

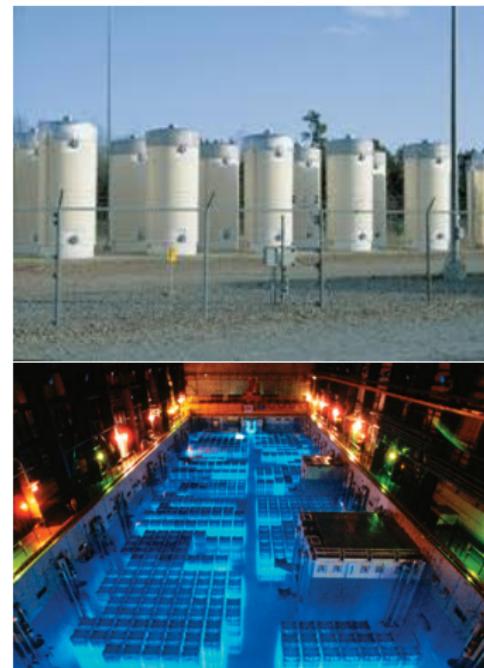
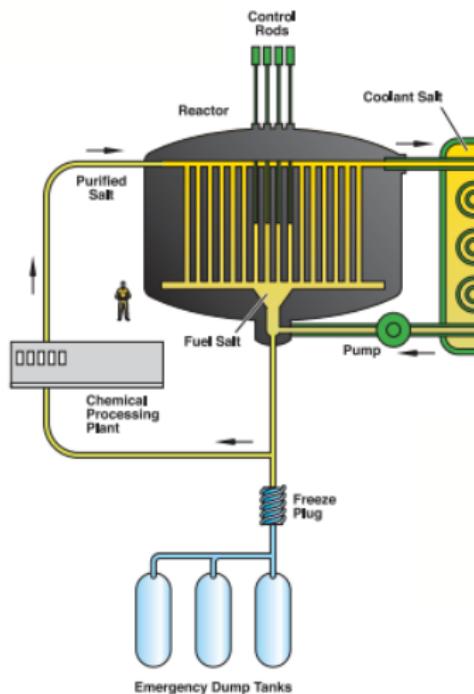
# Advanced Solvers and Radiation Transport



R. N. Slaybaugh, Univ. of Cal. Berkeley

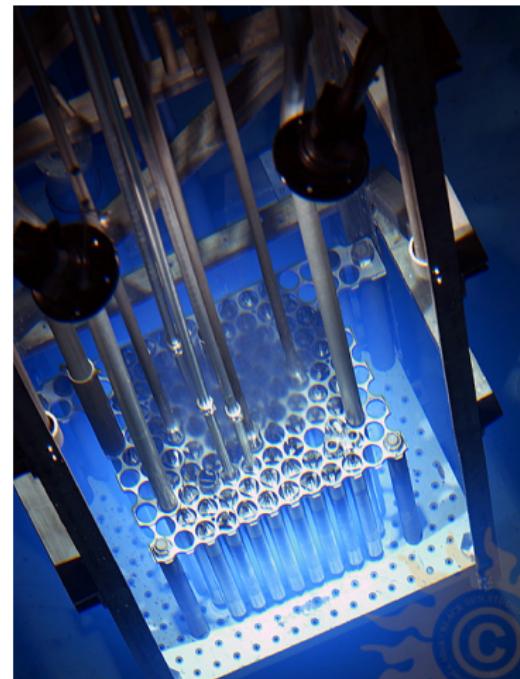
5 December 2017  
UIUC Graduate Colloquium

# NUCLEAR INNOVATION IS NEEDED



# OUTLINE

- ▶ Motivation & Background
- ▶ Hybrid Methods and Strong Anisotropies
  - ▶ Research Objectives
  - ▶ Cadis- $\Omega$  method and highlights
- ▶ Spectrum Shaping for Strategic Research
  - ▶ Research Objectives
  - ▶ Gnowee: Metaheuristic Optimization Algorithm
  - ▶ Coeus: ETA Design Software

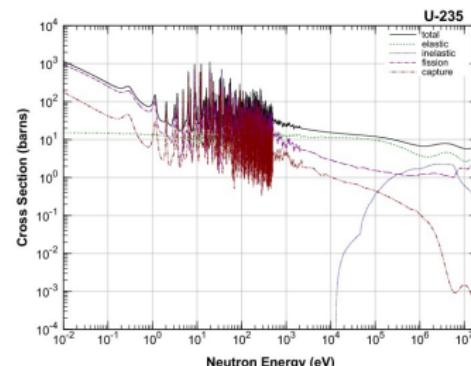


# NUMERICAL METHODS FOR RADIATION TRANSPORT

To facilitate nuclear innovation,  
we need predictive simulation

- ▶ My group builds tools  
(translate applied math into  
code) used to design and  
analyze nuclear systems
- ▶ We focus on high  
performance computing,
- ▶ and inform algorithm  
development with physics of  
problems of interest

$$\begin{aligned} \frac{ds}{(x-a)^r} &= \frac{1}{a^n} \int_{x^{m-h}}^{\infty} dx' \\ &= \frac{2}{n\sqrt{a}} \cos^{-1} \sqrt{\frac{x}{a}} \end{aligned}$$



# SOLVING THE TRANSPORT EQUATION

$$\hat{\Omega} \cdot \nabla \psi(\vec{r}, E, \hat{\Omega}) + \Sigma_t \psi(\vec{r}, E, \hat{\Omega}) = S(\vec{r}, E, \hat{\Omega}) + \int_{4\pi} d\hat{\Omega}' \int_0^\infty dE' \Sigma_s(E', \hat{\Omega}' \rightarrow E, \hat{\Omega}) \psi(\vec{r}, E', \hat{\Omega}')$$

## Monte Carlo

- ▶ *Continuous phase space*
- ▶ Solution has statistical error
- ▶ Localized solutions
- ▶ Optically thick = *slow*

## Deterministic

- ▶ *Discretized phase space*
- ▶ Solution equally valid everywhere
- ▶ Truncation errors
- ▶ Streaming = *ray effects*

