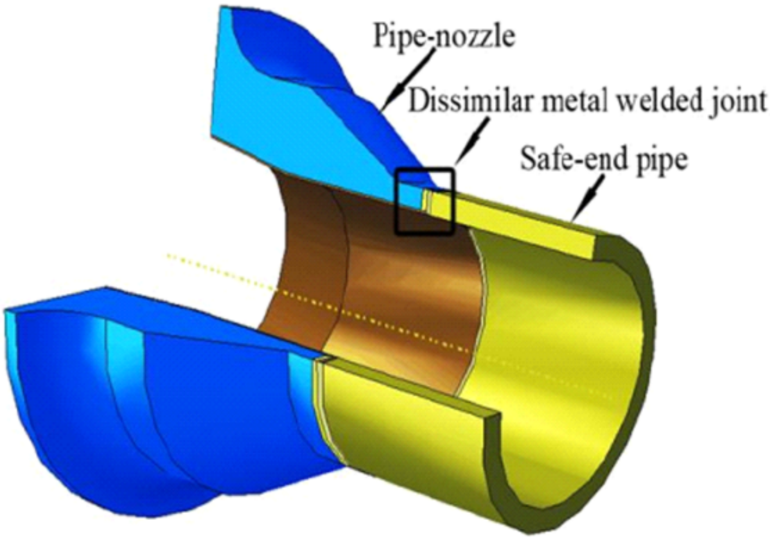
Study of connection between Nozzle and Primary Piping of a PWR



Candidate: Rachele Cottini

October 25, 2024

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1 Introduction

The project involves the study of the nozzle-primary piping connection of a PWR.
The study is divided into sections regarding the different aspects of design and verification.

1.1 Project Data

Internal diameter of piping and nozzle $D_i = 0.8$ m;
Design temperature $T_d = 350$ °C;
Design pressure $p_d = 15$ MPa;
Nozzle stress Intensity $S_m(Td) = 200$ MPa;
Piping stress Intensity $S_m(Td) = 150$ Mpa;

Vessel data:
Inner diameter D_{vi}= 4 m;
Material chosen for the vessel SA508
Young's Modulus E = 200 GPa;
Poisson's Modulus $\nu = 0.3$;
Thermal expansion coefficient $\alpha = 1.2 \times 10^{-6} \quad °C^{-1}$

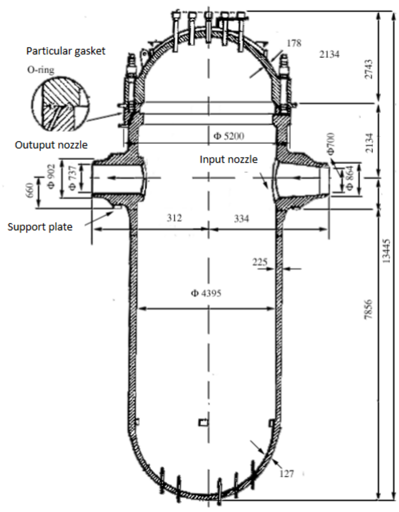


Figure 1: PWR pressure vessel.

2 Sizing of components

Since the value of internal diameter for nozzle and pipe is known, through the application of ASME tentative formula (NB-3324), the **minimum required thickness values** t_r are found:

$$t_r = \frac{pR_i}{S_m - 0.5p} \tag{1}$$

where R_i is the inner radius
From (1) we obtain:

nozzle: $t_{r,n} = 0.031m$
pipe: $t_{r,p} = 0.042m$

As mentioned above, these values are not acceptable as design thicknesses for the components. The wall thicknesses shall never be less than the ones obtained with these equations.

The chosen value for thickness of piping is t_p **= 5 cm**; of course this is the thickness of the region of the nozzle that is connected to the piping through welding.
For what concerns the nozzle, the minimum required thickness is smaller than that of the piping, thanks to the fact that the stress intensity value is higher.

Definition of stress intensity is here recalled:
Stress intensity is defined as twice the maximum shear stress, which is the difference between the algebraically largest principal stress and the algebraically smallest principal stress at a given point.
An acceptable value for nozzle thickness that is also acceptable from the design perspective of the connection would be the same as that of the piping:

$$t_n = 5 \text{ cm}$$

It will be useful for the calculations here after, to find the minimum required thickness for vessel as well. It is possible to use again (1), since the known data is internal diameter D_{vi}.
The required thickness depends on levels of circumferential and axial membrane stresses acting on the cylindrical vessel:

$$t_{r,v,circ} = \frac{pR_{vi}}{S_m - 0.5p} = 16.55cm$$

$$t_{r,v,axial} = \frac{pR_{vi}}{2(S_m - 0.5p)} = 8cm$$

The thickness of vessel must be uniform, so the first value is chosen as the minimum required thickness. Vessel acceptable thickness:

$$t_v = 20 \text{ cm}$$

And the vessel mean radius, that will be useful in the nozzle design is: $R_{mv} = 2.10$ m.

Definitions from ASME III Article 3000 that will be useful for the verification of components:

Primary stress (ASME III NB-3213.8) is any normal stress or shear stress developed by an imposed loading that is necessary to satisfy the laws of equilibrium of external and internal forces and moments. The basic characteristic of a primary stress is that it is not self-limiting.

Secondary stress (ASME III NB-3213.9) is any normal stress or a shear stress developed by the constraint of adjacent material or by self-constraint of the structure. The basic characteristic of a secondary stress is that it is self-limiting. Some example are general thermal stress and bending stress at a gross structural discontinuity.

Membrane stress (NB-3213.6) is the component of normal stress that is uniformly distributed and equal to the average stress across the thickness of the section under consideration.

Bending stress (NB-3213.7) is the component of normal stress that varies across the thickness. The variation may or may not be linear.

2.1 Nozzle

The primary nozzle is a critical component of the reactor vessel. In a four loop-Pressurized Water Reactor, there are height of them: four inlet and four outlet nozzles.
Nozzles connect vessel to primary piping and are designed to withstand the high pressure coolant flow. Furthermore, they are subject to temperature gradient across the thickness.

Composizione percentuale (%)							
Materiale	C	Mn	P	S	Si	Mo	Ni
A533 Gr.B classe1	0.25	1.4	0.015	0.02	0.25	0.5	0.55

Tabella 2.10- Composizione nominale percentuale dell'acciaio A533 Gr.B classe1

The presence of the opening, or more exactly, of the openings, may affect the capacity of the vessel to resist loads, specifically high pressure. Thus, in accordance with ASME NB-3332.1, it must be verified whether the opening on the vessel wall requires reinforcement.
Considering a single opening, no reinforcement is needed if:

$$D_i < 0.2\sqrt{R_{mv}t_v} \tag{2}$$

Where:
$D_i = 0.8$ m is the diameter of the opening (therefore internal diameter of piping and nozzle as well).
$R_{mv} = \frac{D_{vi}}{2} + \frac{t_v}{2} = 2.1$ m is the mean radius of the vessel

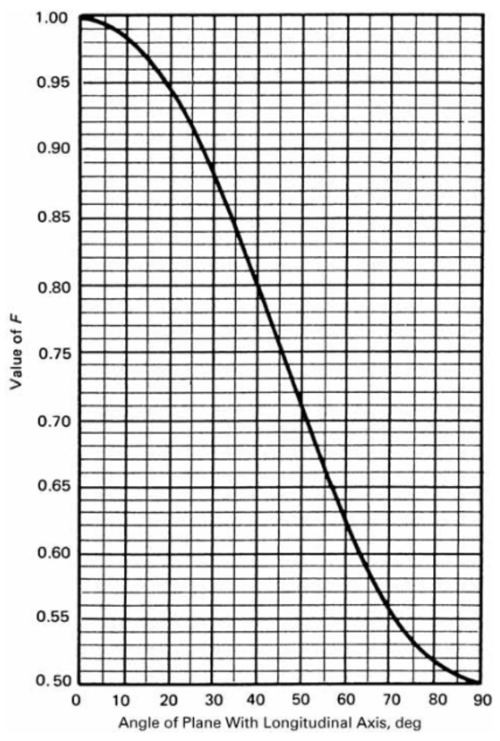
$$0.8 \text{ m} > 0.2\sqrt{2.1 \times 0.2} \text{ m}$$

It is clear that the requirement is not satisfied, reinforcement is needed.
The total cross-sectional area of reinforcement A , required in any given plane for a vessel under internal pressure, shall not be less than:

$$A = Ft_{r,v}D_i \tag{3}$$

F is a correction factor, compensating for the variation in pressure stresses on different plane orientation with respect to the axis of a vessel.
$t_{r,v}$ is the thickness of the vessel which meets the requirements for ASME minimum required thickness, in absence of the opening. It is necessary to use the two values found before for the vessel depending on the considered sectioning.

