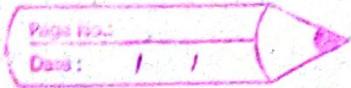


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### 1. Insertion Sort

for an array of size  $n$

for ( $i = 1$ ;  $i < n$ ;  $i++$ )

{

    for ( $j = i$ ;  $j > 0$ ;  $j--$ )

{

        if ( $a[j] < a[j-1]$ )

            temp =  $a[j]$

$a[j] = a[j-1]$

$a[j-1] = \text{temp}$ .

}

}

∴ in the worst case inner loop

executes  $i$  times for every  $i$  in outer loop.

∴ if  $i = n$

The no. of time loop executes in worst time is  $1 + 2 + 3 + \dots + (n-1)$

3

$$\frac{n(n-1)}{2} \Rightarrow \frac{n^2 - n}{2}$$

$O(n^2)$

in worst case  $\approx O(n^2)$

For best case scenario the array should be in ascending order

NAME → RACHIT SONTHALIA

ROLL NO. → 11911079

Page No.: / /  
Date: / /

if the array is in the ascending order. The inner loop will execute only once.

for every i the inner loop will execute 1 time

Time complexity  $\Rightarrow 1 + 1 + \dots$  up to n times  
 $\Rightarrow n - 1$

Time complexity  $\propto O(n)$

Example:

for array size  $n = 5$

for ( $i=1$  ;  $i \leq 5$  ;  $i++$ )

{

    for ( $j=i$  ;  $j \geq 0$  ;  $j--$ )

{

        if ( )

}

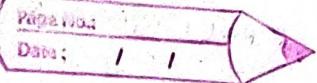
if the code is in ascending order

the inner loop will terminate for

every i in the if condition.

Time complexity  $\Rightarrow O(5)$

In the best case scenario,



In the worst case scenario the array is in the descending order and we have to arrange it in ascending order.

To reduce complexity of insertion sort in worst case we have to increase its space complexity.

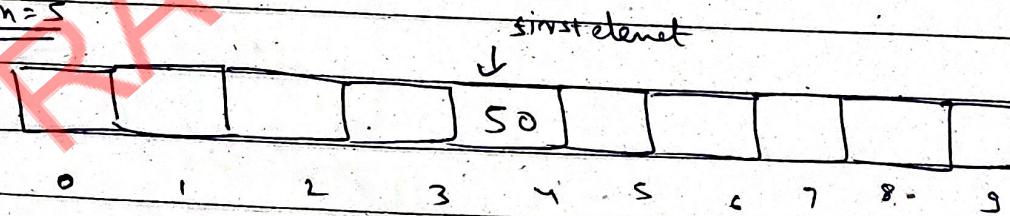
for example array is  $ac[50, 40, 30, 20, 10]$

using insertion sort to arrange ascending order

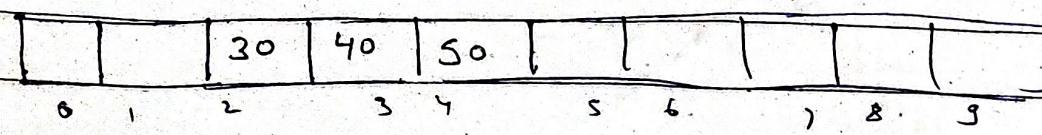
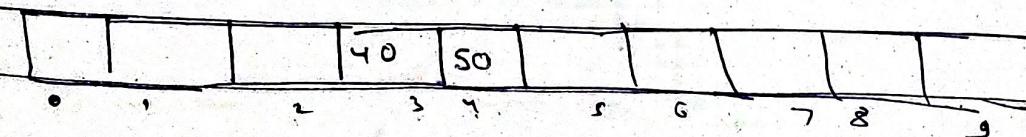
Take array of size  $2m$  if there are  $n$  elements to be sorted

and start filling the elements from  $m-1$  position.

for  $m=5$



array of size = 10



10	20	30	40	50	1	1	1
1	2	3	4	5	6	7	8

10	20	30	40	50	1	1	1
1	2	3	4	5	6	7	8

So here the time complexity for is  $O(n)$  and it was worst case scenario.

