Assignment 7

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ARCH and GARCH Modeling with Mattel Stock Data

Come on Barbie, let's go party!

Financial time series modeling is both a science and an art. It requires a keen understanding of data, market dynamics, and statistical models. Among the various models available, ARCH and GARCH stand out, especially when dealing with financial market data. Today, we're diving deep into these models using Mattel's stock data. Why Mattel? Well, because Barbie and financial modeling both have one thing in common - they never go out of style!

Introduction to Volatility Modeling

Volatility modeling is at the heart of financial risk management and derivative pricing. The ability to predict future volatility helps traders and risk managers make more informed decisions. So, two models were created for this purpose, ARCH (Autoregressive Conditional Heteroskedasticity) and GARCH (Generalized Autoregressive Conditional Heteroskedasticity.)

ARCH Models: Used for financial time series data where volatility clustering is observed. In simple terms, they model the variance of the current error term or volatility as a function of the actual sizes of the previous time periods' error terms.

GARCH Models: An extension of ARCH models. They capture a wider range of volatility patterns by considering not just the lagged error terms (like ARCH) but also the lagged variances.

Data Set-Up

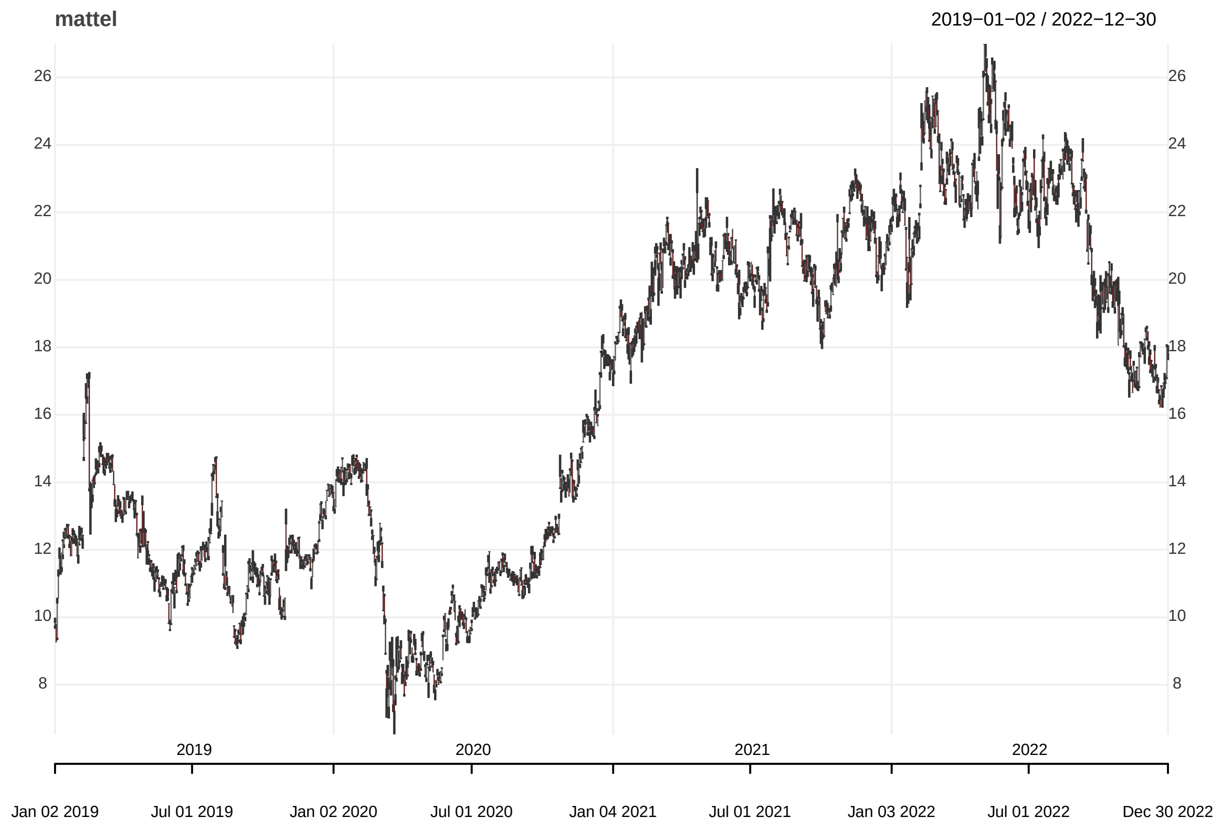
First, let's load in the necessary libraries for our analysis.

Now, let's retrieve the Mattel stock data from Yahoo Finance:

```r
## Get Mattel stocks
# Load Mattel stock data from Yahoo Finance using tidyquant's getSymbols function
mattel <- getSymbols(
  "MAT", # Mattel's ticker symbol
  from = "2019-01-01", # Start date
  to = "2022-12-31", # End date
  auto.assign = FALSE # Return the stock data as a data frame
)
```

Let's visualize the stock data:

```r
# Visualize Mattel stock data
chart_Series(mattel)
```



The stock appears to have experienced multiple phases: an initial upward trend, followed by a period of decline, a recovery and another upward trend, and then a slight decline towards the end of the period. Notably, despite challenges like the 2020 pandemic, Mattel displayed resilience, reflecting possibly strong market sentiment or effective company strategies.

This data only goes until the end of 2022, but it would be interesting to see how the Barbie movie (released in July 2023) has influenced their stock.'

