

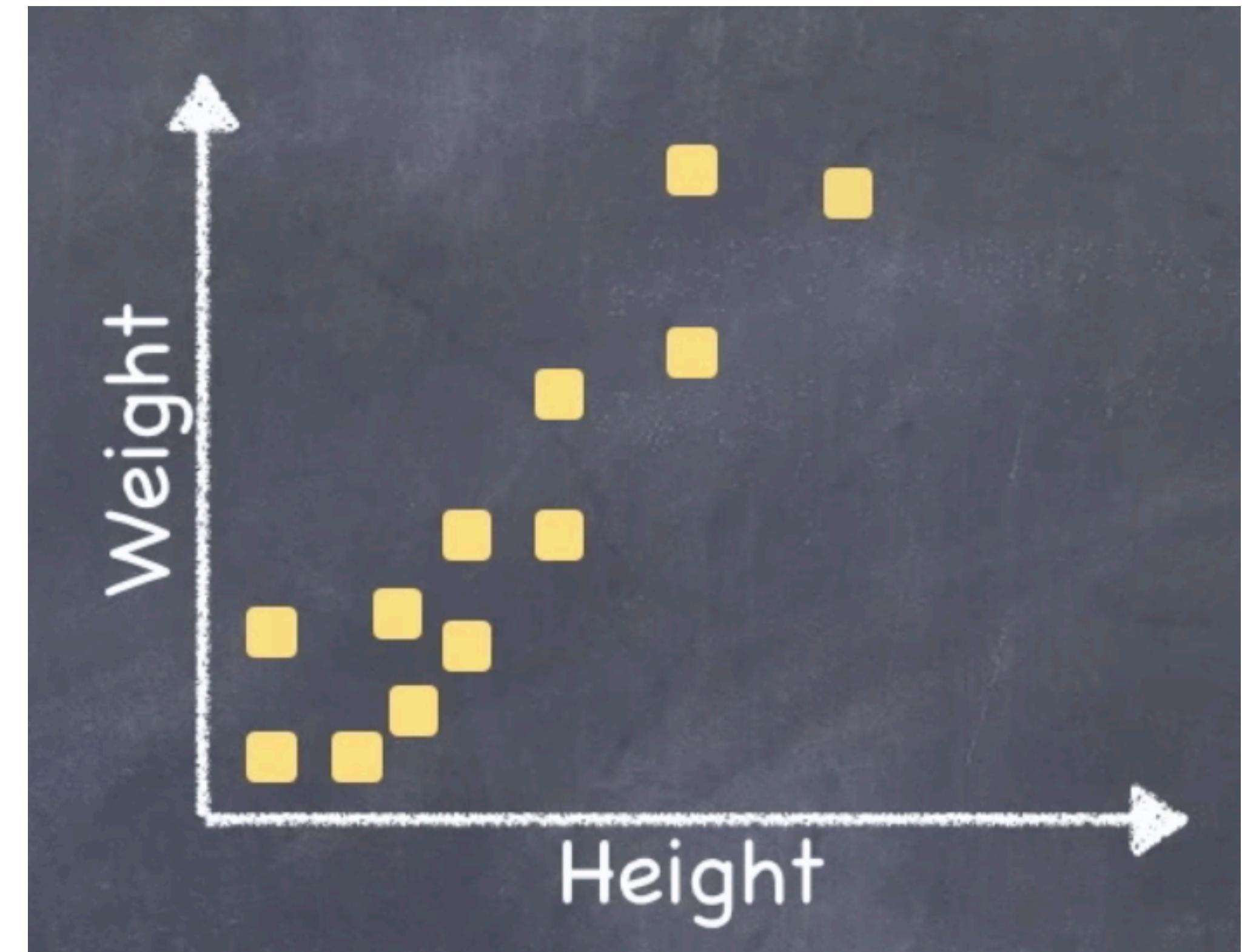
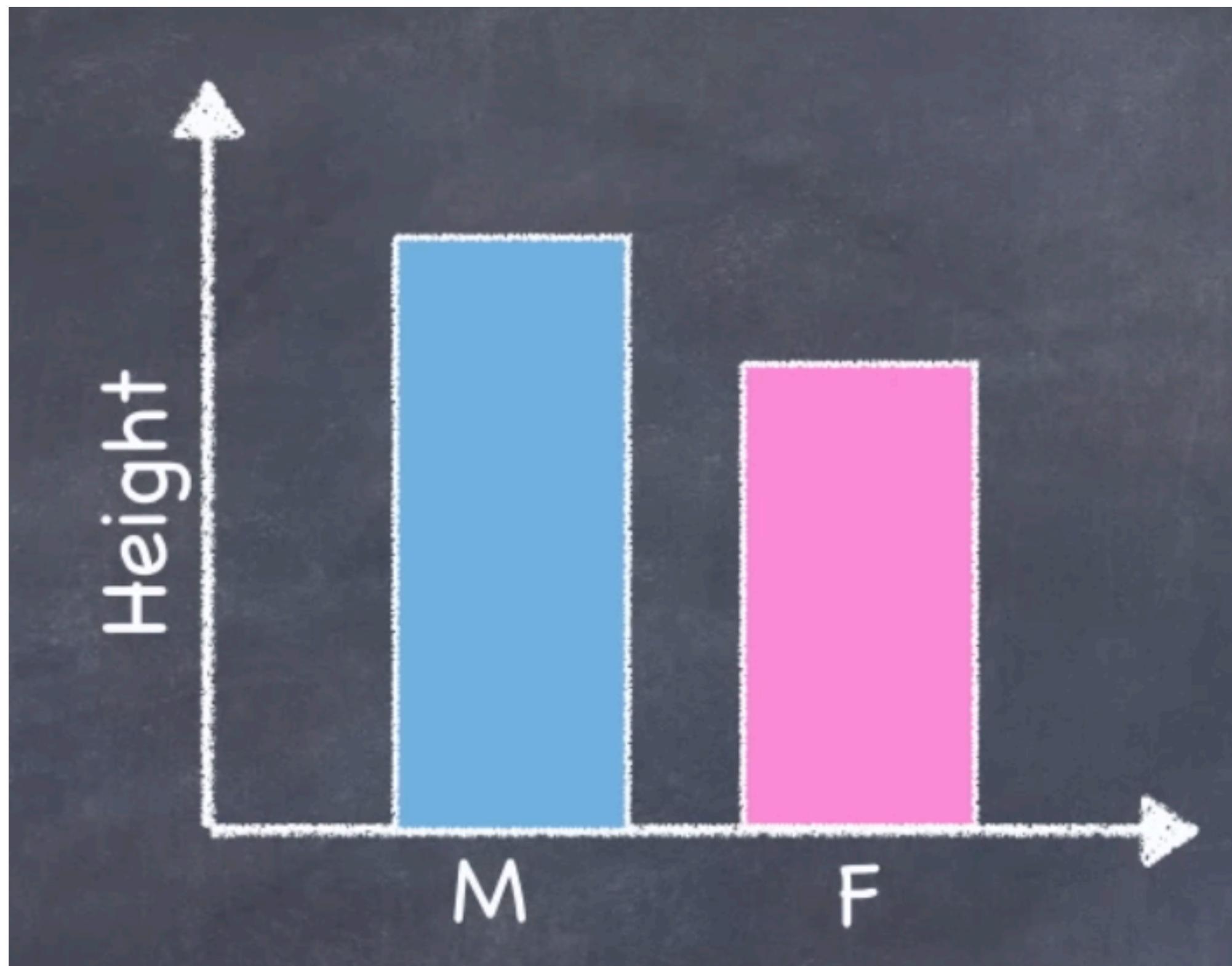
Statistical tests I

Rachel Warnock

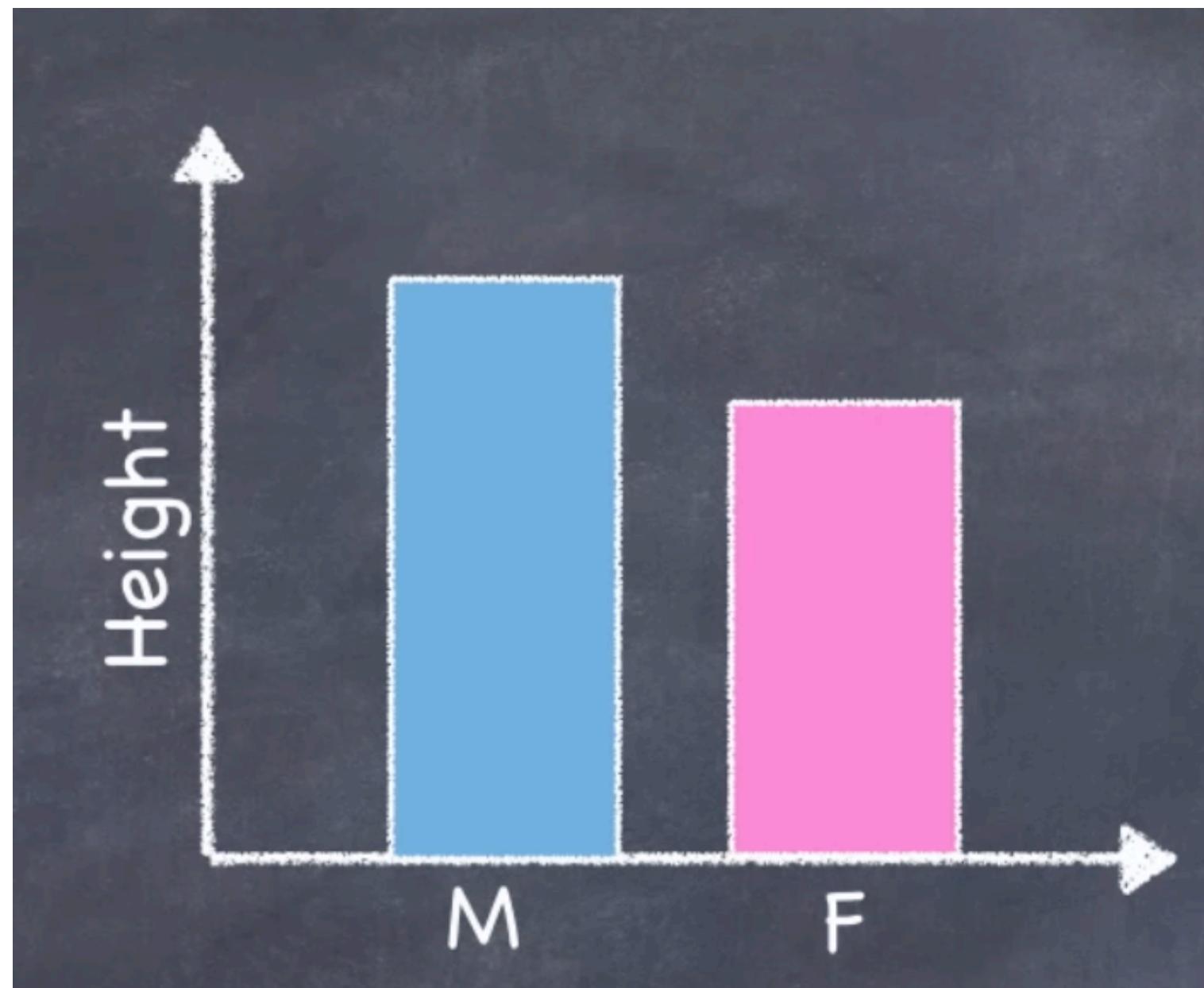
10.04.2024



BEN LASSEN



Today's adventures



compare
the means
of
2 groups → z-test

→ t-test

compare 2+
groups → ANOVA

You do not need to remember all the details of these statistical tests, you only need to get the gist.

