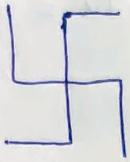
CRYSTALS, SYMMETRY AND TENSORS

Prof. Rajesh Prasad [MLL746]

LECTURE 1 (01/01/2024)

1] Consider the following shape:



It has rotational symmetry
↓
specifically,
4-fold rotational symmetry
⇓ i.e.
"self-coincidence" occurs
by rotation by $\frac{360^\circ}{4} = 90^\circ$

2] Symmetry
↓
An object is said to possess symmetry
↓
if it is brought into self-coincidence
↓
by some operation

* NOTE: A symmetry operation followed by another symmetry operation
↓
is also a symmetry operation

⇒ Property 1 {"Closure" property}

{Here, "followed by" ≡ combination of operations}

3] Symmetry operations of swastika:

90°, 180°, 270°, $360^\circ \equiv 0^\circ$

$360^\circ \equiv 0^\circ$: * NULL operations or Identity operations

Property 2
⇓
NOTE: Every object possesses identity operation ⟶ symbol of identity operation: I/E/e/g.

4] Property ③: Every operation has an inverse operation

$$g \cdot h = h \cdot g = e \text{ (identity)}$$

$$\text{eg: } 90^\circ \cdot 270^\circ = 270^\circ \cdot 90^\circ = e$$

Here, g and h are inverses of each other

(and thus, 90° and 270° are inverses of each other)

* NOTE: 180° is a self-inverse.

5] Property ④: Symmetry operations are associative

$$g_1 \cdot (g_2 \cdot g_3) = (g_1 \cdot g_2) \cdot g_3$$

For example:

$$90^\circ (180^\circ\ 270^\circ) = 90^\circ \cdot 450^\circ$$
$$= 90^\circ \cdot 90^\circ = 180^\circ$$

$$(90^\circ\ 180^\circ)\ 270^\circ = 270^\circ \cdot 270^\circ$$
$$= 540^\circ = 180^\circ$$

6] Let us consider the set of integers ($\mathbb{Z}$) with the operation '+'

We observe:

① $z_1 + z_2 = z_3$, where $z_1, z_2, z_3 \in \mathbb{Z}$

② zero('0') is the identity:

$$z_1 + 0 = 0 + z_1 = z_1$$

③ $z + (-z) = 0$ → i.e. z and $-z$ are inverses

7] Group

A set (G) with a 'binary' operation ($\cdot$) is called a group, if the following properties are satisfied:

① Closure : $a \in G, b \in G \Rightarrow a \cdot b \in G$

② Identity : $\exists\, e \in G$ such that
$$e \cdot a = a \cdot e = a \quad , \forall a \in G$$

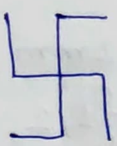
③ Inverse : $\forall a \in G, \exists\, a^{-1} \in G$ such that
$$a \cdot a^{-1} = a^{-1} \cdot a = e$$

multiplication operator

eg: $\mathbb{Z}$, $\times$ is NOT a group
(Integers)

due to the absence of inverses

LECTURE 2 (04/01/2024)

8] 

$$G_{swastika} = \{90°, 180°, 270°, 360°\}$$

{* NOTE: Here +ve angles represent anticlockwise }

We don't need to include 90° clockwise

since that is equivalent to 270° anticlockwise

this can also be written as :

$$\{90°, 180°, 270°, 360° \equiv 0°\}$$

[4^+ 2 4^- 1

2 ⇓ sign not reqd. (since +180° and −180° are the same)

1 ⇓ again, sign is not reqd.

* NOTE: These numbers are written from! $\frac{360°}{\theta}$

this group itself has a name

* it is called group "4" (Hermann-Mauguin Symbol)

(also known as International Symbol)

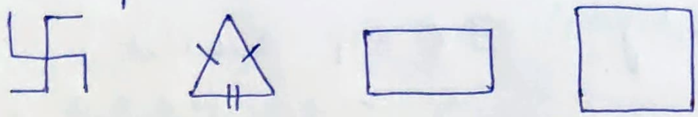
⇓

There is another alternate notation → called Schoenflies Symbol *

that will give : C_4

{ i.e. "Cyclic" 4 } ⇐

9] Some more shapes :



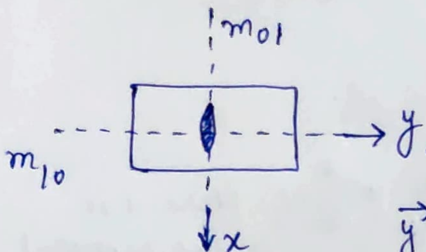
The symmetries in these along with the symmetry groups are as follows :

