Rcpp Portfolio

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2023-03-27

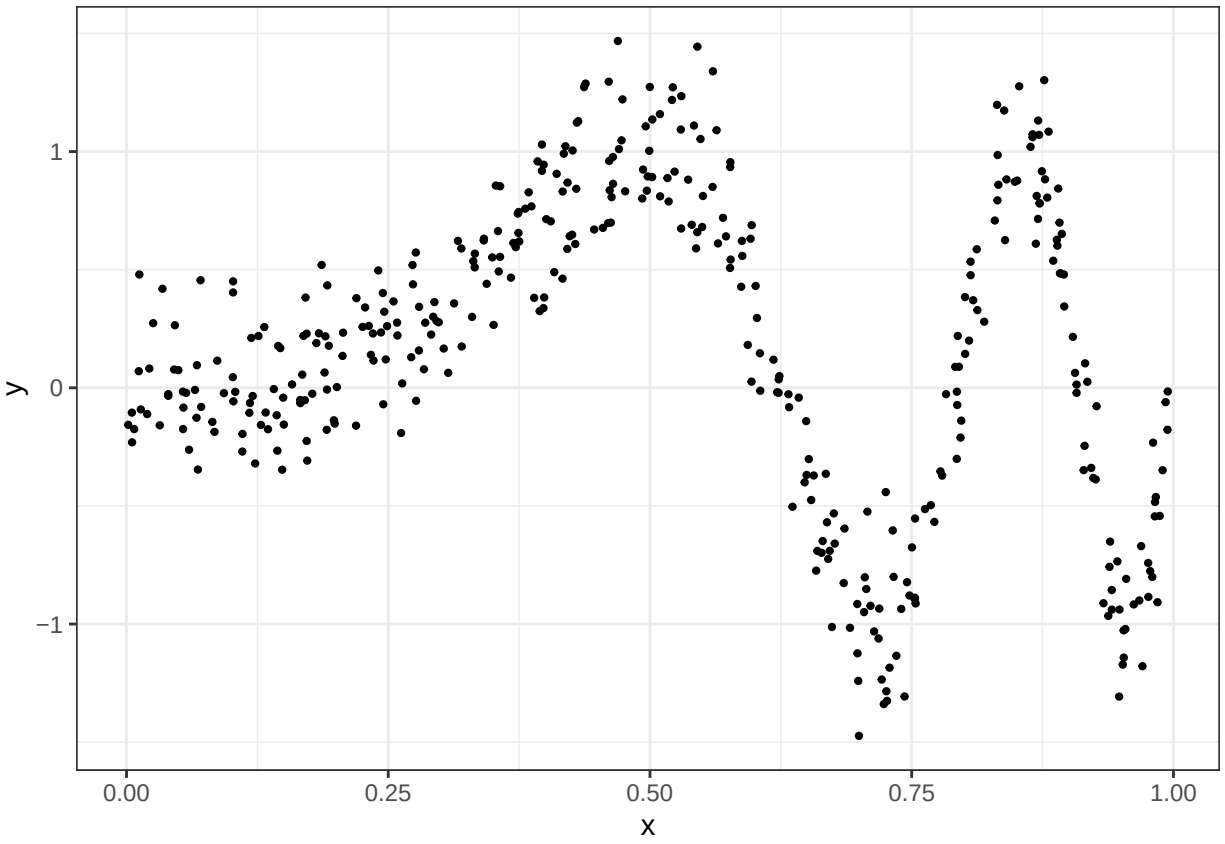
For this portfolio, we use Rcpp to fit an adaptive kernel smoothing regression model.

