

**NE 155**

**Introduction to Numerical Simulations in  
Radiation Transport**

**Lecture 31: Probability and Statics**

R. N. Slaybaugh  
Richard Vasques

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# MAJOR COMPONENTS OF MC ALGORITHM

- **PDFs:** the physical/mathematical system must be described by a set of pdfs.
- **Random number generator:** a source of random #s uniformly distributed on the unit interval.
- **Sampling rule:** prescription for sampling the pdf (given having random #s)
- **Scoring:** the outcomes must be accumulated/tallied for quantities of interest
- **Error estimation:** *an estimate of the statistical error (variance) of the solution*
- **Variance Reduction:** methods for reducing the variance and computation time simultaneously
- **Parallelization:** efficient use of computers

# OUTLINE / LEARNING OBJECTIVES

- ➊ Probability Density Functions
  - ➋ Standard Statistical Quantities
  - ➌ Accuracy vs. Precision
  - ➍ Central Limit Theorem
  - ➎ Relative Error
- ➏ Understand the derivation of basic statistical quantities
  - ➐ Be able to explain the difference between accuracy and precision
  - ➑ Understand how to interpret and apply confidence intervals
  - ➒ Understand derivation and use of relative error

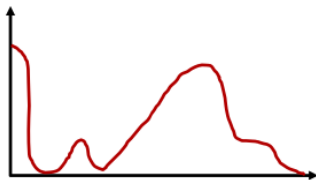
Notes derived from Jasmina Vujic and Paul Wilson

# FUNDAMENTAL CONCEPT

- Many individual particle histories are simulated
- Each physical event is determined by randomly sampling a **probability distribution**
- Each history can contribute to the physical measurement of interest
  - $x_i$  = contribution of history  $i$
  - Different ways to calculate score
    - Does particle cross surface?
    - How much time does particle spend in particular region?

# FUNDAMENTAL CONCEPT

- Set of individual contributions,  $x_i$ , forms a *probability distribution*



- We are interested in the mean value of that contribution,  $\bar{x}_i$ , and its variance,  $S_{\bar{x}}^2$

# TWO ENCOUNTERS WITH PROBABILITY DISTRIBUTIONS

- Probability distributions for the outcome of each physical event
- We use **Random Sampling** techniques to evaluate these at each occurrence
- Underlying probability distribution for each physical measurement of interest
- We estimate the statistical moments of these distributions to get our physical answers

# TO THE BOARD



# TWO TYPES OF MC METHODOLOGY

## Analog

- Natural laws are **preserved**
- The game is the “analog” of the physical problem of interest (the history of each particle is simulated exactly)

## Non-Analog

- To reduce computation time, the strict analog simulation of particles is abandoned (i.e. we CHEAT)
- Variance Reduction techniques:
  - Absorption suppression
  - Russian Roulette (history termination)
  - Splitting (history propagation)
  - Forced collisions
  - Source biasing
  - Hybrid methods



# ANALOG VS. WEIGHTED MC

## Analog

- No alteration of PDFs
- At collision, particle is killed if absorption
- Particle is born with weight 1
- weight unchanged throughout history
- Score when tallying events is 1

## Non-Analog (weighted)

- Alter PDFs to favor events of interest
- Particle can have different birth weight
- Weight is altered if biased PDF is used
- Particle survives “absorption” and weight is changed
- Splitting and RR can change weight
- Score current weight when tallying

# PROBABILITY & STATISTICS SUMMARY

- Rich variety of statistical analysis is possible.
- The difference between accuracy and precision is important
- Accuracy is not always known and can be difficult to improve
- Precision can be improved by more histories in a measurement, but not always more histories in a problem