The most important thing: what test is most appropriate for my data?

Steps for hypothesis testing

1. Define your research question / hypothesis
2. Define your statistical hypothesis (null & alternative)
3. *Find an appropriate test* & sampling distribution
4. Choose the type I error rate

Steps for hypothesis testing

5. Collect the data
6. Calculate test statistics
7. State the **statistical** conclusion
8. Interpret your results

Comparing 2 means – z-test and *t*-test

We often want to know if the average value of some variable is different (or higher or lower) than some out value or group.

CONTINUOUS

measured data, can have ∞ values within possible range.



I AM 3.1" TALL

I WEIGH 34.16 grams

The z-test — the most useless of all statistical tests

Almost never applied in real life (because it relies on a known standard deviation).

It's useful as a stepping stone to understanding the *t*-test.

A simple z-test

You might ask, are students from geology scoring higher than the average student in statistics?

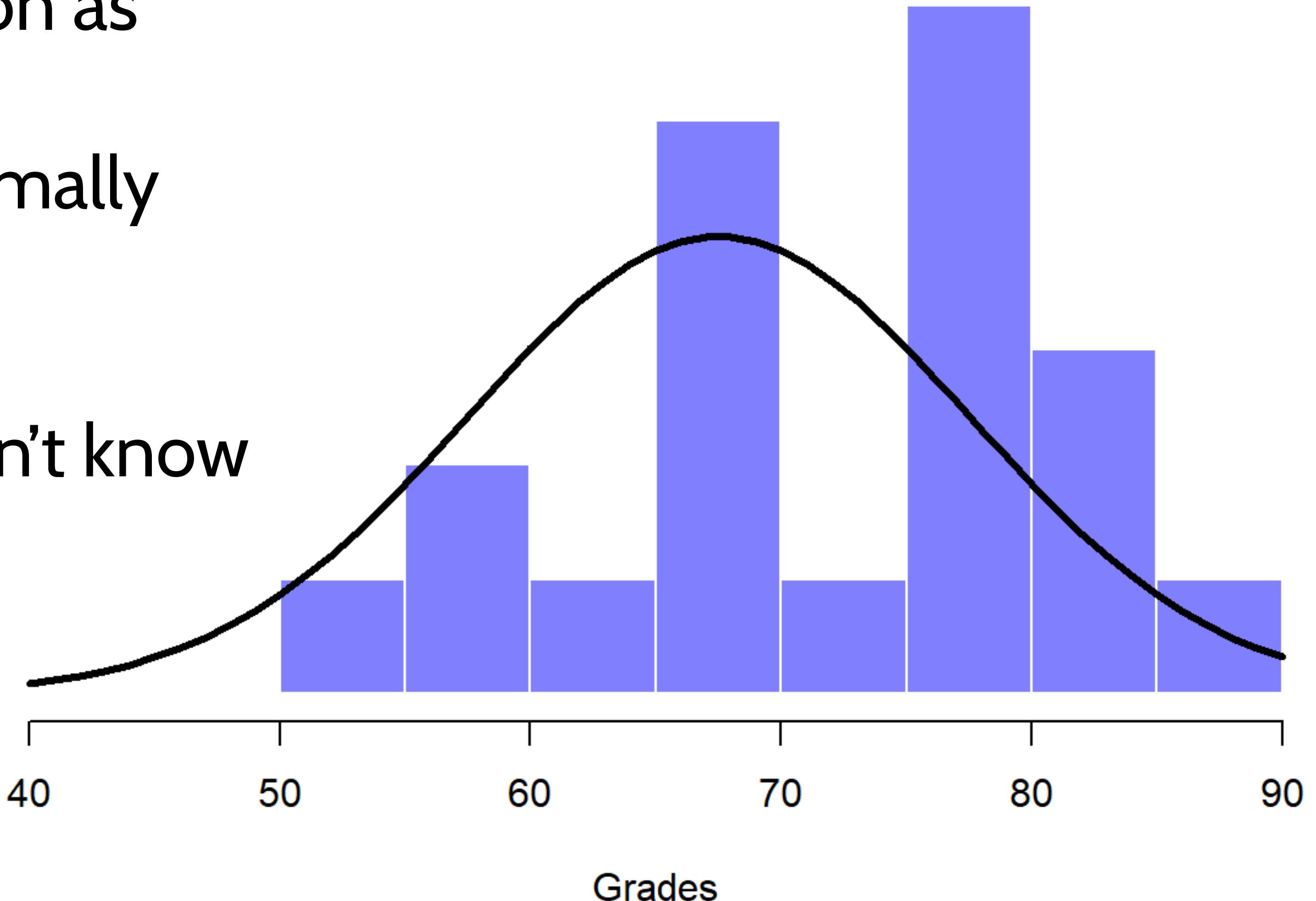
mean average grade = 67.5 and s.d. = 9.5

geology students mean = 73.2, N = 20

Assumptions

- the same standard deviation as the rest of the class
- the student grades are normally distributed

Note: in reality we usually don't know either of these things.

