# Homework 2

## Problem 1

(1)

- Choose

Torte and Pie as $x_1, x_2$.

- to maximize

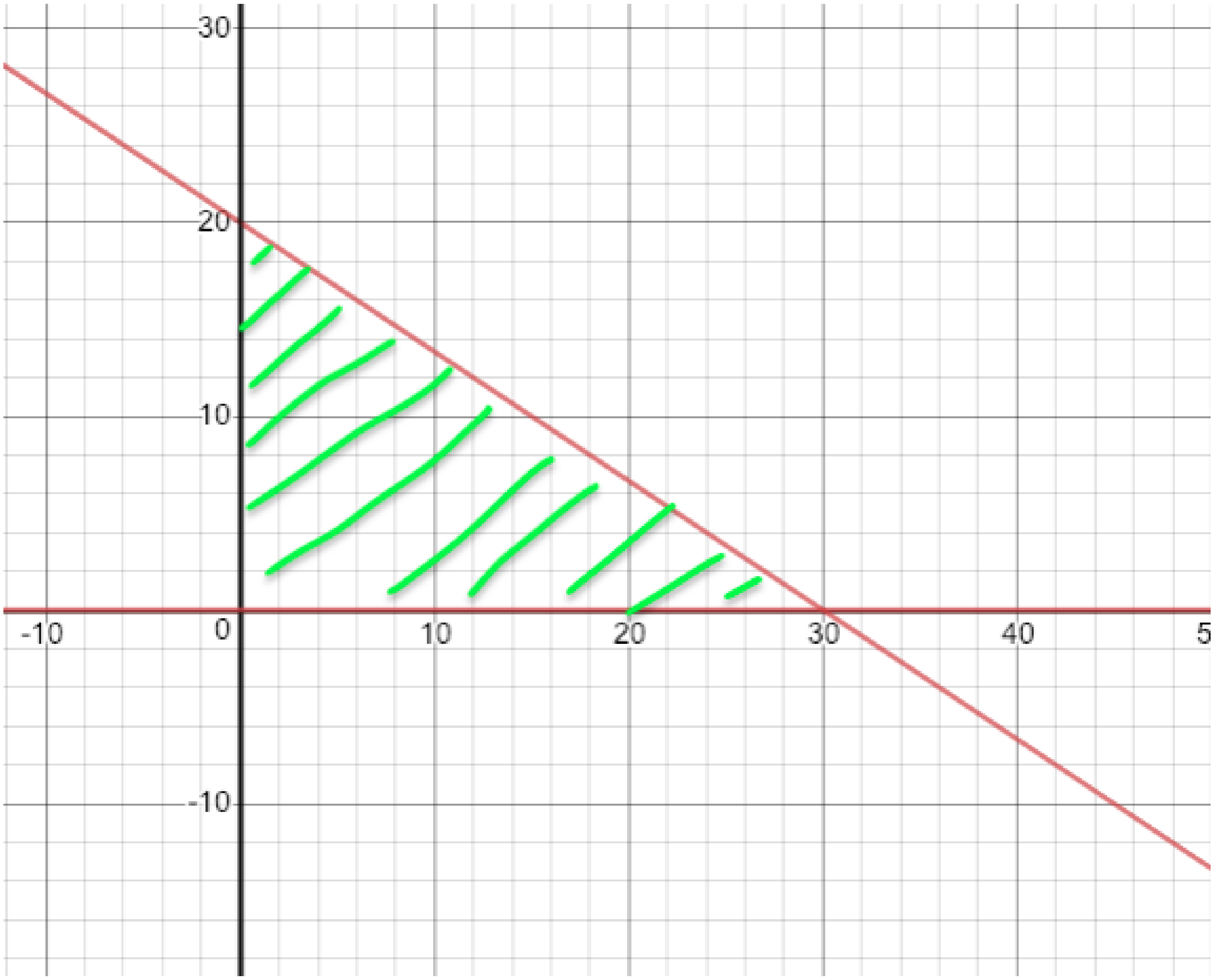
$$4x + 5y$$

- subject to

$$\begin{aligned} 2x + 3y &\leq 60 \\ x, y &\geq 0 \end{aligned}$$

To visualize the problem, the graph is as followed:

```
knitr::include_graphics("Q1.png")
```




The corners are $(0,0), (0,20), (30,0)$, and the corresponding points are $0, 100, 120$. Therefore, when Max eats 30 tortes and 0 pie, he would get the optimized points.

(2)

- Choose

Variant Torte and Pie as $x_1, x_2$.