# SPEEDING UP MONTE CARLO

- ▶ Variance reduction (VR) used to improve Monte Carlo: reduce relative error *and* time by augmenting game
- ▶ Particles are assigned weights that map to impact
- ▶ VR can be used to
  - ▶ set weights at birth
  - ▶ update weights throughout problem
- ▶ Improvement measured as

$$\text{FOM} = \frac{1}{R^2 t} \quad R = \text{relative error}; t = \text{time}$$

**Hybrid Methods:** we use deterministic results to make Monte Carlo VR parameters

[HTTPS://GITHUB.COM/SLAYBAUGHLAB](https://github.com/SlaybaughLab)

- ▶ Hybrid methods
  - ▶ Kelly Rowland: Lagrange Discrete Ordinates
  - ▶ Vanessa Goss and Emily Vu: analysis
  - ▶ Madicken Munk (alumna): Omega methods
- ▶ Deterministic methods
  - ▶ Mario Ortega: eigenvalue acceleration methods
  - ▶ Weixiong Zhang: finite element methods
  - ▶ Josh Rehak: finite element methods
  - ▶ Marissa Ramirez Zweiger: finite element methods
  - ▶ Sam Olivier: finite element methods
- ▶ Other adventures
  - ▶ April Novak: coupled neutronics and TH
  - ▶ Mitch Negus: PWR analysis
  - ▶ Ethan Boado: pelletron experiment design
  - ▶ James Bevins (alumnus): optimization methods
  - ▶ Richard Vasques (alumnus): non-classical transport

# PROJECT 1 MOTIVATION

- ▶ Many important nuclear applications have strong anisotropies
  - ▶ Used fuel casks
  - ▶ Reprocessing facilities
  - ▶ Reactor facilities
  - ▶ Active interrogation
- ▶ New ideas are needed for these problems
  - ▶ Current hybrid methods are only  $f(\vec{x}, E)$
  - ▶ Including angle explicitly is too costly
- ▶ **Goal:** new methods that are easy to use

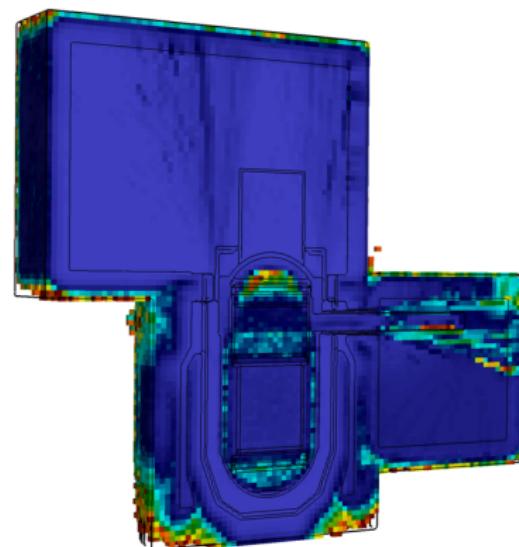


Figure: PWR relative error [1]

# ADJOINT AS AN IMPORTANCE MAP

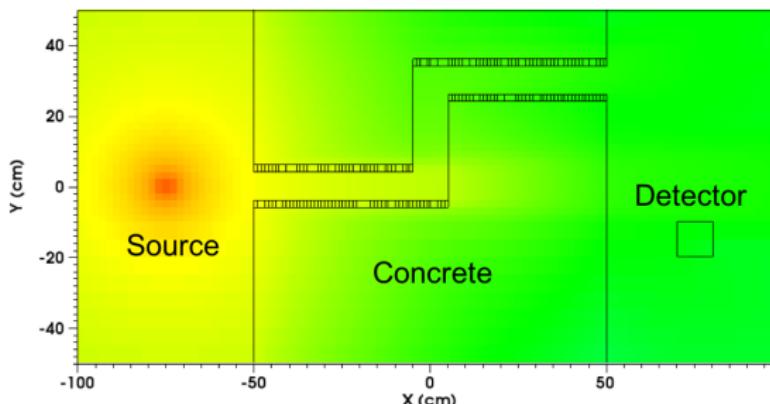
Define response with function  $f(\vec{r}, E)$  in volume  $V_f$  as

$$R = \int_E \int_{V_f} f(\vec{r}, E) \phi(\vec{r}, E) dV dE \quad (1)$$

- ▶ Forward ( $\phi$  or  $\psi$ ): neutrons flow from the source ( $q$ ) to the detector
- ▶ Adjoint( $\phi^\dagger$  or  $\psi^\dagger$ ): particles represent how each part of phase space contributes to the “source” ( $q^\dagger$ )
- ▶  $\phi^\dagger$  represents the expected contribution of a source particle to the response given the source,  $q$ .

# UNDERSTANDING FORWARD FLUX, $\phi(\vec{r}, E)$

10 MeV isotropic point source; NaI detector

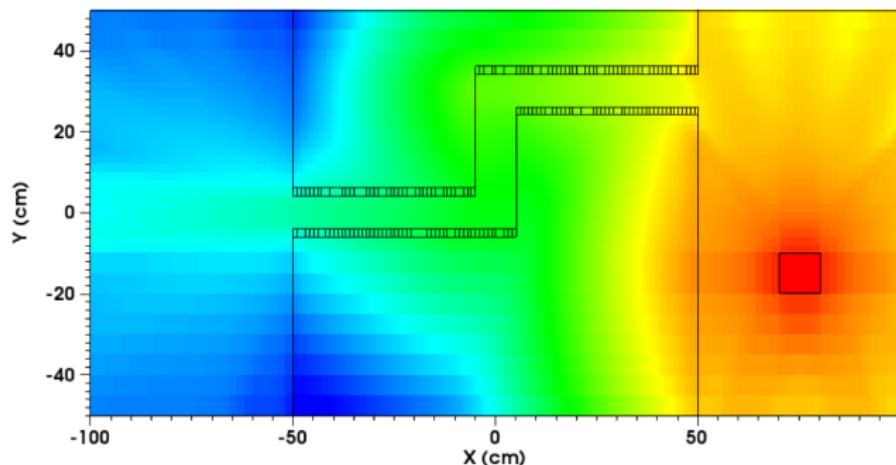


Neutrons in the forward problem will flow from the source to the detector

# UNDERSTANDING ADJOINT FLUX, $\phi^\dagger(\vec{r}, E)$

10 MeV isotropic point source; NaI detector

Adjoint  
measures how  
each part of  
phase space  
contributes to  
the solution:  
  