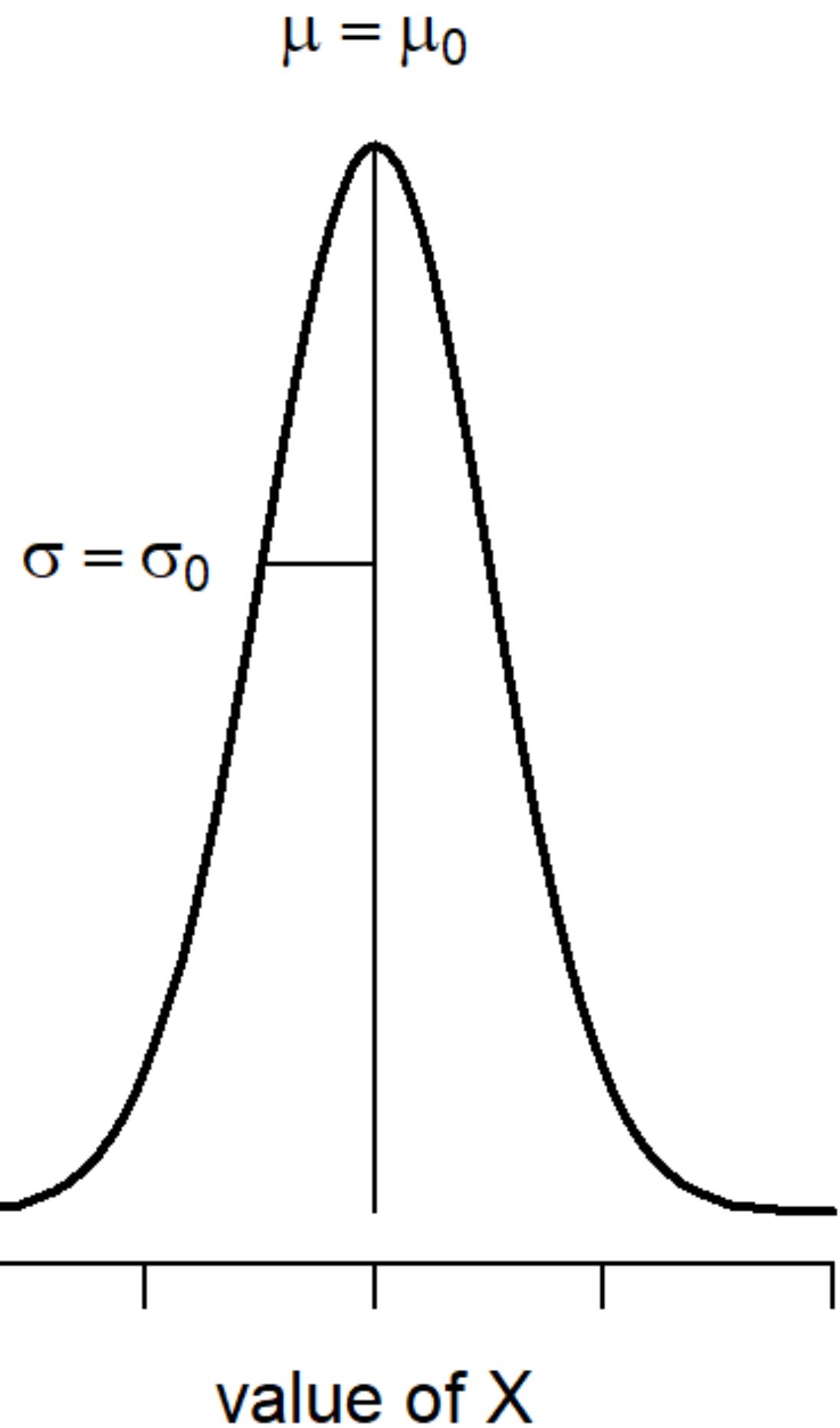


What are our hypotheses? 🤔

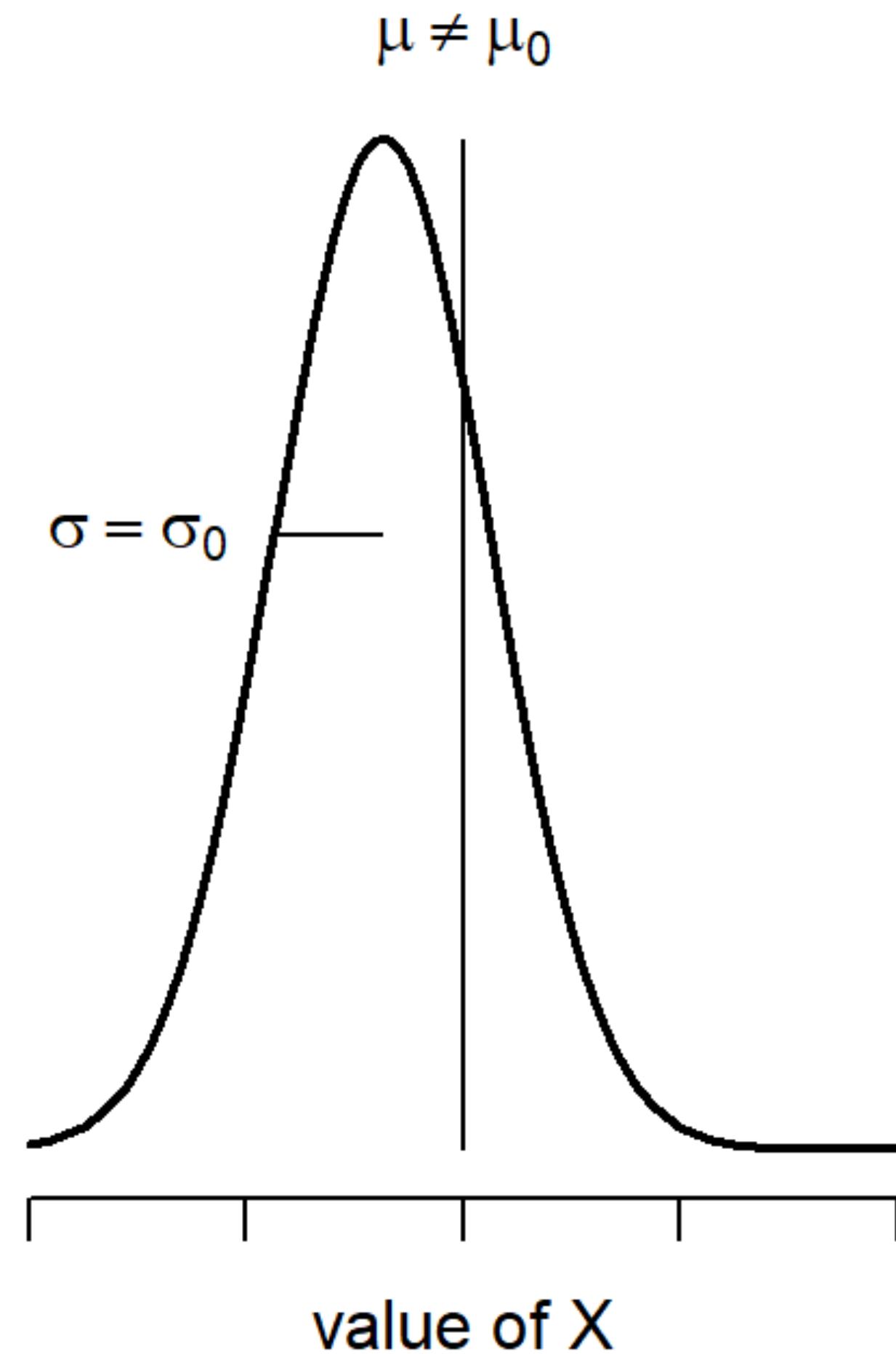
- research
- statistical (null and alternative)

null hypothesis

The hypotheses for
a one-sided or two-
sided z-test



alternative hypothesis



If the null hypothesis is true then the sampling distribution of the mean can be written as:

$$\bar{X} \sim \text{Normal}(\mu_0, \text{SE}(\bar{X}))$$

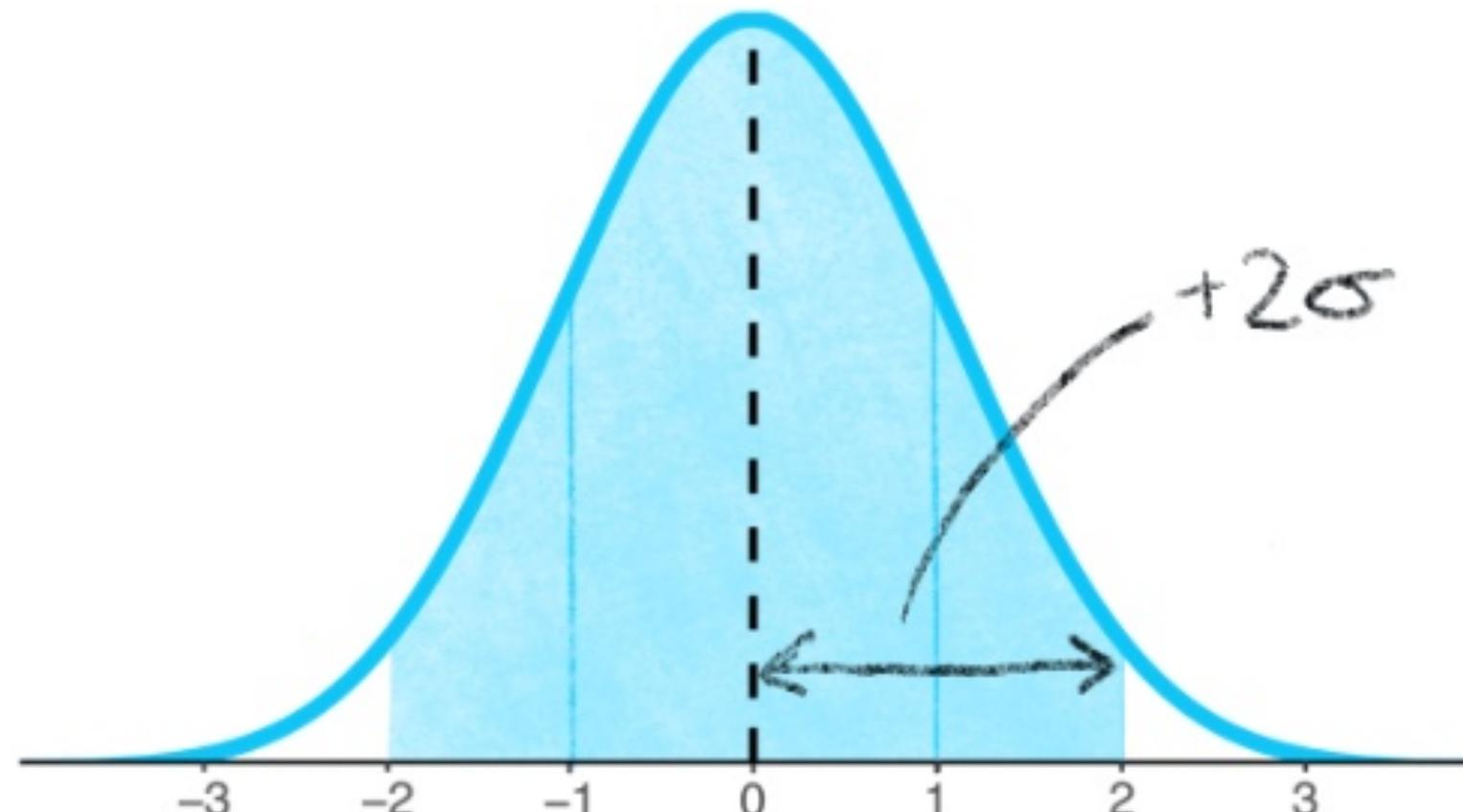
i.e., comes from a distribution with the same mean & SD

If $\mu - \mu_0$ equals or is very close to zero, this indicates support for the null hypothesis.

If $\mu - \mu_0$ is a long way away from zero, then it's looking less likely that we'd accept the null hypothesis.

But how far from zero should it be for us to reject H_0 ?

Standard scores, also known as: z-scores

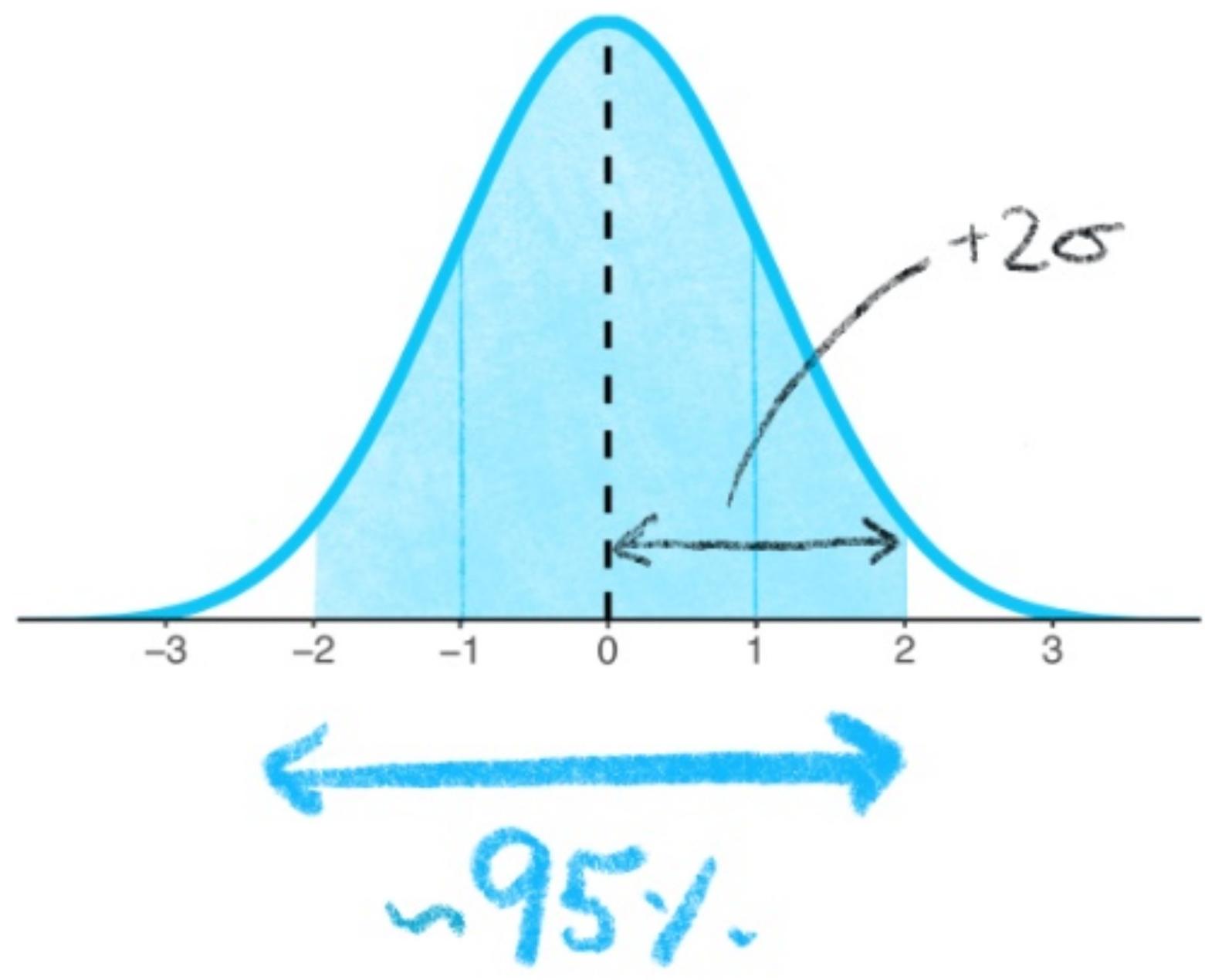


$\sim 95\%$

= number of standard deviations the sample mean is away from the test distribution

$$\text{standard score} = \frac{\text{raw score} - \text{mean}}{\text{standard deviation}}$$

