ECE 302: Probability, Statistics, and Random Processes for EE

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Assignment 1: Basic Concepts of Probability Theory

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Problem 1.2

Solution:

a. The urn experiment had the sample space, S1={0,1,2,3}, and the coin flipped has the sample space, S2={HH,TH,HT,TT}. Therefore, both experiments have four possible outcomes. If the 4 balls in the urn are each labeled with one of the events in S2, then the two experiment can generate the same relative frequencies.

b. Since there are two dice, each one has 6 possible outcomes; therefore, the pair of die tossed once have 36 outcomes. If the urn has 36 labeled balls (labels ranging from 1 to 6 – repeated 6 times) and only two are drawn, then it will have the same relative frequency as if we tossed two dice. Otherwise, 6 labeled balls (labels 1 to 6) can be in the urn, and one selected. Then, the selected one is replaced, and a second draw occurs. This method would also have the same relative frequency.

c. If two cards are drawn with replacement, then there are 52*52=2704 possible solutions. Hence, if we label 52 balls with 52*52 labeled pairs (matching the cards), then two draws can be made from the urn (replacing the 1st drawn card before the second draw) to achieve the same relative frequency. If the two cards are not replaced, then the 52 balls in the urn should be still labeled with 52*52 pairs. Two draws still need to be made, but the first ball is not replaced.

Problem 1.4

Solution:

a. $\boldsymbol{S_{Lisa}} = \{\mathbf{00}, \mathbf{01}, \mathbf{10}\}$
$\boldsymbol{S_{Homer}} = \{\mathbf{10}, \mathbf{11}\}$
$\boldsymbol{S_{Bart}} = \{\mathbf{00}, \mathbf{10}\}$

b. Lisa:
$N_{00}(3) = 1$ $N_{01}(3) = 1$ $N_{10}(3) = 1$

$\boldsymbol{F_{00}(3)} = \frac{\mathbf{1}}{\mathbf{3}}$ $\boldsymbol{F_{01}(3)} = \frac{\mathbf{1}}{\mathbf{3}}$ $\boldsymbol{F_{10}(3)} = \frac{\mathbf{1}}{\mathbf{3}}$

Homer:
$N_{10}(3) = 2$ $N_{11}(3) = 1$
$\boldsymbol{F_{10}(3)} = \frac{\mathbf{2}}{\mathbf{3}}$ $\boldsymbol{F_{11}(3)} = \frac{\mathbf{1}}{\mathbf{3}}$

Bart:
$N_{00}(3) = 2$ $N_{10}(3) = 1$
$\boldsymbol{F_{00}(3)} = \frac{\mathbf{2}}{\mathbf{3}}$ $\boldsymbol{F_{10}(3)} = \frac{\mathbf{1}}{\mathbf{3}}$

Problem 2.3

Solution:

a. $\boldsymbol{S} = \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}\}$

b. $\boldsymbol{A} = \{(\mathbf{1}, \mathbf{4}), (\mathbf{4}, \mathbf{1}), (\mathbf{3}, \mathbf{6}), (\mathbf{6}, \mathbf{3}), (\mathbf{2}, \mathbf{5}), (\mathbf{5}, \mathbf{2})\}$

c. $0 \leq k \leq 5$, |w-n| = k ---- w = number rolled of 1st dice, n = number rolled of 2nd dice

{5} = {(1,6), (6,1)}
{4} = {(1,5), (5,1), (6,2), (2,6)}
{3} = {(1,4), (4,1), (2,5), (5,2), (3,6), (6,3)}
{2} = {(1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,6), (6,4)}
{1} = {(1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (4,5), (5,4), (5,6), (6,5)}
{0} = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

Problem 2.8

Solution:

$U = [0, 1]$

$A =$ $[0,0.25) \cup (0.75,1]$

$1 - U < 0.5$ $\;or\;$ $U > 0.5$ $\quad \therefore B = (0.5, 1]$

$A \cap B = [0,0.25) \cup (0.75,1] \cap (0.5,1] =$ **(0.75,1]**

$A^C \cap B = [[0,0.25) \cup (0.75,1]]^c \cap (0.5,1] = [0.25,0.75] \cap (0.5,1] = [\mathbf{0.5}, \mathbf{0.75}]$