Figure NB-3332.2-1
Chart for Determining Value of *F*



The value obtained from the chart NB-3332.2-1 for an angle of plane orthogonal to longitudinal axis (axial sectioning) is equal to 0.50, while is equal to 1.00 along the axis of the vessel (circumferential sectioning).

Axial reinforcement area for vessel wall:

$$A_1 = \frac{1}{2}t_{r,v,axial}D_i = 0.03m^2$$

Circumferential reinforcement area for vessel wall:

$$A_2 = t_{r,v,circ}D_i = 0.13m^2$$

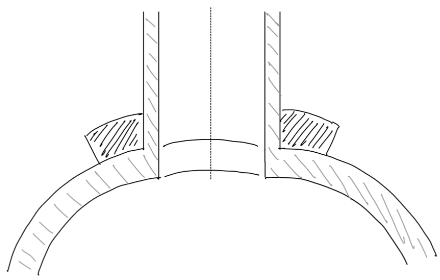


Figure 2: Reinforcement area A_1 obtained by axial sectioning orthogonal to vessel axis

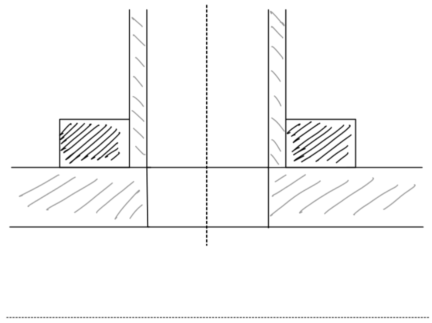


Figure 3: Reinforcement area A_2 obtained by circumferential sectioning parallel to vessel axis

2.1.1 Limits of Reinforcement

These rules offer guidance on how and where the reinforcing material must be allocated around the opening, both in direction of the vessel wall, and normal to it.

1. Limits along the mid surface of the vessel

- 100% of the reinforcement has to be within a distance on each side of the axis of the opening, equal to the greater between:
  - D_i
  - $\frac{D_i}{2} + t_v + t_n$

  Solution is 0.8 m. Reinforcement limit L_1 is within 80 cm around the opening, measuring from the center of it.

- 2/3 of the reinforcement has to be within a distance on each side of the axis of the opening, equal to the greater between:
  - $\frac{D_i}{2} + 0.5\sqrt{R_{mv} t_v} = 0.73$ m
  - $\frac{D_i}{2} + \frac{2}{3}(t_v + t_n) = 0.57$ m

  Two-thirds of reinforcement lies within 73 cm on each side of the center of the opening.

2. Limit along the direction normal to vessel wall.
The chosen geometry for the nozzle corresponds to scheme (d) of Figure NB-3338.2(a)-2 in ASME III. The limits of reinforcement, measured normal to the vessel wall, shall conform to the contour of the surface at a distance from each surface equal to:

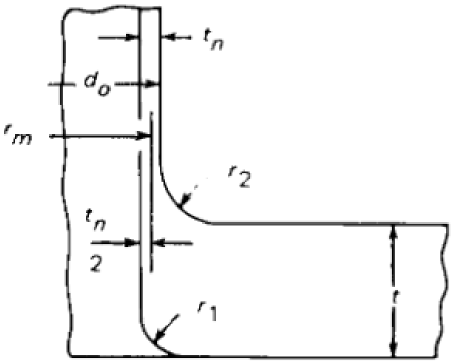
$$L_2 = 0.5\sqrt{r_{m,n} t_n} + 0.5 r_2 = 0.15m$$

$r_{m,n} = r_{i,n} + \frac{t_n}{2} = 0.425$ m
$r_2 = max(\sqrt{D_i t_{min,n}}; t_v/2) = 0.157$ m transition radius between nozzle and wall

Meaning that, measuring from the vessel external wall, the reinforcing material shall contour the piping for 15 cm. From this distance on, the final segment of the nozzle is equal to the primary piping, to which is connected through welding.

To recap, the reinforcement areas A_1, A_2 are known,as well as limits of reinforcement that indicate the regions inside which the reinforcing material must be located.

One thing to bear in mind is that, even though the vessel and the nozzle are designed to be thicker than the minimum required values, these additional thicknesses cannot be considered as participating to the reinforcing material.
This is due to technological reasons, as the material surrounding the nozzle has been subdued to mechanical processes that changed its micro-structure.



(d)

Figure 4: Nozzle geometry

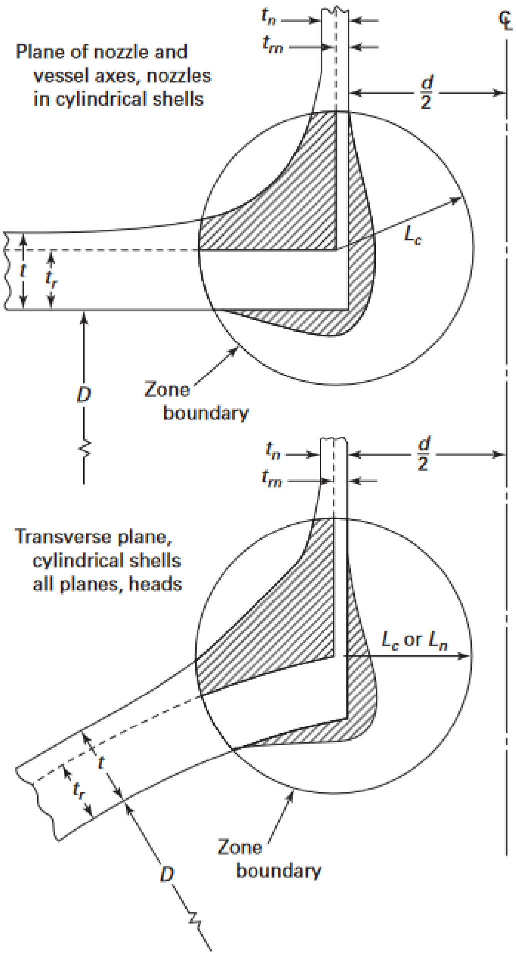


Figure 5: Close up of reinforcing material

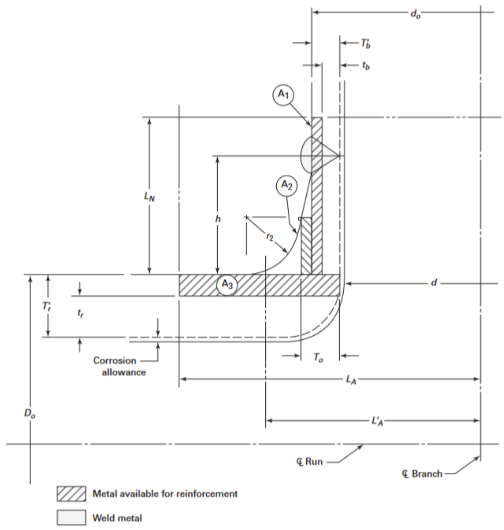


Figure 6: Areas contributing to reinforcement

For the chosen geometry, the thickness of nozzle is constant and equal to 5 cm, so the reinforcing material is generating additional thicknesses t_1^* and t_2^* on the vessel wall, in the axial and circumferential directions.

$$t_1^* = \frac{A_1}{2L_1 - D_i - 2t_n} = 0.04cm$$

$$t_2^* = \frac{A_2}{2L_1 - D_i - 2t_n} = 0.137m = 19cm$$

To uniform the connection, the thickness t_2^* is applied to the vessel wall all around the nozzle. **Thus, in the proximity of the nozzle, the vessel thickness is equal to 39 cm**.

2.1.2 Long cylindrical shell with axisymmetric loading

For the purpose of calculating thermal stresses in the nozzle, this component can be modeled as a **thin cylindrical shell** (in fact $\frac{t_n}{R_m} < 1$), of uniform thickness t_n equal to 5 cm , subject to linear temperature profile and uniform pressure, with clamped edges.

