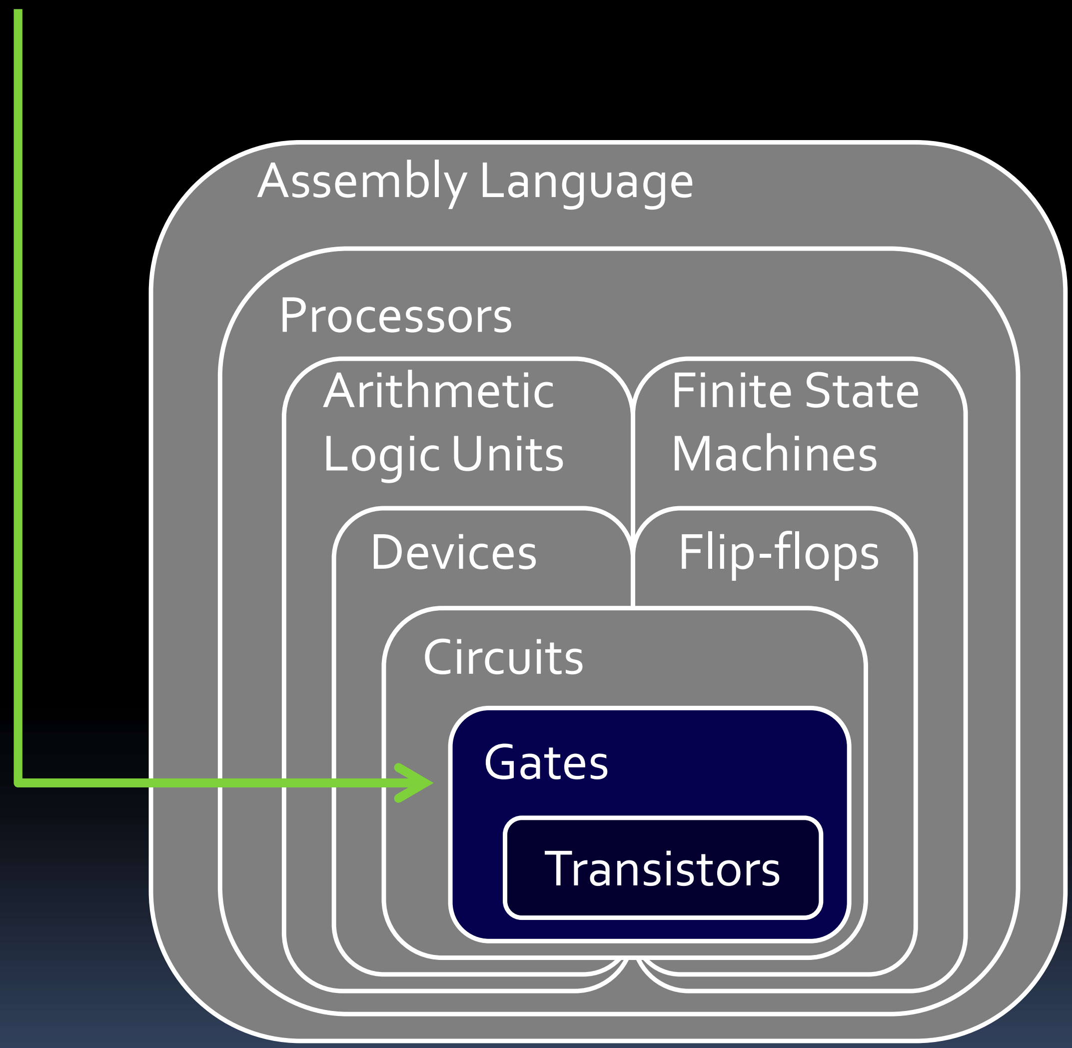
# Circuit Creation

# You are here



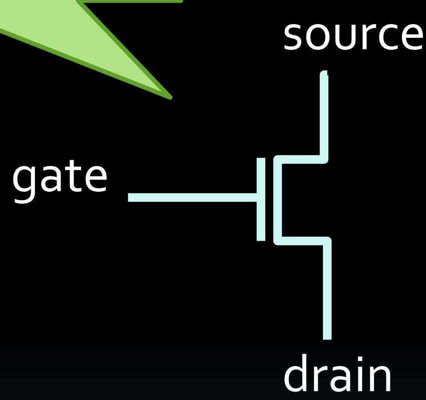
Assembly Language
Processors
Arithmetic Logic Units
Finite State Machines
Devices
Flip-flops
Circuits
Gates
Transistors

# Circuit creation goals

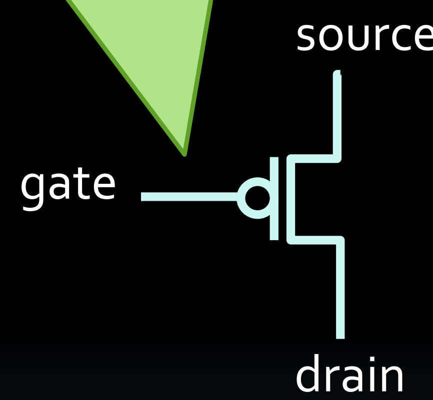
- What does it mean to design a circuit?
  - Given a specified input/output behaviour, connect circuit components to produce this behaviour.
  - Secondary goal: Create the circuit with the lowest possible cost (that uses the fewest components)
- We have seen this already in creating transistor circuits!

# Transistor Circuits

**<u>nMOS transistor:</u>**
**Source and drain are connected when gate input is high (aka "1")**

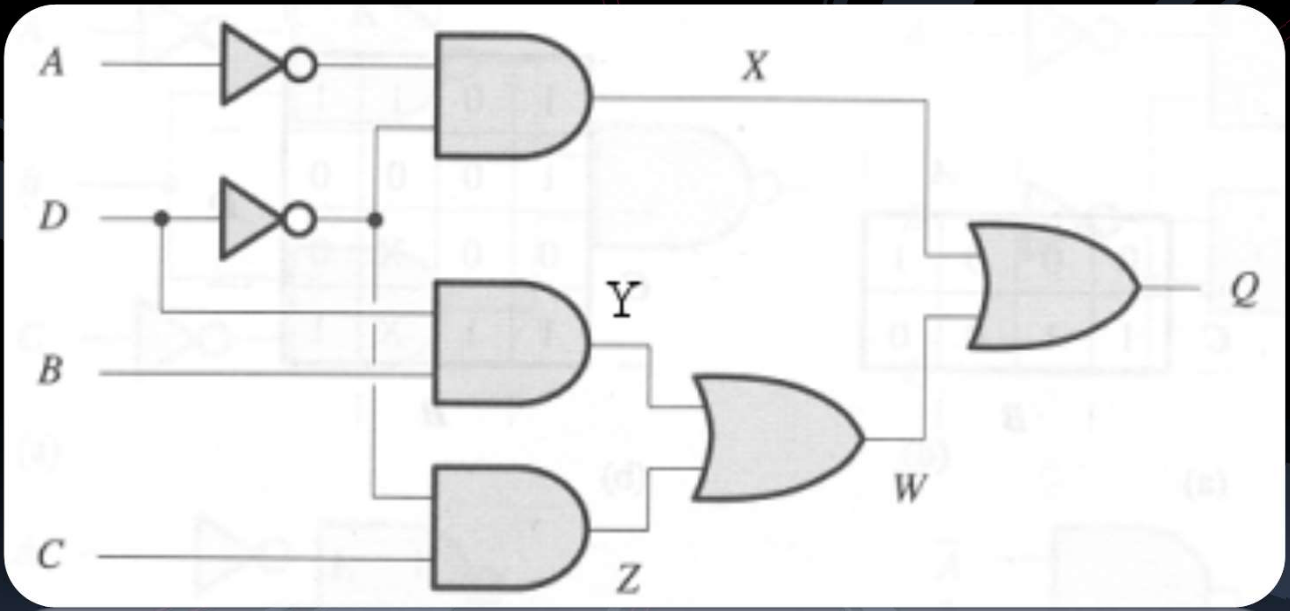



**<u>pMOS transistor:</u>**
**Source and drain are connected when gate input is low (aka "0")**




- Gates are built by constructing a circuit from nMOS and pMOS transistors.
  - What circuits can you make from these gates?

