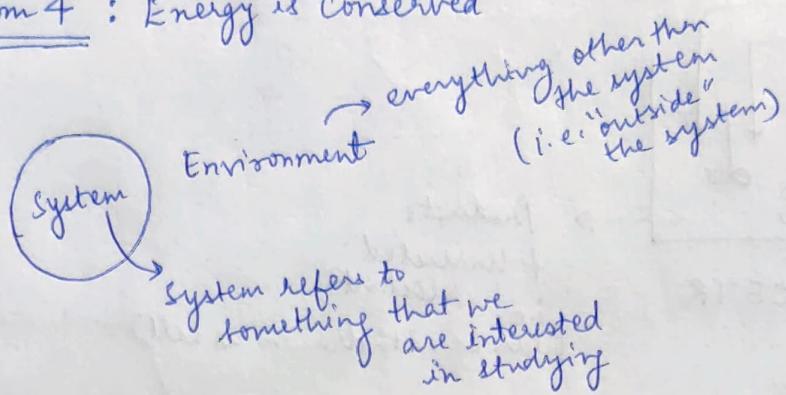


AXIOM 4

67] Axiom 4 : Energy is conserved



Types of systems :

- (i) Isolated system \rightarrow No exchange of Q, W or mass b/w system and environment
 - (ii) Closed system \rightarrow * NOTE: $\Delta U = Q \pm W$ is valid only for closed systems
 - (iii) Open system \rightarrow only mass exchange allowed
 - ↳ here all is not 3 exchanges are possible (i.e. Q, W and mass exchange)
- (But we will be studying open systems)

LECTURE-17

68] For Isolated System

(eg: Universe) ↓

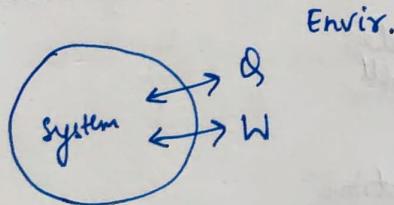
At $t_1 \rightarrow$ Energy is E_1

At $t_2 \rightarrow E_2$

$$\Rightarrow \Delta E = E_2 - E_1 = 0$$

* this is first law of thermodynamics for an isolated system

69] For Closed System

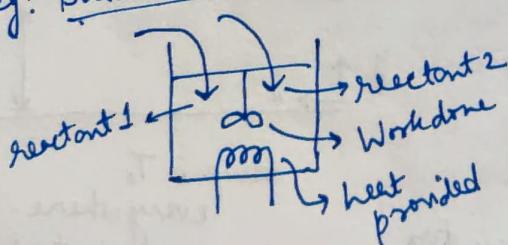


$$E_1 \leftarrow t_1$$

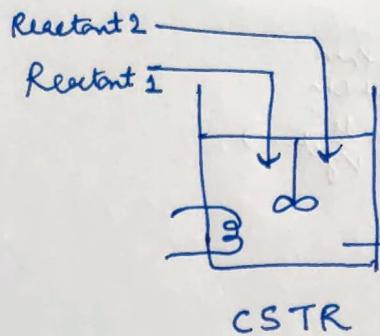
$$E_2 \leftarrow t_2$$

$$\text{Then, } \Delta E = E_2 - E_1$$

eg: Batch Reaction



70] Open system



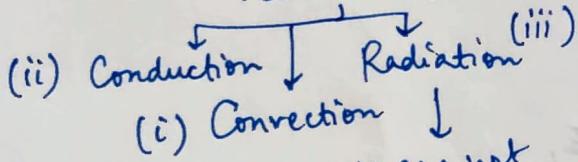
Products
+ Unreacted
reactants
(i.e., mixture)
(can be withdrawn as well)

71] Axiom 4 : Energy is conserved

$$\text{Rate of accumulation of } K.E + I.E. = \text{Rate of inflow of } (K.E + I.E.) \text{ in C.V.} - \text{Rate of outflow of } (K.E + I.E) \text{ from C.V.}$$

$$+ \text{Rate of inflow of heat by convection} - \text{Rate of outflow of heat by conduction into C.V.}$$

* NOTE: Modes of Heat transfer are



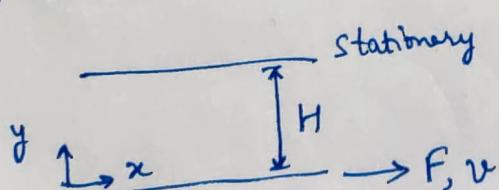
\downarrow
we are not taking into account the effects of radiation in energy conservation equation

\because generally $V.$ small
 \therefore However at $V.$ high Temperatures it becomes important to include this as well

\mp Rate of work done by/on the system C.V.