$\Rightarrow$ This has mirror symmetry $\Rightarrow$ $m = \{1, m\}$

m: this is the name of group

m in $\{1, m\}$: this is name of mirror operation

* NOTE: Every group must have identity operation

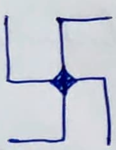


$\vec{y} = (01)$

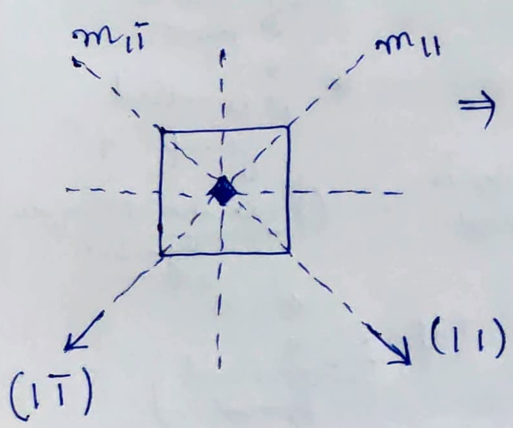
$\vec{x} = (10)$

* this is the convention used

$\Rightarrow$ 2 mirror symmetries + A 2-fold axis $\Rightarrow$ $2mm = \{1, 2, m_{10}, m_{01}\}$

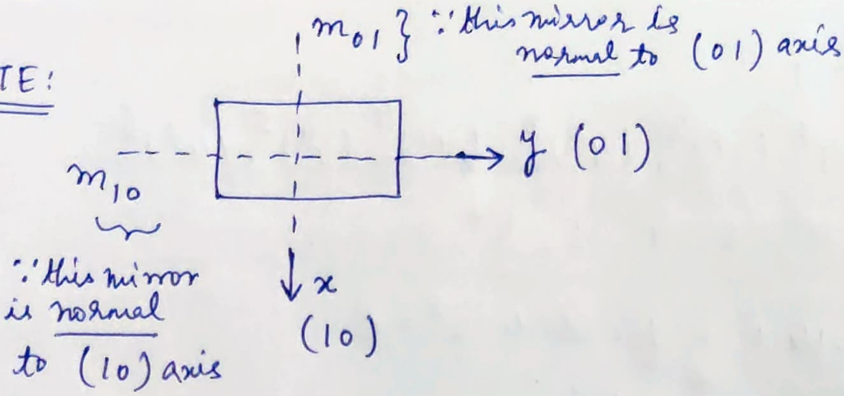


$\Rightarrow$ This has a 4-fold axis $\Rightarrow$ $4 = \{1, 4^+, 4^-, 2\}$

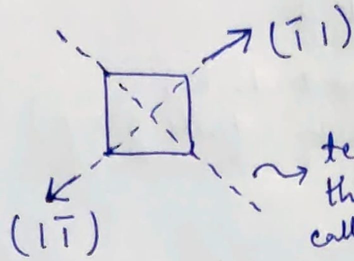


$\Rightarrow$ Has 4-mirror symmetries + A 4-fold axis $\Rightarrow$ $4mm = \{1, 4^+, 2, 4^-, m_{10}, m_{01}, m_{11}, m_{1\bar{1}}\}$

NOTE:



NOTE:



technically this can be called either $m_{1\bar{1}}$ or $m_{\bar{1}1}$

(both are equivalent)

⇒ * However, DO NOT write BOTH in the group

10] **Order of a group**

↓

it is defined as the no. of elements (i.e. operations) in the group

↓

denoted by $|G|$

eg: $|4| = 4$, $|m| = 2$, $|2mm| = 4$, $|4mm| = 8$

11] **Order of an element of a group**

If $a \in G$:

$a, a^2, a^3, a^4, \ldots, a^k, \ldots$ (i.e., if all these elements are distinct ↓ this will be an infinite group)

{ * NOTE: This is not a multiplicative exponent

↓

instead, here: $a \cdot a \equiv a^2$ }

* For a finite group:

↓

for some smallest k

↓

$a^k = 1$ or e (identity)

eg: $(4^+)^4 = 1$

Thus : $4^+, (4^+)^2, (4^+)^3, (4^+)^4, (4^+)^5, (4^+)^6, \dots$

$\Rightarrow 4^+, 2, 4^-, 1, 4^+, 2, \dots$

* Now, the smallest k for which $a^k = e$

↓

is called the **order of the element a**

(if there is no such k, then it is an infinite group)

i.e.

if there is an element for which k does not exist

($a^k \neq e$ for any k)

↓

then the group **is infinite**.