Calculating the z-score



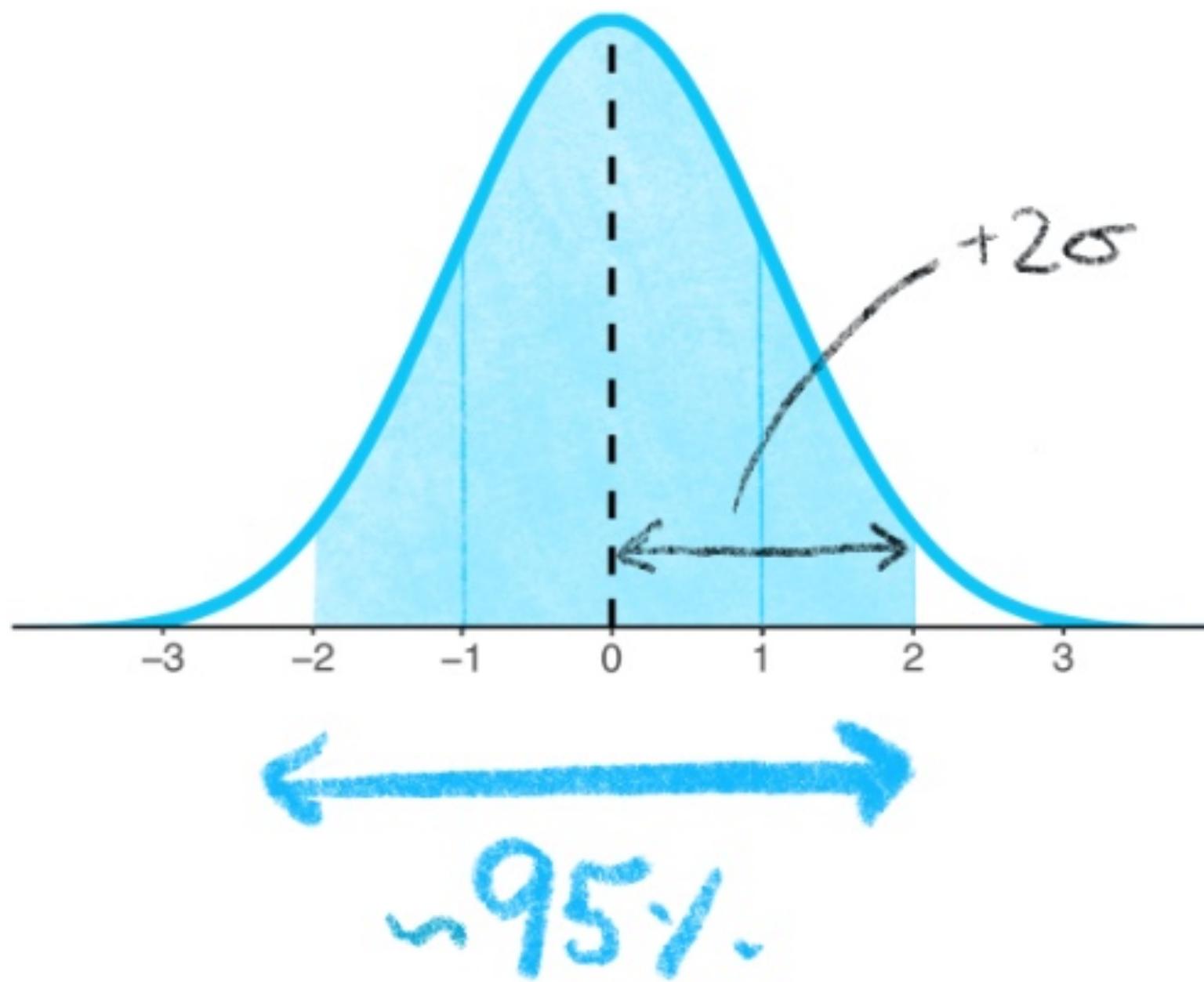
You don't need to know these!
Only here in case of interest!

$$z_{\bar{X}} = \frac{\bar{X} - \mu_0}{\text{SE}(\bar{X})}$$

OR

$$z_{\bar{X}} = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{N}}$$

Calculating the z-score



The z-statistic is equal to the number of standard deviation that separates the observed sample mean μ from the population mean μ_0 predicted by the null hypothesis.

The 5% critical regions for the z-test are always the same.

