AXIOM-1

LECTURE-5 [continued]

32] **Transport Phenomena is based on following five axioms:**

1. Mass is conserved → leads to Equation of continuity
2. Momentum is conserved (essentially, Newton's 2nd Law) → Eq.^{n} of Motion
3. Moment of Momentum is conserved → Gives us $\underset{\sim}{\tau}$ is symmetric
4. Energy is conserved → Eq.^{n} of Energy < Eq.^{n} of mechanical energy / Eq.^{n} of Thermal energy
5. Mass of each component in a multi-component mixture is also conserved.

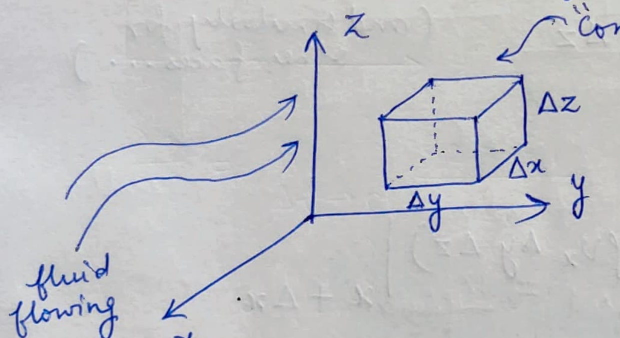
→ Bernoulli Eq.^{n}

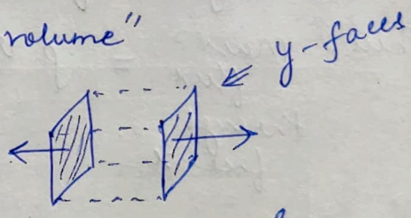
→ Convective diffusion eq.^{n}

[The aim, in other words, ie to get:
Velocity profile | Temperature profile | Concentration profile]

33) **Axiom-1 : Mass is conserved**

Taking cartesian coord. system :





the outer normals of these faces
→ one is in +ve dir.^{n}
other is negative dir.^{n}
↓
this helps us differentiate b/w the two

Writing fluid velocity as :

$\underline{v}(x,y,z,t)$

⇓ 3 components

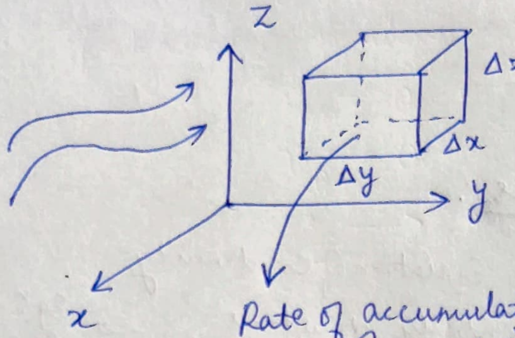
$v_x(x,y,z,t)$

$v_y(x,y,z,t)$

$v_z(x,y,z,t)$

* Here, if we take
$\Delta x \to 0$
$\Delta y \to 0$
and, $\Delta z \to 0$
} → then the control volume collapses to a single point.

Density : $\rho(x,y,z,t)$
↓
(mass / volume)
↓
[If it were mass of a component / volume → this would be called "mass concentration"]



$*$ For conservation of mass :

Rate of accumulation of mass in C.V (control vol^m) = Rate of inflow of mass in C.V − Rate of outflow of mass in C.V.

$$\text{Rate of accumulation of mass in C.V.} = \lim_{\Delta t \to 0} \frac{m|_{t+\Delta t} - m|_t}{\Delta t}$$

$$= \frac{\partial m}{\partial t}$$

$$= \frac{\partial(\rho \Delta x \Delta y \Delta z)}{\partial t} \longrightarrow \text{i.e. the first term}$$

We know, $m = \rho \Delta x \Delta y \Delta z$

Thus, $\text{Mass flux through face-}x = \rho v_x \Delta y \Delta z$ (and similarly for other faces...)

