

NE 155

**Introduction to Numerical Simulations in
Radiation Transport**

Lecture 38: Estimators and Tallies

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OUTLINE

- ➊ Tally & Estimator Basics
- ➋ Surface Current Tally
- ➌ Flux Tallies and Estimators
- ➍ Energy Deposition Tallies and Estimators
- ➎ Pulse Height Tally
- ➏ Detector Tallies

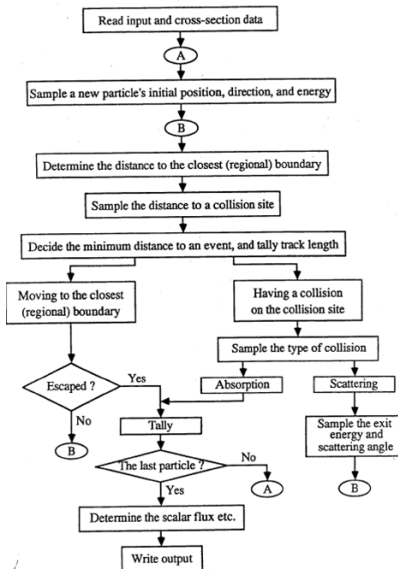
Notes derived from Jasmina Vujic and Paul Wilson

LEARNING OBJECTIVES

- ① Understand relationship between tallies and estimators
- ② Explain derivation of volume and surface track length estimators
- ③ Demonstrate difference between track length energy deposition estimator and pulse height tally
- ④ Understand next-event estimator and its application to point detectors

NOTE: The rest of this is a copy of the xport talk and isn't actually this talk!

Monte Carlo for Transport



POSSIBLE FUTURES FOR A PARTICLE

After we've gotten to **Circle B**, we have a neutral particle:

- At point (x_p, y_p, z_p)
- Moving in direction (u, v, w)
- With energy E

What are possible next events?

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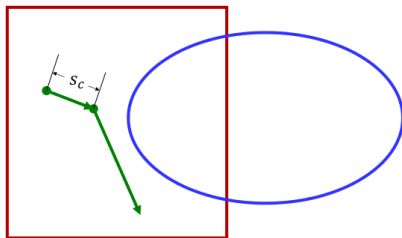


Figure 1: Collision

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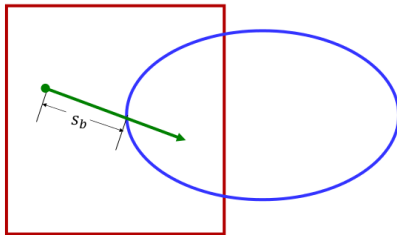


Figure 1: ../figs/Surface Crossing

SAMPLING DISTANCE TO COLLISION

Collisions are probabilistic

- Note that Σ_t , the total macroscopic cross section, will be a function of space if we have multiple materials
- Along a particular path, the *probability of a collision at distance s from the start*:

$$p_c(s) = \Sigma_t(s)e^{-\Sigma_t(s)s}ds$$
$$P_c(n) = \int_0^s \Sigma_t(s)e^{-\Sigma_t(s)s'}ds' = -e^{-\Sigma_t(s)s'}\Big|_0^s = 1 - e^{-\Sigma_t(s)s}$$

- The cross section, $\Sigma_t(s)$, is piecewise constant, but changing

SAMPLING DISTANCE TO COLLISION

- Variable transformation: measure distance in units of *mean free path*:

$$n = \Sigma_t(s)s, \quad dn = \Sigma_t(s)ds$$

- We'll start with the PDF and integrate to get the CDF

$$p_c(n)dn = e^{-n}dn$$

$$P_c(n) = \int_0^n e^{-n'} dn' = -e^{-n'} \Big|_0^n = 1 - e^{-n}$$

- Importantly, this is now independent of the material

SAMPLING DISTANCE TO COLLISION

Randomly sample to determine number of mean free paths until next collision, n_c

- $g(n_c)dn_c = e^{-n_c}dn_c$
- $G(n_c)dn_c = 1 - e^{-n_c}$
- Directly invert to get: $n_c = -\ln(1 - \xi)$
(note $1 - \xi$ is equivalent to ξ)
- In the absence of material boundaries ($\Sigma \neq f(s)$), the distance to a collision, s_c , is