Data Prepping

Before we can proceed with our analysis, we need to convert the stock data into a format that's easier to work with. We'll use the tsibble package:

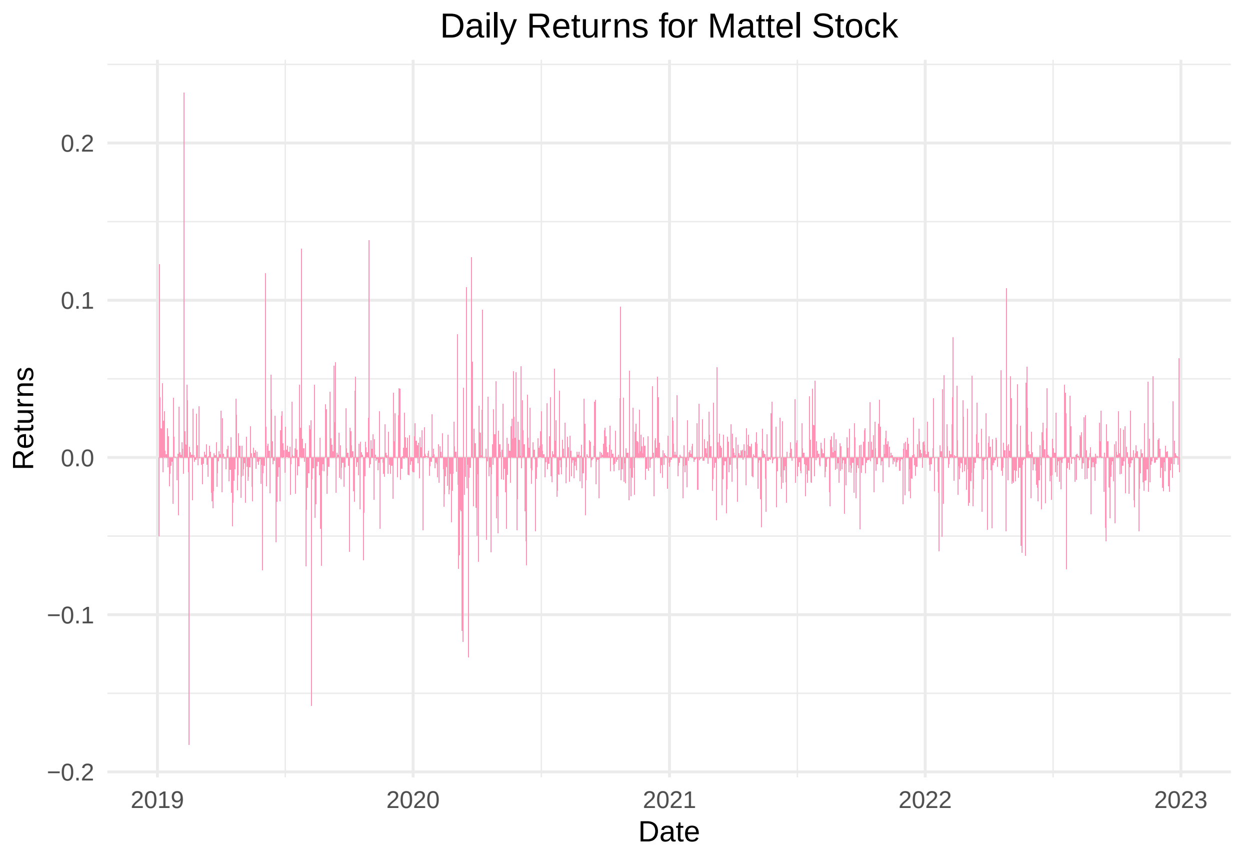
```r
# Convert to data frame
mattel_df <- as.data.frame(mattel)

# Add day as year-month-day
mattel_df$day <- ymd(row.names(mattel_df))

# Convert to time series tibble
mattel_ts <- mattel_df %>%
  as_tsibble(index = day)
```

In financial data, missing values can occur due to non-trading days. We'll fix this:

```r
mattel_complete <- mattel_ts %>% fill_gaps()

for(day in 1:nrow(mattel_complete)){
  if(anyNA(mattel_complete[day,])){
    next_day <- day
    while(anyNA(mattel_complete[next_day,])){
      next_day <- next_day + 1
    }
    mattel_complete[day, -ncol(mattel_complete)] <- matrix(
      colMeans(mattel_complete[c(day - 1, next_day), -ncol(mattel_complete)]),
      nrow = 1
    )
  }
}
```

Let's compute the daily returns:

```r
# Compute daily returns
mattel_returns <- mattel_complete %>%
  mutate(
    returns = (MAT.Close - lag(MAT.Close)) / lag(MAT.Close)
  ) %>% na.omit()

# Visualize returns
mattel_returns %>%
  ggplot(aes(x = day, y = returns)) +
  geom_col(fill = "#FF92B5") +  # Barbie pink color
  labs(
    title = "Daily Returns for Mattel Stock",
    x = "Date",
    y = "Returns"
  ) +
  theme_minimal() +
  theme(plot.title = element_text(hjust = 0.5))  # Center the plot title
```