Thus,

$$\text{Rate Inflow} - \text{Rate outflow} = (\rho v_x \Delta y \Delta z)\Big|_x - (\rho v_x \Delta y \Delta z)\Big|_{x+\Delta x}$$
$$+ (\rho v_y \Delta x \Delta z)\Big|_y - (\rho v_y \Delta x \Delta z)\Big|_{y+\Delta y}$$
$$+ (\rho v_z \Delta x \Delta y)\Big|_z - (\rho v_z \Delta x \Delta y)\Big|_{z+\Delta z}$$

In the resulting eq^n., we can divide both sides by $\Delta x \Delta y \Delta z$:

$$\therefore \frac{\partial \rho}{\partial t} = \frac{(\rho v_x)|_x - (\rho v_x)|_{x+\Delta x}}{\Delta x} + \frac{(\rho v_y)|_y - (\rho v_y)|_{y+\Delta y}}{\Delta y}$$
$$+ \frac{(\rho v_z)|_z + (\rho v_z)|_{z+\Delta z}}{\Delta z}$$

Finally, allowing $\Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0$ we obtain:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho v_x)}{\partial x} - \frac{\partial(\rho v_y)}{\partial y} - \frac{\partial(\rho v_z)}{\partial z}$$

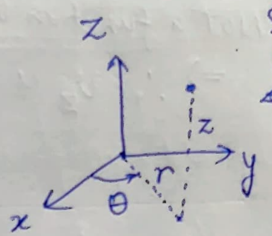
$$\Rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

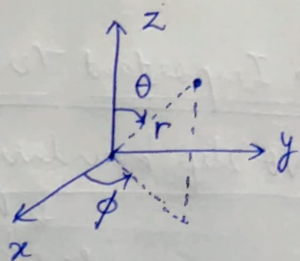
$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot (\rho \underline{v}) = 0}$$

THE CONTINUITY EQUATION

* (Valid in all coordinate systems)

{ Cartesian → (x, y, z)
Cylindrical → (r, z, θ)
Spherical → (r, θ, ϕ) }





(for eg:
In cylindrical: $\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$

In spherical: $\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta)\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$)

* { But no need to memorize these }

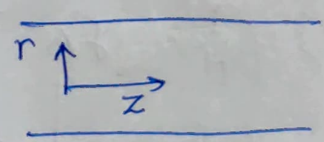
LECTURE-6

34] Steady Flow in a Tube

1) ρ constant
2) Steady flow

← pressure

$p = p_0$, $z = 0$ $\qquad$ $p = p_L$, $z = L$



Let's say: $\underline{v} \Rightarrow \begin{cases} v_r(r, \theta, z, t) \\ v_\theta(r, \theta, z, t) \\ v_z(r, \theta, z, t) \end{cases}$

(technically force)

<u>If pressure is along z-dir^n only</u> :

$v_r = 0$

$v_\theta = 0$

$v_z \longrightarrow$ function of r, z only (NOT θ, t

by symmetry, v_z should not depend on θ

bcz steady state reached (i.e. fully developed flow))

Thus, from continuity eq^n :

$$\frac{\partial (\rho v_z)}{\partial z} = 0$$

$$\Rightarrow \rho \frac{\partial v_z}{\partial z} = 0$$

thus v_z is NOT a function of z

$\Rightarrow \therefore v_z$ is only a function of r

($v_z(r)$)

35) <u>Some Important Terms/Assumptions</u>

(1) <u>No slip boundary condition</u>

i.e. whenever fluid is in contact with a solid

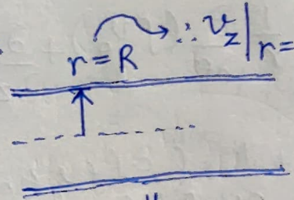
then at interface

$\underline{v}_{solid} = \underline{v}_{fluid}$

Results in Velocity gradients

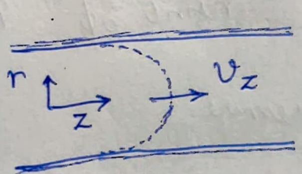
which leads to <u>shear forces</u>

eg:

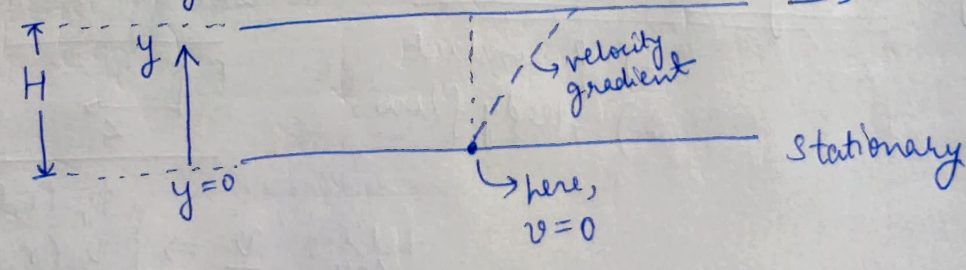


$\therefore v_z\big|_{r=R} = 0$

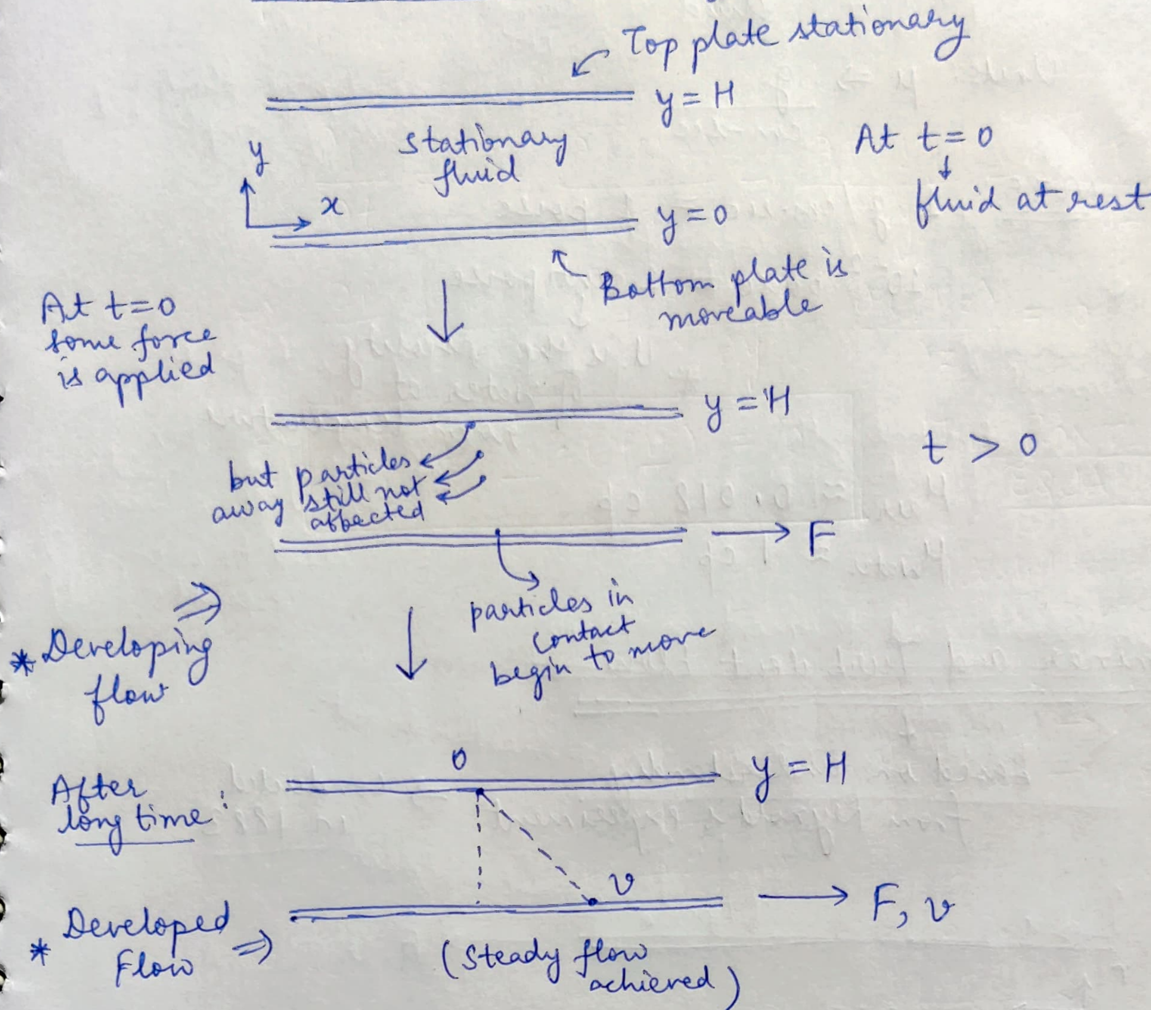
$\Downarrow \therefore v_z$ must be varying with r