# Logic Gate Circuits

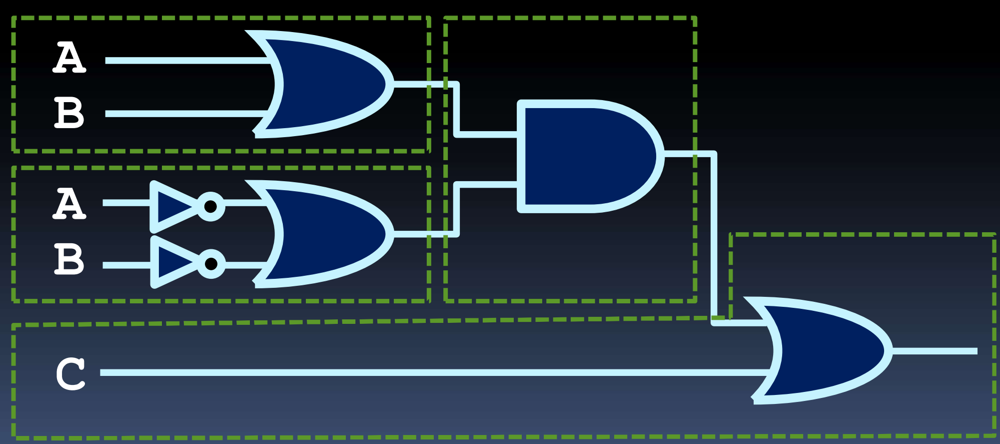


# Boolean expressions

- For Lab 1, you need to represent boolean expressions using logic gates. For example:

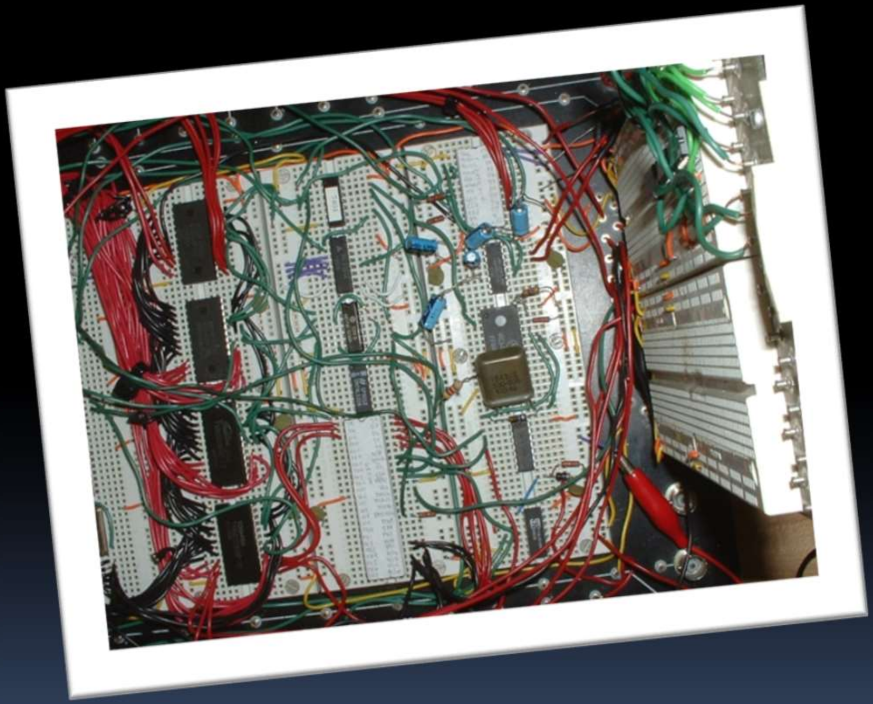
```
Y = (A or B) and (not A or not B) or C
```

- Like so:



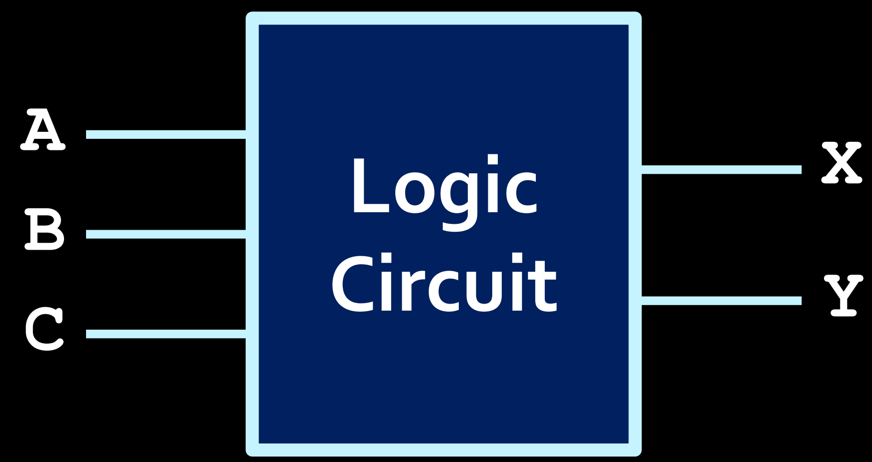
# Creating complex circuits

- What do we do in the case of more complex circuits, with several inputs and more than one output?
  - If you're lucky, a truth table is provided to express the circuit.
  - Usually the behaviour of the circuit is expressed in words, and the first step involves creating a truth table that represents the described behaviour.



# Circuit example

- The circuit on the right has three inputs (A, B and C) and two outputs (X and Y).




- *What logic is needed to set $X$ high when all three inputs are high?*
- *What logic is needed to set $Y$ high when the number of high inputs is odd?*

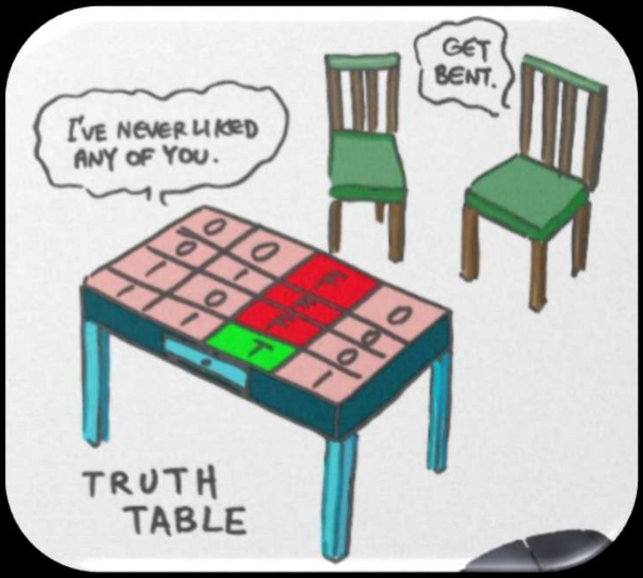
# Combinational circuits

- Small problems can be solved easily.

