A2

2022310853 박연우.

연습문제: 1 3 4 9 10 11 12 15 19 20 21 23 24 25 27 28 29

컴퓨터 연습문제: 1 2

<연습문제>

1. (a) personal

(b) equally likely approach

(c) relative frequency approach

(d) relative frequency approach

(e) relative frequency approach

3. (a) $P(\text{Ace of Spades}) = 1/52$

(b) $P(\text{Ace}) = 4/52 = 1/13$

(c) $P(\text{Red Ace}) = 2/52 = 1/26$

(d) $P(\text{card} \geq 11) = 16/52 = 4/13$

(e) $P(\text{black card} \geq 11) = 8/52 = 2/13$

(f) $P(\text{two Ace cards}) = \binom{4}{2}/\binom{52}{2} = \frac{4\cdot 3}{\frac{52\cdot 51}{2}} = 1/221$

4. $0° \leq \theta \leq 360°$

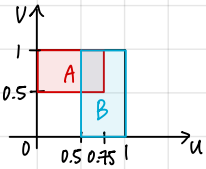
(a) $P(A) = \frac{45}{360} = 0.125$

(b) $P(B) = \frac{67.5-22.5}{360} = 0.125$

(c) $P(A \cap B) = P(22.5 \leq \theta \leq 45) = 1/16 = 0.625$

(d) $P(A \cup B) = P(0 \leq \theta \leq 67.5) = 67.5/360 = 0.1875$

9.



(a) $P(A) = \frac{3/4 \cdot 1/2}{1} = 3/8$

(b) $P(B) = 1/2$

(c) $P(A \cup B) = 3/4$

(d) $P(A \cap B) = 1/8$

10.

company

parts	X	Y	Z	total
A	113	207	342	662
B	57	116	175	348
C	202	83	77	362
total	372	406	594	1372

(a) $P(X) = \frac{372}{1372} = 0.2711$

⇒ 3개의 회사 중 X를 선택하는 확률

(b) $P(B) = \frac{348}{1372} = 0.2536$

⇒ 3개의 부품중 B를 선택할 확률.

(c) $P(B \cap X) = 57/1372 = 0.0415$

⇒ X회사의 B부품을 선택할 확률

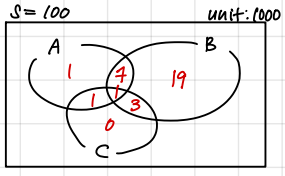
(d) $P(B \cup X) = \frac{(348+372-57)}{1372} = 0.4832$

⇒ X회사의 부품 or 부품 B를 선택할 확률

11. A : 10000 B: 30000 C: 5000 population: 100k

$A \cap B$: 8000 $A \cap C$: 2000 $B \cap C$: 4000 $A \cap B \cap C$: 1000

S = 100 unit: 1000



(a) $|A|+|B|+|C|-|A\cap B|-|B\cap C| - |A\cap C|-|A\cap B\cap C| = 32000$ 명

(b) $|A\cap B|+|B\cap C|+|A\cap C| = 11000$ 명

(c) $|S| - (\text{ans to (a)}) = 68000$ 명.

12. (a) $P(X > Y) = 1/2$

(b) $P(X+Y > 1/2) = 7/8$

(c) $P(\max(X,Y) > 1/2) = 3/4$

(d) $P(XY < 1/4) = 1/4 + 1/2 \ln 2$

$xy < 1/4 \Rightarrow y < 1/4x$

① $x: [0, 1/4]$, $y: [0, 1]$

$\int_0^{1/4}\int_0^1 dy\,dx = \int_0^{1/4} 1\,dx = 1/4$

② $x: [1/4, 1]$ $y: [0, 1]$

$\int_{1/4}^1 \int_0^{1/4x} dy\,dx = 1/4 \int_{1/4}^1 \frac{1}{x}\,dx = 1/4\,(\ln|x|\,|_{1/4}^1)$

$= 1/4(\ln|1| - \ln|1/4|) = 1/4 \cdot 2\ln(2) = 1/2\ln 2$

15. (a) $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{1/8}{1/2} = 1/4$

(b) $P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{1/8}{3/8} = 1/3$

(c) No because $P(A|B) \neq P(A)$ (or $P(B|A) \neq P(B)$)

19. (a) $P(X|C) = \frac{P(X\cap C)}{P(C)} = 202/362 = 0.5580$

(b) $P(Y|C) = \frac{P(Y\cap C)}{P(C)} = 83/362 = 0.2293$

(c) $P(Z|C) = \frac{P(Z\cap C)}{P(C)} = 77/362 = 0.2127$

20.

