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# Optimization Methods

# Optimizing Inter-Hospital Patient Transfer Routing

*Project Report*

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# 1 Introduction

Inter-hospital patient transfers (IHTs), most often undertaken to provide access to specialized care, comprise about 3.5% of all hospital inpatient admissions (1.5M admissions) [1]. Relocating patients to a hospital better suited for their care, this practice is crucial for delivering quality patient outcomes. Further, patient transfers maximize the impact of healthcare providers with unique expertise by providing them with cases that they are best equipped to handle. For these reasons, there exists a significant demand for private companies to provide patient transfer services to hospitals. This project takes the perspective of a fictitious inter-hospital patient transfer company based in Boston, Massachusetts.

# 2 Problem Statement

Despite the clear benefits of IHTs, this practice also introduces additional risks and trade-offs. For example, even for non-emergency IHTs, there exists the possibility that the patient's vital signs become unstable during transit. Further, the abrupt motion of the vehicle could lead to injury or discomfort for the patient, especially for patients with chronic pain. Patients also incur some of the costs of transfer, so patients of low socioeconomic status might elect to remain at a poorly equipped hospital for financial reasons.

The goal of this project is to reduce the direct costs of our fictitious company so that it can invest capital to mitigate the risks of patient transfer and reduce the transfer cost to patients. The hope is that this cost-reduction strategy will be adopted by real-life IHT companies, and the additional capital will result in better and more equitable outcomes for patients across all socioeconomic classes.

The cost reduction in our project was produced by using mixed-integer optimization methods to generate routes that reduce the total distance traveled among all transfer vehicles. According to a former colleague that worked at a real-life IHT company based in Charlotte, North Carolina, the standard procedure is to dispatch vehicles to transfer requests using simple, greedy approaches. This motivates clear opportunities of improvement using optimization.

# 3 Data

To ensure our results translate well to real-life implementations, it was of the utmost importance to use data that is as close to reality as possible. The vast majority of our data comes from public data sets and APIs that allow the exact computation of travel distances and times using the coordinates of Boston hospitals. Because patient transfer demand is not made publicly available, we used metrics on hospital bed capacities to intelligently synthesize this data.

## 3.1 Geographical Data

Using a publicly available government website for hospital locations, we were able to obtain the exact coordinates for each hospital in Boston. The TravelTime API and its corresponding Python SDK allowed the travel times and distances to be computed, and matrices were constructed containing the data for each origin-destination pair.

Next, two locations were selected as *depots* which represent the location where vehicles must begin their route and return at the end. These locations were chosen because they are the locations for real-life ambulance depots, and they are in areas with slightly lower property values than downtown Boston. The orange ambulance icons in Figure 1 in the Appendix depict the locations of the two depots.

## 3.2 Patient Transfer Data

As patient transfer data is not public knowledge, we used information about staffed beds at each hospital and scale that proportionally to build the inter-hospital patient transfer data.

To build this data, we followed the following strategy:

1. Using the resource on staffed beds and some Google searches, we **extracted the number of staffed beds** for the 24 hospitals as shown in Figure 2.
2. **Scaled down the problem** assuming 5% of beds saved for transferred patients coming into the hospital.
3. Using the "transfer beds" found above, we find the **number of patient transfers** by making it proportional to the capacity of a hospital. For instance, Mass General Hospital has 1019 staffed beds, making it 20 beds for transfer patients (based on our assumption). So, based on the capacity of all other hospitals, we proportionally

send patients to MGH such that the total patients does not exceed 20 beds. This patient transfer data can be seen in Figure 3.

# 4 Approach

## 4.1 Main Idea

For a structured approach to the problem, we went ahead with the following strategy:

1. Look at the problem in a scaled-down version
2. Understand the correct and incorrect behaviour in the results from the model.
3. Create a basic optimization framework as an MVP. Start with a few constraints.
4. Build up the problem, make it realistic, add constraints, optimize the model, iterate...

## 4.2 $\epsilon$ Successes

Following each of the above steps, there were “Eureka” moments, where we cracked down a way to add a constraint that made the problem more realistic, mixed with times of intellectual discussion to make this project a success. Here we lay out 3 such phases lined up chronologically to motivate how we started from scratch to building the complete optimization model.

