BRADLEY University

ECE 302: Probability, Statistics, and Random Processes for EE

Fall 2022

Assignment 4: One Random Variables

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Problem 4.62

4.62

$Pr[X \le \pi(r)] = r/100$

a.) $Pr[X \le x] = 1 - e^{-\lambda x} = \frac{r}{100}$

$1 - \frac{r}{100} = e^{-\lambda x}$

$\therefore \pi(r) = x = -\frac{1}{\lambda} \ln\left(\frac{100}{100 - r}\right)$

$\pi(99) \approx \frac{4.6}{\lambda}$

$\pi(95) \approx \frac{3}{\lambda}$

$\pi(90) \approx \frac{23}{\lambda}$

b.) $m = 0 \quad \sigma^2$

$\therefore$ use table

$Pr[X \le x] = 1 - Q\left(\frac{x}{\sigma}\right)$

$1 - Q\left(\frac{x}{\sigma}\right) = 0.90 \Rightarrow \frac{x}{\sigma} = 1.28$

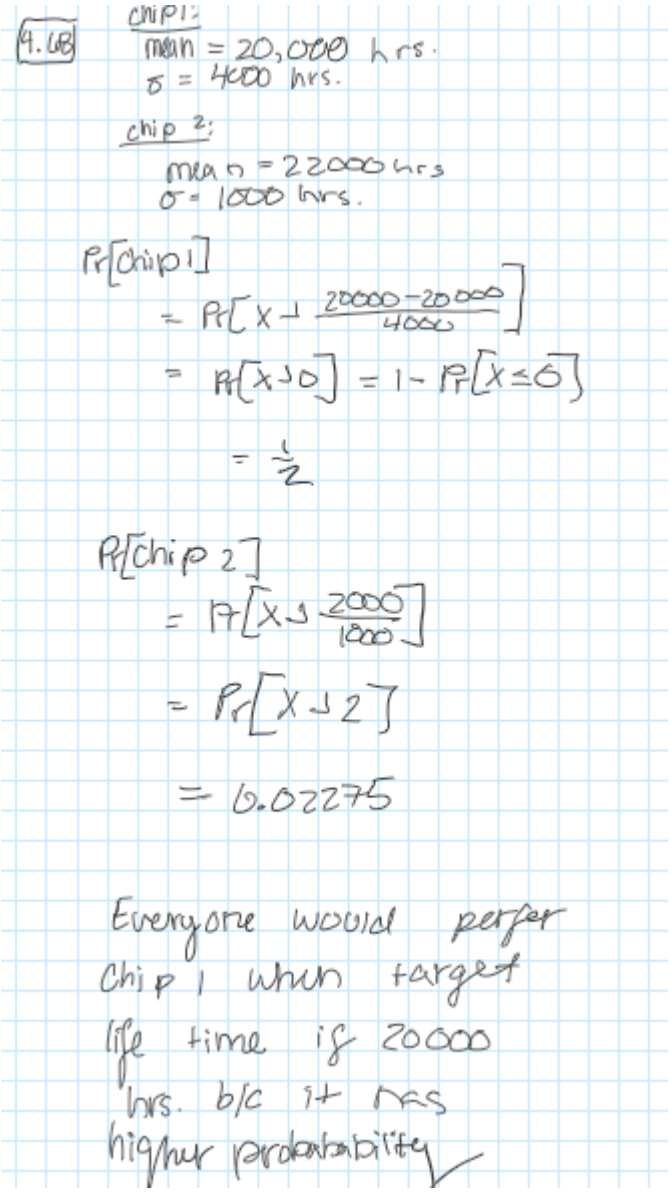
$\pi(90) \approx 1.28\sigma$