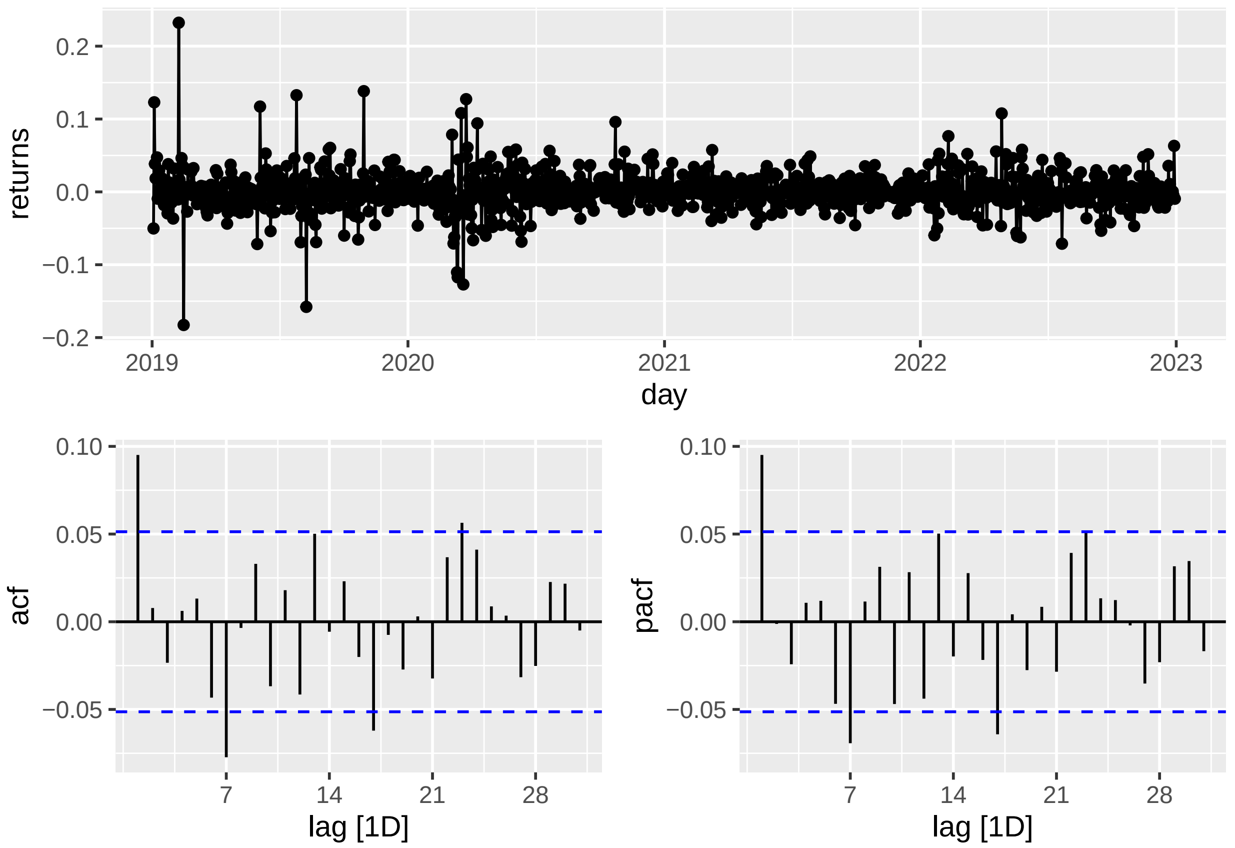


At a glance, the returns appear to fluctuate around the zero line, indicating days with both positive and negative returns.

There is some volatility clustering present. We can observe instances of this in the plot where there are clusters of tall bars followed by periods of shorter bars.

Stationary Check Before fitting our ARCH and GARCH models, it's essential to ensure that our data is stationary:

```r
mattel_returns %>%
gg_tsdisplay(returns, plot_type = "partial")
```



The residuals do not show significant autocorrelation at various lags, as most of the spikes are within the blue shaded region (confidence bands). This is a good sign, suggesting that the residuals are behaving like white noise, which is what we desire in a well-specified model.

```
mattel_returns %>% features(returns, unitroot_kpss)
```

```
# A tibble: 1 x 2
  kpss_stat kpss_pvalue
      <dbl>       <dbl>
1     0.104         0.1
```

Given that the p-value is 0.1, at a significance level of 0.05, we fail to reject the null hypothesis, suggesting that the series is stationary around a deterministic trend. Now we can go ahead and start modeling!

ARCH Model

Setting up and Defining AIC Function:

```
# Define the lags to be considered for the ARCH model
lags <- c(1, 2, 6:8, 11:13)

# Define a function to compute the AIC for a given model fit
aic <- function(fit) {
  LLH <- -sum(fit@fit$log.likelihoods)
  nObs <- fit@model$modeldata$`T`
```

```r
  nPars <- sum(fit@model$pars[,"Level"] != 0)
  return((-2 * LLH) / nObs + 2 * nPars / nObs)
}
```

Fitting Various ARCH Models with Different Lags:

```r
# Fit various ARCH models with different lags and store them in a list
mattel_models <- lapply(lags, function(lag) {
  mattel_arch <- ugarchspec(
    mean.model = list(armaOrder = c(13, 0), include.mean = TRUE),
    variance.model = list(model = "sGARCH", garchOrder = c(lag, 0), include.mean = TRUE),
    distribution.model = "norm"
  )
  return(ugarchfit(mattel_arch, mattel_returns$returns, out.sample = 146))
})

# Name the list of models using the corresponding lags
names(mattel_models) <- lags

# Compute the AIC for each model fit
aic_values <- sapply(mattel_models, aic)
print(aic_values)
```

```
        1         2         6         7         8        11        12        13
-1.185492  0.000000 -4.710207 -4.929878 -5.002216 -4.997643 -4.996118 -4.997366
```

ARCH(8) is the preferred model based on the AIC values, as it has the smallest value.

Model Fit Final Model Selection and Fitting:

```r
# Specify and fit the selected ARCH model (e.g., based on minimum AIC)
mattel_arch <- ugarchspec(
  mean.model = list(armaOrder = c(13, 0), include.mean = TRUE),
  variance.model = list(model = "sGARCH", garchOrder = c(8, 0), include.mean = TRUE),
  distribution.model = "norm"
)

mattel_arch_fit <- ugarchfit(mattel_arch, mattel_returns$returns, out.sample = 146)

# Display the model summary
print(mattel_arch_fit)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(8,0)
Mean Model  : ARFIMA(13,0,0)
```