* **NOTE:** Space groups are infinite groups

↓

since space groups involve translations

NOTE: Order of e is 1

(since $e^1 = e$)

(for eg: $e^1 = e = e^0 = e^2 = e^3$)

NOTE: Since we can define : $a^0 = e$ for all $a \in G$

↓

we will then need to redefine order of an element as :

"The smallest non-zero k for which $a^k = e$"

12] **Smallest Group**

↓

this is a group with one element, i.e. $|G| = 1$

↓

it is given by:

↓

the **identity group** : $\{1\} = 1$

Now, a group of order 2

i.e. $|G| = 2$ ⇒ will be of the form : $\{e, a\}$

this means that

$a^2 = e$

(for example: $m = \{1, m\}$)

13] Cayley's Table (Group Multiplication Table)

Order

1

	e
e	e

multiplication of e and e gives e

OR

	1
1	1

(Here, $G = \{1\}$)

2

	1	m
1	1	m
m	m	$m^2 = 1$

* **NOTE**: Due to closure property

we can never generate anything other than the group members themselves.

3

	e	a	b
e	e	a	b
a	a		
b	b		

to find remaining values ⟹ we can make use of the following two properties :

(i) Two elements in any row are different

(ii) Two elements in any column are different

Proof for (i):

	e …	u …	v …
e			
⋮			
a	a …	b …	c …
⋮			

Here, if $b = c$:

then, $au = av$

Now, by group property, since 'a' has an inverse a^{-1}:

$\Rightarrow \quad a^{-1}au = a^{-1}av$

$\Rightarrow \quad \underbrace{u = v}$

but this is a contradiction, since we assumed that these are different elements.

Hence proved

{ Similarly (ii) can be proved }.

From this property we can now try:

	e	a	b
e	e	a	b
a	a	e	b
b	b		

But then this is invalid

then this must be 'b'

if we try e

$\Downarrow$

Thus:

	e	a	b
e	e	a	b
a	a	b	e
b	b	e	a

This is the final reqd. Cayley Table of order 3

H.W.: Find the groups of order 4 → "abstract groups"

*NOTE: { Till now we found that there's only 1 group of Order 1, 1 of order 2 and 1 of order 3. }

H.W. Find order and cayley table for 4mm group.

LECTURE-3 (08/01/2024)

14] Cayley table for order 4 :

{* NOTE: For each group, there will be one cayley table}

	e	a	b	c
e	e	a	b	c
a	a	(e)	c	b
b	b	c	e/a	a/e
c	c	b	a/e	e/a

Here we have 3 options: e or b or c

i.e. these will be in 2 diff. groups

Since it cannot be a or b, and also not e (as $a \cdot a = e$)

	e	a	b	c
e	e	a	b	c
a	a	(b)		
b	b			
c	c			

	e	a	b	c
e	e	a	b	c
a	a	(c)		
b	b			
c	c			

NOTE : All information about the group is contained in its Cayley table.

eg :

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Order : 2

Order : 2

Order : 2

one such group we have seen is 2mm

* the group is called the cyclic group of order 4

	e	4^+	2	4^-
e	e	...		
4^+	⋮			
2	⋮			
4^-				

C_4

the Swastika group

another example of this is :

Looking at another example : (considering complex no.s with multiplication operation)

	1	−1	i	−i
1	1	−1	i	−i
−1	−1	1	−i	i
i	i	−i	−1	1
−i	−i	i	1	−1

$\Longrightarrow$ this is a group of the form

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	a	e
c	c	b	e	a

15] <u>Cyclic Group of Order 4</u>

$$C_4 \equiv 4 = \{1, 4^+, 2, 4^-\}$$

Schoenflies notation

Hermann-Mauguin (H-M) (International Notation)

in this group, the element 4^+ has the ability to generate all other elements

simply by combining with itself :

4^+

$(4^+)^2 = 4^+ \cdot 4^+ = 2$

$(4^+)^3 = 2 \cdot 4^+ = 4^-$

$(4^+)^4 = 4^- 4^+ = 1$

For this group

* 4^+ and 4^- are both <u>generators</u> of the group

If we tried for 2 instead:

2

$2 \cdot 2 = 1$

$2^3 = 2$

$2^4 = 1$

⋮ so on

$\Rightarrow$ this shows that 2 is NOT a <u>generator</u> of the group

16] <u>Generator</u>

A <u>subset</u> of elements of G

such that all elements of G can be produced

by combination of elements of the subset

Such a subset

is called a generator

17] Cyclic Group

If a group can be generated by a single element

↓

then such a group is called a cyclic group

eg: $4 = \{1, 4^+, 2, 4^-\} = \underbrace{\langle 4^+ \rangle}$

this means, the "group generated by object 4^+"

