Heterogeneous Treatment Effects and Causal Mechanisms

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Various approaches to evaluating mechanisms:

- Heterogeneous treatment effects (HTEs) estimated by treatment $\times$ covariate interactions is very popular

HTEs and mechanisms: a survey

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AJPS (65)	61	41	0.56	0.87
APSR (115)	106	75	0.53	0.90
JoP (83)	142	106	0.55	0.83
Total	309	222	0.55	0.87

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Takeaways:

1. Modal empirical article reports HTEs (treatment $\times$ covariate).
2. 87% of these articles use HTEs to "test mechanisms."

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- Assuming an infinite sample.
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Under-explored problem:

Under what conditions do HTEs provide evidence of mechanism activation?

Outline

Motivating example: Exogenous shocks and voting behavior

- Based on model by Ashworth et al. (2018).
- Shows that HTEs can emerge when associated mechanism is inert.

Outline

Motivating example: Exogenous shocks and voting behavior

Framework: We develop a framework to connect causal mechanisms to HTE with respect to covariates.

- Builds from causal mediation framework (Imai et al., 2010)
- New concepts, assumptions needed for the HTE setting.

Outline

Motivating example: Exogenous shocks and voting behavior

Framework: We develop a framework to connect causal mechanisms to HTE with respect to covariates.

Results: What do we learn about a mechanism from the existence or non-existence of HTE?

- For outcomes that are *directly affected* by mechanism, HTE indicative of mechanism activation under assumptions.
- For outcomes that are *indirectly affected* by mechanism, HTE are not necessarily indicative of mechanism activation.

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Framework: We develop a framework to connect causal mechanisms to HTE with respect to covariates.

Results: What do we learn about a mechanism from the existence or non-existence of HTE?

Discussion: Using these results to inform research design.

Motivating Example

Exogenous shocks and voting

Natural experiment on effect of an exogenous shock, ω, on voter behavior:

- A natural disaster (e.g., Healy and Malhotra, 2010; Huber et al., 2012)
- An economic crisis (e.g., Wolfers, 2002)
- A pandemic (e.g., Achen and Bartels, 2004; Baccini et al., 2021)

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Example: Ashworth, Bueno de Mesquita, Friedenberg (2018):

- Assume our adaption of model is true.
- Suppose we could measure (some) model parameters directly.
  - $\rightarrow$ Characterize causal estimands in terms of these parameters.
- Ask: Can HTEs provide evidence of voter learning mechanism?

Model set-up

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Voters do not observe θ but use governance outcome, g, to update:

$$g = f(\theta, \omega) + \varepsilon.$$

- Higher values of ω correspond to a more adverse shock
- ε is idiosyncratic shock drawn from symmetric, differentiable density, ϕ, that satisfies monotone likelihood ratio property relative to g.

Voter utility

Each voter's utility from a vote for politician, $p \in \{I, C\}$ is given by:

$$u_i^p = \theta^p + v_i \mathbb{I}(p = I)$$

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Variation in the population of voters:

- $v_i \sim U(-1, 1)$ is a valence shock for the incumbent.
- Heterogeneous priors about the incumbent: $\pi_i^I \sim f_\pi$ with support on $(0, 1)$.
- Common prior about the challenger: $\pi^C \in (0, 1)$.

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Voters' posteriors:

$$\beta(\overline{\theta}|\pi_i^I,\ \omega) = \frac{1}{1 + \frac{1-\pi_i^I}{\pi_i^I}\frac{\phi(g-f(\underline{\theta},\ \omega))}{\phi(g-f(\overline{\theta},\ \omega))}}$$

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A voter will vote for the incumbent if:

$$\underbrace{\beta(\overline{\theta}|\pi_i^I, \omega) + v_i}_{E[u_i^I]} \geqslant \underbrace{\pi^C}_{E[u_i^C]}$$

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Treatment: Binary exposure to the shock $\omega \in \{\omega', \omega''\}$

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Mechanism: Voter learning, not valence, since ω enters through voter's posterior.

