CISC 3220 Homework Chapter 7

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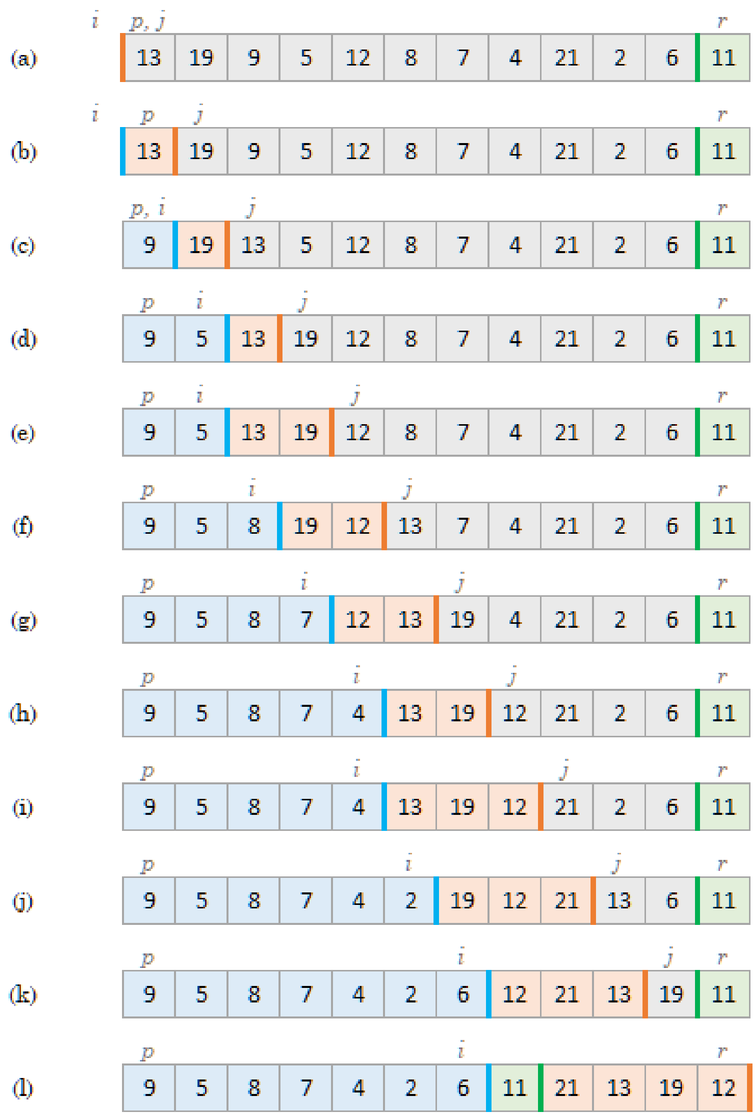
Exercise 7.1

Question 7.1-1



Question 7.1-4

To modify QUICKSORT to sort into nonincreasing order, change the algorithm to check if the element in position j is **greater** than or equal to the pivot element:

```
if A[j] >= A[r]
```

Excercise 7.2

Question 7.2-3

Show that the running time of QUICKSORT is $\Theta(n^2)$ when the array A contains distinct elements and is sorted in decreasing order.

Refer to this part of the Quicksort Algorithm:

```
QuickSort(A, p, r):
    if p < r:
        q = Partition(A, p, r)
        QuickSort(A, p, q - 1)
        QuickSort(A, q + 1, r)
```

In an array of decreasing order, the pivot element is smaller than all the other elements in the array. Thus, after the call to: q = Partition(A, p, r), the pivot element is moved to the first position, and so $q = 1$. Then, the the next two recursive Quicksort calls resolve as follows:

```
QuickSort(A,p,q-1) --> QuickSort(A,1,0)
QuickSort(A,q+1,r) --> QuickSort(A,2,r)
```

The resulting subarrays are of size 0 and of size $n - 1$. The runtime of Partion is $\Theta(n)$. Therefore, the recurrence is:

$$
\begin{aligned}
T(n) &= T(0) + T(n-1) + \Theta(n) \\
&= T(n-1) + \Theta(n) \\
&= \Theta(n^2)
\end{aligned}
$$

Exercise 7.3

Question 7.3-2

When RANDOMIZED-QUICKSORT runs, how many calls are made to the random-number generator RANDOM in the worst case? How about in the best case?

The RANDOMIZED-QUICKSORT function calls RANDOMIZED-PARTITION once, which then calls RANDOM once to pick the random number to be used as the pivot element. Therefore, the running time of this portion is $\Theta(1)$ (used below).

Worst case scenario occurs when partitioning results in one subarray of size 0 and one subarray of size $n-1$. The recurrence for that is:

$$\begin{aligned} T(n) &= T(0) + T(n-1) + \Theta(1) \\ &= T(n-1) + \Theta(1) \\ &= \Theta(n) \end{aligned}$$

Best case scenario occurs when partitioning results in two subarrays of size at most $n/2$. The recurrence for that is:

$$\begin{aligned} T(n) &= 2T(n/2) + \Theta(1) \\ &= \Theta(n) \; \textit{By case 1 of the Master Theorem} \end{aligned}$$