Assignment 5 – part 1

We know that all signals represent the same sine wave sampled at different sampling rates, and that at least one signal has a sampling rate higher than the Nyquist rate. It is therefore safe to assume that the signal with the highest sampling rate – in this case, x1 – has a sampling rate greater than the Nyquist rate. Because of this, we can use the Fast Fourier Transform of x1 to figure out what the frequencies of the sine wave and the Nyquist rate are.

Strangely, frequency components of signal x1 through x5 which were insignificant (i.e. did not show a 'spike' on the FFT figure') had frequencies greater than zero. This was the case when there was and wasn't aliasing. The exact reason for this is unknown, but we think it might not be worth considering (as these frequency components do not really alter the figures).

Signal x1

Signal x1 has a sampling frequency of 1000 Hz. The Fast Fourier transform of x1 is shown in figure 1. The signal is periodic with the only significant frequency component at 133.5 Hz. For the reasons noted above, it is therefore safe to assume the sine wave signals x1 through x5 were sampled from had a single frequency component at 133.5 Hz. The Nyquist rate is therefore of 267 Hz, and any signal with a sampling rate lower than this will show aliasing.
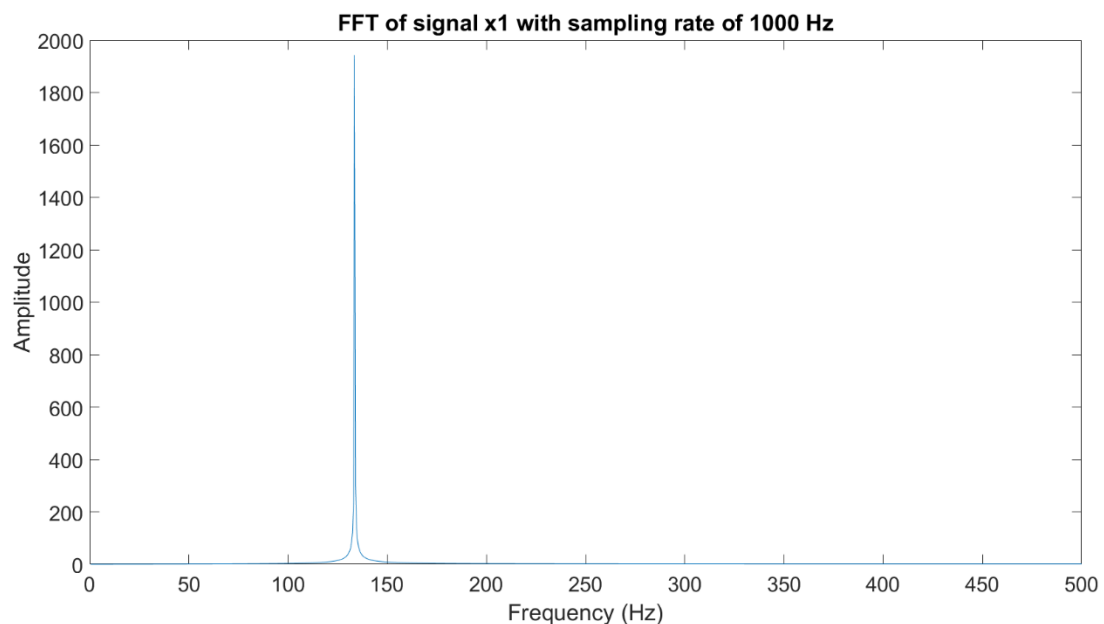

Figure 1: Fast Fourier Transform of signal x1.

