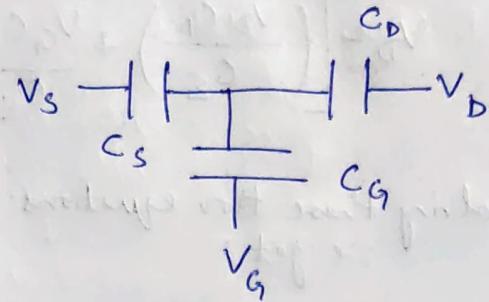
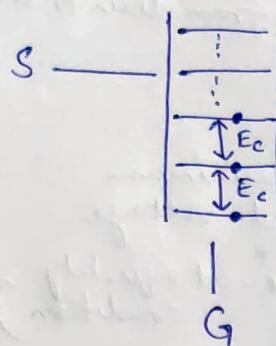


LECTURE 9 (29/01/2024)

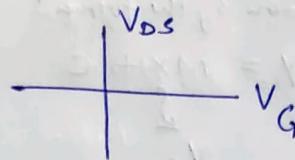
30]



$$\{ C_{\Sigma} = C_S + C_D + C_G \}$$

We are here going to consider
the absence of a source-drain bias

We want to obtain
the plot:



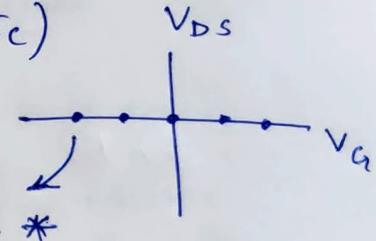
We can see that

whenever we apply V_G such
that the

energy levels of dot align
with source-drain

Current flow occurs

These will
occur at periodic
intervals (of energy gaps
equal to E_C)
these points in the plot
are called
degeneracy points



31] In order for a flow of
current to occur

we know that
voltage at junction needs to
be $\geq \frac{q}{2C_{\Sigma}}$

Now, we will also
need to take into account
the capacitances:

$$\textcircled{1}: \quad \frac{V_D C_D}{C_{\Sigma}} + \frac{V_G C_G}{C_{\Sigma}} > \frac{q}{2C_{\Sigma}}$$

For positive source
drain
i.e. e^- moving from
source to drain

For a reversed polarity case: (negative source drain
 i.e. $V_G < V_D$ moving from drain to source)

$$②: V_D - \left[\left(\frac{V_D C_D}{C_\Sigma} \right) + \frac{V_G C_G}{C_\Sigma} \right] \geq \frac{q}{2 C_\Sigma}$$

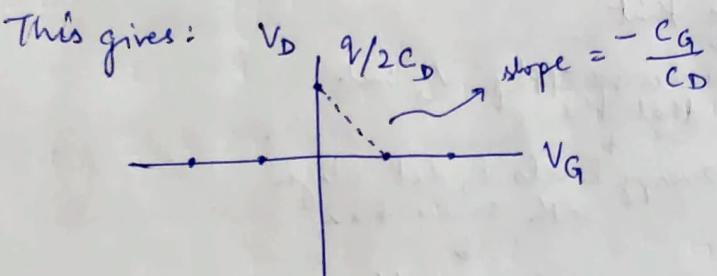
Solving these two equations (far case of minimum required voltage we get:

From ①: $V_D = \frac{q}{2 C_D} - \frac{V_G C_G}{C_D}$ (i.e. at equality condition)

this is a linear equation of the form:

$$Y = MX + C$$

$$V_D \quad V_G$$

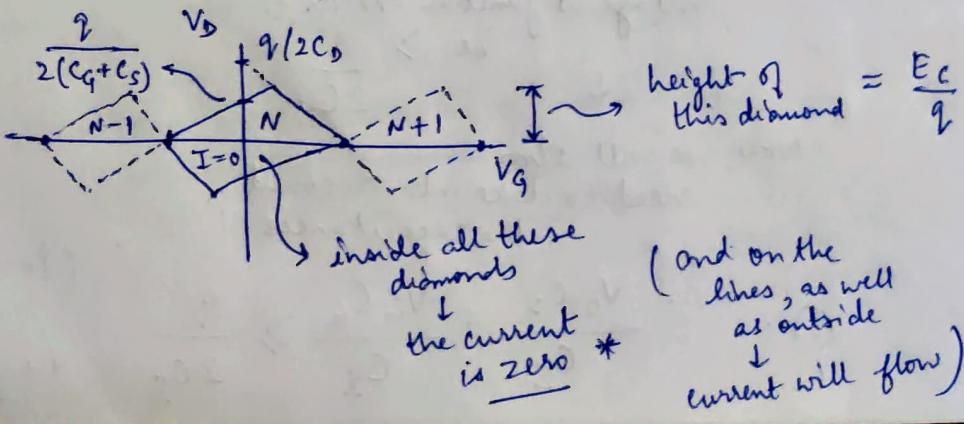


From ②: $V_D = \frac{q}{2 C_\Sigma} + \frac{V_G C_G}{C_\Sigma}$

$$\frac{1 - \frac{C_D}{C_\Sigma}}{\frac{q}{2 C_\Sigma}} = \frac{q}{2(C_G + C_S)} + \frac{V_G C_G}{C_G + C_S}$$