Aside: DAG representation of interactions

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The standard view

$$\begin{tikzcd} & X \arrow[d] \\ Z \arrow[r] & Y \end{tikzcd}$$

$$\frac{\partial Y}{\partial Z} \neq 0$$

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Our notation

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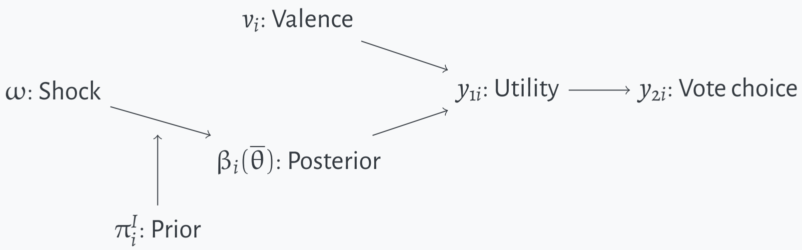

Defining HTEs

To evaluate mechanisms, the empiricist will estimate CATEs at different for different levels of the (candidate) moderators: $x \in \{\pi_i^I, v_i\}$:

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We will evaluate the presence of HTE for:

- Outcomes: $y \in \{$Voter utility for I, Vote for $I\}$
- Potential moderators: $x \in \{$Prior belief about I, Valence$\}$

HTEs and mechanisms (results)

	y_1: Voter utility	y_2: Vote choice
x_1: Prior (π_i)	Mechanism HTE $CATE(\pi) \neq CATE(\pi')$	
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HTE are not necessarily indicative of mechanism activation.

- To what extent is this general?

Framework

Three main components

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An outcome, Y.

- Starting assumption: we measure an outcome that is *directly affected* by a mechanism. Examples:
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A set of pre-treatment covariates, X.

Causal Effects

Mediators as mechanism representations.

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Several causal effects typically described wrt causal mediation.

- Total effect (TE) of Z on Y.
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At the individual/unit level:

$$TE = DE + \sum_{j=1}^{J} IE_j$$

If a mechanism j is activated or present (for any unit), then there exists some unit for which $IE_j \neq 0$.

Estimands

Average treatment effect (ATE):

$$\begin{aligned} ATE &= E_X[Y(z) - Y(z')] \\ &= ADE(z, z'; X) + \sum_{j=1}^{J} AIE(z, z'; X) \end{aligned}$$

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Conditional average treatment effects (CATE): Consider pre-treatment covariate $X_k \in X$. The CATE with respect to $X_k = x$ is:

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Heterogeneous Treatment Effects (HTEs): HTEs exist with respect to pre-treatment covariate $X_k \in X$ iff:

$$\text{CATE}(\text{X}_k = x) \neq CATE(X_k = x')$$

for some $x \neq x' \in X_k$.

Reformulating the question

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More precise version:

Under what conditions are HTEs with respect to X_k sufficient to show that there there exists some unit for which $IE_j \neq 0$?

Relationship to mediation

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Use of HTE:

- Does not require mediators to be measurable, But we need specific measured covariates.
- Invokes a set of exclusion assumptions.
- Seeks to demonstrate that $IE_j \neq 0$ for some unit.

HTEs and Mechanisms:
Directly Affected Outcomes

Concept: Causal Indicator Variable (CIV)

Definition (Causal Indicator Variable)

Pre-treatment variable X_k is a causal indicator variable (CIV) for mechanism 1 if for some $x, x' \in X^k$, $IE_1(X_k = x) \neq IE_1(X_k = x')$.

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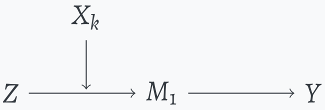
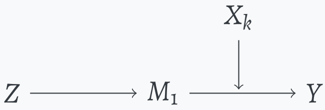
Two possibilities:

- $X_k \in X^{CIV}$ moderates the effect of treatment (Z) on mediator (M_j).
- $X_k \in X^{CIV}$ moderates the effect of the mediator (M_j) on outcome (Y).