The important approximation that will be used here on, **is that of long cylinder**.
In this component, **the effects of constraints are not negligible, thus it is necessary to use the the general analysis, that includes membrane and bending analysis.**

In particular bending stresses are most pronounced at the discontinuities (edges), in this case represented by the junctions between vessel and nozzle edge, and between the other edge and the edge of the stainless steel pipe (this second case will be treated in 3), for now the analysis will be limited to vessel - nozzle edge.
This treatment of the shell is valid for both stainless steel and carbon steel piping. Thermal and mechanical loadings are axysimmetric.
The shell has mean diameter D_m, length L, wall thickness t and is subject to uniform pressure p and constant temperature gradient ΔT.
Inner Temperature $T_i = 350°C$
Outer Temperature $T_o = 330°C$
$c = \frac{t}{2}$
$z \in [-c, +c]$

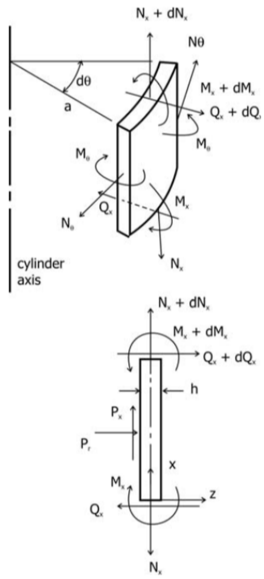


Figure 7: Membrane and bending resultants acting on a cylindrical shell element.

From the balance of forces and moments for the shell, it is possible to write the balance equations, where distributed axial force $p_x = 0$, distributed radial force $p_r = p$:

$$\begin{cases} \frac{dQ_x}{dx} - \frac{N_\phi}{R_m} + p = 0 \\ \frac{dN_x}{dx} = 0 \\ \frac{dM_x}{dx} - Q_x = 0 \end{cases} \tag{4}$$

N_x is constant.
Constitutive equations are needed to solve the general analysis, along with equilibrium equations and congruence equations:

$$\begin{cases} \varepsilon_x = \frac{1}{Et}(N_x - \nu N_\phi) + \frac{N_T}{Et} \\ \\ \varepsilon_\phi = \frac{1}{Et}(N_\phi - \nu N_x) + \frac{N_T}{Et} \end{cases} \tag{5}$$

Thermal stress resultant is given by:

$$N_T = \int_{-c}^{+c} \alpha E \Delta T \, dz = \alpha E \Delta T t \tag{6}$$

Temperature profile is considered linear in the thickness of the pipe (z is the variable of the thickness, with origin in the middle plane):

$$T(z) = \frac{T_i + T_o}{2} - \frac{T_i - T_o}{2} \frac{z}{c} \tag{7}$$

The plane of zero thermal stress is chosen as z = 0, i.e. the middle plane.

Because of this, thermal moment is null:

$$M_T = \int_{-c}^{+c} N_T(z) \, dz = \int_{-c}^{+c} \alpha E \Delta T z \, dz = 0 \tag{8}$$

Solving stress strain relations for the stress resultants:

$$N_x = \frac{Et}{1-\nu^2}(\epsilon_x + \nu\epsilon_\phi) - \frac{N_T}{1-\nu} \tag{9}$$

$$N_\phi = \frac{Et}{1-\nu^2}(\epsilon_\phi + \nu\epsilon_x) - \frac{N_T}{1-\nu} \tag{10}$$

The relations for bending moments resultants are the same as those for plates, because there is no change in radial displacement in circumferential direction:

$$M_x = -K\frac{d^2w}{dx^2} - \frac{M_T}{1-\nu} \tag{11}$$

$$M_\phi = -\nu K\frac{d^2w}{dx^2} - \frac{M_T}{1-\nu} \tag{12}$$

and introducing the flexural rigidity (or bending stiffness) $K = \frac{Et^3}{12(1-\nu^2)}$, the shear resultant:

$$Q_x = -K\frac{d^3w}{dx^3} - \frac{1}{1-\nu}\frac{dM_T}{dx} = -K\frac{d^3w}{dx^3} \tag{13}$$

By doing some passages, the governing equation of the problem is found, it terms of axial displacement w:

$$\frac{d^4w}{dx^4} + \frac{Et}{KR_m^2}w = -\frac{\nu N_x}{KR_m} - \frac{1}{(1-\nu)K}\frac{d^2M_T}{dx^2} + \frac{N_T}{KR_m} + \frac{p}{K} \tag{14}$$

After the fourth-grade differential equation is solved, all the other quantities are found:

$$M_x = -K\frac{d^2w}{dx^2} - \frac{M_T}{1-\nu} = -K\frac{d^2w}{dx^2} \tag{15}$$

$$M_\phi = -\nu K\frac{d^2w}{dx^2} - \frac{M_T}{1-\nu} = -\nu K\frac{d^2w}{dx^2} \tag{16}$$

$$Q_x = -K\frac{d^3w}{dx^3} - \frac{1}{1-\nu}\frac{dM_T}{dx} = -K\frac{d^3w}{dx^3} \tag{17}$$

$$N_x = C \tag{18}$$

$$N_\phi = \frac{Etw}{R_m} + \nu N_x - N_T \tag{19}$$

Axial and circumferential stresses are found from resultants obtained from (14):

$$\sigma_x = \frac{N_x}{t} \pm \frac{6M_x}{t^2} \tag{20}$$

$$\sigma_\phi = \frac{N_\phi}{t} \pm \frac{6M_\phi}{t^2} \tag{21}$$

$$\tau = \frac{3Q}{2t}(1 - \frac{z^2}{t^2}) \tag{22}$$

And the shell rotation in x direction: $\omega(x) = \frac{dw}{dx}$

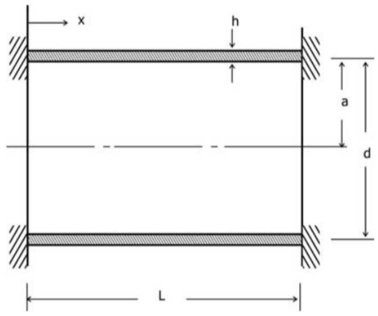


Figure 8: Cylinder with fixed edges

The equation (14), where $p_x = 0$, $N_x = 0$, $M_T = 0$, becomes:

$$\frac{d^4w}{dx^4} + 4\beta^4 w = \frac{N_T}{kR_m} + \frac{p}{K} \tag{23}$$

$\frac{Et}{kR_m^2} = 4\beta^4$, where factor β $\quad \beta = (\frac{3(1-\nu^2)R_m^2}{t^2})^{1/4} = 3.75\frac{1}{m}$
Thermal membrane stress $N_T = \alpha E \Delta T t = 2.4MN/m = 2.4 \times 10^3$ N/mm, constant.

The solution to the displacement equation may be written as the sum of the homogeneous equation and particular solution:

$$w(x) = w_{hom}(x) + w_{part} \tag{24}$$

$$\begin{cases} w_{hom} = e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + e^{\beta x}(C_3 \cos\beta x + C_4 \sin\beta x) \\ w_{part} = \frac{1}{4\beta^4}(\frac{N_T}{kR_m} + \frac{p}{K}) \end{cases} \tag{25}$$

Using the long cylinder approximation, the positive exponential term of (25) must be zero, thus $C_3 = C_4 = 0$:

$$\begin{aligned} w(x) &= e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + \frac{1}{4\beta^4}(\frac{N_T}{KR_m} + \frac{p}{K}) = e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + \frac{N_T R_m}{Et} + \frac{pR_m^2}{Et} \\ &= e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + \frac{pR_m^2}{Et} + \alpha R_m \Delta T \end{aligned} \tag{26}$$

And the expression for the slope or rotation of the shell in x direction:

$$\omega(x) = \frac{dw}{dx} = \beta e^{-\beta x}[(-C_1 + C_2)\cos\beta x - (C_1 + C_2)\sin\beta x] \tag{27}$$

The remaining constants are found from the boundary conditions, considering the shell clamped in x = 0:
- $w(0) = 0$;
- $\omega(0) = 0$.

$$C_1 = C_2 = -[\frac{pR_m^2}{Et} + \alpha R_m \Delta T]$$

The expression for the radial deflection and axial rotation are:

$$w(x) = (\frac{pR_m^2}{Et} + \alpha R_m \Delta T)(1 - e^{-\beta x}(\cos\beta x + \sin\beta x)) \tag{28}$$

$$\omega(x) = \frac{dw}{dx} = 2\beta e^{-\beta x}\sin\beta x(\frac{pR_m^2}{Et} + \alpha R_m \Delta T) \tag{29}$$

For $x = \frac{1}{\beta} = 0.26$ m = 26 cm, called **extinction length**, the effect of bending in the displacement w is damped out:

$$w(\beta) = \frac{pR_m^2}{Et} + \alpha R_m \Delta T = 1.2 \times 10^{-4} m = 0.12mm$$

And the maximum displacement for $\beta = \frac{\pi}{4}$:

$$w_{max}(\frac{\pi}{4}) = (\frac{pR_m^2}{Et} + \alpha R_m \Delta T)(1 - e^{-\beta\frac{\pi}{4}}(\sqrt{2})) = 0.119mm$$