importance  
map



## FORWARD-ADJOINT RELATIONSHIP [2]

Define response with function  $f(\vec{r}, E)$  in volume  $V_f$  as

$$R = \int_E \int_{V_f} f(\vec{r}, E) \phi(\vec{r}, E) dV dE \quad (2)$$

$$\begin{array}{ll} H\phi = q & \text{(forward)} \\ H^\dagger \phi^\dagger = q^\dagger & \text{(adjoint)} \end{array} \quad \begin{array}{l} \langle H\phi, \phi^\dagger \rangle = \langle H^\dagger \phi^\dagger, \phi \rangle, \text{ and therefore} \\ \langle q, \phi^\dagger \rangle = \langle q^\dagger, \phi \rangle \end{array}$$

If we let  $q^\dagger = f(\vec{r}, E)$  then

$$\langle q^\dagger, \phi \rangle = \langle f, \phi \rangle = R = \langle q, \phi^\dagger \rangle \quad (3)$$

Eq. (3) expresses that  $\phi^\dagger$  represents the expected contribution of a source particle to the response.

# ADJOINT AS AN IMPORTANCE MAP

Use *adjoint*: the importance of a source particle to the solution

- ▶ Define  $q^\dagger$  as the response of interest
- ▶ Coarse deterministic calculation to get  $\phi^\dagger$  and  $R$
- ▶ The current state of the art for VR is FW/CADIS [2]

$$\begin{aligned} imp(\vec{r}, E) &= \frac{\phi^\dagger(\vec{r}, E)}{\langle q(\vec{r}, E), \phi^\dagger(\vec{r}, E) \rangle} = \frac{\phi^\dagger(\vec{r}, E)}{R} \\ \hat{q}(\vec{r}, E) &= \frac{\phi^\dagger(\vec{r}, E)q(\vec{r}, E)}{R} \\ w_0(\vec{r}, E) &= \frac{q(\vec{r}, E)}{\hat{q}(\vec{r}, E)} = \frac{R}{\phi^\dagger(\vec{r}, E)} \end{aligned}$$

# CURRENT HYBRID METHODS ARE INSUFFICIENT

Note:  $\phi^\dagger(\vec{r}, E) = \int \psi^\dagger(\hat{\Omega}, \vec{r}, E) d\hat{\Omega}$

- ▶ MC VR parameters created from adjoint deterministic scalar flux that is a function of *space and energy only*
- ▶ Angular dependence of the importance function is not retained, otherwise the map would be
  - ▶ very large (tens or hundreds of GB) and
  - ▶ more costly and complex to use in the MC simulation
- ▶ Drawback: within a given space/energy cell, map provides average importance of a particle moving in *any direction* through the cell—excluding information about how particles move toward the objective

# CURRENT HYBRID METHODS ARE INSUFFICIENT

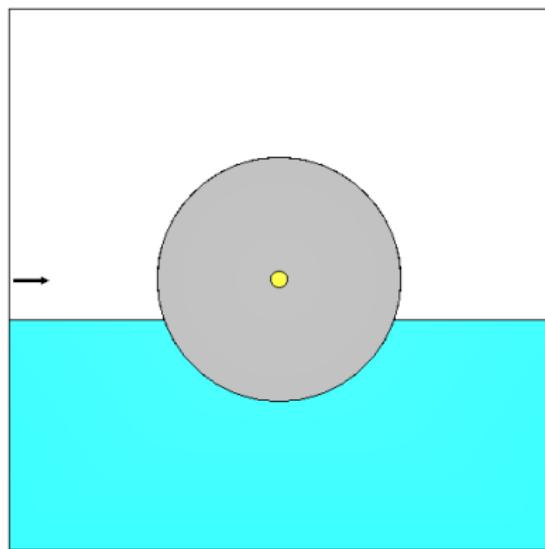


Figure: Spherical boat model with source on left and fissionable material at center

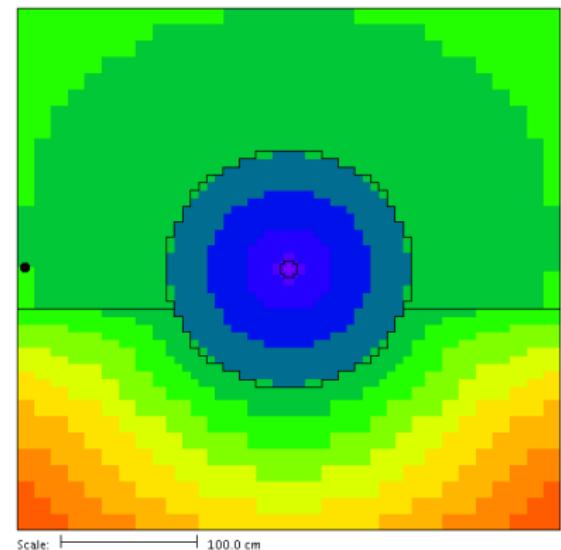


Figure: Target weight window values for 14.1 MeV neutrons

# INTEGRATION WEIGHTING

Different integration plan captures angles in scalar flux creation

$$\phi^\dagger(\vec{r}, E) = \int \psi^\dagger(\hat{\Omega}, \vec{r}, E) d\hat{\Omega} \quad \text{original}$$

$$\phi^\dagger(\vec{r}, E) = \frac{\int \psi(\hat{\Omega}, \vec{r}, E) \psi^\dagger(\hat{\Omega}, \vec{r}, E) d\hat{\Omega}}{\int \psi(\hat{\Omega}, \vec{r}, E) d\hat{\Omega}} \quad \text{new}$$

Major challenges and areas of investigation:

1. Data storage and handling (many GBs)
2. More, less, or differently sensitive to
  - ▶ quality of the discrete ordinates calculation?
  - ▶ ray effects?