$$s_c = \frac{n_c}{\Sigma_t}$$

CALCULATING DISTANCE TO BOUNDARY

- Usually have *more than one material*
- Distance to boundary is deterministic
- Algebra to determine distance between point and surface, s_b
- Convert it to units of mean free path for the current cell's material,

$$n_b = s_b \Sigma_t$$

GEOMETRY REPRESENTATIONS

- Combinatorial Surfaces
 - Define surfaces
 - Boolean operations combine surfaces to create cells
- Combinatorial Solids
 - Choose solid objects
 - Boolean operations combine objects to create regions
- B-Rep (Vertex-Edge)
 - Each object is a single set of vertices and edges connecting them

We're skipping how to find n_b , just know that we can find it

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 - Sample new direction
 - Sample new energy
 - Sample exiting particles
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- Some of these may depend on each other
- Repeat
 - Sample new n_c following collision
 - Calculate new n_b in new direction

OPTION B: CELL BOUNDARY

$$\underline{n_b < n_c:}$$

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OPTION B: CELL BOUNDARY

$n_b < n_c$:

- Boundary is closer than collision
- **Boundary crossing occurs**
- Move particle along ray
 - Update $n_c = n_c - n_b$
- **DO NOT SAMPLE** for new n_c

OPTION B: CELL BOUNDARY

$n_b < n_c$:

- Boundary is closer than collision
- **Boundary crossing occurs**
- Move particle along ray
 - Update $n_c = n_c - n_b$
- **DO NOT SAMPLE** for new n_c
- Calculate new n_b in new cell
 - New set of boundaries
 - New value of Σ_t

SO YOU HAD A COLLISION?

- Sample **target nuclide** for a mixture with J nuclides

$$\Sigma_t = \sum_{j=1}^J N_j \sigma_{t,j}$$

- *Discrete PDF* to determine which nuclide is hit

$$p_i = \frac{\Sigma_{t,j}}{\Sigma_t}$$

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- Sample **reaction type** for an nuclide with R types of reactions

$$\Sigma_{t,j} = \sum_{x=1}^R \Sigma_{x,j}$$

- *Discrete PDF* to determine which reaction occurs

$$p_x = \frac{\Sigma_{x,j}}{\Sigma_{t,j}}$$

OUTCOME OF REACTION

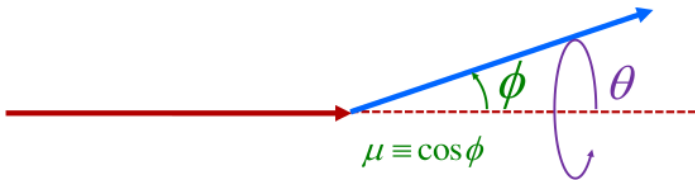
- Particle maybe absorbed
- Particle may continue its history in a *different direction* and with a *different energy*
- Energy-angle distributions are tabulated in different formats
 - Scattering laws have analytic forms with parameters in data tables (Direct inversion or rejection sampling)
 - Tabulated data that describes a piecewise analytic interpolation (Hybrid sampling; we skipped this)

USING A SCATTERING ANGLE

Scattering angles are defined relative to the original direction (considered as the z-axis)

- Polar angle, ϕ , determined by sampling from data
- Azimuthal angle, θ , determined by sampling isotropically
- The new direction is $(\sin(\phi) \cos(\theta), \sin(\phi) \sin(\theta), \cos(\theta))$

$$= (\sqrt{1 - \mu^2} \cos(\theta), \sqrt{1 - \mu^2} \sin(\theta), \mu)$$



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We've developed a general sense of using MC for neutron transport

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- *After a collisions* we need to determine many things associated with the collisions (target, reaction, direction, energy)
- Repeat analysis for collisions/crossing until particle **terminates**

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We've developed a general sense of using MC for neutron transport

- Basic Algorithm
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- Next time: tallying results