### 4.2.1 Very Ambitious Network Flow Flavour

1. **Idea**: Create hospital nodes with ambulances moving between hospitals using binary variables representing whether an ambulance visited transferred a hospital to transfer a patient or not.
2. **Challenge 1**: Single ambulance needs to visit a hospital several times [2] in case of 1+ patients.
3. **Challenge 1.5**: Hospitals (patients) may have multiple mappings to other hospitals.
4. **Challenge 2**: To have a notion of whether a hospital visited or not needed more variables to model leftover patients in the queue at any hospital.
5. **Challenge 3**: Looping through the same two nodes is tricky to model with binary variables.
6. **Challenge 4**: How do we map which patient went to which hospital?
7. **Solution**: Separate out the patients from the hospitals for modelling purposes!

### 4.2.2 Splitting Patients from Hospitals

1. **Idea**: To model the patients with different nodes [3] with patient-hospital-ambulance mapping. Geographically the locations would be the same for two patients at the same hospital but represented with different nodes for ease of index handling. The distance between a patient and its current hospital makes the vehicle automatically switch nodes, for the distance is 0.
2. **Challenge 1**: Generate a patient-hospital mapping. Earlier only the total patients were needed.
3. **Challenge 2**: Mapping patients with a sequence (not ordered) required a time variable.
4. **Reality 1**: A chain of ambulance owners asked by hospitals for patient transfers (Example)
5. **Reality 2**: Different vehicles have a different cost per unit distance.
6. **Solution**: Sequence without time variable using MTZ Formulation [4] and make the model flexible to handle all realistic additions. Model it as a vehicle routing problem!

### 4.2.3 Flexible and Realistic Ambulance Routing Model for Non-Emergency patients

1. **Idea**: Adding all flexibility to handle non-emergency patient transfer. The assignment of patients is pre-assigned by the starting hospital in advance.
2. **Reality 1**: A driver cannot drive more than a certain number of hours in a day. Instead of limiting the number of patient pickups, use a maximum time constraint.
3. **Reality 2**: An ambulance cannot end anywhere except at its depot (or another depot if capacity and vacancy are modelled).
4. **Solution**: The final formulation incorporates all these ideas.

## 4.3 Formulation

We aim at making the formulation as realistic as possible. The following are major ideas considered.

- We look at non-emergency patients; no notion of prioritizing patients over one another.
- The distance & time between nodes is found out using the TravelTime API to realistically depict the objective.
- Patients are pre-assigned to hospitals using generated patient data. This data is sent across to the ambulance chain at the start of the day and the latter takes care of transporting these patients to their respective hospitals.
- Multiple patients can exist at the same hospital with different hospitals as destinations.
- A multi-objective optimizes between the travelling costs, the travelling time and the driver wages which contribute most to the final objective value.
- To ensure that a driver does not travel consecutively for the entire day, we give a max time limit for each driver.
- Electric vehicles have a lower maintenance cost and lesser cost per mile than gas vehicles, however they can travel only for a certain distance before a charge which takes time. We assume that there are sufficient gas station throughout Boston and the gas vehicles can refill almost instantaneously as compared to the electric vehicles. This highlight the trade-off the model would use.
- All costs have been realistically chosen based on the data for Boston available on the internet.
- The vehicles are assigned to a depot and are constrained to come back to the same depot.
- Hourly wages are taken as the average hourly rate for an ambulance driver in Boston [5].

### 4.3.1 Parameters

| | |
|---|---|
| $d_{ij}$ | distance from node $i$ to node $j$ |
| $f_{t^k}$ | fixed cost for vehicle type corresponding to ambulance $k$ |
| $c^k$ | travelling cost per unit distance for ambulance $k$ |
| $d_{elec}$ | distance an electric ambulance can travel in a single charge (60mi) |
| $w$ | hourly wage for an ambulance driver in the region (\$16/hour) |
| $depot_i^k$ | $\begin{cases} 1 & \text{if ambulance } k \text{ is assigned to depot } i \\ 0 & \text{otherwise} \end{cases}$ |
| $t_{ij}$ | travelling time from node $i$ to node $j$ |
| $t_b$ | buffer time to pick up and drop-off patients (20 mins) |
| $t_{max}$ | maximum travelling time for any ambulance (8 hours) |

### 4.3.2 Sets

| | |
|---|---|
| $P$ | set of nodes corresponding to patients to be transferred |
| $K$ | set of integers corresponding to ambulances number |
| $D$ | set of nodes corresponding to ambulance depots |
| $E$ | set of vehicle numbers that correspond to electric vehicles |
| $V$ | set of nodes encapsulating patients, hospitals and depots nodes |