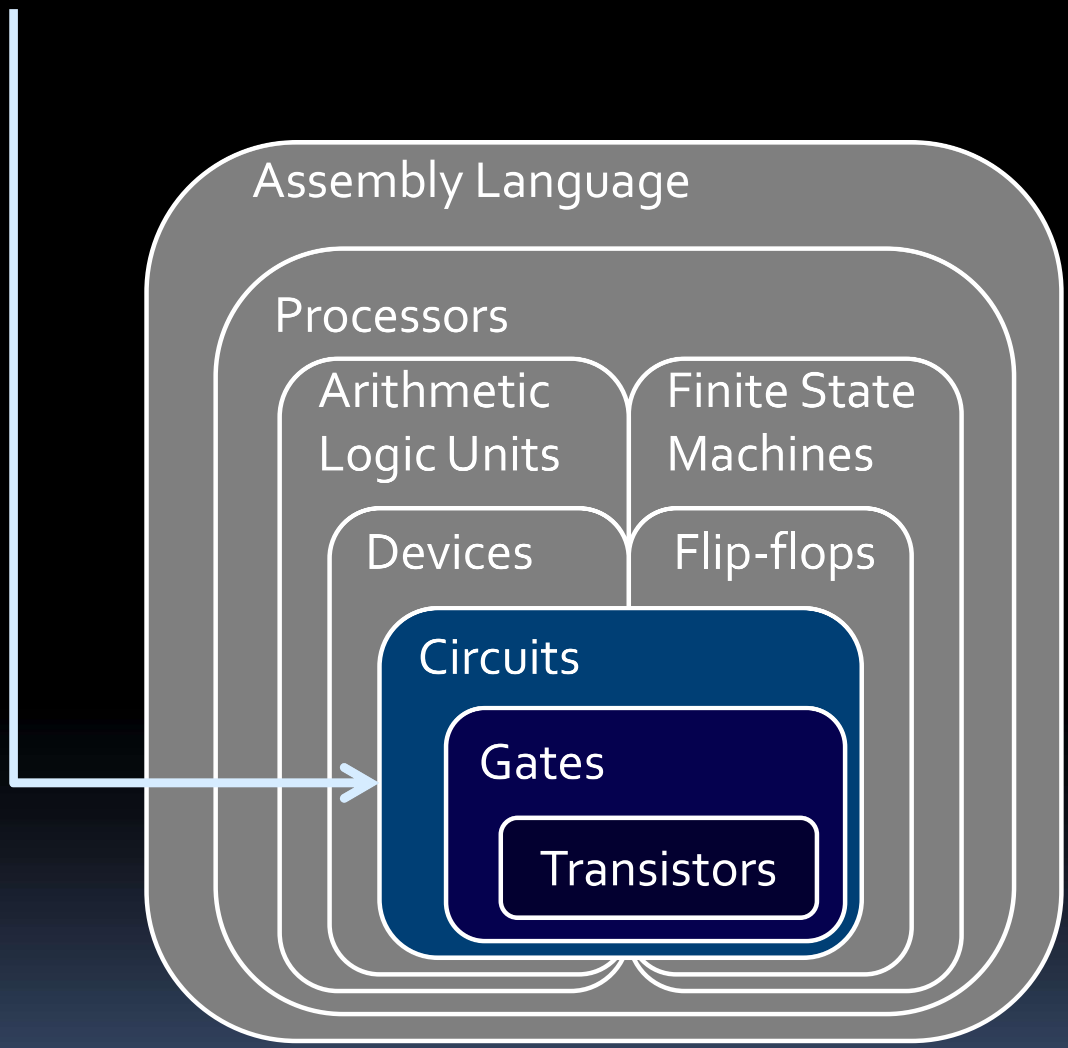

- Larger problems require a more systematic approach.
  - Example: "*Given three inputs A, B, and C, make output Y high in the case where all of the inputs are low, or when A and B are low and C is high, or when A and C are low but B is high, or when A is low and B and C are high.*"

# Creating complex logic

- How do we approach problems like these (and circuit problems in general)?
- Basic steps:
  1. Create truth tables.
  2. Express truth table behaviour as a boolean expression.
  3. Convert this expression into gates.
- The key to an efficient design?
  - Spending extra time on Step #2.



# Now you are here



# Lecture Goals

- After this lecture, you should be able to:
  - Create a truth table that represents the behaviour of a circuit you want to create.
  - Translate the rows in a circuit's truth table into gates that implement that circuit.
  - Use Karnaugh maps to reduce the circuit to the minimal number of gates.

# Circuits as truth tables

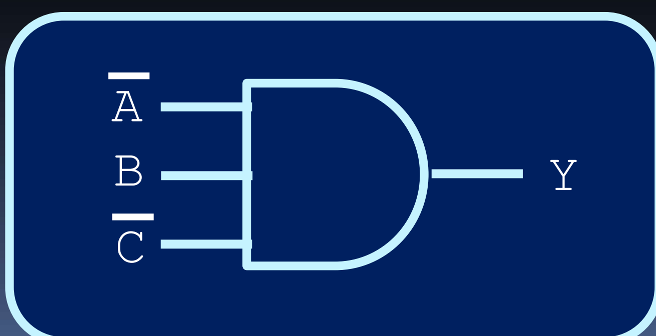
- Consider the following example:
  - *"Given three inputs $A$, $B$, and $C$, make output $Y$ high wherever any of the inputs are low, except when all three are low or when $A$ and $C$ are high."*
- This leads to the truth table on the right.
  - Is there a better way to describe the cases when the circuit's output is high?

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

# A simpler truth table

- What about the simpler truth table on the right?
- The output only goes high in one case, where A=0, B=1 and C=0.
- Translates easily into gates:

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |



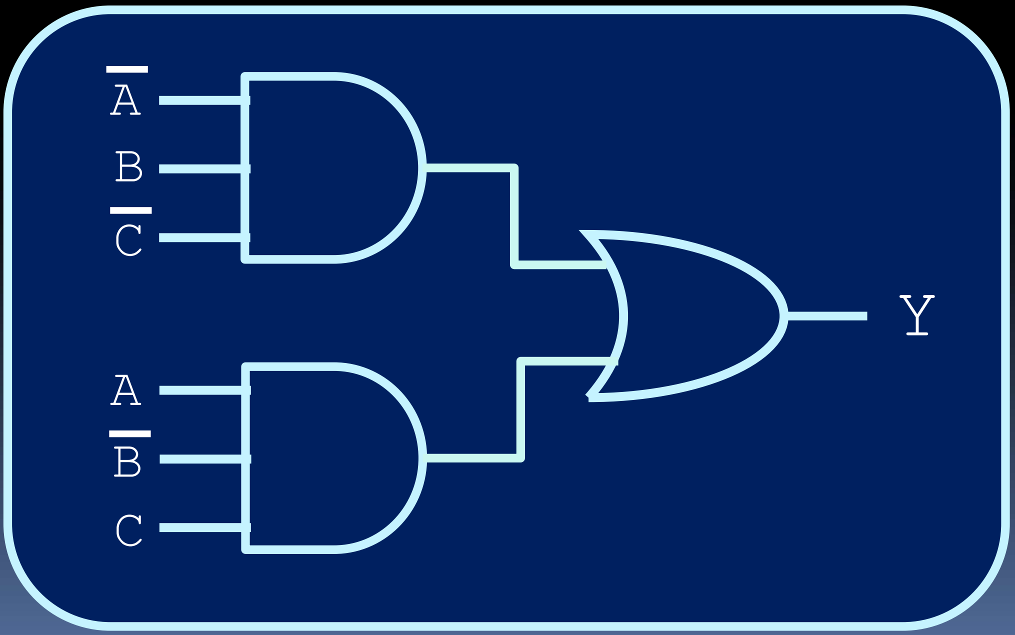

$$Y = \overline{A} \cdot B \cdot \overline{C}$$

$$Y = \overline{A}B\overline{C}$$

# A less simple truth table

- What about the truth table below?
  - The output now goes high in two cases (rows in table):
    - When A=0, B=1 and C=0.
    - When A=1, B=0 and C=1.
- Each case/row can be expressed as a single AND gate:
  - Overall circuit is implemented by combining these AND gates.