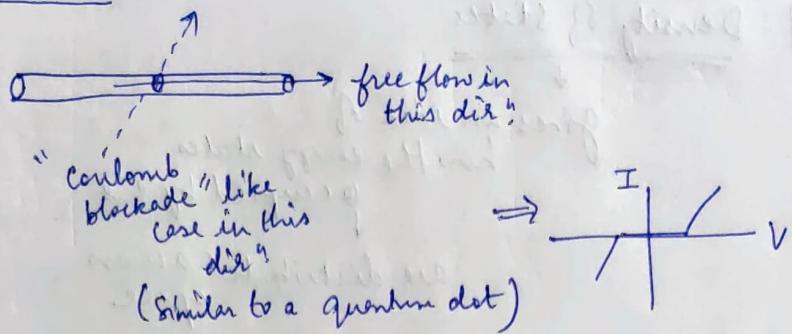
technically this value may be anywhere (above, below or equal to) in comparison to $\frac{q}{2 C_D}$

For example, let's take it to be less than $\frac{q}{2 C_D}$:



32]

1D Nanowires



thus, nanowires show different properties in diff. directions

33]

There can be two kinds of transport

Diffusive
(bulk-like)

+
where the
mean free path = λ

this means that
collisions occur
on average at
lengths of λ

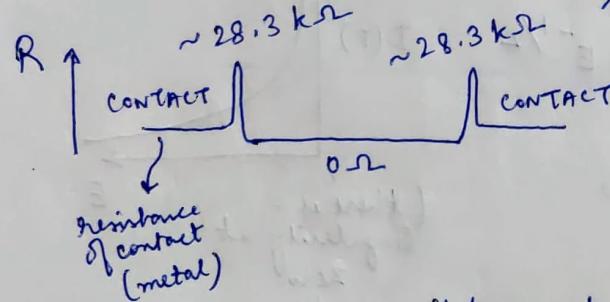
Ballistic

+
this occurs
for the case
when

$$l < \lambda_{\text{bulk}}$$

+
so for such a case
we would effectively
expect there to be

NO COLLISION



+ so we would expect resistance to be zero

But in reality we observe a resistance of $R \sim \frac{2h}{e^2}$

↓
This originates from $\frac{h}{e^2}$ resistance from both contacts

it turns out that when we make QPCs
* (quantum point contacts)

of a 1D material, then contact resistances are created

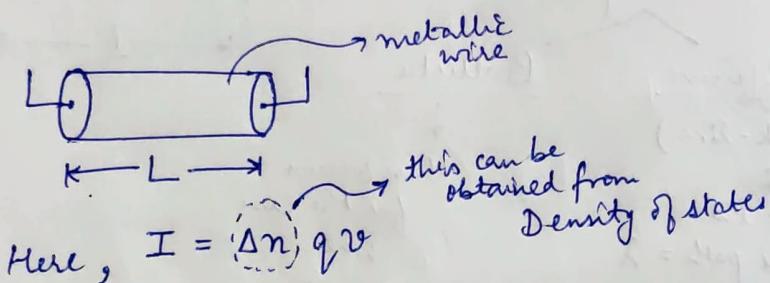
34] Density of States

gives an idea of how the energy states occupied by electrons are distributed across space

For 3D : We saw in the previous course that $D(E) \propto E^{1/2}$

Similarly it can be found for 2D, and 1D
(For 0D it is more complicated)

35]



We can assume (for now) that Ohm's law is valid : $I = G V$

We can write :

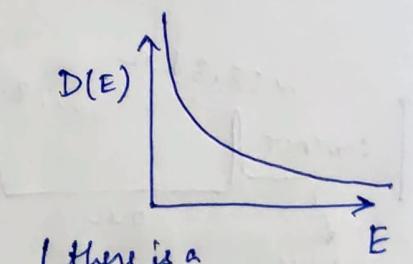
$$\Delta n = \frac{D(E_F)}{L} q V \xrightarrow{\text{voltage}}$$

For the case of 1D, $D(E) \propto E^{-1/2}$

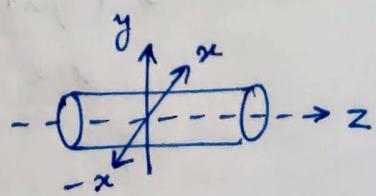
{ NOTE: In reality, for such systems, we have a constraint in two directions, and free motion in one dir! }

↓
and so energy levels are quantized only in 2 dir's!

$$E = \underbrace{E_{0z}}_{\text{free}} + \underbrace{E_{\text{many}}}_{\text{quantized}}$$



(there is a singularity at zero)
↓
this is a characteristic of 1D systems



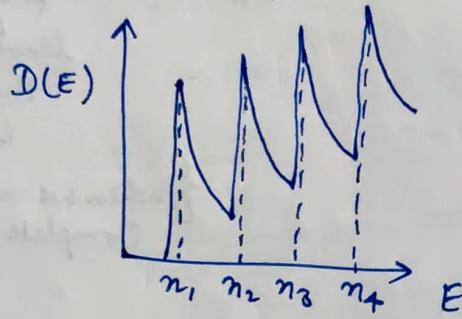
$$E = E_{0z} + \frac{n^2 h^2}{8mL^2} \quad (n^2 = n_x^2 + n_y^2)$$

$$E - E_{0z} = \frac{n^2 h^2}{8mL^2}$$

This implies that actually :