The bending moments, obtained from the equilibrium equations and the solution of (11), have the following expressions:

$$M_x = -K\frac{d^2w}{dx^2} = K(\frac{pR_m^2}{Et} + \alpha R_m \Delta T)2\beta^2 e^{-\beta x}(\sin(\beta x) - \cos\beta x)$$

$$M_\phi = -\nu K\frac{d^2w}{dx^2} = \nu(K(\frac{pR_m^2}{Et} + \alpha R_m \Delta T)2\beta^2 e^{-\beta x}(\sin(\beta x) - \cos\beta x))$$

From which it is possible to find the edge bending moments:

$$M_x(0) = -K(\frac{pR_m^2}{Et} + \alpha R_m \Delta T)2\beta^2 = -24020N = -2.4 \times 10^4 N$$

$$M_\phi(0) = -\nu K(\frac{pR_m^2}{Et} + \alpha R_m \Delta T)2\beta^2 = -7206N = -7.2 \times 10^3 N$$

From (12) to (14), it is possible to find shear force and normal forces:

$$Q_x = 4K\beta^3(\frac{pR_m^2}{Et} + \alpha R_m \Delta T)e^{-\beta x}cos(\beta x)$$

$$N_x = 0$$

$$N_\phi = \frac{Et}{R_m}(\frac{pR_m^2}{Et} + \alpha R_m \Delta T)(1 - e^{-\beta x}(\cos\beta x + \sin\beta x)) - \alpha E \Delta T t$$

As can be seen from figure 9, the internal surface of the shell is the most solicited, with stresses:

$$\sigma_x = \frac{N_x}{t} + \frac{6M_x}{t^2} \tag{30}$$

$$\sigma_\phi = \frac{N_\phi}{t} + \frac{6M_\phi}{t^2} \tag{31}$$

The most stressed points of the region are the ones that belong to the internal surface at coordinate x = 0, i.e. the connection to the vessel.

$$\sigma_x(0) = -6\frac{K(\frac{pR_m^2}{Et} + \alpha R_m \Delta T)2\beta^2}{t^2} = -57.6MPa \tag{32}$$

$$\sigma_\phi(0) = \frac{1}{t}(-\alpha E \Delta T t) - 6\nu\frac{K(\frac{pR_m^2}{Et} + \alpha R_m \Delta T)2\beta^2}{t^2} = -48MPa \tag{33}$$

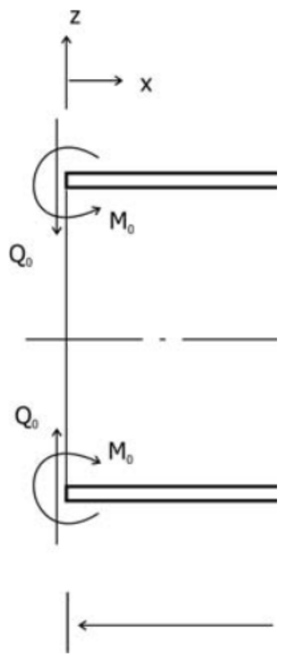


Figure 9: Edge bending moments and shear forces at x = 0

2.1.3 Nozzle Stress Intensity

Thickness t = 5 cm;
Mean radius $R_m = 0.425$ m;

Using Tresca criterion in x = 0:

$$S_{nozzle} = max(|\sigma_x(0) - \sigma_\phi(0)|, |\sigma_x(0)|, |\sigma_\phi(0)|) = 57.6MPa$$

2.2 Primary Piping

The primary piping is the pressure boundary of the NPP. It is subject to uniform pressure and the same temperature gradient considered for the nozzle.
The chosen material is stainless steel 316, austenitic steel with high corrosion resistence.

	C % ≤	Mn % ≤	P % ≤	S % ≤	Si % ≤	Cr %	Ni %	Mo %	altro %
AISI 316	0,07	2	0,045	0,015	1	16,5 ÷ 18,5	10 ÷ 13	2 ÷ 2,5	N ≤ 0,11

Figure 10: Steel composition for SS 316

In this case it is possible to use the membrane theory of shells, because the stresses are evaluated far from the junction with the nozzle. Furthermore, the approximation of long cylinder is valid for this piping too.
Thus, shear forces and bending moments are not considered and the membrane forces in axial and circumferential directions are obtained through the solution of static equilibrium equations for thin cylinders.

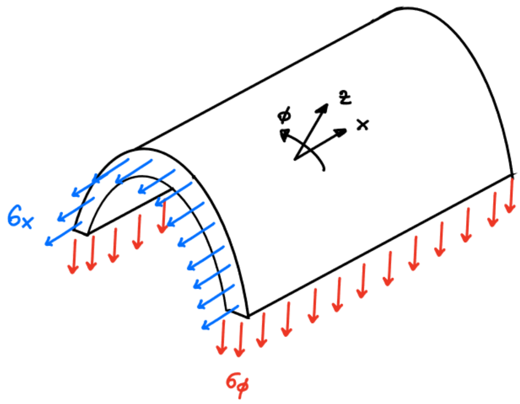


Figure 11: Membrane stresses representation

2.2.1 Membrane stresses evaluation

Membrane stresses:

$$N_{\phi,m} = pR_m = 6.375 MN/m = 6.375 \times 10^3 N/mm \tag{34}$$

$$N_{x,m} = \frac{pR_m}{2} = 3.19 MN/m = 3.19 \times 10^3 N/mm \tag{35}$$

$$\sigma_{\phi,m} = \frac{N_\phi}{t} = 127.5 MPa \tag{36}$$

$$\sigma_{x,m} = \frac{N_x}{t} = 63.8 MPa \tag{37}$$

2.2.2 Primary piping thermal stresses

These stresses are classified as secondary stresses Q in ASME service limits calculation. Temperature profile is considered linear in the thickness of the pipe (z is the variable of the thickness, with origin in the middle plane):

$$T(z) = \frac{T_i + T_o}{2} - \frac{T_i - T_o}{2}\frac{z}{c} \tag{38}$$

Inner Temperature $T_i = 350°C$
Outer Temperature $T_o = 330°C$
$c = \frac{t}{2}$
$z \in [-c, +c]$

The solution of thermal stresses is the same as that found for the nozzle, for infinitely long thin shells:

$$N_T = \int_{-c}^{+c} \alpha E \Delta T \, dz = \alpha E \Delta T t$$

$$\sigma_{\phi,T} = \sigma_{x,T} = E\alpha\Delta T \tag{39}$$

2.2.3 Piping Stress Intensity

$$\sigma_{max,T} = |\sigma_{\phi,T}| = |\sigma_{x,T}| = 72 \quad MPa$$

$$\sigma_{x,max} = \sigma_{x,m} + \sigma_{x,T} = 135.80 MPa$$

$$\sigma_{\phi,max} = \sigma_{\phi,m} + \sigma_{\phi,T} = 199.50 MPa$$

Using Tresca criterion, on the outer surface which results the most stressed, the stress intensity value is calculated:

$$S_{pipe} = max(|\sigma_{x,max} - \sigma_{\phi,max}|, |\sigma_{x,max}|, |\sigma_{\phi,max}|) = 199.50MPa$$

3 Stress state evaluation at nozzle-tube connection

Design pressure $p = 15MPa$;
Temperature at mean surface $T_m = 340°\text{C}$;
Mean cylinder radius $R_m = 0.425$ m;
Cylinders' thickness $t = 0.05$m;
Thermal expansion coefficient Stainless steel pipe $\alpha_p = 1.8 \times 10^{-5} \quad ^\circ C^{-1}$;
Thermal expansion coefficient carbon steel nozzle $\alpha_n = 1.2 \times 10^{-5} \quad ^\circ C^{-1}$;
Poisson's Modulus (of both SS and CS cylinders) $\nu = 0.3$;
Young's Modulus (of SS and CS) $E = 200GPa$;
Elongation stiffness $D = \frac{Et}{1-\nu^2} = 1.10 \times 10^{10}$N/m;
Bending stiffness $K = \frac{Et^3}{12(1-\nu^2)} = 2.29 \times 10^6$Nm ;
Factor β $\beta = (\frac{3(1-\nu^2)R_m^2}{t^2})^{1/4} = 3.75m^{-1}$

The present case can be modeled as two cylinders of different linear thermal expansion coefficients, linked by a welding. The two edges are considered fixed together.
The loads to be considered are internal pressure and thermal stresses. As proven in previous sections, membrane stresses are the same for the two components, thus pressure causes the same displacements.

Since the two cylinders present different thermal expansion, it is necessary to guarantee the equivalence of displacements and of rotations.
To assure congruence is complied with, shear forces and distributed moments, are generated as reactions in the junction.