# METHOD IMPLEMENTATION

- ▶ The space- and energy-dependent importance map is normalized and source biasing parameters are generated in the **same ways** as the current implementation of FW/CADIS
- ▶ Immediately useful; widely applicable
- ▶ We are studying and characterizing the impact
- ▶ Is available currently ADVANTG [3]

# THE NEW METHOD CAPTURES ANISOTROPY

Comparing the original adjoint to CADIS- $\Omega$ ....

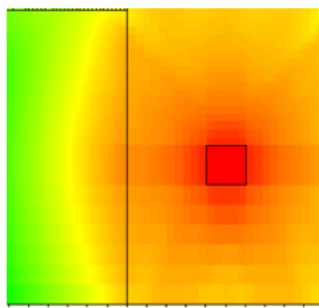


Figure: original adjoint

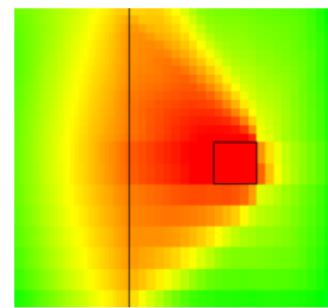


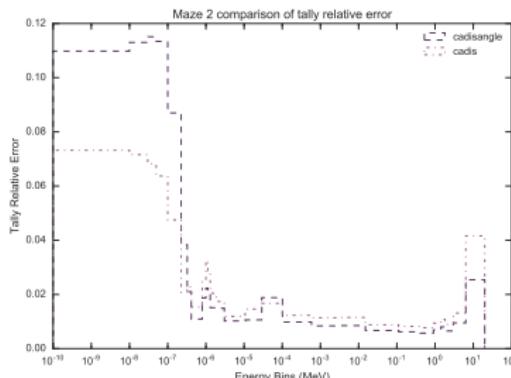
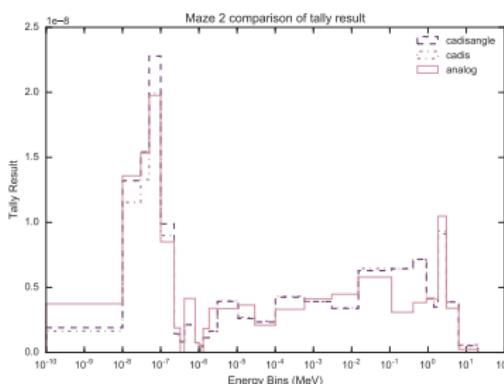
Figure: new adjoint

...shows that the method does incorporate problem physics differently

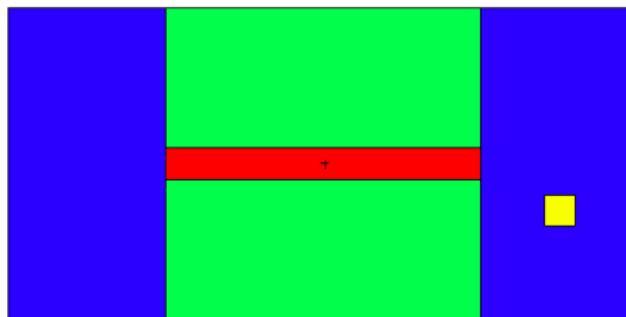
# SINGLE-TURN MAZE RESULTS

- CADIS- $\Omega$  has lower REs at higher energies
- Analog has high RE
- CADIS- $\Omega$  was in the middle for FOM using the worst RE

Run Type	Time (m)	FOM
CADIS	84.4	2.21
CADIS- $\Omega$	237	0.318
analog	11.7	0.0857



# STEEL BEAM IN CONCRETE

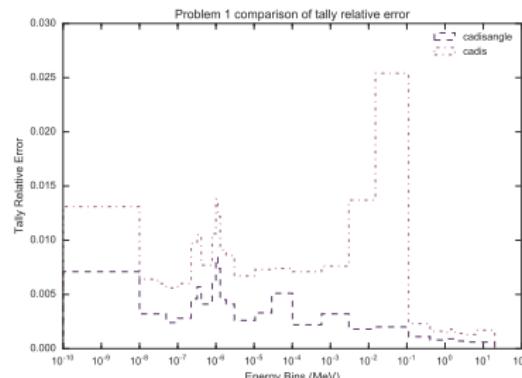
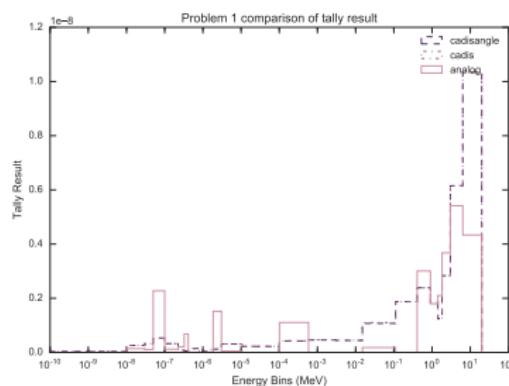


- ▶ Steel plate embedded in concrete; air on each end
- ▶  $^{235}\text{U}$  fission spectrum plane source
- ▶ Steel streams neutrons, concrete scatters them

# STEEL BEAM IN PLATE RESULTS

- ▶ CADIS- $\Omega$  has lower REs at all energies
- ▶ Analog has high RE
- ▶ CADIS- $\Omega$  performed best for all FOMs

Run Type	Time (m)	FOM
CADIS	420	3.69
CADIS- $\Omega$	2,110	<b>6.71</b>
analog	22	0.0448