We first generate data according to the model

$$y_i = \sin(\alpha \pi x^3) + z_i \quad \text{with} \quad z_i \sim \mathcal{N}(0, \sigma^2)$$

In this case we take $\alpha = 4$ and $\sigma = 0.2$.

```
library(dplyr)
library(ggplot2)
n <- 400
alpha <- 4
sigma <- 0.2

x <- runif(n)
y <- sin(alpha * pi * x^3) + rnorm(n, sd = sigma)

data <- tibble(x = x, y =y)

ggplot(data = data, aes(x, y))+
  geom_point(size = 0.8)
```



The Kernel Smoother

We model $\mu(x) = \mathbb{E}(y|x)$ by

$$\hat{\mu}(x) = \frac{\sum_{i=1}^{n} \kappa_\lambda(x, x_i) y_i}{\sum_{i=1}^{n} \kappa_\lambda(x, x_i)}$$

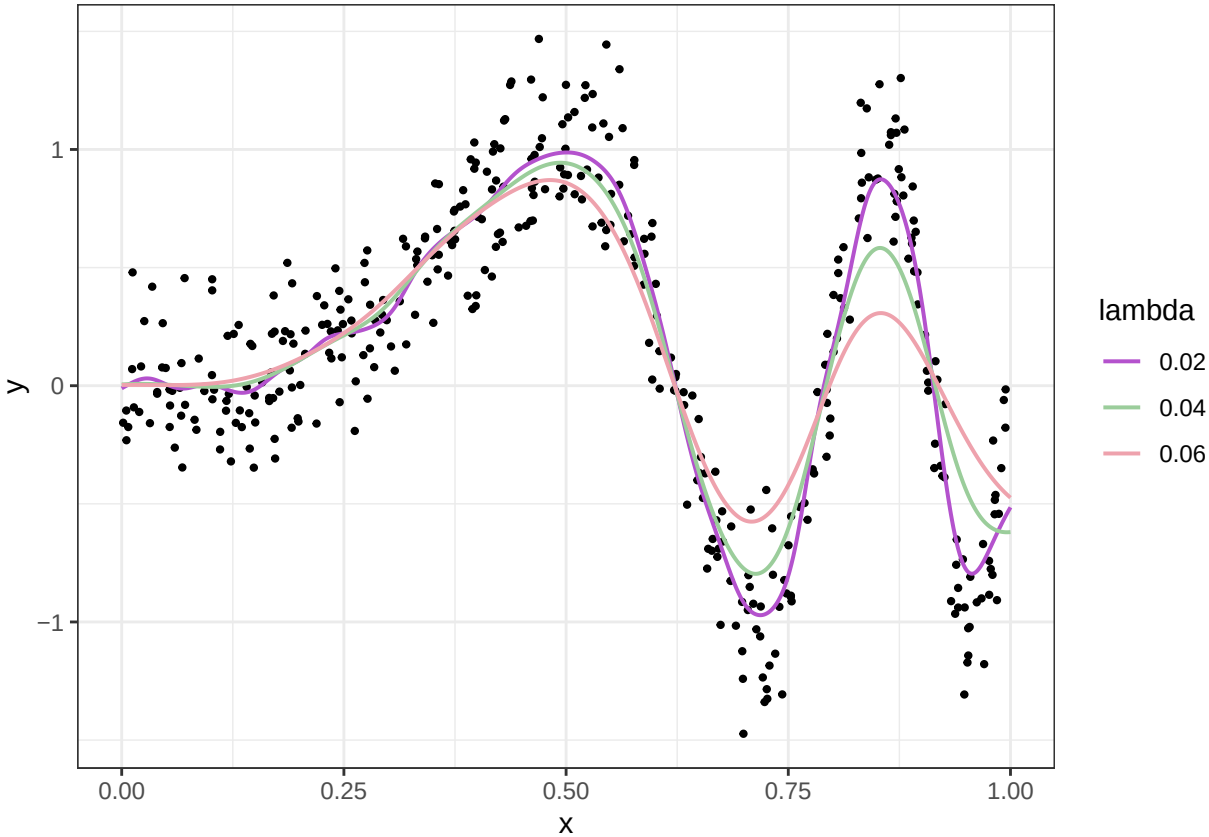
where we take κ_λ to be a Gaussian kernel with variance λ^2.