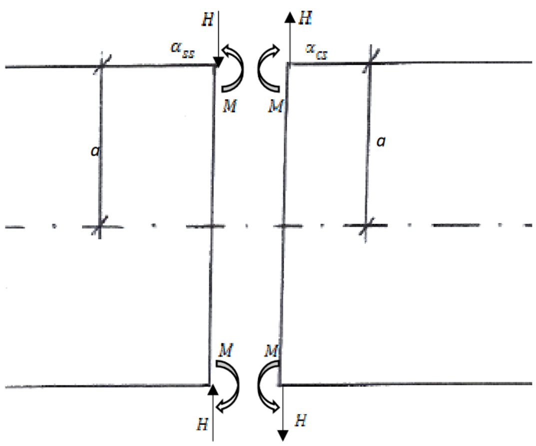


Figure 12: Model of connection

1. Congruence of displacements must be imposed:

$$w_{n,memb} + w_{n,Q_0} + w_{n,M_0} = w_{p,memb} + w_{p,Q_0} + w_{p,M_0} \tag{40}$$

The terms of displacement can be obtained by **superposition of effects**:

- $w_{n,memb}$ is the displacement due to the combination of pressure and thermal effects, it corresponds to (28), calculated in x = 0.

- w_{n,Q_0} is the displacement obtained considering only the shear force acting on the junction. In this case the two shells can be modeled as infinitely long cylinders subject to shear force at the edge.

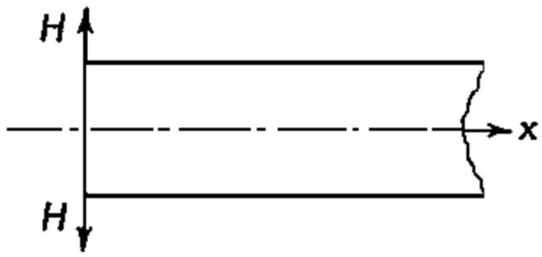


Figure 13: Infinite cylinder subject to shearing force Q_0

- w_{n,M_0} is obtained considering only the moment. In this case the shells are modeled as two infinite cylinders subject to moment M_0 at the edge.

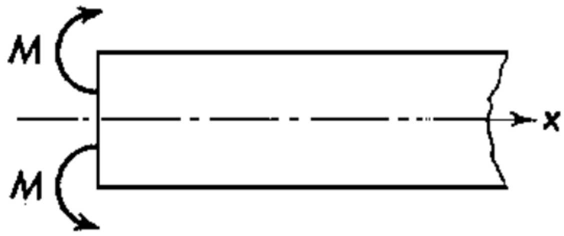


Figure 14: Infinite cylinder subject to moment M_0

Thus obtaining:

$$\begin{cases} w_{n,memb} = \frac{pR_m^2}{Et} + \alpha_n R_m \Delta T \\ w_{n,Q_0} = \frac{Q_0}{2K\beta^3} \\ w_{n,M_0} = \frac{M_0}{2K\beta^2} \end{cases} \qquad \begin{cases} w_{p,memb} = \frac{pR_m^2}{Et} + \alpha_p R_m \Delta T \\ w_{p,Q_0} = -\frac{Q_0}{2K\beta^3} \\ w_{p,M_0} = \frac{M_0}{2K\beta^2} \end{cases} \tag{41}$$

2. Congruence of the rotations:

$$\omega_{n,memb} + \omega_{n,Q_0} + \omega_{n,M_0} = -(\omega_{p,memb} + \omega_{p,Q_0} + \omega_{p,M_0}) \tag{42}$$

But, since $\omega(x) = \frac{dw}{dx}$, the membrane rotation is zero and all the other terms are the same for both components, the equation reduces to:

$$M_0 = 0 \tag{43}$$

Finally, shear force and moment at the junction are found:

$$\begin{cases} Q_0 = 2K\beta^3 \Delta\alpha R_m \Delta T = 12317.7N/m = 12.32N/mm \\ M_0 = 0 \end{cases}$$

The results show that the moment on the junction is equal to zero, while the shear stress imposes a radial stretching to the carbon steel nozzle (that has a lower expansion capability), and a compressive action to the stainless steel pipe.
The two shells behave as the cylinder of Figure (13).
Now, it is useful to define two new coordinates parallel to x: ξ for the primary tube, has the same verse of x, χ for the nozzle with opposite verse.
To calculate the stress characteristics, it is necessary to impose the boundary conditions for both components:

$$\begin{cases} M_\chi(0) = 0 \\ Q_\chi(0) = 2K\beta^3 \Delta\alpha R_m \Delta T \end{cases} \qquad \begin{cases} M_\xi(0) = 0 \\ Q_\xi(0) = -2K\beta^3 \Delta\alpha R_m \Delta T \end{cases} \tag{44}$$

Thus the characteristics are obtained:

- for the carbon steel nozzle:

$$\begin{cases} w(\chi) = \frac{pR_m^2}{Et} + \alpha_n R_m \Delta T + \frac{Q_0}{2K\beta^3} e^{-\beta\chi} \sin(\beta\chi + \frac{\pi}{2}) \\ Q_\chi = \sqrt{2} Q_0 e^{-\beta\chi} \sin(\beta\chi - \frac{\pi}{4}) \\ M_\chi = \frac{Q_0}{\beta} e^{-\beta\chi} \sin(\beta\chi) \\ N_{\phi,\chi} = \frac{Et}{R_m}(\frac{pR_m^2}{Et} + \alpha_n R_m \Delta T + \frac{Q_0}{2K\beta^3} e^{-\beta\chi} \sin(\beta\chi + \frac{\pi}{2})) - \alpha_n E \Delta T t \end{cases}$$

In $\chi = 0$, edge for the nozzle:

$$w(0) = \frac{pR_m^2}{Et} + \alpha_n R_m \Delta T = 3.72 \times 10^{-4} m = 0.37mm$$
$$Q_(\chi = 0) = Q_0 = 12.32 N/mm$$
$$M_0 = 0$$
$$N_{\phi,\chi=0} = \frac{Et}{R_m}(\frac{pR_m^2}{Et} + \alpha_n R_m \Delta T + \frac{Q_0}{2K\beta^3}) - \alpha_n E \Delta T t = 7.6 MN/m = 7.6 \times 10^3 N/mm$$

- for the stainless steel pipe:

$$\begin{cases} w(\xi) = \frac{pR_m^2}{Et} + \alpha_p R_m \Delta T - \frac{Q_0}{2K\beta^3} e^{-\beta\xi} \sin(\beta\xi + \frac{\pi}{2}) \\ Q_\xi = -\sqrt{2} Q_0 e^{-\beta\xi} \sin(\beta\xi - \frac{\pi}{4}) \\ M_\xi = -\frac{Q_0}{\beta} e^{-\beta\xi} \sin(\beta\xi) \\ N_{\phi,\xi} = \frac{Et}{R_m}(\frac{pR_m^2}{Et} + \alpha_p R_m \Delta T - \frac{Q_0}{2K\beta^3} e^{-\beta\xi} \sin(\beta\xi + \frac{\pi}{2})) - \alpha_p E \Delta T t \end{cases}$$

In $\xi = 0$, edge for the pipe:

$$w(0) = \frac{pR_m^2}{Et} + \alpha_p R_m \Delta T = 0.42mm$$
$$Q_(\xi = 0) = Q_0 = 12.32 N/mm$$
$$M_0 = 0$$
$$N_{\phi,\xi=0} = \frac{Et}{R_m}(\frac{pR_m^2}{Et} + \alpha_p R_m \Delta T - \frac{Q_0}{2K\beta^3}) - \alpha_p E \Delta T t = 5.18 MN/m = 5176 N/mm$$