desired α level**two-sided test****one-sided test**

.1

1.644854

1.281552

.05

1.959964

1.644854

.01

2.575829

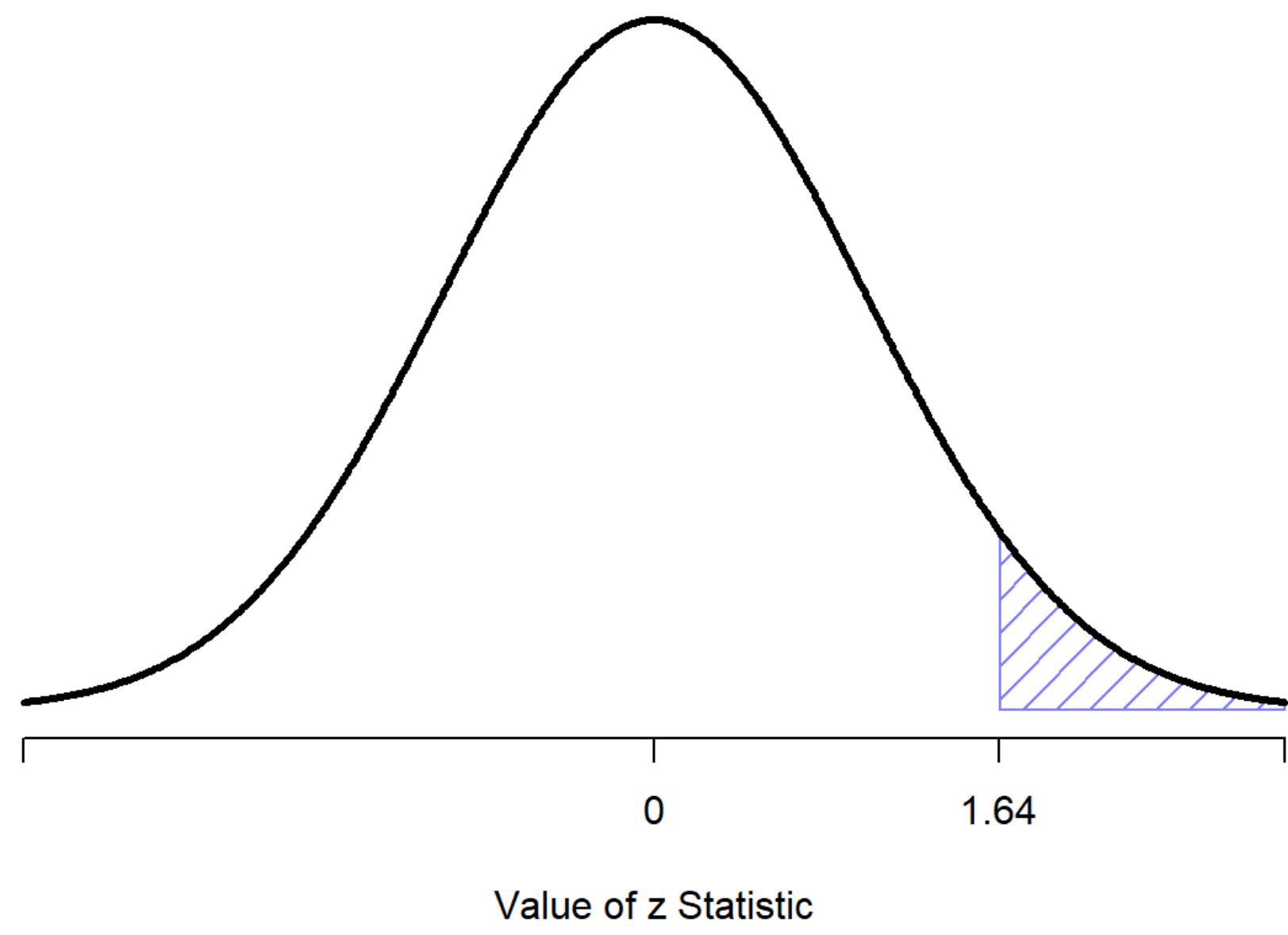
2.326348

.001

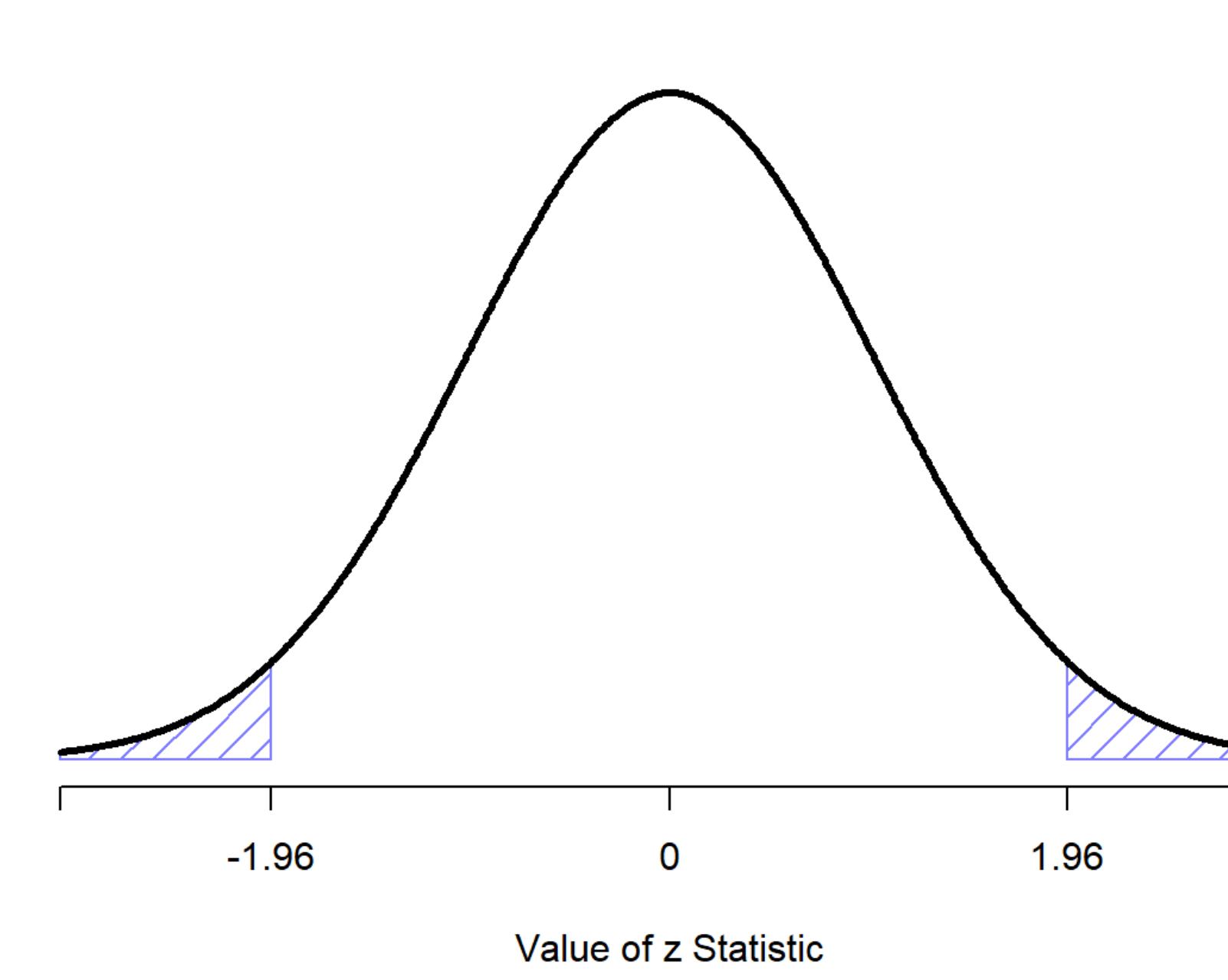
3.290527

3.090232

One Sided Test



Two Sided Test



desired α level	two-sided test	one-sided test
.1	1.644854	1.281552
.05	1.959964	1.644854
.01	2.575829	2.326348
.001	3.290527	3.090232

With a mean grade of 73.2 in the sample of geology students, and assuming a true population standard deviation of 9.5, we get $z = 2.26$.

What is p ?

Assumptions of the z-test

Randomness

Normality

Independence

Known standard deviation

From z-test to *t*-test

Suppose we don't know the true standard deviation?

We only know the **estimated** standard deviation.

See Teacup Giraffes for more on the normal distribution



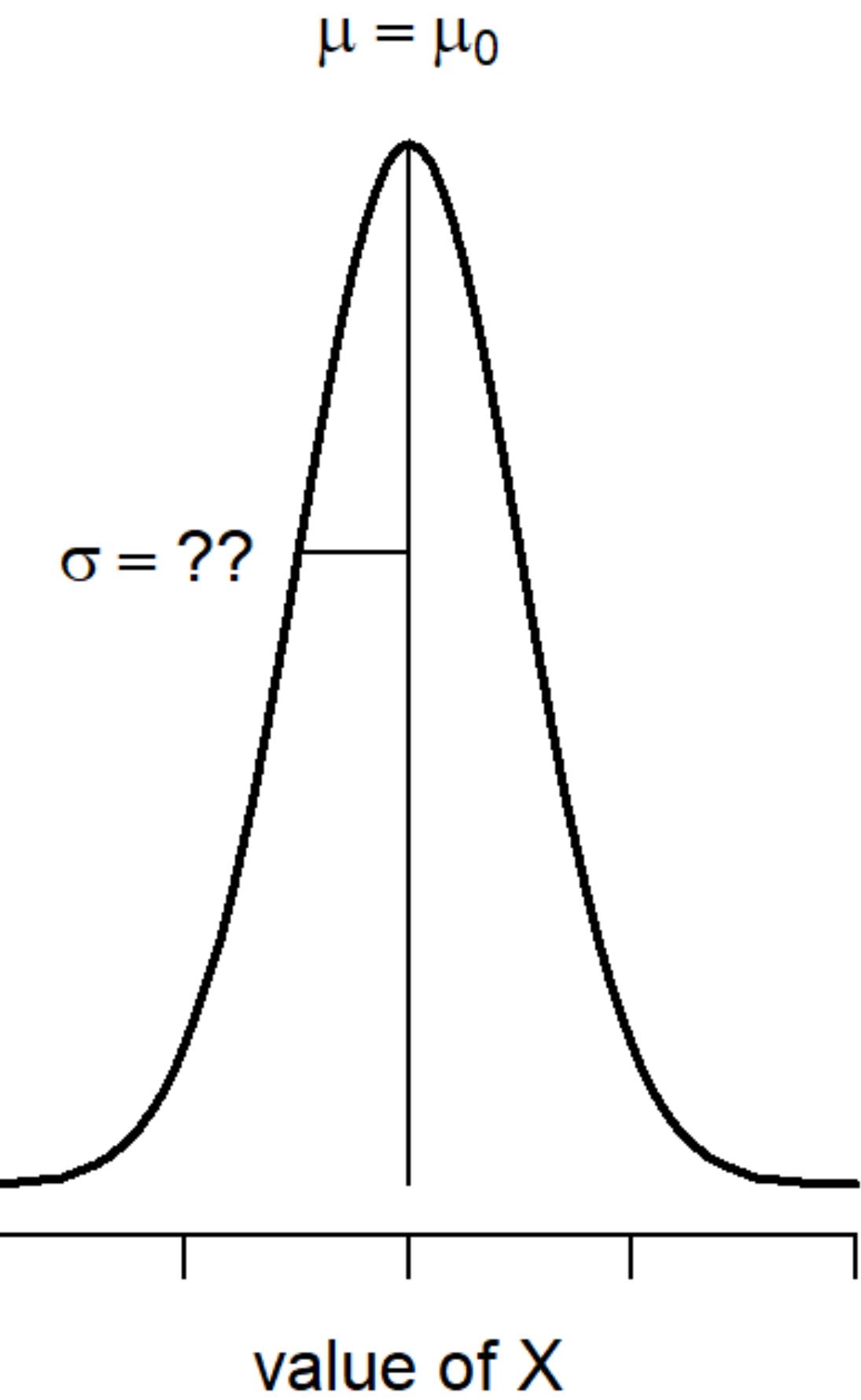
From z-test to *t*-test

We *could* use the estimated s.d. but this wouldn't be strictly appropriate.

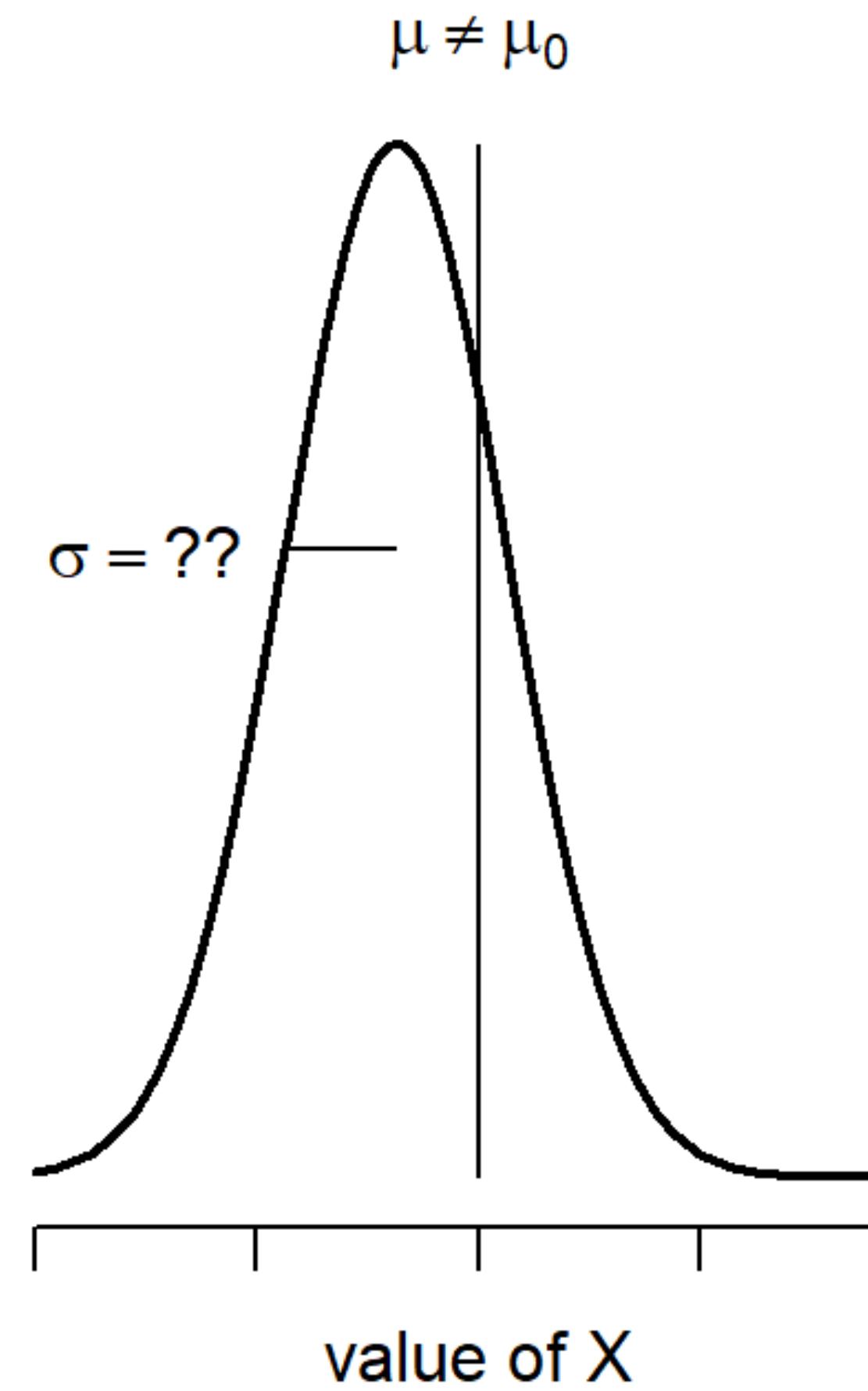
What if the estimate is incorrect?

