Elliptic if $B^2 - 4AC < 0$; **Parabolic** if $B^2 - 4AC = 0$; **Hyperbolic** if $B^2 - 4AC > 0$

$$\Sigma_s(\hat{\Omega}' \cdot \hat{\Omega}) = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} \Sigma_{sl} P_l(\hat{\Omega}') P_l(\hat{\Omega}) \, ; \quad P_0(\hat{\Omega}) = 1 \, , \quad P_1(\hat{\Omega}) = \hat{\Omega}$$

$$D = \frac{1}{3\Sigma_{tr}} = \frac{1}{3(\Sigma_t(\vec{r}) - \Sigma_{s1}(\vec{r}))} \, , \quad \Sigma_{tr} \equiv \Sigma_t - \Sigma_{s1}$$

$$\Sigma_{s1} = \int d\hat{\Omega} \, \hat{\Omega} \Sigma_s \text{ , when azimuthally symmetric } \Sigma_{s1} = \bar{\mu}_0 \Sigma_s$$

$$\vec{J}(\vec{r}) = -\frac{1}{3\Sigma_{tr}} \nabla \phi(\vec{r}) = -D \nabla \phi(\vec{r})$$

$$\tilde{x}_s = x_s + 2D = x_s + \frac{2}{3} \lambda_{tr} \text{ is the extrapolation distance}$$

Taylor expansion of a function about a point:

$$f(x) = \sum_{k=0}^{n-1} \frac{(x-a)^k}{k!} f^{(k)}(a) + R_n \qquad R_n = f^{(n)}(\mu(x)^*) \frac{(x-a)^n}{n!} \qquad \text{for some } \mu(x)^* \in [a, x]$$

If we have $(n+1)$ distinct points $x_0, x_1, \ldots, x_n$ and $f(x)$ is a function defined by these points, then $\exists$ a unique **polynomial**, $P_n(x)$, of degree $\leq n$ that **interpolates** f at the $n+1$ distinct points:

$$f(x_k) = P_n(x_k) \, , \qquad \text{for } k = 0, 1, \ldots, n$$

$$P_n(x) = f(x_0) L_0(x) + \cdots + f(x_n) L_n(x) = \sum_{k=0}^{n} f(x_k) L_k(x)$$

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)} \qquad L_k(x) = \frac{(x-x_0)(x-x_1)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)(x_k-x_1)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$$

If f is $C^{n+1} \in [a, b]$ then for each $x \in [a, b]$ $\exists$ $\xi(x) \in [a, b]$ s.t.

$$f(x) - P_n(x) = R_n(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!} (x-x_0)(x-x_1)\cdots(x-x_n) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!} \prod_{i=0}^{n} (x - x_i)$$

normal equations for m with n unknowns where $n < m$: $\mathbf{A}^T \mathbf{A} \vec{z} = \mathbf{A}^T \vec{b}$

$$\det(\mathbf{A}^T \mathbf{A}) = m \sum_{i=0}^{m} x_i^2 - \big(\sum_{i=0}^{m} x_i \big)^2 = \frac{1}{2} \sum_{i=0}^{m} \sum_{j=0}^{m} (x_i - x_j)^2$$

$$I(f) \approx I_n(f) = \sum_{i=0}^{n} w_i f(x_i)\ , \quad \text{and for } \textbf{Newton-Cotes integration},\ = I(P_n)$$

$$I(P_n(x)) = \sum_{i=0}^{n} \Big[\int_a^b L_i(x)dx\Big] f(x_i)\ , \quad \text{here } w_i = \int_a^b L_i(x)dx = \int_a^b \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)}$$

$$\text{sub in } x = a + sh\ , \quad w_i = (b-a)\frac{1}{n}\int_0^n \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{(s-i)}{(k-i)} ds$$

In **open NC** our real endpoints are x_{-1} and x_{n+1} (so we're doing $\int_{x_{-1}}^{x_{n+1}} f(x)dx$), which gives

$$w_i = \int_a^b L_i(x)dx = \int_a^b \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{(x-x_i)}{(x_k-x_i)} = (b-a)\frac{1}{n+2}\int_0^n \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{(s-i)}{(k-i)} ds$$

For a square matrix, the **first order minor**, M_{ij}, deletes the i^{th} row and j^{th} column and takes the determinant. The corresponding i, j **cofactor** of $\mathbf{A}$ is $C_{ij} = (-1)^{i+j} M_{ij}$. Then

$$\det(\mathbf{A}) = \sum_{j=1}^{n} a_{ij} C_{ij} \text{ for any i} \in \{1, ..., n\} \quad \text{or} \quad = \sum_{i=1}^{n} a_{ij} C_{ij} \text{ for any j} \in \{1, ..., n\}$$

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\mathbf{C}^T$$

$$||\mathbf{A}||_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} |a_{ij}|^2} = \sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2}$$

$$||\mathbf{A}||_1 = \sup_{\vec{x}\neq\vec{0}} \frac{||\mathbf{A}\vec{x}||_1}{||\vec{x}||_1} = \max_{1\leq j\leq n} \sum_{i=1}^{m} |a_{ij}| \qquad \text{max absolute col sum}$$

$$||\mathbf{A}||_\infty = \sup_{\vec{x}\neq\vec{0}} \frac{||\mathbf{A}\vec{x}||_\infty}{||\vec{x}||_\infty} = \max_{1\leq i\leq m} \sum_{j=1}^{n} |a_{ij}| \qquad \text{max absolute row sum}$$

$$||\mathbf{A}||_2 = \sup_{\vec{x}\neq\vec{0}} \frac{||\mathbf{A}\vec{x}||_2}{||\vec{x}||_2} = \text{ the sqrt of the max eigenvalue of } \mathbf{A}^H\mathbf{A}$$

is largest singular value for any matrix, or the spectral radius of a square matrix

The **spectrum** of $\mathbf{A}$: $\sigma(\mathbf{A}) = [\lambda \in \mathbb{C} : \det(\mathbf{A} - \lambda\mathbf{I}) = 0]$