So by this time complexity in best and worst case scenario is  $O(n)$  but for average case if it is still  $O(n^2)$ .

~~BACK TO TOP~~

## Quick Sort (Inplace)

pivot pt is chosen in quick sort

all the values left of the pivot pt  
are less than the value of pivot pt and  
all the values greater than pivot pt are  
on the right of it.

The pivot pt divides array into 2 parts

worst case → when pivot element is smallest or  
largest or all elements are same.

The returned sublist will be of size  $(n-1)$ .

partition (l, m, array)

int start = l

int end = m

pivot = arr[l]

while ( $start < end$ )

{  
    while ( $arr[start] < pivot$ ) && ( $start < m$ )

    start++;

}

    while ( $arr[end] > pivot$ ) && ( $(end) \neq l$ )

        end--;

}  
    if ( $start < end$ )

C

swap (array [start], array [end]).

J

Swap (array [l], array [end])

return end;

If this happens in every then the  $i^{th}$  call will do  $O(n-i)$  work to do the partition.

$$\sum_{i=0}^n O(n-i) = n + (n-1) + (n-2) + \dots + 1$$

$$\rightarrow \frac{n^2+n}{2}$$

$$\rightarrow O\left(\frac{n^2+n}{2}\right)$$

$$\rightarrow O(n^2)$$

Bestcase: In most balanced case: each time we perform a partition we divide the list in two nearly equal pieces. This means each recursive call processes a list of half the size.

Consequently, we can make only  $\log_2 n$  nested calls before we reach a list of size.

each level of calls needs only  $\approx O(n)$  time  
all together.

$$O(n \times \log_2 n)$$

$$\rightarrow O(n \log n)$$

### Example

An array

$$[7 | 6 | 10 | 5 | 3 | 5 | 2 | 1 | 15 | 7]$$

upper  
band

pivot

lower  
band

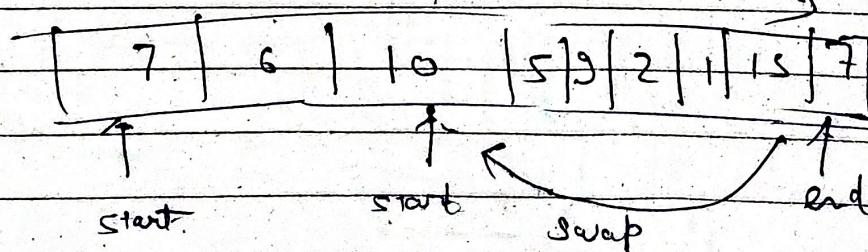
we take first element as pivot and compare it with 6.

$$6 < 7 \text{ (True)}$$

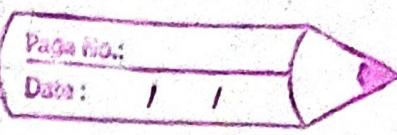
then compare 10

$$10 > 7 \text{ (True) (False)}$$

so,



11 9 11 0 7 9



Now we compare from the end any element which is less than eq to 7 and swap with 10.

now array (after first swap)

