NE 155
Introduction to Numerical Simulations in Radiation Transport
Probability and Statics

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Major Components of MC Algorithm

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- **Parallelization**: efficient use of computers

Outline / Learning Objectives

1. Probability Density Functions
2. Standard Statistical Quantities
3. Accuracy vs. Precision
4. Central Limit Theorem
5. Relative Error

1. Understand the derivation of basic statistical quantities
2. Be able to explain the difference between accuracy and precision
3. Understand how to interpret and apply confidence intervals
4. Understand derivation and use of relative error

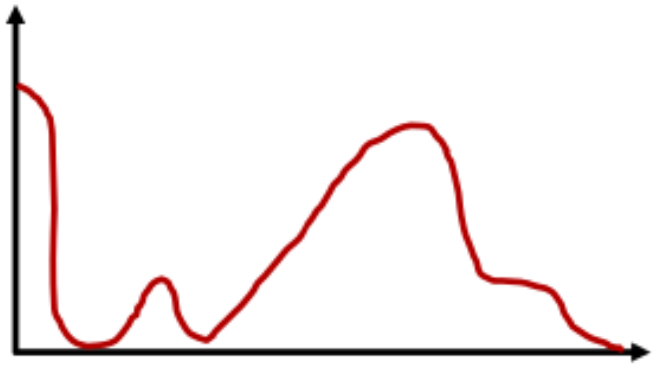
Notes derived from Jasmina Vujic and Paul Wilson

Fundamental Concept

- Many individual particle histories are simulated
- Each physical event is determined by randomly sampling a probability distribution
- Each history can contribute to the physical measurement of interest
  - x_i = contribution of history i
  - Different ways to calculate score
    - Does particle cross surface?
    - How much time does particle spend in particular region?

Fundamental Concept

- Set of individual contributions, x_i, forms a *probability distribution*



- We are interested in the mean value of that contribution, $\overline{x_i}$, and its variance, $S_{\overline{x}}^2$

To the Board

Two Types of MC Methodology

Analog

- Natural laws are **preserved**
- The game is the "analog" of the physical problem of interest (the history of each particle is simulated exactly)

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Non-Analog

- To reduce computation time, the strict analog simulation of particles is abandoned (i.e. we CHEAT)
- Variance Reduction techniques:
  - Absorption suppression
  - Russian Roulette (history termination)
  - Splitting (history propagation)
  - Forced collisions
  - Source biasing
  - Hybrid methods

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- At collision, particle is killed if absorption
- Particle is born with weight 1
- weight unchanged throughout history
- Score when tallying events is 1

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Non-Analog (weighted)

- Alter PDFs to favor events of interest
- Particle can have different birth weight
- Weight is altered if biased PDF is used
- Particle survives "absorption" and weight is changed
- Splitting and RR can change weight
- Score current weight when tallying

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- The difference between accuracy and precision is important
- Accuracy is not always known and can be difficult to improve
- Precision can be improved by more histories in a measurement, but not always more histories in a problem