Signal x2

The fast Fourier Transform of signal x2 can be seen in figure 2. X2 has a sampling frequency of 266 Hz, just 1 Hz below the Nyquist rate. Because of this, the recorded signal is aliased. However, since the sampling frequency of x2 is very close to the Nyquist rate, the effects of aliasing are considerably small. The only significant frequency component is at 133 Hz (instead of at 133.5 Hz, the frequency of the sine wave it was sampled from).
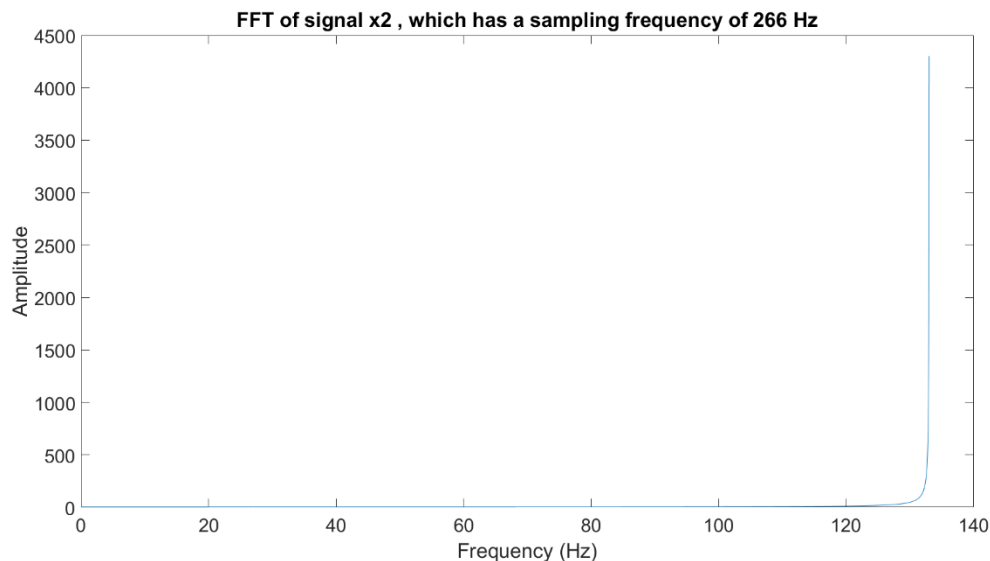

Figure 2: Fast Fourier Transform of signal x2.

Signal x3

Signal x3 and its Fast Fourier Transform are shown in figure 3. X3 has a sampling frequency of 133 Hz, which is approximately half of the Nyquist rate (267 Hz). Because of this, the signal shows aliasing; the frequency component at 133.5 Hz has been 'folded back' at a frequency close to 0 (see bottom panel of figure 3). As can be seen on the top panel of figure 3, the signal seems to behave abnormally; the exact reason why it behaves like this is unknown, but we think it might possibly be due to having a sampling rate approximately half of the Nyquist rate.
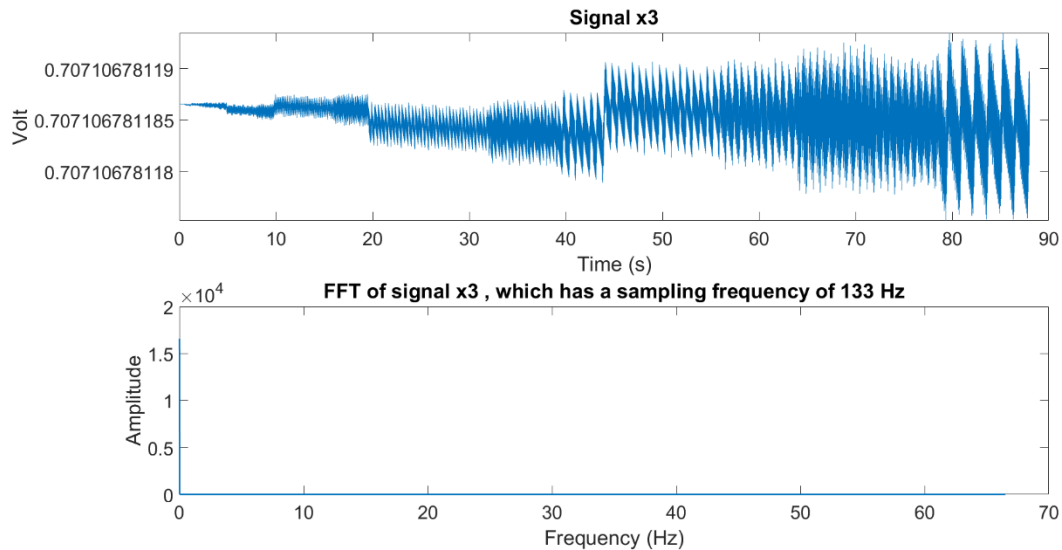
Signal x3

Figure 3: Signal x3 and its Fast Fourier Transform.