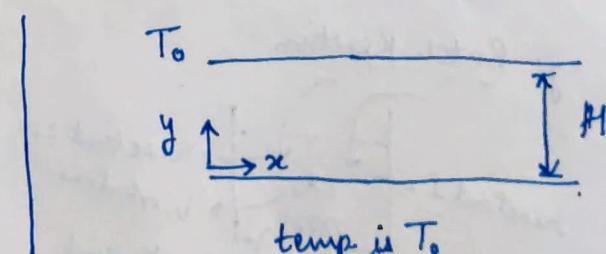
+ Rate of heat addition by some heat source

72] Fourier's Law of Heat Conduction

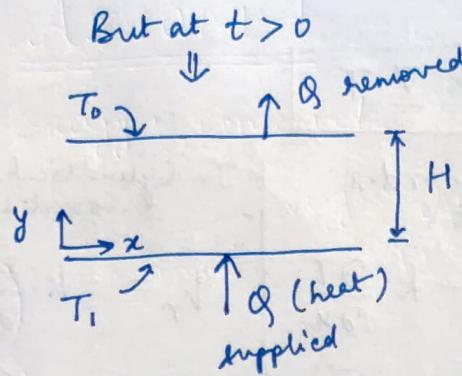


In Newton's Law of Viscosity

$$\text{we obtain: } \tau_{yx} = -\mu \frac{dv_x}{dy}$$



temp is T_0 everywhere at $t=0$



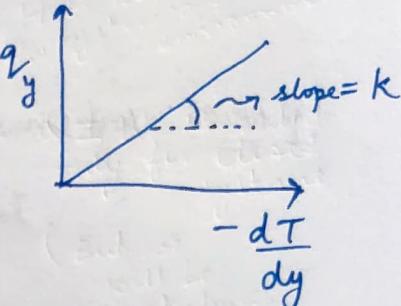
From Fourier's law we see:

$$q_y = \frac{Q}{A}$$

and thus we see, $q_y \propto \frac{dT}{dy}$

Thus,

$$q_y = -k \frac{dT}{dy}$$



(where k = thermal conductivity)

* { NOTE: similarly for mass transfer later on \rightarrow we have Fick's law of diffusion there the coefficient is D_{AB} }

Momentum diffusivity $\rightarrow \mu$ (units: $\frac{g}{cm \cdot sec}$) \downarrow mass diffusivity

Thermal diffusivity $\rightarrow k$ (units: $\frac{cal}{cm \cdot sec \cdot K}$) }

73] We can define the following:

$$\nu = \frac{\mu}{\rho}$$

\leftarrow called
* Kinematic Viscosity
 \downarrow
2 has some units: $\frac{cm^2}{sec}$ \rightarrow thus it is also called
(as in mass diffusivity) * Momentum diffusivity

and, $\alpha = \frac{k}{\rho C_p}$ \leftarrow called
* Thermal diffusivity
(bcz it also has the units: $\frac{cm^2}{sec}$)

74] We can write:

$$\underline{q} = -k \nabla T$$

In rectangular coord.s
we get:

$$q_x = -k \frac{\partial T}{\partial x}$$

$$q_y = -k \frac{\partial T}{\partial y}$$

$$q_z = -k \frac{\partial T}{\partial z}$$

In cylindrical
coord.s:

$$q_r = -k \frac{\partial T}{\partial r}$$

$$q_z = -k \frac{\partial T}{\partial z}$$

$$q_\theta = -k \cdot \frac{1}{r} \frac{\partial T}{\partial \theta}$$

{ NOTE: You can check all these from the tables }

In spherical coord.s:

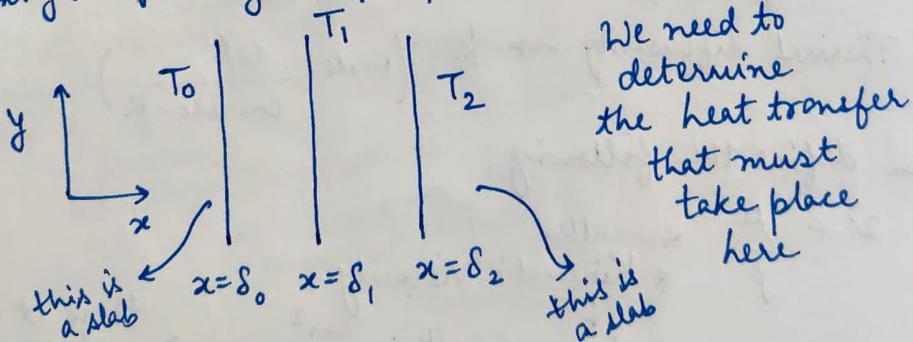
$$q_r = -k \frac{\partial T}{\partial r}$$

$$q_\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta}$$

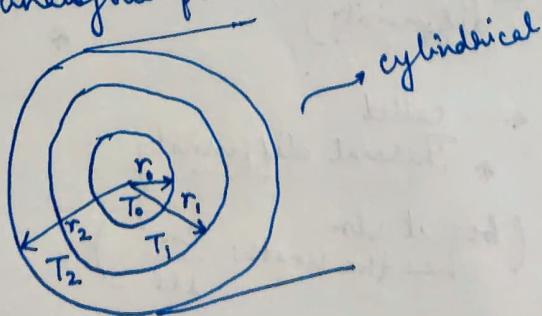
$$q_\phi = -k \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi}$$

75] Conduction in Solids : Shell Energy Balance

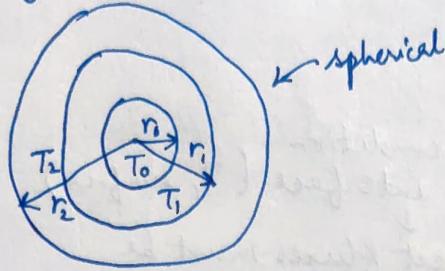
Taking the following example problem:



Another analogous problem is:



Very similarly another analogous problem is:



* NOTE: In all these 3 cases $\rightarrow T_0$ and T_2 are fixed

(thus, they won't be influenced by surroundings)

