CISC 3220 Homework

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Problem 4.5-1 a

$$T(n) = 2T(n/4) + 1$$

$a = 2$

$b = 4$

$f(n) = 1$

$n^{log_b a} = n^{log_4 2} = n^{0.5}$

Case 1: $f(n) = \mathcal{O}(n^{log_b a - \epsilon})$

$0 \leq 1 \leq c \cdot n^{.5-\epsilon}$

Let $\epsilon = .1$

For all asymptotically positive functions, there is a c such that: $0 \leq 1 \leq c \cdot n^{.4}$

So $T(n) = \Theta(n^{0.5})$

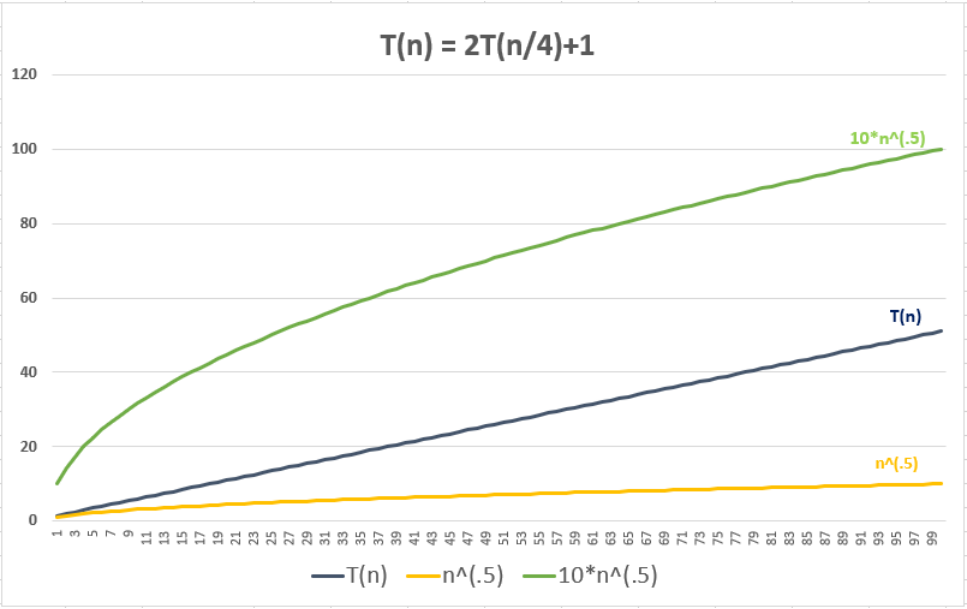
Thus, $c_1 \cdot n^{0.5} \leq 2T(n/4) + 1 \leq c_2 \cdot n^{0.5}$

Let $c_1 = 1$

Let c_2 =???

I can't seem to find a value for c_2 here that is not dependent on n.

What am I doing wrong?? Using 10 for now.



Problem 4.5-1 b

$$T(n) = 2T(n/4) + \sqrt{n}$$

$a = 2$

$b = 4$

$f(n) = \sqrt{n}$

$n^{log_b a} = n^{log_4 2} = n^{0.5}$

Case 2: $f(n) = \Theta(n^{log_b a})$

So $T(n) = \Theta(n^{0.5} \lg n)$

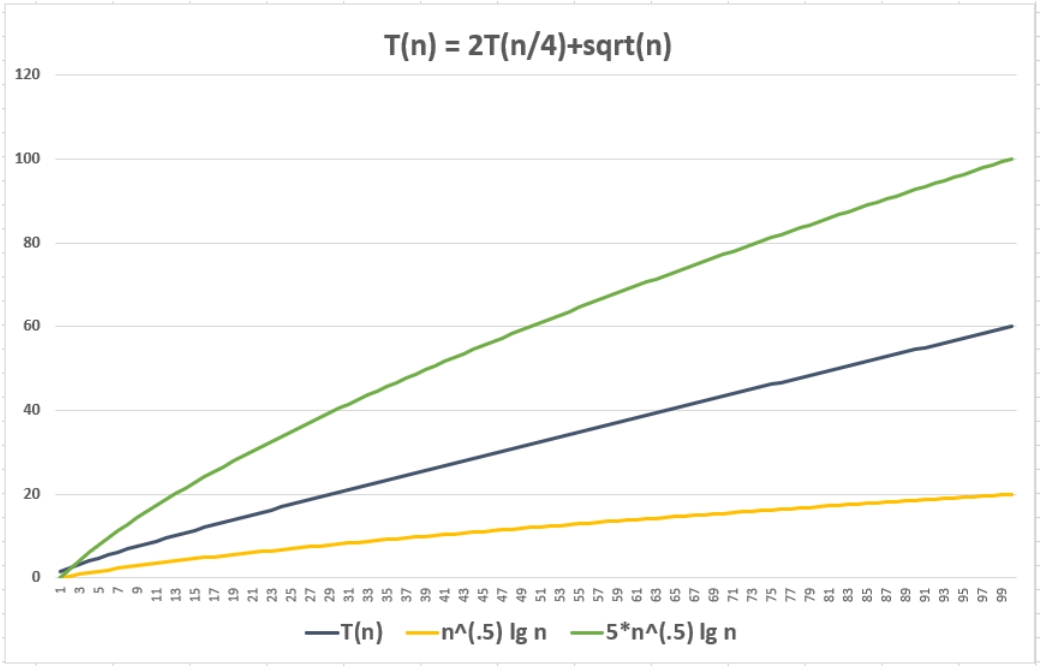
Thus, $c_1 \cdot n^{0.5} \lg n \leq 2T(n/4) + \sqrt{n} \leq c_2 \cdot n^{0.5} \lg n$

Let $c_1 = 1$

Let c_2 =???

