

CISC 3220 Homework Chapter 15

Rachel Friedman

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Exercises 15.2

Question 15.2-1

Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$.

An optimal parenthesization of that sequence is $(A_1A_2)((A_3A_4)(A_5A_6))$ as shown below:

$(A_1A_2) = 5 \times 10 \times 3 = 150$ operations, resulting in a 5×3 matrix

$(A_3A_4) = 3 \times 12 \times 5 = 180$, resulting in a 3×5 matrix

$(A_5A_6) = 5 \times 50 \times 6 = 1500$, resulting in a 5×6 matrix

$(A_3A_4)(A_5A_6) = 3 \times 5 \times 6 = 90$, resulting in a 3×6 matrix

$(A_1A_2)((A_3A_4)(A_5A_6)) = 5 \times 3 \times 6 = 90$.

Total operations = $150 + 180 + 1500 + 90 + 90 = 2100$.

Exercises 15.4

Question 15.4-1

Determine an LCS of $\langle 1, 0, 0, 1, 0, 1, 0, 1 \rangle$ and $\langle 0, 1, 0, 1, 1, 0, 1, 1, 0 \rangle$.

Answer: $\langle 1, 0, 0, 1, 1, 0 \rangle$

	j	0	1	2	3	4	5	6	7	8	9
i	y_i	0	1	0	1	1	0	1	1	0	0
0	x_i	0	0	0	0	0	0	0	0	0	0
1	1	0	0	1	1	1	1	1	1	1	0
2	0	0	1	1	2	2	2	2	2	2	2
3	0	0	1	1	2	2	2	3	3	3	3
4	1	0	1	2	2	3	3	3	4	4	4
5	0	0	1	2	3	3	3	4	4	4	5
6	1	0	1	2	3	4	4	4	5	5	5
7	0	0	1	2	3	4	4	5	5	5	6
8	1	0	1	2	3	4	5	5	6	6	6

Question 15.4-2

Give pseudocode to reconstruct an LCS from the completed c table and the original sequences $X = \langle \dots \rangle$ and $Y = \langle \dots \rangle$, without using the b table.

```
1  PRINT-LCS( $c, X, Y, i, j$ )
2      if  $c[i, j] == 0$ 
3          return
4      if  $X[i] == Y[j]$ 
5          PRINT-LCS( $c, X, Y, i - 1, j - 1$ )
6          print  $X[i]$ 
7      else if  $c[i - 1, j] > c[i, j - 1]$ 
8          PRINT-LCS( $c, X, Y, i - 1, j$ )
9      else
10         PRINT-LCS( $c, X, Y, i, j - 1$ )
```

Exercises 15.5

Question 15.5-1

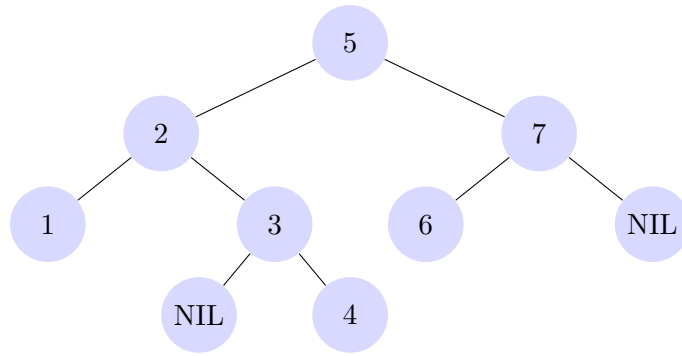
Write pseudocode for the procedure CONSTRUCT-OPTIMAL-BST.root which, given the table root, outputs the structure of an optimal binary search tree.

```
1  CONSTRUCT-OPTIMAL-BST( $root, i, j, last$ )
2      if  $i == j$ 
3          return
4      if  $last == 0$ 
5          print  $root[i, j] + \text{"is\_the\_root"}$ 
6      else if  $j < last$ :
7          print  $root[i, j] + \text{"is\_the\_left\_child\_of"} + last$ 
8      else
9          print  $root[i, j] + \text{"is\_the\_right\_child\_of"} + last$ 
10     CONSTRUCT-OPTIMAL-BST( $root, i, root[i, j] - 1, root[i, j]$ )
11     CONSTRUCT-OPTIMAL-BST( $root, root[i, j] + 1, j, root[i, j]$ )
```

Question 15.5-2

Determine the cost and structure of an optimal binary search tree for a set of $n = 7$ keys with the following probabilities:

i	0	1	2	3	4	5	6	7
p_i		.04	.06	.08	.02	.10	.12	.14
q_i	.06	.06	.06	.05	.05	.05	.05	.05



(For simplification purposes, I eliminated the dummy variables.)
Cost of optimal binary search tree = 3.12.