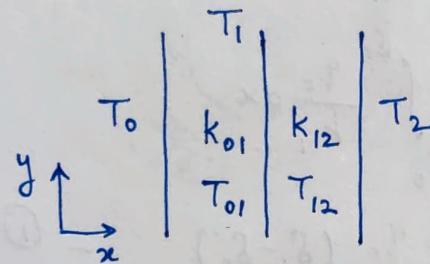
But we can also solve

conjugate problems where T_0 and T_2 may

change due to heat loss to

surroundings (which is at T_{atm})

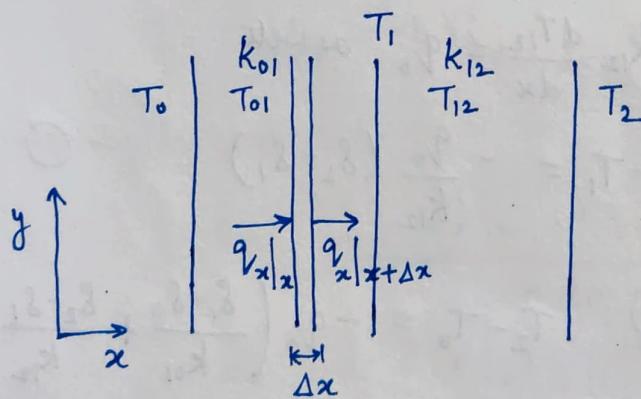
Coming to our problem:



T_0 and T_2 are fixed

Heat must be provided from T_0 side
and removed from T_2 side
(to determine this
heat transfer
in the solution below)

↓ drawing bigger picture
(to take shell)



* Assumptions

Steady state

k_{01} and k_{12} are
constants

$T(x) \approx$ i.e. temp.
is only a
function of x

thus, q_x is there

$$\text{and } q_y = 0 \\ q_z = 0$$

$$\therefore 0 = 0 - 0 + q_x|_x \text{ WH}$$

$$- q_x|_{x+\Delta x} \text{ WH} + 0$$

Dividing by volume of control volume (C.V.) \Rightarrow i.e. $WH \Delta x$

$$\Rightarrow \frac{q_x|_x - q_x|_{x+\Delta x}}{\Delta x} = 0$$

$$\text{Taking } \Rightarrow -\frac{dq_x}{dx} = 0$$

$$\Rightarrow q_x = C_1$$

We know the boundary condition
that at the interface (T_1 , surface)
* heat fluxes must be same

$$\therefore -k_{01} \frac{dT_{01}}{dx} = C_1 \Rightarrow -k_{01} \frac{dT_{01}}{dx} = q_0 \quad \text{Thus we can take:}$$

$$\text{and, } -k_{12} \frac{dT_{12}}{dx} = C_1$$

Boundary condition

$\therefore (q_0 \text{ heat at surface } T_0)$

$$\text{Thus, we get: } dT_{01} = -\frac{q_0}{k_{01}} dx$$

$$\Rightarrow \int_{T_0}^{T_1} dT_{01} = - \int_{\delta_0}^{\delta_1} \frac{q_0}{k_{01}} dx$$

$$\Rightarrow T_1 - T_0 = -\frac{q_0}{k_{01}} (\delta_1 - \delta_0) \quad \text{--- (1)}$$

$$\text{Similarly, as } -k_{12} \frac{dT_{12}}{dx} = q_0 \text{ as well}$$

$$\Rightarrow T_2 - T_1 = -\frac{q_0}{k_{12}} (\delta_2 - \delta_1) \quad \text{--- (2)}$$

$$\text{Adding (1) and (2): } T_2 - T_0 = -q_0 \left(\frac{\delta_1 - \delta_0}{k_{01}} + \frac{\delta_2 - \delta_1}{k_{12}} \right)$$

$$\therefore \Rightarrow q_0 = \frac{T_0 - T_2}{\frac{\delta_1 - \delta_0}{k_{01}} + \frac{\delta_2 - \delta_1}{k_{12}}}$$

Since $\delta_1 - \delta_0 = \text{thickness of slab 1} = \Delta x_1$,

and $\delta_2 - \delta_1 = \text{thickness of slab 2} = \Delta x_2$

$$\text{We can say: } q_0 = \frac{T_0 - T_2}{\frac{\Delta x_1}{k_{01}} + \frac{\Delta x_2}{k_{12}}}$$

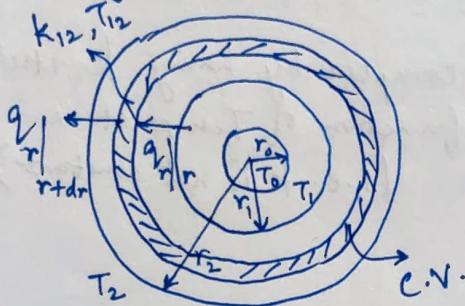
(Similarly if we had 'n' slabs, we would get: $q_0 =$

$$\frac{T_0 - T_2}{\frac{\Delta x_1}{k_{01}} + \frac{\Delta x_2}{k_{12}} + \frac{\Delta x_3}{k_{23}} + \dots + \frac{\Delta x_n}{k_{n-1,n}}}$$

{ this is analogous to:

$$\underbrace{\text{I}}_{V} \xrightarrow[R_1 \quad R_2]{} \Rightarrow I = \frac{V}{R_1 + R_2}$$

76] Similarly for cylindrical problem:



Here we have

$$T(r)$$

thus, q_r

$$q_\theta = 0$$

$$q_z = 0$$

$$0 = 0 - 0 + (q_r 2\pi r L) \Big|_r - (q_r 2\pi r L) \Big|_{r+dr} + 0$$

Dividing by $2\pi r L dr$:

$$0 = \frac{(rq_r) \Big|_r - (rq_r) \Big|_{r+dr}}{r dr}$$

$$\Rightarrow -\frac{1}{r} \frac{d(rq_r)}{dr} = 0$$

$$\Rightarrow rq_r = C_1 = r_0 q_0$$

$$\text{Thus, } r \left(-k_{01} \frac{dT_{01}}{dr} \right) = r_0 q_0$$

$$\Rightarrow \int_{T_0}^{T_1} dT_{01} = \int_{r_0}^{r_i} -\frac{r_0 q_0}{k_{01}} \frac{dr}{r}$$

$$T_1 - T_0 = -\frac{r_0 q_{v0}}{k_{01}} \ln\left(\frac{r_1}{r_0}\right)$$

$$\text{Similarly } \Rightarrow T_2 - T_1 = -\frac{r_0 q_{v0}}{k_{12}} \ln\left(\frac{r_2}{r_1}\right)$$

Thus we get:

$$r_0 q_{v0} = \frac{T_0 - T_2}{\frac{\ln\left(\frac{r_1}{r_0}\right)}{k_{01}} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{k_{12}}}$$

(similarly, if there were more layers,
we would add more terms
in the denominator)

(* NOTE: The problems can be
made more complex by having k itself
as a function of Temperature
(i.e. k is not constant))

77] For spherical case:

$$0 = (q_r 4\pi r^2) \Big|_r - (q_r 4\pi r^2) \Big|_{r+dr}$$

Dividing by $4\pi r^2 dr$:

$$0 = \frac{(r^2 q_r) \Big|_r - (r^2 q_r) \Big|_{r+dr}}{r^2 dr}$$

$$\Rightarrow 0 = -\frac{1}{r^2} \frac{d(r^2 q_r)}{dr}$$

$$\Rightarrow r^2 q_r = C_1 = r_0^2 q_{v0}$$

LECTURE-18

$$\text{So we can write: } -r^2 k_{01} \frac{dT_{01}}{dr} = r_0^2 q_{v0}$$

$$\int_{T_0}^{T_1} dT_{01} = \int_{r_0}^{r_1} -\frac{r_0^2 q_{v0}}{k_{01}} \frac{dr}{r^2}$$

$$T_1 - T_0 = \frac{r_0^2 q_{v0}}{k_{01}} \left(\frac{1}{r_1} - \frac{1}{r_0} \right)$$

$$\text{Similarly, } T_2 - T_1 = \frac{\pi r_0^2 q_0}{k_{12}} \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$\text{Thus: } T_2 - T_0 = \frac{\pi r_0^2 q_0}{k_{12}} \left(\frac{1}{k_{01} r_1} - \frac{1}{k_{01} r_0} + \frac{1}{k_{12} r_2} - \frac{1}{k_{12} r_1} \right)$$

Add in this way we obtain the answer.

78] Boundary Conditions

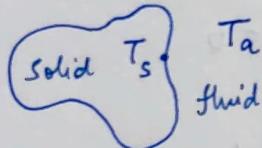
(i) At Interface (e.g.: Solid-fluid, fluid-fluid, etc.)

$$T_I = T_{II}$$

$$q_I = q_{II}$$

(ii) Convective Boundary Conditions

We consider something called
the Heat transfer coefficient *



\downarrow
Related to
Newton's Law of Cooling

$$q \propto (T_s - T_a)$$

$$\Rightarrow q = h(T_s - T_a)$$

\downarrow
heat transfer
coefficient

* This helps us in directly obtaining an expression for q , even in the case for conjugate heat transfer problems

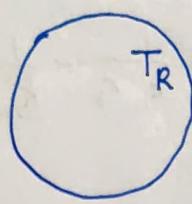
e.g.: where solid is open to air (causing cooling)

\downarrow
Cooling rate calculation is
v. complex to do
directly

(as it involves
the convective K.E.+I.E.
terms)

since cooling depends on
fluid flow, etc.) }

79] (1) Cooling hot sphere



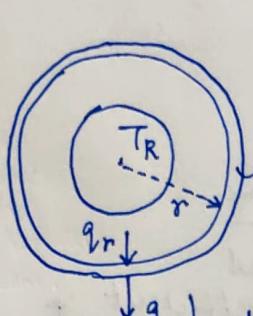
$h_a T_a$

air stationary (in this case)

We take assumption of quasi-steady state

later we might discuss moving air case as well

(i.e. T_R doesn't change)



$h_a T_a$

C.V.

q_r

$q_r|_{r+dr}$

$$T(r) \rightarrow q_r(r)$$

k is constant

{* In reality it should change
↓
since it is an unsteady state problem }
in reality