$A \cup B =$ $[0,0.25) \cup (0.75,1] \cup (0.5, 1] = \boldsymbol{U\ is\ in}\ [\mathbf{0}, \mathbf{0.25})\ \boldsymbol{or}\ (\mathbf{0.5}, \mathbf{1}]$

Problem 2.21

Solution:

a. $S = \{1, 2, 3, 4\ , 5, 6\}$ $\qquad p_k = \lim_{n\to\infty} \frac{N_k(n)}{n}$

$$\boldsymbol{p_1 = \lim_{n\to\infty} \frac{N_1(6)}{6} = \lim_{n\to\infty} \frac{1}{6} = \frac{1}{6}}$$
$$\boldsymbol{p_2 = \lim_{n\to\infty} \frac{N_2(6)}{6} = \lim_{n\to\infty} \frac{1}{6} = \frac{1}{6}}$$
$$\boldsymbol{p_3 = \lim_{n\to\infty} \frac{N_3(6)}{6} = \lim_{n\to\infty} \frac{1}{6} = \frac{1}{6}}$$
$$\boldsymbol{p_4 = \lim_{n\to\infty} \frac{N_4(6)}{6} = \lim_{n\to\infty} \frac{1}{6} = \frac{1}{6}}$$
$$\boldsymbol{p_5 = \lim_{n\to\infty} \frac{N_5(6)}{6} = \lim_{n\to\infty} \frac{1}{6} = \frac{1}{6}}$$
$$\boldsymbol{p_6 = \lim_{n\to\infty} \frac{N_6(6)}{6} = \lim_{n\to\infty} \frac{1}{6} = \frac{1}{6}}$$

b. $\Pr\,[A] = \Pr[4{,}5{,}6] = \Pr[4] + \Pr[5] + \Pr[6] = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \boldsymbol{\frac{3}{6}}$

$Pr[B] = \Pr[1{,}3{,}5] = \Pr[1] + \Pr[3] + \Pr[5] = \boldsymbol{\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}}$

$\Pr[A \cup B] = \Pr[1{,}3{,}5{,}4{,}5{,}6] = \boldsymbol{\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{5}{6}}$

$\Pr[A \cap B] = \Pr[5] = \boldsymbol{\frac{1}{6}}$

$\Pr[A^C] = \Pr[1{,}2{,}3] = \boldsymbol{\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}}$

Problem 2.22

Solution:

a. Sample space: 6*6 = 36 $p_k = \lim_{n\to\infty} \frac{N_k(n)}{n}$

   If die is considered fair:

   $p_1 + p_2 + p_3 + \ldots + p_{36} = 1$

   $\therefore$ ***probabiltiy of each elementary outcome is*** $\mathbf{\frac{1}{36}}$

b. A – number of dots in the first toss (m) is not less than numbers of dots in second toss (n) – m>n

   *** A has total of 21 events

   $\therefore \Pr[A] = \frac{21}{36} = \frac{7}{12} \approx \mathbf{0.5833}$

   B – number of dots in first toss is 6

   $\therefore \Pr[B] = \frac{6}{36} = \frac{1}{6} \approx \mathbf{0.1667}$

   C – number of dots in dice differ by 2

   $\therefore \Pr[C] = \frac{8}{36} = \frac{2}{9} \approx \mathbf{0.2222}$

   $A \cap B^c = \{(1,1), (1,2), (2,2), (1,3), (2,3), (3,3), (1,4), (2,4), (3,4), (4,4), (1,5), (2,5), (3,5), (4,5), (5,5)\}$