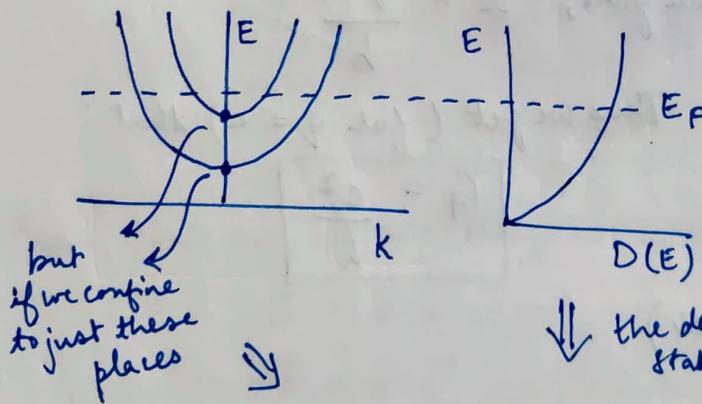
$$D(E) \propto \frac{1}{\sqrt{E - E_{0z}}} \quad \}$$

Hence, there will be a singularity (divergence) at any point where system meets a quantized state :

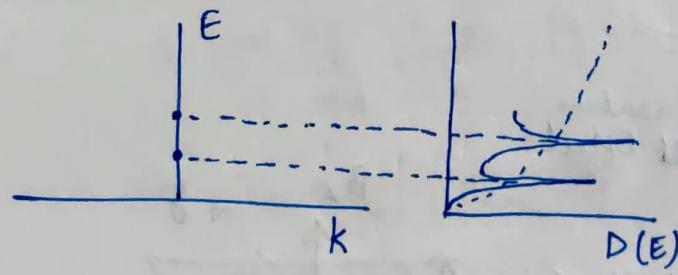


NOTE: The reason for such shapes is:

Without confinement:



↓ the density of states redistribute themselves accordingly



36] Now we have that:

$$\begin{aligned} I &= \Delta n q v \\ &= \frac{D(E_F)}{L} q V q v \\ &= \frac{q^2}{L} D(E_F) v V \end{aligned}$$

* NOTE:

This L comes from within the expression for $D(E)$

↓
we have just written it here separately

(because often people forget to include it inside $D(E)$ itself)

here we know this is of the form:

$$\frac{1}{\sqrt{E}} \cdot v \quad \} \text{ this expression ultimately only leaves behind the planet's constant}$$

{ when we make the complete calculation }

so really $\Rightarrow I = q^2 D(E_F) v V$

which gives, $I = \frac{q^2 V}{h}$

Thus we get (for $q = e$) that:

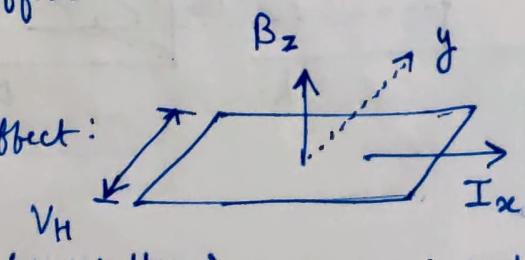
$$G = \frac{e^2}{h}$$

LECTURE 11 (01/02/2024)

36] 2D Systems

↓
involves Quantum Hall Effect

In Classical Hall Effect:



(Hall voltage) \rightarrow develops in such a scenario

Equation of motion of the e^-
which suffers scattering: (i.e. interacts with
electric & magnetic fields)

$$m \frac{d\vec{v}}{dt} = -\frac{m\vec{v}}{\tau} - e\vec{E} - e(\vec{v} \times \vec{B})$$

$\underbrace{\hspace{1cm}}$
scattering

(τ : Relaxation Time)

Here, $\vec{B} = (0, 0, B)$
 $\vec{E} = (E_x, E_y, 0)$

We define a term called
the Cyclotron frequency

$$\omega_c = \frac{eB}{m}$$

This can be expressed
in two eq's:

$$m \frac{dv_x}{dt} = -\frac{mv_x}{\tau} - eE_x + ev_y B$$

$$m \frac{dv_y}{dt} = -\frac{mv_y}{\tau} - eE_y - ev_x B$$

Now, at equilibrium: $m \frac{dv_x}{dt} = 0$ and $m \frac{dv_y}{dt} = 0$

$$\Rightarrow \left. \begin{aligned} \frac{mv_x}{\tau} &= -eE_x + ev_y B \\ \frac{mv_y}{\tau} &= -eE_y - ev_x B \end{aligned} \right] \quad (1)$$

From Ohm's law, we know that:

$$\vec{J} = \sigma \vec{E}$$

$$n e \vec{v} = \sigma \vec{E}$$

$$\Rightarrow \vec{v} = \frac{1}{ne} \sigma \vec{E}$$

$$\Rightarrow v_x = \frac{1}{ne} [\sigma_{xx} E_x + \sigma_{xy} E_y]$$

$$v_y = \frac{1}{ne} [\sigma_{yx} E_x + \sigma_{yy} E_y]$$

(2)

$$\text{, where } \sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

From our previous expressions, we get (i.e. from ①) :

$$v_x = -\frac{e\tau}{m} E_x + \omega_c \tau v_y$$

$$v_y = -\frac{e\tau}{m} E_y - \omega_c \tau v_x$$

Simplifying :

$$v_x = -\frac{e\tau}{m} E_x - \frac{\omega_c e \tau^2}{m} E_y - \omega_c^2 \tau^2 v_x$$

$$\Rightarrow v_x = \frac{-\frac{e\tau}{m} E_x - \frac{\omega_c e \tau^2}{m} E_y}{1 + \omega_c^2 \tau^2} \quad - ③$$

$$\text{and, } v_y = -\frac{e\tau}{m} E_y + \frac{\omega_c e \tau^2}{m} E_x - \omega_c^2 \tau^2 v_y$$

$$\Rightarrow v_y = \frac{-\frac{e\tau}{m} E_y + \frac{\omega_c e \tau^2}{m} E_x}{1 + \omega_c^2 \tau^2} \quad - ④$$