### 4.3.3 Decision Variables

$$x_{ij}^k = \begin{cases} 1 & \text{if ambulance } k \text{ transfers patient from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases}$$

$$y^k = \begin{cases} 1 & \text{if ambulance } k \text{ is used for patient transfers} \\ 0 & \text{otherwise} \end{cases}$$

$$u_i^k = \text{sequence of traversing node } i \text{ by ambulance } k$$

#### 4.3.4 Optimization Model

$$
\begin{aligned}
\min_{x,y,u} \quad & \sum_{k \in K} f_k * y^k + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c^k * d_{ij} * x_{ij}^k + \\
& w * \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} (t_{ij} + t_b) * x_{ij}^k & & (1) \\
\text{s.t.} \quad & \sum_{j \in V} \sum_{k \in K} x_{ij}^k = 1 & \forall\, i \in P \quad & (2) \\
& \sum_{k \in K} x_{i(i+|P|)}^k = 1 & \forall\, i \in P \quad & (3) \\
& \sum_{i \in V} x_{ij}^k - \sum_{i \in V} x_{ji}^k = 1 & \forall\, j \in V, k \in K \quad & (4) \\
& \sum_{i \in V} \sum_{j \in V} x_{ij}^k * (t_{ij} + t_b) \le t_{max} & \forall\, k \in K \quad & (5) \\
& \sum_{i \in V} \sum_{j \in V} x_{ij}^k \le d_{elec} & \forall\, k \in E \quad & (6) \\
& x_{ij}^k = 0 & \forall\, i \in D,\ j \in D,\ k \in K \quad & (7) \\
& x_{ij}^k \le y^k * depot_i^k & \forall\, i \in D,\ j \in V\ k \in K \quad & (8) \\
& u_i^k = 1 & \forall\, i \in D,\ k \in K \quad & (9) \\
& u_i^k - u_j^k + 1 \le (|V| - 1) * (1 - x_{ij}^k) & \forall\, i \in P \cup H,\ j \in P \cup H,\ k \in K \quad & (10) \\
& 2 \le u_i^k \le |V| & \forall\, i \in P \cup H,\ k \in K \quad & (11) \\
& x_{ij}^k \in \{0, 1\} & \forall\, i \in V,\ j \in V,\ k \in K \quad & (12) \\
& y^k \in \{0, 1\} & \forall\, k \in K \quad & (13) \\
& u_i^k \in Z_+ & \forall\, i \in V,\ k \in K \quad & (14)
\end{aligned}
$$

The objective, as shown in (1), adds the cost of maintenance for the ambulance if it is being used to transfer patients, the cost of travelling, and the hourly wages given to the drivers. The cost of maintenance is taken to be only for ambulances that run. This could be considered as the additional cost in maintenance due to running that vehicle. All of these costs are in USD.

Constraints (2)-(3) make sure that the patient is only transferred to a hospital which is designated to it. This mapping is pre-defined by the hospital the patient is currently in. We model the problem such that the patient at node $i$ goes to its designated hospital at node $i + n$. The distance matrix is built to maintain this structure.

Constraint (4) is the flow balance constraint. Thus, if an ambulance transfers a patient to its designated hospital, then it is constrained to leave from that hospital to either pick up another patient or go back to the depot.

Constraint (5) is to ensure that the travelling time for any ambulance is not more than a chosen time limit. This is to ensure that the model does not run a vehicle for more than 8 hours since a driver can only drive for a limited number of hours in a day. This time includes a buffer time of 20 minutes for patient pickup and drop-off.

Constraint (6) is to enforce that the total distance travelled by any electric vehicle is not more than what an electric ambulance can travel on a single charge. The ambulance then needs to go back to its depot, charge and then stay there until required for patient transfer.

Constraint (7) maintains that the ambulances do not travel between different depots.

Constraint (8) is a linking constraint to ensure that an ambulance only goes out from its designated depot and takes a route only if it is being used to transfer patients from hospital to hospital.

Constraints (9)-(11) relate to the MTZ formulation to give a sequence to the nodes traversed by a particular ambulance without using a time variable. This is to ensure that the final sequence of patients could be extracted. The order is important and not the exact point when the ambulance transfers one patient to its assigned hospital.

