Statistical Methods 2

Portfolio 9 – Metropolis-within-Gibbs algorithm

Complete the following task and submit your work on Blackboard by 4pm on Friday 05/05/2023

Consider the same dataset and Bayesian model as in Portfolio 8.

Use a Metropolis-within-Gibbs algorithm to approximate the posterior distribution $\pi(\alpha, \beta|y^0)$ defined in Portfolio 8, where the components of (α, β) are updated one at a time. More precisely, you need

1. To choose the proposal distributions $\{Q_j\}_{j=1}^{p+1}$ used by the $p+1$ Metropolis-Hastings kernels.
2. To implement **yourself** the Metropolis-within-Gibbs algorithm that uses the proposal distributions $\{Q_j\}_{j=1}^{p+1}$ chosen in 1 and that targets $\pi(\alpha, \beta|y^0)$.
3. To assess the convergence of the algorithm (using the acceptance rate[1], trace plots and auto-correlation functions).
4. If needed, modify $\{Q_j\}_{j=1}^{p+1}$ until the convergence behaviour of the algorithm is satisfactory.
5. Plot the marginal posterior distribution for each of the 9 parameters of the model.
6. Compare your results with those obtained in Task 1 with the Metropolis-Hastings algorithm. In particular, mention which of the two algorithms you would recommend to use for approximating $\pi(\alpha, \beta|y^0)$, and explain why.

[1] Theoretical results suggest that the "optimal" overall acceptance rate of a Metropolis-within-Gibbs algorithm is 0.234 (see Neal, Peter, and Gareth Roberts. "Optimal scaling for partially updating MCMC algorithms." *The Annals of Applied Probability* 16.2 (2006): 475-515).