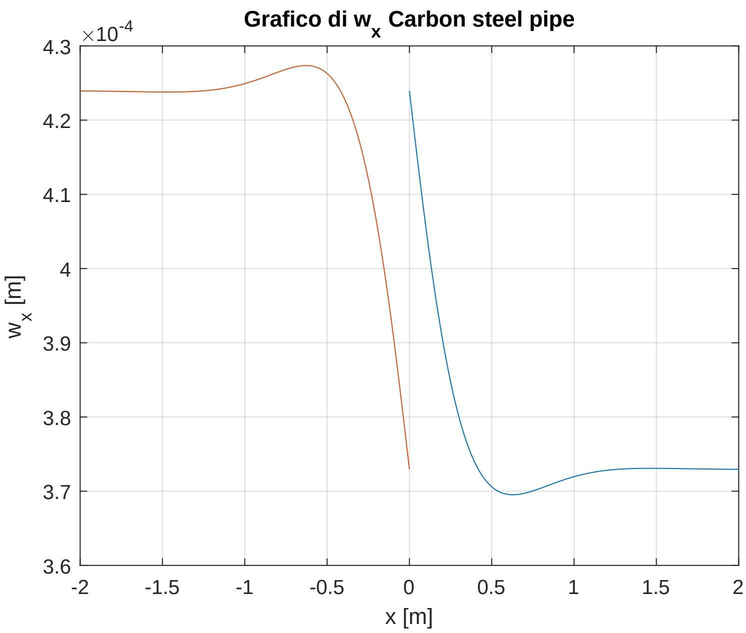
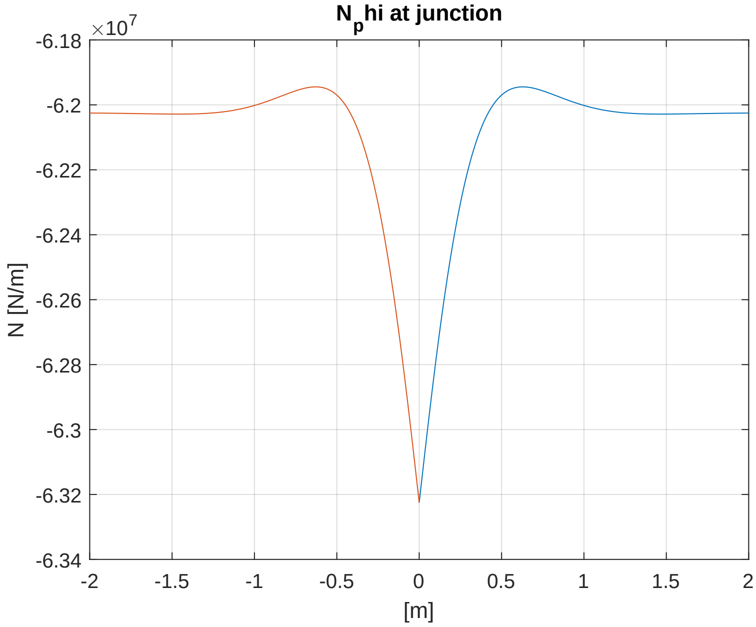
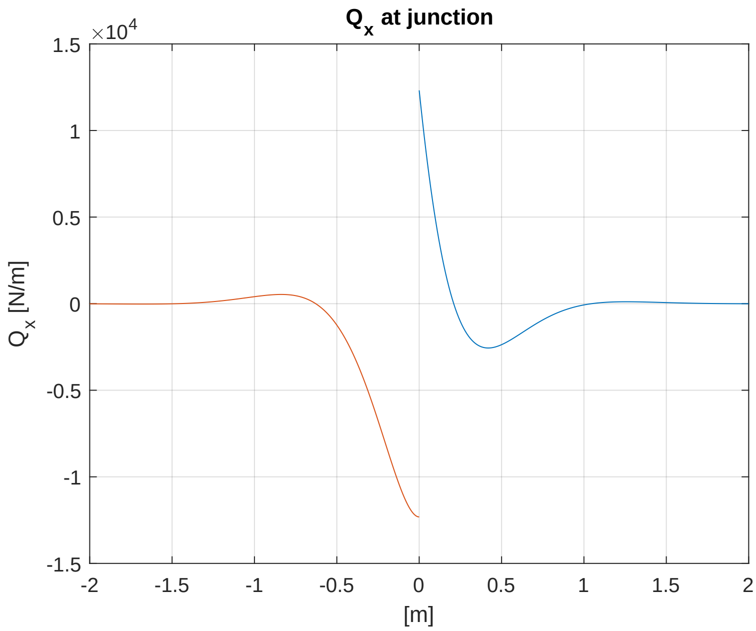


Grafico di w x Carbon steel pipe
×10-4
4.3
4.2
4.1
4
3.9
3.8
3.7
3.6
w x [m]
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
x [m]


N_phi at junction
×10^7
-6.18
-6.2
-6.22
-6.24
-6.26
-6.28
-6.3
-6.32
-6.34
N [N/m]
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
[m]


Q_x at junction
×10^4
1.5
1
0.5
0
-0.5
-1
-1.5
Q_x [N/m]
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
[m]

3.1 Stress Intensity evaluation at the junction

The internal surface of the shell is the most solicited, with stresses:

$$\sigma_x = \frac{N_x}{t} + \frac{6M_x}{t^2}$$
$$\sigma_\phi = \frac{N_\phi}{t} + \frac{6M_\phi}{t^2}$$

At the junction, ie $\chi = \xi = 0$:
Nozzle:

$$\sigma_x(0) = 0$$
$$\sigma_\phi(0) = \frac{1}{t}(\frac{Et}{R_m}(\frac{pR_m^2}{Et} + \alpha_n R_m \Delta T + \frac{Q_0}{2K\beta^3}) - \alpha_n E\Delta T t) = 152MPa$$

Primary pipe:

$$\sigma_x(0) = 0$$
$$\sigma_\phi(0) = \frac{1}{t}(\frac{Et}{R_m}(\frac{pR_m^2}{Et} + \alpha_p R_m \Delta T - \frac{Q_0}{2K\beta^3}) - \alpha_p E\Delta T t) = 103.6MPa$$

The most stressed side of the junction is that of the nozzle.
Using Tresca criterion in x = 0:

$$S_{nozzle} = max(|\sigma_x(0) - \sigma_\phi(0)|, |\sigma_x(0)|, |\sigma_\phi(0)|) = 152MPa$$

In the previous calculation, shear stress has been neglected because it has a negligible contribution to stresses:
$\tau_{max} = \frac{3Q_0}{2t} = 0.37MPa$

3.2 Stress intensity at the point of maximum bending moment

While bending moments are zero at the junction, they may give a significant contribution to stress at a certain distance to the edges of the components.
Therefore, the maximum values of M_x, M_ϕ are calculated:

nozzle: $M_{\chi,n,max} = max(M_\chi) = -1.06 \times 10^3$ N, at $\chi = 0.21$ m.
pipe: $M_{\xi,p,max} = max(M_\xi) = 1.06 \times 10^3$ N, at $\xi = 0.21$ m.

Written in the original coordinate x:
nozzle: $M_{x,n,max} = -1.06 \times 10^3$ N, at $x = -0.21$ m.
pipe: $M_{x,p,max} = 1.06 \times 10^3$ N, at $x = 0.21$ m.

And $M_{\phi,max} = \nu M_{x,max}$:
nozzle: $M_{\phi,n,max} = -318$ N, at $x = -0.21$ m.
pipe: $M_{\phi,p,max} = 318$ N, at $x = 0.21$ m.

Stresses are:

$$\sigma_x = \frac{N_x}{t} + \frac{6M_x}{t^2}$$
$$\sigma_\phi = \frac{N_\phi}{t} + \frac{6M_\phi}{t^2}$$

Nozzle side's stresses at $x = -0.21$ m (21 cm):
$\sigma_{x,n} = (63.8 - 2.54)MPa = 61.26$ MPa
$\sigma_{\phi,n} = (127.5 - 0.76)MPa = 126.74$ MPa

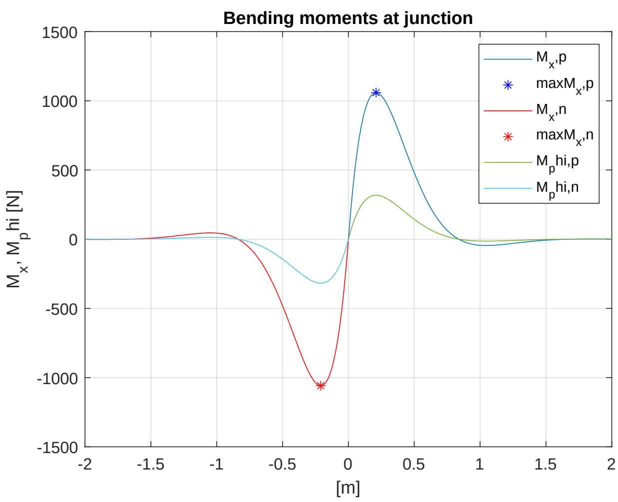
Using Tresca criterion:

$$S_{nozzle} = 126.74 \text{ MPa}$$

Piping side's stresses at $x = 0.21$ m (21 cm):
$\sigma_{x,p} = (63.8 + 2.54)MPa =$ MPa
$\sigma_{\phi,p} = (127.5 + 0.76)MPa = 128.26$ MPa

Using Tresca criterion:

$$S_{pipe} = 128.26 \text{ MPa}$$



4 Limit Analysis of components

In the frame of limit analysis, the material is considered **rigid perfectly plastic**, meaning that it presents no deformations until the yield stress is reached.
In the words of ASME NB-3213.28:
"Limit analysis is used to compute the maximum load that structure assumed to be made of ideally plastic material can carry. At this load, which is termed the collapse load, the deformations of the structure increase without bound."
Defining collapse load as p_0, for load distributions:

$p < p_0$, the structure remains rigid;
$p > p_0$, no equilibrium configuration exists.