Constraints (12)-(14) define the range for decision variables. The sequence variable could be any positive integer that satisfied the previous constraints, while the other decision variables are binary.

# 5 Impact

In order to quantify our impact, a non-optimization baseline formulation was constructed to model the current standard practice for real-life IHT companies, based on my former colleague's guidance. Instead of optimizing over the entire day's demand, a dispatcher assigns patients to routes in real time. Ambulances tend to each be assigned to "regions" such that they are responsible for transfers between only a certain subset of origin-destination pairs. This ensures that ambulances travel smaller distances between transfers.

Our baseline approach utilized the average distance and travel time between hospitals in order to make the baseline depend only on the number of transfers demanded and not the localization of the demand. This average distance and time were divided by the number of ambulances $K$ in order to model the average commute between transfers in that ambulance's region, which is only $1/K$ of the total coverage. Our approach iterates randomly through the demand matrix to simulate a dispatcher assigning to routes in real time as demand is created. If a patient transfer is requested from the same hospital that our ambulance just dropped off a previous patient, the approach bypasses the dispatcher and satisfies this transfer request, since that request requires no additional traveling time between hospitals. If there is no patient ready to be picked up at that same hospital, the ambulance has to travel to a different hospital to perform the next transfer. This entails a distance "penalty" equal to $1/K$ of the average distance as the ambulance needs to travel to the next hospital without any patient. There is also a time penalty equal to $1/K$ of the average travel time plus 20 minutes. The 20 minutes represents the time that the ambulance was waiting for another patient transfer request from the hospital where they are currently located since that is cheaper than immediately traveling to another hospital. Lastly, the distance for traveling to and from the depot was added.

# 6 Results

As mentioned previously, a crucial aspect of our formulation was utilizing the MTZ formulation to extract the sequence of nodes that the ambulance traverses. This is essential to the human interpretation of our results because it provides the order that each route should be taken, i.e., the order that each patient should be transferred. Otherwise, the results would be outputted as the number of times that a route was taken, and the order would have been arbitrary.

In order to visualize our results, these output sequences were converted to Well-Known Text (WKT) format, which is an Open Geospatial Consortium (OGC) standard that allows mapping software to build lines between provided coordinates. This data was passed to Google My Maps, which allowed us to create a map with the routes for each vehicle overlayed. This map was then embedded onto a webpage built with HTML and then hosted the webpage on GitHub. We encourage the reader to explore the results themselves by following the hyperlink above (QR in Fig 6).

One thing that should be noted is that we were required to make adjustments to the scale of the problem due to it not being solvable in a realistic amount of time. Due to the complexity of the formulation, a scale of 70 patients and 144 nodes was not solvable in a realistic time frame with a marginal duality gap. For this reason, the following results will be presented for our scaled-down version with 10 patients. The duality gap here is nil. Although the same could have been done for more patients, what is more vital for the formulation is how it optimizes the global picture instead of taking a greedy or locally optimal approach.

| Formulation | Metric | | | |
|---|---|---|---|---|
| | Fuel Costs | Wage Costs | Fixed Costs | Overall |
| Baseline (USD) | 27.29 | **166.97** | 9.60 | 203.85 |
| Optimized (USD) | **14.52** | 170.40 | **6.10** | **191.02** |
| Percent Improvement (%) | 46.78 | -2.06 | 36.46 | 6.29 |

Table 1: Summary of direct costs broken down by category in comparison with baseline. All costs are given in USD and represent one day of costs with a demand of 10 patients and 4 ambulances

As shown in Table 1, there was a considerable reduction in fuel costs (46.78%) in the optimized strategy versus the baseline. This is particularly exciting because a reduction in fuel costs and travel distance would likely result in reduced emissions, which would have a positive environmental impact. Also, a 6.29% reduction in direct costs is significant, given the specific costs we are considering. For example, wage costs are about an order of magnitude higher than fuel costs, and the majority of the billable time for ambulance drivers in our formulation was actually the buffer time, where patients were being picked up and dropped off. Since reductions in transport time do not have any impact on buffer time, an overall 6.29% reduction is a very promising result.