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |



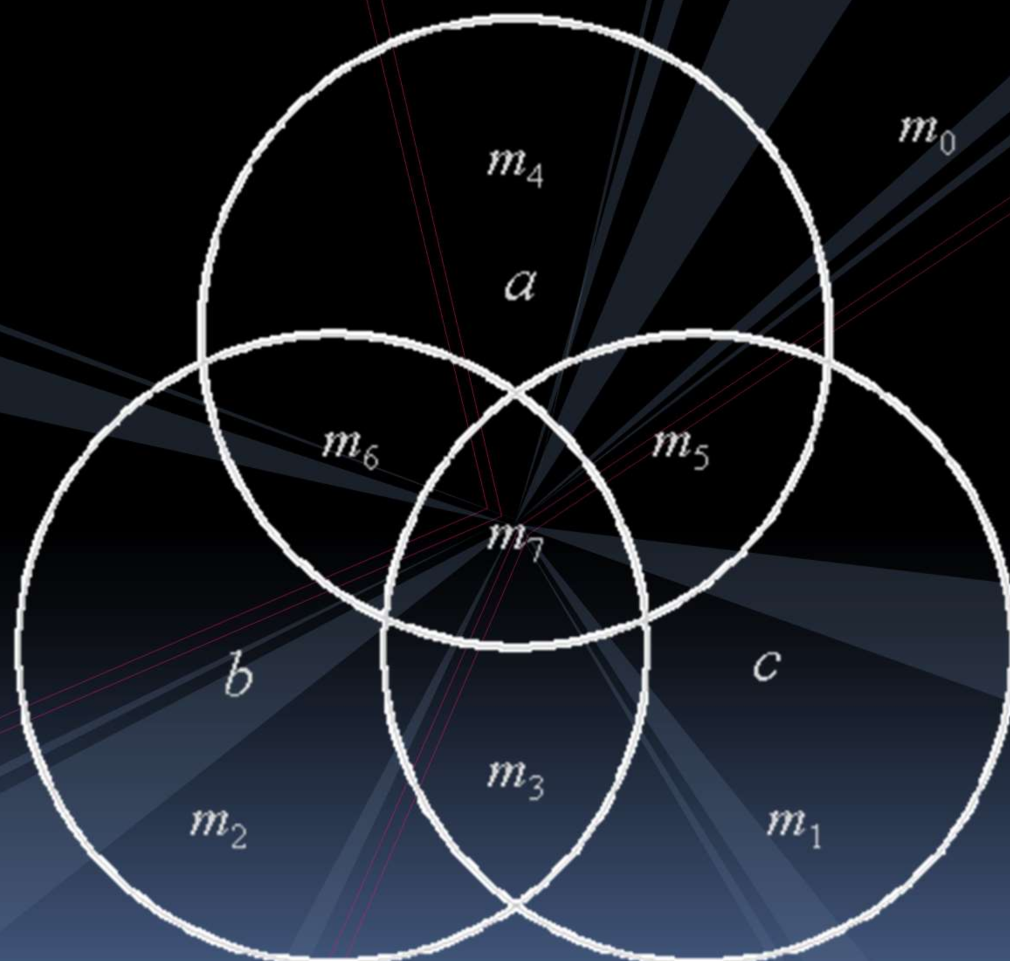
# Minterms

- This method of expressing circuit behaviour assumes the standard truth table format, then specifies which input rows cause high output.
  - The logical expression of these truth table rows (such as $A \cdot \overline{B} \cdot C$) are called minterms.

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

| Minterm | Y |
|---|---|
| $m_0$ | 0 |
| $m_1$ | 1 |
| $m_2$ | 1 |
| $m_3$ | 1 |
| $m_4$ | 1 |
| $m_5$ | 0 |
| $m_6$ | 1 |
| $m_7$ | 0 |

# Minterms and Maxterms



# Minterms and maxterms

- A more formal description:
  - Minterm = an AND expression with every input present in true or complemented form.
  - Maxterm = an OR expression with every input present in true or complemented form.
  - For example, given four inputs (A, B, C, D):
    - Valid minterms:
      - $A \cdot \overline{B} \cdot C \cdot D$, $\overline{A} \cdot B \cdot \overline{C} \cdot D$, $A \cdot B \cdot C \cdot D$
    - Valid maxterms:
      - $A+\overline{B}+C+D$, $\overline{A}+B+\overline{C}+D$, $A+B+C+D$
    - Neither minterm nor maxterm:
      - $A \cdot B+C \cdot D$, $A \cdot B \cdot D$, $A+B$

# Boolean expression notation

- A quick aside about notation:
  - AND operations are denoted in these expressions by the multiplication symbol.
    - e.g. $A \cdot B \cdot C$ or $A*B*C \approx A \wedge B \wedge C$
  - OR operations are denoted by the addition symbol.
    - e.g. $A+B+C \approx A \vee B \vee C$
  - NOT is denoted by multiple symbols.
    - e.g. $\neg A$ or $A'$ or $\bar{A}$
  - XOR occurs rarely in circuit expressions.
    - e.g. $A \oplus B$

# The intuition behind minterms

- To clarify what a mintem means, consider how this expression behaves:

$$m_{15} = A \cdot B \cdot C \cdot D$$

- How do you describe the logical expression above?
- $m_{15}$ describes the case where the output is low at all times, except when A=1, B=1 , C=1 and D=1.

| A | B | C | D | $m_{15}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

# The intuition behind maxterms

- Similarly, consider the following maxterm expression:

$$M_0 = A+B+C+D$$

  - What is this behaviour?
  - $M_0$ is always high, except in the one case where all four input values are low.
- Try it with other input combinations!

| A | B | C | D | $M_0$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Minterm & maxterm notation

- Circuits are often described using minterms or maxterms, as a form of logic shorthand.
  - Given $n$ inputs, there are $2^n$ minterms and maxterms possible (same as the # of rows in the truth table).
  - Naming scheme:
    - Minterms are labeled as $m_x$, maxterms are labeled as $M_x$
      - The $x$ subscript indicates the row in the truth table.
      - $x$ starts at $0$ (when all inputs are low), and ends with $2^n-1$.
  - <u>Example:</u> Given 3 inputs –
    - Minterms are $m_0$ $(\overline{A}\cdot\overline{B}\cdot\overline{C})$ to $m_7$ $(A\cdot B\cdot C)$
    - Maxterms are $M_0$ $(A+B+C)$ to $M_7$ $(\overline{A}+\overline{B}+\overline{C})$

# Minterm & maxterm intuition

- A minterm specifies a row in the truth table where the input values of that row set the output high.
  - Consider: What expression results in a high output for only the first row of the truth table (when inputs are all zero)?

$$Y = \underbrace{\overline{A} \cdot \overline{B} \cdot \overline{C}} = m_0$$

Convert this into a binary # to get the minterm subscript!

