CISC 3220 Homework Chapter 6

Rachel Friedman

April 19, 2020

Exercises 6.1

Question 6.1-1

What are the minimum and maximum numbers of elements in a heap of height h?

A heap is a balanced binary tree where all levels except the last one is completely full. Therefore, the minimum amount of elements is: 2^h and the maximum amount is: $2^{h+1} - 1$.

Question 6.1-2

Show that an n element heap has height $\lfloor \lg n \rfloor$.

From the solution above, we know that the minimum amount of elements in a tree is 2^h and the maximum amount of elements is $2^{h+1} - 1$. Obviously, the minimum amount is less than or equal to the amount of elements which is in turn less than or equal to the maximum amount of elements. Thus:

$$
\begin{aligned}
& 2^h \leq n \leq 2^{h+1} - 1 \\
& h \leq \log n \leq h + 1 && \text{take log on all sides} \\
& h = \lfloor \lg n \rfloor && \text{since } h \text{ is an integer}
\end{aligned}
$$

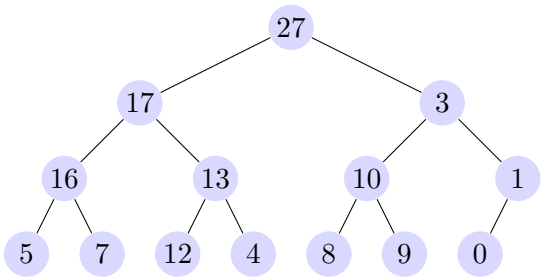
Question 6.1-6

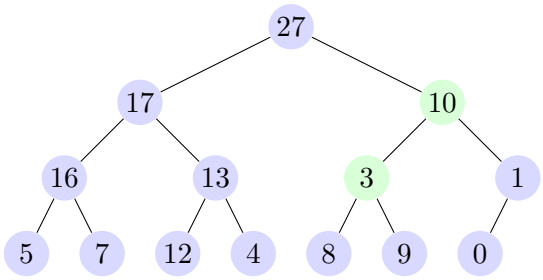
The given array is not a max heap since the subtree with 6 as its root and 7 as its left child violates the max-heap property.

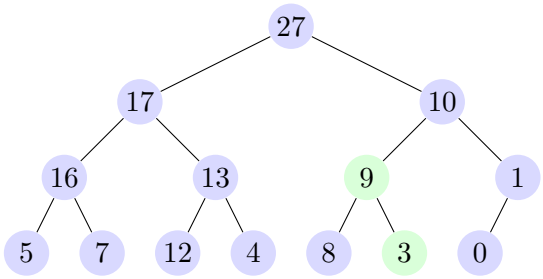
Exercises 6.2

Question 6.2-1

Illustrate **MAX-HEAPIFY(A, 3)** on the array $A = \langle 27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0 \rangle$. (Note to self: top down. Running time: $\mathcal{O}(\lg n)$)







Question 6.2-3

What is the effect of calling MAX-HEAPIFY(A, i) when the element $A[i]$ is larger than its children?

The heap is unchanged.

Exercises 6.3

Question 6.3-1

Illustrate **BUILD-MAX-HEAP** on the array $A = \langle 5, 3, 17, 10, 84, 19, 6, 22, 9 \rangle$.
(Note to self: bottom up. Running time is $\mathcal{O}(n)$.
Need more clarity as to why it's $\mathcal{O}(n)$ if the loop runs from $n/2$ to 1.)