The model to be analyzed for both components, is again that of thin cylindrical shell, with mean radius R_m and length L, subject to uniform pressure and constant temperature gradient.

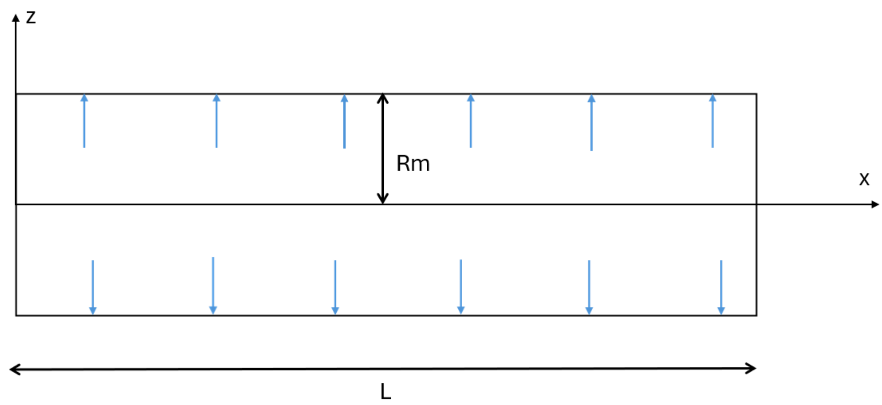


Figure 15: Scheme of cylinder subject to internal pressure p.

The analysis focuses on regions of the two components far from discontinuities, that is after the extinction length for bending effects at the junctions equal to:

- $x = \frac{1}{\beta} = 0.26$ m, at the junction between vessel and nozzle;
- $x = \pm\frac{\pi}{2}\frac{1}{\beta} = \pm 0.41$ m, on each side of the junction between nozzle and primary tube.

By application of equilibrium equations, considering the bending effects null, it is possible to obtain the following results:

$$\begin{cases} \frac{N_\phi}{R_m} - p_r = 0 \\ \frac{dN_x}{dx} = 0 \\ Q_x = 0 \\ M_x = 0 \end{cases}$$

The model is a shell without bottoms, so $p_x = 0$, $p_r = p$ and $N_x = constant = 0$.
In limit analysis, it is customary to work with equations expressed in dimensionless form:

$$n_\phi = \frac{N_\phi}{N_0} \tag{45}$$

$$n_x = \frac{N_x}{N_0} \tag{46}$$

$$s = \frac{Q_x}{N_0} \tag{47}$$

$$d_{int} = \frac{D_{int}}{2\pi N_0 R_m^2} \tag{48}$$

$$d_{ext} = \frac{D_{ext}}{2\pi N_0 R_m^2} \tag{49}$$

Where, $N_0 = \sigma_y t$ is the maximum normal stress up to which the shell can be loaded in uniaxial tensile loading, and $M_0 = \sigma_y \frac{t^2}{4}$ is the maximum bending moment in pure bending (in this case this quantity will not be used).
Concerning kinematic equations for cylindrical shells, because of symmetry of revolution, circumferential displacements v vanish, whereas longitudinal displacement rate and radial displacement rate, are functions of x only.
Thus, the kinematic equations, or the components of the vector strain rate become:

$$\dot{\varepsilon_\phi} = -v_n = -w \tag{50}$$

$$\dot{k_x} = -\frac{d^2 w}{dx^2} = -w'' \tag{51}$$

As already stated, $n_x = 0$, so the yield surface is reduced to:

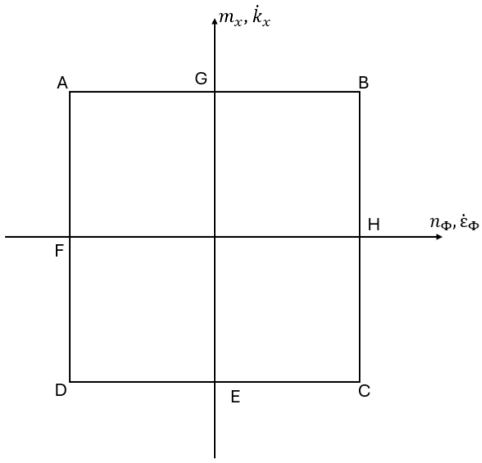


Figure 16: Yield surface for the shell without ends

With the addition of the yield curve to the equilibrium equation and the strain rate vector, it is possible to find the collapse load p_0 for the two components.

Since the shell is subject to internal pressure, it is safe to assume $n_\phi = 1$. In Figure (16), it corresponds to the segment BC.
Also according to the elastic solution, $m_x = 0$.

The two conditions above, correspond to the specific point H in figure (16), and strain rate vector is defined as:

$$\vec{q} = \mu(1, 0) \tag{52}$$

By substituting $n_\phi = 1$ and $m_x = 0$ in the equilibrium equations system, it is possible to find p^-:

$$p^- = \frac{N_0}{R_m}$$

Necessary and sufficient condition that a given load is the collapse load is that it exists for this one a statically allowable stress state stress and a kinematically allowable rate field.

The stress state is statistically allowable for p^-, because it verifies:

- the equilibrium equations;
- the strain condition;
- the boundary conditions.

Thus the load found before corresponds to the lower bound of the collapse load p_0 :

$$p^- \leq p_0.$$

To verify that the velocity field is also kinematically admissible, thus the upper limit theorem is verified as well, it must be true that $D_{int} > 0$:
Internal power:

$$D_{int} = 2\pi R_m \int_0^L (N_\phi \dot{\varepsilon_\phi} + M_x \dot{k_x})\, dx \tag{53}$$

Since $M_x = 0, \quad n_\phi = \frac{N_\phi}{N_0} = 1$:

$$D_{int} = 2\pi R_m \int_0^L (N_0 \dot{\varepsilon_\phi})\, dx = 2\pi R_m N_0 \mu L > 0$$

The condition is verified. By putting also:

$$D_{int} = D_{ext},$$

Where external power is:

$$D_{ext} = 2\pi R_m \int_0^L pw\, dx = 2\pi R_m p \mu R_m$$

The theorem of the upper limit is verified for $p_0^+ = \frac{N_0}{R_m}$, defined as the upper bound of the collapse load. Since two bounds are coincident, p_0 is the collapse load.

It is possible to notice that the collapse load found is independent on length of the cylinder, and on bending moment M_0. This is consistent with the fact that the analysis has been conducted on the central region of the shells, far from the ends where instead the two quantities would play an important role on collapse load quantification.

4.0.1 Nozzle collapse load

$\sigma_y = 350MPa$
$p_0 = \frac{N_0}{R_m} = 42MPa$

Where:
$N_0 = \sigma_y t = 17.5MN/m$

4.0.2 Primary tube collapse load

$\sigma_y = 205MPa$
$p_r = \frac{N_0}{R_m} = 24.10MPa$

Where:
$N_0 = \sigma_y t = 10.25MN/m$

5 Verification of fracture limits at the junction

The appendix G of ASME III provides methods to verify components at LEFM in case of design. **At this point of the study, the component is not built yet.**

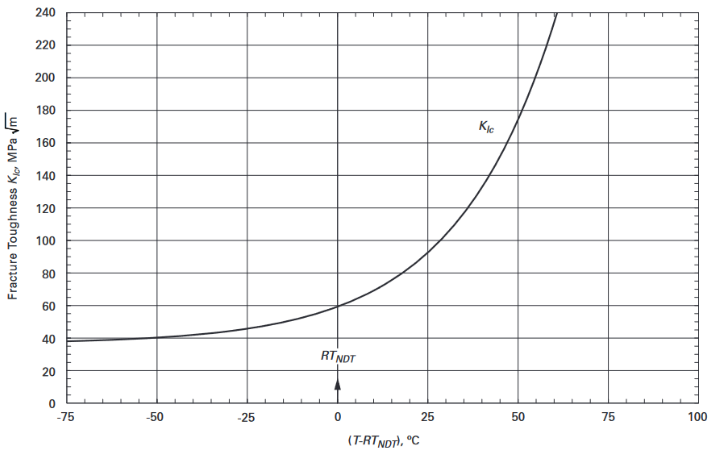
The principles of LEFM are used in this project to verify that the components, in correspondence of the discontinuity, are able to bear the design loads, without reaching the condition for unlimited crack growth.
This limit is represented by the value of K_{Ic}, which can be considered a property of the specific material and is defined as the fracture toughness of the material i.e the resistance opposed by the material to the critical propagation of the defect.