7	6	7	5	9	2	1	15	10
↑	↓	↑	↓	↑	↓	↑	↓	↑

Start              Start              end

Now compare

$$7 < = 7 \quad (\text{Increment start})$$

$$5 < = 7 \quad (\text{Increment start})$$

$$9 < = 7 \quad (\text{False})$$

Start will be at 9

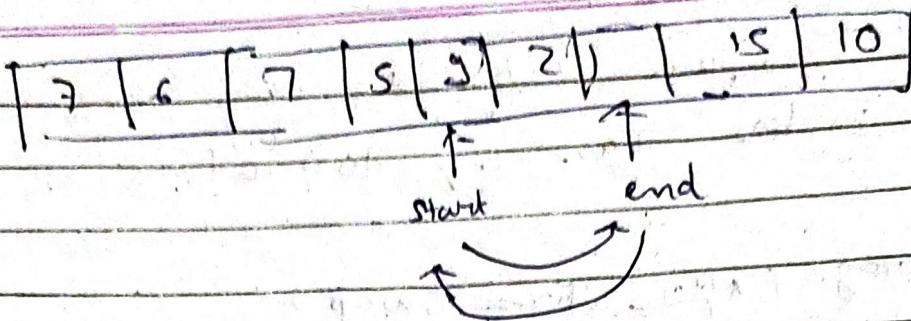
Now compare end

$$7 < = 10 \quad (\text{decrement end})$$

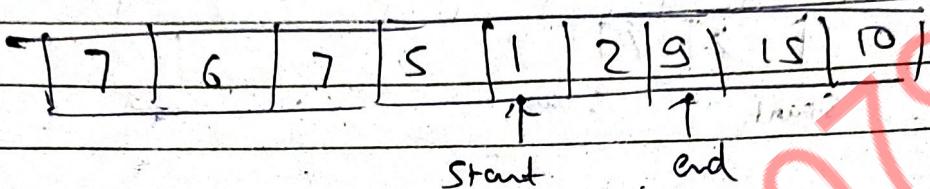
$$7 < = 15 \quad (\text{decreased end})$$

$$7 < = 1 \quad (\text{False})$$

end will be at 1



Now Array (after second swap)



Now compare

$1 <= 7$  (Increment Start)

$2 <= 7$  (Increment Start)

$9 <= 7$  (False)

Start is at '9'

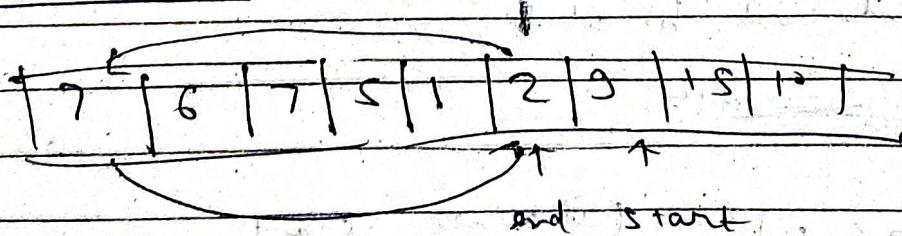
Now compare

$7 <= 9$  (decreased end)

$7 <= 2$  (False)

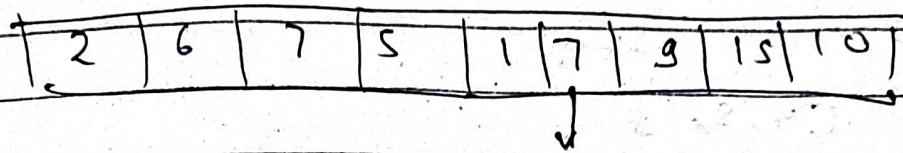
So end at 2

Now array



Now start has crossed end so we will not swap instead we will swap end with pivot element i.e. 7 at o[0]

Now '7' is at correct position. The same procedure will be followed by taking pivot between 2 and 1 & 9 and 10.



In this way Quick Sort works.