*

eg: $2mm = \{1, 2, m_{10}, m_{01}\}$

	1	2	m_{10}	m_{01}
1	1	2	m_{10}	m_{01}
2	2	1	m_{01}	m_{10}
m_{10}	m_{10}	m_{01}	1	2
m_{01}	m_{01}	m_{10}	2	1

⇓ from this we can see that

{ 2 along with any one mirror can act as a generator

i.e. any 2 of these three

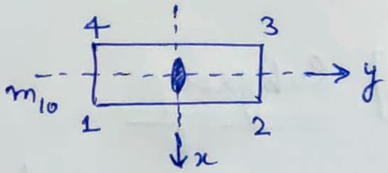
↓

Thus, the group can be written as:

$$2mm = \langle 2, m_{10} \rangle$$
$$= \langle 2, m_{01} \rangle$$
$$= \langle m_{10}, m_{01} \rangle$$

* NOTE:

To find $2m_{10}$:



We know:

m_{10}	2
1 2 3 4	1 2 3 4
↓	↓
4 3 2 1	3 4 1 2

⇓

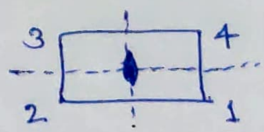
Thus:

1 2 3 4

m_{10} ↳ 4 3 2 1

2 ↳ 2 1 4 3

which is the same for m_{01}



⇓

Thus: $2m_{10} = m_{01}$

* NOTE: If $\langle a, b, c \rangle = G$

then, by definition:

$$a^k b^l c^m \in G$$

NOTE: We can include additional members in a generator

↓

eg: Since $\langle m_{10}, m_{10} \rangle = 2mm$

↓

we can also have

$\underbrace{\langle 2, m_{10}, m_{01} \rangle}_{\text{this is also a generator}} = 2mm$

* This is a Minimal Generator (i.e. we cannot remove any element from it)

NOTE: $2mm = \langle m_{10}, 2 \rangle \Rightarrow$ Not Cyclic (since there is more than one element here)

Hence

$4 = \langle 4^+ \rangle \Rightarrow$ this is Cyclic

18] Subgroup

↓

If a subset of a group is also a group under the same binary operation

↓

it is called a subgroup

eg: for the case of group $4 = \langle 4^+ \rangle = \{1, 2, 4^+, 4^-\}$

$\underbrace{2 = \{1, 2\}}_{\text{this is a subgroup of order 2}}$

NOTE: Symmetries in alphabets:

A	B	C	D	E	F	G
m	m	m	m	m	1	1

H	I	J	K	L	M	N
2mm	2mm	1	m	1	m	2

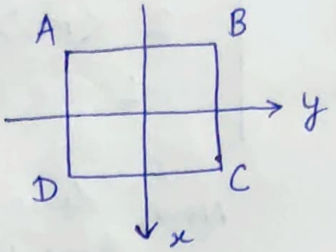
this is an example of an object with only 2 symmetry (another is Z)

LECTURE - 4 (11/01/2024)

19] Cayley Table of 4mm

(Symmetry group of a square)

	1	4^+	2	4^-	m_{10}	m_{01}	m_{11}	$m_{1\bar{1}}$
1								
4^+								
2								
4^-								
m_{10}								
m_{01}								
m_{11}								
$m_{1\bar{1}}$								



↳ solve this yourself (H.W.) *

* **NOTE:** The product of rotation and reflection is always some reflection (reason being, reflection changes the "handedness", whereas rotation does not)

20] Subgroup Diagrams

eg: Within 4mm we see

$$\{\underbrace{1, 2, 4^+, 4^-}_{\text{these are a subgroup}}, m_{01}, m_{10}, m_{11}, m_{1\bar{1}}\}$$