- to maximize

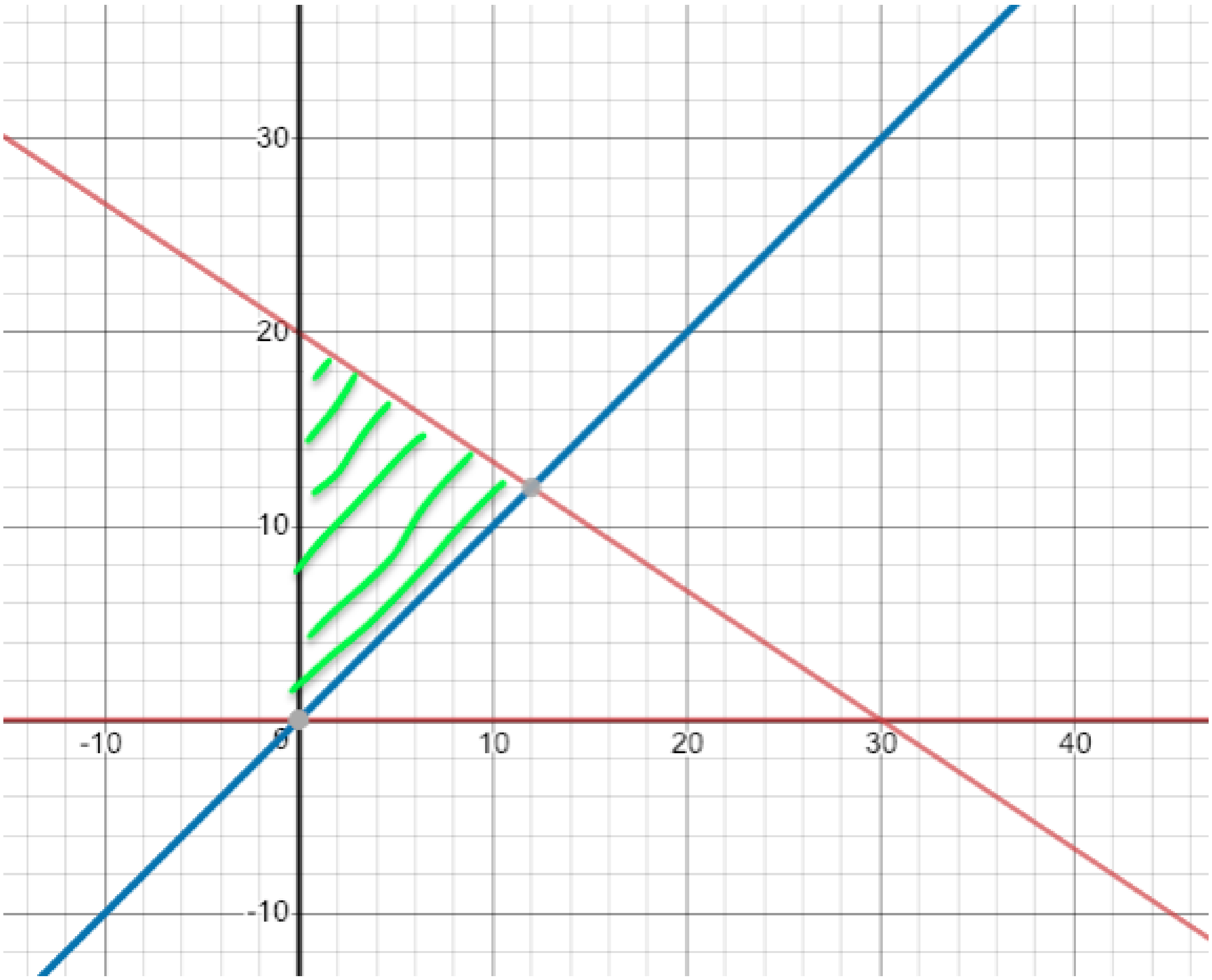
$$4x + 5y$$

- subject to

$$\begin{aligned} 2x + 3y &\leq 60 \\ y &\geq x \\ x, y &\geq 0 \end{aligned}$$

To visualize the problem, the graph is as followed:

```r
knitr::include_graphics("Q1(2).png",dpi=2)
```




The corners are $(0, 0), (0, 20), (12, 12)$, and the corresponding points are $0, 100, 108$. Therefore, when Max eats 12 tortes and 12 pie, he would get the optimized points, and this is 12 points less than the first question.

## Problem 2

(1)

- Choose

wheat and corn as $x, y$.

