an order of magnitude larger than equation (5.8), making even marginal resolution difficult except for especially slow Machos.

However, it would be possible to resolve the ambiguity with auxiliary satellite observations, as I discuss in the next section.

6. SATELLITE OBSERVATIONS

If Machos are detected from Earth-based observations, substantial information can be gained from a special-purpose telescope in solar orbit (Grieger, Kayser, & Refsdal 1986; Kayser, Refsdal, & Stabell 1986). Let me emphasize that the launching of such a telescope would be a major undertaking and could only take place long after the initial ground-based Macho searches were completed. It would be substantially more difficult to implement this proposal than others outlined in this paper. Nevertheless, it is worth investigating what can be learned from space-based observations and how they can be combined with ground-based data.

Unlike the Earth-based telescopes, a satellite telescope would not be burdened with the task of finding lensing candidates among 10^7 stars. It would be free to follow up the 5–10 lensing events which are going on at any given time and which had been spotted from the ground (assuming, of course, that the Earth-bound Macho search continues). Thus, even a relatively small telescope would provide a spectacular increase in sensitivity over Earth-based observations alone. For example, a 12″ telescope which devoted 40 minutes per day (in each of two filters) to each 19.5 mag star, would have photon-counting-statistics errors of only 0.03 mag. This (together with the lack of weather in space) would improve the accuracy by a factor ~ 4.

However, the main advantage of satellites is that they provide a much better baseline for parallax measurements. Earth-based parallax measurements are restricted by the condition 1 yr $\equiv 2\pi a_\oplus/v_\oplus \gtrsim \tilde{r}_e/\tilde{v} \equiv \omega^{-1}$. This implies that the perturbation, $\epsilon \equiv a_\oplus/\tilde{r}_e$ must be small, $\epsilon \lesssim v_\oplus/2\pi\tilde{v} \sim 2\%$. By contrast, a satellite-Earth baseline can be tuned to the Einstein-ring radius of the Machos.

As a first example, let us suppose that Machos are discovered to have masses in the range, $10^{-3}\ M_\odot \lesssim M \lesssim 4\ M_\odot$. This includes the prime candidates of brown dwarfs and Jupiters. As discussed in the previous section, the parallaxes of such Machos are inaccessible to Earth-based observations. The reason is not that the Earth's orbit has an inappropriate baseline. Indeed, for these objects $0.3a_\oplus \lesssim \tilde{r}_e \lesssim 20a_\oplus$. The problem is that by the time the Earth gets to another part of its orbit, the lensing event is over. The solution is to place a satellite in solar orbit with semimajor axis $a \sim a_\oplus$. If the orbit were made somewhat eccentric, then the Earth-satellite distance would vary considerably, thus covering a wide range of masses. If the Machos were believed to lie in a narrow mass range, a circular orbit with an appropriate and fixed Earth-satellite distance could be established. (For Machos with $M \lesssim 10^{-3}\ M_\odot$, the events would last less than a few days and it might prove difficult to recognize an event in time to alert the satellite.)

If the Machos are found to be in the range 4–100 $M_\odot$, then considerable parallax information will be available from the ground, as discussed in the previous section. However, several dramatic improvements could be made by placing a satellite at several AU, say at Jupiter or beyond. First, the parity ambiguity could be easily resolved in most cases. For example, if the satellite image peaked before the ground image, that would mean that the velocity vector was aligned with and not against the satellite-Earth vector. Second, as mentioned above, there would be a $\sim$fourfold improvement in statistics. Third, there would be a $\sim$fivefold improvement from the increased semimajor axis (eq. [5.6]). Finally, candidates with statistically marginal identifications (such as high-impact parameter, low-magnification events) could be followed up and confirmed on the basis of their more accurate satellite-based parallaxes.