⇓

But within this also:

$$\underbrace{1, 2}_{\text{this is also a subgroup}}, 4^+, 4^-$$

Hence to help us see this, we use subgroup diagrams

1 — $\{1\}$

2 — $\{1, 2\}$ — 1

4 — $\{1, 2, 4^+, 4^-\}$ — 2 — 1

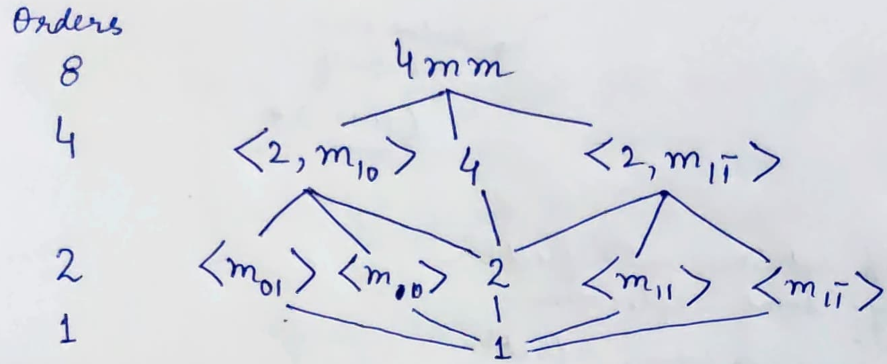
2mm — 2, $\langle m_{10} \rangle$, $\langle m_{01} \rangle$

* We write subgroups of SAME ORDER → at same level

21] Subgroup diagram for 4mm

Order	4mm
4	$4 \quad \langle 2, m_{10} \rangle \quad \langle 2, m_{11} \rangle$
2	$2 \quad \langle m_{10} \rangle \quad \langle m_{01} \rangle \quad \langle m_{11} \rangle \quad \langle m_{1\bar{1}} \rangle$
1	1

Now, drawing this in a tree like structure:



NOTE: $\langle m_{10} \rangle \equiv \{1, m_{10}\}$

$2 \equiv \{1, 2\} \equiv \langle 2 \rangle$

$\vdots$ and so on

22] Cosets

eg: Consider the group, $G = 4mm$

and a subgroup of it, $H = \langle m_{10} \rangle = \{1, m_{10}\}$

Select an element, $g \in G$ and $g \notin H$ { NOTE: In reality this is NOT a real condition (g may belong to H also) }

(i.e., $g \in G - H$)

let's say, we select $g = 2$

If $g \in G$ and $H < G$, then the set

$gH = \{gh \mid h \in H\}$ is called the LEFT coset of H *

i.e., $H = \{h_1, h_2, \cdots, h_m\}$

$gH = \{gh_1, gh_2, \cdots, gh_m\}$

NOTE! gh_i may not be the same as $h_i g$
↓
For groups where they are the same
↓
such groups are called **Abelian groups**

23]

eg: find all left cosets of $H = \langle m_{10} \rangle$ with $g = 2$

$$2\langle m_{10}\rangle = 2\{1, m_{10}\}$$
$$= \{2, m_{01}\}$$

Taking other values of g in G and not in H:

$$4^+\langle m_{10}\rangle = \{4^+, 4^+ m_{10}\}$$
$$= \{4^+, m_{11}\}$$

$$4^-\langle m_{10}\rangle = \{4^-, 4^- m_{10}\}$$
$$= \{4^-, m_{1\bar{1}}\}$$

We are not supposed to take m_{10}
↓
but let's say we try to take it, and see what we get:
↓

$$m_{10}\langle m_{10}\rangle = \{m_{10}, m_{10}^2\}$$
$$= \{m_{10}, 1\}$$
$$= \underbrace{\langle m_{10}\rangle}$$

* we get back one of the cosets that already exists

* (NOTE: we consider the set H itself also as a coset)

24] We can now write G as a union of all its cosets ⇒ effectively, this means we have partitioned the group

eg: $4mm = \langle m_{10}\rangle \cup \{2, m_{01}\} \cup \{4^+, m_{11}\} \cup \{4^-, m_{1\bar{1}}\}$

NOTE: Lagrange's theorem ensures that
↓
for any group
↓
we can partition it into such cosets
↓
and they will have distinct elements

25] We see from the definition,

$$|H| = |gH|$$

{ this means that $gh_i \neq gh_j$, $\forall i \neq j$, $\forall g \in G - H$ }

LECTURE-5 (15/01/2024)

26] **Coset:** Let $H = \{h_1, h_2, \ldots\}$ be a subgroup of G.
Then for $g \in G$ the set :

$$gH := \{gh_1, gh_2, \ldots\} = \{gh \mid h \in H\}$$

is called the <u>left coset of H with representative g</u>.

NOTE: Any elements of a coset can be a coset representative of the coset

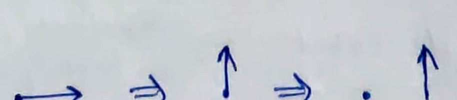
Similarly,

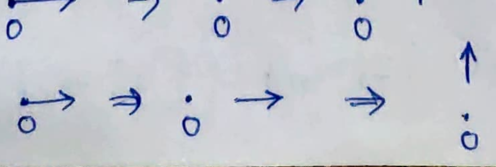
$$Hg := \{h_1 g, h_2 g, \ldots\} = \{hg \mid h \in H\}$$

is called the <u>right coset of H with representative g</u>.