- to maximize

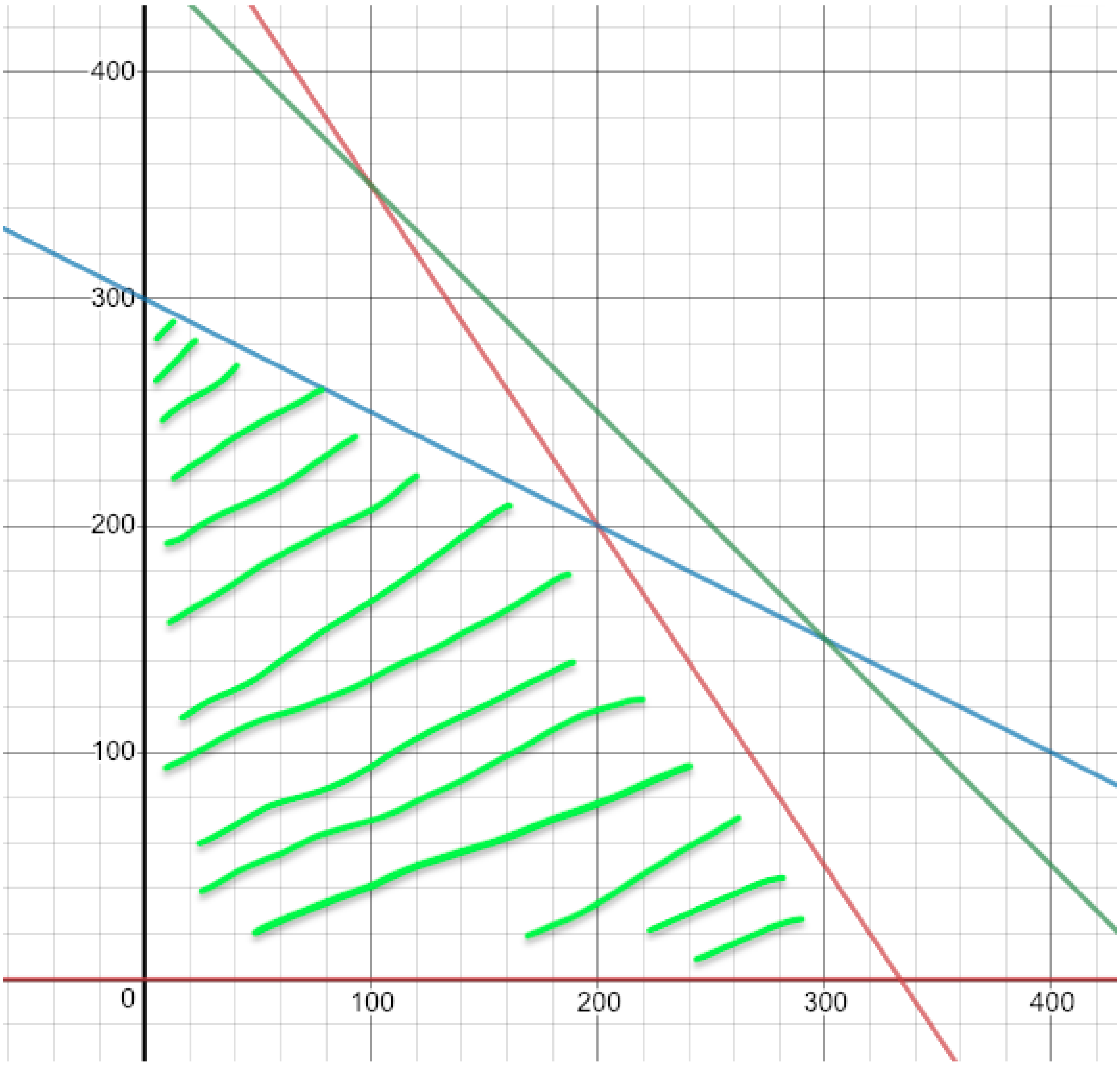
$$2000x + 3000y$$

- subject to

$$3x + 2y \leq 1000$$
$$2x + 4y \leq 1200$$
$$x + y \leq 450$$
$$x, y \geq 0$$

To visualize the problem, the graph is as followed:

```r
knitr::include_graphics("Q2.png")
```




The corners are $(0, 300), (200, 200), (333, 0)$(assume integer), and the corresponding points are $900000, 1000000, 666000$. Therefore, planting 200 acres wheat and 200 acres corn would get the optimized profits.

**Using R to solve this question:**

```r
library('lpSolve')
c = c(2000,3000)
A = matrix(c(3,2,2,4,1,1),3,2,byrow = TRUE)
b = c(1000,1200,450)
dir = rep("<=",3)
s = lp("max",c,A,dir,b)
s$solution
```

```
## [1] 200 200
```