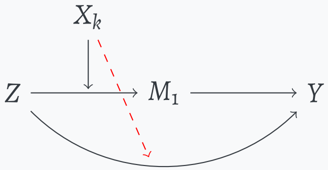


Exclusion assumption I

Assumption (Exclusion I)

Given $z, z' \in Z$ and $x, x' \in X_k$, X_k is excluded to the direct effect such that $ADE(z, z'; X_k = x) = ADE(z, z'; X = x')$.

- Direct effect of Z on Y cannot depend on X_k.

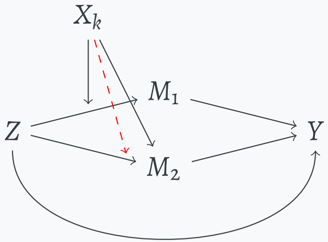


Exclusion assumption II

Assumption (Exclusion II)

Given $z, z' \in Z$ and $x, x' \in X_k$, X_k is excluded to the indirect effect of any other mechanism, $j' \neq j$, $IE_{j'}$, if: $AIE_{j'}(x) = AIE_{j'}(x')$.

- In other words, X_k is not a CIV for M_2.



HTEs as a test of mechanisms (#1 of 2)

Proposition

Suppose that Y is directly affected by mechanism j and Assumptions 1 and 2 hold with respect to X_k. If HTEs exist with respect to X_k, then $X_k \in \boldsymbol{X}^{CIV}$ for mechanism j.

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Implication: By definition of CIV, HTEs imply that $IE_1(X_k = x') \neq IE_1(X_k = x'')$ for some $x', x'' \in X_k$, which indicates that mechanism j is active.

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The usual logic for HTE, but note the assumptions.

HTEs as a test of mechanisms (#2 of 2)

Proposition

Suppose that Y is directly affected by mechanism j and Assumptions 1 and 2 hold. If no HTEs exist with respect to X_k, at least one of the following must be true:

1. $X_k \notin \boldsymbol{X}^{\text{CIV}}$ *for mechanism j.*
2. *No CIV exists.*

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Absence of HTE does not "rule out" a mechanism.

HTE and Mechanisms: Indirectly Affected Outcomes

Why should we care?

Many attitudinal, behavioral outcomes are indirectly affected by mechanisms.

- Example: decisions based on utility maximization.
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Poses challenges for the quantitative detection of mechanisms

- Through HTEs and likely other approaches.

Additional structure, concept

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In our motivating example, for the learning mechanism:

- $\mathbf{X}^{CIV} = \{\pi_i^I\}$
- $\mathbf{X}^R = \{\pi_i^I, v_i\}$

HTEs as a test of mechanisms (#1 of 2)

Proposition

Suppose that observed outcome $L(Y)$ is a non-linear mapping of directly-affected outcome Y and Assumptions 1 and 2 hold. If HTEs exist with respect to X_k, then $X_k \in \boldsymbol{X}^R$.

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Intuition: Using HTE for mechanism detection relies on *additive separability* of X_k from DE and $IE_{\neg j}$ on the latent variable.

- What exclusion assumptions buy us
- But a non-linear $L(\cdot)$ does not preserve additive separability on $L(Y)$.

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Implication: Two possibilities:

- $X_k \in \mathbf{X}^{CIV} \implies$ mechanism j is active.
- $X_k \notin \mathbf{X}^{CIV} \implies$ mechanism j may or may not be active.

Numerical example

Suppose that we are interested in how a mobilization treatment, $Z_i \in \{0, 1\}$, affects voters' turnout decisions. Two covariates in $\mathbf{X}^R$:

- $X_1 \sim \mathcal{N}(0, 1)$
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Turnout—the observed outcome—is given by:

$$L(U(Z, X)) = \begin{cases} 1 & \text{if } (1 + Z)X_1 + X_2 \geqslant 0 \\ 0 & \text{else} \end{cases}$$

HTEs with respect to X_2?

Recall that X_2 is **not** a CIV for the unique mechanism, M_1.

$CATE(X_2 = 1)$ is given by:

$$\begin{aligned} CATE(X_2 = 1) &= E[L(U(Z=1, X)) - L(U(Z=0, X))|X_2 = 1] \\ &= \Pr(2X_1 + 1 > 0) - \Pr(X_1 + 1 > 0) \\ &= \Phi(-1) - \Phi(-\frac{1}{2}) \approx -0.15 \end{aligned}$$

It is straightforward to see that $CATE(X_2 = 0) = \Phi(0) - \Phi(0) = 0$.

