## Simple Induction

From prep 1,

```
a = 1/5
while true
    print a
    a = (1 + a) / 2
```

- " pseudo - Python "
- ignore issues of arithmetic, low-level issues, etc.

- value of a?

> When I say value a, which value do I mean?
> Multiple values of a

Convention:

$a_i$ = value of a just before while cond. is evaluated
for i + 1st time

From prep:

(1)  $\forall n \in \mathbb{N}, a_n < 1$
(2)  $\forall n \in \mathbb{N}, a_{n+1} > a_n$

* Instantiate : "plug in" values for variables LITERALLY, then simplify

e.g. (2) for n = 2 $\longrightarrow$ " $a_{2+1} > a_2$ "

* Expanding : $\forall$ becomes $\land$ ( " and ")

- Trace :

| n | a |
|---|---|
| 0 | 1/5 |
| 1 | 3/5 |
| ⋮ | ⋮ |

- WTP (1) and (2)
  "Want to prove"

( A proof is a way to formalize reasoning

<u>Pf.</u>    WTP    Claim (2):    $\forall n \in \mathbb{N}, \ a_{n+1} > a_n$

Let $n \in \mathbb{N}$.

From Claim (1)

Then, $a_{n+1} = \dfrac{1 + a_n}{2} \quad > \quad \dfrac{a_n + a_n}{2} \quad = \quad a_n$

Using claim (1) is
like a helper function

Don't need to write helper
function first as long as you
have all the pieces

\* This is a **direct proof**, not induction!

WTP    Claim (1):    $\forall n \in \mathbb{N}, \ a_n < 1$

Intuition:

• $a_{n+1} = \dfrac{1 + a_n}{2} < \dfrac{1 + 1}{2} = 1$    from I.H.

• $a_0 = \dfrac{1}{5} < 1$