LECTURE-24

81] ## Magnetism

- magnetization
- dipole
- para / dia magnetism
- moving charges
- → B
- induced
- electromagnets / permanent magnets

82] B and H (NOTE: $\varepsilon \rightarrow$ permittivity)

relation:

$B = \mu H$ ($\mu \rightarrow$ permeability)

$B = \mu_0 (H + M)$ { $M \rightarrow$ Magnetization, $M = \frac{\text{dipole moment}}{\text{Volume}}$ } magnetic

magnetic dipole moment

$m = IA$

also, $M = \chi H$ ($\chi \rightarrow$ susceptibility)

{range: -10^{-6} to 10^{6}}

83] Every material MUST fall into one of these categories

- Diamagnetism
- Paramagnetism
  - Superparamagnetism
- Ferromagnetism
- Ferrimagnetism
- Antiferromagnetism

84] Diamagnetism

We will discuss Classical picture

χ is −ve and small

for Superconductors: $\chi = -1$

i.e perfect diamagnetism

$B = 0 \Rightarrow \mu_0 (H + M) = 0$

implies

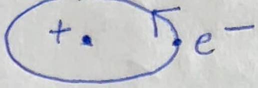
$H = -M$

$\chi = \frac{M}{H} = -1$

85] In order to describe diamagnetism

↓

we take case of **Bound electrons** (* NOTE: Thus, the Classical theory cannot be applied to metals ↓ since they have free e⁻s not bound)



Here, $m = IA = I\pi r^2$ (assuming circular orbits)

↓

or μ_m (magnetic moment)

$$= \frac{e}{t}\pi r^2$$

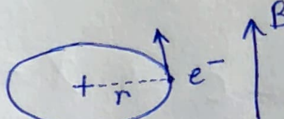
$$= e\nu\pi r^2$$

$$= e\left(\frac{v}{2\pi r}\right)\pi r^2$$

$$= \frac{evr}{2}$$

Now we can consider the case of applying magnetic field B:

↓



we will consider only the instant when B is turned on (so radius doesn't change)

(i.e. changing magnetic field from 0 to B) ↓ will induce an electric field

$$V_e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA)$$

induced emf

In free space:

$$V_e = -\frac{d}{dt}(\mu_0 H A)$$

$$= -\mu_0 \pi r^2 \frac{dH}{dt}$$

$$\Rightarrow E = \frac{V_e}{L} = \frac{V_e}{2\pi r} = \frac{-\mu_0 r}{2}\frac{dH}{dt}$$

Thus, we get: $$m\frac{dv}{dt} = -\frac{\mu_0 e r}{2}\frac{dH}{dt}$$ $\begin{pmatrix} v \to v_1 \text{ to } v_2 \\ H \to 0 \text{ to } H \end{pmatrix}$

This gives us:

$$\Delta v = \frac{-\mu_0 e r H}{2m}$$

this m is mass

Now, we know that:

$$\mu_m = \frac{evr}{2}$$

$$\Rightarrow \Delta \mu_m = \frac{e \Delta v\, r}{2}$$

$$\Delta \mu_m = \frac{-\mu_0 e^2 r^2 H}{4m}$$

$$M = \frac{\Delta \mu_m}{V} = \frac{-\mu_0 e^2 r^2 H}{4mV}$$

Hence, we obtain:

$$\chi = \frac{M}{H} = -\frac{\mu_0 e^2 r^2}{4mV}$$

LECTURE-25

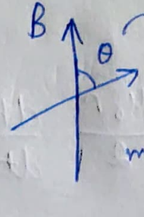
86] Paramagnetism

χ is small and +ve (and we know, $\chi = \frac{M}{H}$)

Origin is based on

- electron orbital moment
- spin moment
  - the application of Pauli's exclusion principle and Hund's rule explains paramagnetism → unpaired spins in the orbitals leads to paramagnetism
  - This is a completely Quantum Mechanical explanation

NOTE:



angle θ decides amt. of Energy needed to align μ with B

μ → mag. moment of e^-

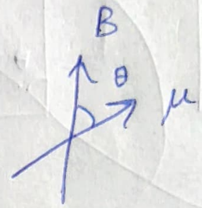
⇓

when μ gets aligned with B → this alignment leads to paramagnetism

NOTE: $\chi = \frac{C}{T}$, Curie's Law $\Rightarrow$ using Classical theory we can try to derive this