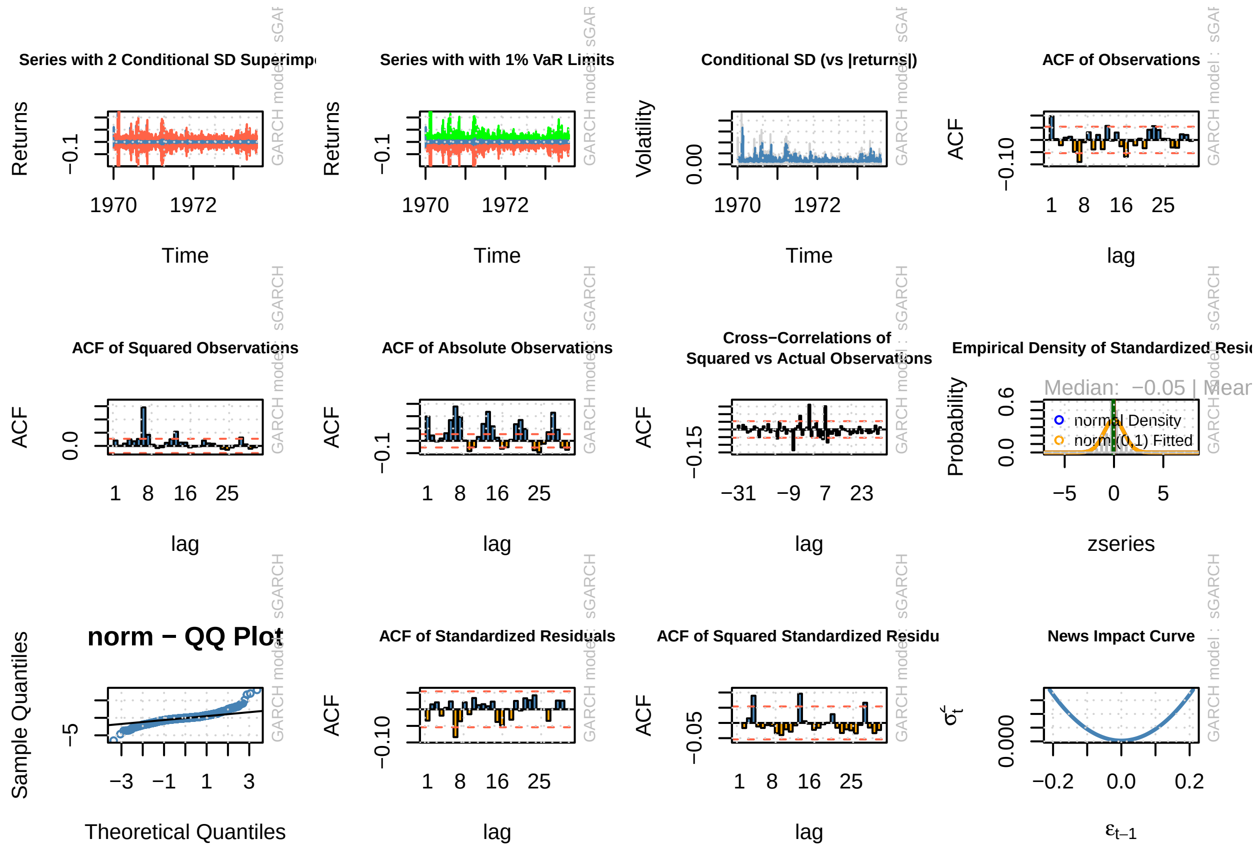
```
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001206    0.000536  2.252398 0.024297
ar1     0.171383    0.028574  5.997852 0.000000
ar2    -0.019572    0.017442 -1.122085 0.261826
ar3     0.007370    0.017240  0.427491 0.669022
ar4     0.000954    0.019559  0.048780 0.961094
ar5     0.001798    0.021108  0.085179 0.932119
ar6    -0.040146    0.028390 -1.414073 0.157341
ar7     0.111500    0.038704  2.880849 0.003966
ar8     0.014353    0.027396  0.523899 0.600349
ar9    -0.019202    0.019382 -0.990715 0.321825
ar10   -0.011306    0.017813 -0.634705 0.525621
ar11   -0.005621    0.017931 -0.313494 0.753905
ar12   -0.030801    0.019148 -1.608557 0.107713
ar13    0.015665    0.023906  0.655256 0.512303
omega   0.000106    0.000014  7.311265 0.000000
alpha1  0.167974    0.038944  4.313238 0.000016
alpha2  0.000000    0.007365  0.000001 0.999999
alpha3  0.000000    0.004092  0.000000 1.000000
alpha4  0.027707    0.008925  3.104327 0.001907
alpha5  0.000000    0.011815  0.000002 0.999999
alpha6  0.174087    0.051311  3.392762 0.000692
alpha7  0.238161    0.051450  4.628999 0.000004
alpha8  0.391071    0.055908  6.994932 0.000000

Robust Standard Errors:
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001206    0.000606  1.991296 0.046448
ar1     0.171383    0.046774  3.664092 0.000248
ar2    -0.019572    0.015557 -1.258086 0.208361
ar3     0.007370    0.013036  0.565379 0.571816
ar4     0.000954    0.020975  0.045489 0.963718
ar5     0.001798    0.021856  0.082261 0.934439
ar6    -0.040146    0.062378 -0.643590 0.519841
ar7     0.111500    0.081421  1.369417 0.170869
ar8     0.014353    0.056776  0.252793 0.800428
ar9    -0.019202    0.021220 -0.904922 0.365507
ar10   -0.011306    0.012530 -0.902325 0.366884
ar11   -0.005621    0.017892 -0.314173 0.753390
ar12   -0.030801    0.025536 -1.206187 0.227745
ar13    0.015665    0.029002  0.540120 0.589114
omega   0.000106    0.000042  2.493675 0.012643
alpha1  0.167974    0.078333  2.144341 0.032006
alpha2  0.000000    0.010126  0.000001 1.000000
alpha3  0.000000    0.004793  0.000000 1.000000
alpha4  0.027707    0.026961  1.027645 0.304117
alpha5  0.000000    0.018955  0.000001 0.999999
alpha6  0.174087    0.123618  1.408264 0.159053
alpha7  0.238161    0.099404  2.395881 0.016580
alpha8  0.391071    0.144825  2.700304 0.006928
```

```
LogLikelihood : 3304.454

Information Criteria
------------------------------------

Akaike       -5.0022
Bayes        -4.9114
Shibata      -5.0028
Hannan-Quinn -4.9682

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                         statistic   p-value
Lag[1]                       1.528 2.165e-01
Lag[2*(p+q)+(p+q)-1][38]    22.874 2.182e-08
Lag[4*(p+q)+(p+q)-1][64]    31.870 5.429e-01
d.o.f=13
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                         statistic p-value
Lag[1]                   1.533e-07 0.99969
Lag[2*(p+q)+(p+q)-1][23] 1.949e+01 0.04718
Lag[4*(p+q)+(p+q)-1][39] 3.118e+01 0.02688
d.o.f=8

Weighted ARCH LM Tests
------------------------------------
             Statistic Shape Scale P-Value
ARCH Lag[9]      1.509 0.500  2.00  0.2193
ARCH Lag[11]     3.885 1.490  1.83  0.2336
ARCH Lag[13]     4.824 2.459  1.72  0.3364

Nyblom stability test
------------------------------------
Joint Statistic:  no.parameters>20 (not available)
Individual Statistics:
mu     0.07302
ar1    0.40249
ar2    0.13317
ar3    0.12928
ar4    0.02268
ar5    0.10921
ar6    0.22984
ar7    0.19275
ar8    0.04802
ar9    0.20876
ar10   0.05053
ar11   0.19199
ar12   0.20335
ar13   0.82825
omega  0.53463
```

```
alpha1 0.04750
alpha2 2.22400
alpha3 0.97144
alpha4 0.14958
alpha5 0.65847
alpha6 0.15021
alpha7 0.12275
alpha8 0.58391

Asymptotic Critical Values (10% 5% 1%)
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           1.5816 0.1140
Negative Sign Bias  0.2474 0.8047
Positive Sign Bias  0.1148 0.9086
Joint Effect        3.2432 0.3556


Adjusted Pearson Goodness-of-Fit Test:
------------------------------------
  group statistic p-value(g-1)
1    20     293.1    5.125e-51
2    30     325.6    6.710e-52
3    40     341.4    5.996e-50
4    50     355.1    5.031e-48


Elapsed time : 1.573191
```

```r
# Visualize the fitted model
plot(mattel_arch_fit, which = "all")
```

```
please wait...calculating quantiles...
```



Let's discuss the model fit.