   $\Pr[A \cap B^c] = \frac{15}{36} \approx \mathbf{0.4167}$

   $A \cap C = \{(3,1), (4,2), (5,3), (6,4)\}$

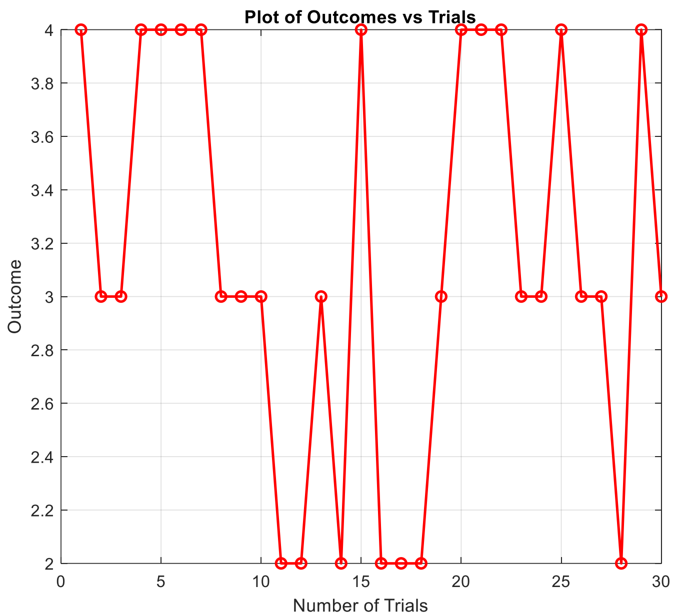
   $\Pr[A \cap C] = \frac{4}{36} \approx \mathbf{0.1111}$

Computer Experiments

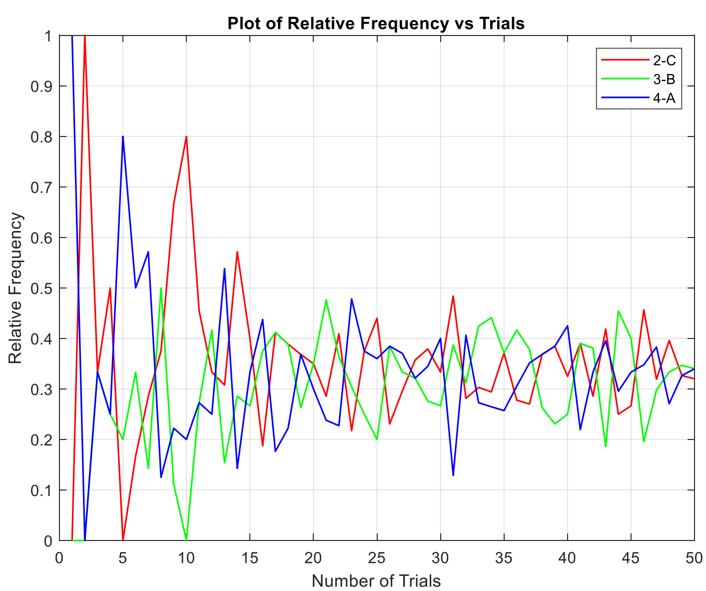
Solution:

1. $S = \{2,3,4\}$

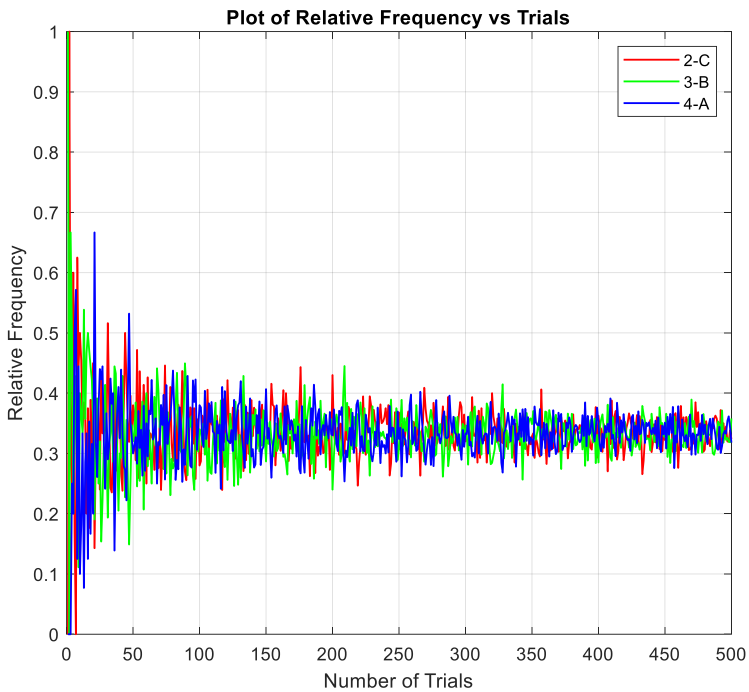
2. Outcome MATLAB Plot:



3. Relative Frequency – N = 50 trials



4. Relative Frequency – N=500 trials



5. $1 = p_A + p_B + p_C \qquad \therefore \Pr[A] = \mathbf{\frac{1}{3}}$

MATLAB Solutions

```
%clear all past figures and data
clc;
clear all;
close all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% #1 --Sample Space

%sample space
s = [2,3,4];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% #2 --Outcomes
%number of trails
n = 30;
%generate random numbers for number of trials (n) either 2,3, or 4 in
% row vector 'out'
out = randi([2,4],1,n);
%vector of 1 to number of trials conducted --- either 30,50,or 500
student = 1:n;

%plot number of students/trials vs Outcomes
figure(1)
plot(student,out, 'r-o','Linewidth', 1.5)
xlabel('Number of Trials')
ylabel('Outcome')
title('Plot of Outcomes vs Trials')
grid on


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% #3 --Relative Frequency
%intializing everything to zero -- f1,f2,f3 are vectors filled with zeros
f2=zeros(1,50);
f3=zeros(1,50);
f4=zeros(1,50);
n2=0;
n3=0;
n4=0;

%setting up vector and q number for the number of trials --changed from 50
%to 50 depending on question 3 or 4
q = 50;
trials = 1:q;

%loop for amount of trials
for i=1:q
    %set up random outcomes from 2,3,4 and put into vector outcome
    outcome = randi([2,4],1,i);

    %getting n value to calculate relative freq - find function gets matrix
    %of all the indexes of the number we are searching for in outcome vector
    %size function gets the size of the second number of the matrix -
    %therefore getting the sum of either 2,3,4 and puts sum in n2,n3, or n4
```

```
    n2= size(find(outcome==2),2);
    n3= size(find(outcome==3),2);
    n4= size(find(outcome==4),2);

    %calc relative freq every loop by diving n2,n3,n4 by trial number (i)
    f2(i)=n2/i;
    f3(i)=n3/i;
    f4(i)=n4/i;
 end


%plot f2,f3,f4 in figure 2, added legend and x-y axias labels
figure(2)
plot(trials,f2,'r','LineWidth', 1)
hold on
plot(trials,f3,'g','LineWidth', 1)
hold on
plot(trials,f4,'b','LineWidth', 1)
hold on
xlabel('Number of Trials')
ylabel('Relative Frequency')
title('Plot of Relative Frequency vs Trials')
legend('2-C','3-B','4-A')
grid on


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% #4 --Relative Frequency
ff2=zeros(1,500);
ff3=zeros(1,500);
ff4=zeros(1,500);
nn2=0;
nn3=0;
nn4=0;

%setting up vector and q number for the number of trials --changed from 50
%to 500 depending on question 3 or 4
qq = 500;
trial = 1:qq;

%loop for amount of trials
for i=1:qq
    %set up random outcomes from 2,3,4 and put into vector outcome
    outcomes = randi([2,4],1,i);

    %getting n value to calculate relative freq - find function gets matrix
    %of all the indexes of the number we are searching for in outcome vector
    %size function gets the size of the second number of the matrix -
    %therefore getting the sum of either 2,3,4 and puts sum in nn2,nn3, or nn4
    nn2= size(find(outcomes==2),2);
    nn3= size(find(outcomes==3),2);
    nn4= size(find(outcomes==4),2);

    %calc relative freq every loop by diving nn2,nn3,nn4 by trial number (i)
```

```
    ff2(i)=nn2/i;
    ff3(i)=nn3/i;
    ff4(i)=nn4/i;
 end


%plot ff2,ff3,ff4 in figure 2, added legend and x-y axias labels
figure(3)
plot(trial,ff2,'r','LineWidth', 1)
hold on
plot(trial,ff3,'g','LineWidth', 1)
hold on
plot(trial,ff4,'b','LineWidth', 1)
hold on
xlabel('Number of Trials')
ylabel('Relative Frequency')
title('Plot of Relative Frequency vs Trials')
legend('2-C','3-B','4-A')
grid on


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% #5 --Pr[A]
%elementary vector of 1
elementary = 1:1;

%dividing elementary vector divided by vector space to get probability of A
%given is that all 3 of sample space are equally likely
disp(length(elementary)/length(s));
```