We implement this with the following function:

```
meanKRS <- function(x, y, xnew, lambda){
  n <- length(x)
  nnew <- length(xnew)

  mu <- numeric(nnew)

  for (i in 1:nnew){
    mu[i] <- sum(dnorm(x,xnew[i], lambda)*y)/ sum(dnorm(x,xnew[i], lambda))
  }

  return(mu)
}
```

We can now compare the fits for different values of λ:

```
library(tidyr)
xnew <- seq(0,1, length.out = 1000)

smooth_large <- meanKRS(x, y, xnew, lambda = 0.06)
smooth_medium <- meanKRS(x, y, xnew, lambda = 0.04)
smooth_small <- meanKRS(x, y, xnew, lambda = 0.02)

plot_data <- tibble(x = xnew) %>%
  mutate("0.06" = smooth_large,
         "0.04" = smooth_medium,
         "0.02" = smooth_small) %>%
  pivot_longer(cols = c("0.06","0.04","0.02"),
               names_to = "lambda",
               values_to = "fitted") %>%
  mutate(lambda = as.factor(lambda))

ggplot() +
  geom_point(data = data,
             aes(x, y), size = 0.8) +
  geom_line(data = plot_data,
            aes(x, fitted, color = lambda), linewidth = 0.7)
```



We now use Rcpp to write a C++ version of `meanKRS()`:

```
library(Rcpp)
```

```
#include <Rcpp.h>
#include <Rmath.h>
using namespace Rcpp;

// [[Rcpp::export(name = "meanKRS_Rcpp")]]

NumericVector meanKRS_Rcpp_I(const NumericVector x, const NumericVector y, const
↪  NumericVector xnew, const double lambda) {
  int n = x.size();
  int nnew = xnew.size();

  NumericVector mu(nnew);

  for (int i = 0; i < nnew; i++){
    mu[i] = sum(dnorm(x,xnew[i], lambda)*y)/ sum(dnorm(x,xnew[i], lambda));
  }

  return mu;
}
```

We check that this function produces the same output as the R version,

```
max(meanKRS(x, y, xnew, lambda = 0.06) - meanKRS_Rcpp(x, y, xnew, lambda = 0.06))
```

```
## [1] 1.332268e-15
```

and compare the performance of the two functions using the `microbenchmark()` function:

```
library(microbenchmark)
microbenchmark("R" = meanKRS(x, y, xnew, lambda = 0.06),
               "Rcpp" = meanKRS_Rcpp(x, y, xnew, lambda = 0.06))
```

```
## Unit: milliseconds
##  expr      min       lq     mean   median       uq      max neval
##     R 17.82321 18.22310 18.73960 18.35359 18.82536 22.45663   100
##  Rcpp 11.54211 11.71761 11.82974 11.79181 11.90058 12.70932   100
```