I can't seem to find a value for c_2 here that is not dependent on n.

What am I doing wrong?? Using 5 for now.



Problem 4.5-1 c

$$T(n) = 2T(n/4) + n$$

$$a = 2$$

$$b = 4$$

$$f(n) = n$$

$$n^{log_b a} = n^{log_4 2} = n^{0.5}$$

Case 3: $f(n) = \Omega(n^{log_b a + \epsilon})$

$$0 \leq n \geq n^{0.5+\epsilon}$$

Let $\epsilon = .1$

$$n \geq n^{.6}$$

Check regularity condition:

Find a c less than 1 for all sufficiently large n, such that:

$$2\frac{n}{4} \leq c_2 \cdot n$$

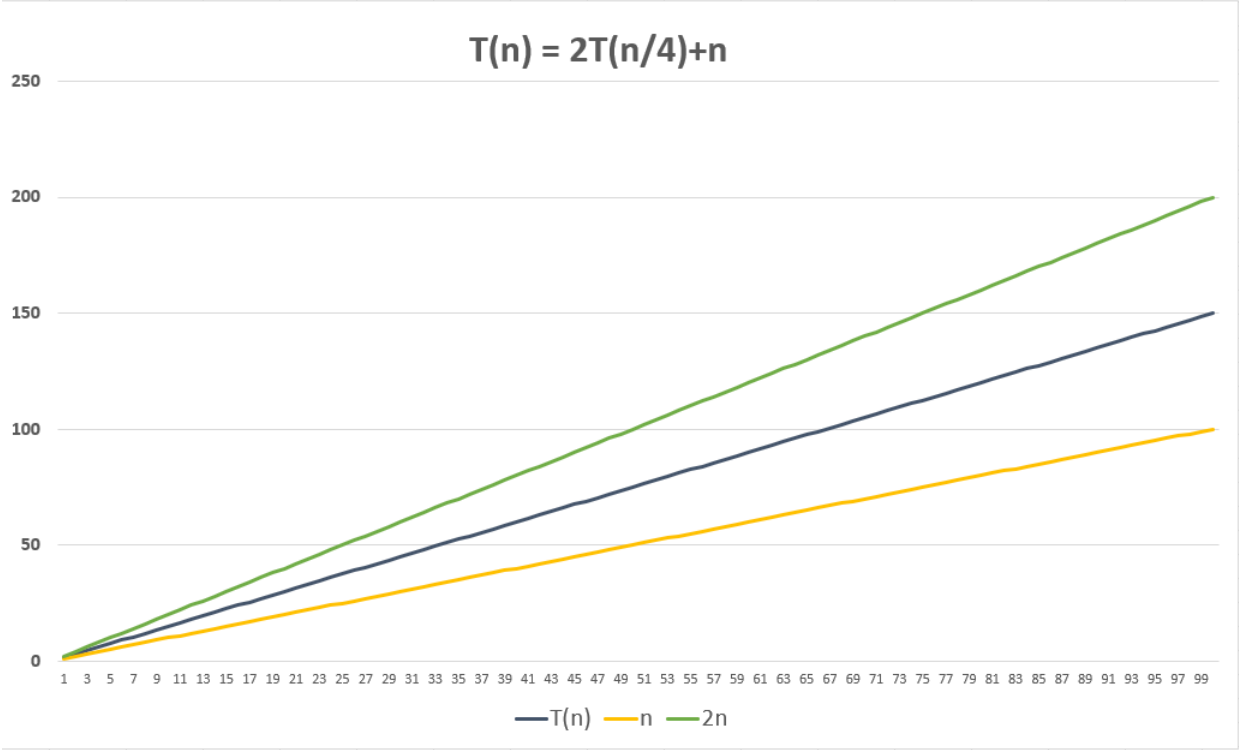
Let $c = \frac{1}{2}$

So $T(n) = \Theta(n)$

$$c_1 n \leq 2T(n/4) + n \leq c_2 n$$

Let $c_1 = 1$

Let $c_2 = 2$



Problem 4.5-1 d

$$T(n) = 2T(n/4) + n^2$$

$$a = 2$$

$$b = 4$$

$$f(n) = n^2$$

$$n^{log_b a} = n^{log_4 2} = n^{0.5}$$

Case 3: $f(n) = \Omega(n^{log_b a + \epsilon})$

$$0 \leq n^2 \geq n^{0.5+\epsilon}$$

Let $\epsilon = .1$

$$n^2 \geq n^{.6}$$

Check regularity condition:

Find a c less than 1 for all sufficiently large n, such that:

$$2\frac{n}{4} \leq c \cdot n^2$$

Let $c = \frac{1}{2}$

So $T(n) = \Theta(n^2)$

$$c_1 n^2 \leq 2T(n/4) + n^2 \leq c_2 n^2$$

Let $c_1 = 0.5$

Let $c_2 = 2$