# RESULTS SUMMARY

We tested a variety of characterization problems

- ▶ Problems with air streaming did not work well for any solver
- ▶ With weight windows
  - ▶ flux changes magnitude too quickly
  - ▶ causes lots of splitting and dramatic weight change
  - ▶ causes high variance and long runtime
- ▶ CADIS- $\Omega$  was great for problems with denser streaming materials
- ▶ These have enough scattering so flux changes more slowly
- ▶ And enough streaming that anisotropy is strong
- ▶ *This is exactly the problem we want to solve*

# PROJECT 1 SUMMARY

- ▶ There are many situations of interest where neutron fluxes have strong anisotropies
- ▶ Current VR methods do not enhance performance sufficiently
- ▶ CADIS- $\Omega$  is one way to capture angular information and shows strong initial promise
- ▶ We're looking at many types of problems and are scaling up to real applications

# INTERLUDE: WHO ARE YOU, ANYWAY?



MECHANICAL AND  
NUCLEAR ENGINEERING



American Nuclear Society



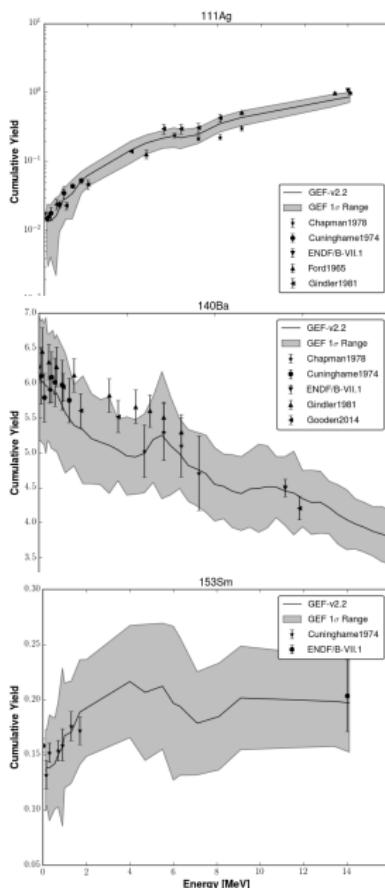
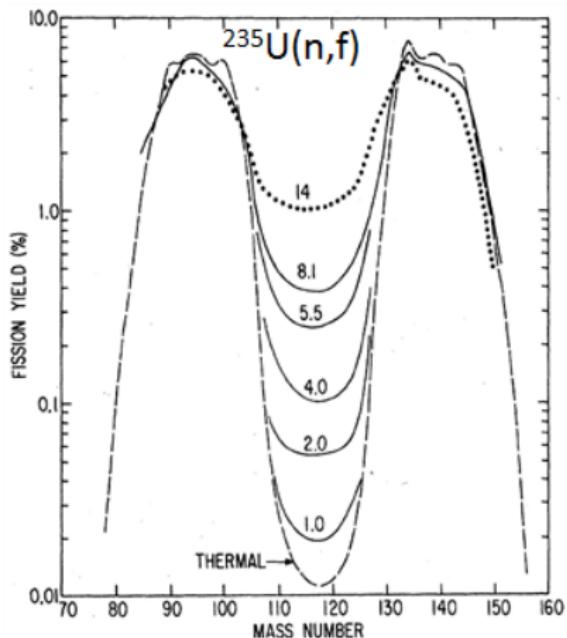
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Nuclear  
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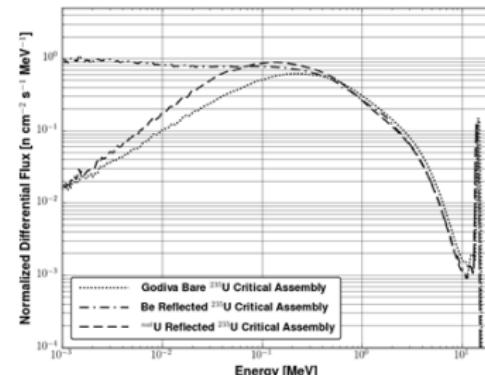
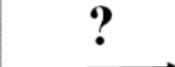
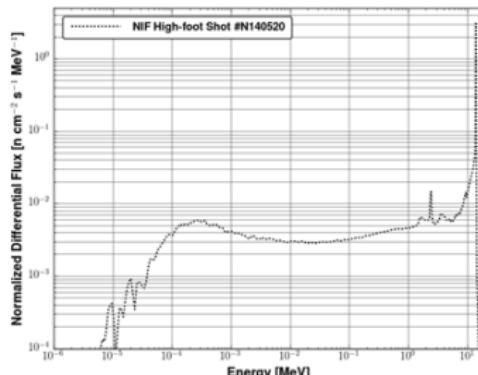
# PROJECT 2 MOTIVATION



# RESEARCH OBJECTIVES

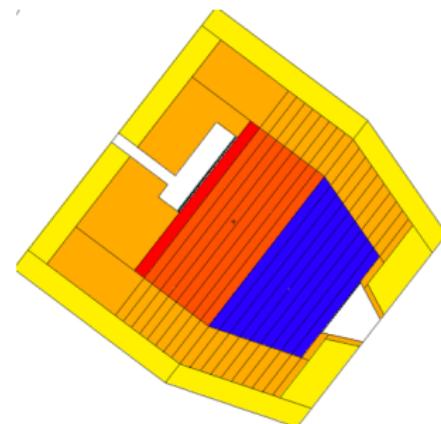
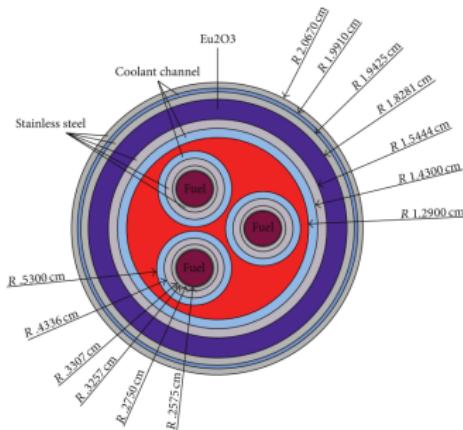
**Develop a capability to design and test custom neutron energy spectra for technical nuclear forensics (TNF)**