Comparing ③ and ④ with ② gives us :

$$\sigma_{xx} = \frac{ne^2 \tau / m}{1 + \omega_c^2 \tau^2} \quad \sigma_{xy} = \frac{ne^2 \omega_c \tau^2 / m}{1 + \omega_c^2 \tau^2}$$

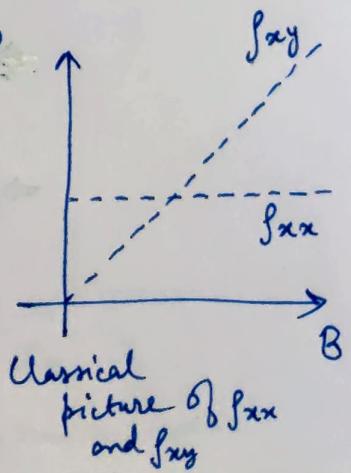
$$\sigma_{yx} = \frac{ne^2 \tau / m}{1 + \omega_c^2 \tau^2} \quad \sigma_{yy} = \frac{ne^2 \omega_c \tau^2 / m}{1 + \omega_c^2 \tau^2}$$

37] If we want f , we get it by taking the matrix inverse of σ :

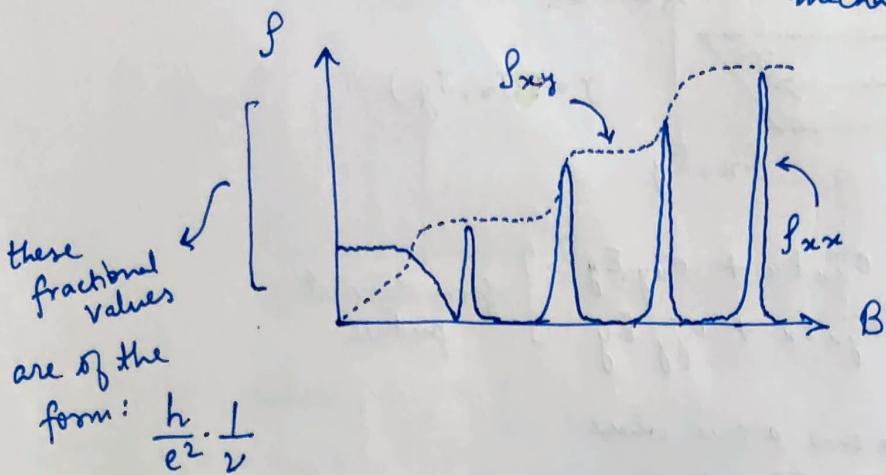
$$f = (\sigma^{-1}) = \frac{1}{ne^2 \tau / m} \begin{bmatrix} 1 & \omega_c \tau \\ -\omega_c \tau & 1 \end{bmatrix}$$

$$\text{i.e. } \Rightarrow f_{xx} = \frac{1}{ne^2 \tau / m}$$

$$f_{xy} = \frac{\frac{eB}{m} \tau}{ne^2 \tau / m} = \frac{B}{ne}$$



However, it turns out that at high values of magnetic field, this picture breaks down (and it can actually only be explained quantum mechanically)



where, v is an integer [Integer Quantum Hall Effect]

NOTE: In reality plateaus may also occur on fractional values for some materials (eg: $v = \frac{3}{5}, \frac{2}{3}, \text{etc...}$)

Experimentally it is found that they neither obey Bose-Einstein nor Fermi-Dirac statistics

this is called Fractional Quantum Hall Effect

(one theory to explain this is by considering that the electron "splits" up into (unobservable) particles called "anyons")

[There are other theories as well]

38] We have the conductivity matrix as:

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \Rightarrow \begin{matrix} \text{taking matrix} \\ \text{inverse:} \end{matrix}$$

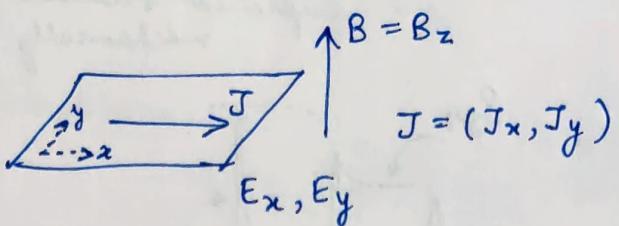
$$G_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

$$\sigma_{xx} = \frac{G_{xx}}{G_{xx}^2 + G_{xy}^2}$$

From the plot we see that there is a region where $G_{xx} = 0$ and $G_{xy} \neq 0 \Rightarrow \sigma_{xx} = 0$ also. But how is this possible?

