Heterogeneous Treatment Effects and Causal Mechanisms

Jiawei Fu and Tara Slough

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Various approaches (qualitative and quantitative) to evaluating mechanisms:

- Heterogeneous treatment effects (HTEs) estimated by treatment-by-covariate interactions is popular in applied work.

HTEs and Mechanisms: A Survey

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AJPS (65)	61	41	0.56	0.87
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Takeaways:

1. Modal empirical article reports HTEs (treatment $\times$ covariate).
2. 87% of articles that report HTE use them to "test mechanisms."

Known vs. under-explored problems

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- Interactions are generally under-powered.
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Under-explored:

Under what conditions do HTEs provide evidence of mechanism activation?

Outline

Motivating example: Exogenous shocks and voting behavior.

- Inspired by a model by Ashworth et al. (2018).
- Shows that HTE can emerge when posited mechanism is inert.

Outline

Motivating example: Exogenous shocks and voting behavior

Framework: We develop a framework to connect causal mechanisms to HTE with respect to covariates.

- Builds from causal mediation framework (Imai et al., 2010)
- New concepts, assumptions necessary for the HTE setting.

Outline

Motivating example: Exogenous shocks and voting behavior

Framework: We develop a framework to connect causal mechanisms to HTE with respect to covariates.

Results: What do we learn from the existence (or non-existence) of HTE with respect to covariates?

- For outcomes that are directly affected by the mechanism, HTE indicative of a mechanism under assumptions.
- For transformations of these directly affected outcomes, HTE are not necessarily indicative of a mechanism, even under these assumptions.

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Framework: We develop a framework to connect causal mechanisms to HTE with respect to covariates.

Results: What do we learn from the existence (or non-existence) of HTE with respect to covariates?

Discussion: Using these results to inform research design.

Motivating Example: Exogenous Shocks and Voting

Exogenous shocks and voting

Natural experiment on effect of an exogenous shock, ω, on voter behavior:

- A natural disaster (e.g., Healy and Malhotra, 2010; Huber et al., 2012)
- An economic crisis (e.g., Wolfers, 2002)
- A pandemic (e.g., Achen and Bartels, 2004; Baccini et al., 2021)

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Example: Ashworth et al., (2018):

1. Assume our adaption of model is true.
2. Suppose we could measure (some) model parameters directly.
    - Characterize causal estimands in terms of these parameters.
3. Ask: Can HTE provide evidence of voter learning mechanism?

Model

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Voters do not observe θ but may use governance outcome, g to update:

$$g = f(\theta, \omega) + \varepsilon.$$

- ω is increasing in the adversity of the shock
- ε is idiosyncratic shock drawn from symmetric, differentiable density, ϕ, that satisfies monotone likelihood ratio property relative to g.

Voter utility

Each voter's utility from a vote for politician, $p \in \{\mathcal{I}, \mathcal{C}\}$ is given by:

$$u_i^p = \theta^p + v_i \mathbb{1}(p = \mathcal{I})$$

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Variation in the population of voters:

- $v_i \sim U(-1, 1)$ is a valence shock for the incumbent.
- Heterogeneous priors about incumbent: $\pi_i^{\mathcal{I}} \sim f_\pi$, $\pi_i^{\mathcal{I}} \in (0, 1)$.
- Common prior about the challenger: $\pi^{\mathcal{C}} \in (0, 1)$.

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$$\beta(\overline{\theta}|\pi_i^I, \omega) = \frac{1}{1 + \frac{1-\pi_i^I}{\pi_i^I}\frac{\phi(g-f(\underline{\theta}, \omega))}{\phi(g-f(\overline{\theta}, \omega))}}$$

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A voter will vote for the incumbent if:

$$\underbrace{\beta(\overline{\theta}|\pi_i^I,\omega) + v_i}_{E[u_i^I]} \geq \underbrace{\pi^C}_{E[u_i^C]}$$

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Treatment: Binary exposure to the shock $\omega \in \{\omega', \omega''\}$

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- Voter utility from the incumbent:

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- Votes for the incumbent:

$$y_{2i} \equiv \mathbb{I}[\beta(\overline{\theta}|\pi_i^I, \omega) + v_i \geq \pi^C]$$

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Mechanism: Voter learning, not valence, since ω enters through voter's posterior.