### Bubble Sort. (Inplace)

It works by repeatedly swapping adjacent elements if they are in wrong order.

example:-

Let we have an array

$$A = [5, 4, 12]$$

for ( $i = 1$ ;  $i < m$ ;  $i++$ )

    for ( $j = 0$ ;  $j < (m - i)$ ;  $j++$ )

        if ( $a[j] > a[j + 1]$ )

$k = a[j]$ ,

$a[j] = a[j + 1]$ ,      swapping  
             $a[j + 1] = k$ ,

11.9.11 079

Page No.:

Date: / /

$$A = [5, 4, 1]$$

first loop :-

$$5 > 4 \quad \text{Swap}$$

$$A = [4, 5, 1]$$

$$5 > 1 \quad \text{Swap}$$

$$A = [4, 1, 5]$$

II loop

$$4 > 1 \quad (\text{swap})$$

$$\rightarrow A = [1, 4, 5]$$

Now the array is sorted but the program  
will still run.

$$4 > 5 \quad (\text{false} \rightarrow \text{No swap})$$

III loop

$$1 > 4 \quad (\text{false} \rightarrow \text{No swap})$$

$$1 > 5 \quad (\text{false} \rightarrow \text{No swap})$$

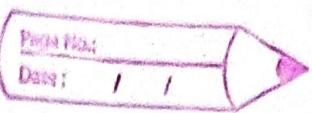
$$A = [1, 4, 5] \quad (\text{sorted})$$

Time complexity:

$$T(n) = O(n-1) \times O(n-1)$$

$$[O(n-1)]^2 \approx O(n^2)$$

11911079



Loop - 1

operates  $(n-1)$  times

Loop - 2

operates  $(n-2)$  times

$$T(n) \approx O(n^2)$$

### Insertion sort

Let there be an array of size  $n$

Main code

```
for (i = 1; i < n; i++)
```

```
{
```

```
    temp = a[i]
```

```
    j = i - 1
```

```
    while (j ≥ 0 & a[j] > temp)
```

```
[
```

```
a[j + 1] = a[j],
```

```
j--
```

```
]
```

```
a[j + 1] = temp
```

```
y
```

for the worst case scenario inner loop

will be executed ' $i$ ' times for every ' $i$ '

in outer loop



11911079

Page No. / /

Date: / /

worst case.

$$1+2+3+\dots+(n-1)$$

$$\frac{n(n-1)}{2} \rightarrow \frac{n^2-n}{2}$$

$$T(n) = O\left(\frac{n^2-n}{2}\right) \approx O(n^2)$$

In place algo are those algo that does not need any extra space and produce an output in the same memory that contains the data by transforming the input 'in place'.

So quick sort and bubble sort are in place whereas merge sort is not, because it requires an extra  $O(n)$  space.

Merge sort is out of place algo

Comparison of Time Complexity

Insertion sort

Best case

$$O(n)$$

worst case

$$O(n^2)$$

Quick sort  $O(n \log n)$

$$O(n^2)$$

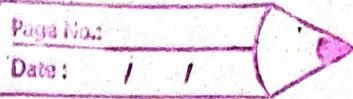
Bubble sort  $O(n)$

$$O(n^2)$$

Merge sort  $O(n \log n)$

$$O(n \log n)$$

11911079



## Merge Sort analysis

void merge sort ( int l, int r, int a )

{

if ( l < r )

{

$$\text{int middle} = \frac{(l+r)}{2};$$

merge sort ( l, middle, a )

merge sort ( middle + 1, r, a );

merge ( l, middle, r, a );

}

}

void merge ( int l, int m, int

[

$$\text{int } m_1 = m - l + 1;$$

$$\text{int } m_2 = r - m;$$

4) The value is entered by the user in the program. n stores the no. of elements and for loop take entry using insert fn.

And using inorder fn print the list in order form after suitable rotation. They are different for for diff operations. Height is used to calculate height (left and right). Rotate right to rotate right same will be rotate left. RR for Right Right rotation. LR for left Right rotation, LL for left Left rotation.

BF to calculate the binary factor and rotate according to it. inorder to print in inorder. insert to insert a new node.

3) The size of array is entered by the user by which I allocate arr size. string is entered by the user. Then merge sort function is used to do a partition of the array and then merge ~~func~~ function is used to merge the array after sorting. Merge fn check the elements of the both array and combines them in ascending order.

The array is partitioned till = very last element and each box contains of single elements.