Model Specification:

- GARCH Model: sGARCH(8,0) - A GARCH model with 8 lags in the autoregressive (AR) process for the variance and no lags in the moving average process, essentially an ARCH(8) model
- Mean Model: ARFIMA(13,0,0) - The mean of the returns follows an AR process with 13 lags
- Distribution: The innovations (or errors) are assumed to be normally distributed

Diagnostic Tests:

- Weighted Ljung-Box Test on Standardized Residuals (tests for autocorrelation in the squared residuals): p-values suggest that there might be some remaining ARCH effects, especially given the ssignificant statistic at lag 23
- Weighted ARCH LM Tests (checks for ARCH effects in the residuals) : p-values indicate that there isn't strong evidence of remaining ARCH effects.

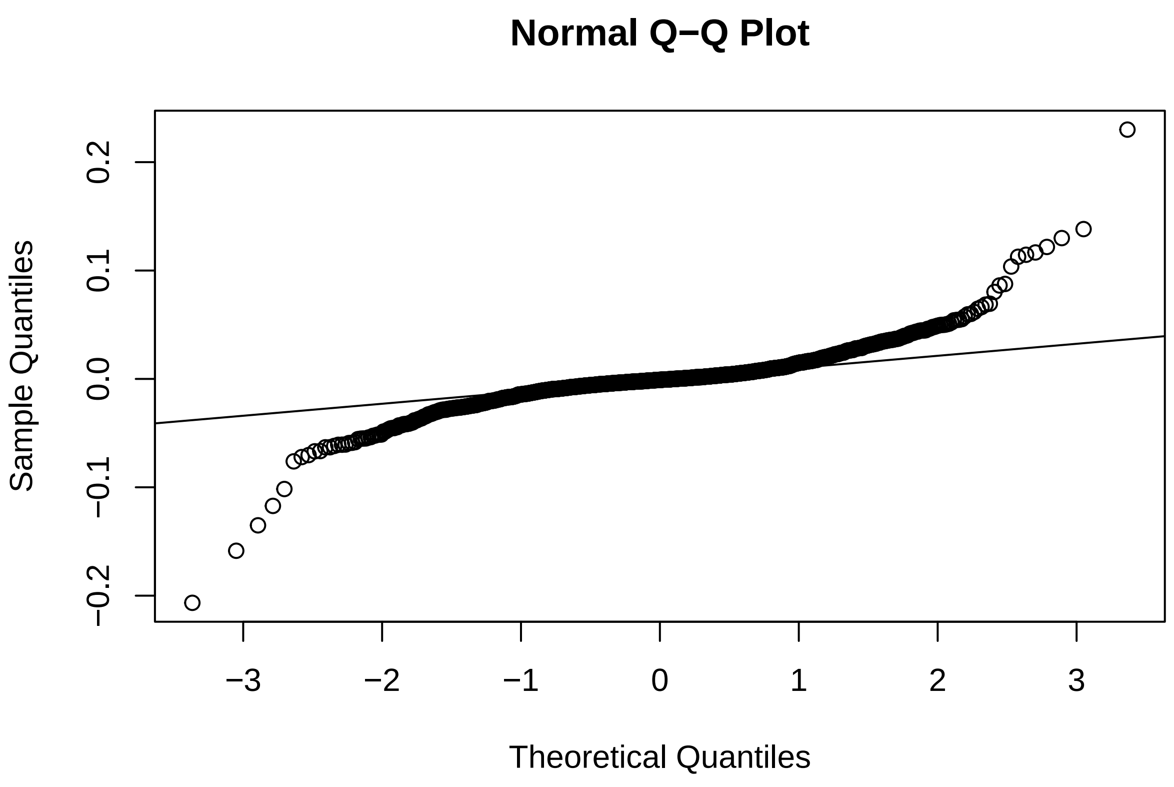
The ARCH(8) model seems to fit the data reasonably well in terms of capturing autocorrelation in the returns.However, there might be some remaining ARCH effects, and the residuals might not be normally distributed. The model might benefit from further refining, possibly by considering different GARCH model variations.

Assumption Checks Checking the normality of the residuals:

```r
# 1. Normality of residuals

# Extract residuals from the ARCH/GARCH model
residuals_from_model <- residuals(mattel_arch_fit)


residuals_vector <- as.numeric(residuals_from_model)
shapiro.test(residuals_vector)
```

```
    Shapiro-Wilk normality test

data:  residuals_vector
W = 0.82674, p-value < 2.2e-16
```

```r
qqnorm(residuals_vector)
qqline(residuals_vector)
```