LECTURE 12 (05/02/2024)

39] We were considering
the case for a 2D material:



$$\begin{aligned} J_x &= \sigma_{xx} E_x + \sigma_{xy} E_y \\ J_y &= \sigma_{yx} E_x + \sigma_{yy} E_y \end{aligned} \quad] \text{ from classical picture}$$

But we saw a case where

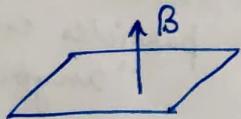
$$\left\{ \begin{array}{l} \sigma_{xx} = 0 \\ g_{xx} = 0 \end{array} \right. \quad (\text{where } g_{xy} \neq 0 \text{ and } \sigma_{xy} \neq 0)$$

it turns out
that this actually is possible

(Technically, this can be seen
properly only from quantum
picture)

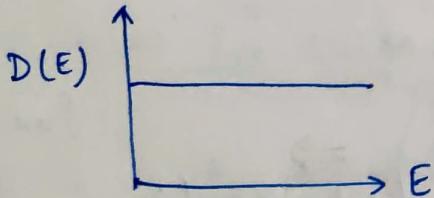
* But conceptually, σ_{xx} being zero tells
us that current flow does not occur,
while g_{xx} tells us that dissipation is zero)

40] Landau Levels

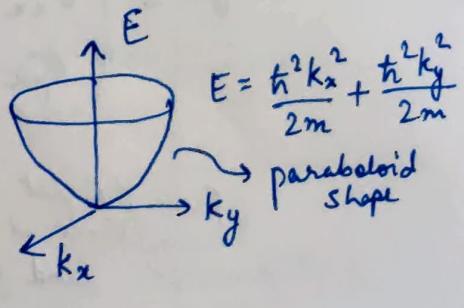


For 2D case
the density of states is:

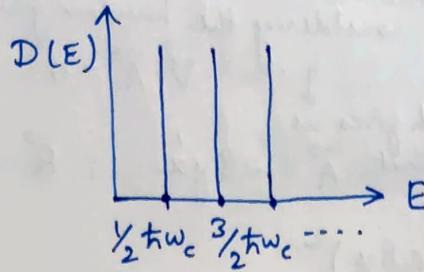
$$D(E) = \text{constant} \propto \frac{\pi m}{h^2} A$$



For the x and y directions,
there is no quantization,
particle moves
freely

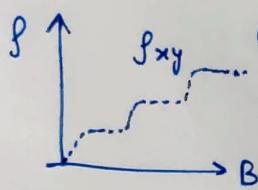


However, when we apply the magnetic field
 ↓ (at all values of magnetic field)
 quantization occurs



{ i.e. the energy levels are :
 ↓
 $\frac{3}{2} \hbar \omega_c$ } these are called Landau levels
 $\frac{1}{2} \hbar \omega_c$ }
 separation between energy levels is $\hbar \omega_c$ }

where, $\omega_c = \frac{eB}{m}$
 { NOTE: $k_B T \ll \hbar \omega_c$ } ↓ i.e. increase in B leads to increase in ω_c



Conceptually, these lead to the plateaus we saw (due to filling of each of these Landau levels)

∴ separation will also increase

41] We have:

$$\mathcal{H} = \frac{\vec{p}_x^2}{2m} + \frac{\vec{p}_y^2}{2m} = \frac{|\vec{p}|^2}{2m}$$

(Hamiltonian originally)

Also, we then applied a magnetic field :

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

and, \mathcal{H} will change to : $\mathcal{H} = \frac{1}{2m} (\vec{p} + e\vec{A})^2$

$$\text{We have taken } \vec{B} = B_z \hat{k}$$

or

$$B \hat{z}$$

We will use the Landau gauge : $\vec{\nabla} \cdot \vec{A} = 0$

This will give us :

$$\boxed{\vec{A} = -B y \hat{x}}$$

LECTURE 13 (07/02/2024)

42] We have :

$$\mathcal{H} = \frac{1}{2m} (\vec{p} + e\vec{A})^2 , \text{ and } \vec{B} = \vec{\nabla} \times \vec{A}$$

where we are considering the Landau gauge:
 $\vec{\nabla} \cdot \vec{A} = 0$
 which gives us
 that $\vec{A} = -By\hat{x}$; $\vec{B} = B\hat{z}$

$$\therefore \mathcal{H} = \frac{1}{2m} p_y^2 + \frac{1}{2m} (p_x - eBy)^2$$

$$\Rightarrow \mathcal{H} = \frac{1}{2m} p_y^2 + \frac{1}{2m} (\hbar k_x - eBy)^2 \quad \{ \text{using } p_x = \hbar k_x \}$$

$$\text{Now, let } y_0 \equiv \frac{\hbar k_x}{eB} \Rightarrow \mathcal{H} = \frac{1}{2m} p_y^2 + \frac{e^2 B^2}{2m} (y - y_0)^2$$

Finally, using $\omega_c = \frac{eB}{m}$:

$$\boxed{\mathcal{H} = \frac{1}{2m} p_y^2 + \frac{1}{2} m \omega_c^2 (y - y_0)^2}$$

Hamiltonian for 1D Harmonic Oscillator



The energy corresponding to this is given by:

$$E_n = (n + \frac{1}{2}) \hbar \omega_c$$

$$\begin{array}{c} \frac{5}{2} \hbar \omega_c \\ \frac{3}{2} \hbar \omega_c \\ \frac{1}{2} \hbar \omega_c \end{array}$$