An alternative to K_{Ic} elaborated in ASME III is the reference fracture toughness K_{Ir} , obtained using both Charpy V and drop weight tests. However, since the latter is more conservative than the former, the results found will be valid for K_{Ic} as well, therefore it will be the reference value of confrontation.

K_{Ic} is calculated as a function of the variable $RT = T - RT_{NDT}$, being:

1. T : operating temperature in ($°F$);
2. RT_{NTD}: reference temperature obtained following the procedure in NB-2331 ASME III).

Figure 17: $K_{I,c}$ for AS508 cl.2 as function of $T - RT_{NDT}$



In the present case, the variable RT is much bigger than the baseline, so the selected value for K_{Ic} is the so defined 'top shelf value', equal to $170ksi\sqrt{in}$, or $186.8MPa\sqrt{m}$ for the chosen steel (AS508 cl.2), and equal to $120ksi\sqrt{in}$ or $131.6MPa\sqrt{m}$ for stainless steel 316.

The relation to be satisfied in presence of discontinuity is the following:

$$2K^P_{I,m} + 2K^P_{I,b} + K^Q_{I,m} + K^Q_{I,b} < K_{I,c} \quad (54)$$

Where:

- $K^P_{I,m} = M_m\sigma_m$ is the stress intensity factor corresponding to primary membrane stress σ_m;
- $K^P_{I,b} = M_b\sigma_b$, stress intensity factor corresponding to primary bending stress;
- $K^Q_{I,m}$, stress intensity factor corresponding to secondary membrane stress;
- $K^Q_{I,b}$, stress intensity factor corresponding to secondary bending stress.

M_m and $M_b = \frac{2}{3}M_m$ are two factors function of $\sqrt{mm}$ and are connected with the type of crack considered.
From LEFM theory, **it is known that the presence of a defect of adequate dimensions causes the critical propagation of the fracture. The crack is assumed normal to the maximum stresses. The sizes of this crack are function of the component thickness, t**.

In case of the junction, $t <$ 4", the critical dimensions are: depth a = 1" = 2.54 cm, length 2c = 15.24 cm.

Componet thickness T (inch)	Crack	
	Depth (a)	Length (2c)
T < 4"	1"	6"
4"< T ≤ 12"	$\frac{1}{4}$T	1.5T
T > 12"	3"	18"

Figure 18: Crack dimensions as function of component thickness

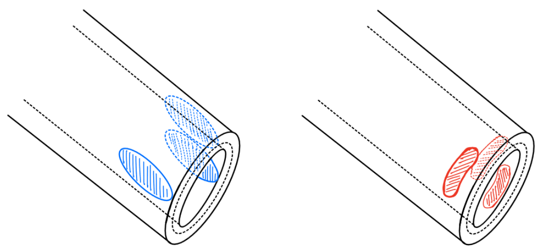


Figure 19: Representation of different types of cracking considered for the shell: in order axial cracking and circumferential cracking.

The value of M_m is function not only of dimension, but also of orientation of the flaw, with respect to the component axis and in case of shells, of position with respect of external and internal surfaces. Therefore, the relation (16) must be verified for all the possible combination of the above crack characteristics, that are listed below together with the corresponding M_m, M_b:

1. Axial cracking
   - Internal surface - $M_m = 0.296\sqrt{mm}$ $\qquad M_b = 0.197\sqrt{mm}$
   - External surface - $M_m = 0.285\sqrt{mm}$ $\qquad M_b = 0.190\sqrt{mm}$
   - Middle surface - $M_m = 0.291\sqrt{mm}$ $\qquad M_b = 0.194\sqrt{mm}$
2. Circumferential cracking
   - Internal surface - $M_m = 0.141\sqrt{mm}$ $\qquad M_b = 0.094\sqrt{mm}$
   - External surface - $M_m = 0.141\sqrt{mm}$ $\qquad M_b = 0.094\sqrt{mm}$
   - Middle surface - $M_m = 0.141\sqrt{mm}$ $\qquad M_b = 0.094\sqrt{mm}$

The value of M_m at middle surface is calculated through linear approximation in the thickness, using the formula:

$$M(z) = \frac{M_{m,int} + M_{m,ext}}{2} - \frac{M_{m,int} - M_{m,out}}{2}\frac{z}{c}$$

$c = \frac{t}{2}$
$z \in [-c, +c]$

It must be kept in mind that pedix "I" in K_I stands for the opening mode of the cracking, that is due to the membrane stress component normal to the length of the crack.
Thus, in the calculation of the stress intensity factors due to membrane stresses, in case of axial cracking σ_ϕ will be used, in case of radial cracking, σ_x.
Finally it is possible to calculate the stress intensity factors using the results found in previous sections.

5.1 Nozzle side of junction

Primary membrane stress intensity factor: $K_{I,m}^P = M_m\sigma_m^P$, σ_m is either σ_x, σ_ϕ ;
Primary bending stress is null;
Secondary membrane stress intensity factor: $K_{I,m}^Q = M_m\sigma_m^Q$;
Secondary bending stress intensity factor: $K_{I,b}^Q = M_b\sigma_{\phi,bending}^Q$

Values obtained in the previous section for stress at the junction:
$\sigma_x(0) = 0$;
$\sigma_{\phi,memb}(0) = -\frac{\alpha_n E\Delta T t}{t} = -48$ MPa;
$\sigma_{\phi,bend}(0) = \frac{1}{t}(\frac{Et}{R_m}(\frac{pR_m^2}{Et} + \alpha_n R_m\Delta T + \frac{Q_0}{2K\beta^3})) = 199.5$ MPa;
$M_x(0) = M_\phi(0) = 0$;
$\tau \approx 0$
(the tangential stress is not considered in the verification as it doesn't participate into the opening mode of the crack);

$$2K_{I,m}^P + K_{I,m}^Q + K_{I,b}^Q < K_{I,c}$$

1. Axial cracking:
   - Internal surface: $(2(-14.21) + 39.30) \quad MPa\sqrt{mm} < 5908.60 \quad MPa\sqrt{mm}$
   - External surface: $(2(-13.68) + 37.91) \quad MPa\sqrt{mm} < 5908.60 \quad MPa\sqrt{mm}$
   - Middle surface: $(2(-13.97) + 37.90) \quad MPa\sqrt{mm} < 5908.6 \quad MPa\sqrt{mm}$

2. Circumferential cracking:
   - Internal surface: $(18.75) \quad MPa\sqrt{mm} < 5908.60 \quad MPa\sqrt{mm}$
   - External surface: $(18.75) \quad MPa\sqrt{mm} < 5908.60 \quad MPa\sqrt{mm}$
   - Middle surface: $(18.75) \quad MPa\sqrt{mm} < 5908.60 \quad MPa\sqrt{mm}$

5.2 Primary pipe side of junction

Values obtained in the previous section for stress at the junction:
$\sigma_x(0) = 0$;
$\sigma_{\phi,memb}(0) = -\frac{\alpha_p E\Delta T t}{t} = -72$ MPa;
$\sigma_{\phi,bend}(0) = \frac{1}{t}(\frac{Et}{R_m}(\frac{pR_m^2}{Et} + \alpha_p R_m\Delta T - \frac{Q_0}{2K\beta^3})) = 175.5$ MPa;
$M_x(0) = M_\phi(0) = 0$;
$\tau \approx 0$;

$$2K_{I,m}^P + 2K_{I,b}^P + K_{I,m}^Q + K_{I,b}^Q < K_{I,c}$$

1. Axial cracking:
   - Internal surface: $(2(-21.31) + 34.57) \quad MPa\sqrt{mm} < 5908.6 \quad MPa\sqrt{mm}$
   - External surface: $(2(-20.52) + 33.35) \quad MPa\sqrt{mm} < 5908.6 \quad MPa\sqrt{mm}$
   - Middle surface: $(2(-20.95) + 34.05) \quad MPa\sqrt{mm} < 5908.6 \quad MPa\sqrt{mm}$

2. Circumferential cracking:
   - Internal surface: $(16.50) \quad MPa\sqrt{mm} < 5908.60 \quad MPa\sqrt{mm}$
   - External surface: $(16.50) \quad MPa\sqrt{mm} < 5908.60 \quad MPa\sqrt{mm}$
   - Middle surface: $(16.50) \quad MPa\sqrt{mm} < 5908.60 \quad MPa\sqrt{mm}$

The two sides of the junction are verified.