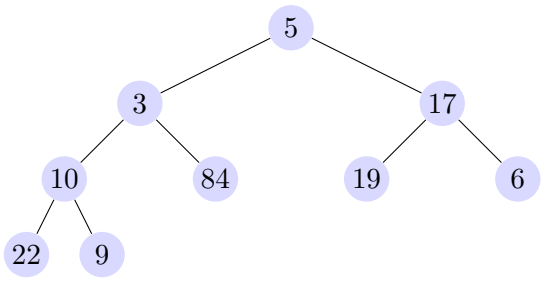


Figure 1: (a)

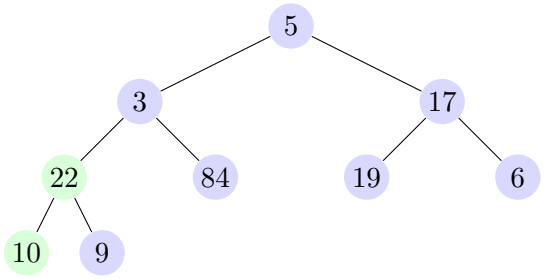


Figure 2: (b)

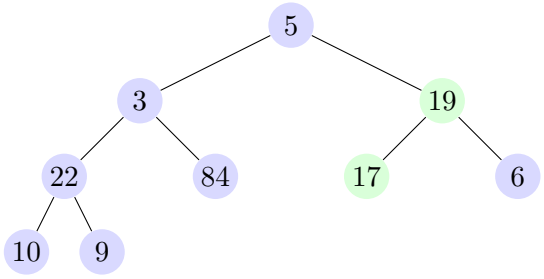


Figure 3: (c)

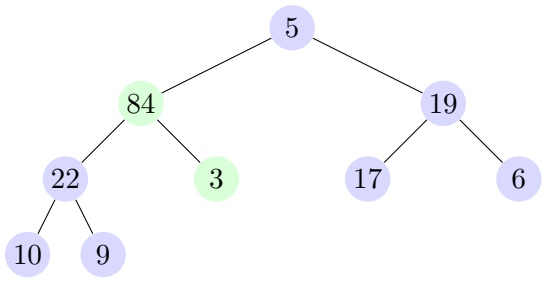


Figure 4: (d)

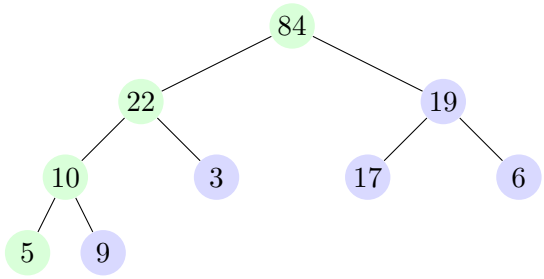


Figure 5: (e)

Exercises 6.4

Question 6.4-1

Illustrate **HEAPSORT** on the array $\langle 5, 13, 2, 25, 7, 17, 20, 8, 4 \rangle$.
(Running time: $\mathcal{O}(n \lg n)$)

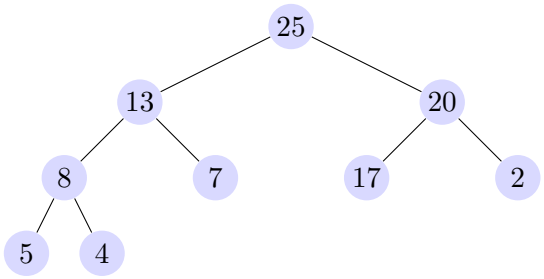


Figure 6: (a) After BUILD MAX-HEAP

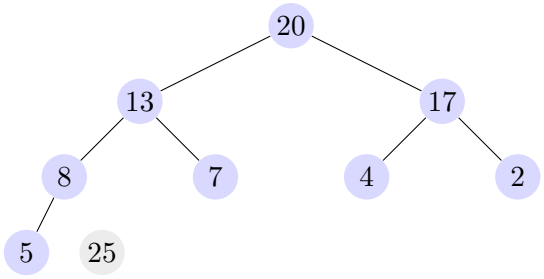


Figure 7: (b)

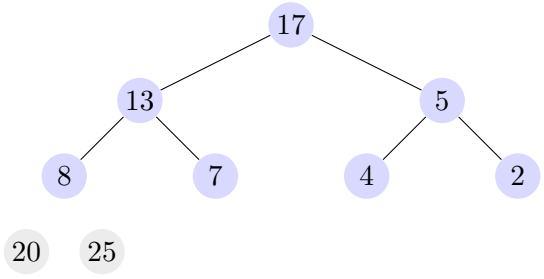


Figure 8: (c)