### 6.1 Interpreting the Optimal Strategy

Upon interpreting the final result, we could make a lot of insights, some of which have been mentioned below:

1. The model chooses ambulances 2 and 3 where the latter is the only electric vehicle in the fleet. The route for ambulance 3 is longer than that of ambulance 2 which makes intuitive sense since electric vehicles have lower per-unit distance travel cost.
2. Initially the electric vehicle was chosen to be at depot 2 and the resulting route was such that this vehicle alone made all the patient transfers. To make it more interesting, we moved this electric vehicle to depot 1. Since an EV gives a big cost reduction over gas vehicles, the model chooses it even though depot 1 is farther from the patient-hospital locations, but picks a petrol vehicle for an optimal solution together.

### 6.2 Uncertain Scenarios

Reflecting upon the idea of making this already complex Vehicle Routing Problem (VRP) more realistic, we realized that the travel time between two nodes is never an exact representation of the reality. There is always uncertainty, at different times of the day, day of the week, season, etc. Simplifying this idea, we model 4 scenarios. We build 3 scenarios, each with 5%, 10%, 50% random noise added to the travelling time. This is equivalent to having some routes with more traffic than the others. We then re-structure our model to run for these 3 scenarios, along with an stochastic or expected cost model where the model would output a route that is optimal under all the scenarios on average. These tweaks in the original formulation are reflected in the Appendix. The new route that is optimal under the weighted scheme of the scenarios is also made available in the website.

The output showing convergence of the optimization model after exploring 567,504 nodes for this problem can be seen in Figure 4. The number of cuts applied for this problem with 10 patients is in Figure 5.

## 7 Scope of Improvement

An outcome that we are particularly proud of is the complexity of the formulation. Not only did it consider decisions of whether to deploy a particular ambulance but it considered the "shortest path" sequence of patient transfers based on a compound objective considering time and distance. Further, it utilized the MTZ formulation to allow us to obtain the order of patient transfers, which was crucial for interpretation. As mentioned previously, a drawback of this is that our full-scale data set did not solve in a realistic period of time. So, the first priority in future improvements would be to modify the formulation to accommodate the full scale.

Once the full scale is accommodated, it would be interesting to quantify the lowering in emissions due to the large reduction in total distance traveled that we achieved and our utilization of electric vehicles. Also, if additional complexity can be added without sacrificing tractability, there are several additions we would like to add to the future model:

1. Allow ambulances to start and end at different depots by incorporating the notion of capacity and vacancy at each depot
2. Time variable to prioritize some patients over others using a queuing structure
3. Notion of an ambulance having to stay until the patient appointment starts
4. Ambulance needs to visit the assigned hospital within a certain time window

## 8 Conclusion

As a result of our efforts, we reduced a substantial proportion of direct costs for our fictitious IHT company. Nearly half of the fuel and electricity costs (46.78%) were eliminated by optimizing the travel paths of each ambulance. These reductions, combined with the fact that we incorporated electric vehicles (EVs) into our fleet, ensured that our implementation would have a positive environmental impact. Perhaps the most exciting aspect of our project is the versatility of our formulation. Despite utilizing medical data in this project, a simple substitution would allow our formulation to optimize any transport problem that requires per-day demand. Further, if one would like to extend our formulation to emergency medical transport, one would only need to include an additional time component. Overall, the prospect of potential implementations and extensions of this project reaffirms the power of optimization.

# 9 Appendix

## 9.1 Reformulation with Uncertainty

We reformulated the existing formulation to introduce 4 scenarios with varied traffic situations. The aim is to find a route that is robust to such changes in traffic which makes the model more realistic. Each scenario corresponds to one of actual, 5%, 10% and 50% uncertainty in the travel time but adding or subtracting a random fraction of the current travel time. Finally, all the 4 scenarios are uniformly weighted ($p_s = \frac{1}{4}$) as motivated by stochastic optimization.

The updated formulation is as seen below. Most of the constraints are same as the original formulations and thus most of the variables are described in detail in Section 4.3. The addition is in the travel time representation. Instead of just having a single time for going from node $i$ to node $j$, there is now a third component which describes which scenario is being considered. Thus, $t^s_{ij}$ is the time taken to go from node $i$ to node $j$ in scenario $s$. Here $s \in S$ where $S = [1, 2, 3, 4]$ for the 4 scenarios mentioned above.