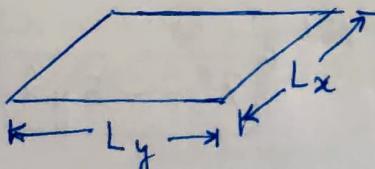
These quantized levels are called LANDAU ENERGY LEVELS

43] Since there is no x variable in the Hamiltonian

↑
 we can say that
 the x values
 can have the same energy
 ↓

Thus, this gives us degeneracy

If y_0 is the point about which the e^- oscillates :



$$l_y l_x = \text{area}$$

$$\text{where, } 0 \leq y_0 \leq l_y$$

$$\Rightarrow 0 \leq \frac{\hbar k_x}{eB} \leq l_y$$

Thus, the max^m value for k_x is :

$$k_x = \frac{eB l_x}{\hbar}$$

Now, degeneracy will be same as density of states

i.e. all values of k_x
will correspond to the same energy level

so we want to count how many values k_x can take

$$\Rightarrow \text{We know: } k_x = \frac{2\pi n_x}{L_x}$$

(to determine degeneracy)

$$\Rightarrow n_x = \frac{L_x k_x}{2\pi} \Rightarrow \text{No. of states}$$

with same energy

is given by max^m value of n_x

A (area)

$$\therefore \text{Degeneracy} = \frac{l_x k_{x \max}}{2\pi} = \frac{l_x eB l_y}{2\pi \hbar} = \frac{eB (l_x l_y)}{\hbar}$$

Using definition of flux, $\phi = BA$:

$$\text{Degeneracy} = \frac{e}{h} \phi$$



Thus, we get:

$$\boxed{\text{Degeneracy} = \frac{\phi}{\phi_0}}$$

* { where $\phi_0 \equiv \frac{h}{e}$ }

Hence, we see that degeneracy
will depend on the magnetic field

low B

\Rightarrow less e's
occupying low energy states

high B

\Rightarrow high no. of e's
occupying energy states of high energy

44] We saw previously:

$$\rho_{xy} = \frac{h}{e^2} \frac{1}{2}, \text{ and also: } \rho_{xy} = \frac{B}{ne}$$

$$\Rightarrow \text{Thus: } n = \frac{eB2}{h} = \frac{1}{A} \cdot \frac{(e)(BA)}{h} 2 \Rightarrow \therefore n = \text{density of states}$$

(i.e. states per unit area)

We can also define:

$$N = \frac{e}{h} 2 BA \Rightarrow N = 2 \frac{\phi}{\phi_0}$$

NOTE: It is the field induced quantization
 which leads to the collapse
 of energy levels

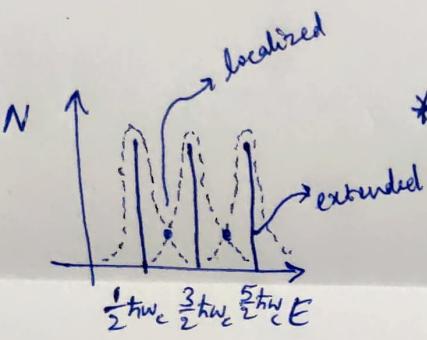
NOTE: Once the lower energy level
 is filled
 e^- 's can't go anywhere until the
 energy cost of $\hbar\omega_c$ is paid
 & thus we get zero f state

45] Why do we observe this
 for a range of magnetic field values

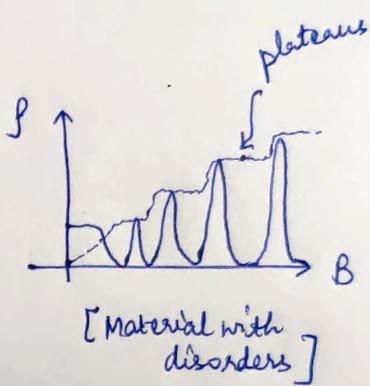
& why not only certain values
 of magnetic field?

Reason: These plateaus
 are observed due to
disorders *

* A clean material
 won't show this effect, &
quantization would occur only
at some integer values



[Material with
 disorders
 starts also
 developing
 localized states]



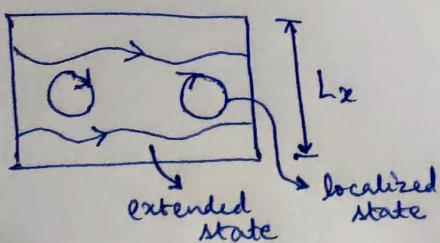
The reasoning here is that these
 energy states start
 to broaden into

extended states & localized states
 ↓ ↓
 these are close to the original these are in the middle
 ↓
 and these localized states form due to disorder & defect

46] Only extended states
 conduct current

whereas localized states do not conduct

→ & thus these
 do NOT contribute
 to resistance



* Here, the no. of defects should follow the conditions:

$v \ll k_B T$] where, v = disorder
 $v \ll \hbar\omega_c$

*
 { Remaining content
 of this lecture
 ↓
 is attached as a
 separate sheet
 on this
 page itself }

[Next Lecture Continues]
 Below

LECTURE 14 (08/02/2024)

46] Nanophotonics

↓
 some of the most initial observations involving this
 ↓
 were objects (such as glass) with metal particles (eg: Cu, Au, Al)
 ↓ embedded
 leading to generation of colours when light transmitted through them