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“Interaction” effect representation not standardized

(Nilsson et al., 2020)

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The standard view

$$\begin{tikzcd} & X \arrow[d] \\ Z \arrow[r] & Y \end{tikzcd}$$

$$\frac{\partial Y}{\partial Z} \neq 0$$

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Our notation

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\begin{tikzcd}
 & & v_i\text{: Valence} \arrow[dr] & & \\
\omega\text{: Shock} \arrow[drr, ""{name=A}] & & & y_{1i}\text{: Utility} \arrow[r] & y_{2i}\text{: Vote choice} \\
 & & \beta_i(\overline{\theta})\text{: Posterior} \arrow[ur] & & \\
 & \pi_i^I\text{: Prior} \arrow[to=A] & & &
\end{tikzcd}
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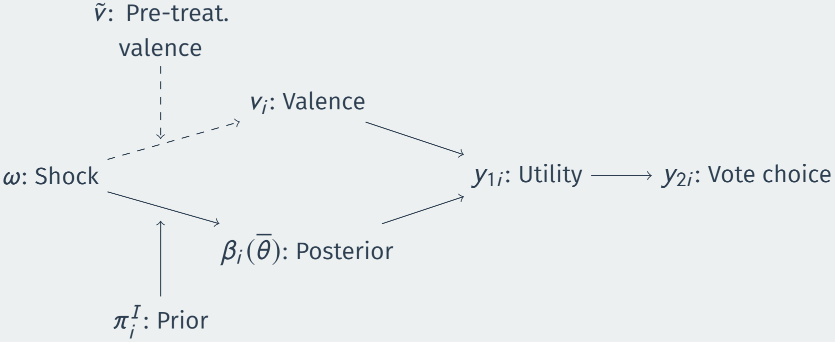

Defining HTEs

To evaluate mechanisms, the empiricist will estimate CATEs for different levels of the (candidate) moderators: $x \in \{\pi_i^I, \tilde{v}\}$:

$$CATE(x') = E[y|\omega = \omega'', x = x'] - E[y|\omega = \omega', x = x']$$

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We will evaluate the presence of HTE for:

- Outcomes: $y \in \{$Voter utility for I, Vote for $I\}$
- Potential moderators: $x \in \{$Prior belief about I, Pre-treatment valence$\}$

HTEs and mechanisms (results)

	y_1: Voter utility	y_2: Vote choice
x_1: Prior (π_i^I)	Mechanism HTE $CATE(\pi') \neq CATE(\pi'')$	
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HTEs and mechanisms: Implication/question

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HTE are not necessarily indicative of mechanism activation.

- To what extent is this general?

Framework

Set-up

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A set of pre-treatment covariates, $\mathbf{X}$.

Causal effects

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Several causal effects typically described wrt causal mediation.

- Total effect (TE) of Z on Y.
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$$TE = DE + \sum_{j=1}^{J} IE_j$$

If a mechanism j is activated or present (for any unit), then there exists some unit for which $IE_j \neq 0$.

Estimands

Average treatment effect (ATE):

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Conditional average treatment effects (CATE): Consider pre-treatment covariate $X_k \in X$. The CATE with respect to $X_k = x$ is:

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Heterogeneous Treatment Effects (HTEs): HTEs exist with respect to pre-treatment covariate $X_k \in X$ iff:

$$CATE(X_k = x) \neq CATE(X_k = x')$$

for some $x \neq x' \in X_k$.

Reformulating the question

Original statement:

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More precise version:

Under what conditions are HTEs with respect to X_k sufficient to show that there there exists some unit for which $IE_j \neq 0$?

Relationship to causal mediation

Mediation:

- Requires mediators to be measurable and measured.
- Assumes sequential ignorability.
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Use of HTE:

- Does not require mediators to be measurable. (But we need specific measured covariates.)
- Invokes a set of exclusion assumptions.
- Seeks to demonstrate that $IE_j \neq 0$ for some unit.

HTEs and Mechanisms

Concept: Causal Indicator Variable (CIV)

Definition (Causal Indicator Variable)

Pre-treatment variable X_k is a causal indicator variable (CIV) for mechanism j if for some $x, x' \in X^k$, $IE_j(X_k = x) \neq IE_j(X_k = x')$.

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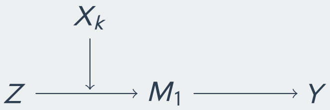
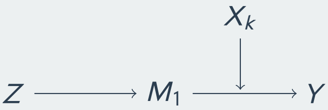
Two possibilities:

- $X_k \in X^{CIV}$ moderates the effect of treatment (Z) on mediator (M_j).
- $X_k \in X^{CIV}$ moderates the effect of the mediator (M_j) on outcome (Y).





Exclusion Assumption I

Assumption (Exclusion I)

For any $x, x' \in X_k$, X_k *is non-linearly excluded to the direct effect such that* $DE(X_k = x) = DE(X_k = x')$.

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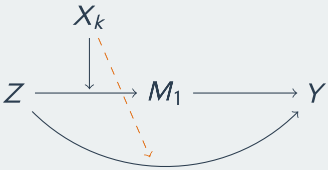
Direct effect of Z on Y cannot depend on X_k.



Exclusion Assumption II

Assumption (Exclusion II)

Given $z, z' \in Z$ *and* $x, x' \in X_k$, X_k *is non-linearly excluded to the indirect effect of mechanism* $j \neq 1$, IE_j, *if:* $IE_j(x) = IE_j(x')$.