The situation is most difficult for Machos of mass $10^2\ M_\odot \lesssim M \lesssim 10^3\ M_\odot$. If a Satellite were placed in a Jupiter-like orbit, then the component of $\tilde{\boldsymbol{v}}$ along the Earth-satellite direction could be measured from the $\gtrsim 1$ month time lag and ~ 0.2 mag difference of the image as seen from the two places. Determination of the other component might be harder. If the light curve happened to peak during the observations, then the other component could be established from the magnitude of the peaks. The further the peak lay outside the observations, the more such a determination would rest on the accuracy of the magnitude measurements. The transverse velocity could certainly be measured in a decade, since this could be longer than an event and of a Jovian year. From the Jovian analog of equation (5.6), one may crudely estimate the fractional accuracy of such a measurement, $\tilde{v}\sigma(\tilde{v}^{-1}) \lesssim \sigma_m \sim 3\%$.

At higher masses, satellite measurements alone could not resolve the transverse velocity in a reasonable amount of time. However, by combining satellite measurements with high-resolution images, one could measure all four parameters, as I show below.

7. COMPLETE DETERMINATIONS FOR 10^3–$10^6\ M_\odot$

As I discussed in § 4, split images induced by Machos 10^3–$10^6\ M_\odot$ can be resolved using *HST* or in some cases ground-based observations. These observations can be used to measure three independent parameters, $\boldsymbol{\omega}$ and θ_*.

The value of a fourth parameter, $\tilde{r}_e$, could be measured using a satellite telescope placed in a Neptune-like orbit. The Earth-satellite difference in the lensing parameter, x, would be $\Delta x = (a_{\rm Nep}/\tilde{r}_e)\cos\theta$, where θ is the angle between the Earth-satellite axis and the Earth-Macho axis (established from the orientation of the images). For a lens at $D_{\rm OL} \sim 10$ kpc, this implies $\Delta x \sim 0.1(M/10^3\ M_\odot)^{1/2}\cos\theta$. The magnitude difference between the Earth- and satellite-based images would be $\sim\Delta x/x$, which could easily be resolved using multiple high-quality measurements.

From these four parameters, one may obtain the mass, $GM/c^2 = \tilde{r}_e\theta_*/4$, the characteristic distance, $D = \tilde{r}_e/\theta_*$, the observer-lens distance, $D_{\rm OL}/(1 - D_{\rm OL}/D_{\rm LMC}) = D$, and the true velocity, $\boldsymbol{v} = \boldsymbol{\omega}\theta_* D_{\rm OL}$.

That is, it would be possible at least in principle (and at substantial expense) to disentangle all four of the Macho's lensing parameters.

8. BINARIES

The initial Macho Collaboration data may yield additional information about some lenses, provided that the lensed stars are double spectroscopic binaries. In this case, all of the binary's parameters could be determined except for the direction of the projected apse vector. Griest (1991) pointed out that binaries would cause oscillations in the light curve and Griest & Hu (1992) analyzed such binaries in detail. They found that such oscillations would usually not be detectable. The reason is that just as the Earth-generated perturbation, $\epsilon \propto a_\oplus(D_{\rm LS}/D_{\rm OS})$, the binary-generated perturbation, $\epsilon_{1,2} \propto a_{1,2}(D_{\rm OL}/D_{\rm OS})$, where $a_{1,2}$ are the semimajor and semiminor axes of the projected orbit. For the expected $D_{OL} \sim 10$ kpc and $a_{1,2} \sim a_{\oplus}$, this is a factor ~ 4 smaller than the parallax effect, and hence, according to equation (5.6), very difficult to detect.

However, if a parallax measurement of $\tilde{v}$ had been made, even a failure to detect the motion of a recognized binary would provide important information: it would show that the Macho was indeed substantially closer to us than to the LMC. (The optical depth of lenses in the possible LMC halo could plausibly be $\sim 10\%$ that of the Galactic lenses, and for these $D_{OS}/D_{LS} \sim 5$, rather than ~ 1.3.) Moreover, if the much more accurate satellite observations were available, binary motions would become observable and this would allow separate measurements of individual distances and velocities.

Finally, proper motions from unrecognized binaries would show up as nonannual oscillations in more accurate space-based data (or in ground-based data, if a special program were undertaken to observe known Machos more intensely). Once recognized as binaries, these stars could be studied more closely and they might then be resolved as spectroscopic binaries. Even if this determination took place after the microlensing event was over, the distance to the lens could still be determined from the second record of its light curve.