- A maxterm specifies a row in the truth table where the input values of that row set the output low.
  - Consider: What expression results in a low output for only the first row of the truth table (when inputs are all zero)?

$$Y = A + B + C = M_0$$

# Quick Exercises

- Given 4 inputs A, B, C and D write:
  - $m_9$
  - $m_{15}$
  - $m_{16}$
  - $M_2$
- Which minterm is this?
  - $\overline{A} \cdot B \cdot \overline{C} \cdot \overline{D}$
- Which maxterm is this?
  - $A+B+C+\overline{D}$

# Minterms into circuits

- How are minterms used for circuits?
  - A single minterm indicates a set of inputs that will make the output go high.
  - <u>Example:</u> $m_2$
    - Output only goes high in the third line of this truth table (assuming 4 inputs).

| A | B | C | D | $m_2$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

# Minterms into circuits

- What if we want to combine minterms?
  - Use an OR operation!
    - The result is an output that goes high in both minterm cases.
  - <u>Example:</u> Consider $m_2+m_8$
    - The third and ninth lines of this truth table result in high output.

| A | B | C | D | $m_2$ | $m_8$ | $m_2+m_8$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |

# Combining minterms & maxterms

- Two canonical forms of boolean expressions:
  - Sum-of-Minterms (SOM):
    - Since each minterm corresponds to a single high output in the truth table, the combined high outputs are a union of these minterm expressions.
    - Expressed in "Sum-of-Products" form.
  - Product-of-Maxterms (POM):
    - Since each maxterm only produces a single low output in the truth table, the combined low outputs are an intersection of these maxterm expressions.
    - Expressed in "Product-of-Sums" form.

$$Y = m_2 + m_6 + m_7 + m_{10} \quad (SOM)$$

| A | B | C | D | $m_2$ | $m_6$ | $m_7$ | $m_{10}$ | Y |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | | |
| 0 | 0 | 0 | 1 | | | | | |
| 0 | 0 | 1 | 0 | | | | | |
| 0 | 0 | 1 | 1 | | | | | |
| 0 | 1 | 0 | 0 | | | | | |
| 0 | 1 | 0 | 1 | | | | | |
| 0 | 1 | 1 | 0 | | | | | |
| 0 | 1 | 1 | 1 | | | | | |
| 1 | 0 | 0 | 0 | | | | | |
| 1 | 0 | 0 | 1 | | | | | |
| 1 | 0 | 1 | 0 | | | | | |
| 1 | 0 | 1 | 1 | | | | | |
| 1 | 1 | 0 | 0 | | | | | |
| 1 | 1 | 0 | 1 | | | | | |
| 1 | 1 | 1 | 0 | | | | | |
| 1 | 1 | 1 | 1 | | | | | |

# Using Sum-of-Minterms

- Sum-of-Minterms is a way of expressing which inputs cause the output to go high.
  - Assumes that the truth table columns list the inputs according to some logical or natural order.
- Minterm and maxterm expressions are used for efficiency reasons:
  - More compact that displaying entire truth tables.
  - Sum-of-minterms are useful in cases with very few input combinations that produce high output.
    - Product-of-maxterms useful when expressing truth tables that have very few low output cases…

$$Y = M_3 \cdot M_5 \cdot M_7 \cdot M_{10} \cdot M_{14} \text{ (POM)}$$

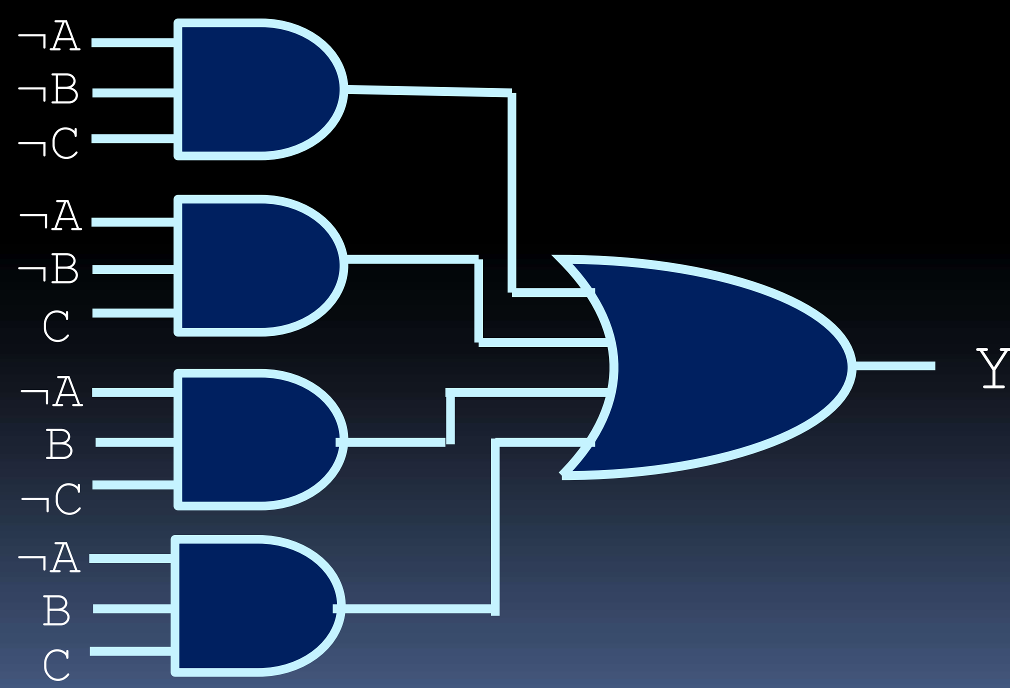
| A | B | C | D | $M_3$ | $M_5$ | $M_7$ | $M_{10}$ | $M_{14}$ | Y |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | | | |
| 0 | 0 | 0 | 1 | | | | | | |
| 0 | 0 | 1 | 0 | | | | | | |
| 0 | 0 | 1 | 1 | | | | | | |
| 0 | 1 | 0 | 0 | | | | | | |
| 0 | 1 | 0 | 1 | | | | | | |
| 0 | 1 | 1 | 0 | | | | | | |
| 0 | 1 | 1 | 1 | | | | | | |
| 1 | 0 | 0 | 0 | | | | | | |
| 1 | 0 | 0 | 1 | | | | | | |
| 1 | 0 | 1 | 0 | | | | | | |
| 1 | 0 | 1 | 1 | | | | | | |
| 1 | 1 | 0 | 0 | | | | | | |
| 1 | 1 | 0 | 1 | | | | | | |
| 1 | 1 | 1 | 0 | | | | | | |
| 1 | 1 | 1 | 1 | | | | | | |

# Converting SOM to gates

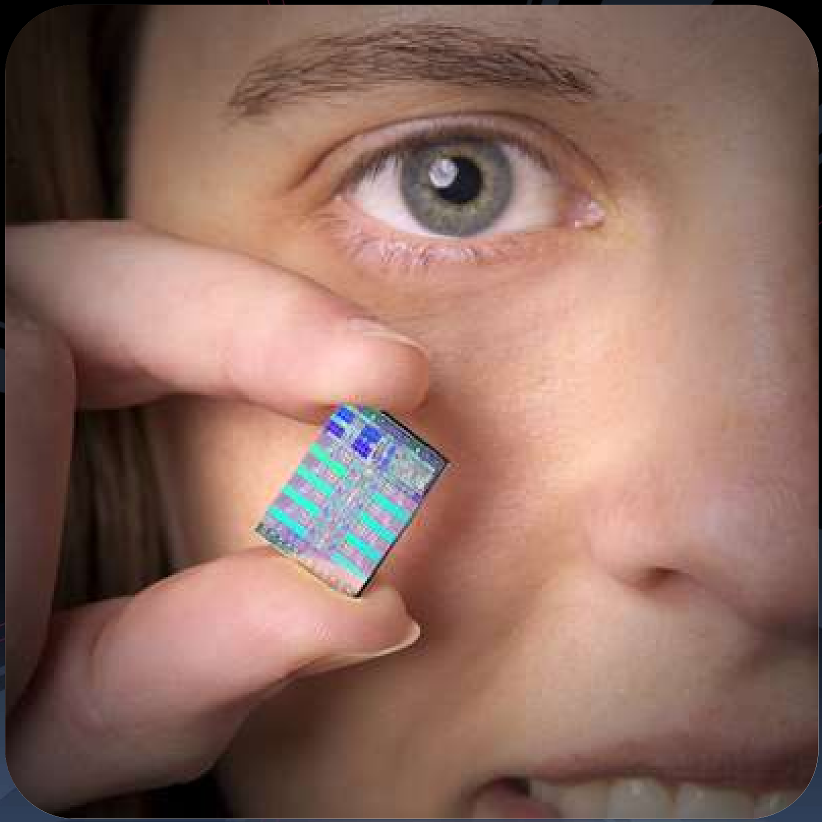
- Once you have a Sum-of-Minterms expression, it is easy to convert this to the equivalent combination of gates:

$$m_0 + m_1 + m_2 + m_3 =$$

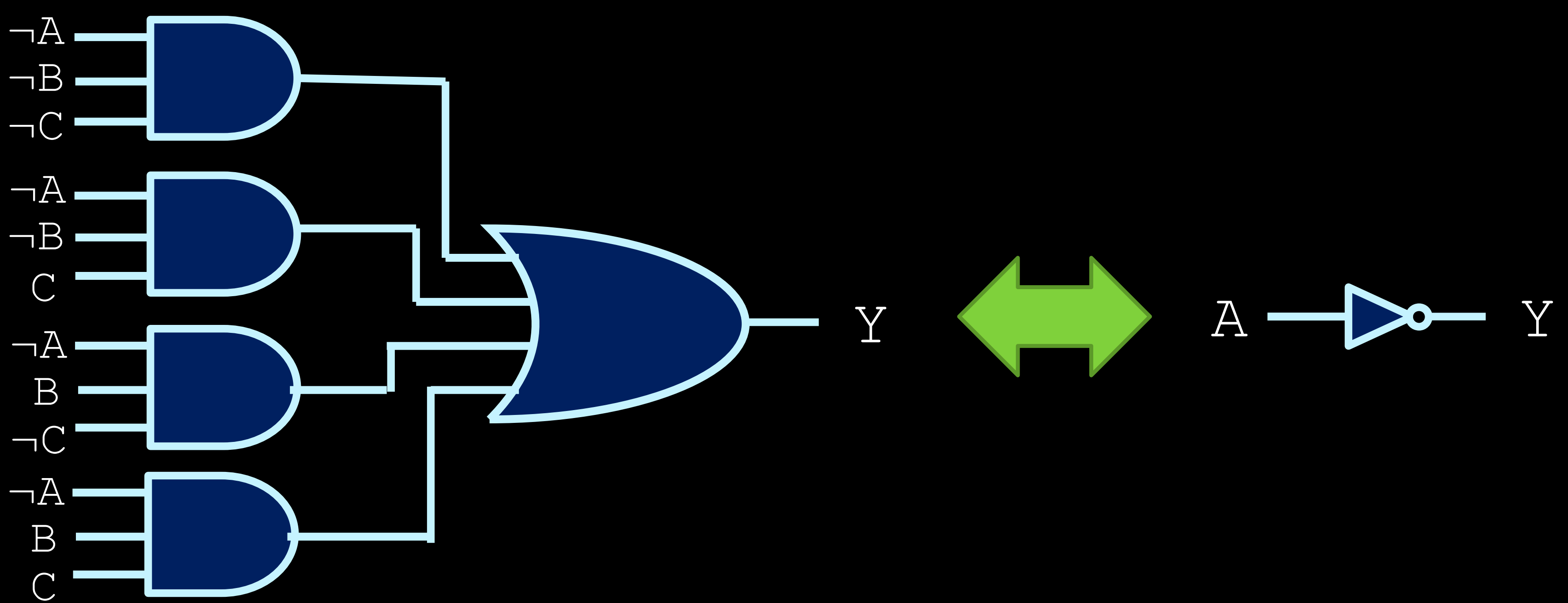
$$\overline{A} \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C + \overline{A} \cdot B \cdot \overline{C} + \overline{A} \cdot B \cdot C =$$



# Reducing circuits



# Reasons for reducing circuits




- Note example of Sum-of-Minterms circuit design.
- To minimize the number of gates, we want to reduce the boolean expression as much as possible from a collection of minterms to something smaller.
- This is where CSC165 skills come in handy ☺

# Boolean algebra review

- Axioms:

$$0 \cdot 0 = 0 \qquad 0 \cdot 1 = 1 \cdot 0 = 0$$
$$1 \cdot 1 = 1 \qquad \text{if } x = 1,\ \overline{x} = 0$$

- From this, we can extrapolate:

If one input of a 2-input AND gate is 1, then the output is whatever value the other input is.

$$x \cdot 0 = \qquad x+1 =$$
$$x \cdot 1 = \qquad x+0 =$$
$$x \cdot x = \qquad x+x =$$
$$x \cdot \overline{x} = \qquad x+\overline{x} =$$
$$\overline{\overline{x}} =$$

If one input of a 2-input OR gate is 0, then the output is whatever value the other input is.

# Other Boolean identities

- Commutative Law:

$$x \cdot y = y \cdot x \qquad\qquad x+y = y+x$$

- Associative Law:

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$
$$x+(y+z) = (x+y)+z$$

- Distributive Law:

$$x \cdot (y+z) = x \cdot y + x \cdot z$$
$$x+(y \cdot z) = (x+y) \cdot (x+z)$$

Does this hold in conventional algebra?

# Consensus Law (via Venn diagram)

- Consensus Law:

$$x \cdot y + \overline{x} \cdot z + y \cdot z = x \cdot y + \overline{x} \cdot z$$

- Proof by Venn diagram:
  - $x \cdot y$
  - $\overline{x} \cdot z$
  - $y \cdot z$
    - Already covered!

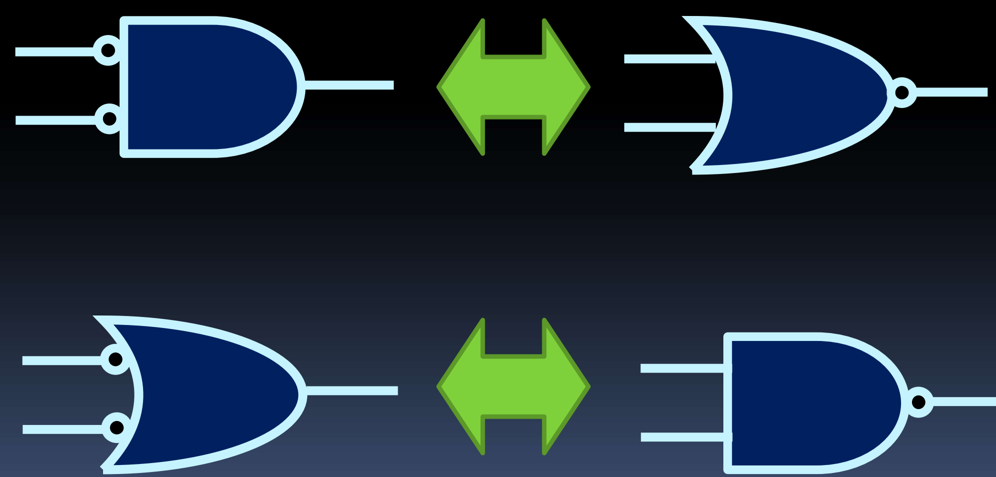
# Other boolean identities

- Absorption Law:

$$x\cdot(x+y) = x \qquad\qquad x+(x\cdot y) = x$$

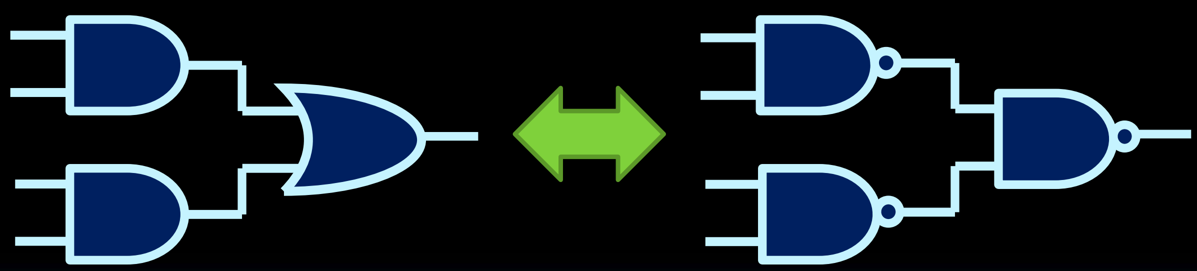
- De Morgan's Laws:

$$\overline{x}\cdot\overline{y} = \overline{x+y}$$
$$\overline{x}+\overline{y} = \overline{x\cdot y}$$

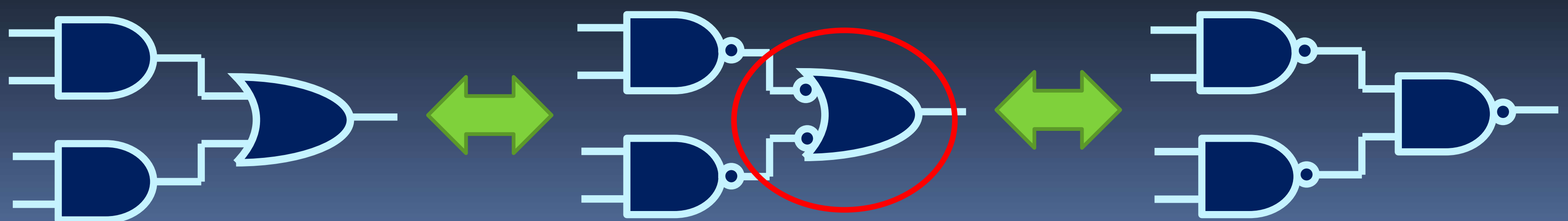


# Converting to NAND gates

- De Morgan's Law is important because out of all the gates, NANDs are the cheapest to fabricate.
  - a Sum-of-Products circuit could be converted into an equivalent circuit of NAND gates:



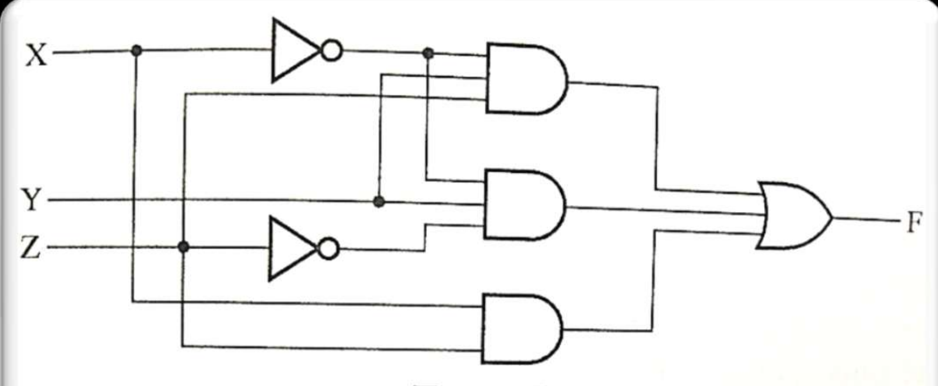
- This is all based on de Morgan's Law:



# Reduction goal: gate cost

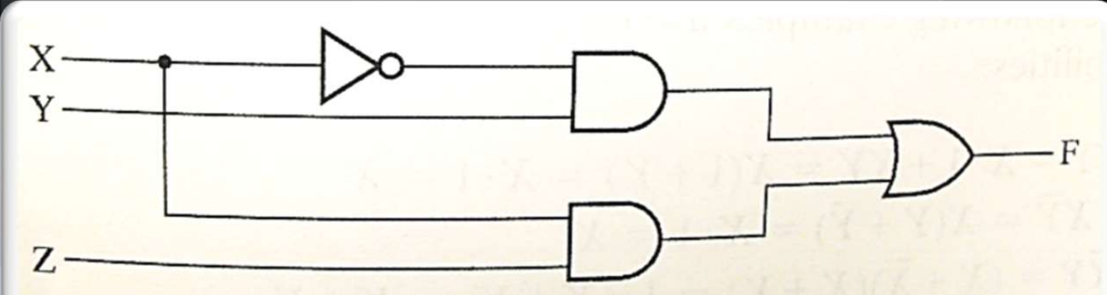
- If these two circuits perform the same operation, which implementation do you prefer? Why?

A.




(a) $F = \overline{X}YZ + \overline{X}Y\overline{Z} + XZ$

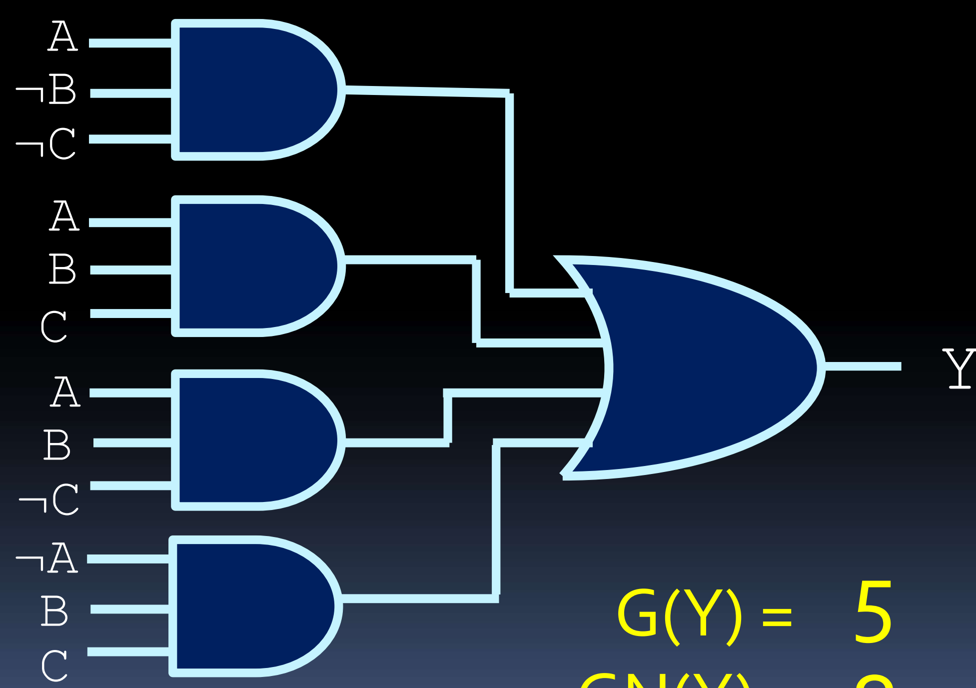
B.




(b) $F = \overline{X}Y + XZ$

# Measuring gate cost

- How do we measure the "simplest" expression?
  - In this case, "simple" denotes the lowest gate cost (G) or the lowest gate cost with NOTs (GN).
  - To calculate the gate cost, simply add all the gates together (as well as the cost of the NOT gates, in the case of the GN cost).