Signal x4

Signal x4 and its Fast Fourier Transform can be seen in figure 4. X4 has a sampling rate of 100 Hz. As we can see, the abnormalities in the signal seen in signal x3 do not seem to be solely due to having a sampling rate much lower than the Nyquist rate, since signal x4 looks normal. However, signal x4 also shows aliasing: it is composed of a single significant frequency component at 33 Hz. Due to being sampled at a rate lower than the Nyquist rate, the signal is composed of a frequency much lower than the 133.5 Hz sine wave it was sampled from (see slide 9 of module 5).
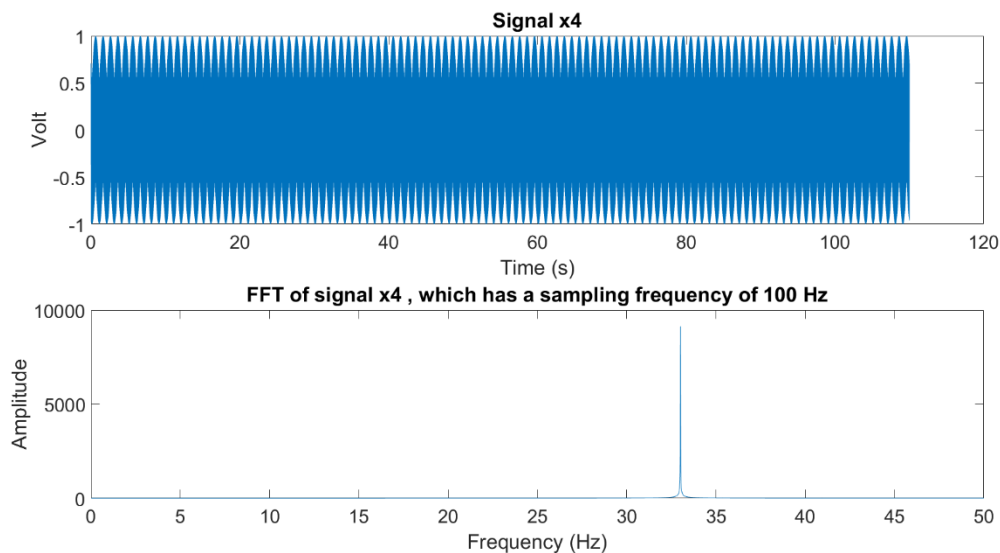

Figure 4: Signal x4 and its Fast Fourier Transform.

Signal x5

The Fast Fourier Transform of signal x5 can be seen in figure 5. X5 has a sampling rate of 25 Hz, which resulted in its estimated frequency component to be even lower than in signal x4. Indeed, the only significant frequency component of signal x5 is at 8 Hz, a frequency very far from the 133.5 Hz sine wave it was sampled from.
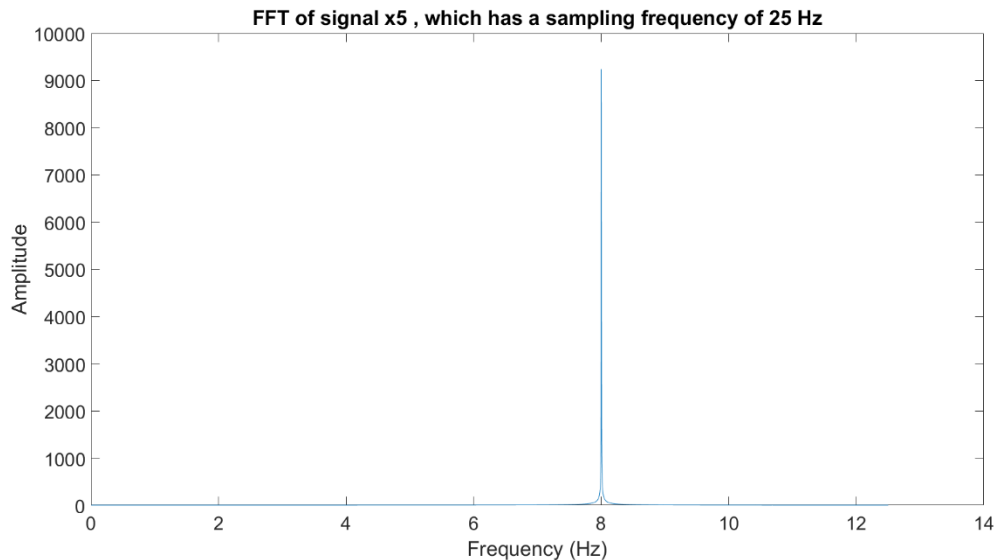

Figure 5: Fast Fourier Transform of signal x5.

