Name: _____

Show your work.

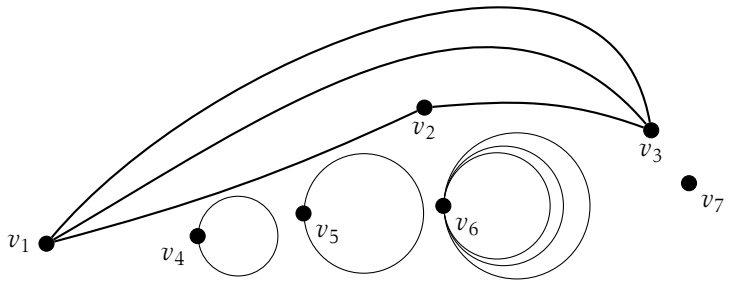
1. Using truth tables, indicate whether each given statement is true or false.

   (a) $\Big(p \to \neg p \Big) \to \Big(q \to \neg q \Big) \Longrightarrow \Big(q \to p \Big)$ (a) _____

   (b) $\Big(p \leftrightarrow \neg p \Big) \vee \neg \Big(q \leftrightarrow \neg q \Big) \Longrightarrow \Big(\neg p \to p \Big)$ (b) _____

   (c) $\neg p \Longleftrightarrow \Big(\Big(\neg p \wedge q \Big) \vee \neg \Big(\neg p \to q \Big) \Big)$ (c) _____

2. Consider the pictured graph $\mathbf{G} = \left(\mathrm{V}, \mathrm{E}, \gamma \right)$:



(a) Supply the adjacency matrix for **G**.

$$\begin{bmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \end{bmatrix}$$

(b) Specify the number of paths from vertex v_1 to vertex v_2 of length 2. (b) _____

(c) Where **M** denotes the adjacency matrix for **G**, specify the value of $\mathbf{M}^7[1,7]$. (c) _____

(d) Where **M** denotes the adjacency matrix for **G**, specify the value of $\left(\mathbf{M}^{\mathrm{T}} \right)^2[2,3]$. (d) _____

(e) Specify the property that function γ satisfies. (e) _____

(A) onto and 1-1 (B) onto and not 1-1 (C) 1-1 and not onto (D) neither onto nor 1-1

3. Consider the function $f : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$ such that for every $m, n \in \mathbb{N}$:

$$f(m,n) \quad = \quad 2^m(2n+1) - 1$$

(a) Indicate whether the following statement is true or false.

(a) _____

For some $s \in \mathbb{N}$, there are no $n, m \in \mathbb{N}$ such that $\Big((m,n), s\Big) \in \text{Graph}(f)$.

(b) Specify the property that function f satisfies.

(b) _____

(A) onto and 1-1 (B) onto and not 1-1 (C) 1-1 and not onto (D) neither onto nor 1-1

(c) Specify whether each given set is finite, countably infinite, or uncountable; if a given set is finite, specify its cardinality.

(i) $\Big(\mathbb{R} \times \mathbb{R} \setminus \mathbb{Q} \times \mathbb{Q}\Big) \setminus \text{Dom}(f)$. (i) _____

(ii) $\mathbb{R} \times \mathbb{R} \setminus \Big(\mathbb{Q} \times \mathbb{Q} \setminus \text{Dom}(f)\Big)$ (ii) _____

(iii) $f\Big(\{0\} \times \mathbb{N}\Big)$. (iii) _____

4. Consider a binary relation S on $\mathbb{R} \times \mathbb{R}$ such that for every $x_1, y_1, x_2, y_2 \in \mathbb{R}$:

$$(x_1, y_1)\,S\,(x_2, y_2) \qquad \text{if and only if} \qquad \text{either} \quad y_1 < y_2 \quad \text{or} \quad y_1 \leq y_2 \ \text{ and } \ x_1 \leq x_2$$

Indicate whether each given statement is true or false.

(a) The relation S is transitive. (a) _____

(b) The relation S is symmetric. (b) _____

(c) The relation S is reflexive. (c) _____

5. Consider sets $A = \{0, \beta, a, ?, @, \wedge, U, \Delta\}$ and $S = \{\$, @, \%, +, !, \#\}$.

(a) How many 3-element subsets of S are there? (a) _____

(b) How many one-to-one functions are there from A to S? (b) _____

(c) How many 2-permutations of A are there? (c) _____

6. An urn has three blue balls and four yellow balls. A set of three balls are to be removed at random from the urn without replacement.

(a) What is the probability that the three balls are all blue? (a) _____

(b) What is the probability that two of the balls are yellow and one of the balls is blue? (b) _____

7. **Quiz 4 Problem**. Let $p(n)$ express the following proposition:

$$\sum_{m=1}^{n} \frac{1}{(3m+1)(3m-2)} \quad = \quad \frac{n}{3n+1}$$

To prove that proposition $p(n)$ is true for every natural number $n \in \mathbb{P}$ by mathematical induction, carry out the following steps.

(a) Specify the applicable base case.

(a) _____

(b) In the space below, establish the base case by mathematical argumentation.

(c) Specify the applicable inductive step.

(c) _____

(d) Specify the inductive hypothesis in the inductive step.

(d) _____

(e) In the space below, establish the inductive step by mathematical argumentation.

Scratch.