Consider motion under electric field. (for an electron):

$$m\ddot{x} = e\vec{E}, \text{ where } \vec{E} = E_0 e^{i\omega t}$$

$$\text{Trying: } x = x_0 e^{i\omega t}$$

$$\Rightarrow -m\omega^2 x_0 = eE_0$$

$$\Rightarrow x_0 = -\frac{eE_0}{m\omega^2}$$

Now, the magnitude of dipole moment is:

$$|p| = ex_0$$

In general, $p = ex_0 e^{i\omega t}$

For n electrons:

$$P = n e x_0 e^{i\omega t}$$

Using the expression we got for x_0 , this gives:

$$P = -\frac{n e^2 E_0}{m \omega^2} e^{i\omega t}$$

Since we also know that, polarization $P = \epsilon_0 \chi E$

$$\Rightarrow \boxed{\chi(\omega) = -\frac{n e^2}{m \omega^2 \epsilon_0}}$$

Using which we define: $\omega_p^2 = \frac{n e^2}{m \epsilon_0}$

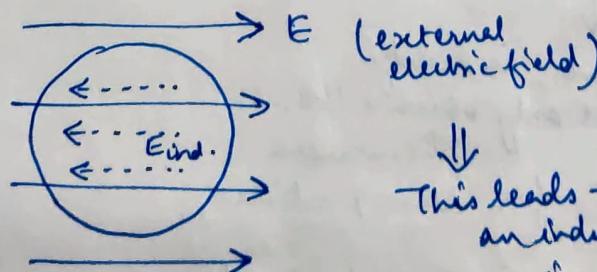
* Plasma Frequency

(Technically, this is the plasma freq. of the bulk, so we could call it ω_{pb})

$$[\text{Thus, } \chi(\omega) = -\frac{\omega_p^2}{\omega^2}]$$

* NOTE: We will see now that, for a sphere we will get the plasma freq. as: $\omega_{sp} = \frac{\omega_{pb}}{\sqrt{3}}$

47]



This leads to an induced electric field inside the sphere (which is a * conductor)

From electromagnetism: $\vec{E}_{ind.} = -\frac{P}{3\epsilon_0}$

$$\therefore \vec{E}_{tot} = \vec{E} + \vec{E}_{ind.} \Rightarrow \therefore E_{tot} = E - \frac{P}{3\epsilon_0}$$

We have:

$$\begin{aligned} P &= \epsilon_0 \chi E_{\text{tot.}} \\ &= \epsilon_0 \chi \left(E - \frac{P}{3\epsilon_0} \right) \end{aligned}$$

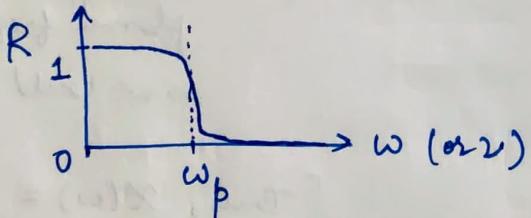
Solving this for P gives us:

$$\begin{aligned} P &= \frac{\epsilon_0 \chi E}{1 + \chi/3} \\ &= \frac{\epsilon_0 \chi(\omega) E}{1 - \frac{ne^2}{(3m\epsilon_0)} \frac{1}{\omega^2}} \end{aligned}$$

From here we get the plasma freq:

$$\omega_{sp}^2 = \frac{ne^2}{3m\epsilon_0} = \frac{\omega_{pb}^2}{3}$$

* NOTE: The reason why the plasma freq. is related to colors, is related to reflectance vs freq. curve (for metals):



* NOTE: Technically the analysis we have performed should apply for any spherical particle (of arbitrary size, since we did not take size into any of our analysis, and it is size independent)

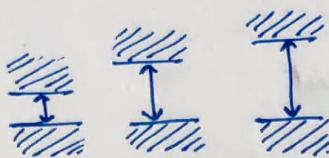
However, in reality we only observe these phenomena for Nanoparticles

The reason for this is that at larger scales, certain damping effects due to electromagnetic fields occur

{ However, we won't study them }
here in this course

NOTE: In the case of semiconductors

Colour variation
is explained due to
variation in band gaps



48] Near-Field Light

Generally, we cannot distinguish b/w points which are arbitrarily close

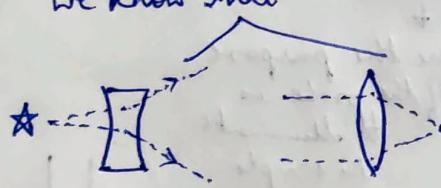
due to the diffraction limit

(which occurs on size $\sim \lambda$)

However, near-field light allows us to go beyond this limit

LECTURE 15 (12/02/2024)

49] We know that



Concave lens
diverges light rays

Convex lens
converges the light rays

But can this convergence occur upto an absolutely small size?