Assignment 5 part 2

To compute the periodograms shown in this section, we segmented the signals into 40 realizations of 1 second each. The periodogram shown is the mean periodogram for each of these realizations. I did my best to get the frequency units right, but I think I might be wrong. I do not know what units were on the y-axis after using the periodogram function, so I put power/frequency (as the function does).

The probability density function for signals x6 though x10 seems to be a bimodal normal density function. For each signal, I estimated the mean and variance of both normal distribution by segmenting the density function into two and computing the mean and variance for values over and below 0, since 0 seemed to be the best threshold to distinguish between the two normal distribution.

Signal x6

The quantization size of signal x6 is of approximately 0.005 Volts and there are approximately 4001 quantization levels (some quantization levels not including any portion of the signal). 17 bits are required to represent the signal up to 10 Volts with 4 decimals, the amount of precision shown in MATLAB ($\log 2(10000) = 16.6$). Figure 6 shows the estimated density function of the signal as well as its periodogram. The density bimodal normal density distribution has estimated means of -4.061 and 4.052 Volts with estimated variances of 3.925 and 3.689, respectively.
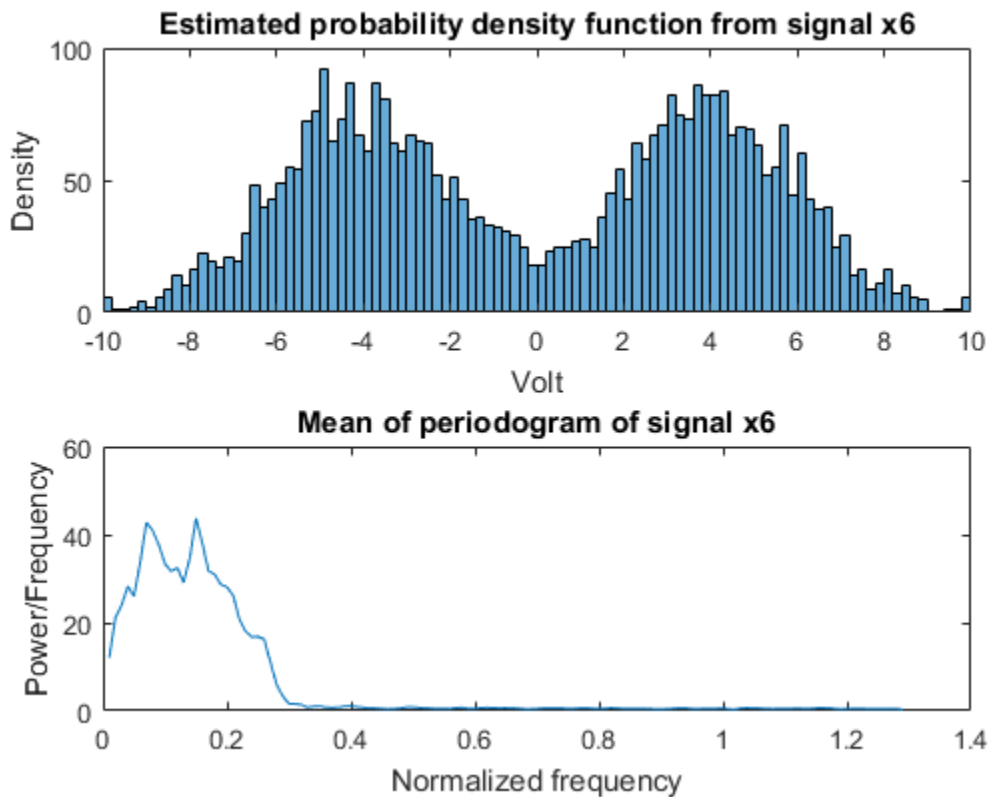

Figure 6: Estimated probability density function and mean periodogram of signal x6.