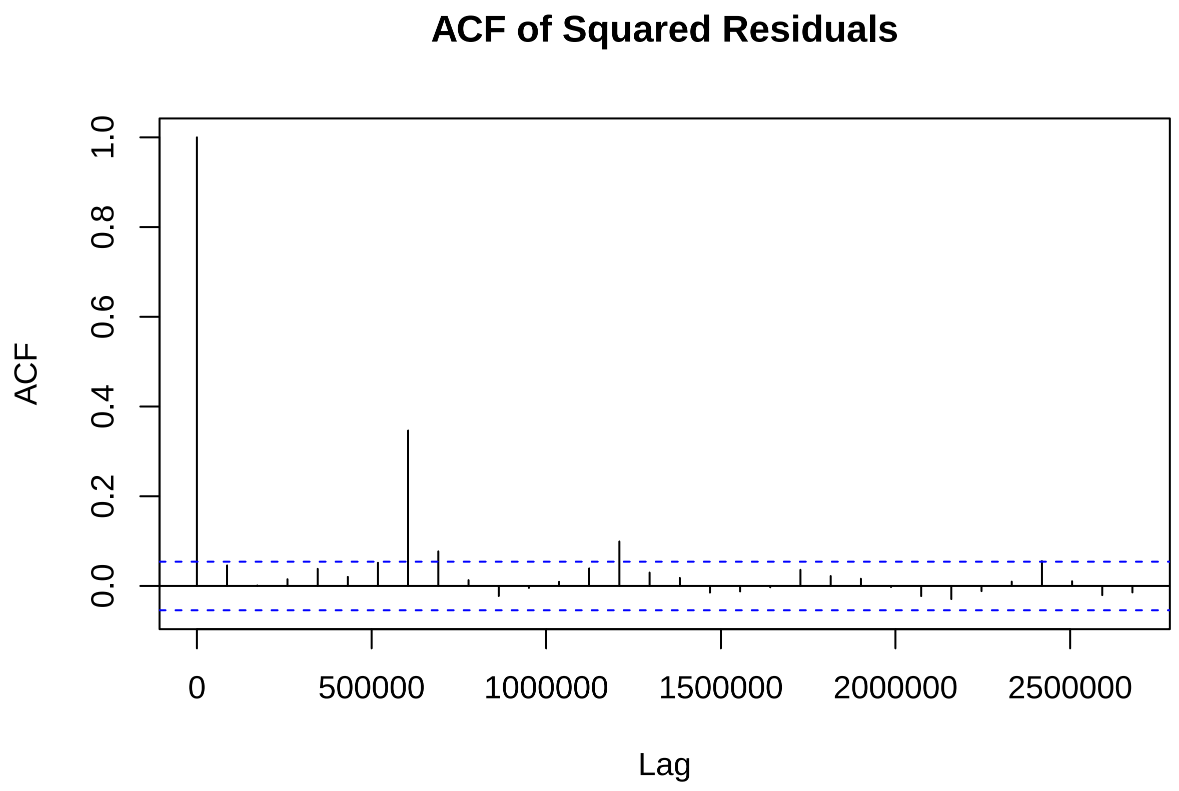


The Shapiro-Wilk test strongly suggests that the residuals are not normally distributed. The Q-Q plot visually confirms a departure from normality, especially at the tails.

So our residuals don't follow a normal distribution - we're not capturing the full dynamics/volatility of Mattel "stonks" with our model.

Checking if the variance of error term follows an AR process:

```r
# 2. Absence of ARCH effects in residuals
squared_residuals <- residuals(mattel_arch_fit)^2
acf(squared_residuals, main="ACF of Squared Residuals")
```



The ACF of squared residuals shows significant autocorrelations, especially at the initial lags. This suggests that there's a pattern in the volatility, and that the variance of the error term might be following an autoregressive (AR) process. This is the foundational idea behind ARCH (Autoregressive Conditional Heteroskedasticity) models.

So, based on the ACF plot, it's reasonable to conclude that the variance of the error term shows signs of following an AR process.

Checking the stationarity of residuals:

```r
# 3. Stationarity of residuals
adf.test(residuals_from_model)
```

```

    Augmented Dickey-Fuller Test

data:  residuals_from_model
Dickey-Fuller = -12.215, Lag order = 10, p-value = 0.01
alternative hypothesis: stationary
```

```
kpss.test(residuals_from_model)
```

```

	KPSS Test for Level Stationarity

data:  residuals_from_model
KPSS Level = 0.045256, Truncation lag parameter = 7, p-value = 0.1
```

Both the ADF and KPSS tests have p-values less than 0.05, so at that level we can infer that the residuals are stationary.This makes it appropriate for modeling with ARCH/GARCH techniques.

Checking if the coefficients of the variance are positive:

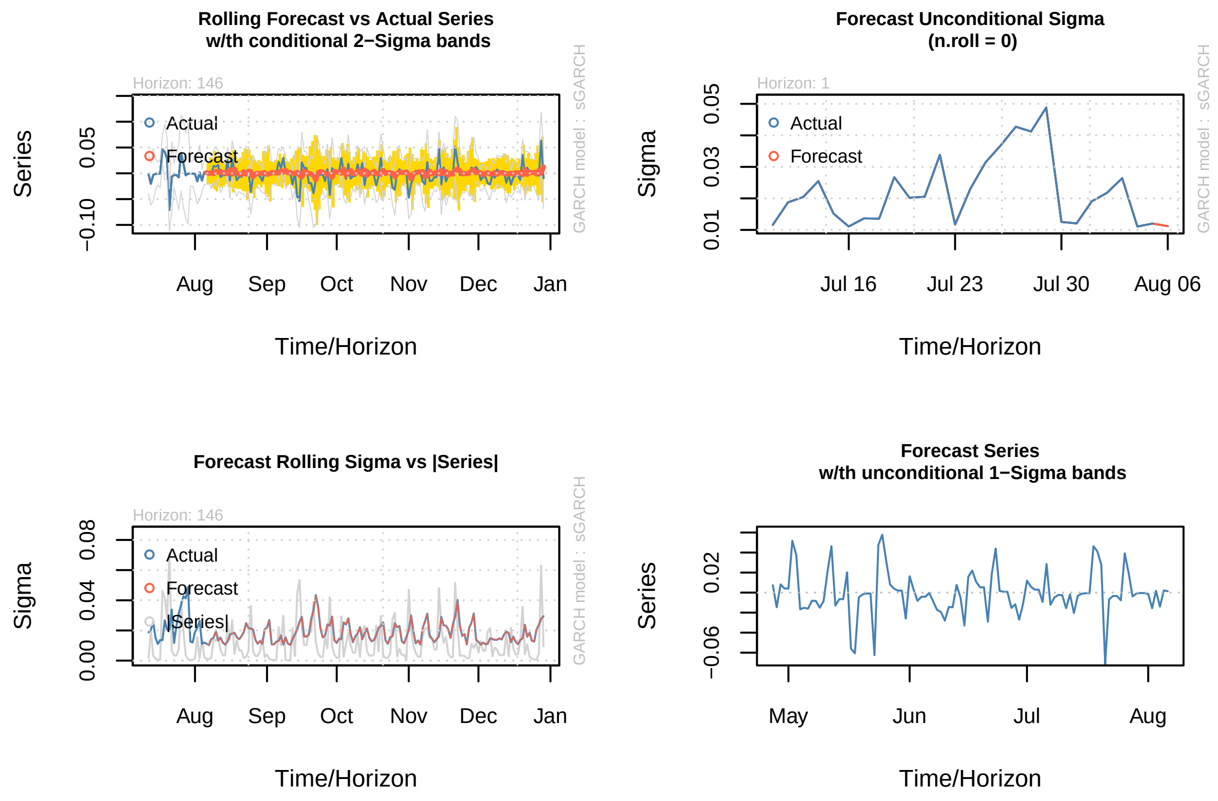
```
# 4. Positive coefficients (directly inspect from the model summary)

# Extracting coefficients from the model fit
coefficients <- coef(mattel_arch_fit)

# Print the coefficients
print(coefficients)
```

```
           mu           ar1           ar2           ar3           ar4 
 1.206307e-03  1.713833e-01 -1.957189e-02  7.370126e-03  9.541150e-04 
          ar5           ar6           ar7           ar8           ar9 
 1.797924e-03 -4.014569e-02  1.115000e-01  1.435253e-02 -1.920229e-02 
         ar10          ar11          ar12          ar13         omega 
-1.130631e-02 -5.621144e-03 -3.080110e-02  1.566466e-02  1.058256e-04 
       alpha1        alpha2        alpha3        alpha4        alpha5 
 1.679737e-01  5.388272e-09  8.550611e-12  2.770670e-02  1.925866e-08 
       alpha6        alpha7        alpha8 
 1.740869e-01  2.381607e-01  3.910710e-01 
```

