1. Show that for every integer $n \in \mathbb{N}^+$:

$$\sum_{i=1}^{n} \frac{1}{(4n+1)(4n-3)} \quad = \quad \frac{n}{4n+1}.$$

Solution. Use mathematical induction to establish that for every integer $n \in \mathbb{N}^+$:

$$\sum_{i=1}^{n} \frac{1}{(4n+1)(4n-3)} \quad = \quad \frac{n}{4n+1}.$$

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Here is a theorem:

Theorem 1. Let $a, b \in \mathbb{R}$. For every integer $n \in \mathbb{N}$:

$$\sum_{m=0}^{n} (a + mb) \quad = \quad \frac{(n+1)(2a+nb)}{2}.$$

$\triangleleft$

Proof. Let $a, b \in \mathbb{R}$. Observe that the identity in question readily follows from the well-known identity for evaluating the sum of the first n natural numbers:

$$\sum_{m=0}^{n} m \quad = \quad \frac{n\big(n+1\big)}{2}.$$

Indeed, for every $n \in \mathbb{N}$:

$$\begin{aligned}
\sum_{m=0}^{n} \big(a + mb\big) \quad &= \quad a \sum_{m=0}^{n} 1 \; + \; b \sum_{m=0}^{n} m \\
&= \quad a\big(n+1\big) \; + \; b \, \frac{n\big(n+1\big)}{2} \\
&= \quad \frac{2a\big(n+1\big) \; + \; nb\big(n+1\big)}{2} \\
&= \quad \frac{\big(n+1\big)\big(2a+nb\big)}{2}.
\end{aligned}$$

$\square$