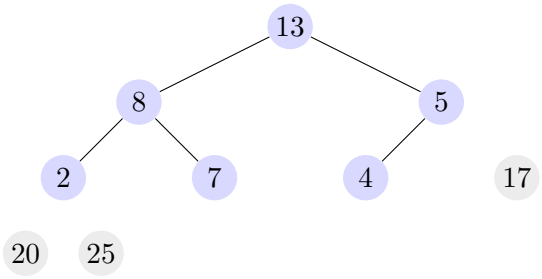


Figure 9: (d)

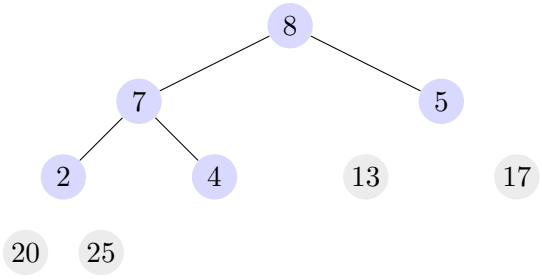


Figure 10: (e)

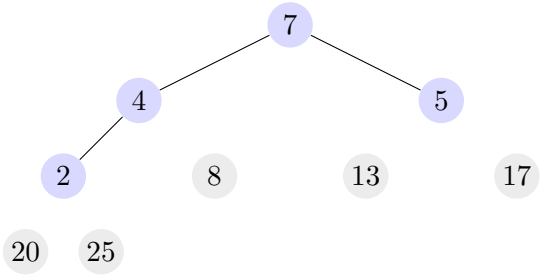


Figure 11: (f)

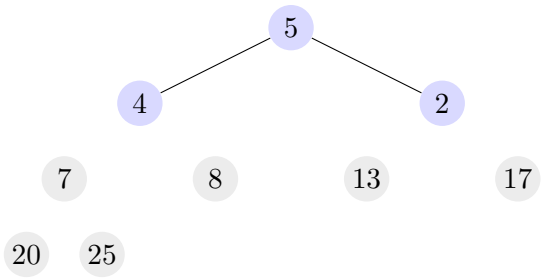


Figure 12: (g)

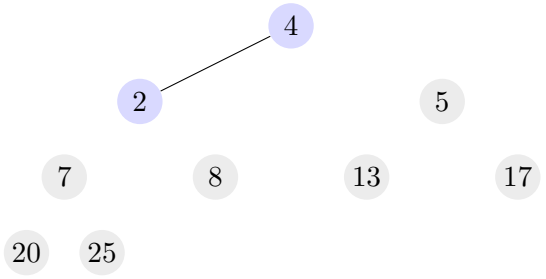


Figure 13: (h)

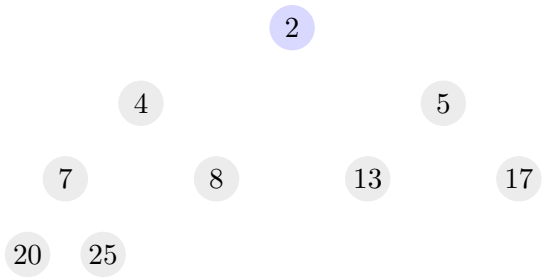


Figure 14: (i)

2	4	5	7	8	13	17	20	25

Figure 15: (j)

Question 6.4-3

What is the running time of HEAPSORT on an array A of length n that is already sorted in increasing order? What about decreasing order?

Even though it's sorted, the first line of HEAPSORT calls BUILD-MAX-HEAP and we know the running time for that is $\mathcal{O}(n)$. It then calls MAX-HEAPIFY for all the nodes, and the running time for MAX-HEAPIFY is $\mathcal{O}(\lg n)$. So the total running time for Heapsort, regardless of the order of the array, is $\mathcal{O}(n \lg n)$.