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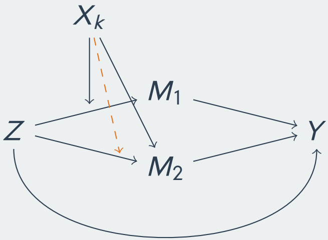
In other words, X_k is not a CIV for any other M_j.



HTEs as a test of the mechanism (#1 of 2)

Proposition

Suppose that Y is directly affected by mechanism 1 and Assumptions 1-2 hold with respect to X_k. If HTE exist with respect to X_k, then $X_k \in \boldsymbol{X}^{CIV}$ for mechanism 1.

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Generalization: Holds for any non-zero linear transformation of Y, $L(Y)$.

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The usual logic for HTE, but note the assumptions.

- Implicit/unstated assumptions $\neq$ the absence of assumptions.

HTEs as test of mechanisms (#2 of 2)

Proposition

Suppose that Y is continuous and Assumptions 1 and 2 hold. If no HTE exist with respect to X_k, at least one of the following must true:

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Alternate explanation: postulated relationship between X_k and M_1 is misspecified.

Absence of HTE less informative than presence of HTE.

Summary (so far...)

Under Assumptions 1-2...

	Outcome variable is: Directly affected by M_1	Indirectly affected by M_1
$\exists$ HTE wrt X_k:	$X_k \in \mathbf{X}^{CIV}$ $\implies$ M_1 is active.	
$\nexists$ HTE wrt X_k:	$X_k \notin \mathbf{X}^{CIV}$ **or** M_1 not active	

Indirectly-affected outcomes

Why should we care:

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- Examples: models of (discrete) choice, Likert scales etc.

Poses challenges for the quantitative evaluation of mechanisms.

One last concept

Useful to define $\mathbf{X}^R$ as the subset of measured covariates with a non-zero effect on directly-affected outcome Y. It is straightforward to see that:

$$\mathbf{X}^{CIV} \subseteq \mathbf{X}^R \subseteq \mathbf{X}$$

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In our motivating example, for the learning mechanism:

- $\mathbf{X}^{CIV} = \{\pi_i^I\}$
- $\mathbf{X}^R = \{\pi_i^I, \widetilde{v}_i\}$

HTEs on indirectly-affected outcomes (#1 of 2)

Proposition

Suppose that observed outcome $L(Y)$ is a non-linear transformation of directly-affected outcome Y and Assumptions 1 and 2 hold. If HTE exist with respect to X_k, then $X_k \in \boldsymbol{X}^R$.

HTEs on indirectly-affected outcomes (#1 of 2)

Proposition

Suppose that observed outcome $L(Y)$ is a non-linear transformation of directly-affected outcome Y and Assumptions 1 and 2 hold. If HTE exist with respect to X_k, then $X_k \in \mathbf{X}^R$.

Implication: Two possibilities:

- $X_k \in \mathbf{X}^{CIV} \implies M_1$ is active.
- $X_k \notin \mathbf{X}^{CIV} \implies M_1$ may or may not be active.

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Intuition: Using HTEs to detect mechanisms requires additive separability of X_k from DE and $IE_{j \neq 1}$.

- What Assumptions 1-2 buy us.
- Non-linear transformation $L(\cdot)$ does not preserve additive separability for indirectly-affected outcomes.

HTEs on indirectly-affected outcomes (#2 of 2)

Proposition

Suppose that observed outcome $L(Y)$ is a non-linear mapping of directly-affected outcome Y and Assumptions 1 and 2 hold. If HTE do not exist with respect to X_k, then $X_k \notin \boldsymbol{X}^R$.

HTEs on indirectly-affected outcomes (#2 of 2)

Proposition

Suppose that observed outcome $L(Y)$ is a non-linear mapping of directly-affected outcome Y and Assumptions 1 and 2 hold. If HTE do not exist with respect to X_k, then $X_k \notin \mathbf{X}^R$.

Implication: We know that $X_k \notin \mathbf{X}^{CIV} \implies M_1$ may or may not be active.

- So no information about mechanism activation.

Summary

Under Assumptions 1-2...

	Outcome variable is: Directly affected by M_1	Indirectly affected by M_1
$\exists$ HTE wrt X_k:	$X_k \in \mathbf{X}^{CIV}$ $\implies$ M_1 is active.	$X_k \in \mathbf{X}^{R}$ M_1 active or inactive
$\nexists$ HTE wrt X_k:	$X_k \notin \mathbf{X}^{CIV}$ **or** M_1 not active	$X_k \notin \mathbf{X}^{R}$ M_1 active or inactive

Is this really an issue?