Signal x7

The quantization size of signal x7 is of approximately 0.0784 Volts and there are approximately 256 quantization levels (some quantization levels not including any portion of the signal). 17 bits are required to represent the signal up to 10 Volts with 4 decimals, the amount of precision shown in MATLAB (log2(10000) = 16.6). Figure 7 shows the estimated density function of the signal as well as its periodogram. The density bimodal normal density distribution has estimated means of -4.06 and 4.05 Volts with estimated variances of 3.924 and 3.687, respectively.
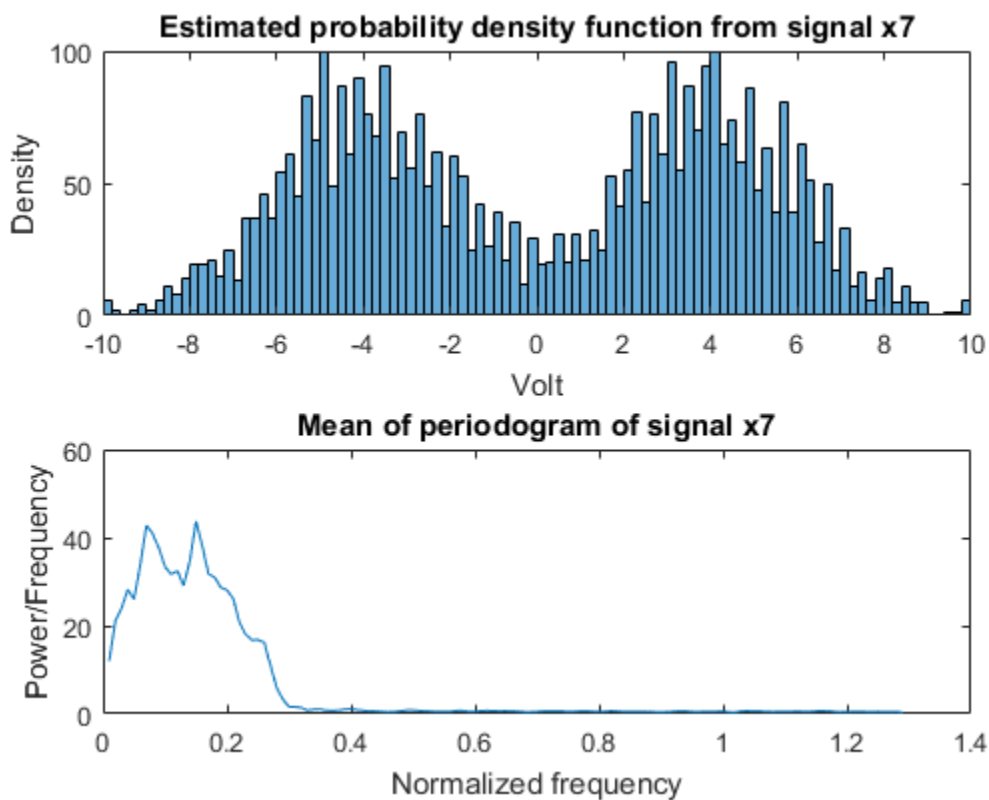

Figure 7: Estimated probability density function and mean periodogram of signal x7.

