CISC 3220 Homework Master Theorem

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Problem 1

$$T(n) = 2T(n/4) + n^{0.58}$$

$$a = 2$$

$$b = 4$$

$$f(n) = n^{0.58}$$

$$n^{log_b a} = n^{log_4 2} = n.5$$

Case 3: $f(n) = \Omega(n^{log_b a + \epsilon})$

$$0 \leq n^{.5+\epsilon} \leq n^{.58}$$

Let $\epsilon = .01$

$$0 \leq n^{.51} \leq n^{.58}$$

Find a $c < 1$ such that $a \cdot f(\frac{n}{b}) \leq c \cdot f(n)$for all sufficiently large n :

$$2 \cdot n^{\frac{.58}{4}} \leq c \cdot n^{.58}$$

Let $c = \frac{1}{2}$

$$2 \cdot n^{.145} \leq \frac{1}{2} \cdot n^{.58}$$

Dividing both sides by $n^{.145}$:

$$2 \leq \frac{1}{2} \cdot n^{.435}$$

which is true for all sufficiently large $n \geq 5$

So $T(n) = \Theta(n^{.58})$

Problem 2

$$T(n) = \sqrt{3}T(n/2) + log_2 n$$

$$a = \sqrt{3}$$

$$b = 2$$

$$f(n) = log_2 n$$

$$n^{log_b a} = n^{log_2 \sqrt{3}} = n^{\frac{log_2 3}{2(log_2 2)}} \approx n^{\frac{1.585}{2}} \approx n^{.8}$$

Case 1: $f(n) = \mathcal{O}(n^{log_b a - \epsilon})$

$$0 \leq log_2 n \leq c \cdot n^{.8-\epsilon}$$

Let $\epsilon = .1$

For all asymptotically positive functions, there is a c such that: $0 \leq log_2 n \leq c \cdot n^{.7}$

So $T(n) = \Theta(n^{log_2 \sqrt{(3)}})$