G(Y) = 5

GN(Y) = 8

# Reducing boolean expressions

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

- Assuming logic specs at left, we get the following:

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

- Now start combining terms, like the last two:

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + \mathbf{A \cdot B}$$

# Reducing boolean expressions

- Different final expressions possible, depending on what terms you combine.
- For instance, given the previous example:

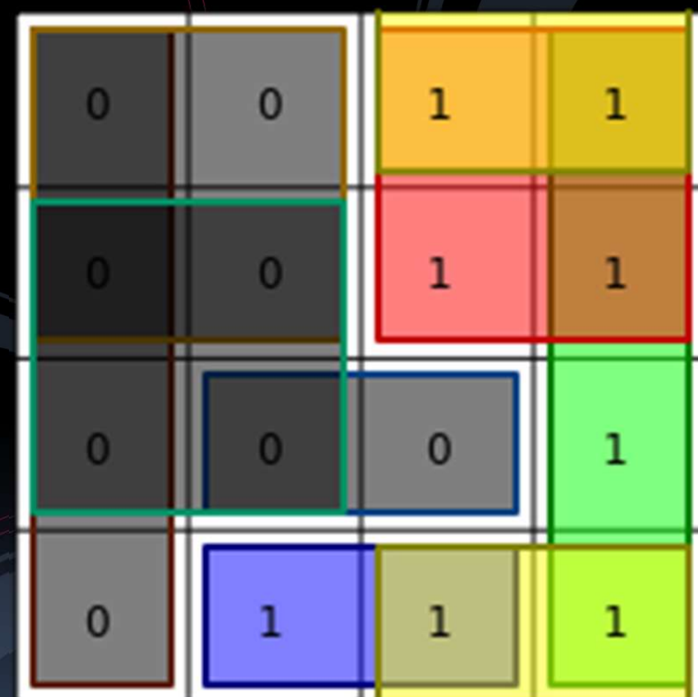
$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot \overline{C} + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$

- But if you combine the end and middle terms…

$$Y = B \cdot C + A \cdot \overline{C}$$

- This reduces the number of gates and inputs ☺
  - But how do we know which terms to combine?

# Karnaugh maps



# Reducing boolean expressions

- In this truth table, what rows could we combine with $m_0$?
  - $m_0+m_1 \rightarrow \overline{A}\cdot\overline{B}$
  - $m_0+m_2 \rightarrow \overline{A}\cdot\overline{C}$
  - $m_0+m_4 \rightarrow \overline{B}\cdot\overline{C}$
  - $m_0+m_1+m_4+m_5 \rightarrow \overline{B}$
- It's not always clear by looking at the truth table, which rows can be combined.
- What if we represent this truth table in a different way?

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | $m_0$ |
| 0 | 0 | 1 | $m_1$ |
| 0 | 1 | 0 | $m_2$ |
| 0 | 1 | 1 | $m_3$ |
| 1 | 0 | 0 | $m_4$ |
| 1 | 0 | 1 | $m_5$ |
| 1 | 1 | 0 | $m_6$ |
| 1 | 1 | 1 | $m_7$ |

# Reducing boolean expressions

- Karnaugh maps (K-maps for short) represent the same information as a truth table, but in a format that helps us see what minterms can be combined.
  - Karnaugh maps are a 2D grid of minterms (see below), arranged so that adjacent minterms in the grid differ by a single literal.
  - Values in the grid are the output for that minterm.

| | $\overline{B}\cdot\overline{C}$ | $\overline{B}\cdot C$ | $B\cdot C$ | $B\cdot\overline{C}$ |
|---|---|---|---|---|
| $\overline{A}$ | 0 | 0 | 1 | 0 |
| $A$ | 1 | 0 | 1 | 1 |

# Karnaugh maps

- Karnaugh maps can be of any size, and have any number of inputs.
  - i.e. the 4-input example here.
- Since adjacent minterms only differ by a single value, they can be grouped into a single term that omits that value.

| | $\bar{C} \cdot \bar{D}$ | $\bar{C} \cdot D$ | $C \cdot D$ | $C \cdot \bar{D}$ |
|---|---|---|---|---|
| $\bar{A} \cdot \bar{B}$ | $m_0$ | $m_1$ | $m_3$ | $m_2$ |
| $\bar{A} \cdot B$ | $m_4$ | $m_5$ | $m_7$ | $m_6$ |
| $A \cdot B$ | $m_{12}$ | $m_{13}$ | $m_{15}$ | $m_{14}$ |
| $A \cdot \bar{B}$ | $m_8$ | $m_9$ | $m_{11}$ | $m_{10}$ |

# Using Karnaugh maps

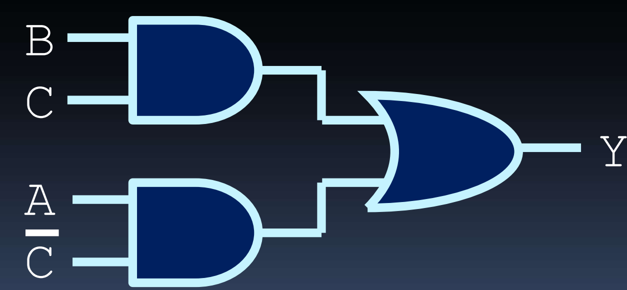
- Once Karnaugh maps are created, draw boxes over groups of high output values.
  - Boxes must be rectangular, and aligned with map.
  - Number of values contained within each box must be a power of 2.
  - Boxes may overlap with each other.
  - Boxes may wrap across edges of map.

| | $\overline{B} \cdot \overline{C}$ | $\overline{B} \cdot C$ | $B \cdot C$ | $B \cdot \overline{C}$ |
|---|---|---|---|---|
| $\overline{A}$ | 0 | 0 | 1 | 0 |
| $A$ | 1 | 0 | 1 | 1 |

# Using Karnaugh maps

| | $\overline{B}\cdot\overline{C}$ | $\overline{B}\cdot C$ | $B\cdot C$ | $B\cdot\overline{C}$ |
|---|---|---|---|---|
| $\overline{A}$ | 0 | 0 | 1 | 0 |
| $A$ | 1 | 0 | 1 | 1 |

- Once you find the minimal number of boxes that cover all the high outputs, create boolean expressions from the inputs that are common to all elements in the box.
- For this example:
  - Vertical box: $B\cdot C$
  - Horizontal box: $A\cdot\overline{C}$
  - Overall equation: $Y = B\cdot C + A\cdot\overline{C}$



# Karnaugh maps and maxterms

- Can also use this technique to group maxterms together as well.
- Karnaugh maps with maxterms involves grouping the zero entries together, instead of grouping the entries with one values.

| | $C+D$ | $C+\overline{D}$ | $\overline{C}+\overline{D}$ | $\overline{C}+D$ |
|---|---|---|---|---|
| $A+B$ | $M_0$ | $M_1$ | $M_3$ | $M_2$ |
| $A+\overline{B}$ | $M_4$ | $M_5$ | $M_7$ | $M_6$ |
| $\overline{A}+\overline{B}$ | $M_{12}$ | $M_{13}$ | $M_{15}$ | $M_{14}$ |
| $\overline{A}+B$ | $M_8$ | $M_9$ | $M_{11}$ | $M_{10}$ |

# Quick Exercise

| | $\overline{C}\,\overline{D}$ | $\overline{C}D$ | $CD$ | $C\overline{D}$ |
|---|---|---|---|---|
| $\overline{A}\,\overline{B}$ | 0 | 0 | 1 | 1 |
| $\overline{A}B$ | 1 | 1 | 0 | 0 |
| $AB$ | 1 | 1 | 0 | 0 |
| $A\overline{B}$ | 0 | 0 | 0 | 0 |

$$F = B \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C$$