

## NE 155, Midterm 2 Review S17

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Here are the topics we've covered and that are fair game for the exam.

The exam will be 50 minutes long and closed book.

You may use a calculator.

I will provide a notes sheet with the exam for your use.

I encourage you to think about what can reasonably be asked about in 50 minutes if there are a few questions. Also think about what can be asked on an exam at all vs. what really requires a computer.

A goal of mine is for you to understand underlying principles and the meaning behind things. If you understand the meaning most other things will come out of it.

### More deterministic methods

Think about what things are reasonable to ask on a 50 minute exam when you can't solve anything with a computer: forms of equations, meaning, trends, etc.

- Direct solutions of linear systems ( $A\vec{x} = \vec{b}$ )
  - diagonal, lower-tri and upper-tri systems
  - LU decomposition
  - tridiagonal systems
- Iterative solutions of linear systems
  - General form of the fixed point iterative method
  - Richardson / Source iteration
  - Jacobi
  - Gauss Seidel
  - SOR
  - convergence

- preconditioning
- Finite difference derivation; application to the 1-D, fixed source diffusion equation
- Finite volume method (1-D and 2-D)
  - Derivation
  - Application to the diffusion equation
  - Vacuum and reflecting boundary conditions
  - Simplification to homogeneous, uniform mesh
- Methods for solving the system of equations we created
  - Directly with Thomas Algorithm
  - Iteratively with Jacobi, GS, or SOR
- Eigenvalue form of the DE
  - form of the equation; applying finite difference
  - applying finite volume method, including BCs
- Solving the Eigenvalue equations
  - determining convergence of  $k$  and  $\phi$
  - calculating  $k$
  - Power Iteration

## **Point Kinetics and Taylor/RK**

- What the point kinetics equation is, including terms and assumptions
- Units of reactivity; what is reactor period
- Simple solutions of the PRKE
- Taylor series and Runge Kutta are two methods for solving the PRKE

## MC: Math with Random Numbers

You can use random numbers to do math in two primary ways:

- sample physical distributions to reproduce physics needed to solve problems
- conduct numerical integration to solve problems

You would choose to do Monte Carlo when

- analytical integration is impossible
- deterministic methods are too slow, require approximations that don't work, you can't get the solution, etc.

A good thing to think about is this comparison of methods

	Monte Carlo	Deterministic
Strengths	<ul style="list-style-type: none"><li>* General geometry</li><li>* Continuous Energy</li><li>* Continuous in Angle</li><li>* Inherently 3-D</li><li>* Easy to parallelize on CPUs</li></ul>	<ul style="list-style-type: none"><li>* Fast</li><li>* Global Solution</li><li>* Solution is of same quality everywhere</li></ul>
Weaknesses	<ul style="list-style-type: none"><li>* Slow</li><li>* Might be memory intensive</li><li>* Solutions have statistical error</li><li>* Local solutions only</li><li>* Must adequately sample phase space</li><li>* Need efficient VR</li></ul>	<ul style="list-style-type: none"><li>* Variable discretization governs solution quality</li><li>* Might be memory intensive</li><li>* Solution contains truncation error</li><li>* Constrained by what you can mesh</li><li>* Ray Effects</li><li>* Can be complicated to parallelize on CPUs</li></ul>

## MC algorithm

Figure 1 shows the algorithm that is basically what happens in MC. We learned the items needed to do all of these steps.

## Statistics

Monte Carlo solution statistics are based on the Central Limit Theorem, which is applicable when samples are taken from the same distribution (identical) and they are independent of one another.

In this case we can assert formulas for the sample mean, error, and variance and relate those to the true mean, error, and variance.

We have the ability to measure precision and criteria for determining how precise an answer must be for it to be acceptable.

## PDFs, CDFs

We learned how to define PDFs and CDFs for continuous and discrete variables; including how to normalize them.