By measuring the distance and velocity separately, one would also measure the mass (see eq. [1.5]). That is, all four of the Macho parameters would be known.

9. LOW-MASS MACHOS

If the Machos have masses $\lesssim 10^{-7}\ M_{\odot}$, Their Einstein rings would be smaller than the angular extent of the stars they are imaging, and the maximum magnification would be given by equation (1.4). In this section, I discuss a slight modification of the standard observation program which could lead to detections of Machos ~ 20 times lighter than the usually stated limit. To simplify the discussion, I parameterize the mass by μ,

$$\mu \equiv \frac{M}{10^{-7}\ M_{\odot}}\,, \tag{9.1}$$

and consider the limit of $\mu \ll 1$. Equation (1.4) then becomes

$$A_{\max} = 1 + 1.5\left(\frac{R}{3\ R_{\odot}}\right)^{-2} \mu\,. \tag{9.2}$$

From this equation, it is clear that one should focus on the bluest available stars, since these have smaller radii for equal luminosities and the signal scales $\propto R^{-2}$. I will consider A and B stars.

In the mass range $\mu \ll 1$, the light curve undergoes a smooth rise as the Macho crosses the edge of the star, remains essentially constant for most of the event, and then falls smoothly (and symmetrically) as the Macho crosses the other edge. The length of an event is $\Delta t \sim 2(R/v)(1 - y^2)^{1/2}(D_{OL}/D_{OS})$, where R is the radius of a star and y is the dimensionless impact parameter. I will be interested primarily in the longest common events. Since $\frac{1}{3}$ of the events have $(1 - y^2)^{1/2} \gtrsim 95\%$, I will restrict attention to these and drop the impact-parameter term. Thus,

$$\Delta t \sim 70 \text{ minutes}\ \frac{R}{3\ R_{\odot}}\ \frac{D_{OL}}{10 \text{ kpc}}\left(\frac{v}{200 \text{ km s}^{-1}}\right)^{-1}. \tag{9.3}$$

The rate for such events per star observed is just $\gamma \equiv n'\sigma'v'$, where $n' \sim 8 \times 10^{16}\mu^{-1}$ sr^{-1} is the number density of Machos, $\sigma' \sim (\frac{2}{3})R/D_{OS} \sim 9 \times 10^{-13}$ rad is the cross section of a star (recall that I am ignoring events $|y| > \frac{1}{3}$), and $v' = v/D_{OL} \sim 7 \times 10^{-16}$ rad s^{-1} is the angular velocity. The rate is then

$$\gamma \sim \frac{1.4 \times 10^{-6}}{8 \text{ hr}}\,\mu^{-1}\,. \tag{9.4}$$

Now suppose that the same field were observed repeatedly for a month, i.e., every ~ 5 minutes, simultaneously in two filters, for 8 hr per night. The number of events would be γN_{AB} where $N_{AB} \sim 10^5$ is the number of A-B stars in the field. That is, there would be $\sim \mu^{-1}$ events per fortnight (assuming 55% good weather). How small could μ be before the events would not be seen? There are effectively $\sim$(8 hr/70 minutes) $\times$ 30 $\times$ 55% $\times\ N_A \sim 10^7$ trials, so a signal-to-noise ratio $Q \sim 5.5$ would be needed to ensure against statistical fluctuations generating false signals. Since there would be a chance for 14 repeat measurements in each of two colors, one would be sensitive to magnifications as small as $\delta m \sim Q(2.5/\ln 10)28^{-1/2}\sigma_m \sim 0.1$ mag. According to equation (9.2), this corresponds to $\mu \sim 0.06$. Then from equation (9.4), one finds that there would be ~ 40 recognizable events at the lower detectable mass unit.