$$
\begin{aligned}
\min_{x,y,u} \quad & \sum_{k \in K} f_k * y^k + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c^k * d_{ij} * x^k_{ij} + \\
& w * \sum_{s \in S} p_s * \left[ \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} (t^s_{ij} + t_b) * x^k_{ij} \right] && && (1) \\
\text{s.t.} \quad & \sum_{j \in V} \sum_{k \in K} x^k_{ij} = 1 && \forall\, i \in P && (2) \\
& \sum_{k \in K} x^k_{i(i+|P|)} = 1 && \forall\, i \in P && (3) \\
& \sum_{i \in V} x^k_{ij} - \sum_{i \in V} x^k_{ji} = 1 && \forall\, j \in V, k \in K && (4) \\
& \sum_{i \in V} \sum_{j \in V} x^k_{ij} * (t^s_{ij} + t_b) \leq t_{max} && \forall\, k \in K,\ s \in S && (5) \\
& \sum_{i \in V} \sum_{j \in V} x^k_{ij} \leq d_{elec} && \forall\, k \in E && (6) \\
& x^k_{ij} = 0 && \forall\, i \in D,\ j \in D,\ k \in K && (7) \\
& x^k_{ij} \leq y^k * depot^k_i && \forall\, i \in D,\ j \in V\ k \in K && (8) \\
& u^k_i = 1 && \forall\, i \in D,\ k \in K && (9) \\
& u^k_i - u^k_j + 1 \leq (|V| - 1) * (1 - x^k_{ij}) && \forall\, i \in P \cup H,\ j \in P \cup H,\ k \in K && (10) \\
& 2 \leq u^k_i \leq |V| && \forall\, i \in P \cup H,\ k \in K && (11) \\
& x^k_{ij} \in \{0, 1\} && \forall\, i \in V,\ j \in V,\ k \in K && (12) \\
& y^k \in \{0, 1\} && \forall\, k \in K && (13) \\
& u^k_i \in Z_+ && \forall\, i \in V,\ k \in K && (14)
\end{aligned}
$$

## 9.2 Figures and Tables

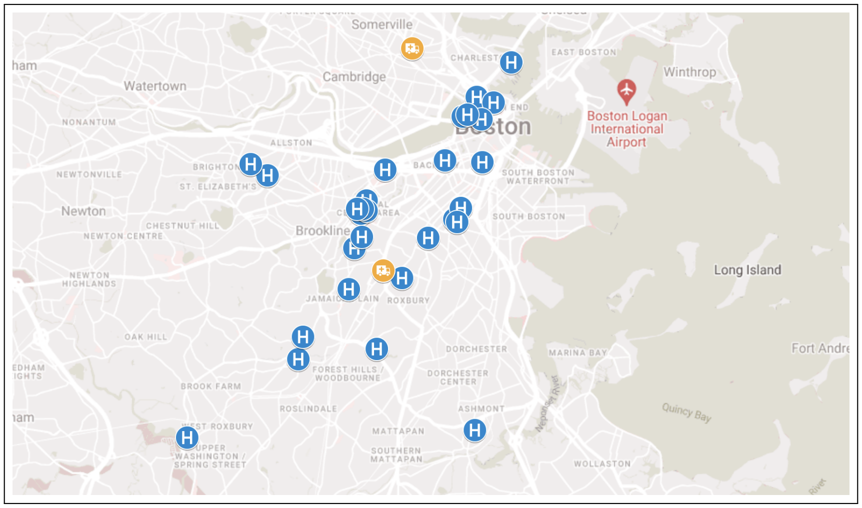


Figure 1: This map depicts the locations of the Boston hospitals utilized in this project (shown as blue **H** icons, as well as the two depots (shown as orange ambulance icons)