$$\therefore (4\pi r^2 q_r)|_r - (4\pi r^2 q_r)|_{r+dr} = 0$$

$$\Rightarrow \frac{(r^2 q_r)|_r - (r^2 q_r)|_{r+dr}}{r^2 dr} = 0$$

$$\Rightarrow \frac{1}{r^2} \frac{d(r^2 q_r)}{dr} = 0$$

$$\Rightarrow r^2 q_r = C_1$$

$$\Rightarrow -r^2 k \frac{dT}{dr} = C_1$$

$$\Rightarrow dT = -\frac{C_1}{kr^2} dr$$

$$\Rightarrow T = +\frac{C_1}{kr} + C_2$$

Boundary conditions: At $r=R$, $T=T_R$
and at $r \rightarrow \infty$, $T=T_a$

$$\therefore C_2 = T_a$$

Now, $T = T_R$ at $r = R$

$$\Rightarrow T_R = \frac{C_1}{kR} + T_a$$

$$\Rightarrow C_1 = kR(T_R - T_a)$$

$$\text{We also know, } q_r = \frac{C_1}{r^2}$$

$$\Rightarrow q_r = \frac{kR(T_R - T_a)}{r^2}$$

$$\text{Thus, } q_r|_{r=R} = \frac{kR(T_R - T_a)}{R^2} = \underbrace{h(T_R - T_a)}_{\substack{\text{from heat transfer} \\ \text{coefficient} \\ \text{equation}}}$$

$$\Rightarrow h = \frac{k}{R} \xrightarrow{\substack{\text{the} \\ \text{obtained}}} \text{heat transfer coefficient}$$

NOTE: This gives us the following known number:

$$* \text{ Nusselt's Number, } N_n = \frac{hD}{k}$$

\downarrow
it is a
dimensionless
quantity

\hookrightarrow definition for N_n

Thus for this problem
we can calculate
Nusselt's no. :

$$N_n = \frac{\frac{k}{R} \cdot D}{k} = \boxed{2} \xrightarrow{\substack{\text{hence the} \\ \text{Nusselt's no.} \\ \text{will be 2} \\ \text{for spherical} \\ \text{problem.}}}$$

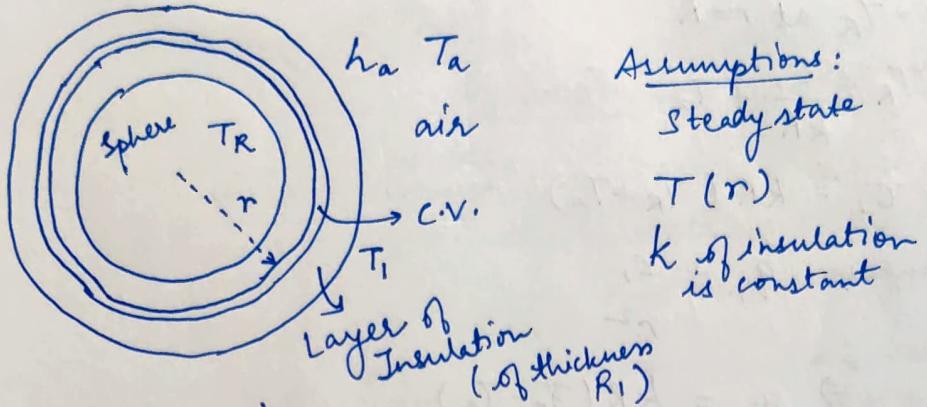
(* In general, Nusselt's no. can be approximated by the formula:

$$\Rightarrow N_n = 2(1 + 4Re^h)$$

Thus, minimum N_n that can be there
is equal to 2)

* \hookrightarrow gives an idea
of the min.
heat transfer that
can occur

(2) Critical Radius of Insulation



Assumptions:
 Steady state
 $T(r)$
 k of insulation
 is constant

We are taking
 C.V. in the insulation:

{ * NOTE: This is
 NOT the exactly
 accurate
 way to solve this
 problem.

$$\therefore (4\pi r^2 q_n)|_r - (4\pi r^2 q_n)|_{r+dr} = 0$$

$$\Rightarrow -\frac{1}{r^2} \frac{d(r^2 q_n)}{dr} = 0$$

$$\Rightarrow r^2 q_n = C_1$$

$$\Rightarrow -k r^2 \frac{dT}{dr} = C_1$$

$$\Rightarrow T = \frac{C_1}{kr} + C_2$$

At $r = R$, $T = T_R$

$$\therefore T_R = \frac{C_1}{kR} + C_2$$

At outer
 radius of insulator: $r = R + R_1$

$$q_n|_{r=R+R_1} = h(T|_{r=R+R_1} - T_R)$$

$$\frac{C_1}{(R+R_1)^2} = h \left(\frac{C_1}{k(R+R_1)} + C_2 - T_R \right)$$

{ NOTE: We could do this more simply by eliminating C_2 :

$$C_2 = T_R - \frac{C_1}{kR}$$

$$\Rightarrow T = \frac{C_1}{kr} + T_R - \frac{C_1}{kR}$$

$$\text{Now, } T_{r=R+R_1} = \frac{C_1}{k(R+R_1)} + T_R - \frac{C_1}{kR} \quad \{$$

* Another way:
that is better

$$\int_{T_R}^{T_1} dT = \int_R^{R+R_1} -\frac{C_1}{kn^2} dr$$

{ * This is
also not the
perfect method.

$$\Rightarrow T_1 - T_R = \frac{C_1}{k} \left(\frac{1}{R+R_1} - \frac{1}{R} \right) \quad -①$$

The
right
method
is given
on the
next page }

Now, $T_1 - T_a = \frac{q_{n=r=R+R_1}}{h}$ \rightarrow this we
already
know: $q_{n=r=R+R_1} = \frac{C_1}{(R+R_1)^2}$

$$\Rightarrow T_a - T_1 = -\frac{q_{n=r=R+R_1}}{h} \\ = -\frac{C_1}{h(R+R_1)^2} \quad -②$$

Thus, adding ① and ②:

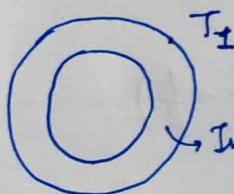
$$T_a - T_R = C_1 \left(\frac{1}{k(R+R_1)} - \frac{1}{kR} \right) - \frac{C_1}{h(R+R_1)^2} \\ = C_1 \left[\frac{-R_1}{k(R+R_1)R} - \frac{1}{h(R+R_1)^2} \right]$$

from here we can find C_1
(we can continue from here later)

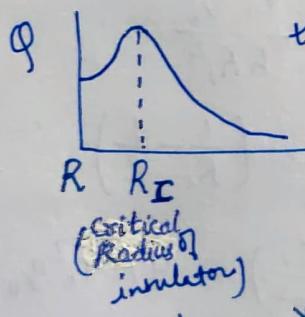
(* NOTE: It turns out
it is actually
v. simple

We had to use
 $C_1 = q_R R^2$
(or $\sigma^2 q_0$,
as we did
previously
too)

* NOTE:



Thus \Rightarrow
net result



We know:

$$Q = q (4\pi r^2)$$

but
this will
go down
rapidly

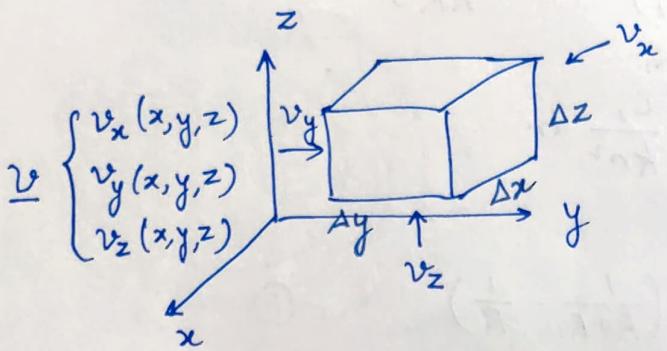
this increases
rapidly

r' refers to
the radius of insulation

(critical
radius of
insulator)

* i.e. upto the critical radius value
we are actually increasing
heat transfer
thus, for good insulation,
we MUST have
critical
radius

80] Axiom 4 : Energy is Conserved



\therefore Rate of accumulation
of $(\hat{U} + k_e)$ in C.V.
 internal energy kinetic energy

$$= \text{Rate of inflow in C.V. of } (\hat{U} + k_e) \text{ by convection}$$

$$+ \text{Net rate of inflow of Heat by conduction}$$

$$\pm \text{Rate of Work done by/on the system}$$

$$+ \text{Rate of Heat addition by some source}$$

LECTURE-19

Continuing the critical radius problem:

$$\int_{T_R}^{T_1} dT = \int_{R}^{R_1} -\frac{C_1}{kr^2} dr$$

$$C_1 = r_0^2 q_0 \quad \text{(this was the crucial step that was missed in the previous attempt to solve this problem)}$$

$$\Rightarrow T_1 - T_R = \frac{C_1}{k} \left(\frac{1}{R_1} - \frac{1}{R} \right) = \frac{r_0^2 q_0}{k} \left(\frac{1}{R_1} - \frac{1}{R} \right)$$

$$\text{And, } T_1 - T_a = \frac{q_r|_{r=R_1}}{h} = \frac{r_0^2 q_0}{h R_1^2}$$

$$\text{Thus, } T_R - T_a = \frac{r_0^2 q_0}{h R_1^2} - \frac{r_0^2 q_0}{k} \left(\frac{1}{R_1} - \frac{1}{R} \right)$$

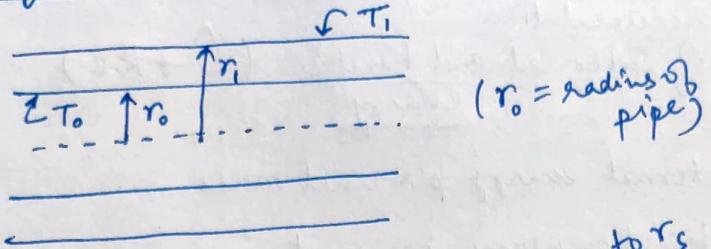
$$\text{Now, } Q = 4\pi r^2 \cdot q_r|_{r=r} = 4\pi r^2 \left(\frac{C_1}{r^2} \right) = 4\pi r_0^2 q_0$$

$$\Rightarrow Q = \frac{4\pi (T_R - T_a)}{\frac{1}{h R_1^2} - \frac{1}{k} \left(\frac{1}{R_1} - \frac{1}{R} \right)}$$

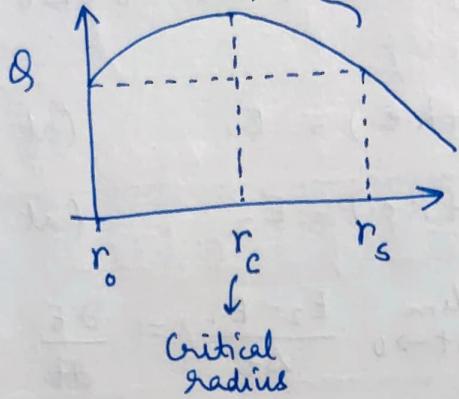
$(r_0$ is basically just R)

{just a mix-up of two notations?}

For cylindrical case : T_a



In this case we will find:



from r_c to r_s
although d is decreasing
it is already
still above the
initial Q value
thus we should
keep $r > r_s$ to
actually reduce heat transfer

80] (*Continued)

{ NOTE: The eqn. of energy is actually made of 2 parts

mechanical energy thermal energy
essentially just the Bernoulli eqn.