In the previous example, if the s.d. changes from 9 to 11, the results become non-significant.

null hypothesis



alternative hypothesis



The one-sample *t*-test



To accommodate the fact we don't know the s.d., we need to subtly change the sampling distribution.

The *t*-test statistic remains very similar:

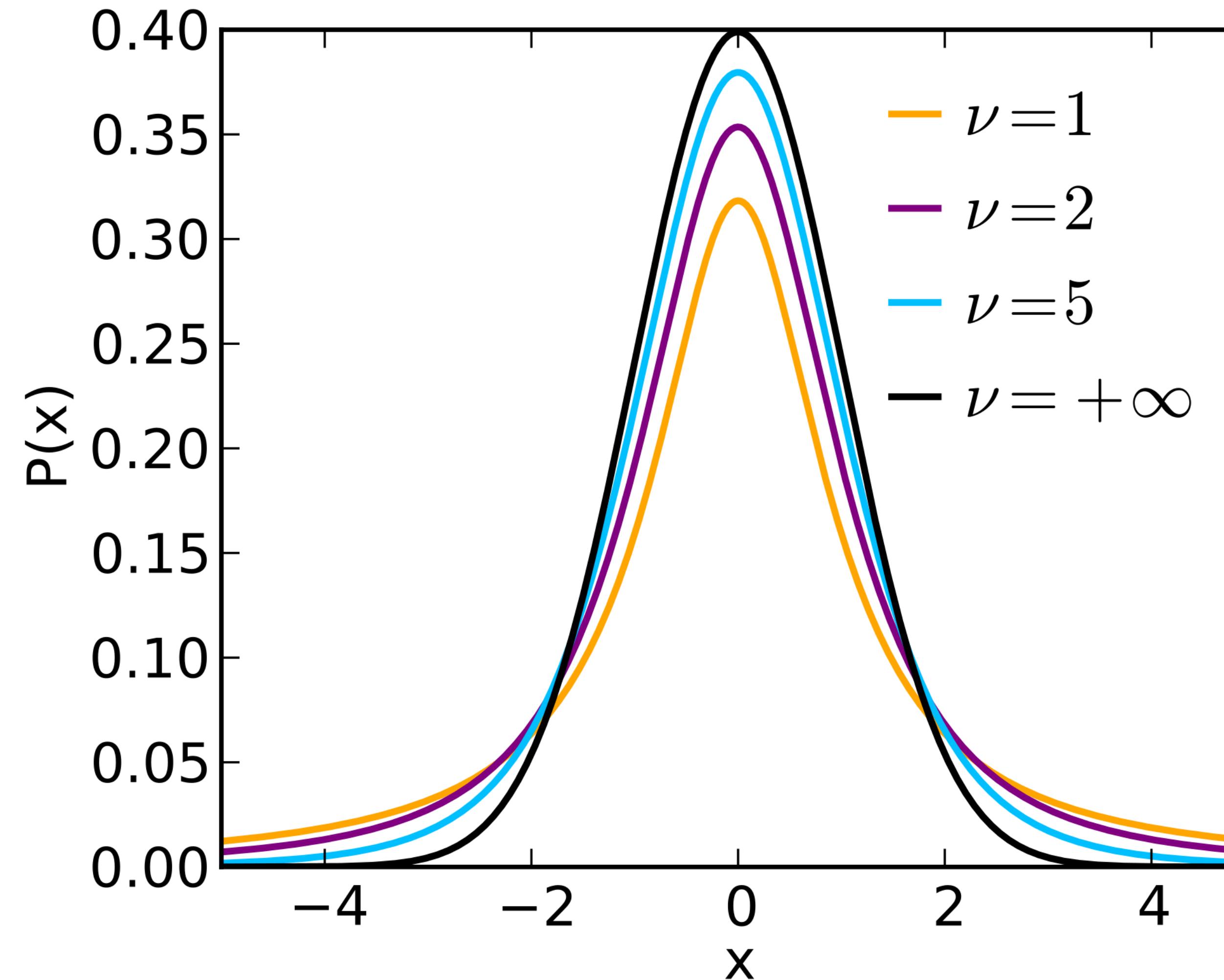
The only diff. is that we use the est. s.d.

$$t = \frac{\bar{X} - \mu}{\hat{\sigma}/\sqrt{N}}$$

If the estimate has been constructed from N observations, then the sampling distribution becomes a *t*-distribution with $N - 1$ degrees of freedom.

From here we can obtain a *p*-value for the *t*-distribution.

The one-sample t -test



Assumptions of the one sample t -test

Randomness

Normality

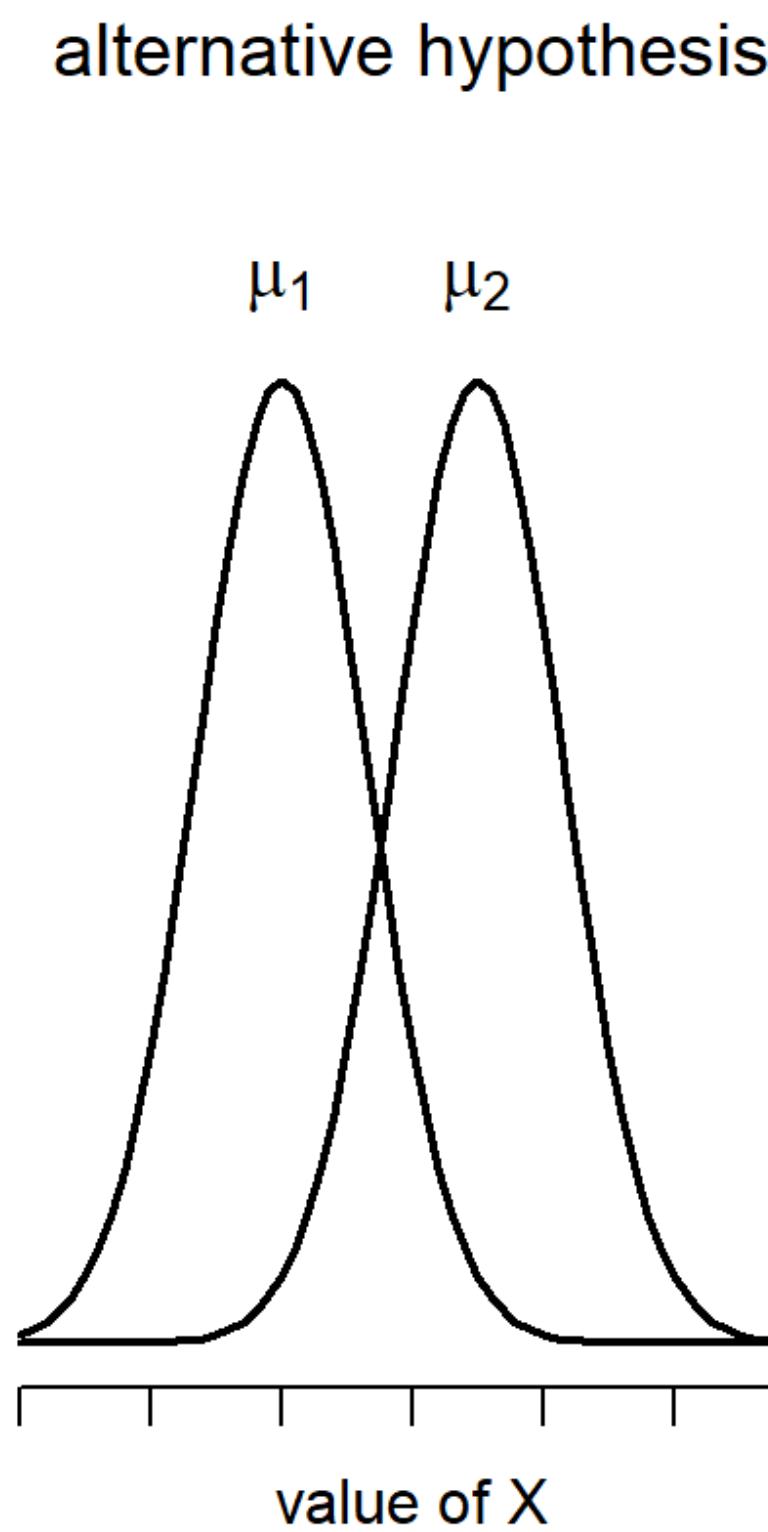
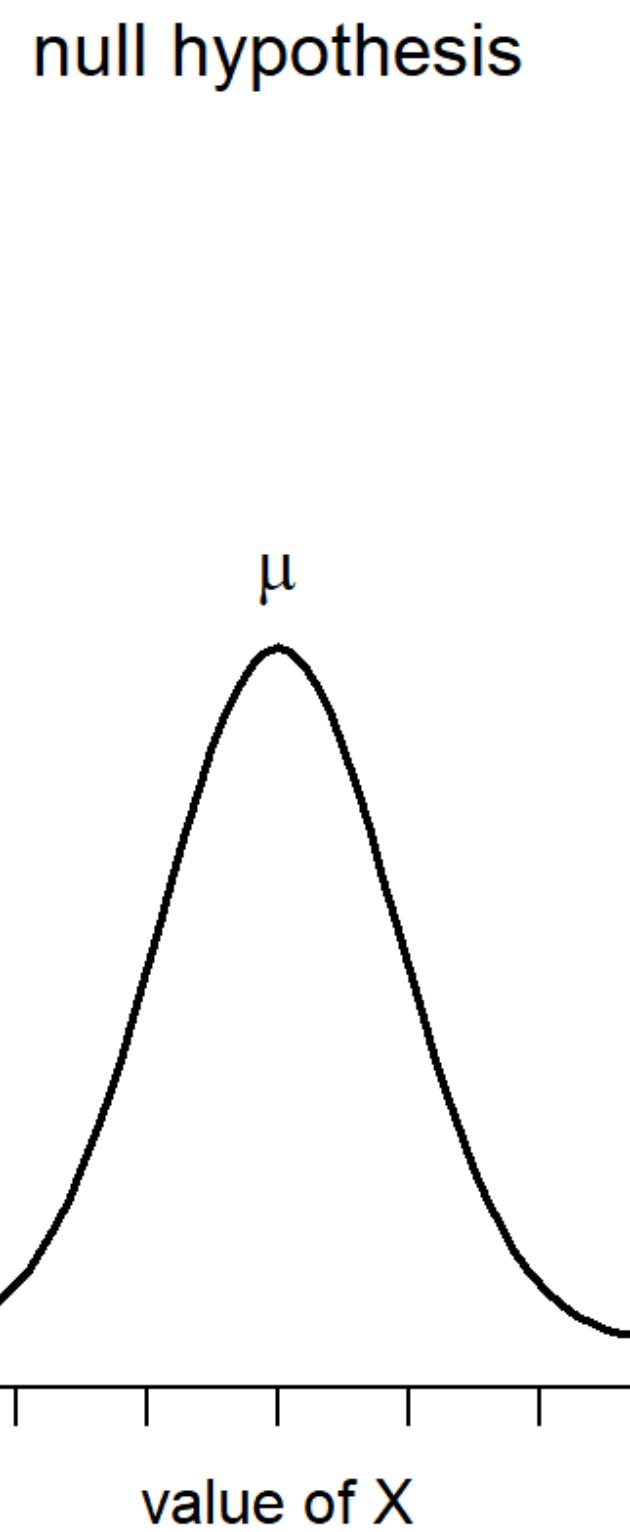
Independence