| Hospital # | Hospital Name | # Staffed Beds | | # Beds for Transfer Patients | %age of Total Beds |
|---|---|---|---|---|---|
| 1 | Arbour Hospital | 136 | 0.020887728 | 3 | 2.34% |
| 2 | Beth Israel Deaconess Medical Center - West Campus | 371 | 0.056980495 | 7 | 5.47% |
| 3 | Beth Isreal Deaconess medical Center - East Campus | 372 | 0.057134081 | 7 | 5.47% |
| 4 | Boston Medical Center - East Newton Campus | 214 | 0.032867455 | 4 | 3.13% |
| 5 | Boston Medical Center - Menino Campus | 214 | 0.032867455 | 4 | 3.13% |
| 6 | Brigham and Women's Hospital | 812 | 0.124712026 | 16 | 12.50% |
| 7 | Carney Hospital | 122 | 0.018737521 | 2 | 1.56% |
| 8 | Children's Hospital Boston | 477 | 0.073260636 | 10 | 7.81% |
| 9 | Dana-Farber Cancer Institute | 30 | 0.004607587 | 1 | 0.78% |
| 10 | Erich Llindemann Mental Health Center | 60 | 0.009215174 | 1 | 0.78% |
| 11 | Faulkner Hospital | 171 | 0.026263247 | 3 | 2.34% |
| 12 | Franciscan Children's Hospital & Rehab Center | 112 | 0.017201659 | 2 | 1.56% |
| 13 | Hebrew Rehabilitation Center | 717 | 0.110121333 | 14 | 10.94% |
| 14 | Jewish Memorial Hospital & Radius Hospital | 207 | 0.031792351 | 4 | 3.13% |
| 15 | Kindred Hospital | 59 | 0.009061588 | 1 | 0.78% |
| 16 | Lemuel Shattuck Hospital | 265 | 0.040700353 | 5 | 3.91% |
| 17 | Mass General Hospital | 1019 | 0.156504377 | 20 | 15.63% |
| 18 | Massachusetts Eye and Ear Infirmary | 41 | 0.006297036 | 1 | 0.78% |
| 19 | New England Baptist Hospital | 75 | 0.011518968 | 2 | 1.56% |
| 20 | Shriner's Burn Institute | 40 | 0.00614345 | 1 | 0.78% |
| 21 | Spaulding Rehabilitation Hospital | 132 | 0.020273384 | 3 | 2.34% |
| 22 | St. Elizabeth's Hospital | 302 | 0.046383044 | 6 | 4.69% |
| 23 | Tufts Medical Center | 405 | 0.062202427 | 8 | 6.25% |
| 24 | VA Bos. Healthcare System - W. Roxbury | 158 | 0.024266626 | 3 | 2.34% |
| | | 6511 | | 128 | 100.00% |

Figure 2: Hospital Data

| # Beds for Transfer | 3 | 7 | 7 | 4 | 4 | 16 | 2 | 10 | 1 | 1 | 3 | 2 | 14 | 4 | 1 | 5 | 20 | 1 | 2 | 1 | 3 | 6 | 8 | 3 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hospital # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | # Patient Transfers |
| 1 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 2 | - | - | - | - | - | 1 | - | 1 | - | - | - | - | 1 | - | - | - | 1 | - | - | - | - | - | - | - | 4 |
| 3 | - | - | - | - | - | 1 | - | 1 | - | - | - | - | 1 | - | - | - | 1 | - | - | - | - | - | - | - | 4 |
| 4 | - | - | - | - | - | 1 | - | - | - | - | - | - | - | - | - | - | 1 | - | - | - | - | - | - | - | 2 |
| 5 | - | - | - | - | - | 1 | - | - | - | - | - | - | - | - | - | - | 1 | - | - | - | - | - | - | - | 2 |
| 6 | - | 1 | 1 | 1 | 1 | - | - | 1 | - | - | - | - | 2 | 1 | - | 1 | 3 | - | - | - | - | 1 | 1 | - | 14 |
| 7 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 8 | - | 1 | 1 | - | - | 1 | - | - | - | - | - | - | 1 | - | - | - | 2 | - | - | - | - | - | 1 | - | 7 |
| 9 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 10 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 11 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 12 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 13 | - | 1 | 1 | - | - | 2 | - | 1 | - | - | - | - | - | - | - | 1 | 2 | - | - | - | - | 1 | 1 | - | 10 |
| 14 | - | - | - | - | - | 1 | - | - | - | - | - | - | - | - | - | - | 1 | - | - | - | - | - | - | - | 2 |
| 15 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 16 | - | - | - | - | - | 1 | - | - | - | - | - | - | 1 | - | - | - | 1 | - | - | - | - | - | - | - | 3 |
| 17 | - | 1 | 1 | 1 | 1 | 3 | - | 2 | - | - | - | - | 2 | 1 | - | 1 | - | - | - | - | - | 1 | 1 | - | 15 |
| 18 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 19 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 20 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 21 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| 22 | - | - | - | - | - | 1 | - | - | - | - | - | - | 1 | - | - | - | 1 | - | - | - | - | - | - | - | 3 |
| 23 | - | - | - | - | - | 1 | - | 1 | - | - | - | - | 1 | - | - | - | 1 | - | - | - | - | - | - | - | 4 |
| 24 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| | | | | | | | | | | | | | | | | | | | | | | | | | 70 |