1. Design energy tuning assembly (ETA) to generate TNF relevant spectrum at NIF
2. Piece-wise application specific validation of ETA design at LBNL 88-Inch Cyclotron
3. Integral test and creation of synthetic debris at NIF



# POTENTIAL APPLICATION AREAS

- ▶ Radiation shielding
- ▶ Radiation effects/damage
- ▶ Medical physics
- ▶ Radio-isotope production
- ▶ Nuclear data
- ▶ Detector calibration and development
- ▶ Fusion blanket design
- ▶ Reactor design



# OPTIMIZATION PROBLEM CLASSES

Optimization problems can be formulated as [5, 6]:

$$\underset{\vec{x} \in \mathbb{R}^d}{\text{Minimize}} \quad f_i(\vec{x}), \quad (i = 1, 2, \dots, I) \quad (4)$$

$$\text{Subject to:} \quad h_j(\vec{x}) = 0, \quad (j = 1, 2, \dots, J) \quad (5)$$

$$g_k(\vec{x}) \leq 0, \quad (k = 1, 2, \dots, K) \quad (6)$$

where  $\vec{x}$  is a vector of the problem design variables

Optimization problems can be classified by [7, 8]:

- ▶ Single or multi-objective
- ▶ Linear or non-linear
- ▶ Constrained or unconstrained
- ▶ Continuous or combinatorial (discrete)
- ▶ Uni-modal or multi-modal

**ETA design is a single objective, non-linear, constrained, continuous and discrete multi-modal optimization problem**

## ETA OPTIMIZATION

For the ETA optimization problem, (4) and (6) are given by [9]:

$$f_1(\vec{x}_p) = \sum_{g=1}^G \left( \frac{\phi_g^O - \phi_g^D(\vec{x}_p)}{\phi_g^O} \right)^2 * \frac{\phi_g^O}{\phi^O} \quad (7)$$

$$g_1(\vec{x}_p) = \sum_{n=1}^N \rho_n V_n - W \leq 0 \quad (8)$$

$$g_2(\vec{x}_p) = N_f^{min} - n\phi V(\sigma_f^{235} + \sigma_f^{238}) \leq 0 \quad (9)$$

Where  $\phi^O$  is the design objective neutron spectrum and  $\phi^D(\vec{x}_p)$  is the neutron spectra corresponding to a candidate design

$\vec{x}_p$  is a vector of the variables for a candidate design given by (in 2-D):

$$\vec{x}_p = \{Cell_1[M_1, \rho_1, IR_1, OR_1, Z1_1, Z2_1], Cell_2[\dots], \dots, Cell_N[M_N, \rho_N, IR_N, OR_N, Z1_N, Z2_N], R_{foil}, Z_{foil}\} \quad (10)$$

# OPTIMIZATION METHODS: METAHEURISTICS [10]

## Hill Climbing

**Intent:** Follow a sequence of local improvements in order to find a locally optimal solution. A single move is performed at each step. If this leads to a better solution, the algorithm then moves on to explore a variant of this new solution, otherwise it remains at the original point and considers a different move.

## Adaptive Memory Programming

**Intent:** Use of memory of past search experience to guide future search.

## Population-Based Search

**Intent:** Multiple, cooperating search processes that are typically executed in parallel.

## Multi-Start

**Intent:** Restart the search process in a different region once it has converged at a local optimum. After this has been repeated a number of times, the best local optimum seen is returned.

## Variable Neighborhood Search

**Intent:** Search different neighborhoods around the location of a known local optimum.

## Directional Search

**Intent:** Identify productive directions within the search space, and then carry out moves accordingly.

## Search Space Mapping

**Intent:** Construct a map to guide search processes across search space

## Intermediate Search

**Intent:** Explore the region between two or more previously visited search points, each of which is known to have a relatively high objective value.

## Neighborhood Search

**Intent:** Find new solutions by exploring those that are a step change – a move – away from the current one. A move could be anything from flipping a single bit to randomly replacing the entire solution.

## Accepting Negative Moves

**Intent:** Allow moves to worse solutions.

PSO EA/GA CS ACO



# GNOWEE: HYBRID METAHEURISTIC OPT.

## General purpose metaheuristic optimization algorithm

- ▶ Handles continuous and discrete variables
- ▶ Robust, complete set of search heuristics
- ▶ Nearly-global convergence
- ▶ Outperforms most other algorithms we tested on all nearly all problems of interest