$1 - Q\left(\frac{x}{\sigma}\right) = .95$

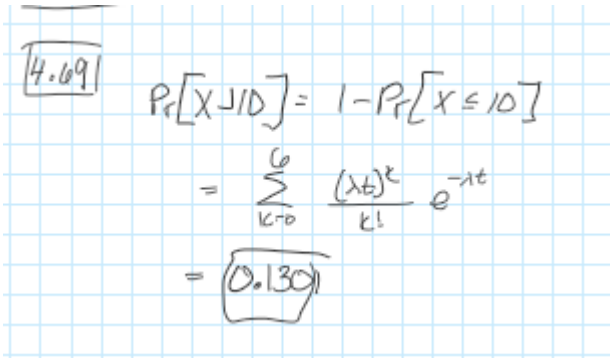
$\pi(95) = 1.65\sigma$

$1 - Q\left(\frac{x}{\sigma}\right) = 2.33\sigma$

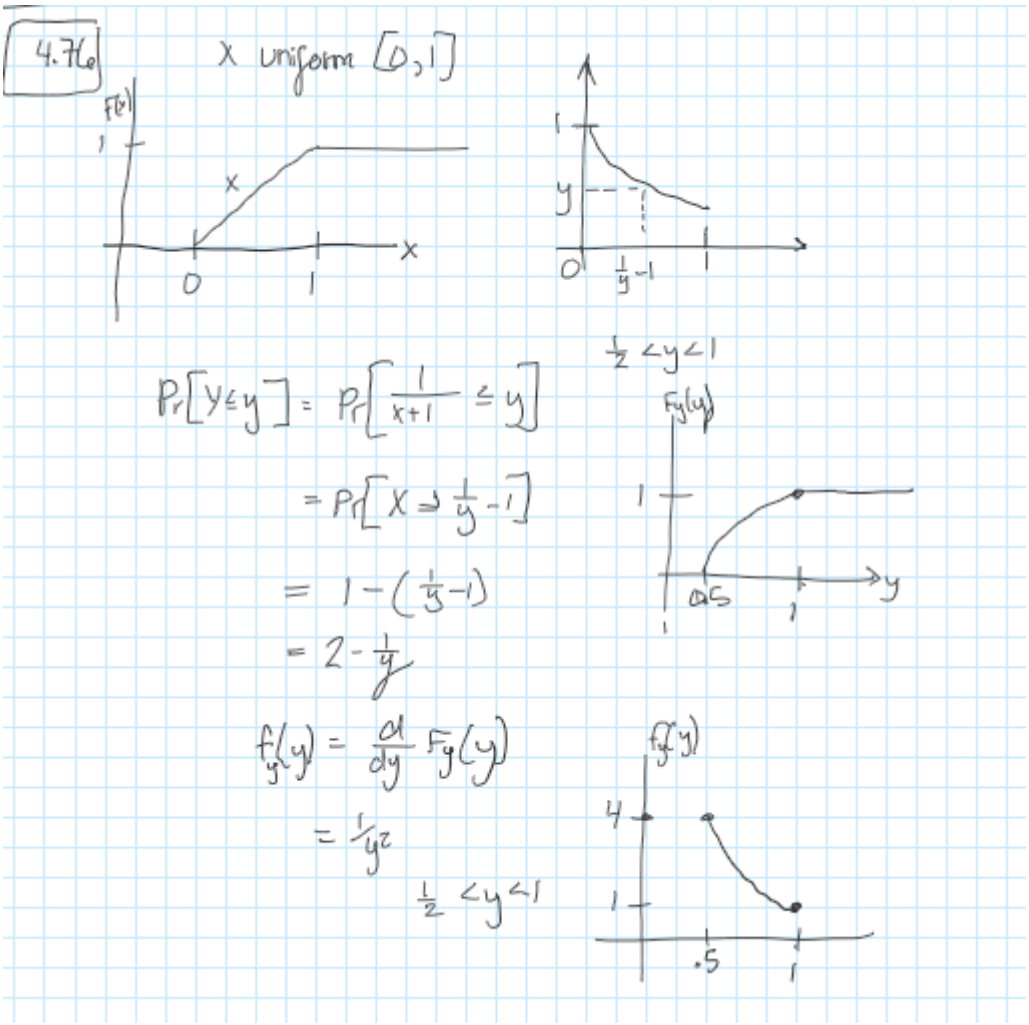
Problem 4.68


4.68
chip 1:
mean = 20,000 hrs.
σ = 4000 hrs.
chip 2:
mean = 22000 hrs
σ = 1000 hrs.
Pr[chip 1]
= Pr[X ≥ (20000-20000)/4000]
= Pr[X ≥ 0] = 1 - Pr[X ≤ 0]
= 1/2
Pr[chip 2]
= Pr[X ≥ 2000/1000]
= Pr[X ≥ 2]
= 0.02275
Everyone would prefer chip 1 when target life time is 20000 hrs. b/c it has higher probability

Problem 4.69



Problem 4.76


4.76
X uniform [0,1]
F(x)
1
x
0
1
x
y
1
0
1/y - 1
1/2 < y < 1
Pr[Y ≤ y] = Pr[1/(X+1) ≤ y]
= Pr[X ≥ 1/y - 1]
= 1 - (1/y - 1)
= 2 - 1/y
f_Y(y) = d/dy F_Y(y)
= 1/y^2
1/2 < y < 1
F_Y(y)
1
.5
1
y
f_Y(y)
4
1
.5
1