HTEs exist with respect to X_2, but we know that $X_2 \in \mathbf{X}^R - \mathbf{X}^{CIV}$.

HTEs as a test of mechanisms (#2 of 2)

Proposition

Suppose that observed outcome $L(Y)$ is a non-linear mapping of directly-affected outcome Y and Assumptions 1 and 2 hold. If HTEs do not exist with respect to X_k, then $X_k \in \mathbf{X}$.

Implication: This is vacuous! Obviously measured covariate X_k is in the set of measured covariates $\mathbf{X}$...

- Without further assumptions about distribution of Y and functional form of $L(\cdot)$ we cannot say anything from a lack of heterogeneity for an indirectly affected outcome!

Summary of results

	Outcome variable is: Directly affected	Outcome variable is: Indirectly affected
$\exists$ HTEs wrt X_k:	$X_k \in \mathbf{X}^{CIV}$	$X_k \in \mathbf{X}^{R}$
	$\implies M_j$ is active.	M_j active or inactive
$\nexists$ HTEs wrt X_k:	$X_k \notin \mathbf{X}^{CIV}$ **and/or** $\nexists$ CIV for mechanism j	$X_k \in \mathbf{X}$ (vacuous)
	M_j active or inactive	M_j active or inactive

Recommendations for Research Design

Recommendation #1: Theoretical questions

Three questions needed to support use of HTEs for mechanism detection:

1. Enumeration of set of candidate mechanisms.

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2. Relationship between a covariate, X_k and each candidate mechanism:
   - $\longrightarrow$ For which mechanism (j) is X_k a candidate CIV?
   - $\longrightarrow$ Is exclusion assumption plausible for every other candidate mechanism?

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2. Relationship between a covariate, X_k and each candidate mechanism:
   - $\longrightarrow$ For which mechanism (j) is X_k a candidate CIV?
   - $\longrightarrow$ Is exclusion assumption plausible for every other candidate mechanism?
3. Classification of mechanisms as *directly affected* or *indirectly affected*
   - $\longrightarrow$ In some theory traditions, requires more theoretical structure.
   - $\longrightarrow$ We should likely focus on HTE for some outcomes but not others.

Recommendation #2: Improving interpretation of HTEs

Absence of HTEs do not "rule out" candidate mechanisms.

- Given a candidate CIV, presence of HTEs is informative only when: exclusion assumptions hold, outcome is directly affected.
- HTEs provide *less information* than is generally asserted by their interpretation.

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Implications of low power for interactions we are less likely to *detect* HTE that do exist.

- Compounds these challenges of interpretation.

Recommendation #3: Improving research design

To use HTEs for mechanism detection, the the more measured candidate CIVs is better.

- We need multiple candidate CIVs to satisfy exclusion assumptions...
- Benefit: mixed results (HTEs in one candidate but not another) resolve ambiguity about existence of CIVs.
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Clearer specification of relationship between mechanisms and outcomes

- Prioritize directly-affected outcomes for mechanism detection
  - $\longrightarrow$ Design measurement instruments to elicit these outcomes.
  - $\longrightarrow$ *Maybe* an argument for latent variable models?

Recommendation #4: Adding assumptions?

Can assumption of monotonicity make HTE more informative in the context of indirectly-observed outcomes?

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Answer: No, not in isolation. We need additional assumptions about:

- The DGP in the form of the empirical distribution of Y or the distribution of any error terms.
- The mapping $L(\cdot)$.

Conclusion

Four takeaways

1. A problem: the use of HTEs for mechanism detection is very popular but under-theorized.
2. HTEs is not a “agnostic” approach to analysis of mechanisms: requires exclusion assumptions.
3. This approach provides information about mechanisms when:
    - Exclusion assumptions hold
    - Outcome is directly affected by a given mechanism.
4. We can better learn about mechanisms HTEs by more carefully approaching these analyses.

Thank You!