There are many varieties of t -test that get round some of these assumptions.

The **one-sample *t*-test** compares the mean of a single sample to a predetermined value to test whether the sample mean is significantly greater or less than that value.

The **independent samples *t*-test** compares the mean of one distinct group to the mean of another group.

The independent-samples *t*-test



If the null is true we expect the difference between the means = 0.
But how close?

Test statistics is:
$$t = (\mu - \mu_0) / SD$$

The independent-samples *t*-test

For the variance, we calculate a “pooled” estimate by taking a weighted average of the variance estimates for each sample.

The weight assigned to each sample is equal to the number of observations in that sample, minus 1.

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}$$



$$w_1 = N_1 - 1$$

$$w_2 = N_2 - 1$$

$$\hat{\sigma}_p = \sqrt{\frac{w_1 \hat{\sigma}_1^2 + w_2 \hat{\sigma}_2^2}{w_1 + w_2}}$$

The independent-samples t -test

Compared to the one sample t -test:

$$t = \frac{\bar{X} - \mu}{\hat{\sigma}/\sqrt{N}} \quad \xrightarrow{\text{red arrow}} \quad t = \frac{\bar{X}_1 - \bar{X}_2}{\text{SE}(\bar{X}_1 - \bar{X}_2)}$$

Again, from here we can obtain a p -value for the t -distribution.

Assumptions of the independent samples *t*-test

Randomness

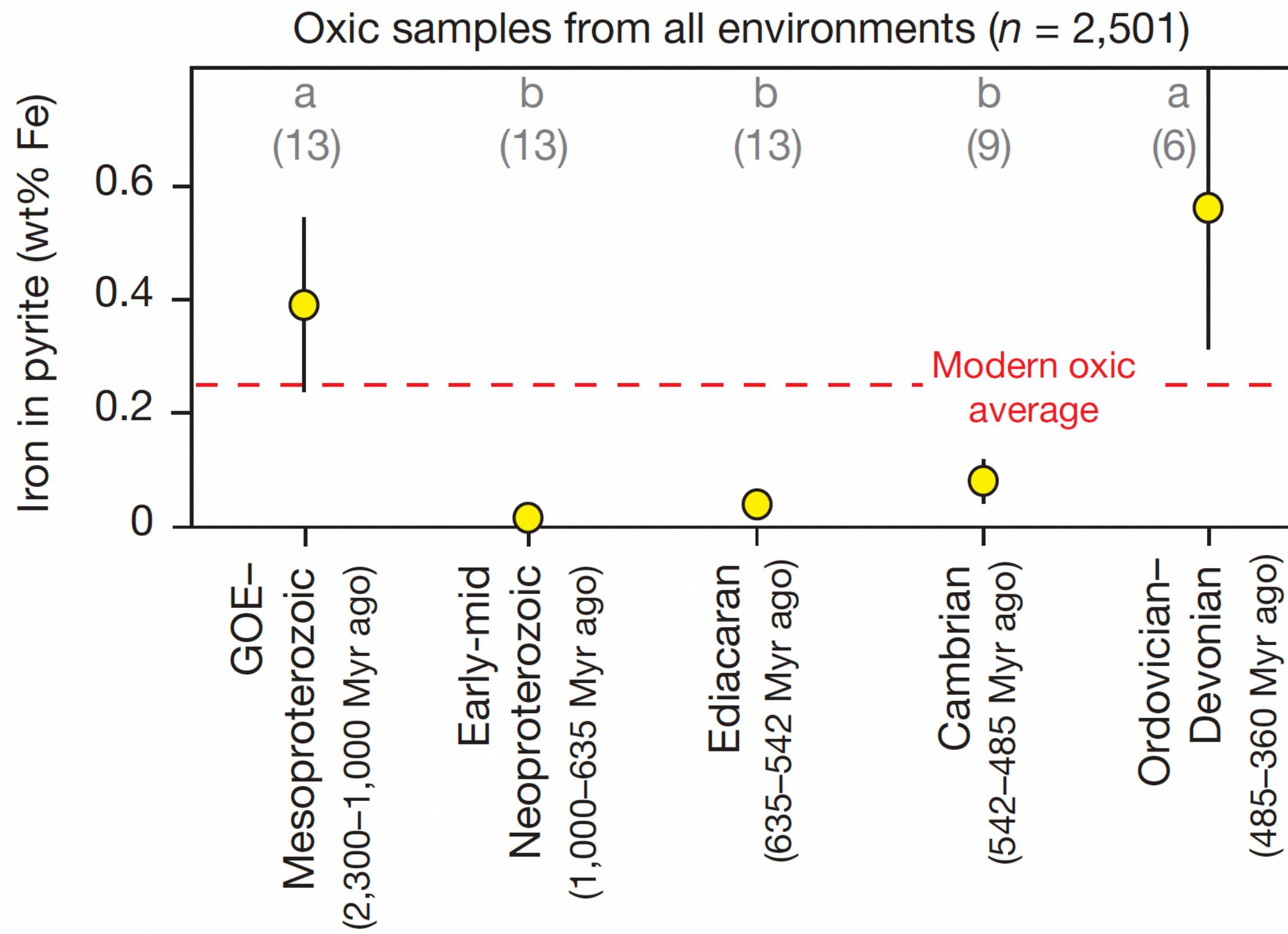
Normality

Independence (between groups)

Homogeneity of variance (the s. d. is the same between both groups)

One-way ANOVA (Analysis of variances)

What if you want to compare more than 2 groups?