**Algorithm 1:** Gnowee Algorithm

**Input :** User defined objective function,  $f$ ; constraints,  $g$  and  $h$ ; and population size,  $n$

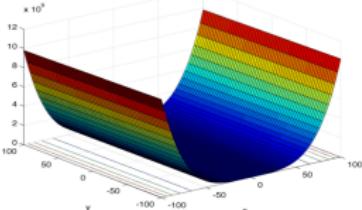
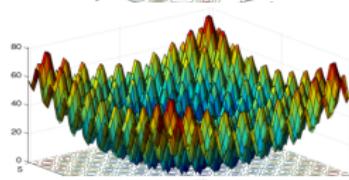
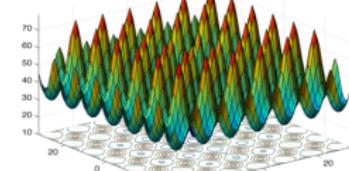
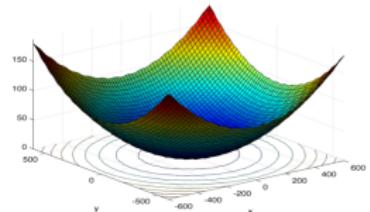
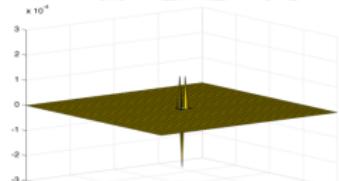
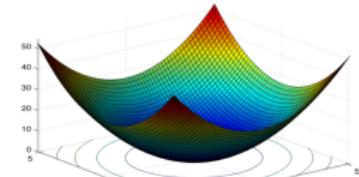
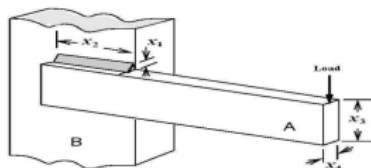
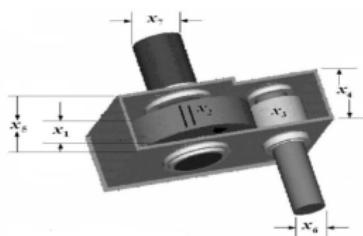
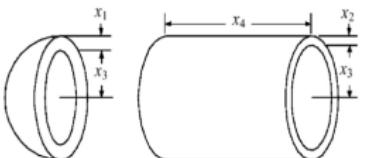
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1 begin
2    $P, \vec{x} \leftarrow \text{Initialization}(n)$  //  $P$  is the parent
      population and  $\vec{x}$  is the design
      variables
3    $P.fit \leftarrow \text{FitCalc}(P, \vec{x})$            //  $fit$  is the
      assessed fitness
4    $C, \vec{x}_d^* \leftarrow \text{Inversion}(P, \vec{x}_d)$ 
5    $P.fit \leftarrow \text{FitCalc}(P, \vec{x})$  while convergence criterion is
      not met do
6      $C, \vec{x}_d^* \leftarrow \text{DiscLévyFlight}(P, \vec{x}_d^*)$  //  $C$  is the
      child population and  $\vec{x}_d^*$  is the
      subset of the design vector
      containing continuous variables
7     for  $i \leftarrow 1$  to  $n$  do
8       if  $f(C_i, \vec{x}_d^*) < P_i.fit$  then
9          $P_i, \vec{x}_d^* \leftarrow C_i, \vec{x}_d^*$ 
10         $P_i.fit \leftarrow f(C_i, \vec{x}_d^*)$  // NOTE: This
          fitness calc and design
          update is performed after
          every procedure but is not
          repeated below for brevity
11     $C, \vec{x}_c^* \leftarrow \text{ContLévyFlight}(P, \vec{x}_c^*)$  //  $\vec{x}$  is the
      subset of the design vector
      containing discrete variables
12     $C, \vec{x}_c^* \leftarrow \text{ContCrossover}(P, \vec{x}_c^*)$ 
13     $C, \vec{x}_d^* \leftarrow \text{Mutation}(P, \vec{x}_d^*)$ 
14     $C, \vec{x}_d^* \leftarrow \text{DiscCrossover}(P, \vec{x}_d^*)$ 
15     $C, \vec{x}_d^* \leftarrow 2\text{-Opt}(P, \vec{x}_d^*)$ 
16     $C, \vec{x}_d^* \leftarrow 3\text{-Opt}(P, \vec{x}_d^*)$ 

```



# GNOWEE: BENCHMARKING [11, 6, 12]





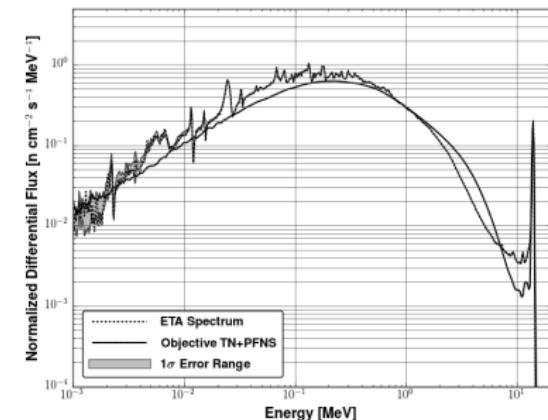
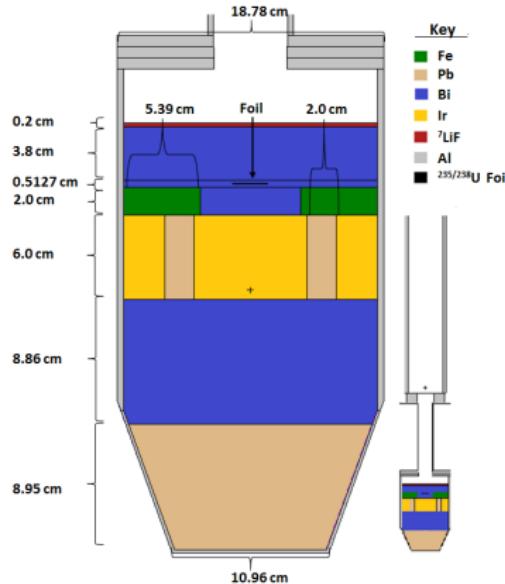
## COEUS: NE OPTIMIZATION SOFTWARE

**ETA design tool to build custom neutron spectra from existing facilities and sources**

- ▶ MCNP-Denovo hybrid radiation transport engine [13, 14, 15]
- ▶ ETA designs generated with Gnowee optimization framework
- ▶ Fully operational on Savio – neutronics design in days
- ▶ Expanding capabilities by adding more objective functions, constraints, and geometric options

# DEVELOPMENT APPROACH

- ▶ 1.2 g HEU foil
- ▶  $\sim 80$  kg
- ▶  $1 \times 10^8 - 1 \times 10^9$  Fissions



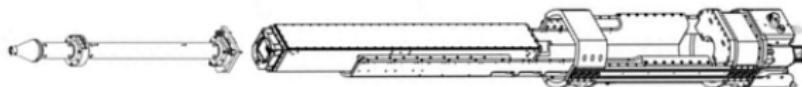