We cannot know in any specific case whether heterogeneity comes from causal heterogeneity or the transformation to the observed outcome.

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But, we can show via simulation in which we control the DGP that it is very easy to generate these dynamics.

Example: Support for greenhouse gas regulation

Tangentially inspired by setup of Little et al. (2021).

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- Accuracy motivates
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Outcome: Binary indicator for "favors increased GHG regulation," assumed to be increasing in dangers of GHGs.

Simulation with real data

ANES 2020 data among declared Democrats and Republicans ($n = 2,883$).

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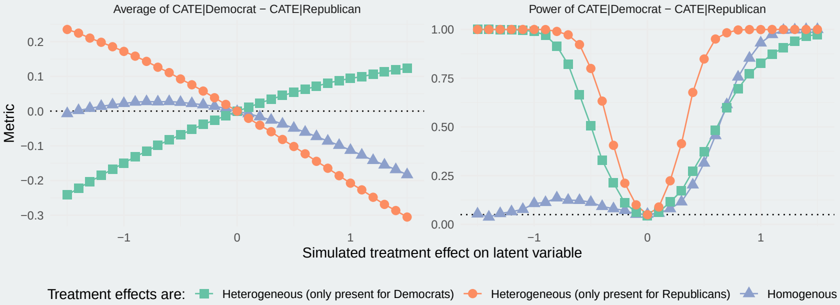
Simulation:

1. Using observed support and matrix of demographic covariates to predict underlying belief in dangers of GHGs. This will be $Y_i(0)$.
2. Simulate treated potential outcomes $Y_i(1)$ by adding treatment effect τ:

$$Y_i(1) = Y_i(0) + \tau \mathbb{1}(\text{Partisanship}_i = P)$$

3. Randomly assign treatment to half the sample to reveal $Y_i(Z)$.
4. Reveal observed outcome $\widetilde{Y}_i \sim \text{Bernoulli}(\text{logit}^{-1}(Y_i(Z)))$.
5. Estimate treatment effects on observed outcome $\widetilde{Y}_i$.

Simulation results



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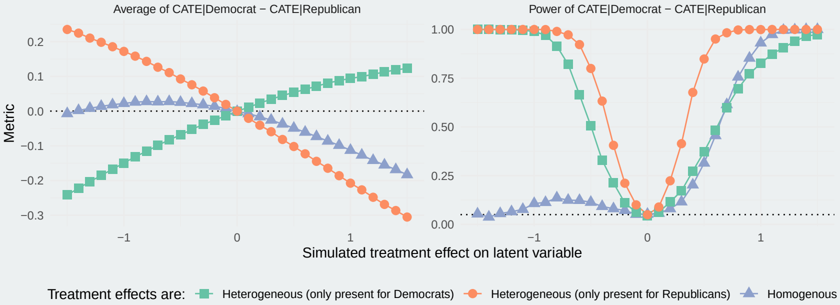

We observe HTE in partisanship for all $\tau \neq 0$ even when treatment effects (on latent variable) are homogenous!

- Magnitude and sign depend on density on the latent variable.

Discussion: Implications for Research Design

Improving the use of HTE: Role of theory

We need more explicit theory to use HTE for mechanism detection.

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Three central questions:

1. What are the candidate mechanisms?
2. What is the relationship between a given covariate, X_k, and each of the candidate mechanisms?
    - For which mechanism (j), is X_k a candidate mechanisms?
    - Are exclusion assumptions plausible for other mechanisms?
3. Do mechanisms directly affect measured outcomes?
    - If so, which outcomes?

Improving the use of HTE: Better research design

Prioritizing different outcomes: Can we measure more directly-affected outcomes?

- When we have the ability to measure directly-affected outcomes, we should do so.
- Possibly more latent-variable estimation for outcomes—mixed feelings here.

Improving the use of HTE: Better research design

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Which covariates should be measured?

- Posit $\mathbf{X}^{CIV}$ before data collection
- Possible implementation: map covariates to candidate mechanisms in pre-analysis plans

Improving the interpretation of HTE

Interpret lack of HTE accurately:

- Does not "rule out" a candidate mechanism (or show that it is inert), even when we have a directly-observed outcome.

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- Does not "rule out" a candidate mechanism (or show that it is inert), even when we have a directly-observed outcome.

Consider implications of lack of power for inferences about the mechanism.

- Absent p-hacking/publication bias etc., low power $\longrightarrow \downarrow$ ability to detect HTE.
- But if lack of HTE has two sources (inert mechanism or misspecified theory), this provides less information.

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Suppose we only observe an indirectly-affected outcome (i.e., vote choice).

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Can we use an assumption of monotonicity to link HTEs to mechanisms?

- In general, not absent further assumptions on the distribution of the directly observed outcome, Y.
- Could impose monotonicity + distributional assumption on Y to facilitate inference about mechanisms.

Thank you!