ECE 302: Probability, Statistics, and Random Processes for EE

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Assignment 6: Extra Credit

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Problem 7.1

$\boxed{7.1}$ $E[X+Y+Z] = E[X]+E[Y]+E[Z]=0$

a.) $var(X+Y+Z) = var(X) + var(Y) + var(Z) + 2cov(X,Y) + 2cov(X,Z) + 2cov(Y,Z)$

$var(X+Y+Z) = 3 + 2(\frac{1}{4}) + 0 + 2(-\frac{1}{4}) = 3$

b.) $var(X+Y+Z) = var(X) + var(Y) + var(Z)$

$= 3$

Problem 7.2

$\boxed{7.2}$

$$\mathrm{cov}(X_i, X_j) = \begin{cases} \sigma^2 & \text{if } i=j \\ \rho\sigma^2 & \text{if } |i-j|=1 \\ 0 & \text{otherwise} \end{cases} \qquad |\rho|<1$$

$$E[S_n] = \sum_{i=1}^{n} E[X_i] = n\mu$$

$$\mathrm{var}(S_n) = \sum_{i=1}^{n} \mathrm{var}(X_i) + \sum_{j=1}^{n}\sum_{i=1}^{n} \mathrm{cov}(X_j, X_i)$$

$$K = \begin{bmatrix} \sigma^2 & \rho\sigma^2 & 0 & \cdots & 0 \\ \rho\sigma^2 & \sigma^2 & \rho\sigma^2 & 0 \cdots & 0 \\ & \ddots & & & \end{bmatrix}$$

$$\therefore \mathrm{var}(S_n) = n\sigma^2 + 2(n-1)\rho\sigma^2$$

Problem 7.22

7.22 $n=100$ $\sigma^2 = npq = 25$

$m = np = 50$

a.)

$Pr[50 \le N \le 55] \approx Q(0) - Q(1) = \frac{1}{2} - 1.159 = \boxed{0.44}$

$n=1000$

b.) $Pr[400 \le N \le 600] \approx Pr\left[\frac{400-500}{\sqrt{250}} \le \frac{N-m}{\sigma} \le \frac{600-500}{\sqrt{250}}\right]$

$\approx \boxed{1 - 2.54(10^{-10})}$

$Pr[500 \le N \le 550] \approx Q(0) - Q(3.162) = \boxed{\frac{1}{2} - 7.3 \cdot 10^{-4}}$

Problem 7.24

$\boxed{7.24}$

$$S = \sum_{i=1}^{20} x_i \longrightarrow E[S] = 20 \cdot E[X] = 20 \times 3.5 = 70$$

$$\text{var}[S] = \sigma^2 = 20\sigma^2 = 20 \times 2.92 = \boxed{58.4}$$

using CLT

$$S \sim N(70, \sqrt{58.4})$$

$$P_r\{60 < S < 80\} = P\left\{\frac{60-70}{7.64} < \frac{S-70}{7.64} < \frac{80-70}{7.64}\right\}$$

$$= 1.20(1.3089) = 0.8094$$

Problem 7.25

$\boxed{7.25}$

$E[X_i] = 36$

$\mathrm{var}(X_i) = 36^2$

$S = X_1 + \dots + X_{16}$ $\quad E(S) = 16(36)^2$

$\mathrm{var}(S) = 16(36^2)$

$$\Pr[S < 600] = \Pr\left[\frac{S - 16(36)}{4(36)} < \frac{600 - 16(36)}{4(36)}\right]$$

$$\approx 1 - Q\left(\frac{1}{6}\right) = 0.592$$

Problem 8.39

8.39

$X_i = m + N_i$

$E[N_i] = 0 \qquad Var[N_i] = 10$

$M_{100} = 100 \qquad \sigma = \sqrt{10}$

$z_{\alpha/2} = 1.96$

$$\left(100 - \frac{1.96\sqrt{10}}{\sqrt{30}},\ 100 + \frac{1.96\sqrt{10}}{\sqrt{30}}\right) = (98.9,\ 101.1)$$