<u>Another example</u> :



(2) Newton's Law of Viscosity



F	v	$\tau_{yx} = \frac{F}{A}$	$\frac{dv_x}{dy} \approx \frac{v}{H}$
F_1	v_1	⋮	⋮
F_2	v_2	⋮	⋮
⋮	⋮	⋮	⋮

Calculated experimentally

↓

we observe the following relation:

$$\tau_{yx} \propto \frac{dv_x}{dy}$$

$$\Rightarrow \boxed{\tau_{yx} = \pm \mu \frac{dv_x}{dy}}$$

* (in Fluid mechanics we take −ve sign)

(eg: $q_y = -k\frac{dT}{dy}$)

↓ heat flows in dir^n opposite to temp. gradient

(similarly, for concentration gradient)

We call : $\mu \rightarrow$ viscosity of the fluid

Units : $\mu \Rightarrow \dfrac{g}{cm\text{-}sec}$

NOTE : $1\ g/cm\text{-}sec = 1$ poise

10^{-2} poise $= 1$ centipoise

$\downarrow$

* it is the viscosity of water at room temperature

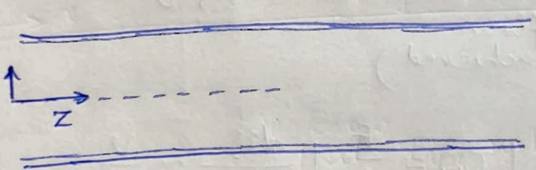
$\mu_{air} \approx 0.018$ cp

$\mu_{water} \approx 1$ cp

(3) Laminar and Turbulent Flow

$\downarrow$

based on understanding from Reynold's experiment $\Rightarrow$ conducted in 1883



injected a dye into the fluid $\Rightarrow$

He measured & compared v_z

$\downarrow$

by taking different R, different ρ and different fluids (i.e. different μ)

$\downarrow$

He noticed

$\downarrow$

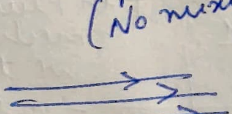
if velocities are small

$\downarrow$

the dye moves in thread-like manner/form

$\Downarrow$

$\because$ The layers retain their individual properties (No mixing)



But for larger velocities

$\downarrow$

as soon as dye injected

$\downarrow$

it spreads out & dissipates, and is not visible

$\Downarrow$

i.e. our assumption that $v_r = 0$ and $v_\theta = 0$ only applies for laminar flow (NOT turbulent flow)

mixing b/w layers takes place

He defined :

$$\boxed{Re = \frac{\rho D v_{z\,avg.}}{\mu}}$$ (Reynold's number)

this is dimensionless

When, $Re \leq 2100 \Rightarrow$ Laminar flow

(beyond 2100, Turbulent flow MAY occur)

(but beyond ~5000, it will surely be turbulent flow)

$$\boxed{\text{Critical Reynold's Number} = 2100}$$

→ valid for any flow

{ Reason: The mathematical form of the non-linear eq^{n}s is such that + they become unstable beyond ~2100 }