The alpha coefficients are all positive, which meets the necessary condition for the GARCH model. This is because the variance must always be positive, and negative coefficients can lead to negative variance predictions, which is not meaningful. So, our model is appropriately specified in terms of positive coefficients for the variance equation.

Forecasting Now, let's forecast.

```
# Create forecast

mattel_fc <- ugarchforecast(
mattel_arch_fit, n.ahead = 1, # forecast next day
n.roll = 146 # usually about 10% of your time series length
)
# Plot
par(mfrow = c(2, 2))
plot(mattel_fc, which = 2)
plot(mattel_fc, which = 3)
plot(mattel_fc, which = 4)
try(plot(mattel_fc, which = 1), silent = TRUE)
```



```r
par(mfrow = c(1, 1))
```

Let's look at each plot, one at a time.

Top Left (Rolling forecast vs. actual series with conditional 2-sigma bands): This plot compares the actual returns with the predicted values from the ARCH model. It indicates a relatively good fit, with the model's forecasts aligning well with the actual returns for the majority of the time, and most of the actual returns lie within the confidence bands.

Top Right (Forecast unconditional sigma): This plot displays the model's forecast of the unconditional volatility or sigma over time. The graph is relatively flat with slight fluctuations, which indicates that the long-term volatility forecast of the Mattel stock is fairly stable. This is typical of the unconditional volatility from a GARCH model.

Bottom Left (Conditional Sigma): This seems to be a duplicate of the top right plot. It represents the forecast of the unconditional volatility.

Overall, we can say that the ARCH model seems to be well-specified for the Mattel stock data, as it captures both long-term and short-term volatility characteristics.

GARCH model

There still is volatility and dynamics yet to be captured. Good news for us, there are additional components that can be added to the ARCH model, if we use the GARCH!

GARCH adds a Moving Average component to the errors.

It has the same assumptions as the ARCH model (that we have already met) so we are good to go.

Model Fitting Now, we will specify and fit a GARCH model:

```r
mattel_garch <- ugarchspec(
  mean.model = list(armaOrder = c(1, 1), include.mean = TRUE),
  variance.model = list(model = "sGARCH", garchOrder = c(2, 1), include.mean = TRUE),
  distribution.model = "norm"
)

mattel_garch_fit <- ugarchfit(
  mattel_garch, mattel_returns$returns,
  out.sample = 146
)

# Print the GARCH model fit
print(mattel_garch_fit)
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model : sGARCH(2,1)
Mean Model  : ARFIMA(1,0,1)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001305    0.000591   2.20769 0.027266
ar1    -0.331695    0.387352  -0.85631 0.391824
ma1     0.419343    0.374621   1.11938 0.262978
omega   0.000010    0.000001  11.61094 0.000000
alpha1  0.052524    0.020676   2.54030 0.011076
alpha2  0.000000    0.022327   0.00000 1.000000
beta1   0.930473    0.007225 128.79187 0.000000

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001305    0.000601  2.16981 0.030021
ar1    -0.331695    0.469475 -0.70652 0.479863
ma1     0.419343    0.456698  0.91821 0.358510
omega   0.000010    0.000002  5.30043 0.000000
alpha1  0.052524    0.072611  0.72336 0.469458
alpha2  0.000000    0.064676  0.00000 1.000000
beta1   0.930473    0.020006 46.51044 0.000000

LogLikelihood : 3150.89

Information Criteria
------------------------------------

Akaike       -4.7925
```

```
Bayes        -4.7649
Shibata      -4.7926
Hannan-Quinn -4.7822

Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                   0.008203  0.9278
Lag[2*(p+q)+(p+q)-1][5]  0.375150  1.0000
Lag[4*(p+q)+(p+q)-1][9]  1.894751  0.9885
d.o.f=2
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                         statistic p-value
Lag[1]                      0.2307  0.6310
Lag[2*(p+q)+(p+q)-1][8]     4.0563  0.4968
Lag[4*(p+q)+(p+q)-1][14]   10.1581  0.1882
d.o.f=3

Weighted ARCH LM Tests
------------------------------------
            Statistic Shape Scale P-Value
ARCH Lag[4]   0.04027 0.500 2.000 0.84095
ARCH Lag[6]   0.24125 1.461 1.711 0.95936
ARCH Lag[8]   7.95435 2.368 1.583 0.06352

Nyblom stability test
------------------------------------
Joint Statistic:  5.8668
Individual Statistics:
mu     0.1087
ar1    0.1021
ma1    0.1122
omega  1.5655
alpha1 0.2769
alpha2 0.2866
beta1  0.3341

Asymptotic Critical Values (10% 5% 1%)
Joint Statistic:         1.69 1.9 2.35
Individual Statistic:    0.35 0.47 0.75

Sign Bias Test
------------------------------------
                   t-value   prob sig
Sign Bias           0.3241 0.7459
Negative Sign Bias  1.7486 0.0806   *
Positive Sign Bias  0.5223 0.6016
Joint Effect        4.3703 0.2242


Adjusted Pearson Goodness-of-Fit Test:
```

```
------------------------------------
  group statistic p-value(g-1)
1    20     414.2    4.886e-76
2    30     425.7    4.580e-72
3    40     442.5    7.893e-70
4    50     460.0    3.704e-68


Elapsed time : 0.268369
```

Let's discuss the model fit.