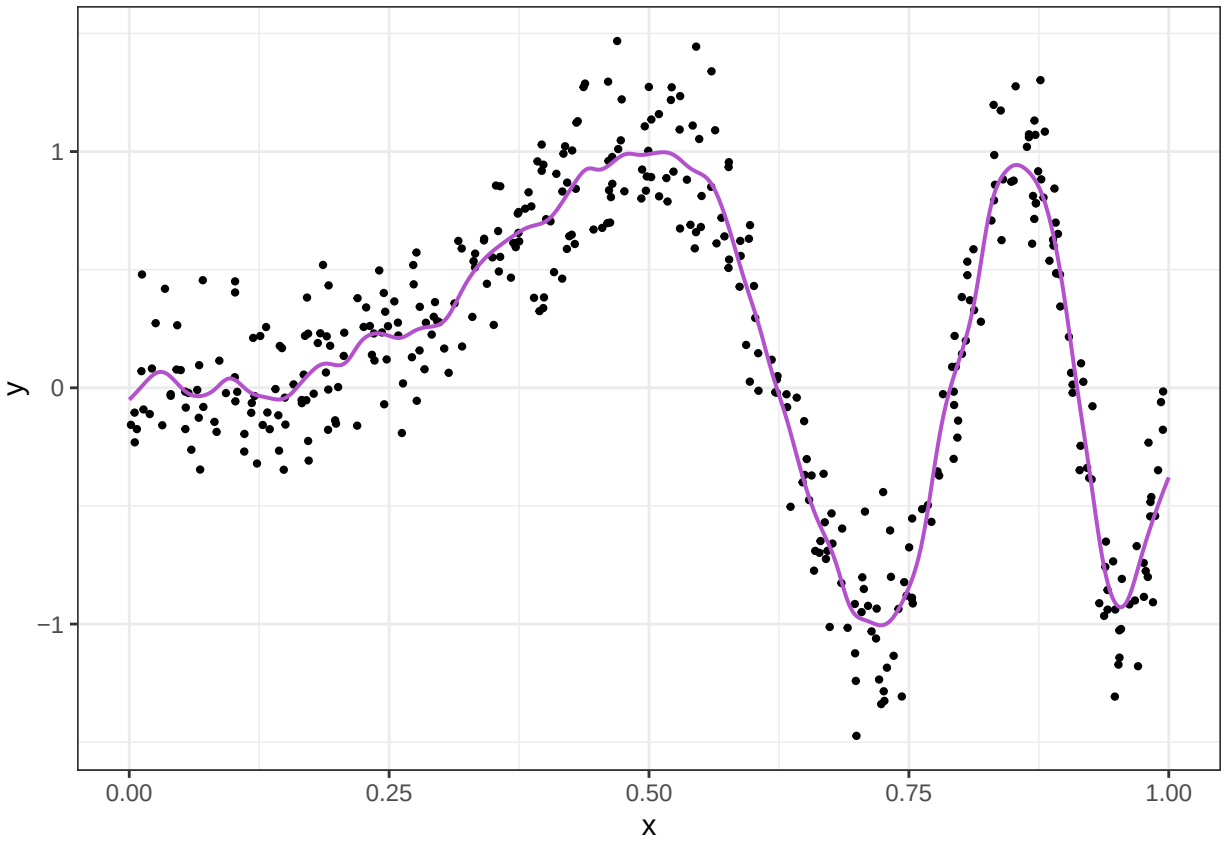
Cross-Validation

We now implement a cross-validation procedure for finding the optimal λ, using the mean squared error of the test set as the metric for determining the fit of our model. We first write the R version of this function:

```
mse_lambda <- function(log_lambda, x, y, x_new, y_new){
  lambda <- exp(log_lambda)

  fitted <- meanKRS(x, y, x_new, lambda)
  return(sum((fitted - y_new)^2))
}
```

```r
lambda_cv <- function(x, y, nfolds = 5){
  n <- length(x)
  groups <- sample(rep(1:nfolds, length.out = n), size = n)

  lambdas <- numeric(nfolds)
  mse <- numeric(nfolds)

  solutions <- vector(mode = "list", nfolds)

  for (i in 1:nfolds){
    x_train <- x[groups != i]
    y_train <- y[groups != i]

    x_test <- x[groups == i]
    y_test <- y[groups == i]

    solution <- optim(par = 0.02, fn = mse_lambda, x = x_train, y = y_train, x_new =
↪   x_test, y_new =  y_test, method = "BFGS")
    lambdas[i] <- exp(solution$par)
    mse[i] <- solution$value
    solutions[[i]] <- solution
  }

 min_ind <- which.min(mse)
 return(lambdas[min_ind])
}
```

```r
hat_lambda <- lambda_cv(x, y)
opt_smooth <- meanKRS(x, y, xnew, hat_lambda)
opt_data <- tibble(xnew = xnew,
         fitted = opt_smooth)
ggplot() +
  geom_point(data = data, aes(x, y), size = 0.8) +
  geom_line(data = opt_data, aes(x = xnew,y = fitted),
            color = "mediumorchid3", linewidth = 0.7)
```



We now write the equivalent function in Rcpp