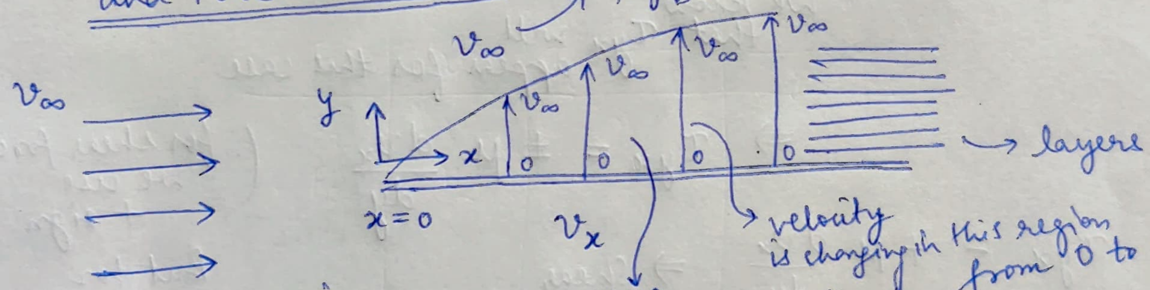
LECTURE-7

36] { * NOTE: Most problems that we'll solve in this course are fully-developed flow.

* Some imp. terms: types of flow → External flow, Internal flow, Boundary layer, Potential flows ... etc.

Boundary Layer and Potential Flows :

in the region velocity is still v_∞ (not yet felt affect of solid)



This is Boundary layer & potential flow

Here v_x is a function of x

velocity is changing in this region from 0 to v_∞

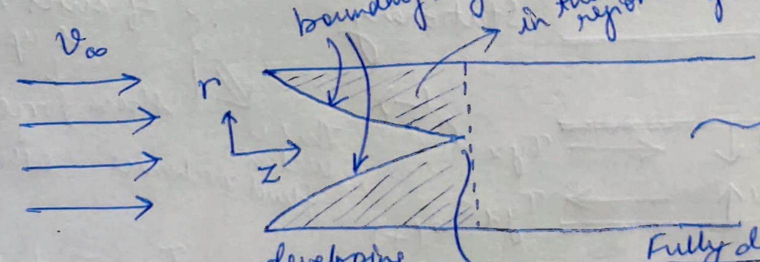
$\Downarrow$

velocity gradients present in this region (shear forces also present here)

37] Internal Flow

boundary layers

in these regions → velocity gradients present → v_z is a function of z *



* Here, v_z is NOT a function of z → Thus, v_r is also ZERO (and, v_θ also zero)

developing flow

Fully developed

Here they merge * ⇒ Thus, from here flow is fully developed

* NOTE: From eq.^{n} of continuity, we can see that if $v_r \neq 0$, then it becomes extremely difficult ↓ thus developing flow is very complicated.

"Entry length" → length of region where developing flow is present

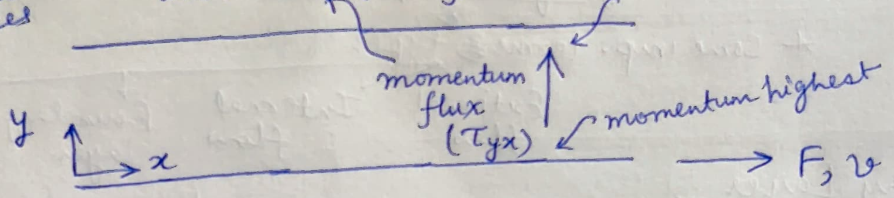
↓

This is usually v. small ↓ relative to length of entire pipe

{ * But for short pipes ↓ using the case for fully developed flow may lead to wrong results }

38] Shear Forces and Momentum Flux

In reality there will be shear forces ⇐ We only work with this idea for τ_{yx}



We know that τ_{yx} will appear for this case

$$\Rightarrow \tau_{yx} = \pm \mu \frac{dv_x}{dy}$$

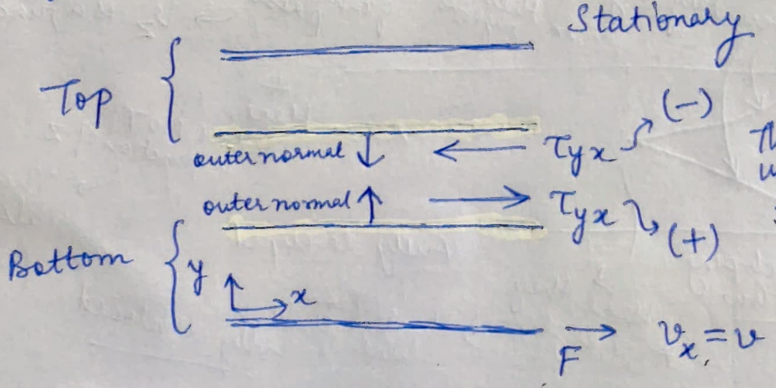
(for shear forces we use + sign. But for our flows from high to low ↓ we use −ve sign)