Problem 8.40

8.40

$$\left(M_n + \frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right) - \left(M_n - \frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right) = \frac{2z_{\alpha/2}\sigma}{\sqrt{n}}$$

a.) 95% confidence

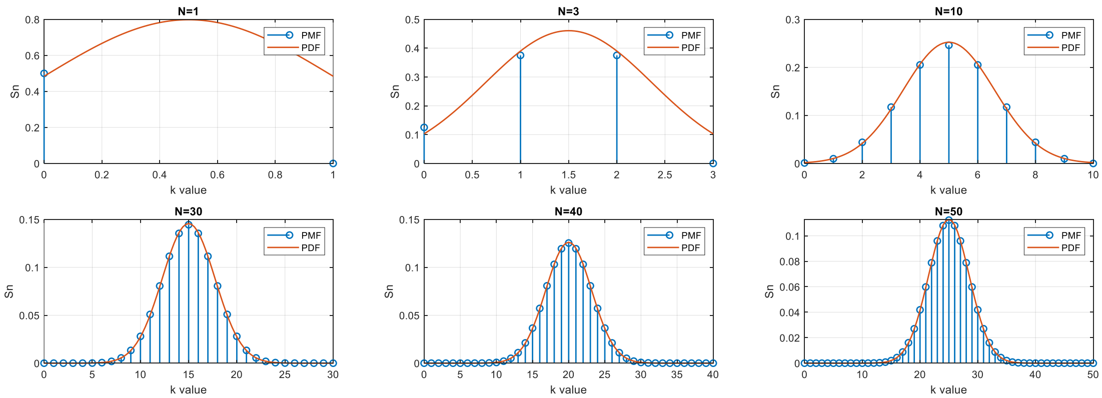
$$\text{width} = \frac{2(1.96)}{\sqrt{n}} = \begin{cases} 1.96 & n=4 \\ 0.98 & n=16 \\ 0.29 & n=100 \end{cases}$$

b.) 99% confidence

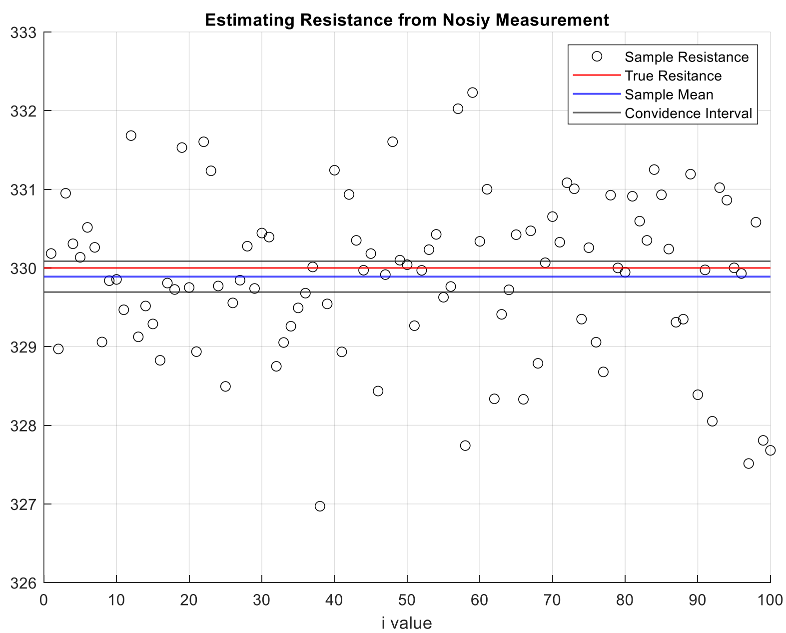
$$\text{width} = \frac{2(2.576)}{\sqrt{n}} = \begin{cases} 2.576 & n=4 \\ 1.288 & n=16 \\ 0.515 & n=100 \end{cases}$$

Computer Experiments

1.



2.