$\chi = \frac{C}{T-\theta}$, Curie-Weiss Law

87] Now, the energy of the magnetic dipole in magnetic field is given by:



$$E_p = -\mu_0 \mu_m H \cos\theta$$
$$= -\vec{\mu}_m \cdot \vec{B}$$

Thus, there is an energy cost associated with alignment of dipole with field

But also, there is thermal energy : $k_B T$
↓
Thus increasing temp. can be used to destroy paramagnetism
↓
bcz the thermal energy will try to randomize the alignment of the moments

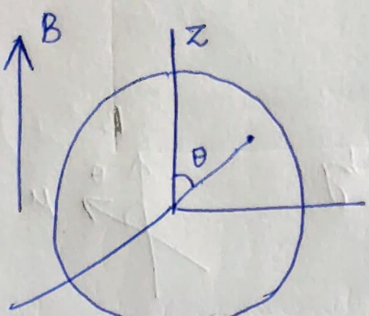
* { In Diamagnetism
↓
we give energy that changes velocity of e^-
↓
we then work with $v_2 - v_1$
Thus, although both v_2 & v_1 are affected by temp.
↓
the effect of temp. cancels out when we work with $v_2 - v_1$ }
↓
that's why Diamagnetism is temperature independent

Now, from statistical mechanics, we get:

$$n \propto \underbrace{e^{-\frac{E_p}{k_B T}}}_{\text{Boltzmann factor}} \Rightarrow n_\theta = N e^{-\frac{E_p}{k_B T}}$$

n → no. of magnetic moments with energy E_p

Now, we will deal with
avg. Magnetic moment
(it will only have z-comp., as B is along z
so others will cancel out in avg.)



$$M_z = N < \mu_m \cos\theta >$$
$$= N\mu_m < \cos\theta >$$

the temp. dependence must be in the $\cos\theta$ term

Thus ⇒ $$M_z = N\mu_m \frac{\int \cos\theta \, e^{-E_p/k_BT} dV}{\int e^{-E_p/k_BT} dV}$$

Finally, the result obtained from this is:

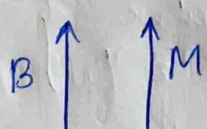
$$M_z = \frac{N\mu_m^2 H}{k_B T}$$

∴ Taking, $\frac{N\mu_m^2}{k_B} = C \Rightarrow \chi = \frac{C}{T}$

88] <u>Ferromagnetism</u>

$\chi \to$ large & +ve

When B is applied & causes alignment of moments:

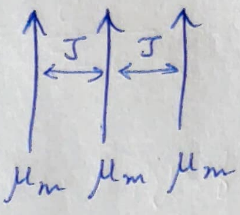


⇓

But the difference here from paramagnetism is that even on removal of field B, the alignment of moments doesn't go to zero

$B = 0$

Explanation is due to interaction energy b/w moments



$\Leftarrow$ interaction energy J

* **NOTE:** In Paramagnetism:

involves interaction b/w $\Rightarrow$ $\mu_B B \sim k_B T$

In Ferromagnetism:

involvement of interaction b/w $\Rightarrow$ $\mu_B B \sim k_B T \sim J$

LECTURE-26

outer electronic configuration:

* **NOTE:** | 1 | 1 | 1 | 1 | 1 |

$\underbrace{\text{1 1 1 1 1}}_{\text{unpaired } e^-\text{s}}$ $\Rightarrow$ thus there will be spin paramagnetism

However, if we have:

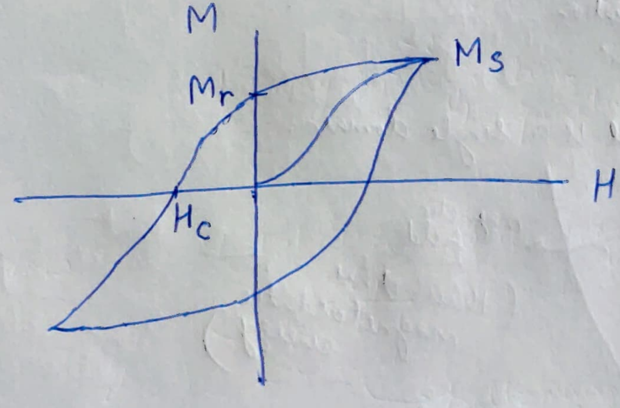
| 1L | 1L | 1L | 1L | 1L |

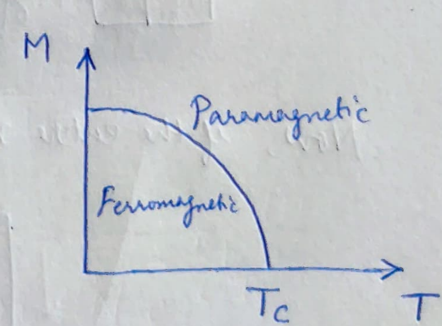
Here spin paramagnetism will not be there