What happens to the decision variables and the total profit when the availability of fertilizer varies from 200 tons to 2200 tons in 100-ton increments?

```r
f_seq = seq(200, 2200, by=100)
for (i in f_seq){
  b = c(1000,i,450)
  s = lp("max",c,A,dir,b)
  print(paste0('when fertilizer is: ',i,' solution is : ', s$solution[1],' ',s$solution[2] ))
  }
```

```
## [1] "when fertilizer is: 200 solution is : 100 0"
## [1] "when fertilizer is: 300 solution is : 150 0"
## [1] "when fertilizer is: 400 solution is : 200 0"
## [1] "when fertilizer is: 500 solution is : 250 0"
## [1] "when fertilizer is: 600 solution is : 300 0"
## [1] "when fertilizer is: 700 solution is : 325 12.4999999999999"
## [1] "when fertilizer is: 800 solution is : 300 50"
## [1] "when fertilizer is: 900 solution is : 275 87.5"
## [1] "when fertilizer is: 1000 solution is : 250 125"
## [1] "when fertilizer is: 1100 solution is : 225 162.5"
## [1] "when fertilizer is: 1200 solution is : 200 200"
## [1] "when fertilizer is: 1300 solution is : 175 237.5"
## [1] "when fertilizer is: 1400 solution is : 150 275"
## [1] "when fertilizer is: 1500 solution is : 125 312.5"
## [1] "when fertilizer is: 1600 solution is : 100 350"
## [1] "when fertilizer is: 1700 solution is : 49.9999999999999 400"
## [1] "when fertilizer is: 1800 solution is : 0 450"
## [1] "when fertilizer is: 1900 solution is : 0 450"
## [1] "when fertilizer is: 2000 solution is : 0 450"
## [1] "when fertilizer is: 2100 solution is : 0 450"
## [1] "when fertilizer is: 2200 solution is : 0 450"
```

When the fertilizer has large quantity, in the question: 1800 tons, the farmer discontinue producing wheat. When the fertilizer has small quantity, in the question: 300, he stop producing corn.

## Problem 3

- Choose

Investment 1,2,3,4,5 as $x_1, x_2, x_3, x_4, x_5$.

- to maximize

$$13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$$

- subject to

$$
\begin{aligned}
11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 &\le 40 \\
3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 &\le 20 \\
0 \le x_1, x_2, x_3, x_4, x_5 && \le 1
\end{aligned}
$$

```
c = c(13,16,16,14,39)
A = matrix(c(11,53,5,5,29,3,6,5,1,34,diag(x=1,5,5)),7,5,byrow = TRUE)
b = c(40,20,1,1,1,1,1)
dir = c(rep("<=",7))
s = lp("max",c,A,dir,b)
s$solution
```

```
## [1] 1.0000000 0.2008600 1.0000000 1.0000000 0.2880835
```

Therefore 1 investment1, 0.201 investment2, 1 investment3, 1 investment4, and 0.288 investment5.

## Problem 4

- Choose

corn, 2% Milk, Wheat Bread as $x, y, z$.

- to minimize

$$0.18x + 0.23y + 0.05z$$

- subject to

$$
\begin{aligned}
x &\le 10 \\
y &\le 10 \\
z &\le 10 \\
2000 \le 72x + 121y + 65z &\le 2250 \\
5000 \le 107x + 500y + 0z &\le 50,000 \\
x, y, z &\ge 0
\end{aligned}
$$

```
library('lpSolve')
c = c(0.18,0.23,0.05)
A = matrix(c(1,0,0,0,1,0,0,0,1,rep(c(72,121,65,107,500,0),2)),7,3,byrow = TRUE)
b = c(10,10,10,2250,50000,2000,5000)
dir = c(rep("<=",5),rep(">=",2))
s = lp("min",c,A,dir,b)
s$solution
```

```
## [1]  1.944444 10.000000 10.000000
```

## Problem 5

- Choose

unit1 year 1,2,3 as $x_1, x_2, x_3$, unit2 year 1,2,3 as $y_1, y_2, y_3$.

- to maximize

$$x_1 + 1.3x_2 + 1.4x_3 + y_1 + 1.2y_2 + 1.6y_3$$

- subject to

$$\begin{aligned}
x_1 + x_2 + x_3 &\leq 2 \\
y_1 + y_2 + y_3 &\leq 3 \\
1.2 \leq x_1 + y_1 &\leq 2 \\
1.5 \leq 1.3x_2 + 1.2y_2 &\leq 2 \\
2 \leq 1.4x_3 + 1.6x_3 &\leq 3 \\
x, y, z &\geq 0
\end{aligned}$$

```
c = c(1,1.3,1.4,1,1.2,1.6)
A = matrix(c(1,1,1,0,0,0,0,0,0,1,1,1,rep(c(1,0,0,1,0,0),2),rep(c(0,1.3,0,0,1.2,0),2),rep(c(0,0,1.4,0,0,
b = c(2,3,2,1.2,1.5,2,3,2)
dir = c(rep("<=",3),rep(">=",2),rep("<=",2),">=")
s = lp("max",c,A,dir,b)
s$solution
```

```
## [1] 0.4615385 1.5384615 0.0000000 1.1250000 0.0000000 1.8750000
```

Thus, cutting unit1/year1 0.46 acre, unit1/year2 1.54 acre, unit1/year3 0 acre. Unit2/year1 1.125 acre, unit2/year2 0 acre, unit2/year3 1.875 acre to maximize the tons of wood.