	1	2	3	tot
10원	10	5	5	20
100원	5	20	10	35
500원	30	15	5	50
total	45	40	20	105

(a) $P(10원) = 1/3(10/45) + 1/3(5/40) + 1/3 \cdot \frac{5}{20}$

$= 0.19907$

(b) $P(100원) = 1/3(5/45 + 20/40 + 10/20)$

$= 0.37037$

(c) $P(500원) = 1/3(\frac{30}{45} + \frac{15}{40} + \frac{5}{20})$

$= 0.43056$

21.

송신(S) \ 수신(R)	0	1
0	0.99	0.01
1	0.005	0.995

$P(S=0) = P(S=1) = 1/2$

$P(r=1) = P(r=1|S=0)\cdot P(S=0) + P(r=1|S=1) \cdot P(S=1) = (0.01\cdot 0.5) + (0.995 \cdot 0.5)$

$= 0.5025$ $P(r=0) = 0.4975$

1) $P(S=0|r=1) = \frac{P(r=1|S=0)P(S=0)}{P(r=1)} = \frac{0.01(0.5)}{0.5025} = 0.0995$

2) $P(S=0|r=0) = \frac{P(r=0|S=0)P(S=0)}{P(r=0)} = \frac{0.99(0.5)}{0.4975} = 0.994497$

3) $P(S=1|r=1) = \frac{P(r=1|S=1)P(S=1)}{P(r=1)} = \frac{0.995(0.5)}{0.5025} = 0.99005$

4) $P(S=1|r=0) = \frac{P(r=0|S=1)P(S=1)}{P(r=0)} = \frac{0.005(0.5)}{0.4975} = 0.00501$

23.

$P(Y_j$ 수신$|X_i$ 송신)

X_i(송신)	Y_j(수신) 0	1	2	3
0	0.96	0.02	0.02	0.01
1	0.006	0.98	0.005	0.01
2	0.01	0.01	0.97	0.01
3	0.02	0.03	0.02	0.93

0.1 0.3 0.3 0.3

$P(Y=1) = 0.1\cdot 0.02 + 0.3(0.98) + 0.3(0.01) + 0.3(0.03)$

$P(Y=2) = 0.1\cdot 0.02 + 0.3(0.005) + 0.3(0.97) + 0.3(0.2)$

(a) $P(X=2|Y=2) = \dfrac{P(Y=2|X=2)P(X=2)}{P(Y=2)} = \dfrac{0.97\cdot 0.3}{0.3005} = 0.9684$

(b) $P(X=1|Y=2) = \dfrac{P(Y=2|X=1)P(X=1)}{P(Y=2)} = \dfrac{0.005(0.3)}{0.3005} = 0.005$

(c) $P(X=2|Y=1) = \dfrac{P(Y=1|X=2)P(X=2)}{P(Y=1)} = \dfrac{0.01\cdot 0.3}{0.308} = 0.00974$

(d) $P(X=3|Y=1) = \dfrac{P(Y=1|X=3)P(X=3)}{P(Y=1)} = \dfrac{0.03\cdot 0.3}{0.308} = 0.0292$

⇒ 신뢰성 : $0.1\cdot 0.96 + 0.3\cdot 0.98 + 0.3\cdot 0.97 + 0.3\cdot 0.93 = 0.96$으로 높은 편이다.

(a)와 (c) 그리고 (b)와 (d)를 보면 수신값 = 전송값의 확률이 높다.

24. $P(D) = 0.01$ D = defect

$P(F|D) = 0.98$ $P(F|D^c) = 0.01$ F = fail

(a)

$P(D|F) = \dfrac{P(F|D)\cdot P(D)}{P(F)} = \dfrac{0.98\cdot 0.01}{0.0197} = 0.4975$

$P(F) = P(F|D)\cdot P(D) + P(F|D^c)\cdot P(D^c)$

$= 0.98\cdot 0.01 + 0.01\cdot 0.99 = 0.0197$

(b) $P(D^c|F^c) = \dfrac{P(F^c|D^c)P(D^c)}{P(F^c)} = \dfrac{0.99\cdot 0.99}{0.9803} = 0.9998$

$P(F^c) = 0.02(0.99) + 0.99(0.99) = 0.9803$

25. D = 약물복용자 T = 검사양성

$P(D) = 0.05$ $P(D^c) = 0.95$

$P(T|D) = 0.98$ $P(T|D^c) = 0.01$

$P(T) = P(T|D)P(D) + P(T|D^c)P(D^c) = 0.98\cdot 0.05 + 0.01\cdot 0.95$

$= 0.0585$

* 양성 반응자가 실제는 비사용자 :

$P(D^c|T) = \dfrac{P(T|D^c)P(D^c)}{P(T)} = \dfrac{0.01(0.95)}{0.0585} = 0.1624$

양성결과이지만 비사용자일 확률이 생각보다 높아서 놀라웠다.