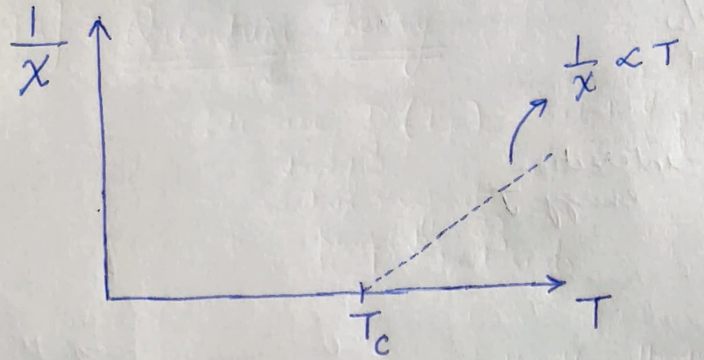
But due to there being no unpaired e^-s

* Orbital paramagnetism will also **NOT** be there

89] Ferromagnetism







90] In Ferromagnetism

↓

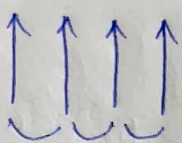
after aligning the moments → even if we remove B

↓

they should still remain aligned

↓

can be explained with an <u>exchange energy</u>



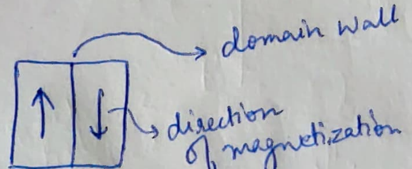
exchange energy

$J \sim k_B T_c$

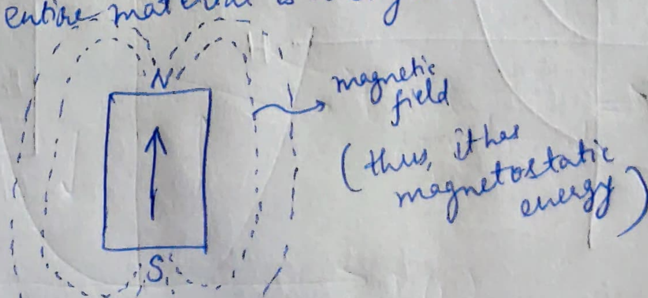
* A very important concept that explains ferromagnetic behaviour

↓

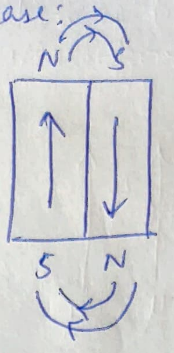
is <u>Ferromagnetic Domains</u>



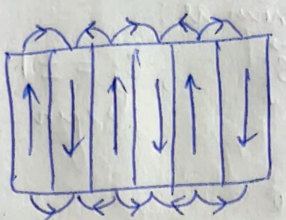
Now, if the entire material is a single domain:



In this case:

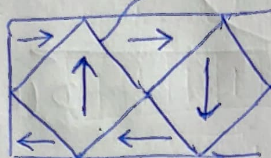


Similarly:

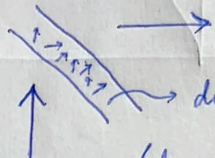


* This magnetostatic energy $\longleftrightarrow$ competes with exchange energy

In general it looks like:



→ if we look at this region
↳ the domain wall is not just a line →



→ domain wall

(here moments are trying to change their alignment)

⇓

If it has <u>never seen a magnetic field</u> *
↓
then we see such a case, where moments of domains cancel out

91] In the presence of magnetic field:

1. Domain growth → can in principle be reversed
2. Domain rotation
   ↓
   doesn't reverse by itself (needs energy to be supplied to reverse it)