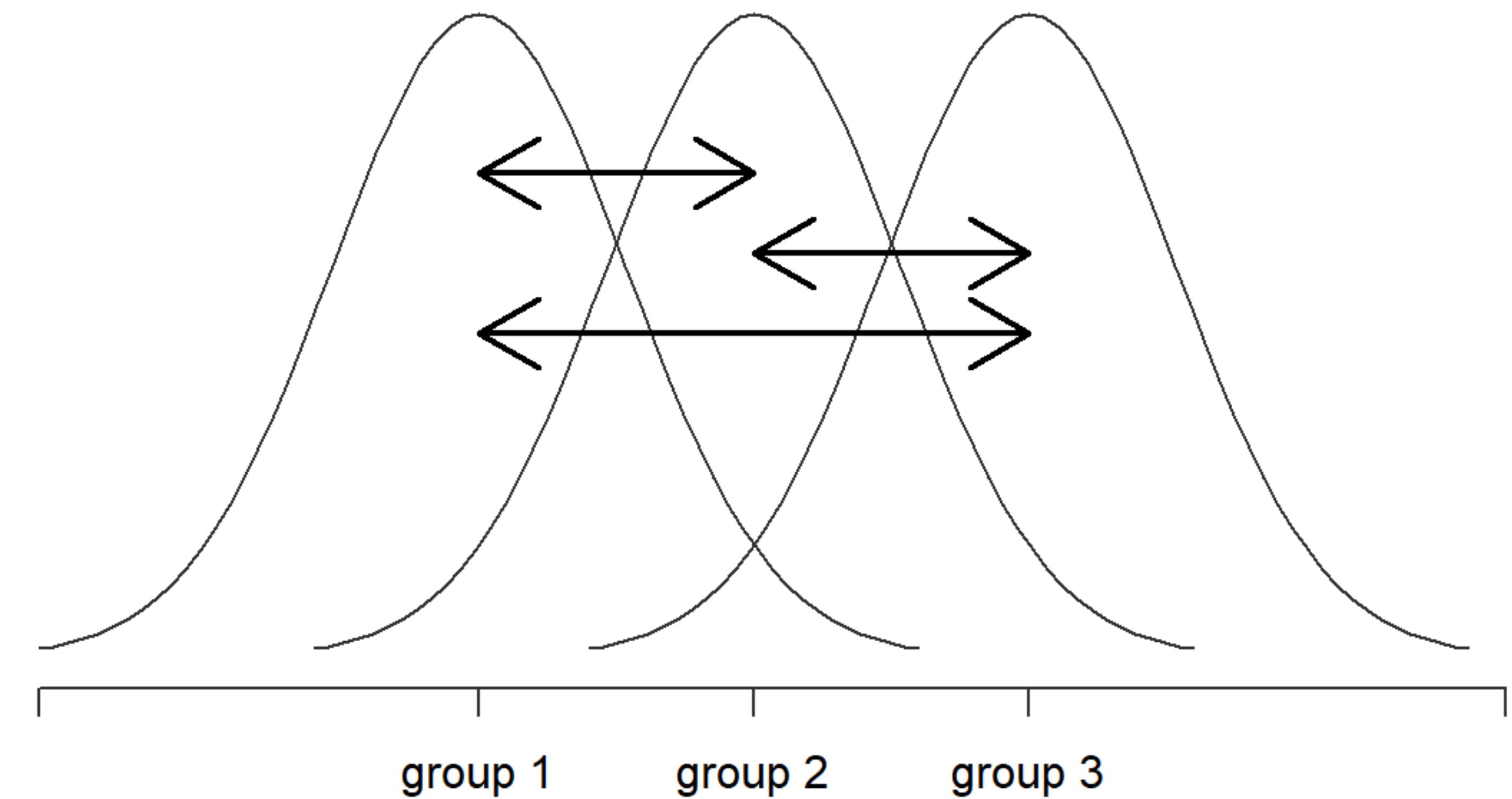


H_0 : it is true that $\mu_P = \mu_A = \mu_J$

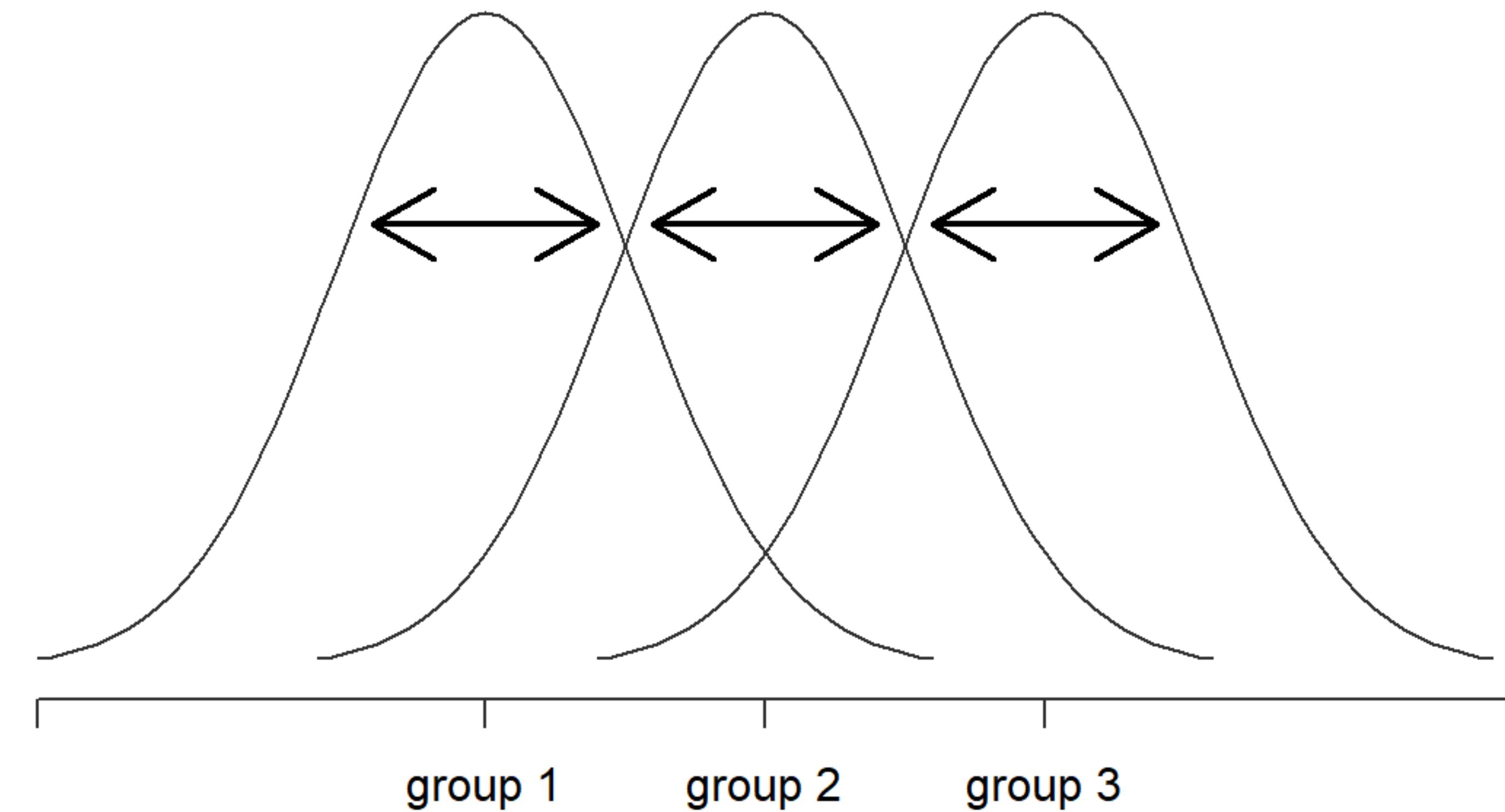
H_1 : it is *not* true that $\mu_P = \mu_A = \mu_J$

Between group variation (SS_b)

i.e., the differences
between group
means



Within group variation (SS_w)



Total group variation (SS_{tot})

SS_{tot} is the sum of “the variation due to the differences in the sample means for the different groups” (SS_b) plus “all the rest of the variation” (SS_w).

$$SS_{tot} = SS_b + SS_w$$

If the null hypothesis is true, we expect all the sample means to be similar and SS_b to be small, with $SS_{tot} = SS_w$.

First we calculate the variance:

$$\text{Var}(Y) = \frac{1}{N} \sum_{k=1}^G \sum_{i=1}^{N_k} (Y_{ik} - \bar{Y})^2 \quad \longrightarrow \quad \text{Var}(Y) = \frac{1}{N} \sum_{p=1}^N (Y_p - \bar{Y})^2$$

Then we calculate the total group (SS_{tot}), within group (SS_w) and between group (SS_b) sum of squares:

$$\begin{aligned} SS_b &= \sum_{k=1}^G \sum_{i=1}^{N_k} (\bar{Y}_k - \bar{Y})^2 \\ &= \sum_{k=1}^G N_k (\bar{Y}_k - \bar{Y})^2 \end{aligned}$$

From sums of squares to the *F*-test via the *F* ratio

We calculate is the degrees of freedom associated with the SS_b and SS_w values.

Next is convert our summed squares value into a “mean squares” value, by dividing by the degrees of freedom.

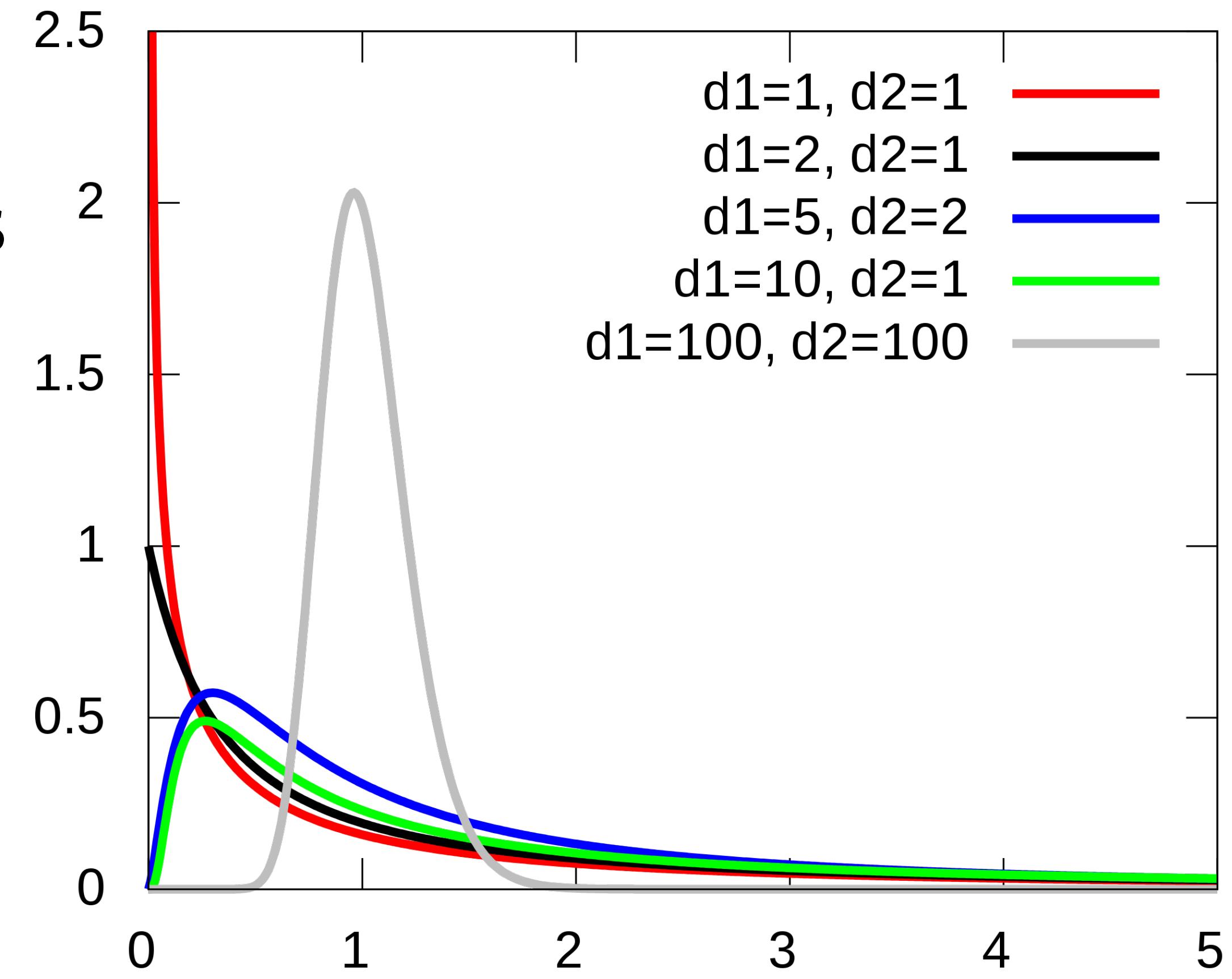
$$\begin{aligned} df_b &= G - 1 \\ df_w &= N - G \end{aligned}$$

$$\begin{aligned} MS_b &= \frac{SS_b}{df_b} \\ MS_w &= \frac{SS_w}{df_w} \end{aligned} \quad \longrightarrow \quad F = \frac{MS_b}{MS_w}$$

The gist Behind the F statistic

The intuition behind the F statistic is straightforward: bigger values of F means that the between-groups variation is large, relative to the within-groups variation.

The larger the value of F , the more evidence we have against the null hypothesis.



Tomorrow's adventures

equivalent to a
one-sample t-
test, non-
parametric

equivalent to
ANOVA, non-
parametric

post hoc
ANOVA test

Tukey-Kramer

Steel Dwass

Wilcoxon test

Kruskal-Wallis



Questions?