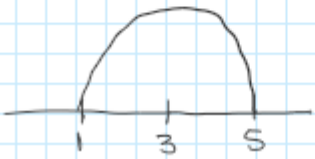
Problem 4.80

4.80 $y = 2x + 3$

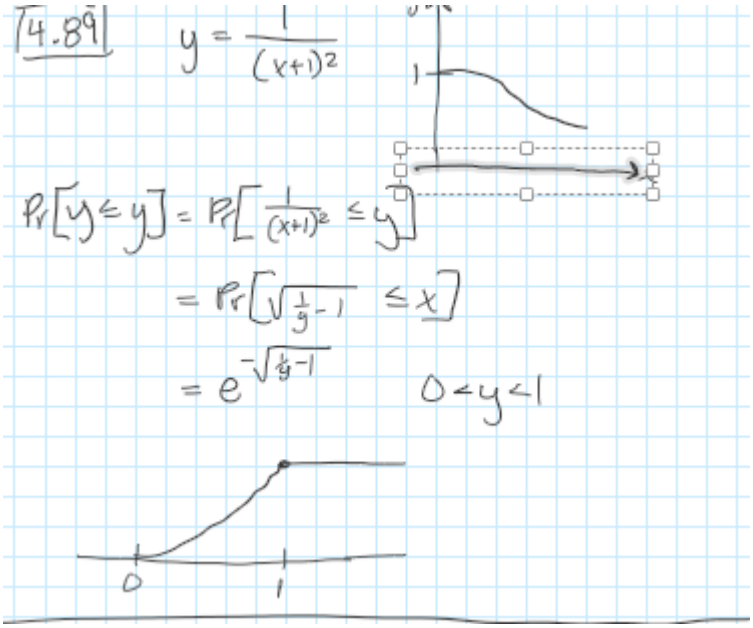
$$F_Y(y) = P_r[2x + 3 \le y] = P_r\left[x \le \frac{y-3}{2}\right]$$
$$= F_x\left(\frac{y-3}{2}\right)$$
$$f_y(y) = \frac{d}{dy} F_y(y) = \frac{d}{dy} F_x\left(\frac{y-3}{2}\right)$$
$$= \frac{1}{2} f_x\left(\frac{y-3}{2}\right)$$

$$f_x(x) = \frac{3}{4}(1 - x^2) \qquad -1 < x < 1$$

$$f_y(y) = \frac{3}{8}\left(1 - \left(\frac{y-3}{2}\right)^2\right) \qquad 1 < y < 5$$

