ECO541 - Industrial Organization - Fall 2019
Profs. Buchholz and Kastl
Problem Set #3
Due 12/2

1. DDC

The goal of this exercise is to familiarize you with some techniques used to compute and estimate single agent dynamic discrete choice models.

Let's suppose we face Zurcher's problem of devising an engine replacement policy. In this case, we already know the parameters governing cost, replacement, and the discount rate. We want to figure out the optimal stopping rule, which specifies when to replace the bus engine given the engine mileage (x_t), mileage transition probabilities ($P(x_{t+1}|x_t)$) and other factors captured in the i.i.d. term ϵ_t.

The exogenous parameters are below.

$$\beta = 0.5$$
$$c(x) = 0.5x$$
$$RC = 3$$

Further, suppose that the transition probabilities are the following:

$$P(x_{t+1} \in [x_t + k, x_t + k + 1)|x_t, k \geq 0) = F(k+1) - F(k)$$

where $F(\cdot)$ is the CDF of the lognormal distribution.
Let the unit of x be $10k$ miles, so that $x = 5$ corresponds to $50,000$ miles.

To answer the following questions, use your favorite software (In this case, I used Matlab). Include images of your plots in your answers and email me your code.

(1) Compute the choice-specific value functions associated with this problem
Hints:
- Define the state space from 10k-500k miles (say, in 100 mile increments). From each state, you'll want to compute transition probabilities of the state updating to each possible future state. Here's the state vector, a 5000 x 1 matrix. Whereas the transition probabilities will be 5000 x 5000.

State vector: $X = \{0, 0.01, 0.02, 0.03, ..., 49.99\}$

- See slide 17 in the notes. At some point you'll use the $\exp(\cdot)$ function. (Note: Matlab delivers $exp(z) = \text{Inf}$ for $z > 708$. You may need to use the identity $log(A + B) = log(A) + log(1 + \frac{B}{A})$ to make things computable.)
- $E_y[\cdot]$ means you're integrating over the transitions to y from x, (i.e., to x_{t+1} from x_t).
- A good starting value for value function iteration is $V_1(x) = 0$ for all x.

(2) Plot both choice-specific value functions and the actual value function, as functions of the state x.

(3) What is the expected mileage x^* at which you replace the engine?

(4) Generate a dataset of observable states and choices for $T = 20000$ periods from the model. (Note that cross-sectional data-points and longitudinal data-point are perfect substitutes here.)

(5) Assume now that you no longer know the parameters or transitions, and that you only observe the data generated by step (4). Use the data from (4) to estimate the model using Rust's MLE/nested-fixed-point algorithm (supposing that you now observe the data and no longer know the parameters in the cost function or replacement costs.).

(6) Using the same data as above, estimate the model using the Hotz and Miller CCP approach. Here is some help:
   (a) Calculate the replacement probabilities at each state using the average replacement rate in the sample. (These estimates are nonparametric estimates of the replacement probabilities.)
   (b) Write down the Hotz-Miller inversion formula for this model. Calculate the differences in choice-specific value functions (making "not replacing" the reference action that is subtracted) using the choice probabilities you estimated above.
   (c) Express per-period utility for each choice in terms of the parameters and the current state.
   (d) Create conditional transition matrices F_0 and F_1 (say, 5×5), which give the transition probabilities of the state conditional on the $\{0, 1\}$ replacement choice. Again, supposing you've observed the data $\{x_t\}$ but do not have knowledge of the true data generating process. You can for example just create quartiles of the x_t as bins to generate the transition matrix.

(e) Using the non-parametric estimates of conditional choice probabilities and the F_0 and F_1 matrices, calculate the (5×5) unconditional transition matrix of states (which takes into account the probability of replacement as well as the transition probabilities).

(f) Write a procedure that takes as inputs (θ_1, R) and (forward) simulates $V_0(a_t)$ and $V_1(a_t)$ based on per-period utilities, unconditional transition probabilities T, the discount factor, and the expectation $E[\epsilon_{kt}|a_t, d_{kt} = 1]$, $k \in \{0, 1\}$ (remember that there is a trick to express this expectation in terms of known objects).

(7) We looked at some other examples of single agent DDC models in class, including one for patent renewal and (sometime in the next few weeks) one for daily labor supply. Come up with another example of a setting where such a model would apply and describe (in words and equations) the problem in a couple of paragraphs.