MATLAB Solutions

```
%Gottschalk, Rachel ECE 302: Assignment #6
close all;
clear all;
clc;


%%%%%%%% Part 1 %%%%%%%%%%
n=1;
[k1,y1]=pmf(n); % send to pmf function
[s1,f1]=pdf(n); % send to pdf function


n=3;
[k2,y2]=pmf(n);  % send to pmf function
[s2,f2]=pdf(n); % send to pdf function

n=10;
[k3,y3]=pmf(n);  % send to pmf function
[s3,f3]=pdf(n); % send to pdf function

n=30;
[k4,y4]=pmf(n); % send to pmf function
[s4,f4]=pdf(n); % send to pdf function

n=40;
[k5,y5]=pmf(n); % send to pmf function
[s5,f5]=pdf(n); % send to pdf function

n=50;
[k6,y6]=pmf(n); % send to pmf function
[s6,f6]=pdf(n); % send to pdf function


% plot all PMF and PDF for different n values
figure(1)
subplot(3,3,1)
stem(k1,y1, "Linewidth", 1.3)
hold on;
plot(s1,f1, "Linewidth", 1.3)
legend('PMF', 'PDF')
grid on;
xlabel('k value')
ylabel('Sn')
title('N=1')

subplot(3,3,2)
stem(k2,y2, "Linewidth", 1.3)
hold on;
plot(s2,f2, "Linewidth", 1.3)
legend('PMF', 'PDF')
grid on;
```

```
xlabel('k value')
ylabel('Sn')
title('N=3')

subplot(3,3,3)
stem(k3,y3, "Linewidth", 1.3)
hold on;
plot(s3,f3, "Linewidth", 1.3)
legend('PMF', 'PDF')
grid on;
xlabel('k value')
ylabel('Sn')
title('N=10')

subplot(3,3,4)
stem(k4,y4, "Linewidth", 1.3)
hold on;
plot(s4,f4, "Linewidth", 1.3)
legend('PMF', 'PDF')
grid on;
xlabel('k value')
ylabel('Sn')
title('N=30')

subplot(3,3,5)
stem(k5,y5, "Linewidth", 1.3)
hold on;
plot(s5,f5, "Linewidth", 1.3)
legend('PMF', 'PDF')
grid on;
xlabel('k value')
ylabel('Sn')
title('N=40')

subplot(3,3,6)
stem(k6,y6, "Linewidth", 1.3)
hold on;
plot(s6,f6, "Linewidth", 1.3)
legend('PMF', 'PDF')
grid on;
xlabel('k value')
ylabel('Sn')
title('N=50')


%%%%%%%% Part 2 %%%%%%%%%%
n = 100;
var = 1; % variance
r = 330; % resistance
i = 1:1:n;
X = zeros(1,length(i));
```

```
for j = 1:length(i) % calculating X values
    X(j) = r + normrnd(0,var);
end

smean = (1/n)*sum(X); % calculate sample mean

% calculating upper and lower bounds
lbound = smean - ((1.96*sqrt(var))/(sqrt(n)));
ubound = smean + ((1.96*sqrt(var))/(sqrt(n)));

% plot the X values, resistance, sample mean, and convidence interval
figure(2)
scatter(i,X, "black")
hold on;
yline(r,'r','LineWidth',1.2)
hold on;
yline(smean, "b", "LineWidth",1.2)
hold on;
yline(lbound, "LineWidth",1)
hold on;
yline(ubound,"LineWidth",1)
grid on;
xlabel('i value')
title("Estimating Resistance from Nosiy Measurement")
legend('Sample Resistance','True Resitance', "Sample Mean", "Convidence Interval")



%%%%%%%%%%%%%%%%%%% Functions %%%%%%%%%%%%%%%%%%%%%%
function [k,y] = pmf(n) %PMF Calulation
    p = 0.5;
    k = 0:1:n;
    y = zeros(1,length(k));
    b = zeros(1,length(k)+1);

    for i = 1:n
        b(i) = nchoosek(n, k(i));
        y(i) = b(i)*(p^k(i))*((1-p)^(n-k(i)));
    end

end


function [s,f] = pdf(n) %PDF Calculation
    p = 0.5;
    s = 0:n/100:n;

    f = normpdf(s,n*p,sqrt(n*(p*(1-p))));

end
```