NOTE: Symmetry operations need not be commutative

eg: $4^+ \vec{T}(0) \neq \vec{T}(0)\, 4^+$

$4^+ \vec{T}(0)$:


$\vec{T}(0)\, 4^+$:


27] **Theorem**: Elements of a coset are all different from one another.

$$gH = \{gh_1, gh_2 \ldots, gh_m\}$$

Proof by contradiction:

Let $gh_i = gh_j$ for some $i \neq j$

Now, since $g \in G$, g^{-1} exists

Thus: $g^{-1}gh_i = g^{-1}gh_j$

$\Rightarrow h_i = h_j$

$\Big\{$ Technically: $g^{-1}(gh_i) = g^{-1}(gh_j)$

By associativity: $(g^{-1}g)h_i = (g^{-1}g)h_j$ $\Big\}$

$\Rightarrow eh_i = eh_j$

But, for $i \neq j$, we should have two different elements of the group H

This is a contradiction.

Hence, $gh_i \neq gh_j$

$\therefore$ All elements of a coset are different.

28] **Theorem**: If $g_1 \in G$, but $g_1 \notin H$, then H and g_1H have no common element.

Proof by contradiction:

Let H and g_1H have a common element x.

Since $x \in H$, let $x = h_i$ of H

Since $x \in g_1H$, let $x = g_1h_j$ for some $h_j \in H$

$\Rightarrow h_i = g_1h_j$

$\Rightarrow h_ih_j^{-1} = g_1h_jh_j^{-1} = g_1e$

$\Rightarrow h_ih_j^{-1} = g_1$

(Since h_j is a member of group H $\Rightarrow$ thus it will have an inverse)

Since $h_i \in H$, $h_j \in H \Rightarrow h_j^{-1} \in H$, and thus by closure $h_ih_j^{-1} \in H \Rightarrow g_1 \in H$

But this is a contradiction.
Hence, H and g_1H cannot have a common element.

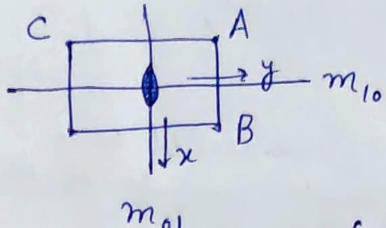
For eg:

$$2mm = \{1, 2, m_{10}, m_{01}\} = G$$

$$H = \langle m_{10} \rangle = \{1, m_{10}\}$$

$$g_1 = 2$$

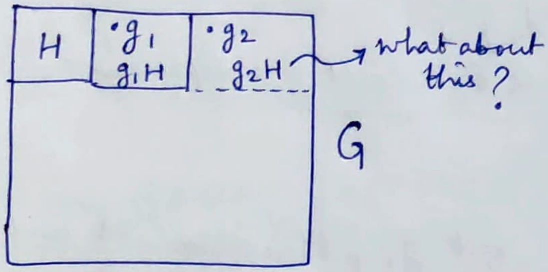
Thus: $g_1H = \{2, 2m_{10}\} = \{2, m_{01}\}$ — we can see that H and g_1H have all different elements (no common)



$2m_{10}(A) = 2(B) = C$ | $m_{01}(A) = C$

* (NOTE: We must apply the operation that is rightmost first)

29]



Consider some element $g_2 \in G$, such that $g_2 \notin H$ and $g_2 \notin g_1H$.
Show that g_2H has no common element with either H or g_1H.

Proof by contradiction: From previous theorem, we know g_2H cannot have common element with H.

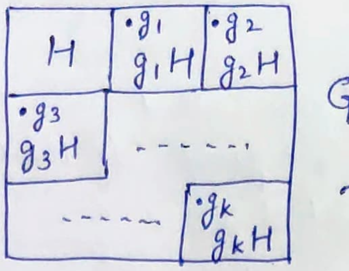
Now, let g_1H and g_2H have a common element x

$$x = g_1h_i = g_2h_j$$

$$\Rightarrow g_2 = g_1h_ih_j^{-1} \in g_1H$$

But that is a contradiction.
Hence Proved.

