Name: ______________________________

Show your work.

1. Consider the alphabet $\Sigma = \{\mathbf{1}, \mathbf{0}\}$. Let $c : \mathbb{N} \longrightarrow \mathbb{N}$ be the function such that the value $c(n)$ for any given $n \in \mathbb{N}$ is the number of Σ-words of length n wherein the string **10** fails to appear.

   (a) Specify the indicated values.

      (i) $c(0)$

      (i) ______________

      (ii) $c(1)$

      (ii) ______________

      (iii) $c(2)$

      (iii) ______________

      (iv) $c(19)$

      (iv) ______________

   (b) Let $p(n)$ express the following proposition:

$$\sum_{i=0}^{n} c(i) \quad = \quad \frac{(n+1)(n+2)}{2}.$$

   To prove that proposition $p(n)$ is true for every natural number $n \in \mathbb{N}$ by mathematical induction, carry out the following steps.

      (i) Specify the applicable base case.

      (i) ______________

(ii) In the space below, establish the base case by mathematical argumentation.

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(iii) Specify the applicable inductive step.

(iii) ______

(iv) Identify the inductive hypothesis in the given inductive step.

(iv) ______

(v) In the space below, establish the inductive step by mathematical argumentation.

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2. Consider the function $\Phi : \mathbb{N} \longrightarrow \mathbb{N}$ for which $\Phi(n) = \min\left(n!, \frac{(2n)!}{2^n}\right)$ for each $n \in \mathbb{N}$. Let $p(n)$ express the following proposition:

$$\Phi(n) \quad = \quad n!$$

To prove that proposition $p(n)$ is true for every natural number $n \in \mathbb{N}$ by mathematical induction, carry out the following steps.

(a) Specify the applicable base case.

(a) ______________________________

(b) In the space below, establish the base case by mathematical argumentation.

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(c) Specify the applicable inductive step.

(c) ______________________________

(d) Identify the inductive hypothesis in the given inductive step.

(d) ______________________________

(e) In the space below, establish the inductive step by mathematical argumentation.

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