* We are interested in mostly just this }

Rate of accumulation of $(\hat{U} + k.e.)$ in C.V. =

①

Net inflow of $(\hat{U} + k.e.)$ by convection

+ ③ Net inflow of heat addition by heat conduction (for by the system, we mean minus sign)

here we will use plus sign only

④ Net rate of Work done on the system by various forces (shear forces, pressure forces, gravity)

+ Rate of heat addition by some heat source

⑤

Now, we have:

$$\textcircled{1} : \frac{\text{Rate of accumulation}}{\text{of Internal and Kinetic energy}} (\hat{U} + \text{k.e.})$$

\hat{U} = Internal energy per unit mass

k.e. = kinetic energy per unit mass

We find $(\hat{U} + \text{k.e.})$ at time t and $t + \Delta t$:

$$\text{Now } \Rightarrow \rho \Delta x \Delta y \Delta z (\hat{U} + \text{k.e.}) = E_1 \quad (\text{at } t)$$

$$\rho \Delta x \Delta y \Delta z (\hat{U} + \text{k.e.}) = E_2 \quad (\text{at } t + \Delta t)$$

$$\text{Now, } \lim_{\Delta t \rightarrow 0} \frac{E_2 - E_1}{\Delta t} = \frac{\partial E}{\partial t}$$

$$= \frac{\partial (\rho \Delta x \Delta y \Delta z (\hat{U} + \text{k.e.}))}{\partial t}$$

Also, instead of k.e. we can simply write $\frac{1}{2} v^2$: $\Rightarrow \frac{\partial (\rho \Delta x \Delta y \Delta z (\hat{U} + \frac{1}{2} v^2))}{\partial t}$

Next, $\textcircled{2}$: Net inflow of $(\hat{U} + \frac{1}{2} v^2)$ by convection

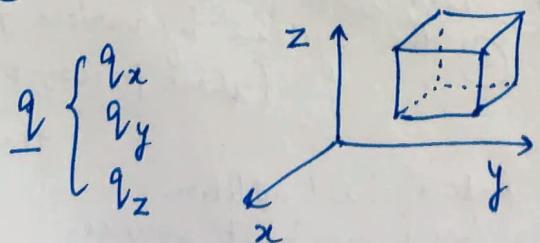
mass flow rate

$$\text{Face x: } \left(\overbrace{\rho v_x \Delta y \Delta z (\hat{U} + \frac{1}{2} v^2)}^{\text{mass flow rate}} \right) \Big|_x - \left(\rho v_x \Delta y \Delta z (\hat{U} + \frac{1}{2} v^2) \right) \Big|_{x+\Delta x}$$

$$\text{Face y: } + \left(\rho v_y \Delta x \Delta z (\hat{U} + \frac{1}{2} v^2) \right) \Big|_y - \left(\rho v_y \Delta x \Delta z (\hat{U} + \frac{1}{2} v^2) \right) \Big|_{y+\Delta y}$$

$$\text{Face z: } + \left(\rho v_z \Delta x \Delta y (\hat{U} + \frac{1}{2} v^2) \right) \Big|_z - \left(\rho v_z \Delta x \Delta y (\hat{U} + \frac{1}{2} v^2) \right) \Big|_{z+\Delta z}$$

For $\textcircled{3}$: Net rate of heat addition by conduction

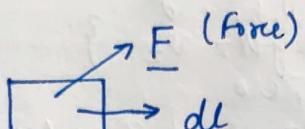


$$\begin{aligned} \text{Thus, we get: } & (q_x \Delta y \Delta z) |_x - (q_x \Delta y \Delta z) |_{x+\Delta x} \\ & + (q_y \Delta x \Delta z) |_y - (q_y \Delta x \Delta z) |_{y+\Delta y} \\ & + (q_z \Delta x \Delta y) |_z - (q_z \Delta x \Delta y) |_{z+\Delta z} \end{aligned}$$

This part till here was easy,
but the main thing is to calculate work done:

(4) : Net rate of work done on system by various forces:

We know:



(displacement)

$$\delta W = \underline{F} \cdot \underline{d\underline{l}}$$

∴ Rate of work done:

$$\frac{\partial W}{\partial t} = \underline{F} \cdot \frac{d\underline{l}}{dt} = \underline{F} \cdot \underline{v}$$

In general:

$$\underline{F} \cdot \underline{v} = F_x v_x + F_y v_y + F_z v_z$$

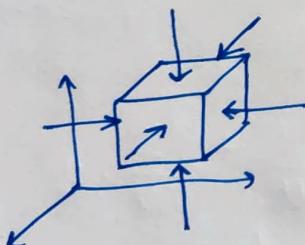
$$\therefore \text{For Gravity: } g \Delta x \Delta y \Delta z \underline{g} \cdot \underline{v}$$

$$= g \Delta x \Delta y \Delta z (g_x v_x + g_y v_y + g_z v_z)$$

For Pressure forces:

they are always trying
to compress the C.V.