$$\tau_{yx} = +\mu \frac{dv_x}{dy} \Rightarrow \text{Shear forces}$$

$$\tau_{yx} = -\mu \frac{dv_x}{dy} \Rightarrow \text{Momentum Flux}$$

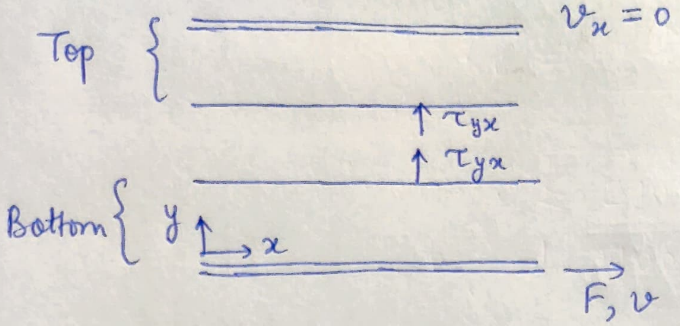
Sign Convention for Momentum Flux and Fluid Mechanics

* For Fluid Mechanics case:



This is used in fluid mechanics ⇐ { If outer normal is upwards (+) ↓ show force as +ve ↓ otherwise −ve

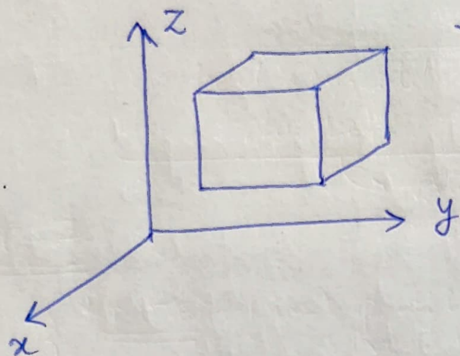
$$\tau_{yx} = \pm \mu \frac{dv_x}{dy}$$



* <u>In our case</u> (<u>using Momentum flux</u>)

So here,
the sign for momentum flux will be −ve !

$$\tau_{yx} = \underset{\substack{\downarrow \\ \text{(always -ve)}}}{-} \mu \frac{dv_x}{dy}$$

39] 

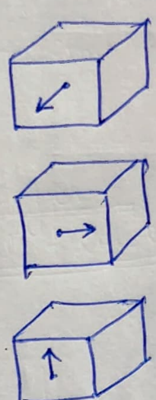
There seem to be
$6 \times 3 = 18$ components
↓
but actually only 9
($\because$ faces on opposite sides have normals along same axis)

$$\begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}$$

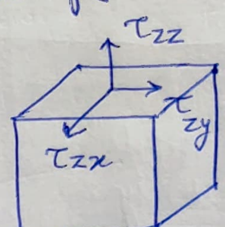
eg: τ_{xx}:

τ_{xy}:

τ_{xz}:



Similarly for z-face:



(and similarly for y-face) ...

40] Using equation of continuity:

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot (\rho \underline{v}) = 0$$

But we know that : $\underline{\nabla} \cdot (s \underline{v}) = s\, \underline{\nabla} \cdot \underline{v} + \underline{v} \cdot \underline{\nabla} s$

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t}} + \rho\, \underline{\nabla} \cdot \underline{v} + \boxed{\underline{v} \cdot \underline{\nabla} \rho} = 0$$

Using definition of Substantial derivative $\Rightarrow$

$$\boxed{\frac{D\rho}{Dt} + \rho\, \underline{\nabla} \cdot \underline{v} = 0}$$

* Alternate form of <u>Equation of Continuity</u>

<u>LECTURE-8</u>

* <u>NOTE</u>: Momentum flux (τ_{yx} for instance) $\xrightarrow{\text{implies}}$ Momentum is along x but the transfer of momentum (i.e. flux) is along y.