## Principle of Simple Induction

$$P(0) \land ( \forall n, P(n) \Rightarrow P(n+1)) \Rightarrow \forall n, P(n)$$

what we prove

what we conclude

> Induction!

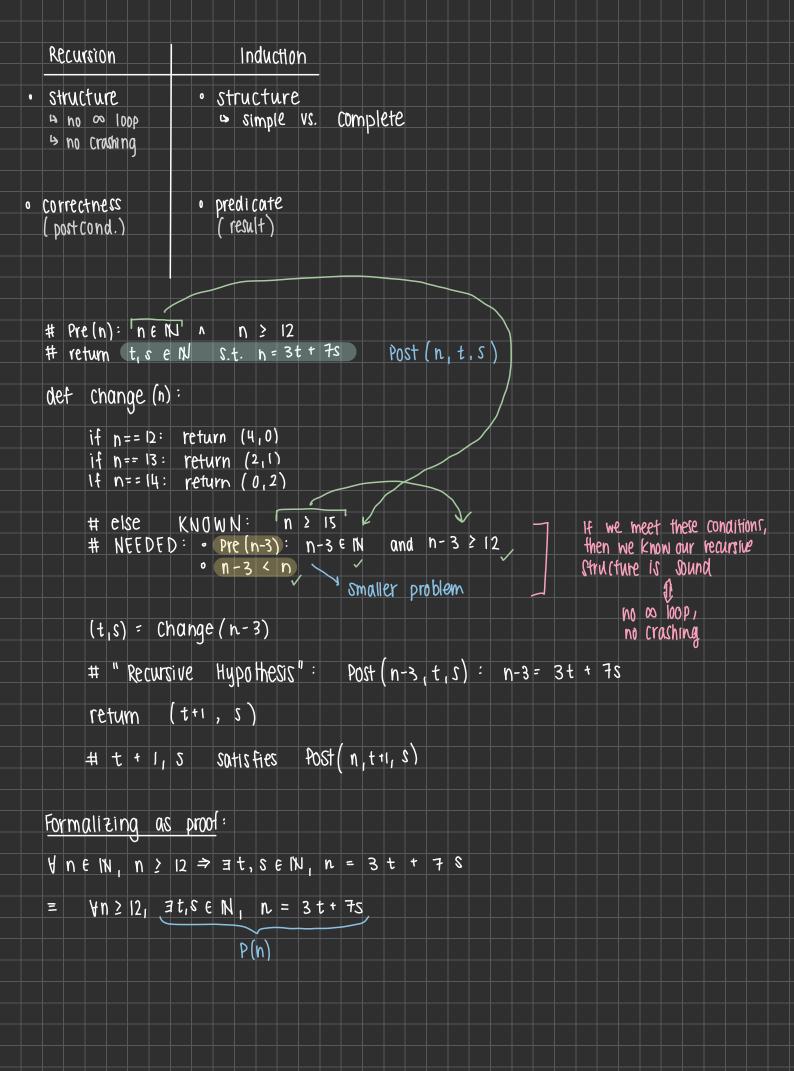
Formalize

Predicate:

$$\int Yn\xi iN, \quad \exists t,s \in \mathbb{N}, \quad n = 3t + 7s$$

Cannot quantify ninside, or else it becomes a dummy variable

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WTP: \\ \text{Yn212, P(n)}
     (\forall n \in \mathbb{N}, n \ge 12 \Rightarrow P(n))
Inductive Structure:
 P(12) 1
 P(13)
 P(14) \wedge \forall n \ge 15, P(n-3) \Rightarrow P(n)
                                    equivalent statements!
          ¥n212, P(n) ⇒ P(n+3)
                                     (see prep 2)
Homework: write the proof
Formalize as code:
# pre (n): n \in \mathbb{N} and n \ge 12
                            (pre-condition)
(post-condition)
def Change (n):
   if n = = 12: return (4,0)
   if n = = 13: return (2,1)
   if n = 14 : return (0,2)
   # else (n ≥ 15)
   (t,s) = change(n-3)
   return t+1,s
```



Pf. Algorithm starts w/ n that meets Pre (n) Two equally good ways to prove Let new with n 2 12. • Case n= 12: Then, 12=3.4 + 7.0 • CASE n=13: Let t=2, S=1. Then, t,S &N and 3++75 = 3.2 +7.1 = 6+7 = 13 · Case n = 14: (Similarly) · Case n ≥ 15: This is the justification in algorithm. Assume P(n-3). (Inductive hypothesis) Let  $t_1 s \in IN$  witness P(n-3), so n-3=3t+7s. So, h = 3t + 3 + 7S = 3(t+1) + 7Sso, t+1, s & N and witness P(n)