30]



{NOTE: $|H| = |gH|$
$\forall g \in G - H$ }

Thus from here we see clearly that this process of selecting a new element
&
and constructing a new coset
&
can continue till no element of the group is left (assuming finite group)

We then have a partition of the finite group into k (some integer) cosets &
each having equal number of elements (equal to the order of the subgroup, say m)

Let the order of the group be $|G| = n$

Then, $n = km$

OR

$$\boxed{|G| = k\,|H|}$$

i.e. the order of the group is divisible by the order of the subgroup

This is called LAGRANGE'S THEOREM

Lagrange's Theorem:

The order of subgroup divides the order of the group.

eg: $H = G \Rightarrow \dfrac{|G|}{|H|} = 1$

eg: for $H = \{e\} \Rightarrow \dfrac{|G|}{|H|} = \dfrac{|G|}{1} = |G|$

* NOTE: The reverse of this theorem, however, is NOT true necessarily.

31] Conjugate element

Two elements h and h' are called Conjugate elements of a group if there is some $g \in G$ such that :

$$h' = g h g^{-1}$$

eg: $2mm = \{1, 2, m_{10}, m_{01}\}$

Consider $h = m_{10}$

For $g = 1$: $1 m_{10} 1^{-1} = 1 m_{10} 1 = m_{10}$

* (NOTE: Every element is a conjugate with itself)

For $g = 2$: $2 m_{10} 2^{-1} = 2 m_{10} 2$
$= 2 m_{01}$
$= m_{10}$

{ Here, we find out that $m_{10} 2 = m_{01}$ and then make use of it }

For $g = m_{10}$: $m_{10} m_{10} m_{10}^{-1} = m_{10} m_{10} m_{10}$
$= m_{10} 1$
$= m_{10}$

For $g = m_{01}$: $m_{01} m_{10} m_{01}^{-1} = m_{01} m_{10} m_{01}$
$= m_{01} 2$
$= m_{10}$

LECTURE 6 (18/01/2024)

32] Symmetry operations are a mapping of space to itself

Mapping : A function which maps any given point P of space to another point $\tilde{P}$ (called the image point)

[i.e. $\tilde{P}$ is the image of P]

For a symmetry operation

↓

the mapping is of the form of

↓

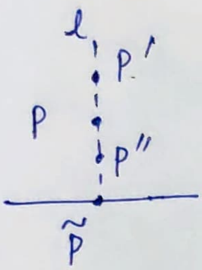
a one to one correspondence, or <u>bijection</u>, b/w points in space

For example:

A 2D projection of a point on a line

↓

is NOT one to one ⇝ ∴ it CANNOT be a symmetry operation



$\Rightarrow$ Here $M(P) = \tilde{P}$

but also $M(P') = \tilde{P}$, $M(P'') = \tilde{P}$

(and so on, $\forall$ points Q on line l, $M(Q) = \tilde{P}$)

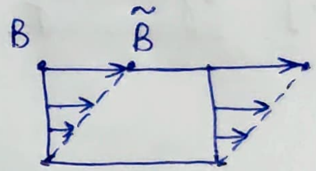
33] <u>Shear operation</u>

↓

Here, the points move parallel to a fixed line

↓

the magnitude by which they move is proportional to the distance of that point from the line



* Can this be a symmetry operation?

↓

<u>Answer</u>: Yes

34] <u>Isometry</u> : A mapping which does not change distances (also called rigid motion)

↓

A kind of operation with the property

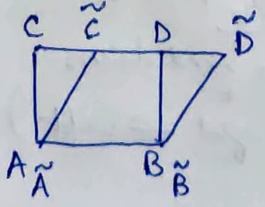
↓

that || lines are mapped to || lines

↓

but it can't change the distance

eg: Shear:



Here $AB \parallel CD$, $AC \parallel BD$

and also

$\tilde{A}\tilde{B} \parallel \tilde{C}\tilde{D}$, $\tilde{A}\tilde{C} \parallel \tilde{B}\tilde{D}$

But, since $|AC| \neq |\tilde{A}\tilde{C}|$ $\Rightarrow$ thus, Shear is <u>NOT an isometry</u> *

An isometry is a special kind of affine transformation

* **NOTE:** Affine transformations
↓
are those, where points, lines and planes are preserved
↓
and parellel lines remain parellel after transformation

(∵ points $\xrightarrow{\text{map to}}$ points
lines → lines
planes → planes)

* **NOTE:**

Mappings
- Affine
  - Isometry
  - Not isometry
- Not affine

35] **Symmetry** : A kind of isometry
↓
which brings an object into self-coincidence
↓
is called a symmetry

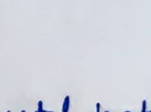
36] **Crystal** : A periodic arrangement of atoms (translationally periodic)