<u>LECTURE - 27</u>

92] <u>Ferromagnetism</u>

$$\underbrace{\uparrow \ \uparrow \ \uparrow \ \uparrow}_{J}$$

$J \to$ +ve

<u>Antiferromagnetism</u>

$$\underbrace{\uparrow \ \downarrow}_{J} \ \uparrow \ \downarrow \ \uparrow$$

→ not nearest neighbour → it is ferromagnetic couple

$J \to$ -ve

→ there are only <u>nearest neighbour interactions</u> *

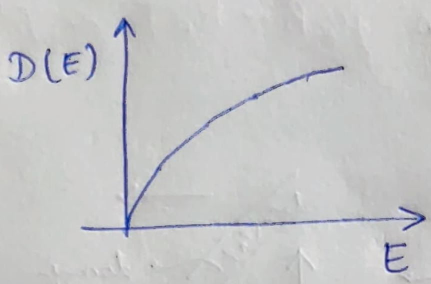
93] **Quantum Theory**

We use $D(E_F)$ ~ density of states near fermi level

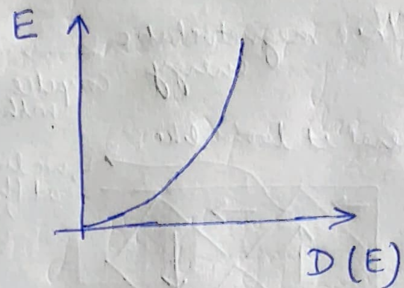
to define certain magnetic properties of materials

*[**NOTE** (in 92]) : We can write the Hamiltonian as :
$$\mathcal{H} = -J\,\vec{S_i}\cdot\vec{S_j}\ ,\quad S_i = \pm 1 \text{ (spin)}$$
]

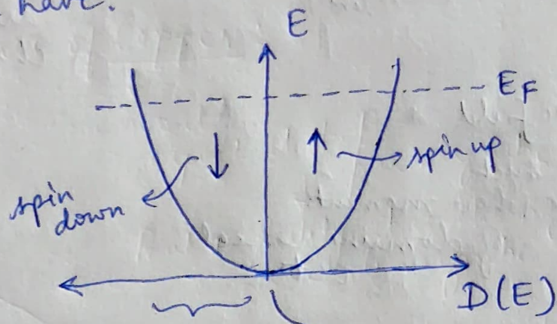
We know :



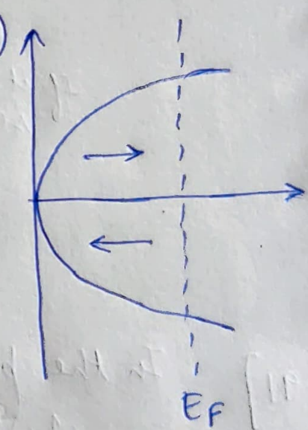
$\Rightarrow$



Here we have :



$\Rightarrow$ equivalently



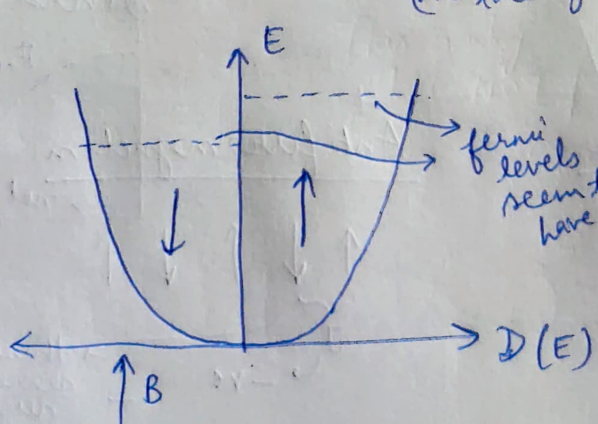
D(E) is still +ve here also

This is NOT zero for D(E)

B

magnetic field applied this way

$\Downarrow$ Some spin down will change to spin up (and take high energy states which were empty previously)

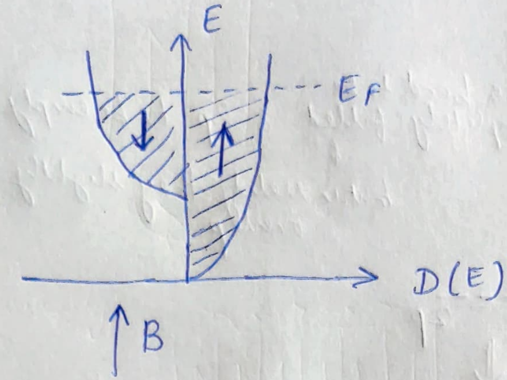


fermi levels seem to have changed $\Rightarrow$ But since finally we need to have SAME fermi level

$\Downarrow$ thus

{next page}

⇓ we get:



94] Now, to calculate susceptibility:

$$\chi = \frac{M}{H}$$ ⟶ this is the paramagnetic susceptibility here

(∵ we are allowing the spins to rotate and align themselves)