Signal x8

The quantization size of signal x8 is of approximately 1.333 Volts and there are 16 quantization levels. 17 bits are required to represent the signal up to 10 Volts with 4 decimals, the amount of precision shown in MATLAB (log2(10000) = 16.6). Figure 8 shows the estimated density function of the signal as well as its periodogram. The density bimodal normal density distribution has estimated means of -4.048 and 4.071 Volts with estimated variances of 3.968 and 3.812, respectively.
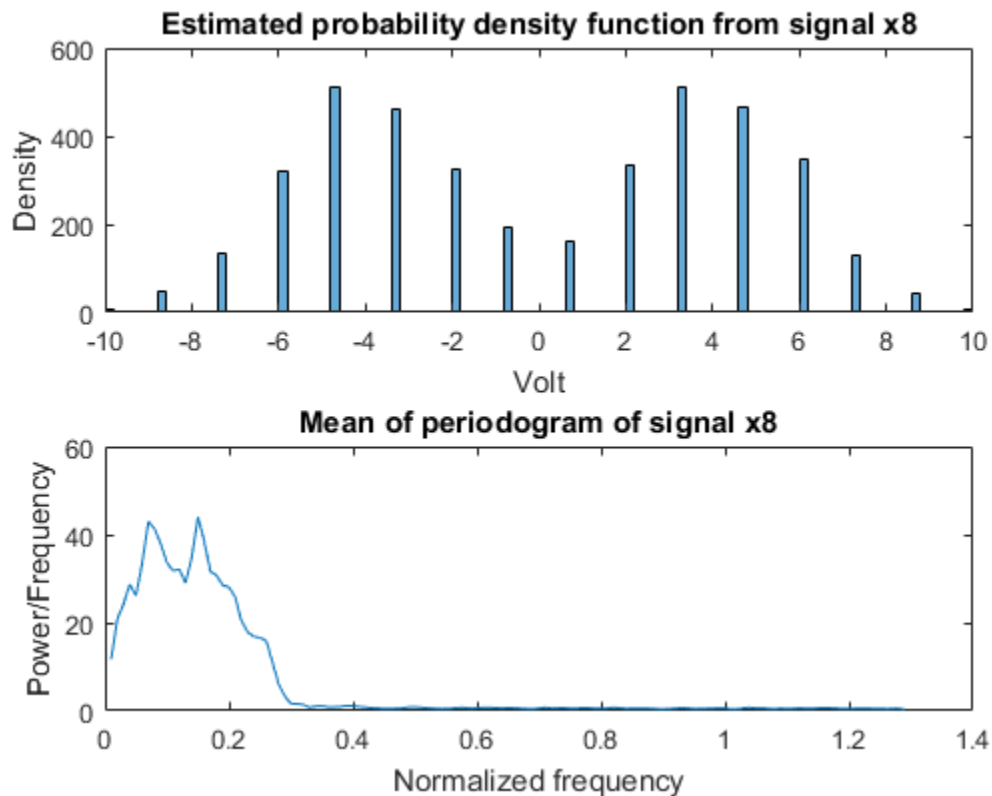

Figure 8: Estimated probability density function and mean periodogram of signal x8.

Signal x9

The quantization size of signal x9 is of approximately 6.666 Volts and there are 4 quantization levels. 17 bits are required to represent the signal up to 10 Volts with 4 decimals, the amount of precision shown in MATLAB (log2(10000) = 16.6). Figure 9 shows the estimated density function of the signal as well as its periodogram. The density bimodal normal density distribution has estimated means of -3.957 and 3.939 with estimated variances of 3.772 and 3.675, respectively.
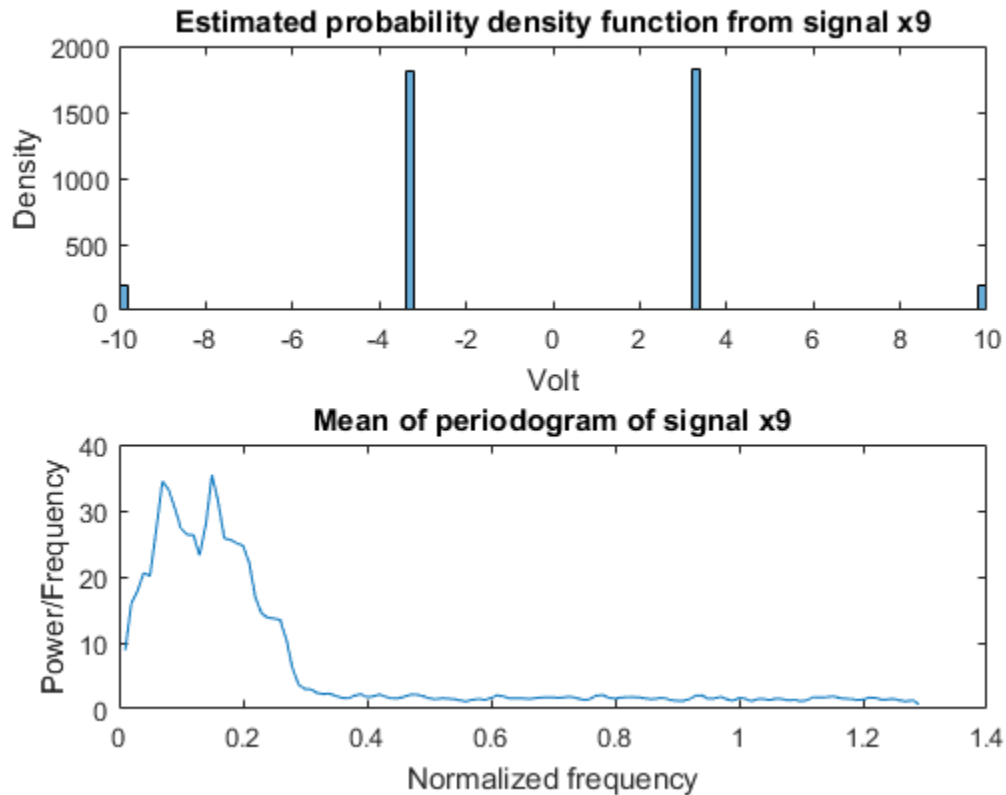