## Sampling

We learned how to sample different kinds of CDFs. An example of direct inversion is determining the **distance to the next interaction** (we also did this as a function of mean free path only):

- $\Sigma_t$  = total macroscopic cross section of material

$$\Sigma_T = \sum_{j=1}^J N_j \sigma_t^j$$

- The PDF for distance to collision is probability of interaction per unit distance  $\times$  probability of traveling distance  $s$  without interacting

$$p(s) = \Sigma_t \exp(-\Sigma_t s)$$

- We integrate and normalize to get the CDF

$$P(s) = 1 - \exp(-\Sigma_t s)$$

- To actually sample this, we invert it and get a random number

$$s = \frac{-\ln(\xi)}{\Sigma_t}$$

If we're in a multi-region problem, we figure out if we intersect a boundary and if so move the particle to that boundary and determine how much farther it goes into the next material before having a collision.

After finding the location of the collision and the isotope collided with, we need to determine **what type of collision** occurs.

- $\Sigma_t = \Sigma_{elastic} + \Sigma_{inelastic} + \Sigma_{capture} + \Sigma_{fission} + \dots$
- The probability of reaction of type  $i$  for a given isotope is

$$p_i = \frac{\Sigma_i}{\Sigma_t}$$

- This gives a set of discrete probabilities, which we can sample
  - directly: generate  $\xi$ , determine  $k$  s.t.  $P_{k-1} \leq \xi \leq P_k$ , return  $i = i_k$ .
  - or by making an alias table and sampling.

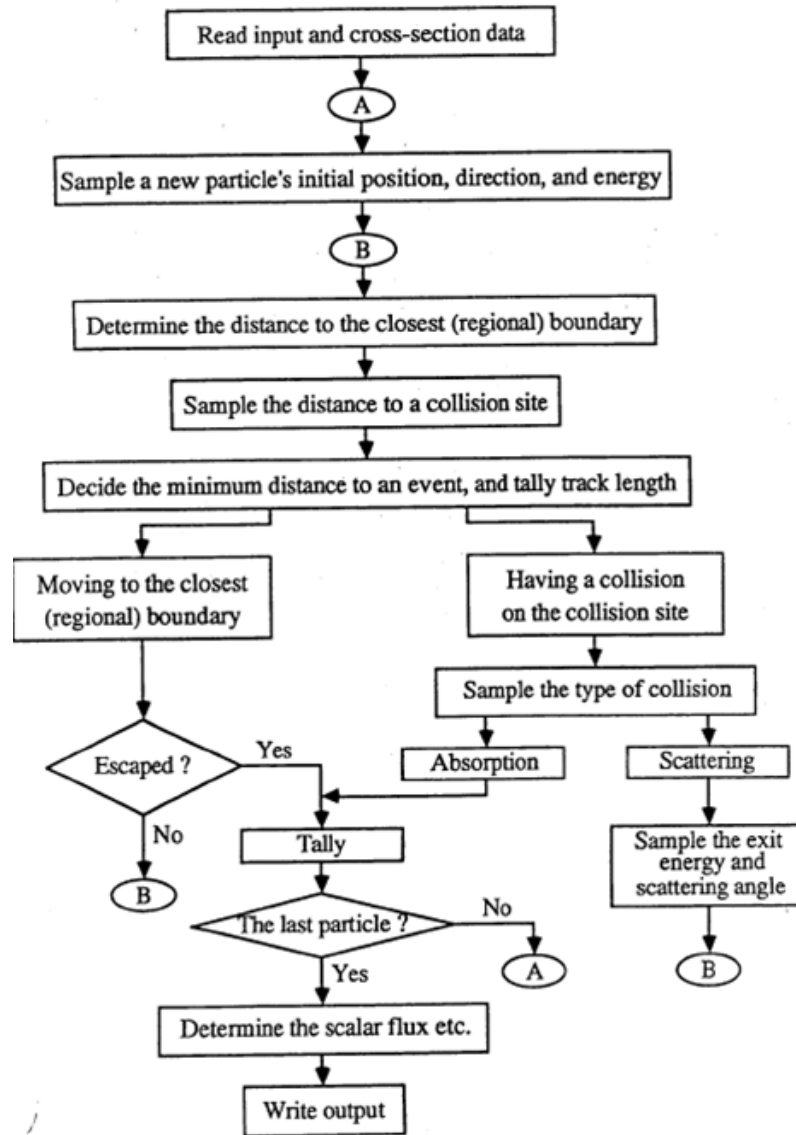


Figure 1: Monte Carlo neutral particle transport algorithm