Figure 3: Inter-Hospital Patient Transfer Data

```
Root relaxation: objective 1.558327e+02, 116 iterations, 0.00 seconds (0.00 work units)

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0  155.83267    0   25          -  155.83267      -     -    0s
H    0     0                     371.7234811  155.83267  58.1%     -    0s
H    0     0                     359.4849433  155.83267  56.7%     -    0s
H    0     0                     342.5872844  155.83267  54.5%     -    0s
H    0     0                     289.5476856  156.51501  45.9%     -    0s
     0     0  156.51501    0   42  289.54769  156.51501  45.9%     -    0s
     0     0  157.32071    0   42  289.54769  157.32071  45.7%     -    0s
H    0     0                     273.1761933  157.34409  42.4%     -    0s
     0     0  157.49449    0   42  273.17619  157.49449  42.3%     -    0s
     0     0  157.52469    0   42  273.17619  157.52469  42.3%     -    0s
H    0     0                     253.1011933  157.96991  37.6%     -    0s
H    0     0                     230.0083044  157.96991  31.3%     -    0s
     0     0  158.30983    0   24  230.00830  158.30983  31.2%     -    0s
     0     0  159.15715    0   22  230.00830  159.15715  30.8%     -    0s
     0     0  159.15715    0   22  230.00830  159.15715  30.8%     -    0s
H    0     0                     215.0366300  159.15715  26.0%     -    0s
     0     2  159.15715    0   22  215.03663  159.15715  26.0%     -    0s
H   36    40                     208.1723522  159.15715  23.5%  14.4    0s
* 2057  1244              87     201.3322300  159.15715  20.9%   8.5    0s
*14236  7714              67     197.7023333  167.09543  15.5%   7.6    2s
*18358  9475              53     196.8010311  168.37673  14.4%   7.7    2s
H18634  8461                     191.4676978  168.41749  12.0%   7.7    2s
 30016 12246  175.25496   28   32  191.46770  170.32913  11.0%   7.9    5s
 61869 15644  186.83024   38   48  191.46770  170.32913  11.0%   8.2   10s
 111212 26806  175.91996   48   19  191.46770  173.85944  9.20%   8.3   15s
 161943 36588 infeasible   44       191.46770  176.08098  8.04%   8.3   20s
H161950 36459                     191.2043644  176.08266  7.91%   8.3   20s
 212637 41245  183.84442   53   27  191.20436  177.79046  7.02%   8.3   25s
 270523 45333 infeasible   58       191.20436  179.47236  6.14%   8.3   30s
H325103 47894                     191.2043505  180.72462  5.48%   8.2   34s
 329745 48003  184.03396   49   22  191.20435  180.82886  5.43%   8.2   35s
H334472 48004                     191.2043453  180.91979  5.38%   8.2   35s
 386629 46148  190.08536   56   29  191.20435  181.98450  4.82%   8.1   40s
 444138 39449  186.11791   57   24  191.20435  183.32686  4.12%   8.1   45s
 500502 30093  187.22813   52   15  191.20435  184.92049  3.29%   8.0   50s
 561619  4045     cutoff   50       191.20435  189.27032  1.01%   8.0   55s
```

Figure 4: The model output showing convergence

```
Cutting planes:
  Gomory: 12
  Lift-and-project: 5
  Implied bound: 20
  MIR: 1
  Flow cover: 19
  Relax-and-lift: 2

Explored 567504 nodes (4505187 simplex iterations) in 55.37 seconds (70.02 work units)
Thread count was 8 (of 8 available processors)
```

Figure 5: The complexity of the problem with 10 patients

Figure 6: QR Code for Webpage visualizing vehicle routes

Use link to access our webpage if the above QR code expires.

# References


[1] M. K. W. N. Hernandez-Boussard T, Davies S, "Interhospital facility transfers in the united states: A nationwide outcomes study," *J Patient Saf*, vol. 13, no. 4, pp. 187–191, 2017.

[2] P. Fermin Cueto, I. Gjeroska, A. Vilalta, and M. Anjos, "A solution approach for multi-trip vehicle routing problems with time windows, fleet sizing, and depot location," *Networks*, vol. 78, 02 2021.

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