27. $P(\text{everyone has a diff bday}) = \dfrac{365!}{(365-25)!} = 0.4313$

적어도 2명 이상 생일이 같을 prob $= 1 - 0.4313 = 0.5687$

28. 4장의 Ace : $\binom{4}{4} = 1$

Ace 4개 외의 한장 : $\binom{48}{1} = 48$

$\therefore \dfrac{48}{\binom{52}{5}} = 1.8469\cdot 10^{-5}$

29. (a) $\dfrac{13C_{13}}{52C_{13}} = 1.5748\times 10^{-12}$

(b) $\dfrac{4\cdot {}_{13}C_{13}}{52C_{13}} = 6.2991\times 10^{-12}$

<컴퓨터 실습 문제1>

1.

Editor - /Users/rachelie/Documents/MATLAB/a2_1.m

a2_1.m

```
function a2_1()
    clc;
    fprintf('=== Probability Calculator for Events A and B ===\n');

    % Get number of total outcomes
    N = input('Enter total number of outcomes: ');

    % Get probability for each outcome
    fprintf('Enter the probabilities for each of the %d outcomes (as a row vector):\n', N);
    probabilities = input('Example: [0.1 0.2 0.05 ...]: ');

    % Input indices for event A
    A_indices = input('Enter indices for event A (e.g., [1 2 3]): ');
    A_indices = unique(A_indices);

    % Input indices for event B
    B_indices = input('Enter indices for event B (e.g., [3 4 5]): ');
    B_indices = unique(B_indices);

    % Compute event sets
    A = A_indices(:);
    B = B_indices(:);

    % Intersection and union
    A_and_B = intersect(A, B);
    A_or_B = union(A, B);

    % Compute probabilities
    P_A = sum(probabilities(A));
    P_B = sum(probabilities(B));
    P_AB = sum(probabilities(A_and_B));
    P_AorB = sum(probabilities(A_or_B));

    % Conditional probabilities (protect against divide-by-zero)
    P_A_given_B = P_AB / P_B;
    P_B_given_A = P_AB / P_A;

    % Display results
    fprintf('\n--- Results ---\n');
    fprintf('P(A)         = %.4f\n', P_A);
    fprintf('P(B)         = %.4f\n', P_B);
    fprintf('P(A ∩ B)     = %.4f\n', P_AB);
    fprintf('P(A ∪ B)     = %.4f\n', P_AorB);
    fprintf('P(A | B)     = %.4f\n', P_A_given_B);
    fprintf('P(B | A)     = %.4f\n', P_B_given_A);
end
```

example run:

```
=== Probability Calculator for Events A and B ===
Enter total number of outcomes: 5
Enter the probabilities for each of the 5 outcomes (as a row vector):
Example: [0.1 0.2 0.05 ...]: [0.1 0.2 0.3 0.1 0.3]
Enter indices for event A (e.g., [1 2 3]): [1 2 3]
Enter indices for event B (e.g., [3 4 5]): [3 4 5]

--- Results ---
P(A)         = 0.6000
P(B)         = 0.7000
P(A ∩ B)     = 0.3000
P(A ∪ B)     = 1.0000
P(A | B)     = 0.4286
P(B | A)     = 0.5000
>>
```

2.

a2_1.m a2_2.m

```
function a2_2(trials)
    if nargin < 1
        trials = 1e6; % default number of simulations
    end

    rng('shuffle');

    % (a) At least 1 six in 6 dice rolls
    countA = 0;
    % (b) At least 2 sixes in 12 dice rolls
    countB = 0;
    % (c) At least 3 sixes in 18 dice rolls
    countC = 0;

    for i = 1:trials
        rollsA = randi(6, 1, 6);     % 6 dice
        rollsB = randi(6, 1, 12);    % 12 dice
        rollsC = randi(6, 1, 18);    % 18 dice

        if sum(rollsA == 6) >= 1
            countA = countA + 1;
        end
        if sum(rollsB == 6) >= 2
            countB = countB + 1;
        end
        if sum(rollsC == 6) >= 3
            countC = countC + 1;
        end
    end

    % Estimate probabilities
    pA = countA / trials;
    pB = countB / trials;
    pC = countC / trials;

    fprintf('Estimated probabilities after %d trials:\n', trials);
    fprintf('(a) P(at least 1 six in 6 rolls)     = %.4f\n', pA);
    fprintf('(b) P(at least 2 sixes in 12 rolls)  = %.4f\n', pB);
    fprintf('(c) P(at least 3 sixes in 18 rolls)  = %.4f\n', pC);
end
```

example run with 1 million trials

```
>> a2_2
Estimated probabilities after 1000000 trials:
(a) P(at least 1 six in 6 rolls)     = 0.6646
(b) P(at least 2 sixes in 12 rolls)  = 0.6191
(c) P(at least 3 sixes in 18 rolls)  = 0.5971
```