Figure 9: Estimated probability density function and mean periodogram of signal x9.

Signal x10

The quantization size of signal x10 is of approximately 20 Volts and there are 2 quantization levels. Technically, only 4 bits are necessary to represent the signal (log2(10) = 3.3). Figure 10 shows the estimated density function of the signal as well as its periodogram. The density bimodal normal density distribution has estimated means of -10 and 10 with estimated variances of 0 and 0.
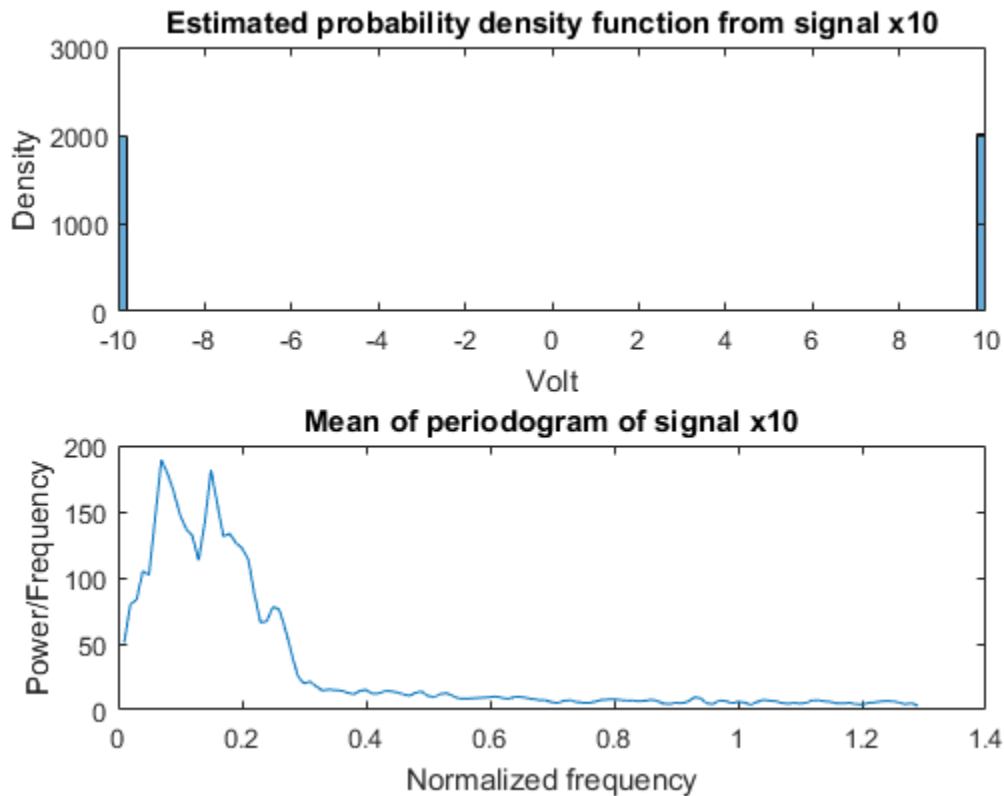

Figure 10: Estimated probability density function and mean periodogram of signal x10.

Larger quantization sizes lead to worse resolution of the estimated probability function. This is the property of the signal most affected by the quantization size. The periodogram was also affected quite dramatically by the quantization sizes of signal x9 and x19; even though the shape remained the same, the values for the y-axis are totally different. Surprisingly, the mean and variance of the two normal distributions were only moderately affected by the quantization sizes. Differences in means and variances are noticeable starting from signal x9, with a quantization size of 6.666 Volts, and even then, these differences are far from drastic. Of course, with a quantization size of 20 Volts (signal x10), the estimated means and variances are completely off.