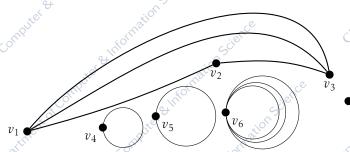
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2. Consider the pictured graph  $G = (V, E, \gamma)$ :



(a) Supply the adjacency matrix for G.

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(b) Specify the number of paths from vertex  $v_1$  to vertex  $v_2$  of length 2.

(b) \_\_\_\_\_\_\_

(c) Where M denotes the adjacency matrix for G, specify the value of  $M^7[1,7]$ .

- (c) \_\_\_\_0
- (d) Where M denotes the adjacency matrix for G, specify the value of  $(M^T)^2[2,3]$ .
- (d) \_\_\_\_\_2

(e) Specify the property that function  $\gamma$  satisfies.

(e) D

- (A) onto and 1-1
- (B) onto and not 1-1
- (C) 1-1 and not onto
- (D) neither onto nor 1-1

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3. Consider the function  $f : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$  such that for every  $m, n \in \mathbb{N}$ :

$$f(m,n) = 2^m(2n+1) - 1$$

(a) Indicate whether the following statement is true or false.

(a) False

For some  $s \in \mathbb{N}$ , there are no  $n, m \in \mathbb{N}$  such that  $((m, n), s) \in Graph(f)$ .

(b) Specify the property that function f satisfies.

(b) \_\_\_\_A

- (A) onto and 1-1 (B) onto and not 1-1 (C) 1-1 and not onto (D) neither onto nor 1-1
- (c) Specify whether each given set is finite, countably infinite, or uncountable; if a given set is finite, specify its cardinality.
  - (i)  $(\mathbb{R} \times \mathbb{R} \setminus \mathbb{Q} \times \mathbb{Q}) \setminus \text{Dom}(f)$ .

(i) uncountably infinite

(ii)  $\mathbb{R} \times \mathbb{R} \setminus (\mathbb{Q} \times \mathbb{Q} \setminus \text{Dom}(f))$ 

(ii) uncountably infinite

(iii)  $f(\{0\} \times \mathbb{N})$ 

(iii) countably infinite

4. Consider a binary relation S on  $\mathbb{R} \times \mathbb{R}$  such that for every  $x_1, y_1, x_2, y_2 \in \mathbb{R}$ :

$$(x_1, y_1) S(x_2, y_2)$$
  $\forall$  if and only if either  $y_1 < y_2$  or  $y_1 \le y_2$  and  $x_1 \le x_2$ 

Indicate whether each given statement is true or false.

(a) The relation S is transitive.

(b) The relation S is symmetric.

(c) The relation S is reflexive.

- 5. Consider sets  $A = \{0, \beta, a, ?, @, \land, U, \Delta\}$  and  $S = \{\$, @, \%, +, !, \#\}$ .
  - (a) How many 3-element subsets of S are there?

(a) 
$$\binom{6}{3} = 20$$

(b) How many one-to-one functions are there from A to S?

(c) How many 2-permutations of A are there?

$$(c) \frac{P(8,2) = 50}{}$$

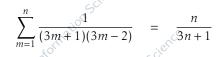
- 6. An urn has three blue balls and four yellow balls. A set of three balls are to be removed at random from the urn without replacement.
  - (a) What is the probability that the three balls are all blue?

(a) 
$$\frac{1}{35}$$

(b) What is the probability that two of the balls are yellow and one of the balls is blue?

(b) 
$$\frac{3 \cdot \binom{4}{2}}{35} = \frac{18}{35}$$

7. **Quiz** 4 **Problem**. Let p(n) express the following proposition:



To prove that proposition p(n) is true for every natural number  $n \in \mathbb{P}$  by mathematical induction, carry out the following steps:

(a) Specify the applicable base case.



(b) In the space below, establish the base case by mathematical argumentation.

Given  $n \in \mathbb{P}$ , let L(n) denote the expression on the lefthand side of equation p(n), and let R(n) denote the expression on the righthand side of equation p(n):

$$L(n) = \sum_{m=1}^{n} \frac{1}{(3m+1)(3m-2)}$$

$$R(n) = \frac{n}{3n+1}.$$

Evidently, L(1) = 1/4 = R(1), as desired. This establishes the base case.

- (c) Specify the applicable inductive step.
- (c) For every  $k \in \mathbb{P}$ , if p(k) is true, then p(k+1) is true
- (d) Specify the inductive hypothesis in the inductive step.



Let  $k \in \mathbb{P}$ . Suppose proposition p(k) is true — that is, equality L(k) = R(k) obtains. To verify that L(k+1) = R(k+1), observe:

$$L(k+1) = \sum_{m=1}^{k+1} \frac{1}{(3m+1)(3m-2)}$$

$$= L(k) + \frac{1}{(3(k+1)+1)(3(k+1)-2)}$$

$$R(k) + \frac{1}{(3(k+1)+1)(3(k+1)-2)}$$

$$= \frac{k}{3k+1} + \frac{1}{(3k+4)(3k+1)}$$

$$\frac{k(3k+4)+1}{(3k+4)(3k+1)}$$

$$= \frac{3k^2 + 4k + 1}{(3k+4)(3k+1)}$$

$$= \frac{(k+1)(3k+1)}{(3k+4)(3k+1)}$$

$$\frac{k+1}{3k+4}$$

$$= \frac{k+1}{3(k+1)+1}$$

$$=$$
  $R(k+1)$ .

This establishes that for every  $k \in \mathbb{P}$ , if proposition p(k) is true, then proposition p(k+1) is true.

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