Answer: No
(it will be limited due to diffraction)

In principle the small separation that can be resolved

Can be obtained from:

$$\Delta = 200 \text{ nm}$$

taking

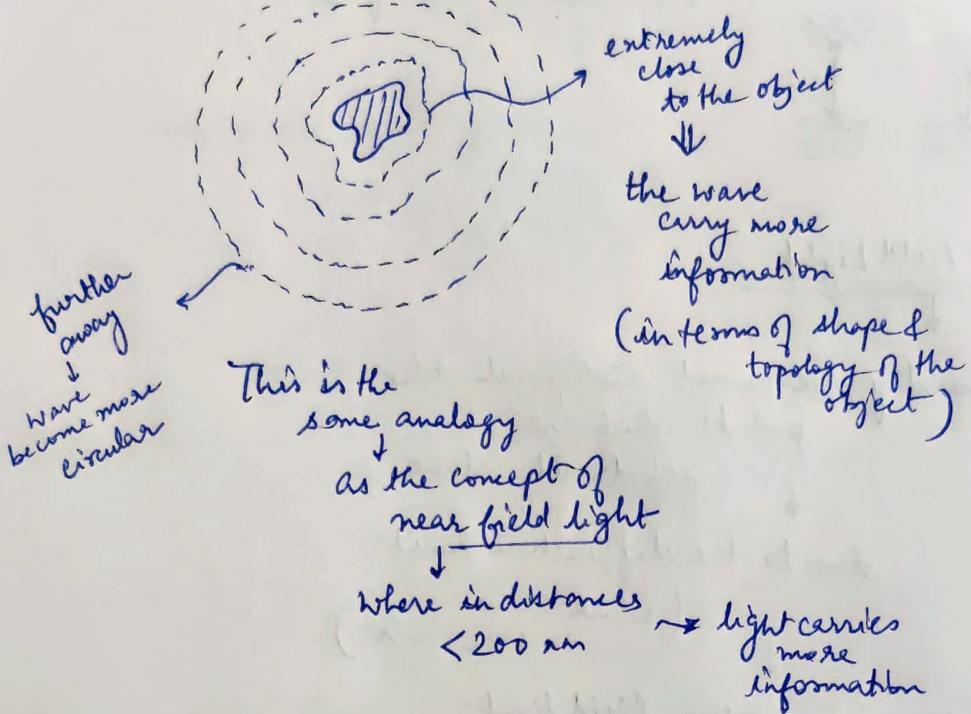
$$\lambda \approx 400 \text{ nm}$$

$$\Delta = \frac{1.22 \lambda}{2n \sin \theta}$$

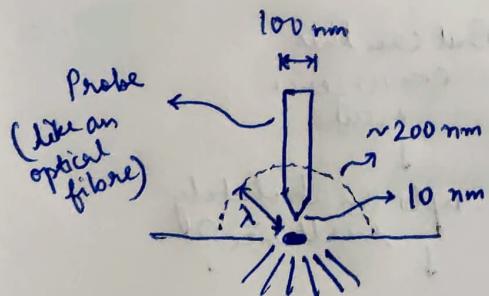
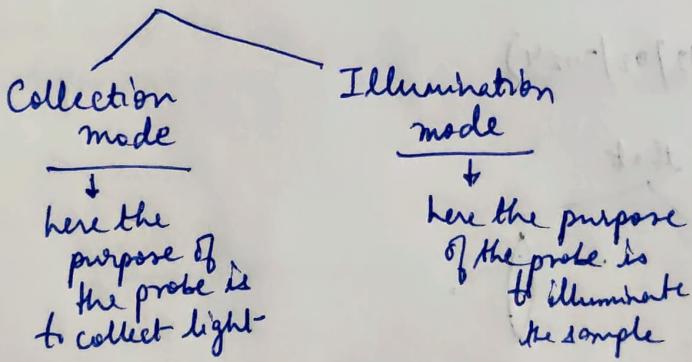
$$(\text{i.e., } \Delta \sim \frac{\lambda}{2})$$



NOTE: For the case of an object producing ripples in water:

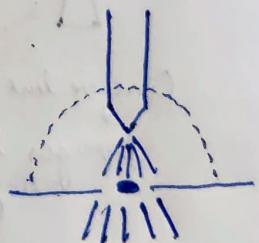


50] There are two main modes involved



{ in either case distance between probe & sample should be within range of 20 nm }

(For near-field light phenomena)



NOTE: STM_z (Scanning Transmission Microscopes) are used to detect how close we are moving the probe to the sample.

and then the tip of the STM is replaced with a probe

which is why this concept was discovered later

(i.e. moving the probe about precisely)

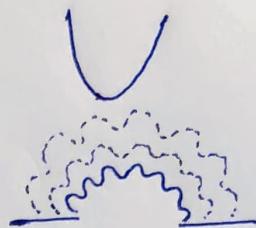
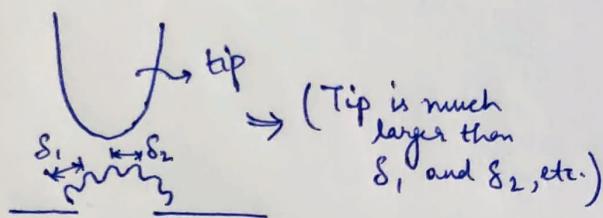
51] The advantage of Near Field Imaging versus AFM

(only height profiles)

In AFM
also we
try to get
the shape

However if the
AFM tip is
larger than
surface morphological
features:

e.g:



Then here AFM
will not give us
the information

* NOTE: In near-field
we go close enough
such that the effects of
dispersion and interference
are minimized

NOTE:



further away
becomes "circular"
(dispersion effect)

closer
wavefronts
contain more light
in higher frequency
ranges as well
(whereas
further away
the higher freq.
light
diminishes)

This frequency
change is what is captured
by the Near-field
Imaging

In Near-field Imaging

+
we directly
see the light
from the object
(i.e. the
wavefront
itself)

+
and this contains
the information
of the
sample surface
morphology