We need to find M :

$$M = \Delta N \mu_{ms}$$

and we can write :

$$\Delta N = \Delta E \, D(E_F)$$

where, $\Delta E = \mu_0 \mu_{ms} H$

$$\Rightarrow \Delta N = \mu_0 \mu_{ms} H \, D(E_F)$$

and thus, $M = \mu_0 \mu_{ms}^2 H \, D(E_F)$

Which means:

$$\chi_p = \mu_0 \mu_{ms}^2 D(E_F)$$

We can write the diamagnetic susceptibility as:

$$\chi_{dia} \propto Z r^2$$

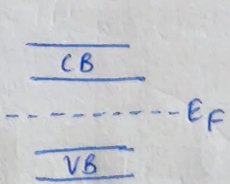
95] Taking the example of Silicon

↓

$D(E_F) = 0$ for Si

↓

Thus, $\chi_p = 0$



* { Thus, doping will alter all properties of the material

↓

electrical, optical as well as magnetic }

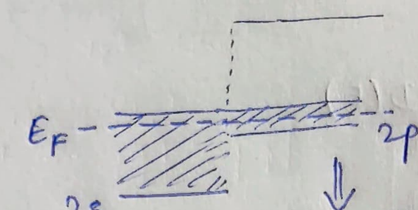
eg: here it can be made paramagnetic by doping

96] Taking the example of Beryllium:

$1s^2 2s^2$

we expect 2s band to be completely filled & 2p to be empty

but in reality they slightly overlap



Thus, E_F is actually very low in the band

which means $D(E_F)$ will be very small

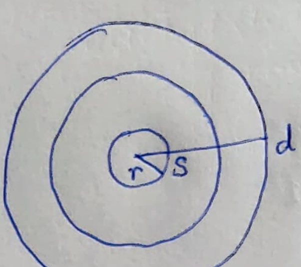
$\therefore$ Thus χ_p is also extremely small

* This is why Be shows diamagnetic behaviour

97] In the case of Copper:

we would expect it to be paramagnetic

But in reality it is showing a weak diamagnetic response



(s $\bar{e}$'s are generally much closer than d $\bar{e}$'s)

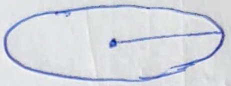
Cu : $3d^{10} 4s^1$

and, $\chi_{dia} \propto Z r^2$

Hence, $\chi_{dia} > \chi_p$ for Cu

This is why it is NOT paramagnetic

98] Magnetic moment of an electron



$$\mu_m = \frac{evr}{2}$$

Quantization : $mvr = n\hbar$ or $\frac{nh}{2\pi}$

(this condition can be easily obtained by solving the Schrödinger equation)

Thus, we get:

$$\mu_m = \frac{e}{2} \frac{nh}{2\pi m} = n \frac{eh}{4\pi m}$$

For $n=1 \Rightarrow \mu_m = \boxed{\frac{eh}{4\pi m} = \mu_B} \rightarrow$ called Bohr Magneton *

{ Generally, μ_m need not be integer multiples of μ_B in materials }

$\Downarrow$

Atomic spins $\xrightarrow[\text{integer multiples}]{\text{are}}$ $n\mu_B$ $(n = 1, 2, 3, \ldots)$

Magnetic moment of solid $\rightarrow$ $\underbrace{(\ )}_{\text{can be non-integral values}}\mu_B$

99] Ferromagnetism (FM)

N spins

$[\underset{\mu_B}{\uparrow}\ \underset{\mu_B}{\uparrow}\ \underset{\mu_B}{\uparrow}\ \underset{\mu_B}{\uparrow} \ldots]$

$\therefore \Rightarrow M = N\mu_B$

Ferrimagnetism

$\downarrow$

* Consists of 2 different types of spins (i.e. of diff. magnitudes):

$\underset{2\mu_B}{\uparrow}\ \underset{\mu_B}{\downarrow}\ \underset{2\mu_B}{\uparrow}\ \underset{\mu_B}{\downarrow}\ \underset{2\mu_B}{\uparrow}$ * (partial cancellation)

eg: material containing both Fe^{2+}, Fe^{3+}

Anti-ferromagnetism (AFM)

Here the alternating spins cancel out

$\Rightarrow M = 0$

LECTURE-28

100] Ferromagnetism

↳ exchange field : explains the basis for ferromagnetism

eg: He : $1s^2$

excited state: $1s^1$ $\boxed{\uparrow}$ $2s^1$ $\boxed{\uparrow}$ ⇒ in reality we see this