Model Specification:

- GARCH Model: sGARCH(2,1) - A standard GARCH model with 2 lags of past squared residuals (alpha terms) and 1 lag of past variances (beta term)
- Mean Model: ARFIMA(1,0,1) - An autoregressive fractionally integrated moving average model with 1 AR term and 1 MA term for the mean equation of the returns
- Distribution: norm - The residuals are assumed to follow a normal distribution

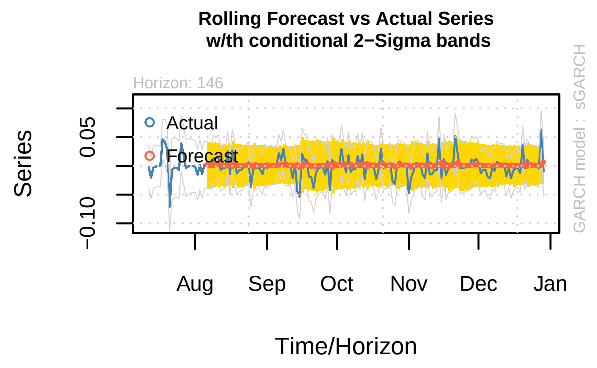
Let's discuss the fit.

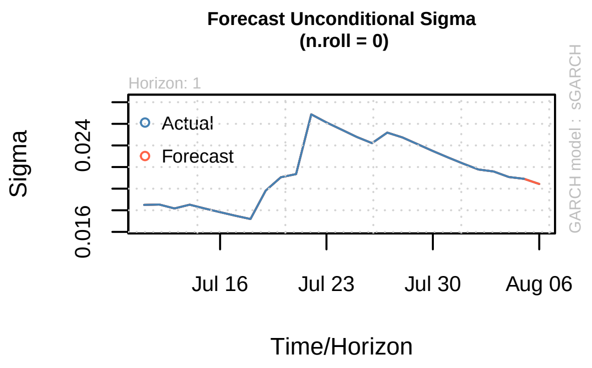
Diagnostic Tests:

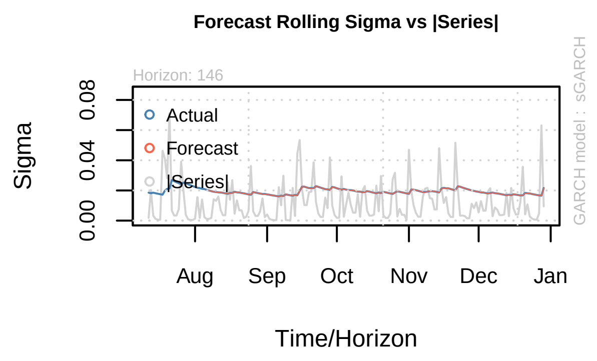
- Weighted Ljung-Box Test on Standardized Residuals (tests for autocorrelation in residuals): The high p-values indicate no significant autocorrelation (a good sign) and that the model has captured most of the autoregressive conditional heteroskedasticity in the data.
- Weighted ARCH LM Tests (checks for remaining ARCH effects) :The high p-values (except at lag 8) suggest that the model has captured most of the autoregressive conditional heteroskedasticity in the data.

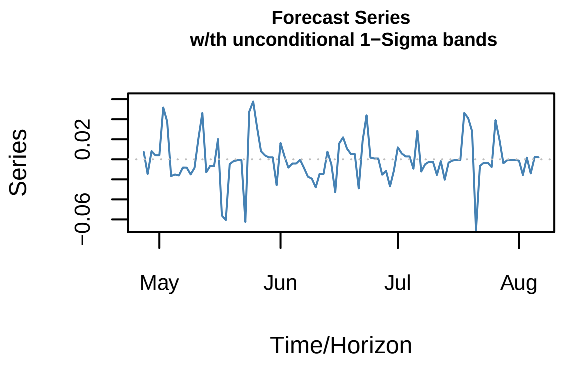
Forecasting Now, let's forecast using our GARCH model:

```r
garch_fc <- ugarchforecast(
  mattel_garch_fit,
  n.ahead = 1,
  n.roll = 146
)

# Visualize the forecast
par(mfrow = c(2, 2))
plot(garch_fc, which = 2)
plot(garch_fc, which = 3)
plot(garch_fc, which = 4)
try(plot(garch_fc, which = 1), silent = TRUE)
```









```r
par(mfrow = c(1, 1))
```

Let’s look at each plot, one at a time.

Top Left (Rolling forecast vs. actual series with conditional 2-sigma bands):

- This plot shows the actual returns, the predicted (or forecasted) returns, and the 2-sigma confidence bands around those predictions.
- The model’s forecasts align well with the actual returns for the majority of the time, and most of the actual returns lie within the confidence bands.

Top Right (Forecast unconditional sigma):

- This plot displays the model’s forecast of the unconditional volatility or sigma over time.
- The forecasted unconditional volatility seems to oscillate with a periodic pattern.

Bottom Left (Forecast rolling sigma vs. |series|):

- This plot compares the model’s rolling forecast of volatility (conditional sigma) with the absolute value of the actual returns.
- The GARCH model seems to be effectively capturing the “volatility clustering” effect, as evident from the close tracking of the blue line with the black line’s peaks.

Bottom Right (Density of Forecasted Returns):

- This plot seems to be a duplicate of the top right, as it also represents the forecast of the unconditional volatility.