Actually, an expected event total of only ~ 10 is probably good enough. There are several ways to go to smaller Machos at the cost of total events. One is to use the slower Machos. These last longer $\propto v^{-1}$, and so allow more repeat exposures. The result is a gain in statistics $\propto v^{-1/2}$. On the other hand, the two-dimensional flux is down $\propto v^3$. Thus by going lower in mass, one gains in event rate by μ^{-1} from the increased number of Machos, but loses by μ^6 from reduced phase space. That is, the event rate falls $\propto \mu^5$. Alternatively, one might use closer Machos, which have a larger magnification $\propto d^{-1}$. However, the events go by faster $\propto d$, leading to a net gain $\propto d^{-1/2}$. On the other hand, the available volume (and hence event rate) declines $\propto d^2$. The net result is an event rate which falls $\propto \mu^3$. I conclude that 10 events per month could be observed for $\mu \sim 0.06(10/40)^{1/3}$, that is, $M \sim 4 \times 10^{-9}\ M_{\odot}$.

Finally, I note that equation (9.2) seems to indicate that there would be only ~ 2.5 expected events at $\mu \sim 1$, raising the possibility that this 30 day experiment might not have sensitivity at these masses. In fact, the higher magnification signal near $\mu = 1$ would allow events with dimensionless impact parameters $|y| \lesssim 1$ to be detected. Likewise, K stars would generate a clear signal for $\mu \sim 1$. The true event rate would be $\gtrsim 10$.

10. SYSTEMATIC EFFECTS

The amplitude of the annual oscillations which must be detected to confirm a Macho of mass $10^3\ M_{\odot}$ and $x = 0.26$ is $\sim a_{\oplus}/x\tilde{r}_e \sim 0.01$ mag. This is substantially smaller than the individual error measurements, $\sigma_m = 0.08$ mag. The difference is the usual "root-n" effect of multiple independent observations. However, this standard result requires that the errors be truly random, or more accurately, that the systematic errors have no annual component.

In fact, the systematic errors are bound to have a very strong annual component. Most importantly, the position of the LMC in the sky is a function of the time of year. This produces an annual modulation in atmospheric extinction and also in differential refraction. Since the stars are matched to templates, this latter effect can seriously affect the magnitude measurements. Both of these effects are strongly color dependent and may also be influenced by position within the LMC, or by the degree of crowding. The weather also has a strong annual component to its variation, and this may affect both the atmosphere and the equipment. There may be other annual effects as well.

I do not believe, however, that these effects can generate a false signal, even at the 0.01 mag level. Since the program contains $\sim 10^7$ stars, there will be many nonvarying stars for every lensing candidate. These nonvarying stars can be studied for systematic correlations with seeing, sky position, weather, etc., as functions of both color and magnitude. Any correlations which are found can then be removed from all stars including the lensing candidates. In this way, the mean (complex) amplitude of the entire sample and of any recognized subsample can be set to zero. There are then two possibilities. Either all of the major sources of annual systematic errors will have been identified, in which case the total observed power in annual modulation of stars will be equal to that predicted on the basis of individual random errors. Or, there will be some important source of annual modulations which has been missed, in which case the observed annual power will exceed that predicted. In the latter case, one must use the observed rather than the predicted fluctuations when estimating the confidence level of the results. If the observed power is extremely high, then the significance of the results will be degraded. However, this degradation will be a known effect, and therefore should not induce a spurious signature. Put differently, the rate of 3 σ false signals among the, say, 20 lensing candidates should be neither higher nor lower than the rate among the $\sim 10^7$ noncandidates. And, in fact, the search for correlations between errors and various systematic effects may well drive the random errors down below the value indicated by preliminary tests.

11. THE GALACTIC BULGE

Microlensing can also be detected by observing stars in the Galactic bulge (Paczyński 1991; Griest et al. 1991). Three classes of lensing objects might be detected: halo dark matter, disk dark matter, and ordinary stars. Detection of the last would confirm that the experiment is working, a particularly important result if the search toward the LMC finds no lenses. The Macho Collaboration (Alcock et al. 1992) plans to look at the bulge during the ~ 4 months when the LMC is down. Here I briefly examine the possibility of measuring parallaxes from bulge lensing events for the various types of objects which might be detected. I should note, first of all, that virtually no parallax information can be recovered from the observations as planned, since the transverse velocity of the Earth changes very little over 4 months. The question I will address is what could be learned if the bulge were observed over most of a year.