Another possibility: $1s^1$ $\boxed{\uparrow}$ $2s^1$ $\boxed{\downarrow}$

{ **NOTE:** $\boxed{\uparrow\uparrow}$

⇓

this is not possible as same state means there will be v. high repulsion

thus in reality we see: $\boxed{\uparrow\downarrow}$ }

Now when excited state is formed ⇒ $\boxed{\uparrow}$ $\boxed{\uparrow}$

this is found to be most stable (i.e. parallel spins are more stable)

↓ based on exchange energy

↓ thus this explains the basis for ferromagnetism

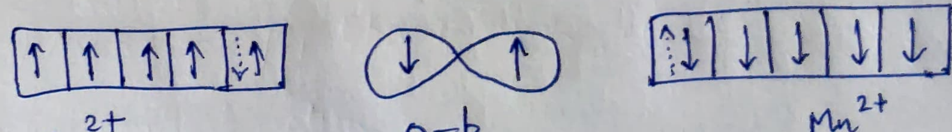
(* **NOTE:** Technically ↑ and ↓ are interchangeable

⇓

so: $\boxed{\downarrow}$ $\boxed{\downarrow}$ is also a valid excited state)

101] Taking the example of MnO

↳ involves superexchange mediated by O



Mn^{2+} O-p Mn^{2+}

↳ mediator/donor

102] Exchange field was defined primarily by Weiss

Weiss → molecular field

$$\boxed{H_\omega = \gamma M} \rightarrow \text{for FM (Ferromagnetism)}$$

$$\Rightarrow H_{tot} = H + \gamma M$$

Since, $\frac{M}{H_{tot}} = \frac{C}{T} \Rightarrow \frac{M}{H+\gamma M} = \frac{C}{T}$

$$\Rightarrow \frac{\chi}{1+\gamma\chi} = \frac{C}{T}$$

$$\Rightarrow \chi T = C + \gamma \chi C$$

$$\Rightarrow \chi (T - \gamma C) = C$$

$$\Rightarrow \chi = \frac{C}{T - \underbrace{\gamma C}_{\theta}}$$

θ : Curie temperature

* NOTE: This is valid for

$$\underset{\text{(mag. energy)}}{\mu_B H_\omega} \simeq \underset{\text{(thermal energy)}}{k_B \theta}$$

→ which can be used to calculate H_ω

If thermal energy is larger → ferromagnetism is not possible

Similarly, $\boxed{H_\omega = -\gamma M}$ → for AFM (Anti Ferromagnetism)

which gives:

$$\chi = \frac{C}{T + \underbrace{\gamma C}_{\theta_N}}$$

θ_N : Neel's temperature

For Ferrimagnetism:

$$\begin{matrix} \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow & \uparrow & \downarrow \\ A & B & A & B & A & B & A & B \end{matrix}$$

∴ We can consider M_A, M_B

Here, we can write the following expressions:

$$H_w^A = \gamma_{AA} M_A - \gamma_{AB} M_B$$

$$H_w^B = \gamma_{BB} M_B - \gamma_{AB} M_A$$

(from here it can be obtained that for AFM case, the eq^n $H_w = -\gamma M$ is true)

So now:

$$H_{tot} = H + H_w^A + H_w^B$$

$$\frac{M}{H_{tot}} = \frac{C}{T}$$

(and from here we can solve to get χ as we did previously)

LECTURE-29

[Major Syllabus]

→ 25% Before Minor, 75% after Minor

→ Do not need to know derivations for topics before Minor, only concepts are important

→ Most questions will require to think & derive (will have multiple parts)

Optical properties

1) Should know 3 types of differential eq^{n}s and how to solve them (* definitely will get 1 question from this)

2) Classical and quantum theory (Bands, etc.)

3) LASERs → population inversion, etc.

4) Emission & absorption

Electrical properties

1) Density of states
* (important)

Magnetic properties

1) Diamagnetism
↳ derivations for χ

2) Paramagnetism
↳ Q.M. derivation for χ

3) Ferromagnetic materials
↳ Domain formation
↳ Role of defects in Hysteresis loop

4) FM, AFM, Ferri

FM ↓ $\chi = \frac{C}{T-\theta}$

AFM ↓ $\chi = \frac{C}{T+\theta}$

Ferri ↳ Calculate χ for this as well *

{* Bring calculators, not to use mobiles}