thus it is opposite to
normal on every face



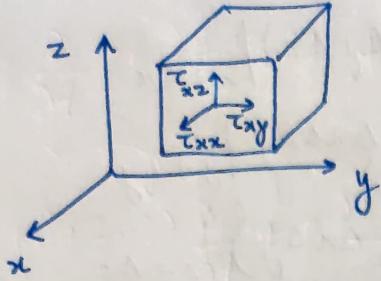
$$\begin{aligned} & [v_x (P \Delta y \Delta z)] |_x - [v_x (P \Delta y \Delta z)] |_{x+\Delta x} \\ & + [v_y (P \Delta x \Delta z)] |_y - [v_y (P \Delta x \Delta z)] |_{y+\Delta y} \\ & + [v_z (P \Delta x \Delta y)] |_z - [v_z (P \Delta x \Delta y)] |_{z+\Delta z} \end{aligned}$$

For Shear forces:

We have: $\tau = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$

forces acting on x-face

forces in x-dir. in y-dir. in z-dir.



$$\begin{aligned}
 & -(\tau_{xx})_f \Delta y \Delta z v_x \\
 & -(\tau_{xy})_f \Delta y \Delta z v_y \\
 & -(\tau_{xz})_f \Delta y \Delta z v_z
 \end{aligned}$$

* { NOTE: For momentum flux we will write:

$$\begin{aligned}
 & -(-\tau_{xx}) \Delta y \Delta z v_x \\
 & -(-\tau_{xy}) \Delta y \Delta z v_y \\
 & -(-\tau_{xz}) \Delta y \Delta z v_z \quad \}
 \end{aligned}$$

$$\text{Thus, } \Rightarrow \left(\tau_{xx} \Delta y \Delta z v_x + \tau_{xy} \Delta y \Delta z v_y + \tau_{xz} \Delta y \Delta z v_z \right) \Big|_x$$

$$\text{At } x + \Delta x \Rightarrow \left(-\tau_{xx} \Delta y \Delta z v_x - \tau_{xy} \Delta y \Delta z v_y - \tau_{xz} \Delta y \Delta z v_z \right) \Big|_{x+\Delta x}$$

Similarly
on y-face:

$$\begin{aligned}
 & \Delta x \Delta z (\tau_{yx} v_x + \tau_{yy} v_y + \tau_{yz} v_z) \Big|_y \\
 & - \Delta x \Delta z (\tau_{yx} v_x + \tau_{yy} v_y + \tau_{yz} v_z) \Big|_{y+\Delta y}
 \end{aligned}$$

Lastly, on
z-face:

$$\begin{aligned}
 & \Delta x \Delta y (\tau_{zx} v_x + \tau_{zy} v_y + \tau_{zz} v_z) \Big|_z \\
 & - \Delta x \Delta y (\tau_{zx} v_x + \tau_{zy} v_y + \tau_{zz} v_z) \Big|_{z+\Delta z}
 \end{aligned}$$

Finally, ⑤ Rate of heat addition by some
heat source

Let S_c be heat rate
per unit volume

$$\therefore \Rightarrow S_c \Delta x \Delta y \Delta z$$

Now, we put all eqn's ①, ②, ③, ④ and ⑤ together
and divide by $\Delta x \Delta y \Delta z$

and take,

$$\begin{aligned}
 & \Delta x \rightarrow 0 \\
 & \Delta y \rightarrow 0 \\
 & \Delta z \rightarrow 0
 \end{aligned}$$

This gives us the final ultimate formula:

$$\begin{aligned}
 \frac{\partial}{\partial t} \rho (\hat{U} + \frac{1}{2} v^2) = & - \frac{\partial}{\partial x} \rho v_x (\hat{U} + \frac{1}{2} v^2) - \frac{\partial}{\partial y} \rho v_y (\hat{U} + \frac{1}{2} v^2) \\
 & - \frac{\partial}{\partial z} \rho v_z (\hat{U} + \frac{1}{2} v^2) \\
 & - \left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right) \\
 & + \rho (v_x g_x + v_y g_y + v_z g_z) \\
 & - \left(\frac{\partial}{\partial x} \rho v_x + \frac{\partial}{\partial y} \rho v_y + \frac{\partial}{\partial z} \rho v_z \right) \\
 & - \left[\frac{\partial}{\partial x} (\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz} v_z) \right. \\
 & \quad + \frac{\partial}{\partial y} (\tau_{yx} v_x + \tau_{yy} v_y + \tau_{yz} v_z) \\
 & \quad \left. + \frac{\partial}{\partial z} (\tau_{zx} v_x + \tau_{zy} v_y + \tau_{zz} v_z) \right] \\
 & + S_c
 \end{aligned}$$

In vector-tensor form this simplifies to:

$$\begin{aligned}
 \frac{\partial}{\partial t} \rho (\hat{U} + \frac{1}{2} v^2) = & - \underline{\nabla} \cdot \rho \underline{v} (\hat{U} + \frac{1}{2} v^2) \\
 & - \underline{\nabla} \cdot \underline{q} + \rho \underline{v} \cdot \underline{g} - \underline{\nabla} \cdot \rho \underline{v} \\
 & - \underline{\nabla} \cdot (\underline{\tau} \cdot \underline{v}) + S_c
 \end{aligned}$$

EQUATION OF ENERGY

LECTURE-20

81) Applying the identity: $\underline{\nabla} \cdot \underline{s} \underline{w} = \underline{s} \underline{\nabla} \cdot \underline{w} + \underline{w} \cdot \underline{\nabla} \underline{s}$

We can transform:

$$\underline{\nabla} \cdot \underbrace{\rho \underline{v}}_{\underline{w}} (\hat{U} + \frac{1}{2} v^2) \quad \underbrace{\underline{s}}_S$$