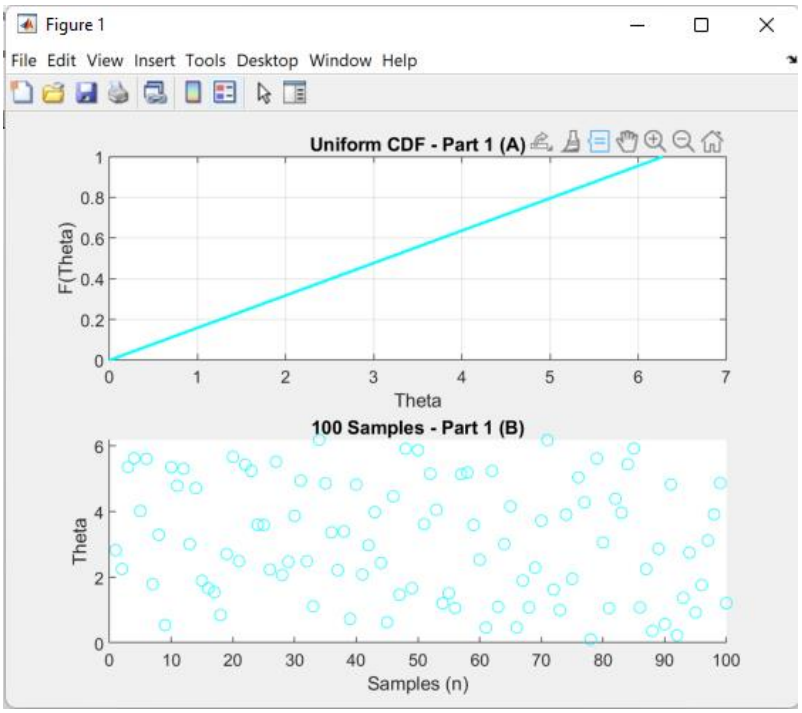


Problem 4.89

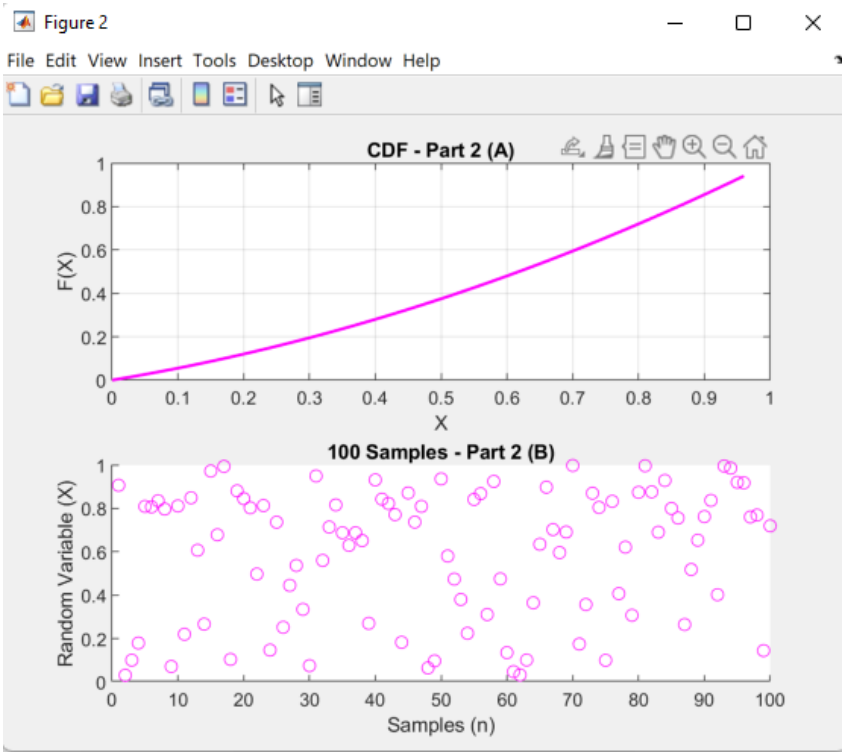

4.89
$y = \frac{1}{(x+1)^2}$
1
$P_r[y \le y] = P_r\left[\frac{1}{(x+1)^2} \le y\right]$
$= P_r\left[\sqrt{\frac{1}{y}-1} \le x\right]$
$= e^{-\sqrt{\frac{1}{y}-1}}$
$0 < y < 1$
0
1

Computer Experiments

1.



2.



3.