Lattice : A translationally periodic arrangement of points

* { Crystal = Lattice + Motifs }

Crystallographic Symmetry: Symmetry of an ideal crystal pattern

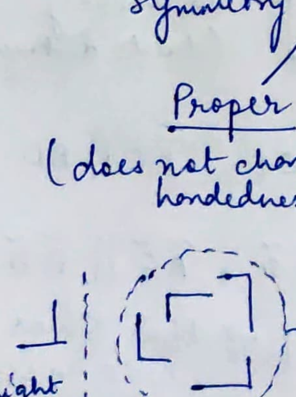
37] Symmetry operations
- **Proper** (does not change handedness)
- **Improper** (changes handedness
↓
Right ⇌ Left) ⟶ eg: Reflection :



(it changes handedness)

(such an object that can occur in left and right handed forms) ⟶ called * **Chiral**

NOTE:



* **NOTE:** Some synonyms are:

Type I ≡ "Direct" ≡ "Proper" ⇝ No change in handedness

Type II ≡ "Opposite" ≡ "Improper" ⇝ Change in handedness

38] **List of Crystallographic Symmetry Operations**

S.No.	NAME	TYPE I/II	FIXED POINTS	ORDER
1	Identity	I	All Space	1
2	Translation	I	None	∞
3	Rotation	I	Axis (Line)	$2, 3, 4, \cancel{5}, 6, \cancel{7}, \ldots$ [As per Crystallographic Restriction Theorem] ↓ to be proved later
4	Screw Rotation (Rotation + Translation ∥ to the axis)	I	None	∞

LECTURE 7 [25/01/2024]: 288th Birth Anniversary of Lagrange

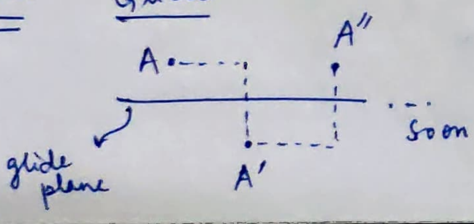
(above table continued)...

39]

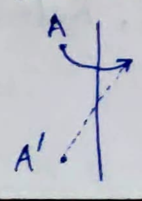
S.No.	NAME	TYPE I/II	FIXED POINTS	ORDER
5	Reflection	II	Mirror plane	2
6	Glide (Reflection + Translation ∥ to m)	II	None	∞
7	Inversion	II	Centre of inversion	2
8	Rotoinversion (Rotation + Inversion in centre lying on the rotation axis)	II	Centre of inversion	$\cancel{\bar{1}}, \cancel{\bar{2}}, \bar{3}, \bar{4}, \bar{6}$ ($\bar{1}$ ≡ inversion) ($\bar{2} \equiv m$)

* **NOTE:**

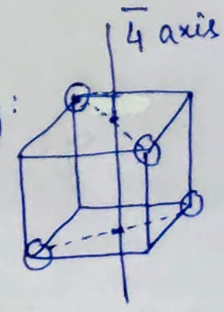
Glide



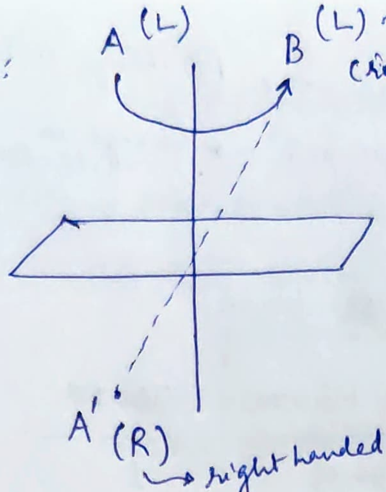
Rotoinversion



⇒ eg:



NOTE:



⟹ Thus we see that $\bar{2} \equiv$ mirror reflection (m)

40] Affine Transformation

↓

Most affine transformations (hence symmetric operations) that we will study

↓

are a rotation or rotoinversion in an axis

↓

through the origin, followed by a translation

although this is NOT true in general always (eg: shear)

[This can be proved by us later]

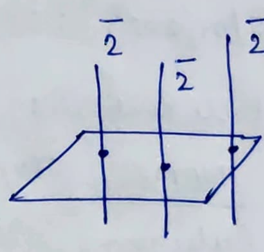
NOTE: Although $\bar{2} \equiv m$

↓

a mirror will be unique

↓

but $\bar{2}$ axis can be chosen anywhere perpendicular to the mirror!



NOTE: In an affine transformation:

Line → Line || Line → || Line

Plane → Plane || Plane → || Plane

41] Linear Transformation

↓

is an affine transformation

↓

which leaves at least one point fixed

eg: Rotation, Rotoinversion