Energy Range	Target Normalized Differential Fluence	% Fluence Achieved
0-3 keV	$6.24 \times 10^{-5}$	$103.4 \pm 2.0\%$
3-100 keV	$3.41 \times 10^{-2}$	$140.7 \pm 0.1\%$
0.1-6 MeV	$8.46 \times 10^{-1}$	$96.0 \pm 0.0\%$
6-10 MeV	$1.65 \times 10^{-2}$	$117.3 \pm 0.2\%$
10-16 MeV	$1.01 \times 10^{-1}$	$119.4 \pm 0.0\%$

ETA vs Objective Spectrum

# NIF EXPERIMENT: TNF VALIDATION

## Experimental Overview:

- ▶  $\sim 1.0 \times 10^{15}$  neutrons in  $4\pi$
- ▶ Minimize  $\rho R$  in direction of the ETA DIM
- ▶ ETA fielded as snout on DIM DLP located 75 mm from TCC

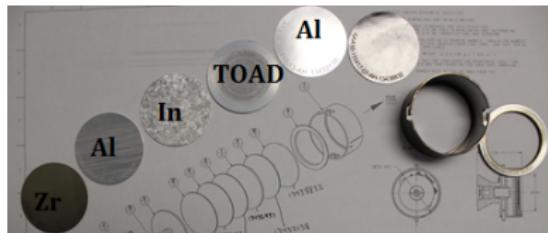


- ▶ No on-line DIM diagnostics required
- ▶ Radio-chemistry and gamma spectroscopy facilities required post-shot
- ▶ NTOF, FNADS, and MRS required to measure the source term

# NIF EXPERIMENT: TNF VALIDATION

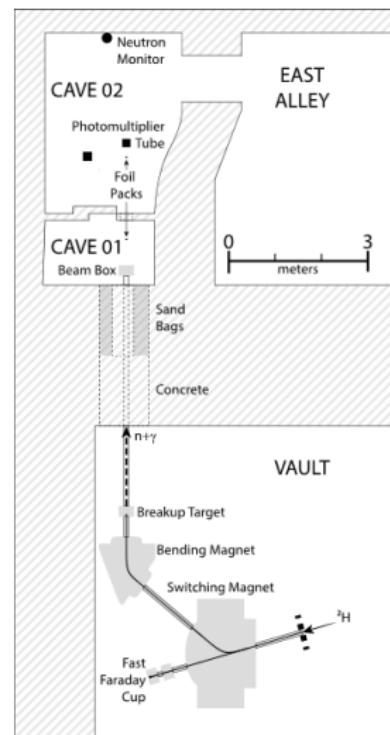
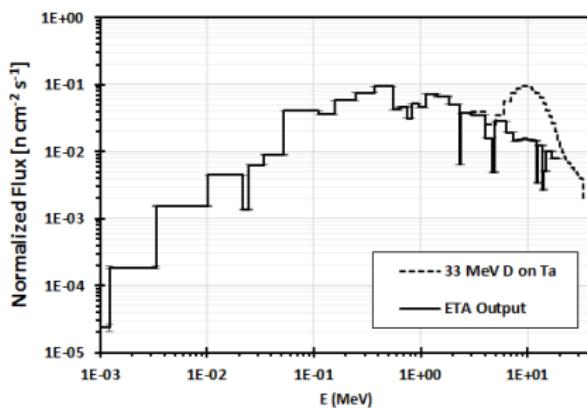
## Expected Experimental Outcomes:

- ▶ Generation of realistic FP distribution
- ▶ Quantification of spectrum through fission splits
- ▶ Unfolding of spectrum using activation analysis



# 88-INCH EXPERIMENTS: TNF VALIDATION

- ▶ 29/33 MeV D-breakup on Ta
- ▶ Field NIF ETA design
- ▶ Field partial ETA stackups
- ▶ EJ-309 detectors, activation, and fission products measurements



## PROJECT 2 SUMMARY

- ▶ Spectral shaping methods can be used to expand the capabilities of existing facilities to cover new mission spaces
- ▶ Coeus provides an efficient capability to design and optimize ETAs for spectral shaping
  - ▶ Not input or output specific
  - ▶ Further development to improve user flexibility underway
- ▶ Experimental validation of TNF application at LBNL 88-Inch Cyclotron executed
- ▶ Planning underway for NIF shot
  - ▶ Scoping study has shown feasibility
  - ▶ Partial funding/support from DNDOD/NTNFC, DTRA, and LANL



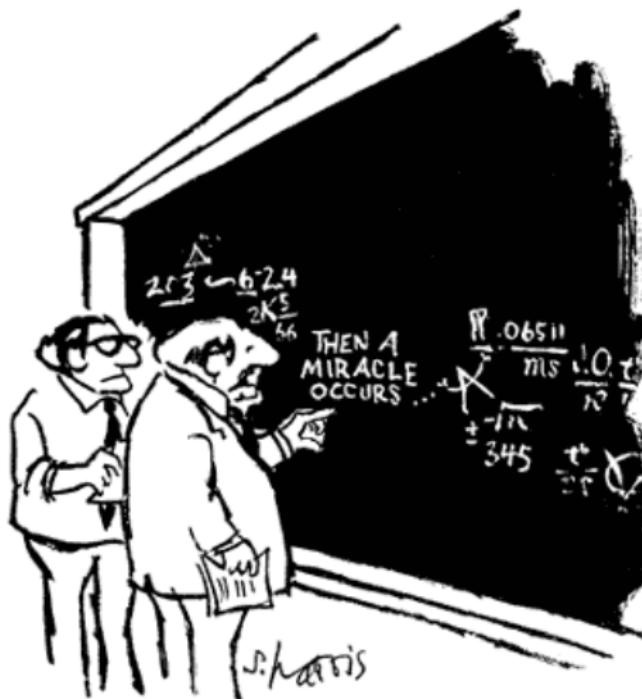
We would accomplish many more things if we did not think of them as impossible.

- Vince Lombardi

# SUMMARY

- ▶ Innovation is needed for many nuclear technologies
- ▶ Predictive simulation can play a key role
- ▶ We're developing better hybrid methods
  - ▶ for problems with strong anisotropies
  - ▶ and to provide evaluative flexibility
- ▶ Energy tuning assemblies can provide strategic investigative tools
  - ▶ for technical nuclear forensics
  - ▶ as well as many other applications

# QUESTIONS?



"I think you should be more explicit here in step two."

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## DISCLAIMERS

The views expressed in this research are those of the author and do not reflect the official policy or position of the United States Air Force, the Department of Defense, or the United States Government.

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