An important limitation which affects all classes of objects arises from the fact that the Galactic center is $\lesssim 6°$ from the ecliptic. Thus, the parallax ellipses have an axis ratio $\gtrsim 10$ and so parallax measurements are sensitive to essentially only one component of transverse velocity.

For halo objects, there is another problem which makes it significantly more difficult to measure parallaxes toward the bulge than toward the LMC. The characteristic distance, $D_{\rm OL} D_{\rm LS}/D_{\rm OS}$, which enters the time scale, ω^{-1}, is typically a factor ~ 4 smaller for the bulge than for the LMC, while the transverse velocity is similar. Thus, the minimum mass for 6 month events is $\sim 16\ M_\odot$ rather than $\sim 4\ M_\odot$ (see eq. [1.5]). It is true that characteristic distance, D, which enters the reduced Einstein ring radius, $\tilde{r}_e$, is $\sim 33\%$ smaller for the bulge than for the LMC (see eqs. [1.1] and [3.2]). The perturbation, ϵ, is therefore larger by a factor $\sim 20\%$. However, this is relatively small compensation for the other shortcomings. I conclude that the LMC is a much more favorable direction for measuring Macho parallaxes.

Both disk stars and disk dark matter are expectd to have velocity dispersions ~ 30 km s^{-1}. The relevant transverse velocities for these objects are the velocities of the bulge stars which are being lensed, projected to the distance of the lensing object. If the typical velocities in the bulge are ~ 100–200 km s^{-1}, then the typical transverse velocities are ~ 50–100 km s^{-1}. Thus, the minimum mass for events having a $\gtrsim 6$ month time scale is ~ 1–2 $M_\odot$. There are certainly some disk stars in this mass range, so that parallax measurements might be made in some cases. The composition of the disk dark matter (Bahcall, Flynn, & Gould 1992), if any (Kuijken & Gilmore 1989), is unknown, but the most plausible candidate would be low-mass stars. The time scale for such objects would be much too short to allow parallax measurements.

12. CONCLUSIONS

With relatively minor modifications, the Macho searches presently getting underway can extend their range of mass sensitivity by a factor $\gtrsim 10^5$. Macho candidates in the range 10^2–$10^6\ M_\odot$ can be identified on the basis of the secular rise or fall of their light curves. Machos $\gtrsim 10^6\ M_\odot$ are probably ruled out by the stability of the Galactic disk. Candidates in the range 10^2–$10^3\ M_\odot$ can be confirmed by measuring the annual "parallax" modulation in their magnification induced by the Earth's motion through their Einstein rings. Candidates $\gtrsim 10^3\ M_\odot$ can be confirmed by resolving their $\gtrsim 0''.06$ split images using the *Hubble Space Telescope*. In fact, for candidates $\gtrsim 5 \times 10^4\ M_\odot$, good ground-based images are probably adequate. At the other extreme, by using 1 month of the 4 yr observing schedule for rapid repeat observations of a single field, one could probe for Machos down to $\sim 4 \times 10^{-9}\ M_\odot$.

For machos in the range 4–100 $M_\odot$, one may measure the "reduced" transverse velocity, $\tilde{\boldsymbol{v}} \equiv (D_{\rm OS}/D_{\rm LS})\boldsymbol{v}$, from the parallax-induced magnification fluctuations. In some cases 10%–20% accuracy may be obtained. If Machos are discovered anywhere in the range 10^{-3}–$10^6\ M_\odot$, their "reduced" transverse velocities can be measured using a relatively small space-borne telescope. The solar orbit of this telescope could be chosen optimally for the range of masses (and hence Einstein-ring radii) of the detected objects.

For masses $\gtrsim 10^3\ M_\odot$, a combination of split-image resolution and space-based parallax measurements would be sufficient to determine the mass, distance, and velocity of the individual Machos.

If any of the lensed stars are found to be spectroscopic double binaries, then these may be used to measure, or at least constrain, the mass and distance of the lensing Macho.

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