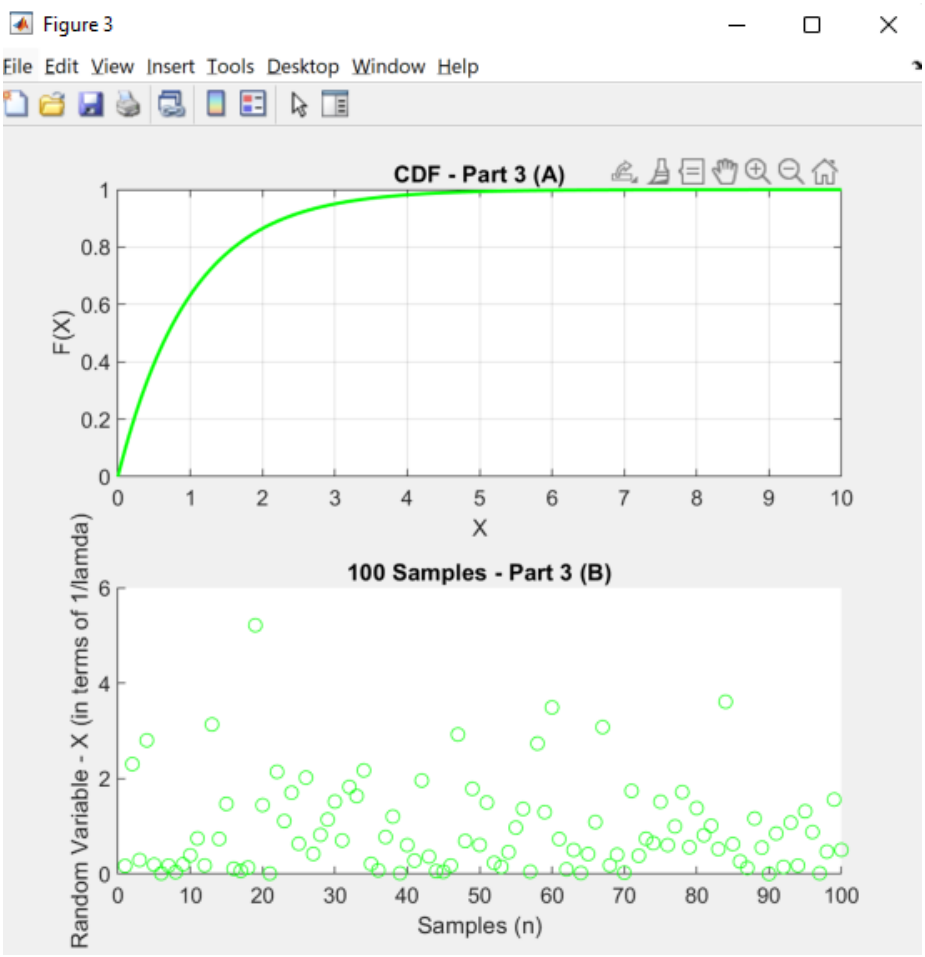


Figure 3
File Edit View Insert Tools Desktop Window Help
CDF - Part 3 (A)
F(X)
X
100 Samples - Part 3 (B)
Random Variable - X (in terms of 1/lamda)
Samples (n)

MATLAB Solutions

```
%Gottschalk, Rachel ECE 302: Assignment #4
close all;
clear all;
clc;


%%%%%%%%%%%%%%%%%%%% Part 1 %%%%%%%%%%%%%%%%%%%%
% A - CDF
x = 0:((2*pi)/100):(2*pi); % x values for part 1 A
a = 0; % lower bound
b = 2*pi; % upper bound
y1= cdf1(x,a,b); % sends variables to function

%%%%%%%%%%%%%%%%%%%% Part 2 %%%%%%%%%%%%%%%%%%%%
y=0:0.03:0.98; % x values for part 2 A
x2=cdf2(y); % sends variables to function

%%%%%%%%%%%%%%%%%%%% Part 3 %%%%%%%%%%%%%%%%%%%%
x=0:0.02:10; % x values for part 3 A
x3=cdf3(x); % sends variables to function



%%%%%%%%%%%%%%%%%%%% Functions %%%%%%%%%%%%%%%%%%%%%%%%
function y = cdf1(x,a,b)
    for i = 1:length(x) % finds length of vector x and loops through the length
        if x(i)<=0 % if x value is less than 0 then y=0
            y(i)=0;
        elseif x(i)>=a && x(i)<b % if x between 0 and 2pi then y is calulated
            y(i)=((x(i)-a)/(b-a));
        else % if x if greater than or equal to 2pi then y=1
            y(i)=1;
        end
    end

    y1 = rand(100,1); % creates 100 random samples
    x1=2*pi*y1; % puts random samples into inverse function to calculate x
    sample=1:100; % vector of 1 to 100

    %plots figure 1 with Uniform CDF and Inverse Function
    figure(1);
    subplot(2,1,1)
    plot(x,y,LineWidth=1.5,Color='cyan'); grid on;
    title('Uniform CDF - Part 1 (A)')
    xlabel("Theta")
    ylabel("F(Theta)")
    subplot(2,1,2)
    scatter(sample,x1,"cyan")
    title("100 Samples - Part 1 (B)")
    xlabel("Samples (n)")
    ylabel("Theta")
```

```
end

function x2 = cdf2(x)
    for i = 1:length(x) %finds length of vector x and loops through the length
        y(i) = ((x(i))^2+x(i))/2; % calculates the CDF of RV X
    end

    sample=1:100; % creates 100 random samples vector
    x2=sqrt(2*(rand(100,1))+1/4)-1/2; % puts random samples into inverse function to
calculate x

    %plots figure 2 with Uniform CDF and Inverse Function
    figure(2);
    subplot(2,1,1)
    plot(x,y,LineWidth=1.5,Color='magenta'); grid on;
    title('CDF - Part 2 (A)')
    xlabel("X")
    ylabel("F(X)")
    subplot(2,1,2)
    scatter(sample,x2,"magenta")
    title("100 Samples - Part 2 (B)")
    xlabel("Samples (n)")
    ylabel("Random Variable (X)")

end

function x3 = cdf3(x)
    y=expcdf(x); %built in matlan function that calculates the exponential cdf of vector

    sample=1:100; % creates 100 random samples vector
    x3=-(log(1-rand(100,1))); % puts random samples into inverse function to calculate x

    %plots figure 1 with Expoential CDF and Inverse Function
    figure(3);
    subplot(2,1,1)
    plot(x,y,LineWidth=1.5,Color='green'); grid on;
    title('CDF - Part 3 (A)')
    xlabel("X")
    ylabel("F(X)")
    subplot(2,1,2)
    scatter(sample,x3,"green")
    title("100 Samples - Part 3 (B)")
    xlabel("Samples (n)")
    ylabel("Random Variable - X (in terms of 1/lamda)") %note lamda is just variable with no
given values, so write RV X in terms of lamda

end
```